DEPARTMENT OF THE INTERIOR

U.S. GEOLOGICAL SURVEY

BING--A BASIC computer program for modeling unsteady flow of Bingham viscoplastic material

by

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Open-File Report 89-367

Although this program has been extensively tested, the U.S. Geological Survey cannot guarantee that it will give accurate results for all applications nor that it will work on all computer systems.

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1989

INTRODUCTION

A solution for transient flow of a Bingham viscoplastic material between parallel flat plates has been programmed in BASIC and presented here. This program, BING, gives shear stress and velocity in a viscoplastic layer where shear stress exceeds material strength and in a rigid central layer where shear stress is below material strength. The results of the solution can be applied to accelerating and decelerating flow of materials which can be modeled as a Bingham viscoplastic, such as debris flows.

In what follows, we present the theoretical background for the computer program beginning with a brief introduction to the concept of a Bingham viscoplastic material. We then give a brief summary of the analytic solution and follow this with a description of the BASIC program (appended to this report). Finally, an example calculation is presented.

BINGHAM VISCOPLASTIC MATERIALS

Bingham viscoplastic materials, unlike Newtonian fluids, can sustain non-zero deviatoric stresses in a state of rest (Bingham, 1922; Oldroyd, 1947; Prager, 1961). For this material, no flow will occur when J_2, the second invariant of the deviator stress tensor, is less than or equal to the square of a constant yield stress, K. When $J_2 > K^2$, flow occurs.

These relationships are expressed formally in standard tensor notation (Prager, 1961) in the constitutive equations,

$$2\mu D_{ij} = \begin{cases} 0 & J_2 \leq K^2, \quad (1a) \\ \left[1 - \frac{K}{J_2^{1/2}}\right] S_{ij} & J_2 > K^2, \quad (1b) \end{cases}$$

where $J_2 = 1/2\, S_{ij}S_{ij}$.

In equations (1a) and (1b), μ is the viscosity,

$$D_{ij} = 1/2\left[\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right] \quad (2)$$

is the strain rate tensor (expressed in terms of the velocity components), and

$$S_{ij} = \sigma_{ij} - 1/3\, \delta_{ij}\sigma_{kk} \quad (3)$$

is the deviator stress tensor, σ_{ij} is the stress tensor, and δ_{ij} is the Kronecker delta.

Since $S_{ii} = 0$, equation (1b) requires that

$$D_{ii} = \frac{\partial v_i}{\partial x_i} = 0 \quad , \quad (4)$$

that is, this viscoplastic material is incompressible. In addition to satisfying continuity (equation 4), the velocity field must satisfy the equations of motion

$$\rho\left[\frac{\partial v_k}{\partial t} + v_j \frac{\partial v_k}{\partial x_j}\right] = \frac{\partial \sigma_{jk}}{\partial x_j} \quad . \tag{5}$$

RECTILINEAR FLOW OF BINGHAM MATERIAL IN RESPONSE
TO A TIME-VARYING PRESSURE GRADIENT

Consider a rectilinear flow of a Bingham material under a pressure gradient $\partial P/\partial x_1$ in a region $0 < x_2 < h$ bounded by flat rigid plates at $x_2 = 0$ and $x_2 = h$ (fig. 1).

It can be shown that the constitutive equations and the equations of motion, respectively, reduce in this case to

$$\frac{\partial v_1}{\partial x_2} = 0 \qquad |\sigma_{12}| \leq K, \text{ and} \tag{6a}$$

$$\mu \frac{\partial v_1}{\partial x_2} = \sigma_{12} - K(\sigma_{12}/|\sigma_{12}|) \qquad |\sigma_{12}| > K \tag{6b}$$

and

$$\frac{\partial^2 \sigma_{12}}{\partial x_2^2} = \frac{\rho}{\mu} \frac{\partial \sigma_{12}}{\partial t} \quad . \tag{7}$$

Equations (6b and 7) apply only when the rate of deformation $\frac{\partial v_1}{\partial x_2}$ is non-vanishing; where $|\sigma_{12}| > K$. Where $|\sigma_{12}| \leq K$, $\frac{\partial v_1}{\partial x_2} = 0$, v_1 is a function of t only, and equilibrium is satisfied by

$$\sigma_{12} = \rho \frac{\partial v_1(t)}{\partial t} x_2 + f(t) \quad . \tag{8}$$

The Bingham material between the plates (fig. 1) remains at rest until the absolute value of shear stress created by the pressure gradient exceeds the shear strength. Because of symmetry, shear stress will always be zero midway between the plates ($x_2 = h/2$) and shear stress will first exceed shear strength at $x_2 = 0$ and $x_2 = h$. This leads to initiation of viscoplastic flow near the upper and lower plates and the development of a central rigid plug moving at a velocity equal to the velocity where $|\sigma_{12}| = K$. This continues until the shear stress everywhere drops below the yield strength, the rigid plug fills the entire space between the plates, and flow stops.

