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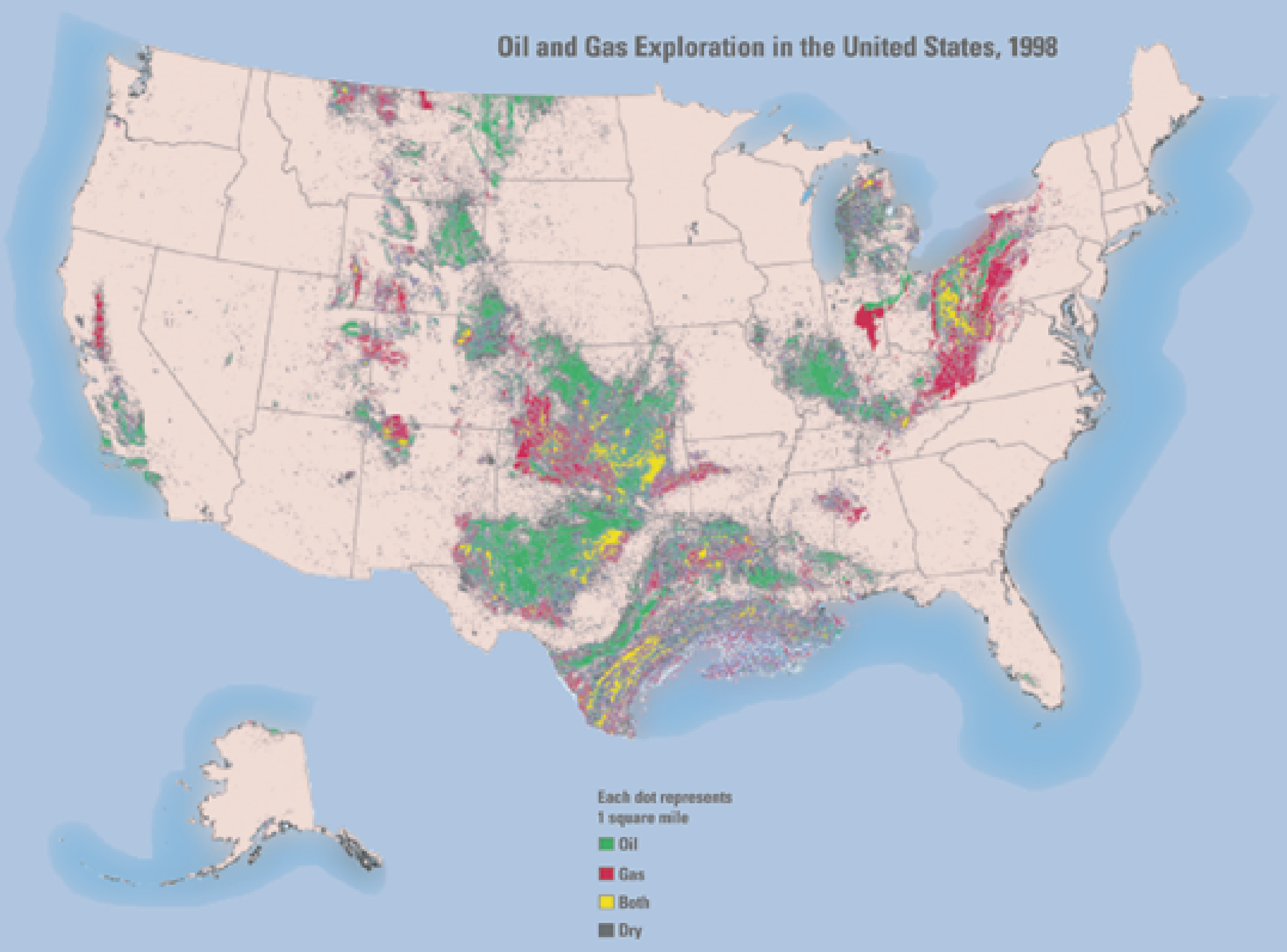
Modified Arrington Method for Calculating Reserve Growth—A New Model for United States Oil and Gas Fields

Chapter D *of*
Geologic, Engineering, and Assessment Studies of Reserve Growth

U.S. Geological Survey Bulletin 2172–D



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Cover. This map represents historical oil and gas exploration and production data for the conterminous United States. It was derived from data used in U.S. Geological Survey Geologic Investigations Series I-2582.* The map was compiled using Petroleum Information Corporation's (currently IHS Energy Group) database of more than 2.2 million wells drilled in the U.S. as of June 1993. The area of the U.S. was subdivided into 1 mi^2 grid cells for which oil and gas well completion data were available. Each colored symbol represents a 1 mi^2 cell (to scale) for which exploration has occurred. Each cell is identified by color as follows: red, a gas-producing cell; green, an oil-producing cell; yellow, an oil- and gas-producing cell; gray, a cell that has been explored through drilling, but no production has been reported. See Mast and others (1998) for details on map construction.

*Mast, R.F., Root, D.H., Williams, L.P., Beeman, W.R., and Barnett, D.L., 1998, Areas of historical oil and gas exploration and production in the conterminous United States: U.S. Geological Survey Geologic Investigations Series I-2582, one sheet.

Modified Arrington Method for Calculating Reserve Growth—A New Model for United States Oil and Gas Fields

By Mahendra K. Verma

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Edited by T.S. Dyman, J.W. Schmoker, and Mahendra Verma

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Abstract

Reserve (or field) growth is an appreciation of total ultimate reserves through time and is observed throughout the productive lives of oil and gas fields in all petroleum provinces—but most especially in mature petroleum provinces (like many in the United States) when the rate of finding new discoveries reduces to a low level. The importance of forecasting reserve growth accurately in a mature petroleum province made it necessary to develop improved growth functions, and a critical review of the original Arrington method was undertaken.

A modification of the pioneering Arrington (1960) method for estimating reserve growth suggests that, as a basis for optimum reserve growth functions, cumulative growth factor smoothing produces a better match with known volume data than does annual growth factor smoothing. Cumulative growth factor smoothing is thus the basis for the building of reserve growth functions in this study. Estimates of oil and gas growth during 1992–1996 based on the modified Arrington method are closer to the actual volumes than those based on the functions from 1995 National Assessment. The new growth functions predict an average annual reserve growth of 0.42 percent per year for oil and 0.90 percent per year for gas over a 30-year (1996–2026) period.

Introduction

Since the completion of its 1995 National Petroleum Assessment (Root and others, 1995), the U.S. Geological Survey (USGS) has undertaken a project to study field growth by identifying the range in variation of reserve (or field) growth estimates using different datasets and growth factors. The project also identifies the geologic, engineering, economic, and other controls governing the growth of fields through time. This report analyzes a portion of that variation using a method developed by the USGS, based on modifications to an original method developed by Arrington (1960). The results are not intended to represent official estimates of reserve growth for the Nation. Rather, they are presented to compare national growth estimates of the USGS 1995 National Assessment with new estimates based on a modification of the Arrington method, using the 1996 version of the U.S. Energy Information Administration (EIA)'s Oil and Gas Integrated Field File of reserves data from 1977 through 1996. Other chapters in this volume also analyze reserve growth using different datasets and different growth functions.

