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Quantifying favorableness for occurrence of a mineral deposit type using fuzzy logic - an example from Arizona

by

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Abstract

An application of possibility theory from fuzzy logic to the quantification of favorableness for quartz-carbonate vein deposits in the southern Santa Rita Mountains of southeastern Arizona is described. Three necessary but probably not sufficient conditions for the formation of these deposits were defined as the occurrence of carbonate bearing rocks within hypabyssal depths, significant fracturing of the rocks, and proximity to a felsic intrusive. The quality of data available to evaluate these conditions is variable over the study area. The possibility of each condition was represented as a fuzzy set enumerated in a grid over the area. The intersection of the sets measures the degree of simultaneous occurrence of the necessary factors and provides a measure of the possibility of deposit occurrence. Using fuzzy set techniques, the effect of one or more fuzzy sets relative to the others in the intersection can be controlled. and logical combinations of the sets can be used to impose a time sequential constraint on the necessary conditions. Other necessary conditions, and supplementary conditions such as variable data quality or intensity of exploration can be included in the analysis by their proper representation as fuzzy sets.

Introduction

Mineral resource estimation is one example from a large class of earth science problems (others are, for example, source distributions of geophysical anomalies, the distribution of igneous rocks in space and time, and the uplift of mountain ranges) which are characterized by a dependence on many factors. The form of this dependence for some or all of the factors may be imprecisely or poorly known, and there may be undiscovered factors important to the problem. Moreover, mechanisms relating the factors to the outcome are generally poorly known. Consequently, solutions to these problems are imprecise and nonunique.

In this paper, we do not attempt resource estimation; instead, we assert that any estimation process will find a quantified estimate of the possibility of occurrence (defined here as the favorableness) of a particular deposit type of great utility. We report here the results of an attempt to produce a map of favorableness for a test area using fuzzy logic.

Mineral deposits are often classified into types based upon common characteristics or properties (cf. Cox and Singer, 1986). If the classification is based upon ore genesis and environment of deposition, one can often define conditions both spatially and temporally which are necessary (but not necessarily sufficient) for the occurrence of a deposit. The purpose of the work reported here was to embody these conditions in an objective rather than subjective way into an evaluation technique such that the relative influence of each condition on the evaluation could be examined. Our goal is thus to move toward a theory of possibility of existence of a type of deposit which can be applied to a particular area using geologic, geophysical and geochemical data to define the conditions.

Several avenues of investigations could be taken to build such a theory; here we restrict ourselves to an application of a quantitative theory of possibility put forward by Zadeh as an application in a series of papers beginning in 1965 (Zadeh, 1965). The more general branch of mathematics of which this theory is a part is referred to as fuzzy logic (Zadeh, 1965), and for this work we have used a chapter from Kandel (1982) as our principal reference on the algebra of fuzzy logic.

A fuzzy set is defined as a subset of objects from a large set whose membership in the subset may not be complete. The grade of membership is large (traditionally 1) for objects which completely belong to the fuzzy set; it is small (traditionally 0) for objects which do not belong to the fuzzy set. For a particular object, the more certainly it belongs to the fuzzy set, the closer its membership grade is to 1. Grade of membership is usually represented by a membership function which need not be linear or even continuous; indeed many interesting fuzzy sets have extremely nonlinear membership functions. It is important to realize that conventional set theory, now called "crisp" set theory because membership is either total or absent, is a particular case of fuzzy set theory where the membership function has only two values, 1 and 0.

As in conventional set theory, set-theoretic operations can be defined on fuzzy sets, including equality, containment, union, and intersection, all of which have meanings analogous to their crisp set equivalents. Two fuzzy sets combined by one of these operations yields a fuzzy set. If the range of the membership function is [0,1] for both sets then the resulting membership function has the same range in the sense of a 0 value meaning absence from the resultant fuzzy set and a value of 1 means full membership. Other operations can be defined which yield fuzzy sets also; one used in this work is the product, wherein one multiplies the corresponding membership function values. Two special cases used here are termed the concentration and the dilation operations which yield the concentrated and dilated fuzzy sets respectively. In both cases, the membership function from one fuzzy set is used twice to form the two sets to be operated upon. The membership function values of a concentrated fuzzy set are defined as the square of the unconcentrated fuzzy set membership values; the membership function values for a dilated fuzzy set are defined as the square root of the membership values of the undilated fuzzy set. Because the membership function ranges only over [0,1], concentration emphasizes the objects whose membership function values are near 1, whereas the dilation emphasizes the objects whose membership function values are near 0. These two operations can be used to relatively weight the importance of a fuzzy set representing a particular condition. In terms of possibility theory, dilation corresponds to a modifier like "more or less", in the sense that it will increase relatively the importance of the membership values which are small. Similarly, concentration corresponds to a modifier "very" and relatively enhances membership values which are large. It is important to note that possibility is nonstatistical in nature, and possibility cannot be inferred from probability nor vice-versa (Kandel, 1982).

For the application studied here, the large set is the set of all rocks. The subset is the rocks of the area being studied; all fuzzy sets will cover this area and be spatially registered in the same way. Practically, this amounts to laying a grid over the area and specifying the membership function in each grid cell. Each necessary condition for a mineral deposit type is specified by a fuzzy set, the estimates of whose membership values quantify the degree of membership of the grid cell for that condition. The possibility of existence of a mineral deposit is then a series of operations combining the fuzzy sets representing the necessary conditions for a mineral deposit. Probably the simplest combination which expresses the necessity of all the conditions is the intersection operator which yields a fuzzy set whose membership function values express the degree to which all the necessary conditions are met from the available data.The fuzzy intersection is defined as the minimum value of the membership functions of the fuzzy sets being intersected in each grid cell. This constitutes an objective, quantified measure of favorableness within the study area. If uniform and objective procedures could be applied to all areas and deposit types, the absolute value favorableness measure would be quantitative; otherwise, the measure is only a relative one and is based on the area and data studied.

