Joint Inversion of Acoustic and Resistivity Data for the Estimation of Gas Hydrate Concentration

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Abstract

Downhole log measurements, such as acoustic or electrical resistivity logs, are frequently used to estimate in situ gas hydrate concentrations in the pore space of sedimentary rocks. Usually the gas hydrate concentration is estimated separately based on each log measurement. However, measurements are related to each other through the gas hydrate concentration, so the gas hydrate concentrations can be estimated by jointly inverting available logs. Because the magnitude of slowness of acoustic and resistivity values differs by more than an order of magnitude, a least-squares method, weighted by the inverse of the observed values, is attempted. Estimating the resistivity of connate water and gas hydrate concentration simultaneously is problematic, because the resistivity of connate water is independent of acoustics. In order to overcome this problem, a coupling constant is introduced in the Jacobian matrix. In the use of different logs to estimate gas hydrate concentration, a joint inversion of different measurements is preferred to the averaging of each inversion result.

Introduction

Gas hydrate has become an important research topic, because of its significance as a potential resource and because it is a controlling element in global warming, as well as a factor relevant to sea floor stability (Sloan and others, 1999). In this context, the estimation of in situ gas hydrate amounts present in sediments is important, and downhole well-log measurements are frequently used to quantify accumulations.

Gas hydrate in the pore space increases the elastic velocities and electrical resistivities of the host sediments. Therefore, downhole acoustic and electrical resistivity logs are most commonly used to identify and quantify natural gas hydrate resources.

Collett (1983, 1995) used downhole measurements extensively not only to identify the presence of gas hydrate in sediments and but also to quantify the amount of gas hydrate. Mathews (1986) used electrical resistivity logs to estimate gas hydrate saturation at the NW-Eileen #2 well, Alaska, and the deep sea drilling project Site 570 at Blake Ridge. Recently, Collett and Ladd (2000) also used resistivity logs to estimate gas hydrate amounts at Blake Ridge for Ocean Drill Program leg 164, Sites 994, 995, and 997, as did Miyairi and others (1999) at the Mallik 2L-38 gas hydrate research well, northeastern

Canada. Acoustic data have likewise been used to estimate hydrate concentrations. (For example, see Collett, 1983; 1995; Guerin and others, 1999; Lee and Collett, 1999; Lee, 2000.)

These estimates of gas hydrate were all derived based on measurements from only one log type, except that Miyairi and others (1999) used a statistical inversion of the logs. Because log measurements—*P*-wave, *S*-wave, and resistivity—are related to each other through the gas hydrate in the pore space, gas hydrate concentrations can be estimated by jointly inverting acoustic and resistivity logs.

This paper presents a method of estimating gas hydrate concentration by jointly inverting acoustic slowness (*P*-wave and *S*-wave slowness) and resistivity. Theoretical predictions of slownesses are computed from a weighted equation (Lee and others, 1996), and resistivities of sediments are computed from Archie's equation (1942). This inversion method was applied to the log data acquired at the Mallik 2L-38 gas hydrate research well, Mackenzie Delta, Canada (Dallimore and others, 1999).

Acknowledgments

Theory

Inversion Equation

Gas hydrate concentration can be estimated by a joint inversion of the acoustic (*P*- and *S*-wave) and resistivity data. Theoretical development of geophysical inversion methods has been extensive (Aki and Richards, 1980; Lines and Treitel, 1984; Tarantola, 1987). The derivation of the inversion method in this article closely follows Lee (1990). The method can be formulated by the Taylor series expansion of acoustic and resistivity equations, assuming that the unknown variables are the gas hydrate concentration (c) and the resistivity of the connate water (R_w).

Column vectors O and T are defined as follows:

O = column vector of the slownesses of P-wave (S_p), and S-wave (S_s), and the resistivity (R_t).

T = column vector of the computed or theoretical slowness of P-wave, slowness of S-wave, and resistivity.

Acoustics and resistivity can be approximated by the following equation:

$$T(m^{n+1}) = T(m^n) + \left(\frac{\partial T}{\partial c}\right)_n \Delta c + \left(\frac{\partial T}{\partial R_w}\right)_n \Delta R_w + ..., \text{ or}$$

$$\begin{bmatrix} S_p(m^{n+1}) \\ S_s(m^{n+1}) \\ R_t(m^{n+1}) \end{bmatrix} = \begin{bmatrix} S_p(m^n) \\ S_s(m^n) \\ R_t(m^n) \end{bmatrix} + \begin{bmatrix} \partial S_p(m^n)/\partial c \\ \partial S_s(m^n)/\partial c \\ \partial R_t(m^n)/\partial c \end{bmatrix} \Delta c + \begin{bmatrix} 0 \\ 0 \\ \partial R_t(m^n)/\partial R_w \end{bmatrix} \Delta R_w \text{ or,}$$

$$\begin{bmatrix} S_p(m^{n+1}) \\ S_s(m^{n+1}) \\ R_t(m^{n+1}) \end{bmatrix} = \begin{bmatrix} S_p(m^n) \\ S_s(m^n) \\ R_t(m^n) \end{bmatrix} + \begin{bmatrix} \partial S_p(m^n)/\partial c, & 0 \\ \partial S_s(m^n)/\partial c, & 0 \\ \partial R_t(m^n)/\partial c, & \partial R_t(m^n)/\partial R_w \end{bmatrix} \begin{bmatrix} \Delta c \\ \Delta R_w \end{bmatrix} \text{ or,}$$

$$T(m^{n+1}) = T(m^n) + G\Delta m \qquad (1)$$

where m is a parameter vector consisting of c and R_w, G is a 3×2 Jacobian matrix,

$$\Delta c = c^{n+1} - c^n; \Delta R_w = R_w^{n+1} - R_w^n ; \text{ and } \Delta m = m^{n+1} - m^n,$$

which is a column vector of $(\Delta c, \Delta R_w)$.

The least-squares solution of Δm in Equation (1) for a given observation O is attained by minimizing the mean-squared error (E): $E = (O - T)^2$. Because the magnitude of resistivity is different from the values of slowness, a weighted least-squares method is attempted; the error function is defined as

$$E_i = ((O_i - T_i)/O_i)^2$$

where $i = 1, 2,$ and 3, which are values that represent indices for the P-wave slowness, S-wave slowness, and resistivity respectively; E_i is the component of the error function; T_i is the component of the computed value; and O_i is the component of the observed value. Then the solution can be written as

$$\Delta m = G_{we}^{-1}(I_3 - T_i/O_i) = G_{we}^{-1}\mathrm{E} \qquad (2)$$

where G_{we}^{-1} is the generalized inverse of the 3×2 weighted Jacobian matrix (element of G_{we} is the element of G in Equation (1) divided by the corresponding observed value), and I_3 is a 3×1 column vector consisting of 1's. The generalized inverse of G_{we}^{-1} can be obtained using a singular value decomposition (SVD), and the solution can be written as

$$m^{n+1} = m^n + V\Lambda^{-1}U^T(I_3 - T_i/O_i) \qquad (3)$$

where V is a 2×2 matrix of orthonormal eigenvectors, U is a 3×3 matrix of orthonormal eigenvectors, Λ is a diagonal matrix of two singular values, and the superscript T denotes a matrix transpose.

