A Finite-Element Model for Simulation of Two-Dimensional Steady-State Ground-Water Flow in Confined Aquifers

By Eve L. Kuniansky

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ILLUSTRATIONS

FACTORS FOR CONVERTING INCH-POUND UNITS TO INTERNATIONAL SYSTEM (SI) OF METRIC UNITS

Multiply inch-pound units	By	To obtain metric units
inch (in.)	25.4	millimeter (mm)
cubic foot per second (ft^3/s)	0.02832	cubic meter per second (m^3/s)
foot (ft)	0.3048	meter (m)
gallons per minute (gal/min)	0.01666	cubic meter per second
square foot per day (ft^2/d)	0.09290	square meter per day (m^2/d)

National Geodetic Vertical Datum of 1929 (NGVD of 1929): A geodetic datum derived from a general adjustment of the first-order level nets of both the United States and Canada, formerly called Sea Level Datum of 1929. NGVD of 1929 is referred to as sea level in this report.

A FINITE-ELEMENT MODEL FOR SIMULATION OF TWO-DIMENSIONAL STEADY-STATE GROUND-WATER FLOW IN CONFINED AQUIFERS

By

Eve L. Kuniansky

ABSTRACT

A computer program based on the Galerkin finite-element method was developed to simulate two-dimensional steady-state ground-water flow in either isotropic or anisotropic confined aquifers. The program may also be used for unconfined aquifers of constant saturated thickness. Constant head, constant flux, and head-dependent flux boundary conditions can be specified in order to approximate a variety of natural conditions, such as a river or lake boundary, and pumping well. The computer program was developed for the preliminary simulation of ground-water flow in the Edwards-Trinity Regional aquifer system as part of the Regional Aquifer-Systems Analysis Program. Results of the program compare well to analytical solutions and simulations from published finite-difference models. A concise discussion of the Galerkin method is presented along with a description of the program. Provided in the Supplemental Data section are a listing of the computer program, definitions of selected program variables, and several examples of data input and output used in verifying the accuracy of the program.

INTRODUCTION

Mathematical modeling has become a useful tool in examining many problems involving flow through porous media. This report documents a simple two-dimensional finite-element-method computer program originally developed by the author in 1981 and updated to include anisotropy. The computer program was developed specifically for preliminary simulation of ground-water flow in the Edwards-Trinity aquifer system in west-central Texas as part of the Regional Aquifer-Systems Analysis (RASA) program. A key part of the Edwards-Trinity RASA project is the development of a digital ground-water flow model of the system in order to gain a better understanding of ground-water movement within the aquifer system. Part of the aquifer system is anisotropic with the principal directions of anisotropy varying over the area of the aquifer system. The finite-element method can approximate anisotropic conditions of this type; thus, it was selected as the numerical method to apply to this project. The purpose of this report is to document the computer program and demonstrate that the computer program works accurately.

MODEL DEVELOPMENT

Steady-state ground-water flow can be described by the partial differential equation derived from the principal of conservation of mass and the assumptions that water is incompressible and of constant viscosity (Bear, 1979, p. 93; Bouwer, 1978, p. 202; Raudkivi and Callander, 1976, p. 43). In reality, flow through most porous media is three dimensional, but most aquifers are one to two orders of magnitude thinner in the vertical direction than in the horizontal direction. Therefore, many ground-water flow problems can be approximated mathematically as two-dimensional horizontal flow (Bear and Verruijt, 1987, p. 21). To obtain a unique solution to the equation, boundary conditions must be specified. There are three basic types of boundary condition: Dirichlet or constant head, Neumann or constant flux, and Cauchy or head-dependent flux (Bear, 1979, p. 96-98). The computer program was designed for specifying any combination of these three conditions at any point or boundary. The program allows for specifying anisotropic transmissivity.

The program can be used for confined aquifers or unconfined aquifers by computing the transmissivity as the average saturated thickness of the aquifer multiplied by permeability. If unconfined aquifers are simulated, the saturated thickness determined for computing transmissivity must not be much different from that after simulation around new pumping centers.

The main advantage of the finite-element method compared to the finite-difference method is that the orientation of the principal axes of anisotropy can vary in direction over the flow domain. With the finite-difference method the axes of anisotropy must be aligned along the cartesian-coordinate axes. Another advantage of the finite-element method is that the shapes of irregular external or internal boundaries can be approximated more closely with fewer nodes and that flux from point sources or sinks can be applied directly at a point.

Governing Equation

The two-dimensional, steady-state, ground-water flow equation solved by this computer program is:

$$\frac{\partial}{\partial x} T_{xx}(x,y) \frac{\partial h(x,y)}{\partial x} + \frac{\partial}{\partial y} T_{yy}(x,y) \frac{\partial h(x,y)}{\partial y} = 0 \qquad (1)$$

Subject to the boundary conditions of the following type:

$$h(x,y) = H_0(x,y), \text{ Dirichlet or constant head;} \qquad (1a)$$

$$T\frac{\partial h(x,y)}{\partial n} = q_0 \text{ , Neumann or constant flux; and} \qquad (1b)$$

$$C(H_b - h(x,y)) = q_b, \text{ Cauchy or head-dependent flux;} \qquad (1c)$$

where $T_{xx}(x,y)$ is the transmissivity in the x-direction;
- $T_{yy}(x,y)$ is the transmissivity in the y-direction;
- $h(x,y)$ is the vertically averaged hydraulic head at a point in the aquifer;
- H_0 is a constant known value of hydraulic head at a point on the boundary;
- q_0 is a constant known flux;
- q_b is a head-dependent flux;
- H_b is a known hydraulic head outside of the flow domain;
- C is a known hydraulic conductance (permeability of a semiconfining unit divided by its thickness);
- n is the outward normal vector; and
- (x,y) are the cartesian coordinates.

It is assumed in equation 1 that the axis is aligned along the principal direction of anisotropy (Bear, 1979, p. 104-105).

Application of the Galerkin Finite-Element Technique

The Galerkin finite-element method has been widely used to solve equation 1 numerically and its application is described in numerous texts, including: Bathe and Wilson, 1976; Bear and Verruijt, 1987; Huyakorn and Pinder, 1983; Pinder and Gray, 1977; Reddy, 1986; Remson and others, 1971; Wang and Anderson, 1982; and Zienkiewicz, 1977. The finite-element technique differs from the finite-difference method in that it involves piecewise approximation of the flow domain. The following discussion of the Galerkin approach is a composite of the many different texts describing the method and is limited to the simplest two-dimensional finite element, the three-nodal trangular element. For more detailed information about the finite-element method and other similar approaches to solving equation 1, the reader is encouraged to obtain the cited references.

There are five basic steps to solving a ground-water flow problem by the finite-element method (Huyakorn and Pinder, 1983, p. 26; Reddy, 1986, p. 344). The steps are:

1. Division of the flow domain into discrete finite elements that are connected at points called nodes. Figure 1 is an example of a finite-element mesh showing the numbering of nodes and elements. Nodes are located at the vertices of the triangular elements and are assigned numbers. Three nodes define each element and are assigned numbers in counterclockwise order. The node number and its cartesian coordinates and the sequence of nodes defining each triangular element are required data for the finite-element model.

2a. Derive the approximating functions called basis functions, shape functions, or interpolation functions. Generally, the approximating functions are algebraic polynomial functions used for interpolating the unknown function in terms of its values. For a three-nodal triangular element the three basis functions are selected by finding functions that equal 1 at the given node and 0 at the other remaining nodes in the triangle, such that the approximate value for the hydraulic head anywhere within the triangle equals the sum of the head at each node times its basis function (see equation 2 below).

$$\hat{h}(x,y) \approx \sum_{m=1}^{3} h_m \phi_m \qquad (2)$$

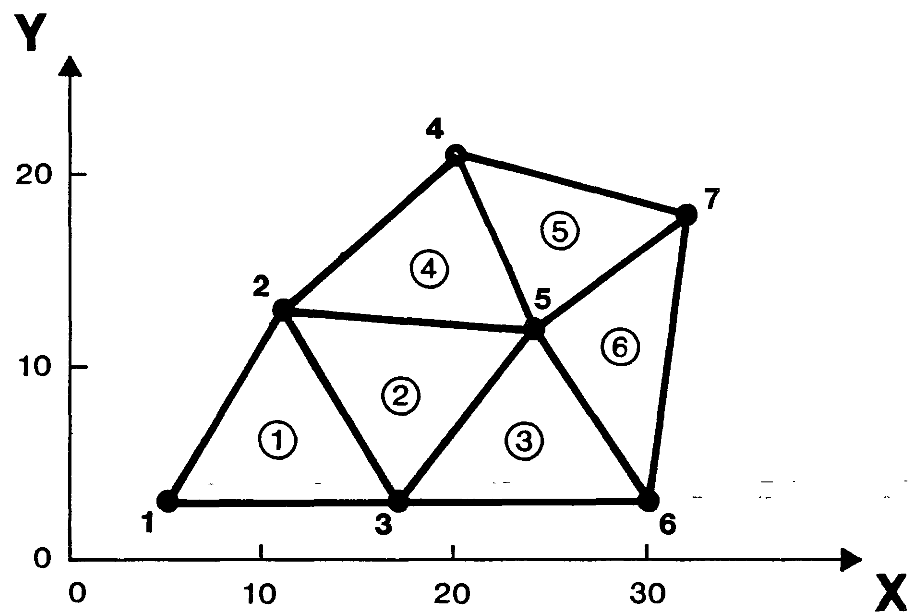
where $\hat{h}(x,y)$ is the approximate potentiometric head;
h_m are the heads at the nodes of the element; and
ϕ_m are the basis functions for each node.

A typical triangular element with nodes i, j, and k is shown in figure 2. To efficiently incorporate anisotropy into the computations, the coordinates for the vertices are converted from the global coordinate system to a local coordinate system. The local coordinate system is rotated such that the principal direction of anisotropy, for the element, is parallel to the local x-axis. For two-dimensional flow, this results in a second-rank transmissivity tensor that has only two nonzero components on the diagonal (Bear, 1979, p. 72; Papadopulous, 1965, p. 22). The local coordinate system is also translated such that the origin is at the centroid of each element. The translation helps reduce roundoff errors. Equations 3 below are the equations for transforming the coordinate system.

$$x'=(x-x_0)\cos\alpha + (y-y_0)\sin\alpha$$
$$y'=(y-y_0)\cos\alpha + (x-x_0)\sin\alpha \qquad (3)$$

where x',y' are the local coordinates;
x_0, y_0 are the coordinates of the centroid of the triangle;
x,y are the global coordinates; and
α is the angle (measured counterclockwise) from the global x-axis and aligned with the major axis of anisotropy.

EXAMPLE MESH



a. node coordinate data

node number	x coordinate	y coordinate
1	5	3
2	10	13
3	17	3
4	20	21
5	24	12
6	30	3
7	32	18

b. element connection data

element number	node connections *i*	*j*	*k*
①	1	3	2
②	2	3	5
③	3	6	5
④	4	2	5
⑤	4	5	7
⑥	7	5	6

Figure 1.--Finite-element mesh, node coordinate data, and element connection data.

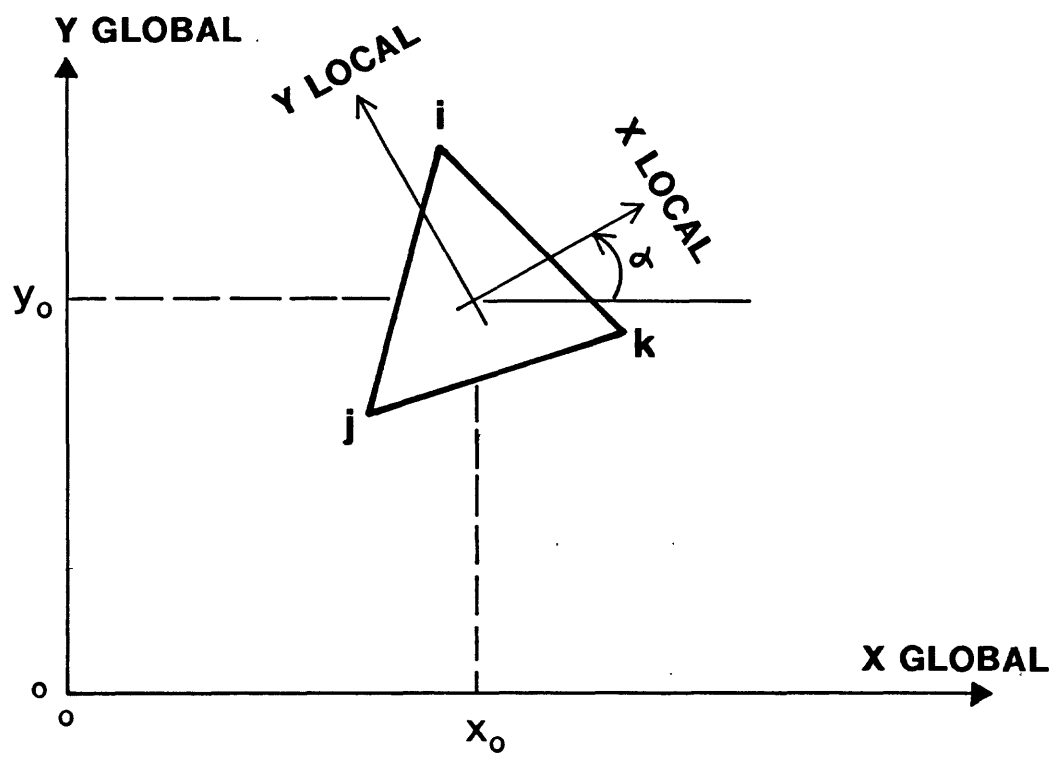


Figure 2.--Triangular element showing the global and local coordinate systems

Now that the coordinates are defined, the basis functions can be derived. The basis functions are derived from the algebraic polynomial expression for a line.

$$\phi_m = a_m + b_m x_m + c_m y_m \tag{4a}$$

where x_m, y_m are the local coordinates at the vertices of the element;
a_m, b_m, and c_m are the coefficients for each basis function; and
m is equal to i, j, and k (the indices for each node).

To determine the coefficients for the three basis functions, a_m, b_m, and c_m, we use the fact that at the given node the function must equal 1 and at the other nodes of the triangle the function must equal 0. This results in a set of three equations with three unkowns for each basis function.

$$\begin{array}{lll} \phi_i & \phi_j & \phi_k \\ 1 = a_i + b_i x_i + c_i y_i & 1 = a_j + b_j x_j + c_j y_j & 1 = a_k + b_k x_k + c_k y_k \\ 0 = a_i + b_i x_j + c_i y_j & 0 = a_j + b_j x_i + c_j y_i & 0 = a_k + b_k x_i + c_k y_i \\ 0 = a_i + b_i x_k + c_i y_k & 0 = a_j + b_j x_k + c_j y_k & 0 = a_k + b_k x_j + c_k y_j \end{array} \tag{4b}$$

These sets of equations are solved for the coefficients of the basis functions and the resulting solutions are:

$$
\begin{aligned}
a_i &= \frac{1}{2A}(x_j y_k - x_k y_j) & b_i &= \frac{1}{2A}(y_j - y_k) & c_i &= \frac{1}{2A}(x_k - x_j) \\
a_j &= \frac{1}{2A}(x_k y_i - x_i y_k) & b_j &= \frac{1}{2A}(y_k - y_i) & c_j &= \frac{1}{2A}(x_i - x_k) \\
a_k &= \frac{1}{2A}(x_i y_j - x_j y_i) & b_k &= \frac{1}{2A}(y_i - y_j) & c_k &= \frac{1}{2A}(x_j - x_i)
\end{aligned} \tag{4c}
$$

where A is the area of the triangle equal to

$$A = [(x_i y_j - x_j y_i)+(x_k y_i - x_i y_k)+(x_j y_k - x_k y_j)]/2 \tag{4d}$$

2b. Use the Galerkin weighted residual method to completely formulate a set of linear algebraic equations for the head at the nodes of the individual element. The approximate solution of the partial differential equation is obtained by substitution of equation 2 into equation 1, resulting in some residual error, ε.

$$\varepsilon = \frac{\partial}{\partial x} T_{xx} \frac{\partial \hat{h}(x,y)}{\partial x} + \frac{\partial}{\partial y} T_{yy} \frac{\partial \hat{h}(x,y)}{\partial y} \tag{5}$$

Minimize the residual error by using the basis function obtained in step 2a as the weighting function for the residual error and setting this integral over the area of the element equal to 0.

$$\iint_A [(T_{xx}\frac{\partial}{\partial x}\frac{\partial \hat{h}(x,y)}{\partial x} + T_{yy}\frac{\partial}{\partial y}\frac{\partial \hat{h}(x,y)}{\partial y})\phi_m] \, dx \, dy = 0 \tag{6a}$$

where m = i, j, and k

T_{xx} is the transmissivity along the local x-axis and is a constant for the element; and

T_{yy} is the transmissivity along the local y-axis and is a constant for the element;

and

$$\frac{\partial \hat{h}(x,y)}{\partial x} = \frac{\partial \phi_i}{\partial x} h_i + \frac{\partial \phi_j}{\partial x} h_j + \frac{\partial \phi_k}{\partial x} h_k \tag{6b}$$

$$\frac{\partial \hat{h}(x,y)}{\partial y} = \frac{\partial \phi_i}{\partial y} h_i + \frac{\partial \phi_j}{\partial y} h_j + \frac{\partial \phi_k}{\partial y} h_k$$

$$\frac{\partial \phi_i}{\partial x} = b_i \quad \frac{\partial \phi_j}{\partial x} = b_j \quad \frac{\partial \phi_k}{\partial x} = b_k \tag{6c}$$

$$\frac{\partial \phi_i}{\partial y} = c_i \quad \frac{\partial \phi_j}{\partial y} = c_j \quad \frac{\partial \phi_k}{\partial y} = c_k$$

After completing the derivatives and integrating equation 6a, the differential equation 1 is reduced to a linear set of equations for the unknown head at each node of the triangular element. In matrix form the resulting equation is:

$$A\begin{bmatrix}(T_{xx}b_ib_i+T_{yy}c_ic_i) & (T_{xx}b_ib_j+T_{yy}c_ic_j) & (T_{xx}b_ib_k+T_{yy}c_ic_k)\\(T_{xx}b_jb_i+T_{yy}c_jc_i) & (T_{xx}b_jb_j+T_{yy}c_jc_k) & (T_{xx}b_jb_k+T_{yy}c_jc_k)\\(T_{xx}b_kb_i+T_{yy}c_kc_i) & (T_{xx}b_kb_j+T_{yy}c_kc_k) & (T_{xx}b_kb_k+T_{yy}c_kc_k)\end{bmatrix}\begin{Bmatrix}h_i\\h_j\\h_k\end{Bmatrix}=\begin{Bmatrix}0\\0\\0\end{Bmatrix} \quad (7)$$

This is called the element coefficient matrix.

3. Assemble the element coefficient matrices into a matrix for the entire flow domain called the global coefficient matrix. Figure 3 shows how each element matrix is incorporated into the global matrix. As can be seen on figure 3, each element surrounding a node contributes terms to the global matrix based on the node numbers of the other vertices of the element. The contributions from each element are summed together.

4. Impose boundary conditions. This will be discussed in more detail in the section on boundary conditions. The boundary conditions change the values in the right-hand side of equation 7 and some of the coefficients of the global matrix.

5. Solve the system of equations. This can be done using a direct method of solving the equations, such as Gaussian elimination. This computer program employs simple Gaussian elimination. Different methods of solving linear systems of equations are given in Conte and de Boor (1980, chapter 4).

