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Simple models for the estimation and measurement of frictional heating by an earthquake

by

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Abstract

A review of the simplest models of frictional heating provides a useful order-of-magnitude perspective on coseismic and post-seismic thermal effects and their relation to earthquake parameters. In addition to fault slip and friction, which determine the energy dissipation, coseismic temperatures are sensitive to event duration, slip velocity, and width of the shear zone. Although coseismic heating may differ by several hundred degrees for events with the same slip and friction, the associated post-seismic temperature anomalies will generally be indistinguishable a few days after faulting. As suggested by McKenzie and Brune, the product of slip and friction might be recovered by thermal observations in a well drilled into the slip zone of moderate to large earthquakes. Precision temperature logs obtained within a few months of the event should help identify the slip zone, and subsequent temperature-time series within it could permit separation of the thermal disturbances caused by faulting, drilling, and the possible occurrence of water movements sometimes invoked in faulting models.

INTRODUCTION

The purpose of this note is to pull together some results for the simplest models of frictional heating by an earthquake to permit rapid estimates of coseismic and post-seismic thermal effects from simple graphs and rules of thumb. In this form, they are useful for back-of-the-envelope estimates of the role of frictional heat in the faulting process, and to evaluate the feasibility of various procedures to determine the frictional component of the earthquake energy budget from temperature measurements in boreholes. The possibility of making such observations, previously suggested by McKenzie and Brune (1972), is now receiving increased attention in connection with discussions of deep drilling for scientific purposes [e.g., Zoback et al., 1984].

When fault surfaces move past one another an amount 2u against an average frictional (or viscous) resistance τ, work is converted to heat in the amount $2\tau u$ per unit area of fault. This results in a local coseismic temperature rise that will subsequently dissipate, presumably by conduction. This effect has been discussed in relation to the dynamics of faulting with analytical results for an infinitesimally thin fault by Sibson [1975] McKenzie and Brune [1972], and Richards [1972]. Cardwell et al. [1978] discussed the problem in terms of a numerically integrated result for the fault of finite width illustrated in figure 1. A simple form of the solution for the fault of figure 1, applied by Lachenbruch (1980) to a related problem, is herein adapted to simple representations for estimating the coseismic temperature rise and its post-seismic dissipation. For intuitive convenience, graphical results are presented in a general dimensional form. The mathematical results are all well known and have generally been used in the previous discussions of various parts of this problem.

COSEISMIC FRICTIONAL HEATING

Let t^* be the duration of the slip event and $2v = 2u/t^*$ be the slip velocity (see fig. 1). We assume conditions vary only in the x-direction, heat is transferred exclusively by conduction, deformation is homogeneous in the fault zone, and slip velocity and fault width are constant during the event. The simplest case occurs when the fault-width 2a is so large that little heat escapes by conduction from the fault zone during the short time t^* of the event. (This condition is $a > \sqrt{4\alpha t^*}$ where the right side is the effective distance heat can diffuse in time t^* in a medium of thermal diffusivity α which we assume to be 0.01 cm²/sec). Denoting density by ρ and heat capacity by c and assigning them reasonable values ($\rho = 2.65$ g/cm³, $c = 0.95$ J g^{-1} K^{-1} whence $\rho c = 2.5$ J cm^{-3} K^{-1} $\sim$25 bars/K), we can write the temperature rise θ in terms of stress (τ) and strain (u/a).

$$\theta = \frac{\tau}{\rho c}\frac{vt}{a} \equiv \frac{\tau}{\rho c}\frac{u}{a} \tag{1a}$$

$$\sim 4°\mathrm{C} \times \frac{u}{a} \text{ if } \tau = 100 \text{ bars} \tag{1b}$$

$$\sim 40°\mathrm{C} \times \frac{u}{a} \text{ if } \tau = 1000 \text{ bars} \tag{1c}$$

Thus for fault strains greater than unity ($u/a \gtrsim 1$) coseismic heating can be large.