In this analysis, the derivatives for the Jacobian G are obtained from a weighted equation for the acoustic data (Lee and others, 1996) and the Archie equation (Archie, 1942) for the resistivity measurement.

Derivatives for the Acoustics

A weighted equation (WE) can be written as follows, using slowness of the constituents (based on Lee and others, 1996):

$$S_p = W\phi(1-c)S_{p1} + [1 - W\phi(1-c)]S_{p2} \qquad (4)$$

where

S_p = slowness by the WE
S_{p1} = slowness by the Wood equation
S_{p2} = slowness by the time average equation
W = a weighting factor
ϕ = porosity
c = gas hydrate concentration.

The slowness by Wood equation (S_{p1}) and slowness by time average equation (S_{p2}) are given by

$$S_{p1} = \sqrt{\frac{\rho\phi(1-c)S_w^2}{\rho_w} + \frac{\rho\phi c S_h^2}{\rho_h} + \frac{\rho(1-\phi)S_m^2}{\rho_m}}$$

$$S_{p2} = \phi(1-c)S_w + \phi c S_h + (1-\phi)S_m \qquad (5)$$

where ρ, ρ_w, ρ_h, and ρ_m are densities of gas-hydrate-bearing sediment, water, gas hydrate, and modified matrix, respectively; S_w, S_h, and S_m are the slowness of water, gas hydrate, and modified matrix, respectively.

Shear-wave velocity is given by the following formula (Lee and others, 1996):

$$S_s = \frac{S_p}{\alpha(1-\phi) + \beta c} \qquad (6)$$

where S_s is the slowness of the S-wave, α and β are the ratios of S-wave velocity to P-wave velocity for the matrix of non-gas-hydrate-bearing sediments and for gas hydrate, respectively.

Using Equations (4), (5) and (6), the derivatives can be obtained as

$$\frac{\partial S_p}{\partial c} = -W\phi S_{p1} + W\phi S_{p2} + W\phi(1-c)\frac{\partial S_{p1}}{\partial c} + [1 - W\phi(1-c)]\frac{\partial S_{p2}}{\partial c} \qquad (7)$$

where

$$\frac{\partial S_{p1}}{\partial c} = 0.5[-\rho\phi S_w^2/\rho_w + \rho\phi S_h^2/\rho_h]/S_{p1}, \text{ and}$$

$$\frac{\partial S_{p2}}{\partial c} = -\phi S_w + \phi S_h$$

$$\frac{\partial S_s}{\partial c} = \frac{(\partial S_p)/\partial c)[\alpha(1-\phi) + c\beta\phi] - \beta\phi S_p}{[\alpha(1-\phi) + c\beta\phi]^2} \qquad (8)$$

Derivatives for the Resistivity

Water saturation (C_w) from the Archie equation is given by

$$C_w = \left(\frac{aR_w}{\phi^m R_t}\right)^{1/n} \qquad (9)$$

where a, m, and n are constants, R_t is the resistivity of the formation, R_w is the resistivity of the connate water, and c,

which is given by $c = (1 - C_w)$, is the gas hydrate concentration. Using Equation (9),

$$\frac{\partial R_t}{\partial c} = \frac{nR_t}{1 - c}$$

$$\frac{\partial R_t}{\partial R_w} = \frac{R_t}{R_w} \qquad (10)$$

Modified Joint Inversion (MJI)

The solution in Equations (2) or (3) can be written in the following way using the element of Jacobian matrix (G_{ij}).

$$c^{n+1} = c^n + \frac{G_{11}E_1 + G_{21}E_2}{G_{11}^2 + G_{21}^2}$$

$$R_w^{n+1} = R_w^n + \frac{-G_{31}G_{11}E_1 - G_{31}G_{21}E_2 + (G_{11}^2 + G_{21}^2)E_3}{G_{32}(G_{11}^2 + G_{21}^2)} \qquad (11)$$

The solution based on Equation (11) indicates that the resistivity measurement does not contribute directly to the estimation of gas hydrate concentration using the inversion scheme shown in Equation (1); there is no E_3 component in the estimation of c. The only contribution of resistivity measurement to the joint inversion is through R_w, not through c; accordingly, the estimated c from the P- and S-wave data is affected indirectly from the computed resistivity value, which provides minimum total error. In other words, the preceding joint inversion scheme computes the gas hydrate concentration using P-wave and S-wave data and attempts to fit the observed resistivity as closely as possible by changing R_w.

The reason for decoupling resistivity in Equation (11) for the estimation of c comes from the fact that the element of G_{12} and G_{22}, which are elements of the Jacobian matrix G shown in Equation (1), are zeros. In order to avoid this unwanted behavior, the elements of G_{12} and G_{22} are modified by adding an arbitrary constant (γ). As can be seen from the numerical example given later, the magnitude of the constants controls the coupling between acoustics and resistivity in the joint inversion. The joint inversion with $G_{12} = \gamma$ and $G_{22} = \gamma$, where γ is a non-zero constant, is called a modified joint inversion (MJI), and γ is called a coupling constant in this report. The inversion scheme using the original formulation, shown in Equation (1), is called the original joint inversion (OJI) to differentiate from MJI.

Inversion Examples

Original Joint Inversion (OJI)

