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In Situ Geomechanics of Crystalline and Sedimentary Rocks, Part I: The Long-Term Deformation and Time-Temperature Correspondence of Viscoelastic Rock--An Alternative Theoretical Approach

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Preface

This report is the first of a series summarizing the results of the U.S. Geological Survey's research program in geomechanics aimed at investigating and assessing the potential of crystalline and sedimentary rock masses as geologic repositories for nuclear waste.

Introduction

A number of investigators have suggested that a relationship exists between the effects of time and temperature on the deformation of rock. Heard (1963, 1967, 1972) has shown experimentally that the differential stress corresponding to a given strain is decreased both by increasing the temperature and by decreasing the strain rate. Similarly, laboratory studies of creep behavior of rock (Goetze, 1971; Rummel, 1968) show that creep rates increase with increasing temperature.

To explain the apparent relationship between the temperature and time deformation effects, these authors appeal to the rate process theories of solid state physics. In this report we will consider the relationship between temperature and time from a somewhat different viewpoint. We will consider rocks to be nonlinear thermoviscoelastic solids; that is, we will use a continuum mechanics approach. An advantage of this approach is that general three-dimensional time- and temperature-dependent relationships between stress and strain can be developed from experimental data. Another advantage, which has been exploited principally in polymer technology, is that the theory allows extrapolation of the results of short-term tests to long-term behavior.

This last point is of importance to both engineering and geology. The theory allows the prediction of the long-term behavior of underground excavations where temperatures may become high, such as nuclear waste repositories. It also allows for greater understanding of long-term crustal processes such as the evolution of geologic structures.

In what follows we will briefly review some theoretical results for nonlinear thermoviscoelasticity, consider its application to the experimental data referenced above, and discuss the limitations and implications of using this approach.

Theoretical formulation

Biot (1954, 1958) was the first to develop a unified theory of linear viscoelasticity based on irreversible thermodynamics. Schapery (1964, 1968) extended Biot's work to develop general three-dimensional constitutive equations for nonlinear thermoviscoelastic solids. His equations, which are given here for isothermal conditions in one-dimensional convolution form, are

$$\varepsilon = g_0 D_0 \sigma + g_1 \int_0^t \Delta D(\psi - \psi') \frac{dg_2\sigma}{d\tau} d\tau \qquad (1),$$

and the inverse relation,

$$\sigma = h_e E_\infty \varepsilon + h_1 \int_0^t \Delta E(\xi - \xi') \frac{dh_2\varepsilon}{d\tau} d\tau \qquad (2),$$

where σ and ε are uniaxial stress and strain, $\Delta D \equiv D(\psi)-D_0$ is the transient component of the creep compliance with D_0 being the initial compliance, and $\Delta E \equiv E(\xi)-E_\infty$ represents the transient component of the relaxation modulus with E_∞ as the long-term or equilibrium modulus. The parameters ψ and ξ are reduced times defined by

$$\psi = \int_0^t \frac{dt'}{a_{\sigma T}(\sigma, T)} \qquad (3),$$

and

$$\xi = \int_0^t \frac{dt'}{a_{\varepsilon T}(\varepsilon, T)} \qquad (4).$$

The parameters $a_{\sigma T}$ and $a_{\varepsilon T}$ are the so-called thermomechanical shift factors. They are related to entropy production in Schapery's thermodynamic theory and reflect the fact that stress, strain, and especially temperature (T) each have a strong influence on relaxation times in thermoviscoelastic materials. The parameters g_0, g_1, g_2, and h_e, h_1, and h_2 in equations 1 and 2 are related to free energy in the thermodynamic theory and are dependent on the level of stress or strain and in some instances on temperature.

For a creep test at constant stress and temperature, equation 1 reduces to

$$\varepsilon = g_0 D_0 \sigma + g_1 g_2 \Delta D \left(\frac{t}{a_{\sigma T}}\right)\sigma \qquad (5).$$

Division of equation 5 by σ gives the nonlinear creep compliance,

$$D(t) = \frac{\varepsilon}{\sigma} = g_0 D_0 + g_1 g_2 \, \Delta D \left(\frac{t}{a_{\sigma T}}\right) \qquad (6).$$

For a relaxation test at constant strain and temperature, equation 2 becomes

$$\sigma = h_e E_\infty \varepsilon + h_1 h_2 \Delta E\left(\frac{t}{a_{\varepsilon T}}\right)\varepsilon \qquad (7).$$

Division of equation 7 by ε gives the nonlinear relaxation modulus

$$E(t) = \frac{\sigma}{\varepsilon} = h_e E_\infty + h_1 h_2 \, \Delta E \left(\frac{t}{a_{\varepsilon T}}\right) \qquad (8).$$

When stresses or strains are small the temperature dependence of $\alpha_{\sigma T}$ and $a_{\varepsilon T}$ often predominates. If it is assumed that the effect of increasing the temperature is to shorten the time required for relaxation, then curves representing the moduli or compliances as functions of time for various temperatures can be shifted horizontally into a single curve at a reference temperature (Morland and Lee, 1960). Such a curve is called a master curve and can be interpreted as representing the moduli or compliance at the reference temperature for very long times. As an example, the master curve for polypropylene, constructed from the stress-relaxation data given in figure 1, is shown in figure 2. Note the long relaxation time for the master curve in figure 2. Media which can be shifted into master curves such as that in figure 2 are called thermorheologically simple.

