PERFORMANCE OF BED LOAD TRANSPORT EQUATIONS IN MOUNTAIN GRAVEL-BED RIVERS: A RE-ANALYSIS

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Abstract: Our recent examination of bed load transport data from mountain gravel-bed rivers in the western United States shows that the data can be fit by a simple power function of discharge, with the coefficient being a function of drainage area (a surrogate for basin sediment supply) and the exponent being a function of supply-related channel armoring (transport capacity in excess of sediment supply) (Barry et al., 2004). We also compared the performance of our proposed equation to that of five commonly used bed load transport equations. Here, we explore the sensitivity of equation performance to differences in how the statistical assessment of error is parameterized. We also consider the performance of these bed load transport equations in terms of geomorphic significance. Over the past two decades numerous studies have assessed the performance of various equations for predicting bed load transport; however, these analyses have been based on paired observations of measured and predicted bed load transport, the majority of which were taken at low flows (including Barry et al. (2004)). Consequently, formula performance is weighted toward low discharges which may not have geomorphic significance. Here, we consider equation performance at a number of gravel-bed rivers in mountain basins of the western United States in terms of the accuracy with which the equations are able to predict the effective discharge.

INTRODUCTION

In previous work, we examined bed load transport in 24 mountain gravel bed-rivers in central Idaho, using an extensive dataset recently compiled by King et al (2004). This data set included over 2,100 bed load transport observations collected over a range of flows from 2 to 181% of the 2-year flood flow (Q_2). We showed that the observed bed load transport could be fit by a simple power function of discharge and that the coefficient of this equation is a function of drainage area (a surrogate for basin sediment supply), while the exponent is a function of supply-related armoring, parameterized by Dietrich et al.'s (1989) q^* ratio (Barry et al., 2004). We evaluated the performance of this bed load transport equation and four other commonly used equations in terms of their ability to predict the observed bed load transport rates at 17 test sites outside of Idaho, thereby providing a test of our equation independent from the sites from which it was developed. The 17 test sites are mountain gravel-bed rivers in Oregon, Wyoming and Colorado, and are further described by Barry et al. (2004). The selected bed load transport equations were those of 1) Meyer-Peter and Müller (1948), 2) Ackers and White (1973) (as modified by Day (1980)), 3) Bagnold (1980), 4) Parker (1990), and Barry et al. (2004).

To examine formula performance, we calculated the critical error, e^*, at each of the 17 test sites, where e^* is the smallest amount of error that will lead to adequate model performance (i.e., acceptance of the null hypothesis of equal distributions of observed and predicted bed load transport rates assessed via Freese's (1960) χ^2 test at a significance level of 0.05). Hence, we are asking how much error would have to be tolerated to accept a given bed load transport equation (Reynolds, 1984). Freese's (1960) χ^2 test is calculated as

$$\chi^2 = \frac{\sum_{i=1}^{n} (x_i - \mu_i)^2}{\sigma^2} \tag{1}$$

where x_i is the i^{th} predicted value, μ_i is the i^{th} observed value, n is the number of observations, and σ^2 is the required accuracy defined as

$$\sigma^2 = \frac{E^2}{(1.96)^2} \tag{2}$$

where E is the user-specified acceptable error, and 1.96 is the value of the standard normal deviate corresponding to a two-tailed probability of 0.05. We evaluate χ^2 using log-transformed values of bed load transport, with ε added to both x_i and μ_i prior to taking the logarithm, and E defined as one log unit (i.e., ± an order of magnitude error). We use ε in our analysis because the Meyer-Peter and Müller (1948), Ackers and White (1973) and Bagnold (1980) equations contain a transport threshold. Formulae of this sort often erroneously predict zero transport at low to moderate flows that are below the predicted threshold for transport. To include the incorrect zero-transport predictions in our log-transformed assessment of formula performance, we added a constant ε (equal to the lowest predicted transport rate of $1 \cdot 10^{-15}$ kg/m·s) to all predicted bed load transport rates. We find that both the Meyer-Peter and Müller (1948) equation and the Bagnold (1980) equation typically under predict total transport due to the large number of incorrect zero predictions, with the magnitude of this under prediction set by ε. Figures 1 and 2 (modified from Barry et al. (2004)) show the prediction error and the critical error, e^*, at each of the 17 test sites. The effect of ε set to $1 \cdot 10^{-15}$ kg/m·s is illustrated by the extent of under-prediction, with the magnitude of the under-prediction set by ε (Figure 1), and in the high values of critical error associated with both the Meyer-Peter and Müller (1948) and the Bagnold (1980) equations (Figure 2). Because the effect of ε is evident in the both the prediction errors (Fig. 1) and the critical errors (Fig. 2) for the Meyer-Peter and Müller (1948) and the Bagnold (1980) equations, we examine the sensitivity of formula performance to ε in this paper.

