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**Interactive inversion of dipole loop-loop electromagnetic data for layered earth models  
using numerical integration and complex image theory (Version 1.0)**

**by**

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## SYSTEM REQUIREMENTS

### Hardware Needed

The minimum hardware is: a 386 (DX or SX) IBM compatible PC with at least 3 MB RAM (i.e., 1 MB conventional and 2 MB extended or XMS memory), a 387 math coprocessor, an EGA or VGA video adapter and monitor, a hard disk with more than 3 MB free space, and a 1.44 MB floppy disk drive. The current version of this program requires 1,268,940 bytes of free XMS memory.

### Software Needed

MS-DOS 3.0 or higher, and RSoft (TM) Inc. RPlot Scientific Graphics (Version 2.05). See Appendix 3 for an order form for the RPlot package, currently available at \$149 plus \$5 shipping.

## INTRODUCTION

### Overview

This report is provided as a reference guide to define the basic graphical user interface (GUI) responses needed to perform interactive forward or inverse modeling of dipole loop-loop electromagnetic (EM) data using an IBM compatible personal computer (PC). The underlying model used is a one-dimensional (1-D) horizontal layered earth with parameters being the layer resistivities and thicknesses below the earth's surface. Mathematical details of the layered earth modeling are summarized below and in Appendix 1 that describes two general methods: numerical integration and complex image theory to compute forward or inverse sounding curves in the frequency-domain. Any combination of 5-different dipole loop-loop geometries can be selected in the modeling. The program in this report is named INVDP (for INVersion of DiPole data).

### Scope of This Guide

Ideally, a program using a GUI should not require a detailed reference guide, provided the user is familiar with the subject matter of the application program. A good GUI-type program should be sufficiently intuitive with program prompts and graphical displays so that the user has a visual reminder of the required response in order to properly solve the problem at hand. If the GUI is completely comprehensive, then a reference guide would be redundant. However, as will be evident in what follows, a lot of compromises to this ideal situation are sometimes necessary in practice, and thus a few basic "rules" need to be defined first.

This report has been developed to be about 95 percent reference material, with the first 5 percent devoted to getting started using a few basic definitions. Thus, for a user in a hurry, it is only necessary to read the following material down to the section METHOD OF COMPUTATION. The rest of the report is mainly detailed material for reference purposes (e.g., HotKey options, post plotting options, and troubleshooting), and may not be needed initially for all experienced users.

### Conventions Used in This Guide

The notation: <Key> (e.g., <Enter>, <y> or <n>) used in prompt messages means that a user should respond by pressing the appropriate key, without pressing any additional key; i.e., <Key> is a "HotKey", and requires only a single key-stroke. The <y> or <n> key is used to answer "yes" or "no" to various questions. Other HotKey's are <Esc>, <Home>, <End>, <Up-Arrow>, <Down-Arrow>, and various function keys: <F1>, <F2>, ..., etc. Note that <AnyKey> means that any key can be pressed, except a control key: <Ctrl>, <Alt>, or <Shift>.

Prompts for numerical values must *always* be followed by pressing the <Enter> or <Return> key, e.g., 100 <Enter>. Certain parameter changes require syntax of the form: r2=100 <Enter> to change the resistivity of layer 2, for example; a long version is also allowed as rho2=100 <Enter>, or rh2=100 <Enter> if you want to type more letters. Layer parameters are denoted as "rho" (resistivity in ohm-m) and "h" (thickness in m), and "nl" is the number of layers.

The interaction screen has four subdivisions (or windows--defined later) for text messages and user responses. Text windows are temporarily saved in memory and replaced by a full screen when curves are drawn graphically. After a graph is displayed, pressing <AnyKey> will restore the text windows as previously displayed. A limitation in the present version of this GUI program does not allow graphical and text windows to be placed simultaneously on the viewing screen. This is mainly due to the hardware and software presently employed. A HotKey is available to flip back-and-forth between text and graphic screens to partially overcome this limitation.

## GETTING STARTED

### Installing Software

The supplied diskettes contains the executable program INVDP.EXE, less the required RPlot Scientific Graphics Package. The diskettes also contains test data sets (see Appendix 2 for names of the distributed files). To read all files to a current subdirectory, simply insert the supplied diskette (#1 of 2) in drive b: and type: copy b:\*. \* (Repeat for disk #2.)

Remove the diskette. To run program INVDP from any other directory, make sure the installed directory (and RPlot directory) is in the PATH statement in your AUTOEXEC.BAT file at boot time. (If necessary, see your DOS manual for an explanation of the PATH statement.) If a new PATH is edited in the AUTOEXEC.BAT file, then reboot your system before running INVDP for the first time.

### Quick Start

To run a test problem using the data file TEST1.DAT (supplied on the distributed disk: a free-field file described in more detail later), type the command:

```
invdp
```

Read the Welcome Screen, and press <AnyKey> to continue. At this point, four windows are displayed with the titles:

- |    |                |                       |
|----|----------------|-----------------------|
| 1. | Mode Selection | (Top Left of Screen)  |
| 2. | Parameters     | (Middle of Screen)    |
| 3. | Messages       | (Bottom of Screen)    |
| 4. | Layered Model  | (Top Right of Screen) |

Look at the information in the Message window for what to do next.

A detailed tutorial on running any forward or inverse problem is given in the section STEPS TO RUN SOFTWARE; however, you could try various options on your own, by following the prompts in the Parameter window, but always looking at the Message window for reminders on active <Keys>, etc. If you enter incorrect responses, an "{Error}: ..." message (in green) will appear in the Parameter window indicating what went wrong, and followed by a repeat prompt to continue. Refer to the section TROUBLESHOOTING for additional information on specific error messages.

Some words of caution: Don't try to type ahead (or too fast), and be sure to read each message completely--at least until you become familiar with all the prompt options.

Note that the Layered Model window will be updated when each model parameter is entered. To correct a value before <Enter> is pressed, use the <BackSpace> key. Also, all run information for forward and/or inverse modeling will be saved in an output file for any option selected, and serves as a history file of all action taken; in one case, the output file also becomes an input file for subsequent processing. Similarly, various RPlot files are also saved in your subdirectory for possible post plotting (e.g., laser printing, importing to word processors, etc.; see the section POST PLOTTING OPTIONS for more details.)

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*The rest of this guide is for detailed reference purposes; however, a user unfamiliar with forward or inverse EM modeling methods may want to continue reading.*

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## METHOD OF COMPUTATION

The main idea of using a GUI-type program is to graphically display prompts, user responses, and multiple curves drawn of all computed EM response functions. An attempt has been made in program INVDP to minimize the amount of interactive typing required by assuming needed information about the problem that can be inferred from any observed data read; e.g., number of observations, frequency range, plotting scales, etc.

### Forward Mode

The EM solution for a magnetic dipole loop source and a receiver loop (coil) on or above a 2-layer earth model was developed by Frischknecht (1967), which was an extension of the original work by Wait (1954, 1955) for a homogeneous earth. Subsequent generalization to an n-layered earth was published by Anderson (1979) using the normalization and notation established by Frischknecht (1967). INVDP uses numerical integrations involving many Hankel transforms with a digital filtering algorithm (Anderson, 1979) to efficiently compute the EM fields for any combination of 5 different loop-loop configurations, including a loop and grounded wire type. Thus the mutual coupling ratios for the five dipole loop configurations used in this report are:

1. Horizontal coplanar loops
2. Perpendicular loops
3. Vertical coplanar loops
4. Vertical coaxial loops
5. Horizontal coplanar loop and wire element

In addition to the numerical integration method defined above, I implemented a new fast approximation method using complex image theory originally developed by Bannister (1969, 1986, 1987) for computing the field responses for a homogeneous half-space. Using a technique proposed by Wait (1970), the closed-form (integral-free) image theory formulas for a half-space are easily extended to a layered earth model using a Q-factor correction applied as a normalizer to the half-space propagation term. The layered earth approximation using the modified image theory formulas is described in more detail in Appendix 1.

Comparison of the simplified formulas in Appendix 1 with the numerical integration formulas in Anderson (1979) imply that a much faster computation is afforded by the image method over the integration method. This is discussed more in the Inverse Mode section below.

It should be noted that I have used the quasi-static approximation (i.e., neglecting displacement currents) in computing all forward EM field responses for any loop-loop configuration. No tests are made for violation of this assumption in program INVDP.

Because the exact mathematical solution using numerical integration discussed by Anderson (1979; p.1292-3) is readily available in the literature, I will not repeat these formulas here. However, the approximate solution using image theory, generalized for a layered earth, has not appeared before, and therefore is summarized in detail in Appendix 1.

In program INVDP, the layered earth parameters of interest are denoted as  $\rho_i$  and  $h_i$ ,  $i=1, \dots, nl$ , where for layer  $i$ ,  $\rho_i$  is the resistivity (in ohm-m),  $h_i$  is the thickness (in m), and  $nl$  is the number of horizontal isotropic layers. Allowable range is  $1 \leq nl \leq 10$  in INVDP. In addition to the above 5 loop-loop types, forward sounding curves can be computed and displayed graphically with the following ordinates derived from the mutual coupling ratios  $Z/Z_0$  (Frischknecht, 1967) as:

1. Amplitude
2. Phase
3. In-phase (Real-part)
4. Quadrature (Imaginary-part)
5. Conductivity
6. Tilt angle
7. Ellipticity

The geometry of the loop-loop configuration is specified by the  $r$ -separation distance, transmitter loop height, and receiver loop height, where either or both loops can be on the ground or in the air. Parametric sounding (varying frequency) or geometric sounding (varying  $r$ -separation) can be performed selectively. The fields computed can be expressed either as mutual coupling ratios  $Z/Z_0$  or as percent, e.g., a Max-Min type sounding. If percents are selected via an OPTIONAL DEFAULT SET FILE described below, the formulas used in the computations are,

$$\text{In-phase} = 100 [\text{Re} (Z/Z_0) - 1] (\%),$$

$$\text{Quadrature} = 100 [\text{Im} (Z/Z_0)] (\%).$$

Selecting type 5, conductivity, will compute and display the ordinate in milli-Siemens/m (mS/m) from only the quadrature component of either the horizontal coplanar or vertical coplanar loop systems using the formula (Frischknecht et al., 1991),

$$\sigma_a = 4 \text{Im} (Z/Z_0) / \mu_0 \omega r^2.$$

Types 6 and 7, tilt angle and ellipticity, require both horizontal and perpendicular loop configurations to give the polarization ellipse parameters as expressed by many authors (e.g., Smith and Ward, 1974). The tilt angle and ellipticity formulas given in Smith and Ward are used with the vertical and radial magnetic field components ( $H_z$  and  $H_r$ ) in program INVDP.

In selecting the Forward Mode, one can either generate families of curves, by varying any model parameter(s) at each interaction, or superimpose a set of computed curves over a given observed data set. At any time in the forward modeling, one can choose to Flip Modes (see below) to Inverse Mode via the <Home> key, provided an observed data set was first loaded as an option.