Reserve growth is a term used to refer to estimated increases in the total technically and economically recoverable petroleum reserves of a field that commonly occur through time because (1) additional reservoir and geologic information leads to increases in estimates of hydrocarbons-in-place of existing reservoirs or pools; (2) new reservoirs or pools are discovered in existing fields; and (3) improvements take place in the hydrocarbon recovery factor owing to better understanding of reservoir characteristics and behavior through use of 3D/4D seismic interpretation, better geophysical logging tools, and improved reservoir simulation techniques. Additionally, application of horizontal-well drilling technology and enhanced recovery methods improve the hydrocarbon recovery factors significantly, resulting in increased estimates of reserves, particularly in oil reservoirs.

Several technical papers have been published to show that the phenomenon of reserve growth is applicable throughout the productive lives of oil and gas fields in all petroleum provinces but most especially in mature petroleum provinces (like many in the U.S.) when the rate of finding new discoveries reduces to a low level. Arrington (1960) was probably the first to publish the significance of the concept and proposed a method to estimate reserve growth when only a window of historical reserve and production record is available for analysis. Attanasi and Root (1994) published their reserve growth work within the conterminous U.S. and found that the reserve growth contributed to more than 90 percent of the U.S. proved reserves during 1978–1991. Root and others (1995) reported a significant increase in technically recoverable conventional oil resources (including existing and undiscovered accumulations) over what was reported in the previous assessment by Mast and others (1989) for onshore U.S. fields and those located in State waters. Lore and others (1996) also developed a field-growth model for the U.S. offshore Gulf Coast fields. Fisher (1991) indicated substantial potential of U.S. oil and natural gas through discovery and reserve growth. Sem and Ellerman (2000), and Watkins (2000) recorded reserve appreciation (same as reserve growth) in both the United Kingdom and Norwegian sectors of the North Sea. Verma and others (2001) reported on reserve growth in the Volga-Ural province of Russia.

The estimation of reserve growth has always been a complex problem, particularly when only a limited record of historical reservoir and production data is available for analysis. Although other approaches for estimating reserve growth have been proposed over the years, Arrington's method still remains one of the most useful because of its simplicity and comparable results with other methods. However, parts of the Arrington method require modifications in order to improve results. Accordingly, the objective of this study is to examine the Arrington method for its possible use in developing reserve growth functions (models), and to modify the procedure where necessary for the purpose of more accurately forecasting U.S. oil and gas reserves.

Acknowledgments

The author thanks Ronald R. Charpentier and Thaddeus S. Dyman for their thorough review of the paper and valuable comments. Appreciation is also due to other USGS staff for their help in preparing this report.

Background

Arrington (1960) evaluated the success of exploration programs by establishing a correlation between the cost of finding oil and the amount of oil found. During this process, he formulated a method for estimating the annual reserve growth rate when reserve data were available for only a limited time period. His analysis revealed the need to maintain a systematic record of an individual field's reserves since its discovery year to allow for the best estimation of reserve growth. In the Arrington method, the annual growth factor is based on a 3-year moving average of annual reserve increases. Because of this moving average, the first-year growth rate is not calculated, and the average is therefore determined by plotting the annual reserve increases against years since discovery and then extrapolating the average curve. Marsh (1971) used the Arrington method to calculate reserve growth, working with only four annual reserve estimates for oil discovered in the U.S. for each of the 10 years between 1960 and 1969, and he also introduced the concept of a cumulative growth function. Dolton and others (1981) used Arrington's method in the 1980 National Oil and Gas Assessment but did not apply the 3-year moving average of annual reserve increase. Robert Megill, in a series of articles, described Arrington's approach to reserve growth (1989a–d). Attanasi and Root (1994) published the results of their study on U.S. reserve growth using the Oil and Gas Integrated Field File (OGIFF) dataset, developed by the EIA of the U.S. Department of Energy.

Analysis Guidelines and Assumptions

The main guidelines for data handling and processing in this study are as follows:

1. Only oil and gas fields within the conterminous U.S. are included; however, Federal Offshore and continuous fields are excluded because (a) Federal Offshore fields fall under the jurisdiction of Minerals Management Service; and (b) continuous fields, which are the hydrocarbon accumulations without well-defined hydrocarbon-water contacts, require different assessment methods (Schmoker, 1996); continuous fields were likewise excluded from the USGS 1995 National Assessment of U.S. Oil and Gas Resources (Root and others, 1995). Alaskan fields are also not included; they are relatively less mature and their development strategies are different from those in the Lower 48 States, owing to different operating environments.
2. The 1996 version of the OGIFF dataset (which includes production data and reserve estimates made during 1977–1996) was used for the development of oil and gas growth functions, based on a modified Arrington method.
3. The 1991 version of OGIFF dataset was used for comparison of the reserve growths based on the modified Arrington method with the growth functions from the USGS 1995 National Assessment of U.S. Oil and Gas Resources (Attanasi and Root, 1994).
4. The total estimated ultimate reserve is defined here as known petroleum volume (KPV) to highlight the fact that the reserve estimates change through time; thus, calling them ultimate reserves is somewhat misleading. The KPV is the sum of cumulative production and remaining estimated reserves as of the date of reporting or evaluation.
5. The KPV data for the years prior to 1900 are not included in the analysis because of uncertainty in the accuracy of the values reported for those years.
6. Twenty years (1977–1996) of KPV data allow inclusion of 19 estimates of annual growth factor (AGF) for each age since discovery; 1 year after discovery, 2 years after discovery, and so on.
7. Seventy-eight AGFs were obtained for oil, because the data for oil fields were available for fields discovered in 1900 through 1977, but only 66 AGFs were obtained for gas because of sparse data on gas fields in early years. The 1996 version of the OGIFF database reveals that fields discovered in 1900 showed reserve growth through 1995. Based on this observation, I decided to extrapolate the cumulative growth factor (CGF) curves for both oil and gas to the 95th year since discovery. No growth was assumed beyond 95 years, because of uncertainty from lack of data support.

Methodology

The OGIFF dataset, which contains the U.S. annual production, cumulative production, and the remaining reserve data by field, has been used in the present analysis. Because of confidentiality of the field-level data, only a summary of OGIFF was published by the EIA (1990). In the present analysis using a modified Arrington method, fields were classified on the basis of gas-oil ratio: less than 20,000 standard cubic feet/stock tank barrel (SCF/STB) for oil fields, and more than 20,000 SCF/STB for gas fields. (One STB is equal to 42 U.S. gallons.)

The total proved reserve (or KPV) is the summation of cumulative production and remaining reserves at the time of reporting. KPVs, which are the basic data required for the estimation of reserve growth, were extracted from the 1996 version of the OGIFF dataset (includes reserve data from 1977 through 1996). They are tabulated in such a way that the original and subsequently revised KPVs of all the fields for each year since discovery are shown in columns from left to right in table 1. To explain the basics of the proposed modified Arrington method, KPVs extracted for the fields discovered in 1900 through 1902 are given in the first three rows in table 1, and the KPVs extracted for the fields discovered in 1960 through 1996 are given in subsequent rows. The procedure, which is similar in both the original Arrington and the modified Arrington method for calculating annual growth factor, is described herein in a manner similar to that used by D.H. Root (in Dolton and others, 1981):

1. Let $W(I, J)$ be the KPV for the fields discovered in the year "I" and KPV reporting year "J." In table 1, discovery year (I) is shown in the leftmost column (range: 1900–1902, 1960–1996), and KPV reporting year (J) is shown in the top row (range: 1977–1996).