An example: Silver-lead vein-type deposits of the southern Santa Rita Mountains, Arizona

We have selected silver-lead vein deposits in the southern Santa Rita Mountains in southeastern Arizona as a test case for several reasons (a geologic map of the area due to Drewes (in press) is displayed in Figure 1). First, the area is known to the authors and relatively well studied from an exploration and evaluation standpoint (see Drewes, in press, for a summary and a bibliography); second, the deposits are areally small, and prospective areas are not generally obvious from the regional-scale geological, geophysical and geochemical data usually available for evaluation. Finally, the deposits are of the quartz-carbonate vein type, and, based upon the deposit-type description of Cox (1986), we assume that one necessary condition is the availability of carbonate-bearing rocks. In the evaluation of this area using another technique (Bultman and Gettings, in press) the assessment team decided to approximate the silver-lead vein deposits in this area with a descriptive model of polymetallic veins (Cox, 1986), and this model was used to derive the necessary conditions used in this work.

Three necessary conditions for the existence of polymetallic veins were selected: 1) the presence of carbonate-bearing rocks within hypabyssal depths; 2) significant fracturing of the host rocks; and 3) close proximity to felsic intrusive rocks. In this case the temporal sequence of the conditions could be ignored because, except in one case of minor map extent in the test area, the carbonate rocks are all older than the felsic intrusives and in all cases, fracturing is at least contemporaneous with intrusion if not pre-existing. In other areas and deposit types, the temporal sequence would have to be explicitly included; this is easily done with modern Geographic Information System (GIS) software which makes provision for cell-by-cell logical tests.

For each factor, a map was compiled from existing geologic, geophysical, geochemical, LANDSAT imagery, and side-looking radar imagery data (du Bray, in press) consisting of outlined areas to each of which a membership function value was assigned. In the case of carbonates, the possibility of carbonates being present was assigned based on the geologic description of the map unit. For example, if a unit was described as "sandstone with minor limestone" a membership value of 0.1 was assigned, and "predominately limestone" was assigned a value of 0.9, and so on. For all cases, a likelihood of about one half for the condition was assigned a membership value of 0.5. Geophysical data were used to assist in projecting geologic relations beneath basin fill and to estimate depths to bedrock. Areas of deep fill were given low membership values since the possibility of a deposit within 1-2 kilometers of the surface, a usual exploration maximum depth, is small. A better treatment of exploration depth would have been to include it as a separate necessary condition (fuzzy set). Projections of stratigraphic and structural information were used to estimate the possibility of carbonates in the subsurface. Fracturing was estimated primarily from mapped faults and photo-interpreted textures and lineaments on the side-looking radar data and is assumed to extend into the subsurface.The fracturing map was the most difficult to estimate owing to a lack of data, and could be improved by fieldwork and examination of aerial photography. The three maps were drawn without any reference to the location of known deposits, in order that the known deposits could serve as a partial test of the ability of the method to delineate favorable terrain.

The linework from three maps of necessary conditions was then digitized and entered in a GIS system (ARC/INFO in this case) and the membership function values were assigned to the resulting polygons. The GIS system then converted the vector polygon maps to a grid (raster or cell) map, and using the grid functions of the GIS system, various fuzzy set operations were completed. Figures 2-4 display respectively the resulting grids of the carbonate possibility, fracture density, and felsic intrusive fuzzy sets. The location of known and possible mineral occurrences of the quartz-carbonate vein type are shown on all figures for reference.

Figure 5 displays the fuzzy set of possibility for quartz-carbonate veins computed by taking the fuzzy intersection of the three conditions. The membership values on Fig. 5 represent a semi-quantitative measure of the favorableness for deposits of quartz-carbonate vein type in the study area based upon the descriptive model and the available geoscience information. Because the fuzzy intersection is computed as the minimum of the membership values of the conditions for each grid cell, the resulting membership (possibility) value is reasonable. If, for example, two of the three membership values are very large but the third small the resulting membership value is the small one and the method does not overestimate possibility.

Discussion

The resulting possibility map (Fig. 5) displays several interesting features. First, only about 6 percent of the map area has a resulting possibility value of 0.5 or larger, and about 94 percent of the study area has a value of 0.3 or less. This result agrees with experience; truly favorable ground is quite rare. Examination of the known deposits distribution shows two deposits in the south-central area of the map which occur in a very low possibility area. This case demonstrates the dependence of the results of this (or any other) method on both data quality and the geologic model. In assigning membership values to the carbonate fuzzy set (Fig. 2) the large Tertiary-Cretaceous granite TKg ("A" in Fig. 1) west of the two deposits was assumed to extend to great depth as was the Jurassic granite to the east (Fig. 1), and thus the carbonate possibility was assigned a small value. Obviously another equally viable geologic model is that the TKg is a relatively thin sill overlying stratified rocks and thus the possibility of carbonate rocks beneath the TKg unit would be increased. Applying this to the TKg unit lying along the west side of the Jurassic granite (Jg, Fig. 1) would raise the possibility value in all of the low possibility areas of Fig. 5. Because of the ease with which the fuzzy sets can be edited with a GIS system, it is easy to try various scenarios improve the match with known deposits and/or minimize the effects of errors inherent in the various sets.

In a general sense, the use of a membership function to represent necessary conditions allows one to objectively include poorly understood and incomplete data into the analysis. A fuzzy set could be included that represented the reliability or availability of data, which is generally variable over the map area. Because modern GIS systems include the capability for logical comparisons and branches on a cell-by-cell basis, it is possible to include time constraints based on sets giving the age of the various conditions, and to change the membership function value based on another set such as data quality or information availability.