In an investigation of the performance of the joint inversion using more than one type of measurements, parameters shown in table 1 were used. The initial values for c and R_w for the inversion were 0 and 0.4 ohm-meters (ohm-m), and both c and R_w were simultaneously estimated. Figure 1 shows the result of the joint inversion using P-wave, S-wave, and resistivity (PSR inversion) from the original inversion scheme shown
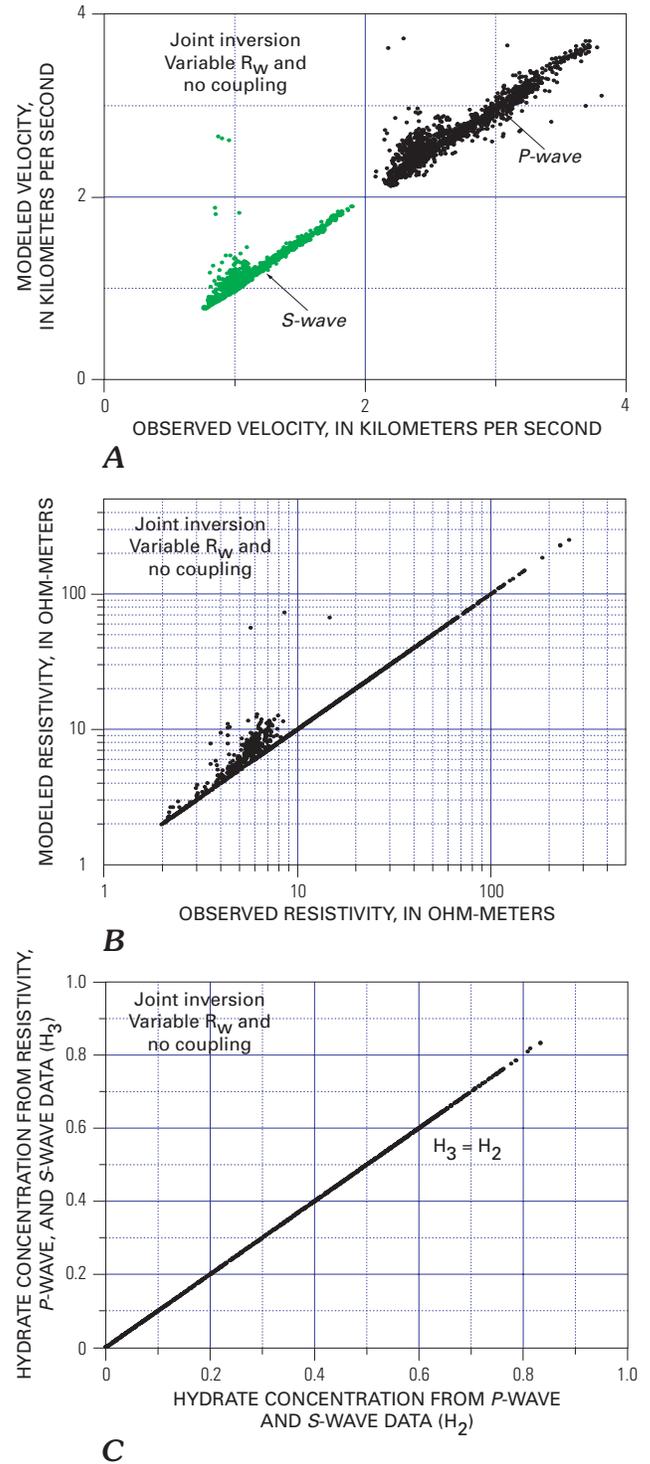
Figure 1. Graph showing the result of the original joint inversion of acoustics and resistivity. Both gas hydrate concentration (c) and the resistivity of connate water (R_w) were simultaneously estimated. *A*, Cross plot of modeled velocities versus observed velocities. *B*, Cross plot of modeled resistivity versus observed resistivity. *C*, Cross plot of gas hydrate concentration estimated from acoustics versus that estimated from the joint inversion of acoustics and resistivity.

in Equation (1). Figure 1*A* shows the comparison of the observed velocity versus the modeled (computed) velocity. Most of the scattering is about a 45° line and the average computed P-wave velocity is about 1 percent higher than the

Table 1. Values used in the inversion.

Meaning	Symbol	Value	Remarks
Slowness of hydrate (s/m)	S_h	0.303	Type 1 gas hydrate
Slowness of water (s/m)	S_w	0.667	
Slowness of modified matrix (s/m)	S_m	0.203	30% volume clay content
Ratio of V_s/V_p	α	0.56	Non-hydrate-bearing sediment
Ratio of V_s/V_p	β	0.51	Type 1 gas hydrate
Density of gas hydrate (g/cm^3)	ρ_h	0.91	Type 1 gas hydrate
Density of water (g/cm^3)	ρ_w	1.0	
Density of modified matrix (g/cm^3)	ρ_m	2.65	
Weighting factor	W	1.44	
Cementation factor	m	1.95	Archie's Equation
Archie's parameter	a	1.05	Archie's Equation
Exponent	n	1.9386	Archie's Equation
Resistivity of water (ohm-m)	R_w	0.4	

observed velocity; the average computed *S*-wave velocity is 3 percent higher than the observed velocity (table 1). When only *P*-wave and *S*-wave velocities are used in the joint inversion (PS inversion), the modeled velocities are 2.72 ±0.40 kilometers per second (km/s) and 1.21±0.27 km/s for the *P*-wave and *S*-wave velocities, respectively. The modeled *P*-wave velocity from PS inversion more closely matches the observed velocity, but there is little difference in *S*-wave velocities.

Figure 1*B* shows the modeled and observed resistivity (from PSR inversion). Unlike the acoustic case, the computed resistivities are almost identical to those observed when R_t is greater than about 9 ohm-m. Modeled resistivities are higher than the observed resistivities when R_t is less than 9 ohm-m.

The comparison between estimated gas hydrate concentrations using only acoustic data and estimates based on combined acoustic and resistivity data is shown in figure 1*C*. As the theory predicts, the estimates are almost identical. As indicated in Equation (11), the gas hydrate concentration is estimated mostly from *P*- and *S*-wave data, and R_w is estimated by matching the observed R_t with the theoretical prediction.

Figure 2, showing the estimated R_w for the preceding example and the observed resistivity (R_t) with respect to depth, indicates that the majority of estimated R_w values falls between 0.2 ohm-m and 1 ohm-m. A cross plot between the R_w and the observed resistivity (R_t) indicates that estimated R_w is roughly proportional to the observed resistivity. Figure 2 also indicates that the estimated R_w appears to decrease with respect to the increasing depth. A least-squares fitting curve for the estimated R_w is given by $R_w = 2.72-0.00214D$, where *D* is depth in meters (m). At $D = 1,000$ m, the estimated R_w is 0.58 ohm-m. The accuracy of the estimated R_w is discussed later.

Figure 3 shows the PSR inversion result when R_w is assumed to be a constant of 0.4 ohm-m. When R_w is a constant,

Equation (11) cannot be used for the inversion solution, and only the gas hydrate concentration, *c*, is estimated. Therefore, the Jacobian matrix shown in Equation (1) is a 3×1 column matrix. In this case, the solution is given by:

$$c^{n+1} = c^n + \frac{G_{11}E_1 + G_{21}E_2 + G_{31}E_3}{G_{11}^2 + G_{21}^2 + G_{31}^2} \qquad (12)$$

If the original solution form is used, the results of the PSR inversion using a constant R_w approach the results of the resistivity inversion (R inversion) as shown later.

The modeled resistivity is similar to the observed resistivity when resistivity is greater than 8 ohm-m (fig. 3*B*), and modeled acoustics show a large scattering around a 45° line (fig. 3*A*). Table 2 indicates that the average of the modeled *P*-wave velocity is 2.84 ±0.46 km/s, which is about 4 percent higher compared to the observed value, and 1.35 ±0.33 km/s for the *S*-wave velocity, which is about 15 percent higher. The estimated gas hydrate concentrations using acoustic and resistivity data are much higher than those based on acoustic data only (PS inversion), as shown in figure 3*C* and table 3.