If the fault is narrower than the distance heat can diffuse during the event (i.e., if $a < \sqrt{4\alpha t^*}$) or equivalently if the event lasts longer than the time constant ($\lambda = a^2/4\alpha$) for the fault, then the temperature rise on the axial plane of the shear zone depends on time and velocity independently, and (1a) is replaced by the more general expression (Lachenbruch, 1980, equation 45a)

$$\theta(0,t) = \frac{\tau}{\rho c}\frac{vt}{a}\left[1 - 4i^2 \operatorname{erfc}\frac{a}{\sqrt{4\alpha t}}\right], \quad \text{all } a, \qquad 0 < t < t^* \tag{2a}$$

$$\cong \frac{\tau}{\rho c}\frac{vt}{a} \qquad , \quad a/\sqrt{4\alpha t} > 1 \tag{2b}$$

$$\rightarrow 2\frac{\tau}{\rho c}\frac{v\sqrt{t}}{\sqrt{\pi\alpha}} \qquad , \quad a/\sqrt{4\alpha t} \rightarrow 0 \tag{2c}$$

This is illustrated in figure 2 which gives the rise in axial plane temperature as a function of time during faulting for a range of event durations (up to 100 sec) and fault widths (0 to 40 m) that probably brackets most earthquakes. The temperature scale, based on a slip velocity 2v of 25 cm/sec and friction τ of 100b, can be adjusted proportionally for any other choice, e.g., for 2v = 50 cm/sec and τ = 1000 b multiply the temperature scale by 20.

Figure 2 illustrates how wide-fault behavior ($\theta \propto t$, equation 2b) can pass to narrow-fault behavior ($\theta \propto t^{\frac{1}{2}}$, equation 2c) as the event proceeds. With uncertainties of an order of magnitude in friction τ, and at least that much in fault-width 2a, an enormous range of temperatures is possible even if slip velocity 2v and duration t* are known. If friction, fault width, and the slip 2u were known, but duration and velocity were not, a range of temperature is still possible on narrow faults. For example, slip of 50 cm may have resulted from a 1-second event slipping at 50 cm/sec or a 10-second event at 5 cm/sec. For τ = 100b, a 4-mm-wide fault would warm by 500°C in the first case and by 260°C in the second according to figure 2. (In the first case, the ordinate scale is multiplied by 2, and in the second, it is divided by 5 to accommodate the two different velocities.) It is seen that the curve labeled "4 cm" remains linear to t ∿100 sec, probably longer than the duration of local slip in most earthquakes. Hence as pointed out by Cardwell et al. [1978], equation (1) should be useful for predicting coseismic temperature rise on the axis of faults whenever shear zones are at least a few cm wide. If the slip (2u) were 50 cm and fault width (2a) were 5 cm, then according to (1) or (2) θ would be 40°C for 100 bars friction, 400°C for 1000b. Narrower faults attain much higher temperatures, but as we shall see, they decay very rapidly after the event and are soon indistinguishable from effects of wider faults with the same slip and friction.

POSTSEISMIC DECAY OF TEMPERATURE DISTURBANCE

The postseismic conductive decay of temperatures in the fault zone depends upon two time constants: the duration of local seismic slip t^*, and the time λ required for heat to diffuse across the half width of the heated shear zone ($\lambda = a^2/4\alpha$). The complete expression for temperature decay ($t>t^*$) within a uniform fault zone ($x <a$) and beyond it ($x >a$) is (from minor modifications of Carslaw and Jaeger, 1959, equations 9 and 10, p. 80)

$$\theta(x,t) = \frac{\tau}{\rho c}\frac{v}{a}\{t[1 - 2i^2 \operatorname{erfc}\frac{a-x}{\sqrt{4\alpha t}} - 2i^2 \operatorname{erfc}\frac{a+x}{\sqrt{4\alpha t}}]$$

$$-(t-t^*)[1 - 2i^2 \operatorname{erfc}\frac{a-x}{\sqrt{4\alpha(t-t^*)}} - 2i^2 \operatorname{erfc}\frac{a+x}{\sqrt{4\alpha(t-t^*)}}]\} \qquad x < a \tag{3a}$$

$$= \frac{\tau}{\rho c}\frac{v}{a}\{t[i^2 \operatorname{erfc}\frac{x-a}{\sqrt{4\alpha t}} - i^2 \operatorname{erfc}\frac{x+a}{\sqrt{4\alpha t}}]$$

$$-(t-t^*)\ [i^2 \operatorname{erfc}\frac{x-a}{\sqrt{4\alpha(t-t^*)}} - i^2 \operatorname{erfc}\frac{x+a}{\sqrt{4\alpha(t-t^*)}}]\} \qquad x > a \tag{3b}$$

where $i^2 \operatorname{erfc}\beta$ is the second integral of the error function of β [e.g., Carslaw and Jaeger, 1959, Appendix II). The general expression for the coseismic ($t<t^*$) temperature rise is obtained by deleting the second lines in equations (3a) and (3b), i.e., by formally setting $t \equiv t^*$.