### Inverse Mode

The inversion method used to obtain 1-D layered earth models from dipole loop data follows an algorithm by Anderson (1982) that describes an adaptive nonlinear least-squares method for constrained minimization problems. The original algorithm for unconstrained solutions was originally published by Dennis et al. (1979; 1981), and was used by Anderson (1982) to perform general constrained nonlinear regression. The latter approach is used in a somewhat modified form for program INVDP.

The forward calculation used in the inverse solution is the same as that described above. Note that repeated forward solutions are needed for estimating partial derivatives required in the Inverse Mode, however, the user may ignore the details here. More information on the adaptive least-squares algorithm can be found in Anderson (1982) and Dennis et al. (1979; 1981).

In an attempt to relieve the user of complicated options in performing an inverse solution for a given observed data set, I have devised a set of default control parameters to handle many typical situations. Thus the program automatically obtains as much information about the given observed data and parameters as possible, such as data ranges and parameter bounds, and proceeds with an inversion attempt with minimal user input. For difficult or special cases, some control parameters can be overridden as described in the section OPTIONAL DEFAULT SET FILE.

The Inverse Mode requires a list-directed (free-field) observed data file that must be stored in the current directory before running INVDP. An example of an observed data file is given in a later section. Requirements of the observed data file are listed in section I below.

Nonlinear least-squares inversion can be time-consuming using the numerical integration method, even with fast digital filters for all Hankel transforms, therefore an option to Flip Methods to the image theory formulas can be useful to quickly find an

acceptable starting model as a first approximation of the layered parameters. The image method results are surprisingly good in many cases, and may be sufficient for a final approximate solution. Note that one could always Flip Methods back to numerical integration at any interactive stage to refine an image theory inverse solution.

While fitting a layered earth model to observed data, parameters can be entered or changed at various stages after attempting a least-squares solution. Options are provided to hold any parameter(s) fixed or unfixed at certain points. Trying to find a good starting model might be facilitated graphically by first going into Forward Mode, and then use the flip to Inverse Mode option. In some cases, one could flip back to Forward Mode if unsuccessful with finding a good starting estimate for inverse modeling.

Program INVDP is primarily designed to find a graphical forward and/or inverse solution for parametric (frequency varying) or geometric (r-separation varying) sounding data. Joint inversion of multiple component data at a single site for a layered earth model is easily done by concatenating each component observations in a combined data file. A special case allows for processing EM-31 and/or EM-34 geometric sounding data, discussed in the next section.

## **STEPS TO RUN SOFTWARE**

In general, the following steps (I-V) may be used to run program INVDP for any given forward and/or inverse problem. Step I is required only as a preprocessing step to prepare data for options requiring an observed data input file. The other steps noted are dependent on the option(s) selected at run time.

In all steps, the user is prompted for an output file name, where a summary of all forward and/or inverse activity is listed. The output file has more information about the modeling, especially in Inverse Mode, and is an ASCII formatted file for possible printing later.

### **I. Free-field Observed Data Files**

Program INVDP uses only an ASCII free-field observed data file format. The data file must have a separate record corresponding to each unique frequency or r-separation. The only format restriction is that each data record cannot have any missing values, depending on the observed data type as defined below, and that the frequency or r-separation column is in ascending order and greater than 0. Each data element must be separated by a "space" or "," in each record; values with a decimal point are not necessarily required. It is also helpful if all data files are named with the .DAT suffix so that one of the HotKey's will function properly--see HotKey Options below.

The free-field observed data file must have  $n$ -records ( $2 < n \leq 500$ ), each with 6-columns, and ordered as follows:

$(y(i), (x(i,j), j=1,5), i=1, n)$ , where

$y(i)$  is the Observed value specified by  $x(i,2)$ ,

$x(i,1)$  is the Frequency  $> 0$  (Hz),

$x(i,2)$  defines the "yType" in  $y(i)$  as a decimal number, [loops].[iobs], where:

[loops]=1 for horizontal coplanar loops,

[loops]=2 for perpendicular loops,

[loops]=3 for vertical coplanar loops,

[loops]=4 for vertical coaxial loops,

[loops]=5 for horizontal coplanar loop & wire element, and

[iobs]=1 for amplitude,

[iobs]=2 for phase (deg.),

[iobs]=3 for in-phase (real-part),

[iobs]=4 for quadrature (imag-part),

[iobs]=5 for conductivity (mS/m),

[iobs]=6 for tilt angle (deg.),

[iobs]=7 for ellipticity.

$x(i,3)$  is the r-Separation  $> 0$  (m),

$x(i,4)$  is the Transmitter (Tx) loop elevation  $\geq 0$  (m), and

$x(i,5)$  is the Receiver (Rx) loop or wire elevation  $\geq 0$  (m).

Note that it is very important that  $x(i,2)=yType$  be restricted to a decimal number where the whole part is between 1 and 5, and the decimal fraction is between .1 and .7; e.g., the number 1.1 in  $x(i,2)$  means that  $y(i)$  is an observed amplitude for a horizontal coplanar loop, etc. Some combinations, such as  $x(i,2)=2.5$ , are not allowed because only loop configuration 1 or 3 can be used when a conductivity observation is denoted by the decimal part of yType (i.e., only 1.5 or 3.5 could be used in this case). Similarly, only loop configuration 1 can be used with .6 or .7 decimal parts in yType. All other combinations of whole and decimal parts are possible, but care should be taken to avoid erroneous yType values in  $x(i,2)$  corresponding to the given observed  $y(i)$  value.

To prepare a parametric (frequency) sounding file for a fixed yType in  $x(i,2)$ , make sure that  $x(i,3)$  is constant and  $x(i,1)$  is in ascending order for  $i=1,2,\dots,n$ . Similarly, to prepare a geometric (r-separation) sounding file for a fixed yType in  $x(i,2)$ , make sure that

$x(i,1)$  is constant and  $x(i,3)$  is in ascending order for  $i=1,2,\dots,n$ . An exception to the latter rule is allowed only for EM-31 or EM-34 geometric sounding data, where  $x(i,2)=1.5$  or  $3.5$ ; i.e., in this case,  $x(i,3)$  must be given in ascending r-separation order, but for each  $i$ ,  $x(i,1)$  changes and is the EM-31 or EM-34 frequency corresponding to the r-separation in  $x(i,3)$ , as illustrated by the example in the next paragraph.

To indicate multiple subset data in one file (for joint inversion), simply concatenate each subset data records as defined above, but do not let the total number of observations ( $\Sigma n$ ) exceed 500, which is an arbitrary limit that is considered more than adequate for most dipole loop-loop forward or inverse modeling. An example of amplitude and phase subset data in one file is given in the section EXAMPLE TEST RESULTS. A typical example of a joint data set for EM-31 and EM-34 (containing yType 1.5 and 3.5 subsets) is the following:

```

17.0 39200 1.5 3.66 .75 .75
25.0 6400 1.5 10.0 .0 .0
41.0 1600 1.5 20.0 .0 .0
93.0 400 1.5 40.0 .0 .0
13.5 39200 3.5 3.66 .75 .75
18.5 6400 3.5 10.0 .25 .25
34.0 1600 3.5 20.0 .25 .25
56.0 400 3.5 40.0 .25 .25

```

To run INVDP with a data set having certain observations missing, copy the original file to another file name and delete the appropriate records. Also, a user could write a preprocessor program to create selected data sets for use in INVDP; e.g., some data acquisition systems may produce the observed data in some fixed format, but could be easily reformatted to the free-field data matrix as defined above. The approach used in INVDP was to provide flexibility and freedom in the observed data file, but not restrict the user to a rigid or system-dependent fixed format. In addition, the data record order above allows easy joint inversions without asking the user for many additional on-line prompts during program execution.

Several example data sets for different cases are provided with the supplied diskette, and can be listed to see how the above requirements must be used prior to forward or inverse modeling.

## II. Observed Data Plot Only

This Mode selection requires an observed data file. The data is always listed in the output file when this option is used. It is helpful if all data files have the .DAT suffix so that the <F10> key can be used to invoke the command DIR \*.DAT in the Parameter window. When this option is chosen, the program returns to the main menu after viewing the plot(s). Another Mode must be selected to continue.

### III. Forward Mode

There are two possible options in Forward Mode:

#### (1) Forward: Curves Only

This option does not require an observed data file. The main use of this option is to run program INVDP to produce a family of forward curves without displaying discrete observed data. Because no observed data are used, the user is prompted for the loop and sounding type(s), a minimum x-axis frequency or distance (x1), maximum frequency or distance (xm), and number of points per decade (nx). Selecting the <Home> key to flip to Inverse Mode is disallowed for this option. Note that several stacked or multiple curve sets are possible in this mode, with many multiple curves on separate or nested screens as selected. To avoid many interactive prompts for this option, the next option is advised using an existing data file.

#### (2) Forward: Curves & Obs.Data

This option requires an observed data file. This option can be useful to try various approximate starting models superimposed over the observed data prior to attempting the Inverse Mode. After a few parameter changes (including varying nl), one could try the Flip Mode <Home> key to use the last model as a starting guess for inverse modeling. Note that you can always flip back to Forward Mode from Inverse Mode as well.

In addition, comparison of numerical integration and complex image methods can be performed using the Flip Method <F6> key when noted in the Message window. Each curve method and mode are denoted by symbols and line-types in the plot legend.

### IV. Inverse Mode

There are two possible options in Inverse Mode:

#### (1) Inverse: Initialize & Start

This option requires an observed data file. This is the primary starting point with an observed data set and an initial attempt of inverse modeling. The observed data can be viewed (and reviewed) prior to entering a starting model guess. Because it helps to see the curves while estimating layered parameters, a HotKey <F1> is available at various points to refresh the screen between text and graphical screens.

As mentioned in the METHOD OF COMPUTATION section, many control parameters for typical problems are set to reasonable default values (e.g., NITER=25 sets

the maximum number of nonlinear least-squares iterations allowed to converge). Also the data set itself is used to determine the number of observations (n) by counting records until end-of-file, etc. Each data set is scanned for possible data errors before allowing the run to continue; however, not all errors can be easily detected, and this is one of the reasons why a prior data plot is useful.

After the data file is accepted by INVDP, a starting model is required. It is suggested that a minimum number of layers be used to try to graphically find a "best-fit". Subsequent modeling can be used to insert (or delete) additional layers as deemed necessary. When the program begins computing an inverse solution, the message

... INVDP is running ... Please wait ...

means a pause in response time should be expected. The computation time is very dependent on the starting estimate of the model parameters. If a poor estimate is given, and the default numerical integration option is selected, then for any  $nl > 1$ , the CPU-time could be large. However, if one sets the parameter "image=1", as described in the OPTIONAL DEFAULT SET FILE below, then a much faster time can be expected using the image theory method. The default value "image=0" uses the numerical integration method, unless overridden initially in file INVDP.SET, or after running an inversion, the Flip Method <F6> key is pressed. (Note that <F6> cannot be pressed for the very first run.)

