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Sensitivity Analysis for Seismic Risk

Using a Fault-Rupture Model

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Abstract

A model for ground motion at a site resulting from finite-length ruptures along a fault is analyzed. The model assumes that rupture length is a function of earthquake magnitude, that ruptures occur with equal probability at all allowable rupture locations, and that ground motion (acceleration) at a site is a function of the closest distance from the site to the causative rupture. Ruptures must be wholly contained within the fault limits, in contrast with the model of Der Kiureghian and Ang (1977) which permits a rupture to extend beyond the fault by one-half the rupture length.

Equations are developed for determining the average frequency with which ruptures along a fault will cause specified accelerations to be exceeded at a given site. It is shown that rupture length plays a large role in the exceedance rates obtained and that minimum and maximum earthquake magnitudes and overall fault length may also affect the values significantly. The effects of varying the parameters differ with site location and acceleration level.

The usual relationship between median rupture length and magnitude is assumed, $\log(l)=a+bm$. For each magnitude, rupture lengths are lognormally distributed about the median value with standard deviation σ_l. Using "long" (those corresponding, for example, to $1.5\sigma_l$) versus "short" ($-1.5\sigma_l$) rupture lengths for each magnitude may change the calculated exceedances of an acceleration by a factor of twenty or more. The expected number of exceedances obtained by integrating over rupture length may, in many cases, be approximated by using only a single rupture length per magnitude corresponding to a mean rupture length (which may differ considerably from the median length for the magnitude).

A series expansion permits acceleration exceedances to be separated into "point source" and "rupture length" contributions. For an Esteva attenuation of the form $\log a = c_1 + c_2 m + c_3 \log R$ the first "point source" term (assuming infinite maximum magnitude) is equivalent to Cornell's (1968) result for point sources (zero-length ruptures) on infinite faults; this term may account for only a small fraction of the total exceedances obtained when mean or median rupture lengths are used.

If accelerations are regarded as being lognormally distributed with standard deviation σ_a, the expected exceedances calculated for a fixed acceleration at a site may be significantly higher than those obtained when only the log-mean acceleration is used for each magnitude and distance.

Calculated ground motion exceedance rates may differ by a factor of two or more at sites a few kilometers apart near the end of a fault, and fault location, if not known exactly, should perhaps be treated probabilistically.

Introduction

Computer programs (Bender and Perkins, 1982, and McGuire 1976, 1978) that have been used extensively for seismic hazard analysis and mapping assume that earthquakes occur as points within source zones or as finite length ruptures along faults. This paper concerns only the latter situation, and develops equations for the rupture model used. The analytic development (as contrasted with the numerical approach of the computer programs) facilitates an examination of the model and enables a better understanding of the roles of the various parameters and interactions between them.

The rupture model to be investigated is a line-rupture model, meaning earthquakes occur as finite length breaks on a fault line. Acceleration at a site resulting from a single rupture of a given magnitude and location is a function of closest distance from the site to the rupture. This distance obviously is not equivalent to epicentral or hypocentral distance, and authors who use the latter rather than closest distance are in effect modeling point sources on a line.

Accelerations are evaluated using an attenuation relationship giving acceleration as a function of earthquake magnitude and distance. It does not seem possible to use hypocentral distance and adjust the attenuation function to take rupture length into account since, for example, in a rectangular coordinate system, a linear rupture with hypocenter at (0,0) and end points at (0,0), (X,0) would be at a closest distance of zero from a site at (X,0); a rupture with the same hypocenter but having end points at (0,0), (-X,0) would be at closest distance X from the same site. An attenuation relationship based on hypocentral distance would not be able to distinguish between the two cases in the example. For a magnitude 6.5 earthquake, a median rupture length given by Bonilla and Buchanan (1970) was 28 km, representing a non-neglible difference between possible hypocentral and closest distances.

Additionally, when acceleration resulting at a site from a rupture of finite length on a fault is regarded as a function of closest distance from the site to the rupture, a long rupture will tend to produce a higher acceleration at a site than will a short rupture, since the closest distance from a long segment to a site is likely to be less than that from a shorter segment.

The model to be analyzed will henceforth be referred to as the *fcr* or fault contained rupture model; it will be compared with the fault rupture model of Ang and Der Kiureghian (suggested by Ang, 1974, and expanded by Der Kiureghian and Ang, 1975, 1977). In the *fcr* model, ruptures must be wholly contained within the fault, whereas in the model of Der Kiureghian and Ang only rupture centers must be located on the fault. It will be shown that, especially for sites near the fault, either fault rupture model tends to give substantially more exceedances of an acceleration level than does the model (Cornell, 1968) in which earthquakes are regarded as point sources.

Equations are developed in this paper for determining exceedance rates of specified accelerations at a site using a general attenuation function and then using a particular (Esteva, 1969) form, $\log a = c_1 + c_2 \cdot m + c_3 \log R$. It will be shown that rupture length, fault length and earthquake magnitude range affect acceleration exceedance rates in a complicated manner depending on acceleration level and site location relative to the fault. Some sample results will be provided to illustrate the various effects and to compare models.

Model assumptions will first be listed, and then it will be shown how they are implemented in two computer programs. This will help to introduce some of the basic ideas, integrations, probability distributions, and geometry of the model.

Model Assumptions

Assumption 1. In an X-Y coordinate system, the fault is a single line segment of length L, located on the X axis extending from (0,0) to $(L,0)$. The site is located at (X,P).

Assumption 2. Rupture lengths are lognormally distributed as a function of magnitude with standard deviation σ_l. (See, for example, Bonilla and Buchanan, 1970). If the mean value of the log of the length of a rupture of a magnitude m earthquake is

$$\ln b_m = a + g\, m \tag{1}$$

where b_m = break or rupture length

m = Richter magnitude,

$a > 0$, $g > 0$ (constants),

the variation in $\ln b_m$ is normally distributed with standard deviation σ_l. The probability of a break of length b_m in the range

$$\exp[a + g\, m + fr\, \sigma_l] \le b \le \exp[a + g\, m + (fr + \varepsilon)\, \sigma_l] \tag{2}$$

is the area under the normal probability integral

$$pr(fr) = \frac{1}{\sqrt{2\pi}\sigma_l} \int_{fr\,\sigma_l}^{(fr+\varepsilon)\sigma_l} \exp\left(-\frac{x^2}{2\sigma_l^{\,2}}\right)dx \tag{3}$$

Assumption 3. A break of length b_m may with equal probability have its center at any point $(C,0)$ on the fault such that

$$\left(\frac{b_m}{2},0\right) \le (C,0) \le \left(L - \frac{b_m}{2},0\right).$$

The break must be wholly contained within the fault, and so break length cannot exceed fault length. [Ang's model requires instead that the break be centered on the fault: $(0,0) \le (C,0) \le (L,0)$ with a maximum break length of $2L$.]

Assumption 4. Peak acceleration (or velocity or some other measure of ground motion) at the site (X,P) is a function of earthquake magnitude and closest distance from the break to the point. Acceleraticns for each magnitude are distributed lognormally around the mean peak (log) value with standard deviation σ_a.

Assumption 5. Earthquakes are restricted to occur in the magnitude range $m_0 \leq m \leq m_{max}$. Their density is the truncated negative exponential distribution:

$$f(m) = \begin{cases} \dfrac{b \exp[-b\,(m - m_0)]}{1 - \exp[-b\,(m_{max} - m_0)]} & \text{for } m_0 \leq m \leq m_{max} \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

Assumption 6. The mean occurrence rate r per year of earthquakes for magnitudes in the range $m_0 \leq m \leq m_{max}$ for the fault remains constant during the time periods of interest.

Assumption 7. Earthquake occurrences have a Poisson distribution, that is, the probability of exactly k occurrences during time period t is

$$p(k) = \frac{(r\,t)^k \exp(-r\,t)}{k!}$$

where r = average rate per unit time.

Note that earthquakes are regarded as independent events; the fact that an earthquake did or did not occur at a specific time has no influence on subsequent earthquake arrivals.

Assumptions 1-6 are used to determine the yearly rate of exceedance $Pex(a_i)$ of specified levels of ground motion a_i at a site. Given $Pex(a_i)$, from assumption 7, the probability that no earthquake occurring on the fault during a period of t years will produce an acceleration $a \geq a_i$ is

$$P(a < a_i) = \exp[-Pex(a_i)\,t] \tag{5}$$

and the probability that one or more earthquakes will produce an acceleration $a \geq a_i$ is $P(a \geq a_i) = 1 - \exp[-Pex(a_i)\,t]$.

The computer program for seismic hazard evaluation, SEISRISK II (Bender and Perkins, 1982), finds accelerations that have specified probabilities of being exceeded during given t-year time periods. The seismic hazard program, FRISK (McGuire, 1978), determines instead accelerations corresponding to time periods t (which McGuire calls return periods) for which

$$t = \frac{1}{1-\exp[-Pex(a_i)]}.$$

The computation of exceedance rates $Pex(a_i)$ of various accelerations a_i in each of the computer programs will be described briefly in the following section. The programs differ in their approaches but yield consistent values of $Pex(a_i)$ when the same parameter values and attenuation function are used.

Current Computational Procedures

Both computer programs permit a fault to be composed of several connected straight-line segments, and allow contributions to $Pex(a_i)$ from a set of faults to be summed.

I. FRISK (Mc Guire, 1978).

FRISK divides the magnitude range $m_0 \leq m \leq m_{max}$ into a number (n_{mag}) of intervals Δm apart, and assumes that all earthquakes for which

$$m_j - \frac{\Delta m}{2} \leq m < m_j + \frac{\Delta m}{2} \qquad (1 \leq j \leq n_{mag})$$

occur at m_j, the midpoint of the j^{th} interval. A set of (n_{bl}) rupture length are selected for each magnitude so that the i^{th} length for the m^{th} magnitude:

$$b_{m_j} = \exp(a + g\, m_j + f_i \sigma_l) \tag{6}$$
$$(fr_{i-1}\sigma_l < f_i < fr_i \sigma_l)$$

occurs with probability p_i, where

$$p_i = \frac{1}{\sqrt{2\pi}} \int_{fr_{i-1}}^{fr_i} \exp(\frac{-x^2}{2})dx \qquad \text{and} \qquad \sum_{i=1}^{n_{bl}} p_i = 1. \tag{7}$$

Ruptures of length $b_{m_j}(i)$ have their centers at each of n evenly spaced locations Δl apart on the fault at distances along the fault

$$\frac{b_{m_j}(i)}{2}, \frac{b_{m_j}(i)}{2}+\Delta l, \frac{b_{m_j}(i)}{2}+2\Delta l, \ldots, L-\frac{b_{m_j}(i)}{2}$$

with probability $\frac{1}{n}$ for each location.

Since all n locations for each magnitude and rupture length are equally likely, the fractional seismicity associated with a single rupture $b_{m_j}(i)$ is

$$s[b_{m_j}(i)] = \frac{\rho_j p_i}{n} \tag{8}$$

where (from equation 4)

$$\rho(j) = \frac{\exp[-b(m_j - m_0)][\exp(\frac{b\,\Delta m}{2}) - \exp(\frac{-b\,\Delta m}{2})]}{1-\exp[-b(m_{max} - m_0)]} \tag{9}$$

= probability of an earthquake in the j^{th} magnitude interval.

The closest distance from one segment $b_{m_j}(i)$ to the site is determined and the mean acceleration for a magnitude m earthquake at that distance is computed from an assumed functional relationship between acceleration, earthquake magnitude and distance from site to source. If a_1 is the calculated mean log acceleration, the probability that the actual acceleration will exceed the value a_0 is (by model assumption 4),

$$pr(a \geq a_0) = \varphi^*[\frac{a_0 - a_1}{\sigma_a}] \tag{10}$$

where

$$\varphi^*(w) = \frac{1}{\sqrt{(2\pi)}} \int_x^{\infty} \exp(\frac{-x^2}{2})dx \tag{11}$$

= the complement of the standardized normal distribution.

The probability that an earthquake in the range $m_0 \leq m \leq m_{max}$ produces an acceleration $a \geq a_0$ at (X,P) then is the probability that $a \geq a_0$, given that a particular break location, break length and earthquake magnitude occur, weighted by the probability of that occurrence and summed over all cases.

$$pr[a \geq a_0] = \sum_{m=1}^{n_{mag}} \sum_{i=1}^{n_{bl}} \sum_{k=1}^{n} pr[a \geq a_0 | m_j, b_{m_j}(i,k), s(b_{m_j}(i))] \tag{12}$$

where $b_{m_j}(i,k) = i^{th}$ break length for magnitude m_j at k^{th} rupture location (rupture center on fault at distance $\frac{b_{m_j}(i)}{2} + (k-1)\,\Delta l$ from end).

The annual exceedance rate $Pex(a_i)$ then is $pr[a \geq a_i]$ multiplied by the annual rate of earthquakes in the range $m_0 \leq m \leq m_{max}$. FRISK calculates $Pex(a_i)$ for a selected set of a_i and interpolates to find the accelerations for the desired return periods.

In using the program for hazard mapping, calculating $\varphi^*[\frac{a_0 - a_i}{\sigma_a}]$ for each break for each magnitude is quite time consuming. The interpolation may be poor if the selected a_i are too far from the solution values. Choosing a small but efficient set of a_i may be difficult if a large geographical area is mapped and accelerations vary widely. Choosing a large set of a_i can result in wasted computation. These difficulties are largely avoided in the second program, which builds a histogram of acceleration occurrences at a site and then determines acceleration exceedances using the histogram.

II. SEISRISK II (Bender and Perkins, 1982)

In SEISRISK II (revised from the original undocumented program of S.T. Algermissen and D.M. Perkins, 1972), a table of 100 acceleration values is constructed such that the k^{th} entry a_k represents accelerations in the range $a_{k-1} \leq a < a_k$. Associated with these a_k's are accumulators into which fractional expected acceleration occurrences are summed. For a given magnitude, acceleration a in the range $a_{k-1} \leq a < a_k$ will be produced only if the point of a rupture closest to the site is within an appropriate distance range. Therefore, for each magnitude used, distances that correspond to the acceleration boundaries a_k are determined (by interpolating in a table of acceleration as a function of distance and magnitude). Let $d_k(j) =$ distance at which a magnitude m_j earthquake causes acceleration a_k.

For a specified rupture midpoint on the fault and rupture length, there is a unique closest distance from the rupture to the site. There may, therefore, be an interval of length $x[b_{m_j}(i)]$ along the fault such that ruptures with midpoints in this interval are at distance $d_k(j) < d \leq d_{k-1}(j)$ from (X,P). The fraction of breaks $b_{m_j}(i)$ yielding $a_{k-1} \leq a < a_k$ then becomes $\frac{x[b_{m_j}(i)]}{L - b_{m_j}(i)}$. Summing over magnitude and rupture length gives the contribution to accelerations in the range $a_{k-1} \leq a < a_k$.

$$C_k = \sum_{j=1}^{n_{mag}} \sum_{i=1}^{n_{bl}} \frac{x[b_{m_j}(i)]\, s[b_{m_j}(i)]}{L - b_{m_j}(i)} \tag{13}$$

where $s[b_{m_j}(i)]$ was defined in equation 8.

Since the accelerations a_k are mean values from a log normal distribution with standard deviation σ_a, to account for acceleration variability each C_k is redistributed into the set of accumulators so that the fraction placed in the j'^{th} accumulator for accelerations $a_{j-1}' \leq a < a_j'$ is proportional to an area under the normal probability curve. Repeating for all k gives the new entry C_j':

$$C'_j = \frac{1}{\sqrt{2\pi}\sigma_a} \sum_{k=1}^{n_{ac}} C_k \int_{\ln(a_{j-1}')}^{\ln(a_j')} \exp \frac{[-(x - V_k)]^2}{2\sigma_a{}^2} dx \tag{14}$$

= (new) fractional earthquake occurrences in range $a_{j-1} \leq a < a_j'$.

where

n_{ac} = subscript corresponding to the maximum mean log acceleration possible at (X,P)

$V_k = \frac{\ln(a_{k-1}) + \ln(a_k)}{2}$ = mean log acceleration for $a_{k-1} \leq a < a_k$.

A yearly exceedance rate $Pex(a_k)$ of accelerations a_k may be calculated dirctly as

$$Pex(a_k) = rate \sum_{j=k+1}^{k_{max}} C'_j \tag{15}$$

where $rate$ = expected total earthquakes in one year in the range $m_0 \leq m \leq m_{max}$

k_{max} = subscript corresponding to highest acceleration

after redistribution ($k_{max} \geq n_{ac}$).

Note that because σ_a is treated as independent of magnitude and distance, acclerations G_k may be "spread out" only after the acceleration histogram at $(\dot{X},P)$ has been completed. As a result, when earthquakes from a number of sources produce accelerations at the site, SEISRISK II evaluates the normal probability integral much less frequently than does FRISK, and the larger number of accelerations at which $Pex(a_i)$ is computed reduces the interpolation errors.

The two computer programs were run for a variety of parameter combinations and geometries to investigate the properties of the fault-rupture model. Both programs, however, are subject to some discretization error (from treating a continuous distribution of magnitudes, rupture length and locations as if they were concentrated at a set of distinct points) and the numerical procedures cause a certain lack of smoothness in the results. These inaccuracies complicate making sensitivity studies using the programs and might cause us to attribute to the model properties that are really properties of the approximations made.

In the analytic formulation, we shall be concerned principally with $Ex(a_k)$, exceedances of acceleration a_k before acceleration variability is taken into account, that is from equation 13,

$$Ex(a_k) = rate \sum_{j=k+1}^{100} G(j).$$

We begin by developing the appropriate equations for $Ex(a_k)$ for the fault-rupture model defined by model assumptions 1-6.

Analytic Development of Fault Rupture Equations

Integrals for acceleration exceedance rates at a site will be developed using a general attenuation function $a = f(m, R)$ in which acceleration at a site increases with earthquake magnitude m and decreases with distance R from the source. R is defined here as the distance from the site to the nearest point on the rupture.

If the nearest point on the rupture is at depth d with coordinates $(X_0, 0, -d)$, the distance to the site at (X, Y) is

$$R = \sqrt{(X-X_0)^2 + Y^2 + d^2} \tag{16}$$

This distance is identical to the distance from a surface rupture $(d = 0)$ to a point at (X, P) where

$$P = \sqrt{Y^2 + d^2}.$$

Because the attenuation function in this case involves only the equivalent of a surface distance R, we shall hereafter, without loss of generality, assume the site lies at (X, P). Define

$R_1(m)$ = distance at which an earthquake of magnitude m produces acceleration a_1 at (X, P).

All ruptures on the fault of magnitude m earthquakes that have at least one point closer to (X, P) than $R_1(m)$ will produce accelerations greater than a_1.

For each magnitude in the range of permitted magnitudes $m_0 \le m \le m_{max}$, the fraction of possible ruptures on the fault that will be closer than $R_1(m)$ to (X, P) is sought. This fraction depends partly on rupture length, which is a function of magnitude.

Since by model assumption 2, rupture lengths are lognormally distributed for each magnitude, define rupture length for magnitude m as a function of $fr\,\sigma_l$, that is

$$b_m(fr\,\sigma_l) = \exp(a + fr\,\sigma_l)\exp(g\,m) = H(fr\,\sigma_l)\exp(g\,m). \tag{17}$$

We shall calculate the rate of accelerations exceeding a_1 using, initially, a fixed value of fr in the rupture length-magnitude relationship; then we shall integrate over fr. Since $fr\,\sigma_l$ is constant for the time being, we shall simplify the notation by writing $b_m = H\exp(g\,m)$ for $b_m(fr\,\sigma_l) = H(fr\,\sigma_l)\exp(g\,m)$. H is now a constant multiplicative factor which stretches or shrinks the rupture length around its log-mean value for each magnitude, depending on the choice of fr. We first seek $Ex(a_1|H)$, the expected rate of exceedance of acceleration a_1 at (X,P) given H.

Two cases will be considered: the perpendicular from (X,P) intersects the extended fault line $(Y=0)$ beyond the end of the fault segment ($X<0$ or $X>L$); or the perpendicular from (X,P) intersects the fault segment itself ($0 \leq X \leq L$).

Case 1. $X<0$ (Site beyond end of fault.) (By symmetry $X>L$ could equivalently be considered.)

The point on the fault $x_l(m)$ that is at distance $R_1(m)$ from (X,P) is the solution to

$$R_1(m)^2 = [x_l(m) - X]^2 + P^2 \tag{18}$$

or

$$x_l(m) = \sqrt{R_1(m)^2 - P^2} + X.$$

Because the distance from (X,P) to a point on the fault $(l,0)$ increases as l increases, all ruptures of magnitude m earthquakes that have their closest endpoints at $0 \leq l \leq x_l(m)$ will produce acceleration $a \geq a_1$ at (X,P). Equivalently, all ruptures with midpoints $l = l_{mid}$ on the fault for which

$$\frac{b_m}{2} \leq l_{mid} \leq x_l(m) + \frac{b_m}{2} \tag{19}$$

will contribute to $Ex(a_1|H)$ (figure 1A).

