GEOLOGICAL SURVEY CIRCULAR 164

BOOK 3 - TOPOGRAPHIC INSTRUCTIONS

# Part 3 C - Photogrammetric Mapping

# Chapter 3 C 7 a-e MULTIPLEX PROCEDURE

By Edward I. Loud, Jr.

UNITED STATES DEPARTMENT OF THE INTERIOR
Oscar L. Chapman, Secretary

GEOLOGICAL SURVEY
W. E. Wrather, Director

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TOPOGRAPHIC INSTRUCTIONS

BOOK 3

# Part 3 C
# Photogrammetric Mapping

# Chapter 3C7 a-e
# MULTIPLEX PROCEDURE

By Edward I. Loud, Jr.

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# INTRODUCTION TO THE MULTIPLEX

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# INTRODUCTION TO THE MULTIPLEX

## ABSTRACT

By direct projection of overlapping photographs, printed on glass plates, the multiplex produces an exact optical model, in miniature, of the terrain to be mapped. To create the model, the multiplex projectors must be properly positioned and oriented so that they duplicate the orientation of the aerial camera at the instant of each exposure. By means of a floating mark, horizontal and vertical measurements can be made in the model, and planimetry and contours can be drawn. The applicability of the multiplex to a given mapping project depends largely on the contour interval and compilation scale required, and also depends, to a lesser extent, on the vegetation and terrain cover as it may affect accuracy requirements. The steps in multiplex procedure are orientation, stereotriangulation, and compilation of detail. In orientation, the projectors are arranged so that the projected images form a stereoscopic model which can be adjusted to fit horizontal and vertical control points. In stereotriangulation, three or more multiplex projectors are oriented so that the consecutive models fit existing control, permitting the establishment of additional or intermediate control. In compilation, the features appearing in the model are delineated on the map manuscript.

## THE MULTIPLEX METHOD

### 1. General principles of multiplex

In the multiplex method of mapping, the natural terrain features are re-created at a miniature scale by the optical projection of overlapping aerial photographs properly oriented to each other and to the datum. Each photograph, in the form of a miniature print on a glass plate (called the diapositive), is positioned on the focal plane stage of a projector. Light from the projector light source passes through the diapositive and the optical system of the projector so as to project the image of the terrain in a manner which is geometrically similar to the original photographic process but opposite in direction. The parallelism that exists between the geometry of the aerial photography and the geometry of the multiplex reconstruction, or "model"[2], is suggested in figure 1.

In order that the model may be an exact duplicate, in miniature, of the actual terrain, the projected photographs must be accurately positioned and oriented so as to reproduce exactly the conditions existing at the instant each photograph was taken. It is therefore necessary to recover the space coordinates of the exposure stations, at the model scale, and the angular

[1] The number 3 C 7-a signifies Book 3, Part C, Chapter 7-a of the Geological Survey loose-leaf manual of Topographic Instructions. For a table of contents, see Chapter 1 A 2 (Circular 92).

[2] That portion of the reconstructed terrain formed by a single pair of overlapping photographs constitutes one "model". The several successive models formed by consecutive photographs in the line of flight, constitute a multiplex "strip".

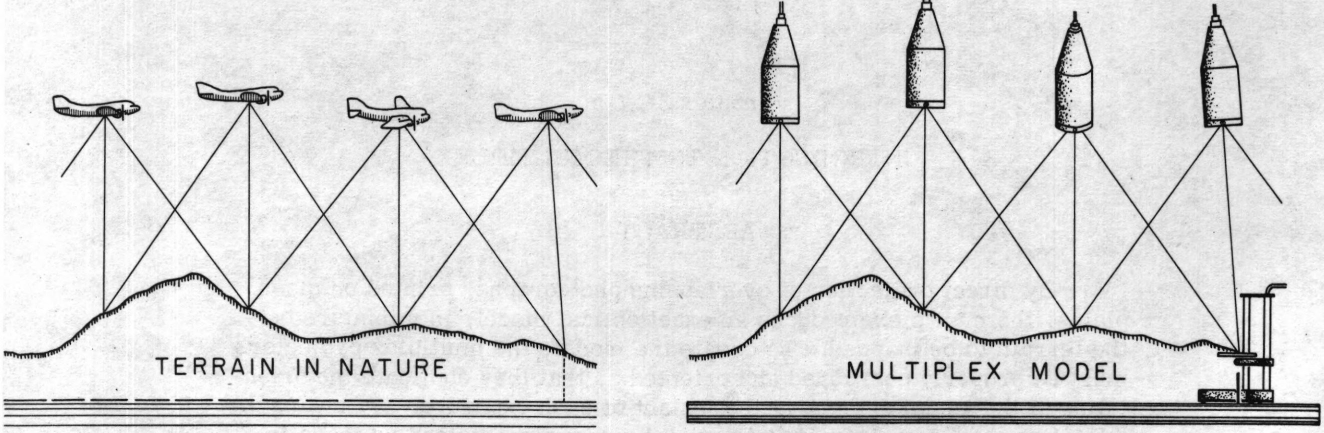


Figure 1.--Geometric parallelism between aerial photography and multiplex reconstruction.

orientation of the camera at each station. This is accomplished by adjustment of the multiplex projectors which have translational motions along three mutually perpendicular axes, and rotational motions about each of these axes. When the projectors are properly oriented, all corresponding image rays from a pair of projectors intersect in space. The integration of all such points represents an optical model similar in all respects to the actual terrain.

In order that the observer may see the model in three dimensions it is necessary that he view one of the photographs with one eye, and the second photograph with the other eye. This is accomplished by projecting the images in complementary colors and observing them through correspondingly colored spectacles. By means of a floating mark, accurate horizontal and vertical measurements can be made in the model. The floating mark is contained in the surface of a viewing screen, or platen, mounted on a tracing table. To examine any portion of a model, the tracing table is moved into such position that the projected images of that portion fall on the platen.

Vertical motion of the floating mark is obtained by raising or lowering the platen by means of an elevating wheel and screw forming a part of the tracing table. This motion can be measured by means of a height-indicating scale. Horizontal motion of the floating mark is obtained by sliding the tracing table over the map surface. A pencil is mounted vertically below the floating mark so that map detail observed in the model can be drawn on the map sheet.

The multiplex is simple in principle and construction. Because of this simplicity, it is especially well adapted to the teaching of the basic principles of photogrammetry and to the training of personnel in the basic methods of stereoscopic mapping. Experience has shown that persons trained in the operation of the multiplex are able to comprehend the operation of other stereoscopic instruments more easily and rapidly than would otherwise be possible.

A feature which makes the multiplex unique among instruments is the manner in which multiplex stereotriangulation is performed. Several successive photographs can be placed in simultaneous orientation to form a continuous, bridge-like strip of models for the purpose of spanning areas having little or no control. The inadequately controlled models, being members of the bridge, are consequently fully oriented when the bridge as a whole is oriented. Further supplemental procedures are required only to the extent that errors, which are present to some degree in

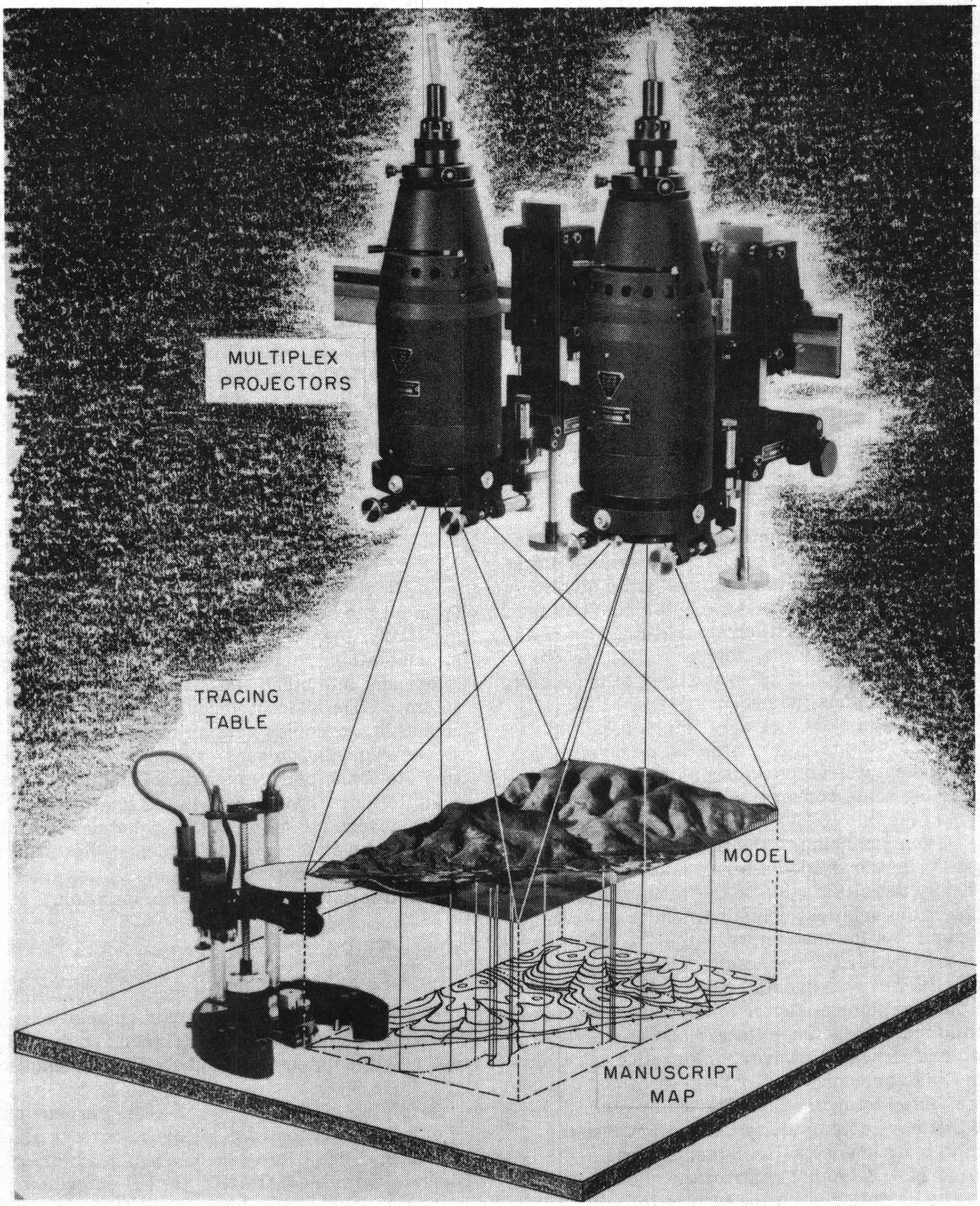


Figure 2.--Principle of the multiplex.

any plotting system, may make the strip deviate slightly from this ideal condition. Some simple analytical correction may then be advisable. Both positions and elevations can be established by multiplex stereotriangulation, but the vertical accuracy requirements, being generally more rigid than the horizontal accuracy requirements, may limit and often preclude the practice of vertical stereotriangulation. Supplemental horizontal positions, however, are regularly established by multiplex stereotriangulation, using from 2 to 11 or 12 models, although as many as 19 or 20 models may be set up. Insofar as the accuracy requirements permit, the limit depends on the projector capacity of the particular multiplex supporting frame being used.

2. Applicability of the multiplex

The mapping assignments of which the multiplex is capable—as expressed by the scales, flight heights, and contour intervals that are suitable for multiplex compilation--are determined by the more or less fixed relationship that must be maintained among these three factors[3]. Thus the contour interval, which is often the controlling factor, roughly determines the necessary flight height, and this in turn rather closely determines the model scale (which is the same as the compilation scale, except when a pantograph or other device is used for scale conversion).

When referring to the flight height-contour interval relationship, the term "C-factor" is often used. By definition, this is the ratio of the flight height to the smallest contour interval that can be obtained from the instrument within the generally accepted standard of accuracy for topographic maps. In theory it should be possible to calculate the maximum C-factor for any stereoscopic plotting instrument when its various characteristics are known. However, in practice the maximum C-factor cannot easily be determined because it is affected to such a large extent by other factors that have no particular connection with the plotting instrument itself. Some of these are: the amount and type of control; the accuracy standard to be complied with; the type of terrain; atmospheric conditions; quality of photography; and the likelihood of errors in other phases of the process. The actual C-factors used for compilation vary over wide limits for the same reason, and for other reasons such as the necessity of planning flights within prescribed boundaries, or the necessity of utilizing photography not specifically planned for a project. As a result of Geological Survey experience, a value of 600 is usually considered to be a safe limiting value for the multiplex C-factor, under average conditions. Once the flight height is established, the multiplex model scale is governed solely by the projection distance of the multiplex projector which distance can be varied only insofar as the depth of focus of the lens permits.

[3]See Chapter 3 B 3, Planning vertical photography, for a detailed discussion of this relationship.

## STEPS IN MULTIPLEX PROCEDURE

3. The major phases

The steps in the compilation of a map by the multiplex are grouped into two or, in some instances, three major phases. The basic phase is orientation, or the process by which the model is brought into being and made to conform to a datum. When stereotriangulation is required, orientation becomes both an initial part of the stereotriangulation phase and, usually, an intermediate phase between stereotriangulation and compilation. Compilation is the final phase, which covers the delineation on the manuscript of the features seen in the model. When stereotriangulation is not required, the initial orientation is followed directly by compilation.

4. Orientation

The orientation phase can be divided into interior orientation and exterior orientation. Exterior orientation can be further subdivided into relative orientation and absolute orientation.

a. Interior orientation. --The purpose of interior orientation is to recover, insofar as possible, a projected cone of rays geometrically identical with the cone of rays that entered the camera lens to make the original exposure. Interior orientation imposes two requirements: (1) The diapositive must represent a reduction from the original aerial negative in the ratio of the focal length of the aerial camera to the principal distance[4] of the projector, the principal distance being the length of the perpendicular from the interior perspective center of the projection lens to the plane of the diapositive; and (2) this perpendicular must intersect the plane of the diapositive at the principal point of the photograph. The first requirement is satisfied in the diapositive printing operation by proper setting of the reduction ratio of the printer. Interior orientation, to the stereocompiler, means only satisfying the second requirement. This is accomplished in the operation known as "centering" the diapositive.

b. Relative orientation.--The purpose of relative orientation is to reconstruct the same perspective conditions between a pair of photographs that existed when the photographs were taken. As it is not ordinarily known just what these conditions were, the multiplex procedure of relative orientation is founded on the fact that each point on the ground was the origin (and hence an intersection) of a pair of conjugate rays, one to each exposure station. Therefore, their reprojected counterparts in the multiplex must also be made to intersect, pair by pair. Following interior orientation, relative orientation is accomplished by a systematic procedure of applying rotational and translational movements to the projectors, at the same time observing the images on the platen of the tracing table, until conjugate images are made to coincide over the entire model area. This procedure of relative orientation is familiarly referred to as "clearing y-parallax."

c. Absolute orientation.--As a final step, the model which has been created as a result of interior and relative orientations must now be "absolutely" oriented with reference to a vertical and a horizontal datum, if map-making measurements are to be readily obtained from it. The vertical datum is provided by the top surface of the multiplex supporting table. The slight difference between this surface, which is plane, and the datum in nature, which conforms to the curvature of the earth, is not significant within the area usually covered by a single model. The horizontal datum is provided by the control points plotted with reference to the map projection on the base sheet. This base sheet is laid flat upon the multiplex table top, where the tracing table can slide upon it.

Absolute orientation procedure comprises three operations: bringing the model to the proper size, or "scaling" the model; bringing the model datum parallel to the table top, called either "leveling" or "horizontalizing" the model; and "positioning" the model with reference to the horizontal datum.

The multiplex model is spatial in concept, embracing not only the visible surface of the terrain but also the space above and below this surface. In addition to the terrain surface, therefore, another surface can be conceived as having been reconstructed as part of the model, a surface representing the sea-level datum of nature. This surface is called the "model datum." When the interior and relative orientations of the photographs have been correctly recovered, the model datum is, for all practical purposes, a plane or flat surface over the extent of one model. When absolute orientation is also recovered, the model datum becomes parallel to the table top, or "level." If photographic distortions or orientation errors cause this model datum to be deformed, or "warped," the datum cannot be leveled. In speaking of the model datum, the word "datum" is often omitted, hence it is customary to refer to the model as being either "warped," or "flat but not level," or "level," as the case may be. Although the terrain surface of the model may also be referred to as flat, or level, the context will always make the reference clear.

5. Stereotriangulation

A stereotriangulated strip that is supported by control at both ends is called a "bridge;" a strip that is supported at only one end is called an "extension." Bridging is the more usual type of stereotriangulation, because an extension will not provide the accuracy needed in most domestic mapping. Bridging and extending procedures are further qualified as "horizontal" or "vertical" according to their primary purpose, which may be either to establish supplemental positions (horizontal pass points) or to establish supplemental elevations (vertical pass points). In the absence of ground control, these pass points are

[4] See figure 6, Multiplex projector optical system, in Chapter 3 C 7-b.

needed to control each individual model during the compilation of detail; pass points also afford a means of foreseeing and adjusting differences between adjacent strips.

It was mentioned previously that certain small errors--arising particularly in the various lenses through which the image must pass--are responsible for a difference between the theory and the practice of stereotriangulation. Disregarding these errors for the moment, the basic procedure of stereotriangulation is as follows.

After the absolute orientation of the first and second of a series of projectors has been attained, forming a model also in absolute orientation, it is possible to orient a third projector relative to the second, forming a new model; it is further possible to bring the new model to the same scale as the original--both operations utilizing motions of the third projector alone, without disturbing the first two projectors or the original model. When this procedure is carried out, the third projector will perforce become absolutely oriented like the others, and both models will exist simultaneously in a state of absolute orientation. This process of attaching additional models to previously oriented models can be continued up to the limit imposed by the length of the multiplex frame. In effect, the several models then constitute one continuous strip model, which is all in absolute orientation inasmuch as the initial model was in absolute orientation. If stereotriangulation practice were like stereotriangulation theory, the compilation of detail could now proceed without further delay.

In actual stereotriangulation, however, progressive discrepancies within the length of the strip will be encountered, owing to the cumulative effects of the aforementioned small errors. In practice, therefore, stereotriangulation procedure must include steps for minimizing these discrepancies.

6. Compilation of detail

When the model is in absolute orientation (or reorientation, as the case may be) it is ready for the compilation of detail. In compilations, the planimetric features are almost always compiled first. These are then inked, to be followed finally by the compilation and inking of the contours.

GEOLOGICAL SURVEY

TOPOGRAPHIC INSTRUCTIONS

Chapter 3C 7-b

# MULTIPLEX EQUIPMENT

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Chapter 3 C 7-b[1]

# MULTIPLEX EQUIPMENT

## ABSTRACT

The several types of multiplex equipment may be classified according to make, degree of tilt that can be accommodated, angular field, and principal distance. The main components of multiplex plotting equipment are the projector, the tracing table, the supporting frame, the supporting table, the filter spectacles, the electrical system, and the cooling system. The diapositive printer is a necessary complement of multiplex equipment, although it is not a part of the plotting instrument itself. All of these are described in detail. Procedures for the calibration and adjustment of multiplex equipment are divided into routine procedures performed by the stereocompiler and laboratory procedures performed by skilled technicians with special tools, but laboratory procedures are beyond the scope of this chapter. Routine procedures, and methods of roughly checking laboratory calibration are described in detail.

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## TYPES OF MULTIPLEX EQUIPMENT

### 1. Different makes

The multiplex is manufactured in the United States by the Bausch and Lomb Optical Company. Among the foreign-made multiplex instruments are the Williamson and Ross type (England); the Zeiss type (Germany); and the Nistri type (Italy). In addition, different models of the same manufacture increase the total number of different designs of multiplex equipment. Although these differences alter the superficial appearance of the equipment, they do not change the fundamental principles; and although these differences may have an important bearing on the operational accuracy of the instrument, they have relatively little effect on operational procedures. The discussion of multiplex equipment in this chapter applies particularly to the Bausch and Lomb instrument of U. S. Geological Survey design.

### 2. Degree of tilt accommodated

The multiplex is primarily adapted for use with vertical photography. Small tilts can readily be accommodated (the mechanical limit of the tilt of a multiplex projector ranges from 10° about the x-axis to approximately 20° about the y-axis). During the war, some multiplex projectors were converted to utilize the high-oblique photography of the trimetrogon cameras by a simple change in their supporting brackets that gave them an initial tilt of 60° from the vertical. Subsequently, a limited number of projectors have been manufactured specifically for use with oblique photography. These projectors embody changes in their optical design to provide a better image, and other minor modifications. Except for these minor changes, oblique equipment is similar to vertical multiplex equipment. This chapter, however, deals specifically with the vertical multiplex.

### 3. Angular field

The earlier multiplex projectors had an angular field of about 70° which matched the

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[1]The number 3 C 7-b signifies Book 3, Part C, Chapter 7-b of the Geological Survey loose-leaf manual of Topographic Instructions. For a table of contents, see Chapter 1 A 2 (Circular 92).

maximum angular coverage afforded by single-lens aerial cameras at the time. This type of projector, called the "normal-angle" projector, is obsolete, having been replaced by the wide angle type which projects a cone slightly in excess of 90°. This wider angular coverage permits the use of more favorable base-height ratios, thus increasing the accuracy of the instrument despite the possibility of some slight loss in the definition of the projected image. The wider angular coverage also increases the model area, thus decreasing the amount of control required. Although the procedures employed in operating either type of projector are almost identical, the wide-angle type is specifically assumed in the present discussion.

4. Principal distance

An important geometrical property of a multiplex projector is its principal distance (the perpendicular distance from the internal perspective center of the projector lens to the plane of the emulsion side of the diapositive). This distance has a direct bearing on the ratio of reduction from negative to diapositive scale and on the ratio of enlargement from diapositive to model scale.

The nominal principal distances of the different types of multiplex projectors used in this country are:

| | |
|---|---|
| B&L wide angle--USGS type . . . . | 30 mm |
| B&L wide angle--US Army type . . | 28 mm |
| B&L normal angle . . . . . . . . . | 46 mm |
| Zeiss wide angle. . . . . . . . . . | 22 mm |
| Zeiss normal angle . . . . . . . . | 46 mm |

MULTIPLEX DIAPOSITIVE PRINTER

5. Functions of diapositive printer

A prerequisite to any understanding of the multiplex is an understanding of the functions of the multiplex diapositive printer. This printer (fig. 1) is essentially a reducing camera of the highest precision, which is used to prepare the diapositives (positive transparencies of the aerial photographs on glass plates) required by the multiplex projectors. Although the printer is necessarily considered an item of multiplex equipment, the operation of diapositive printing is not regarded as a multiplex operation--the latter begins with the orientation of the completed diapositives in the multiplex projector[2].

The multiplex printer has three important functions: reduction of negative size; distortion compensation; and addition of the principal point mark. These functions are particularly important in enabling the multiplex projectors to achieve the geometrically parallel reproduction of nature which is an essential principle of the multiplex.

6. Reduction of negative size

The diapositive printer reduces the aerial photograph approximately according to the ratio:

$$\frac{\text{focal length of camera}}{\text{principal distance of projectors}}$$

More precisely, the reduction setting of the printer is based on the principal distance[3] of the aerial negative, rather than on the focal length of the aerial camera. Thus the effect of film shrinkage is taken into account.