Observe that the updated least-squares solution will appear in the Layered Model window. Parameters can be held fixed (or unfixed) at certain times as noted in the Message window; this may be necessary if too many layers are attempted that are not detectable with the given observed data.

At this point, either an acceptable least-squares solution has been obtained (at least graphically as a good fit), or other starting models may be indicated. In the latter case, one could Flip Modes to the forward case in an attempt to find forward curves better suited as a new starting model; then, you could Flip Modes back again to the Inverse Mode with a better starting guess. Again, you could also Flip Methods back and forth between runs to save time, provided the computation Method is selected *before* the Mode change.

## (2) Inverse: Modify & Restart

This option requires a previous output file, which is always generated, and in turn lists the observed data file that was used. At the prompt to enter the "Restart file name", you must supply the exact file name, which must exist in the current directory. If a suffix of .OUT was used, then the <F9> key can be used to give a directory listing of \*.OUT files in the Parameter window.

If the Restart file exists, program INVDP asks if you want to view the last plot, and then asks for a RUN number of the curve or inverse solution to modify and restart. This RUN number must correspond to an inverse run--not a forward run; if the inverse RUN number is valid, the last solution layered parameters will be updated in the Layered Model window. The numerical method for the selected inversion RUN will also be restored, but can be changed via the <F6> key before restarting the inversion.

At this point, any model parameter (or nl) must be changed as defined in the Message window to continue, and then proceed as in step IV(1) with additional inverse runs.

## V. HotKey Options

As already noted, many HotKey's are used during forward or inverse modeling to perform some immediate action. All active HotKey's will be summarized in the bottom Message window at various stages during the interaction. Most of the user responses to enter information in the Parameter window should be obvious, after reading the Message window definitions. However, for completeness, the following table lists all HotKey's possible:

<u>HotKey</u>	<u>Meaning or Action Taken</u>
< AnyKey >	Continue by pressing any non-control key.
< Enter >	Continue by pressing only the "Enter" key.
< UpArrow >	Scroll Mode window upward to highlight option.
< DownArrow >	Scroll Mode window downward to highlight option.
< Esc >	Cancel operation (returns for new Mode).
< End >	End program INVDP (returns to DOS).
< Home >	Flip Mode (to opposite Mode: Forward <-> Inverse).
< y >	Answer "yes" (do not press <Enter> after <y>).
< n >	Answer "no" (do not press <Enter> after <n>).
< [key] >	Select sounding type [key] defined in the Message window.
< F1 >	Replot (replace text screen with graphic screen).
< F2 >	Delete curve(s).
< F3 >	Undelete curve(s).
< F4 >	Fix/Unfix a parameter.
< F5 >	More changes to make.
< F6 >	Flip Method (to opposite Method: Integration <-> Image).
< F9 >	List file names via DOS command: DIR *.OUT
< F10 >	List file names via DOS command: DIR *.DAT

### Plot and Replot Options

Function keys <F1>, <F2>, and <F3> provide ways to modify the graphic screen at certain times during the interaction. For example, when many curves are displayed for a large number of RUNs, one can use <F2> to ask for a range of RUNs to be temporarily removed from plotting. Any curve or individual set of curves can also be undeleted using <F3> to restore deleted curves again. Note that no additional computations are done via <F2> or <F3> in this manner, but only a control plot file(s) (suffix .PC1, .PC2, ...) is modified in this case. All curves can always be restored after exiting INVDP (see POST PLOTTING OPTIONS section below).

### Flip Modes and Flip Methods

Perhaps the most useful HotKey options are to Flip Modes (<Home> key) between forward or inverse interactions and to Flip Methods (<F6> key) between numerical integration and complex image theory. These options are active immediately after a forward or inverse curve has been displayed. At this point, if the <Home> or <F6> key is pressed, you are asked if you want to flip to the opposite mode or method. If the answer is <n> (in case you made a mistake), then control returns back to the present state. However, if the answer is <y> after pressing <Home>, then the Mode window will reflect the flipped mode with an "\*" character in front of the new mode. One can then proceed with the new mode as if originally selected. Using the Flip Mode or Flip Method options interactively can be a very useful way to investigate hard to fit data sets.

## OPTIONAL DEFAULT SET FILE

If a file named INVDP.SET exists in the working subdirectory where INVDP is run, then certain control parameters can be set (i.e., overridden) from the default values used for typical cases. Refer to Anderson (1982) for a complete description of some of the inversion parameters used below. The remaining control parameters are for special cases as defined in the following table:

Parameter	Default	Action of SET File Control Parameter
iprt	-1	iprt=-1 gives a listing of all data points; iprt<-1 suppresses all calculated data points; iprt>0 gives detailed Inverse Mode lines and iprt=0 gives shorter lines. All listings are given in the output file.

Parameter	Default	Action of SET File Control Parameter
nspline	0	nspline=0 means no spline interpolation is used to plot all computed curves; nspline>0 splines all computed curve plots. nspline=2 or 3 is usually adequate to produce smooth curve plots.
niter	25	maximum number of iterations allowed in Inverse Mode if no convergence obtained. If this limit is reached, then no error statistics are given in the output file since a covariance matrix was not computed.
iwt	2	iwt=2 will perform statistical weighting when in Inverse Mode so that all observations are weighted uniformly (this is highly recommended). One can use iwt=0 to treat all data without weights in the least-squares inversion. Note iwt=1 is not allowed.
sp	3	sp $\geq$ 3 selects an upper/lower bound option in the adaptive nonlinear algorithm defined by Anderson (1982; table 1); sp=3 is the constrained regression option. sp<3 is not recommended for program INVDP. Also sp=3 is usually better than sp=4 here.
eps	.1E-5	requested integration Method accuracy for all Hankel transforms via digital filtering. To obtain the maximum accuracy possible, as described by Anderson (1979), set eps=0 (usually not needed).
ipct	0	ipct=0 computes mutual coupling ratios $Z/Z_0$ , and ipct=1 converts $Z/Z_0$ to percent (e.g., for Max-Min soundings). Plots are annotated via parameter ipct.
ishift	0	ishift=0 means no shift parameter is to be included when in Inverse Mode; ishift=1 will include a shift multiplier (or additive tilt correction) parameter during inversion. An estimate of the shift will be prompted when ishift=1. Note that the shift can be negative for tilt angle data.

Parameter	Default	Action of SET File Control Parameter
iratio( )	0, 0	iratio(i), i=1,2, is a special case to compute ratios of various loop-loop configurations. Normally iratio=0,0 (default) is used to compute $Z/Z_0$ for a selected yType in x(i,2); however, one can set iratio=2,1 (for example) to force INVDP to compute the ratio of loop type 2 to 1. Note that iratio(i)>0, i=1,2, must be set in file INVDP.SET in order to select this option.
rotate( )	0, 0	rotate(i), i=1,2, is a special case to compute a rotation adjustment for a tilted transmitter loop to avoid misorientation errors. The default values rotate=0,0 ignores this option. If $\theta_i = \text{rotate}(i) \neq 0$ (deg.), where $\theta_1$ is the Tx loop tilt from horizontal and $\theta_2$ is the azimuth toward the Rx loop, then the following rotation formulas are used: $Z/Z_0[1] = Z/Z_0[1]\cos(\theta_1) + Z/Z_0[2]\sin(\theta_1)\sin(\theta_2)$ $Z/Z_0[2] = Z/Z_0[2]\cos(\theta_1) - 2 Z/Z_0[4]\sin(\theta_1)\sin(\theta_2)$ $Z/Z_0[3] = Z/Z_0[3]\sin(\theta_1)\cos(\theta_2).$
image	0	image=0 will use the numerical integration Method in either Forward Mode or Inverse Mode. Using image=1 in the SET file will change the initial Method to complex image theory. The <F6> HotKey can also be used to change the initial image control value after the first interaction.
ider	1	ider=1 uses finite difference derivatives in Inverse Mode, and is always required for this version of INVDP. ider=0 may be used in future versions to indicate analytic derivatives are available.
rfactor	100	rfactor > 1 is used to automatically determine upper and lower bounds about each rho(i) when $sp \geq 3$ in Inverse Mode as: $bl(i) \leq \rho(i) \leq bh(i)$ , where $bl(i) = \text{pow}10(\rho(i)/\text{rfactor})/10$ , $bh(i) = \text{pow}10(\rho(i)*\text{rfactor})$ , and pow10 is the nearest power-of-10.

Parameter	Default	Action of SET File Control Parameter
hfactor	10	hfactor > 1 is used to automatically determine upper and lower bounds about each h(i) when $sp \geq 3$ in Inverse Mode as: $bl(i) \leq h(i) \leq bh(i)$ , where $bl(i) = \text{pow}10(h(i)/\text{hfactor})/10$ , $bh(i) = \text{pow}10(h(i)*\text{hfactor})$ , and pow10 is the nearest power-of-10.

The syntax of file INVDP.SET is in NAMELIST format, where parameters can appear in any order (see any Fortran manual). Parameters not given assume the assigned default values; e.g., to override parameters `iprt`, `nspline`, and `image`, use:

```
$set iprt= -2,nspline= 2,image= 1$
```

Note that parameter  $iprt \leq -2$  can be used to reduce the length of the output files by suppressing the calculated data matrix lines. The other default control parameters generally are adequate for "typical" observed data sets. However, changes might be required in some cases. In particular, to process an inversion for any special case noted in the above table, one must *always* include the appropriate control parameters in a INVDP.SET file in the working directory where INVDP is run. In general, as indicated above, I recommend that "image=1" be chosen in the SET file to speed up any initial Inverse Mode run; and then the Flip Method <F6> key can be used later to refine a final inverse solution.

If file INVDP.SET does not exist in the working directory, then the above default control parameters will always be assumed. The output file will always contain a NAMELIST listing of the control parameters as assumed by default or overridden in a INVDP.SET file. If problems occur for special cases, the user should always consult the listing first to verify that correct SET parameter values were used.

### EXAMPLE TEST RESULTS

An example problem, using the observed data file TEST1.DAT on the supplied diskette, is listed below. File TEST1.DAT was generated from a previous Forward Mode run using a known model for this example. To save space, only the output file (called T1.OUT), which also lists the input file contents, is shown as first run in Forward Mode (RUN 1), followed by a flip to Inverse Mode (RUN 2), and finally a flip Method from image to numerical integration (RUN 3). For this example, the INVDP.SET file contains the override parameters (note that the image Method is used initially by setting `image=1`):

```
$set iprt= -2,nspline= 3,image= 1$
```

Only the final graphic screen plot showing the observed data and all three RUNs is reproduced below; also various text screens during the interaction will not be given here. The interested user may try to duplicate these results directly on his/her PC if desired.