The slip duration t^* will generally be only a few seconds and can therefore be neglected for computing the temperature hours, days or weeks after faulting. Neglecting slip-duration t^* relative to post-seismic observation time ($t-t^*$), we can treat the faulting as an instantaneous source of strength $\tau u/\rho c a$ distributed throughout the shear zone (x < a) and (3) is replaced by [Carslaw and Jaeger, equation 3, p. 54]

$$\theta(x,t) = \frac{1}{2}\frac{\tau}{\rho c}\frac{u}{a}[\operatorname{erf}\frac{a-x}{\sqrt{4\alpha t}} + \operatorname{erf}\frac{a+x}{\sqrt{4\alpha t}}] \quad , \qquad \text{all } x,\ t >> t^* \tag{4}$$

Finally, if our observation time is large relative to the time constant λ of the shear zone (for a 4-cm wide shear zone λ is ~100 seconds, but for a 4-m shear zone, it is ~10 days, see inset, fig. 1) the fault can be treated as an instantaneous plane (zero-width) source of strength $2\tau u/\rho c$. This leads to the further simplification (Carslaw and Jaeger, 1959, equation 4, p. 259):

$$\theta(x,t) = \frac{\tau}{\rho c}\frac{u}{\sqrt{\pi\alpha t}}e^{-\frac{x^2}{4\alpha t}} \qquad \text{all } x,\ t >> t^*,\quad t >> \lambda \tag{5}$$

Temperature rise computed from (5) is shown in figure 3 for a fault slip $2u$ of 1 m opposed by frictional resistance τ of 100b. The figure shows that the high axial plane temperatures in the previous numerical examples are short-lived. If the slip were 2 m, the temperatures of figure 3 would be multiplied by 2, if the friction were 1kb, they would be multiplied by 10. The one-day curve is valid only for faults for which the time constant ($\lambda \equiv a^2/4\alpha$) is much less than a day, e.g., for $2a \lesssim 40$ cm in which case $\lambda \lesssim 3$ hrs; fault with widths up to a meter comply with (5) and Figure 3 by $\lesssim 1$ week (see inset, fig. 1). Temperatures earlier in the postseismic period can be obtained for a fault of any width from the more complete expressions (4) or (3).

ESTIMATION OF FRICTION FROM POST-SEISMIC TEMPERATURE MEASUREMENTS

The areas under all of the curves in Figure 3 must be the same and equal to the frictional source strength $2\tau u/\rho c$. If this area could be determined from a post-seismic temperature profile across the fault, it would provide an opportunity to recover friction τ from an estimate of slip 2u. The method is robust, as it is independent of model geometry or the details of faulting as long as their effects remain one-dimensional. The principal difficulty with it is that observation wells that predate the earthquake will generally be blocked in the fault zone, and drilling a hole after the earthquake may cause a thermal disturbance that could mask much of the frictional anomaly sought.

As an alternate (or supplement) to integrating the temperature disturbance normal to the fault, friction might be determined by monitoring the post-seismic rise and fall of temperature at a fixed distance, x, from the fault axis; in fact, this has been done successfully in the laboratory by Lockner and Okubo [1983]. Whether the drilling disturbance from a post-seismic observation well will interfere with such measurements of frictional heat depends on the relative magnitudes and time constants for the two anomalies. Behavior of the drilling anomaly has been discussed elsewhere [e.g., Lachenbruch and Brewer, 1959]; it can generally be minimized by special drilling procedures and calculated and corrected out for times exceeding the duration of drilling at the measurement depth, possibly a few days to weeks. The behavior of the frictional anomaly will usually be described adequately by the instantaneous-plane-fault approximation (equation 5) for distances x and post-seismic measurement times t likely to be convenient. The temperature for this case, expressed in normalized form, is described by the following result and illustrated in Figure 4.

$$\frac{\theta}{\theta_{max}} = \sqrt{2}\, e^{\frac{1}{2}} \sqrt{\frac{\delta}{t}}\, e^{-\delta/t} \tag{6a}$$

Here the normalization factor for time is the "wall time constant" δ given by

$$\delta \equiv \frac{x^2}{4\alpha} \tag{6b}$$

and the normalization factor for temperature rise is the maximum post-seismic value θ_m attained at point x; it is obtained by differentiation of (5).

$$\theta_m = \sqrt{\frac{2}{\pi}}\, e^{-\frac{1}{2}}\, \frac{\tau u}{\rho c}\, \frac{1}{x} \tag{6c}$$

or

$$\theta_m \text{ in } ^\circ\text{C} \cong \frac{\tau \text{ in bars}}{100} \times \frac{2u}{x} \tag{6d}$$

The dimensional result (6d) is a convenient rule of thumb that follows from $\rho c \cong 25$ bars/°C (equation 1a). For measurements at a distance from the fault equal to the slip, the maximum temperature rise will be 1°C for each 100 b of friction; at a distance of 10 × slip, it will be 1°C for each kilobar. The anomalous temperature grows from 10% θ_m to θ_m between the times $t \cong \frac{1}{4}\delta$ and $t_m = 2\delta$ (equation 6a, Figure 4).