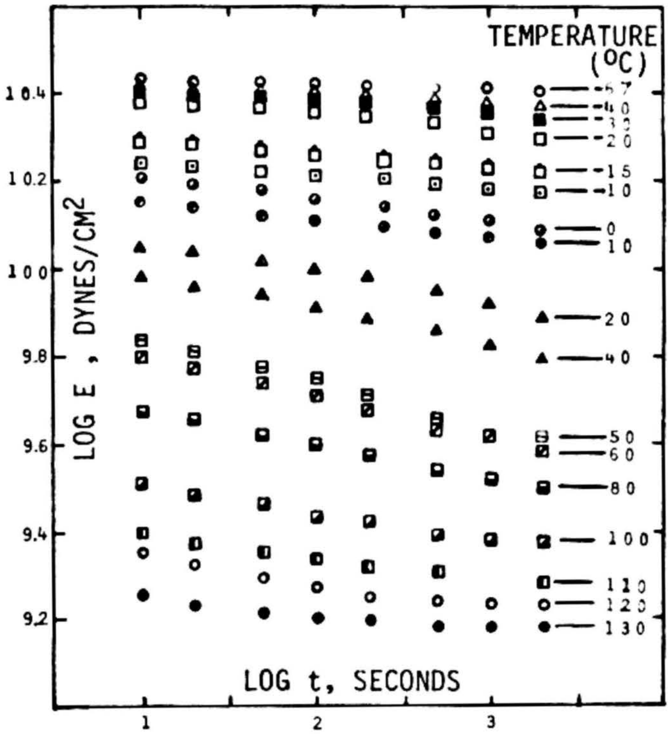


Figure 1.--Stress-relaxation data for polypropylene modified from Passaglia and Knox, 1964. The points represent relaxation data at different temperatures where the temperatures are in °C.

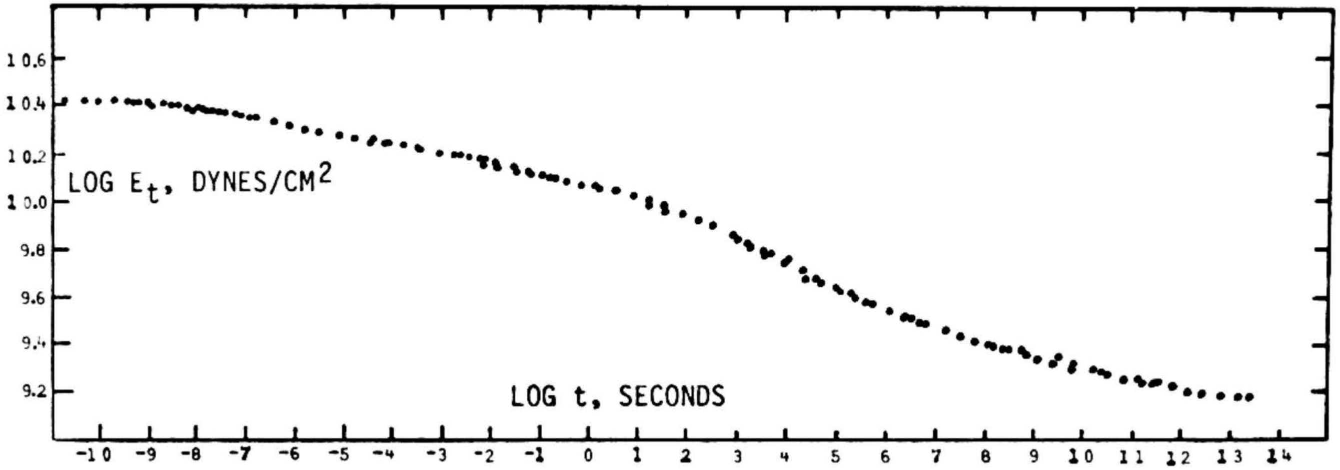


Figure 2.--Master curve for polypropylene at 20°C. Modified from Passaglia and Knox, 1964.

If the free energy is not strongly affected by temperature the parameters g_0, g_1, g_2 and h_e, h_1, and h_2 in equations 1 and 2 depend only on the level of stress or strain. We will find this to be the case in applying the theory to the results of constant strain rate and creep tests on rocks in the next section. In fact, it will be found that the effects of stress and strain can be accounted for by simple vertical shift factors, so separate master curves can be constructed for each stress or strain level.

Schapery's theory states that when the stress is sufficiently small $g_0 = g_1 = g_2 = 1$. Also, at a small stress (the reference stress, σ_R) and at a reference temperature (T_R), $a_{\sigma T}(\sigma_R, T_R) = 1$, by definition, and so $\psi \equiv t$ (see equation 3) and equation 1 becomes the usual linear viscoelastic relation

$$\epsilon = D_0 \sigma + \int_0^t \Delta D\,(t - \tau)\,\frac{d\sigma}{d\tau}\,d\tau \qquad (9).$$

Similarly, at a small reference strain, ϵ_R, and a reference temperature, T_R, equation 2 becomes the linear relation

$$\sigma = E_\infty \epsilon + \int_0^t \Delta E\,(t - \tau)\,\frac{d\epsilon}{d\tau}\,d\tau \qquad (10).$$

Application of the theory to test data

We now consider the application of equations 1 through 10 to the experimental data of Heard (1963, 1967, 1972) and Goetze (1971). Since Heard's data is obtained from constant strain rate tests on Yule marble (trade name for Leadville Limestone of Colorado) and polycrystalline halite, we must first discuss how a relaxation modulus is derived from such a test.