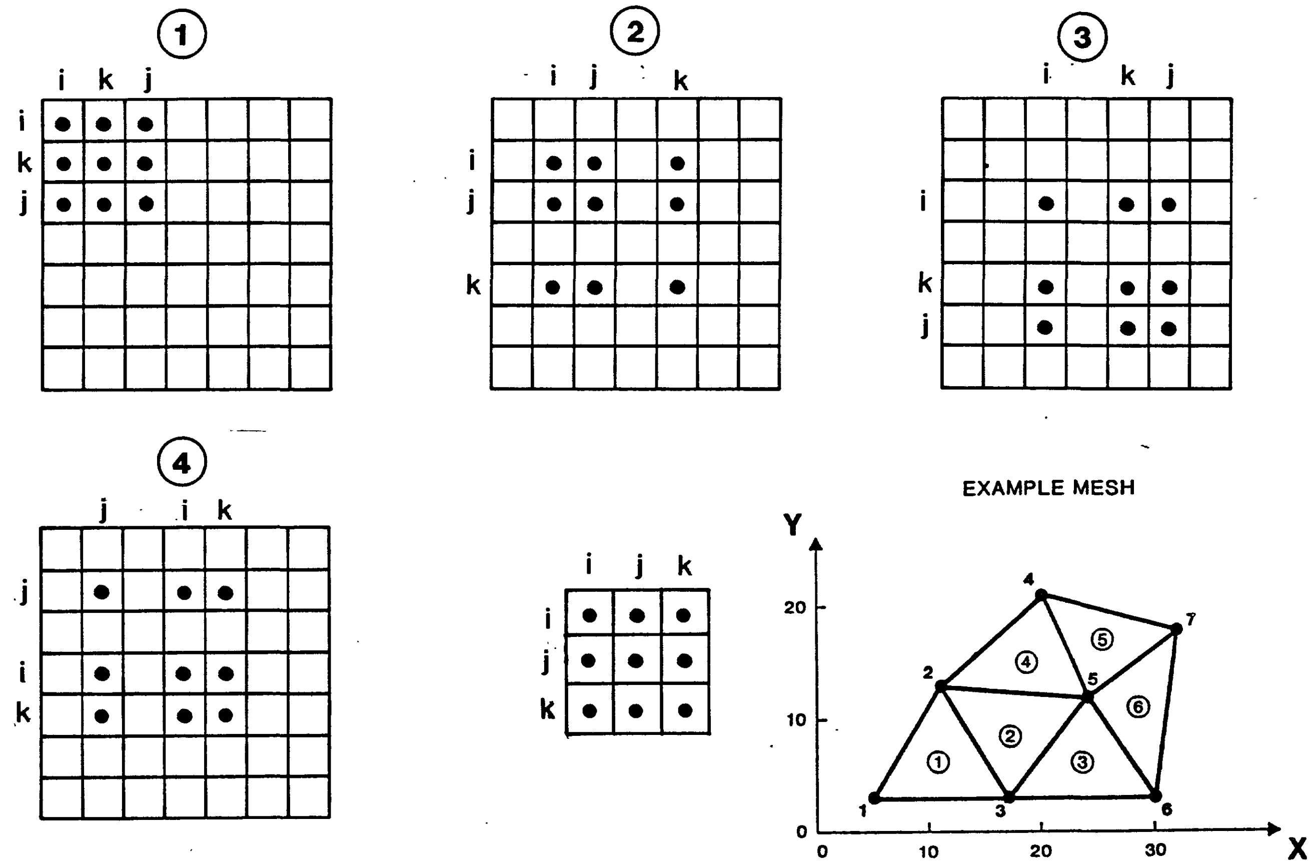
Boundary Conditions

The three basic boundary conditions are constant head, constant flux, and head-dependent flux. These three conditions can be incorporated mathematically into the finite-element equations to simulate most physical boundary conditions.

Constant-head conditions can be simulated by eliminating the equation for the node with the known head or by forcing the solution for the head to be the known head by multiplying the diagonal term of the global matrix at the constant-head node by a large number and adding to the solution vector the known head multiplied by the same large number (Wang and Anderson, 1982, p. 128). This forces the equation solved to become a trivial equation that will solve for $h = H_o$. This computer program does the latter.

Constant head nodes can represent many naturally occurring conditions, such as; a lake incised into the aquifer, a river incised into the aquifer, or a spring pool altitude. An important consideration in specifying a constant-head node is whether or not there is direct hydraulic connection between the aquifer and the surface-water body. Generally, there will be some intervening unit of differing permeability between the aquifer and the lake or river. If this intervening unit is thick enough or of low permeability, a head-dependent flux should be considered.

As mentioned previously, constant-flux terms for flow across the outer boundary of the mesh are directly incorporated into the right-hand-side vector of the equations at the location in the vector for that node (positive for



Element connection data

element number	node connections i	j	k
①	1	3	2
②	2	3	5
③	3	6	5
④	4	2	5
⑤	4	5	7
⑥	7	5	6

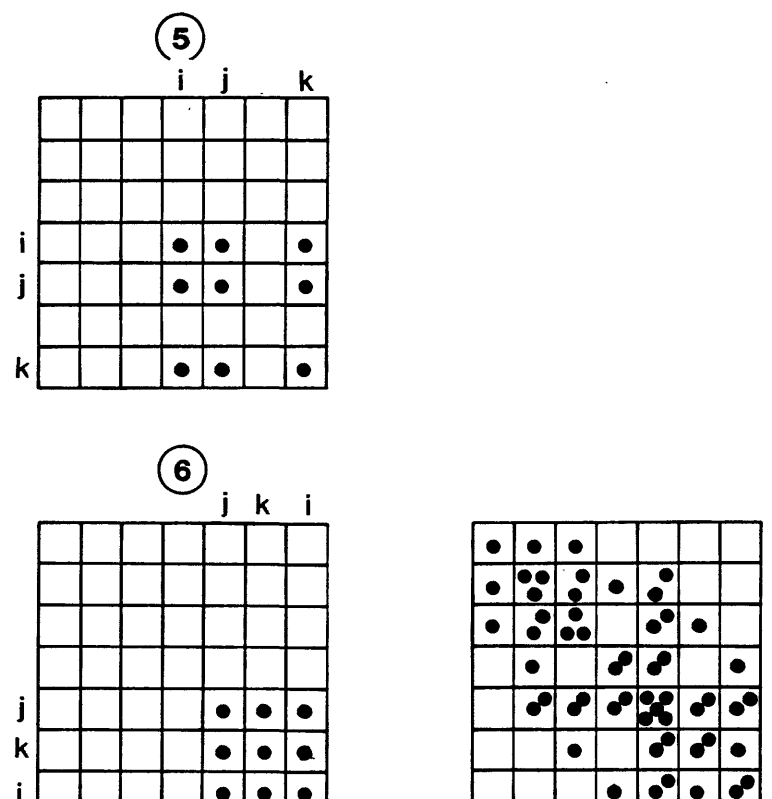


Figure 3.--Relation of element coefficient matrices to the global coefficient matrix. (modified from Wang and Anderson, 1982, figure 6.4)

recharge and negative for discharge). For linear triangles, the terms at each node due to a constant flux, q_o, across an element side is given by:

$$F_n = \int_L q_o \phi_n \, ds = \frac{q_o L}{2} \qquad (8)$$

where F_n is the flux at the nodes on the side; and

L is the length of the line segment between node n and the adjacent node.

It is important to note that by using the finite-element approximation, if no flux is specified at the boundary of the flow domain, then the boundary is automatically a no-flow boundary (Bear, 1979, p. 150-151). This is known as the natural boundary condition and results from integration by parts of equation 6.

Other constant-flux conditions such as areally distributed recharge or point sources or sinks can be simulated by adding these flux terms directly to the right-hand-side vector. Recharge can be computed as a constant flux over an element that is redistributed to each node. A discharge well is a point sink. Integration of these terms are discussed in Pinder and Gray (1977, p. 100-102).

Head-dependent flux conditions from a point source, such as leakage from a small pond or segment of a partially penetrating river, are incorporated into the set of equations by adding the known head multiplied by the hydraulic conductance term to the solution vector of the equation for that node and adding the hydraulic conductance term to the diagonal term of the global coefficient matrix. The hydraulic conductance is given by equation 9 below.

$$C = (K'A)/B \qquad (9)$$

where C is hydraulic conductance;

K' is the permeability of the intervening layer;

A is the area of the source or sink, length times width of the river or the area of the simulated pond; and

B is the thickness of the intervening layer.

As the hydraulic conductance term, C, increases in value, the solution approaches the trivial equation formulated for a constant-head node. Head dependent fluxes from areal sources, such as the flow through a leaky confining bed where the head external to the modeled aquifer is known, require integration of the boundary condition equation, 1c, over the element and are not currently incorporated in this computer program.

This program is not sophisticated; the three types of boundary conditions are available, but the user must make sure that they are used properly. Packages are not included for converting recharge rates in length per time units to specified fluxes at each node. For further discussions regarding application of boundary conditions see: Franke and others (1987); Bear (1979, chapter 5); and Bear and Verruijt (1987, chapter 4).

COMPUTER PROGRAM

The computer program was originally written in Fortran IV (Kuniansky, 1982) and has been modified to run on most computers. The program is structured such that the main program calls many subroutines. Variables are passed to the subroutines in common blocks and in arguments. Variable dimensioning is not used. Thus, to increase the number of triangular elements to be solved, care must be taken to increase all arrays in the dimension statements.

Main Program

The main program first calls the subroutine INPUT, next initializes the global matrix and right-hand-side vector, then begins formulating the element coefficient matrices and assembles these into the global coefficient matrix. These operations occur within a do loop which calls the subroutines LOCAL, ELEM, and ASEMB. It's important to note that the main program converts the angle of anisotropy for each element given in degrees to radians. After the assembly of the global coefficient matrix, subroutine BOUND is called and the boundary conditions are put into the coefficient matrix and right-hand-side vector. Next, subroutines REDUCE and SOLVE are called. REDUCE is where the method of Gaussian elimination is applied to the matrix and SOLVE is the back-solving subroutine for determining the unkown nodal heads. Subroutine OUTPUT sends the title and computed heads to a file in ASCII format (see flow chart, fig. 4).

Subroutine INPUT

Subroutine INPUT initializes boundary-condition arrays, and reads in the title and units, number of nodes, elements, and three types of boundary conditions. Also in the INPUT subroutine, the band width of the matrix to be solved is computed. If the band width is greater than the dimensioning of the global matrix, a warning is written to the output file and the program stops.

Subroutine LOCAL

Subroutine LOCAL is where the coordinate system of each element is converted from the global system to the local system. The coefficients of the basis functions and the integrals of the basis functions are computed in this subroutine.

Subroutine ELEM

Subroutine ELEM takes the coefficients computed in the LOCAL subroutine for the element and computes the full element coefficient matrix based on transmissivity and the coefficients for the integrals of the basis functions.

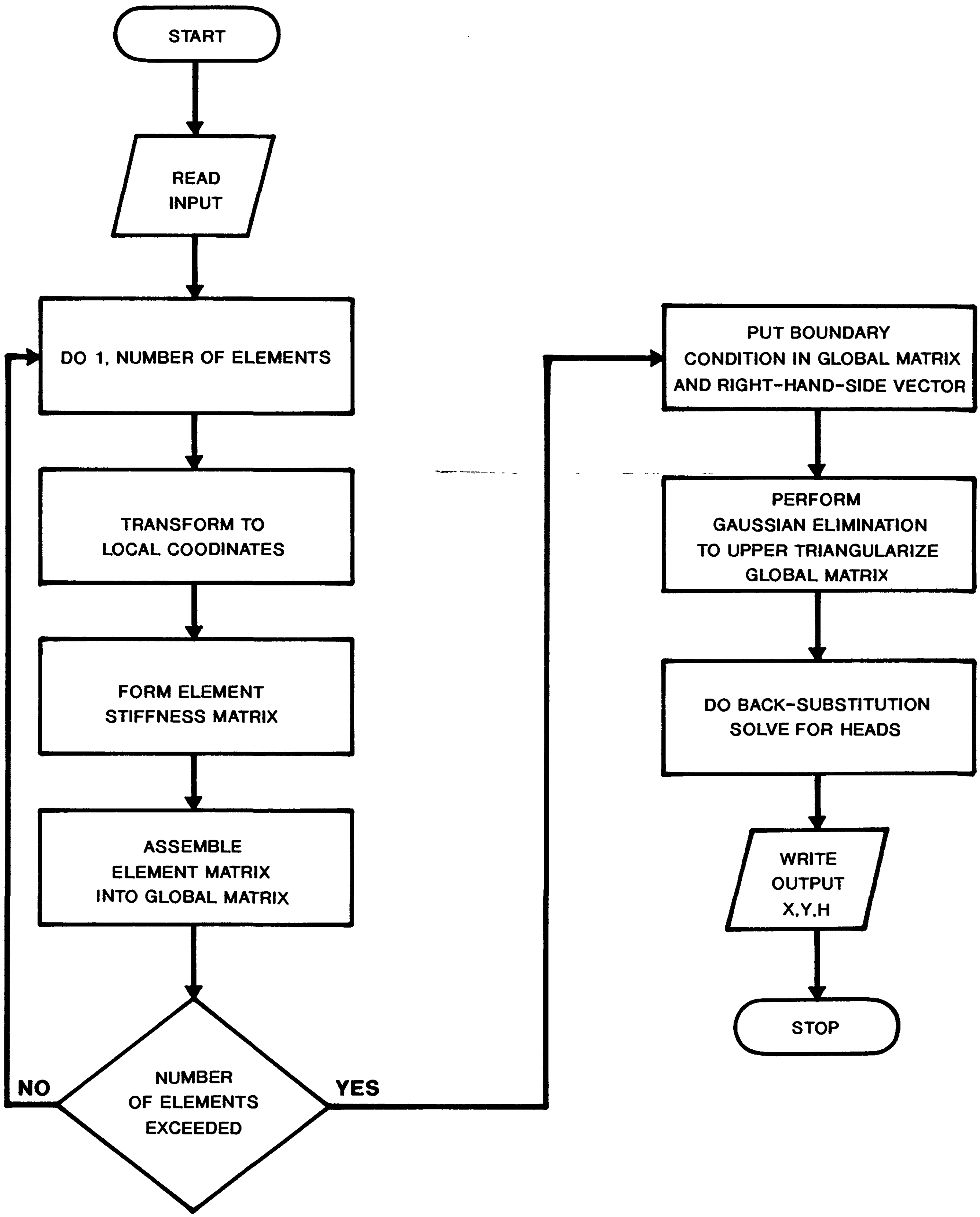


Figure 4.--Generalized flow chart of the computer program.

Subroutine ASEMB

The element coefficient matrices are assembled into the global coefficient matrix by subroutine ASEMB. In this program, the matrix is stored in a banded rectangular array to save computer storage memory.

Subroutine BOUND

Subroutine BOUND inserts the three boundary condition types into the global coefficient matrix and right-hand-side vector. The values were read in during the INPUT subroutine. This is the first subroutine called for after the do loop in the main program.

Subroutine REDUCE

Subroutine REDUCE is where the first step in solving the system of linear algebraic equations by Gaussian elimination is accomplished. The alogorithm reduces the matrix to its upper-triangular form. It is written for a matrix stored in banded form.

Subroutine SOLVE

Back substitution is done to the upper-triangular matrix by subroutine SOLVE. The unknown heads are written over the solution vector in this subroutine as the back substitution proceeds.

Subroutine OUTPUT

Subroutine OUTPUT writes the title cards and unknown heads to an output file. All output is written to a file opened on Fortran unit 6. The output file stores the cartesian coordinates and the unknown heads, such that post-processing programs can be used to display the simulated heads.

EVALUATION OF THE ACCURACY OF THE MODEL

Three simulations were run with the computer code to insure that no errors were made in converting the numerical algorithm to Fortran computer language. The first was a comparison of the computer-simulated solution to the analytical solution for steady flow from a pumping well in an isotropic aquifer, known as the Theim equation (Bear, 1979, p. 306). The second was a comparison of the computer-simulated solution to the analytical solution of the Theim equation for steady flow to a pumping well in an anisotropic aquifer. The third test simulation was a comparison of the solution results from the computer program to the solution results of a finite-difference model of a pumping well in a square confined aquifer (Reilly and others, 1987, Appendix 3).

The mesh shown in figure 5 was designed for simulating radial flow to a pumping well and compared to the solution of the Theim equation below:

$$h(r) = H_o - \frac{Q}{2\pi T}\log\left(\frac{R}{r}\right) \qquad (10)$$

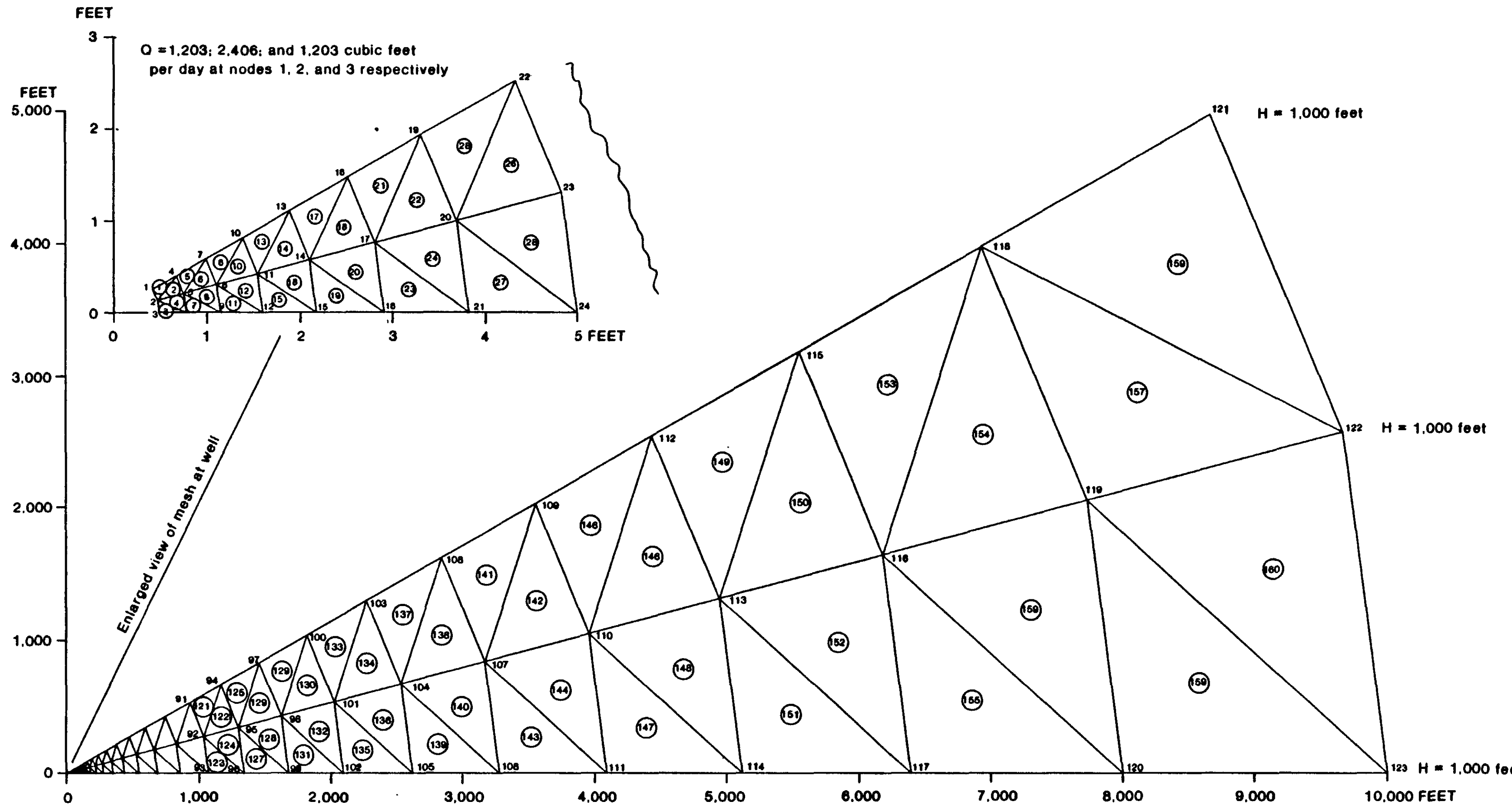


Figure 5.--Finite-element mesh used for simulating radial flow to a pumping well in an isotropic aquifer.

where R is the radius of influence of the well;
r is the distance from the well
h(r) is the head at the distance r;
H_0 is the initial head in the aquifer at R;
T is aquifer transmissivity; and
Q is the discharge rate of the pump.

Because this is radially symmetric flow, the mesh was designed as a 30-degree wedge with no flux boundaries along the sides. The radius of influence of the pumping well was determined iteratively from the Theim equation and the mesh extends out to the radius of influence of 10,000 ft. A constant-head boundary was set at the radius of influence and constant-flux boundary was set at the three nodes defining the modeled part of the circumference of the well. The well was pumping at a rate of 300 gal/min (57,754 ft^3/d) in an aquifer having a transmissivity of 5,000 ft^2/d. The radius of the well was 0.5 ft. At the radius of influence the head in the aquifer was 1,000 ft above sea level. The comparison of the simulated head versus the analytical solution is shown in figure 6. The mean absolute error is 0.095 ft.