In principle, we might hope to make independent estimates of both friction and distance to the fault at each point where we obtain a time series of precision temperature measurements in a borehole near a fault after an earthquake. The time constant δ, and hence x would be determined from the time scale of the observed disturbance (equation 6a), and θ_m and hence the frictional work $2\tau u$ would be determined from the temperature scale (equation 6c). Whether such determinations are feasible depends (among other things) upon numerical values of θ_m and δ under plausible observation conditions.

The constraints on such observations imposed by theory can be judged from Figure 5 where the three dashed lines and left-hand ordinate scale give the maximum temperature for a large (2u = 2.5 m), moderate (2u = 50 cm) and small (2u = 10 cm) earthquake, respectively. The two solid lines and right-hand ordinate scale show the interval of time over which the temperature anomaly rises from 10% to its full value θ_m at any point x.

If, as suggested by McKenzie and Brune [1972], a hole were drilled to intersect a fault promptly after an earthquake, satisfactory measurement conditions might obtain about 10 m from the fault. There, the disturbance would not rise to 10% for about 2½ months (Figure 5) by which time the drilling might be completed and its thermal effects largely dissipated. The maximum faulting disturbance would be reached in about 1½ years; its value would range from 0.25°C to 0.05°C for the large and moderate earthquakes of Figure 5 if the friction were 100 b; if friction were 1 kb, the range would be 2½° to 0.5°C. Hence at a distance of about 10 m from the fault slip event, useful results might be obtained for a variety of conditions with a monitoring system with about 0.01°C precision. After larger earthquakes, the temperature rise will still be appreciable ($\gtrsim$0.1°C, Figure 5) 20 or 30 m from the fault if friction is significant, but it will be very slow and continue for many years (Figure 4 inset, Figure 5). Such long observation periods place a severe demand on the stability of the monitoring system. Within 10 m of the fault, the signal will be stronger and rapidly changing; characteristics that should help identify the locus of slip from precision thermal logs taken with a few months of faulting. It is likely that precision temperature monitoring at several points in this region (say a 25-m interval containing the fault plane) could provide the redundancy needed to separate the drilling and faulting disturbances and to identify any heat that might be transported by the hypothetical water movements sometimes postulated for models of faulting [e.g., Hanks and O'Neal, 1980].

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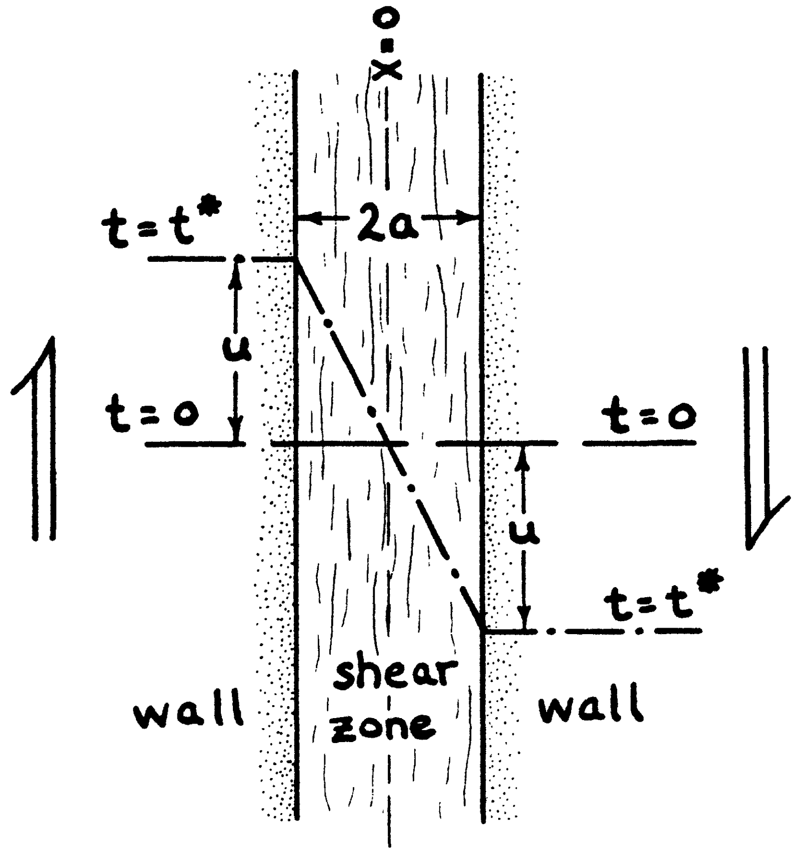


Figure 1. The simple fault model. Dashed line (t=0) is deformed into dash-dot line (t=t*) during slip of amount 2u across shear zone of width 2a in an event of duration t*.