### Output File Listing

The output file, generated for all Modes, contains a summary of all activity performed by the user during the entire interaction. The Forward Mode output is very simple, and mainly lists a few derived and input parameters, and possibly computed curve values if default parameter `iprt=-1` is not changed--see above .SET file options. The Inverse Mode output is very similar to the format defined by Anderson (1982); the user is referred to the latter reference for all output parameter definitions used by the adaptive nonlinear least-squares algorithm (Dennis et al., 1979; 1981).

The output file listing for the above example (file name = T1.OUT) follows:

Program INVDP -- Forward: Curves & Obs.Data -- DATE: 10/05/92

Obs.Data File: test1.dat

<Obs.Data>: RUN = 0 FILE(s) = t1.\*

-----

y(i) Obs	x(i,1) Freq	x(i,2) yType	x(i,3) r_Sep	x(i,4) h_Tx	x(i,5) z_Rx	i
0.117489E+01	0.500000E+01	1.1	0.500000E+03	0.000000E+00	0.000000E+00	1
0.121080E+01	0.792447E+01	1.1	0.500000E+03	0.000000E+00	0.000000E+00	2
0.123477E+01	0.125594E+02	1.1	0.500000E+03	0.000000E+00	0.000000E+00	3
0.124220E+01	0.199054E+02	1.1	0.500000E+03	0.000000E+00	0.000000E+00	4
0.123385E+01	0.315479E+02	1.1	0.500000E+03	0.000000E+00	0.000000E+00	5
0.121533E+01	0.500000E+02	1.1	0.500000E+03	0.000000E+00	0.000000E+00	6
0.119336E+01	0.792447E+02	1.1	0.500000E+03	0.000000E+00	0.000000E+00	7
0.117234E+01	0.125594E+03	1.1	0.500000E+03	0.000000E+00	0.000000E+00	8
0.115385E+01	0.199054E+03	1.1	0.500000E+03	0.000000E+00	0.000000E+00	9
0.113773E+01	0.315479E+03	1.1	0.500000E+03	0.000000E+00	0.000000E+00	10
0.112259E+01	0.500000E+03	1.1	0.500000E+03	0.000000E+00	0.000000E+00	11
0.110444E+01	0.792447E+03	1.1	0.500000E+03	0.000000E+00	0.000000E+00	12
0.107170E+01	0.125594E+04	1.1	0.500000E+03	0.000000E+00	0.000000E+00	13
0.995105E+00	0.199054E+04	1.1	0.500000E+03	0.000000E+00	0.000000E+00	14
0.828130E+00	0.315479E+04	1.1	0.500000E+03	0.000000E+00	0.000000E+00	15
0.556664E+00	0.500000E+04	1.1	0.500000E+03	0.000000E+00	0.000000E+00	16
0.446862E+01	0.500000E+01	1.2	0.500000E+03	0.000000E+00	0.000000E+00	17
0.339523E+01	0.792447E+01	1.2	0.500000E+03	0.000000E+00	0.000000E+00	18
0.183964E+01	0.125594E+02	1.2	0.500000E+03	0.000000E+00	0.000000E+00	19
0.980436E-01	0.199054E+02	1.2	0.500000E+03	0.000000E+00	0.000000E+00	20
-0.150759E+01	0.315479E+02	1.2	0.500000E+03	0.000000E+00	0.000000E+00	21
-0.279394E+01	0.500000E+02	1.2	0.500000E+03	0.000000E+00	0.000000E+00	22
-0.381087E+01	0.792447E+02	1.2	0.500000E+03	0.000000E+00	0.000000E+00	23
-0.479715E+01	0.125594E+03	1.2	0.500000E+03	0.000000E+00	0.000000E+00	24
-0.608526E+01	0.199054E+03	1.2	0.500000E+03	0.000000E+00	0.000000E+00	25
-0.808442E+01	0.315479E+03	1.2	0.500000E+03	0.000000E+00	0.000000E+00	26
-0.113574E+02	0.500000E+03	1.2	0.500000E+03	0.000000E+00	0.000000E+00	27
-0.167615E+02	0.792447E+03	1.2	0.500000E+03	0.000000E+00	0.000000E+00	28
-0.256137E+02	0.125594E+04	1.2	0.500000E+03	0.000000E+00	0.000000E+00	29

```
-0.396043E+02  0.199054E+04  1.2  0.500000E+03  0.000000E+00  0.000000E+00  30
-0.597722E+02  0.315479E+04  1.2  0.500000E+03  0.000000E+00  0.000000E+00  31
-0.840411E+02  0.500000E+04  1.2  0.500000E+03  0.000000E+00  0.000000E+00  32
```

```
&SET RFACTOR=100.000,HFACTOR=10.0000,ROTATE=0.000000,0.000000,
EPS=0.100000E-05,IRATIO=0,0,SP=3,IWT=2,IDER=1,IPRT=-2,IPCT=0,
NITER=25,NSPLINE=3,ISHIFT=0,IMAGE=1 &END
```

```
<forward>:  RUN = 1  FILE(s) = t1.*
```

```
-----
nl=  2  x1=  5.000000  nx=  5  xm=  5000.000  image= 1
```

```
rho()=
  0.3000000E+03  0.3000000E+01
h()=
  0.3000000E+03
```

```
RUN= 1 CPU= 0.060 (s) = 0.001 (m)
```

```
Program INVDP -- Inverse: Initialize & Start -- DATE: 10/05/92
```

```
<inverse>:  RUN = 2  FILE(s) = t1.*
```

```
-----
n= 32  k= 3  ip= 0  m= 5
iwt= 2  ider= 1  iprt= -2  niter= 25
sp= 3  image= 1
```

```
parameter lower bounds: bl=
```

```
0.99999997E-05  0.10000000E-02  0.10000000E+02
```

```
initial parameters: b=
```

```
0.33333334E-02  0.33333334E+00  0.30000000E+03
```

```
parameter higher bounds: bh=
```

```
0.10000000E+01  0.10000000E+03  0.10000000E+05
```

```
** itr_dp (ider=0) or nl2sno (ider=1) called **
```

i	initial x(i)	d(i)			
1	0.576806E-01	0.523E+03			
2	0.576806E-01	0.216E+03			
3	0.171214E+00	0.165E+03			
it	nf	f	df	cosmax	var
0	1	0.747E+02		0.843E+00	
2	4	0.347E+01	0.753E+01	0.659E+00	0.261E+02
4	6	0.132E+01	0.141E+00	0.174E-01	0.272E+01
6	7	0.132E+01	0.579E-03	0.100E-02	0.833E-04

```
***** variability convergence *****
```

function	0.132191E+01	variability	0.832618E-04
func. evals	7	grad. evals	6
grad. norm	0.831732E+00	cosmax	0.100043E-02

i	final x(i)	d(i)	g(i)
1	0.639531E-01	0.396E+03	0.263E+00

```

2      0.622798E-01      0.452E+03      -0.736E+00
3      0.135623E+00      0.593E+03      0.285E+00

```

covariance = scale \* (j\*\*t \* j)\*\*-1

```

row 1      0.5908E-06
row 2     -0.1086E-06  0.1887E-05
row 3     -0.1019E-06  0.1294E-05  0.1162E-05

```

\*\* rmserr= 0.14386610E+01 ave|res.err|= 0.180036E+02

covariance matrix

```

1  0.5908E-06
2 -0.1086E-06  0.1887E-05
3 -0.1019E-06  0.1294E-05  0.1162E-05

```

correlation matrix

```

1  0.1000E+01
2 -0.1028E+00  0.1000E+01
3 -0.1230E+00  0.8741E+00  0.1000E+01

```

\*\*parm\_sol. std\_error rel\_error % error \*\*

```

1  0.4094E-02  0.7687E-03  0.1202E-01  0.1202E+01
2  0.3884E+00  0.1374E-02  0.2206E-01  0.2206E+01
3  0.1926E+03  0.1078E-02  0.7948E-02  0.7948E+00

```

=====

INVERSE SOLUTION SUMMARY: RUN = 2 FILE(s) = t1.\* image = 1

nl	rho	(% error)	h	(% error)	depth (sum h)
1	0.2442370E+03	(0.120E+01)	0.1926278E+03	(0.795E+00)	0.1926278E+03
2	0.2574851E+01	(0.221E+01)			

=====

RUN= 2 CPU= 1.150 (s) = 0.019 (m)

<inverse>: RUN = 3 FILE(s) = t1.\*

-----

```

n=      32      k=      3      ip=      0      m=      5
iwt=    2      ider=    1      iprt=    -2      niter=   25
sp=     3      image=    0

```

parameter lower bounds: bl=

```

0.99999997E-05  0.10000000E-02  0.10000000E+02

```

initial parameters: b=

```

0.40943832E-02  0.38837200E+00  0.19262776E+03

```

parameter higher bounds: bh=

```

0.10000000E+01  0.10000000E+03  0.10000000E+05

```

\*\* itr\_dp (ider=0) or nl2sno (ider=1) called \*\*

```

i      initial x(i)      d(i)
1      0.639531E-01      0.358E+03
2      0.622798E-01      0.409E+03
3      0.135623E+00      0.395E+03

```

it	nf	f	df	cosmax	var
0	1	0.576E+01		0.784E+00	
2	3	0.446E-05	0.603E-01	0.872E+00	0.290E+02
4	5	0.689E-07	0.745E-07	0.420E+00	0.942E+01
5	6	0.689E-07	-0.393E-07	0.420E+00	0.705E+01

\*\*\*\*\* x-convergence \*\*\*\*\*

function	0.689031E-07	variability	0.705395E+01
func. evals	6	grad. evals	5
grad. norm	0.661143E-01	cosmax	0.420394E+00

i	final x(i)	d(i)	g(i)
1	0.706991E-01	0.371E+03	-0.757E-02
2	0.706973E-01	0.321E+03	-0.502E-01
3	0.138350E+00	0.426E+03	0.424E-01

covariance = scale \* (j\*\*t \* j)\*\*-1

row 1	0.3526E-13		
row 2	-0.1247E-13	0.2519E-12	
row 3	-0.9323E-14	0.1719E-12	0.1436E-12

\*\* rmserr= 0.12037246E-03 ave|res.err|= 0.208192E-02

covariance matrix

1	0.3526E-13		
2	-0.1247E-13	0.2519E-12	
3	-0.9323E-14	0.1719E-12	0.1436E-12

correlation matrix

1	0.1000E+01		
2	-0.1324E+00	0.1000E+01	
3	-0.1310E+00	0.9040E+00	0.1000E+01

\*\*parm\_sol. std\_error rel\_error % error \*\*

1	0.5000E-02	0.1878E-06	0.2656E-05	0.2656E-03
2	0.5000E+00	0.5019E-06	0.7099E-05	0.7099E-03
3	0.2000E+03	0.3789E-06	0.2739E-05	0.2739E-03