2. This method requires summation of the same year KPVs, that is, KPV of fields at their discovery year, 1 year after discovery, 2 years after discovery, and so on. Calculation of annual growth factor (AGF) requires the ratio of two summations. The equation for the first AGF (1 year after discovery) is given below along with calculation details, using KPV data from table 1.

$$AGF(1) = \frac{\sum_{a=1977}^{1995} W(a, a+1)}{\sum_{a=1977}^{1995} W(a, a)} = \frac{2,310}{1,167} = 1.97943 \qquad (1)$$

where "W" in the numerator is the KPV for the discovery year "a" and KPV reporting year "a+1," and in the denominator it is for the discovery year "a" and KPV reporting year "a." The value of "a" varies, from 1977 through 1995.

The denominator in the preceding equation is the summation of 19 KPV values, one for each of the discovery years from 1977 through 1995, which are shown inside the double-lined boxes in table 1. The sum (1,167 million barrels), also inside a double-lined box, is shown in the column with heading "1st Sum." The numerator is the summation of KPV values, shown in the dashed-line boxes, for the same group as in the denominator but a year later. The sum (2,310 million barrels) for the numerator is shown in the column with heading "2nd Sum." Ratio of the two sums (1.97943; last column in table 1) is AGF (1), representing annual reserve growth during the first year after discovery.

The equation for the AGF (2), representing reserve growth 2 years after discovery, is given next along with summation and AGF values.

$$AGF(2) = \frac{\sum_{a=1976}^{1994} W(a, a+2)}{\sum_{a=1976}^{1994} W(a, a+1)} = \frac{2,864}{2,478} = 1.15577 \qquad (2)$$

where "W" in the numerator is the KPV for the discovery year "a" and KPV reporting year "a+2," and in the denominator it is for the discovery year "a" and KPV reporting year "a+1." The value of "a" varies, from 1976 through 1994.

The denominator is the summation of 19 KPV values, one for each group that was discovered a year earlier—that is, in 1976 through 1994 but reported in 1977 through 1995. In this case, the denominator sums to 2,478 million barrels and the numerator sums to 2,864 million barrels (table 1). Ratio of the two sums (1.15577) is AGF (2), representing annual reserve growth 2 years after discovery.

Applying a similar concept, a generalized equation for the nth AGF (n years after discovery) can be written as

$$AGF(n) = \frac{\sum_{a=1978-n}^{1996-n} W(a, (a+n))}{\sum_{a=1978-n}^{1996-n} W(a, (a+(n-1)))} \qquad (3)$$

where "W" in the numerator is the KPV for fields discovered in the year "a" and KPV reporting year being "a+n," and in the denominator it is for the fields discovered in year "a" and KPV reporting year being "a+(n–1)." The value of "a" ranges from 1978–n (lower summation limit) to 1996–n (upper summation limit), respectively. In equation 3, "n" is the number of year(s) after discovery.

For calculating successive AGFs, the procedure is repeated on the next line above (table 1). This process of calculating AGFs is continued until we reach the line that includes the KPV for the fields discovered in year 1900 and the KPV reporting being year 1977. The last line will be reached when "n" is equal to 78, and the lower limit of "a" (year 1900) is reached in the above mathematical expression. Data for the years prior to 1900 have been ignored because of the uncertainty of the accuracy of data. This procedure therefore resulted in 78 AGF values. Each AGF is based on 19 years of data. A similar procedure was repeated for gas fields, where only 66 AGFs were obtained because few gas fields were discovered prior to 1912.

Basis for New Reserve Growth Functions

Because the cumulative growth factors (CGF) are used to estimate potential additional reserves, the proposed modified Arrington method (based on regressing CGF data) should provide better results than the original Arrington method, which requires first the calculation and regression of AGFs,

Table 1. Known petroleum volumes (KPVs) for conterminous U.S., excluding Federal Offshore and continuous fields.

[KPVs in million barrels. Data source: 1977–1996 Oil and Gas Integrated Field File. AGF, annual growth factor; D. Year, discovery year]

D. Year	KPV 77	KPV 78	KPV 79	KPV 80	KPV 81	KPV 82	KPV 83	KPV 84	KPV 85	KPV 86	KPV 87	KPV 88	KPV 89	KPV 90	KPV 91	KPV 92	KPV 93	KPV 94	KPV 95	KPV 96	1st Sum	2nd Sum	AGF
1900	649	659	669	674	677	700	702	718	731	714	724	732	731	740	743	748	748	755	763	755			
1901	2,289	2,265	2,367	2,474	2,515	2,542	2,608	2,812	2,944	2,983	3,030	3,263	3,435	3,480	3,475	3,479	3,539	3,578	3,621	3,651			
1902	945	948	940	968	948	980	995	1,040	1,039	1,056	1,124	1,135	1,221	1,188	1,201	1,190	1,170	1,174	1,175	1,165			
							Data Break								Data Break								
1960	793	811	802	851	843	861	889	905	914	915	922	937	937	939	954	955	967	974	988	998			
1961	369	374	386	393	408	427	438	454	471	468	479	479	492	497	506	513	510	510	514	521			
1962	547	552	563	581	600	614	651	659	668	673	687	685	679	684	694	700	699	715	724	734			
1963	355	358	366	383	411	423	433	449	457	466	471	485	494	521	532	540	542	552	565	565			
1964	481	463	469	474	490	504	516	529	547	551	558	560	567	582	602	621	622	624	658	681			
1965	461	473	491	503	512	520	532	546	556	561	568	575	585	593	608	609	613	620	628	630			
1966	474	464	485	498	514	486	499	506	512	519	515	516	528	528	529	522	534	538	537	547			
1967	345	355	366	385	384	388	402	407	421	426	438	450	451	453	460	463	462	470	480	487			
1968	314	321	319	330	340	353	367	370	384	386	386	393	397	407	415	419	427	429	434	449			
1969	328	341	350	365	372	382	388	396	409	417	422	425	429	437	438	447	450	453	465	467			
1970	784	805	812	873	892	922	925	958	977	983	990	1,009	1,018	1,023	1,024	1,040	1,076	1,112	1,116	1,153			
1971	170	180	187	199	208	221	240	232	245	245	247	256	259	268	292	306	297	287	299	309			
1972	413	441	442	470	483	503	518	536	561	566	578	629	638	680	652	653	654	659	664	671			
1973	155	161	176	197	220	222	229	235	243	243	246	247	248	252	256	260	262	271	275	278			
1974	390	410	410	446	445	462	505	528	543	542	549	646	591	606	600	613	604	613	620	614			
1975	185	193	205	232	233	246	251	263	276	283	293	294	304	311	312	312	316	325	337	342			
1976	239	287	338	381	381	384	429	476	467	467	487	491	491	514	537	549	549	571	572	577			
1977	114	176	226	268	323	336	348	381	394	397	407	429	477	507	504	508	509	517	543	554			
1978	0	51	101	130	153	159	186	208	242	229	238	243	248	256	261	267	276	287	291	298	9235	9505	1.02924
1979	0	0	69	158	191	264	292	312	326	360	369	406	442	460	452	466	477	485	488	525	8798	8967	1.01921
1980	0	0	0	116	325	331	363	353	343	368	372	388	390	403	414	421	417	403	427	435	8724	8864	1.01605
1981	0	0	0	0	83	161	186	219	270	259	276	289	304	311	315	337	335	336	347	351	8409	8528	1.01415
1982	0	0	0	0	0	128	225	245	259	257	259	275	282	302	302	318	314	315	316	327	8144	8381	1.02910
1983	0	0	0	0	0	0	74	145	186	178	212	209	220	229	246	232	246	243	248	252	7764	7915	1.01945
1984	0	0	0	0	0	0	0	92	168	174	183	184	200	203	211	218	226	229	236	251	7373	7554	1.02455
1985	0	0	0	0	0	0	0	0	92	159	190	221	226	227	235	258	264	276	284	284	6987	7183	1.02805
1986	0	0	0	0	0	0	0	0	0	37	69	77	86	94	93	96	97	98	109	109	6493	6751	1.03974
1987	0	0	0	0	0	0	0	0	0	0	45	91	104	99	104	114	122	121	126	126	6137	6305	1.02737
1988	0	0	0	0	0	0	0	0	0	0	0	48	79	90	97	98	93	95	98	102	5663	5911	1.04379
1989	0	0	0	0	0	0	0	0	0	0	0	0	32	71	89	107	88	84	86	87	4708	4966	1.05480
1990	0	0	0	0	0	0	0	0	0	0	0	0	0	33	70	84	81	73	67	70	4379	4608	1.05230
1991	0	0	0	0	0	0	0	0	0	0	0	0	0	0	41	67	81	86	84	90	3968	4056	1.02218
1992	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	26	56	71	78	79	3663	3892	1.06252
1993	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19	38	44	54	2971	3327	1.11982
1994	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	36	80	78	2478	2864	1.15577
1995	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	31	71	1167	2310	1.97943
1996	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	29			