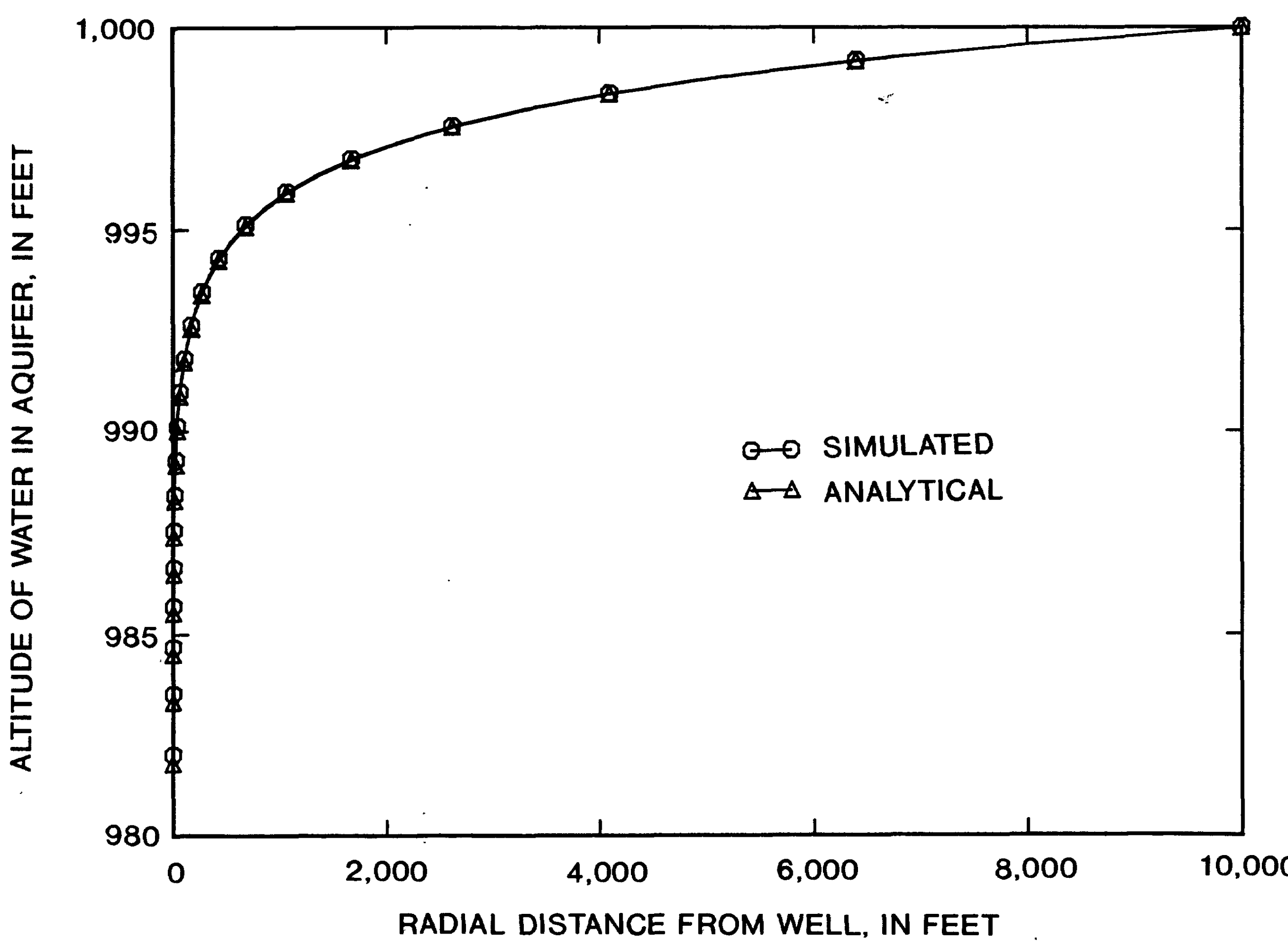


Figure 6.--Water level along the well radius from the simulation and the analytical solution of a pumping well in an isotropic aquifer.

The mesh designed for the second simulation was similar to the wedge for the first simulation, except that it extends around a full circle of radius 20,000 ft. For this simulation, the total flow to the well of radius 0.5 ft was about 300 gal/min, transmissivity in the principal direction was 5,000 ft^2/d and 500 ft^2/d in the minor direction. The angle of anisotropy with respect to the global coordinate system was 30 degrees. The initial head in the aquifer was 1,000 ft. The flow to the well was distributed equally to the nodes at the center of the mesh for the model simulation and a constant head boundary of 1,000 ft was placed at each node on the outside boundary of the mesh. The analytical solution was derived by using the Theim equation with the computation of radial distance from the well computed with the transformation for the radius to account for anisotropy and the transmissivity term in equation 10 modified (Papadopulos, 1965; Glover and Moody, 1974) in equation 11 below:

$$\text{Radius} = \left\{ \frac{TX\, y^2 + TY\, x^2}{TX\, TY} \right\}^{.5} \qquad (11a)$$

$$T = \{(TX)(TY)\}^{.5} \qquad (11b)$$

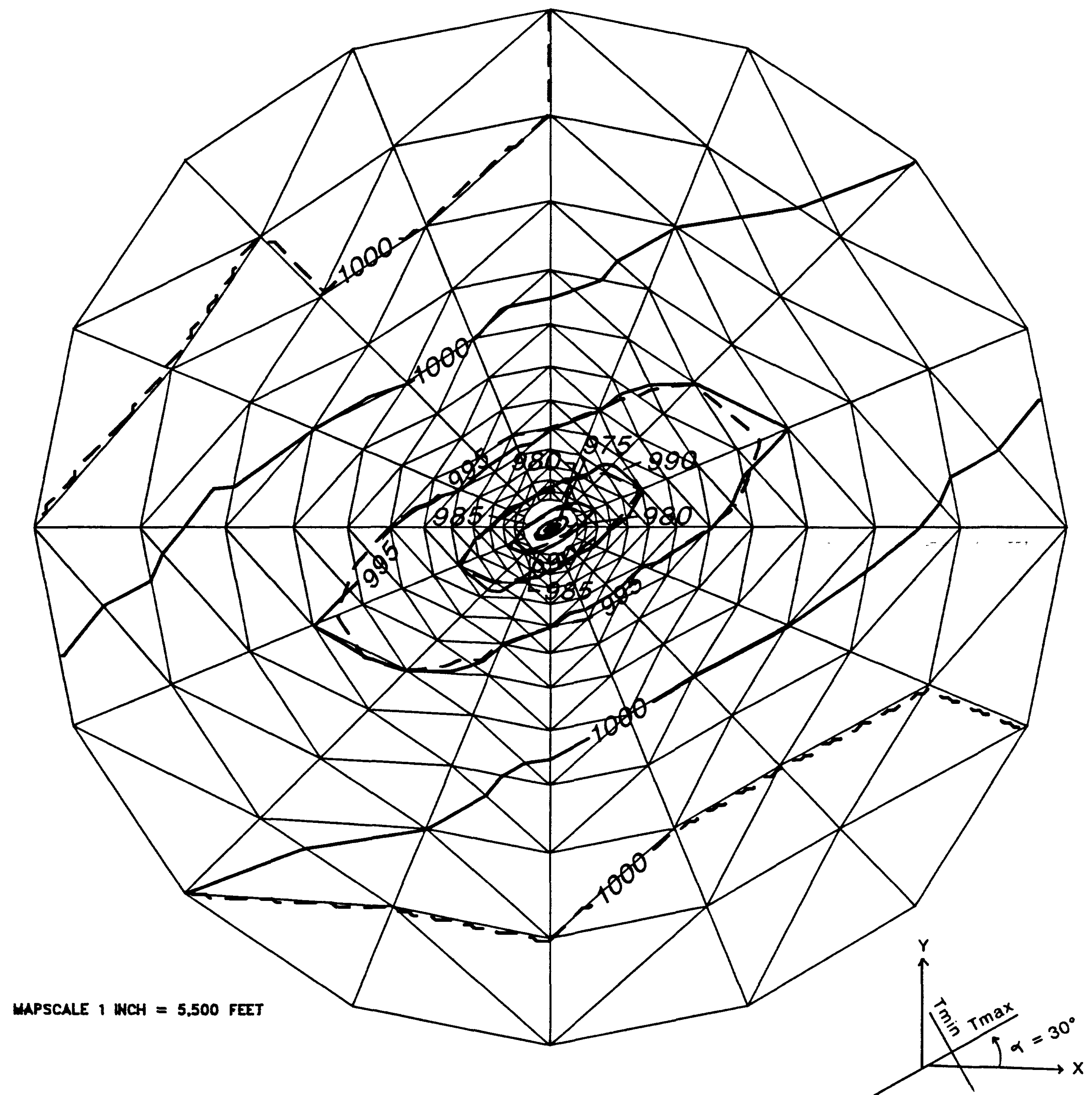
where x,y are the coordinates rotated to the direction of the major and minor axes of anisotropy;
TX is 5,000 ft^2/d; and
TY is 500 ft^2/d.

The Theim equation was solved using the head at the well and the radius at the well in equation 10.

The finite element mesh and the lines of equal head from the simulation and analytical solution are shown in figure 7. The mean absolute error was 2.21 ft. The cause of this amount of error between the two solutions is due to the fact that the analytical solution is not truly a two-dimensional solution. As can seen from equations 11a and b, the radius and transmissivity are transformed to account for the assumption of a transmissivity ellipse (Papadopulos, 1965), but equation 10 is a one-dimensional equation.

The mesh designed for the third test simulation was set up identically to the finite-difference grid for problem 1 in Reilly and others (1987, p. 9-12). The finite-element mesh with lines of equal head from both the finite-difference and finite-element solutions are shown in figure 8. In the problem example there is a square confined aquifer 10,000 ft by 10,000 ft with a transmissivity of 0.0155 ft^2/s. On the northern and southern boundaries are impermeable bedrock. On the eastern boundary is a fully incised lake with a stage of 200 ft. On the western boundary is a fully incised river with a stage of 0 ft. In plan view, as shown in figure 8 there is a pumping well located at 3,500 ft from the eastern boundary and 5,000 ft from the southern boundary pumping at a rate of 3.1 ft^3/s. The published finite-difference solution (Reilly and others, 1987, appendix 3) was to the nearest foot only. The mean absolute error between the two solutions was 0.53 ft. This error was computed after rounding the simulated heads to the nearest foot.

There is good agreement between the Galerkin finite-element simulations and results of both of the analytical solutions and the finite-difference



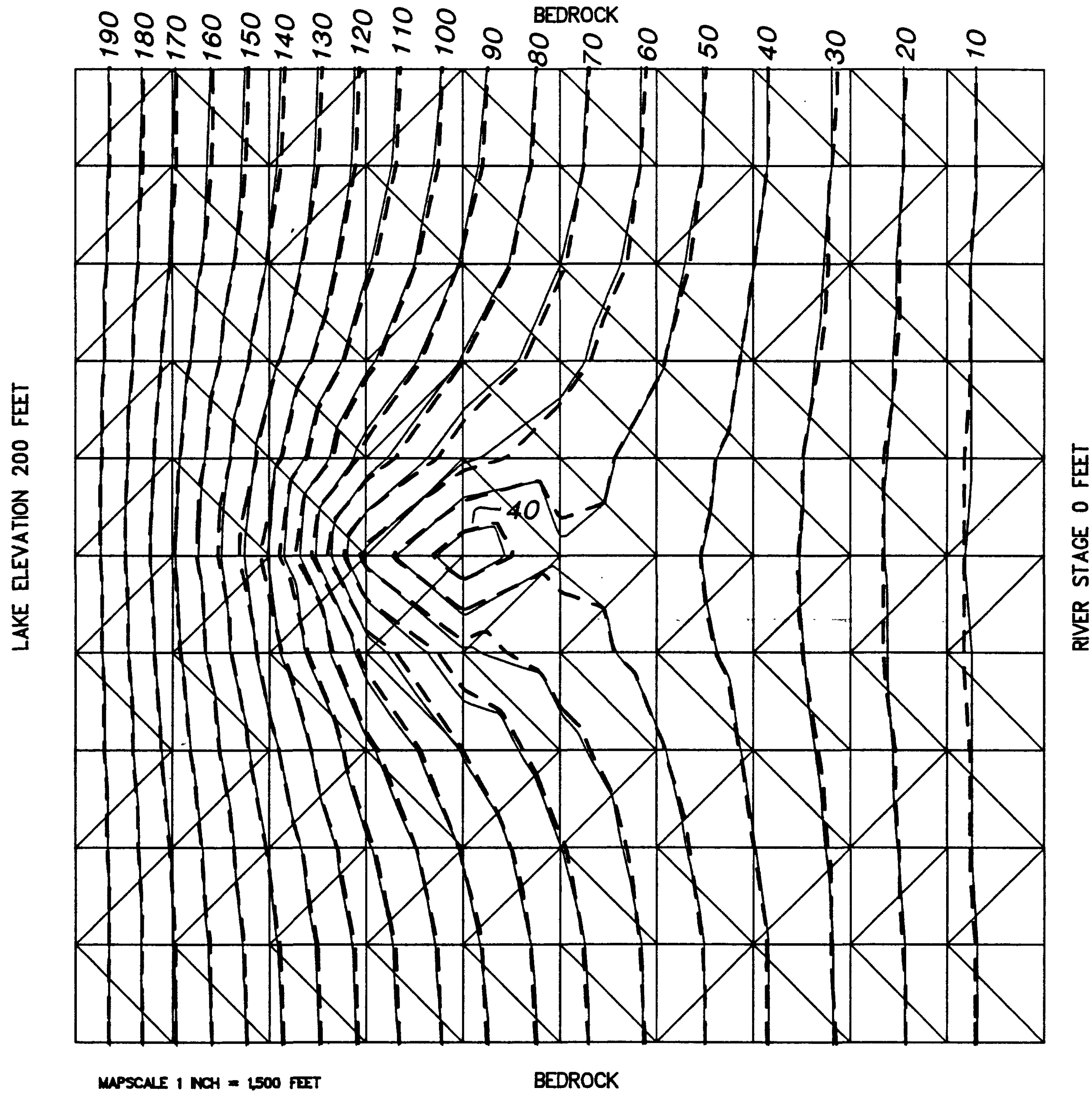
EXPLANATION

- - 995 - — LINE OF EQUAL HEAD--Shows elevation of water levels from finite-element simulation of flow to a well. Contour interval is 5 feet.

— 995 — LINE OF EQUAL HEAD--Shows elevation of water levels from analytical solution of flow to a well. Contour interval is 5 feet.

——— FINITE ELEMENT MESH--Shows how aquifer is represented in model. At a distance 20,000 feet from the well the head is 1000 feet. The well at the center of the mesh is pumped at a rate of 300 gallons per minute. The angle of anisotropy is 30 degrees and transmissivity is 5,000 square feet per day in the principal direction and 500 square feet per day in the minor direction.

Figure 7.--Finite-element mesh and lines of equal head from the simulation and analytical solution of a pumping well in an isotropic aquifer.



EXPLANATION

— — 190 — — LINE OF EQUAL HEAD—Shows elevation of water levels from finite-element simulation of flow to a well. Contour interval is 10 feet.

—— 190 —— LINE OF EQUAL HEAD—Shows elevation of water levels from finite-difference simulation of flow to a well. Contour interval 10 feet.

———— FINITE ELEMENT MESH—Shows how aquifer is represented in model. Transmissivity is .0155 square feet per second. A well is located at x= 3,500 feet and y = 5,000 feet pumping at a rate of 3.1 cubic feet per second. The bedrock boundaries are simulated as no flow. The lake and river boundaries are simulated as constant head.

Figure 8.--Finite-element mesh and lines of equal head from the finite-element and finite difference simulations of a well pumping in an aquifer bounded by a lake and a river.

model. These simulations indicate that the Galerkin finite-element technique was formulated properly and the program will adequately approximate two-dimensional ground-water flow.

MESH DESIGN CONSIDERATIONS

Because the computer program uses Gaussian elimination technique for banded matrices to directly solve the system of equations, care should be used in optimally numbering the nodes of the triangles to minimize the global matrix bandwidth. This is accomplished by minimizing the maximum difference between node numbers of contiguous nodes. If the process of assembling the global matrix from the element matrices is studied, it will be clear that large differences in the node numbers of connected triangles lead to a sparse global matrix, that is, a matrix with many zeroes between numbers in a row. A sparse matrix generally leads to more numerical error because of the increased number of computations. Creating a mesh with a large number of triangles connected at the same node also increases sparseness.

With the finite-element approximation of the ground-water flow equation, the geometry of the individual elements influences the solution process. The element matrices formed within subroutines LOCAL and ELEM, in general, will have positive diagonal terms and negative off-diagonal terms. This is the main reason the Galerkin finite-element approximation is so stable for the ground-water flow equation. Triangles with an obtuse angle have a positive off-diagonal term (Torak and Davis, oral commun., 1987). After checking different triangular shapes, it was determined that isosceles triangles with the larger angle approaching 90 degrees also had a positive off-diagonal term, usually an order of magnitude less than the diagonal term of the element matrix when assuming a transmissivity of 1. If a large group of these triangles are connected, then the off-diagonal term in the global matrix may approach the magnitude of the diagonal term. This could lead to a less numerically stable set of equations.

SUMMARY

The computer program described in this report can be used to solve many steady-state ground-water flow problems that can be properly approximated as two-dimensional flow. The test simulations indicate that the Galerkin finite-element numerical algorithm was properly formulated and the program is accurate. To properly simulate ground-water flow problems, boundary conditions must be incorporated into the model. This program allows for incorporating the three types of boundary conditions.

The computer program, list of selected variables, and examples of data input and output for the three test simulations are included in the Supplemental Data section.

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S U P P L E M E N T A L   D A T A

(PAGE 23 FOLLOWS)

SUPPLEMENTAL DATA

Supplemental Data I. Fortran Program Listing

Main Program

```
C********************************************************************
C    STEADY-STATE 2-DIMENSIONAL 3-NODAL TRIANGULAR ELEMENT
C     GALERKIN FINITE-ELEMENT PROGRAM   by Eve L. Kuniansky
C Program originally developed as a subroutine of a parameter
C estimation scheme May, 1982 and was updated for the
C Prime mini computer 10-28-88.  Last update 4-26-89.
C********************************************************************
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3), B(3), C(3)
      DIMENSION X(5000),Y(5000),T(9000),S(5000,51)
      DIMENSION ICON(9000,3),F(5000),SE(3,3)
      DIMENSION XL(3),YL(3)
      DIMENSION ALPHA(9000),ANIS(9000)
      DIMENSION NDBC(1000),DBC(1000)
      DIMENSION NBCN(1000),BCN(1000)
      DIMENSION NBCM(1000),BCMH(1000),BCMR(1000)
      DIMENSION ITITLE(20), IUNITS(20)
C
      COMMON/NAME/ITITLE,IUNITS
      COMMON/INT/NNODE,NELEM
      COMMON/BC/NDBC,DBC,IDBC,NBCN,BCN,IBCN,NBCM,BCMH,BCMR,IBCM
      COMMON/LCL/XL,YL,A,B,C,AREA,XB,YB
      COMMON/ASEM/SE,S,ICON,IUBW,IB
      COMMON/ANO/ALPHA,ANIS
      COMMON/OUT/X,Y
       CALL INPUT(T)
C
C      INITIALIZE ARRAYS
C
        DO 10 I=1,NNODE
         F(I)=0.
         DO 10 J=1,IUBW
          S(I,J)=0.
  10  CONTINUE
C
C     BEGIN CALLING SUBROUTINES
         PII=3.141592653589793238462643
      DO 30 N=1,NELEM
       ALP=(ALPHA(N)*PII)/180.
       ANI=ANIS(N)
       CALL LOCAL(N,ALP)
        TEL=T(N)
       CALL ELEM(N,TEL,ANI)
       CALL ASEMB(N)
  30  CONTINUE
        CALL BOUND(F)
        CALL REDUCE(F)
        CALL SOLVE(F)
        CALL OUTPUT(F)
      STOP
       END
```

Subroutine INPUT

```
      SUBROUTINE INPUT(T)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION ITITLE(20),IUNITS(20)
      DIMENSION A(3), B(3), C(3)
      DIMENSION X(5000),Y(5000),T(9000),S(5000,51)
      DIMENSION ICON(9000,3),F(5000),SE(3,3)
      DIMENSION XL(3),YL(3)
      DIMENSION ALPHA(9000),ANIS(9000)
      DIMENSION NDBC(1000),DBC(1000)
      DIMENSION NBCN(1000),BCN(1000)
      DIMENSION NBCM(1000),BCMH(1000),BCMR(1000)
      COMMON/NAME/ITITLE,IUNITS
      COMMON/INT/NNODE,NELEM
      COMMON/BC/NDBC,DBC,IDBC,NBCN,BCN,IBCN,NBCM,BCMH,BCMR,IBCM
      COMMON/LCL/XL,YL,A,B,C,AREA,XB,YB
      COMMON/ASEM/SE,S,ICON,IUBW,IB
      COMMON/ANO/ALPHA,ANIS
      COMMON/OUT/X,Y
C
  100 FORMAT(20A4)
  110 FORMAT(5I5)
C     INITIALIZE ARRAYS
C
       DO 120 K=1,1000
        NDBC(K)=0
        NBCN(K)=0
        NBCM(K)=0
        DBC(K)=0.
        BCN(K)=0.
        BCMH(K)=0.
        BCMR(K)=0.
  120 CONTINUE
       DO 130 L=1,9000
        ALPHA(L)=0.
        ANIS(L)=1.
          DO 121 J=1,3
             ICON(L,J) = 0
  121   CONTINUE
        T(L)=0.
  130 CONTINUE
       DO 140 M=1,5000
        X(M)=0.
        Y(M)=0.
  140 CONTINUE
      READ(5,100) (ITITLE(I),I=1,20)
      READ(5,100) (IUNITS(I),I=1,20)
      READ(5,110) NNODE,NELEM,IDBC,IBCN,IBCM
C
       DO 210 J=1,NNODE
        READ(5,*) JJ,X(J),Y(J)
  210 CONTINUE
```

```
C
      DO 220 J=1,NELEM
       READ(5,*) (ICON(J,N),N=1,3),T(J),ALPHA(J),ANIS(J)
  220 CONTINUE
C
C    COMPUTE BAND WIDTH
      ITEST=0
      DO 240 I=1,NELEM
       DO 230 J=1,3
       DO 230 JJ=1,3
        ITEST1=IABS(ICON(I,J)-ICON(I,JJ))
         IF(ITEST1.GT.ITEST) ITEST=ITEST1
  230 CONTINUE
  240 CONTINUE
       IUBW=2*ITEST+1
       IB=(IUBW+1)/2
C
C    READ IN DIRICHLET BC (constant head)
      IF(IDBC .LT. 1) GO TO 260
        DO 250 K=1,IDBC
         READ(5,*) NDBC(K),DBC(K)
  250 CONTINUE
  260  CONTINUE
C    READ IN NEUMAN BC (constant flux)
     IF(IBCN .LT. 1) GO TO 280
       DO 270 K=1,IBCN
        READ(5,*)NBCN(K),BCN(K)
  270 CONTINUE
  280 CONTINUE
C
C    READ IN CAUCHY OR MIXED BC (head dependent flux)
      IF(IBCM .LT. 1) GO TO 291
       DO 290 K=1,IBCM
        READ(5,*)NBCM(K),BCMH(K),BCMR(K)
  290 CONTINUE
  291  CONTINUE
     IF(IUBW.LT.52) GO TO 292
      WRITE(6,*) 'ERROR...BAND WIDTH TOO LARGE'
     STOP
  292 CONTINUE
      RETURN
       END
```