Moreover, our previous analysis considered formula performance based on paired observations of measured and predicted bed load transport, similar to approaches used in other studies (e.g., Gomez and Church, 1989; Yang and Huang, 2001). Because the majority of bed load transport measurements are typically taken during low flows, the assessment of formula performance may be biased toward low discharges which generally do not have geomorphic significance. Geomorphically significant sediment transport in sand- and gravel-bed rivers typically occurs at bankfull flow, which is recognized as both the effective discharge (Q_e, that which transports the most sediment over time (Wolman and Miller, 1960; Andrews and Nankervis, 1995)) and the channel forming discharge (that which controls channel morphology (e.g., Henderson, 1963; Parker, 1978)).

Here, we examine sensitivity of formula performance to selection of ε, and we explore the geomorphic performance of bed load transport equations in terms of their ability to accurately predict the effective discharge.

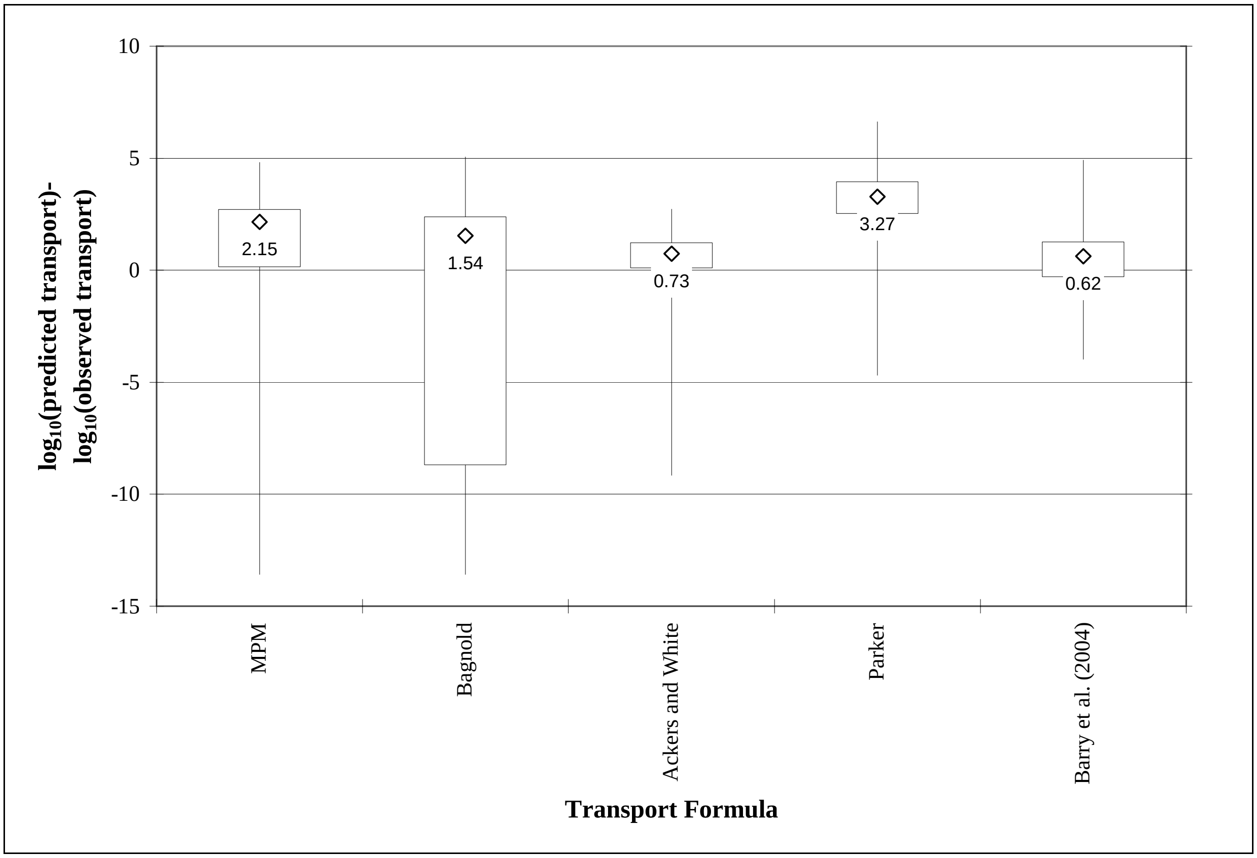

Figure 1 Box plots of the distribution of $\log_{10}$ differences between observed and predicted bed load transport rates for Barry et al.'s (2004) 17 test sites. Median values are specified. Extent of whiskers indicates maximum and minimum values. Upper and lower ends of each box indicate the inter-quartile range. MPM stands for Meyer-Peter and Müller.

FORMULA PERFORMANCE AND THE SELECTION OF ε

Figure 3 illustrates the effect of varying ε between values of $1 \cdot 10^{-15}$ kg/m·s to 10 kg/m·s, demonstrating that formula performance is sensitive to ε, particularly for the Meyer-Peter and Müller (1948) and Bagnold (1980) equations. The performance of these equations improves significantly as ε increases up to values of $1 \cdot 10^{-4}$ kg/m·s. In contrast, performance of the Ackers and White (1973), Parker (1990) and Barry et al. (2004) equations does not respond until ε becomes greater than $1 \cdot 10^{-4}$ kg/m·s, after which the median value of the critical error, e^*, for all five equations begins to increase. The difference in behavior between the two sets of equations has to do with the number of incorrect zero bed load transport rates predicted by each equation. Significant numbers of incorrect zero predictions make e^* a function of ε, rather than an indicator of actual formula performance. This is particularly evident for the results of the Meyer-

Peter and Müller (1948) and Bagnold (1980) equations due to the high number of incorrect zero predictions for those equations at our study sites (Fig. 3, Barry et al. 2004). In contrast, the Ackers and White (1973), Parker (1990) and Barry et al. (2004) equations predict some degree of transport at most discharges, which makes their e^* values less susceptible to choice of ε (at least up to values of $1{\bullet}10^{-4}$ kg/m·s). Prediction of transport at most discharges agrees with Paintal's (1971) notion that there is no critical shear stress for incipient motion, just different degrees of motion as discharge is increased. However, transport rates in these equations become vanishingly small at low discharges.