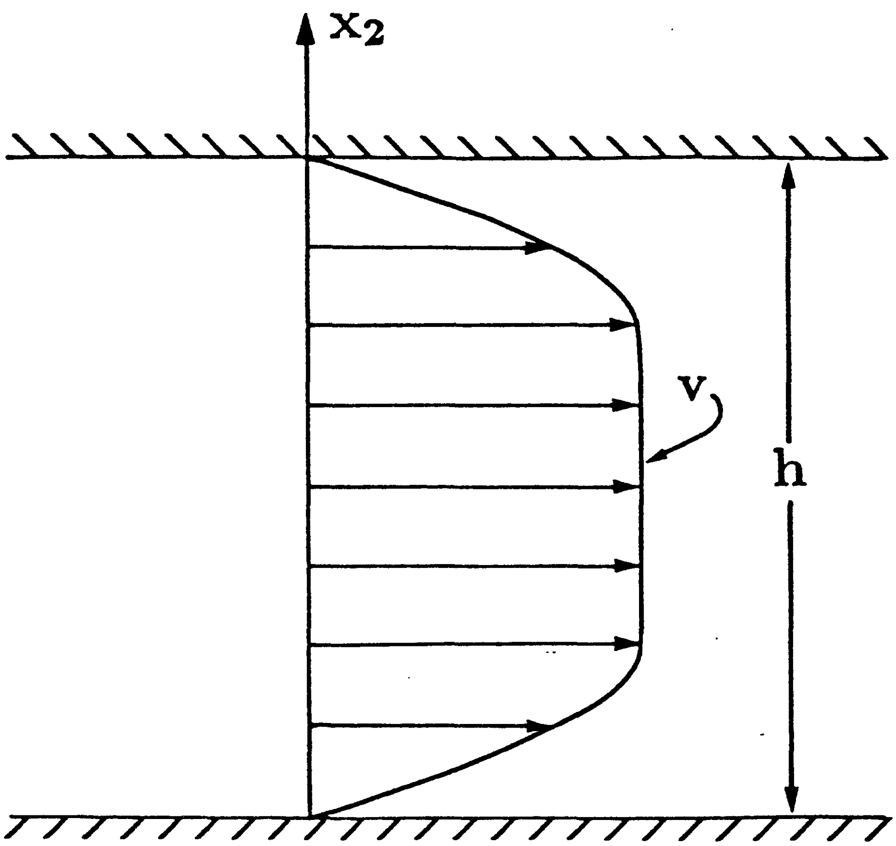

Figure 1. Rectilinear flow of a Bingham material between flat rigid walls at $x_2 = 0$ and $x_2 = h$.

These conditions on shear stress and velocity are all satisfied by taking the initial distribution of shear stress to be $\sigma_{12} = K[1 - \frac{2x_2}{h}]$. This distribution of shear stress assures that $|\sigma_{12}| = K$ at $x_2 = 0$ and $x_2 = h$ in the instant before flow begins. The assumed initial shear stress condition then is

$$\sigma_{12} = K[1 - \frac{2x_2}{h}] \qquad 0 \leq x_2 \leq h \text{ and } t=0 \tag{9}$$

and the boundary condition on shear stress is

$$\sigma_{12} = 0 \text{ for } x_2 = h/2 \text{ and } t>0 \quad . \tag{10}$$

Note that condition (10) is a consequence of symmetry and, also because of symmetry, only the shear stress (which is always positive) and velocity distributions for $0 \leq x_2 \leq h/2$ will be presented below.

The solution to equation (7) satisfying conditions (9) and (10) is

$$\sigma_{12} = K[1 - \frac{2x_2}{h}] +$$

$$\frac{4\mu}{\rho h} \sum_{n=1}^{\infty} (-1)^{n-1} \sin\left[\frac{(2n-1)\pi(x_2-h/2)}{h}\right] \int_0^t \left[\exp-\left(\frac{\mu(2n-1)^2\pi^2(t-\lambda)}{\rho h^2}\right)\right] \left[\frac{\partial P}{\partial x_1} + \frac{2K}{h}\right] d\lambda \quad . \tag{11}$$

Equation (11) applies only where $|\sigma_{12}| > K$. When $|\sigma_{12}| \leq K$, shear stress is given by equation (9), which can be written as

$$\sigma_{12} = \frac{[x_2/h - 1/2]}{[\xi(t)/h - 1/2]} \tag{12}$$

Here, $\xi(t)$ represents the x_2 values where $|\sigma_{12}| = K$ during flow.

Velocities when $|\sigma_{12}| > K$, are given by substituting equation (11) in equation (6b) and integrating. This gives

$$v_1 = -\frac{Kx_2^2}{\mu h} +$$

$$\frac{4}{\rho\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)} \cos\frac{[(2n-1)\pi(x_2-h/2)]}{h} \int_0^t \left[\exp+\left(\frac{\mu(2n-1)^2\pi^2(t-\lambda)}{\rho h^2}\right)\right] \left[\frac{\partial P}{\partial x_1} + \frac{2K}{h}\right] d\lambda \tag{13}$$

Velocities when $|\sigma_{12}| \leq K$ (plug velocities) depend on time only and are obtained by substituting $\xi(t)$ for x_2 in equation (13). Values of $\xi(t)$ are obtained by setting $\sigma_{12} = K$ in equation (11) and solving the resulting equation for $\xi(t)$.