Subroutine LOCAL

```
      SUBROUTINE LOCAL(NEL,ALPHA)
        IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3), B(3), C(3)
      DIMENSION X(5000),Y(5000),S(5000,51)
      DIMENSION ICON(9000,3),SE(3,3)
      DIMENSION XL(3),YL(3)
      COMMON/LCL/XL,YL,A,B,C,AREA,XB,YB
      COMMON/ASEM/SE,S,ICON,IUBW,IB
      COMMON/OUT/X,Y
C
C           COMPUTING CENTROID OF TRIANGLE
       XB=(X(ICON(NEL,1))+X(ICON(NEL,2))+X(ICON(NEL,3)))/3.
       YB=(Y(ICON(NEL,1))+Y(ICON(NEL,2))+Y(ICON(NEL,3)))/3.
C
C     COMPUTE LOCAL COORDINATES
        DO 300 L=1,3
      XL(L)=(X(ICON(NEL,L))-XB)*COS(ALPHA)+(Y(ICON(NEL,L))-YB)*
     &SIN(ALPHA)
      YL(L)=(Y(ICON(NEL,L))-YB)*COS(ALPHA)-(X(ICON(NEL,L))-XB)*
     &SIN(ALPHA)
  300 CONTINUE
C      CALCULATION OF AREA OF TRIANGLE
        O=XL(2)*YL(3)-XL(3)*YL(2)
        P=XL(1)*YL(3)-XL(3)*YL(1)
        U=XL(1)*YL(2)-XL(2)*YL(1)
       AREA=ABS(O-P+U)/2.
C
C      CALCULATION OF COEFFICIENTS USED IN STIFFNESS MATRIX
         A(1)=O
         A(2)=-P
         A(3)=U
         B(1)=YL(2)-YL(3)
         B(2)=YL(3)-YL(1)
         B(3)=YL(1)-YL(2)
         C(1)=XL(3)-XL(2)
         C(2)=XL(1)-XL(3)
         C(3)=XL(2)-XL(1)
      RETURN
      END
```

Subroutine ELEM

```
      SUBROUTINE ELEM(NEL,TEL,ANIS)
       IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(3), B(3), C(3)
      DIMENSION ICON(9000,3),SE(3,3),S(5000,51)
      DIMENSION XL(3),YL(3)
      COMMON/LCL/XL,YL,A,B,C,AREA,XB,YB
      COMMON/ASEM/SE,S,ICON,IUBW,IB
C
C      CALCULATE TX AND TY
        TX=TEL
        TY=TEL/ANIS
C
      EX=TX/(4.*AREA)
      EY=TY/(4.*AREA)
C
C     COMPUTE ELEMENT STIFFNESS MATRIX TERMS
      SE(1,1)=EX*(B(1)**2)+EY*(C(1)**2)
      SE(2,2)=EX*(B(2)**2)+EY*(C(2)**2)
      SE(3,3)=EX*(B(3)**2)+EY*(C(3)**2)
      SE(1,2)=EX*(B(1)*B(2))+EY*(C(1)*C(2))
      SE(1,3)=EX*(B(1)*B(3))+EY*(C(1)*C(3))
      SE(2,3)=EX*(B(2)*B(3))+EY*(C(2)*C(3))
      SE(2,1)=SE(1,2)
      SE(3,1)=SE(1,3)
      SE(3,2)=SE(2,3)
       RETURN
       END
```

Subroutine ASEMB

```
      SUBROUTINE ASEMB(NEL)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION SE(3,3),S(5000,51),ICON(9000,3)
      COMMON/ASEM/SE,S,ICON,IUBW,IB
C
       DO 400 JJ=1,3
        DO 400 J=1,3
         NP=ICON(NEL,JJ)
         KK=ICON(NEL,J)
         LL=KK-NP
      S(NP,IB+LL)=S(NP,IB+LL)+SE(JJ,J)
  400 CONTINUE
      RETURN
      END
```

Subroutine BOUND

```
      SUBROUTINE BOUND(F)
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION S(5000,51)
      DIMENSION ICON(9000,3),F(5000),SE(3,3)
      DIMENSION NDBC(1000),DBC(1000)
      DIMENSION NBCN(1000),BCN(1000)
      DIMENSION NBCM(1000),BCMH(1000),BCMR(1000)
      COMMON/BC/NDBC,DBC,IDBC,NBCN,BCN,IBCN,NBCM,BCMH,BCMR,IBCM
      COMMON/ASEM/SE,S,ICON,IUBW,IB
C
C    INPUT SPECIFIED FLUX AT NODE -DISCHARGE +RECHARGE-NEUMAN BC
      IF(IBCN .LT. 1) GO TO 510
       DO 500 K=1,IBCN
         F(NBCN(K))=F(NBCN(K))+BCN(K)
  500 CONTINUE
  510 CONTINUE
C
C     INPUT CONSTANT HEAD USING NUMERICAL TRICK-DIRICHLET BC
      IF(IDBC .LT. 1) GO TO 530
      DO 520 J=1,IDBC
       F(NDBC(J))=F(NDBC(J))+S(NDBC(J),IB)*DBC(J)*10.E9
       S(NDBC(J),IB)=S(NDBC(J),IB)*10.E9
  520 CONTINUE
  530 CONTINUE
C    INPUT HEAD DEPENDENT FLUX-CAUCHY OR MIXED BC
      IF(IBCM .LT. 1) GO TO 550
       DO 540 L=1,IBCM
        F(NBCM(L))= F(NBCM(L))+BCMH(L)*BCMR(L)
        S(NBCM(L),IB)=S(NBCM(L),IB)+BCMR(L)
  540 CONTINUE
  550 CONTINUE
      RETURN
      END
```

Subroutine REDUCE

```
      SUBROUTINE REDUCE(F)
       IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION S(5000,51)
      DIMENSION ICON(9000,3),F(5000),SE(3,3)
      COMMON/INT/NNODE,NELEM
      COMMON/ASEM/SE,S,ICON,IUBW,IB
C
      IB1=IB+1
       DO 33 N=1,NNODE
        LL=IB
       DO 22 L=IB1,IUBW
       I=N+L-IB
        IF(I.GT.NNODE) GO TO 33
        LL=LL-1
        IF(S(I,LL) .EQ. 0) GO TO 22
       C=S(I,LL)/S(N,IB)
        J=LL
       DO 11 K=IB1,IUBW
       J=J+1
   11 S(I,J)=S(I,J)-C*S(N,K)
       F(I)=F(I)-C*F(N)
   22 CONTINUE
   33 CONTINUE
      RETURN
      END
```

Subroutine SOLVE

```
      SUBROUTINE SOLVE(F)
       IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION S(5000,51)
      DIMENSION ICON(9000,3),F(5000),SE(3,3)
      COMMON/INT/NNODE,NELEM
      COMMON/ASEM/SE,S,ICON,IUBW,IB
C
      IB1=IB+1
      F(NNODE)=F(NNODE)/S(NNODE,IB)
       DO 55 M=2,NNODE
         N=NNODE+1-M
       DO 44 L=IB1,IUBW
        IF(S(N,L) .EQ. 0) GO TO 44
       K=N+L-IB
        IF(K.GT.NNODE) GO TO 55
       F(N)=F(N)-S(N,L)*F(K)
   44 CONTINUE
        F(N)=F(N)/S(N,IB)
   55 CONTINUE
      RETURN
      END
```

Subroutine OUTPUT

```
      SUBROUTINE OUTPUT(F)
       IMPLICIT REAL*8 (A-H,O-Z)
       DIMENSION ITITLE(20),IUNITS(20)
       DIMENSION F(5000),Y(5000),X(5000)
       COMMON/OUT/ X,Y
       COMMON/NAME/ITITLE,IUNITS
       COMMON/INT/NNODE,NELEM
C
      WRITE(6,10) (ITITLE(I),I=1,20)
      WRITE(6,10) (IUNITS(I),I=1,20)
   10 FORMAT(20A4)
   20 FORMAT(2E15.7,F10.2)
      DO 100 I=1,NNODE
       WRITE(6,20) X(I),Y(I),F(I)
  100 CONTINUE
      RETURN
      END
```

Supplemental Data II. Definition of Variables within the Common Blocks

ITITLE --alphanumeric, 80 characters for title of problem

IUNITS --alphanumeric, 80 characters for units of problem

NNODE --integer, total number of nodes in mesh

NELEM --integer, total number of triangular elements

IDBC --integer, number of constant head nodes

NDBC --integer vector array, contains node number for constant head

DBC --real vector array, contains values of constant head

IBCN --integer, total number of nodes with constant flux

NBCN --integer vector array, contains node number for constant flux

BCN --real vector array, contains values of constant flux

IBCM --integer, total number of nodes with head-dependent flux

NBCM --integer vector array, contains node number for head-dependent flux

BCMH --real vector array, contains values of known head outside of system

BCMR --real vector array, contains values of conductance term

XL --real vector array, local nodal x-coordinates

YL --real vector array, local nodal y-coordinates

A --real vector array, geometric coefficient for approximation

B --real vector array, geometric coefficient for approximation

C --real vector array, geometric coefficient for approximation

AREA --real, area of triangular element

XB --real, x-coordinate of centroid of triangular element

YB --real, y-coordinate of centroid of triangular element

SE --real 3X3 array, element stiffness matrix

S --real rectangular array, global stiffness matrix

ICON --integer rectangular array, element connection matrix

IUBW --integer, computed in INPUT, the band width of the global matrix

IB	--integer, upper band width of the global matrix
ALPHA	--real vector array, angle in degrees of anisotropy for each element
ANIS	--real vector array, ratio of maximum to minimum transmissivity
TEL	--real vector array, transmissivity in major direction
X	--real vector array, global x-coordinate for each node
Y	--real vector array, global y-coordinate for each node

Supplemental Data III. Data Input Examples for Test Problems

Example 1.

Data input for the first test problem: radial flow to a well in a confined aquifer.

```
pumping well h=1000, rw=.5, T=5000 ft2/d, Q=300 gpm        (ITITLE)
units feet and day                                          (IUNITS)
 123  160    3     3    0          (NNODE,NELEM,IDBC,IBCN,IBCM)
   1      0.433     0.250          (node number, X(n),Y(n))
   2      0.483     0.129
   3      0.500     0.000
   4      0.687     0.396
   5      0.766     0.205
   6      0.793     0.000
   7      1.004     0.579
   8      1.119     0.300
   9      1.159     0.000
  10      1.399     0.808
  11      1.560     0.418
  12      1.615     0.000
  13      1.906     1.100
  14      2.126     0.570
  15      2.201     0.000
  16      2.539     1.466
  17      2.833     0.759
  18      2.932     0.000
  19      3.331     1.924
  20      3.716     0.996
  21      3.847     0.000
  22      4.346     2.509
  23      4.847     1.299
  24      5.018     0.000
  25      5.613     3.241
  26      6.261     1.678
  27      6.481     0.000
  28      7.198     4.155
  29      8.028     2.151
  30      8.311     0.000
  31      9.225     5.326
  32     10.290     2.757
  33     10.652     0.000
  34     11.760     6.790
  35     13.117     3.515
  36     13.580     0.000
  37     14.928     8.619
  38     16.652     4.462
  39     17.238     0.000
  40     18.889    10.906
  41     21.070     5.645
  42     21.812     0.000
  43     23.959    13.833
  44     26.724     7.161
  45     27.666     0.000
```

46	30.297	17.492
47	33.793	9.054
48	34.984	0.000
49	38.224	22.065
50	42.629	11.422
51	44.131	0.000
52	48.358	27.919
53	53.939	14.452
54	55.839	0.000
55	61.032	35.237
56	68.076	18.240
57	70.474	0.000
58	76.875	44.384
59	85.747	22.975
60	88.768	0.000
61	96.680	55.818
62	107.837	28.894
63	111.636	0.000
64	121.522	70.161
65	135.546	36.318
66	140.321	0.000
67	152.575	88.089
68	170.182	45.598
69	176.178	0.000
70	191.390	110.499
71	213.477	57.199
72	220.998	0.000
73	240.060	138.599
74	267.764	71.744
75	277.198	0.000
76	300.898	173.723
77	335.622	89.926
78	347.447	0.000
79	376.945	217.629
80	420.446	112.654
81	435.259	0.000
82	472.004	272.512
83	526.474	141.063
84	545.023	0.000
85	593.362	342.578
86	661.838	177.332
87	685.156	0.000
88	741.580	428.341
89	827.525	221.726
90	856.681	0.000
91	927.286	535.544
92	1034.633	277.219
93	1071.088	0.000
94	1159.512	669.603
95	1293.625	346.613
96	1339.207	0.000
97	1449.794	837.177
98	1617.363	433.356
99	1674.355	0.000
100	1812.643	1046.645

101	2022.034	541.785
102	2093.290	0.000
103	2266.438	1308.616
104	2528.137	677.392
105	2617.233	0.000
106	2833.674	1636.081
107	3160.761	846.901
108	3272.161	0.000
109	3543.502	2045.869
110	3952.420	1059.023
111	4091.737	0.000
112	4430.770	2558.103
113	4941.985	1324.177
114	5116.207	0.000
115	5539.830	3198.397
116	6178.929	1655.618
117	6396.794	0.000
118	6926.431	3998.947
119	7725.443	2070.015
120	7997.894	0.000
121	8660.254	5000.000
122	9659.260	2588.200
123	10000.000	0.000

1	2	4	5000.0	.0	1.0	
4	2	5	5000.0	.0	1.0	
2	3	6	5000.0	.0	1.0	(ICON(i,j),j=1,3,TEL,ALPHA,ANIS)
2	6	5	5000.0	.0	1.0	
4	5	7	5000.0	.0	1.0	
7	5	8	5000.0	.0	1.0	
5	6	9	5000.0	.0	1.0	
5	9	8	5000.0	.0	1.0	
7	8	10	5000.0	.0	1.0	
10	8	11	5000.0	.0	1.0	
8	9	12	5000.0	.0	1.0	
8	12	11	5000.0	.0	1.0	
10	11	13	5000.0	.0	1.0	
13	11	14	5000.0	.0	1.0	
11	12	15	5000.0	.0	1.0	
11	15	14	5000.0	.0	1.0	
13	14	16	5000.0	.0	1.0	
16	14	17	5000.0	.0	1.0	
14	15	18	5000.0	.0	1.0	
14	18	17	5000.0	.0	1.0	
16	17	19	5000.0	.0	1.0	
19	17	20	5000.0	.0	1.0	
17	18	21	5000.0	.0	1.0	
17	21	20	5000.0	.0	1.0	
19	20	22	5000.0	.0	1.0	
22	20	23	5000.0	.0	1.0	
20	21	24	5000.0	.0	1.0	
20	24	23	5000.0	.0	1.0	
22	23	25	5000.0	.0	1.0	
25	23	26	5000.0	.0	1.0	
23	24	27	5000.0	.0	1.0	
23	27	26	5000.0	.0	1.0	

25	26	28	5000.0	.0	1.0
28	26	29	5000.0	.0	1.0
26	27	30	5000.0	.0	1.0
26	30	29	5000.0	.0	1.0
28	29	31	5000.0	.0	1.0
31	29	32	5000.0	.0	1.0
29	30	33	5000.0	.0	1.0
29	33	32	5000.0	.0	1.0
31	32	34	5000.0	.0	1.0
34	32	35	5000.0	.0	1.0
32	33	36	5000.0	.0	1.0
32	36	35	5000.0	.0	1.0
34	35	37	5000.0	.0	1.0
37	35	38	5000.0	.0	1.0
35	36	39	5000.0	.0	1.0
35	39	38	5000.0	.0	1.0
37	38	40	5000.0	.0	1.0
40	38	41	5000.0	.0	1.0
38	39	42	5000.0	.0	1.0
38	42	41	5000.0	.0	1.0
40	41	43	5000.0	.0	1.0
43	41	44	5000.0	.0	1.0
41	42	45	5000.0	.0	1.0
41	45	44	5000.0	.0	1.0
43	44	46	5000.0	.0	1.0
46	44	47	5000.0	.0	1.0
44	45	48	5000.0	.0	1.0
44	48	47	5000.0	.0	1.0
46	47	49	5000.0	.0	1.0
49	47	50	5000.0	.0	1.0
47	48	51	5000.0	.0	1.0
47	51	50	5000.0	.0	1.0
49	50	52	5000.0	.0	1.0
52	50	53	5000.0	.0	1.0
50	51	54	5000.0	.0	1.0
50	54	53	5000.0	.0	1.0
52	53	55	5000.0	.0	1.0
55	53	56	5000.0	.0	1.0
53	54	57	5000.0	.0	1.0
53	57	56	5000.0	.0	1.0
55	56	58	5000.0	.0	1.0
58	56	59	5000.0	.0	1.0
56	57	60	5000.0	.0	1.0
56	60	59	5000.0	.0	1.0
58	59	61	5000.0	.0	1.0
61	59	62	5000.0	.0	1.0
59	60	63	5000.0	.0	1.0
59	63	62	5000.0	.0	1.0
61	62	64	5000.0	.0	1.0
64	62	65	5000.0	.0	1.0
62	63	66	5000.0	.0	1.0
62	66	65	5000.0	.0	1.0
64	65	67	5000.0	.0	1.0
67	65	68	5000.0	.0	1.0
65	66	69	5000.0	.0	1.0

65	69	68	5000.0	.0	1.0
67	68	70	5000.0	.0	1.0
70	68	71	5000.0	.0	1.0
68	69	72	5000.0	.0	1.0
68	72	71	5000.0	.0	1.0
70	71	73	5000.0	.0	1.0
73	71	74	5000.0	.0	1.0
71	72	75	5000.0	.0	1.0
71	75	74	5000.0	.0	1.0
73	74	76	5000.0	.0	1.0
76	74	77	5000.0	.0	1.0
74	75	78	5000.0	.0	1.0
74	78	77	5000.0	.0	1.0
76	77	79	5000.0	.0	1.0
79	77	80	5000.0	.0	1.0
77	78	81	5000.0	.0	1.0
77	81	80	5000.0	.0	1.0
79	80	82	5000.0	.0	1.0
82	80	83	5000.0	.0	1.0
80	81	84	5000.0	.0	1.0
80	84	83	5000.0	.0	1.0
82	83	85	5000.0	.0	1.0
85	83	86	5000.0	.0	1.0
83	84	87	5000.0	.0	1.0
83	87	86	5000.0	.0	1.0
85	86	88	5000.0	.0	1.0
88	86	89	5000.0	.0	1.0
86	87	90	5000.0	.0	1.0
86	90	89	5000.0	.0	1.0
88	89	91	5000.0	.0	1.0
91	89	92	5000.0	.0	1.0
89	90	93	5000.0	.0	1.0
89	93	92	5000.0	.0	1.0
91	92	94	5000.0	.0	1.0
94	92	95	5000.0	.0	1.0
92	93	96	5000.0	.0	1.0
92	96	95	5000.0	.0	1.0
94	95	97	5000.0	.0	1.0
97	95	98	5000.0	.0	1.0
95	96	99	5000.0	.0	1.0
95	99	98	5000.0	.0	1.0
97	98	100	5000.0	.0	1.0
100	98	101	5000.0	.0	1.0
98	99	102	5000.0	.0	1.0
98	102	101	5000.0	.0	1.0
100	101	103	5000.0	.0	1.0
103	101	104	5000.0	.0	1.0
101	102	105	5000.0	.0	1.0
101	105	104	5000.0	.0	1.0
103	104	106	5000.0	.0	1.0
106	104	107	5000.0	.0	1.0
104	105	108	5000.0	.0	1.0
104	108	107	5000.0	.0	1.0
106	107	109	5000.0	.0	1.0
109	107	110	5000.0	.0	1.0

```
  107  108  111     5000.0          .0         1.0
  107  111  110     5000.0          .0         1.0
  109  110  112     5000.0          .0         1.0
  112  110  113     5000.0          .0         1.0
  110  111  114     5000.0          .0         1.0
  110  114  113     5000.0          .0         1.0
  112  113  115     5000.0          .0         1.0
  115  113  116     5000.0          .0         1.0
  113  114  117     5000.0          .0         1.0
  113  117  116     5000.0          .0         1.0
  115  116  118     5000.0          .0         1.0
  118  116  119     5000.0          .0         1.0
  116  117  120     5000.0          .0         1.0
  116  120  119     5000.0          .0         1.0
  118  119  122     5000.0          .0         1.0
  118  122  121     5000.0          .0         1.0
  119  120  123     5000.0          .0         1.0
  119  123  122     5000.0          .0         1.0
121 1000.                                          (NDBC(n),DBC(n))
122 1000.
123 1000.
  1 -1605.7                                        (NBCN(n),BCN(n))
  2 -1605.7
  3 -1605.7
```

Example 2.