=====

INVERSE SOLUTION SUMMARY: RUN = 3 FILE(s) = t1.\* image = 0

nl	rho	(% error)	h	(% error)	depth (sum h)
1	0.2000002E+03	(0.266E-03)	0.1999998E+03	(0.274E-03)	0.1999998E+03
2	0.2000108E+01	(0.710E-03)			

=====

RUN= 3 CPU= 6.370 (s) = 0.106 (m)

END of RUN(s). Date & Times:

-----

10/05/92

Start = 09:51:13.02  
Stop = 09:55:23.70

Elapsed = 4.178 (m)  
CPU Sum = 0.126 (m) -- for all RUNs

## Discussion of Results

Referring to the above output file listing, several comments concerning the interactive run are noted as follows:

1. The main menu Mode "Forward: Curves & Obs.Data" was selected.
2. The observed data file name was TEST1.DAT (selected output files named T1.\*).
3. The observed data contained joint yType observations denoted by  $x(i,2)=1.1$  for  $i=1,16$  concatenated by  $x(i,2)=1.2$  for  $i=17,32$ . Note that the frequency column  $x(i,1)$  ranges from 5 to 5000 Hz and is repeated for  $i=1,16$  in  $i=17,32$ . The horizontal loop-loop geometry was  $r\_Sep=500$ ,  $h\_Tx=0$ , and  $z\_Rx=0$  (i.e., on the surface).
4. The number of layers (nl) was 2, and the resistivity ( $r[ho]$ ) and thickness (h) values chosen for RUN 1 were:

nl	rho	h
1	300	300
2	3	

5. After step 4 finished, the <Home> key was pressed to Flip Modes from forward to inverse. Here I answered <y> to continue, and also answered <y> to use the last model (RUN 1) as a starting model for the inversion (RUN 2). (Note that I could have modified any parameters before starting RUN 2, but the RUN 1 curves looked somewhat near to the observed data plots.) I then pressed <Enter> to begin the inversion for RUN 2.

6. The inverse solution for RUN 2 was displayed in the Layered Model window as:

nl	rho	h
1	244.2370	192.6278
2	2.574851	18.0 % misfit

Note that 7-figures are printed to handle most ranges, but not all decimal places are significant here. The output file gives the parameter %-errors, but are linear statistics for a nonlinear least-squares solution, provided convergence was obtained. When  $nl \geq 6$ , the Layered Model window is compressed into 6-columns, and only 2-figures are printed under multiple rho and h columns. Again, one should see the output file for complete results about any solution. The %-misfit above is rounded to 1-decimal place, and is the same as the  $ave | \%res.err |$  value (E-format) in the output file. Refer to Anderson (1982) for all notations and definitions given in the output file listing above.

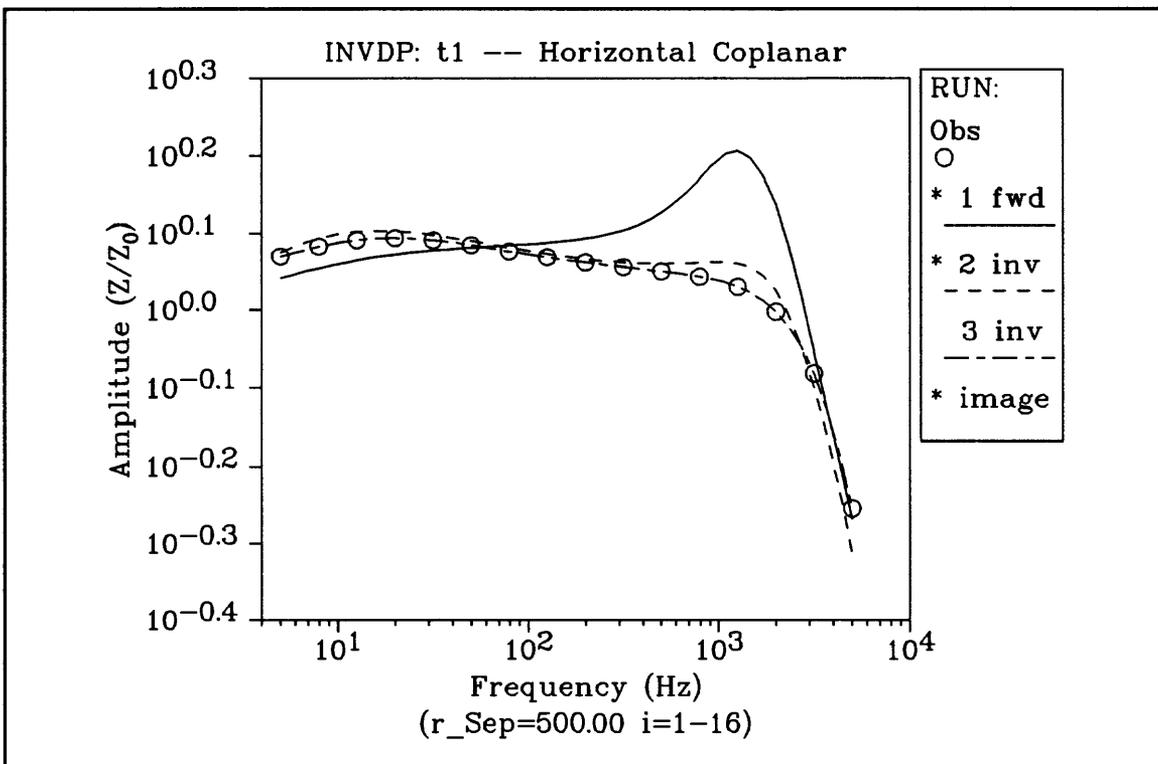
7. At this point, the image Method solution in step 6 showed an 18 % misfit error, but the calculated curves (see plots below) were very close to the observed data, which in this case, I knew that a known model was used by numerical integration to generate the

"observed data". To try to get a better misfit error, I could either attempt adding more layers (e.g.,  $nl \geq 3$ ) or flip to the numerical integration Method. I chose to press the <F6> key to Flip Methods, and then hit <Enter> to continue with another  $nl=2$  fit using numerical integration. As expected, the exact known model was recovered (but with longer CPU-time) in RUN 3 as:

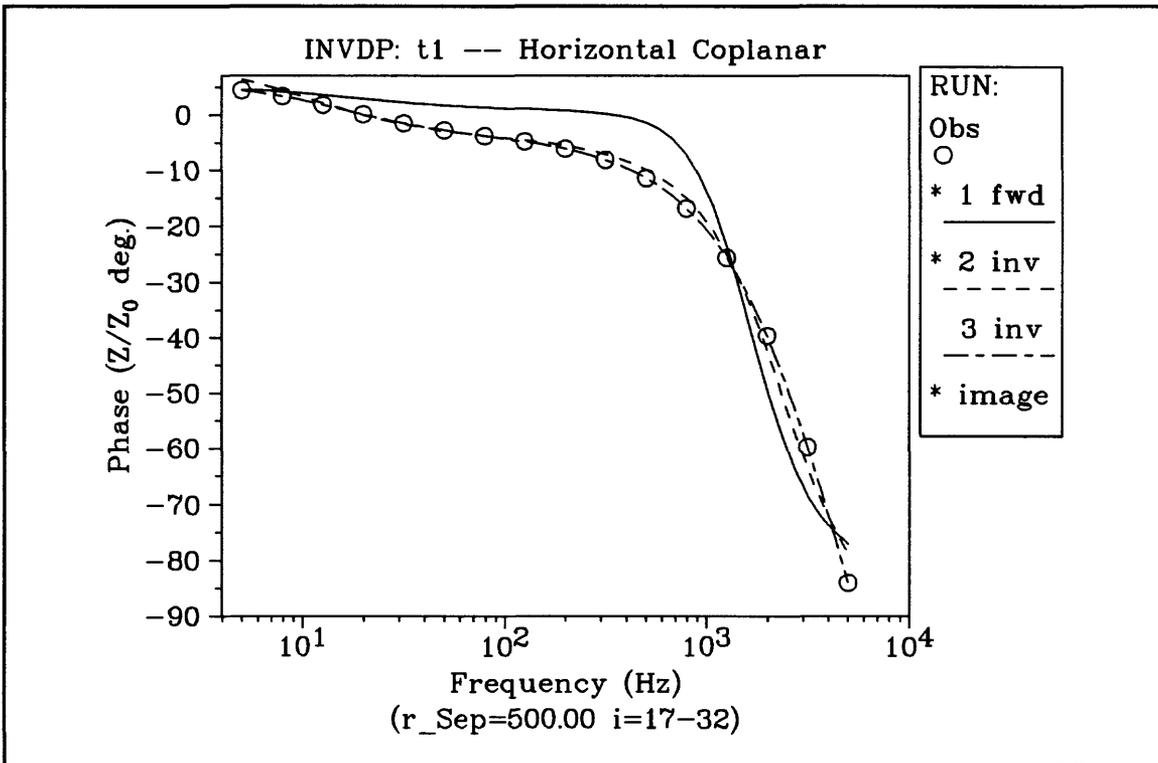
nl	rho	h
1	200.0002	199.9998
2	2.000108	0.0 % misfit

I could have arrived at approximately the same least-squares solution using  $image=0$  initially in the SET file. However, the total CPU-time would probably be much longer than with using  $image=1$  initially as illustrated by this example.

8. The final graphical displays showing the observed data, RUN 1 forward curve, and RUNs 2 and 3 inverse solutions (calculated curves) are reproduced below (imported RPlot file via WordPerfect).



The corresponding phase results to the above amplitude curves are plotted separately in the next graph.



The upper-right legend in each graph denotes the RUN numbers with a following notation "fwd" (forward) and "inv" (inverse) Modes. The first notation is "Obs" (observed), and refers to the plotted observed points denoted by "○". RUN numbers marked with an "\*" notes that the image Method was used (no "\*" means numerical integration Method). Each RUN number curve has a different line-type immediately below the RUN number in the legend, and is displayed in color on a VGA monitor. The color and line-type scheme for the first 3 RUNs are:

RUN	Color	Line-Type
1	Yellow	Solid
2	Light Green	Uniform Dash
3	Light Red	Variable Dash

If more than about 9 RUNs are used, the bottom of the legend may be clipped, unless <F2> is used to temporarily delete selected curves.

9. More details on the inverse solution for RUNs 2 and 3 are given in the output file T1.OUT under the heading: "INVERSE SOLUTION SUMMARY". The summary lines are located in the above output file listing between parallel "===...===" lines.