Finally, for large times and a final constant pressure gradient, equations (11), (12), and (13) give Prager's (1961) equations for shear stress, velocity, and flow thickness of a Bingham material in steady flow between parallel plates. These are,

$$\sigma_{12} = \frac{\partial P}{\partial x_1}[x_2 - h/2] \tag{14}$$

and

$$v_1 = \frac{1}{2\mu}\frac{\partial P}{\partial x_1}[x_2 - h]x_2 - \frac{Kx_2}{\mu} \tag{15}$$

which apply for $|\sigma_{12}| > K$. The steady flow plug velocity (where $|\sigma_{12}| \leq K$) is given by

$$v_1 = -\frac{1}{2\mu}\frac{\partial P}{\partial x_1}\xi^2 \tag{16}$$

where ξ is given by

$$\xi = h/ \quad K/\frac{\partial P}{\partial x_1} \quad . \tag{17}$$

NONDIMENSIONALIZATION

Before proceeding further, it is useful to define some dimensionless quantities. Let

$$v^* = v_1/v_o \quad , \tag{18a}$$

$$t^* = tv_o/x_o \quad , \tag{18b}$$

$$x_1^* = x_1/x_o \quad , \tag{18c}$$

$$x_2^* = x_2/x_o \quad , \tag{18d}$$

and

$$\rho^* = \rho/\rho_o = 1 \quad , \tag{18e}$$

where the zero-subscripted quantities, respectively, represent typical fixed velocities, times, lengths, and densities for the flow. The term ρ^* in equation (18e) is unity because the flow is incompressible (equation 4) and the typical fixed length x_o is taken to be the distance h between the plates in figure 1. The typical velocity, v_o, can be conveniently taken as the steady flow plug velocity, equation (16).

Also

$$\tau^* = \sigma_{12}/K \quad , \tag{19a}$$

$$P^* = P/K \quad , \tag{19b}$$

$$Re = \frac{\rho_o v_o x_o}{\mu} \tag{19c}$$

is the Reynolds number, and

$$Bi = \frac{K x_o}{\mu v_o} \tag{19d}$$

is the Bingham number. The Reynolds number compares kinetic energy to viscous dissipation and the Bingham number compares plastic and viscous dissipation.

The ratio $Bi/Re(= K/\rho_o v_o^2)$ compares plastic dissipation to kinetic energy in the flow.

Equations (11) and (12) in nondimensional form are

$$\tau^*(x_2^*,t) = 1-2x_2^* - \frac{4}{Re} \sum_{n=1}^{\infty} (-1)^{n-1}$$

$$\cdot \sin\left[(2n-1)\pi/2\,(1-2x_2^*)\right] \int_0^{t^*} \left(\exp\left(\frac{-(2n-1)^2 \pi^2 \lambda^*}{Re}\right)\right] P_1^*(t^*-\lambda^*)d\lambda^* \tag{20}$$

where $P_1^* = 2 + \frac{\partial P^*}{\partial x_1^*}$ and

$$\tau^* = \left[\frac{x_2^* - 1/2}{\xi^*(t^*)-1/2}\right] \tag{21}$$

Equation (20) applies only where $|\tau^*| > 1$ and equation (21) applies where $|\tau^*| \leq 1$. Nondimensional velocities for $|\tau^*| > 1$ from equation (13) are

$$v^*(x_2^*,t^*) = -Bi x_2^{*2} - \frac{Bi}{Re}\frac{4}{\pi} \cdot$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cos[(2n-1)\pi/2(1-2x_2^*)]}{(2n-1)} \int_0^{t^*} \exp \frac{-(2n-1)^2 \pi^2 \lambda^*}{Re} P_1^*(t^* - \lambda^*)d\lambda^* . \tag{22}$$

Velocities when $|\tau^*| \leq 1$ (plug velocities) depend on time only and are obtained by substituting $\xi^*(t)$ for x_2^* in equation (22).

STARTING AND STOPPING FLOW

The program BING.BAS listed in the Appendix gives results for the case where a constant pressure gradient, $\frac{\partial P^*}{\partial x_1^*} = P_o^*$, is maintained for a fixed time, $t_1^* = t_1 v_o / x_o$. Then with

$$\frac{\partial P^*}{\partial x_1^*} = \begin{cases} 0 & t^* < 0 \\ P_o^* & 0 < t^* < t_1^* \\ 0 & t^* > t_1^* \end{cases}$$

shear stress and velocity for $|\tau^*| > 1$ are given by

$$\tau^* = 1 - 2x_2^* - \frac{4(2+P_o^*)}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \sin[(2n-1)\pi/2[1-2x_2^*]]}{(2n-1)^2} \{H(t^*) - H(t^*-t_1^*)$$

$$-H(t^*) \exp[\frac{-(2n-1)^2 \pi^2 t^*}{Re}] + H(t^*-t_1^*) \exp[\frac{-(2n-1)^2 \pi^2 (t^*-t_1^*)}{Re}]\} \qquad (23)$$

and

$$v^* = \frac{-4Bi(2+P_o^*)}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \cos[(2n-1)\pi/2(1-2x_2^*)]}{(2n-1)^3} \{H(t^*) - H(t^*-t_1^*)$$

$$-H(t^*) \exp[\frac{-(2n-1)^2 \pi^2 t^*}{Re}] + H(t^*-t_1^*) \exp[\frac{-(2n-1)^2 \pi^2 (t^*-t_1^*)}{Re}]\} - Bi x_2^{*2} \qquad (24)$$

where $H(t^*)$ is the unit step function. Equations (23) and (24) are obtained by integration of equations (20) and (22).