For a constant strain rate test, the strain is given in a uniformly strained specimen by

$$\varepsilon = \dot{\varepsilon}_0$$

where $\dot{\varepsilon}_0$ is the constant strain rate.

The one-dimensional linear viscoelastic constitutive equation

$$\sigma = \int_0^t E\ (t - \tau)\ \frac{d\varepsilon}{d\tau}\ d\tau \qquad ,$$

becomes in this case

$$\sigma = \dot{\varepsilon}_0 \int_0^t E\ (t - \tau) d\tau$$

or

$$\sigma = \dot{\varepsilon}_0 \int_0^t E\ (v) dv \tag{11}$$

with $v = t - \tau$. Here $E\ (t)$ is the relaxation modulus.

The secant modulus is by definition

$$E_S \equiv \frac{\sigma}{\varepsilon}$$

or $\sigma = E_S\varepsilon$. Then equation 11 can also be written

$$\sigma = \varepsilon E_s = \dot{\varepsilon}_0 \int_0^t E\ (v)\ dv$$

or, since $\varepsilon = \dot{\varepsilon}_0 t$,

$$E_s = \frac{1}{t} \int_0^t E\ (v) dv \qquad (12).$$

Differentiating 12 we find the relaxation modulus

$$E\ (t) = E_s + t\ \frac{dE_s}{dt} \qquad (13).$$

Marble and halite

Now consider the results of constant strain rate tests on Yule marble (1-cylinders; Heard, 1963, 1967) and on polycrystalline halite (Heard, 1972). Both tests are under conditions of triaxial extension with a confining pressure of 5 kbar for the marble and 2 kbar for the salt. The secant moduli $(\frac{\sigma}{\varepsilon})$ at strain levels of 2, 5, and 10 percent for the marble and 4, 6, 8, and 10 percent for the halite are calculated from the experimental data, and the time required to reach each strain level is calculated from $t = \frac{\varepsilon}{\dot{\varepsilon}_0}$. Resulting plots of $\log_{10} E_s(t)$ versus $\log_{10}(t)$ for each strain level for temperatures of 300° through 800°C for the marble and temperatures of 23°, 100°, 200°, 248°, 300°, and 440°C for the halite are then found to shift horizontally into master curves, four of which are shown in figures 3 through 6. The reference temperature for the marble is 300°C and that for the halite is 23°C.

The horizontal shift function for each strain level is found to be described by the Arrhenius expression

$$\log_{10} a_T = \frac{Q}{2.303R}\left[\frac{1}{T} - \frac{1}{T_R}\right] \qquad (14),$$

where Q is the activation energy, R is the gas constant, and T_R is the reference temperature. Note that $a_T = 1$ when $T = T_R$. The activation energy used for the marble is Heard's (1963) value of 62.4 kcal and that for the halite is 30 kcal, which is close to Heard's (1972) values. This is not surprising in view of what is known about rate-controlling molecular processes, creep, and stress relaxation in these rocks. For a detailed discussion see Heard (1963, 1972).

The vertical shift function is found by comparing the master curves in figures 3 through 6 for the different strain levels. It is found to be given approximately by

$$h = \left(\frac{\epsilon}{\epsilon_R}\right)^{-2/3} \qquad (15),$$

which is 1 when $\epsilon = \epsilon_R$. For the marble the reference strain is 2 percent and for the halite, $\epsilon_R = 4$ percent.

The solid curves in figures 3 through 6 are an approximate analytic representation of the master curves obeying the modified power law

$$E_s(\xi) = h \left[E_{s\infty} + \frac{E_{so} - E_{s\infty}}{\left[1 + \frac{\xi}{\tau_o}\right]^n}\right] \qquad (16).$$

Here, since $a_{\epsilon T}$, the thermomechanical shift function, is assumed to depend only on temperature, which is timewise constant, equation 4 gives $\xi = \frac{t}{a_T}$. For the marble E_{so}, the short term secant modulus, is 60 kbar and $E_{s\infty}$, the long-term modulus, is zero. The time constant τ_0 for the marble is 5X10^7 seconds, and the exponent n is 0.11. For the halite E_{so} = 7 kbar, $E_{s\infty}$ = 80 bars, τ_0 = 5X10^8 sec, and n = 0.15.