The relative importance of a condition relative to the others in the intersection can be varied by the operations of dilation and concentration discussed above. We applied dilation and concentration to the carbonate fuzzy set (Fig.2) before computing the intersection. Figure 6 shows the result of dilation of the carbonate fuzzy set on the intersection, and Figure 7 shows the result of concentration. As is shown by comparison with the uniformly weighted intersection of Fig. 5, the results are quite sensitive to these operations, and correspond nicely to geological statements such as "carbonate presence is the most important factor" (dilation) and "carbonate presence is more or less important" (concentration). Note that the association of "most" with dilation and "more or less" with concentration here applies to the resulting fuzzy set intersection and not the set it is applied to; this terminology is confusing in the literature.

Another fuzzy set was added to the analysis constructed using a clay and hydrous ferric oxide alteration map created from a standard analysis of a LANDSAT image. Inclusion of this set in the analysis left the fuzzy intersection (Fig. 5) unchanged as the altered rock areas correlate strongly with one or more of the other factors.

Conclusions

This application of the methods of fuzzy logic to favorableness estimation is based on the premise that it is easier to evaluate the possible presence or absence of individual conditions necessary for a mineral deposit, such as fracturing, lithology, etc., than to estimate the possibility of the occurrence of the mineralized system. The operations of combining the fuzzy sets then assign a grade to all parts of the area of evaluation which quantifies in a relative sense the degree to which all the factors are present in the proper order. Because of the "fuzzy" rather than present-or-absent nature of the membership function approach, it may identify localities previously unrecognized by other methods. The fuzzy sets themselves constitute a valuable source of data for other analyses which utilize some or all of the quantified factors.

The method is objective and repeatable, yet can utilize vague, ambiguous, or varying reliability data which commonly occur in geological descriptions. Moreover, through the use of modifiers, the relative weight of a particular fuzzy set upon the outcome can be controlled, and as a result the relative importance of each condition upon the outcome can be assessed. If additional information or ideas become available, they can be easily incorporated into the evaluation. New criteria can be added as additional fuzzy sets, and logical combinations between sets and (or) at a cell-by-cell level can be used to refine the evaluation model. If, rather than relative quantification of favorableness or possibility, the purpose of the mineral resource estimate is to estimate the number of undiscovered deposits, the procedure presented here forms an objective basis for definition of tracts and helps guide the estimates of deposit numbers within the tracts.

Finally, the method as carried out here is reasonably expedient. Disregarding time spent learning about fuzzy logic and how to perform the desired operations with the GIS software, only about two man-days were required for the fuzzy set evaluation presented here. The major part of this time was taken up by the digitizing and polygon-attributing tasks. Both authors were familiar with the geology of the study area, however, and evaluation of an area unknown to the investigators would require more time to digest and formulate the geoscience data into the appropriate fuzzy sets.

References cited

Bultman, M.W., and Gettings, M.E., in press, Chapter J: Quantitative Mineral Resource Assessment of Coronado National Forest *in* du Bray, E.A., editor, Mineral Resource Potential of the Coronado National Forest, Arizona and New Mexico: U.S. Geological Survey Bulletin, 600 ms p.

Cox, D.P., 1986, Descriptive model of polymetallic veins *in* Cox, D.P., and Singer, D.A., eds., Mineral deposit models: U.S. Geological Survey Bulletin 1693, p. 125.

Drewes, Harald, in press, Chapter B: Geology of Coronado National Forest *in* du Bray, E.A., editor, Mineral Resource Potential of the Coronado National Forest, Arizona and New Mexico: U.S. Geological Survey Bulletin, 600 ms p.

du Bray, E.A., editor, Mineral Resource Potential of the Coronado National Forest, Arizona and New Mexico: U.S. Geological Survey Bulletin, 600 ms p.

Kandel, Abraham, 1982, Chapter 2: The algebra of inexactness *in*

Fuzzy techniques in pattern recognition: John Wiley and Sons, Inc., U.S.A., p 22-90.

Zadeh, L.A., 1965, Fuzzy sets: IEEE Information and Control, v. 8, p. 338-353.

Figure captions

Figure 1. Geologic map of the southern Santa Rita Mountains area, southeastern Arizona, modified from Drewes (in press). Faults are shown by bold lines, contacts by lighter lines, and veins by light lines with a black dot on them, Geologic symbols, from Drewes (in press): QTg, Holocene to Miocene gravel, sand and conglomerate; Tr, Tertiary rhyolitic extrusive rocks; Tri, Tertiary rhyolitic intrusive rocks; TKg, Late Cretaceous to Tertiary granitic intrusive rocks; TKa, Late Cretaceous to Tertiary andesitic intrusive rocks; TKvs, Late Cretaceous to Tertiary andesitic to rhyolitic volcanic and volcaniclastic sedimentary rocks;Ksv, Late Cretaceous undivided volcaniclastic sedimentary and andesitic to rhyolitic volcanic rocks; Kbl, Early Cretaceous undivided rocks, andesitic to rhyolitic rocks, conglomerate and sandstone; Kb, Early Cretaceous shale and siltstone, some sandstone, conglomerate and limestone; Kr, Cretaceous rhyolitic volcanic rocks; Ka, Cretaceous andesitic volcanic rocks; JTrvs, Jurassic and Triassic rhyolitic to andesitic volcanic rocks and eolian sandstones and redbeds; Jg, Jurassic granitic intrusive rocks; PPn, Permian and Pennsylvanian limestone and dolomite, some siltstone, sandstone and marlstone; Pzl, Lower Paleozoic limestone and dolomite, some sandstone, shale, and conglomerate; Yg, Middle Proterozoic granitic intrusive rocks; YXgn, Middle and Early Proterozoic gneissic granite and older schist or gneiss. Solid squares are locations of known carbonate-quartz vein deposits; open squares are locations of possible carbonate-quartz vein deposits.

Figure 2. Fuzzy set representing the possibility of the presence of carbonate bearing

rocks within hypabyssal depths in the area of Figure 1. A value of 1.0 means the possibility is a certainty, and a value of 0.0 means impossibility, with intermediate values representing increasing possibility. Solid squares are locations of known carbonate-quartz vein deposits; open squares are locations of possible carbonate-quartz vein deposits.