If the cone of rays emanating from a projector is to be similar to the original cone that entered the camera at the instant of exposure, the principal distance of the diapositive must be equal to the principal distance of the projectors. Therefore, provision is made for setting the reduction ratio precisely. If the reduction ratio were not accurately set, the projected cone of rays would be either slightly elongated or slightly flattened, with the result that the vertical scale

[2] A full description of the diapositive printer and its operation is contained in Chapter 3 C 6, Diapositive preparation.

[3] Principal distance is the perpendicular distance from the internal perspective center to the plane of a particular finished negative or print. This distance is equal to the calibrated focal length corrected for both the enlargement or reduction ratio and the film or paper shrinkage or expansion and maintains the same perspective angles at the internal perspective center to points on the finished negative or print, as existed in the taking camera at the moment of exposure. This is a geometrical property of each particular finished negative or print.

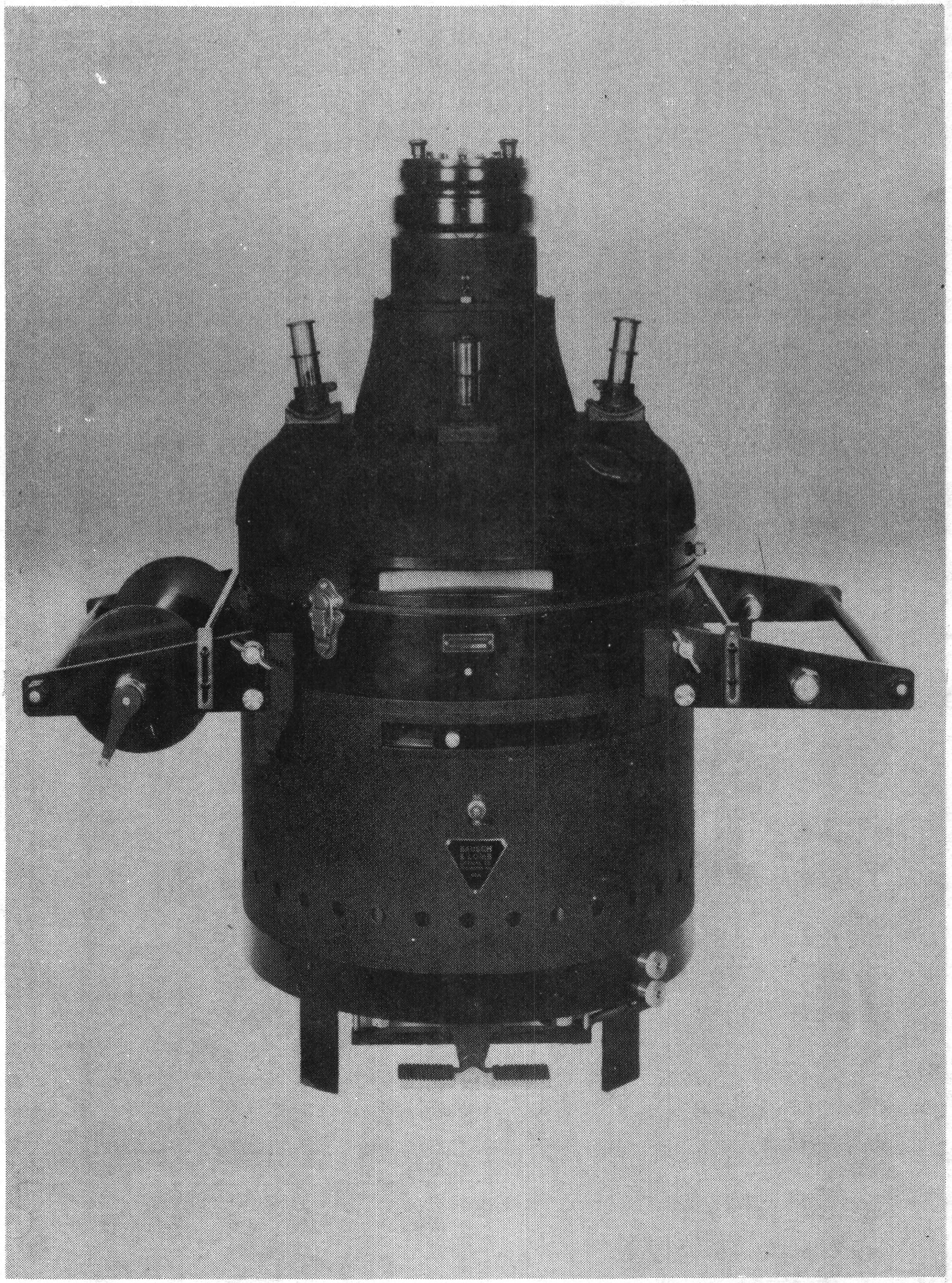

Figure 1. --Multiplex diapositive printer.

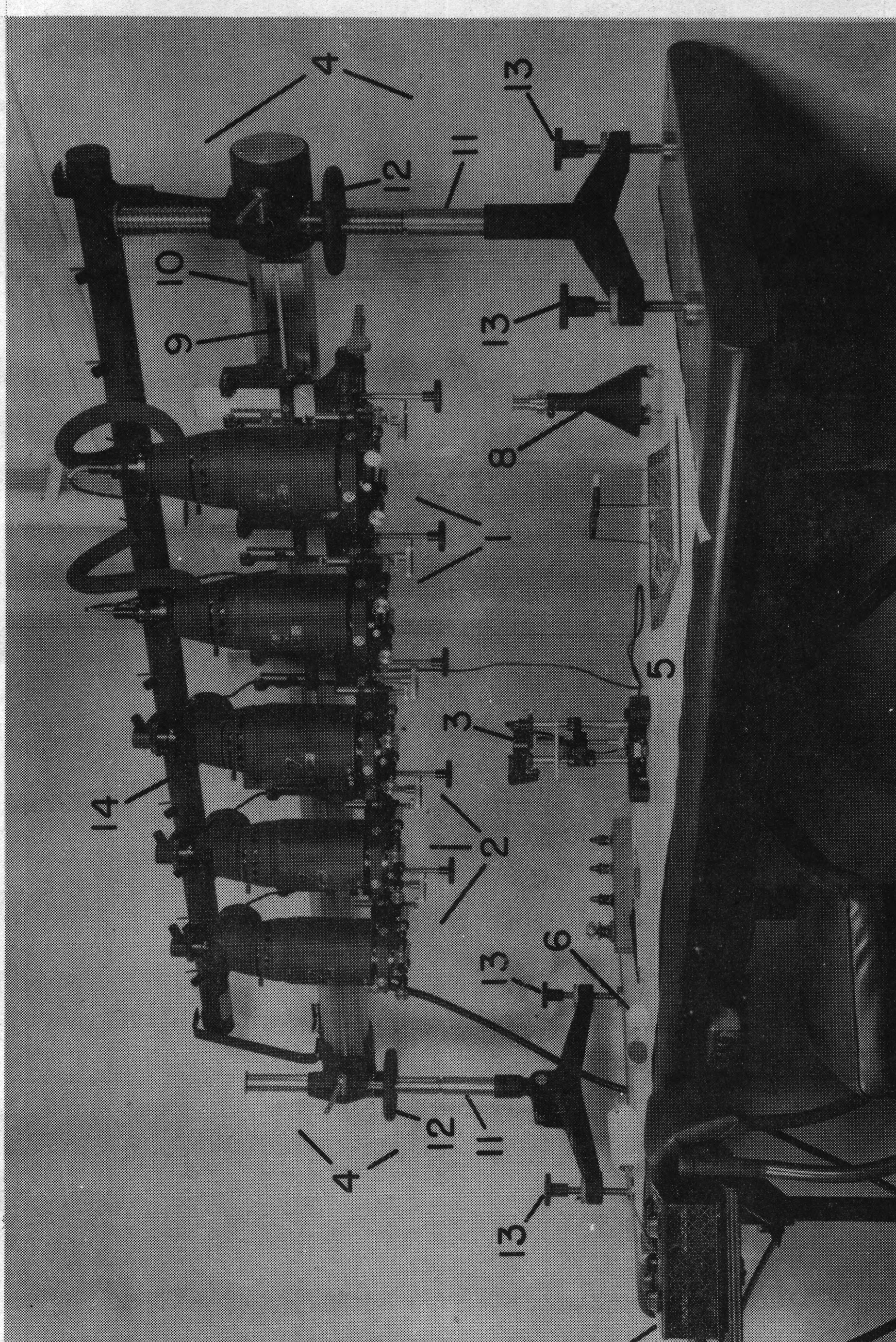


Figure 2. --Multiplex plotting equipment.

of the model would be slightly in error with respect to the horizontal scale. With the proper reduction ratio, the vertical and horizontal scales will be equal.

7. Distortion compensation

Another function of the multiplex diapositive printer is distortion compensation. The optical system of the printer introduces a calculated distortion such that the resultant distortion of camera, printer, and projector combined is as near zero as possible. Figure 3 illustrates the functions of distortion compensation and reduction of negative size in recovering the same angle between two projected rays as existed between the corresponding rays in nature.

8. Addition of the principal-point mark

The multiplex printer is the source of the small cross (or circle), printed on the diapositive, that marks the principal point of the photograph. The original of this mark is etched on a glass plate in the negative plane of the printer. It is the responsibility of the printer operator to position the negative before a diapositive is exposed, by alining the fiducial marks on the negative with certain collimating lines on the glass plate.

## MULTIPLEX PLOTTING EQUIPMENT

9. Main components

The multiplex in readiness for operation is shown in figure 2. The principal items that may be seen in the illustration are: the projectors (1 and 2)--two different designs are shown; the tracing table (3); the supporting frame (4); the supporting table (5); the filter spectacles (6); the electrical control box (7); and the principal point microscope (8).

10. The projector

a. Essential parts.--Of the two designs of the Geological Survey-type multiplex projector that are illustrated in figure 2, projector 1 is at present the more widely used, while projector 2 is of more recent design. Projector 2, which is shown in more detail in figures 5 and 6, is used as a basis for this description, although the

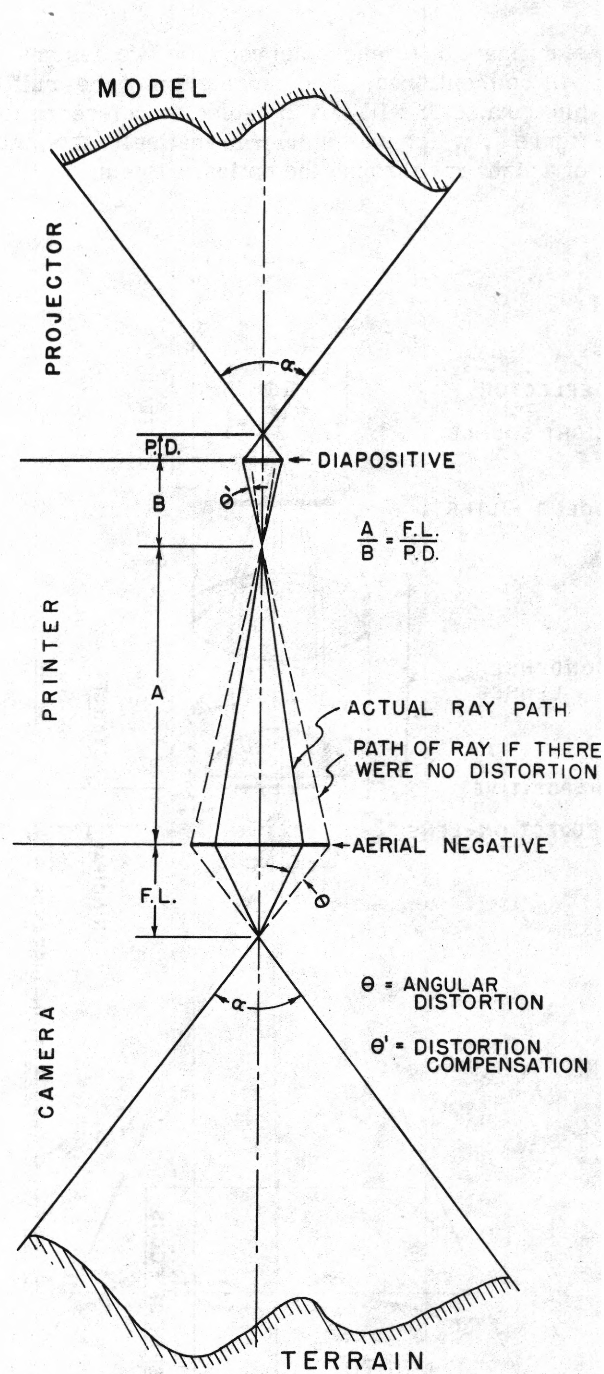


Figure 3.--Relationship of camera, printer, and projector in the multiplex system.

significant differences between the two designs will be mentioned. The description of the multiplex projector will also be aided by reference to figure 4, which shows diagrammatically the path of a light ray through the optical system.

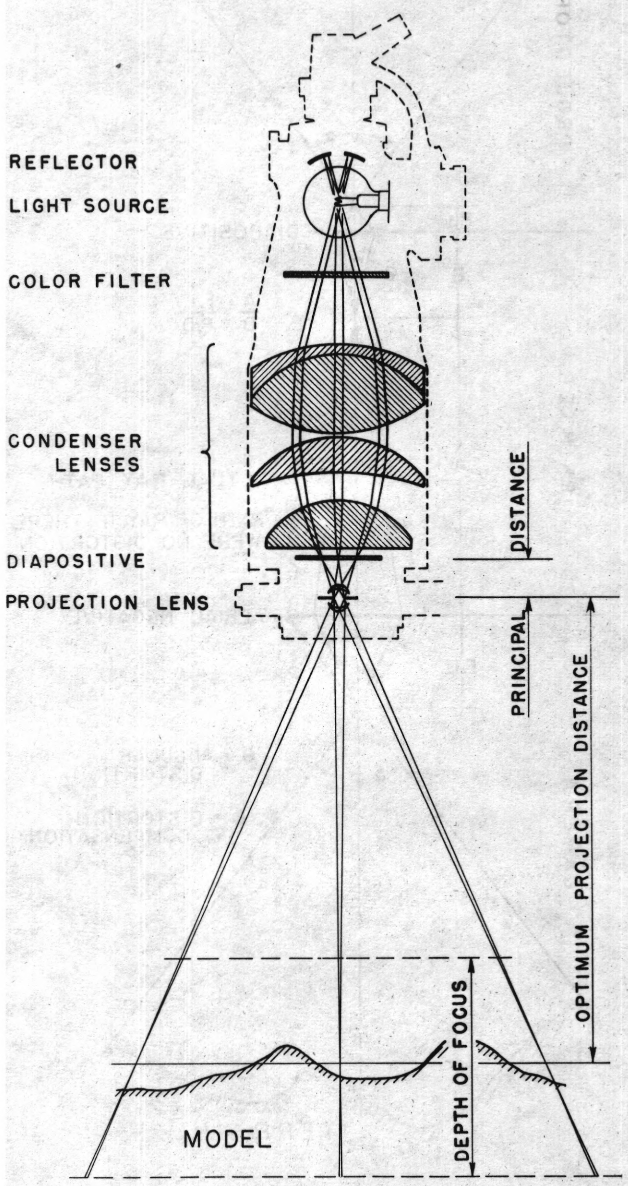


Figure 4.--Multiplex projector optical system.

The essential parts of the multiplex projector are: the light source (1, fig. 5); the color filter (2, fig. 5); the condensing system in the condenser housing (3, fig. 5); the projector supporting bracket (4, fig. 5); and the camera body (fig. 6) containing the projection lens (1) and the diapositive stage (2). The camera body is sometimes called "projector cone" or "lens cone." In a discussion of multiplex operation it is convenient to distinguish between the two main assemblies: the lamp-filter-condenser assembly (5, fig. 5), which is quite generally termed the "lamp house;" and the camera body-supporting bracket assembly (6, fig. 5), which is variously referred to as the "base," the "projector body," or simply as the "projector." The components of this base portion of the projector will be described first.

b. The projection lens.--The multiplex projection lens is small and essentially distortion-free. Its focal length and aperture opening have been selected so as to bring the image of the diapositive to optimum focus in a plane 360 millimeters below the projector, with an approximate range from 290 to 460 millimeters within which the loss of definition is hardly perceptible. This depth of focus is necessary because the tracing table platen on which the projected image is viewed must be raised and lowered in accordance with the relief of the stereoscopic model.

c. The diapositive stage and centering mechanism.--The diapositive is supported in the projector at a fixed distance above the projection lens. In the Geological Survey-type projector, the diapositive is supported upon four bosses, one under each corner, which duplicate those used to support the diapositive in the multiplex diapositive printer. In the later-model projectors these bosses are mounted in a frame (2, fig 6) which surrounds the diapositive and in which the diapositive can be securely fastened. Centering adjustment screws (3, fig. 6) slide this frame in its own plane (perpendicular to the lens axis) for the purpose of alining the principal point of the diapositive with the principal point of the projector ("centering" the diapositive).

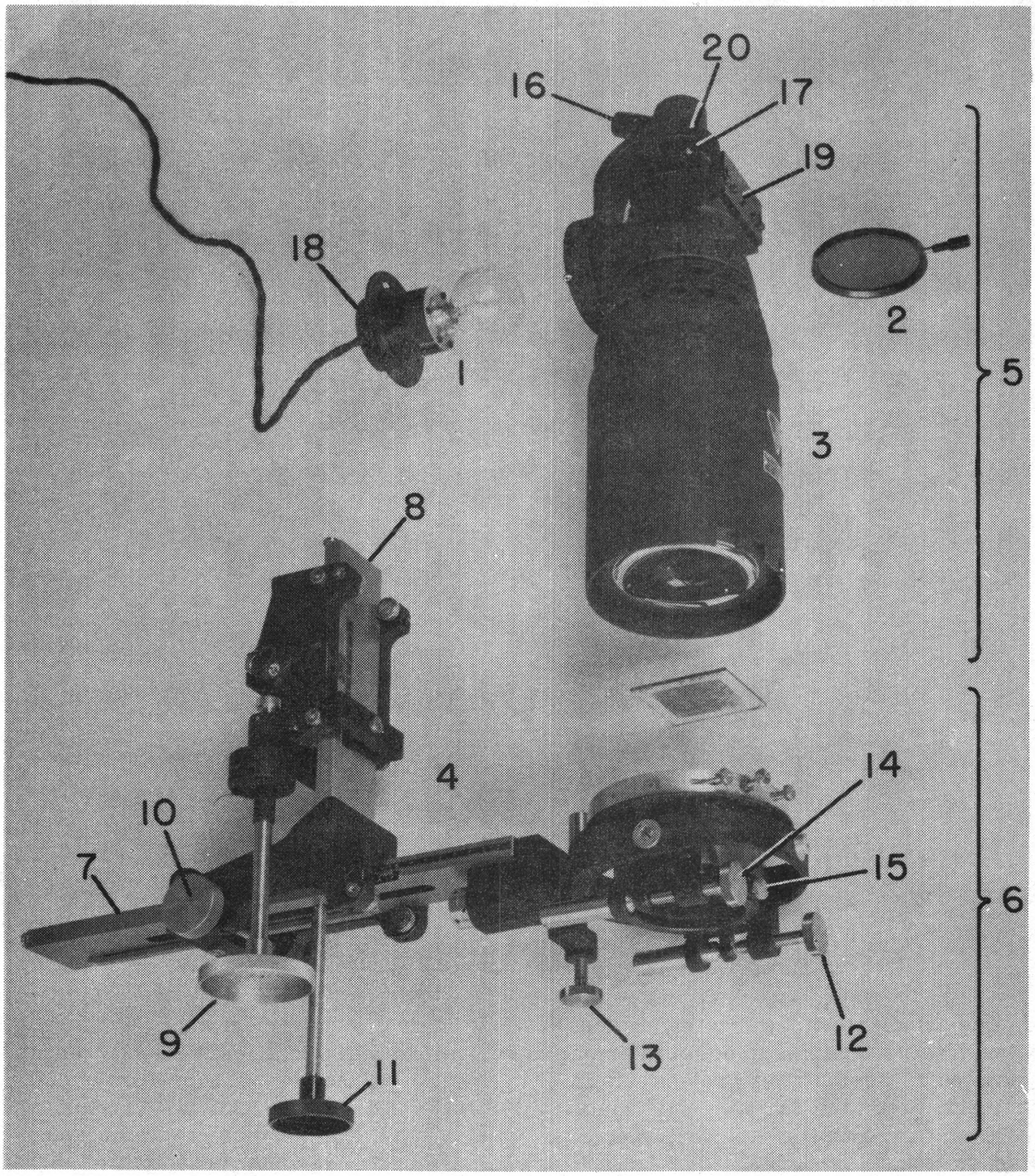


Figure 5. --Multiplex projector.

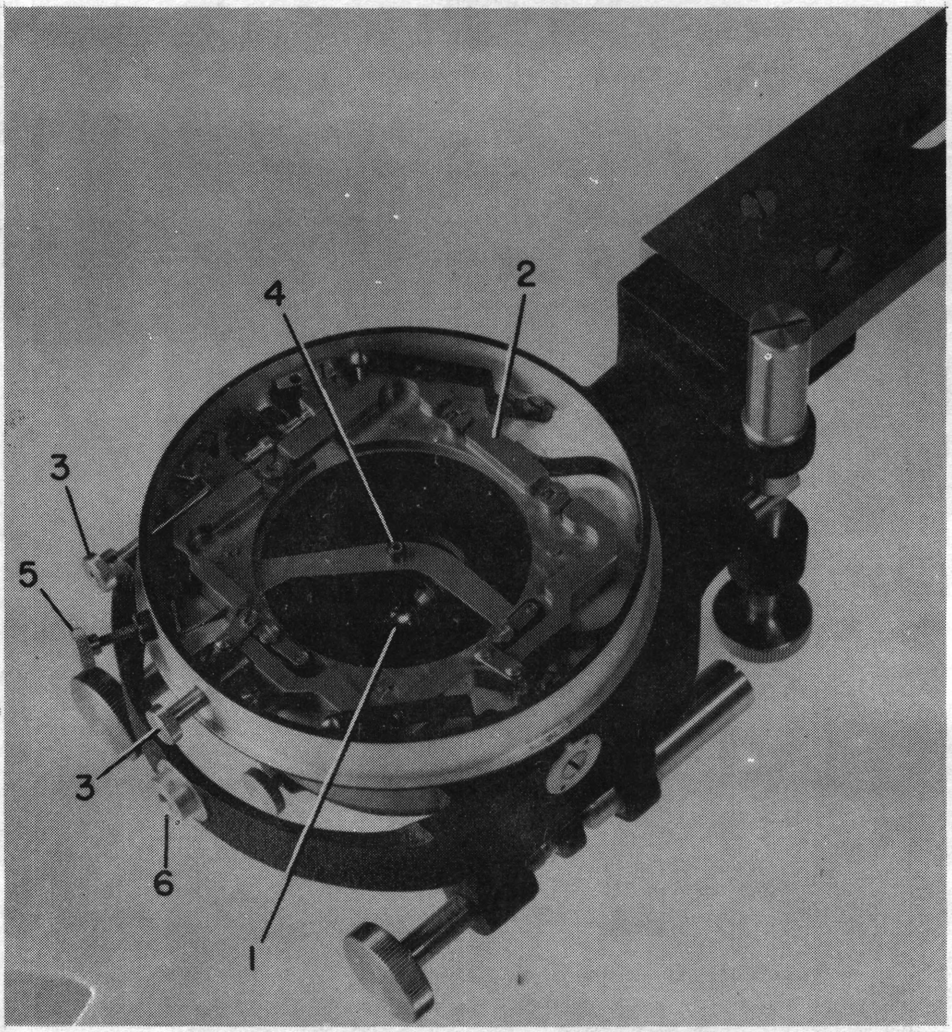


Figure 6.--Multiplex projector camera body.

d. The principal-point indicating device.--One of the great differences to be found among recent designs of the multiplex projector is in the device employed to indicate the principal point. Three different indicating devices are briefly described.