Shear stresses and velocities when $|\tau^*| \leq 1$ are given, respectively, by equation (21) and by substituting $\xi^*(t^*)$ for x_2^* in equation (24). Values of $\xi^*(t^*)$ are obtained by setting $\tau^* = 1$ in equation (23) and solving the resulting equation:

$$\xi^*(t) + \frac{2(2+P_o^*)}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}\sin[(2n-1)\pi/2[1-2\xi^*(t^*)]]}{(2n-1)^2} \{H(t^*)-H(t^*-t_1^*)$$

$$-H(t^*) \exp[\frac{-(2n-1)^2\pi^2 t^*}{Re}] + H(t^*-t_1^*) \exp[\frac{-(2n-1)^2\pi^2(t^*-t_1^*)}{Re}]\} = 0 \qquad (25)$$

for $\xi^*(t^*)$. This is accomplished in BING.BAS by Newton-Raphson iteration.

DESCRIPTION OF BING.BAS

A listing of BING.BAS is given in the Appendix. The program is written in QuickBASIC 4.5. The executable program requires approximately 64K bytes of memory. The ASCII source code requires 12.2K bytes of disk storage. The source code and executable code may be downloaded from the USGS-ISD Denver Service Center Bulletin Board. To access via a 1200 baud modem, dial Area Code 303-236-4739 or 303-236-4740. Calculations of the location of the rigid plug boundary, $\xi^*(t^*)$, the shear stress distribution, τ^*, and velocity distributions, v^*, are carried out as described in the previous sections. Newton-Raphson iteration for $\xi^*(t^*)$ is accomplished in subroutine NEWT. Inputs to BING.BAS are, in order, the number of time steps, the number of x_2^*-steps, the minimum value of x_2^*, the maximum value of x_2^*, the minimum value of t^*, the maximum value of t^*, the maximum number of terms, Kmax, for the series solutions for τ^* and v^*, the constant dimensionless pressure gradient, $\partial P^*/\partial x_1^*$, Reynolds's number, Bingham's number, and the duration of pressure gradient application, t_1^*. Note that the dimensionless pressure gradient $\partial P^*/\partial x_1^*$ will be a negative number. Also, note that Kmax must be large (>50) for convergence of the Fourier series at small times.

To run the program, type BING. The input menu then appears with the cursor on the first line ready for input. The question on the line with the cursor is printed in inverse video. Enter the value and press the return or down arrow. This causes the cursor to move to the next input line. Continue this procedure until all input has been entered. The input values can be edited on the screen by using the up, down, and left arrows. The backspace key has the same function as the left arrow, and the return or the enter key has the same function as the down arrow key. Editing can be carried out at any time before execution.

Press the spacebar to execute the program. At this time, the user will be requested to enter an output file name. If no name is entered, the default file name is BAS010.DAT. This output file contains all values of x_2^*, t^*, $\xi^*(t^*)$, v^*, and τ^*. After execution, the input menu reappears on the screen allowing input values to be edited and the program to be rerun with new values.

EXAMPLE CALCULATION

We conclude with an example calculated with BING.BAS. The input values as they appear on the screen and the output values as they appear in the output file follow.

Values of $\xi^*(t^*)$ and $v^*(t^*)$ for 50 x_2^* steps, but otherwise identical input values, are shown in figures 2 and 3.

It is seen in figure 2 that for times less then 0.3, the thickness of the viscoplastic layer asymptotically approaches the limiting value of 0.3, the steady state value for a constant pressure gradient of -5. Following removal of the pressure gradient at $t^* = 0.3$, the thickness of the viscoplastic layer becomes vanishingly small as dimensionless time approaches 1.0.

For the velocities shown in figure 3, note the decrease in thickness of the rigid plug and increase in velocity during application of the pressure gradient. Also note the subsequent increase in plug thickness and decrease in velocity after removal of the pressure gradient.

```
Enter number of time steps. 5
Enter number of X steps.    5
Enter X minimum.            0
Enter X maximum.            .5
Enter T minimum.            0
Enter T maximum.            .5
Enter K maximum.            50
Enter p zero.               -5
Enter re.                   1
Enter bi.                   1
Enter Enter T1.             .3
```