Figure 3. Fuzzy set representing the possibility of significantly fractured rocks within hypabyssal depths in the area of Figure 1. Value definitions and symbols as in Fig.2. Fracturing possibility was estimated from geologic map data and textures and lineaments on side-looking radar images of the area. Estimated degree of fracturing on the surface was assumed to extend vertically downward for this work.

Figure 4. Fuzzy set representing the possibility of the presence of felsic intrusive rocks within hypabyssal depths in the area of Figure 1. Value definitions and symbols as in Fig.2.

Figure 5. Fuzzy set resulting from the intersection of the three fuzzy sets of Figs. 2-4 and representing the relative possibility of quartz-carbonate vein deposit occurrence from the criteria of carbonate rock availability, significant fracturing, and felsic intrusive rocks as necessary conditions. Value definitions and symbols as in Fig.2. Solid squares are locations of known carbonate-quartz vein deposits; open squares are locations of possible carbonate-quartz vein deposits. For each grid cell

of the area, the fuzzy intersection is calculated as the minimum value for that grid cell from the corresponding cell values of the fuzzy sets being intersected.

Figure 6. Fuzzy set representing the intersection of the fuzzy sets of Figs. 2-4 but with the influence of the set of Fig. 2 (carbonate bearing rock availability) enhanced by the operation of dilation. Value definitions and symbols as in Fig.2. Note increase of both area and value of possibility relative to Fig. 5. Solid squares are locations of known carbonate-quartz vein deposits; open squares are locations of possible carbonate-quartz vein deposits.

Figure 7. Fuzzy set representing the intersection of the fuzzy sets of Figs. 2-4 but with the influence of the set of Fig. 2 (carbonate bearing rock availability) de-emphasized by the operation of concentration. Value definitions and symbols as in Fig.2. Note decrease of both area and value of possibility relative to Fig. 5. Solid squares are locations of known carbonate-quartz vein deposits; open squares are locations of possible carbonate-quartz vein deposits.