(1) Centering arm.--The centering arm device may be seen in figure 6. The principal point of this projector is indicated by a small dot etched on a glass reticle mounted on a movable arm (4). Normally out of the field of view, this arm is pivoted at one end so that it may be swung out beneath the diapositive by an external release screw (5). Releasing the arm until it hits a stop automatically places the dot at the principal point of the projector.

(2) Principal-point microscope.--The earlier and more widely-used model of the Geological Survey-type projector (1, fig. 2) requires the use of an accessory instrument known as a principal point microscope. Figure 7 shows this instrument (1) together with the testing fixture (4) and gauge (3) required in its calibration. A point on the line of collimation is defined in the microscope by cross lines etched on a glass reticle. To center a diapositive, the microscope

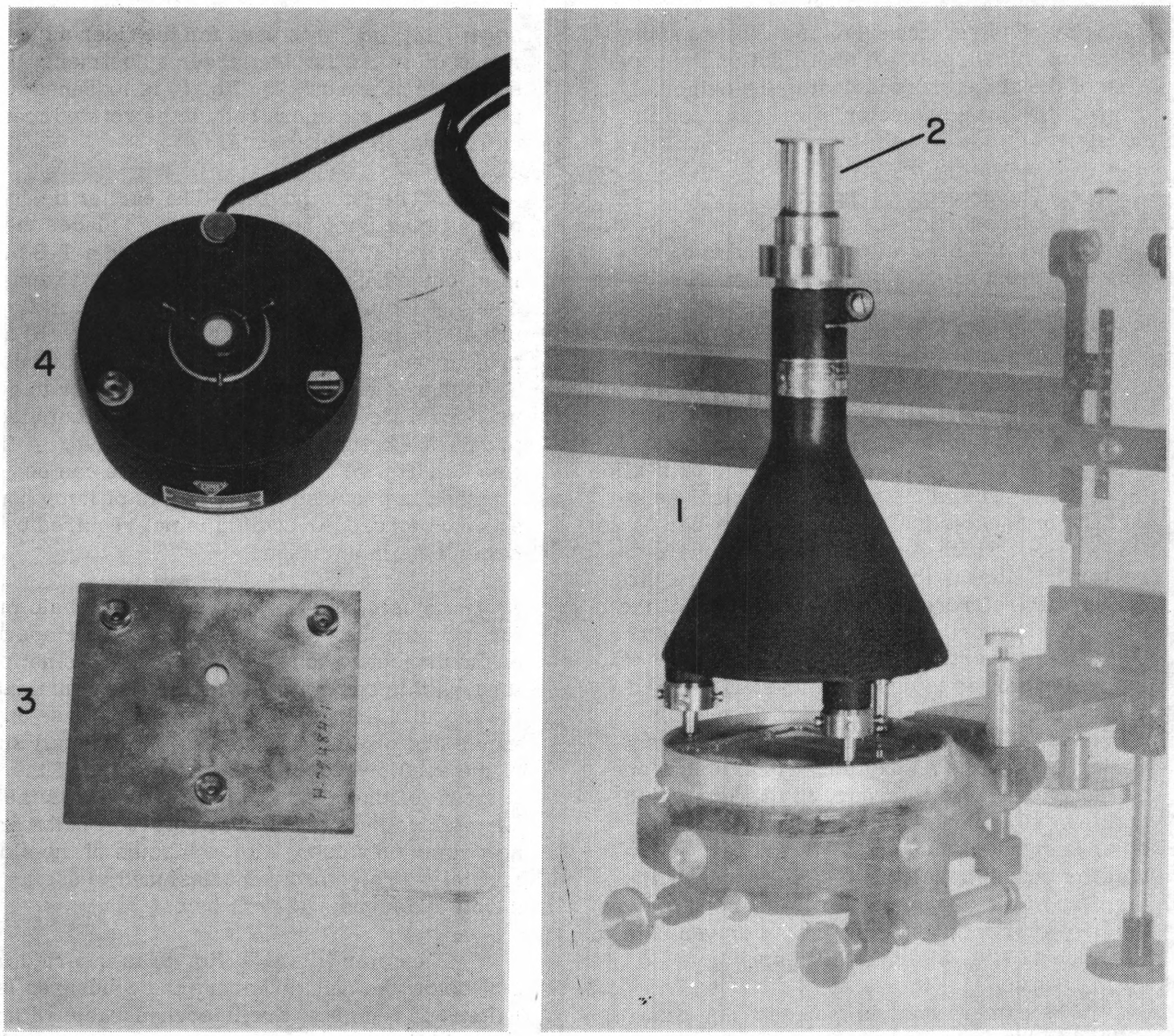


Figure 7.--Principal-point microscope and testing equipment.

must be placed on the camera body as shown. Three foot pads are located beyond the diapositive frame just inside the rim of the camera body to receive the feet of the microscope. One of these foot pads is conical, one is V-shaped, and one is flat, to insure a positive position of the microscope such that its line of collimation accurately indicates the principal point of the projector. The use of the testing equipment is described below in article 22.

(3) Stage plate.--Both in the Zeiss-Geological Survey and in the Bausch and Lomb-U. S. Army types of projectors, the diapositive is supported, not upon bosses, but upon a glass plate called a stage plate. Spring clamps force the diapositive down against this plate; the centering mechanism moves only the diapositive itself. A disadvantage of this arrangement is that in time the stage glass may become scratched from the diapositive sliding upon it. The advantage

of the stage plate, however, is in the marking of the principal point. A small dot on the upper side of the stage glass indicates the principal point of this type projector simply and continuously.

e. The projector supporting bracket.--In addition to supporting the projector on the multiplex frame, the bracket provides the camera body with six freedoms of motion (three of translation and three of rotation), which are essential to the complete recovery of the photograph orientation.

The supporting bracket provides translational motions through an L-shaped arrangement of two sliding members: a horizontal arm or y-slide (7, fig. 5) and a vertical arm, or z-slide (8, fig. 5). The whole assembly is designed to be hung on the x-slide of the multiplex bar (9, fig. 2) along which it can be moved in the x-direction. Although the orientation of these three slides with reference to some fixed part of the multiplex supporting table changes slightly with tilt of the supporting frame, the orientation of the slides serves to designate the x-, y-, and z-directions sufficiently well for descriptive purposes. In certain instances, it may be convenient to assume an explicit orientation of one or more axes with reference to the supporting table, or to the model air base, or to some other designated element. The knobs that control the translational motions are grouped at the rear of the projector: Knob 9 (fig. 5) drives the projector in x; knob 10 (fig. 5) drives it in y; and knob 11 (fig. 5) drives it in z.

The camera body is mounted in a gimbal affixed to the front end of the y-slide. This mount permits the camera body to be rotated about three axes that normally are perpendicular to one another. Rotation of the camera body within the inner ring of the gimbal is about a z-axis and is called swing. Rotation of the inner ring is about an x-axis (secondary axis) and is called x-tilt. Rotation of the outer ring is about a y-axis (primary axis) and is called y-tilt. The x-tilt and the y-tilt are controlled by tangent screws (12, fig. 5; and 13, fig. 5, respectively) which permit the camera body to be tilted from its normal position approximately 10° crosswise to the line of flight and almost double that amount parallel to the flight direction. A third tangent screw (14, fig. 5) is used for the finer adjustments of swing; for the larger adjustments the swing clamp screw (15, fig. 5) is loosened, permitting the camera body to be rotated manually through a full circle.

f. The lamp house.--The earlier design of multiplex lamp house (1, fig. 2) places the bulb socket at the very top, in which a T-8 bulb is mounted base up. The condensing system does not include the reflector indicated in figure 4. In the newer style projector (fig. 5) a prefocused G-16$\frac{1}{2}$ bulb is mounted horizontally. This change not only permits a reflector to be used for added concentration of light, but also provides a better ventilation of the bulb. A fitting (16, fig. 5) to which an air hose can be attached is provided on both types of lamp houses, although forced air cooling is not required by the newer type lamp house.

Adjustments are provided on the bulb mount for focusing the maximum light upon the aperture of the projection lens, as light that does not pass through this opening does not help to illuminate the model. (Instructions for making these adjustments are given in article 24.) The small size of the aperture opening limits the size of the filament of the bulb, and indirectly limits the filament voltage rating. Twenty-volt bulbs are now commonly used, although some of the older multiplex projectors were designed to use a 6-volt and some a 12-volt bulb.

The color filter (2, fig. 5) is inserted in a slot below the bulb and above the condenser lenses. Filters of red glass are interchangeable with filters of blue glass, so that complementary colors for the projection of the stereoscopic pair of photographs can always be provided.

11. The tracing table

a. Function.--The multiplex tracing table shown in figure 8, provides the means of relating the stereoscopic model and the manuscript. It forms the connecting link between these two. Essentially, the tracing table provides a screen (1) on which the projected model can be viewed; a floating mark (2); a height indicating scale (3); a plotting pencil (4); and a horseshoe-shaped base to support the whole.

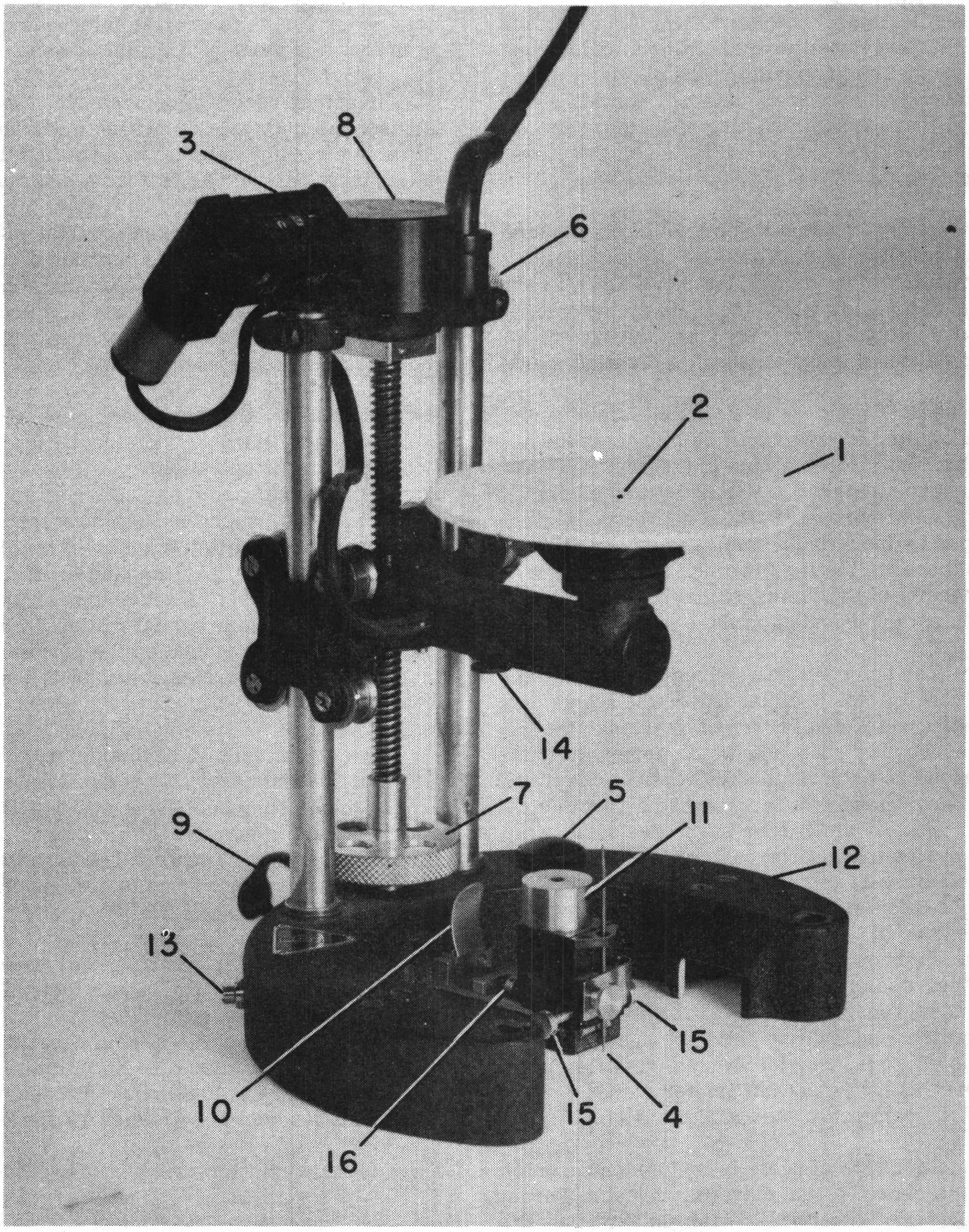


Figure 8.--Multiplex tracing table.

b. The screen and floating mark.--The viewing screen is a circular platen about $3\frac{1}{2}$ inches in diameter. If desired, its white surface can be made even whiter by igniting a small ribbon of magnesium and holding the platen, which is removed from the tracing table, above the flame so that a smooth coating of the powdery white magnesium oxide is deposited on the viewing surface.

A feature of some tracing tables is a platen that can be tilted as much as 30° toward the observer as an aid to viewing the more distant parts of the model. This tilt of the platen in no way affects the plotting geometry since the axis of tilt is designed to pass through the floating mark.

The floating mark is a single, luminous dot, formed by light from a small bulb beneath the platen shining through a hole in the center of the platen approximately 0.004 inch in diameter. Recently, an improved floating mark has been developed by mounting a small sapphire in this hole to disperse the light through a wider angle. The intensity of the luminous mark can be varied by a control (5) mounted on the base.

c. The height-indicating scale.--When the lock screw (6, fig. 8) is loosened, the height of the floating mark can be varied by turning the elevating wheel (7, fig. 8), which drives the platen carriage up or down the two vertical columns. The vertical movement is indicated by a scale, several types of which have been developed. One simple device (not shown) is a millimeter scale engraved directly on one of the columns. A vernier affixed to the platen carriage affords a minimum reading of 0.1 millimeter. The vernier can be adjusted through a range of about 10 millimeters for the purpose of "indexing" or setting the zero reading at a convenient value. Conversions between metric and English units are readily made from tables, prepared in advance, that list for each foot the equivalent millimeter values at each of the commonly used model scales.

Another type of scale is a "Veeder" counter (3, fig. 8) geared to the elevating thread. Three drums record the vertical motion in tens, units, and tenths of millimeters. This counter can easily be set to any index reading by lifting the gear box cover (8, fig. 8) and loosening a lock nut to disengage the gears.

A recent development of the counter type scale is a gear box equipped with interchangeable gears, so as to provide for readings directly in feet at a variety of model scales.

Another recent development is the Williamson tracing table of British manufacture. Differing in many details, this tracing table has as its most outstanding feature a scale engraved on a glass bar, magnified by optical projection onto a glass screen at the rear of the platen. No vernier is employed. A number of interchangeable glass scales enable this table to read elevations in the stereoscopic model not only in millimeters, but also in feet at many different model scales.

d. The plotting pencil and base.--The plotting pencil is supported directly below the floating mark so that it will mark upon the manuscript the orthographic projection of any point in the model at which the floating mark is placed. The pencil can be lifted from or lowered to the manuscript by means of the ring (9, fig. 8) at the rear of the base. A thumb bar (10, fig. 8) helps to steady the lowering motion. The weight (11, fig. 8) can be placed on the pencil chuck to increase the pressure as the point of the pencil wears.

A 6-8 volt bulb in each side of the base (12, fig. 8) lights a small area of manuscript around the plotting pencil. A switch (13, fig. 8) controls these lights and the light provided to illuminate the height scale. These three lights are usually wired in series to operate from the 20-volt supply.

The tracing table slides easily over the manuscript sheet on small agate feet. Provision is made for adjusting these feet, together with the plotting pencil, to insure that the pencil is perpendicular to the table and centered below the floating mark at all times. These adjustments are described in articles 19 and 20.

12. The supporting frame

a. Function.--The multiplex supporting frame consists of a horizontal bar (10, fig. 2) and one or more uprights (11, fig. 2) that support this bar over the multiplex table. In addition to supporting the projectors, one of the usual functions of the frame is to tilt all the projectors as a unit, thereby providing a simple means of leveling either a model or a strip.

Inasmuch as the top surface of the multiplex supporting table is the vertical datum of the instrument, the term "level" is used with reference to the model and to all parts of the multiplex except the table top itself to designate parallelism with the table top, which may or may not be truly horizontal[4].

b. Bar.--The multiplex bar spans the table lengthwise. Attached to and extending the full length of the bar is a smooth track (9, fig. 2) upon which the projectors can be hung and along which they may be slid in an x-direction. A rack running the full length of this x-slide engages a pinion gear on each projector to provide a means of driving the projectors along the slide.

The length of multiplex bars ranges from 3 to 14 feet, but 6 feet and 7 feet are the most common lengths. The multiplex unit pictured in figure 2 has a 7-foot bar. Some 7-foot bars are so made that two can be joined together to form a longer bar. When this is done, a cantilever upright is needed to give additional support to the bar at the junction of the sections. Such a frame is more easily transported than one with a single 14-foot section, but it has an operational disadvantage in that the center support makes leveling by means of the frame more difficult.

c. Uprights.--Because the space beneath the projectors must be unobstructed, the multiplex bar is supported either by uprights at its ends or in cantilever fashion by one or more uprights at the back of the table. Three types of uprights are described, the inverted T-frame, the A-frame, and the nontilting frame.

The "Inverted-T" frame is the type of upright shown in figure 2. Each end of the bar is supported on a vertical, threaded post. This bar can be inclined lengthwise by handwheels (12) which regulate the height of each end independently. The base of each upright is supported on foot screws (13) by which the entire frame can be tilted in any direction. One of the posts is usually pivoted at its junction with the base; otherwise the four-point support of the frame would make it possible to put an undesirable twisting stress on the bar. The handwheels, foot screws, and pivot can all be securely locked when the orientation of the model is completed.

The "A" frame uprights are the simplest and most rigid type. They have no handwheels, all the tilting being done by means of the four foot screws. One of the uprights is usually pivoted at its junction with the bar. Without a pivot to release any twisting stress, extra care must be taken in co-ordinating the adjustments of the foot screws.

The nontilting frame is suitable for those multiplex users who never set up more than one or two models at a time. This is a small multiplex frame and table in one integral unit, having a 3-foot bar supported in cantilever fashion by a rigid framework that extends up from the rear of the table. Although this frame cannot be tilted, the slab forming the table top rests upon an adjustable support so that it can be tilted instead. As explained in Chapter 3C 7-c, however, it is always possible to level a model entirely with the projector motions, without ever using these adjustments of the slab or frame.

13. The supporting table

a. The table top.--Multiplex table tops are uniformly about 3 feet wide (measuring from front to back) but come in lengths ranging from 3 to 7 feet to suit the various lengths of supporting frames. Longer supporting frames can be placed on two tables that have been placed end to end and carefully alined. The primary requirement of the table top is that its upper surface be flat within a few thousandths of an inch, as this surface is the datum plane of the instrument. The material of which the table top is made is therefore important; slate, aluminum, and glass have been used. Of these, slate is by far the best.

Slate tops, in addition to being the sturdiest and most stable of the three materials, are also the most easily surfaced, the most consistently available, and the least expensive. A 3- by 7-foot slate top having a thickness of 3 inches is sufficiently rigid to remain flat without critical dependence upon its supports. Such a slab,

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[4] See article 3, Chapter 3 C 7-a.

however, weighs about 1,100 pounds; therefore, when portability of the equipment is a factor, the purchaser turns to aluminum or glass.

Aluminum table tops are much thinner than those of slate, being reinforced by heavy ribs cast on the underside. No separate frame is needed, as the legs can be fastened directly to the top casting. The aluminum top is not as rigid as the slate, however, and each leg (of which there are five) must be carefully adjusted after the table is set up to eliminate any warp or bending of the top surface. Aluminum tops have been preferred for setting up long tables because of the ease with which they can be clamped and splined together to form a continuous flat surface.

Glass tops are the least desirable and the least used. Large glass plates would require several intermediate adjustable supports. The use of glass is best limited to small multiplex tables (for one or two models only).

b. The table frame.--A slate top or a glass top must be supported on a frame, preferably one of steel, as the weight of the table top plus the weight of the multiplex bar and projectors is considerable. The multiplex table frame is provided with adjustable feet, so that all the legs can be brought to bear equally upon the floor. If desired, the plane of the table top can be established as a truly horizontal plane by means of these foot adjustments. It is not necessary, however, to have the table top exactly leveled except in unusual instances when some gravity-dependent device such as a plumb bob or a spirit level is to be used in connection with the orientation of the projectors or model.

14. Filter spectacles

The filter spectacles (6, fig. 2) which the observer must wear in order to see the stereoscopic model have one red lens and one blue lens. As a rule, the greater the observer's stereoscopic ability, the greater is his preference for densities and shades of these colors that subdue rather than completely obscure the alternate image. Beginners, however, will benefit by the darker shades which provide a more solid though less brilliant model. The red lens is always plane; the blue lens is now made with a slight negative correction to balance the refraction of the two colors. For the observer's convenience in viewing consecutive models without having to exchange the projector filters, the spectacles either are made reversible or are supplied in sets of two, one with the red lens on the left and one with the blue lens on the left.

Experiments have been made using polarized light to project the model and polarizing eye filters to obtain stereoscopic perception. A special surface for the tracing-table platen is required to provide a diffused but still polarized reflection, but such a surface has been difficult to obtain. The usual result is that either the back of the model appears too dim, or the texture of the platen surface is too evident. Although the method is workable it is not, at least at the present time, as satisfactory as the complementary colors.

15. The electrical system

The multiplex electrical system usually consists of a transformer, a control box (7, fig. 2) containing two or more selector switches and two or more rheostats, a multiple-outlet strip along the top or back of the horizontal bar, and the interconnecting wiring.

The function of the transformer is to reduce the line voltage to the maximum required by the various light bulbs (commonly 20-volt, although both 6-volt and 12-volt multiplex projectors were once used). The selector switches are wired to the multiplex outlets--except one or two constant voltage outlets reserved for the tracing table connection--so that any desired pair of projectors can be lighted. The rheostats regulate the brightness of each image independently. (For good stereoscopic perception, each image must be about equally bright at the part of the model being viewed.)

A slightly different method of voltage control is employed in the case of the 3-foot multiplex frame and table mentioned above. This unit is equipped with two "Variac" controls, which combine the function of transformer and rheostat in each control.

16. The cooling system

Forced air cooling for the multiplex projectors of newer design (2, fig. 2) is optional, but the projectors of older design (1, fig. 2) are operated without cooling only at the expense of shorter bulb life and at the risk of damaging the condenser lens. A fitting (16, fig. 5) for an air hose is provided on both types of lamp house, and the supporting frame carries a header (14, fig. 2) to which several air hoses can be connected. A large hose connects the header either to an individual fan or, in laboratories having several multiplex units, to an air duct connecting all the units to a central fan.