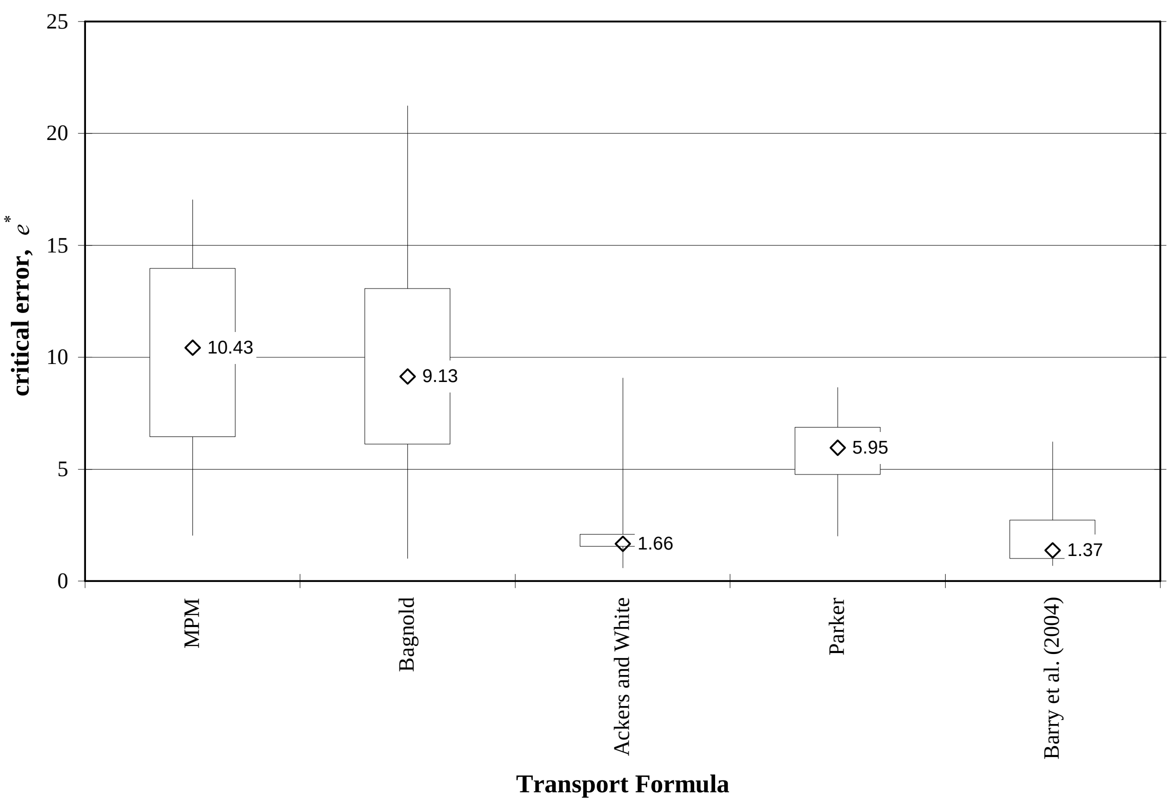

Figure 2 Box plots of the distribution of critical error, e^*, for the 17 test sites. Median values are specified. Extent of whiskers indicates maximum and minimum values. Upper and lower extents of box illustrate the inter-quartile range. MPM stands for Meyer-Peter and Müller.

Both the Ackers and White (1973) and Barry et al. (2004) equations out-perform the other equations included in this analysis until the value of ε increases to 1 kg/m·s. Furthermore, as ε increases to values greater than 10 kg/m·s all equations show similar values of e^*. The similarity in critical error as ε increases to 10 kg/m·s and larger is because such large values of ε are greater than the majority of the observed transport rates. Consequently, the magnitude of error (i.e., the degree of over-prediction) is similar for all equations and is set by the value of ε.

Figure 3 illustrates that the influence of ε on the median critical errors, e^*, of the Meyer-Peter and Müller [1948] and the Bagnold [1980] equations is least when ε is between $1{\bullet}10^{-3}$ and $1{\bullet}10^{-4}$

kg/m·s. Similarly, ε only begins to influence the critical errors, e^*, of the other equations when ε is greater than $1 \cdot 10^{-4}$ kg/m·s. Together, these observations indicate that an ε value between $1 \cdot 10^{-3}$ and $1 \cdot 10^{-4}$ kg/m·s is perhaps a more appropriate value than $1 \cdot 10^{-15}$ kg/m·s, as was selected in Barry et al. (2004).