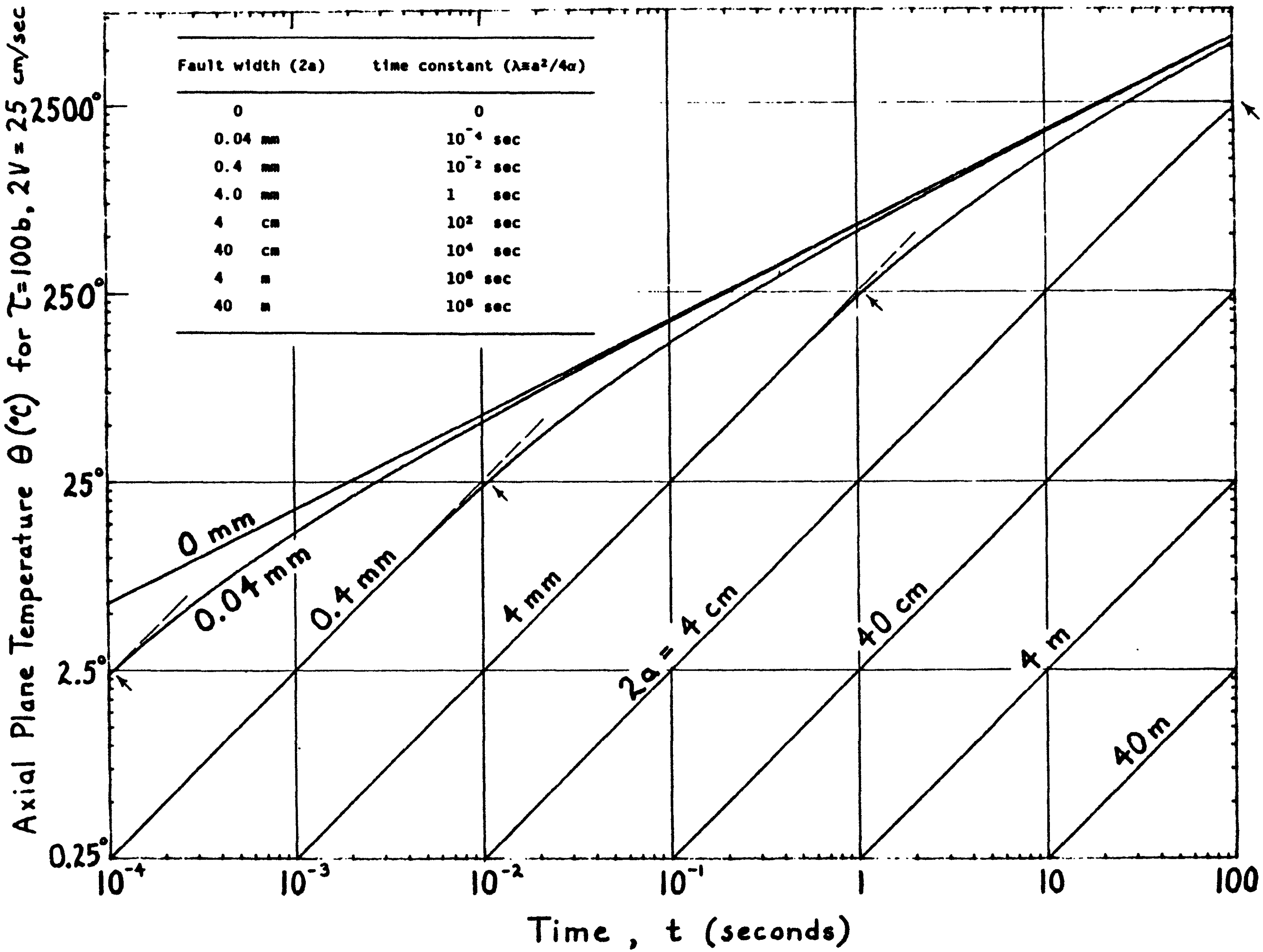


Figure 2. Coseismic temperature rise on axial plane as a function of time since initiation of faulting. Numbers on curves give fault width 2a; inset table gives corresponding time constants. Temperatures are shown for friction τ of 100 bars and slip velocity 2v of 25 cm/sec. They can be scaled to any other values, e.g., for $2v = 50$ cm/sec and $\tau = 1000$ bars multiply temperatures by 20. To the left of arrows (at $t=\lambda$) temperature rises linearly with time (equations 1, 2b).