BAS010.DAT

X	T	Xi	V	TAU
0.0000	0.0000	0.0000	0.0000	1.0000
0.1000	0.0000	0.0000	0.0000	0.8000
0.2000	0.0000	0.0000	0.0000	0.6000
0.3000	0.0000	0.0000	0.0000	0.4000
0.4000	0.0000	0.0000	0.0000	0.2000
0.5000	0.0000	0.0000	0.0000	0.0000
0.0000	0.1000	0.2325	0.0000	2.0409
0.1000	0.1000	0.2325	0.0804	1.5691
0.2000	0.1000	0.2325	0.1152	1.1335
0.3000	0.1000	0.2325	0.1152	0.7476
0.4000	0.1000	0.2325	0.1152	0.3738
0.5000	0.1000	0.2325	0.1152	0.0000
0.0000	0.2000	0.2783	0.0000	2.3251
0.1000	0.2000	0.2783	0.1084	1.8395
0.2000	0.2000	0.2783	0.1684	1.3635
0.3000	0.2000	0.2783	0.1684	0.9023
0.4000	0.2000	0.2783	0.1684	0.4512
0.5000	0.2000	0.2783	0.1684	0.0000
0.0000	0.3000	0.2923	0.0000	2.4311
0.1000	0.3000	0.2923	0.1188	1.9402
0.2000	0.3000	0.2923	0.1882	1.4492
0.3000	0.3000	0.2923	0.1882	0.9632
0.4000	0.3000	0.2923	0.1882	0.4816
0.5000	0.3000	0.2923	0.1882	0.0000
0.0000	0.4000	0.1810	0.0000	1.4297
0.1000	0.4000	0.1810	0.0323	1.2087
0.2000	0.4000	0.1810	0.0323	0.9405
0.3000	0.4000	0.1810	0.0323	0.6270
0.4000	0.4000	0.1810	0.0323	0.3135
0.5000	0.4000	0.1810	0.0323	0.0000
0.0000	0.5000	0.0777	0.0000	1.1602
0.1000	0.5000	0.0777	0.0000	0.9472
0.2000	0.5000	0.0777	0.0000	0.7104
0.3000	0.5000	0.0777	0.0000	0.4736
0.4000	0.5000	0.0777	0.0000	0.2368
0.5000	0.5000	0.0777	0.0000	0.0000

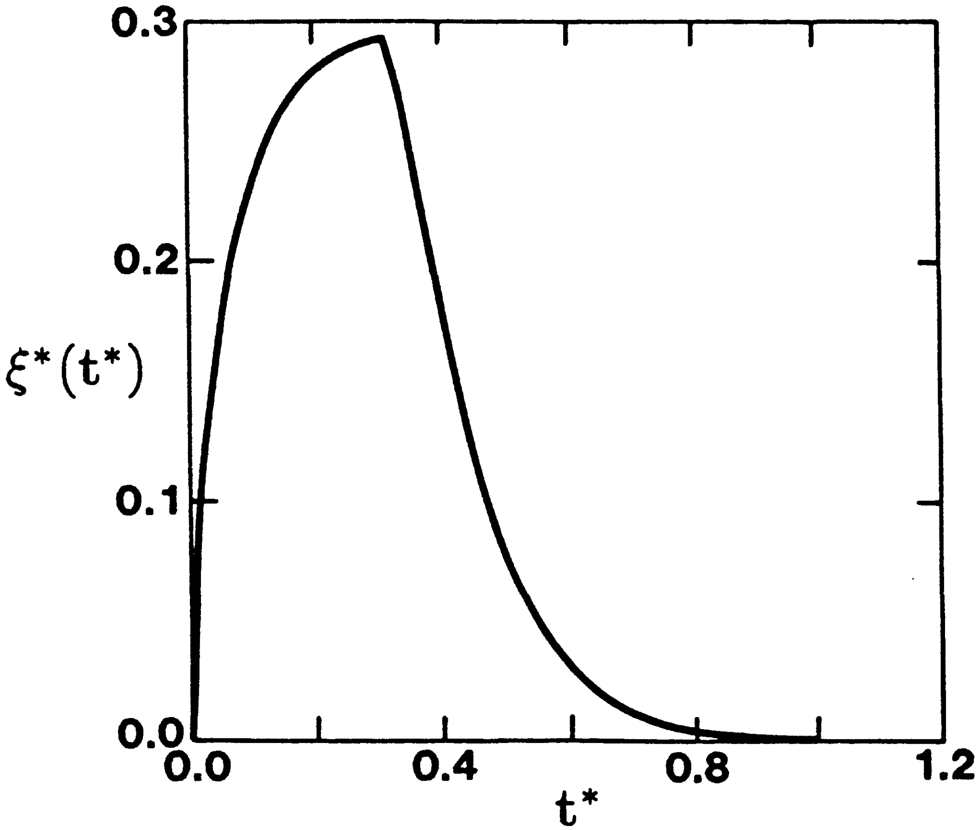

Figure 2. Variation with time, t^*, of the boundary, $\xi^*(t^*)$, between the rigid plug and viscoplastic region, for $P_o^* = -5$, Re = Bi = 1, $t_1^* = .3$.