Modified Joint Inversion Scheme (MJI)

Result of the MJI, estimating *c* and R_w simultaneously with γ=1, is shown in figure 4. Figure 4*A* shows the comparison of the observed velocity versus the modeled (computed) velocity. Like results shown in figure 1*A*, the scattering between the modeled and observed velocities is about the 45° line, and the average computed *P*-wave velocity is about 2 percent higher than the observed velocity; the average computed *S*-wave velocity is about 5 percent higher than the observed velocity (table 2).
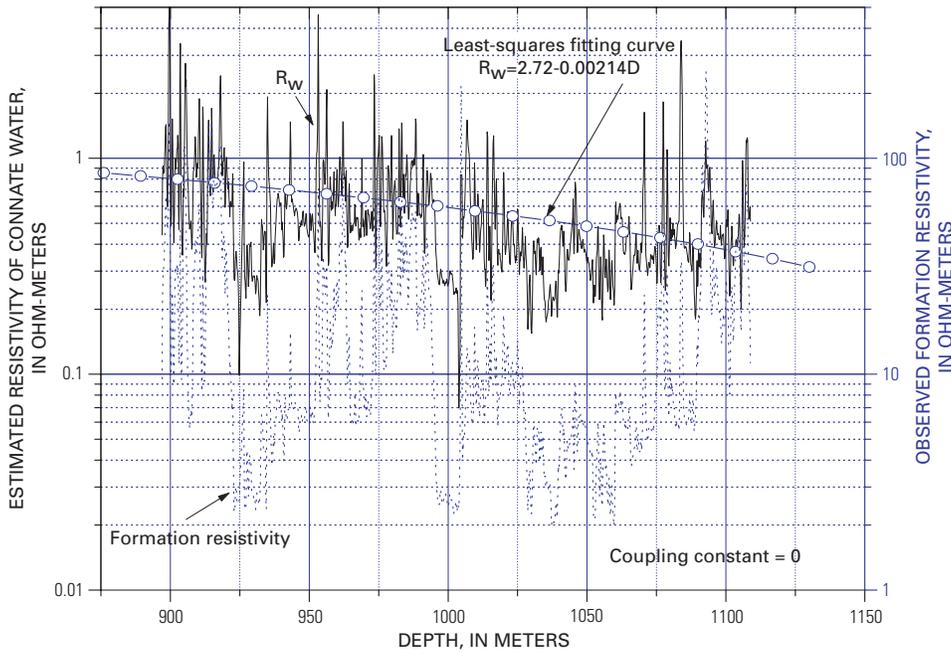
Figure 2. Graph showing estimated resistivity of connate water (R_w) using original joint inversion and observed formation resistivity with respect to depth. *D*, depth in meters.

Table 2. Modeled velocities and resistivity from the joint inversion.

[Percentage (%) in parentheses is the fractional error between observed and modeled values. Model 1, result from the joint inversion estimating c and R_w simultaneously using the original equation; Model 2, result from joint inversion estimating c with a constant $R_w = 0.4$ ohm-m; Model 3, result from the modified joint inversion with $\gamma = 1$ and estimating c and R_w simultaneously]

	Observed	Model 1 (OJI)	Model 2 (Constant R_w)	Model 3 (MJI)
V_p (km/s)	2.72±0.39	2.75±0.38 (1%)	2.84±0.46(4%)	2.78±0.40(2%)
V_s (km/s)	1.17±0.26	1.21±0.27(3%)	1.35±0.33(15%)	1.23±0.28 (5%)
R_t(ohm-m)	17.97±24.99	18.44±23.93 (3%)	17.99±23.32 (0%)	18.26±23.89 (2%)

Table 3. Results of estimated gas hydrate concentrations using various methods.

Measurement	Estimation[1]	Estimation[2]	Remarks
P-wave	0.29±0.25	0.39±0.23	
S-wave	0.28±0.24	0.36±0.21	
Resistivity	0.33±0.30	0.49±0.24	Constant R_w= 0.4 ohm-m
P- and *S*-wave	0.28±0.24	0.37±0.22	Joint inversion
P-, *S*-wave, and resistivity	0.28±0.24	0.37±0.22	OJI with variable R_w
P-, *S*-wave, and resistivity	0.30±0.26	0.39±0.23	MJI with γ=1 and variable R_w
P-, *S*-wave, and resistivity	0.32±0.30	0.47±0.24	MJI with γ=5 and $R_w = 0.4$ ohm-m
P-, *S*-wave, and resistivity	0.32±0.30	0.47±0.24	OJI with $R_w = 0.4$ ohm-m
P- and *S*- wave	0.28±0.25	0.36±0.22	Average
P-, *S*-wave and resistivity	0.30±0.26	0.36±0.24	Average