The cooling system can be either a blower or an exhaust system. An advantage of a system that blows air into the projector is that a filter can be used on the air intake to catch the dust that otherwise would be sucked into the lamp house, where it would accumulate on the filter and upper condenser lens, and rapidly decrease the efficiency of the lighting system.

## CALIBRATION AND ADJUSTMENT OF MULTIPLEX EQUIPMENT

17. Distinction between routine and laboratory procedures

For the purpose of this discussion, the procedures involved in calibrating and adjusting multiplex equipment will be divided into two classes: routine procedures, or those which are the responsibility of the stereocompiler and, hence, a routine part of operation; and laboratory procedures, or those requiring the use of special tools and equipment in the hands of skilled technicians. The procedures in the routine class will be described in detail, but a discussion of the laboratory procedure is beyond the scope of this chapter--instead, there will be described methods that are available to the stereocompiler for roughly checking the calibration performed in the laboratory.

The stereocompiler will make the adjustments or perform the checking operations listed below, either at such times as are suggested, or whenever the need is apparent. When a complete adjustment and check of equipment is made, however, it is important that the order in which the operations are listed be followed rather closely.

(1) Adjust the floating mark bulb.

(2) Check the verticality of the tracing table columns.

(3) Center the plotting pencil.

(4) Focus the principal-point microscope (when this instrument is used).

(5) Check the collimation of the principal-point microscope (when this instrument is used).

(6) Orient the axis of the filament coil (in the newer type lamp house only).

(7) Position the projector bulb.

(8) Check the adjustment of the reflector (in the newer type lamp house only).

(9) Check the principal distance of the projector.

(10) Check the principal point of the projector.

Of these operations, (2), (3), (5), (9), and (10) are related to the calibration of multiplex equipment, inasmuch as they involve adjustments made to fixed standards upon which the proper geometric functioning of the instrument depends. The remaining operations pertain to adjustments that are not highly critical, but that primarily affect the convenience with which the visual observations can be made. Indirectly, anything hindering the stereocompiler's observations may result in errors in the map compilation fully as serious as errors due to faulty calibration of the equipment.

18. Adjusting the floating mark bulb

The position of the bulb beneath the hole in the platen of the tracing table affects the visibility of the floating mark, especially at the far edge of the model. Whenever this bulb is replaced, therefore, it should be repositioned as follows:

Loosen the screw (14, fig. 8) on the underside of the platen carriage. Move the bulb back and forth by sliding this screw in its slot, at the same time viewing the floating mark at a low angle.

When the position of the bulb affording the brightest light is found, lock the bulb in position by retightening the screw.

19. Checking the verticality of the tracing table columns

To insure that the vertical movement of the floating mark is perpendicular to the supporting table top, the three agate feet of the tracing table are so mounted that each one can be adjusted up or down independently, until the columns on which the platen carriage rides are perpendicular to the plane of the feet. This is a laboratory adjustment, which the stereocompiler can check as follows:

Lower the platen as far as it will go. Illuminating one projector, place the floating mark on any well-defined image point and plot this position with the tracing table pencil, first making sure that the pencil lead is sharpened to a carefully centered point.

Without disturbing the elevation of the platen, turn the tracing table 180° and replot the same image point. If there is a discrepancy between the two plotted positions, tentatively adjust the pencil point to a mid-point position as described in step (3) of article 20.

Raising the platen to its highest point, repeat the above procedure of plotting and checking a point, which may be either the same or some other well-defined image point. If a discrepancy is now apparent at this elevation, it indicates that the columns are not plumb. This adjustment needs to be checked only occasionally.

20. Centering the plotting pencil

This adjustment of the tracing table is the only adjustment affecting the calibration of multiplex equipment that the stereocompiler is charged with as a routine part of multiplex operation. The purpose of the adjustment is to center the pencil directly beneath the floating mark. The pencil lead should at all times be sharpened to a carefully centered point. This adjustment presupposes that the columns of the tracing table are perpendicular to the plane of the feet of the tracing table.

Lock the platen at a fixed height (preferably at about the average plane of a model). Illuminating one projector, place the floating mark on any well-defined image point and plot this position with the tracing table pencil.

Turn the tracing table 180° and replot the same image point. If there is a discrepancy between the two plotted positions, mark a mid-point between the two.

Keeping the floating mark on the same image point, loosen the two adjusting screws (15, fig. 8) at the sides of the pencil chuck. Shift the chuck by means of these screws and the one behind the chuck (16, fig. 8) until the pencil point moves to the mid-point position. Tighten the side screws to lock the chuck in position.

Check the adjustment by repeating the procedure.

The stereocompiler should check this adjustment at least once a day until he is confident that the pencil will hold its adjustment for a longer period.

21. Focusing the principal-point microscope

a. Eyepiece.--Each stereocompiler who uses the principal-point microscope should refocus the eyepiece (2, fig. 7) if necessary, to suit his own vision. The eyepiece, which is threaded into the microscope tube, should be turned until the cross lines on the reticle appear sharp. Focusing the eyepiece does not disturb the collimation of the microscope.

b. Objective.--The objective lens of the principal-point microscope is properly focused upon the plane of the diapositive at the factory, and should not need refocusing. The stereocompiler can check the focus of this lens while using the microscope to center a diapositive, by moving his eye from side to side across the diameter of the eyepiece lens. If the cross lines on the microscope reticle appear to move relative to the diapositive image, the objective is not properly focused. Correcting the focus of the objective lens should be done in the instrument laboratory, since it is almost certain to necessitate the readjustment of the instrument's line of collimation.

## 22. Checking the collimation of the principal-point microscope

The test equipment shown in figure 7 is used in the laboratory adjustment of the line of collimation of the principal-point microscope. Two requirements have to be met: First, the feet of the microscope must define an equilateral triangle of a certain size; second, the cross lines of the reticle must define a line of sight that passes through the exact center of this triangle. The gauge plate (3, fig. 7) is used first to set the feet in the proper triangle, after which the illuminated testing fixture (4, fig. 7) is used to adjust the line of sight. Once the target on the testing fixture has been properly set, subsequent collimations of the principal-point microscope can be made on this fixture alone.

The following procedure of checking the adjustment of the line of collimation will be used by the stereocompiler each time he centers a diapositive in a projector.

With the microscope placed on the projector in one of the three alternative positions, center the diapositive as described in article 4 (b), chapter 3 C 7-c.

Check this centering by successively placing the microscope in each of the other positions. A need for recollimation of the microscope is indicated if a discrepancy in one or both of the check positions is noted. (A tolerance of 0.001 inch or about the width of one of the lines forming the principal-point mark is permissible.)

## 23. Orienting the axis of the filament coil of the projector bulb

This adjustment applies only to lamp houses such as the one shown in figure 5, in which the bulb is mounted horizontally. With this orientation of the bulb, the axis of the filament coil revolves in a vertical plane when the socket mount is rotated. The best illumination is obtained when the filament coil axis is vertical, or parallel to the axis of the optical system. To adjust the axis of the filament coil:

Hold an auxiliary lens of short focal length, such as a pocket magnifier, below the projector, and move this lens up or down until it forms an image of the lighted filament upon the table top. (If an auxiliary lens is not available, the projector lens itself can be made to focus the filament image by carefully lifting the lamp house from the base a distance approximately equal to the principal distance of the projector. The auxiliary lens method is preferred, however.)

Rotate the bulb by the knurled ring (18, fig. 5) at the back of the bulb socket until the filament image appears doughnut-shaped.

Whenever a bulb is replaced after this adjustment is once made, the filament coil axis will always come to a vertical position if the three-holed flange on the socket assembly is always replaced with the same hole at the top. Therefore, the stereocompiler needs to check this adjustment only when he suspects that it may have been disturbed.

## 24. Positioning the projector bulb

Before this adjustment can be made, either a diapositive or a clear glass plate of equal thickness must be in place in the projector.

With the lamp house in place, turn on the light to maximum intensity. Remove the color filter. Hold a white screen (the back of a contact print will do) close beneath the projector lens so that the whole field of illumination can be viewed.

Using the two lateral adjusting screws on the bulb mount (17, fig. 5), center the filament so that any color appearing in the illuminated area is symmetrically disposed. (This color is caused by chromatic aberrations of the condensing system.) If the filament were now to be viewed by the method described above of holding a small auxiliary lens beneath the projector, the filament should appear well centered in the aperture opening, which appears as a faint circle surrounding the filament image.

Using the knurled ring (20, fig. 5) at the top of the lamp house, adjust the bulb vertically until the illuminated field is as color-free as possible. In particular, any bluish cast should be avoided--a tinge of yellow, on the other hand, is not undesirable, and a residual ring of light yellow around the outer edge of the field may often have to be accepted.

25. Checking the adjustment of the reflector

The reflector in the newer-style lamp house (fig. 5) is properly adjusted at the factory and should seldom need readjusting. Its adjustment is a laboratory procedure. The adjusting screws (19, fig. 5) should not be disturbed by the stereocompiler. The stereocompiler can check the adjustment of the reflector as follows:

Position the bulb carefully by the procedure given in article 24.

Grasping the lamp house firmly with both hands, raise it from the projector base until the image of the filament is seen on the table. (When this test is made in the laboratory, a special jig is used, which fits upon the projector base and provides a platform to support the lamp house at the proper height above the projector lens.)

Displace the lamp house horizontally to include the image formed by the reflector. (Because of the hole in its center, the reflector images only the oblique rays.)

If the reflector is properly adjusted, the reflected image should be of the same size as the direct (and brighter) one formed by the condenser lenses alone, and the two images should be in superposition. Adjustment of the reflector is therefore needed if one image is readily distinguishable from the other.

As an improperly positioned bulb could also be the source of a displacement between the direct and the reflected images of the filament, it is suggested that the bulb be repositioned by trial, when a displacement is noted, until the two images are brought together. With the lamp house then replaced on the base, if the illuminated field shows an undesirable amount of color, the lamp house should be sent to the optical laboratory for readjustment of the reflector.

26. Checking the principal distance of the projector

The two procedures that are here given for checking the principal distance will afford only a rough check, for the purpose of discovering a relatively large error, as they are capable of but one-fifth to one-tenth the accuracy required in the laboratory calibration of the principal distance. (In the laboratory calibration, it is attempted to set the principal distance to its nominal value with a precision of approximately 1 part in 3,000.)

The principal distance is accurately calibrated before the projector leaves the factory, and seldom needs recalibration. The stereocompiler will, therefore, check the principal distance only when there are consistent indications that it may be in error. The indication of an error of principal distance is basically an inequality between the horizontal and vertical scale of the model[5].

If the stereocompiler should find it necessary to check a principal distance, he will need one accessory that is not required in the everyday operation of the multiplex, a diapositive having calibrated graduations as indicated in figure 9. A glass plate etched with a precise grid is commonly used in the laboratory

[5]See article 5, Chapter 3 C 7-c.

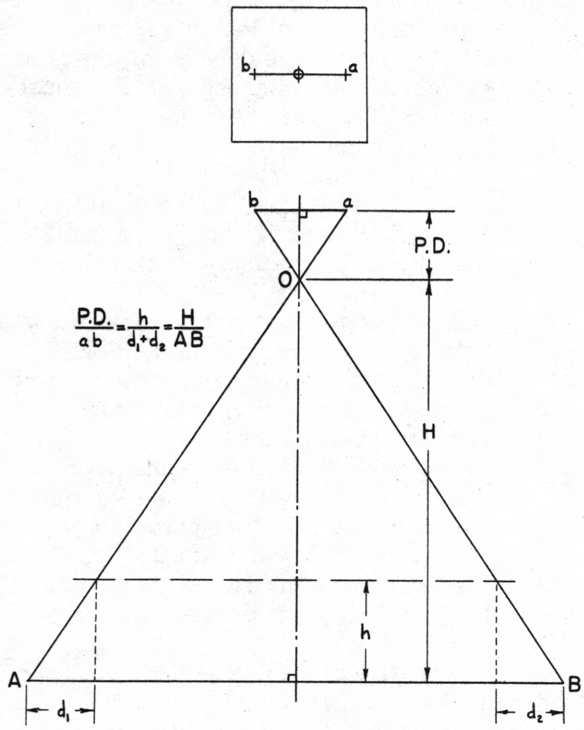


Figure 9. --Checking principal distance by projected grid.

calibration of the principal distance, and carefully made photographic copies of such a grid can be used in the following procedures.

a. Suggested procedure. --The preferred procedure for checking principal distance is as follows:

Place the grid plate in the projector and center one of the grid intersections. Swing the camera body until the grid lines are roughly parallel to the x- and y-axes of the projector.

Viewing the projected grid on the table top, tilt the projector (x- and y-tilt) until the pattern is precisely rectangular. Triangle OAB (figure 9) is now similar to triangle Oab. The grid plate is also parallel to the table top provided the optical axis of the projection lens is a straight line.

On the grid line passing through the principal point in the x-direction, select two grid intersections, one on each side of the principal point and each approximately 20 millimeters from the principal point. Using a needle in place of the tracing table pencil, and having checked the calibration of the tracing table with the needle in place, plot the positions of the selected grid intersections, once with the platen at its lowest possible elevation and again with the platen at its maximum height. Calculate the sum of the two displacements, $d_1 + d_2$.

Compute the principal distance (P.D.) according to the relationship

$$P.D. = \frac{h \times \overline{ab}}{d_1 + d_2} \qquad (1)$$

where h is the vertical range of the platen and $\overline{ab}$ is the distance on the grid plate between the two grid intersections.

b. Alternative procedure. --On the front of the outer gimbal ring is a conical boss (6, fig. 6). The following procedure makes the assumption that the center of this boss is exactly at the height of the exterior perspective center when the y-slide of the projector is supported in a level position. The assumption of this condition is warranted, for it is a condition which the manufacturer attempts to attain very closely. (Subsequent adjustments of the principal distance will disturb this relationship slightly, as the principal distance is altered physically by a movement of the lens in relation to the rest of the camera body while the diapositive plane remains fixed.)

Tilt the supporting frame until the y-slide of the projector is parallel to the top of the multiplex supporting table. The y-slide may be leveled by means of a bubble, provided the table top has also been leveled.

Place the grid plate in the projector and center one of the grid intersections.

Viewing the projected grid on the table top, tilt the projector as described in the second step of the procedure described under a, until the pattern is precisely rectangular.

Measure the distance $\overline{AB}$ (fig. 9) on the table between two grid intersections in line with the principal point and approximately 20 millimeters either side of the principal point. The distance $\underline{ab}$ between the same two points on the grid plate should also be measured, if not known.

Measure the height $\underline{H}$ of the center of the conical boss above the multiplex table.

Compute the principal distance (P.D.) from the relationship

$$P.D. = \frac{H \times \overline{ab}}{\overline{AB}} \qquad (2)$$

27. Checking the position of the principal point of the projector

The two procedures that are given here for checking the principal point will afford only a rough check, for the purpose of discovering a relatively large error, as they are capable of approximately one-fifth to one-tenth the accuracy required in the laboratory calibration of the principal point. (In the laboratory calibration it is attempted to set the principal point mark to define a perpendicular from the interior perspective center to the diapositive plane with an angular error no greater than 15 seconds.)

The principal point is accurately calibrated before the projector leaves the factory, and seldom needs recalibration. The stereocompiler will therefore check the principal point only when there are consistent indications that its position may be in error. A position error of the principal point is often indicated by its effect on y-parallax, and sometimes by its effect on elevations in the model. Like a principal distance error, the effect of a given error increases in proportion to the relief in the model[6].

a. Suggested procedure.--This procedure requires the use of the precise grid plate described in article 26.

Place the grid plate in the projector and carefully center one of the grid intersections. (If the centering is done by means of a principal-point microscope, the adjustment of this instrument must have been previously checked.)

Viewing the projected grid on the table top, tilt the projector (x- and y-tilt) until the pattern is precisely rectangular.

Using a tracing table that has been carefully calibrated with respect to the verticality of its columns, place the floating mark on the principal-point image. Without moving the base of the tracing table, move the platen up and down through its full range. If the floating mark and the principal-point image do not remain coincident, that is, if the principal-point ray is not plumb, the principal point is in error.

b. Alternative procedure.--This procedure does not require the use of a grid plate, but it does make the assumption that the optical axis of the projector lens is a straight line and that the diapositive plane is perpendicular to the mechanical axis of swing.

Carefully center a diapositive in the projector. If a principal-point microscope is used, it should be in accurate adjustment.

Observing the principal-point mark on either the multiplex table or the platen, slowly rotate the projector by the swing motion through 180°, being careful not to disturb any of the other motions. If the principal-point mark does not remain stationary, an error in the position of the principal point is indicated.

[6] See article 5, Chapter 3 C 7-c.

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GEOLOGICAL SURVEY

TOPOGRAPHIC INSTRUCTIONS

Chapter 3C 7-c

# MULTIPLEX ORIENTATION

## Contents

## Illustrations

Chapter 3 C 7-c[1]

# MULTIPLEX ORIENTATION


## ABSTRACT

Orientation procedure is divided into interior orientation and exterior orientation. The latter is subdivided into relative orientation and absolute orientation. Alternative methods are available for each phase of orientation; once the method is selected, a fixed routine is followed. Where alternative methods are given, the exposition is accompanied by a discussion of when to use each method. Interior orientation procedure varies according to the means provided for centering the diapositive; the means may be a centering arm, principal-point microscope, or stage plate. Relative orientation can be accomplished either by the "one projector" method or the "swing-swing" method, depending on whether it is desired to limit the adjustments to one projector or apply them to both projectors. In absolute orientation procedure, positioning of the model can be effected with the base sheet either fixed or movable, and leveling of the model can be accomplished by adjusting either the projectors or the frame. Difficulties arising from special problems and the effects of errors are treated in connection with each phase of orientation. Minor variations from the described procedures are permissible, but are not given in detail.


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## SCOPE OF MULTIPLEX ORIENTATION

### 1. Procedures and objectives

The broad steps of multiplex orientation, and the relation of orientation to the complete chain of multiplex mapping operations, are indicated in a preceding chapter[2]. The objects of the various orientation procedures may be briefly restated as follows:

a. Interior orientation. --To recover a projected cone of rays geometrically identical with the cone of rays making the original exposure.

b. Exterior orientation. --To recover the position of each camera station and the angular orientation of each photograph. Exterior orientation is divided into two parts:

(1) Relative orientation--To reconstruct the same perspective conditions between a pair of photographs that existed when the photographs were taken, thus establishing a model.

(2) Absolute orientation--To orient the model with reference to a horizontal and vertical datum.

This chapter is devoted to detailed instructions for effecting each orientation procedure.

[1]The number 3 C 7-c signifies Book 3, Part C, Chapter 7-c of the Geological Survey loose-leaf manual of Topographic Instructions. For a table of contents, see Chapter 1 A 2 (Circular 92).

[2]Chapter 3 C 7-a, Introduction to the multiplex.

## INTERIOR ORIENTATION PROCEDURE

2. Preliminary steps

Remove the lamp houses from all the projectors. As the bottom rims of the lamp houses are likely to leave black marks wherever they are set down, care should be taken never to set the lamp houses directly on the compilation sheet.

Loosen each swing lock screw and rotate the projectors until all the diapositive centering mechanisms are similarly oriented. When the projectors being used are of the types having external screws for the centering of the diapositive, it is essential that the projectors all be turned with these screws to the front. When the centering screws are internal (i.e., in the type projector employing a principal-point microscope), this step is not essential, but is a matter of convenience.

Before inserting any diapositives, carefully blow or brush any dust or chips of glass from the projector lens. (In the Bausch and Lomb-U. S. Army type projector, dust is kept from the lens by the stage plate. Dusting the stage plate is especially important as dust or glass chips here may make a permanent scratch upon the glass, as well as being a potential source of error by preventing the diapositive from lying in its proper plane.)

3. Inserting the diapositive

For convenience, it is suggested that all the diapositives be placed in the projectors in such order and orientation that it will not be necessary to shift the order of the projectors on the bar or to rotate any projector 90° or 180°. One way to do this is to lay out the contact prints in the desired order and with the overlapping portions of the photographs adjoining each other. Lay out the diapositives, emulsion side down, in the same order and with the same orientation. Then simply rotate each diapositive 180° when placing in the projector to allow for the fact that in projection the image is reversed left for right and top for bottom.

Having placed each diapositive in its projector, emulsion side down, clamp it securely. Whatever manner of stops or springs is used to hold the diapositive in place, care should be taken to see that the diapositive is secure but not subject to any undue pressure that might bend or break the glass between the supporting bosses.

Check the diapositive for contact with the bosses by tapping it lightly on each corner. If the contact is good, no clicking of the glass against the boss will be heard. Checking the diapositive for contact with a glass stage plate (as is required in the U. S. Army type projector) is best done by looking for the interference patterns, or "Newton's rings," in the light reflected from the two surfaces in contact. The presence of these "rings" is an indication of sufficient contact.

If the projector is one of the newer Geological Survey types in which the diapositive is fastened in a surrounding frame, it is well to tap this frame lightly to see that it in turn rests securely on its supporting bosses.

4. Centering the diapositive

The diapositive centering procedure will depend on which of the three previously described devices is used for indicating the principal point of the projector.

a. Centering arm.--Replace the lamp house and release the centering arm until it comes up against its stop. Either on the platen or on the table, observe the image of the black dot on the reticle of the centering arm, and the image of the white cross or circle marking the principal point of the diapositive. Bring these marks into concentric alinement by means of the centering screws, being careful to make the final twist of the screws a positive motion, against their opposing springs.

b. Principal-point microscope.--Do not replace the lamp house, but place the microscope in position on the projector by first placing one foot (any one of the three may be used) in the cone-shaped foot pad, then revolving the second foot into the V-shaped pad, and finally lowering the third foot onto the flat pad. Place a light source or a light-colored surface below the projector lens for a background. Observing the principal-point mark of the diapositive through the microscope, aline it with the cross lines of the microscope reticle, always being careful to make the final adjustment against the spring. Check this alinement with the microscope successively placed in all three possible orientations. If the centering checks, remove the microscope and replace the lamp house.

c. <u>Stage plate</u>.--Replace the lamp house. Observing the projected images of the principal-point marks, the one on the stage plate and the one on the diapositive, aline them by means of the centering screws, being careful to make the final movement of the diapositive against the two springs.

5. Effects of interior orientation errors

Figure 1 shows the deformations of a model caused by small errors of principal distance and of principal-point location. Principal-distance and principal-point errors have no effect upon a model that is a flat surface, but their effects increase in proportion to the relief in the model. Only the vertical effects are important; the horizontal displacements are seldom large enough to be significant. In figure 1, the dashed lines indicate the normal shape of a model which, in this illustration, is assumed to be a block with rectangular faces. The solid lines then picture the nature of each particular deformation. The magnitudes of the deformations shown are greatly exaggerated in proportion to those that would likely occur in practice.

Both principal-distance error and principal-point error can result from improper calibration of either the aerial camera, the multiplex printer, or the multiplex projector. Principal-point error can also result from a failure on the part of the diapositive maker to aline each negative properly in the printer, or from a failure on the part of the stereocompiler to center each diapositive correctly in its projector.