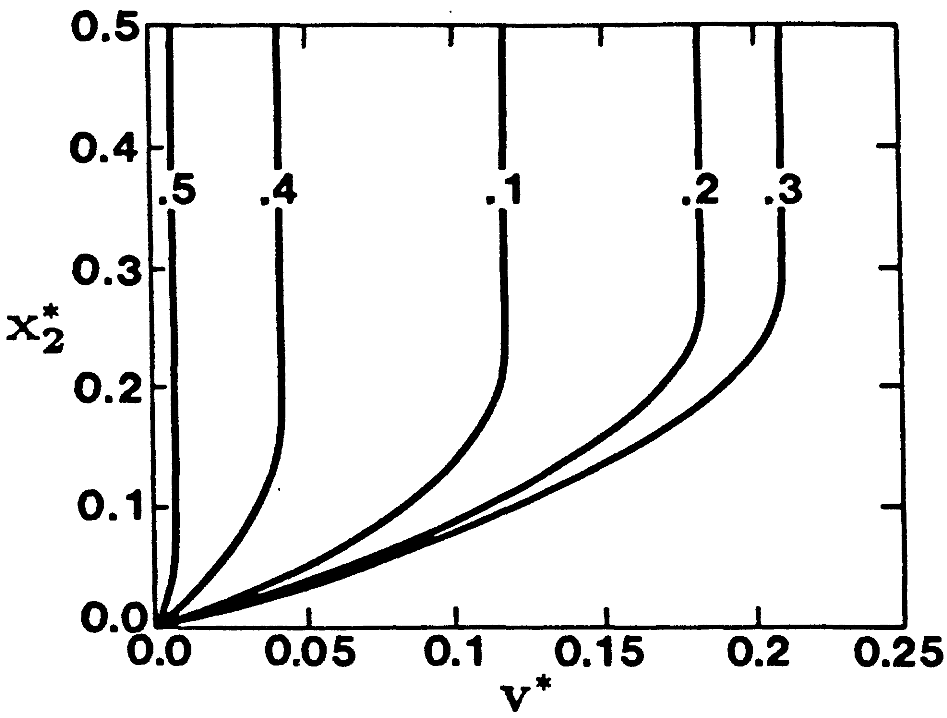

Figure 3. Variation with x_2^* of velocity, v^*, for various times when $P_o^* = -5$, Re = Bi = 1, and $t_1^* = 0.3$. The numbers on the curves are values of t^*.

REFERENCES CITED

Bingham, E.C., 1922, Fluidity and Plasticity: New York, McGraw-Hill Book Company, Inc., 440 p.

Oldroyd, J.G., 1947, Two-dimensional plastic flow of a Bingham solid-A plastic boundary layer theory for slow motion: Proceedings, Cambridge Philosophical Society, v. 43, p. 383-395.