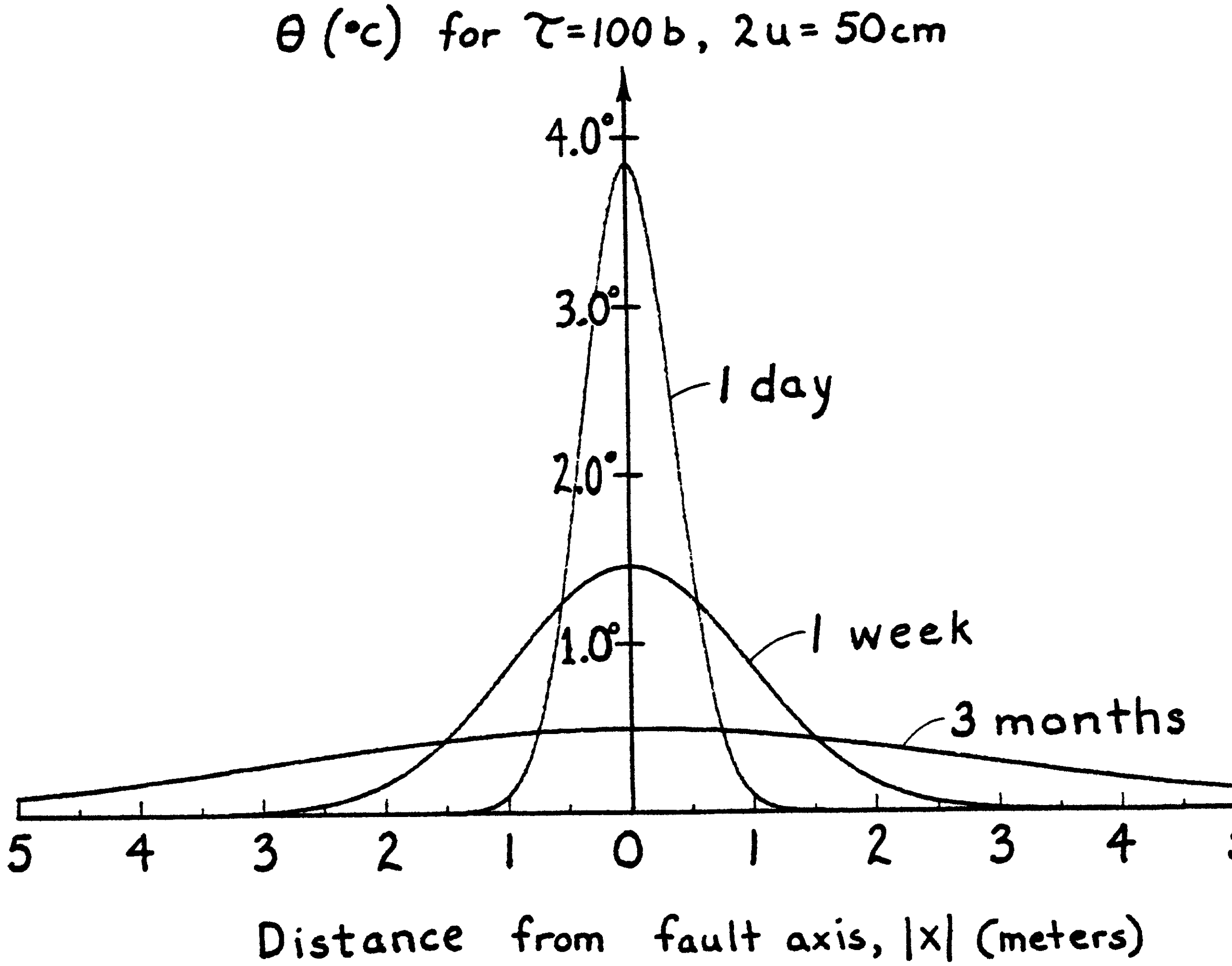