There is a magnitude m_{lo} below which a_1 will not be produced at (X,P) by a rupture anywhere on the fault. This corresponds to $x_l(m_{lo}) = 0$, and

$$R_1(m_{lo}) = \sqrt{X^2 + P^2}.$$

Case 1. Site at (X,P) beyond end of fault, $X < 0$.

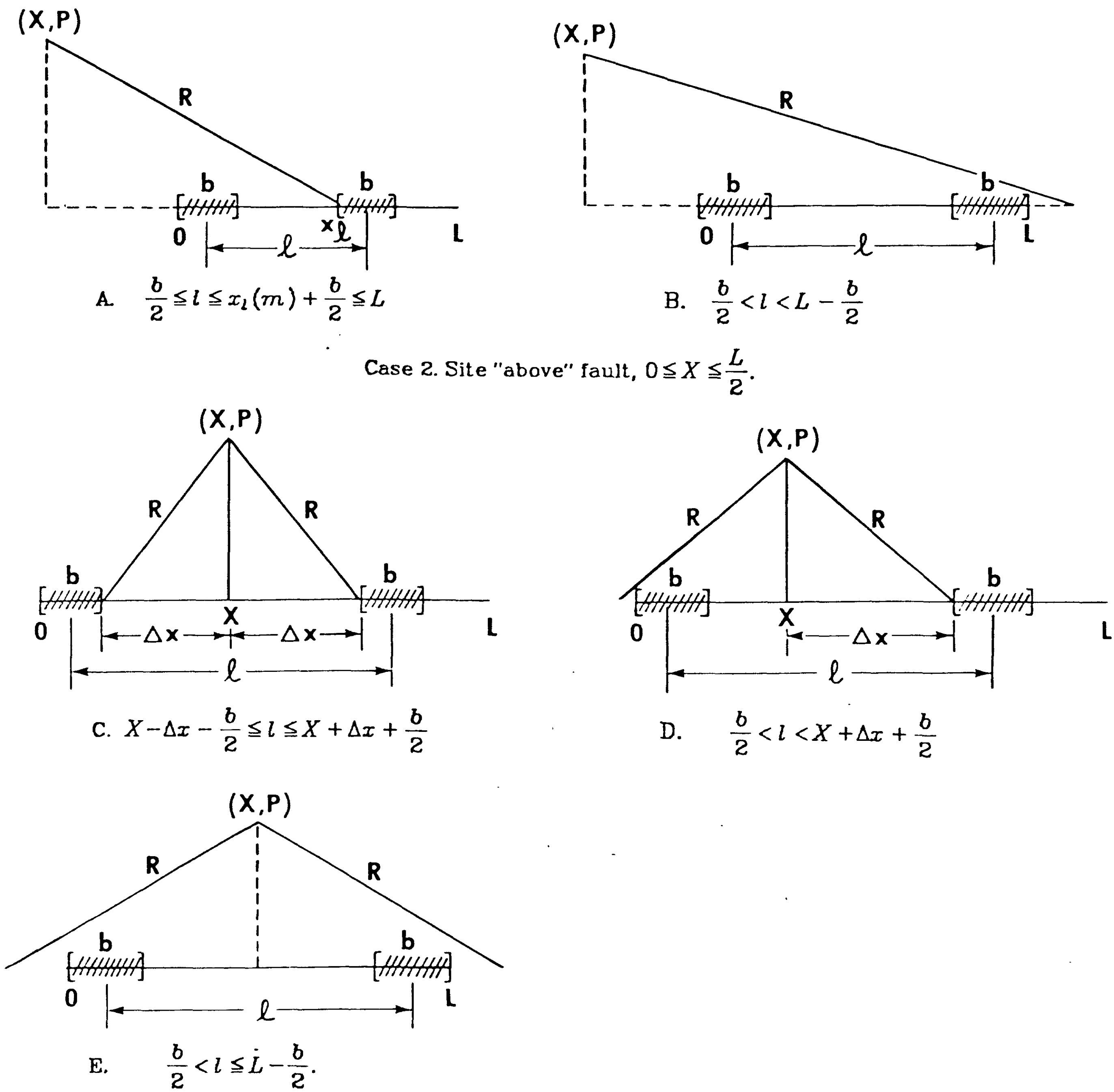

Figure 1. Locations on a fault of centers of magnitude m earthquakes yielding accelerations $a \geq a_1$ at (X,P). Fault extends on x-axis from $x=0$ to $x=L$. Ruptures of magnitude m earthquakes centered on fault at $x=l$ will cause accelerations $a \geq a_1$ at (X,P) as indicated.

$R = R_1(m) =$ radius within which a magnitude m earthquake will produce acceleration $a \geq a_1$ at (X,P).

l_{mid} = possible rupture-center locations on a fault of length L of a magnitude m earthquake of ruptures of length $b = b_m$. $\frac{b_m}{2} \leq l_{mid} \leq L - \frac{b_m}{2}$.

$x_l(m) =$ point(s) on fault intersected by a circle of radius $R_1(m)$ centered at (X,P).

Since magnitudes below m_0 are not permitted, set

$$m_{min} = \max(m_0, m_{l_0}). \tag{21}$$

If $m_{min} > m_{max}$, where m_{max} is the maximum permitted magnitude, a_1 does not occur at (X,P). Therefore, we may assume $m_{min} < m_{max}$.

There is a magnitude m_z above which all ruptures regardless of location on the fault will yield $a > a_1$ (figure 1B). This is the magnitude $m = m_z$ for which

$$x_l(m) = L - br(m) \tag{22}$$

where $$br(m) = \min(b_m, L)$$

(If rupture length b_m as given by equation 17 would exceed fault length L, rupture length is set equal to fault length). For $m = m_z$

$$L - br(m_z) = \sqrt{R_1(m_z)^2 - P^2} + X, \tag{23}$$

(There is a unique solution m_z since both $br(m)$ and $R_1(m)$ are nondecreasing as m increases.)

Set

$$m_u = \begin{cases} m_{min} & m_z < m_{min} \\ m_z & m_{min} \leq m_z \leq m_{max} \\ m_{max} & m_{max} < m_z . \end{cases} \tag{24}$$

The fraction of earthquakes of magnitude m that yields $a \geq a_1$ at (X,P) is :

$$pr(a \geq a_1 | H, m) = \begin{cases} 0 & m < m_{min} \\ \dfrac{\sqrt{R_1(m)^2 - P^2} + X}{L - H\exp(gm)} & m_{min} \leq m \leq m_u \\ 1 & m_u < m \leq m_{max} \end{cases} \tag{25}$$

Using equation 25 together with model assumptions 2 and 5 gives for $X < 0$:

$$Ex(a_1|H) = rate \int_{m_0}^{m_{max}} f(m)\, pr(a \geq a_1 | H, m) = \tag{26}$$

$$\frac{rate\, b\exp(b\, m_0)}{1 - \exp[-b(m_{max} - m_0)]} \left\{ \int_{m_{min}}^{m_u} \exp(-b\, m) \frac{\sqrt{R_1(m)^2 - P^2} + X}{L - H\exp(g\, m)} dm + \int_{m_u}^{m_{max}} \exp(-b\, m)\, dm \right\}$$

$$= \qquad I_u \qquad + \qquad I_{all}$$

(Either integral may have a zero range of integration.)

Case 2. $0 \le X \le \frac{L}{2}$ (Site "above" fault)

Let (X,P) be a site such that a perpendicular from the site to the extended fault line intersects the fault at some interior point. Assume $0 \le X \le \frac{L}{2}$. (By symmetry, $\frac{L}{2} < X \le L$ could equivalently be considered.) As in Case 1, we wish to determine the fraction of the fault on which earthquakes producing accelerations $a \geqq a_1$ at (X,P) may originate.

Again let m_{lo} = minimum magnitude at which a_1 is felt at (X,P). This corresponds to $R_1(m_{lo}) = P$, and the point on the fault line that is at distance $R_1(m_{lo})$ from (X,P) is at $(X,0)$, that is, $x_l(m_{lo}) = X$. Let $m_{\min} = \max(m_0, m_{lo})$.

For $m = m_{lo}$, only ruptures on the fault intersecting $(X,0) = [x_l(m_{lo}),0]$ will produce a_1 at (X,P). For magnitudes $m > m_{lo}$, there will be two points $x_{l_1}(m)$ and $x_{l_2}(m)$ on the extended fault line for which

$$R_1(m) = \sqrt{\Delta x(m)^2 + P^2} \tag{27}$$

that is,

$$x_{l_1}(m) = X - \Delta x(m) \quad \text{and} \quad x_{l_2}(m) = X + \Delta x(m)$$

for some $\Delta x(m)$.

For a magnitude m earthquake all ruptures with midpoints l_{mid} in the range

$$X - \Delta x(m) - \frac{b_m}{2} \le l_{mid} \le X + \Delta x(m) + \frac{b_m}{2}$$

will contribute to exceedances of a_1 at (X,P). However, since rupture length cannot exceed fault length and ruptures are required to be wholly contained within the fault l_{mid} is further restricted to lie within

$$\frac{b_m}{2} \le l_{mid} \le L - \frac{b_m}{2}$$

or

$$\max(X - \Delta x(m) - \frac{b_m}{2}, \frac{b_m}{2}) \le l_{mid} \le \min(X + \Delta x(m) + \frac{b_m}{2}, L - \frac{b_m}{2}). \tag{28}$$

Several cases must, therefore, be considered.

(a) Magnitudes m for which $X - \Delta x(m) - b_m \geq 0$.

For magnitudes m for which this holds, the fault is so long that even if the fault were longer, any additional ruptures of length b_m would be too far away from the site (X, P) to produce accelerations $a \geq a_1$ (figure 1C).

Let m_{x_1} be the magnitude for which equality holds $X = \Delta x(m) + b_m$. Define m_{u_1} as in equation 24. For $m_{\min} \leq m \leq m_{u_1}$, all ruptures with midpoints l_{mid} in the range

$$X - \Delta x_l - \frac{b_m}{2} \leq l_{mid} \leq X + \Delta x_l + \frac{b_m}{2}$$

will contribute to $Ex(a_1|H)$. This corresponds to a fraction of possible magnitude m earthquakes

$$fr(m) = \frac{2\Delta x(m) + b_m}{L - b_m} \tag{29}$$

giving for $m_{\min} \leq m \leq m_{u_1}$,

$$pr(a \geq a_1, |H, m) = \frac{2\sqrt{R_1(m)^2 - P^2} + H \exp(g\, m)}{L - H \exp(g\, m)} \tag{30}$$

(b) Magnitudes $m > m_{u_1}$ for which $X + \Delta x(m) + b_m \leq L$.

At these magnitudes, additional ruptures of length b_m would contribute to accelerations $a > a_1$ if the fault extended beyond (0,0) in the negative x-direction. Adding fault length beyond $(L, 0)$ in the positive x-direction would not increase the possible ruptures that produce $a \geq a_1$ at (X,P). (Recall $X \leq \frac{L}{2}$, figure 1D). Let m_{x_2} be the magnitude for which $L - X = \Delta x(m_{x_2}) + b_{m_{x_2}}$.

($m_{x_2} \geq m_{x_1}$ since $L - X \geq X$ based on the assumption that $X \leq \frac{L}{2}$.)

Define m_{u_2} as in equation 24. For $m_{u_1} \leq m \leq m_{u_2}$, all magnitude m ruptures for which

$$\frac{b_m}{2} \leq l_{mid} \leq X + \Delta x(m) + \frac{b_m}{2}$$

will contribute for a fraction

$$fr(m) = \frac{X + \Delta x(m)}{L - b_m} \tag{31}$$

giving for $m_{u_1} \leq m \leq m_{u_2}$

$$Pr(a \geq a_1 | H,m) = \frac{\sqrt{R_1(m)^2 - P^2} + X}{L - H\exp(g\,m)} \tag{32}$$

(c) Magnitudes $m_{u_2} \leq m \leq m_{\max}$

All earthquakes regardless of their location on the fault will contribute to exceedances of a_1 (figure 1E).

For $m_{u_2} \leq m \leq m_{\max}$

$$Pr(a \geq a_1 | H, m) = 1. \tag{33}$$

Combining these results yields, for $0 \leq X \leq \frac{L}{2}$,

$$Ex(a_1|H) = \frac{b\ rate\ \exp(b\,m_0)}{1 - \exp[-b\,(m_{\max} - m_0)]} \left\{ \int_{m_{\min}}^{m_{u_1}} \exp(-b\,m) \frac{2\sqrt{R_1(m)^2 - P^2} + H\exp(g\,m)}{L - H\exp(g\,m)}\,dm \right.$$

$$\left. + \int_{m_{u_1}}^{m_{u_2}} \exp(-b\,m) \frac{\sqrt{R_1(m)^2 - P^2} + X}{L - H\exp(g\,m)} + \int_{m_{u_2}}^{m_{\max}} \exp(-b\,m)\,dm \right\} \tag{34}$$

$$= \quad I_{u_1} \quad + I_{u_2} \quad + I_{all}.$$

Equations 26 and 34 are the general fault-rupture equations.

A closer examination of these equations for $Ex(a_1|H)$ will provide some insight into the effect of magnitude limits m_0, m_u, and $m_{\max}$ and the role of fault length, break length and site location. Along with the general results, we shall note some specific properties of the model when the widely used Esteva (1969) attenuation relationship for (log) acceleration as a function of distance and magnitude is used to define $R_1(m)$. We shall use the form

$$\ln a = c_1 + c_2 m + c_3 \ln R; \quad c_1 > 0, \quad c_2 > 0, \quad c_3 < 0. \tag{35}$$

In this case,

$$R_1(m) = a_1^{\frac{1}{c_3}} \exp(-\frac{c_1}{c_3} - \frac{c_2}{c_3} m). \tag{36}$$

(1) Minimum magnitude, m_0.

The choice of m_0 may significantly affect the exceedance rate of some accelerations. If m_{l_0} the minimum magnitude yielding acceleration a_1 at (X,P) is less than m_0, the minimum assumed, then including additional earthquakes in the range $m_{l_0} \leq m < m_0$ will increase the exceedances of a_1 by an amount equal to the change in $Ex(a_1 | H)$ resulting when $m_{min} = m_{l_0}$ rather than $m_{min} = m_0$ is used as the lower limit of integration in equations 26 and 34. Loss of acceleration exceedances due to minimum magnitude cutoff should be suspected when a magnitude less than m_0 will produce accelerations greater than the acceleration of interest for some possible site-to-source distances.

More importantly, even when the minimum magnitude cutoff does not cause loss of acceleration exceedances at levels of interest when a single acceleration is assigned to each magnitude and distance, it may do so when acceleration variability is taken into account. If a lognormal distribution of acceleration with standard deviation σ_a is assumed, accelerations are in effect "spread out" around their (log) mean values. Since there may be fewer accelerations $a < a_1$ to redistribute when magnitudes below m_0 are eliminated, ignoring these magnitudes may reduce the rate (after redistribution) of some accelerations $a > a_1$.

For some reasonable values of c_1, c_2 and c_3 in the Esteva attenuation (equation 35), using $m = 4.5$ as opposed to $m = 3.0$ (where the rate for earthquakes with magnitudes $m > 4.5$ remains the same in both cases) changes the frequency of accelerations up to .2g at distances of 10-20 km from the fault. The difference is greater when acceleration variability is taken into account than when only (log) mean values are used (figure 2).

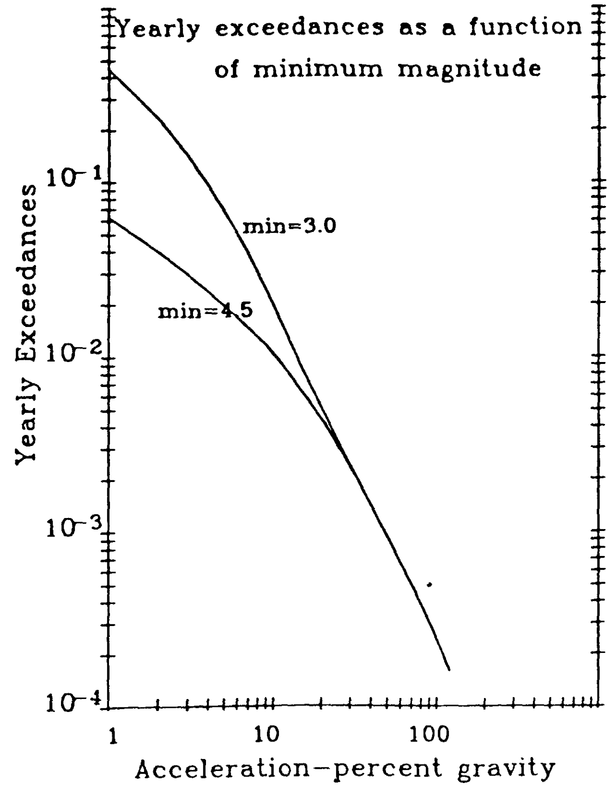

Figure 2. Acceleration exceedances at a site 10 km from the center of a 400 km fault when magnitudes are restricted to the range $4.5 \leq m \leq 7.5$ and when additional magnitudes $3.0 \leq m < 4.5$ are also included. Acceleration variability $\sigma_a = .6$
Attenuation function $\ln a = 3.4 + .89m - 1.17 \ln R$;
$rate = .1$ earthquakes per year in the range $4.5 \leq m \leq 7.5$; $b = 2$.

(2) m_u or (m_{u_s}): magnitude above which all ruptures occurring anywhere on the fault will yield acceleration $a > a_1$ at (X, P) for H fixed.

The fraction $fr(m)$ of magnitude m earthquakes that contributes to acceleration a_1 increases steadily with magnitude in the range

$$0 \leq fr(m) < 1 \qquad m_{\min} \leq m < m_j$$

and remains constant

$$fr(m) = 1 \qquad\qquad m_j \leq m \leq m_{\max} \tag{37}$$

where

$$m_j = \begin{cases} m_u \text{ for site at } (X,P), & X < 0, \text{ or } X > L \\ m_{u_s} \text{ for site at } (X,P), & 0 \leq X \leq L . \end{cases}$$

Thus for fixed m_{min} and m_{max}, as m_j decreases the fraction of earthquakes $m_{min} \leq m \leq m_{max}$ that cause acceleration a_1 to be exceeded at (X,P) increases. But the value of m_j depends upon rupture length, site location, acceleration level and fault length. An understanding of the roles of each of these parameters will help make clear certain behaviors of the model.

(a) $Ex(a_1|H)$ as a function of fault length and of the x-coordinate of site location.

Consider sites at $(0,P)$ and $(\frac{L}{2},P)$ at distance P perpendicular to the end to the center of the fault, respectively. The same minimum magnitude m_{min} will produce acceleration a_1 at both these sites. Yet $Ex(a_1|H)$ at $(\frac{L}{2},P)$ may be (perhaps considerably) more than twice the value of $Ex(a_1|H)$ at $(0,P)$ or even equal to it, depending on fault length L and acceleration a_1 (figure 3).

To see this, note that if the fault is sufficiently long, the entire contribution to $Ex(a_1|H)_{(\frac{L}{2},P)}$ is from integral I_{u_1} and to $Ex(a_1|H)_{(0,P)}$ is from integral I_{u_2} in equation 34, yielding

$$Ex(a_1|H)_{(\frac{L}{2},P)} \geq 2Ex(a_1|H)_{(0,P)}. \tag{38}$$

(Equality holds only at $H = 0$, that is for point sources or zero-length ruptures.)

On the other hand, the acceleration a_1 and fault length may be such that the entire contribution to the exceedances of a_1 at both sites is from the integral I_{all}. In this case,

$$Ex(a_1|H)_{(\frac{L}{2},P)} = Ex(a_1|H)_{(0,P)} \tag{39}$$

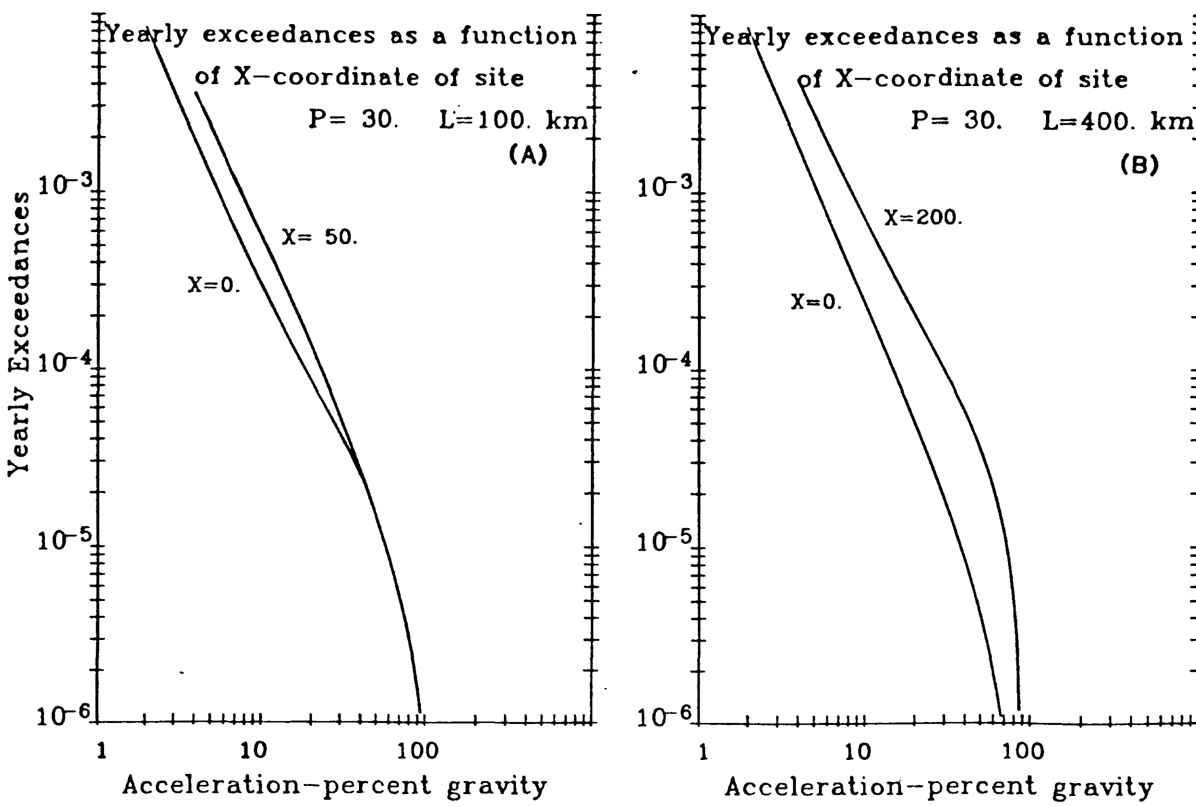

Figure 3. Relative acceleration exceedances at sites 30 km from the center and 30 km from the end of faults 100 and 400 km long; $4.0 \leq m \leq 8.5$; $b = 2$.