Prager, W., 1961, Introduction to Mechanics of Continua: Boston, Ginn and Company, 230 p.

```
REM   This program was written by W. Savage
REM   and P. Powers of the U. S. G. S.  MS 966;
REM   POBox 25046; Denver, CO.  80225
REM   The program is written in QuickBASIC 4.5
REM   for use with an IBM compatible microcomputer.
REM
DECLARE SUB openfiles (file3$)
DECLARE SUB newt (t, t1, kmax, stc, xii, re)
DECLARE SUB length (TMP$, ll!, var1!(), row!)
DIM var1!(15), da$(15)
COMMON SHARED filexist%, file3$
filexist% = 0
CLS
DATA "Enter number of time steps.  "
DATA "Enter number of X steps.     "
DATA "Enter X minimum.             "
DATA "Enter X maximum.             "
DATA "Enter T minimum.             "
DATA "Enter T maximum.             "
DATA "Enter K maximum.             "
DATA "Enter p zero.                "
DATA "Enter re.                    "
DATA "Enter bi.                    "
DATA "Enter Enter T1.              "
DATA "  "
DATA "Press S P A C E B A R to run program."
DATA "Press ESC to end."
FOR i = 1 TO 14
   READ que$
   da$(i) = que$
NEXT i
GOSUB InitialScreen:
DO
CH$ = INKEY$
IF CH$ <> "" THEN
  ascval = ASC(RIGHT$(CH$, 1))
  IF ascval = 75 THEN GOTO lagn:
  IF ascval = 80 THEN GOSUB downarrow:
  IF ascval = 72 THEN GOSUB uparrow:
  IF ascval = 13 THEN GOSUB downarrow:
  IF ascval = 27 THEN GOTO tend:
  IF ascval = 32 THEN GOSUB runpg:
  IF ascval = 8 THEN GOSUB leftarrow:
  IF ascval > 32 THEN
     nc% = nc% + 1
     COLOR 0, 0
     LOCATE row, col
     PRINT CHR$(95);
     COLOR 7, 0
     IF TCH$ = "0" THEN TCH$ = ""
     TCH$ = TCH$ + CH$
     LOCATE row, col
     PRINT TCH$;
     COLOR 31, 0
     PRINT CHR$(95);
  END IF
```

```
        END IF

lagn:

        LOOP

uparrow:

        GOSUB normcol:
        IF TCH$ <> "" THEN
           GOSUB sc
        END IF
        CH$ = ""
        TCH$ = ""
        KEY(11) OFF
        nc% = 0
        COLOR 0, 0
        IF var1!(row) < 0 THEN
           LOCATE row, col + 2
        ELSE
           LOCATE row, col
        END IF
        PRINT CHR$(95);
        COLOR 7, 0
        LOCATE row, 28
        PRINT var1!(row);
        COLOR 31, 0
        IF row = 1 THEN
             LOCATE row, col
        ELSE
             row = row - 1
        IF var1!(row) = 0 THEN
           LOCATE row, 29
        ELSE
           CALL length(TCH$, l, var1!(), row)
           l = LEN(STR$(var1!(row)))
           LOCATE row, 29
           COLOR 7, 0
           PRINT TCH$;
        END IF
          END IF
          COLOR 31, 0
          PRINT CHR$(95);
          col = 29
          GOSUB revcol:
        RETURN

downarrow:
        GOSUB normcol:
        maxrow% = 11
        IF TCH$ <> "" THEN
           GOSUB sc
        END IF
        CH$ = ""
        TCH$ = ""
        COLOR 0, 0
```

```
        col = col + nc%
        nc% = 0
        IF var1!(row) < 0 THEN
           LOCATE row, col + 1
        ELSE
           LOCATE row, col
        END IF
        PRINT CHR$(95);
        COLOR 7, 0
        LOCATE row, 28
        PRINT var1!(row);
        COLOR 31, 0
        IF row = 25 THEN
           LOCATE row, col
        ELSE
           IF row < maxrow% THEN row = row + 1
           CALL length(TCH$, 1, var1!(), row)
           LOCATE row, 29
           COLOR 7, 0
           IF TCH$ <> "0" THEN
              PRINT TCH$;
           END IF
        END IF
        COLOR 31, 0
        PRINT CHR$(95);
        col = 29
        GOSUB revcol:
        RETURN

leftarrow:

        IF TCH$ <> "" THEN
           GOSUB sc
        END IF
        CH$ = ""
        TCH$ = ""
        CALL length(TCH$, 1, var1!(), row)
        COLOR 0, 0
        leftmost = 29 + 1
        IF leftmost < 29 THEN leftmost = 29
        LOCATE row, leftmost
        PRINT CHR$(95);
        IF var1!(row) = 0 THEN
           LOCATE row, 29
        ELSE
           LOCATE row, 29
           COLOR 7, 0
           IF TCH$ <> "0" THEN
              TCH$ = MID$(TCH$, 1, 1 - 1)
           END IF
           PRINT TCH$;
        END IF
        COLOR 31, 0
        PRINT CHR$(95);
        var1!(row) = VAL(TCH$)
        RETURN
```

```
        RESTORE
        COLOR 7, 1
        LOCATE 25, 1
        PRINT "                    (Use Up ARROW, Down ARROW/[RETURN
], Left Arrow)          ";
        LOCATE 1, 1
        COLOR 7, 0
        FOR i = 1 TO 14
           PRINT da$(i)
        NEXT i
        FOR i = 1 TO 11
          LOCATE i, 28
          PRINT var1!(i)
        NEXT i
        CALL length(TCH$, 1, var1!(), row)
        IF TCH$ <> "0" THEN
           LOCATE 1, 29 + 1
        ELSE
           LOCATE 1, 29
        END IF
        col = POS(0)
        row = CSRLIN
               COLOR 31, 0
               LOCATE row, col
               PRINT CHR$(95);
               CH$ = ""
               TCH$ = ""
        RETURN

revcol:

        COLOR 0, 7
        LOCATE row, 1
        PRINT da$(row)
        COLOR 7, 0
        RETURN

normcol:

        LOCATE row, 1
        COLOR 7, 0
        PRINT da$(row)
        COLOR 0, 7
        RETURN

sc:

        var1!(row) = VAL(TCH$)
        RETURN

InitialScreen:

        RESTORE
        SCREEN 0
        WIDTH 40
        COLOR 14, 4
```

```
        CLS
        LOCATE 10, 16
        PRINT "B I N G"
        LOCATE 20, 8
        PRINT "Press any key to continue."
        DO WHILE INKEY$ = ""
        LOOP
        SCREEN 0
        WIDTH 80
        COLOR 7, 0
        CLS
        COLOR 7, 1
        LOCATE 25, 1
        PRINT "             (Use Up ARROW, Down ARROW/[RETURN],
 Left Arrow/[BS])    ";
        LOCATE 1, 1
        COLOR 7, 0
        FOR i = 1 TO 14
          IF i = 1 THEN
             row = 1
             GOSUB revcol:
             row = 0
          ELSE
            PRINT da$(i)
          END IF
        NEXT i
        FOR i = 1 TO 11
          LOCATE i, 28
          PRINT var1!(i)
        NEXT i