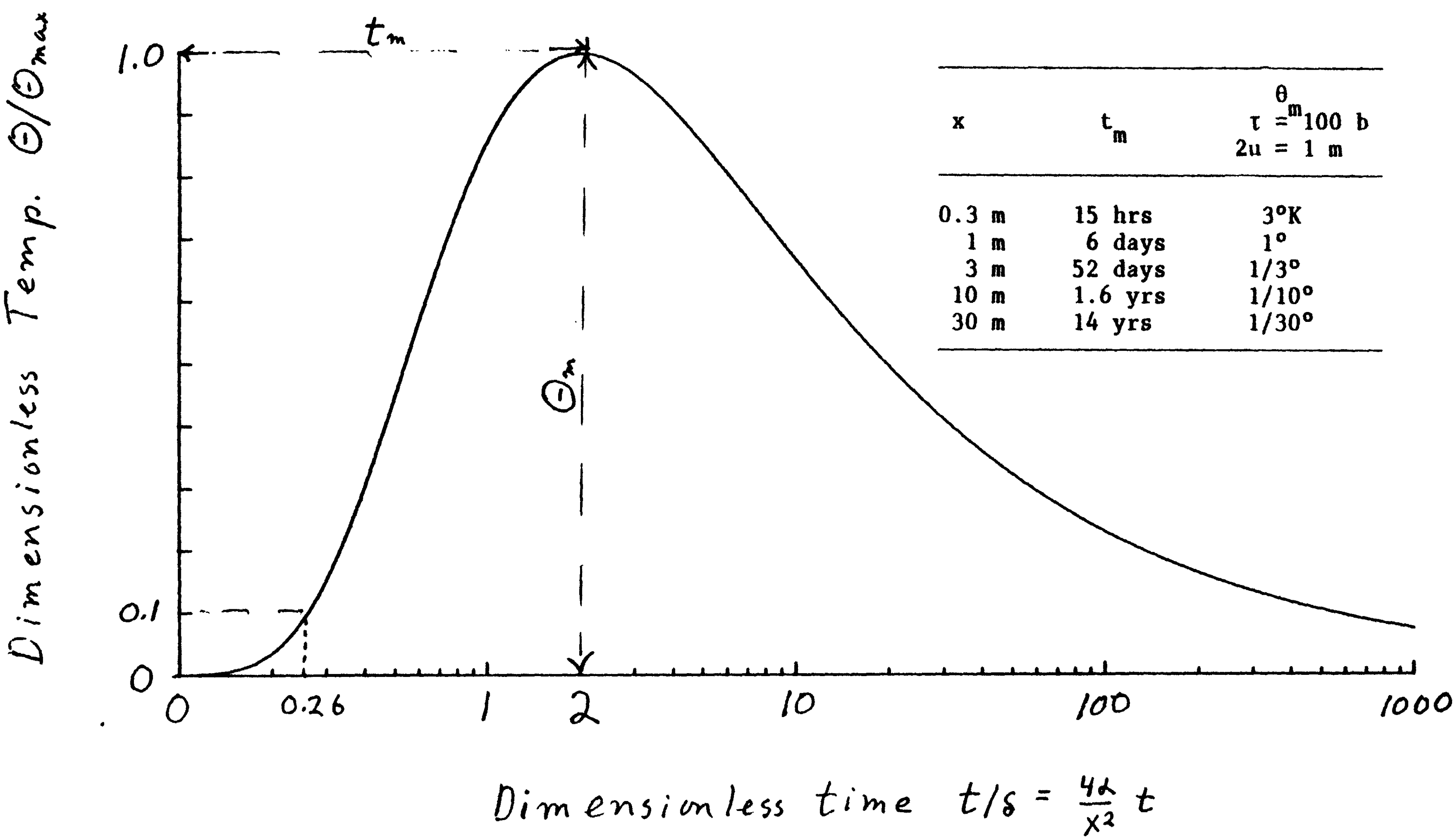