and then calculation of CGFs. Also, because the original Arrington method requires a few additional steps to calculate CGFs, it is more likely for CGF to deviate more from the actual data. To compare the two approaches, the first fifteen years (1977–1991) out of twenty years (1977–1996) of data from the 1996 version of the OGIFF database were used. The last five years (1992–1996) of data were used for performance comparison of the two models.

AGF Smoothing

AGFs for oil are calculated using the previously described methodology. The data are plotted against years since discovery and are regressed to obtain a best-fit curve. Of all the functions for regression (such as exponential, power, reciprocal, and logarithmic), the reciprocal function provided the best results, for which a generalized equation can be written as:

$$AGF = \alpha + \beta / YSD \tag{4}$$

where α and β are the constants. *YSD* is the Year Since Discovery, and its range is 1–78.

However, a curve based on the reciprocal function showed a poor match with the actual data when all the AGF data were included in the regression, but a significantly improved match when the first AGF data point was excluded. These regression analysis results are shown in figure 1.

As can be seen in figure 1, curve 1 resulting from the regression of all the data points shows a lower AGF (1) value (about 1.7579) relative to the first data point (1.9794), higher AGFs values than the data for the years 2 through 13 (for example, whereas curve 1 shows AGF (2) and AGF (3) as 1.3692 and 1.2397, respectively, the corresponding values of the data points are 1.1564 and 1.1226, respectively), and AGF values less than 1 for all the years beyond 40 years. An AGF value of less than 1 means reduction in reserves. Curve 2, which was obtained by regressing all but the first data point, shows excellent match with the actual AGF data. Because the AGF data points for the early years are critical in shaping the cumulative growth function, curve 2 is considered more appropriate for developing reserve growth functions.

Using curve 2 produces two options for determining the first AGF value to be applied in developing reserve growth functions:

1. Extrapolate curve 2, as was done in the Arrington method (in the original Arrington method, there was no choice but to extrapolate the curve, because first AGF was not available).
2. Accept the first AGF data (1.97943) as the first point for curve 2.

For option 1, the extrapolated value of the first AGF (1.25) is substantially different from the actual data (1.9794). Therefore, I decided to go with option 2, using the first data point (1.97943) as the first AGF of regression curve. For option 2, the values of constants, alpha and beta, obtained from regression of data are 1.0018 and 0.3088, respectively. The AGFs from the regression curve are then used to calculate the cumulative growth factor (CGF), as per the following equation.

$$CGF(n) = AGF(1) \times AGF(2) \times AGF(3)......AGF(n) \tag{5}$$

where "*n*" is the year since discovery.

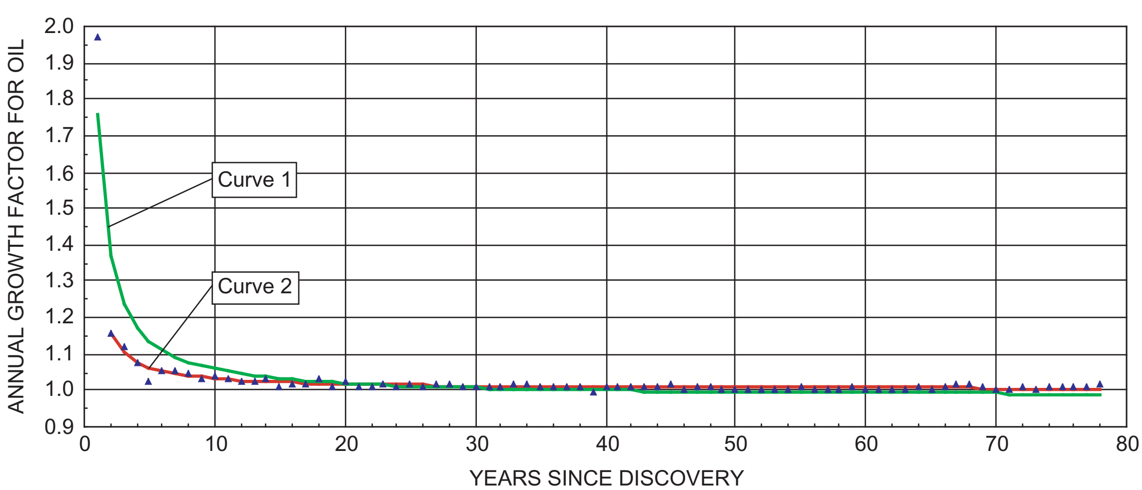

Figure 1. Regression analysis of annual growth factors for oil in conterminous U.S. (based on first 15 years of data from OGIFF 1977–1996), using reciprocal function: Curve 1 (green line) is based on regressing all data points; curve 2 (red line) excludes the first data point (1.97943); data points plotted as blue triangles.

CGF Smoothing

Alternatively, CGFs are calculated from the AGF data, using equation 5, and the calculated CGF data are regressed using a power function for a satisfactory match. Of all the functions for producing regression curves (such as reciprocal, hyperbolic, power, and logarithmic), the power function gave the best results. The resulting equation for calculating CGF is

$$CGF = 1.7378(YSD)^{0.3152} \qquad (6)$$

where *YSD* is the Year Since Discovery, and its range is 1–78.

The CGFs from the regression curve are the final CGFs to be used in developing growth functions.

To establish a procedure for developing growth models with improved forecasting capability, the reserve forecasts from the two approaches—AGF smoothing and CGF smoothing—were compared with the actual oil volumes over a 5-year period from 1992 through 1996 (fig. 2). From the plot, it can be seen that the CGF smoothing provides a better match with the data than the AGF smoothing; thus, CGF smoothing was used as the basis for building reserve growth functions (modified Arrington method) for this report.

