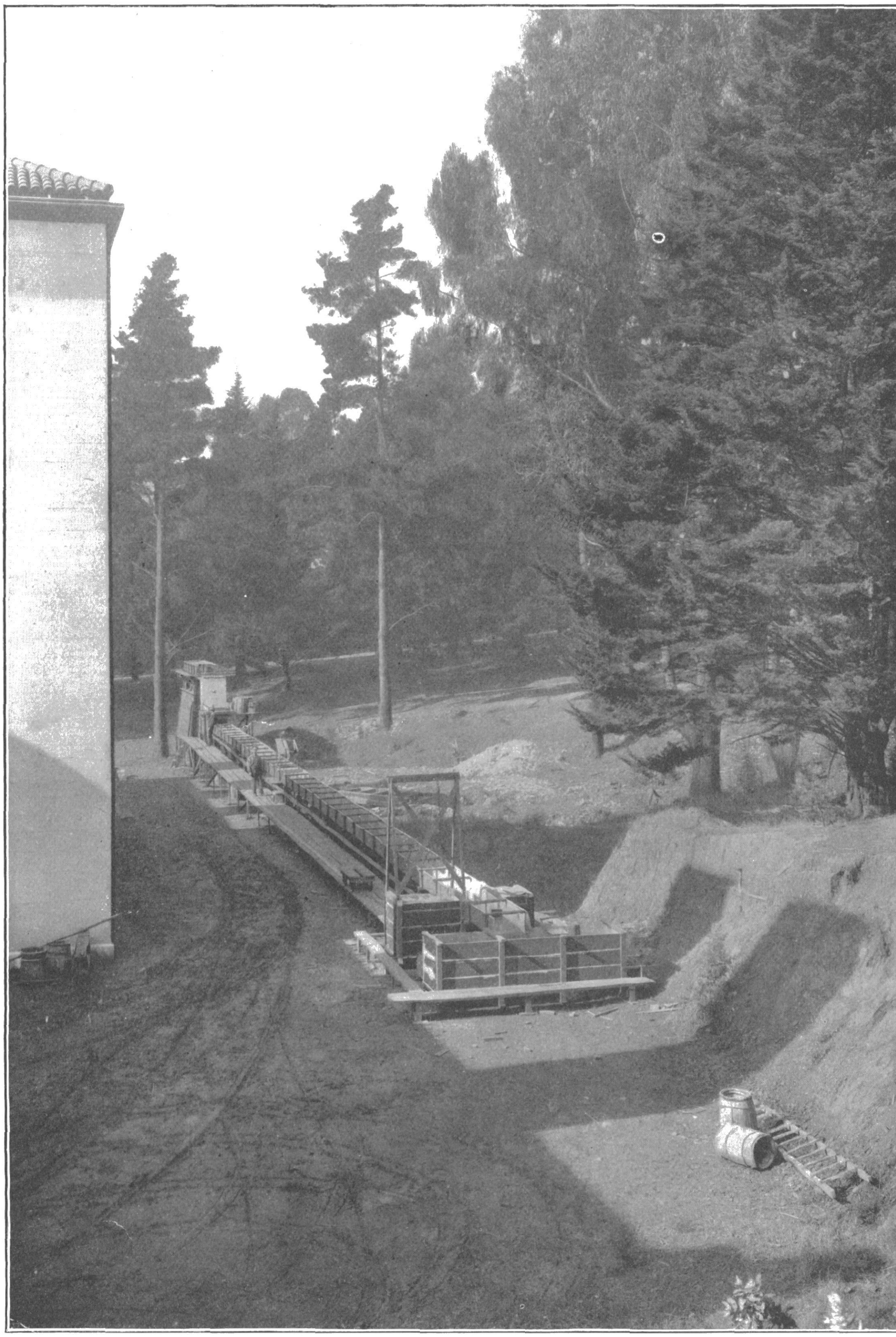

APPARATUS ON CAMPUS OF UNIVERSITY OF CALIFORNIA.