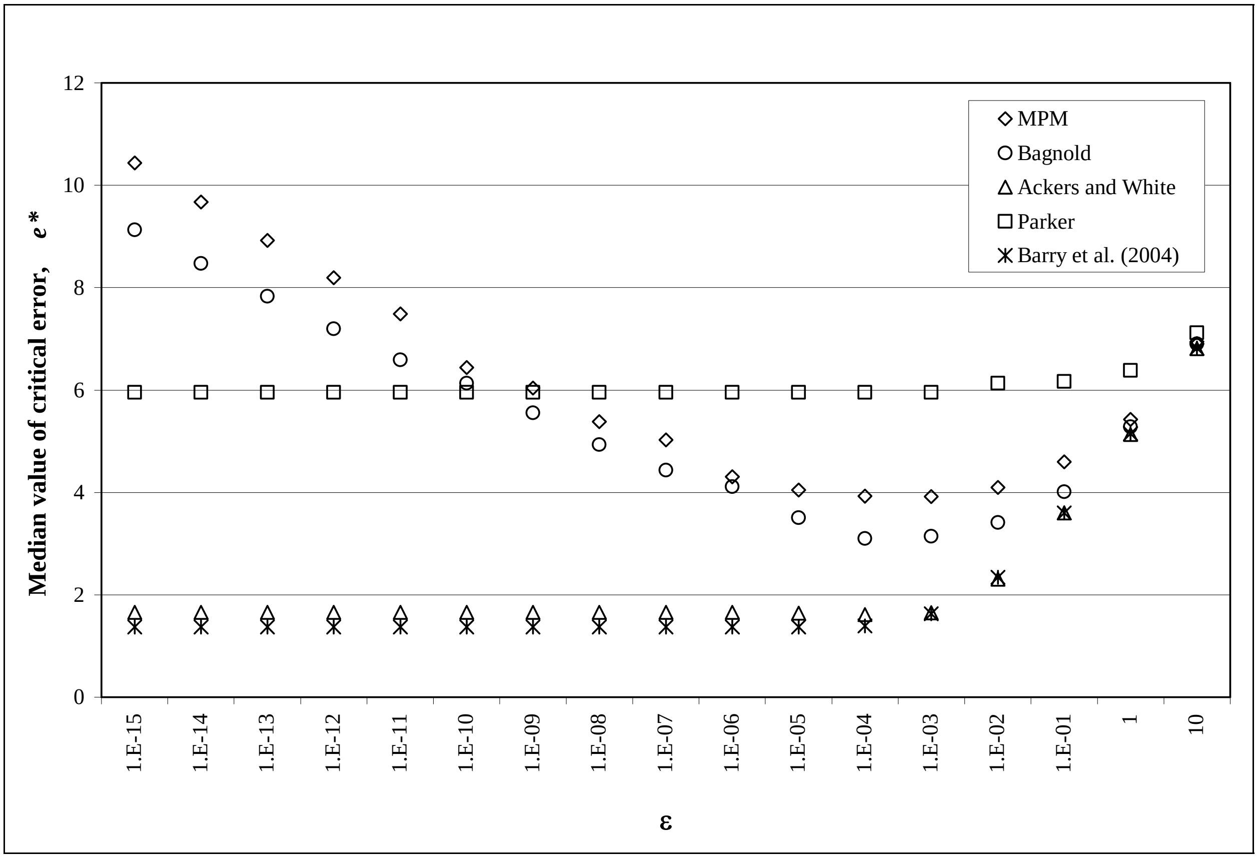

Figure 3 Sensitivity of median critical error values, e^*, to changes in ε. MPM stands for Meyer-Peter and Müller.

ASSESSMENT OF GEOMORPHIC SIGNIFICANCE

In this portion of the paper, we assess the accuracy with which each equation is able to predict the effective discharge (Q_e) (Wolman and Miller, 1960) at each site. The observed bed load rating curve at each site was used to determine Q_e, with the rating curve expressed as a power function of discharge (Barry et al., 2004)

$$q_b = \alpha Q^{\beta} \tag{3}$$

where q_b is bed load transport per unit width, and α and β are empirical values (Leopold et al., 1964, Smith and Bretherton, 1972; Vanoni, 1975). We only included those sites where 1) the observed bed load transport data are well described by Equation (3) (i.e., where the correlation coefficient (r^2) of the rating curve is greater than 0.70, and there is no obvious non-linearity to the observed transport data in $\log_{10}$ space) and 2) where the observed record of discharge covers at least 10 years (Biedenharn et al., 2001). Only 22 of the 41 sites examined by Barry et al. (2004) met these criteria and were examined here.

At each of the 22 sites, we followed the method proposed by Biedenharn et al. (2001) to calculate flow frequency distributions. This approach divides the range of observed discharges into 25 arithmetic discharge class intervals. The "true" bed load transport rate for each discharge interval is determined by applying the site-specific rating curve to each discharge class. Similarly, predicted transport rates for each discharge class are determined from the five transport equations discussed above. Shear stress and other necessary equation parameters were determined for each discharge following an approach similar to that used by Barry et al. (2004). The product of the bed load transport and flow frequency within each discharge interval is the total bed load transport for that interval. The effective discharge occurs where this product is maximized.