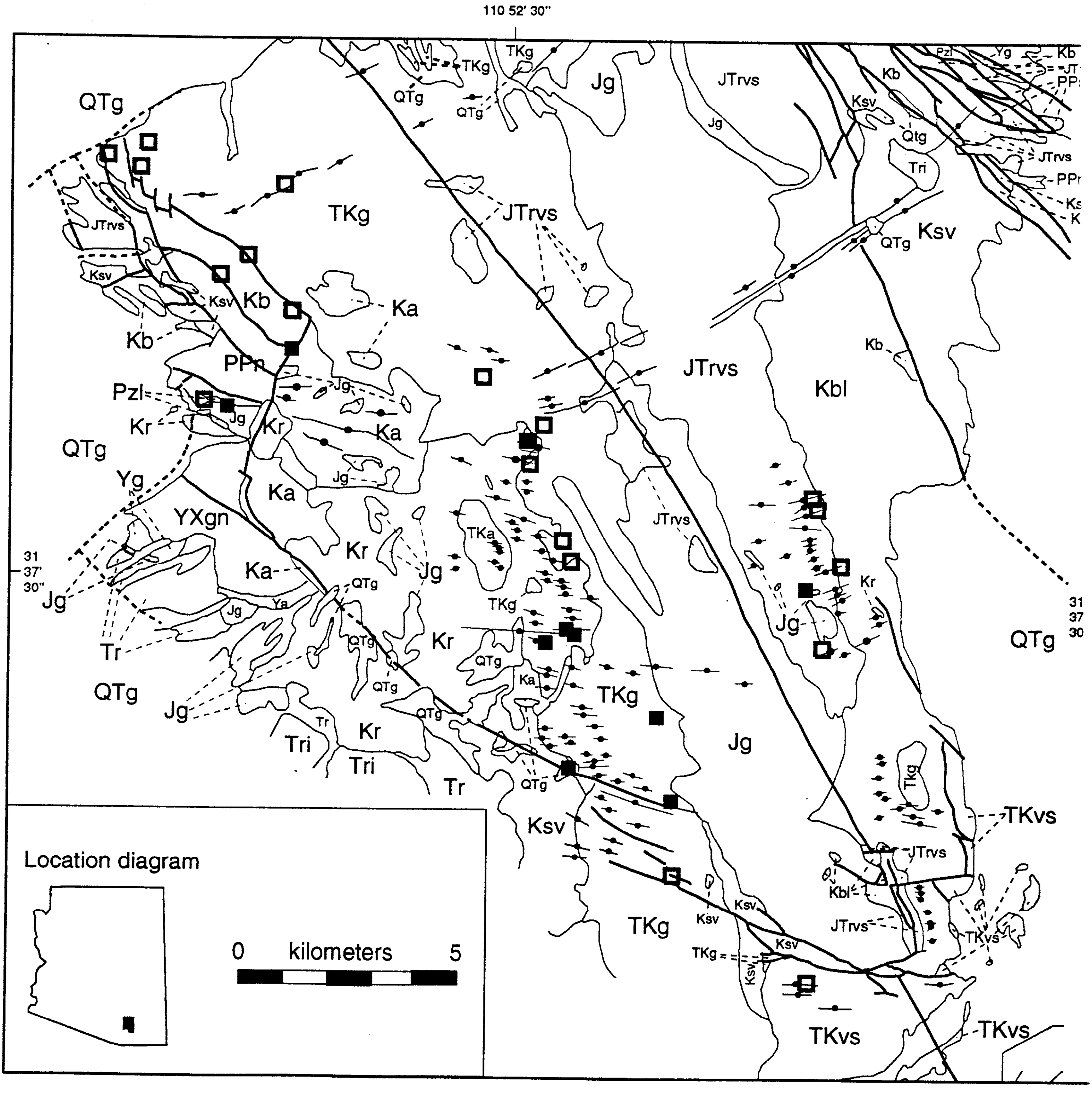

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Figure 1

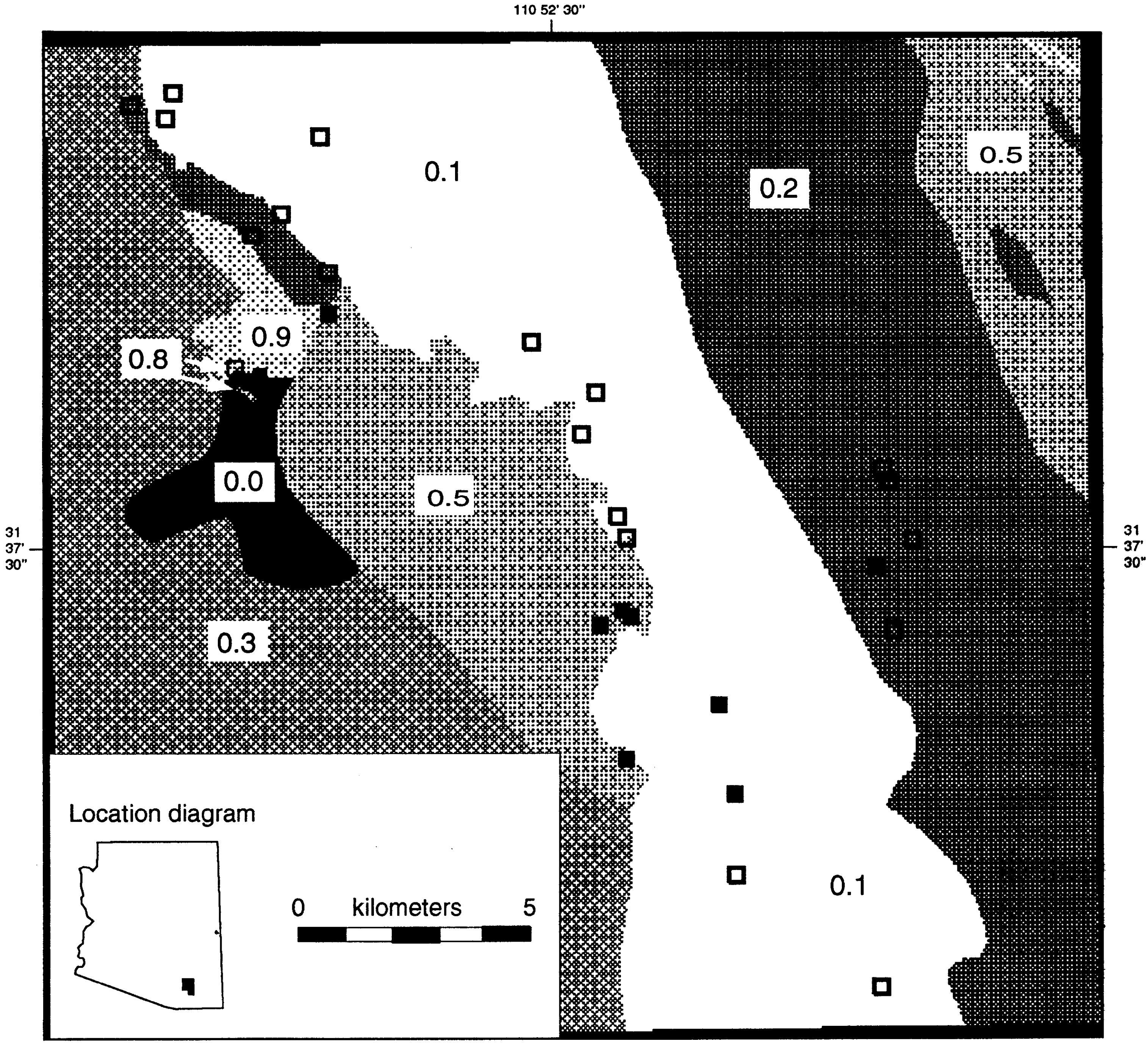
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Figure 2