Reserve Growth Functions Based on the Modified Arrington Method

As just indicated, the CGF smoothing was chosen as the method for data smoothing in the modified Arrington method. Of all the functions for producing regression curves (such as reciprocal, hyperbolic, power, and logarithmic), power functions gave the best results and were therefore utilized for regression of CGF data for both oil and gas fields. A generalized equation for the power function can be written as

$$CGF = \alpha(YSD)^{\beta} \qquad (7)$$

For oil fields, the values of α and β, obtained from the regression of CGF data, are 1.75752 and 0.30050, respectively, and *YSD* is Years Since Discovery and ranges from 1 through 95.

Since the new growth functions based on the modified Arrington method are used for forecasting hydrocarbon reserve growth, I decided to check the precision of regression, based on the procedure described by Draper and Smith (1966), and the strength of correlation, based on the procedure by Walpole and Myers (1978). The regression precision, which has a significant impact on the models' prediction capabilities, can be judged by the ratio (R^2) of the sum of squares due to regression to the sum of squares about the mean. The value of this ratio (R^2) ranges from 0 to 1, with 0 for the poorest and 1 for the best (ideal) regression curve. A high value (0.939) of ratio (R^2) indicates a high degree of precision in data regression, and the high correlation coefficient (0.996) shows a high confidence in correlation for the equation. Therefore, we expect that the oil growth functions should result in a reliable forecast, so long as the economic conditions during the forecast period remain similar to the conditions in the historical-data period.

The cumulative growth factors (functions) for U.S. oil fields are shown in figure 3 and table 2. For comparison, the oil growth functions from the USGS 1995 National

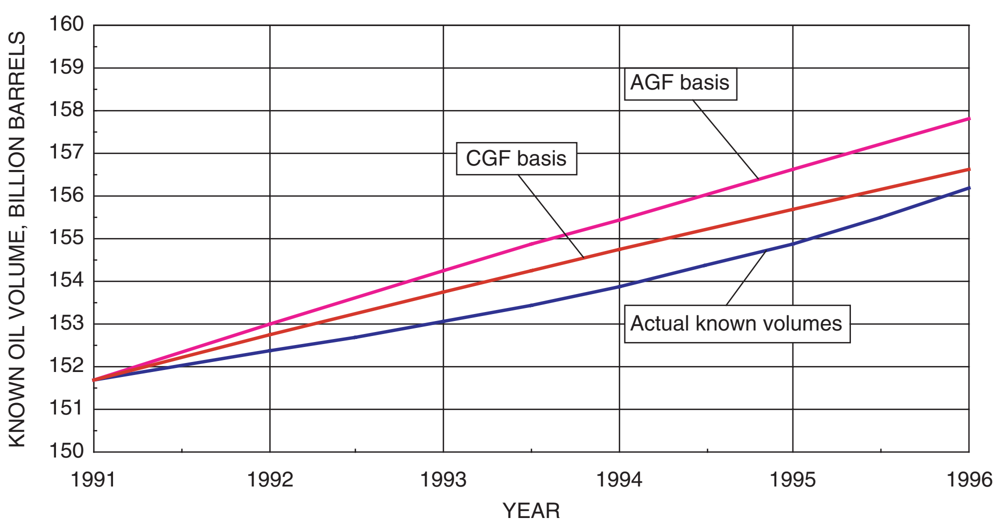

Figure 2. Forecasts of reserve growth based on smoothing of annual growth factors (AGFs) and cumulative growth factors (CGFs), compared with known oil volumes in conterminous U.S. for period 1992–1996.

Assessment of U.S. Oil and Gas Resources, as reported by Attanasi and others (1994), are also plotted in figure 3.

For gas fields, the initial attempt to regress the data with a power function did not yield as good a match as for the oil field data. Therefore, the CGF data were split into two segments—one segment covering the first 20 years and the second segment the rest of the growth period, resulting in an excellent match. Figure 4 clearly demonstrates the improvement brought about by splitting the data into two segments and regressing them separately.

Values of α and β for the curve regressing all the data are 1.56639 and 0.36060, respectively, and for the curve with two segments are as follows:

	α	β
First segment with YSD range of 1–20	1.75590	0.30220
Second segment with YSD range of 21–95	1.14183	0.44670

For gas growth functions, the ratio (R^2) for the curve regressing all data was calculated as 0.895, and for the two-segments curve as 0.995, demonstrating the relative superiority of the two-segments curve. Based on the preceding, the two-segments curve was accepted for the gas growth functions. To check the strength of correlation, the correlation coefficient (r) for the equation based on the two-segments curve was calculated as 0.994, indicating excellent correlation between the dependent and independent variables. The composite cumulative growth values based on regressions of two separate segments are shown in figure 5 and table 3. For comparison, the gas growth functions from the USGS 1995 National Assessment of U.S. Oil and Gas Resources, as reported by Attanasi and Root (1994), are also plotted in figure 5.

Validation of the New Growth Functions

To validate the new growth functions (based on the modified Arrington method) against the earlier functions based on the least-square method from the 1995 National Assessment, a comparison of reserve growth predictions was made, based on these two functions. Earlier growth functions were developed using the 1991 version of the OGIFF dataset (reserve estimates made in 1977 through 1991). Therefore, it was essential that we use the same 1991 version of the OGIFF dataset for developing the new growth functions to maintain a common basis for performance comparison. The forecasts from the two functions were compared with the actual reported KPV values for the 5-year period from 1992 through 1996.

Because of the subsequent revision of data by EIA, the KPV values for the year 1991 were found to be different in the two versions of OGIFF dataset owing to revision of the data by EIA. The 1991 KPV values in the 1996 version of the OGIFF dataset (containing reserve estimates made in 1977 through 1996) were found to be 0.4 percent higher for oil in oil fields, 3.1 percent higher for gas in oil fields, 2.1 percent lower for gas in gas fields, and 0.7 percent lower for liquid in gas fields. In view of these differences in KPVs, I considered it appropriate to use the 1991 version of the OGIFF dataset for developing growth functions, but then to use the 1996 version of the OGIFF dataset for 1991 KPVs to ensure consistency of data when grown oil and gas volumes were compared with actual reported KPV values. The results are plotted in figure 6 for oil and figure 7 for gas in the conterminous U.S., excluding Federal Offshore and continuous fields.

As can be seen from the two illustrations, the modified Arrington method yields much better results for the oil in oil fields (fig. 6) and slightly better results for the gas in gas