[1] Estimate of gas hydrate concentration averaged from depth range 897–1,109 meters including zero gas hydrate concentration values.
[2] Estimate of gas hydrate concentration averaged from depth range 897–1,109 meters excluding zero gas hydrate concentration values.
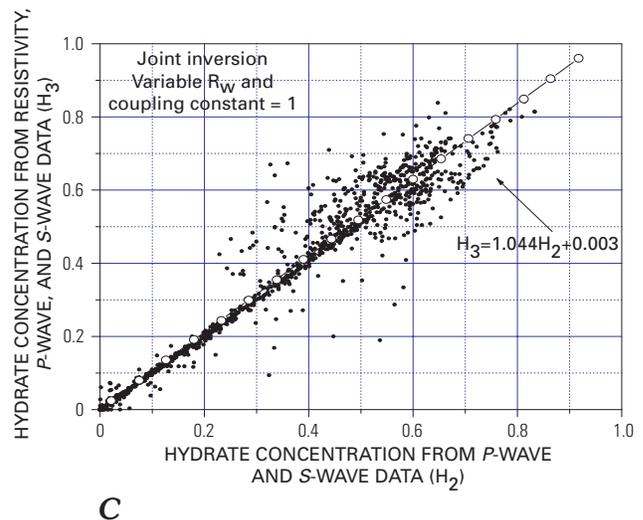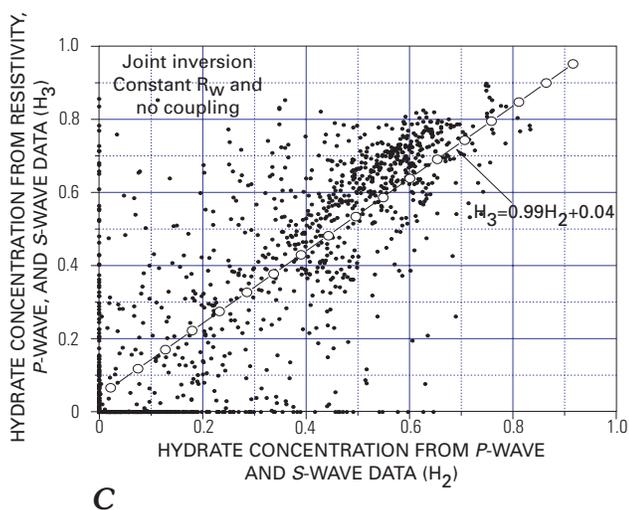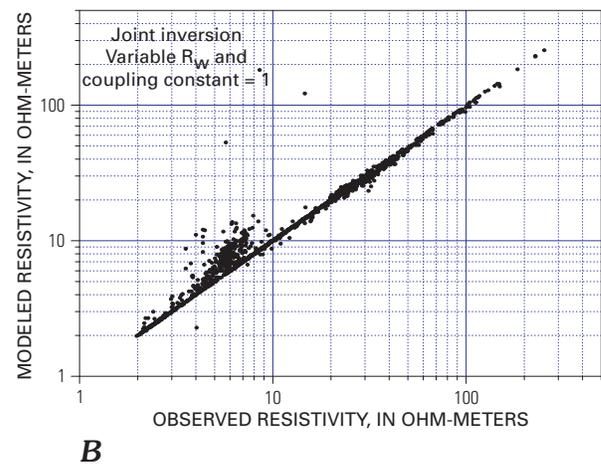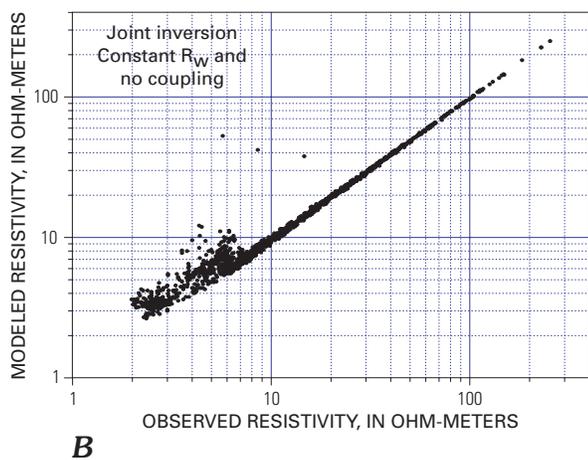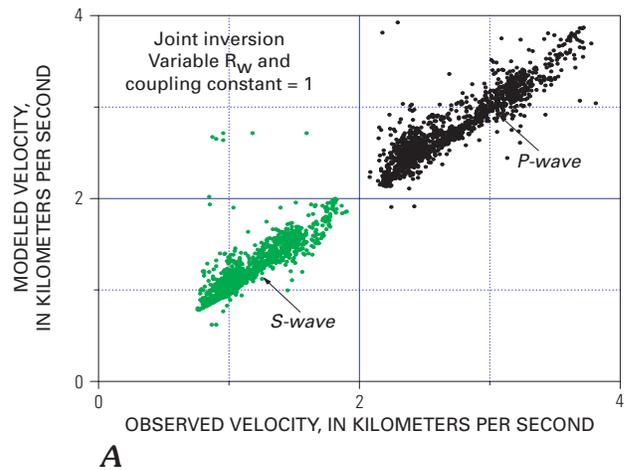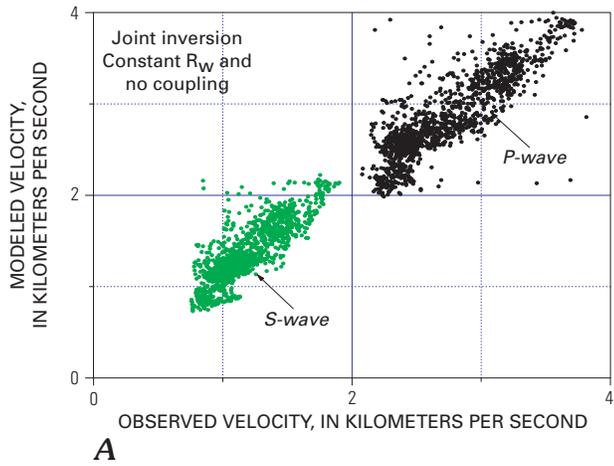
Figure 3. Graph showing the result of joint inversion of acoustics and resistivity. Only gas hydrate concentration (c) with a constant resistivity of connate water R_W=0.4 ohm-meter (ohm-m) was estimated. A, Cross plot of modeled velocities versus observed velocities. B, Cross plot of modeled resistivity versus observed resistivity. C, Cross plot of gas hydrate concentration estimated from acoustics versus that estimated from the joint inversion of acoustics and resistivity.

Figure 4. Graph showing the result of the modified joint inversion of acoustics and resistivity with a coupling constant γ=1. Both gas hydrate concentration (c) and the resistivity of connate water (R_W) were simultaneously estimated. A, Cross plot of modeled velocities versus observed velocities. B, Cross plot of modeled resistivity versus observed resistivity. C, Cross plot of gas hydrate concentration estimated from acoustics versus that estimated from the joint inversion of acoustics and resistivity.

The accuracy of the predicted velocity from MJI is between those of OJI with variable R_w and OJI with a constant R_w.

Figure 4B shows the modeled and observed resistivity (from PSR inversion). Like the results for the velocities, the accuracy of predicted resistivity is between that from Model 1 (OJI with a variable R_w) and that from Model 2 (OJI with a constant R_w).

A comparison of estimated gas hydrate concentration using only acoustic data (PS inversion) and acoustic and resistivity data (PSR inversion) is shown in figure 4C. A least-squares fitting curve indicates that the gas hydrate concentration from PSR inversion is about 4 percent higher than that from the PS inversion.

Figure 5 shows the estimated R_w for figure 4's example, and the observed resistivity (R_t) with respect to depth. The general trend of the estimated R_w with respect to the depth is similar to the result of OJI, but the estimated R_w is somewhat less than that from OJI. A least-squares fitting curve for the estimated R_w is given by $R_w = 1.2 - 0.00071D$, where D is depth in meters. At $D = 1,000$ m, the estimated R_w is 0.49 ohm-m.

Table 2 and figure 4 show that the result of MJI is between the results from Model 1 and Model 2, which indicates that the resistivity measurement contributed to the estimation of gas hydrate concentration using MJI with $\gamma = 1$ when c and R_w are simultaneously estimated. The degree of contribution of the resistivity to the inversion result is somewhat controlled by the magnitude of the coupling constant (γ), which will be addressed later.