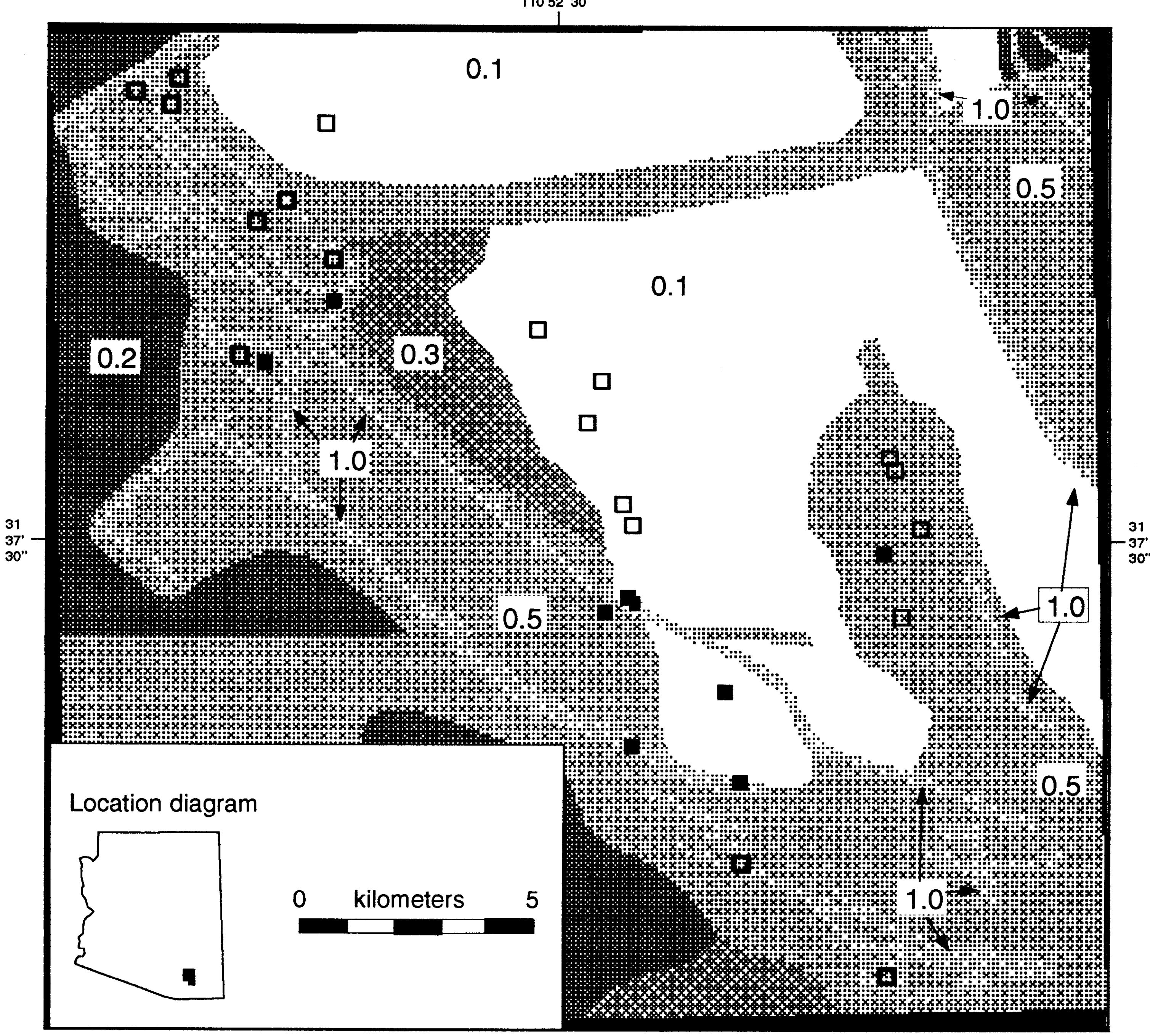

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Figure 3

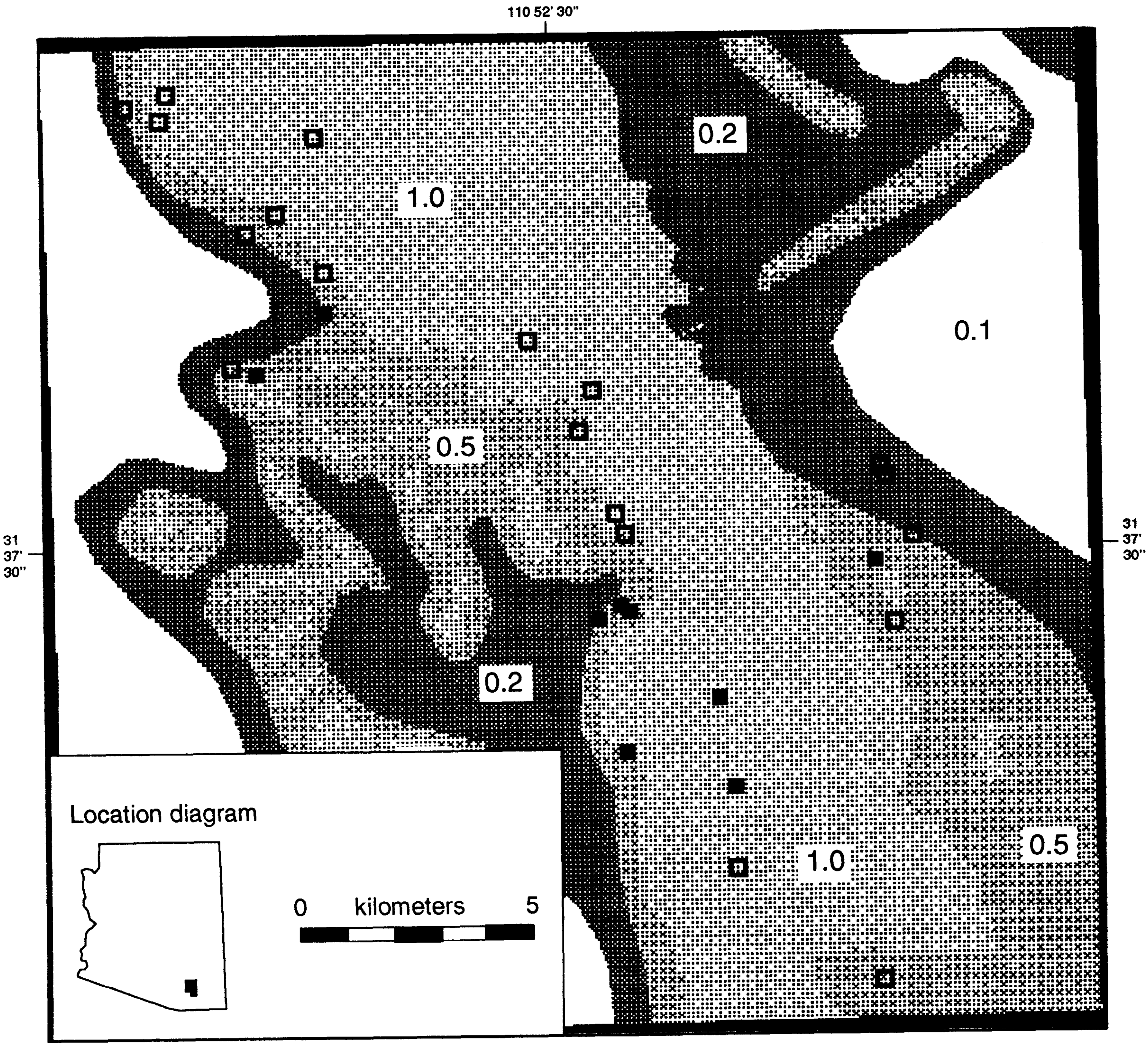
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Figure 4