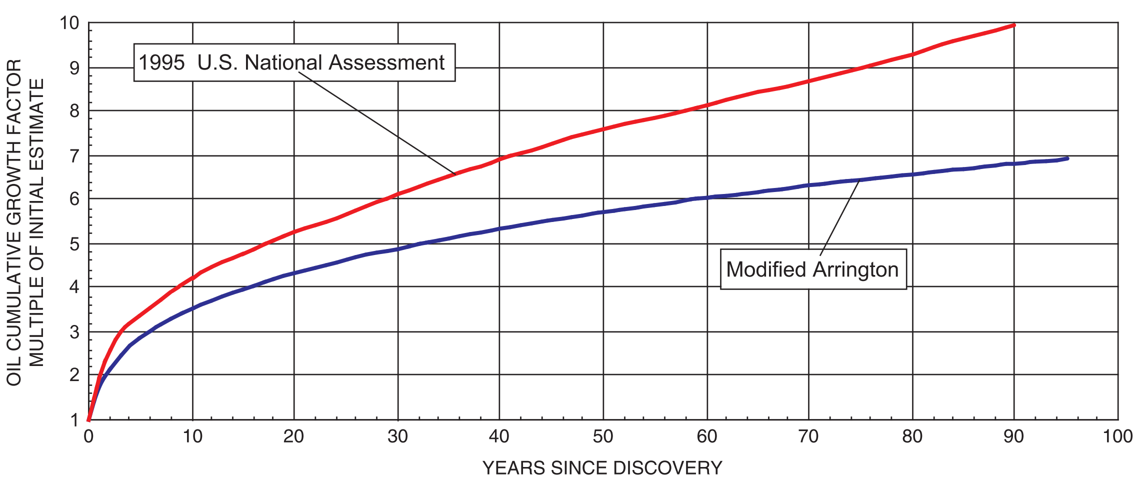

Figure 3. Cumulative growth functions for oil fields in conterminous U.S., excluding Federal Offshore and continuous fields, based on modified Arrington method using data from 1977–1996 Oil and Gas Integrated Field File (OGIFF) database. For comparison, the growth functions resulting from USGS 1995 National Assessment of U.S. Oil and Gas Resources (Attanasi and Root, 1994), using 1977–1991 OGIFF database, are also shown.

Table 2. Oil cumulative growth factor, based on modified Arrington method using 1977–1996 Oil and Gas Integrated Field File.

[These functions are for the U.S. Lower 48 States, excluding Federal Offshore and continuous fields]

Years since discovery	Modified Arrington cum. growth factor	Years since discovery	Modified Arrington cum. growth factor
0	1.00000	50	5.69430
1	1.75752	51	5.72829
2	2.16451	52	5.76181
3	2.44498	53	5.79489
4	2.66575	54	5.82753
5	2.85063	55	5.85975
6	3.01116	56	5.89156
7	3.15393	57	5.92298
8	3.28306	58	5.95402
9	3.40134	59	5.98468
10	3.51075	60	6.01498
11	3.61276	61	6.04494
12	3.70846	62	6.07454
13	3.79874	63	6.10382
14	3.88429	64	6.13278
15	3.96566	65	6.16142
16	4.04332	66	6.18975
17	4.11765	67	6.21778
18	4.18899	68	6.24553
19	4.25761	69	6.27298
20	4.32374	70	6.30017
21	4.38760	71	6.32708
22	4.44937	72	6.35373
23	4.50920	73	6.38012
24	4.56724	74	6.40625
25	4.62361	75	6.43215
26	4.67842	76	6.45780
27	4.73178	77	6.48322
28	4.78378	78	6.50840
29	4.83449	79	6.53337
30	4.88399	80	6.55811
31	4.93235	81	6.58264
32	4.97964	82	6.60695
33	5.02590	83	6.63106
34	5.07118	84	6.65497
35	5.11555	85	6.67868
36	5.15904	86	6.70219
37	5.20169	87	6.72552
38	5.24355	88	6.74865
39	5.28463	89	6.77161
40	5.32499	90	6.79438
41	5.36465	91	6.81698
42	5.40364	92	6.83940
43	5.44198	93	6.86166
44	5.47971	94	6.88375
45	5.51684	95	6.90567
46	5.55340		
47	5.58940		
48	5.62488		
49	5.65984		

fields (fig. 7), as explained herein. Over the 5-year period from 1992 to 1996, the new reserve growth functions based on the modified Arrington method predicted growths of 5.1 billion barrels of oil in oil fields and 29.7 trillion cubic feet (TCF) of gas in gas fields for the conterminous U.S., excluding Federal Offshore and continuous fields. For the same period, the growth functions from the 1995 National Assessment gave reserve growths of 6.3 billion barrels of oil and 28.1 TCF of gas. A comparison of these predicted growths with actual growths (4.5 billion barrels of oil and 42.2 TCF of gas) indicates that, whereas the modified Arrington method predicts 11.9 percent higher oil growth and 29.8 percent lower gas growth, the least-squares based functions (models) from the 1995 National Assessment predict 39.3 percent higher oil growth and 33.6 percent lower gas growth. Based on these results, the new growth functions provide more accurate forecasts, particularly for oil. The work by Attanasi and others (1999), who concluded that the functions based on the

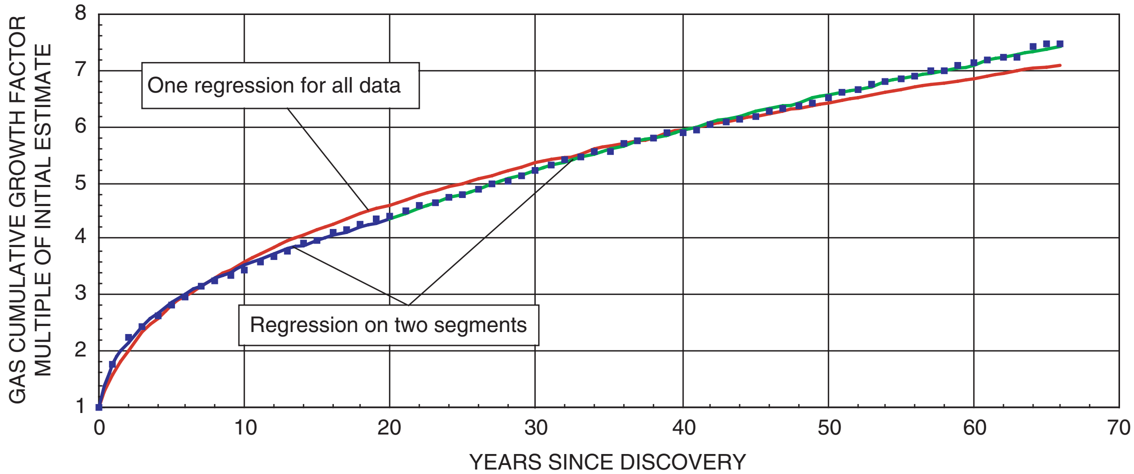

Figure 4. Results of regressing the cumulative growth factor (CGF) data for conterminous U.S. gas fields: one curve based on regression of all data (red line); second curve based on splitting data into two segments (one including the first 20 years shown as blue line, and the other the remainder of the growth period shown as green line), and regressing the two segments separately. Data points are shown as blue squares.

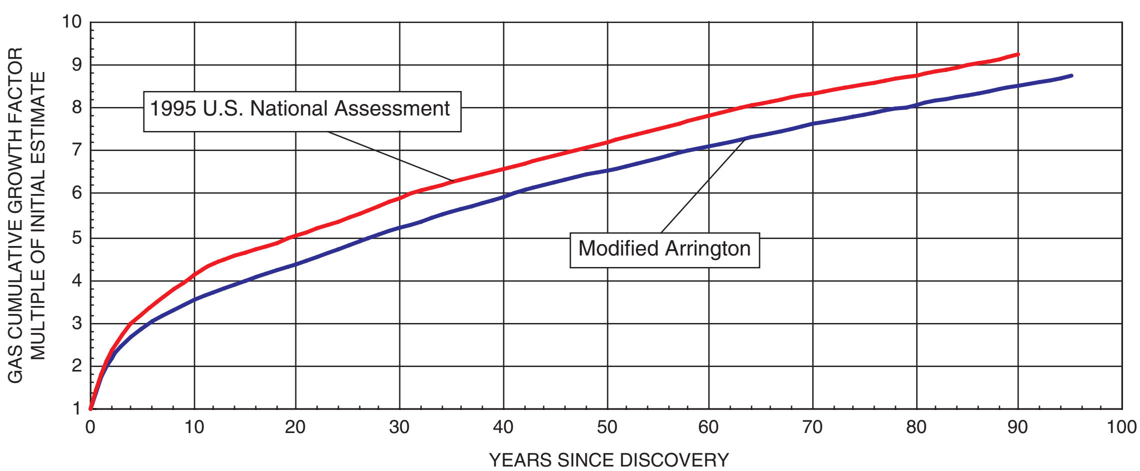

Figure 5. Gas cumulative growth functions for gas fields in conterminous U.S., excluding Federal Offshore and continuous fields, based on modified Arrington method using data from 1977–1996 Oil and Gas Integrated Field File (OGIFF) database. For comparison, the growth functions resulting from USGS 1995 National Assessment of U.S. Oil and Gas Resources (Attanasi and Root, 1994), using 1977–1991 OGIFF database, are also shown.