(b) Acceleration exceedances as a function of P (perpendicular distance) of the site from the fault and fixed m_{lo} (for the Esteva attenuation.)

For two sites with the same X-coordinate and different P, (X,P_1) and (X,P_2), let m_{lo} (fixed) be the lowest magnitude producing acceleration a_1 at (X,P_1) and a_2 at (X,P_2). We shall show that if $P_2 > P_1$, a higher fraction of the earthquakes in the range $m_{lo} < m < m_{max}$ contributes to $Ex(a_1 \mid H)_{(X,P_1)}$ than contributes to $Ex(a_2 \mid H)_{(X,P_2)}$. For the Esteva attenuation, a_i is the acceleration for which

$$\ln a_i = c_1 + c_2 m_{lo} + c_3 \ln P_i, \qquad i = 1,2. \tag{40}$$

and for a rupture at distance $R_i = k\ P_i$, where $k =$ a constant, a_i is produced by the magnitude m_i for which

$$\ln a_i = c_1 + c_2 m_i + c_3 \ln(k P_i) \qquad i=1,2$$

from which it follows that $-\frac{c_3}{c_2}\ln k = m_i - m_{lo}$, $i=1,2$ Therefore, $m_1 = m_2$ and d_i, the length of fault on which magnitude m_i earthquakes will produce acceleration a_i at (X, P_i) is

$$d_i = \sqrt{R_i^{\,2} + P_i^{\,2}} = P_i\sqrt{k^2 - 1} \tag{41}$$

Hence a larger fraction of earthquakes at magnitudes $m_{lo} < m$ will contribute to $Ex(a_1 | H)_{(X,P_1)}$ than to $Ex(a_2 | H)_{(X,P_2)}$, and for a fixed magnitude range, the more distant site is affected by earthquakes along a higher fraction of the fault (figure 4).

(3) Maximum magnitude m_{max}

Because high accelerations can result from high magnitude earthquakes, uncertainty in specifying the "true" m_{max} has been of considerable concern in risk analysis. We shall assume the rate of earthquake occurrences remains fixed for $m_o \le m_{max}(old)$ and investigate the effect of adding additional earthquakes in the range $m_{max}(old) < m \le m_{max}(new)$. That is, we shall assume

$$f(m) = \begin{cases} \dfrac{b\exp[-b(m-m_0)]}{1-\exp[-b(m_{max}(old)-m_0)]}dm & \text{for } m_0 \le m \le m_{max}(new) \\ 0 & \text{elsewhere} \end{cases}$$

The precise effect of m_{max} on exceedances of a given acceleration at a particular site can be determined by adjusting the integration limits involving m_{max} and possibly m_u, m_{u_1} m_{u_2} in equation 26 or 34 and evaluating the integrals for the respective values. As illustrated in figure 5A and 5B, the relative effect of changing m_{max} on the exceedances of a given acceleration level is not the same at different sites. The relative effect on different accelerations at the same site is also non-uniform (figure 5C). We shall give a heuristic argument why this is so.

The highest acceleration a_{max} possible at a site is produced by a magnitude m_{max} earthquake in which the rupture overlaps the point on the fault closest to

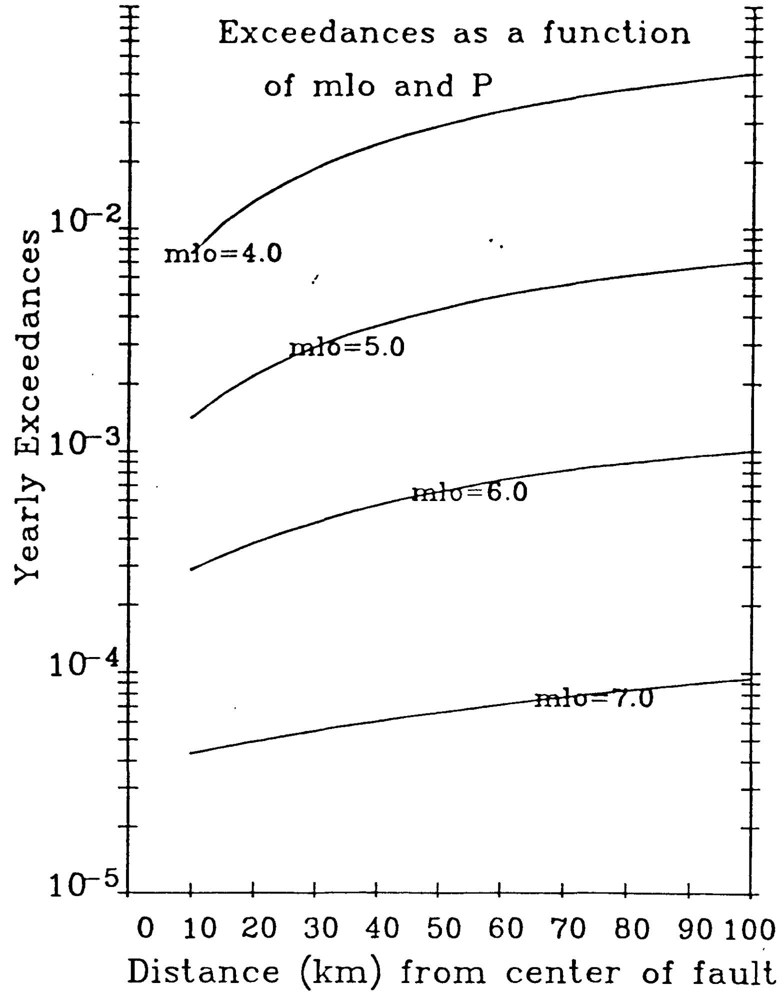

Figure 4. Acceleration exceedances as a function of m_{lo}, P.
m_{lo} = lowest magnitude that produces acceleration a_p at (X, P); $m_{max} = 7.5$.
In this example, an Esteva attenuation, $\ln a_p = c_1 + c\, m_{lo} + c_3 \ln P$ is used; the site is at $(200, P)$; the fault extends from (0,0) --(400,0). For m_{lo} fixed, the same magnitude range $m_{lo} \leq m \leq m_{max}$ yields more exceedances of a_p as P increases, or the site is affected by more earthquakes along the fault in a given magnitude range as the perpendicular distance from the site to the fault increases.

the site. In this case, $m_{min} = m_{max}$. Occurrences of accelerations near a_{max} will obviously be greatly affected by small changes in m_{max}. However, if an acceleration a_1 is considerably less than a_{max}, then m_{min} is considerably less than m_{max}, and changing m_{max} by a small amount will have little effect on the exceedances of a_1. That is, if $m_{max}(new)$ is such that

$$\tau = \frac{m_{max}(new) - m_{max}(old)}{m_{max}(old) - m_{min}} \ll 1,$$

even including all the additional earthquakes at magnitudes $m_{max}(old) < m \leq m_{max}(new)$ does not substantially affect the exceedances of a_1,

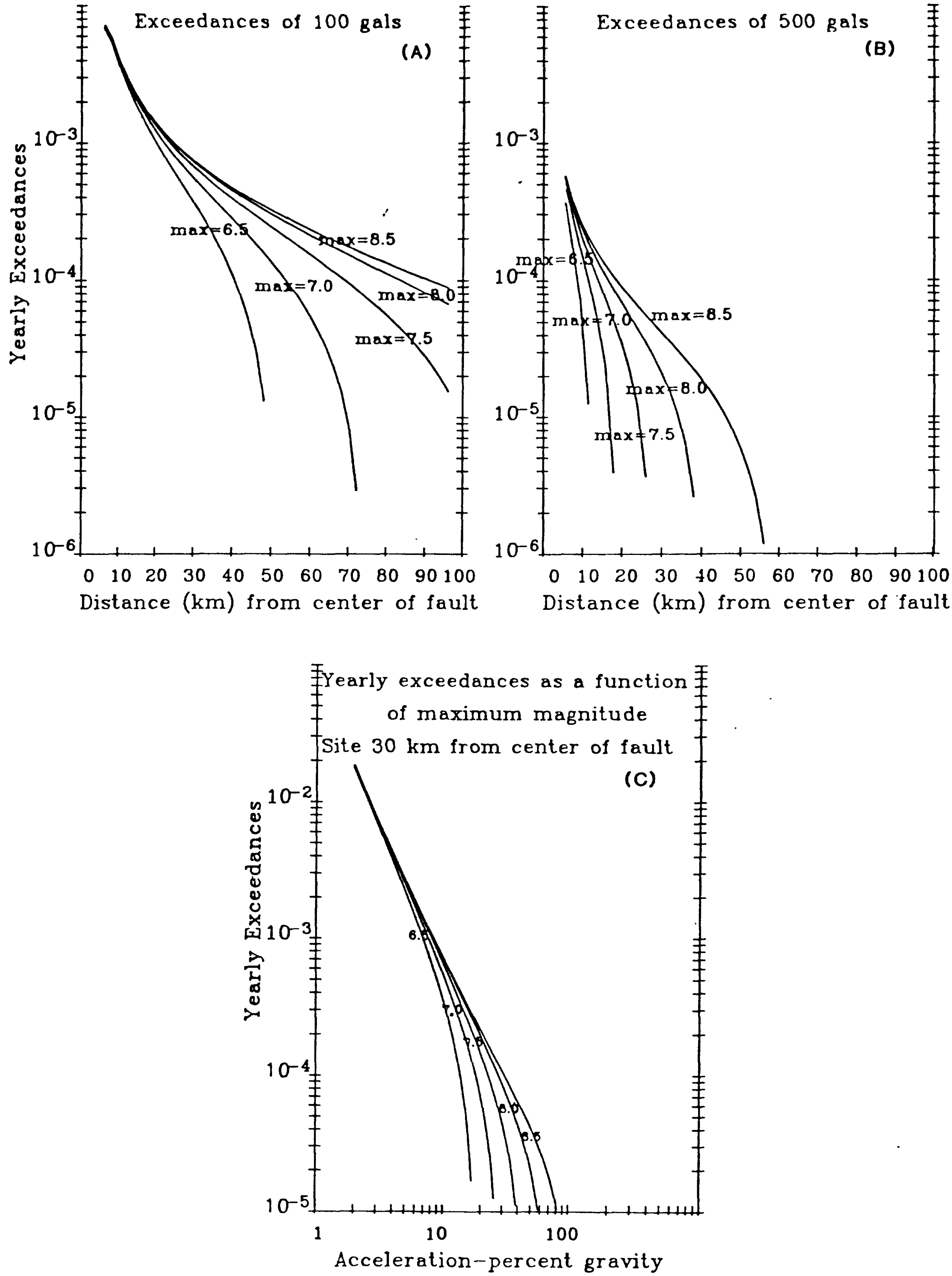

Figure 5. Acceleration exceedances as a function of maximum magnitude.
A, B: Exceedances of .1g and .5 g for sites at varying perpendicular distances P from the center of a 400 km fault.
C: Exceedances of various acceleration levels at a site 30 km. from the center of the fault.

since earthquake occurrences decrease exponentially with magnitude and the large number at lower magnitudes (near m_{min}) will dominate. However, as m_{min} increases (corresponding at a fixed site to a higher a_1), small absolute changes in m_{max} become larger fractional changes in τ and cause correspondingly larger fractional changes in $Ex(a_1|H)$.

Now let $a_1 = a_{max}(old)$ at a site. We shall look at how exceedances of a_1 change when $m_{max}(new) = m_{max}(old) + \Delta m$ at sites at $(\frac{L}{2}, P)$ and $(0, P)$.

Using equations 26 and 34, the incremental exceedances $\Delta Ex(a_1|H)$ resulting from earthquakes in the range $m_{max}(old) < m < m_{max}(new)$ is approximated by

$$\Delta Ex(a_1|H)_{(\frac{L}{2},P)} = \min \begin{cases} Q \dfrac{[2\Delta d + H \exp(gm)] \Delta m}{L - H \exp(g\, m)} \\ Q \end{cases} \tag{42}$$

$$\Delta Ex(a_1|H)_{(0,P)} = \min \begin{cases} \dfrac{Q\, \Delta d\, \Delta m}{L - H \exp(g\, m)} \\ Q \end{cases} \tag{43}$$

$$Q = \frac{rate \exp[-b\,(m - m_0)]}{1 - \exp[-b\,(m - m_0)]}, \qquad \Delta d = \sqrt{R_1(m)^2 - P^2}. \tag{44}$$

Since $R_1[m_{max}(old)] = P$, Δd is small when Δm is small and the "break length term" $H \exp(g\, m)$ may contribute most of the exceedances of a_1 at $(\frac{L}{2}, P)$. This term vanishes for a site at $(0, P)$. Thus at the highest accelerations possible at (X, P) the incremental absolute effect of a small change Δm in maximum magnitude may depend upon rupture length and site location.

Some additional remarks may be made when the Esteva attenuation function, (equation 35) is used. In this case, the maximum acceleration possible at (X, P) increases exponentially with m_{max}. Now a_{max} is that value of a for which

$$\ln a_{max} = c_1 + c_2 m_{max} + c_3 \ln CD\ , \tag{45}$$

where

$$CD = \begin{cases} P & 0 \le X \le \frac{L}{2} \\ \sqrt{P^2 + X^2} & 0 < X \end{cases}$$

and

$$a_{max}(new) = a_{max}(old)\exp[c_2(m_{max}(new) - m_{max}(old))]\,. \tag{46}$$

If the acceleration range at a site is defined as

$$Ar = a_{max} - a_0 \tag{47}$$

where

$$\ln a_0 = c_1 + c_2 m_0 + c_3 \ln CD, \tag{48}$$

for sites at (X, P_1) and (X, P_2)

$$\frac{a_{max(X,P_1)}}{a_{0(X,P_1)}} = \frac{a_{max(X,P_2)}}{a_{0(X,P_2)}} \tag{49}$$

Because the maximum acceleration possible at the site decreases as P increases, the acceleration range becomes smaller as P increases. It follows that the effect of maximum magnitude on acceleration a_1 at (X, P) depends upon where in the acceleration range a_1 lies for that site.

(4) Fault length.

Let the earthquake rate r per year per kilometer of fault be fixed. If the fault is short enough, adding fault length will increase the number of earthquakes that may be felt and hence increase the rate of exceedance of most accelerations at a site. Beyond some point, in this model, however, adding fault length will actually cause the exceedances calculated to decrease (to some limit). The length at which acceleration exceedances begin to decrease depends upon the site location, attenuation function, acceleration level and other parameters. The role of fault length may be explained as follows:

Consider faults of length L and length $k\,L$, where $k > 1$. Let the faults extend from $(0,0)-(L,0)$ and $(0,0)-(k\,L,0)$ respectively. For simplicity assume the site is either at $(0,P)$, above the end of the fault, or above the midpoint at $(\frac{L}{2},P)$ or

$(\frac{k\,L}{2}, P)$.

If the integral I_{all} does not contribute to $Ex(a_1|H)$, the only quantities that change in equation 34 when fault length is increased are those involving fault length and rate, and since

$$\frac{r\,L}{L - H\exp(g\,m)} > \frac{r\,k\,L}{k\,L - H\exp(g\,m)}$$

where r =earthquake rate per unit of fault length

$r\,L$ (or $r\,k\,L$)= total earthquakes along fault (of length L of $k\,L$),

the shorter fault will yield more exceedances of a_1 at (X, P) than will the longer fault.

On the other hand, if for both faults all earthquakes contribute to $Ex(a_1|H)$ regardless of location (the only integral involved in both cases is I_{all}), then ruptures on the longer fault will produce k times as many exceedances of a_1 at (X,P) as will those on the shorter fault.

Figure 6A illustrates a case in which there are more exceedances of accelerations at all levels at a site near the center a short (100 km) fault than near the center of a longer (400 km) fault having thc same rate of earthquakes per unit length. The situation is clearly reversed at a site further away (at a greater distance P) from the center of the fault. Figure 6B illustrates exceedances of a fixed acceleration at a site as a function of fault length where, again, earthquake rate per unit length is constant.

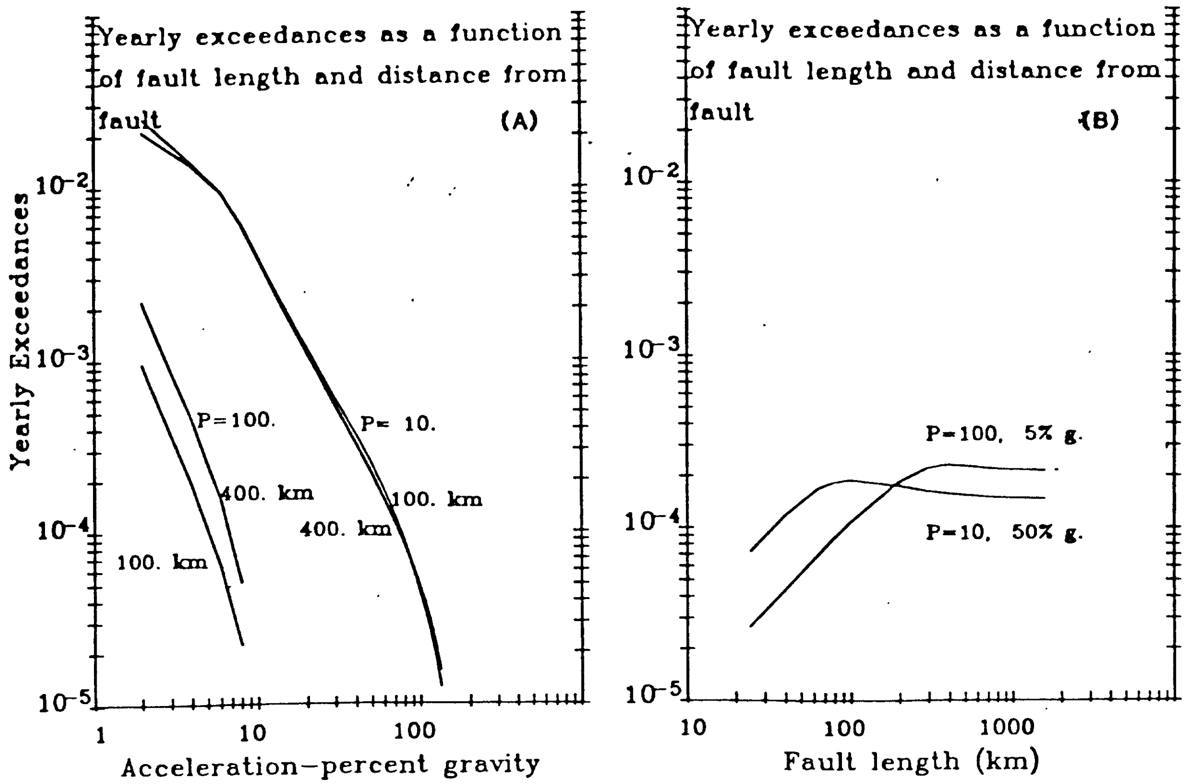

Figure 6. Acceleration exceedances as a function of fault length when earthquake rate per unit length remains constant.
A: Sites 10 km and 100 km from the centers of faults 100 km and 400 km long.
B: Relative exceedances of .05 g at sites 100 km and of .5 g at sites 10 km from the centers of faults that vary in length from 20 km to 1000 km.

(5) Rupture Length.

Rupture lengths for three magnitude rupture length relationships (for a log normal distribution with standard deviation $\sigma_l = 1.20$) are illustrated in Table 1. For each relationship, a great variation in length may be seen particularly at higher magnitudes between the longest and shortest ruptures for each magnitude.