B

C. D. Walcott.

DEPARTMENT OF THE INTERIOR

UNITED STATES GEOLOGICAL SURVEY

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PROFESSIONAL PAPER 86

THE

TRANSPORTATION OF DÉBRIS BY RUNNING WATER

BY

GROVE KARL GILBERT

BASED ON EXPERIMENTS MADE WITH THE ASSISTANCE OF

EDWARD CHARLES MURPHY

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PREFACE.

Thirty-five years ago the writer made a study of the work of streams in shaping the face of the land. The study included a qualitative and partly deductive investigation of the laws of transportation of débris by running water; and the limitations of such methods inspired a desire for quantitative data, such as could be obtained only by experimentation with determinate conditions. The gratification of this desire was long deferred, but opportunity for experimentation finally came in connection with an investigation of problems occasioned by the overloading of certain California rivers with waste from hydraulic mines. The physical factors of those problems involve the transporting capacity of streams as controlled by various conditions. The experiments described in this report were thus instigated by the common needs of physiographic geology and hydraulic engineering.

A laboratory was established at Berkeley, Cal., and the investigation became the guest of the University of California, to which it is indebted not only for space, within doors and without, but for facilities of many kinds most generously contributed.

Almost from the beginning Mr. E. C. Murphy has been associated with me in the investigation and has had direct charge of the experiments. Before the completion of the investigation I was compelled by ill health to withdraw from it, and Mr. Murphy not only made the remaining series of experiments, so far as had been definitely planned, but prepared a report. This report did not include a full discussion of the results but was of a preliminary nature, it being hoped that the work might be continued, with enlargement of scale, in the near future. When afterward I found myself able to resume the study, there seemed no immediate prospect of resuming experimentation, and it was thought best to give the material comparatively full treatment. It will readily be understood from this account that I am responsible for the planning of the experimental work as well as for the discussion of results here contained, while Mr. Murphy is responsible for the experimental work. It must not be understood, however, that in assuming responsibility for the discussion I also claim sole credit for what is novel in the generalizations. Many conclusions were reached by us jointly during our association, and others were developed by Mr. Murphy in his report. These have been incorporated in the present report, so far as they appeared to be sustained by the more elaborate analysis, and specific credit is given only where I found it practicable to quote from Mr. Murphy's manuscript.

Mr. J. A. Burgess was for a short time a scientific assistant in the laboratory, and his work is described in another connection. Credit should be given to Mr. L. E. Eshleman, carpenter, and Mr. Waldemar Arntzen, mechanician, for excellent work in the construction of apparatus. I recall with sincere gratitude the cordial cooperation of several members of the university faculty, and the investigation is especially indebted to the good offices and technical knowledge of Prof. S. B. Christy and Prof. J. N. Le Conte.

Portions of my manuscript were read by Dr. R. S. Woodward and Dr. Lyman J. Briggs, and the entire manuscript was read by Mr. C. E. Van Orstrand and Mr. Willard D. Johnson. To these gentlemen and to members of the editorial staff of the Geological Survey I am indebted for criticisms and suggestions leading to the elimination of some of the crudities of the original draft.

While the aid which my work has received from many colleagues has been so kindly and efficient that individual mention seems invidious, my gratitude must nevertheless be expressed for valuable assistance by Mr. François E. Matthes in the examination of foreign literature, and for the unfailing encouragement and support of Mr. M. O. Leighton, until recently in charge of the hydrographic work of the Survey.

G. K. G.

ABSTRACT.

Scope.—The finer débris transported by a stream is borne in suspension. The coarser is swept along the channel bed. The suspended load is readily sampled and estimated, and much is known as to its quantity. The bed load is inaccessible and we are without definite information as to its amount. The primary purpose of the investigation was to learn the laws which control the movement of bed load, and especially to determine how the quantity of load is related to the stream's slope and discharge and to the degree of comminution of the débris.