Table 3. Gas cumulative growth factor, based on modified Arrington method using 1977–1996 Oil and Gas Integrated Field File.

[These functions are for the U.S. Lower 48 States, excluding Federal Offshore and continuous fields]

Years since discovery	Modified Arrington cum. growth factor	Years since discovery	Modified Arrington cum. growth factor
0	1.00000	50	6.55435
1	1.75590	51	6.61259
2	2.16507	52	6.67020
3	2.44729	53	6.72719
4	2.66958	54	6.78360
5	2.85580	55	6.83943
6	3.01757	56	6.89470
7	3.16147	57	6.94943
8	3.29165	58	7.00363
9	3.41092	59	7.05732
10	3.52127	60	7.11050
11	3.62417	61	7.16319
12	3.72073	62	7.21541
13	3.81183	63	7.26717
14	3.89816	64	7.31847
15	3.98029	65	7.36934
16	4.05868	66	7.41977
17	4.13372	67	7.46978
18	4.20575	68	7.51937
19	4.27503	69	7.56857
20	4.34181	70	7.61737
21	4.44872	71	7.66579
22	4.54214	72	7.71384
23	4.63323	73	7.76151
24	4.72216	74	7.80883
25	4.80905	75	7.85579
26	4.89405	76	7.90241
27	4.97726	77	7.94869
28	5.05878	78	7.99463
29	5.13870	79	8.04026
30	5.21711	80	8.08556
31	5.29409	81	8.13055
32	5.36970	82	8.17524
33	5.44402	83	8.21963
34	5.51711	84	8.26372
35	5.58901	85	8.30752
36	5.65979	86	8.35104
37	5.72949	87	8.39427
38	5.79815	88	8.43724
39	5.86582	89	8.47993
40	5.93253	90	8.52236
41	5.99833	91	8.56453
42	6.06325	92	8.60645
43	6.12732	93	8.64811
44	6.19056	94	8.68953
45	6.25302	95	8.73070
46	6.31472		
47	6.37567		
48	6.43592		
49	6.49547		

least-squares method from the 1995 National Assessment projected 33 percent higher oil growth and 25 percent lower gas growth over a 5-year period (1992–1996), confirmed that the new reserve growth functions give better results.

Results and Discussion

The new reserve growth functions based on the modified Arrington method for oil and gas in the conterminous U.S., excluding Federal Offshore and continuous fields, are in figures 3 and 5, respectively. These illustrations show that, on average, an oil field will grow by a factor of 6.9 and a gas field by a factor of 8.7 over the 95-year period. This higher growth for gas at first seems contrary to expectations, but apparently other factors, such as oil and gas prices, outweighed the impact of reservoir characteristics (Attanasi and others, 1999).

Using the 1991 version of the OGIFF dataset, forecasts of oil and gas reserve growth in the U.S. over the next

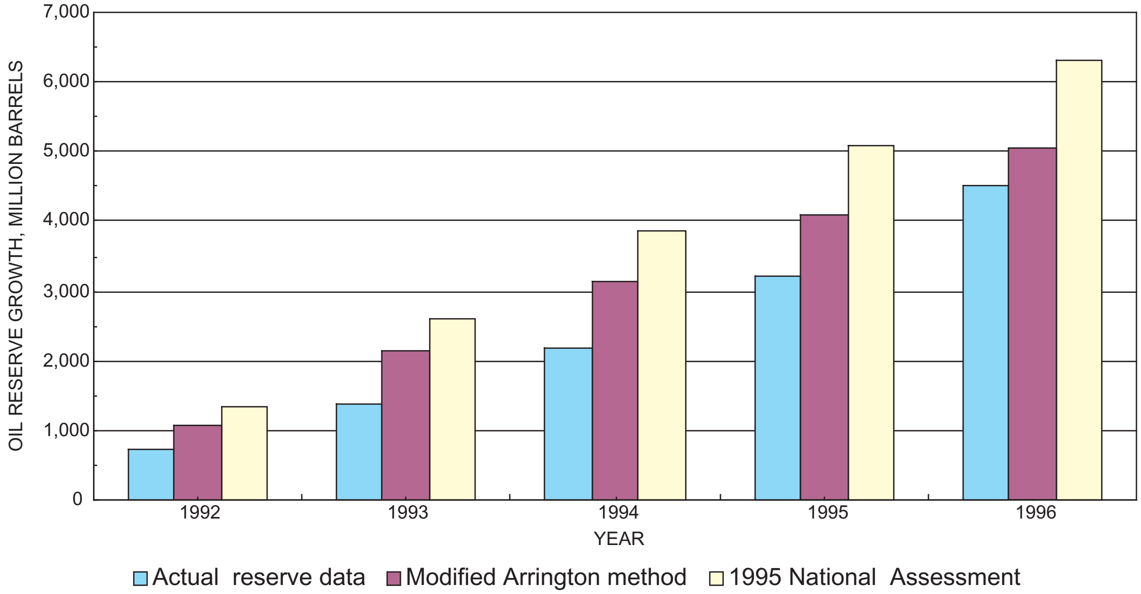

Figure 6. Forecasts of oil reserve growth in conterminous U.S., excluding Federal Offshore and continuous fields, based on modified Arrington method and growth functions from USGS 1995 National Assessment of U.S. Oil and Gas Resources (Attanasi and Root, 1994), compared with actual growth values over period 1992–1996. Both growth functions are based on 1977–1991 Oil and Gas Integrated Field File (OGIFF) dataset to maintain a common basis, but the 1991 known oil volumes are taken from 1977–1996 OGIFF dataset to project growth.