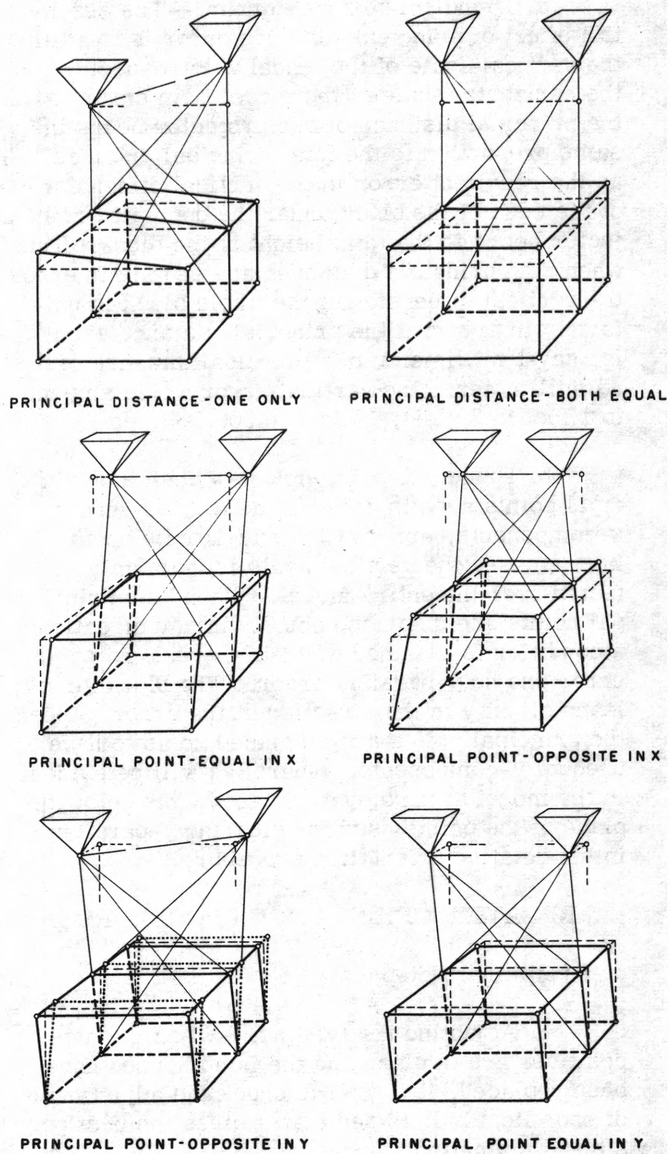


Figure 1.--Effects of interior orientation errors.

Those errors having their source in the optical or mechanical performance of the aerial camera or diapositive printer are very likely to be duplicated in each projection. The errors of negligent operation are more likely to occur in one projection only.

a. Principal-distance error.--The essential effect of principal-distance error is to alter the vertical scale of the model with respect to the horizontal scale of the model. An error in the principal distance of one projector bears the same proportion to the total principal distance as the resultant error in the vertical dimension of the edge of the block under the opposite projector bears to the total height of the block. Thus, when both principal distances are equally in error, the vertical dimension of the whole block is uniformly in error; if the principal-distance error is, say 1 millimeter in a principal distance of 30 millimeters, the vertical error would amount to 1 foot in 30 feet, 10 feet in 300 feet, etc.

b. Principal-point error.--When the principal points are displaced so as to have unequal x-components, errors of vertical scale occur. In this case, the vertical scale is uniformly affected over the entire model. When both principal points are displaced equally in any direction, a condition is obtained wherein there is no scale error and no y-parallax error. The block simply leans slightly in the direction of the error. When the principal points are displaced so as to have unequal y-components, y-parallax will be evident in the model at all elevations above and below the plane of the points used for clearing y-parallax in the relative orientation procedure.

## RELATIVE ORIENTATION PROCEDURE

### 6. Preliminary steps

a. Adjusting the lighting.--When the diapositives are in place and the lamp houses have been replaced, it is well to check the adjustment of each light bulb to see that it gives the best possible illumination[3].

b. Preliminary orientation.--The time required to perform orientation (both relative and absolute) may often be decreased if at the start the projectors are roughly oriented according to the best estimate of what their ultimate orientations will be. If nothing else is known, it may at least be assumed that the flight line is level, straight, and parallel with the multiplex bar and that each photograph is truly vertical. Accordingly, it is good practice to commence by leveling the multiplex frame and each projector, not necessarily measuring these adjustments, but at least estimating them with care. Set all the y-slides to a uniform value, likewise the z-slides. Space the projectors apart an average amount (200 to 240 millimeters).

### 7. The perception of y-parallax

When corresponding images from a pair of projectors not in relative orientation are observed on the tracing table platen, components of separation in both the x- and y-directions will ordinarily be seen. (For the purpose of this discussion of relative orientation, the model air base will be considered to be parallel to the x-direction unless otherwise stated.) The x-component at any one point can be eliminated either by altering the air base (x-motion of the projectors) or by changing the height of the platen. The separation then remaining at the point is strictly y-parallax (often referred to among multiplex photogrammetrists simply as parallax), which can be eliminated only by some motion of the projectors other than x-motion.

When relative orientation is first begun, y-parallax will be observed without the filter spectacles and will appear as a distinct separation between corresponding images. The color filters should be in place but only as an aid in distinguishing one image from the other. Images affording good contrast and ready identification should be selected, such as cleared fields surrounded by woods, or prominent roads or fence patterns.

As relative orientation progresses and the y-parallax separation diminishes to something less than a millimeter it becomes preferable to view the parallax stereoscopically, using the filter spectacles. Viewed in this way, y-parallax appears no longer as a separation of the two images, for they are fused into one three-dimensional model, but as a y-direction split in the floating mark, the apparent separation between the two dots being equal to the disagreement of the actual images on the platen. The smallest y-parallax appears finally as a slight fuzziness of the dot or perhaps as a slight fringe of colors upon its upper and lower edges. Moving the mark about a small area to obtain different backgrounds, an experienced observer can detect a y-parallax of 0.1 millimeter or even less.

[3] This adjustment is described in article 24, Chapter 3 C 7-b, Multiplex equipment.

8. Suggested procedures of clearing y-parallax

The many alternative procedures of clearing y-parallax are but slight variations of one fundamental procedure of relative orientation. The procedures given in this article are two of the standard variations which the multiplex beginner should memorize, practising them at first using a pair of specially made diapositives having lines or dots to mark the positions numbered 1 to 6 in figure 2.

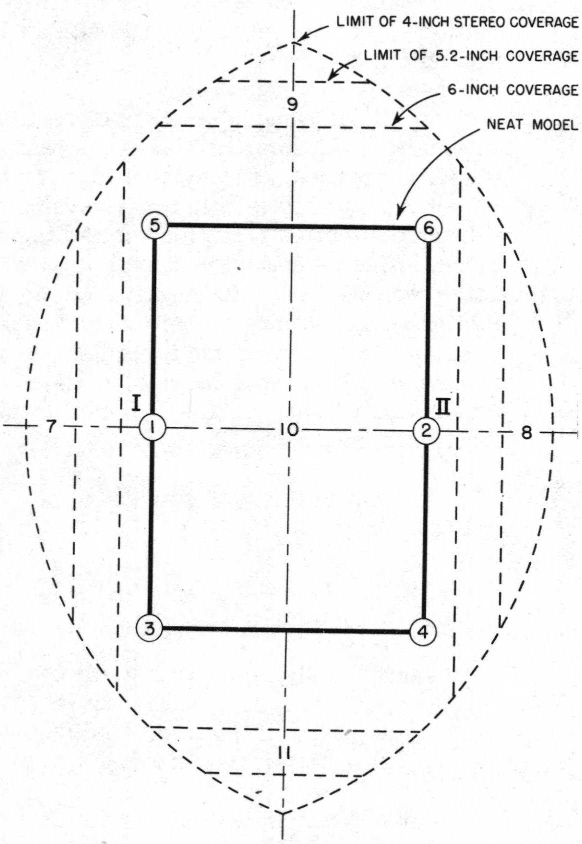


Figure 2.--Locations in the model at which to observe y-parallax.

The possible variations of procedure can more readily be appreciated if the six motions of the multiplex projector are examined, as the effects of these motions dictate the general procedure of relative orientation. Figure 3 shows how these motions would appear to distort the projected image of a grid. The plane of projection is assumed originally to be parallel to the diapositive grid plate, and the projected grid in this position is represented by dashed lines. The numbers locate the critical positions in a model at which y-parallax will be observed and cleared, as indicated in figure 2.

a. <u>"One-projector" method of relative orientation (sometimes called "y-swing" method)</u>.--Refer to the numbered positions given in figure 2. All the motions are applied to projector II:

Clear y-parallax at 2 with y-motion.

Clear y-parallax at 1 with swing.

Clear y-parallax at 4 with z-motion.

Clear y-parallax at 3 with y-tilt. If the amount of tilt is large it is ofttimes convenient, although never necessary, to restore the model approximately to its original projection distance by a suitable x-motion.

Overcorrect approximately $1\frac{1}{2}$ times the y-parallax at 6 with x-tilt.

Repeat the procedure until no y-parallax remains at any of the five positions.

Check position 5. The y-parallax here indicates that some of the previous positions have not been completely cleared of y-parallax.

If projector I had been used, the requirement obviously would have been to reverse positions 1, 3, and 5 with positions 2, 4, and 6 respectively. Positions 3 and 4 may be reversed with positions 5 and 6 respectively in any procedure. This is a matter of choice.

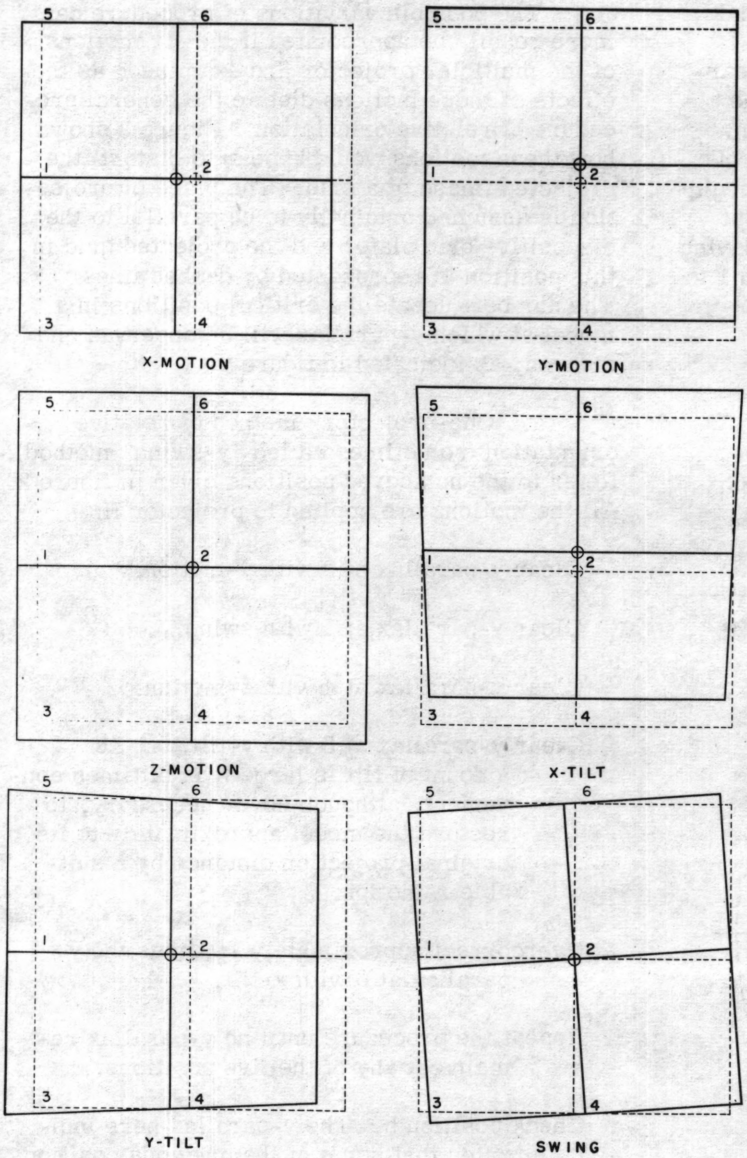


Figure 3.--Effects of the six projector motions.

In addition to these possible substitutions of positions, the order in which the motions are adjusted may also be varied to some extent. From the different combinations of these factors that are possible, the stereocompiler can perhaps choose an alternative procedure that he will prefer to the one given above. For example, an alternative that is preferred by some stereocompilers consists of adjusting the z-motion in the third step of the foregoing procedure so as to distribute y-parallax equally between positions 4 and 6. The x-tilt is next adjusted, overcorrecting the y-parallax at either 4 or 6 approximately 4 times. If these adjustments are suitably made, positions 2, 4, and 6 can be cleared of y-parallax by a readjustment of the y-motion alone. Finally, y-tilt is adjusted at position 3.

b. "Swing-swing" method of relative orientation.--Insofar as relative orientation is concerned, it is evident that identical swings applied to both projectors have the same effect as y-motion applied to one alone. Also, identical y-tilts applied to both projectors have the same effect as z-motion applied to one alone. If these substitutions are made in the foregoing procedure, a method of achieving relative orientation without the use of translational motions is obtained:

Clear y-parallax at 1 with swing of projector II.

Clear y-parallax at 2 with swing of projector I.

Clear y-parallax at 3 with y-tilt of projector II. (Follow with x-motion if desired. See step 4, one-projector method.)

Clear y-parallax at 4 with y-tilt of projector I. (Follow with x-motion if desired.)

Overcorrect ($1\frac{1}{2}$ times) the y-parallax at either 5 or 6 with x-tilt of either projector.

Repeat the procedure until all positions have been cleared of y-parallax.

9. When to use each method

When one model is being added to another already in relative orientation, as when a strip of models is being oriented in the process of stereotriangulation, it is necessary to use the "one-projector" method to orient the added projector. Otherwise it is always possible to use either method.

As the "swing-swing" method does not make use of the translational motions, the y- and z-settings of the projectors will remain unchanged during the process of relative orientation by this method. If some particular setting of these elements is desired in the ultimate model, the "swing-swing" method makes it possible to preset these values and then accomplish relative orientation without disturbing them. The common application of the "swing-swing" method is in the relative orientation of the initial model of a strip, or of any single model, wherein the projectors should be preset so that the air base is parallel to the bar. The subsequent leveling of the model will be simplified by maintaining the air base approximately level during relative orientation. The adjustments of both relative and absolute orientation are simplified by maintaining the x-tilt axes of the two projectors as nearly as possible in line with each other.

When it is permissible to use either method, the stereocompiler may, if he desires, combine parts of both methods. Thus, for example, he might use the y-motion of one projector but not the z-motion; or the z-motion but not the y-motion. This intermixing of methods is often practiced, especially when the adjustments to be made are small.

10. Possible use of positions outside the neat model

There are times when it is advantageous to observe y-parallax at positions outside the neat model, after the best solution has been obtained using the positions 1 through 6 (which have been assumed to lie on the edges of the neat model).

Positions 7 and 8 obviously offer the largest possible base for determining the swing settings. These settings can be checked most accurately by looking for y-parallax here. Positions 9, 10, and 11 offer the largest possible base for determining the relative x-tilt setting. A small error in this setting may be noticed by careful observations at these three positions.

The strongest determination of the relative y-tilt setting would seem to be made at positions outside of 3, 4, 5, and 6, but as various residual image distortions are likely to have their greatest effect in these areas it is actually necessary to take care not to observe y-parallax beyond the limits of the neat model in the corners. (Although distortion might exist at positions 7, 8, 9, 10, and 11, it would not create y-parallax. Distortion would not create y-parallax at positions 7, 8, and 10 because its direction at these positions would have no y-component. At positions 9 and 11 any distortions would be symmetrical, hence their y-components would be equal and would cancel each other in effect.)

11. Effects of errors of relative orientation

The effects on the model datum of a small error in the setting of each motion of the multiplex projector are pictured in figure 4. (These effects, as indicated by the differences between the dashed and the solid outlines, are greatly exaggerated for illustrative purposes. Assuming a maximum residual y-parallax of 0.1 millimeter, the magnitude of any effect is but a few tenths of a millimeter, excepting, of course, the effect of x-motion, which involves no y-parallax.) These errors affect elevations in the model more seriously than they do horizontal positions. The horizontal deformation resulting from a residual y-parallax of 0.1 or 0.2 millimeter is not significant in a single model.

As indicated in figure 4, errors of relative orientation affect the vertical datum of the model essentially as follows:

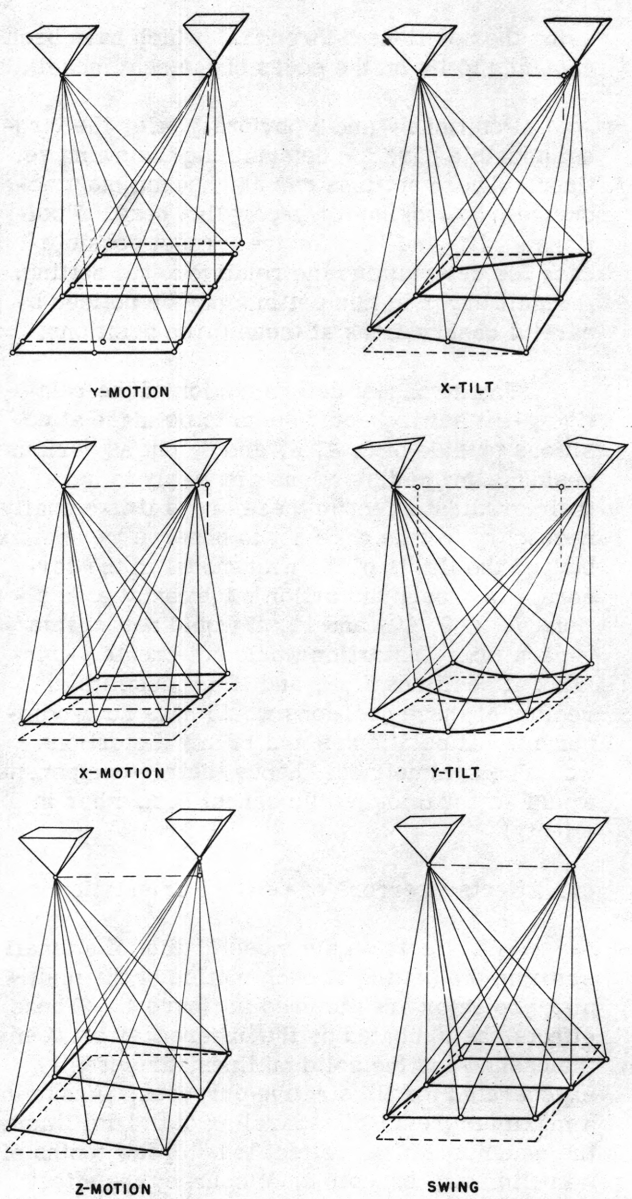


Figure 4.--Effects of relative orientation errors.

Error from y-motion.--The error has no effect upon the model datum.

Error from x-motion.--The effect of this error is to change the scale of the model. The uniform raising or lowering of the model datum with respect to the table top, or instrument datum, is offset by re-indexing the scale of the tracing table.

Error from z-motion.--The principal effect of this error is to tilt the model datum about a y-axis. As the datum remains flat, this effect is offset by the subsequent leveling of the model when absolute orientation is performed. The maximum tilt of the neat model permitted by 0.1-millimeter y-parallax in the neat model is 0.2 millimeter[4].

Error from x-tilt.--The principal effect of this error is to produce a hyperboloidal warping of the model datum. The maximum curvature of the datum occurs along the diagonals of the model. In the x- and in the y-directions the datum remains uncurved; that is, any xz- or yz-section is a straight line. Maximum warp permitted by 0.1-millimeter y-parallax in the neat model (deviation of one corner above or below a plane passing through the other three corners) is 0.8 millimeter. (See footnote 4).

Error from y-tilt.--The principal effect of this error is to produce a cylindrical warping of the model datum. The maximum curvature of the datum occurs in the x-direction, the datum remaining uncurved in the y-direction. Convergent y-tilt lowers the y-direction center-line of the model with respect to the left and right edges, as shown in figure 4; divergent y-tilt causes this center-line of the model to be raised. Maximum warp permitted by 0.1-millimeter y-parallax in the neat model (deviation of the center of the model above or below a plane passing through the four corners) is 0.1 millimeter. (See footnote 4).

Swing error.--The principal effect of this error is to tilt the model datum about an x-axis. As the datum remains flat, this effect is offset by the subsequent leveling of the model when absolute orientation is performed. Maximum tilt permitted by 0.1-millimeter y-parallax in the neat model is 0.44 millimeter. (See footnote 4).

[4] Approximate values. Computed for a base-height-width ratio of 2:3:3.

To summarize, it is seen that of the six errors only two, namely x-tilt error and y-tilt error, have a significant effect. Each of these warps the model datum in a characteristic manner, by which the error may be identified. It is also to be noted that no relative orientation error permits of any curvature of the model datum in the y-direction.

Errors of relative orientation can result not only from failure of the stereocompiler to clear y-parallax completely, but also from a photographic distortion. As the y-components of distortion at each of the four corner locations where y-parallax is observed usually differ between the two photographs of the overlapping pair, erroneous projector settings will undoubtedly result from clearing y-parallax completely at these positions. Fortunately, however, the effect of distortion in present-day multiplex projector-printer-camera systems is not appreciable in a single model.

As relative orientation is but a means to an end, the ultimate objective of all the orientation processes being absolute orientation, it is permissible to use the flatness of the model datum instead of y-parallax as a means of adjusting the x-tilt and the y-tilt settings. In fact, the ultimate settings of these two motions usually must depend on their effect on model flatness as revealed during the course of leveling. The x-tilt motion especially creates a warp of the model datum more critical than its y-parallax effect (see article 18).

12. Relative orientation problems

The terrain model is never a perfectly rectangular, plane surface, as has been assumed, and the projector axes are not often truly parallel to the x-, y-, and z-directions respectively. Yet in most cases, the actual model is regular enough to respond normally to the standard procedures of clearing y-parallax.

The attainment of a parallax-free model becomes more difficult, however, when the model presents any of the following to an extreme degree: rugged terrain, tilt of the photographs, unequal settings of the projector y-slides, or water areas.

The nature of the problem presented by each of these conditions is outlined below and a course of action suggested, but the stereocompiler must deduce the exact steps to be taken in any particular situation.

a. Rugged terrain.--The effect of topographic relief upon relative orientation procedure depends both on the magnitude of the relief and on the distribution of the areas of high and low relief within the model. As an example of this, two extreme cases are illustrated in figure 5.

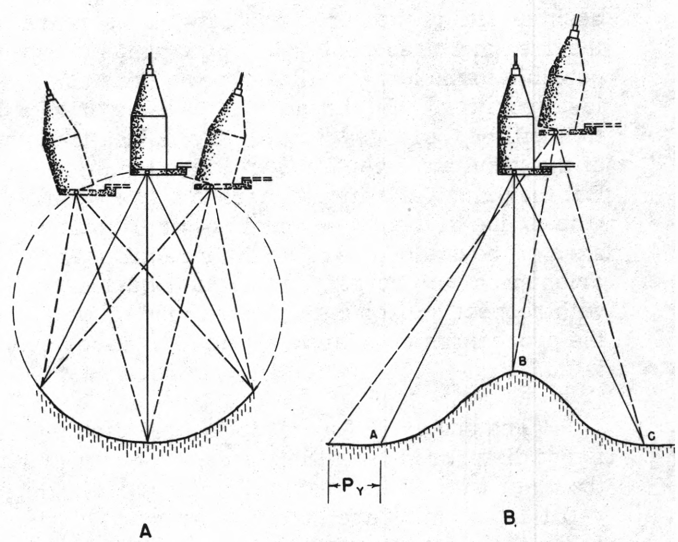


Figure 5.--Examples of the effect of relief.