In the summary, the linear parameter resolution (or % errors) are noted in parentheses following the nonlinear parameter solution, but expressed in Fortran exponential format (instead of the g-format used in the Layered Model window). By looking at the plot and the resolution statistics for RUN 2, the fit appears good, and as mentioned earlier, RUN 3 was used to recover the exact known 2-layer forward model previously run with the parameters:

nl	rho	h	(exact known model)
1	200	200	
2	2		

I leave it as an exercise for the user (if desired) to see what would happen if trying  $nl=3$  after RUN 2. In this case, I would suggest using the main menu Mode "Inverse: Modify & Restart" with output file T1.OUT as a Restart file, and select the inverse solution for RUN 2 to modify; then insert a new layer (e.g., after layer 1) with some reasonable guesses for the new second layer parameters. At the Change Parameter prompt, begin by typing:  $nl=3$  <Enter>. Note that INVDP will automatically reorder the previous values in the Layered Model window, so that reentering existing values are not necessary; also a new layer can be inserted immediately after the surface by using an index of 0; layers can be inserted/deleted interactively by using a different  $nl$  than last used, and responding to the prompts for new values.

### Other Observed Data Files

The distributed diskette also contains a few "real observed data" files taken from a loop-loop field survey at Medicine Lake, California, and published by Anderson et al. (1983). These files are supplied as additional test data that was previously interpreted using another method, and can be used to further test program INVDP for comparison purposes (or for practice). The following additional data files are provided on the supplied diskette:

<u>File Name</u>	<u>From Anderson et al. (1983, Table, p.19)</u>
MED2.DAT	Line 2, Sta.1C, Tilt-&-Ellipticity, p.23-25
MED15.DAT	Line 15, Sta.6C, Ratio $H_r/H_z$ , p.62-64
MED26.DAT	Line 26, Sta.8B, Field $E_x$ , p.95-97
MED27.DAT	Line 27, Sta.9B, Ellipticity, p.98-100

It may be instructive to try your hand at using any of the above "real data sets" without looking at the previously determined least-squares solutions published by Anderson et al. (1983). The Medicine Lake data used some of the control parameters previously

defined. The control parameters that would be required in the INVDP.SET file to run any of the above data sets are given in the following table:

File Name	rotate( )	iratio( )	ishift
MED2.DAT	-.3, 5.9	0, 0	0
MED15.DAT	-.7, 34.3	2, 1	0
MED26.DAT	0, 0	0, 0	1
MED27.DAT	2.5, 58.96	0, 0	0

### Conclusions About Inversion

I would like to point out that "real" field data measured over a limited frequency range that contains instrumental noise would always make finding proper (or acceptable) least-squares solutions much more difficult than in the above example. In many cases, it may be necessary to hold some parameters fixed that cannot be sufficiently resolved due to insufficient data or when trying too many layers.

Perhaps the usefulness of this program to view graphically the observed data along with a search for a probable 1-D layered model may be its greatest advantage. Certainly the ability to flip between forward and inverse modes (and/or image and numerical integration methods) at any time should make the search easier.

### POST PLOTTING OPTIONS

#### RPlot System (CopyRight by RSoft, Inc.)

Various post plotting capabilities are easily provided using the RPlot Graphical Package. After a final curve plot is obtained from INVDP on-line, it is possible to modify some of the generated plot files for subsequent hard copy output to laser printers, changing fonts, importing to word processors (as done in the above section), etc. The RPlot package is very versatile, and has many special options for controlling a printer, plot resolution, paper orientation, etc.

It is beyond the scope of this report to indicate all possible RPlot usage as a "stand-alone" plot package. Note, however, that INVDP produces as a by-product several RPlot-type files in the current directory; specifically, the file name prefix (denoted [prefix] below) is taken the same as in the output file name. The RPlot-type files would be:

[prefix].PC1, [prefix].PC2, ..., etc. (max. 35 possible .PCx files)

[prefix].100, [prefix].200, ..., (copy of obs.data points for RPlot)  
 [prefix].101, [prefix].201, ..., (possible splined curves for RUN 1)  
 [prefix].102, [prefix].202, ..., (possible splined curves for RUN 2)

... etc ...

The above RPlot files with a suffix .PCx (with x=1,2,...,9,A,B,...,Z) are automatically generated by program INVDP to contain standard fonts and other RPlot control plot options. It is very easy to change these files to produce other RPlot effects after running INVDP. For example, to just view the very last graphic screen(s) after exiting INVDP, simply type:

plot @[prefix].PC1 (or .PC2, etc.)

where [prefix] is the output file prefix name (without the [ ]'s).

Refer to the RPlot User's Manual on how to modify any .PCx file for special effects. One easy exclude/include modification to a .PCx file is to insert/delete a "!" character at the beginning of a line (i.e., anything after an ! is treated as a comment). For example, this is how curves are deleted or undeleted while running INVDP. Thus, all curves that were deleted while running INVDP can easily be replotted after exit by removing the ! on all RUN lines (near the end of the file). Instead of commenting out a complete line, any line can be edited for special effects; e.g., in the above example plots in this report, the .PCx files were modified by removing the text after \n (but not the last "" character) in the line:

```
/tb'(r\_Sep=500.00 i=1-16)\nPress <AnyKey> to Continue'
```

as

```
/tb'(r\_Sep=500.00 i=1,16)'
```

One very useful RPlot post plotting option is the ability to import a final graphic file from INVDP directly into most current word-processing files for IBM compatible PCs. RPlot can generate an HPGL, CGM, or EPS formatted file for this purpose. For example, any .PCx file must first be renamed with a .PCS suffix, and comment line !/pf can be changed to /pf; then issue the plot command "plot @[prefix]" (without a suffix) to produce an HPGL file with the suffix .PLT; this file can then be imported directly into WordPerfect, or other word processors. Again, refer to the RPlot User's Manual for more information.

## TROUBLESHOOTING

A multitude of built-in error messages are provided by program INVDP. Most user entry errors are detected immediately after the associated prompt in the Parameter window; the syntax of the error message is:

{Error}: ... message here ...

The error message is displayed in "green" in the Parameter window, and is usually followed by another prompt to reenter with a correct value or HotKey. In some cases, it may be necessary to cancel the Mode using the <Esc> key, or to try another option defined in the Message window. Sometimes, pressing <Enter> a second time may be necessary to make active a previous set of option keys--use this approach only to get out of a jam! Also, <Ctrl-C> can be used to abort the DOS run at certain times.

### Some Specific Error Messages and Possible Solutions

{Error}: BAD Syntax, ...TRY Again...

This is a catch-all when invalid numbers or names are entered. Do not enter non-numeric characters when entering numbers, although + or . characters are allowed in entered numbers.

When making parameter changes, the correct syntax must be:

< name > [layer] = < value > < Enter >

where [layer] is required (without the [ ]'s) only for a layer parameter name; e.g., r2=100<Enter> -or- rho2=100<Enter> -and- nl=3<Enter>

Note that the "=" character must always be used here.

When [un]fixing a layer parameter, the correct syntax must be:

[-]< name > < layer > < Enter >

where [-] is optionally used to unfix a layer parameter name; e.g., r2<Enter> will FIX r2, and -r2<Enter> will UNFIX r2. Note that the "=" character cannot be used here.

**{Error}: Invalid KEY, ...TRY Again...**

Only keys listed in the Message window can be pressed. Generally, this error is when `<y>`, `<n>`, or another `<[key]>` was required. The only way out, in this case, is to enter one of these keys (without a following `<Enter>` key!).

**{Error}: file name has 0 chars.**

**{Error}: file name missing a "."**

**{Error}: file name missing prefix.**

**{Error}: file name missing prefix. >9 chrs.**

**{Error}: file name suffix cannot begin with ".pc"**

This group of messages all pertain to entering an invalid file name. The rule under MS-DOS is that a file name prefix must have 8 or fewer characters before the "." separator ( "." always required in program INVDP), followed by 3 or fewer characters for the file name suffix; e.g., 12345678.123 is a valid file name, but usually a letter is used to start the file name prefix.

In program INVDP, file names without the "." separator are not allowed, because the program uses the `[prefix.]` to create all the RPlot associated files. It is suggested that all observed data files have a .DAT suffix so that the `<F10>` HotKey is functional, and output files have a suffix different than .DAT (e.g, .OUT or .LIS).

The last file name error above (".PC" suffix) is to avoid any conflicts with RPlot .PCx (x=1,2,...) plot files. Simply choose another file name with a different suffix to continue.

**{Error}: No changes were made for this RUN (?)...TRY Again, or Quit**

This error occurs when continuing by pressing `<Enter>` without making any parameter changes from the previous RUN. Sometimes this is really not an error, but an oversight by the user; other times this can occur when flipping Modes. In the latter case, simply change any parameter before ending the changes with a null `<Enter>`.

**{Error}: Cannot open Restart file**

**{Error}: BAD Restart file; no Obs.Data File line (?)**

**{Error}: BAD Restart file; no Obs.Data File name (?)**

**{Error}: BAD Restart file: Curve type wrong!**

A previous "output" file must be used as a Restart "input" file. If a suffix of .OUT was used, then the `<F9>` key can be pressed to display all \*.OUT file names in the

current directory. Note that this Mode (Inverse: Modify & Restart) can only be restarted from an existing output file that contained at least one Inverse Mode run. This also implies that all associated RPlot files must exist in the current directory in order to replot the previous results prior to restarting.

{Error}: Cannot open Obs.Data file!  
 {Error}: BAD Syntax in Obs.Data file!!  
 {Error}: Improper numeric fields in Obs.Data file  
 {Error}: nobs > 500 in Obs.Data file  
 {Error}: x(i,j) not increasing for constant x(i,k) [j or k = 1 or 3]  
 {Error}: Less than 3 Obs in joint data at x(i,j)  
 {Error}: x(i,j) ≤ 0 [j = 1 or 3]; i in [1,n], n=nobs  
 {Error}: x(i,j) < 0 [j = 4 or 5]  
 {Error}: x(i,2) < 1.1 or > 5.7  
 {Error}: x(i,2)=loops.iobs with iobs=0 or > 7 not allowed  
 {Error}: x(i,2)=loops.iobs has iobs ≥ 6 but loops.ne.1  
 {Error}: x(i,2)=loops.iobs has iobs=5 & loops.ne.1 or 3

These errors all pertain to an invalid or unknown observed data file. See the section STEPS TO RUN SOFTWARE (Free-field Observed Data Files) for the proper file syntax to use. Also, list any of the \*.DAT files on the supplied diskette for examples of correctly ordered data sets.

### Other Undocumented Problems

If other undocumented errors occur using INVDP with seemingly proper observed data files, then I would appreciate receiving feedback information from users. Even though several tests were made during a beta-testing stage, errors of omission or other unknown errors may still exist in the present version.

### Future Plans

Version 1.0 of INVDP should be called about 1.5 due to the changes already made while developing this program. I intend to look for economical ways to display text and graphic screens simultaneously, possibly using MS-Windows and another Fortran compiler; if and when this happens, perhaps an improved Version 2.0 (or 3.0) will become available in the future. Meanwhile, the present version will hopefully serve as a good starting point for interactive inversion using a 386 (or 486) 32-bit PC.

## ACKNOWLEDGMENTS

I would like to thank Bob Horton and Adel Zohdy, both from the USGS, for their review comments and/or running an earlier version of this program during the beta-testing and development stages. Many of their suggestions for improvement to the present version were adopted where feasible.