        CALL length(TCH$, 1, var1!(), row)
        IF TCH$ <> "O" THEN
           LOCATE 1, 29 + 1
        ELSE
           LOCATE 1, 29
        END IF
        col = POS(0)
        row = CSRLIN
               COLOR 31, 0
               LOCATE row, col
               PRINT CHR$(95);
               CH$ = ""
               TCH$ = ""
        RETURN

runpg:

        CALL openfiles(file3$)
        CLS
        LOCATE 5, 25
        COLOR 31, 0
        PRINT " W O R K I N G "
        COLOR 7, 0
        nts = var1!(1)
        nxs = var1!(2)
        xmin = var1!(3)
```

```
        xmax = var1!(4)
        tmin = var1!(5)
        tmax = var1!(6)
        kmax = var1!(7)
        pzero = var1!(8)
        re = var1!(9)
        bi = var1!(10)
        t1 = var1!(11)
        pi = 3.14159
        LOCATE 10, 1
        COLOR 14, 0
        PRINT "                         T                Xi   "
        tinc = (tmax - tmin) / nts
        xinc = (xmax - xmin) / nxs
        nxmax% = nxs
        ntmax% = nts
        n% = 0
        m = 0
        stc = 4! * (2! + pzero) / (pi ^ 2)
        vtc = stc / pi
        t = tmin
        f% = 0
        ainc% = 0
        DO
           CALL newt(t, t1, kmax, stc, xii, re)
           x = xmin
           DO
              st = 0!
              vt = 0
              stt1 = 0!
              vtt1 = 0!
              cx = 1! - (2! * x)
              IF x >= xii THEN
                 tau = (x - .5) / (xii - .5)
                 v = v1
                 GOTO 100
              END IF
              k = 0
175           k = k + 1
              ck1 = 2! * k - 1
              ck2 = (COS((k - 1) * pi)) / (ck1 ^ 2)
              ck3 = ck2 / ck1
              st1 = ck2 * SIN(.5 * pi * ck1 * cx)
              st2 = 1! - EXP(-ck1 ^ 2 * pi ^ 2 * t / re)
              IF t = 0! THEN st2 = 0!
              st = st + stc * st1 * st2
              vt1 = ck3 * COS(.5 * pi * ck1 * cx)
              vt = vt + vtc * vt1 * st2
              IF t >= t1 THEN
                 st3 = EXP(-ck1 ^ 2 * pi ^ 2 * (t - t1) / re)
                 stt1 = stt1 + stc * st1 * (st3 - (1 - st2))
                 vtt1 = vtt1 + vtc * vt1 * (st3 - (1 - st2))
                 st = stt1
                  vt = vtt1
              END IF
              IF k <= kmax THEN GOTO 175
```

```
125              tau = cx - st
                 vl = -bi * (x ^ 2 + vt)
                 IF t = 0! THEN vl = 0!
                 v = vl
100              PRINT #3, USING "  ######.####    ######.####
######.####      ######.####  ######.####"; x; t; xii; v; tau
                 n% = n% + 1
                 x = x + xinc
              LOOP WHILE n% <= nxmax%
              n% = 0
              m = m + 1
              t = t + tinc
           LOOP WHILE m <= ntmax%
           COLOR 7, 0
           LOCATE 20, 1
           PRINT "      Press any key to return to initial input
screen."
           DO WHILE INKEY$ = ""
           LOOP
           CLS
           CLOSE #3
           GOSUB InitialScreen
           RETURN

tend:
           CLS
           CLOSE #3
           BEEP
           COLOR 7, 0
           PRINT "                                  D O N E"
           END

er1:

           CLS
           filexist% = 1
           CLOSE #3
           RESUME NEXT

'
SUB length (TMP$, ll, var1!(), row)

      TMP$ = STR$(var1!(row))
      TMP$ = RTRIM$(TMP$)
      TMP$ = LTRIM$(TMP$)
      ll = LEN(TMP$)

END SUB

'
SUB newt (t, t1, kmax, stc, xii, re)

           pi = 3.14159
150        k = 0!
           fti = 0!
           ftf = 0!
```

```
        cxi = 1! - 2! * xi
165     k = k + 1
        ck1 = 2 * k - 1
        ck2 = (COS((k - 1) * pi)) / (ck1 ^ 2)
        cf1 = ck2 * SIN(.5 * pi * ck1 * cxi)
        cf2 = 1! - EXP(-ck1 ^ 2 * pi ^ 2 * t / re)
        cf3 = ck1 * ck2 * COS(.5 * pi * ck1 * cxi)
        IF t <= t1 THEN
                fti = fti + .5 * stc * cf1 * cf2
                ftf = ftf + .5 * pi * stc * cf3 * cf2
        ELSE
                cf4 = EXP(-(ck1 ^ 2 * pi ^ 2 * (t - t1) / re)
)
                cf5 = 1! - cf2
                cf6 = cf4 - cf5
                fti = fti + .5 * stc * cf1 * cf6
                ftf = ftf + .5 * pi * stc * cf3 * cf6
        END IF
        IF k <= kmax THEN GOTO 165
160     xii = xi - (xi + fti) / (1! - ftf)
        IF ABS(xi - xii) <= .000001 THEN GOTO 200
        xi = xii
        GOTO 150
200     PRINT TAB(18); USING " #####.#####   #####.#####"; t;
 xii

END SUB

'
SUB openfiles (file3$)

        COLOR 7, 0
        filexist% = 0

agn:

        CLS
        IF file3$ = "" THEN
            tfile3$ = "bas010.dat"
        ELSE
            tfile3$ = file3$
        END IF
        PRINT "Enter the output file name with extension.  [R
ETURN] = "; tfile3$; "  ";
        LINE INPUT file3$
        file3$ = LTRIM$(file3$)
        IF file3$ = "" THEN
           file3$ = tfile3$
        END IF
        REM An error means file is new.
        ON ERROR GOTO er1:
        OPEN file3$ FOR INPUT AS #3
        ON ERROR GOTO 0:
        IF filexist% = 1 THEN GOTO agn2:

agn1:
```

```
        choice$ = ""
        PRINT "There is already a file by that name.  Enter O
=overwrite, N=new name."
        DO WHILE choice$ = ""
           choice$ = INKEY$
        LOOP
        choice$ = UCASE$(LEFT$(choice$, 1))
        IF choice$ = "O" THEN
           CLOSE #3
        END IF
        IF choice$ = "N" THEN
           CLOSE #3
           GOTO agn:
        END IF
        IF choice$ <> "O" AND choice$ <> "N" THEN
           PRINT "Please enter O or N"
           PRINT "Press any key to proceed."
           DO WHILE INKEY$ = ""
           LOOP
           CLS
           GOTO agn1:
        END IF

agn2:

        ON ERROR GOTO 0
        OPEN file3$ FOR OUTPUT AS #3
        PRINT #3, "           X             T              Xi
         V            TAU"

END SUB
```