To facilitate comparison of predicted and "true" values of effective discharge across watersheds of widely varying size, we normalized both values by drainage area, producing values of unit effective discharge for each site. Figure 4 shows the distribution of the differences between the predicted and "true" values of unit effective discharge across the 22 sites using the 5 equations included here. The median error is close to zero for all equations included in this analysis. However, there are substantial differences between the 5 equations in terms of both the width of the inter-quartile ranges and the 95% prediction intervals. To illustrate the potential absolute error in the predicted value of effective discharge at a site, an error of 0.05 in unit effective discharge at the Selway River site would translate to an absolute error of 248 m^3/sec. By way of comparison, the "true" value of effective discharge at this site is 677 $m^3 s^{-1}$.

CONCLUSION

We find that formula performance is sensitive to ε. In particular, the statistical assessment of both the Meyer-Peter and Müller (1948) and Bagnold (1980) equations is in large part determined by ε due to the large number of incorrect predictions of zero transport for those equations at our sites. The influence of ε is minimized between $1 \bullet 10^{-3}$ and $1 \bullet 10^{-4}$ kg/m·s, and with ε set between these values the assessment of formula performance differs from that presented in Barry et al. (2004). That is, both the Meyer-Peter and Müller (1948) and Bagnold (1980) equations out-perform that of Parker (1990). However, the equations of Ackers and White (1973) and Barry et al. (2004) out-perform the other equations included in this analysis, and this result is insensitive to changes to the value of ε between $1 \bullet 10^{-15}$ and $1 \bullet 10^{-1}$ kg/m·s.