In summary, the preceding results imply that the joint inversion estimating c and R_w simultaneously is feasible. The result of OJI estimating c and R_w simultaneously from acoustic and resistivity data demonstrates that OJI is closely similar to a sequential estimation of gas hydrate concentration based on using acoustic and R_w from resistivity data using the estimated gas hydrate concentration as input. However, the results of MJI indicate the possibility of a way to control the contribution of acoustic and resistivity measurement to the inversion, even though the coupling between the acoustic and resistivity is arbitrary.

Discussion

Weighted Error

The solution of the modified joint inversion using arbitrary non-zero constants for the G_{12} and G_{22} can be written as

$$c^{n+1} = c^n + Q_{11}E_1 + Q_{12}E_2 + Q_{13}E_3$$

$$R_w^{n+1} = R_w^n + Q_{21}E_1 + Q_{22}E_2 + Q_{23}E_3 \tag{13}$$

where Q_{ij} represents the element of the generalized inverse of G_w. Based on the data set from the Mallik 2L-38 gas hydrate research well, the average slowness of the observed P-wave is 0.375 ± 0.052 seconds per kilometer (s/km), the average slowness of S-wave is 0.894 ± 0.184 s/km, and the average resistivity is 18.60 ± 26.43 ohm-m. Let $\Delta S_p / S_p$, $\Delta S_s / S_s$, and $\Delta R_t / R_t$ be fractional errors in P-wave slowness, S-wave slowness, and resistivity respectively. If the fractional errors are of the same magnitude or similar, then $\Delta R_t \gg \Delta S_s > \Delta S_p$, because the magnitude of the resistivity is much higher than the magnitude of the slowness and S-wave slowness is higher than the P-wave
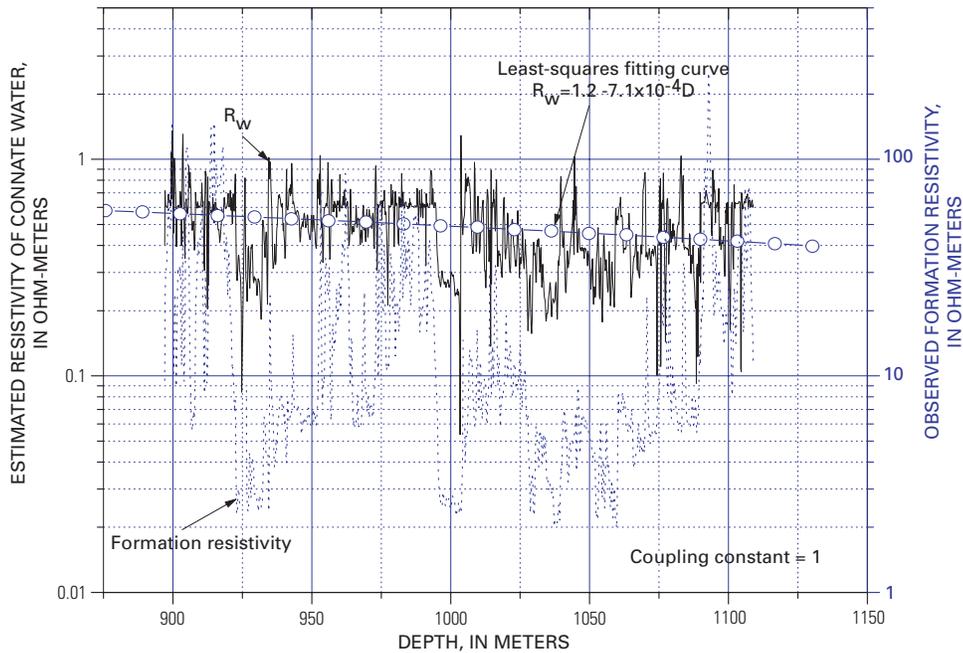
Figure 5. Graph showing estimated resistivity of connate water (R_w) using modified joint inversion with a coupling constant $\gamma = 1$ and observed formation resistivity with respect to depth. D, depth in meters.

slowness. If a least-squares error is not weighted, the solutions shown in Equation (13) are affected mostly by E_3, which is ΔR_t. So the solution can be approximated by

$$c^{n+1} \approx c^n + Q_{13}\Delta R_t$$

$$R_w^{n+1} \approx R_w^n + Q_{23}\Delta R_t \qquad (14)$$

Therefore, when an unweighted error is used for the joint inversion, Equation (14) implies that P-wave and S-wave do not contribute much to the joint inversion, and the estimation of hydrate concentration is dominated by the resistivity measurement. At the same time, Equation (14) indicates that R_w and c are not well constrained, because the error in R_t controls both the c and R_w simultaneously.

When the weighted error is used, the magnitudes of E_i's are on the same order of magnitude. Therefore, each measurement contributes to the estimation of gas hydrate concentration and resistivity of connate water.

Simultaneous Estimation of c and R_w

Estimating c and R_w by simultaneously using the joint inversion method is problematic as mentioned previously, mainly because only the resistivity, and not the acoustics, depends on both c and R_w. In consequence, some of the elements of the Jacobian matrix are zero. This leads to the decoupling of the resistivity from the estimation of gas hydrate concentration as shown in Equation (11). In order to overcome this problem, a modified joint inversion formula was introduced. However, because the coupling constant (γ) is not based on a physical principle and c depends on R_w, the result of R_w estimation should be checked against other estimates.

Miyairi and others (1999) estimated the R_{wa}, which is defined by $R_{wa} = R_t \phi^m / a$, from the resistivity logs. The R_{wa} should be equal to the R_w in clean water-bearing formations. According to Miyairi and others (1999), the resistivity of connate water for the non-gas-hydrate-bearing sediment at the Mallik 2L-38 varies between 0.2 and 0.5 ohm-m and decreases as depth increases. As shown in figure 2, the majority of estimated R_w from OJI varies between 0.2 and 1.0 ohm-m and decreases with depth. Miyairi and others (1999) used R_w=0.48, 0.38, and 0.29 ohm-m for the depth intervals of 814–933 m, 933–1,076 m, and 1,076–1,150 m, respectively. The estimated R_w from OJI is 0.85, 0.57, and 0.34 ohm-m for the average depth interval of 873 m, 1,004 m, and 1,113 m respectively. Comparing the result of OJI inversion and Miyairi and others (1999), the rate of resistivity decrease with depth is similar, but R_w values differ by 0.05–0.4 ohm-m. The result from MJI indicates that the estimated R_w values vary slightly from the OJI values, being 0.58, 0.49, and 0.41 ohm-m for the depths of 873 m, 1,004 m, and 1,113 m respectively, which are about 30 percent higher than those used by Miyairi and others (1999).

Behavior of the Joint Inversion Method

Inversion results indicate that the coupling constant (γ) in the Jacobian matrix controls the amount of coupling between the acoustics and resistivity in the estimation of gas hydrate concentration. When γ=0, the modified joint inversion reverts to the original joint inversion method, and the gas hydrate estimation is controlled mostly by the acoustical properties, receiving minimal contributions from the resistivity measurements, as indicated in figure 1.