Recall that the analysis thus far (equations 26 and 34) has assumed a constant value of $f\tau$

$$H = H(f\tau\,\sigma_l) = \exp(a + f\tau\,\sigma_l)$$

and that $f\tau$ is a stretching factor--ruptures become longer at each magnitude as $f\tau$ increases. Recall also that given $f\tau$, $m_j(f\tau\,\sigma_l)$ is the magnitude above

Table 1.--Typical rupture lengths when variability is included in three magnitude-rupture length relationships. The standard deviation given by Bonilla and Buchanan (1970) $\sigma_l = 1.20$ is used for all three relationships. $Fr = 0$ corresponds to the median rupture length. The other rupture lengths are those for $fr = \pm 1, \pm 2$ corresponding to lengths between the "short" $(-2\sigma_l)$ and "long" $(+2\sigma_l)$ extremes.

			fr			
mag	-2.0	-1.0	0.0	1.0	2.0	
4.0	0.3	0.9	3.0	9.8	32.4	k
4.5	0.4	1.4	4.6	15.3	50.8	i
5.0	0.7	2.2	7.2	24.0	79.4	l
5.5	1.0	3.4	11.3	37.5	124.3	o
6.0	1.6	5.4	17.7	58.7	194.5	m
6.5	2.5	8.4	27.8	91.9	304.4	e
7.0	4.0	13.1	43.5	143.9	476.4	t
7.5	6.2	20.5	68.0	225.2	745.6	e
8.0	9.7	32.1	106.4	352.4	1166.8	r
8.5	15.2	50.3	166.5	551.4	1826.0	s

A. Bonilla and Buchanan (1970) fit for world-wide surface faulting (used extensively by McGuire)
$ln(l) = -2.498 + .896m + 1.20fr$; $(\log_{10}(l) = -1.085 + .389m + .52fr)$

			fr			
mag	-2.0	-1.0	0.0	1.0	2.0	
4.0	0.3	1.1	3.5	11.6	38.3	k
4.5	0.6	1.9	6.2	20.5	68.0	i
5.0	1.0	3.3	11.0	36.5	120.9	l
5.5	1.8	5.9	19.6	64.9	214.9	o
6.0	3.2	10.5	34.8	115.3	381.7	m
6.5	5.6	18.7	61.9	204.9	678.4	e
7.0	10.0	33.2	109.9	364.1	1205.5	t
7.5	17.8	59.0	195.4	647.0	2142.4	e
8.0	31.7	104.9	347.2	1149.8	3807.3	r
8.5	56.3	186.4	617.1	2043.3	6766.2	s

B. A relationship used (with $fr=0$) by Der Kiureghian and Ang, 1977, in their risk analysis $ln(l) = -3.350 + 1.150m + 1.20fr$

			fr			
mag	-2.0	-1.0	0.0	1.0	2.0	
4.0	0.0	0.1	0.3	0.9	3.0	k
4.5	0.1	0.2	0.6	1.9	6.5	i
5.0	0.1	0.4	1.3	4.2	13.8	l
5.5	0.2	0.8	2.7	8.9	29.5	o
6.0	0.5	1.7	5.8	19.1	63.1	m
6.5	1.1	3.7	12.3	40.8	134.9	e
7.0	2.4	7.9	26.3	87.1	288.6	t
7.5	5.1	17.0	56.3	186.3	617.0	e
8.0	11.0	36.3	120.3	398.4	1319.4	r
8.5	23.5	77.7	257.3	852.0	2821.1	s

C. Curve A of Wallace, 1970. Earthquake recurrence intervals in the San Andreas fault (used by Algermissen and Perkins, 1976).
$\ln(l) = -7.370 + 1.520m + 1.20fr$

which all earthquakes regardless of location on the fault contribute to $Ex[a_1 | H(fr\, \sigma_l)]$ at a specified site. As fr increases and ruptures become longer at all magnitudes $m_j(fr\, \sigma_l)$ decreases, and

$$m_j(-u\sigma_l) > m_j(fr) > m_j(u\, \sigma_l), \qquad -u < fr < u \,. \tag{50}$$

As $m_j(fr\, \sigma_l)$ decreases, a higher fraction of earthquakes along the fault contributes to exceedances of a_1. Because all earthquakes at magnitudes $m > m_j(-u\, \sigma_l)$ contribute to $Ex[a_1 | H(fr\, \sigma_l)]$ for all $-u < fr \leqq +u$, the difference in $Ex[(a_1 | H(fr\, \sigma_l)]$ as a function of $H(fr\sigma_l)$ is due to earthquakes having magnitudes $m < m_j(-u\, \sigma_l)$. In the extreme case, if $m_j(-u\, \sigma_l) = m_{\min}$, all earthquakes contribute to $Ex(a_1)$ for all magnitudes $m \geq m_{\min}$ and all rupture lengths.

Table 2 illustrates for $m_{max=8.5}$, $m_{lo} = 6.0$, at a site above the center of the fault (at $X = \frac{L}{2}$), the value of $m_j(fr\, \sigma_l)$ for $fr = -2$, 0, +2 (short, median and long) break lengths as a function of distance P and fault length L. As shown, $m_j(-u\, \sigma_l)$ decreases as P increases, indicating that for m_{lo} fixed, rupture length has less effect at more distant sites.

However, in many cases of interest, particularly for sites nearer the fault, I_{all} does not contribute and rupture length is quite important. We can see precisely how break length affects the exceedances of a_1, that is how $Ex(a_1 | H)$ varies as a function of H, provided that rupture length remains less than fault length for all magnitudes and I_{all} is zero or negligible relative to the other integrals in equation 26 or 34.

In this case the denominator $[L - H \exp(g\, m)]$ can be expanded in a series and the corresponding integrals in equations 26 and 34 approximated by a sum of terms. Using

$$\frac{1}{L - H \exp(g\, m)} = \frac{1}{L\,[1 - \frac{H}{L}\exp(g\, m)]} = \frac{1}{L(1-Z)} = \frac{1}{L}(1 + Z + Z^2 + Z^3 + ..) \tag{51}$$

Table 2 Example showing magnitude limit m_{u_1} as a function of fault length, rupture length and distance from the site to the fault.

At each site, for a given value of m_{lo}, there is an acceleration which is produced only by magnitudes $m_{lo} \leq m$. At magnitudes $m_{u_1} \leq m \leq m_{max}$, all earthquakes on the fault regardless of location will contribute to that acceleration. As m_{u_1} decreases, the fraction of earthquakes yielding the acceleration increases. Tabular values are for sites at $(\frac{L}{2}, P)$ above the center of a fault L km long, for short, median and long ruptures, that is for $f_r = -2$, 0, +2 in the rupture length-magnitude relationship of table 1A..
$m_{max} = 8.5$, $m_{lo} = 6.0$. Attenuation function: $\ln a = 3.4 + .89m - 1.17 \ln R$.

$f_r = -2$

	P (km)									
L	10	20	30	40	60	80	100	150	200	300
100	7.90	7.19	6.81	6.57	6.32	6.20	6.14	6.06	6.04	6.02
200	8.50	8.01	7.56	7.25	6.84	6.60	6.44	6.23	6.14	6.07
400	8.50	8.50	8.41	8.07	7.60	7.27	7.04	6.66	6.45	6.24
800	8.50	8.50	8.50	8.50	8.46	8.11	7.84	7.36	7.05	6.67
1600	8.50	8.50	8.50	8.50	8.50	8.50	8.50	8.20	7.85	7.37

$f_r = 0$

	P (km)									
L	10	20	30	40	60	80	100	150	200	300
100	6.76	6.52	6.36	6.26	6.14	6.09	6.06	6.03	6.02	6.01
200	7.53	7.23	7.00	6.82	6.57	6.41	6.30	6.16	6.10	6.05
300	8.32	8.03	7.78	7.58	7.28	7.03	6.85	6.55	6.37	6.20
800	8.50	8.50	8.50	8.41	8.08	7.82	7.61	7.22	6.95	6.61
1600	8.50	8.50	8.50	8.50	8.50	8.50	8.46	8.04	7.73	7.29

$f_r = +2$

	P (km)									
L	10	20	30	40	60	80	100	150	200	300
100	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
200	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00
400	6.02	6.01	6.01	6.01	6.00	6.00	6.00	6.00	6.00	6.00
800	6.76	6.72	6.69	6.65	6.58	6.53	6.47	6.37	6.29	6.18
1600	7.54	7.49	7.45	7.41	7.34	7.27	7.20	7.06	6.93	6.73

---

we have

$$NT \int_{m_0}^{m_{max}} 2 \exp(-b\,m) \frac{\sqrt{R_1(m)^2 - P^2}}{L - \exp(g\,m)}\, dm \approx \sum_{n=0}^{N_1} I_n(H) \qquad (52)$$

where

$$I_n(H) = \frac{NT}{L} \int_{m_{min}}^{m_{max}} 2 \exp(-b\,m) \sqrt{R_1(m)^2 - P^2} \left(\frac{H}{L}\right)^n \exp(n\,g\,m)\, dm\,, \qquad (53)$$

and

$$Nt \int_{m_0}^{m_{max}} \frac{H \exp[(-b+g)m]}{L - \exp(g\,m)}\, dm \approx \sum_{n=0}^{N_2} J_n(H) \tag{54}$$

$$J_n(H) \approx NT \frac{H}{L} \int_{m_{min}}^{m_{max}} \exp[-(b+g)m]\,(\frac{H}{L})^n \exp(n\,g\,m)\, dm \tag{55}$$

and

$$NT = \frac{b \exp(b\, m_0)\, rate}{1 - \exp[-b\,(m_{max} - m_0)]} \tag{56}$$

Using these relationships, from equation 34, for a site at $(\frac{L}{2}, P)$,

$$Ex(a_1 | H)_{(\frac{L}{2},P)} \approx \sum_{n=0}^{N_1} I_n(H) \; + \sum_{n=0}^{N_2} J_n(H) \tag{57}$$

and from equation 26, for a site at $(0, P)$,

$$Ex(a_1 | H) \approx \frac{1}{2} \sum_{n=0}^{N_1} I_n(H)\, . \tag{58}$$

For the Esteva attenuation function (equation 35), under certain conditions, the $I_n(H)$ terms are of the form

$$I_n(H) = \frac{Q_n(H)\,\Gamma(\frac{3}{2})\,\Gamma(\frac{b-ng}{c} - \frac{1}{2})}{\Gamma(\frac{b-ng}{c} + 1)} - \int_0^{(\frac{P}{R_{max}})^2} w^{\frac{b-ng-\frac{3}{2}}{c}}\, dw \tag{59}$$

(See Appendix F for definitions, derivation and discussion.)

The first term of $I_0(H)$ involving gamma functions

$$\frac{Q_0(H)\,\Gamma(\frac{3}{2})\,\Gamma(\frac{b}{c} - \frac{1}{2})}{\Gamma(\frac{b}{c} + \frac{1}{2})}$$

is equivalent to (within a normalizing factor) Cornell's (1968) result for earthquakes regarded as point sources (zero length ruptures) occurring on an infinite fault with infinite maximum magnitude. The second term of I_0 corrects for the fact that a finite maximum magnitude is assumed. Since $R_1(m_{max})$ is a function of acceleration as well as magnitude, at the highest accelerations at a site, $R_1(m_{max})$ can be very close to P. In this case, the correction may be nearly as large as the Cornell term. However, if $R_1(m_{max})$ is large compared with P, then $\left[\frac{P}{P_1(m_{max})}\right]^2$ is small and the correction for finite maximum magnitude is small.

While the I_0 term gives acceleration exceedances for point sources, the J_0 term gives the extra exceedances due to rupture length. The higher order terms $I_k(H)$, $J_k(H)$, $0<k$, give the correction to the "point source" and "rupture length" terms due to to the fact that the fault is not infinitely long and ruptures must be wholly contained within the fault.

If the denominator is originally L (as in the Der Kiureghian and Ang model) instead of $[L - H \exp(g\, m)]$ only the term $k=0$ is required. The $I_k(H)$, $J_k(H)$ terms for $k>0$ give the difference in exceedances calculated by the two models for a site above the center of the fault at $(\frac{L}{2}, P)$. For a site at $(0,P)$ for the model of Der Kiureghian and Ang, $Ex(a_1|H)$ is given by $Ex(a_1|H)=\frac{1}{2}(I_0+J_0)$. (See equation A15, Appendix A).

Some sample results for four sites and several values of a_1 and fr are given in Appendix D for an Esteva attenuation. We have assumed in equations 53 and 55 that the integral I_{all} is not present, that is that $m_j(fr\, \sigma_l)=m_{max}$. In Appendix D we have relaxed the restriction that $m_{max}>m_j(fr\, \sigma_l)$, changed the integration limits in equations 53 and 55 from m_{max} to $m_j(fr\, \sigma_l)$ and added in the contribution (if any) from I_{all}. Contributions from I_0, I_1, J_0, and I_{all} plus the five-term total ("Sum" column) are shown. The column marked "analytic" gives the results obtained by evaluating the original integrals by Gaussian quadrature. "Sum" may be seen to give a good approximation to the "analytic" result for the usual break lengths, becoming less accurate as fr increases to correspond to extremely long breaks.

Note that even when $fr=0$ (median break length) J_0 the principal "break length term" may be larger than I_0 the "point source approximation" and cannot be neglected. Hence treating earthquakes as point sources can give results quite different from those obtained when rupture length is taken into account, and the term I_0 gives a good approximation to the sum only under some conditions. The differences are particularly dramatic when longer ruptures are permitted.

Between $fr = -2$ and $fr = +2$, (short and long extremes of rupture length), $Ex[a_1 | H(fr\ \sigma_l)]$ may vary by factors as high as ten or twenty or more. However, the "extreme" situations corresponding to very long or very short break lengths occur infrequently and the real interest lies in the result of integrating over break length weighted by probability of occurrence, that is in the expected value $Ex(a_1)$.

We shall investigate the effect of integrating over break length and show that exceedances of $Ex[a_1 | H(0)]$ calculated using only the median ($fr = 0$) length tends to underestimate the integrated $Ex(a_1)$. That is, longer ruptures have a greater effect than their frequency of occurrence might indicate, and indeed the mean rupture length (which is greater than the median length) more nearly approximates the integrated result. We shall show this using an approximation, which is valid when the length of all ruptures remains less than the fault length, and when $m_j(fr\ \sigma_l) < m_{max}$ for all fr considered.

Rupture Length Integration

The normal probability integral has limits that extend from $-\infty$ to $+\infty$. The formula for rupture length as a function of magnitude

$$b_m = \exp(a + g\ m + fr\ \sigma_l)$$

would result in an unbounded length if fr were permitted to increase indefinitely. Obviously such long ruptures are impossible and so we shall truncate the distribution assuming all ruptures occur for fr within some range

$-u \leq fr \leq +u$, where u is of the order of say 1.5 or 2. The denominator of equation 60 is a renormalization to correct for the fact that the integration limits are $-u\,\sigma_l$ to $+u\,\sigma_l$ rather than $-\infty$ to $+\infty$.

Integrating over break length gives

$$Ex(a_1) = \frac{\int_{-u\sigma_l}^{+u\sigma_l} Ex[a_1 \,|\, H(y)]\, p(y)\, dy}{\frac{1}{\sqrt{2\pi}} \int_{-u}^{+u} \exp(-\frac{y^2}{2})\, dy} \qquad (60)$$

where $p(y) = \frac{1}{\sqrt{2\pi}\sigma_l} \exp(\frac{-y^2}{2\sigma_l^2})$

$H(y) = \exp(a + y), \quad y = fr\,\sigma_l$ (see model assumption 2.)

$Ex(a_1)$ may be calculated directly by integrating equation 26 or 34 for $Ex(a_1 \,|\, H(y)]$ and then numerically integrating over $H(y)$. However, when the series expansion of equations 53 and 55 for $I_n[H(y)]$ and $J_n[H(y)]$ is valid, (rupture length remains less than fault length throughout and $m_j = m_{\max}$) the integration over $H(y)$ may be performed analytically for each n. Since the integrals $I_n[H(y)]$, $J_n[H(y)]$ are of the form

$$I_n[H(y)] = H(y)^n \int_{m_{\min}}^{m_{\max}} f_n(m)\, dm \qquad (61)$$

$$J_n[H(y)] = H(y)^{n+1} \int_{m_{\min}}^{m_{\max}} g_n(m)\, dm \qquad (62)$$

the integration over y of $I_n[H(y)]$ may be written

$$\frac{\int_{-u\sigma_l}^{+u\sigma_l} I_n[H(y)]\, p(y)\, dy}{\int_{-u\sigma_l}^{+u\sigma_l} p(y)\, dy} = \frac{\int_{-u\sigma_l}^{+u\sigma_l} H(y)^n\, p(y)\, dy}{\int_{-u\sigma_l}^{u\sigma_l} p(y)\, dy} \int_{m=m_{\min}}^{m_{\max}} f_n(m)\, dm \qquad (63)$$

and similarly for $J_n[H(y)]$.

It can be shown that

$$T(H^n)=\frac{\int_{-u\sigma_l}^{+u\sigma_l} H(y)^n p(y)\,dy}{\int_{-u\sigma_l}^{+u\sigma_l} p(y)\,dy} = \frac{\exp[n\,a+\frac{(n\sigma_l)^2}{2}]\int_{-u-n\sigma_l}^{+u-n\sigma_l}\exp(\frac{-x^2}{2})\,dx}{\int_{-u}^{+u}\exp(\frac{-z^2}{2})\,dz} \tag{64}$$

Instead of actually integrating over y (or $f\tau\,\sigma_l$) we may find a single value of y that gives the same result. By the mean value theorem, there is a value y_0 in the range $-u\,\sigma_l \leq y_0 \leq +u\,\sigma_l$ such that

$$T(H^n)=H(y_0)^n=\exp(a+f\tau_n\,\sigma_l)^n\,. \tag{65}$$

Equations 64 and 65 for $T(H^n)$ can be solved for $f\tau_n$. Table 3 presents values of $f\tau_n$ for $\sigma_l=1.20$ and $\sigma_l=.30$ and integration range $-1.5\leq f\tau\leq 1.5$, and $-2\leq f\tau\leq +2$.

Table 3

	$\sigma_l=1.20$			$\sigma_l=.30$	
	u=1.5	u=2.0		u=1.5	u=2.0
n	$f\tau_n$	$f\tau_n$	n	$f\tau_n$	$f\tau_n$
1	.313	.438	1	.082	.115
2	.554	.774	2	.163	.228
3	.721	1.005	3	.240	.337

These results enable us, when the series expansion is valid, to write for a site at $(\frac{L}{2},P)$,

$$Ex(a_1)\approx I_0+\sum_{n=0}^{N_2}J_n[H(f\tau_{n+1}\sigma_l)]+\sum_{n=1}^{N_1}I_n[H(f\tau_n\,\sigma_l)]\,. \tag{66}$$

But since $0<f\tau_1<f\tau_2<......$, and since $I_n[H(f\tau\,\sigma_l)]$ and $J_n[H(f\tau\sigma_l)]$ are increasing functions of $f\tau_n$, using $f\tau_n=f\tau_1$ in equation 66 does not overestimate $Ex(a_1)$.

For $n=1$, $T(H)$ represents a first moment and $f\tau_1$ is such that $T(H)\exp(g\,m)=\exp(a+g\,m+f\tau_1\sigma_l)$ equals the mean rupture length. Hence using only the median length (or length corresponding to $f\tau=0$, the mean of the log of the rupture length) gives a lower value and therefore a worse approximation to $Ex(a_1)$.

Tables in Appendix E evaluated at rupture lengths corresponding to various values of fr illustrate $Ex[a_1 \mid H(fr\,\sigma_l)]$ when a single length is used for each magnitude and show the integrated result $Ex(a_1)$. In many cases a single length corresponding to fr_1 gives a good approximation to $Ex(a_1)$, even when the conditions used in developing the series expansion are not met completely. Thus, for example, for $u = 1.5$, $\sigma_l = 1.20$, $Ex[a_1 \mid H(.313\sigma_l)]$ evaluated for the single break length corresponding for $fr = .313$ (the mean length for lengths in the range to $\pm 1.5\sigma_l$) frequently gives a good approximation to the integrated result, while the median $fr = 0$ length underestimates $Ex(a_1)$. Since integration over break length is time consuming (numerical integrations in SEISRISK II and FRISK use four or five lengths per magnitude) a considerable savings results if one rupture length per magnitude suffices. Figure 7 illustrates acceleration exceedance values at three sites when very short (point source), median length and mean length ruptures are assumed.

Essentially the same arguments may be made when the Der Kiureghian and Ang model, which requires ruptures to have their midpoint on the fault, is used. The denominator then is L instead of $[L - H\exp(g\,m)]$ and only the principal "break length term" J_0 need be integrated. (The I_0 term is independent of $H(y)$.) Formulas for this alternate model are developed in Appendix A.

Probabilistic Accelerations.

Thus far we have associated a single value of acceleration with each magnitude and distance, when actually, accelerations have some distribution of values, and should be treated probabilistically. The computer programs SEISRISK II and FRISK assume accelerations are lognormally distributed around their median values, that is around the mean value of the log-acceleration with standard deviation (in log-acceleration) σ_a. Note that σ_a is regarded as constant, independent of magnitude and acceleration.