Data input for flow to a well in an anistropic aquifer.

```
RADIAL FLOW TEST2 Q=57805FT3/D TMAX=5000 TMIN 500 FT2/D AL=30 AN=10 (ITITLE)
UNITS FEET DAY                                                      (IUNITS)
  336  640   16   16    0                        (NNODE,NELEM,IDBC,IBCN,IBCM)
    1 20000.462 20000.191                        (node number,X(n),Y(n))
    2 20000.191 20000.462
    3 19999.809 20000.462
    4 19999.538 20000.191
    5 19999.538 19999.809
    6 19999.809 19999.538
    7 20000.191 19999.538
    8 20000.462 19999.809
    9 20000.500 20000.000
   10 20000.000 20000.500
   11 19999.500 20000.000
   12 20000.000 19999.500
   13 20000.354 20000.354
   14 19999.646 20000.354
   15 19999.646 19999.646
   16 20000.354 19999.646
   17 20049.126 20020.348
   18 20020.348 20049.126
   19 19979.652 20049.126
   20 19950.874 20020.348
   21 19950.874 19979.652
   22 19979.652 19950.874
   23 20020.348 19950.874
   24 20049.126 19979.652
   25 20053.174 20000.000
   26 20000.000 20053.174
   27 19946.826 20000.000
   28 20000.000 19946.826
   29 20037.600 20037.600
   30 19962.400 20037.600
   31 19962.400 19962.400
   32 20037.600 19961.083
   33 20119.016 20000.000
   34 20000.000 20119.016
   35 19880.984 20000.000
   36 20000.000 19880.984
   37 20109.957 20045.545
   38 20045.545 20109.957
   39 19954.455 20109.957
   40 19890.043 20045.545
   41 19890.043 19954.455
   42 19954.455 19890.043
   43 20045.545 19890.043
   44 20109.957 19954.455
   45 20084.158 20084.158
   46 19915.842 20084.158
   47 19915.842 19915.842
   48 20084.158 19912.879
```

49	20185.873	20076.991
50	20076.991	20185.873
51	19923.009	20185.873
52	19814.127	20076.991
53	19814.127	19923.009
54	19923.009	19814.127
55	20076.991	19814.127
56	20185.873	19923.009
57	20201.188	20000.000
58	20000.000	20201.188
59	19798.812	20000.000
60	20000.000	19798.812
61	20142.262	20142.262
62	19857.738	20142.262
63	19857.738	19857.738
64	20142.262	19852.721
65	20283.202	20117.306
66	20117.306	20283.202
67	19882.694	20283.202
68	19716.798	20117.306
69	19716.798	19882.694
70	19882.694	19716.798
71	20117.306	19716.798
72	20283.202	19882.694
73	20306.536	20000.000
74	20000.000	20306.536
75	19693.464	20000.000
76	20000.000	19693.464
77	20216.754	20216.754
78	19783.246	20216.754
79	19783.246	19783.246
80	20216.754	19775.595
81	20438.220	20000.000
82	20000.000	20438.220
83	19561.780	20000.000
84	20000.000	19561.780
85	20404.863	20167.699
86	20167.699	20404.863
87	19832.301	20404.863
88	19595.137	20167.699
89	19595.137	19832.301
90	19832.301	19595.137
91	20167.699	19595.137
92	20404.863	19832.301
93	20309.869	20309.869
94	19690.131	20309.869
95	19690.131	19690.131
96	20309.869	19679.188
97	20602.826	20000.000
98	20000.000	20602.826
99	19397.174	20000.000
100	20000.000	19397.174
101	20556.939	20230.691
102	20230.691	20556.939
103	19769.309	20556.939

104	19443.061	20230.691
105	19443.061	19769.309
106	19769.309	19443.061
107	20230.691	19443.061
108	20556.939	19769.309
109	20426.263	20426.263
110	19573.737	20426.263
111	19573.737	19573.737
112	20426.263	19558.678
113	20751.596	20311.321
114	20311.321	20751.596
115	19688.679	20751.596
116	19248.404	20311.321
117	19248.404	19688.679
118	20751.596	19688.679
119	20813.522	20000.000
120	20000.000	20813.522
121	19186.478	20000.000
122	20000.000	19186.478
123	20575.247	20575.247
124	19424.753	20575.247
125	19424.753	19424.753
126	19688.679	19248.403
127	20311.321	19248.403
128	20575.247	19404.427
129	20994.918	20412.108
130	20412.108	20994.918
131	19587.892	20994.918
132	19005.082	20412.108
133	19005.082	19587.892
134	20994.918	19587.892
135	21076.892	20000.000
136	20000.000	21076.892
137	18923.108	20000.000
138	20000.000	18923.108
139	19587.892	19005.081
140	20412.108	19005.081
141	20761.478	20761.478
142	19238.522	20761.478
143	19238.522	19238.522
144	20761.478	19211.612
145	21406.104	20000.000
146	20000.000	21406.104
147	18593.896	20000.000
148	20000.000	18593.896
149	21299.071	20538.092
150	20538.092	21299.071
151	19461.908	21299.071
152	18700.929	20538.092
153	18700.929	19461.908
154	19461.908	18700.929
155	20538.092	18700.929
156	21299.071	19461.908
157	20994.266	20994.266
158	19005.734	20994.266

159	19005.734	19005.734
160	20994.266	18970.593
161	21688.385	20699.352
162	20699.352	21688.385
163	19300.648	21688.385
164	18311.615	20699.352
165	18311.615	19300.648
166	21688.385	19300.648
167	21827.495	20000.000
168	20000.000	21827.495
169	18172.505	20000.000
170	20000.000	18172.505
171	19300.648	18311.614
172	20699.352	18311.614
173	21292.235	21292.235
174	18707.765	21292.235
175	18707.765	18707.765
176	21292.235	18662.089
177	22354.234	20000.000
178	20000.000	22354.234
179	17645.766	20000.000
180	20000.000	17645.766
181	22175.029	20900.926
182	20900.926	22175.029
183	19099.074	22175.029
184	17824.971	20900.926
185	17824.971	19099.074
186	22175.029	19099.074
187	19099.074	17824.970
188	20900.926	17824.970
189	21664.696	21664.696
190	18335.304	21664.696
191	18335.304	18335.304
192	21664.696	18276.460
193	22783.334	21152.894
194	21152.894	22783.334
195	18847.106	22783.334
196	17216.666	21152.894
197	17216.666	18847.106
198	22783.334	18847.106
199	23012.659	20000.000
200	20000.000	23012.659
201	16987.341	20000.000
202	20000.000	16987.341
203	22130.272	22130.272
204	17869.728	22130.272
205	17869.728	17869.728
206	18847.106	17216.665
207	21152.894	17216.665
208	22130.272	17794.422
209	23543.714	21467.854
210	21467.854	23543.714
211	18532.146	23543.714
212	16456.286	21467.854
213	16456.286	18532.146

214	23543.714	18532.146
215	23835.689	20000.000
216	20000.000	23835.689
217	16164.311	20000.000
218	20000.000	16164.311
219	22712.242	22712.242
220	17287.758	22712.242
221	17287.758	17287.758
222	18532.146	16456.284
223	21467.854	16456.284
224	22712.242	17191.876
225	24889.167	20000.000
226	20000.000	24889.167
227	15110.833	20000.000
228	20000.000	15110.833
229	24517.002	21871.003
230	21871.003	24517.002
231	18128.997	24517.002
232	15482.998	21871.003
233	15482.998	18128.997
234	24517.002	18128.997
235	23457.164	23457.164
236	16542.836	23457.164
237	16542.836	16542.836
238	18128.997	15482.997
239	21871.003	15482.997
240	23457.164	16420.617
241	26206.015	20000.000
242	20000.000	26206.015
243	13793.985	20000.000
244	20000.000	13793.985
245	25733.611	22374.939
246	22374.939	25733.611
247	17625.061	25733.611
248	14266.389	22374.939
249	14266.389	17625.061
250	25733.611	17625.061
251	24388.316	24388.316
252	15611.684	24388.316
253	15611.684	15611.684
254	17625.061	14266.387
255	22374.939	14266.387
256	24388.316	15456.542
257	27254.372	23004.859
258	23004.859	27254.372
259	16995.141	27254.372
260	12745.628	23004.859
261	12745.628	16995.141
262	27254.372	16995.141
263	25552.256	25552.256
264	14447.744	25552.256
265	14447.744	14447.744
266	27852.076	20000.000
267	20000.000	27852.076
268	12147.924	20000.000