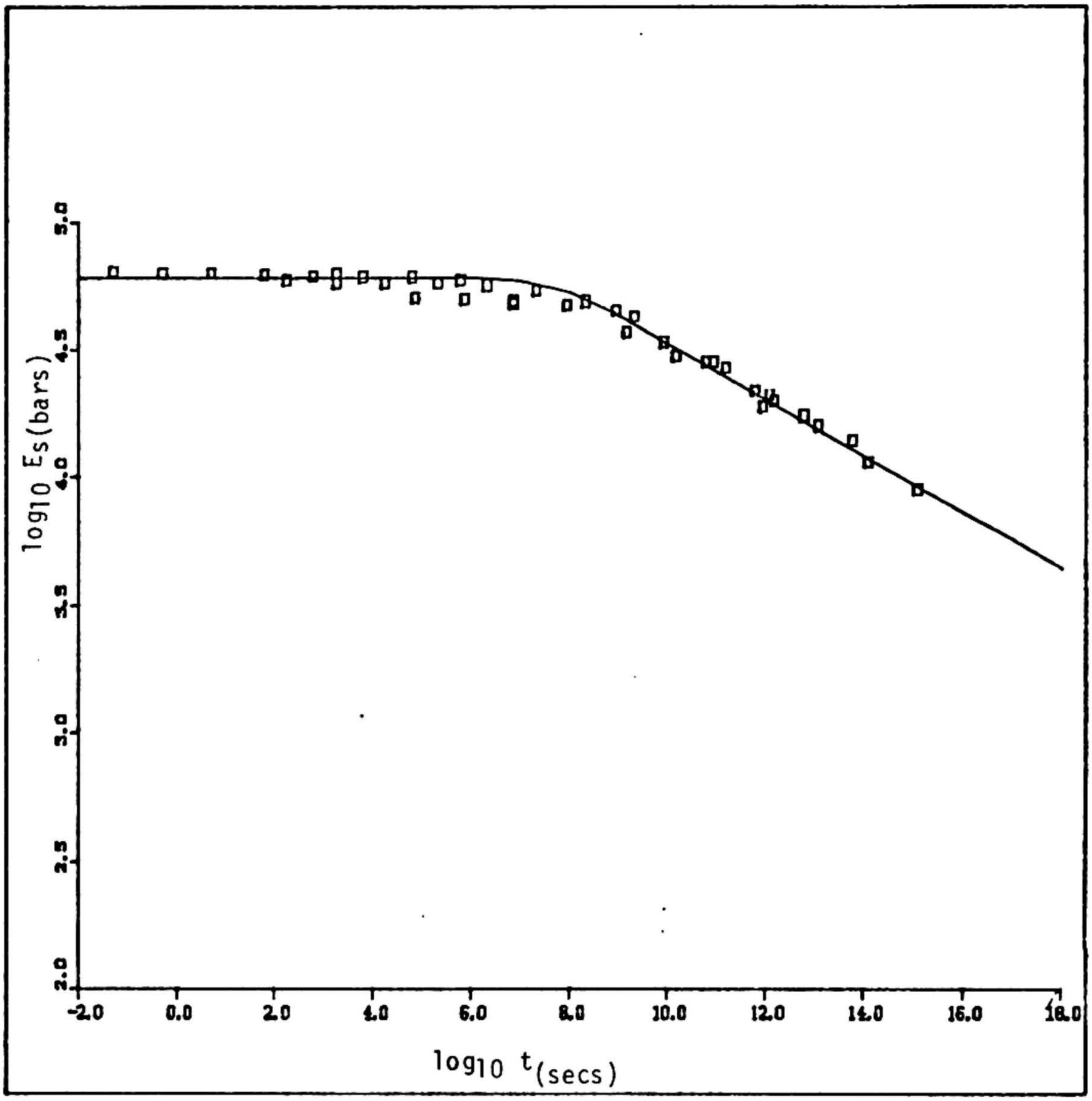


Figure 3.--Master curve based on data (Heard, 1963) for Yule marble at 2 percent strain and a reference temperature of 300°C.