We also find that prediction of the effective discharge is not particularly sensitive to one's choice of bed load transport equation (at least for those examined here). This result corroborates the analytical results of Goodwin (2004), demonstrating why the effective discharge estimate tends to be a reliable and robust indicator. That is, even when one cannot predict the absolute value of sediment transport accurately it is possible to estimate the "channel forming" or "effective discharge". Consequently, the selection of an appropriate sediment transport equation depends on the intended application. For example, in channel restoration work, estimates of the cross-sectional area are usually obtained from empirical relations based on the effective discharge, rather than the magnitude of sediment transport at different flow conditions. However, if one were interested in modeling landscape evolution or the effective storage life of a dam, accurate prediction of the magnitude of sediment transport is critical and, therefore, more care may be needed in selecting an appropriate transport equation.

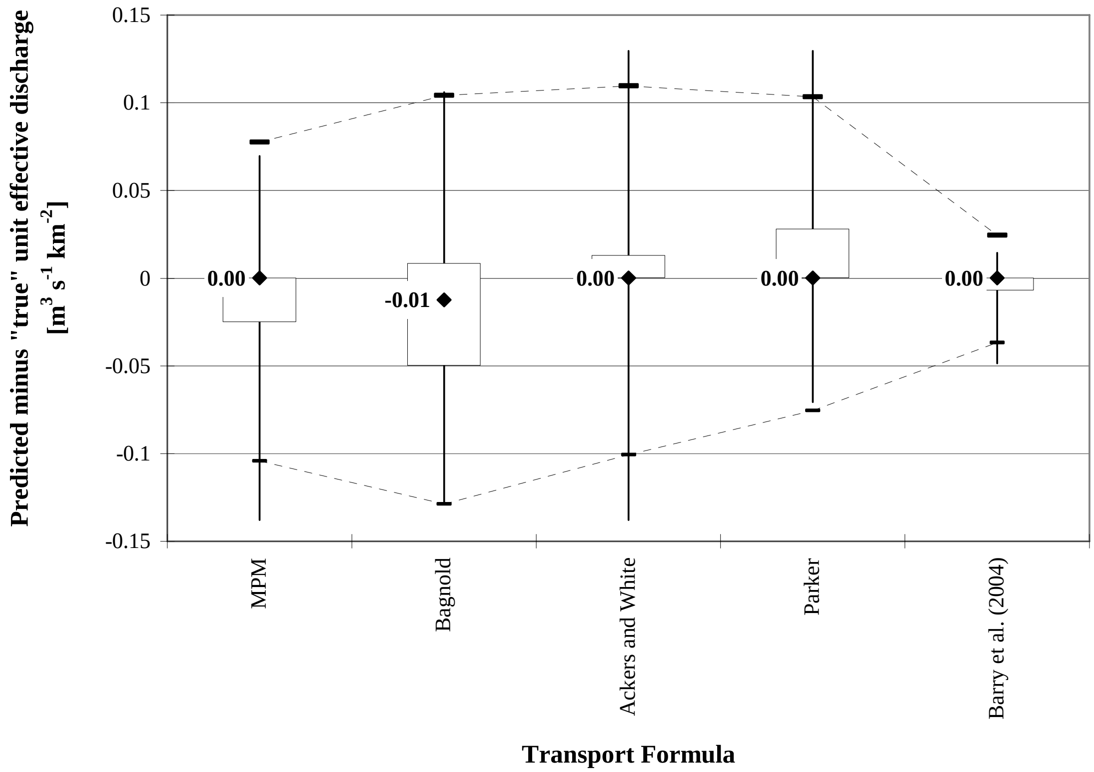

Figure 4 Box plots of the distribution of differences between predicted and "true" unit effective discharge. Median values are specified. Extent of whiskers indicates maximum and minimum values. Upper and lower extents of box illustrate the inter-quartile range. Also shown (dashed lines) are the 95% prediction intervals [Neter et al., 1974; Zar, 1974]. MPM stands for Meyer-Peter and Müller.

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