## REFERENCES

- Anderson, W.L., 1979, Numerical integration of related Hankel transforms of orders 0 and 1 by adaptive digital filtering: *Geophysics*, **44**, 1287-1305.
- , 1982, Adaptive nonlinear least-squares solution for constrained or unconstrained minimization problems (Subprogram NLSOL): U.S. Geological Survey Open-File Report 82-68.
- Anderson, W.L., Frischknecht, F.C., Raab, P.V., Bradley, J.A., and Turnross, J., 1983, Results and preliminary inversion of loop-loop frequency-domain electromagnetic soundings near Medicine Lake, California: U.S. Geological Survey Open-File Report 83-830.
- Bannister, P.R., 1969, The image theory quasi-static fields of antennas above the earth's surface: Navy Underwater Systems Center, Rep. 1061.
- , 1986, Applications of complex image theory: *Radio Science*, **21**, 605-616.
- , 1987, Extension of finitely conducting earth-image-theory results to any range, *in* Simplified expressions for the electromagnetic fields of elevated, surface, or buried dipole antennas: Naval Underwater Systems Center. (Also avail. as Navy Underwater Systems Center, Rep. 6885, 1984.)
- Dennis, J.E., Gay, D.M., and Welsh, R.E., 1979, An adaptive nonlinear least-squares algorithm: Univ. of Wisconsin MRC Tech. Sum. Rept. 2010 (also available as NTIS Rept. AD-A079-716).
- , 1981, An adaptive nonlinear least-squares algorithm: *ACM Transactions on Mathematical Software*, **7**, 348-368.
- Frischknecht, F.C., 1967, Fields about an oscillating magnetic dipole over a two-layer earth and application to ground and airborne electromagnetic surveys: *Quart. Colo. School of Mines*, **62**.

Frischknecht, F.C., Labson, V.F., Spies, B.R., and Anderson, W.L., 1991, Profiling methods using small sources, *in* Electromagnetic methods in applied geophysics, Vol. 2, applications: Soc. of Expl. Geophys., Tulsa, OK, Nabighian, M.N., ed.

Smith, B.D., and Ward, S.H., 1974, On the computation of polarization ellipse parameters: *Geophysics*, **39**, 867-869.

Wait, J.R., 1954, Mutual coupling of loops lying on the ground: *Geophysics*, **19**, 290-296.

-----, 1955, Mutual electromagnetic coupling of loops over a homogeneous ground: *Geophysics*, **20**, 630-637.

-----, 1970, *Electromagnetic waves in stratified media*, Pergamon Press, Inc.

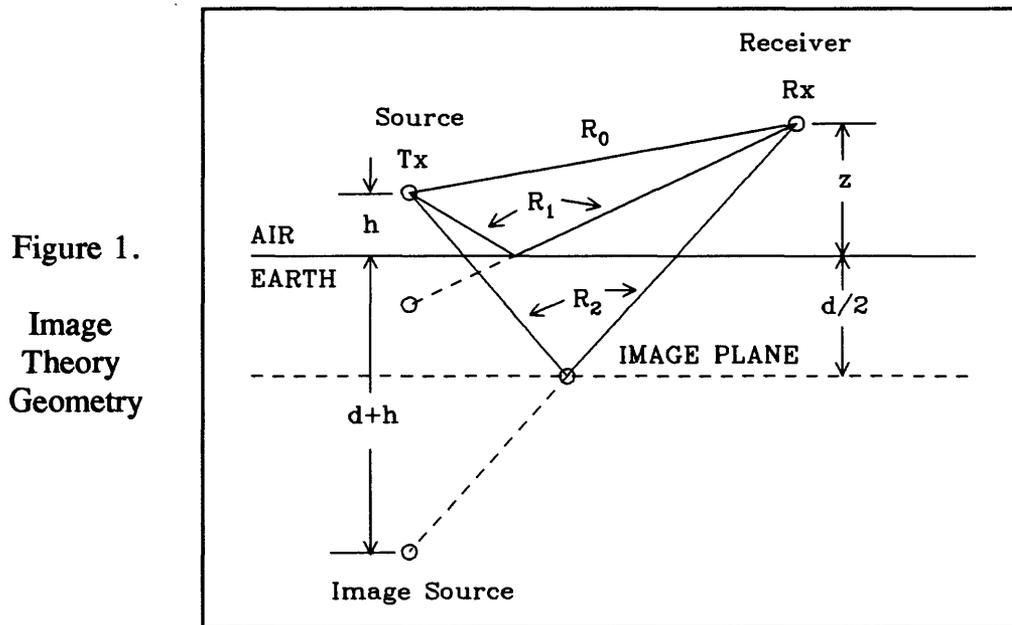
## APPENDIX 1. -- Complex image theory extended to layered earth models

The purpose of this appendix is to (a) summarize Bannister's (1987) image theory formulas for five loop-loop configurations on or above a homogeneous half-space, and (b) to give new formulas for extending the half-space image solution to any layered earth model.

### (a) Half-space image theory approximation

Bannister (1987) presents a detailed derivation of the complex image theory solution for various dipole sources on, above, or in a conductive half-space. In the formulas needed for program INVDP, I only consider solutions on or above the surface, and in particular, only for a vertical magnetic dipole (VMD), a horizontal magnetic dipole (HMD), or a horizontal electric dipole (HED). An interesting summary of the complex image theory is also presented by Bannister (1986), and may be more readily available than the 1987 reference. In this appendix, only the pertinent formulas for five loop-loop configurations will be discussed.

Figure 1 shows the geometry of a transmitter (Tx) loop source and a receiver (Rx) loop or coil on or above the surface of a homogeneous half-space.



As described by Bannister (1987), an image source is placed at a complex depth  $(d+h)$ , where  $d=2/\gamma_1$  is defined from the depth  $d/2$  of the image plane. The other symbols used in Figure 1 and remaining formulas are given in the following table:

## GLOSSARY OF SYMBOLS

A	$A = (1 + \Gamma_{11})/2 + F(w)(1 - \Gamma_{11})/2$ , used in equation (4)
d	$d = 2/\gamma_1$ , complex image depth
F(w)	$F(w) = 1 - i(\pi w)^{1/2} \exp(-w) \operatorname{erfc}(iw^{1/2})$ , used in A
h	height ( $h \geq 0$ m) of transmitter loop ( $h_{Tx}$ )
i	$i = (-1)^{1/2}$
m	magnetic dipole moment (Amp-m <sup>2</sup> )
$\eta$	$\eta = \gamma_1/\gamma_0$ , index of refraction
p	electric current moment (Amp-m)
r	$r = (x^2 + y^2)^{1/2}$ , Tx-Rx r-separation in xy-plane
$R_0$	$R_0 = [r^2 + (z-h)^2]^{1/2}$ (see Figure 1)
$R_1$	$R_1 = [r^2 + (z+h)^2]^{1/2}$ (see Figure 1)
$R_2$	$R_2 = [r^2 + (d+z+h)^2]^{1/2}$ , complex distance
w	$w = -\gamma_0 R_1 (\sin \psi_1 + \Delta)^2 / 2$ , used in F(w)
z	height ( $z \geq 0$ m) of receiver loop ( $z_{Rx}$ )
$\Gamma_{11}$	$\Gamma_{11} = (\sin \psi_1 - \Delta) / (\sin \psi_1 + \Delta)$ , used in A
$\gamma_0$	$\gamma_0 = (-\omega \mu_0 \epsilon_0)^{1/2}$ , propagation in air
$\gamma_1$	$\gamma_1 = (i\omega \mu_0 \sigma_1)^{1/2}$ , quasi-static propagation in half-space
$\Delta$	$\Delta = 1/\eta = \gamma_0/\gamma_1$ , refraction index reciprocal
$\epsilon_0$	$\epsilon_0 = 10^{-9}/36\pi$ , permittivity of free space (Farads/m)
$\sigma_1$	conductivity of lower half-space (Siemens/m)
$\phi$	$\phi = \tan^{-1}(y/x)$ , Tx-Rx azimuth in xy-plane
$\mu_0$	$\mu_0 = 4\pi \cdot 10^{-7}$ , permeability of free space (Henries/m)
$\psi_0$	$\psi_0 = \tan^{-1}[(z-h)/r]$ , elevation angle
$\psi_1$	$\psi_1 = \tan^{-1}[(z+h)/r]$ , elevation angle
$\psi_2$	$\psi_2 = \tan^{-1}[(d+z+h)/r]$ , complex elevation angle
$\omega$	$\omega = 2\pi f$ , angular frequency (Radians/s), $f > 0$ Hz.

The complex image theory formulas from Bannister (1987; Table 2) for the magnetic field components on or above a homogeneous earth for five common loop-loop types are:

1. The  $H_z$  field for a VMD (loop type 1, horizontal coplanar),

$$H_z = -m \left\{ [(1 + \gamma_0 R_0 + \gamma_0^2 R_0^2) - \sin^2 \psi_0 (3 + 3\gamma_0 R_0 + \gamma_0^2 R_0^2)] \exp(-\gamma_0 R_0) / R_0^3 - [(1 + \gamma_0 R_2 + \gamma_0^2 R_2^2) - \sin^2 \psi_2 (3 + 3\gamma_0 R_2 + \gamma_0^2 R_2^2)] \exp(-\gamma_0 R_2) / R_2^3 \right\} / 4\pi \quad (1)$$

2. The  $H_r$  field for a VMD (loop type 2, perpendicular),

$$H_r = m \left\{ [(3 + 3\gamma_0 R_0 + \gamma_0^2 R_0^2) \sin \psi_0 \cos \psi_0] \exp(-\gamma_0 R_0) / R_0^3 - [(3 + 3\gamma_0 R_2 + \gamma_0^2 R_2^2) \sin \psi_2 \cos \psi_2] \exp(-\gamma_0 R_2) / R_2^3 \right\} / 4\pi \quad (2)$$

3. The  $H_\phi$  field for a HMD (loop type 3, vertical coplanar),

$$H_\phi = -m \cos \phi \left\{ (1 + \gamma_0 R_0 + \gamma_0^2 R_0^2) \exp(-\gamma_0 R_0) / R_0^3 + (1 + \gamma_0 R_2) \exp(-\gamma_0 R_2) / R_2^3 + [\Gamma_{11} + (1 - \Gamma_{11}) F(w)] \gamma_0^2 R_1^2 \exp(-\gamma_0 R_1) / R_1^3 \right\} / 4\pi \quad (3)$$

4. The  $H_r$  field for a HMD (loop type 4, vertical coaxial),

$$H_r = m \sin \phi \left\{ [(2 + 2\gamma_0 R_0) - \sin^2 \psi_0 (3 + 3\gamma_0 R_0 + \gamma_0^2 R_0^2)] \exp(-\gamma_0 R_0) / R_0^3 + [(2 + 2\gamma_0 R_2 + \gamma_0^2 R_2^2) - \sin^2 \psi_2 (3 + 3\gamma_0 R_2 + \gamma_0^2 R_2^2)] \exp(-\gamma_0 R_2) / R_2^3 - [(2 + 2\gamma_0 R_1 + \gamma_0^2 R_1^2) - \sin^2 \psi_1 (3 + 3\gamma_0 R_1 + \gamma_0^2 R_1^2)] \exp(-\gamma_0 R_1) / R_1^3 + [(2 + 2\gamma_0 R_1 A) - \sin^2 \psi_1 (3 + 3\gamma_0 R_1 + \gamma_0^2 R_1^2)] \exp(-\gamma_0 R_1) / R_1^3 \right\} / 4\pi \quad (4)$$

5. The  $H_z$  field for a HED (loop type 5, horizontal coplanar and wire element),

$$H_z = p \sin \phi \left\{ (1 + \gamma_0 R_0) \cos \psi_0 \exp(-\gamma_0 R_0) / R_0^2 - (1 + \gamma_0 R_2) \cos \psi_2 \exp(-\gamma_0 R_2) / R_2^2 \right\} / 4\pi \quad (5)$$

Note that in equations (1)-(5), only complex exponential or trigonometric functions are required to approximate the magnetic field components. Therefore, one should expect a much faster computation time using the image Method (image=1) over the numerical integration Method (image=0) in program INVDP. The only restriction in using (1)-(5) is that  $|\eta^2| \geq 10$ , which is usually true for typical earth models.