269	20000.000	12147.924
270	16995.141	12745.625
271	23004.859	12745.625
272	25552.256	14251.449
273	29959.033	20000.000
274	20000.000	29959.033
275	10040.967	20000.000
276	20000.000	10040.967
277	29200.947	23811.157
278	23811.157	29200.947
279	16188.843	29200.947
280	10799.053	23811.157
281	10799.053	16188.843
282	29200.947	16188.843
283	27042.100	27042.100
284	12957.900	27042.100
285	12957.900	12957.900
286	16188.843	10799.050
287	23811.157	10799.050
288	27042.100	12708.931
289	28904.404	28904.404
290	11095.596	28904.404
291	11095.596	11095.596
292	32592.729	20000.000
293	20000.000	32592.729
294	7407.271	20000.000
295	20000.000	7407.271
296	31634.165	24819.029
297	24819.029	31634.165
298	15180.971	31634.165
299	8365.835	24819.029
300	8365.835	15180.971
301	31634.165	15180.971
302	15180.971	8365.831
303	24819.029	8365.831
304	28904.404	10780.782
305	35884.849	20000.000
306	20000.000	35884.849
307	4115.151	20000.000
308	20000.000	4115.151
309	31232.285	31232.285
310	8767.715	31232.285
311	8767.715	8767.715
312	34675.688	26078.869
313	26078.869	34675.688
314	13921.131	34675.688
315	5324.312	26078.869
316	5324.312	13921.131
317	34675.688	13921.131
318	13921.131	5324.307
319	26078.869	5324.307
320	31232.285	8370.596
321	40000.000	20000.000
322	20000.000	40000.000
323	.000	20000.000

```
324 20000.000       .000
325 38477.591 27653.669
326 27653.669 38477.591
327 12346.331 38477.591
328  1522.409 27653.669
329  1522.409 12346.331
330 38477.591 12346.331
331 34142.136 34142.136
332  5857.864 34142.136
333  5857.864  5857.864
334 12346.331  1522.402
335 27653.669  1522.402
336 34142.136  5357.864
  9   25   17    5000.0        30.0        10.0 (ICON(i,j),j=1,3,TEL,ALPHA,ANIS)
  9   17    1    5000.0        30.0        10.0
 25   33   37    5000.0        30.0        10.0
 25   37   17    5000.0        30.0        10.0
 33   57   49    5000.0        30.0        10.0
 33   49   37    5000.0        30.0        10.0
 57   73   65    5000.0        30.0        10.0
 57   65   49    5000.0        30.0        10.0
 73   81   65    5000.0        30.0        10.0
 65   81   85    5000.0        30.0        10.0
 81   97   85    5000.0        30.0        10.0
 85   97  101    5000.0        30.0        10.0
 97  119  101    5000.0        30.0        10.0
101  119  113    5000.0        30.0        10.0
119  135  113    5000.0        30.0        10.0
113  135  129    5000.0        30.0        10.0
135  145  129    5000.0        30.0        10.0
129  145  149    5000.0        30.0        10.0
145  167  149    5000.0        30.0        10.0
149  167  161    5000.0        30.0        10.0
167  177  161    5000.0        30.0        10.0
161  177  181    5000.0        30.0        10.0
177  199  181    5000.0        30.0        10.0
181  199  193    5000.0        30.0        10.0
199  215  193    5000.0        30.0        10.0
193  215  209    5000.0        30.0        10.0
215  225  209    5000.0        30.0        10.0
209  225  229    5000.0        30.0        10.0
225  241  229    5000.0        30.0        10.0
229  241  245    5000.0        30.0        10.0
241  266  245    5000.0        30.0        10.0
245  266  257    5000.0        30.0        10.0
266  273  257    5000.0        30.0        10.0
257  273  277    5000.0        30.0        10.0
273  292  277    5000.0        30.0        10.0
277  292  296    5000.0        30.0        10.0
292  305  296    5000.0        30.0        10.0
296  305  312    5000.0        30.0        10.0
305  321  312    5000.0        30.0        10.0
312  321  325    5000.0        30.0        10.0
  1   17   13    5000.0        30.0        10.0
 13   17   29    5000.0        30.0        10.0
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17	37	29	5000.0	30.0	10.0
29	37	45	5000.0	30.0	10.0
37	49	45	5000.0	30.0	10.0
45	49	61	5000.0	30.0	10.0
49	65	61	5000.0	30.0	10.0
61	65	77	5000.0	30.0	10.0
65	85	77	5000.0	30.0	10.0
77	85	93	5000.0	30.0	10.0
85	101	93	5000.0	30.0	10.0
93	101	109	5000.0	30.0	10.0
101	113	109	5000.0	30.0	10.0
109	113	123	5000.0	30.0	10.0
113	129	123	5000.0	30.0	10.0
123	129	141	5000.0	30.0	10.0
129	149	141	5000.0	30.0	10.0
141	149	157	5000.0	30.0	10.0
149	161	157	5000.0	30.0	10.0
157	161	173	5000.0	30.0	10.0
161	181	173	5000.0	30.0	10.0
173	181	189	5000.0	30.0	10.0
181	193	189	5000.0	30.0	10.0
189	193	203	5000.0	30.0	10.0
193	209	203	5000.0	30.0	10.0
203	209	219	5000.0	30.0	10.0
209	229	219	5000.0	30.0	10.0
219	229	235	5000.0	30.0	10.0
229	245	235	5000.0	30.0	10.0
235	245	251	5000.0	30.0	10.0
245	257	251	5000.0	30.0	10.0
251	257	263	5000.0	30.0	10.0
257	277	263	5000.0	30.0	10.0
263	277	283	5000.0	30.0	10.0
277	296	283	5000.0	30.0	10.0
283	296	289	5000.0	30.0	10.0
296	312	289	5000.0	30.0	10.0
289	312	309	5000.0	30.0	10.0
312	325	309	5000.0	30.0	10.0
309	325	331	5000.0	30.0	10.0
13	29	18	5000.0	30.0	10.0
13	18	2	5000.0	30.0	10.0
29	45	38	5000.0	30.0	10.0
29	38	18	5000.0	30.0	10.0
45	61	50	5000.0	30.0	10.0
45	50	38	5000.0	30.0	10.0
61	77	66	5000.0	30.0	10.0
61	66	50	5000.0	30.0	10.0
77	93	86	5000.0	30.0	10.0
77	86	66	5000.0	30.0	10.0
93	109	102	5000.0	30.0	10.0
93	102	86	5000.0	30.0	10.0
109	123	114	5000.0	30.0	10.0
109	114	102	5000.0	30.0	10.0
123	141	130	5000.0	30.0	10.0
123	130	114	5000.0	30.0	10.0
141	157	150	5000.0	30.0	10.0

141	150	130	5000.0	30.0	10.0
157	173	162	5000.0	30.0	10.0
157	162	150	5000.0	30.0	10.0
173	189	182	5000.0	30.0	10.0
173	182	162	5000.0	30.0	10.0
189	203	194	5000.0	30.0	10.0
189	194	182	5000.0	30.0	10.0
203	219	210	5000.0	30.0	10.0
203	210	194	5000.0	30.0	10.0
219	235	230	5000.0	30.0	10.0
219	230	210	5000.0	30.0	10.0
235	251	246	5000.0	30.0	10.0
235	246	230	5000.0	30.0	10.0
251	263	258	5000.0	30.0	10.0
251	258	246	5000.0	30.0	10.0
263	283	278	5000.0	30.0	10.0
263	278	258	5000.0	30.0	10.0
283	289	297	5000.0	30.0	10.0
283	297	278	5000.0	30.0	10.0
289	309	313	5000.0	30.0	10.0
289	313	297	5000.0	30.0	10.0
309	331	326	5000.0	30.0	10.0
309	326	313	5000.0	30.0	10.0
2	18	10	5000.0	30.0	10.0
10	18	26	5000.0	30.0	10.0
18	38	26	5000.0	30.0	10.0
26	38	34	5000.0	30.0	10.0
38	50	34	5000.0	30.0	10.0
34	50	58	5000.0	30.0	10.0
50	66	58	5000.0	30.0	10.0
58	66	74	5000.0	30.0	10.0
66	86	82	5000.0	30.0	10.0
66	82	74	5000.0	30.0	10.0
86	102	98	5000.0	30.0	10.0
86	98	82	5000.0	30.0	10.0
102	114	120	5000.0	30.0	10.0
102	120	98	5000.0	30.0	10.0
114	130	136	5000.0	30.0	10.0
114	136	120	5000.0	30.0	10.0
130	150	146	5000.0	30.0	10.0
130	146	136	5000.0	30.0	10.0
150	162	168	5000.0	30.0	10.0
150	168	146	5000.0	30.0	10.0
162	182	178	5000.0	30.0	10.0
162	178	168	5000.0	30.0	10.0
182	194	200	5000.0	30.0	10.0
182	200	178	5000.0	30.0	10.0
194	210	216	5000.0	30.0	10.0
194	216	200	5000.0	30.0	10.0
210	230	226	5000.0	30.0	10.0
210	226	216	5000.0	30.0	10.0
230	246	242	5000.0	30.0	10.0
230	242	226	5000.0	30.0	10.0
246	258	267	5000.0	30.0	10.0
246	267	242	5000.0	30.0	10.0

258	278	274	5000.0	30.0	10.0
258	274	267	5000.0	30.0	10.0
278	297	293	5000.0	30.0	10.0
278	293	274	5000.0	30.0	10.0
297	313	306	5000.0	30.0	10.0
297	306	293	5000.0	30.0	10.0
313	326	322	5000.0	30.0	10.0
313	322	306	5000.0	30.0	10.0
10	26	19	5000.0	30.0	10.0
10	19	3	5000.0	30.0	10.0
26	34	39	5000.0	30.0	10.0
26	39	19	5000.0	30.0	10.0
34	58	51	5000.0	30.0	10.0
34	51	39	5000.0	30.0	10.0
58	74	67	5000.0	30.0	10.0
58	67	51	5000.0	30.0	10.0
74	82	67	5000.0	30.0	10.0
67	82	87	5000.0	30.0	10.0
82	98	87	5000.0	30.0	10.0
87	98	103	5000.0	30.0	10.0
98	120	103	5000.0	30.0	10.0
103	120	115	5000.0	30.0	10.0
120	136	115	5000.0	30.0	10.0
115	136	131	5000.0	30.0	10.0
136	146	131	5000.0	30.0	10.0
131	146	151	5000.0	30.0	10.0
146	168	151	5000.0	30.0	10.0
151	168	163	5000.0	30.0	10.0
168	178	163	5000.0	30.0	10.0
163	178	183	5000.0	30.0	10.0
178	200	183	5000.0	30.0	10.0
183	200	195	5000.0	30.0	10.0
200	216	195	5000.0	30.0	10.0
195	216	211	5000.0	30.0	10.0
216	226	211	5000.0	30.0	10.0
211	226	231	5000.0	30.0	10.0
226	242	231	5000.0	30.0	10.0
231	242	247	5000.0	30.0	10.0
242	267	247	5000.0	30.0	10.0
247	267	259	5000.0	30.0	10.0
267	274	259	5000.0	30.0	10.0
259	274	279	5000.0	30.0	10.0
274	293	279	5000.0	30.0	10.0
279	293	298	5000.0	30.0	10.0
293	306	298	5000.0	30.0	10.0
298	306	314	5000.0	30.0	10.0
306	322	314	5000.0	30.0	10.0
314	322	327	5000.0	30.0	10.0
3	19	14	5000.0	30.0	10.0
14	19	30	5000.0	30.0	10.0
19	39	30	5000.0	30.0	10.0
30	39	46	5000.0	30.0	10.0
39	51	46	5000.0	30.0	10.0
46	51	62	5000.0	30.0	10.0
51	67	62	5000.0	30.0	10.0

62	67	78	5000.0	30.0	10.0
67	87	78	5000.0	30.0	10.0
78	87	94	5000.0	30.0	10.0
87	103	94	5000.0	30.0	10.0
94	103	110	5000.0	30.0	10.0
103	115	110	5000.0	30.0	10.0
110	115	124	5000.0	30.0	10.0
115	131	124	5000.0	30.0	10.0
124	131	142	5000.0	30.0	10.0
131	151	142	5000.0	30.0	10.0
142	151	158	5000.0	30.0	10.0
151	163	158	5000.0	30.0	10.0
158	163	174	5000.0	30.0	10.0
163	183	174	5000.0	30.0	10.0
174	183	190	5000.0	30.0	10.0
183	195	190	5000.0	30.0	10.0
190	195	204	5000.0	30.0	10.0
195	211	204	5000.0	30.0	10.0
204	211	220	5000.0	30.0	10.0
211	231	220	5000.0	30.0	10.0
220	231	236	5000.0	30.0	10.0
231	247	236	5000.0	30.0	10.0
236	247	252	5000.0	30.0	10.0
247	259	252	5000.0	30.0	10.0
252	259	264	5000.0	30.0	10.0
259	279	264	5000.0	30.0	10.0
264	279	284	5000.0	30.0	10.0
279	298	284	5000.0	30.0	10.0
284	298	290	5000.0	30.0	10.0
298	314	290	5000.0	30.0	10.0
290	314	310	5000.0	30.0	10.0
314	327	310	5000.0	30.0	10.0
310	327	332	5000.0	30.0	10.0
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46	52	40	5000.0	30.0	10.0
62	78	68	5000.0	30.0	10.0
62	68	52	5000.0	30.0	10.0
78	94	88	5000.0	30.0	10.0
78	88	68	5000.0	30.0	10.0
94	110	104	5000.0	30.0	10.0
94	104	88	5000.0	30.0	10.0
110	124	116	5000.0	30.0	10.0
110	116	104	5000.0	30.0	10.0
124	142	132	5000.0	30.0	10.0
124	132	116	5000.0	30.0	10.0
142	158	152	5000.0	30.0	10.0
142	152	132	5000.0	30.0	10.0
158	174	164	5000.0	30.0	10.0
158	164	152	5000.0	30.0	10.0
174	190	184	5000.0	30.0	10.0
174	184	164	5000.0	30.0	10.0

190	204	196	5000.0	30.0	10.0
190	196	184	5000.0	30.0	10.0
204	220	212	5000.0	30.0	10.0
204	212	196	5000.0	30.0	10.0
220	236	232	5000.0	30.0	10.0
220	232	212	5000.0	30.0	10.0
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236	248	232	5000.0	30.0	10.0
252	264	260	5000.0	30.0	10.0
252	260	248	5000.0	30.0	10.0
264	284	280	5000.0	30.0	10.0
264	280	260	5000.0	30.0	10.0
284	290	299	5000.0	30.0	10.0
284	299	280	5000.0	30.0	10.0
290	310	315	5000.0	30.0	10.0
290	315	299	5000.0	30.0	10.0
310	332	328	5000.0	30.0	10.0
310	328	315	5000.0	30.0	10.0
4	20	11	5000.0	30.0	10.0
11	20	27	5000.0	30.0	10.0
20	40	27	5000.0	30.0	10.0
27	40	35	5000.0	30.0	10.0
40	52	35	5000.0	30.0	10.0
35	52	59	5000.0	30.0	10.0
52	68	59	5000.0	30.0	10.0
59	68	75	5000.0	30.0	10.0
68	88	83	5000.0	30.0	10.0
68	83	75	5000.0	30.0	10.0
88	104	99	5000.0	30.0	10.0
88	99	83	5000.0	30.0	10.0
104	116	121	5000.0	30.0	10.0
104	121	99	5000.0	30.0	10.0
116	132	137	5000.0	30.0	10.0
116	137	121	5000.0	30.0	10.0
132	152	147	5000.0	30.0	10.0
132	147	137	5000.0	30.0	10.0
152	164	169	5000.0	30.0	10.0
152	169	147	5000.0	30.0	10.0
164	184	179	5000.0	30.0	10.0
164	179	169	5000.0	30.0	10.0
184	196	201	5000.0	30.0	10.0
184	201	179	5000.0	30.0	10.0
196	212	217	5000.0	30.0	10.0
196	217	201	5000.0	30.0	10.0
212	232	227	5000.0	30.0	10.0
212	227	217	5000.0	30.0	10.0
232	248	243	5000.0	30.0	10.0
232	243	227	5000.0	30.0	10.0
248	260	268	5000.0	30.0	10.0
248	268	243	5000.0	30.0	10.0
260	280	275	5000.0	30.0	10.0
260	275	268	5000.0	30.0	10.0
280	299	294	5000.0	30.0	10.0
280	294	275	5000.0	30.0	10.0
299	315	307	5000.0	30.0	10.0

299	307	294	5000.0	30.0	10.0
315	328	323	5000.0	30.0	10.0
315	323	307	5000.0	30.0	10.0
11	27	21	5000.0	30.0	10.0
11	21	5	5000.0	30.0	10.0
27	35	41	5000.0	30.0	10.0
27	41	21	5000.0	30.0	10.0
35	59	53	5000.0	30.0	10.0
35	53	41	5000.0	30.0	10.0
59	75	69	5000.0	30.0	10.0
59	69	53	5000.0	30.0	10.0
75	83	69	5000.0	30.0	10.0
69	83	89	5000.0	30.0	10.0
83	99	89	5000.0	30.0	10.0
89	99	105	5000.0	30.0	10.0
99	121	105	5000.0	30.0	10.0
105	121	117	5000.0	30.0	10.0
121	137	117	5000.0	30.0	10.0
117	137	133	5000.0	30.0	10.0
137	147	133	5000.0	30.0	10.0
133	147	153	5000.0	30.0	10.0
147	169	153	5000.0	30.0	10.0
153	169	165	5000.0	30.0	10.0
169	179	165	5000.0	30.0	10.0
165	179	185	5000.0	30.0	10.0
179	201	185	5000.0	30.0	10.0
185	201	197	5000.0	30.0	10.0
201	217	197	5000.0	30.0	10.0
197	217	213	5000.0	30.0	10.0
217	227	213	5000.0	30.0	10.0
213	227	233	5000.0	30.0	10.0
227	243	233	5000.0	30.0	10.0
233	243	249	5000.0	30.0	10.0
243	268	249	5000.0	30.0	10.0
249	268	261	5000.0	30.0	10.0
268	275	261	5000.0	30.0	10.0
261	275	281	5000.0	30.0	10.0
275	294	281	5000.0	30.0	10.0
281	294	300	5000.0	30.0	10.0
294	307	300	5000.0	30.0	10.0
300	307	316	5000.0	30.0	10.0
307	323	316	5000.0	30.0	10.0
316	323	329	5000.0	30.0	10.0
5	21	15	5000.0	30.0	10.0
15	21	31	5000.0	30.0	10.0
21	41	31	5000.0	30.0	10.0
31	41	47	5000.0	30.0	10.0
41	53	47	5000.0	30.0	10.0
47	53	63	5000.0	30.0	10.0
53	69	63	5000.0	30.0	10.0
63	69	79	5000.0	30.0	10.0
69	89	79	5000.0	30.0	10.0
79	89	95	5000.0	30.0	10.0
89	105	95	5000.0	30.0	10.0
95	105	111	5000.0	30.0	10.0

105	117	111	5000.0	30.0	10.0
111	117	125	5000.0	30.0	10.0
117	133	125	5000.0	30.0	10.0
125	133	143	5000.0	30.0	10.0
133	153	143	5000.0	30.0	10.0
143	153	159	5000.0	30.0	10.0
153	165	159	5000.0	30.0	10.0
159	165	175	5000.0	30.0	10.0
165	185	175	5000.0	30.0	10.0
175	185	191	5000.0	30.0	10.0
185	197	191	5000.0	30.0	10.0
191	197	205	5000.0	30.0	10.0
197	213	205	5000.0	30.0	10.0
205	213	221	5000.0	30.0	10.0
213	233	221	5000.0	30.0	10.0
221	233	237	5000.0	30.0	10.0
233	249	237	5000.0	30.0	10.0
237	249	253	5000.0	30.0	10.0
249	261	253	5000.0	30.0	10.0
253	261	265	5000.0	30.0	10.0
261	281	265	5000.0	30.0	10.0
265	281	285	5000.0	30.0	10.0
281	300	285	5000.0	30.0	10.0
285	300	291	5000.0	30.0	10.0
300	316	291	5000.0	30.0	10.0
291	316	311	5000.0	30.0	10.0
316	329	311	5000.0	30.0	10.0
311	329	333	5000.0	30.0	10.0
15	31	22	5000.0	30.0	10.0
15	22	6	5000.0	30.0	10.0
31	47	42	5000.0	30.0	10.0
31	42	22	5000.0	30.0	10.0
47	63	54	5000.0	30.0	10.0
47	54	42	5000.0	30.0	10.0
63	79	70	5000.0	30.0	10.0
63	70	54	5000.0	30.0	10.0
79	95	90	5000.0	30.0	10.0
79	90	70	5000.0	30.0	10.0
95	111	106	5000.0	30.0	10.0
95	106	90	5000.0	30.0	10.0
111	125	126	5000.0	30.0	10.0
111	126	106	5000.0	30.0	10.0
125	143	139	5000.0	30.0	10.0
125	139	126	5000.0	30.0	10.0
143	159	154	5000.0	30.0	10.0
143	154	139	5000.0	30.0	10.0
159	175	171	5000.0	30.0	10.0
159	171	154	5000.0	30.0	10.0
175	191	187	5000.0	30.0	10.0
175	187	171	5000.0	30.0	10.0
191	205	187	5000.0	30.0	10.0
187	205	206	5000.0	30.0	10.0
205	221	206	5000.0	30.0	10.0
206	221	222	5000.0	30.0	10.0
221	237	222	5000.0	30.0	10.0

222	237	238	5000.0	30.0	10.0
237	253	238	5000.0	30.0	10.0
238	253	254	5000.0	30.0	10.0
253	265	254	5000.0	30.0	10.0
254	265	270	5000.0	30.0	10.0
265	285	270	5000.0	30.0	10.0
270	285	286	5000.0	30.0	10.0
285	291	286	5000.0	30.0	10.0
286	291	302	5000.0	30.0	10.0
291	311	302	5000.0	30.0	10.0
302	311	318	5000.0	30.0	10.0
311	333	318	5000.0	30.0	10.0
318	333	334	5000.0	30.0	10.0
6	22	28	5000.0	30.0	10.0
6	28	12	5000.0	30.0	10.0
22	42	36	5000.0	30.0	10.0
22	36	28	5000.0	30.0	10.0
42	54	60	5000.0	30.0	10.0
42	60	36	5000.0	30.0	10.0
54	70	76	5000.0	30.0	10.0
54	76	60	5000.0	30.0	10.0
70	90	84	5000.0	30.0	10.0
70	84	76	5000.0	30.0	10.0
90	106	100	5000.0	30.0	10.0
90	100	84	5000.0	30.0	10.0
106	126	122	5000.0	30.0	10.0
106	122	100	5000.0	30.0	10.0
126	139	138	5000.0	30.0	10.0
126	138	122	5000.0	30.0	10.0
139	154	148	5000.0	30.0	10.0
139	148	138	5000.0	30.0	10.0
154	171	170	5000.0	30.0	10.0
154	170	148	5000.0	30.0	10.0
171	187	180	5000.0	30.0	10.0
171	180	170	5000.0	30.0	10.0
187	206	202	5000.0	30.0	10.0
187	202	180	5000.0	30.0	10.0
206	222	218	5000.0	30.0	10.0
206	218	202	5000.0	30.0	10.0
222	238	228	5000.0	30.0	10.0
222	228	218	5000.0	30.0	10.0
238	254	244	5000.0	30.0	10.0
238	244	228	5000.0	30.0	10.0
254	270	269	5000.0	30.0	10.0
254	269	244	5000.0	30.0	10.0
270	286	276	5000.0	30.0	10.0
270	276	269	5000.0	30.0	10.0
286	302	295	5000.0	30.0	10.0
286	295	276	5000.0	30.0	10.0
302	318	308	5000.0	30.0	10.0
302	308	295	5000.0	30.0	10.0
318	334	324	5000.0	30.0	10.0
318	324	308	5000.0	30.0	10.0
12	28	7	5000.0	30.0	10.0
7	28	23	5000.0	30.0	10.0

28	36	23	5000.0	30.0	10.0
23	36	43	5000.0	30.0	10.0
36	60	43	5000.0	30.0	10.0
43	60	55	5000.0	30.0	10.0
60	76	55	5000.0	30.0	10.0
55	76	71	5000.0	30.0	10.0
76	84	71	5000.0	30.0	10.0
71	84	91	5000.0	30.0	10.0
84	100	91	5000.0	30.0	10.0
91	100	107	5000.0	30.0	10.0
100	122	107	5000.0	30.0	10.0
107	122	127	5000.0	30.0	10.0
122	138	127	5000.0	30.0	10.0
127	138	140	5000.0	30.0	10.0
138	148	140	5000.0	30.0	10.0
140	148	155	5000.0	30.0	10.0
148	170	155	5000.0	30.0	10.0
155	170	172	5000.0	30.0	10.0
170	180	172	5000.0	30.0	10.0
172	180	188	5000.0	30.0	10.0
180	202	188	5000.0	30.0	10.0
188	202	207	5000.0	30.0	10.0
202	218	207	5000.0	30.0	10.0
207	218	223	5000.0	30.0	10.0
218	228	223	5000.0	30.0	10.0
223	228	239	5000.0	30.0	10.0
228	244	239	5000.0	30.0	10.0
239	244	255	5000.0	30.0	10.0
244	269	255	5000.0	30.0	10.0
255	269	271	5000.0	30.0	10.0
269	276	271	5000.0	30.0	10.0
271	276	287	5000.0	30.0	10.0
276	295	287	5000.0	30.0	10.0
287	295	303	5000.0	30.0	10.0
295	308	303	5000.0	30.0	10.0
303	308	319	5000.0	30.0	10.0
308	324	319	5000.0	30.0	10.0
319	324	335	5000.0	30.0	10.0
7	23	16	5000.0	30.0	10.0
16	23	32	5000.0	30.0	10.0
23	43	32	5000.0	30.0	10.0
32	43	48	5000.0	30.0	10.0
43	55	48	5000.0	30.0	10.0
48	55	64	5000.0	30.0	10.0
55	71	64	5000.0	30.0	10.0
64	71	80	5000.0	30.0	10.0
71	91	80	5000.0	30.0	10.0
80	91	96	5000.0	30.0	10.0
91	107	96	5000.0	30.0	10.0
96	107	112	5000.0	30.0	10.0
107	127	112	5000.0	30.0	10.0
112	127	128	5000.0	30.0	10.0
127	140	128	5000.0	30.0	10.0
128	140	144	5000.0	30.0	10.0
140	155	144	5000.0	30.0	10.0

144	155	160	5000.0	30.0	10.0
155	172	160	5000.0	30.0	10.0
160	172	176	5000.0	30.0	10.0
172	188	176	5000.0	30.0	10.0
176	188	192	5000.0	30.0	10.0
188	207	192	5000.0	30.0	10.0
192	207	208	5000.0	30.0	10.0
207	223	208	5000.0	30.0	10.0
208	223	224	5000.0	30.0	10.0
223	239	224	5000.0	30.0	10.0
224	239	240	5000.0	30.0	10.0
239	255	240	5000.0	30.0	10.0
240	255	256	5000.0	30.0	10.0
255	271	256	5000.0	30.0	10.0
256	271	272	5000.0	30.0	10.0
271	287	272	5000.0	30.0	10.0
272	287	288	5000.0	30.0	10.0
287	303	288	5000.0	30.0	10.0
288	303	304	5000.0	30.0	10.0
303	319	304	5000.0	30.0	10.0
304	319	320	5000.0	30.0	10.0
319	335	320	5000.0	30.0	10.0
320	335	336	5000.0	30.0	10.0
16	32	24	5000.0	30.0	10.0
16	24	8	5000.0	30.0	10.0
32	48	44	5000.0	30.0	10.0
32	44	24	5000.0	30.0	10.0
48	64	56	5000.0	30.0	10.0
48	56	44	5000.0	30.0	10.0
64	80	72	5000.0	30.0	10.0
64	72	56	5000.0	30.0	10.0
80	96	92	5000.0	30.0	10.0
80	92	72	5000.0	30.0	10.0
96	112	108	5000.0	30.0	10.0
96	108	92	5000.0	30.0	10.0
112	128	118	5000.0	30.0	10.0
112	118	108	5000.0	30.0	10.0
128	144	134	5000.0	30.0	10.0
128	134	118	5000.0	30.0	10.0
144	160	156	5000.0	30.0	10.0
144	156	134	5000.0	30.0	10.0
160	176	166	5000.0	30.0	10.0
160	166	156	5000.0	30.0	10.0
176	192	186	5000.0	30.0	10.0
176	186	166	5000.0	30.0	10.0
192	208	198	5000.0	30.0	10.0
192	198	186	5000.0	30.0	10.0
208	224	214	5000.0	30.0	10.0
208	214	198	5000.0	30.0	10.0
224	240	234	5000.0	30.0	10.0
224	234	214	5000.0	30.0	10.0
240	256	250	5000.0	30.0	10.0
240	250	234	5000.0	30.0	10.0
256	272	262	5000.0	30.0	10.0
256	262	250	5000.0	30.0	10.0

272	288	282	5000.0	30.0	10.0
272	282	262	5000.0	30.0	10.0
288	304	301	5000.0	30.0	10.0
288	301	282	5000.0	30.0	10.0
304	320	317	5000.0	30.0	10.0
304	317	301	5000.0	30.0	10.0
320	336	330	5000.0	30.0	10.0
320	330	317	5000.0	30.0	10.0
8	24	9	5000.0	30.0	10.0
9	24	25	5000.0	30.0	10.0
24	44	25	5000.0	30.0	10.0
25	44	33	5000.0	30.0	10.0
44	56	33	5000.0	30.0	10.0
33	56	57	5000.0	30.0	10.0
56	72	57	5000.0	30.0	10.0
57	72	73	5000.0	30.0	10.0
72	92	81	5000.0	30.0	10.0
72	81	73	5000.0	30.0	10.0
92	108	97	5000.0	30.0	10.0
92	97	81	5000.0	30.0	10.0
108	118	119	5000.0	30.0	10.0
108	119	97	5000.0	30.0	10.0
118	134	135	5000.0	30.0	10.0
118	135	119	5000.0	30.0	10.0
134	156	145	5000.0	30.0	10.0
134	145	135	5000.0	30.0	10.0
156	166	167	5000.0	30.0	10.0
156	167	145	5000.0	30.0	10.0
166	186	177	5000.0	30.0	10.0
166	177	167	5000.0	30.0	10.0
186	198	199	5000.0	30.0	10.0
186	199	177	5000.0	30.0	10.0
198	214	215	5000.0	30.0	10.0
198	215	199	5000.0	30.0	10.0
214	234	225	5000.0	30.0	10.0
214	225	215	5000.0	30.0	10.0
234	250	241	5000.0	30.0	10.0
234	241	225	5000.0	30.0	10.0
250	262	266	5000.0	30.0	10.0
250	266	241	5000.0	30.0	10.0
262	282	273	5000.0	30.0	10.0
262	273	266	5000.0	30.0	10.0
282	301	292	5000.0	30.0	10.0
282	292	273	5000.0	30.0	10.0
301	317	305	5000.0	30.0	10.0
301	305	292	5000.0	30.0	10.0
317	330	321	5000.0	30.0	10.0
317	321	305	5000.0	30.0	10.0

321 1000. (NDBC(n),DBC(n))
322 1000.
323 1000.
324 1000.
325 1000.
326 1000.
327 1000.

```
328 1000.
329 1000.
330 1000.
331 1000.
332 1000.
333 1000.
334 1000.
335 1000.
336 1000.
1 -3612.8125                              (NBCN(n),BCN(n))
2 -3612.8125
3 -3612.8125
4 -3612.8125
5 -3612.8125
6 -3612.8125
7 -3612.8125
8 -3612.8125
9 -3612.8125
10 -3612.8125
11 -3612.8125
12 -3612.8125
13 -3612.8125
14 -3612.8125
15 -3612.8125
16 -3612.8125
```

Example 3.

Data input for the third test problem: comparison to finite-difference simulation results (Reilly and others, 1984).