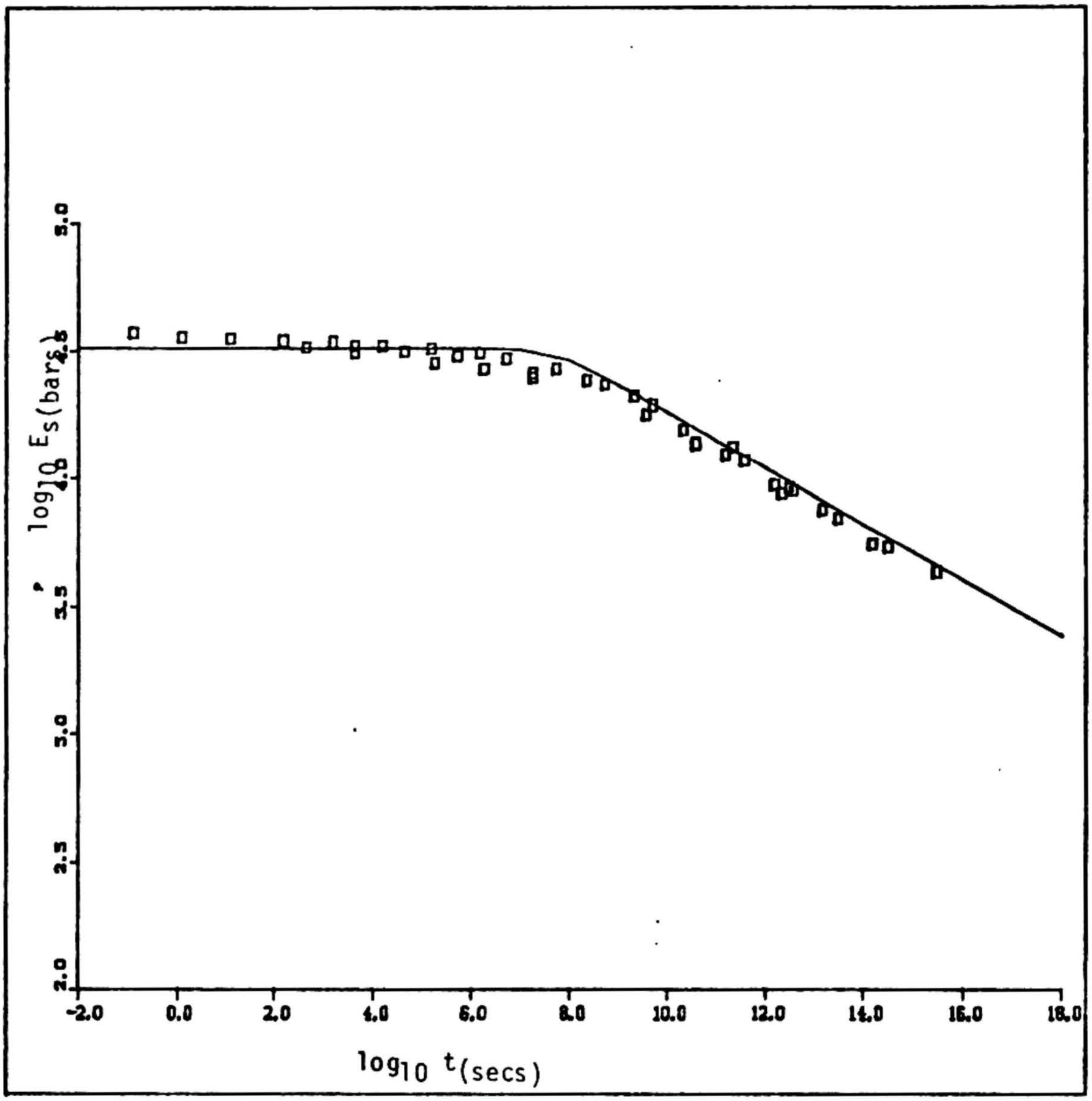


Figure 4.--Master curve based on data (Heard, 1963) for Yule marble at 5 percent strain and a reference temperature of 300°C.

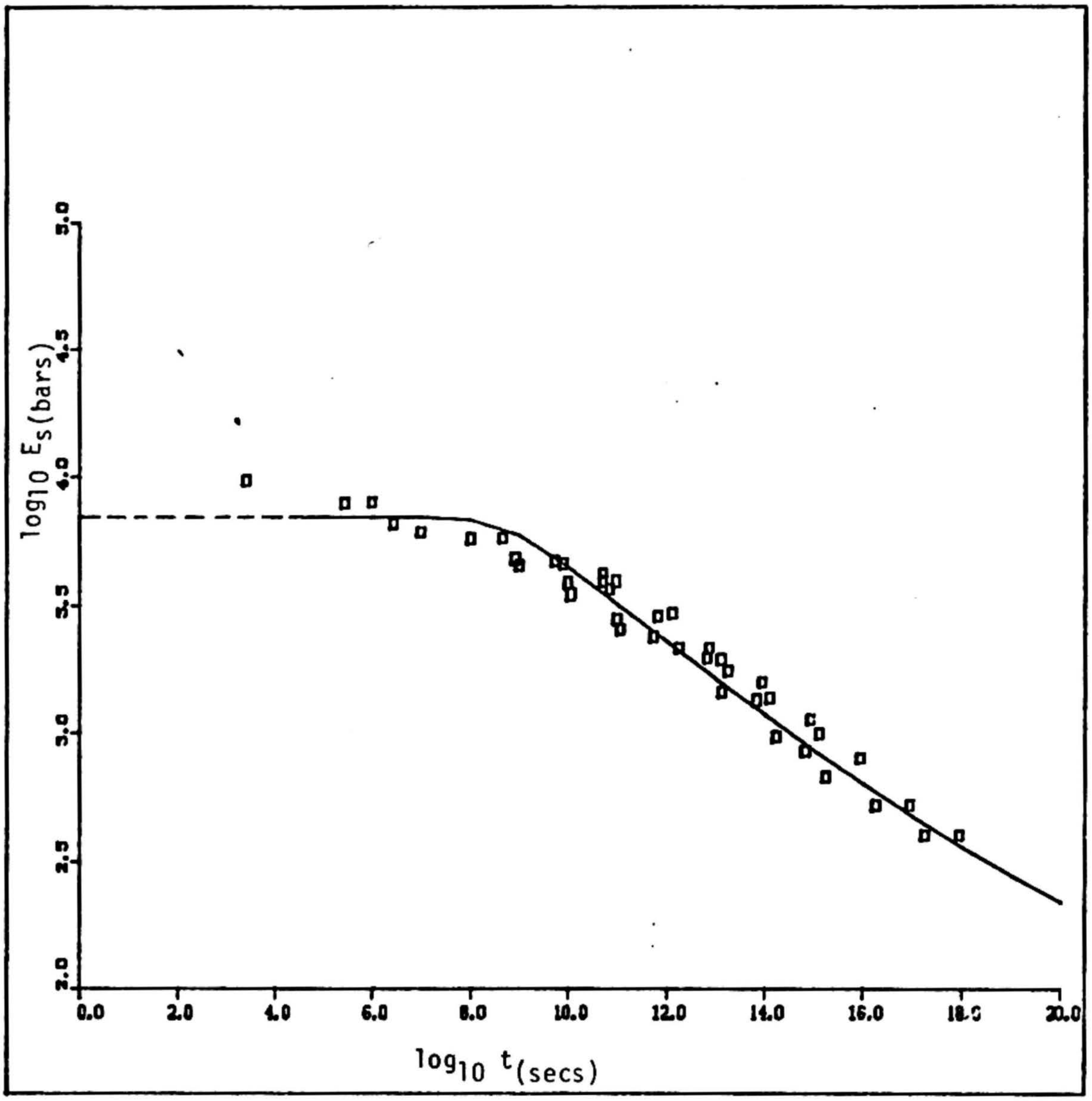


Figure 5.--Master curve based on data (Heard, 1972) for polycrystalline halite at 4 percent strain and a reference temperature of 23°C.