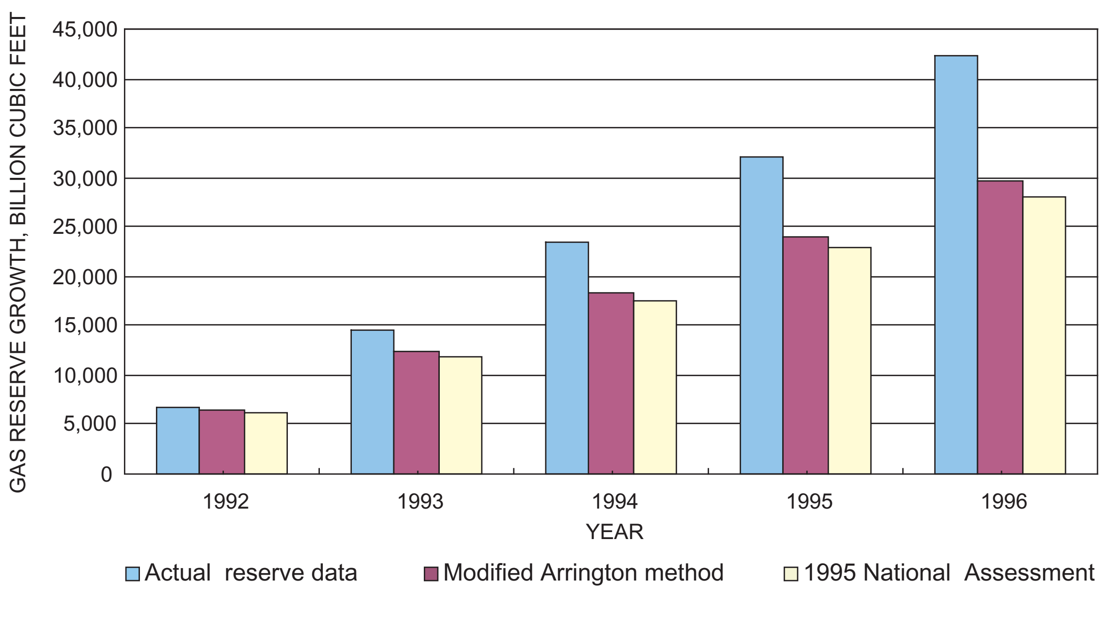

Figure 7. Forecasts of gas reserve growth in conterminous U.S., excluding Federal Offshore and continuous fields, based on modified Arrington method and growth functions from USGS 1995 National Assessment of U.S. Oil and Gas Resources (Attanasi and Root, 1994), compared with actual growth values over period 1992–1996. Both growth functions are based on 1977–1991 Oil and Gas Integrated Field File (OGIFF) dataset to maintain a common basis, but the 1991 known gas volumes are taken from 1977–1996 OGIFF dataset to project growth.

Table 4. Reserve growth forecasts based on growth functions from modified Arrington method and USGS 1995 National Assessment for the Lower 48 States, excluding Federal Offshore and continuous fields.

[Data source: 1977–1991 Oil and Gas Integrated Field File (OGIFF) dataset. Gas volumes have been converted to equivalent oil volumes using a factor of 6,000 SCF/barrel; SCF, standard cubic feet; TCF, trillion cubic feet]

Hydrocarbon type	Known petroleum volume -1991	30-year reserve increase		80-year reserve increase	
		Mod. Arrington	95 Assessment	Mod. Arrington	95 Assessment
Oil in oil fields, billion bbl	151.026	22.896	31.403	31.387	45.463
Gas in oil fields, TCF	248.142	44.380	59.146	63.355	90.372
Gas in gas fields, TCF	534.194	144.758	138.616	232.635	222.601
Oil in gas fields, billion bbl	7.003	1.727	1.662	2.717	2.609
Total oil equivalent, billion bbl	288.418	56.146	66.025	83.436	100.234

Table 5. Reserve growth forecasts based on growth functions from modified Arrington method for the Lower 48 States, excluding Federal Offshore and continuous fields.

[Data source: 1977–1996 Oil and Gas Integrated Field File (OGIFF) dataset. Gas volumes have been converted to equivalent oil volumes using a factor of 6,000 SCF/barrel. SCF, standard cubic feet; TCF, trillion cubic feet]

Hydrocarbon type	Known petroleum volume 1996	Modified Arrington method Potential reserve increase	
		30-year	80-year
Oil in oil fields, billion bbl	156.186	19.512	26.894
Gas in oil fields, TCF	270.235	39.393	56.759
Gas in gas fields, TCF	565.068	148.332	243.760
Oil in gas fields, billion bbl	7.280	1.745	2.800
Total oil equivalent, billion bbl	302.683	52.545	79.781

30-year and 80-year periods were made using the modified Arrington method, and these were then compared with the forecasts based on functions developed during the USGS 1995 National Assessment of U.S. Oil and Gas Resources. Such comparisons show that the growth functions based on a modified Arrington method gave reserve increases of 22.9 billion barrels of oil and 44.4 TCF of gas in oil fields, whereas the earlier functions from the 1995 National Assessment gave corresponding values of 31.4 billion barrels of oil and 59.1 TCF of gas, over a 30-year period (1991–2021). For gas fields, the new growth functions predicted reserve increases of 144.8 TCF of gas and 1.7 billion barrels of liquid hydrocarbons, whereas the functions from the 1995 National Assessment gave corresponding increases of 138.6 TCF of gas and 1.7 billion barrels for liquid hydrocarbons. Thus, the growth functions from the 1995 National Assessment predicted 37 percent higher oil and 4 percent lower gas reserves. These values are in line with the results of Attanasi and others (1999), who reported that the earlier growth functions tended toward higher oil and lower gas growths. The end result is that in comparison with the growth functions from the USGS 1995 National Assessment of Oil and Gas Resources (Attanasi and Root, 1994), the modified Arrington method gave relatively much lower oil reserves but somewhat similar gas reserves.

Table 4 lists the reserve increases for oil and gas in both oil and gas fields estimated for the conterminous U.S., excluding Federal Offshore and continuous fields, over 30-year (1991–2021) and 80-year (1991–2071) periods, based on the new growth functions as well as the functions from the 1995 National Assessment. The table also lists reserve growth estimates in terms of barrels of oil equivalent (BOE), which have been calculated using a factor of 6,000 standard cubic feet (SCF) of gas per barrel, based on British thermal unit (Btu) values of oil and gas, to convert gas to oil volumes.

The new reserve growth functions for oil and gas fields based on the modified Arrington method, using the 1996 version of the OGIFF dataset, are shown in figures 3 and 5 and in tables 2 and 3. Reserve growth forecasts for 30-year (1996–2026) and 80-year (1996–2076) periods are given in table 5. The average annual increases amount to 0.42 percent for oil and 0.49 percent for gas in oil fields, and to 0.88 percent for gas and 0.80 percent for liquid hydrocarbons in gas fields over the 1996–2026 period. Higher growth for gas compared to oil is in line with the trend of growth functions, as shown in figure 3 for oil and in figure 5 for gas. Table 5 also shows total growth in terms of barrels of oil equivalent (BOE), using a factor of 6,000 SCF per barrel, based on Btu values of oil and gas, to convert gas to oil volumes.

Conclusions

1. The new oil and gas growth functions (modified Arrington method) are based on a high degree of precision in their data regression.
2. The new reserve growth functions show that oil fields in the U.S., over a 95-year period, would grow 6.9 times and gas fields 8.7 times the initial reserve estimates made at the time of discovery.
3. Compared to the growth functions from the USGS 1995 National Assessment of U.S. Oil and Gas Resources, the new growth functions provide a more conservative estimate of oil and a somewhat similar estimate for gas.
4. For a short-term forecast—for example, the 5-year period from 1992 through 1996—the modified Arrington method predicts growths in oil and gas reserves that are closer to the actual growths than that predicted by the growth functions from the USGS 1995 National Assessment, particularly for oil.
5. The new oil and gas growth functions predict an average potential reserve growth of 0.42 percent per year for the oil in oil fields and 0.88 percent per year for the gas in gas fields over the 30-year period from 1996 through 2026.

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