Figure 3. Post-seismic temperature distribution. Temperatures are shown for friction τ of 100 bars and slip 2u of 1 m; they can be scaled proportionally for any other values. Curves are good approximations as long as the post-seismic time they represent is large relative to the fault's time constant $\lambda(a)$; see inset, Figure 2.



x	t_m	θ_m τ = 100 b $2u$ = 1 m
0.3 m	15 hrs	3°K
1 m	6 days	1°
3 m	52 days	1/3°
10 m	1.6 yrs	1/10°
30 m	14 yrs	1/30°

Figure 4. Temperature rise as a function of time t at distance x from a fault event at t=0, x=0. Temperature is normalized by the maximum value at x, θ_m (equation 6) and time is normalized by the wall time constant $\delta \equiv x^2/4\alpha$. Inset table gives θ_m (for 100 b friction and 1 m slip) and t_m for selected distances x from fault.

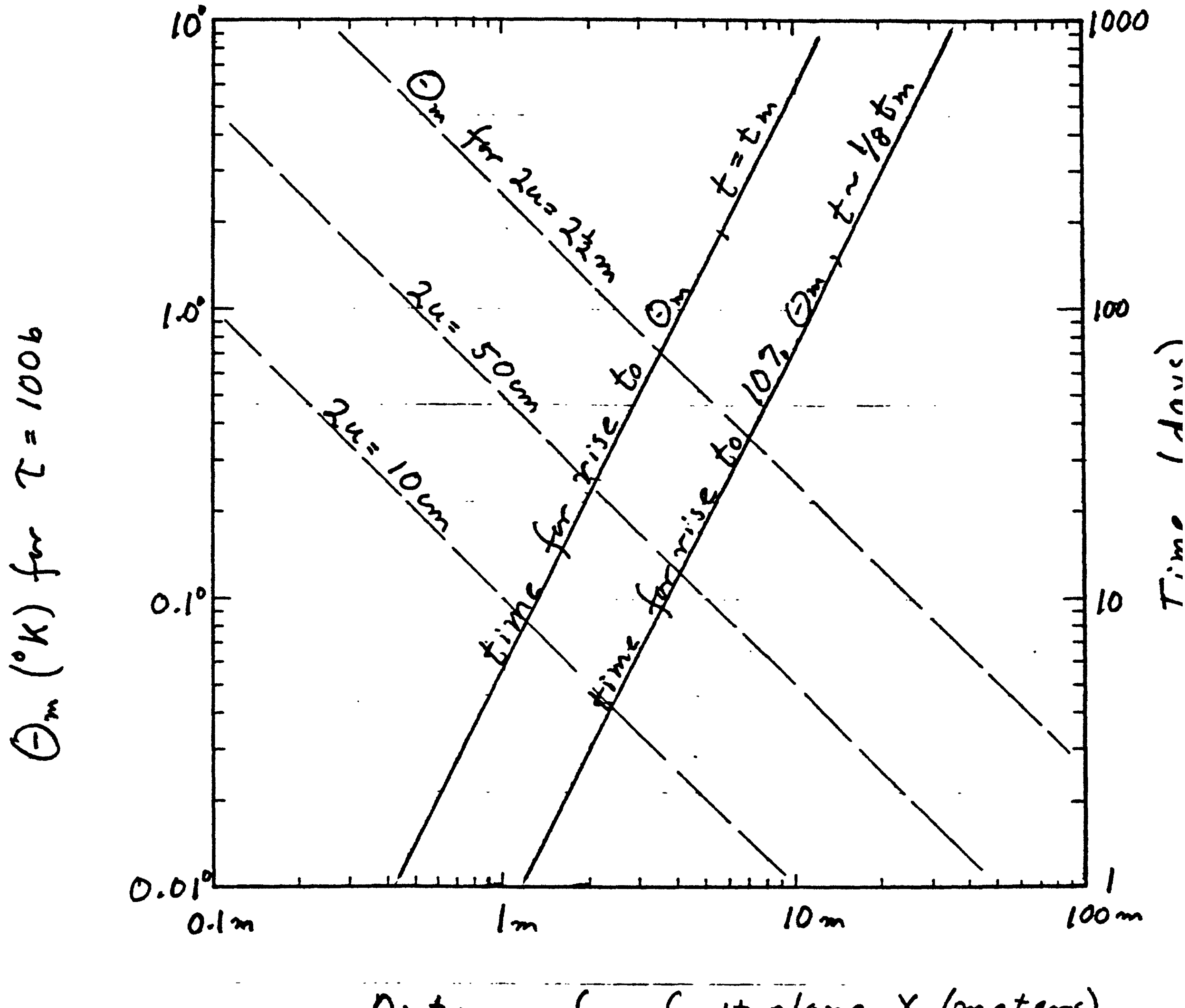


Figure 5. Strength and timing of the post-seismic temperature anomaly. Dashed lines and left-hand ordinate scale give maximum temperature, θ_m, at distance x from a large (2u = 2½ m), medium (2u = 50 cm) and small (2u = 10 cm) earthquake with friction τ of 100 b. For different τ scale, temperature proportionally. Solid curves and right-hand ordinate scale give time for temperature to rise of 10% θ_m (t ≈ ¼δ) and to the full value θ_m (t = 2δ).