```
PROBLEM 1 OF84-459, T=.0155 FT3/S                  (ITITLE)
FEET SECOND                                        (IUNITS)
 121  200   22    2    0                 (NNODE,NELEM,IDBC,IBCN,IBCM)
   1      0.000 10000.000
   2      0.000  9000.000
   3      0.000  8000.000
   4      0.000  7000.000                (node number,X(n),Y(n))
   5      0.000  6000.000
   6      0.000  5000.000
   7      0.000  4000.000
   8      0.000  3000.000
   9      0.000  2000.000
  10      0.000  1000.000
  11      0.000     0.000
  12   1000.000 10000.000
  13   1000.000  9000.000
  14   1000.000  8000.000
  15   1000.000  7000.000
  16   1000.000  6000.000
  17   1000.000  5000.000
  18   1000.000  4000.000
  19   1000.000  3000.000
  20   1000.000  2000.000
  21   1000.000  1000.000
  22   1000.000     0.000
  23   2000.000 10000.000
  24   2000.000  9000.000
  25   2000.000  8000.000
  26   2000.000  7000.000
  27   2000.000  6000.000
  28   2000.000  5000.000
  29   2000.000  4000.000
  30   2000.000  3000.000
  31   2000.000  2000.000
  32   2000.000  1000.000
  33   2000.000     0.000
  34   3000.000 10000.000
  35   3000.000  9000.000
  36   3000.000  8000.000
  37   3000.000  7000.000
  38   3000.000  6000.000
  39   3000.000  5000.000
  40   3000.000  4000.000
  41   3000.000  3000.000
  42   3000.000  2000.000
  43   3000.000  1000.000
  44   3000.000     0.000
  45   4000.000 10000.000
  46   4000.000  9000.000
  47   4000.000  8000.000
```

48	4000.000	7000.000
49	4000.000	6000.000
50	4000.000	5000.000
51	4000.000	4000.000
52	4000.000	3000.000
53	4000.000	2000.000
54	4000.000	1000.000
55	4000.000	0.000
56	5000.000	10000.000
57	5000.000	9000.000
58	5000.000	8000.000
59	5000.000	7000.000
60	5000.000	6000.000
61	5000.000	5000.000
62	5000.000	4000.000
63	5000.000	3000.000
64	5000.000	2000.000
65	5000.000	1000.000
66	5000.000	0.000
67	6000.000	10000.000
68	6000.000	9000.000
69	6000.000	8000.000
70	6000.000	7000.000
71	6000.000	6000.000
72	6000.000	5000.000
73	6000.000	4000.000
74	6000.000	3000.000
75	6000.000	2000.000
76	6000.000	1000.000
77	6000.000	0.000
78	7000.000	10000.000
79	7000.000	9000.000
80	7000.000	8000.000
81	7000.000	7000.000
82	7000.000	6000.000
83	7000.000	5000.000
84	7000.000	4000.000
85	7000.000	3000.000
86	7000.000	2000.000
87	7000.000	1000.000
88	7000.000	0.000
89	8000.000	10000.000
90	8000.000	9000.000
91	8000.000	8000.000
92	8000.000	7000.000
93	8000.000	6000.000
94	8000.000	5000.000
95	8000.000	4000.000
96	8000.000	3000.000
97	8000.000	2000.000
98	8000.000	1000.000
99	8000.000	0.000
100	9000.000	10000.000
101	9000.000	9000.000
102	9000.000	8000.000