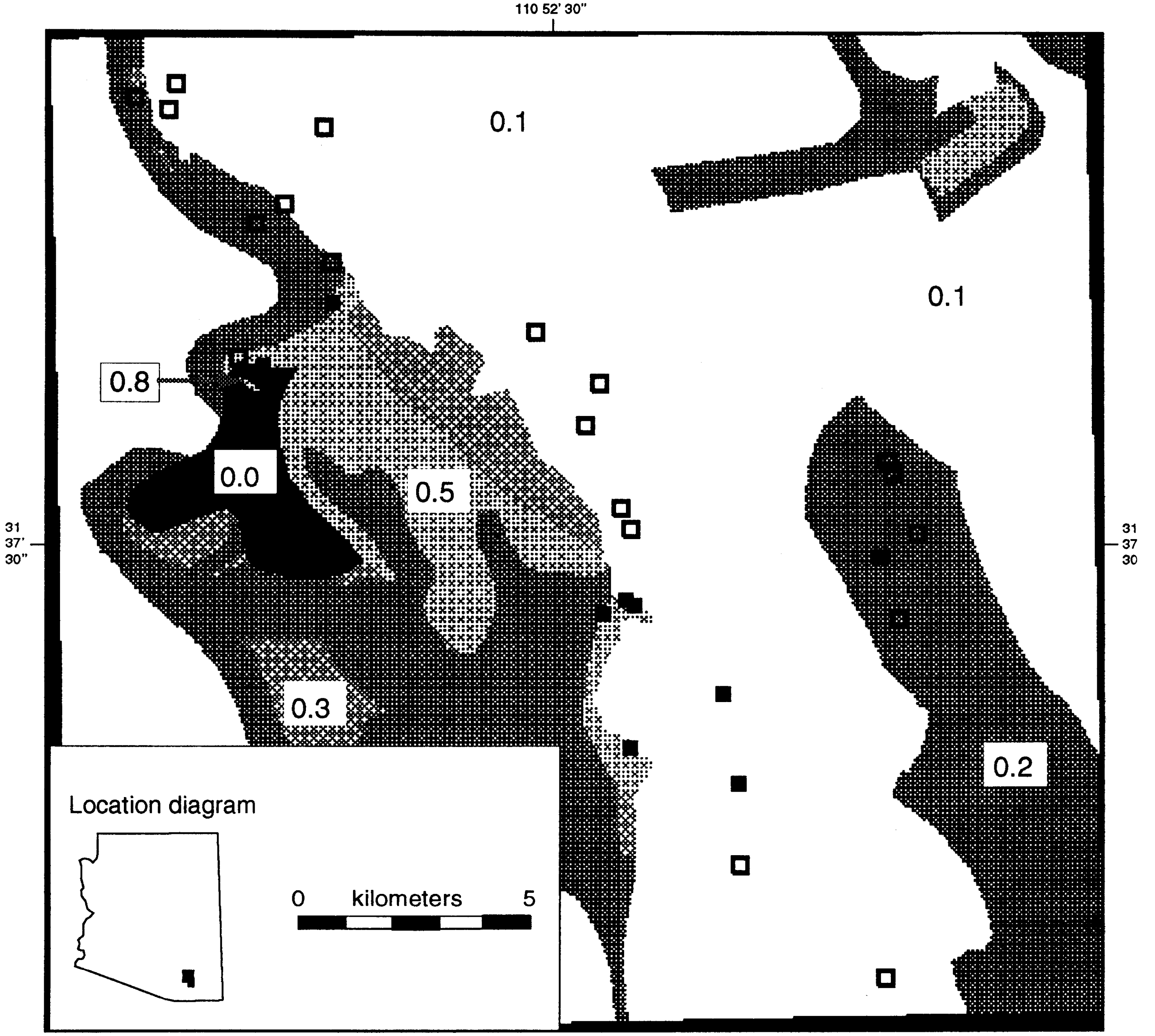
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Figure 5