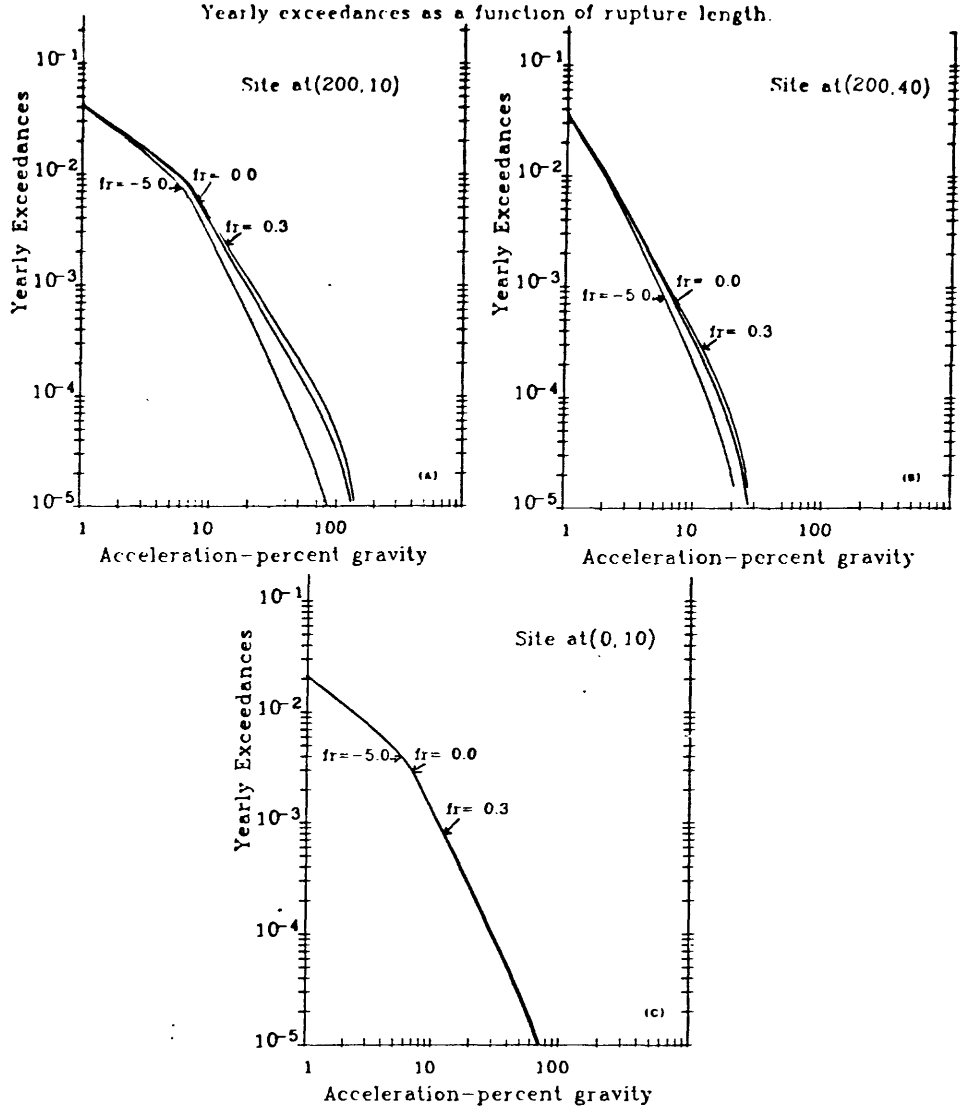

Figure 7. Acceleration exceedances at three sites as a function of rupture length. The curves labeled $fr=-5$ give exceedances when very short rupture lengths (almost point sources) are used; the curves $fr=0$ correspond to median length ruptures and those for $fr=.3$ to mean length ruptures for $\sigma_l = 1.2$ (in $\ln_e$).
The fault extends along the x axis from (0,0)--(400,0); magnitude range is $4.0 \leq m \leq 7.5$

Under this assumption, using earthquake $rate = 1$, the probability that a random earthquake in the range $m_{\min} \leq m \leq m_{\max}$ will cause acceleration a_0 to be exceeded at (X,P) is

$$Pex(a_0) = \int_{a_{\min}}^{a_{\max}} \varphi^* \left[\frac{\ln a_0 - \ln a}{\sigma_a} \right] pr(a)\, da \qquad (67)$$

where φ^* = complement of the cumulative normal distribution

a_{min} = lowest acceleration for which $pr(a) \geq 0$ at $(X<P)$

a_{max} = highest acceleration for which $pr(a) \geq 0$ at (X,P)

$pr(a)$ = density of median acceleration a at (X,P).

The inclusion of acceleration variability may significantly affect acceleration exceedances. Figure 8 illustrates exceedance values at three sites for median length ruptures for several values of σ_a (where $\sigma_a = 0$ represents no acceleration variability).

In order to gain some insight into how acceleration variability affects the expected exceedances of a_0, we shall show that $Pex(a_0)$ can be expressed as the sum of two terms

$$Pex(a_0) = Ex(a_0) + T(a_0, \sigma_a) \, , \tag{68}$$

where

$Ex(a_0)$ = probability that acceleration a_0 is exceeded assuming median acceleration values

$T(a_0, \sigma_a)$ = additional exceedances of a_0 due to acceleration variability.

To show this, note that

$$Pex(a_0) \approx Ex(a_2) + \int_{a_1}^{a_2} \varphi^* \left[\frac{\ln a_0 - \ln a}{\sigma_a} \right] pr(a)\, da \tag{69}$$

where

$$\ln a_1 = \ln a_0 - n\ \sigma_a \tag{70}$$

$$\ln a_2 = \ln a_0 + n\ \sigma_a \tag{71}$$

for some number n of standard deviations from a_0. (Accelerations with log-mean values $a < a_1$ are ignored in this approximation.)

Next, letting $w = \ln a - \ln a_0$ and using the relationship $\varphi^*(-x) = 1 - \varphi^*(x)$ yields

$$Pex(a_0) \approx Ex(a_0) + a_0 \int_0^{n\sigma_a} \varphi^*\left(\frac{w}{\sigma_a}\right) \left\{ \exp(-w)\, pr[a_0 \exp(-w)] - \exp(w) pr[a_0 \exp(w)] \right\} dw \tag{72}$$

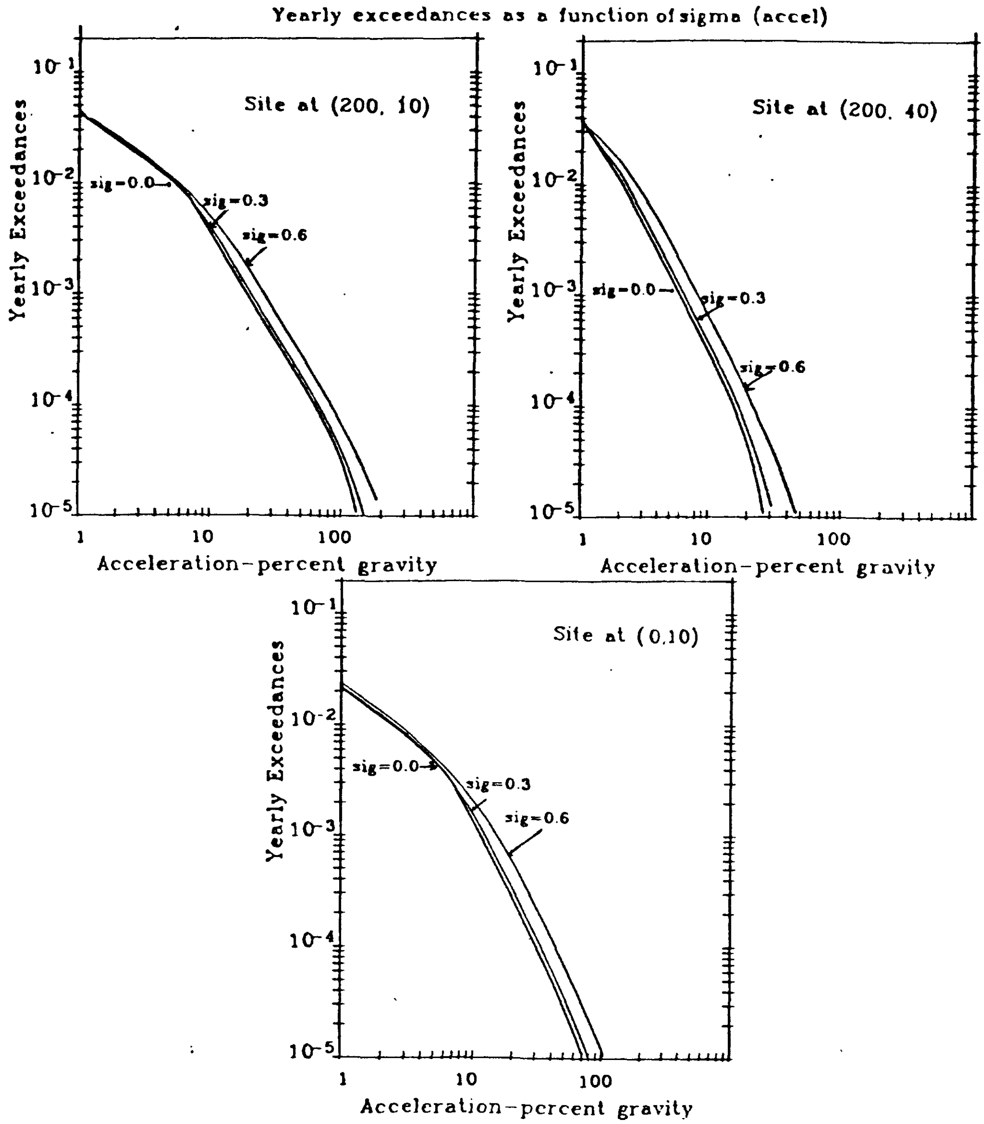

Figure 8. Acceleration as a function of σ_a, standard deviation in log-acceleration (using median length ruptures) at three sites; $\sigma_a = 0$ corresponds to using only the median value of log- acceleration. (Values of σ_a shown represent variability in the natural logarithm of acceleration, $\ln_e(a)$.)

The fault extends along the x-axis from (0,0)--(400,0); magnitude range is $4.0 \leq m \leq 7.5$.

Thus, the effect of acceleration variability on the probability of exceeding acceleration a_0 is related to the difference in acceleration density at acceleration levels $a_0 \exp(-w)$ and $a_0 \exp(w)$ for $0 \leq w \leq n\ \sigma_a$.

At the higher accelerations possible at a site, including variability can have particularly dramatic effects on the expected exceedances. Since a_{max} = highest acceleration for which $pr(a) > 0$ at (X,P), $pr[a_0 \exp(w)] = 0$ whenever

$a_0 \exp(w) > a_{max}$. There may, therefore, be a dearth of acceleration densities to subtract off at the higher accelerations at a site. If $a_0 \geq a_{max}$, the second term in the integral in equation 72 vanishes, but the first term will continue to contribute whenever $a_{min} \leq a_0 \exp(-w) \leq a_{max}$. This corresponds on the high end to accelerations $a_0 \leq a_{max} \exp(n\, \sigma_a)$. Thus, the highest acceleration felt at (X,P) increases by a factor $\exp(n\, \sigma_a)$ and there will be exceedances of all accelerations below $a_{max} \exp(n\, \sigma_a)$. Similarly, restricting magnitudes to $m_0 \leq m$ affects accelerations that would be produced by magnitudes $m < m_0$, causing, in some cases, $Pex(a_0)$ to be lower than it would be if magnitudes $m < m_0$ were permitted.

Formulas for the density $pr(a)$ are derived in Appendix C for the Esteva attenuation and fcr model. It can be seen that there are a number of different expressions depending on magnitude limits $m_{lo}, m_{min}, m_u, m_{u_1}, m_{u_2}$ and the site location. Since the magnitude limits are a function of acceleration, several different expressions for $pr(a)$ may be required over the range $a_1 \leq a \leq a_2$. This makes a detailed analysis quite difficult. (A numerical derivative $pr(a) = \frac{-[Ex(a+\varepsilon) - Ex(a)]}{\varepsilon}$ may, of course, be used for computational purposes.)

One situation, which is easy to analyze, occurs when the site is at a distance R from a point source, and the Esteva attenuation is used. (The point source case is equivalent to the case when only the integral I_{all} contributes to exceedances of a_0; that is, all ruptures regardless of location on the fault produce accelerations $a \geq a_0$.) In this case, as shown in Appendix B, a single expression for $pr(a)$ is valid throughout the range and

$$Pex(a_0) = Ex(a_0) + k_2 a_0^{-(\frac{b}{c_2})} \int_0^{n\sigma_a} \varphi^* \left[\frac{w}{\sigma_a}\right] [\exp(\frac{b\, w}{c_2}) - \exp(\frac{-b\, w}{c_2})]\, dw \qquad (73)$$

$$= Ex(a_0) + 2k_2 a_0^{-\frac{b}{c_2}} \sigma_a \int_0^{n} \varphi^*(w) \sinh(\frac{b\, \sigma_a w}{c_2})\, dw$$

where

$$k_2 = \frac{\exp[b(\frac{c_1}{c_2} + m_0)] R^{(\frac{b c_3}{c_2})}}{c_2[1 - \exp(m_{max} - m_0)]} \tag{74}$$

so long as $a_{min} \leq a_1$, $a_2 \leq a_{max}$ (equations 69 and 70).

For this situation,

a) $Pex(a_0)$ increases as σ_a increases (obvious since $\sinh(x)$ increases as x increases.)

b) For fixed σ_a, the integral in equation 73 is constant so that $Pex(a_0)$ is of the form

$$Pex(a_0) \approx Ex(a_0) + a_0^{-\frac{b}{c_3}} \cdot \text{constant}. \tag{76}$$

An expression for $Pex(a_0)$ for the point source case was derived by Cornell (1971). A different derivation is given in Appendix B.

Applications

Tables in Appendix E illustrate how acceleration exceedances of 100 gals (.1g) vary as a function of rupture length and site location for an Esteva-type attenuation with parameters

$$\ln a = 3.4 + .89\, m - 1.17 \ln R \tag{77}$$

using both the fcr model (denominator $[L - H \exp(g\, m)]$, and alternate model (denominator L); and for the Schnabel and Seed (1973) (SS) attenuation curves using the fcr model. The tables are provided only to indicate some general behaviors of the the models and to provide some comparisons between them. The actual numbers depend, of course, on specific parameters (b-value, fault length, attenuation, rate, and so forth), and are not significant; only relative values are of interest. The tables show $Ex[a\,|H(fr\,\sigma_l)]$ for several values of fr (short to long rupture lengths), $Ex(a)$ (exceedances integrated over rupture length), and $Pex[a\,|H(fr\,\sigma_l)]$, (exceedances when acceleration variability is included, fr fixed).

Column headings in the tables below and in Appendix E represent the X-coordinate, row headings the P-coordinate of a site at (X,P). The fault extends 400 km along the x-axis, $0 \leq x \leq 400$. Magnitude range is $4.0 \leq m \leq 7.5$ for exceedances of 100 gals; $4.0 \leq m \leq 8.5$ for exceedances of 500 gals.

(1) Site location differences.

For a given model and attenuation function, and for the P-coordinate of the site fixed, $Ex[a \mid H(fr\,\sigma_l)]$, $Ex(a)$ and $Pex[a \mid H(fr\,\sigma_l)]$ tend to change considerably at sites near the end of the fault a few kilometers apart in the x-direction. For $P = 10$, a site at $X = +10$ (km) tends to have exceedance values that are two to four times as large as those at a site at $X = -10$ (km). The difference decreases as P increases, but generally the values remain larger by at least a factor 1.5 or 2 for $P < 65$. (See Appendix E.)

Figure 10 illustrates acceleration exceedances, for the attenuation in equation 77, at a number of sites having the same P-coordinate and different X-coordinates. Figure 10A assumes minimum magnitude $m_0 = 4.$; figure 10C assumes $m_0 = 1$.

The sharp decrease in acceleration exceedances calculated for a site a few kilometers past the end of the fault compared with those calculated for a site near the end of the fault may be an unrealistic consequence of the model.

(2) Model Differences:

Tables (4) and (5) illustrate the ratio of $Ex(a)$ obtained using the fcr model (ruptures wholly contained within the fault) to $Ex(a)$ calculated at the same site for the alternate model (in which only rupture midpoints are required to be on the fault) for $a = 100$ gals and $a = 500$ gals. (Zero ratio implies that the acceleration is not felt at the site for at least one model.)

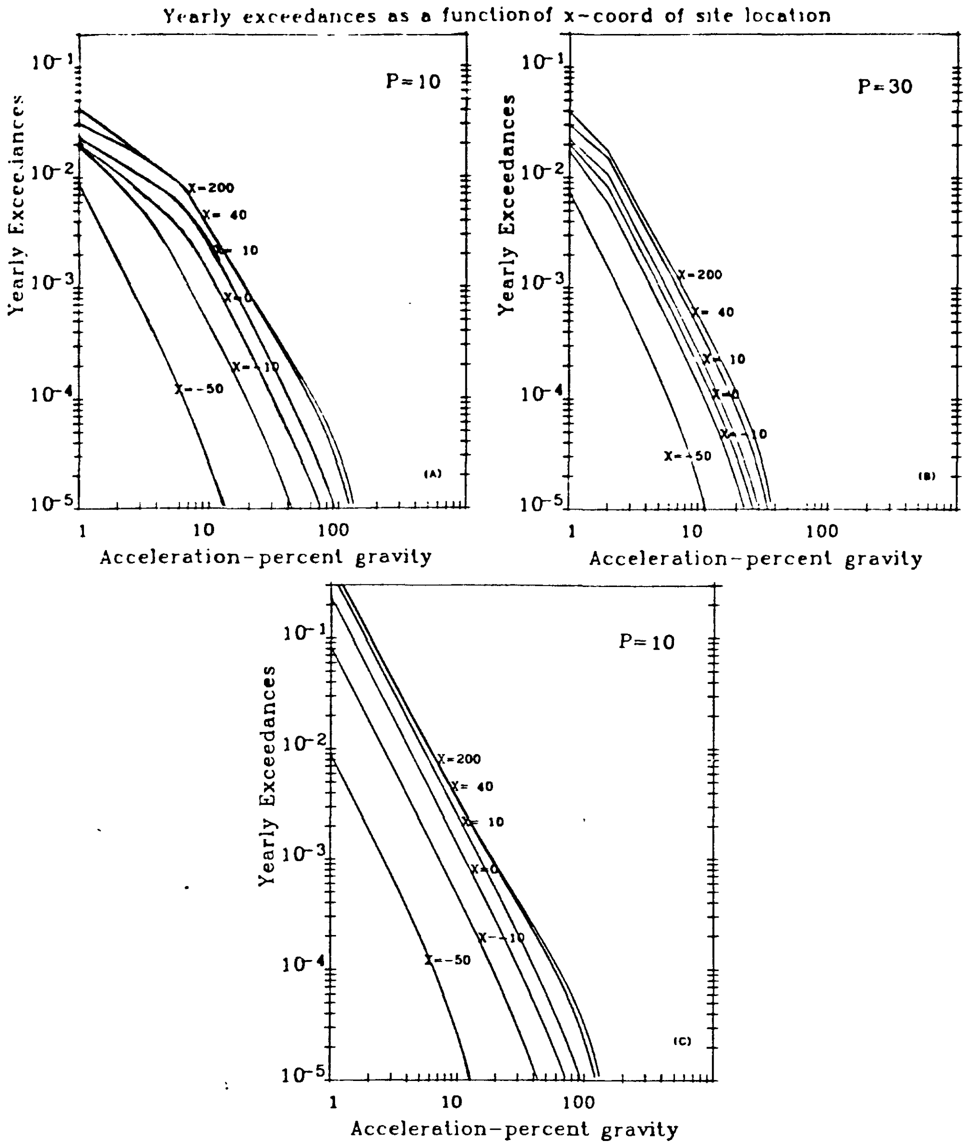

Figure 10. Acceleration exceedances as a function of the X-coordinate of the site for a fixed P-coordinate; $m_{max} = 7.5$. In (A) and (B) $m_0 = 4.0$; in (C) $m_0 = 1.0$. The fault extends from (0,0)--(400,0).