Figure 5A illustrates a hypothetical model in which the terrain surface has the shape of a cylinder whose axis is parallel to the air base and is at such a distance from it that the extension of the cylindrical surface would include the air base. The dashed lines show how a projector can assume various orientations and still permit all pairs of conjugate image rays to intersect. Thus, relative orientation cannot be determined solely by the usual y-parallax effects. Unless the relative x-tilt is properly set, however, the model datum will be warped, as indicated in figure 4. In lieu of y-parallax observations, therefore, the x-tilt setting must be determined by adjusting the motions until elevations in each of the four corners of the model read correctly.

Figure 5B illustrates the opposite condition in which the terrain is at a much higher elevation directly under the flight line than it is along the front and back edges of the model. This condition increases the accuracy of the x-tilt setting as determined by y-parallax, because for a given increment of x-tilt it creates a greater than normal y-parallax differential between $\overline{AB}$ and $\overline{BC}$. For the condition shown in figure 5B the required overcorrection of x-tilt in the routine procedure would be but one-half, approximately, of the amount of y-parallax, Py.

b. Excessively tilted photographs.--If excessive tilts of the aerial photographs decrease the area of stereoscopic overlap so that it is not possible to make y-parallax observations at the ideal positions, the difficulty of relative orientation will be increased accordingly. The strength of the resultant orientation will also be diminished. But the principal difficulty caused by excessive tilts is due to the displacement of the rotational axes of the projectors from their rectilinear orientation with respect to the air base and/or with respect to each other, conditions on which the procedures of relative orientation were postulated.

The one axis of a multiplex projector that is not disturbed by rotation of the projector about the other axes is the y-axis (primary axis). The y-tilt adjustments are therefore always observed in the model at the same positions relative to the y-axes.

The x-tilt axis can become displaced so that components of swing are introduced by the x-tilt motion and the apparent overcorrection of y-parallax by this motion must be varied to allow for this. Trial and error will soon reveal whether more or less correction is required.

The swing axis can become displaced in both the x- and the y-directions, but as the swing adjustments are made prior to any other, this is not of particular consequence. If it is desired that the y-parallax cleared by the swing motions should not be reintroduced by the succeeding y-tilt adjustments, swing parallax must be cleared primarily at each nadir point rather than at the principal points. However, the swing motions (or y-motion and swing) undoubtedly will be slightly interaffecting at the nadir positions and successive trials of slight over- or under-corrections of the swing settings probably will be required.

c. Unequal settings of the y-slides.--Though it is not attributable to tilts of the aerial photographs, an excessive difference in the settings of the projector y-slides also violates the rectilinear orientation of the rotational axes with respect to the air base. The y-tilt in effect introduces components of x-tilt, and conversely, x-tilt introduces components of y-tilt. Relative orientation procedure is complicated by this condition which should be avoided whenever possible. If the air base must be set at an excessive angle to the bar, however, the logical procedure is to use, in place of x-tilt alone or y-tilt alone, a combination of the two motions calculated to produce a resultant tilt in the desired direction. Simple estimation of the proportions of each tilt motion together with a few extra runs through the procedure will be all that ordinarily is required.

d. Extensive water areas.--The orientation of photography over coastal areas often is difficult because it is not possible to see y-parallax in the water portions of the model. The least that can be done, of course, is to clear the y-parallax over the extent of the land area, but such a solution will obviously not be as strong as it would have been if images had been available over the full model for y-parallax observation. Limited land areas for y-parallax observation will thus lessen the accuracy of stereotriangulation, although they will not, as a rule, lessen the accuracy obtainable from the individual model, inasmuch as the area of the model from which detail can be compiled is limited to exactly the same extent.

Nevertheless, advantage should be taken whenever possible of any effects that erroneous relative orientation has upon the flatness of the model datum. These effects are illustrated in figure 4. If water-level readings can be taken along shore lines at certain critical positions in the model, they provide a means of readily detecting warping of the model datum.

As examples of the use of elevation readings in achieving relative orientation, figure 6 represents models wherein excessive water areas preclude the making of y-parallax observations at one or more critical positions.

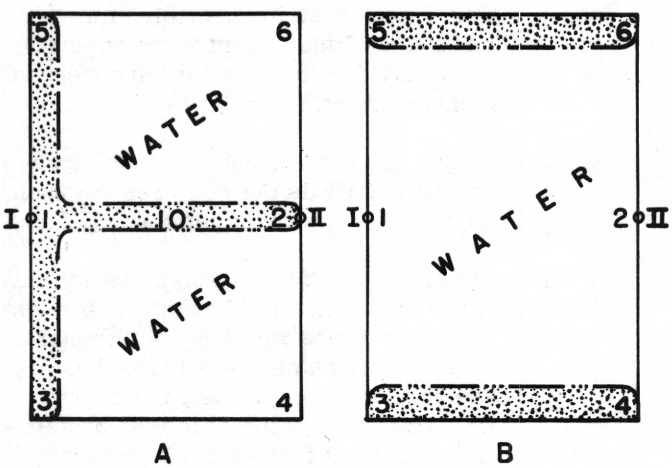


Figure 6. --Examples of extensive water areas.

Missing in the model in figure 6A is a position at which to observe the y-tilt adjustment of projector I (step 4 of the "swing-swing" method). In place of step 4, therefore, projector I should be y-tilted until shore-line readings at positions 1, 10, and 2 indicate no curvature of the model datum in this direction. (These elevations may not necessarily indicate a level datum, however.)

If projector II were to be used alone, its y-tilt adjustment would precede rather than follow the adjustment of the z-motion. Step 3 of the "one-projector" method would then be the adjustment of the y-tilt of projector II using the three shore-line readings across the width of the model as described above. Step 4 would be the z-motion adjustment, observing y-parallax at position 3 (or 5) instead of position 4 (or 6).

Missing in the model in figure 6B are positions at which to observe the swing adjustments. A procedure similar to this might be used:

Swing projector II until the y-parallaxes at 5 and 3 are symmetrical with respect to the x-axis of the model.

Swing projector I until the y-parallaxes at 4 and 6 are symmetrical.

Clear y-parallax at 3 by y-tilt of projector II. This should simultaneously clear y-parallax at 5.

Clear y-parallax at 4 by y-tilt of projector I. This should simultaneously clear y-parallax at 6.

Then x-tilt either projector until shore-line readings at positions 3, 4, 5, and 6 indicate no warping of the model datum (elevation difference between 3 and 5 equals elevation difference between 4 and 6).

Repeat the procedure until the model datum is flat and until at the same time no y-parallax remains at any position.

## ABSOLUTE ORIENTATION PROCEDURE

### 13. Preliminary steps

a. Identification of horizontal control. -- The horizontal control to which the model will be positioned may have been identified in various ways[5]. Positions that have been established by ground surveys either are marked on the faces of the contact prints, or are identified solely by descriptions of the points. In the latter case, the identification of several points in a group (as, for example, stations along a traverse line) can often be verified by the way in which, as a group, they correlate with their plotted positions.

Horizontal pass points, or supplemental positions established by stereotriangulation[6] for controlling individual models, will have been plotted on the base sheet either as actual points or as lines delineating small portions of planimetric features. When the stereotriangulator has used points, he will have marked the images on the faces of the contact prints; when he has used lines, identification of the images will usually be self-evident once the general locations of the features are known. The stereotriangulator

[5] See Chapter 2 F 1, Supplemental control planning.

[6] See Chapter 3 C 7-d, Multiplex stereotriangulation.

may have encircled these general areas on the prints; or if the principal points are included among the pass points--and they should be--these usually serve to position the model well enough that identification of the other pass points can readily be made. The points selected for horizontal pass points will not necessarily be the same ones used for vertical control, although both should be located approximately at each corner of the neat model.

b. Identification of vertical control. --Supplemental control points are usually circled on the faces of the contact prints, each point being designated by a letter. Brief descriptions and the elevations of the points are shown on the back of the prints[7].

c. Conversion of elevations. --Prior to leveling a model, all control elevations must be converted to millimeter values unless the multiplex tracing table is one of the newer types capable of direct readings in feet. Millimeter conversions are most conveniently obtained if a table is prepared beforehand to suit the plotting scale and the range of elevations anticipated.

14. Establishing scale and horizontal position

The scale of the model now in relative orientation is governed solely by the length of the air base between projectors. To scale a model, therefore, one of the two projectors must be moved in a direction parallel to the air base. (If the air base is not parallel to the direction of x-motion, appropriate amounts of y-motion and/or z-motion must be included in order that relative orientation of the model will not be disturbed.) The model can then be shifted bodily until it rests in its correct position over the control. This can be done either by positioning the model with respect to the base sheet or by positioning the base sheet with respect to the model. Although establishing scale and establishing horizontal position can thus be conceived as separate operations, with scaling being prerequisite to positioning, the two operations are combined in one procedure inasmuch as the control plotted on the base sheet must be the basis for determining both scale and position.

[7] See Chapter 2 F 1, Supplemental control planning.

a. Base sheet not moved.--If it is desired that the base sheet not be moved, the procedure is as follows:

(1) Select two horizontal control positions that are widely separated within the model. For convenience in describing this procedure, call them point a and point b. Place the floating mark at the observed height of point a.

(2) Without changing this height, move the tracing table until its pencil is resting upon the plotted position of point a.

(3) Move both projectors equally in x and equally in y until the image of point a is brought into coincidence with the floating mark. (The x-motions can be equalized by keeping the height of the point constant; the y-motions can be equalized by keeping y-parallax from being introduced into the model.)

(4) Move the tracing table to point b, note the discrepancy between the model position and the plotted position of point b.

(5) Estimate the component of this discrepancy parallel to a line between a and b as a proportion of the total distance between the points. Shrink or expand the model as required by altering the length of the air base in similar proportion. (An x-motion will have to be accompanied by small components of y-motion and/or z-motion if the air base is not parallel to the multiplex bar.)

(6) Repeat steps (1), (2), (3), and (4).

(7) This time, estimate the component of discrepancy that is perpendicular to a line between a and b. This is a measure of the horizontal angle through which the model must be rotated. Swing one projector through this angle.

(8) Using motions of the other projector to clear the y-parallax thus introduced, regain relative orientation. (Ordinarily only a y-motion and a swing motion will be necessary.)

(9) Repeat the entire procedure if necessary until the model fits both control points at once. The model will satisfy all horizontal control points when it is scaled and positioned to any two, provided that the model is approximately level and that there are no other errors. Read the elevation of a vertical control point in each corner of the model. If the model is level within a millimeter, the orientation is sufficiently close for planimetry; if not, level the model approximately, as described in article 16, and repeat the entire scaling and positioning procedure. If the model is being oriented for contouring, level accurately, rather than approximately, before bringing the model to its final scale and position.

The model can be positioned very conveniently, if only tentatively, by using the two principal points, which were plotted when the strip was stereotriangulated, as points a and b in the above procedure. The model should, of course, be finally positioned upon a mean of the pass points and other control regardless of whether this position satisfies the principal points or not.

b. Base sheet moved.--Positioning is somewhat easier when the base sheet can be moved. The above procedure is followed with two exceptions:

(1) In step (3), instead of moving the projectors, slide the base sheet, and with it the tracing table which is set over point a, until the floating mark is returned to coincidence with the image of point a.

(2) In place of steps (7) and (8), set the floating mark first at the observed height of point b, then moving the tracing table without changing this height, set the tracing pencil upon the plotted position of point b. Pinning the base sheet to the table with one finger over point a, rotate the base sheet, moving the tracing table with it, until the floating mark is brought as closely as possible into coincidence with the image of point b.

c. When to use each method.--It is occasionally necessary, for various reasons, to set up two nonconsecutive models on the same frame at the same time. When this must be done, at least one model must be positioned and leveled entirely by movements of the projectors alone. Ordinarily, however, it is permissible both to move the base sheet and to tilt the frame by reasonable amounts, and it is not only somewhat simpler, but actually preferable to move the projectors as little as possible after they have been approximately placed in a convenient working position. If the parallelism between the air base and the multiplex bar, which was purposely established at the outset of relative orientation, is to be retained through the process of absolute orientation, any rotating of the model about a vertical axis with respect to horizontal control must be done by moving the base sheet.

15. Difficulty in positioning a model

Occasionally a model will be encountered which, because of one error or another, cannot be made to fit satisfactorily upon all the pass points and control positions. Unless the error is so large as to suggest a need for re-stereotriangulation, it may be advisable to divide the model into segments, and to compile detail from one segment at a time, shifting the position of the model slightly for each segment. This highly practical procedure of "breaking" the model, as it is commonly called, can also be applied with respect to the level of the model during the contour compilation. It disposes most easily of small errors which accrue despite the best efforts of everyone to build and to operate equipment without errors; however, it should never be used as a justification for relaxing these efforts.

The position or positions that are finally decided upon should be marked well by additional or supplementary pass points for use in checking the position of the model during the compilation process.

16. Leveling the model

Leveling is accomplished in two steps, one being leveling in the x-direction and the other leveling in the y-direction. The model may be leveled in either direction first. Two methods of leveling also are available to the stereo-compiler, who may choose either or combine the two. Method one utilizes only the motions of the projectors, the multiplex frame remaining stationary. Method two utilizes only the motions of the frame, the projectors moving in space inasmuch as they are attached to the frame and must move with it.

a. Leveling in the y-direction, using the projectors. --A vertical control point will ordinarily be located in each of the four corners of the model (positions 3, 4, 5, and 6 in figure 2).

Index on any one of the four points. Assume point 5 is used.

Read the elevation of point 3 and note the error. This may be confirmed by reading the elevations at 6 and 4.

Then x-tilt both projectors equally until further readings of the elevations indicate no slope of the model datum in the y-direction.

A suggested method of estimating the proper amount to tilt the projectors in the above procedure is:

Set the floating mark either on point 3 or 4 or on any point on a line between the two.

Raise or lower the floating mark (according to whether the readings indicate the front of the model needs to be raised or lowered) an amount equal to one-half the error (at point 3 with respect to point 5). Then, when x-tilting the projectors in the third step above, tilt them until point 3 is seen to arrive at the height of the floating mark. As point 5 changes equally but in the opposite direction, the whole error should thus be tilted out.

The two x-tilt motions can be temporarily equalized by clearing with one the y-parallax introduced by the other. The ultimate settings of each projector in x-tilt, however, will probably be dependent on flattening and leveling the model datum, rather than on y-parallax observations (see discussions of warped models in articles 11 and 19).

If the x-tilt rotations of the two projectors are not about a common axis, they will disturb some of the other elements of relative orientation slightly, causing y-parallaxes of these elements to appear. These must be removed by reverting temporarily to the procedures of relative orientation.

b. Leveling in the x-direction using the projectors. --Referring again to figure 2:

Index on any one of the four elevations. Assume point 3 is used.

Read point 4 and note the error. Assume this indicates the right side of the model is too high with respect to the left side.

Lower projector II (or raise projector I) with z-motion an amount equal to the error, measuring its movement on the projector z-scale. (The z-motion of a projector can also be gauged by measuring with the tracing table the accompanying rise or fall of a point in the model under the opposite projector.)

Regain relative orientation by clearing the y-parallax thus introduced, following the "swing-swing" method. Ordinarily use of only the two y-tilt motions will be necessary.

Repeat the procedure if necessary until the model is satisfactorily leveled.

c. Alternative procedure of leveling in the x-direction, using the projectors. --The equalization of the two y-tilt motions and their proper coordination with the z-motion are often difficult when they are dependent solely on the recovery of the original y-parallax solution. There are alternative procedures that make use of elevation readings to achieve this coordination. For example, assume the right side of the model is again too high:

Select some well defined image point near position 1 and observe its elevation with the tracing table.

Next y-tilt projector II until the point is raised 13/4 the amount by which the right side of the model was in error.

Then y-tilt projector I until the point is brought down 9/4 the amount of the original error.

Lower the point the remaining 4/4 to its original height with the z-motion of projector II.

The values in this procedure are based on a B/H ratio of 2/3.

Except for the introduction of a small amount of y-parallax, a small z-motion of one projector alone has the effect of simply tilting the model datum slightly in the x-direction as shown in figure 4. Because the amount of y-parallax introduced by a z-motion of 0.1 or 0.2 millimeter is not objectionable, a simple z-motion of one projector often affords a practical means of removing a residual slope of this magnitude in the x-direction when the projectors must be used and when difficulty is encountered in coordinating the three usual projector adjustments because of their small magnitude.

d. Leveling with the frame. --Because leveling a model by means of the frame in no way disturbs the relative orientation of the projectors, it is generally considered the simpler of the two methods.

Loosen the lock screw or lock nut (if the frame is of a type having a pivoted junction) to obviate the possibility of a twisting stress being put on the bar.

Read elevations as before to determine the slope of the model datum in the x- and y-directions.

Tilt the frame in the y-direction as required, using equal turns of the two front foot screws or equal turns of the two back foot screws.

Tilt the frame in the x-direction as required, using either handwheel. Instead of the handwheels, the two foot screws at either end of the frame, preferably the pivoted end, however, could just as well be used, turning both equally. Where "A"-frame end uprights are used, the foot screws are the only means of tilting the frame, in either the x- or the y-direction.

With a little experience, it is easy to estimate the number of turns of the foot screws or handwheels required to raise or lower one edge of the model 0.1 millimeter with respect to the opposite edge. The required number of turns is dependent upon the dimensions of the frame and upon the pitch of the screw threads, as well as upon the dimensions of the model. It is in no way dependent upon the distance of the model from either end of the frame. The inexperienced stereocompiler will find it helpful to calibrate and mark the handwheels and foot screws in terms of this 0.1 millimeter differential across an average model.

e. Leveling with the projectors vs. leveling with the frame. --When two non-consecutive models must be set up on the same frame at the same time, one model, at least, must be leveled using the projector motions exclusively. Otherwise, the choice between using the projectors or using the frame for leveling is a matter of the stereocompiler's preference. In any case, it will generally be found expedient to use the projectors for the larger initial adjustments in order to avoid tilting the frame excessively.

17. Effect of leveling on the position of the model

Aside from the angular rotation of the model, which is the purpose of the leveling operation, the total movement of the model in space in the x-, y-, and z-directions depends upon the position of the axis of rotation with respect to the model. If this axis is considerably to one side, as it is when the frame is used for leveling in the x-direction, then the angular change will be accompanied by a considerable vertical movement of the whole model. This is compensated by the re-indexing of the tracing table scale, which must be done in any case after each leveling operation and before each rereading of the control elevations.

If the axis of rotation is much above the plane of the model, as it is when the projectors are used, leveling will be accompanied by a considerable horizontal shift of the model, making it mandatory that the leveling operations, when the projectors are used, precede the final positioning of the model.

18. Warped models

A model datum that is not flat obviously cannot be leveled. It was shown in article 11 that only two errors of relative orientation affect the flatness of the model datum, y-tilt error and x-tilt error. Of these, x-tilt error is the more difficult to avoid; a model that can be leveled upon all four corner control elevations without further adjustment of the relative x-tilt being the exception rather than the rule. As an x-tilt motion of one projector has its maximum effect upon the elevations along the opposite side of the model and has a minimum effect on the elevations along the side directly under the projector (see fig. 4), projector I in the procedure of article 16 (a) should be x-tilted primarily according to the indications of elevations 4 and 6, while projector II should be x-tilted primarily according to the indications of elevations 3 and 5.

19. Difficulty in leveling a model

Sometimes a model cannot be made to agree exactly with all of the vertical control. The procedure in such a case depends on the cause of the difficulty, of which there are these possibilities:

(1) Misidentification of a control point.

(2) An error in copying elevation figures.

(3) An imperfect clearing of y-parallax.

(4) An image distortion of some sort.

(5) An error in the control itself.

Possibilities (1) and (2) should be eliminated first by checking the identification of all points and by checking all figures, especially the conversions of feet into millimeters.

If the errors of the multiplex readings on the vertical control suggest a warp of the model datum of a kind that might be due to an error of relative orientation, the stereocompiler should eliminate this possibility by further adjusting the relative tilt settings of the projectors in an attempt to flatten the datum. It is seldom possible, however, to curve a model datum more than 0.1 or 0.2 millimeter by y-tilt, or to warp it more than 0.4 or 0.5 millimeter by x-tilt without introducing objectionable y-parallax. If greater errors than these exist, or if their pattern is irregular, the control elevations themselves should be suspected of error.

Although the control elevations may be suspected of error, it is also possible that image distortion may be the source of the difficulty. The stereocompiler cannot, therefore, use the model implicitly to prove the existence of an error in the control. If he has access to the field survey notes he should check them. If no error is discovered there, it is suggested that he try setting up other models using different exposures of the same flight. Similar difficulty in leveling these models would definitely point to image distortion as the source of error; a reflight should be obtained if possible. When the difficulty is an isolated instance, however, one of the following courses of action must be decided upon:

(1) Suspend compilation until the suspected elevations can be field checked.

(2) Determine and use a multiplex elevation.

(3) Accept the control elevations and force the model to fit them.

If a multiplex elevation of the disputed point is to be established and if the point lies along an edge or at one corner of the model, the multiplex elevation should be established by setting up adjoining models and determining the elevation of the point from all the models in which the point is visible.

If forcing the model to the given elevations is decided upon, there are two possible methods.

One is to divide or "break" the model into two or more triangular segments bounded by control, and to level and contour each as one would separate models. The other method is to "establish a local index." This is an especially convenient method for compensating small random errors in closely grouped points. The procedure is simply to index the scale or counter of the tracing table upon one control elevation, and then adjust it gradually to the index of adjacent control elevations as the contouring progresses toward them. As a rule, the stereocompiler will not continually reset the index, but will simply add or subtract small increments from each contour setting.

## PROGRESSIVE ACHIEVEMENT OF EXTERIOR ORIENTATION

### 20. Effect of repetition of orientation procedures on degree of refinement

In general, the operations of exterior orientation are performed in the sequence in which they have been described. It is seldom possible, however, to make any setting of y-parallax, model position, or model level with positive assurance that it will not be affected by some succeeding adjustment. If at any point in the procedure of relative or absolute orientation, any of the preceding settings appear to have been disturbed too greatly it is advisable to stop immediately and revert to the earliest setting affected and repeat the procedure from that point on. It is better still, many stereocompilers feel, to go through the entire procedure of exterior orientation two or three times, quickly at first, then more carefully, bringing a higher degree of perfection to the orientation with each repetition. As an example, it is suggested that a rough orientation be made first, which will clear y-parallax (without the use of filter spectacles) within 1 or 2 millimeters, level the model within 3 or 4 millimeters and position it within 5 or 6 millimeters. A second, improved orientation will clear y-parallax within 0.5 millimeter, level the model within 1 millimeter and position it within 2 millimeters. These are very general values, given only to suggest an approximate degree of refinement for each repetition. The stereocompiler will take but a few minutes to perform these rough orientations. The adjustments of the final, precise repetition should then be small enough not to interaffect each other noticeably, and all settings will be made as accurately as possible.