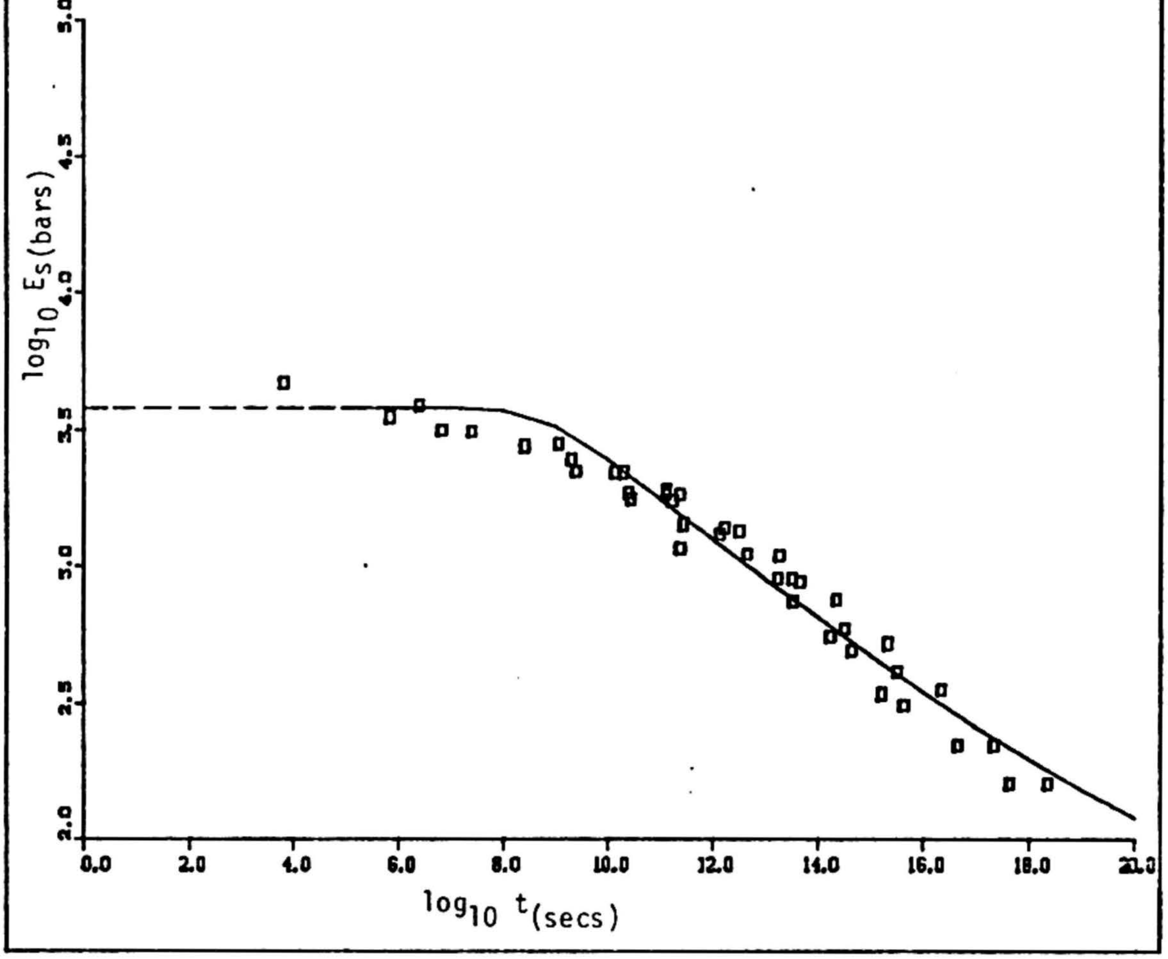


Figure 6.--Master curve based on data (Heard, 1972) for polycrystalline halite at 10 percent strain and a reference temperature of 23°C.

Equation 16 covers a broad spectrum of behavior of marble and halite over extended ranges of time. For example, the time required for relaxation of the secant modulus from 60 to 10 kbar in marble under 5 kbar confining pressure, a constant extensile strain of 2 percent, and a temperature of 300°C is predicted to be 10^{16} seconds or 316 million years. In contrast, at the higher temperature of 600°C and the same pressure conditions, the same relaxation occurs in about 3.75 years. Because the vertical shift factor is independent of temperature, analogous effects will occur at the higher strain levels.

Finally, applications usually require the actual relaxation modulus derived from equation 13. Writing equation 16 in terms of time, substituting in equation 13, and reverting to reduced time we arrive at

$$E(\xi) = h\ E_{s\infty} + \frac{h\ [E_{so} - E_{s\infty}]}{[1 + \frac{\xi}{\tau_o}]^n}\,[1 - n\,[\frac{\frac{\xi}{\tau_o}}{1 + \frac{\xi}{\tau_o}}]\,] \quad .$$

Since n is small and the expression $[\frac{\frac{\xi}{\tau_o}}{1 + \frac{\xi}{\tau_o}}]$ is always less than 1, the actual relaxation moduli are given to a sufficient approximation for the marble and halite by equation 16.