It may be seen that, at 100 gals (.1g) for sites perpendicular to the center of the fault (X=200), both models give very similar results at lower P values. with the relative difference or ratio increasing as P increases. As the X-coordinate of the site is shifted parallel to the fault past the end of the fault (as X decreases from X=200 to X=-50), the alternate model begins to give higher results (about the same at X=50) and increases with decreasing X until at $X=-50$ it gives values that are three or four (or more) times as high as the fcr

Table 4

	$Ex(100)_{fcr}/Ex(100)_{alt}$							
	X							
P	200	100	50	10	0	-5	-10	-50
10	1.06	1.04	1.01	.84	.71	.61	.53	.32
20	1.10	1.07	.99	.80	.71	.65	.59	.32
30	1.14	1.08	.96	.79	.72	.68	.63	.33
40	1.17	1.08	.95	.79	.73	.69	.65	.33
50	1.20	1.09	.95	.80	.74	.70	.66	.32
65	1.22	1.10	.96	.80	.74	.70	.66	.27
80	1.24	1.11	.96	.79	.72	.67	.63	.17
100	1.29	1.15	.95	.68	.56	.49	.41	.00

Table 5

	$Ex(500)_{fcr}/Ex(500)_{alt}$							
	X							
P	200	100	50	10	0	-5	-10	-50
10	1.25	1.13	.93	.59	.43	.32	.25	.04
20	1.30	1.15	.95	.68	.57	.50	.43	.04
30	1.27	1.16	1.01	.81	.73	.67	.60	.00
40	1.25	1.16	1.03	.87	.80	.75	.68	.00
50	1.24	1.16	1.04	.88	.81	.75	.64	.00
65	.00	.00	.00	.00	.00	.00	.00	.00

model. At X=-50, (where the ratio in table 5 is zero, $Ex(500)=0$ or not felt in the fcr model), $Ex(500)_{(-50,P)}$ is as high as 10%-35% of the value of $Ex(500)_{(200,P)}$ in the alternate model. In the alternate model, for P fixed, $Ex[a \mid H(fr\ \sigma_l)]$ and $Ex(a)$ tend to be almost exactly twice as high at $X=\frac{L}{2}$ as at $X=0$, since the fault is treated as the sum of two independent segments of length $\frac{L}{2}$. In the fcr model, the difference is closer to a factor 2.5 or 3.

Thus while the model makes relatively little difference in the calculated risk at lower accelerations for sites above the fault, the choice of model can have a substantial fractional effect on exceedances calculated at sites some distance away beyond the end of the fault. However, the absolute values of the numbers may become very small, and large fractional differences may be unimportant. Thus, in examples for $Ex(500)$, a tabular value of 100 for the assumed rate of $r=.1$ earthquake per year corresponds to a return period of 100,000 years, and changing the result by a factor ten may have little influence on the earthquake hazard assessment.

(3) Attenuation function.

Table (6) illustrates the ratio of $Ex(100)$ computed using the $f\tau$ model with the Schnabel and Seed attenuation curves to $Ex(100)$ for the same model and the Esteva attenuation (equation 77). Comparison is complicated by the inclusion of d=depth in equation 77 where $R^2 = X^2 + P^2$, and $P^2 = Y^2 + d^2$, whereas in the SS equations, $P = Y$. Tables shown assume $d = 0$, but major discrepancies and shape differences remain, even after changing depth or shifting in the P direction. (The ratios reflect the difference in the attenuation assumed at distances other than those where the predominant number of significant strong-motion records are available.)

At $a = 500$ gals, the Schnabel and Seed curves give nonzero exceedances only to $P = 10$, whereas the Esteva attenuation gives some exceedances to $P = 50$ (not shown).

Table 6

	$Ex(100)_{SS}/Ex(100)_{Esteva}$							
	X							
P	200	100	50	10	0	-5	-10	-50
10	.43	.42	.39	.34	.45	.61	.82	.43
20	.97	.98	.97	.95	1.02	1.08	1.13	.38
30	1.27	1.30	1.35	1.30	1.22	1.15	1.07	.31
40	.97	.98	1.00	.92	.84	.79	.73	.19
50	.67	.66	.65	.59	.56	.54	.51	.07
65	.39	.38	.37	.31	.27	.25	.22	

(4) Effect of integration over rupture length.

Tables in Appendix E show $Ex(a)$ integrated over $-1.5 < f\tau < +1.5$, and $Ex[a \mid H(f\tau\ \sigma_l)]$ ranging from low ($f\tau = -2$) to high ($f\tau = +2$) values (very short to very long break lengths). As previously noted, there may be a difference in exceedances of factors of ten or twenty between the extremes. As discussed in the text, for $\sigma_l = 1.20$, a single value of $f\tau$, ($f\tau = .313$) gives a good approximation to the integrated result in many cases. Ratios

$\frac{Ex(a)_{(-1.5 \le f\tau \le 1.5)}}{Ex[a\,|H(f\tau\,\sigma_l)]}$ for $f\tau=0$, $f\tau=.313$ are shown below in tables 7-10 for $a=100$, 500 gals, for the $fc\tau$ model using equation 77.

Table 7

$Ex(100)_{integrated}/Ex[(100)|H(0\sigma_l)]$

P	X: 200	100	50	10	0	-5	-10	-50
10	1.14	1.13	1.10	1.01	1.02	1.03	1.04	1.15
20	1.17	1.14	1.08	1.03	1.04	1.05	1.06	1.16
30	1.19	1.14	1.06	1.06	1.07	1.07	1.08	1.17
40	1.21	1.13	1.07	1.09	1.09	1.10	1.10	1.19
50	1.22	1.12	1.08	1.11	1.12	1.12	1.13	1.22
65	1.23	1.12	1.12	1.16	1.16	1.17	1.18	1.27
80	1.25	1.13	1.17	1.21	1.22	1.23	1.24	1.38
100	1.35	1.14	1.24	1.34	1.37	1.39	1.41	

Table 8

$Ex(100)_{integrated}/Ex[100|H(.313\sigma_l)]$

P	X: 200	100	50	10	0	-5	-10	-50
10	1.01	1.00	.99	.98	1.01	1.01	1.01	1.08
20	1.01	1.00	.98	1.01	1.01	1.02	1.02	1.08
30	1.01	.99	.97	1.02	1.03	1.03	1.03	1.09
40	1.01	.99	.99	1.04	1.04	1.04	1.05	1.11
50	1.01	.98	1.01	1.05	1.06	1.06	1.06	1.13
65	.99	.98	1.04	1.08	1.08	1.09	1.09	1.17
80	.97	.98	1.08	1.12	1.13	1.13	1.14	1.26
100	.98	.96	1.13	1.22	1.25	1.26	1.26	

Table 9

$Ex(500)_{integrated}/Ex[500|H(0\sigma_l)]$

P	X: 200	100	50	10	0	-5	-10	-50
10	1.39	1.28	1.11	1.17	1.27	1.40	1.62	13.77
20	1.28	1.21	1.25	1.52	1.69	1.85	2.10	21.91
30	1.12	1.18	1.45	1.92	2.23	2.47	2.79	
40	.98	1.20	1.55	2.21	2.68	3.04	3.48	
50	.89	1.22	1.63	2.63	3.47	4.21	5.27	

Table 10

$Ex(500)_{integrated}/Ex[500|H(.313\sigma_l)]$

P	X: 200	100	50	10	0	-5	-10	-50
10	1.02	.98	.92	1.10	1.16	1.26	1.41	9.67
20	.91	.94	1.10	1.31	1.44	1.56	1.74	15.14
30	.79	.95	1.19	1.55	1.78	1.96	2.19	
40	.70	.93	1.19	1.68	2.02	2.28	2.59	
50	.68	.88	1.17	1.88	2.46	2.98	3.71	

Cases in which the ratio $\frac{Ex(a)}{Ex[a|H(fr\sigma_a)]} < 1$ result when all earthquakes contribute and $Ex[a|H(fr\sigma_a)]$ is a constant for all $fr > fr_v$, for some $fr_v < u$. In this case, the mean value $Ex(a)$ must be less than the value $Ex[a|H(fr_v\sigma_l)]$ and must occur at $fr < fr_v$.

The large ratios, those cases where $\frac{Ex(a)}{Ex[a|H(fr\sigma_l)]} >> 1$ occur when only magnitudes near m_{max} contribute to exceedances of a, $R_1(m)^2 - P^2$ is small, and most of the contribution to the integrated result is due to the effect of long ruptures.

Table 11

$Pex[100|H(.313\sigma_l)]/Ex[100|H(.313\sigma_l)]$

P \ X	200	100	50	10	0	-5	-10	-50
10	1.34	1.34	1.35	1.34	1.53	1.82	2.08	3.19
20	1.82	1.84	1.90	2.13	2.13	2.24	2.27	3.35
30	1.87	1.90	2.01	2.24	2.23	2.33	2.38	3.63
40	1.94	1.98	2.14	2.33	2.37	2.49	2.55	4.12
50	2.05	2.12	2.33	2.53	2.60	2.74	2.82	4.98
65	2.39	2.48	2.80	3.10	3.25	3.46	3.61	7.99
80	3.24	3.36	3.89	4.55	4.92	5.32	5.69	22.17
100	11.34	11.22	14.07	20.58	25.52	30.56	37.50	17.66

Table 12

$Ex(100)_{SS}/Ex(100)_{Esteva}$

P \ X	200	100	50	10	0	-5	-10	-50
10	1.49	1.52	1.64	2.04	2.02	2.11	2.16	21.45
20	1.49	1.61	1.94	2.06	2.10	2.23	2.33	53.73
30	1.59	1.88	2.12	2.29	2.42	2.61	2.80	15.22
40	1.95	2.36	2.59	3.02	3.34	3.73	4.21	12.02
50	3.62	4.12	4.69	6.35	7.78	9.68	12.68	9.12

(5) Effect of integration over acceleration variability.

The integration over acceleration gives a value $Pex(a)$ larger than the corresponding $Ex(a|H(fr\sigma_l)]$ obtained using just the single mean peak acceleration, and allows higher accelerations to be felt at (X,P). Both the maximum possible acceleration and $Pex(a)$ tend to increase at (X,P) as σ_a increases. Tables 11 and 12 give $Pex(a|H(fr\sigma_a)]$ for several situations and illustrate these effects.

The large ratios may be explained as follows: If a_{max} = acceleration at (X,P) resulting when an earthquake of magnitude m_{max} occurs on the fault as near as possible to the site, $Ex(a) = 0$ for $a > a_{max}$, and including acceleration variability means that some of the exceedances of lower accelerations are "spread upward", possibly causing $Pex(a) > 0$ even though $Ex(a) = 0$.

Conclusions

Maximum and minimum earthquake magnitude, fault length and rupture length may have important effects on the calculated exceedances of an acceleration level; however, these effects vary with site location and acceleration, and generalizations based upon looking at a particular site or acceleration may be misleading.

For sites perpendicular to the interior of the fault, acceleration exceedance values obtained using mean length ruptures for each magnitude may be three to five times as high as those obtained using point sources. For sites beyond the end of the fault perpendicular to the extended fault line, the ratio may be considerably less, depending in part upon whether rupture length is large relative to fault length at some magnitudes. If a log-normal distribution of rupture lengths is assumed, a good approximation to acceleration exceedance values calculated by integrating over rupture length is generally obtained by using only the mean rupture length for each magnitude. It is emphasized that the mean length is longer than the median length (or mean log-length) which is frequently used in calculations and gives lower values.

Assumptions regarding maximum magnitude have a large effect on the higher accelerations calculated at a site, but may be unimportant at the lower accelerations. The minimum magnitude cutoff may cause a loss of exceedances of some accelerations. If there were no magnitude restrictions and infinite (or semi-infinite) fault lengths were permitted the exceedance curves would vary more smoothly as a function of acceleration.

Exceedances of an acceleration level may be two to five times as high at a site near the center of the fault as at a site near the end at the same perpendicular distance P from the fault. Sites beyond the end of the fault are particularly sensitive to site location, a change of 10 km in the x-direction resulting in up to a factor two change in exceedances of a. As P increases, accelerations become less sensitive to the X-coordinate of the site.

This model and that of Ang and Der Kiureghian give similar results for sites near the center of the fault. Relative differences in exceedances of a calculated using the two models become larger as the distance from the fault increases and as the acceleration level a increases.

Different attenuation functions can yield very different acceleration exceedance values. The fractional difference in exceedances of an acceleration calculated using two attenuation functions is not the same at different sites. Including acceleration variability rather than assuming a single value of acceleration for each magnitude and distance will cause a significant increase in the higher accelerations at a site. Depending upon the attenuation function, including variability if σ_a is large may have a greater effect on the acceleration exceedance values than effects of including rupture length. Attenuation function effects can and should be distinguished from model effects.

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Appendix A

Derivation of $Ex(a_1|H)$ for the rupture model of Der Kiureghian and Ang (1975).

This rupture model requires only that the rupture midpoint be located on the fault; the rupture is permitted to extend by one half its length beyond the end of the fault and thus rupture length may be twice the fault length. The derivation of $Ex(a_1|H)$ for this case parallels the development of equations 26 and 34 for the fcr (fault contained rupture) model, and we assume familiarity with the notation and derivation of $Ex(a_1|H)$ for the fcr model. As before, earthquake magnitudes are restricted to $m_0 \le m \le m_{max}$.

m_{l_0} = lowest magnitude that can produce acceleration a_1 at (X,P)

$m_{min} = \max(m_0, m_{l_0})$

$b_m = H\exp(g\,m)$ = rupture length for magnitude m ruptures.

Again, two cases will be considered.

Case 1. $X < 0$ (Site beyond end of fault.)

Let $x_l(m)$ be the point on the fault which is at distance $R_1(m)$ from (X,P):

$$x_l(m) = \sqrt{R_1(m)^2 - P^2} + X. \tag{A1}$$

If $(l,0)$ is any point on the fault, $(0 \le l \le L)$, all ruptures with midpoints $l = l_{mid}$ on the fault for which

$$0 \le l_{mid} \le x_l(m) + \frac{b_m}{2} \tag{A2}$$

will contribute to $Ex(a_1|H)$. The lowest magnitude m_{l_0} that could produce a_1 at the site is the magnitude at which $x_l(m) + \frac{b_m}{2} = 0$. Set $m_{min} = \max(m_0, m_{l_0})$

Assume $m_{min} < m_{max}$. (Otherwise $Ex(a_1|H) = 0$.)

There is a magnitude m_s above which all ruptures regardless of location on the fault will yield $a \ge a_1$; this is the magnitude m_s for which

$$x_l(m_s) = L - \frac{b_T(m_s)}{2} \tag{A3}$$

where

$$br(m) = \min(b_m, 2L) \tag{A4}$$

(that is, if rupture length b_m would exceed twice the fault length L, rupture length is set equal to twice the fault length). Then

$$L - \frac{br(m_x)}{2} = \sqrt{R_x(m_x)^2 - P^2} + X \,, \tag{A5}$$

for a unique $m = m_x$ since both $br(m)$ and $R_1(m)$ are nondecreasing as m increases. Set

$$m_u = \begin{cases} m_{\min} & m_x \leq m_{\min} \\ m_x & m_{\min} < m_x \leq m_{\max} \\ m_{\max} & m_{\max} < m_x \end{cases} \tag{A6}$$

The fraction of earthquakes of magnitude m which yield $a > a_1$ at (X, P) is

$$pr(a \geq a_1 | H, m) = \begin{cases} 0 & m \leq m_{\min} \\ \dfrac{x_l(m) + \dfrac{b_m}{2}}{L} & m_{\min} < m \leq m_u \\ 1 & m_u < m \leq m_{\max} \end{cases} \tag{A7}$$

Thus for $X < 0$,

$$Ex(a_1|H) = rate \int_{m_0}^{m_{\max}} f(m) pr(a \geq a_1 | H, m)\, dm = \frac{b \; rate \exp(b\, m_0)}{1 - \exp[-b\,(m_{\max} - m_0)]} \tag{A8}$$

$$\cdot \left\{ \int_{m_{\min}}^{m_u} \exp(-b\, m) \frac{\sqrt{R_1(m)^2 - P^2} + X + \dfrac{H}{2} \exp(g\, m)}{L} dm + \int_{m_u}^{m_{\max}} \exp(-b\, m)\, dm \right\}.$$

(Either integral may have a zero range of integration.)

Case 2. $0 \leq X \leq \frac{L}{2}$ (Site "above" fault.)

For a magnitude m earthquake all ruptures with midpoints l_{mid} in the range

$$X - \Delta x(m) - \frac{b_m}{2} \leq l_{mid} \leq X + \Delta x(m) + \frac{b_m}{2}$$

where $$\Delta x(m) = \sqrt{R_1(m)^2 - P^2}$$

will contribute to exceedances of a_1 at (X,P). Rupture midpoints are restricted to lie within

$$\max[X-\Delta x(m)-\frac{b_m}{2},\ 0] \le l_{mid} \le \min[X+\Delta x(m)+\frac{b_m}{2},\ L]. \tag{A9}$$

Several cases must, therefore, be considered.

(a) Magnitudes m for which $X - \Delta x(m) - \frac{b_m}{2} \ge 0$.

For these magnitudes, even if the fault were longer, any additional ruptures of length b_m would be too far away from the site at (X,P) to produce accelerations $a \ge a_1$.

Let m_{x_1} be the magnitude for which equality holds $X = \Delta x(m) + \frac{b_m}{2}$. Define m_{u_1} as in equation A6. For $m_{\min} \le m \le m_{u_1}$, all ruptures with l_{mid} in the range

$$X - \Delta x(m) - \frac{b_m}{2} < l_{mid} < X + \Delta x(m) + \frac{b_m}{2}$$

will contribute to $Ex(a_1 | H)$. This corresponds to a fraction of possible magnitude m earthquakes

$$fr(m) = \frac{2\Delta x(m) + b_m}{L} \tag{A10}$$

For $m_{\min} \le m \le m_{u_1}$

$$pr(a \ge a_1 | H, m) = \frac{2\sqrt{R_1(m)^2 - P^2} + H \exp(g\ m)}{L} \tag{A11}$$

(b) Magnitudes $m > m_{u_1}$ for which $X + \Delta x(m) + \frac{b_m}{2} \le L$.

At these magnitudes, additional ruptures of length b_m would contribute to accelerations $a \ge a_1$ if the fault extended beyond (0,0) in the negative x-direction. Adding fault length beyond $(L,0)$ in the positive x-direction would not increase the possible ruptures which produce $a > a_1$ at (X,P), since $X \le \frac{L}{2}$.

Let $m = m_{x_2}$ be the magnitude for which $L - X = \Delta x(m) + \frac{b_m}{2}$. Define m_{u_2} as in equation A6. For $m_{u_1} < m \le m_{u_2}$, all magnitude m ruptures for which

$$0 \le l_{mid} \le X + \Delta x(m) + \frac{b_m}{2}$$

will contribute for a fraction

$$fr(m) = \frac{X + \Delta x(m) + \frac{b_m}{2}}{L} \qquad \text{(A12)}$$

giving for $m_{u_1} < m \leq m_{u_2}$

$$Pr(a > a_1 | H, m) = \frac{\sqrt{R_1(m)^2 - P^2} + X + \frac{H}{2}\exp(g\,m)}{L} \qquad \text{(A13)}$$

(c) Magnitudes $m_{u_2} < m \leq m_{max}$ (if $m_{u_2} < m_{max}$).

All earthquakes regardless of location on the fault will contribute to exceedances of a_1.