GEOLOGICAL SURVEY

TOPOGRAPHIC INSTRUCTIONS

Chapter 3C 7-d

# MULTIPLEX STEREOTRIANGULATION

Contents



Illustrations

Chapter 3 C 7-d[1]

# MULTIPLEX STEREOTRIANGULATION

## ABSTRACT

Multiplex stereotriangulation affords a means of establishing supplementary horizontal and vertical control points when relatively few ground survey control points are available. Horizontal bridging (stereotriangulation between control points at both ends of a strip) is accomplished by (1) orienting an initial model accurately to given control, (2) bridging successive models, each to the preceding model, and (3) adjusting the scale of the entire strip so that the scale is uniform and the set-up fits all the horizontal control. When the bridge is complete, pass points are selected and marked; when two adjoining parallel strips are bridged, the pass points common to the two strips are adjusted to give a single solution. Vertical bridging is accomplished by (1) establishing a horizontal bridge, (2) reading the elevations on pass points and ground survey control points, (3) plotting the BZ curves, and (4) adjusting the multiplex elevations according to the BZ curves. Because of the more rigid requirements for vertical control, vertical stereotriangulation is not used as extensively as horizontal stereotriangulation. Extension of control by stereotriangulation beyond ground survey control is readily performed, but there is no way of checking the accuracy of the extended portion and the method is therefore used only when there is no other alternative. Other procedures for multiplex stereotriangulation include a leap-frog method using only two projectors, and the use of auxiliary information or devices such as cross flights, altimetric data, tilt data, and shoran control.

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## HORIZONTAL BRIDGING

### 1. Purpose of horizontal bridging

Photographic coverage of large areas is supplied in the form of parallel flights or strips. Each strip must be bridged between the lines of horizontal control that are usually placed crosswise to the flight direction at certain intervals. The exact number of models necessary to span the control interval depends on the ground distance between control and on the flight height of the photography.

When each strip is bridged, pass points are plotted in the common side-lap area between adjoining strips. Finally, the pass points are adjusted; that is, a unique position for each pass point is determined wherever slight discrepancies between adjoining strips are indicated. The adjusted pass points are to be used for the subsequent repositioning of each model for the compilation of detail.

### 2. Planning the procedure

One of the first decisions the stereotriangulator must make is the order in which the several strips will be set up. The factors that enter into his decision are the amount and distribution of control and the spacing and arrangement of the

[1] The number 3 C 7-d signifies Book 3, Part C, Chapter 7-d of the Geological Survey loose-leaf manual of Topographic Instructions. For a table of contents, see Chapter 1 A 2 (Circular 92).

flights. Preferably those strips having the most control, or the most positively identifiable control, should be oriented first. A study of the control plan and the flight index usually suggests a logical procedure.

The next decision to be made is which model of the strip to orient first. Preferably, it should be a model near the center of the strip so that the bridging of successive models can proceed in both directions from it. In this way, any progressive accumulation of error in the absolute orientation of successive models will be kept at a minimum. It is also highly desirable that the initial model be one with sufficient control so that it can be completely and independently leveled, scaled, and positioned at the outset. As the horizontal control is usually confined to the end models, it is often more practical to commence orientation at either end and to proceed through the strip entirely in one direction. A compromise procedure that is sometimes advisable is to assume a scale for the initial central model, correcting this scale later when the bridge has progressed to the nearest controlled model. Once one strip has been scaled, its pass points can be used to furnish the initial scale and position of any model of the adjoining strips.

3. Orientation of the initial model

a. Short strips.--For strips of moderate length (7 or 8 models), both projectors of the initial model should be set at the mid-points of their y-slides and also at the mid-points of their z-slides. This is done to allow for a maximum deviation of the flight line from a straight line in any direction. Relative orientation of the model is completed by the "swing-swing" method.

b. Long strips.--Setting the azimuth of the initial model of long strips (14 or 16 models) so that the flight line will not "run off the bar" often requires careful attention. The range of the z-motion is not as likely to be exceeded as the range of the y-motion, for the altitude of an airplane can be maintained constant more easily than can its course. Before setting up the initial model of a long strip it is suggested that the base sheet be oriented in azimuth as follows:

Estimate the position of the principal point of one of the end photographs with respect to the horizontal control and spot this position approximately on the base sheet.

In the same manner spot the approximate position on the base sheet of a principal point at the opposite end of the strip.

Position each end of the sheet in the y-direction so that each of these positions would fall directly below a projector that has its y-slide set at the mid-point.

The initial model should now be oriented and positioned without moving the base sheet. This model must, of course, be one with sufficient control or pass points to enable its absolute orientation.

4. Accuracy of leveling required

It is not essential that the initial model be precisely leveled. Even if it were to be carefully leveled, it is unlikely that the succeeding models would maintain level datums. The probability is that the datum of each model would show a rather consistent downward "break" from the preceding one, so that the datum of the whole strip would be arched upward in the center. This vertical bow is chiefly the result of a small residual lens distortion and is the most persistent deformation of a multiplex strip.

The only requirement of leveling is that no model of the strip be out of level more than the limit required for good planimetric accuracy which, depending on the amount of relief in a model, may be as much as 10 millimeters across the width of a model. Because of the severity of vertical bow in some instances, it is recommended, especially if the initial model is not a central model, that the multiplex frame be used to level the strip as a whole between vertical control at either end of the strip. This should be done after all the models have been brought to a uniform and approximately correct scale.

5. Bridging successive models

When the initial model has been oriented as well as possible with whatever control is available, the adjoining models are successively brought into relative orientation by the "one projector" method. In each case, motions are applied only to the added projector, which forms a model with one of the projectors of the preceding model. The orientation of preceding models is not disturbed. After each model is cleared of y-parallax, and before the relative orientation of the next one is begun, the newly oriented model should be brought to the same scale as the preceding one by the following procedure, commonly called "bridging." (In its larger and more appropriate sense, the term "bridging" designates the entire procedure of stereotriangulation between control.)

Viewing first the initial model (whose scale is not to be changed), select some well-defined image point common to both models, the elevation of which can be accurately read. This point should be as near as possible to the center of that area which is overlapped by both models; it will normally be close to the principal point of the center photograph. Place the floating mark at the elevation of this point.

View the second model and adjust the length of its air base by an x-motion (and components of y-motion and/or z-motion if needed) of the third projector until the elevation of the point is made equal with its elevation in the first model, as indicated by the floating mark, which is not moved.

In this manner all the models in the strip are bridged, each in turn, to the preceding one.

6. Adjusting the scale of the strip

Because a very small error in the scale of the first model will accumulate as a much greater error in the long extension of several models, it is likely that, even though the initial model was most carefully scaled, the length of a strip will need further adjusting in order to fit between the control at the ends. This adjustment of the overall scale of a multiplex strip is similar to the method of scaling an individual model, with the exception that the adjustment must be apportioned among the several models, as described in the following procedure:

(1) Select one well defined control point in each end model. For convenience in describing this procedure, call them point a and point b.

(2) Set the floating mark at the observed height of point a, then move the tracing table and place its pencil over the plotted position of point a.

(3) Slide the base sheet and the tracing table with it until the floating mark is returned to coincidence with the image of point a. Point a is now directly over its plotted position.

(4) Set the floating mark at the observed height of point b, then move the tracing table and place its pencil over the plotted position of point b.

(5) Pinning the base sheet to the table with one finger over point a, rotate the sheet and the tracing table with it until the floating mark is in line with points a and b. Note the discrepancy then remaining between the model position and the plotted position of point b.

(6) Divide this discrepancy by the number of model widths between a and b. Lengthen (or shorten, as required) the air base of one model by this amount. Presumably the model containing point a will be used, although any model could be used.

(7) Rebridge all the models, bringing them to the scale of the model adjusted in the previous step.

(8) Repeat the procedure if necessary until the strip is scaled between point a and point b.

The adjustment in step (6) above is extremely critical and any method of increasing its accuracy is worthwhile. A change in the length of an air base probably can be most accurately gaged by measuring the resultant vertical movement of the model. An alternative procedure to step (6) is: Viewing any well-defined image point in the model, change the air base by an amount such that the point is raised or lowered, as required, an amount equal to Δz, which is given by the expression

$$\Delta z = \frac{H \times E}{L} \qquad (1)$$

where E is the error in length at point b; H is the average projection distance; and L is the distance between point a and point b.

By reference to figure 1, Δz can also be determined graphically as follows: From the intersection of the arc representing the number of model widths between a and b with the arc representing the average air base length, follow a radial line toward the origin of the graph until it intercepts the known value of the total error, E, on the abscissa scale. Read the datum adjustment, Δz, at this point on the ordinate scale.

7. Deformations of the strip datum caused by relative orientation errors

The cumulative effect upon the datum of the strip of a recurring error in each of the elements of relative orientation, including x-motion, is shown in figure 2. (Although x-motion is not strictly an element of relative orientation, it is included because an error of its setting results in a bridging error, the effect of which is similar to that of z-motion error.) It should be clearly understood that these deformations of the strip datum refer to the way in which the control points of the strip fit upon their plotted positions; they do not refer to the actual configuration of the terrain surface, nor do they have any connection with the shape of the strip as it is governed by the path of the airplane in flight.

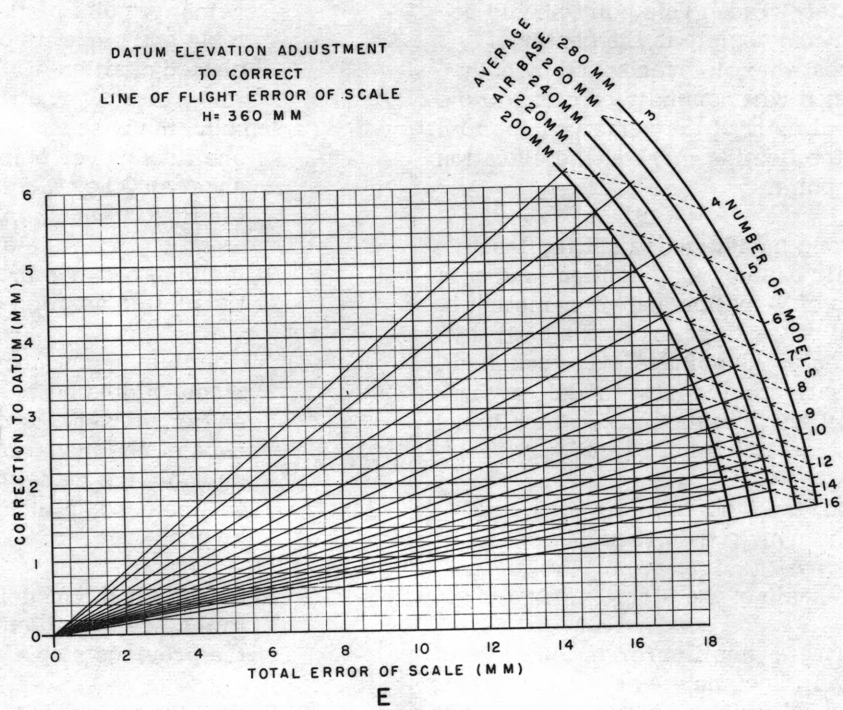


Figure 1.--Scale correction by adjustment of datum elevation.

The magnitudes of the deformations, as indicated by the differences between the solid and the dashed outlines, are greatly exaggerated in figure 2. The actual magnitude of a particular deformation depends upon three factors: The magnitude of the causative error; the rate of recurrence of the error in the strip; and the number of models in the strip.

a. Vertical deformations.--The principal errors affecting the vertical datum of the strip are x-tilt error and y-tilt error.

Error in x-tilt causes the datum of the strip to "corkscrew" or "twist"--that is, to slope progressively more and more in the y-direction. This slope reverses its trend across

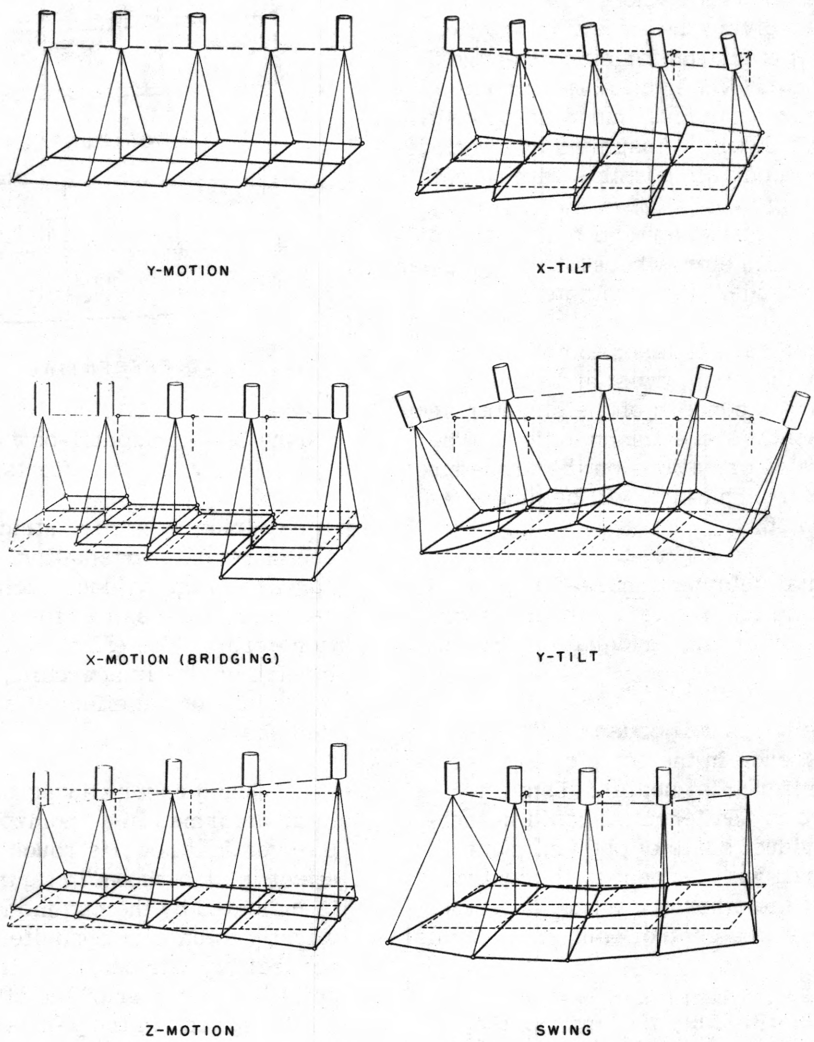


Figure 2.--Cumulative effects of relative orientation errors.

the width of each model, but the twisting shear (called cross-tilt) between successive models overbalances this reversal. Assuming every model were warped the maximum possible with a limit of 0.1 millimeter of y-parallax in the neat area, the change in the y-slope of the strip would average approximately 0.8 millimeter per model[2].

Error in y-tilt causes "vertical bow". The datum of the strip as a whole bows up or down in the center giving the effect, in profile, of a number of short chords forming an approximately circular arc. The negligible curvature of the datum of each individual model is opposite to that of the strip datum. Assuming every model to be warped the maximum possible with a limit of 0.1 millimeter of y-parallax in the neat area the middle ordinate of the vertical bow is approximately 0.05 $N^2$ millimeter, where N is the number of models in the bridge. (See footnote 2.)

If reasonable care is taken to obtain an average level for the strip, twist and vertical bow effects are not a concern of the stereotriangulator doing horizontal stereotriangulation. The influence of these deformations on the procedures of vertical stereotriangulation will be discussed in articles 15 and 16.

b. Horizontal deformations.--The principal errors affecting the horizontal datum of the strip are: z-motion error, bridging error, and swing error.

Error in z-motion will cause a slight but progressive difference in the scale of each successive model (differential scale). This effect is shown in figure 3. Differential scale is indicated by the individual scale of one end model appearing too small with respect to its control while the scale of the other end model appears too large. If there is a control point in the center of the strip, differential scale will cause a discrepancy there in the x-direction. Assuming a limit of 0.1 millimeter of y-parallax in every neat model, the maximum amount a point in the center of the strip could be in error is 0.033 $N^2$ millimeter, where N is the number of models in the strip. (See footnote 2.)

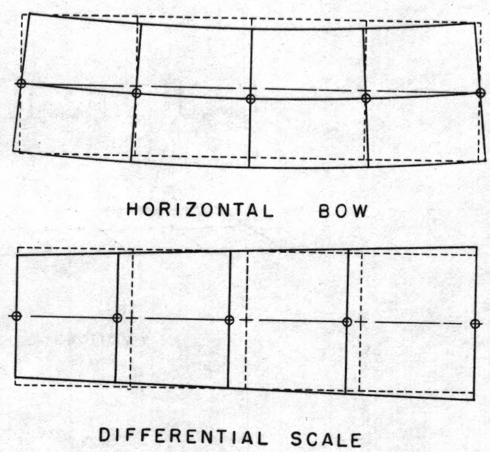


Figure 3.--Horizontal-bow and differential-scale effects.

Bridging error, strictly speaking, is not one of relative orientation. However, a repeated "break" in the bridge, such as shown, will have essentially the same effect as a repeated z-motion error. The effect of a 0.1-millimeter break in each bridge is approximately one-fourth the magnitude of the effect of z-motion error specified above.

Swing error causes the datum of the strip to be deformed in a "horizontal bow", which, like vertical bow, is roughly circular. This effect is also shown in figure 3. Horizontal bow is indicated by the end models appearing to be slightly swung, in opposite directions, with respect to their control. If there is a control point in the center of the strip, bow will cause a discrepancy in the y-direction there. With a limit of 0.1-millimeter of y-parallax in the neat area of every model, the maximum possible middle ordinate of the horizontal bow is approximately 0.025 $N^2$ millimeter, where N is the number of models in the strip. (See footnote 2.)

[2] These values are based on a base-width height ratio of 2:3:3. They are approximate, being intended only to give a basis for estimating the magnitude of the deformations. The possibility that a residual y-parallax of 0.1 millimeter (or an equivalent image distortion) might recur in the most unfavorable pattern in every model of a strip must be taken into consideration.

An error in y-motion has no cumulative effect. There are, therefore, but four different deformations of the multiplex strip: two vertical--twist and vertical bow; and two horizontal--differential scale and horizontal bow.

8. Procedure when horizontal deformations are evident

There is no reason to suppose that errors on the part of the stereotriangulator in orienting and bridging the models will consistently be of a kind that would accumulate to produce an appreciable horizontal deformation, although such is not impossible. One known cause of both bow and differential scale is tangential distortion in either the camera lens or the printer lens. Every precaution is taken in the assembly of these lenses to prevent this distortion; nevertheless it is occasionally encountered. Therefore, when the strip has been scaled between two points, all the other horizontal control points that can be seen in the strip should be examined, looking particularly for indications of a consistent trend in any discrepancies. If the strip does not agree with all the control, but appears to be deformed in either or both of the patterns of figure 3, it is suggested that the stereotriangulator eliminate his orientation as a possible cause by testing each model for the presence of those residual y-parallaxes and/or errors of bridging that might be capable of causing the particular horizontal deformation that is evident. Assuming that the relative orientation and bridging operations were done to the best of his ability in the first place, the stereotriangulator must do more than simply take another look at the models. He must test them by actually moving each projector very slightly, being desirous of accumulating a small amount of swing, z-motion, or bridging error in a direction calculated to oppose the observed deformation of the strip. By rocking each motion through the range within which he can see no change in the y-parallax (because of the limitation of visual acuity), he can adjust each motion just short of the point where y-parallax commences to be discernible. What he achieves is an orientation of the strip that is very slightly prejudiced in a manner tending to offset the effect of the error. This procedure should not be allowed to introduce any noticeable y-parallax, nor to overcompensate the deformation, although undercompensation may have to be accepted. If some deformation still remains, it can safely be attributed to a distortion beyond the stereotriangulator's control. If there had been any intermediate control points, bow and differential scale errors might have been minimized by "breaking" the strip, that is, by scaling it in two or more segments using each intermediate point as the end of a separate bridge rather than simply as a pier in a longer bridge. Finally, the procedure of adjusting the pass points to correct for differences between adjacent strips must be relied upon to eliminate any remaining error.

9. Selecting and marking pass points

When the strip is scaled and the optimum-control solution has been obtained, pass points are selected and plotted on the base sheet. These points should be located in the corners of each model and at such other positions as are necessary to provide points in the corners of the models of the adjoining strips. It is best, therefore, to select these points after an examination of the photographs of all these strips. Pass points may be actual points such as lone trees or intersections of lines, or they may be carefully drawn outlines of well-defined features such as sharp bends in roads or streams. The principal point of each photograph should also be plotted at this time. These are useful if the models are not to be preserved intact and so have to be reoriented later for compilation of detail.

Pass points should be plotted and drawn with utmost care. Although the absolute elevations of the points may be far from correct, the floating mark must be placed exactly on the model surface when plotting or checking the position of any point. If this is not done, the perspective nature of the image projection will result in an erroneous position being marked.

10. Adjustments between adjoining strips

All the pass points except those around the perimeter of a project will have their positions determined from two adjoining strips, and small discrepancies in these duplicate positions are likely to be noted. These discrepancies should not be large--their magnitude depending more or less on the length of the strip--but nevertheless a unique position for each pass point must be decided upon. A median or simple average position should be marked unless one strip is felt to be stronger than another owing to the distribution of control, or unless an analysis of the discrepancies between several adjoining strips suggests a more plausible adjustment. In such cases, the position finally marked for each pass point must be a weighted average.

## VERTICAL BRIDGING

### 11. Orientation of the models

Whether the purpose of multiplex stereotriangulation is to establish supplemental elevations or supplemental horizontal positions, the initial steps of orienting the models, leveling the strip, and bringing it to scale are much the same.

In relative orientation, the ultimate accuracy of vertical stereotriangulation is dependent chiefly on the precision and uniformity with which this operation is performed; therefore, no attempt should be made to bias the y-parallax solution because of horizontal bow or differential scale. Nor should the y-parallax solution be biased in any attempt to flatten a vertical deformation of the strip datum. Slight vertical deformations are virtually unavoidable and any attempt to regulate them would undoubtedly require the introduction of visible amounts of y-parallax into each model, a procedure that cannot be controlled with as much precision as the complete removal of y-parallax. The correction of these errors must be done by analytical means.

In leveling a strip, the very presence of vertical bow makes anything but an approximate leveling impossible. The analytical method of eliminating errors due to vertical bow and other deformations compensates at the same time for the manner in which the strip was leveled. However, for best results, the average slope of the strip datum, both from end to end and from front to back, should be made as nearly level as possible.

In scaling the strip, it is not necessary that a perfect fit be obtained between the two end positions, point a and point b. The range of relief will be relatively small compared to the overall length of a strip, so that an error in length of 2 or 3 millimeters would seldom be responsible for as much as 0.1 millimeter of error in any elevation reading.

### 12. Reading the elevations

The elevations to be read (vertical pass points) are selected in the same general locations as the horizontal pass points; thus they will include one in the corner of each model plus any other points needed to control the corners of the models of adjacent strips. Points to be read also include all of the vertical control. The readings on these points indicate the probable nature of the vertical deformation of the strip datum and hence the required corrections to the readings on other points. All readings are made without changing the index of the tracing table scale. The index may be set on any point.