Granite

Goetze (1971) used time-temperature superposition to interpret transient creep in Westerly Granite under a confining pressure of 4-5 kbar and temperatures of 430°, 595°, 657°, 711°, and 715°C. In figure 6 of his paper he gives what is essentially a master curve for $\frac{\varepsilon'}{\varepsilon_0}$, where ε' is the transient creep strain and ε_0 is the initial elastic strain, at 711°C and an axial load of 640 bars. His results can be predicted by using equation 6 when it is written in the form

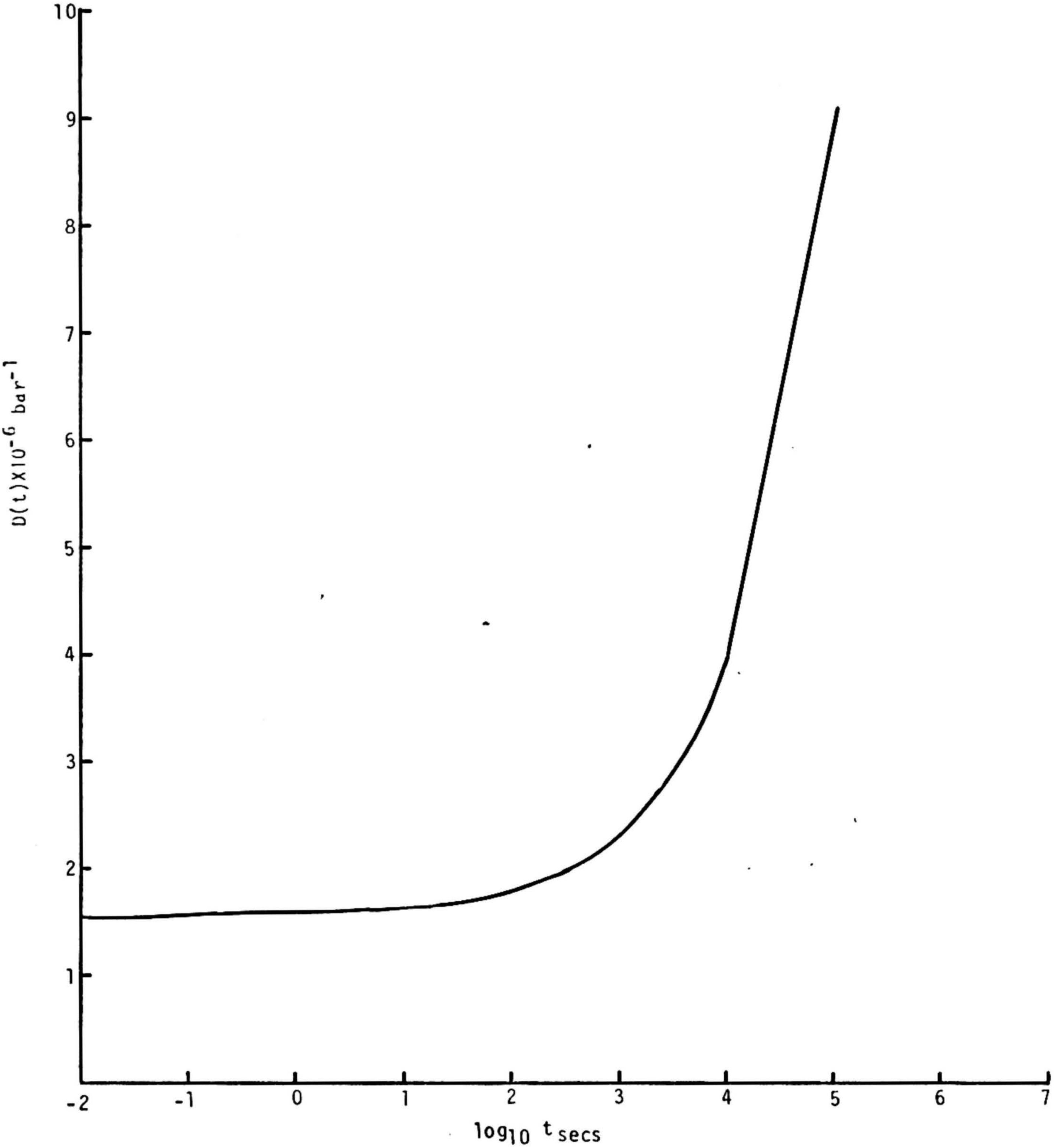
$$D(t) = gD_0[1 + (\frac{t}{a_T\tau_0})^{1/2}] \tag{17}$$

where $g = (\frac{\sigma}{\sigma_R})^{2/3}$, $\tau_0 = 4.66X10^3$, and a_T is given by equation 14.

The initial compliance, $D_0 \equiv \frac{\varepsilon_0}{\sigma}$ at 711°C and 640 bars, is estimated from Goetze's figures 6 and 7 in the following way: from his figure 6 we obtain $\frac{\varepsilon'}{\varepsilon_0} = 5X10^{-2}$ at 10 seconds; and from his figure 7, $\varepsilon' = 5X10^{-5}$ at 10 seconds, which gives $\varepsilon_0 = 1X10^{-3}$. Then $D_0 = \frac{1X10^{-3}}{640 \text{ bars}} = 1.56X10^{-6}$ bar^{1}. Since $E_0 = \frac{1}{D_0}$ this last value corresponds to an initial elastic modulus (E_0) of 640 kilobars.