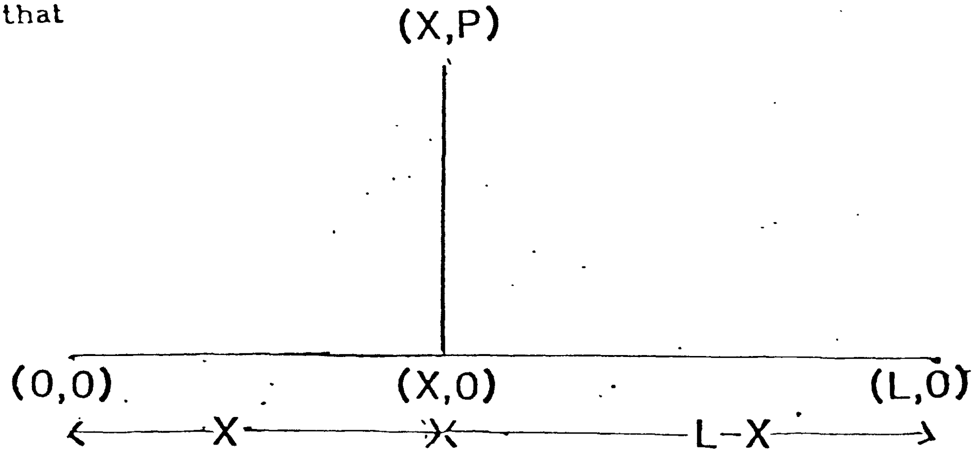
For $m_{u_2} < m \leq m_{max}$

$$Pr(a > a_1 | H, m) = 1 . \qquad \text{(A14)}$$

Combining these results yields, for $0 \leq X \leq \frac{L}{2}$

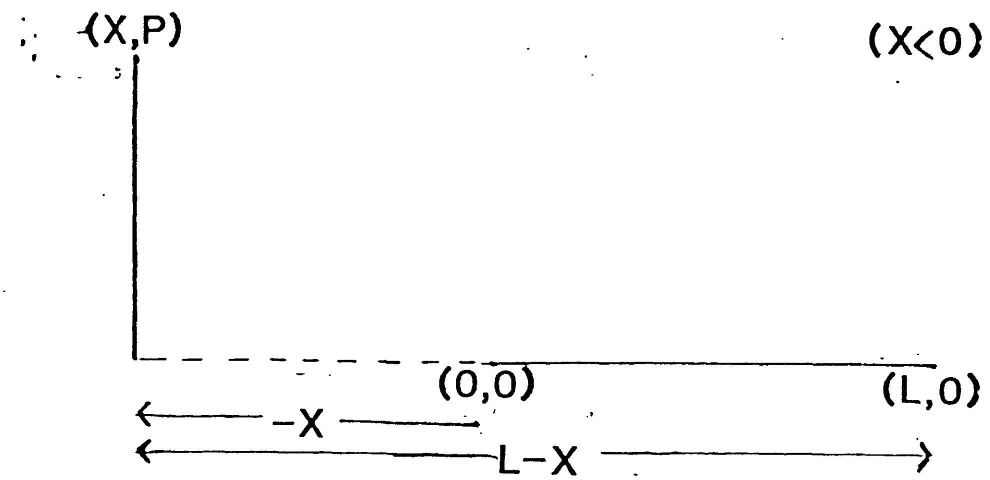
$$Ex(a_1|H) = \frac{b\,rate\,\exp(b\,m_0)}{1 - \exp[-b\,(m_{max} - m_0)]}\Bigg[\int_{m_{min}}^{m_{u_1}} \exp(-b\,m)\frac{2\sqrt{R_1(m)^2 - P^2} + H\exp(g\,m)}{L}\,dm$$
$$+ \int_{m_{u_1}}^{m_{u_2}} \exp(-b\,m)\frac{\sqrt{R_1(m)^2 - P^2} + X + \frac{H}{2}\exp(g\,m)}{L}\,dm + \int_{m_{u_2}}^{m_{max}} \exp(-b\,m)\,dm\Bigg] \qquad \text{(A15)}$$

Der Kiureghian and Ang (1977) have pointed out that for a site at (X, P) where $0 \leq X \leq \frac{L}{2}$, the computation of $Ex(a_1|H)$ may be done instead for a site located at $(0, P)$ for two segments of length X and $L - X$ and results combined so that



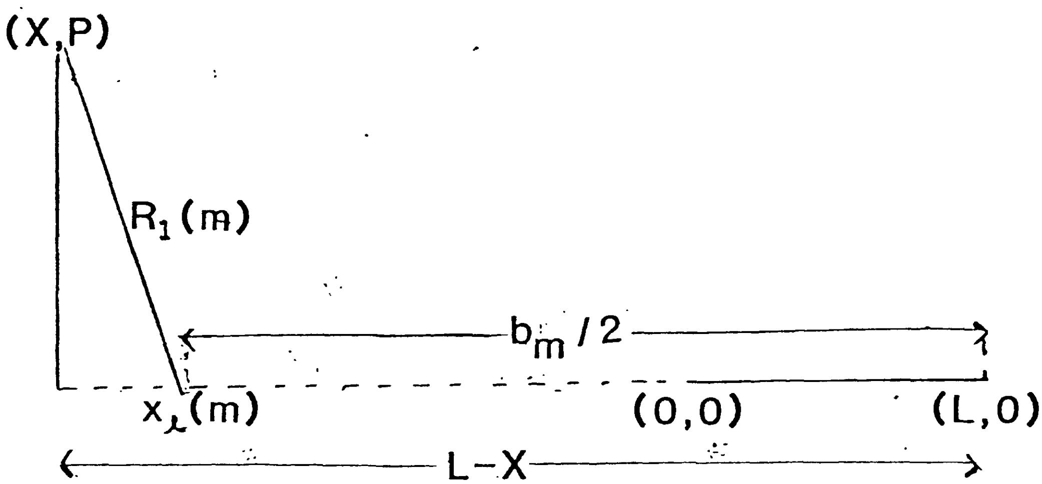
$$Ex(a_1|H)_{(seg\,L)} = Ex(a_1|H)_{(seg\,X)}\frac{X}{L} + Ex(a_1|H)_{(seg\,L-X)}\frac{L-X}{L} . \qquad \text{(A16)}$$

(The fault may not be treated as two segments in the fcr model, because in that model possible rupture center locations depend upon both rupture length and fault length.) Der Kiureghian and Ang also asserted that for a site beyond the end of the fault, $X < 0$ or $X > L$, a similar decomposition holds.



$$Ex(a_1 \mid H)_{(segL)} = Ex(a_1 \mid H)_{(segL-X)} \frac{L-X}{L} - Ex(a_1 \mid H)_{(seg\,-X)} \frac{(-X)}{L}. \quad \text{(A17)}$$

While this is true for most rupture lengths, there is one case in which it is not correct. This formulation permits rupture lengths $\frac{b_m}{2} = L - X$, on segment $(L-X)$.



In the case shown,

$$x_l(m) + \frac{b_m}{2} = L - X\ ,$$

where $$x_l(m) = \sqrt{R_1(m)^2 - P^2},$$

so all magnitude m earthquakes on $(L-X)$, on $(-X)$ and by equation A17 on (L) contribute to $Ex(a_1 \mid H)_{(segL)}$. In reality, however, the maximum length rupture permitted for segment L is $b_m = 2L$ and $X + x_l + L < 0$, so magnitude m does not contribute to $Ex(a_1|H)_{(seg L)}$.

Appendix B

Exceedances of acceleration a_0 when variability is taken into account for point source ruptures.

$Pex(a_0)$ is defined as the probability that a random earthquake in the range $m_0 \leq m' \leq m_{max}$ will cause an acceleration greater than a_0 at a site when accelerations are assumed (for each magnitude) to be normally distributed around the (log) mean acceleration with standard deviation σ_a. That is, $Pex(a_0)$ is is given by:

$$Pex(a_0) = \int_{a_1}^{a_2^*} \left[\frac{\ln(a_0) - \ln(a)}{\sigma_a} \right] pr(a)\, da \qquad \text{(B1)}$$

where

$pr(a) =$ density of (log-mean) acceleration a

$a_1 =$ lowest (log-mean) acceleration for which $pr(a) > 0$ at the site;

$a_2 =$ highest (log-mean) acceleration for which $pr(a) > 0$ at the site.

For earthquakes in the range $m_0 \leq m \leq m_{max}$, using the Esteva attenuation gives

$$\ln(a) = c_1 + c_2 m + c_3 \ln(R)$$

Let

$$\ln(a_1) = \ln(a_0) - n_1 \sigma_a$$

$$\ln(a_2) = \ln(a_0) + n_2 \sigma_a$$

for the proper n_1, n_2.

Let

$$w = \ln(a) - \ln(a_0).$$

Then

$pr(a) = pr[a_0 \exp(w)]$, $\quad da = a_0 \exp(w)\, dw$, and

$$Pex(a_0) = \int_{-n_1\sigma_a}^{n_2\sigma_a} \varphi^*\left(-\frac{w}{\sigma_a}\right) pr[a_0 \exp(w)]\, a_0 \exp(w)\, dw. \qquad \text{(B2)}$$

For the point source case, the probability that an acceleration $a_1 > a$ is produced is

$$Pr(a_1 > a) = \frac{b}{1-\exp[-b(m_{max}-m_0)]} \int_{m_a}^{m_{max}} \exp[-b(m-m_0)]\,dm \tag{B3}$$

where for the Esteva attenuation

$$m_a = \frac{\ln(a) - c_1 - c_3 \ln(R)}{c_2} \tag{B4}$$

from which (see Appendix C)

$$pr(a) = \frac{k\, b\, \exp[-b(m_a - m_0)]}{c_2 a}\, da = k_2 b\, a^{(-\frac{b}{c_2}-1)}\, da \tag{B5}$$

where

$$k = \frac{1}{1-\exp[-b(m_{max}-m_0)]}$$

$$k_2 = (\frac{k}{c_2}) \exp[b \frac{c_1}{c_2} + m_0] R^{(b \frac{c_3}{c_2})}.$$

Thus

$$Pex(a_0) = k_2 b\, a_0^{(-\frac{b}{c_2})} \int_{-n_1\sigma_a}^{n_2\sigma_a} \sigma_a^{*}(-\frac{w}{\sigma_a}) \exp(-b \frac{w}{c_2})\, dw. \tag{B6}$$

Using the relationships

$$\varphi^{*}(x) = \frac{1}{2}[1 - \mathrm{erf}(\frac{x}{\sqrt{2}})]$$ and

$$\exp(a x) \cdot \mathrm{erf}(b x) = \exp(a x)\ \mathrm{erf}(b x) - \frac{1}{a}\exp(\frac{a^2}{4b^2})\,\mathrm{erf}(b x - \frac{a}{2b})$$

yields

$$Pex(a_0) = (1-k)\,\varphi^{*}(-n_2) + k\,\varphi^{*}(n_1) \tag{B7}$$

$$+k R^{(b \frac{c_3}{c_2})} a_0^{-\frac{b}{c_2}} \exp[b \frac{c_1}{c_2} + b\, m_0 + b^2 \frac{\sigma_a}{2c_2^2}]\,[\varphi^{*}(-n_2 - \frac{b\,\sigma_a}{c_2}) - \varphi^{*}(n_1 - \frac{b\,\sigma_a}{c_2})].$$

This is equivalent to Cornell's (1971) result.

Appendix C

Calculation of $pr(a)$ from equations 26 and 34.

To calculate the acceleration density $pr(a)$ from the exceedances of a as given by equations 26 and 34, we note that

$$pr(a) = \frac{\partial[1-Ex(a)]}{\partial a} \tag{C1}$$

and differentiate under the integral sign using the following theorem:

If

$$Q(a) = \int_{u_0(a)}^{u_1(a)} f(m,a)\,dm \tag{C2}$$

where u_0 and u_1 are differentiable functions in a closed interval, (a_0, a_1); $f(x,a)$ and $f_a(m,a)$ are continuous in the region $a_0 < a < a_1$ and $u_0(a) \leq m < u_1(a)$, then

$$\frac{dQ}{da} = \int_{u_0(a)}^{u_1(a)} \frac{df(m,a)}{da}dm - f(u_0,a)\frac{du_0}{da} + f(u_1,a)\frac{du_1}{da}\,. \tag{C3}$$

The quantities m_u in equation 26 and m_{u_1} and m_{u_2} in equation 34 enter as the upper limit of integration in one integral and lower limit in the succeeding integral. Hence derivatives of m with regard to a (the second and third terms in equation C3) cancel. The only derivative of m with regard to a that remains occurs at $m = m_{lo}$.

We shall determine the expression for $pr(a)$ specifically for the Esteva attenuation $\ln a = c_1 + c_2 m + c_3 \ln R$. In this case

$$\left.\frac{\partial m}{\partial a}\right|_{m=m_0} = \frac{\partial}{\partial a}\left[\frac{\ln a - c_1 - c_3 \ln CD}{c_2}\right] = \frac{1}{c_2 a} \tag{C4}$$

$$CD = \begin{cases} \sqrt{P} & X > 0 \\ \sqrt{P^2 + X^2} & 0 \leq X \end{cases}.$$

and

$$\frac{\partial R^2}{\partial a} = \frac{2R^2}{c_3 a}. \tag{C5}$$

Recall that earthquakes are restricted to occur in the magnitude range $m_0 \leq m \leq m_{max}$, and that

m_{lo} = lowest magnitude which produces acceleration a at (X,P).

$m_{min} = \max(m_0, m_{lo})$

The expression for $pr(a)$ depends upon site location and magnitude limits m_{lo}, and m_u (or m_{u_1}, m_{u_2}). (The limits m_u, m_{u_1} and m_{u_2} are defined as in equations 26 and 34 and relate to magnitudes above which all earthquakes on portions of the fault produce acceleration a or greater at (X,P).)

Case 1. Site at (X,P), $0<X$.

Using equations 25, C4 and C5, we obtain for $pr(a)$

$$pr(a) = \begin{cases} -NT \displaystyle\int_{m_{min}}^{m_u} \frac{\exp(-bm)\,(R^2+P^2)^{-\frac{1}{2}} R^2}{[L - H\exp(g\,m)]\,c_3 a} dm; & m_{min} < m_u;\ m_{min} = \max(m_0, m_{lo}) \\ NT \dfrac{\exp(-b\,m_{min})}{c_2 a} & m_{min} = m_u;\ m_0 < m_{lo} \\ 0 & m_{min} = m_u;\ m_{lo} < m_0. \end{cases} \tag{C6}$$

where

$$NT = \frac{rate\ \exp(b\,m_0)}{1-\exp[-b\,(m_{max} - m_0)]}.$$

Case 2. $0 \leq X \leq \frac{L}{2}$ Using equation 34, C5 and C6 and setting

$$T_1 = -\int_{m_{min}}^{m_{u_1}} \frac{2\exp(-bm)\,(R^2 - P^2)^{-\frac{1}{2}} R^2}{[L - H\exp(g\,m)]c_3 a} dm$$

$$T_2 = -\int_{m_{u_1}}^{m_{u_2}} \frac{\exp(-b\,m)\,(R^2 - P^2)^{-\frac{1}{2}} R^2}{[L - H\exp(gm)]c_3 a} dm\ ,$$

we obtain for $pr(a)$

$$pr(d) = \begin{cases} NT\,(T_1 + T_2) & m_{\min} = m_0;\ m_{\min} < m_{u_1} \\ NT\,(T_1 + T_2 + \dfrac{H \exp[m_{\min}(-b+g)]}{c_2 a\,[L - H\exp(g\,m)]}) & m_{\min} = m_{lo};\ m_0 < m_{lo} < m_{u_1} \\ NT \cdot T_2 & m_{\min} = m_0 = m_{u_1};\ m_{lo} < m_0 \\ NT(T_2 + \dfrac{\exp(-b\,m_{\min})X}{c_2 a\,[L - H\exp(g\,m)]}) & m_{lo} = m_{\min} = m_{u_1};\ m_0 < m_{lo} \\ 0 & m_0 = m_{\min} = m_{u_2};\ m_{lo} < m_0 \\ NT\,\dfrac{\exp(-b m_{\min})}{c_2 a} & m_{\min} = m_{lo} = m_{u_2};\ m_0 < m_{lo}\ . \end{cases} \qquad \text{(C7)}$$

Appendix D

Decomposition of acceleration exceedances into "point source" and "rupture length" contributions and comparison of results of series expansion with direct integration for some cases.

Sample values of $Ex(a \mid fr\, \sigma_l)$ using the series expansion for I_0, I_1, J_0, J_1 from equations F12 and F17 for an Esteva attenuation are shown.

Table D1 illustrates the contributions of I_0, I_1, J_0, J_1 to the total exceedances of a_1 (for several values of a_1) at a felt point above the center of the fault at $(\frac{L}{2}, P)$ as a function of break length

$$b_m = \exp(a + fr\,\sigma_a)\exp(g\,m) = H(fr\,\sigma_l)\exp(g\,m).$$

I_0, I_1, J_0, J_1, are evaluated using equations F11 and F17 for an Esteva attenuation $\ln a = 3.4 + .89m - 1.17 \ln R$. (Correction terms from equation F16 for $j = 0, 1, 2$ the upper limit of integration have been applied to I_0, I_1).

I_0 represents accelerations that would occur of breaks were of zero length or point sources.

J_0 is the integral of the "break length term" using dènominator L.

I_1 and J_1 are first order corrections to I_0 for the finite break length $H \exp(g\,m)$ in the denominator $[L - H \exp(g\,m)]$.

The magnitude range is $4.0 \leq m \leq 7.5$.

The minimum magnitude column gives the lowest magnitude earthquake (occurring at distance P) that could produce the specified acceleration at (X, P). $M_u(fr\,\sigma_1)$ is the minimum of m_{max} and the magnitude beyond which an earthquake occurring anywhere on the fault would cause a_1. The column I_{all} represents all earthquakes in the range $m_u(fr\,\sigma_l) \leq m \leq m_{max}$.

The "sum" column gives the total

$$\sum = I_0 + I_1 + J_0 + J_1 + I_{all}.$$

The "analytic" column gives the results of numerically integrating the original equation.

Table D2 is similar to table D1 for a felt point above the end of the fault at $(0,P)$. In this case, $J_0 = J_1 = 0$. I_0 and I_1 do not contain the factor 2 included for a felt point above the center of the fault since now earthquakes occur only to one side of (X,P).

Table D1: Illustrating the decomposition of acceleration exceedances into point and rupture source contributions for a site 10 km or 50 km from the center of a 400 km fault.

f_T	I_0	J_0	I_1	J_1	I_{all}	sum	analytic	m_{min}	m_u
			Site at (200,10) a_1=100 gals						
-2	2.58e-3	7.77e-5	6.99e-6	1.93e-7	0.00e0	2.66e-3	2.66e-3	4.38	7.50
-1	2.58e-3	2.57e-4	2.31e-5	2.11e-6	0.00e0	2.86e-3	2.86e-3	4.38	7.50
0.	2.58e-3	8.52e-4	7.67e-5	2.32e-5	0.00e0	3.53e-3	3.53e-3	4.38	7.50
1.	2.49e-3	2.74e-3	2.18e-4	2.18e-4	2.07e-4	5.87e-3	5.93e-3	4.38	6.91
2.	2.06e-3	7.76e-3	4.46e-4	1.54e-3	2.35e-3	1.42e-2	1.48e-2	4.38	5.86
			Site at (200,10) a_1=200 gals						
-2	5.16e-4	3.14e-5	2.39e-6	1.32e-7	0.00e-0	5.50e-4	5.50e-4	5.16	7.50
-1	5.16e-4	1.04e-4	7.91e-6	1.45e-6	0.00e-0	6.29e-4	6.29e-4	5.16	7.50
0	5.16e-4	3.44e-4	2.62e-5	1.59e-5	0.00e-0	9.02e-4	9.05e-4	5.16	7.50
1	4.88e-4	1.09e-3	7.40e-5	1.50e-4	1.12e-4	1.91e-3	1.97e-3	5.16	7.10
2	2.80e-4	2.63e-3	9.35e-3	7.47e-4	1.96e-3	5.45e-3	5.88e-3	5.16	5.94
			Site at (200,10) a_1=300 gals						
-2	1.94e-4	1.80e-5	1.20e-6	1.01e-7	0.00e-0	2.14e-4	2.14e-4	5.62	7.50
-1	1.94e-4	5.95e-5	3.97e-6	1.11e-6	0.00e-0	2.59e-4	2.59e-4	5.62	7.50
0	1.94e-4	1.97e-4	1.32e-5	1.22e-5	0.00e-0	4.17e-4	4.19e-4	5.62	7.18
1	1.80e-4	6.13e-4	3.65e-5	1.14e-4	8.25e-5	1.03e-3	1.07e-3	5.62	7.18
2	5.08e-5	8.21e-4	2.10e-5	3.31e-4	1.81e-3	3.03e-3	3.28e-3	5.62	5.98
			Site at(200,10) a_1=500 gals						
-2	5.20e-5	8.33e-6	4.46e-7	6.61e-8	0.00e-0	6.08e-5	6.08e-5	6.19	7.50
-1	5.20e-5	2.76e-5	1.48e-6	7.25e-7	0.00e-0	8.18e-5	8.18e-5	6.19	7.50
0	5.20e-5	9.14e-5	4.89e-6	7.95e-6	0.00e-0	1.56e-4	1.58e-4	6.19	7.50
1	4.54e-5	2.73e-4	1.29e-5	7.23e-5	5.93e-5	4.63e-4	4.99e-4	6.19	7.25
2					1.16e-3	1.16e-3	1.16e-3	6.19	6.19
			Site at (200,50) a_1=100 gals						
-2	1.18e-4	5.20e-6	1.19e-6	4.90e-8	0.00e-0	1.24e-4	1.24e-4	6.50	7.50
-1	1.18e-4	1.72e-5	3.93e-6	5.37e-7	0.00e-0	1.39e-4	1.40e-4	6.50	7.50
0	1.18e-4	5.70e-5	1.30e-5	5.89e-5	0.00e-0	1.94e-4	1.96e-4	6.50	7.50
1	6.42e-5	1.22e-4	1.90e-5	3.48e-5	1.51e-4	3.91e-4	4.14e-4	6.50	7.01
2				5.87e-4	5.73e-4	5.87e-4	5.73e-4	6.50	6.50
			Site at (200,50) a_1=200 gals						
-2	4.81e-6	7.20e-7	7.77e-8	1.01e-6	0.00e-0	5.61e-6	5.85e-6	7.28	7.50
-1	4.81e-6	2.39e-6	2.24e-7	1.10e-7	0.00e-0	7.53e06	7.80e-6	7.28	7.50
0	4.81e-6	7.90e-6	7.43e-7	1.21e-6	0.00e-0	1.47e-5	1.53e-5	7.28	7.50
1		3.88e-6		1.81e-6	4.33e-5	4:77e:5	5.12e-5	7.28	7.31
2					5.17e-5	6.75e-5	5.17e-5	7.28	7.28

Table D2. Illustrating the decomposition of acceleration exceedances into point and rupture sources for sites 10km and 50 km from the end of a 400 km fault.