The result of modified joint inversion using γ=5, shown in figure 6, is almost identical to that from the R inversion (table 3). Numerical tests indicate that as γ increases, the estimated gas hydrate concentration from a joint inversion of acoustic and resistivity approaches the result of inversion using only resistivity, and the estimated R_w approaches a constant value, which is the initial value.

Conversely, as γ decreases, the result of MJI approaches that of OJI. These observations indicate that the coupling constant controls the degree of resistivity contribution to the estimation of gas hydrate concentration using the joint inversion. However, it is difficult to predict the general behavior of inversion results with respect to the coupling constant, except for low and high constant values.

Understanding the Coupling Constant

The modified joint inversion formula can be better understood if we assume that erroneous P-wave and S-wave values are used in the following way when computing the Jacobian matrix:

$$S_p^e = S_p^t + \gamma R_w$$

$$S_s^e = S_s^t + \gamma R_w \qquad (15)$$

where the superscript t indicates the true computed values from the theory and the superscript e indicates the altered values used in the inversion. Then, $G_{12} = \gamma$ and $G_{22} = \gamma$. Note that only elements of the Jacobian matrix, not the theoretical values, are altered.

If γ becomes small, the erroneous theoretical values used in the modified joint inversion approach the true values. In this case, the solution of the modified inversion approaches the solution of the original inversion shown in Equation (11).

If γ is large, the acoustic model fails to fit the observation accurately and errors become large. When the error of computed slowness is large, Equation (1) indicates that the relative contribution of G_{11} and G_{21} compared to G_{12} and G_{22} to the overall inversion is insignificant. Therefore, $G_{11} \approx 0$ and $G_{21} \approx 0$ as γ becomes large. In this case, the solution can be written as

$$c^{n+1} \approx c^n + \frac{2\gamma^2 G_{31}E_3 - \gamma G_{31}G_{32}(E_1 + E_2)}{2\gamma^2 G_{31}^2}$$

$$\approx c^n + \frac{E_3}{G_{31}} \text{ as } \gamma \to \infty \text{ and}$$

$$R_w^{n+1} \approx R_w^n + \frac{E_1 + E_2}{2\gamma} \approx R_w^n \text{ as } \gamma \to \infty \qquad (16)$$

Equation (16) indicates that as the coupling constant becomes larger, E_3 or the resistivity measurement dominates the estimation of c, and the change of R_w is insignificant or R_w approaches the initial value.
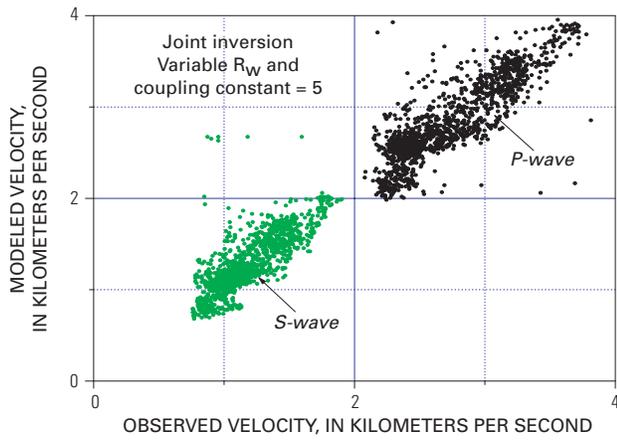
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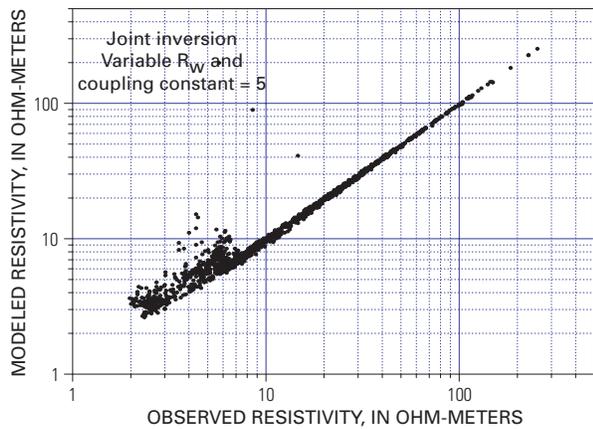
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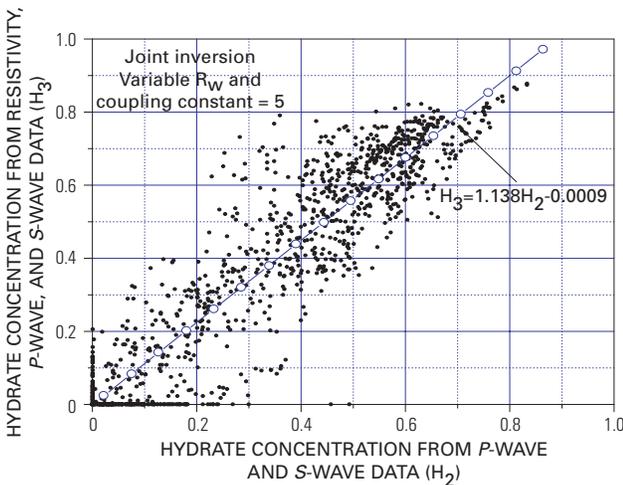
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Figure 6. Graph showing the result of the modified joint inversion of acoustics and resistivity with a coupling constant $\gamma=5$. Both gas hydrate concentration (c) and the resistivity of connate water (R_w) were simultaneously estimated. *A,* Cross plot of modeled velocities versus observed velocities. *B,* Cross plot of modeled resistivity versus observed resistivity. *C,* Cross plot of gas hydrate concentration estimated from acoustics versus that estimated from the joint inversion of acoustics and resistivity.

Note that the solution of MJI with a large γ approaches the result of OJI with a constant R_w. Equation (11) indicates that when there is no update of R_w in the inversion, the following relation should be valid:

$$G_{31}(G_{11}E_1 + G_{21}E_2) = (G_{11}^2 + G_{21}^2)\,E_3 \qquad (17)$$

Substituting Equation (17) into Equation (11) for c, the result is

$$c^{n+1} = c^n + \frac{E_3}{G_{31}} \qquad (18)$$

This is identical to the inversion solution of the resistivity and identical to Equation (16).

Comparison of Various Inversion Results

Each measurement—P-wave, S-wave, and resistivity—can be used separately in the inversion to estimate gas hydrate concentrations. When only R_t is used in the inversion, R_w should be an input, because of the presence of two unknowns with one observation. Table 3 shows results of the estimation of gas hydrate at the Mallik 2L-38 well from various combinations of measurements. Among all the estimates, those using only resistivity measurements yield the highest average hydrate concentrations.