A master curve for the creep compliance of Westerly Granite at an axial load of 640 bars, a temperature of 711°C, and a confining pressure of 4.5 kilobars is shown in figure 7. Creep compliances at other temperatures and axial loads are given by equation 17 with appropriate values for the vertical shift factor, g, which is related to stress level and the horizontal shift factor a_T, which depends on temperature. Some examples are given in figure 8. We see again that temperature has a pronounced effect; creep at lower temperatures requires greater times.

Figure 7.--Master curve based on data (Goetze, 1971) for Westerly Granite at σ=640 bars and T=711° C.



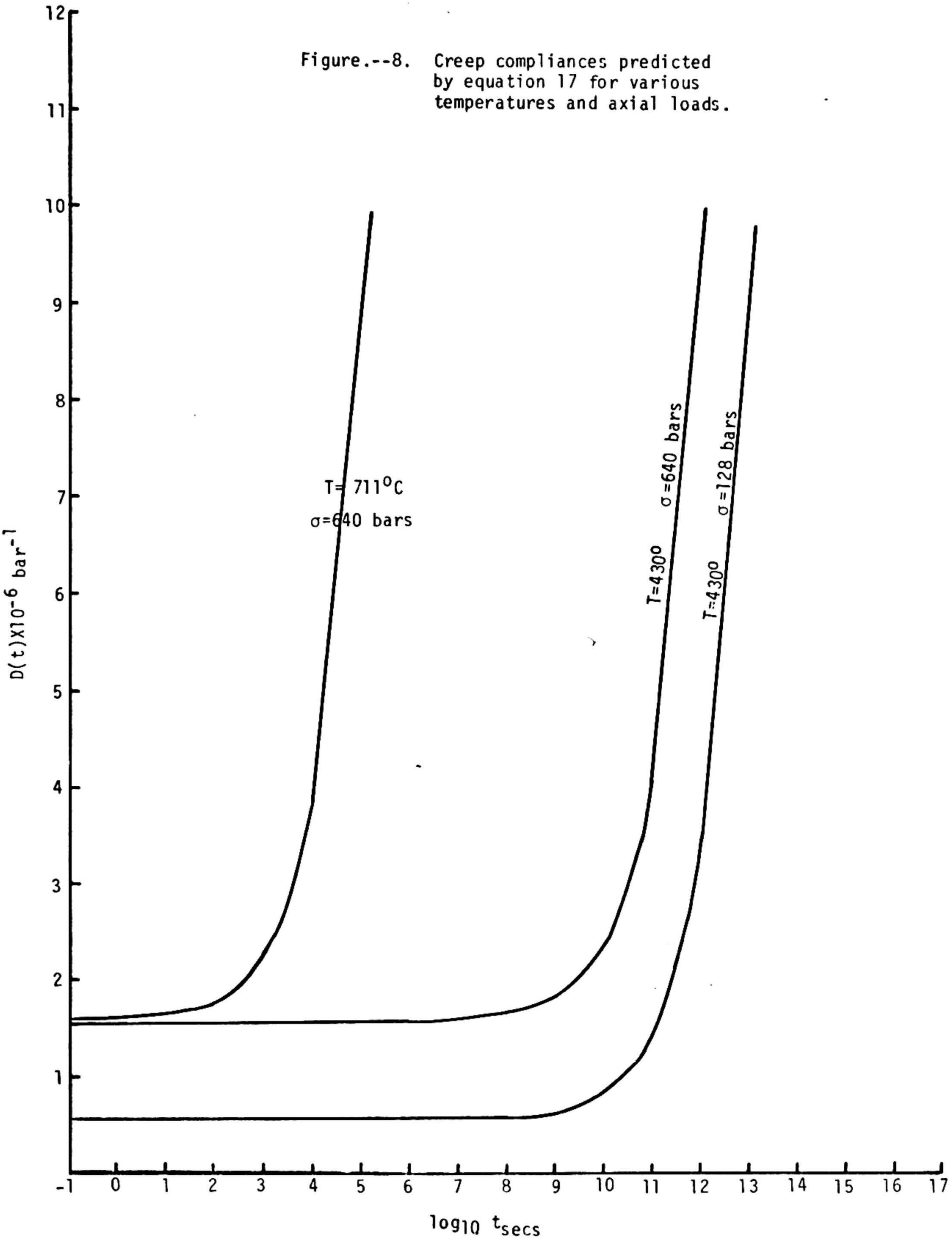


Figure.--8. Creep compliances predicted by equation 17 for various temperatures and axial loads.

Discussion

The significance of the master curves produced here for various rock types lies in the fact that measurements made at short times can be extrapolated to predict behavior at very long and geologically significant times. The value of a capability by which long-term rock properties can be predicted for any temperature or strain is obvious, particularly with regard to the design of nuclear waste repositories in geologic environments.

Unfortunately, constant strain-rate or creep tests at elevated temperatures and the moderate to low confining pressures more typical of the anticipated overburden depth are scarce. The effects of other environmental factors, such as moisture and fluid pressure on long-term rock behavior, are also little understood. The proper evaluation of potential geologic repositories must await further developments in experiments and the use of more realistic rheological models.

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