f_T	I_0	I_1	I_{all}	sum	analytic	m_{min}	m_u
			Site at (X,0)	$a_{1=100}$ gals			
-2	1.29e-3	3.50e-6	0.00e+0	1.29e-3	1.29e-3	4.38	7.50
-1	1.29e-3	1.16e-5	0.00+0	1.30e-3	1.30e-3	4.38	7.50
0	1.29e-3	3.83e-5	0.00+0	1.33e-3	1.33e-3	4.38	7.50
1	1.29e-3	1.27e-4	0.00e+0	1.41e-3	1.45e-3	4.38	7.50
2.	1.21e-3	3.30e-4	4.18e-4	1.96e-3	2.25e-3	4.38	6.64
			Site at (0,10)	a_1=200 gals			
-2	2.58e-4	1.19e-6	0.00e+0	2.59e-4	2.59e-4	5.16	7.50
-1	2.58e-4	3.96e-6	0.00e+0	2.62e-4	2.62e-4	5.16	7.50
0	2.58e-4	1.31e-5	0.00e+0	2.71e-4	2.72e-4	5.16	7.50
1	2.58e-4	4.34e-5	0.00e+0	3.01e-4	3.16e-4	5.16	7.50
2	2.23e-4	9.95e-5	3.48e-4	6.70e-4	8.39e-4	5.16	6.71
			Site at (0,10)	a_1=300 gals			
-2	9.71e-5	6.00e-7	0.00e+0	9.77e-5	9.77e-5	5.62	7.50
-1	9.71e-5	1.99e-6	0.00e+0	9.91e-5	9.92e-5	5.62	7.50
0	9.71e-5	6.58e-6	0.00e+0	1.04e-4	1.04e-4	5.62	7.50
1	9.71e-5	2.18e-5	0.00e+0	1.19e-4	1.28e-4	5.62	7.50
2	7.46e-5	4.31e-5	3.23e-4	4.41e-4	5.57e-4	5.62	6.74
			Site at (0,10)	a_1=500 gals			
-2	2.60e-5	2.23e-7	0.00e-0	2.62e-5	2.62e-5	6.19	7.50
-1	2.60e-5	7.39e-7	0.00e-0	2.67e-5	2.68e-5	6.19	7.50
0	2.60e-5	2.45e-6	0.00e-0	2.84e-5	2.87e-5	6.19	7.50
1	2.60e-5	8.10e-6	0.00e-0	3.41e-5	3.87e-5	6.19	7.50
2	1.35e-5	1.04e-5	3.01e-4	3.25e-4	3.86e-4	6.19	6.77
			Site at(0,50)	a_1=60 gals			
-2	2.44e-4	1.81e-6	0.00e-0	2.46e-4	2.46e-4	5.92	7.50
-1	2.44e-4	5.98e-6	0.00e-0	2.50e-4	2.50e-4	5.92	7.50
0	2.44e-4	1.98e-5	0.00e-0	2.64e-4	2.66e-4	5.92	7.50
1	2.44e-4	6.56e-5	0.00e-0	3.10e-4	3.42e-4	5.92	7.50
2	1.33e-4	8.42e-5	4.61e-4	6.78e-4	8.59e-4	5.92	6.60
			Site at (0,50)	a_1= 100 gals			
-2	5.89e-5	5.93e-7	0.00e-0	5.95e-5	5.95e-5	6.50	7.50
-1	5.89e-5	1.97e-6	0.00e-0	6.09e-5	6.10e-5	6.50	7.50
0	5.89e-5	6.51e-6	0.00e-0	6.54e-5	6.63e-5	6.50	7.50
1	5.89e-5	2.15e-5	0.00e-0	8.05e-5	9.49e-5	6.50	7.50
2	1.09e-5	9.13e-6	3.48e-4	3.68e-4	4.32e-4	6.50	6.71
			Site at (0,50)	a_1=200 gals			
-2	2.41e-6	3.39e-8	0.00e-0	2.44e-6	2.56e-6	7.28	7.50
-1	2.41e-6	1.12e-7	0.00e-0	2.52e-6	1.65e-6	7.28	7.50
0	2.41e-6	3.71e-7	0.00e-0	2.78e-6	2.99e-6	7.28	7.50
1	2.41e-6	1.23e-6	0.00e-0	3.64e-6	5.25e-6	7.28	7.50
2	-8.20e-7	-1.39e-6	5.17e-5	4.95e-5	5.17e-5	7.28	7.28

Appendix E

Tables to illustrate how acceleration exceedances vary as a function of site location, rupture model, attenuation function and rupture length.

Tabular entries show yearly exceedance rates of 100 gals (.1g) multiplied by 10^6. Absolute values are not intended to be significant; only relative values are meaningful for comparison between models, etc. Results will be given for three cases.

Case 1: fcr (fault contained rupture) model: ruptures wholly contained within fault limits; Esteva attenuation

$$\ln a = 3.4 + .89m - 1.17\ln R \qquad \text{(E1)}$$

Case 2: der Kiureghian and Ang model: rupture midpoints only must be on the fault; ruptures may extend beyond end of fault by one-half rupture length. Same attenuation function as Case 1.

Case 3: fcr model using Schnabel and Seed (1973) attenuation curves.

Results are for a fault 400 km long, located on the X-axis from (0,0)--(400,0). Site is located at (X,P). Column headings give X coordinate of site (from 200 km, directly above the center of the fault to -50 km beyond end of fault.) Row headings give P coordinate where

$$P = \sqrt{Y^2 + d^2} \text{ if equation E1 is used; } R = \sqrt{X^2 + P^2}$$

$$P = Y \text{ if Schnabel and Seed attenuation is used.}$$

Input values $b = 2$ (see model assumption 4), rate=.1 earthquakes per year, and magnitude range $4.0 \leq m \leq 7.5$ are used. The rupture length-magnitude relationship is from Bonilla and Buchanan (1970):

$$\log_{10} = -1.085 + .389m + .52fr$$

where $.52 = \sigma_l$, standard deviation in log rupture length.

$Ex(a \mid fr\,\sigma_l)$ shown correspond to $fr = -2$ (short), $fr = 0$ (median), $fr = .313$ (average) and $fr = 2$ (long) ruptures. The result integrated over rupture length $-1.5 \leq fr \leq 1.5$ is also given. $[Ex(a \mid .313\sigma_l)]$ may be compared with the integrated result $Ex(a)$.]

Two results $Pex(a\,|.313\sigma_l)$ are also shown for Case 1 when integration is done over acceleration variability, for $\sigma_a=.6$ and $\sigma_a=.3$ for accelerations lognormally distributed around their median (mean log) values. (Rupture length $3.13\sigma_l$ is "average" or mean length.) These integrated results should be compared with $Ex(a\,|.313\sigma_l)$ for Case 1 in which $\sigma_a=0$.

Case 1. fcr model, Esteva attenuation
(1a) Exceedances of 100 gals for very short ruptures (-2 σ_l)

$Ex(100\,\vert -2\sigma_l)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	2660	2660	2632	2179	1291	762	434	23
20	811	810	786	565	392	301	222	20
30	382	382	362	245	184	153	124	16
40	211	211	196	129	101	87	74	12
50	124	124	114	74	60	52	45	8
65	57	57	52	33	27	24	21	4
80	23	23	22	14	11	10	8	1
100	3	3	3	2	1	1	1	0

(1b) Exceedances of 100 gals for median length ruptures (0 σ_l).

$Ex(100\,\vert 0\,\sigma_l)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	3533	3513	3410	2443	1328	790	455	26
20	1136	1117	1028	607	413	319	237	23
30	559	542	470	265	198	165	135	18
40	320	307	252	142	111	96	81	14
50	196	186	145	83	66	58	50	9
65	96	90	66	38	31	27	24	4
80	43	40	28	16	13	11	10	1
100	6	6	4	2	1	1	1	0

(1c) Exceedances of 100 gals for "mean" length ruptures (.313 σ_l).

$Ex(100\,\vert .313\sigma_l)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	3989	3946	3790	2521	1348	805	466	28
20	1312	1273	1138	623	425	329	245	24
30	657	622	515	275	206	172	241	20
40	383	353	272	149	117	101	86	15
50	238	214	156	87	70	62	54	10
65	120	103	71	41	33	29	26	5
80	55	46	30	17	14	12	10	1
100	9	8	5	2	2	1	1	0

(1d) Exceedances of 100 gals for very long ruptures $(2\sigma_l)$.

	$Ex(100 \mid 2\sigma_l)\,10^6$							
	X							
P	200	100	50	10	0	-5	-10	-50
10	14804	13282	9542	3719	2247	1570	1125	339
20	5272	4008	2614	1416	1104	950	812	323
30	2429	1868	1368	888	756	689	624	300
40	1129	1047	862	636	567	530	494	263
50	587	587	559	468	432	412	391	182
65	249	249	249	249	249	247	239	94
80	106	106	106	106	106	105	102	37
100	18	18	18	18	18	18	17	0

(1e) Exceedances of 100 gals integrated over rupture length, $-1.5 \leq fr \leq 1.5\,fr$.

	$Ex(100)\,10^6$							
	X							
P	200	100	50	10	0	-5	-10	-50
10	4024	3963	3763	2465	1356	811	472	30
20	1330	1274	1110	629	431	334	251	26
30	667	617	500	281	212	177	145	21
40	388	347	268	154	121	105	90	16
50	240	209	157	92	74	65	57	11
65	118	101	74	44	36	32	28	5
80	53	45	33	19	16	14	12	2
100	9	7	5	3	2	2	1	0

Case 2. der-Kiureghian and Ang model. Esteva attenuation

(2a) Exceedances of 100 gals for very short ruptures $(-2\sigma_l)$

	$Ex(100 \mid -2\sigma_l)\,10^6$							
	X							
P	200	100	50	10	0	-5	-10	-50
10	2561	2650	2625	2195	1325	793	458	25
20	806	806	784	574	403	312	232	22
30	379	379	361	250	190	159	129	18
40	209	209	196	132	104	90	77	13
50	123	123	114	76	61	54	47	9
65	56	56	52	34	28	25	22	4
80	23	23	22	14	12	10	9	1
100	3	3	3	2	1	1	1	0

(2b) Exceedances of 100 gals for median length ruptures $(0\sigma_l)$.

	$Ex(100 \mid 0\sigma_l)\,10^6$							
	X							
P	200	100	50	10	0	-5	-10	-50
10	3425	3420	3366	2712	1712	1142	713	58
20	1073	1068	1022	720	536	443	353	51
30	516	512	474	321	258	227	195	42
40	290	287	258	173	145	131	117	32
50	175	173	151	102	87	80	73	23
65	84	83	71	48	42	39	36	13
80	37	37	31	21	18	17	16	6
100	5	5	5	3	3	2	2	

(2c) Exceedances of 100 gals for "mean" length ruptures $(.313\,\sigma_l)$

$Ex(100\,\vert\,.313\,\sigma_l)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	3812	3802	3732	2954	1906	1328	859	80
20	1206	1197	1136	789	603	510	418	70
30	584	577	527	255	292	261	229	57
40	331	325	287	196	166	151	137	44
50	201	196	168	115	100	93	86	32
65	97	95	79	55	49	46	42	18
80	43	43	35	24	22	20	19	9
100	7	7	6	4	3	3	3	1

(2d) Exceedances of 100 gals for very long ruptures $(2\sigma_l)$

$Ex(100\,\vert\,2\,\sigma_l)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	11684	11410	10544	7117	5953	5371	4789	1356
20	3776	3528	2919	2181	1995	1903	1810	1067
30	1804	1600	1309	1066	1004	973	942	694
40	947	848	721	615	588	574	561	450
50	568	495	435	382	368	361	355	298
65	249	235	215	195	189	187	186	160
80	106	105	100	94	91	90	89	79
100	18	18	18	18	18	17	17	15

(2e) Exceedances of 100 gals integrated over rupture length, $-1.5 \leqq fr \leqq 1.5$.

$Ex(100)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	3813	3798	3720	2921	1907	1340	893	94
20	1206	1192	1125	785	603	511	421	82
30	584	572	519	355	292	261	230	66
40	330	320	281	194	166	151	137	50
50	200	192	165	115	100	93	86	36
65	97	92	77	55	49	46	42	20
80	43	41	34	24	22	20	19	9
100	7	6	5	4	3	3	3	1

Case 3. fcr model, Schnabel and Seed attenuation

(3a) Exceedances of 100 gals for very short ruptures $(-2\sigma_l)$

$Ex(100\vert-2\sigma_l)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	1157	1157	1143	775	564	455	355	10
20	827	827	816	548	402	328	257	7
30	479	479	471	326	230	181	136	5
40	182	182	178	119	86	70	54	2
50	69	69	68	43	33	27	22	0
65	15	15	15	10	7	6	4	0
100	0	0	0	0	0	0	0	0

(3b) Exceedances of 100 gals for median ruptures $(0\sigma_l)$

$Ex(100\,|\,0\sigma_l)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	1519	1510	1407	818	589	476	373	11
20	1117	1109	1024	578	422	345	271	9
30	703	696	634	349	245	193	146	5
40	298	294	252	131	94	76	59	2
50	123	120	93	48	36	31	25	1
65	34	34	24	11	8	6	5	0
80	0	0	0	0	0	0	0	0

(3c) Exceedances of 100 gals for mean length ruptures $(.313\sigma_l)$

$Ex(100\,|\,.313\sigma_l)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	1714	1687	1517	836	603	488	382	12
20	1274	1249	1108	593	434	355	280	9
30	826	805	700	360	253	200	152	6
40	211	211	196	129	101	87	74	2
50	155	143	100	51	39	33	27	1
65	45	41	26	12	9	7	5	0
80	0	0	0	0	0	0	0	0

(3d) Exceedances of 100 gals for very long ruptures $(+2\sigma_l)$

$Ex(100\,|\,-2\sigma_l)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	6167	4738	3125	1743	1385	1206	1037	299
20	4773	3548	2405	1401	1137	1006	874	283
30	3469	2556	1761	1038	844	745	651	214
40	1348	1185	925	632	545	499	454	122
50	415	415	416	380	356	341	324	47
65	109	109	109	109	109	107	100	0
100	0	0	0	0	0	0	0	0

(3e) Exceedances of 100 gals integrated over rupture length, $(-1.5 \leqq fr \leqq 1.5)$

$Ex(100)\,10^6$								
	X							
P	200	100	50	10	0	-5	-10	-50
10	1741	1684	1483	842	609	494	387	13
20	1297	1245	1075	600	439	360	284	10
30	846	801	674	365	258	205	155	7
40	377	342	267	141	102	83	65	3
50	160	137	102	54	42	35	23	1
65	46	39	28	14	10	8	6	0
100	0	0	0	0	0	0	0	0

Exceedances of 100 gals integrated over variability in acceleration for $f\,c\tau$ model. Esteva attenuation.

(4a) Exceedances of 100 gals integrated over acceleration variability ($\sigma_a = .3$) using "mean" rupture length ($.313\,\sigma_l$)

$Pex(100\|.313\,\sigma_l)\,10^6$ σ_a=.3								
	X							
P	200	100	50	10	0	-5	-10	-50
10	4443	4409	4256	2895	1565	986	575	39
20	1512	1474	1335	754	511	407	306	35
30	763	727	614	336	252	217	178	30
40	451	419	333	185	146	130	111	24
50	286	259	196	112	91	83	72	18
65	152	133	96	57	47	43	39	11
80	81	68	48	29	24	23	20	6
100	32	26	18	11	9	9	8	3

(4b) Exceedances of 100 gals integrated over acceleration variability ($\sigma_a = .6$) using "mean" rupture length ($.313\sigma_l$)

$Pex(100\|.313\sigma_l)\,10^6$ σ_a =.6								
	X							
P	200	100	50	10	0	-5	-10	-50
10	5348	5302	5101	3378	2059	1468	967	89
20	2390	2334	2167	1330	905	735	557	81
30	1227	1181	1033	609	460	402	334	71
40	742	699	583	346	277	252	219	61
50	489	452	363	221	183	170	15	50
65	286	257	199	126	108	102	93	37
80	178	155	118	78	68	65	60	27
100	99	85	64	44	39	37	35	18

Appendix F

Discussion and derivation of $I_n(H)$ (equaton 53) for the Esteva attenuation function.

From equation 52,

$$NT \int_{m_{min}}^{m_{max}} \frac{2\exp(-b\,m)\sqrt{R_1(m)^2-P^2}}{L-H\exp(-g\,m)}\,dm \approx \sum_{n=0}^{N_1} I_n(H) \qquad \text{(F1)}$$

where

$$I_n(H) = \frac{2NT\,H^n}{L^{n+1}} \int_{m_{min}}^{m_{max}} \exp(-b\,m)\sqrt{R_1(m)^2-P^2}\exp(n\,g\,m)\,dm \qquad \text{(F2)}$$

$$NT = \frac{b\exp(b\,m_0)\,rate}{1-\exp[-b\,(m_{max}-m_0)]}$$

For the Esteva attenuation function

$$\ln a = c_1 + c_2 m + c_3 \ln R(m); \qquad c_3 < 0. \qquad \text{(F3)}$$

and

$$R_1(m)^2 = K\exp(c\,m) \qquad \text{(F4)}$$

where

$$K = a_1^{-\frac{2}{c_3}}\exp\left(\frac{-2c_1}{c_3}\right); \qquad c = \frac{-2c_2}{c_3}. \qquad \text{(F5)}$$

For m_{min}, the lowest magnitude producing acceleration a_1 at a site at (X,P),

$$R_1(m_{min})^2 = K\exp(c\,m_{min}) = P^2. \qquad \text{(F6)}$$

and

$$R_1(m_{max})^2 = K\exp(c\,m_{max}). \qquad \text{(F7)}$$

Define a new variable w:

$$w = \left[\frac{P}{R}\right]^2 = \frac{P^2\exp(-c\,m)}{K} \qquad \text{(F8)}$$

Then (assuming $P \neq 0$), equation F2 may be written:

$$I_n(H) = 2Q_n(H) \int_{\left(\frac{P}{R_{max}}\right)^2}^{1} w^{\frac{b-ng}{c}-\frac{3}{2}}\sqrt{1-w}\;dw \qquad \text{(F9)}$$

where

$$Q_n(H) = NT\,\frac{H^n}{L^{n+1}}\left|\frac{K}{P^2}\right|^{\frac{b-ng}{c}}\frac{P}{c}. \qquad \text{(F10)}$$

Using the relationship, which is valid for $m,\ n > 0$,

$$\int_0^1 x^{m-1}(1-x)^{n-1}\,dx = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \qquad \text{(F11)}$$

if $\frac{b-ng}{c} > \frac{1}{2}$ (an inequality which, depending on parameter values, may possibly hold only for n as low as zero or one) equation F9 may be written:

$$I_n(H) = \frac{2Qn(H)\,\Gamma(\frac{3}{2})\,\Gamma(\frac{b-ng}{c}-\frac{1}{2})}{\Gamma(\frac{b-ng}{c}+1)} - \int_0^{(\frac{P}{R_1(m_{max})})^2} w^{(\frac{b-ng}{c}-\frac{3}{2})}\sqrt{1-w}\;dw \qquad \text{(F12)}$$

The second term in equation F12 would vanish if an infinite maximum magnitude were permitted. The upper limit of integration $\left|\frac{P}{R_1(m_{max})}\right|^2$ is actually a function of magnitude difference $m_{max} - m_{min}$ since

$$\frac{P}{R_1(m_{max})}^{2} = \exp\left[\frac{2c_2}{c_3}(m_{max} - m_{min})\right]. \qquad \text{(F13)}$$

As $m_{min} \rightarrow m_{max}$, $\frac{P}{R_1(m_{max})} \rightarrow 1$ and the "correction" term approaches the principal term. However, if $R_1(m_{max}) \gg P$, the correction term is small, $w = \left|\frac{P}{R}\right|^2$ remains small, and a few terms of the series give a good approximation to

$$\sqrt{1-w} \approx 1 - \frac{w}{2} - \frac{w^2}{8} + \cdots \sum_{j=0}^{J} s_j\,w^j. \qquad \text{(F14)}$$

and the second term of equation F12 may be approximated by

$$\int_0^{(\frac{P}{R_1(m_{max})})^8} w^{(\frac{b-ng}{c}-\frac{3}{8})}\sqrt{1-w}\;dw \approx \sum_{j=0}^{J}\int_0^{(\frac{P}{R_1(m_{max})})^2} w^{\frac{b-ng}{c}-\frac{3}{2}}\,w^j s_j\,dw \qquad \text{(F15)}$$

$$= \sum_{j=0}^{J} s_j \frac{\left(\frac{P}{R_1(m_{max})}\right)^{2(\frac{b-ng}{c}+j-\frac{1}{2})}}{\frac{b-ng}{c}+j-\frac{1}{2}}.$$

The ratio of the third term to the second in the summation is less than

$\frac{1}{4}\left[\frac{P}{R_1(m_{max})}\right]^2$ and for $\frac{P}{R_1(m_{max})} < .6$, for example, using as few as two terms of the series will give less than 10% error.

Note that these approximations require that $\frac{P}{R_1(m_{min})} = 1$, or that $m_{min} = m_0$. Again observe that for accelerations which would be obtained at magnitudes $m \leqq m_0$, the artificial restriction that all earthquakes occur at magnitudes $m \geqq m_0$ complicates the mathematics and may distort the results.

(b) Integration of $J_n(H)$--equation 55. The J_n terms may be integrated directly giving

$$J_n(H) = \frac{\exp[(-b+(n+1)g)m]}{-b+(n+1)g}\left[\frac{H}{L}\right]^{(n+1)} NT \Big|_{m=m_{min}}^{m=m_{max}} \tag{F16}$$