The average hydrate concentrations estimated from P-wave and S-wave slownesses are similar, but the estimated amount is less than that from resistivity by about 5 percent when all zero concentration values are included and by about 12 percent when zero concentration values are excluded. The average gas hydrate concentration estimated from the PS inversion is smaller than that from the PSR inversion with a constant R_w, or a variable R_w using MJI with $\gamma=1$, but the estimates are similar to those of PSR inversion with a variable R_w using OJI.

The results of the simultaneous estimations of c and R_w from MJI with $\gamma=1$ indicate that the estimates of gas hydrate concentration are slightly higher, on the order of 2 percent, than that from estimates based on acoustics. Estimated gas hydrate concentrations indicate that MJI with $\gamma=1$ still favors acoustics, but the contribution of resistivity to the estimates is apparent, and the amount may be controlled by the value of γ. However, it is difficult to predict the behavior of the inversion result with respect to the coupling constant.

Gas hydrate concentrations by averaging each individual estimation are also shown in table 3. The gas hydrate concentrations estimated from the average of P- and S- wave estimates are similar to that of the PS inversion. Estimates based on the PSR inversion using MJI with $\gamma=1$ are similar to the averages of three individual estimates. But the cross plots for the modeled and observed quantities are substantially different. Figure 7 shows the result of averaging gas hydrate concentrations from separate estimations from P-wave, S-wave, and resistivity data. The modeled velocities are similar to those from MJI with $\gamma=1$; in fact, average velocities of modeled P- and S-waves are 2.78±0.38 km/s and 1.23± 0.26 km/s, respectively. The only difference in the velocity estimates is the amount of scattering of the modeled velocities. However, the modeled resistivity differs markedly from the observed value (fig. 7*B*), being about 50 percent lower.
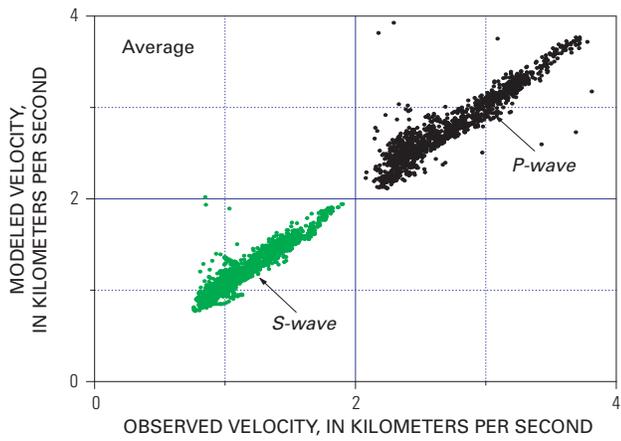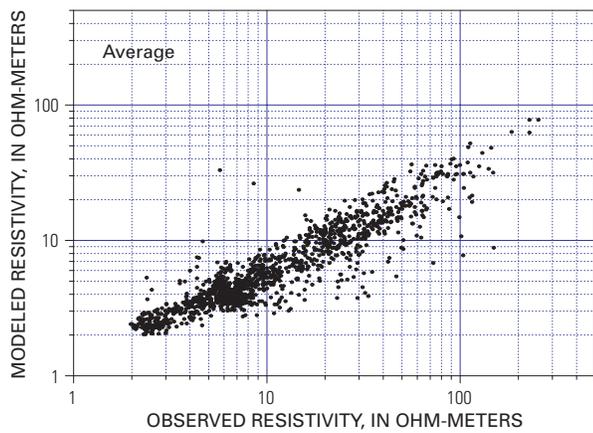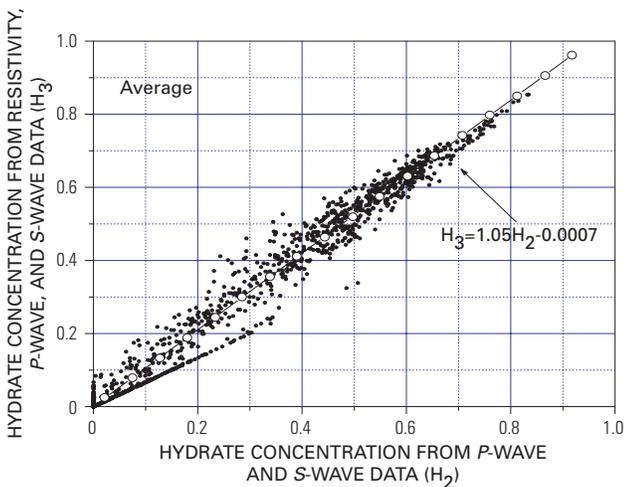
Figure 7. Graph showing the result of averaging the individual gas hydrate estimates from *P*-wave, *S*-wave, and resistivity data. *A*, Cross plot of modeled velocities versus observed velocities. *B*, Cross plot of modeled resistivity versus observed resistivity. *C*, Cross plot of gas hydrate concentration estimated from acoustics versus that estimated from the joint inversion of acoustics and resistivity.

Various inversion methods provide different estimations of gas hydrate concentrations. Judging which estimate is the most accurate one is difficult, because gas hydrate concentrations in sediments at the Mallik 2L-38 well were not accurately measured. The difference of gas hydrate estimation between resistivity and acoustics may be real, because the lateral resolution (or depth of investigation) of each logging tool is different, or the difference may be an artifact owing to the inaccuracy of acoustic models based on the weighted equation or erroneous parameters of the Archie's constants.

Conclusion

The following conclusions can be drawn from a joint inversion of acoustics and resistivity measurements for gas-hydrate-bearing sediments.

1. Simultaneously estimating the gas hydrate concentration and the resistivity of connate water (R_w) is problematic, because the original inversion equation for the estimation of gas hydrate concentration uses only acoustics decoupled from the resistivity measurement.

2. A modified joint inversion method, implemented by inserting a coupling constant where the element of Jacobian matrix is zero, partly compensates the adverse effect of decoupling. However, because the coupling constant is not based on the physical principles, predicting the effect of the coupling constant is difficult except at the large values.

3. The contribution of resistivity to gas hydrate estimates can be controlled in part by the magnitude of the coupling constant. As the value of the coupling constant (γ) increases, the modified joint inversion favors the resistivity measurement; when γ decreases, MJI favors acoustics.

4. Even though R_w is constrained by *P*- and *S*-wave data through *c*, the estimated R_w from the modified joint inversion should be checked against R_w values estimated by other methods, because R_w is not constrained physically by acoustics.

5. When a constant R_w is used in the gas hydrate estimation, the joint inversion method has no clear advantage over the individual inversion. However, when *c* and R_w are estimated simultaneously, the modified joint inversion method may be preferred.

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