```
103  9000.000  7000.000
104  9000.000  6000.000
105  9000.000  5000.000
106  9000.000  4000.000
107  9000.000  3000.000
108  9000.000  2000.000
109  9000.000  1000.000
110  9000.000     0.000
111 10000.000 10000.000
112 10000.000  9000.000
113 10000.000  8000.000
114 10000.000  7000.000
115 10000.000  6000.000
116 10000.000  5000.000
117 10000.000  4000.000
118 10000.000  3000.000
119 10000.000  2000.000
120 10000.000  1000.000
121 10000.000     0.000
  1    2   12     .0155          .00         1.00 (ICON(i,j)j=1,3),TEL,ALPHA,ANIS)
 12    2   13     .0155          .00         1.00
  2    3   13     .0155          .00         1.00
 13    3   14     .0155          .00         1.00
  3    4   14     .0155          .00         1.00
 14    4   15     .0155          .00         1.00
  4    5   15     .0155          .00         1.00
 15    5   16     .0155          .00         1.00
  5    6   16     .0155          .00         1.00
 16    6   17     .0155          .00         1.00
  6    7   17     .0155          .00         1.00
 17    7   18     .0155          .00         1.00
  7    8   19     .0155          .00         1.00
  7   19   18     .0155          .00         1.00
  8    9   19     .0155          .00         1.00
 19    9   20     .0155          .00         1.00
  9   10   20     .0155          .00         1.00
 20   10   21     .0155          .00         1.00
 10   11   22     .0155          .00         1.00
 10   22   21     .0155          .00         1.00
 12   13   24     .0155          .00         1.00
 12   24   23     .0155          .00         1.00
 13   14   25     .0155          .00         1.00
 13   25   24     .0155          .00         1.00
 14   15   26     .0155          .00         1.00
 14   26   25     .0155          .00         1.00
 15   16   27     .0155          .00         1.00
 15   27   26     .0155          .00         1.00
 16   17   28     .0155          .00         1.00
 16   28   27     .0155          .00         1.00
 17   18   28     .0155          .00         1.00
 28   18   29     .0155          .00         1.00
 18   19   30     .0155          .00         1.00
 18   30   29     .0155          .00         1.00
 19   20   30     .0155          .00         1.00
 30   20   31     .0155          .00         1.00
```

20	21	31	.0155	.00	1.00
31	21	32	.0155	.00	1.00
21	22	33	.0155	.00	1.00
21	33	32	.0155	.00	1.00
23	24	34	.0155	.00	1.00
34	24	35	.0155	.00	1.00
24	25	35	.0155	.00	1.00
35	25	36	.0155	.00	1.00
25	26	37	.0155	.00	1.00
25	37	36	.0155	.00	1.00
26	27	38	.0155	.00	1.00
26	38	37	.0155	.00	1.00
27	28	39	.0155	.00	1.00
27	39	38	.0155	.00	1.00
28	29	39	.0155	.00	1.00
39	29	40	.0155	.00	1.00
29	30	41	.0155	.00	1.00
29	41	40	.0155	.00	1.00
30	31	41	.0155	.00	1.00
41	31	42	.0155	.00	1.00
31	32	42	.0155	.00	1.00
42	32	43	.0155	.00	1.00
32	33	44	.0155	.00	1.00
32	44	43	.0155	.00	1.00
34	35	45	.0155	.00	1.00
45	35	46	.0155	.00	1.00
35	36	47	.0155	.00	1.00
35	47	46	.0155	.00	1.00
36	37	47	.0155	.00	1.00
47	37	48	.0155	.00	1.00
37	38	48	.0155	.00	1.00
48	38	49	.0155	.00	1.00
38	39	49	.0155	.00	1.00
49	39	50	.0155	.00	1.00
39	40	50	.0155	.00	1.00
50	40	51	.0155	.00	1.00
40	41	52	.0155	.00	1.00
40	52	51	.0155	.00	1.00
41	42	53	.0155	.00	1.00
41	53	52	.0155	.00	1.00
42	43	53	.0155	.00	1.00
53	43	54	.0155	.00	1.00
43	44	55	.0155	.00	1.00
43	55	54	.0155	.00	1.00
45	46	57	.0155	.00	1.00
45	57	56	.0155	.00	1.00
46	47	58	.0155	.00	1.00
46	58	57	.0155	.00	1.00
47	48	59	.0155	.00	1.00
47	59	58	.0155	.00	1.00
48	49	60	.0155	.00	1.00
48	60	59	.0155	.00	1.00
49	50	61	.0155	.00	1.00
49	61	60	.0155	.00	1.00
50	51	61	.0155	.00	1.00

61	51	62	.0155	.00	1.00
51	52	63	.0155	.00	1.00
51	63	62	.0155	.00	1.00
52	53	63	.0155	.00	1.00
63	53	64	.0155	.00	1.00
53	54	64	.0155	.00	1.00
64	54	65	.0155	.00	1.00
54	55	66	.0155	.00	1.00
54	66	65	.0155	.00	1.00
56	57	68	.0155	.00	1.00
56	68	67	.0155	.00	1.00
57	58	69	.0155	.00	1.00
57	69	68	.0155	.00	1.00
58	59	70	.0155	.00	1.00
58	70	69	.0155	.00	1.00
59	60	71	.0155	.00	1.00
59	71	70	.0155	.00	1.00
60	61	72	.0155	.00	1.00
60	72	71	.0155	.00	1.00
61	62	72	.0155	.00	1.00
72	62	73	.0155	.00	1.00
62	63	74	.0155	.00	1.00
62	74	73	.0155	.00	1.00
63	64	74	.0155	.00	1.00
74	64	75	.0155	.00	1.00
64	65	76	.0155	.00	1.00
64	76	75	.0155	.00	1.00
65	66	77	.0155	.00	1.00
65	77	76	.0155	.00	1.00
67	68	79	.0155	.00	1.00
67	79	78	.0155	.00	1.00
68	69	79	.0155	.00	1.00
79	69	80	.0155	.00	1.00
69	70	81	.0155	.00	1.00
69	81	80	.0155	.00	1.00
70	71	82	.0155	.00	1.00
70	82	81	.0155	.00	1.00
71	72	83	.0155	.00	1.00
71	83	82	.0155	.00	1.00
72	73	83	.0155	.00	1.00
83	73	84	.0155	.00	1.00
73	74	85	.0155	.00	1.00
73	85	84	.0155	.00	1.00
74	75	85	.0155	.00	1.00
85	75	86	.0155	.00	1.00
75	76	87	.0155	.00	1.00
75	87	86	.0155	.00	1.00
76	77	87	.0155	.00	1.00
87	77	88	.0155	.00	1.00
78	79	90	.0155	.00	1.00
78	90	89	.0155	.00	1.00
79	80	91	.0155	.00	1.00
79	91	90	.0155	.00	1.00
80	81	91	.0155	.00	1.00
91	81	92	.0155	.00	1.00

81	82	93	.0155	.00	1.00
81	93	92	.0155	.00	1.00
82	83	94	.0155	.00	1.00
82	94	93	.0155	.00	1.00
83	84	94	.0155	.00	1.00
94	84	95	.0155	.00	1.00
84	85	96	.0155	.00	1.00
84	96	95	.0155	.00	1.00
85	86	97	.0155	.00	1.00
85	97	96	.0155	.00	1.00
86	87	97	.0155	.00	1.00
97	87	98	.0155	.00	1.00
87	88	99	.0155	.00	1.00
87	99	98	.0155	.00	1.00
89	90	101	.0155	.00	1.00
89	101	100	.0155	.00	1.00
90	91	101	.0155	.00	1.00
101	91	102	.0155	.00	1.00
91	92	103	.0155	.00	1.00
91	103	102	.0155	.00	1.00
92	93	104	.0155	.00	1.00
92	104	103	.0155	.00	1.00
93	94	105	.0155	.00	1.00
93	105	104	.0155	.00	1.00
94	95	105	.0155	.00	1.00
105	95	106	.0155	.00	1.00
95	96	107	.0155	.00	1.00
95	107	106	.0155	.00	1.00
96	97	107	.0155	.00	1.00
107	97	108	.0155	.00	1.00
97	98	108	.0155	.00	1.00
108	98	109	.0155	.00	1.00
98	99	110	.0155	.00	1.00
98	110	109	.0155	.00	1.00
100	101	112	.0155	.00	1.00
100	112	111	.0155	.00	1.00
101	102	112	.0155	.00	1.00
112	102	113	.0155	.00	1.00
102	103	114	.0155	.00	1.00
102	114	113	.0155	.00	1.00
103	104	115	.0155	.00	1.00
103	115	114	.0155	.00	1.00
104	105	115	.0155	.00	1.00
115	105	116	.0155	.00	1.00
105	106	116	.0155	.00	1.00
116	106	117	.0155	.00	1.00
106	107	118	.0155	.00	1.00
106	118	117	.0155	.00	1.00
107	108	119	.0155	.00	1.00
107	119	118	.0155	.00	1.00
108	109	119	.0155	.00	1.00
119	109	120	.0155	.00	1.00
109	110	121	.0155	.00	1.00
109	121	120	.0155	.00	1.00

1 200. (NDBC(n),DBC(n))

2	200.	
3	200.	
4	200.	
5	200.	
6	200.	
7	200.	
8	200.	
9	200.	
10	200.	
11	200.	
111	0.	
112	0.	
113	0.	
114	0.	
115	0.	
116	0.	
117	0.	
118	0.	
119	0.	
120	0.	
121	0.	
39	-1.55	(NBCN(n),BCN(n))
50	-1.55	

Supplemental Data IV. Data Output for Test Problems

Example 1.

pumping well h=1000, rw=.5, T=5000 ft2/d, Q=300 gpm
units feet and day

0.4330000E+00	0.2500000E+00	981.95
0.4830000E+00	0.1290000E+00	982.01
0.5000000E+00	0.0000000E+00	981.95
0.6870000E+00	0.3960000E+00	982.81
0.7660000E+00	0.2050000E+00	982.82
0.7930000E+00	0.0000000E+00	982.81
0.1004000E+01	0.5790000E+00	983.50
0.1119000E+01	0.3000000E+00	983.50
0.1159000E+01	0.0000000E+00	983.50
0.1399000E+01	0.8080000E+00	984.10
0.1560000E+01	0.4180000E+00	984.10
0.1615000E+01	0.0000000E+00	984.10
0.1906000E+01	0.1100000E+01	984.66
0.2126000E+01	0.5700000E+00	984.66
0.2201000E+01	0.0000000E+00	984.66
0.2539000E+01	0.1466000E+01	985.19
0.2833000E+01	0.7590000E+00	985.19
0.2932000E+01	0.0000000E+00	985.19
0.3331000E+01	0.1924000E+01	985.68
0.3716000E+01	0.9960000E+00	985.68
0.3847000E+01	0.0000000E+00	985.68
0.4346000E+01	0.2509000E+01	986.16
0.4847000E+01	0.1299000E+01	986.16
0.5018000E+01	0.0000000E+00	986.16
0.5613000E+01	0.3241000E+01	986.63
0.6261000E+01	0.1678000E+01	986.63
0.6481000E+01	0.0000000E+00	986.63
0.7198000E+01	0.4155000E+01	987.08
0.8028000E+01	0.2151000E+01	987.08
0.8311000E+01	0.0000000E+00	987.08
0.9225000E+01	0.5326000E+01	987.53
0.1029000E+02	0.2757000E+01	987.53
0.1065200E+02	0.0000000E+00	987.53
0.1176000E+02	0.6790000E+01	987.97
0.1311700E+02	0.3515000E+01	987.97
0.1358000E+02	0.0000000E+00	987.97
0.1492800E+02	0.8619000E+01	988.41
0.1665200E+02	0.4462000E+01	988.41
0.1723800E+02	0.0000000E+00	988.41
0.1888900E+02	0.1090600E+02	988.84
0.2107000E+02	0.5645000E+01	988.84
0.2181200E+02	0.0000000E+00	988.84
0.2395900E+02	0.1383300E+02	989.27
0.2672400E+02	0.7161000E+01	989.27
0.2766600E+02	0.0000000E+00	989.27
0.3029700E+02	0.1749200E+02	989.70
0.3379300E+02	0.9054000E+01	989.70
0.3498400E+02	0.0000000E+00	989.70
0.3822400E+02	0.2206500E+02	990.12

0.4262900E+02	0.1142200E+02	990.12
0.4413100E+02	0.0000000E+00	990.12
0.4835800E+02	0.2791900E+02	990.55
0.5393900E+02	0.1445200E+02	990.55
0.5583900E+02	0.0000000E+00	990.55
0.6103200E+02	0.3523700E+02	990.97
0.6807600E+02	0.1824000E+02	990.97
0.7047400E+02	0.0000000E+00	990.97
0.7687500E+02	0.4438400E+02	991.39
0.8574700E+02	0.2297500E+02	991.39
0.8876800E+02	0.0000000E+00	991.39
0.9668000E+02	0.5581800E+02	991.81
0.1078370E+03	0.2889400E+02	991.81
0.1116360E+03	0.0000000E+00	991.81
0.1215220E+03	0.7016100E+02	992.23
0.1355460E+03	0.3631800E+02	992.23
0.1403210E+03	0.0000000E+00	992.23
0.1525750E+03	0.8808900E+02	992.64
0.1701820E+03	0.4559800E+02	992.64
0.1761780E+03	0.0000000E+00	992.64
0.1913900E+03	0.1104990E+03	993.06
0.2134770E+03	0.5719900E+02	993.06
0.2209980E+03	0.0000000E+00	993.06
0.2400600E+03	0.1385990E+03	993.47
0.2677640E+03	0.7174400E+02	993.47
0.2771980E+03	0.0000000E+00	993.47
0.3008980E+03	0.1737230E+03	993.88
0.3356220E+03	0.8992600E+02	993.88
0.3474470E+03	0.0000000E+00	993.88
0.3769450E+03	0.2176290E+03	994.29
0.4204460E+03	0.1126540E+03	994.29
0.4352590E+03	0.0000000E+00	994.29
0.4720040E+03	0.2725120E+03	994.70
0.5264740E+03	0.1410630E+03	994.70
0.5450230E+03	0.0000000E+00	994.70
0.5933620E+03	0.3425780E+03	995.12
0.6618380E+03	0.1773320E+03	995.12
0.6851560E+03	0.0000000E+00	995.12
0.7415800E+03	0.4283410E+03	995.52
0.8275250E+03	0.2217260E+03	995.52
0.8566810E+03	0.0000000E+00	995.52
0.9272860E+03	0.5355440E+03	995.93
0.1034633E+04	0.2772190E+03	995.93
0.1071088E+04	0.0000000E+00	995.93
0.1159512E+04	0.6696030E+03	996.34
0.1293625E+04	0.3466130E+03	996.34
0.1339207E+04	0.0000000E+00	996.34
0.1449794E+04	0.8371770E+03	996.74
0.1617363E+04	0.4333560E+03	996.74
0.1674355E+04	0.0000000E+00	996.74
0.1812643E+04	0.1046645E+04	997.15
0.2022034E+04	0.5417850E+03	997.15
0.2093290E+04	0.0000000E+00	997.15
0.2266438E+04	0.1308616E+04	997.56
0.2528137E+04	0.6773920E+03	997.56

0.2617233E+04	0.0000000E+00	997.56
0.2833674E+04	0.1636081E+04	997.96
0.3160761E+04	0.8469010E+03	997.96
0.3272161E+04	0.0000000E+00	997.96
0.3543502E+04	0.2045869E+04	998.37
0.3952420E+04	0.1059023E+04	998.37
0.4091737E+04	0.0000000E+00	998.37
0.4430770E+04	0.2558103E+04	998.78
0.4941985E+04	0.1324177E+04	998.78
0.5116207E+04	0.0000000E+00	998.78
0.5539830E+04	0.3198397E+04	999.19
0.6178929E+04	0.1655618E+04	999.19
0.6396794E+04	0.0000000E+00	999.19
0.6926431E+04	0.3998947E+04	999.59
0.7725443E+04	0.2070015E+04	999.59
0.7997894E+04	0.0000000E+00	999.59
0.8660254E+04	0.5000000E+04	1000.00
0.9659260E+04	0.2588200E+04	1000.00
0.1000000E+05	0.0000000E+00	1000.00

Example 2.

RADIAL FLOW PROBLEM Q=57805FT3/D TMAX=5000 TMIN 500 FT2/D AL=30 AN=10
UNITS FEET DAY

0.2000046E+05	0.2000019E+05	961.84
0.2000019E+05	0.2000046E+05	961.87
0.1999981E+05	0.2000046E+05	961.90
0.1999954E+05	0.2000019E+05	961.88
0.1999954E+05	0.1999981E+05	961.84
0.1999981E+05	0.1999954E+05	961.87
0.2000019E+05	0.1999954E+05	961.90
0.2000046E+05	0.1999981E+05	961.88
0.2000050E+05	0.2000000E+05	961.86
0.2000000E+05	0.2000050E+05	961.89
0.1999950E+05	0.2000000E+05	961.86
0.2000000E+05	0.1999950E+05	961.89
0.2000035E+05	0.2000035E+05	961.84
0.1999965E+05	0.2000035E+05	961.89
0.1999965E+05	0.1999965E+05	961.85
0.2000035E+05	0.1999965E+05	961.90
0.2004913E+05	0.2002035E+05	968.17
0.2002035E+05	0.2004913E+05	970.74
0.1997965E+05	0.2004913E+05	973.45
0.1995087E+05	0.2002035E+05	972.09
0.1995087E+05	0.1997965E+05	968.25
0.1997965E+05	0.1995087E+05	970.90
0.2002035E+05	0.1995087E+05	973.24
0.2004913E+05	0.1997965E+05	971.91
0.2005317E+05	0.2000000E+05	970.38
0.2000000E+05	0.2005317E+05	972.71
0.1994683E+05	0.2000000E+05	970.52
0.2000000E+05	0.1994683E+05	972.36
0.2003760E+05	0.2003760E+05	968.98
0.1996240E+05	0.2003760E+05	973.30
0.1996240E+05	0.1996240E+05	969.10

0.2003760E+05	0.1996108E+05	973.25
0.2011902E+05	0.2000000E+05	974.46
0.2000000E+05	0.2011902E+05	976.81
0.1988098E+05	0.2000000E+05	974.57
0.2000000E+05	0.1988098E+05	976.65
0.2010996E+05	0.2004554E+05	972.00
0.2004554E+05	0.2010996E+05	975.17
0.1995446E+05	0.2010996E+05	977.45
0.1989004E+05	0.2004554E+05	976.35
0.1989004E+05	0.1995446E+05	972.08
0.1995446E+05	0.1989004E+05	975.12
0.2004554E+05	0.1989004E+05	977.32
0.2010996E+05	0.1995446E+05	976.23
0.2008416E+05	0.2008416E+05	972.92
0.1991584E+05	0.2008416E+05	977.31
0.1991584E+05	0.1991584E+05	972.94
0.2008416E+05	0.1991288E+05	977.31
0.2018587E+05	0.2007699E+05	974.76
0.2007699E+05	0.2018587E+05	978.01
0.1992301E+05	0.2018587E+05	980.17
0.1981413E+05	0.2007699E+05	979.06
0.1981413E+05	0.1992301E+05	974.82
0.1992301E+05	0.1981413E+05	977.93
0.2007699E+05	0.1981413E+05	980.08
0.2018587E+05	0.1992301E+05	978.99
0.2020119E+05	0.2000000E+05	977.12
0.2000000E+05	0.2020119E+05	979.56
0.1979881E+05	0.2000000E+05	977.19
0.2000000E+05	0.1979881E+05	979.43
0.2014226E+05	0.2014226E+05	975.68
0.1985774E+05	0.2014226E+05	980.04
0.1985774E+05	0.1985774E+05	975.66
0.2014226E+05	0.1985272E+05	980.09
0.2028320E+05	0.2011731E+05	977.07
0.2011731E+05	0.2028320E+05	980.32
0.1988269E+05	0.2028320E+05	982.31
0.1971680E+05	0.2011731E+05	981.40
0.1971680E+05	0.1988269E+05	977.10
0.1988269E+05	0.1971680E+05	980.17
0.2011731E+05	0.1971680E+05	982.31
0.2028320E+05	0.1988269E+05	981.37
0.2030654E+05	0.2000000E+05	979.13
0.2000000E+05	0.2030654E+05	981.58
0.1969346E+05	0.2000000E+05	979.17
0.2000000E+05	0.1969346E+05	981.65
0.2021675E+05	0.2021675E+05	977.94
0.1978325E+05	0.2021675E+05	982.22
0.1978325E+05	0.1978325E+05	977.87
0.2021675E+05	0.1977559E+05	982.32
0.2043822E+05	0.2000000E+05	981.19
0.2000000E+05	0.2043822E+05	983.51
0.1956178E+05	0.2000000E+05	981.20
0.2000000E+05	0.1956178E+05	983.53
0.2040486E+05	0.2016770E+05	978.92
0.2016770E+05	0.2040486E+05	982.14

0.1983230E+05	0.2040486E+05	984.15
0.1959514E+05	0.2016770E+05	983.15
0.1959514E+05	0.1983230E+05	978.92
0.1983230E+05	0.1959514E+05	982.06
0.2016770E+05	0.1959514E+05	984.17
0.2040486E+05	0.1983230E+05	983.15
0.2030987E+05	0.2030987E+05	979.82
0.1969013E+05	0.2030987E+05	984.04
0.1969013E+05	0.1969013E+05	979.74
0.2030987E+05	0.1967919E+05	984.17
0.2060283E+05	0.2000000E+05	982.91
0.2000000E+05	0.2060283E+05	985.19
0.1939717E+05	0.2000000E+05	982.90
0.2000000E+05	0.1939717E+05	985.20
0.2055694E+05	0.2023069E+05	980.60
0.2023069E+05	0.2055694E+05	983.79
0.1976931E+05	0.2055694E+05	985.82
0.1944306E+05	0.2023069E+05	984.80
0.1944306E+05	0.1976931E+05	980.58
0.1976931E+05	0.1944306E+05	983.74
0.2023069E+05	0.1944306E+05	985.84
0.2055694E+05	0.1976931E+05	984.82
0.2042626E+05	0.2042626E+05	981.50
0.1957374E+05	0.2042626E+05	985.70
0.1957374E+05	0.1957374E+05	981.42
0.2042626E+05	0.1955868E+05	985.84
0.2075160E+05	0.2031132E+05	982.18
0.2031132E+05	0.2075160E+05	985.36
0.1968868E+05	0.2075160E+05	987.40
0.1924840E+05	0.2031132E+05	986.37
0.1924840E+05	0.1968868E+05	982.14
0.2075160E+05	0.1968868E+05	986.41
0.2081352E+05	0.2000000E+05	984.52
0.2000000E+05	0.2081352E+05	986.77
0.1918648E+05	0.2000000E+05	984.49
0.2000000E+05	0.1918648E+05	986.78
0.2057525E+05	0.2057525E+05	983.08
0.1942475E+05	0.2057525E+05	987.27
0.1942475E+05	0.1942475E+05	982.98
0.1968868E+05	0.1924840E+05	985.32
0.2031132E+05	0.1924840E+05	987.43
0.2057525E+05	0.1940443E+05	987.43
0.2099492E+05	0.2041211E+05	983.67
0.2041211E+05	0.2099492E+05	986.83
0.1958789E+05	0.2099492E+05	988.86
0.1900508E+05	0.2041211E+05	987.83
0.1900508E+05	0.1958789E+05	983.62
0.2099492E+05	0.1958789E+05	987.89
0.2107689E+05	0.2000000E+05	986.01
0.2000000E+05	0.2107689E+05	988.24
0.1892311E+05	0.2000000E+05	985.96
0.2000000E+05	0.1892311E+05	988.27
0.1958789E+05	0.1900508E+05	986.81
0.2041211E+05	0.1900508E+05	988.92
0.2076148E+05	0.2076148E+05	984.56

0.1923852E+05	0.2076148E+05	988.74
0.1923852E+05	0.1923852E+05	984.44
0.2076148E+05	0.1921161E+05	988.91
0.2140610E+05	0.2000000E+05	987.43
0.2000000E+05	0.2140610E+05	989.63
0.1859390E+05	0.2000000E+05	987.37
0.2000000E+05	0.1859390E+05	989.70
0.2129907E+05	0.2053809E+05	985.09
0.2053809E+05	0.2129907E+05	988.23
0.1946191E+05	0.2129907E+05	990.24
0.1870093E+05	0.2053809E+05	989.23
0.1870093E+05	0.1946191E+05	985.02
0.1946191E+05	0.1870093E+05	988.24
0.2053809E+05	0.1870093E+05	990.34
0.2129907E+05	0.1946191E+05	989.31
0.2099427E+05	0.2099427E+05	985.97
0.1900573E+05	0.2099427E+05	990.12
0.1900573E+05	0.1900573E+05	985.81
0.2099427E+05	0.1897059E+05	990.33
0.2168839E+05	0.2069935E+05	986.49
0.2069935E+05	0.2168839E+05	989.60
0.1930065E+05	0.2168839E+05	991.59
0.1831161E+05	0.2069935E+05	990.59
0.1831161E+05	0.1930065E+05	986.40
0.2168839E+05	0.1930065E+05	990.68
0.2182750E+05	0.2000000E+05	988.82
0.2000000E+05	0.2182750E+05	990.98
0.1817250E+05	0.2000000E+05	988.74
0.2000000E+05	0.1817250E+05	991.11
0.1930065E+05	0.1831161E+05	989.67
0.2069935E+05	0.1831161E+05	991.72
0.2129224E+05	0.2129224E+05	987.36
0.1870776E+05	0.2129224E+05	991.47
0.1870776E+05	0.1870776E+05	987.15
0.2129224E+05	0.1866209E+05	991.70
0.2235423E+05	0.2000000E+05	990.16
0.2000000E+05	0.2235423E+05	992.28
0.1764577E+05	0.2000000E+05	990.07
0.2000000E+05	0.1764577E+05	992.45
0.2217503E+05	0.2090093E+05	987.84
0.2090093E+05	0.2217503E+05	990.91
0.1909907E+05	0.2217503E+05	992.88
0.1782497E+05	0.2090093E+05	991.89
0.1782497E+05	0.1909907E+05	987.75
0.2217503E+05	0.1909907E+05	992.00
0.1909907E+05	0.1782497E+05	991.10
0.2090093E+05	0.1782497E+05	993.02
0.2166470E+05	0.2166470E+05	988.70
0.1833530E+05	0.2166470E+05	992.76
0.1833530E+05	0.1833530E+05	988.43
0.2166470E+05	0.1827646E+05	993.00
0.2278333E+05	0.2115289E+05	989.17
0.2115289E+05	0.2278333E+05	992.19
0.1884711E+05	0.2278333E+05	994.10
0.1721667E+05	0.2115289E+05	993.15

0.1721667E+05	0.1884711E+05	989.06
0.2278333E+05	0.1884711E+05	993.26
0.2301266E+05	0.2000000E+05	991.46
0.2000000E+05	0.2301266E+05	993.52
0.1698734E+05	0.2000000E+05	991.36
0.2000000E+05	0.1698734E+05	993.67
0.2213027E+05	0.2213027E+05	990.01
0.1786973E+05	0.2213027E+05	993.99
0.1786973E+05	0.1786973E+05	989.80
0.1884711E+05	0.1721667E+05	992.36
0.2115289E+05	0.1721667E+05	994.23
0.2213027E+05	0.1779442E+05	994.22
0.2354371E+05	0.2146785E+05	990.47
0.2146785E+05	0.2354371E+05	993.41
0.1853215E+05	0.2354371E+05	995.26
0.1645629E+05	0.2146785E+05	994.35
0.1645629E+05	0.1853215E+05	990.37
0.2354371E+05	0.1853215E+05	994.46
0.2383569E+05	0.2000000E+05	992.71
0.2000000E+05	0.2383569E+05	994.70
0.1616431E+05	0.2000000E+05	992.61
0.2000000E+05	0.1616431E+05	994.84
0.2271224E+05	0.2271224E+05	991.29
0.1728776E+05	0.2271224E+05	995.16
0.1728776E+05	0.1728776E+05	991.11
0.1853215E+05	0.1645628E+05	993.58
0.2146785E+05	0.1645628E+05	995.39
0.2271224E+05	0.1719188E+05	995.38
0.2488917E+05	0.2000000E+05	993.95
0.2000000E+05	0.2488917E+05	995.84
0.1511083E+05	0.2000000E+05	993.85
0.2000000E+05	0.1511083E+05	995.98
0.2451700E+05	0.2187100E+05	991.79
0.2187100E+05	0.2451700E+05	994.62
0.1812900E+05	0.2451700E+05	996.36
0.1548300E+05	0.2187100E+05	995.52
0.1548300E+05	0.1812900E+05	991.69
0.2451700E+05	0.1812900E+05	995.62
0.2345716E+05	0.2345716E+05	992.58
0.1654284E+05	0.2345716E+05	996.27
0.1654284E+05	0.1654284E+05	992.42
0.1812900E+05	0.1548300E+05	994.78
0.2187100E+05	0.1548300E+05	996.49
0.2345716E+05	0.1642062E+05	996.48
0.2620601E+05	0.2000000E+05	995.14
0.2000000E+05	0.2620601E+05	996.89
0.1379398E+05	0.2000000E+05	995.05
0.2000000E+05	0.1379398E+05	997.03
0.2573361E+05	0.2237494E+05	993.10
0.2237494E+05	0.2573361E+05	995.77
0.1762506E+05	0.2573361E+05	997.37
0.1426639E+05	0.2237494E+05	996.61
0.1426639E+05	0.1762506E+05	993.00
0.2573361E+05	0.1762506E+05	996.71
0.2438832E+05	0.2438832E+05	993.84

0.1561168E+05	0.2438832E+05	997.30
0.1561168E+05	0.1561168E+05	993.70
0.1762506E+05	0.1426639E+05	995.93
0.2237494E+05	0.1426639E+05	997.49
0.2438832E+05	0.1545654E+05	997.48
0.2725437E+05	0.2300486E+05	994.40
0.2300486E+05	0.2725437E+05	996.85
0.1699514E+05	0.2725437E+05	998.26
0.1274563E+05	0.2300486E+05	997.62
0.1274563E+05	0.1699514E+05	994.32
0.2725437E+05	0.1699514E+05	997.70
0.2555226E+05	0.2555226E+05	995.09
0.1444774E+05	0.2555226E+05	998.20
0.1444774E+05	0.1444774E+05	994.95
0.2785208E+05	0.2000000E+05	996.29
0.2000000E+05	0.2785208E+05	997.84
0.1214792E+05	0.2000000E+05	996.21
0.2000000E+05	0.1214792E+05	997.97
0.1699514E+05	0.1274562E+05	997.02
0.2300486E+05	0.1274562E+05	998.36
0.2555226E+05	0.1425145E+05	998.36
0.2995903E+05	0.2000000E+05	997.39
0.2000000E+05	0.2995903E+05	998.68
0.1004097E+05	0.2000000E+05	997.32
0.2000000E+05	0.1004097E+05	998.79
0.2920095E+05	0.2381116E+05	995.75
0.2381116E+05	0.2920095E+05	997.87
0.1618884E+05	0.2920095E+05	999.01
0.1079905E+05	0.2381116E+05	998.52
0.1079905E+05	0.1618884E+05	995.68
0.2920095E+05	0.1618884E+05	998.58
0.2704210E+05	0.2704210E+05	996.34
0.1295790E+05	0.2704210E+05	998.97
0.1295790E+05	0.1295790E+05	996.22
0.1618884E+05	0.1079905E+05	998.03
0.2381116E+05	0.1079905E+05	999.09
0.2704210E+05	0.1270893E+05	999.09
0.2890440E+05	0.2890440E+05	997.58
0.1109560E+05	0.2890440E+05	999.54
0.1109560E+05	0.1109560E+05	997.48
0.3259273E+05	0.2000000E+05	998.40
0.2000000E+05	0.3259273E+05	999.34
0.7407271E+04	0.2000000E+05	998.35
0.2000000E+05	0.7407271E+04	999.42
0.3163417E+05	0.2481903E+05	997.12
0.2481903E+05	0.3163417E+05	998.76
0.1518097E+05	0.3163417E+05	999.56
0.8365835E+04	0.2481903E+05	999.26
0.8365835E+04	0.1518097E+05	997.06
0.3163417E+05	0.1518097E+05	999.30
0.1518097E+05	0.8365831E+04	998.91
0.2481903E+05	0.8365831E+04	999.60
0.2890440E+05	0.1078078E+05	999.62
0.3588485E+05	0.2000000E+05	999.29
0.2000000E+05	0.3588485E+05	999.79

0.4115151E+04	0.2000000E+05	999.26
0.2000000E+05	0.4115151E+04	999.84
0.3123228E+05	0.3123228E+05	998.80
0.8767715E+04	0.3123228E+05	999.87
0.8767715E+04	0.8767715E+04	998.74
0.3467569E+05	0.2607887E+05	998.52
0.2607887E+05	0.3467569E+05	999.49
0.1392113E+05	0.3467569E+05	999.87
0.5324312E+04	0.2607887E+05	999.78
0.5324312E+04	0.1392113E+05	998.49
0.3467569E+05	0.1392113E+05	999.80
0.1392113E+05	0.5324307E+04	999.59
0.2607887E+05	0.5324307E+04	999.89
0.3123228E+05	0.8370596E+04	999.90
0.4000000E+05	0.2000000E+05	1000.00
0.2000000E+05	0.4000000E+05	1000.00
0.0000000E+00	0.2000000E+05	1000.00
0.2000000E+05	0.0000000E+00	1000.00
0.3847759E+05	0.2765367E+05	1000.00
0.2765367E+05	0.3847759E+05	1000.00
0.1234633E+05	0.3847759E+05	1000.00
0.1522409E+04	0.2765367E+05	1000.00
0.1522409E+04	0.1234633E+05	1000.00
0.3847759E+05	0.1234633E+05	1000.00
0.3414214E+05	0.3414214E+05	1000.00
0.5857864E+04	0.3414214E+05	1000.00
0.5857864E+04	0.5857864E+04	1000.00
0.1234633E+05	0.1522402E+04	1000.00
0.2765367E+05	0.1522402E+04	1000.00
0.3414214E+05	0.5357864E+04	1000.00

Example 3.

PROBLEM 1 OF84-459, T=.0155 FT3/S
FEET SECOND

0.0000000E+00	0.1000000E+05	200.00
0.0000000E+00	0.9000000E+04	200.00
0.0000000E+00	0.8000000E+04	200.00
0.0000000E+00	0.7000000E+04	200.00
0.0000000E+00	0.6000000E+04	200.00
0.0000000E+00	0.5000000E+04	200.00
0.0000000E+00	0.4000000E+04	200.00
0.0000000E+00	0.3000000E+04	200.00
0.0000000E+00	0.2000000E+04	200.00
0.0000000E+00	0.1000000E+04	200.00
0.0000000E+00	0.0000000E+00	200.00
0.1000000E+04	0.1000000E+05	171.24
0.1000000E+04	0.9000000E+04	170.71
0.1000000E+04	0.8000000E+04	169.07
0.1000000E+04	0.7000000E+04	166.35
0.1000000E+04	0.6000000E+04	162.96
0.1000000E+04	0.5000000E+04	160.60
0.1000000E+04	0.4000000E+04	162.96
0.1000000E+04	0.3000000E+04	166.35
0.1000000E+04	0.2000000E+04	169.07

0.1000000E+04	0.1000000E+04	170.71
0.1000000E+04	0.0000000E+00	171.24
0.2000000E+04	0.1000000E+05	143.56
0.2000000E+04	0.9000000E+04	142.51
0.2000000E+04	0.8000000E+04	139.22
0.2000000E+04	0.7000000E+04	133.36
0.2000000E+04	0.6000000E+04	124.88
0.2000000E+04	0.5000000E+04	116.48
0.2000000E+04	0.4000000E+04	124.88
0.2000000E+04	0.3000000E+04	133.36
0.2000000E+04	0.2000000E+04	139.22
0.2000000E+04	0.1000000E+04	142.51
0.2000000E+04	0.0000000E+00	143.56
0.3000000E+04	0.1000000E+05	117.98
0.3000000E+04	0.9000000E+04	116.55
0.3000000E+04	0.8000000E+04	111.95
0.3000000E+04	0.7000000E+04	102.99
0.3000000E+04	0.6000000E+04	86.73
0.3000000E+04	0.5000000E+04	55.56
0.3000000E+04	0.4000000E+04	86.73
0.3000000E+04	0.3000000E+04	102.99
0.3000000E+04	0.2000000E+04	111.95
0.3000000E+04	0.1000000E+04	116.55
0.3000000E+04	0.0000000E+00	117.98
0.4000000E+04	0.1000000E+05	95.25
0.4000000E+04	0.9000000E+04	93.77
0.4000000E+04	0.8000000E+04	89.04
0.4000000E+04	0.7000000E+04	79.91
0.4000000E+04	0.6000000E+04	63.51
0.4000000E+04	0.5000000E+04	32.27
0.4000000E+04	0.4000000E+04	63.51
0.4000000E+04	0.3000000E+04	79.91
0.4000000E+04	0.2000000E+04	89.04
0.4000000E+04	0.1000000E+04	93.77
0.4000000E+04	0.0000000E+00	95.25
0.5000000E+04	0.1000000E+05	75.46
0.5000000E+04	0.9000000E+04	74.25
0.5000000E+04	0.8000000E+04	70.54
0.5000000E+04	0.7000000E+04	64.10
0.5000000E+04	0.6000000E+04	55.12
0.5000000E+04	0.5000000E+04	46.51
0.5000000E+04	0.4000000E+04	55.12
0.5000000E+04	0.3000000E+04	64.10
0.5000000E+04	0.2000000E+04	70.54
0.5000000E+04	0.1000000E+04	74.25
0.5000000E+04	0.0000000E+00	75.46
0.6000000E+04	0.1000000E+05	58.09
0.6000000E+04	0.9000000E+04	57.24
0.6000000E+04	0.8000000E+04	54.75
0.6000000E+04	0.7000000E+04	50.85
0.6000000E+04	0.6000000E+04	46.36
0.6000000E+04	0.5000000E+04	43.52
0.6000000E+04	0.4000000E+04	46.36
0.6000000E+04	0.3000000E+04	50.85
0.6000000E+04	0.2000000E+04	54.75

0.6000000E+04	0.1000000E+04	57.24
0.6000000E+04	0.0000000E+00	58.09
0.7000000E+04	0.1000000E+05	42.40
0.7000000E+04	0.9000000E+04	41.87
0.7000000E+04	0.8000000E+04	40.36
0.7000000E+04	0.7000000E+04	38.17
0.7000000E+04	0.6000000E+04	35.96
0.7000000E+04	0.5000000E+04	34.86
0.7000000E+04	0.4000000E+04	35.96
0.7000000E+04	0.3000000E+04	38.17
0.7000000E+04	0.2000000E+04	40.36
0.7000000E+04	0.1000000E+04	41.87
0.7000000E+04	0.0000000E+00	42.40
0.8000000E+04	0.1000000E+05	27.78
0.8000000E+04	0.9000000E+04	27.49
0.8000000E+04	0.8000000E+04	26.66
0.8000000E+04	0.7000000E+04	25.52
0.8000000E+04	0.6000000E+04	24.45
0.8000000E+04	0.5000000E+04	23.98
0.8000000E+04	0.4000000E+04	24.45
0.8000000E+04	0.3000000E+04	25.52
0.8000000E+04	0.2000000E+04	26.66
0.8000000E+04	0.1000000E+04	27.49
0.8000000E+04	0.0000000E+00	27.78
0.9000000E+04	0.1000000E+05	13.76
0.9000000E+04	0.9000000E+04	13.63
0.9000000E+04	0.8000000E+04	13.27
0.9000000E+04	0.7000000E+04	12.78
0.9000000E+04	0.6000000E+04	12.35
0.9000000E+04	0.5000000E+04	12.17
0.9000000E+04	0.4000000E+04	12.35
0.9000000E+04	0.3000000E+04	12.78
0.9000000E+04	0.2000000E+04	13.27
0.9000000E+04	0.1000000E+04	13.63
0.9000000E+04	0.0000000E+00	13.76
0.1000000E+05	0.1000000E+05	0.00
0.1000000E+05	0.9000000E+04	0.00
0.1000000E+05	0.8000000E+04	0.00
0.1000000E+05	0.7000000E+04	0.00
0.1000000E+05	0.6000000E+04	0.00
0.1000000E+05	0.5000000E+04	0.00
0.1000000E+05	0.4000000E+04	0.00
0.1000000E+05	0.3000000E+04	0.00
0.1000000E+05	0.2000000E+04	0.00
0.1000000E+05	0.1000000E+04	0.00
0.1000000E+05	0.0000000E+00	0.00