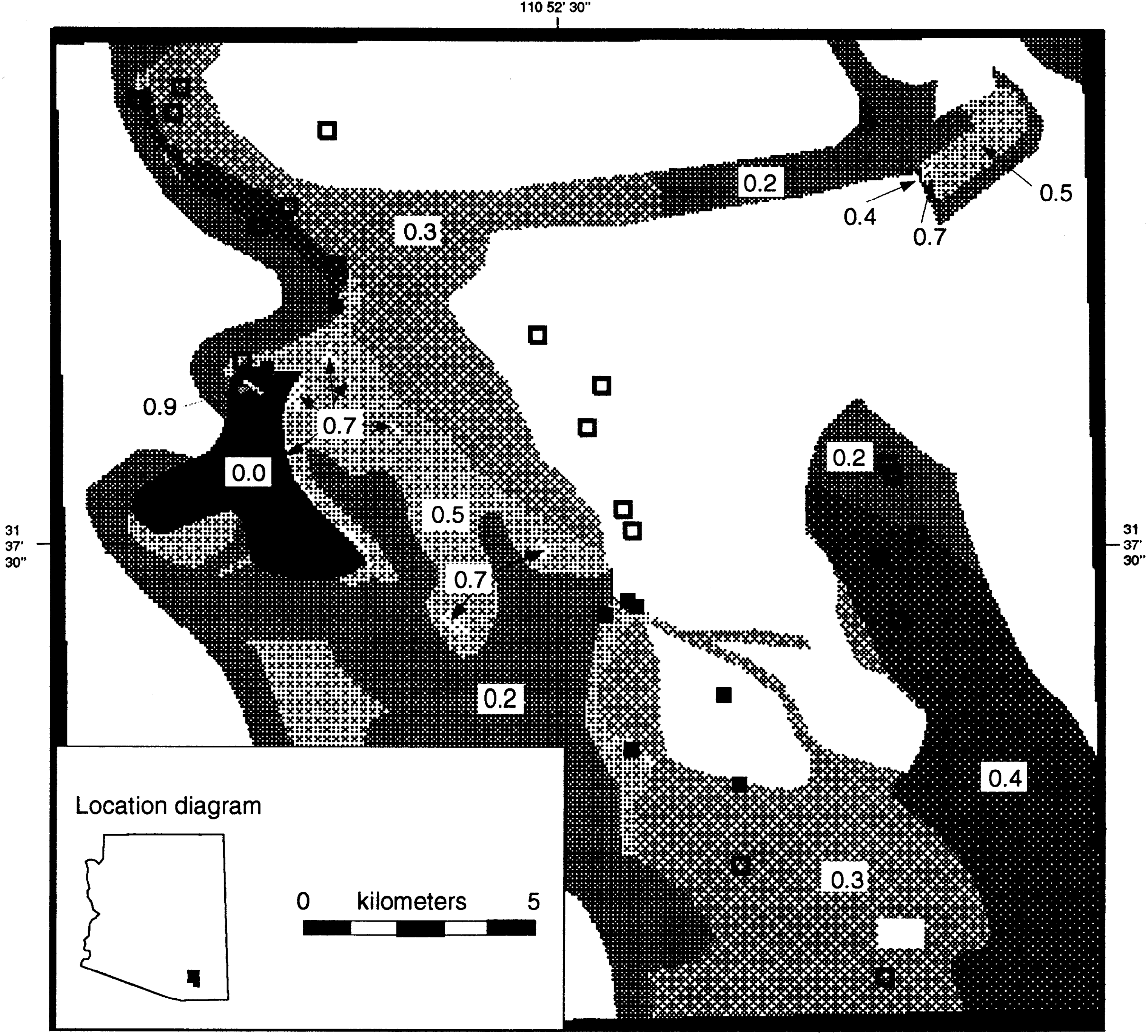
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Figure 6

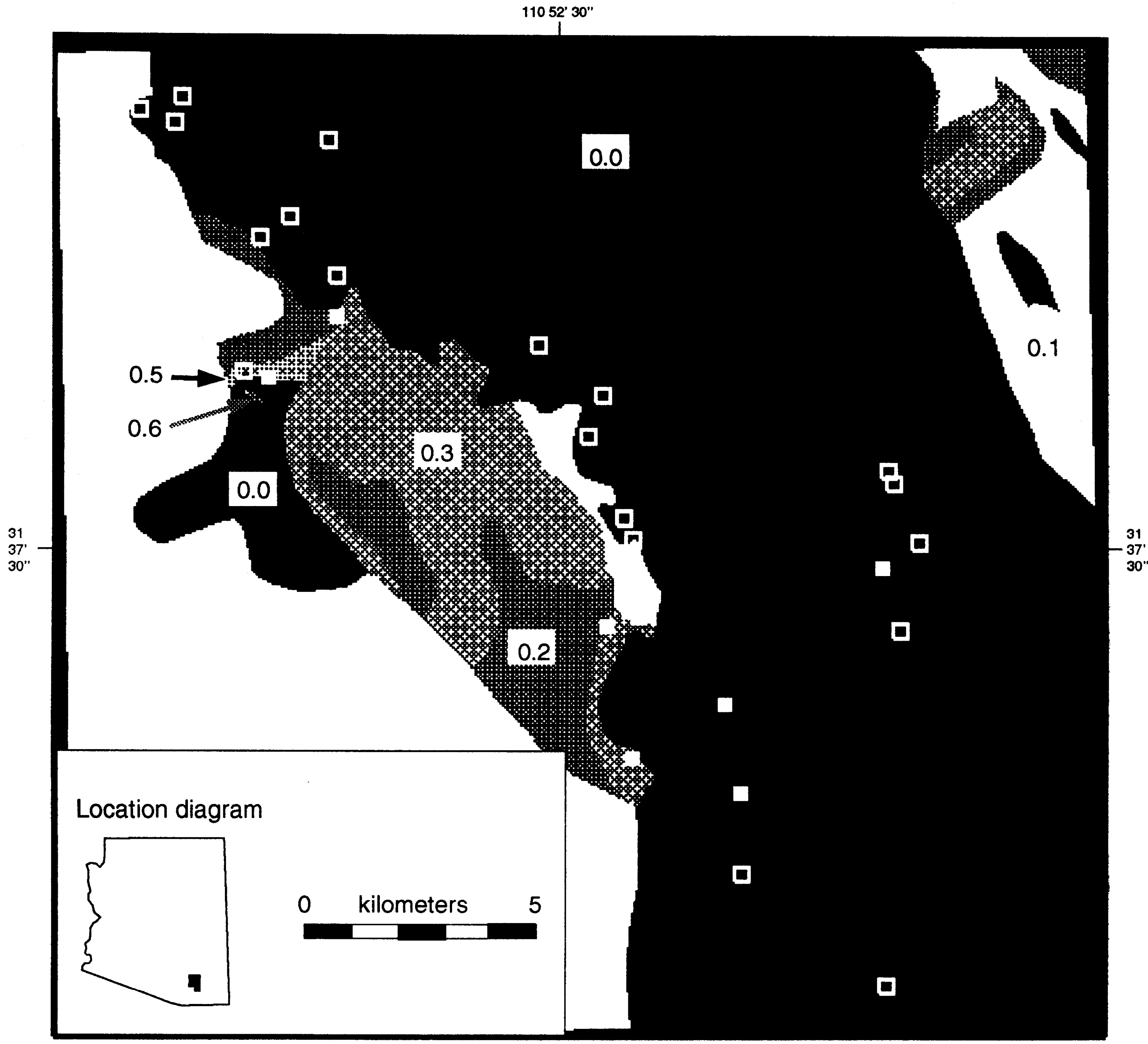
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Figure 7