Equations (1)-(5) give the total magnetic fields (i.e., primary plus the secondary fields), and thus the mutual coupling ratios can easily be computed as

$$Z/Z_0 = (\text{primary} + \text{secondary}) / \text{primary} = \text{total} / \text{primary}.$$

Assuming  $m=p=1$  and  $\phi=\pi/2$ , the primary fields used in INVDP are:

INVDP loop-type	primary field used in $Z/Z_0$
1	$H_{zp} = -[3(z-h)^2/(4\pi R_0^5) - 1/(4\pi R_0^3)]$
2	same as in loop-type 1
3	$H_{\phi p} = -1/(4\pi R_0^3)$
4	$H_{rp} = \{2-3\sin\psi_0[(z-h)/R_0]^2\}/(4\pi R_0^3)$
5	$H_{zp} = \cos\psi_0/(4\pi R_0^2)$

(b) Layered earth image theory approximation

It is relatively easy to extend the half-space image solution given in section (a) above using a technique discussed by Wait (1970). Wait showed that a Q-factor correction can be applied as a normalizer to the half-space propagation term ( $\gamma_1$ ) to produce the response of any layered earth model.

Let the earth have any number of horizontal stratified isotropic layers ( $nl \geq 1$ ) with the parameters

$$\begin{aligned} h_j &= \text{layer-}j \text{ thickness (m), } j=1,2,\dots,nl-1, \\ \sigma_j &= \text{layer-}j \text{ conductivity (mhos/m), or } \rho_j = 1/\sigma_j \text{ (ohm-m), } j=1,2,\dots,nl, \text{ and} \\ \mu_j &= \mu_0 \text{ assumed for all } j=1,2,\dots,nl. \end{aligned}$$

The propagation term in each layer has the form (neglecting displacement currents)

$$\gamma_j^2 = i\omega\mu_0\sigma_j, \quad j=1,2,\dots,nl.$$

Using backward recurrence, starting from the bottom layer ( $j=nl$ ), a Q-factor can be propagated upward to the first layer as

$$Q^{(j-1)} = [\gamma_{j-1}Q^{(j)} + \gamma_j \tanh(\gamma_{j-1}h_{j-1})] / [\gamma_j + \gamma_{j-1}Q^{(j)} \tanh(\gamma_{j-1}h_{j-1})], \quad (6)$$

where  $j=nl,\dots,2$ ,  $Q^{(nl)}=1$ , and the last  $Q^{(1)}$  in equation (6) is used to modify  $\gamma_1$  as

$$\gamma_1^* = \gamma_1/Q^{(1)}. \quad (7)$$

Equation (7) is the Q-factor correction for a layered earth to be used in the half-space equations (1)-(5) with  $\gamma_1^*$  replacing  $\gamma_1$ . It should be noted that the  $\tanh(Z)$  complex function in equation (6) is evaluated using an  $\exp(Z)$  complex function as

$$\tanh(Z) = [1 - \exp(-2Z)] / [1 + \exp(-2Z)], \quad Z = \gamma_{j-1} h_{j-1}.$$

Observe that when  $n_l=1$  (a half-space) equation (6) reduces to  $Q^{(1)}=1$ , and hence equation (7) is not needed. Application of equations (6) and (7) for  $n_l > 1$  prior to using equations (1)-(5) completes the derivation of the layered earth image theory approximation method.

## **APPENDIX 2. -- Source and Executable Codes (less RPlot Software)**

The source code for program INVDP (less the RPlot Software, available via Appendix 3) was written in extended Fortran-77 language. The Fortran compiler, window library, and protected mode 386 operating system used to generate INVDP.EXE was:

Lahey F77L-EM/32 (Version 5.01)

Lahey SpinDrift Windows Library (Version 2.01)

Lahey/Phar Lap 386|DOS-Extender (Version 4.1L)

The Lahey Fortran system allows calling an MS-DOS command internally and returning like a normal subprogram call. In this way, I was able to interface the RPlot Software (commercially available from RSoft, Inc.) without needing additional plotting graphics in the source file INVDP.PC. It should be noted that the RPlot software (written mostly in C) is not a subroutine package, but rather, a separate "stand-alone" plot system that utilizes a full-screen in graphics mode. Also note that Lahey's Windows Library does not support a mouse, so only the <ArrowKey>'s can be used to select menu objects in text windows.

If desired, this program can be executed using MS-Windows (3.0 or higher) in enhanced mode via the DOS compatibility box or by double-clicking the DOS icon; however, the RPlot graphics cannot be entirely rescaled using the present version of RPlot, and will be clipped if resizing the DOS window.

Note that the distributed protected mode executable program (INVDP.EXE) has been bound with the Phar Lap 386|DOS-Extender and is royalty-free to end users. The only additional software that is required to execute INVDP.EXE is the RPlot Graphics Package available from RSoft, Inc. (see Appendix 3 for an ordering form).

### **Method of Ordering**

The source and executable code for program INVDP (less RPlot Software) can be ordered by the open-file report number(s) indicated on the title page. Copies of this report and diskette (described below) can be obtained for a nominal cost (current cost unknown) by contacting:

U.S. Geological Survey  
Open-File Services Section  
Mail Stop 902  
Box 25425, Federal Center  
Denver, CO 80225

Phone: (303) 236-7476  
FAX: (303) 236-1972

Floppy Diskette Files

The following files are supplied on two 1.44 Mb (high-density) diskettes for this report:

READ.ME -- List of files on this diskette, and any last minute notes.  
 INVDP.PC -- Complete PC Fortran-77 source file (less RPlot Software).  
 INVDP.EXE -- Linked and bound protected mode executable program.  
                   (Requires the RPlot system from Appendix 3 at run-time).

INVDP.WP -- WordPerfect 5.1 file used to produce this report.  
                   (This file can be used to generate additional copies, etc.)

TEST1.DAT -- Test data for the example problem in this report.  
 MED2.DAT -- Medicine Lake observed data from Anderson (1983, p.23)  
 MED15.DAT -- Medicine Lake observed data from Anderson (1983, p.62)  
 MED26.DAT -- Medicine Lake observed data from Anderson (1983, p.95)  
 MED27.DAT -- Medicine Lake observed data from Anderson (1983, p.98)

T1.OUT -- Output file from the example problem in this report.  
 T1.PCx -- RPlot control files from the example problem. [x=1,2]  
 T1.x00 -- RPlot Obs.Data files from the example problem. [x=1,2]  
 T1.x01 -- RPlot RUN = 1 files from the example problem. [x=1,2]  
 T1.x02 -- RPlot RUN = 2 files from the example problem. [x=1,2]  
 T1.x03 -- RPlot RUN = 3 files from the example problem. [x=1,2]

WELCOME.PCS -- Welcome screen for INVDP (an RPlot .PCS file).

### APPENDIX 3. -- RPlot Software Order Form

Program INVDP cannot function without having the RPlot Graphics Library and executables on the hard disk stored in a suitable subdirectory and in the DOS PATH. The RPlot Graphics (Version 2.05) is available from RSoft, Inc. using the attached RSoft order form. Permission was granted by RSoft, Inc. to include a current copy of their order form in this report.

RSoft, Inc., RPlot Graphics Order Form (Version 2.05) starts on the next page:

**SYSTEM REQUIREMENTS:**

RPlot will run on an IBM PC, XT, AT, PS/2 or compatible computer with 256K free RAM, a CGA, EGA, VGA or Hercules video adapter, and MS-DOS version 3.0 or higher. The software supports Epson compatible dot matrix printers, HP LaserJet/DeskJet and compatible printers, as well as PostScript printers. Note that an ASCII text editor must be used in conjunction with this program (e.g. the line editor EDLIN which is supplied with DOS, or preferably any screen-oriented text editor). Use of a math coprocessor is recommended.

**GUARANTEE AND SUPPORT:**

RPlot is guaranteed for 30 days and is 100% refundable. RSoft provides technical support free of charge to any registered user. Users who discover a software error in a particular situation will receive a free updated version as soon as it becomes available.

**PRICING:**

RPlot is available for only \$149.00. For educational institutions, RPlot is discounted to \$99.00. Call RSoft for other discounting and site licensing information.

**ACKNOWLEDGMENTS:**

All product names referred to above are trademarks or registered trademarks of their respective manufactures.

**RPlot Order Form**

	<b>Ship To:</b>	<b>Bill To:</b>
Name	_____	_____
Affiliation	_____	_____
Address	_____	_____
City, State, Zip	_____	_____
Phone	_____	_____

<b>Quantity</b>	<b>Price (see above)</b>	<b>Amount</b>	<b>Diskette Size:</b>
Rplot _____ x _____	=	_____	<input type="checkbox"/> 5 ¼ "
Tax (NY residents add appropriate sales tax)		_____	<input type="checkbox"/> 3 ½ "
Shipping & Handling (\$5.00 + \$3.00 ea addnl; add \$2.00 extra for UPS 2nd-day service)		_____	
	<b>TOTAL</b>	_____	

**Payment Method:**

- Check or Money Order (made payable to RSoft, Inc.)  
 Purchase Order (recognized institutions only) PO # \_\_\_\_\_

**Note to Purchaser:**

Name \_\_\_\_\_ How you heard of RPlot \_\_\_\_\_

**Send Order To:**

**RSoft™, Inc.** • 345 Riverside Drive • Suite 2G • New York, NY 10025  
 Phone: (212) 666-0959 • FAX: (212) 666-3962