### 13. Adjusting for cross tilt and bridging error

Each corner point will be read in each of two adjoining models of the strip. These readings will reveal any relative tilt between the two model datums, also any slight error in the bridging of the two models. Differences between models of plus or minus 0.2 millimeter are very common. It is not practical to attempt to reduce these differences beyond this amount by further adjustments of the projectors. Rather, the readings are corrected by either of two methods, the first and simpler method being to adopt an average value wherever the readings on a point differ between the two models. The second method is to compute mathematically the unique value for each point that would be obtained if the differences were to be removed by carefully bridging all the models in succession, starting at one end and, with x-motion only, bridging each model exactly to the preceding one at both the front corner and the back corner of the model, assuming that such a procedure is possible. This method is illustrated in figure 4, wherein all the original readings are shown adjusted to the datum of the model at the left end of the strip.

In practice, there is little to recommend one method over the other. After adjusted values have been obtained, they are used throughout the remainder of the process, and the original multiplex readings are discarded.

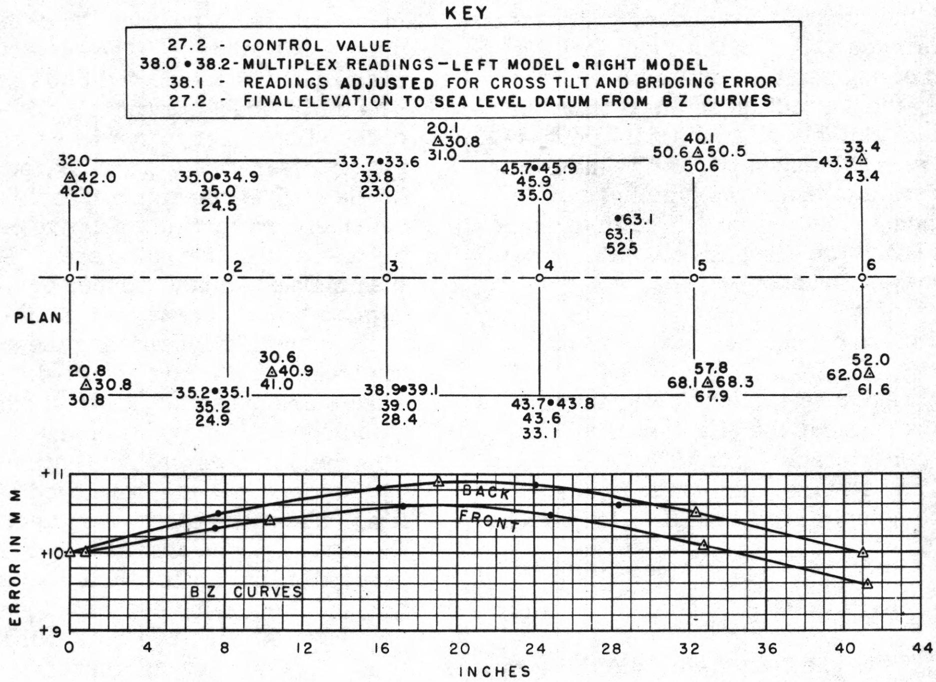


Figure 4.--Example of use of BZ curves.

14. Measuring the x- and y-coordinates of the points

A straight line is now lightly drawn on the base sheet along the center of the strip. This is a reference axis along which the x-coordinate of each vertical control and pass point should be measured. (Units of inches to the nearest half-inch are convenient.) The y-coordinate of each point should also be approximately measured. The latter is needed only when the slope of the strip in the y-direction (due to twist) in combination with excessive variation in the y-coordinates of the points makes it inadmissible to assume that all points lie at a uniform y-distance from the reference axis. (See article 16.)

15. Plotting the BZ curve to correct for vertical bow

Selecting suitable scales, plot on graph paper the x-coordinates of the vertical control points as abscissae. As ordinates, plot the errors evident at each control point between the adjusted multiplex readings and the true values. Through these points draw a smooth curve. A spline will be found helpful in drawing this curve, which is known as the BZ curve. The ordinates under the BZ curve then give the corrections that must be subtracted from all the adjusted multiplex readings to correct for vertical bow and to reduce the readings to a sea level datum.

16. Procedure when the strip datum twists or slopes in the y-direction

When the elevation readings indicate that the strip datum either twists or slopes in the y-direction, two correction curves must be used. These may be either two BZ curves representing separately the error along the front edge and the error along the back edge of the strip, or they may be one BZ curve representing the error along the center line of the strip plus an auxiliary curve showing the additional error along the front and back edges of the strip due to the slope or twist. Both curves in the former instance and the auxiliary curve in the latter instance will represent the error of the strip at a uniform y-distance from the reference axis. In the event that the y-coordinate of any point differs too greatly from this distance, the correction obtained from the curves must be suitably interpolated. Interpolated values should be used in plotting the BZ or the twist curves as well as in obtaining corrections from them. Figure 4 provides an example of the use of BZ curves in correcting the errors of vertical bow and twist.

17. Effect of earth curvature

When strips covering long ground distances are stereotriangulated, the effect of earth curvature may be considerable. Its effect is to cause the datum of any strip to arch upward in a vertical bow. The height of this arch, V, is given by the expression

$$V = 0.167\, L^2 \qquad (2)$$

where V is in feet, and L is the length of the strip in miles. However, no separate calculation of this error is necessary, since the above-described method of correcting vertical bow and twist errors automatically corrects all vertical deformations of the datum, regardless of their source.

18. Adjustments between adjoining strips

Some discrepancies are to be expected in the values obtained by vertical stereotriangulation of adjoining strips. Just as a single position for each horizontal pass point was obtained by adopting a weighted average, a single value for the elevation of each vertical point is also decided upon by compromising its two values. This adjusted value is determined by weighting each of the two discrepant figures after giving due consideration to the distribution of control and the probable "strength" of each strip as revealed by an examination of the discrepancies between the several strips.

## EXTENSION

19. Stereotriangulation with control at only one end of a strip

Stereotriangulation beyond control--called "cantilever extension" or simply "extension"--is more simple to perform than bridging-type stereotriangulation. The results obtained, however, cannot be expected to be as accurate. The initial model or models will be at one end of the strip. They must be carefully and completely cleared of y-parallax, positioned, and leveled. The remaining models are then successively brought into relative orientation and bridged to the preceding ones. Pass points are then marked without further delay, because without control at the far end of the strip there is no possibility of refining the overall scale or of determining if any bow or differential scale effects are present. Vertical pass points can also be read if they are desired. BZ curves must be based on what indications are provided by the models at the "anchored" end of the strip. If no indications can be obtained, a center line BZ curve may be assumed on the basis of the orientation of another strip by the same stereotriangulator, using controlled photography taken by the same camera. Finally, the adjustment of pass point discrepancies between adjacent strips is made in the same manner as in the case of bridged strips.

## SPECIAL PROCEDURES OF STEREOTRIANGULATION

20. Stereotriangulation with two projectors

Although the length of a strip that can be set up is definitely limited by the length of the multiplex frame, it is not necessarily limited by the number of projectors available. If necessary, stereotriangulation can be performed by using only two projectors by "leap-frogging" one projector past the other as the successive models are oriented. The scale of the initial model is passed along by bridging successive models as usual, except that the elevation of the tie point used in each bridging operation must be read and recorded in one model before the trailing projector is removed to be used in orienting the succeeding model. "Leap-frog" procedure also necessitates the marking of pass points from each model as it is set up. Errors of closure upon the control at the terminal end of the strip must be distributed among the intermediate positions and/or elevations by analytical methods. Graphical methods employing correction curves for each coordinate component of error are preferred, but least-square computations similar to those used in adjusting closure errors in traverse lines can be used.

21. Stereotriangulation aids

When stereotriangulation of sparsely controlled areas must be performed, there are a number of means that can be employed to supplement ground control. These include flights crossing the regular pattern at intervals; altimetric data; tilt data; and shoran. At present, none of these methods or devices are capable of the accuracy of ground surveys; therefore, they should be employed only when accuracy requirements permit. The methods of applying the information furnished by these aids are not as firmly established as are the more routine methods of multiplex operation. When and if the stereotriangulator has occasion to use any of them he will probably desire to develop his own methods to a large extent. A few comments can be made regarding the functions of these aids.

a. Cross flights.--Flight strips crossing the main flight pattern serve to knit the regular or compilation strips more closely together. They supplant the usual lines of control, making only an occasional control point necessary within a large area.

b. Altimetric data.--Data concerning the altitude of each exposure station can be recorded from an altimeter or a statoscope carried in the airplane. Fairly accurate indications of the relative heights of the exposure stations are sometimes obtained. A comparison of these data with the measured heights of the oriented multiplex projectors is used to adjust the overall level of a strip (in the x-direction) and to furnish the necessary information for plotting a BZ curve. One ground control elevation is desirable for an index point, as the absolute altitudes of the aircraft may be indicated with a relatively low degree of accuracy.

c. Tilt data.--Tilt data, such as are furnished by a horizon camera or by gyroscopic means are most readily applicable in the operation of instruments such as the stereoplanigraph, which have calibrated scales indicating the tilts of the projectors. Multiplex-projector tilts can be measured, however, either by optical means that record the tilt of the principal ray, or by a level bubble placed upon the projector. The tilt values of all the projectors in a stereotriangulated strip should be employed to determine the level and the curvature of the strip. Tilt data furnish a vertical datum both in the x- and in the y-directions.

d. Shoran[3].--Shoran can be used to determine the position of the aircraft at each exposure station. These positions, plotted on the base sheet, are nadir points directly above which each corresponding projector should be oriented. There are a number of devices that indicate the plumb ray from each projector lens, such as vertical collimators, level bubbles, and plumb bobs. Because of random errors in the determination of individual positions by shoran, it is best to scale the strip as a unit, selecting the best average scale and position. This procedure of using the plumb rays of the projectors requires that the models be carefully leveled; therefore, allowances must be made for the slopes of some models due to the vertical bow of the strip.

[3]A full description of the shoran method is given in Chapter 3 C 20, Stereotriangulation with shoran-controlled photography.

GEOLOGICAL SURVEY

TOPOGRAPHIC INSTRUCTIONS

Chapter 3C 7-e

# MAP COMPILATION BY MULTIPLEX

## Contents



## Illustration

Chapter 3 C 7-e[1]

# MAP COMPILATION BY MULTIPLEX


## ABSTRACT

Multiplex maps are compiled in two phases, planimetry and contours; planimetry is always compiled first in each model. For planimetry, the model must be accurately positioned but only approximately leveled, while for contouring it must be both positioned and leveled accurately. Planimetric features are compiled in a definite order, according to importance. In tracing planimetry, the height of the platen must be continuously regulated so as to keep the floating mark in contact with the model surface. Repeated reference to contact prints is necessary to distinguish obscure features. Inking is done piecemeal as the compilation progresses. Compilation of contours requires special visual ability. In tracing contours, the platen is clamped at the height of the chosen contour and the tracing table is guided so as to keep the floating mark always on the ground. Special measures are taken in contouring certain types of difficult areas. Minor drainage is drawn in pencil to serve as a guide in alining contours.


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## MULTIPLEX COMPILATION ROUTINE

### 1. Order of compilation

The compilation of a topographic map by multiplex is divided into two principal phases: compilation of planimetry, and compilation of contours. For a given model, both phases are completed before the next model is begun. Within each model, the planimetry is always compiled first so that the contours can subsequently be shaped to conform with planimetric features. The woodland outline is a part of the planimetry, although it is sometimes compiled on a separate transparent overlay.

## COMPILATION OF PLANIMETRY

### 2. Orientation requirements for planimetric compilation

Reorienting individual models for planimetric compilation is required unless it has been possible to retain the orientation of each stereotriangulated strip while adjoining strips were being scaled. If adjusted pass points cannot be obtained, or are not desired, then planimetry of a strip can be compiled as soon as stereotriangulation is completed, without the need of resetting the models. When a model is reset, the orientation procedures (interior, relative, and absolute) are performed in the usual way, with one permissible exception: it is not necessary that the model be absolutely leveled. Although greater tolerances might be permissible, a level solution within 1.0 millimeter across the model in both directions is quickly obtainable and affords sufficient accuracy in all cases. If contour compilation is to follow the compilation of planimetry, the stereocompiler may choose to complete the leveling of the model before starting any compilation.

The position of the model should be checked at frequent intervals throughout the compilation process. Changes in humidity cause some basesheet materials to expand or shrink appreciably, thus affecting the scale and position of the model

[1]The number 3 C 7-e signifies Book 3, Part C, Chapter 7-e of the Geological Survey loose-leaf manual of Topographic Instructions. For a table of contents, see Chapter 1 A 2 (Circular 92).

with respect to the control base. If the model should be found out of position, either it or the base sheet must be repositioned and the features already traced checked for errors. Under extreme conditions, it may be necessary to rescale the model.

3. Order of compilation of features

It is preferable to compile all the features of each kind at one time; in this way the chances of overlooking and omitting any detail are minimized. A suggested procedure is to compile all main roads first, following these in order by secondary roads, buildings, obvious drainage, fence lines, and woodland outlines. Railroads, shore lines, and wide streams, if they occur, should take precedence over all of these. The particular details to be compiled depend upon the type of map being prepared. A general rule to follow is to compile first those features whose exact positioning or alinement is most important.

4. The tracing operation

The tracing operation is simply stated: Place the floating mark upon the image of the feature to be traced; lower the pencil to the paper; slide the tracing table so as to follow the outline of the image with the floating mark (see fig. 1). The floating mark may be guided through the more intricate detail by resting the edge of each hand upon the map sheet and using the fingers to move the tracing table. The height of the platen must be continuously regulated with one index finger, so as to keep the floating mark always in contact with the model surface.

Some suggestions with respect to the tracing operation follow:

Each feature should be completely traced or outlined with a continuous line. Retracing a line in the opposite direction provides a good check on the accuracy of the tracing.

No attempt should be made to symbolize the features as they are traced with the tracing table; symbols can be added when the lines are repenciled or inked.

Care should be taken when plotting objects having height, such as buildings and trees, to avoid tracing their shadows instead of their true positions. High buildings in cities may have to be plotted by their roof lines, as the photograph perspective may cause their bases to be partially obscured. It is most important to have the floating mark in contact with the feature being traced, at the correct elevation.

Planimetry should not be compiled beyond the limits of the neat model, which in most cases is approximately the area bounded by the pass points. In exceptional circumstances, planimetric detail falling completely outside the stereoscopic area may be desired, but the difficulty of determining the proper heights at which to intercept the perspective image rays may make detail thus plotted extremely unreliable.

5. Reference to contact prints

It will be necessary when compiling the smaller and more obscure details of planimetry such as dense collections of buildings, railroad yards, trails, and streams in heavily wooded areas, to refer repeatedly to the contact prints. The brilliance and clarity of the model seen through a simple lens stereoscope far surpass that of a multiplex model and many details can be seen that might be overlooked if the multiplex models were used alone. When details have been identified by the use of the prints, they usually can be seen well enough in the multiplex model to be plotted accurately.

6. Inking

From time to time as the compilation progresses, the completed planimetry should be either repenciled or inked. This is necessary because the constant sliding of the tracing table over the manuscript will gradually smear and rub out the original lines. Inked lines are generally preferred; they are more permanent and provide clearer photographic copy.


Figure 1.--Multiplex compilation.

Because inking must be done piecemeal as the compilation progresses, it is not convenient to assign the operation to a special draftsman. The multiplex stereocompiler therefore, being burdened with the task, takes it as an opportunity to smooth out some lines, reshape others as required, and add the proper symbols[2].

## COMPILATION OF CONTOURS

### 7. Visual skill required

Skill in contouring comes only after weeks of practice and even then not to everyone. A special kind of visual ability is required, which, if lacking in the beginning, may prove impossible to develop. Multiplex trainees, therefore, should be selected only after a test of their innate stereoscopic ability. This test should not only measure their ability to perceive a depth difference between two points in space but also their ability to perceive the shapes of larger masses, as demonstrated by their ability to trace a reasonably approximate contour line.

When making any stereoscopic measurement with a floating mark, but most especially when contouring, the observer must strive for an awareness of the model around, above, and below the mark itself. Staring at the floating mark is a tendency which beginners often have to overcome.

### 8. Orientation requirements for contouring

Contouring requires that the model be both positioned and leveled as accurately as possible. Even though all the models of a strip remain in orientation, the impracticability of accurately leveling more than one model at a time makes it compulsory to compile contours model by model, leveling each model in turn.

During the compilation of contours, the stereocompiler should frequently check both the position and the level of the model. He should even more frequently check the consistency of his elevation reading on a given point (index reading), as his personal "index" may be subject to a variation of 0.1 to 0.3 millimeter. Nearly all observers experience this "change in index" at one time or another. When it does occur, it usually takes place during the first minutes of stereoscopic observation. As a person's stereoscopic experience increases, this tendency of his "index" to change diminishes.

### 9. Contour tracing

When he is satisfied with the orientation of the model and has checked his index reading, the stereocompiler may begin drawing the contours. The platen is set and clamped at the height of any suitable contour and the contour traced as a continuous line by guiding the floating mark wherever it must go to keep it always "on the ground." When enough of one contour line has been drawn, the platen is raised or lowered an amount equal to the contour interval and another line is traced. In this manner, contours are compiled over the whole model. The compilation can begin at any place in the model, but it is best to begin either at the top or at the bottom of a slope rather than at random in between. Depending on the character of the topography, it is usually preferable to compile by features, or by small groups of features, rather than to thread one contour across a whole model before starting to trace the next contour. In steep terrain, it is often unnecessary to trace every contour from the model. Every other contour, or in some cases every fourth or fifth contour will be sufficient. The intermediate contours can be sketched in afterward when the whole compilation is shaped or retouched to give the proper "expression" to the lines.

### 10. Contouring difficult areas

Difficulty in contouring is often experienced in areas of relatively flat terrain, in densely wooded areas, in shadows, and in areas that offer either too much or too little image contrast. Ordinarily, it will not be necessary for the stereocompiler to leave these areas uncompleted if he makes good use of the contact prints and if he applies a knowledge of topography. There are occasional situations, however, in which it is impossible to draw the contours with a sufficient degree of reliability. In such cases, the area in question should be left blank except for the notation, "Complete in field."

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[2] All symbols should be drafted in accordance with Chapter 3 C 19, Photogrammetric compilation symbols.

a. Flat terrain.--If flat terrain has any effect on the accuracy with which a multiplex elevation can be read, it is to better the accuracy. In contouring, however, when the average slope of the ground becomes too small--say, less than one foot in a hundred--the horizontal position of a contour line and hence its shape become difficult to determine. Rather than trace a contour as a continuous line, the stereocompiler may find it necessary to move the floating mark at right angles to the general direction of the contour in an attempt to determine a band within which no separation between the mark and the ground can be noticed. After a number of bands have been located, each contour line can be sketched in the most logical position within its own band.

b. Wooded areas.--When the ground is completely obscured by trees, accurate contours cannot be traced. Whenever possible, therefore, aerial photography is taken when leaves are not on the trees, so that no difficulty is ordinarily experienced in seeing the ground except in areas of evergreen growth. When evergreens or leafed-out trees must be reckoned with, contouring is not altogether impractical provided scattered glimpses of ground can be had through openings in the trees; if these openings are numerous enough, reasonably accurate contours can be drawn from one opening to the next by carrying the floating mark at a uniform distance below the tops of the trees. In places where the tree heights would logically be expected to be greater or less than average, an estimated allowance must be made for the difference in height.

c. Shadows.--Large areas in shadow cannot be contoured unless detail can be seen in the shadow. Contours can be run through small shadows, however, with little difficulty if the stereocompiler does not let himself be deceived into mistaking the impression of relief naturally associated with shadows for a true stereoscopic impression. An example of this deception may be seen in the spotty shadows of partly leafed-out trees, which often give one the impression that the ground is a very uneven surface although in reality it may be quite smooth.

d. Images with insufficient contrast.--Unbroken fields of grass or grain or stretches of sand or snow often present such a monotone that it is impossible to make a stereoscopic reading upon them. If such areas are not too large, however, it is practical to sketch the contours across them with the aid of the contact prints, which usually resolve the texture of the terrain.

e. Images with excessive contrast.--It is a natural tendency for white or very bright images to appear slightly higher or nearer to the eye than dark-toned images. This can be demonstrated if one single photograph is printed on two diapositives, being offset on one so that a stereoscopic model of the pair may be set up. Although this model has no relief, being formed by two identical images, there is a strong tendency to see alternate light and dark areas as an uneven surface. The stereocompiler should bear this phenomenon in mind when reading control elevations as well as when contouring.

## 11. Drawing the drainage

Regardless of the skill with which the floating mark is used in tracing the contour lines, some smoothing, respacing or reshaping of the lines will be necessary if they are to express the characteristic features of the terrain with utmost clarity and proper emphasis. As most land forms have arisen to some extent through the effects of erosion, the representation of drainage is of great importance in obtaining the proper expression of these forms. Drafting the proper expression into a multiplex compilation will always be aided if all the drainage, large and small, is lightly drawn in prior to the final inking or repenciling of the contours. Only the flatter drains will have to be traced from the model; other drains can be sketched through the contour re-entrants. Then, when the stereocompiler is repenciling or inking the contours, the drainage lines help him to aline the contours precisely and to shape them with the proper expression.

## 12. Spot elevations[3]

Spot elevations are approximately correct elevations of certain topographic and cultural features that are published in order to furnish

[3]Revised February 1953.

map users with more specific elevations of these features than may be interpolated from the contours. Spot elevations are published for such objects as roadforks; summits of mountains and prominent hills; water surfaces of lakes, ponds, and wide rivers; stream forks; prominent high points in roads; bottom elevations in depressions; large flat areas; and section corners, boundary markers, and other well-defined points.

Spot elevations should be recorded by the stereocompiler wherever needed to supplement the contour information or to supplement spot elevations that may have been obtained in the course of field surveys. They should not be shown on steep slopes. On maps to be published as 15-minute quadrangles, the minimum density should be about one per square mile, and for $7\frac{1}{2}$-minute quadrangles, three per square mile. Maximum density will depend on the nature of the terrain. The objective should be to show elevations of as many prominent features as practicable but not to an extent that would clutter the map.

The recording of such spot elevations is an integral part of multiplex compilation. Each elevation is determined by averaging repeated readings in the multiplex model. All elevations determined by reading in the stereo-models must be followed by the letter T when shown on the manuscript, thus: 678T. When the map location of a spot elevation is in doubt, the point is indicated on the map sheet by a small right-angle cross.

13. Inking or repenciling

Inking or repenciling the completed contours and desired drainage lines are the remaining steps in multiplex compilation. Inking or repenciling is preferably done piecemeal as the compilation of the model progresses, the size of an area that should be compiled at one time depending on the density of the contours, the clarity of the original penciled lines, and the stereocompiler's working convenience.

Inking is more practical than repenciling, since the lines must withstand considerable rubbing by the tracing table. The stereocompiler may wish to repencil first and then ink the lines. This affords him an opportunity to smooth and shape the contours to his satisfaction before committing them to the permanence of ink. Direct inking without prior repenciling saves considerable time, but requires more skill in achieving the proper topographic expression.

If adjoining models are to be compiled subsequently on the same manuscript, the stereocompiler will want to leave a narrow band of contours around the joining edges uninked, so that he can compromise minor discrepancies in compilation along these edges without tedious erasures of inked work.

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