Method.—To this end a laboratory was equipped at Berkeley, Cal., and experiments were performed in which each of the three conditions mentioned was separately varied and the resulting variations of load were observed and measured. Sand and gravel were sorted by sieves into grades of uniform size. Determinate discharges were used. In each experiment a specific load was fed to a stream of specific width and discharge, and measurement was made of the slope to which the stream automatically adjusted its bed so as to enable the current to transport the load.

The slope factor.—For each combination of discharge, width, and grade of débris there is a slope, called competent slope, which limits transportation. With lower slopes there is no load, or the stream has no capacity [1] for load. With higher slopes capacity exists; and increase of slope gives increase of capacity. The value of capacity is approximately proportional to a power of the excess of slope above competent slope. If S equal the stream's slope and σ equal competent slope, then the stream's capacity varies as $(S-\sigma)^n$. This is not a deductive, but an empiric law. The exponent n has not a fixed value, but an indefinite series of values depending on conditions. Its range of values in the experience of the laboratory is from 0.93 to 2.37, the values being greater as the discharges are smaller or the débris is coarser.

The discharge factor.—For each combination of width, slope, and grade of débris there is a competent discharge, κ. Calling the stream's discharge Q, the stream's capacity varies as $(Q-\kappa)^o$. The observed range of values for o is from 0.81 to 1.24, the values being greater as the slopes are smaller or the débris is coarser. Under like conditions o is less than n; or, in other words, capacity is less sensitive to changes of discharge than to changes of slope.

The fineness factor.—For each combination of width, slope, and discharge there is a limiting fineness of débris below which no transportation takes place. Calling fineness (or degree of comminution) F and competent fineness ϕ, the stream's capacity varies with $(F-\phi)^p$. The observed range of values for p is from 0.50 to 0.62, the values being greater as slopes and discharges are smaller. Capacity is less sensitive to changes in fineness of débris than to changes in discharge or slope.

The form factor.—Most of the experiments were with straight channels. A few with crooked channels yielded nearly the same estimates of capacity. The ratio of depth to width is a more important factor. For any combination of slope, discharge, and fineness it is possible to reduce capacity to zero by making the stream very wide and shallow or very narrow and deep. Between these extremes is a particular ratio of depth to width, ρ, corresponding to a maximum capacity. The values of ρ range, under laboratory conditions, from 0.5 to 0.04, being greater as slope, discharge, and fineness are less.

Velocity.—The velocity which determines capacity for bed load is that near the stream's bed, but attempts to measure bed velocity were not successful. Mean velocity was measured instead. To make a definite comparison between capacity and mean velocity it is neces-

[1] *Capacity* is defined for the purposes of this paper as the maximum load of a given kind of débris which a given stream can transport. See page 35.

sary to postulate constancy in some accessory condition. If slope be the constant, in which case velocity changes with discharge, capacity varies on the average with the 3.2 power of velocity. If discharge be the constant, in which case velocity changes with slope, capacity varies on the average with the 4.0 power of velocity. If depth be the constant, in which case velocity changes with simultaneous changes of slope and discharge, capacity varies on the average with the 3.7 power of velocity. The power expressing the sensitiveness of capacity to changes of mean velocity has in each case a wide range of values, being greater as slope, discharge, and fineness are less.

Mixtures.—In general, débris composed of particles of a single size is moved less freely than débris containing particles of many sizes. If fine material be added to coarse, not only is the total load increased but a greater quantity of the coarse material is carried.

Modes of transportation; movement of particles.—Some particles of the bed load slide; many roll; the multitude make short skips or leaps, the process being called saltation. Saltation grades into suspension. When particles of many sizes are moved together the larger ones are rolled.

Modes of transportation; collective movement.—When the conditions are such that the bed load is small, the bed is molded into hills, called dunes, which travel downstream. Their mode of advance is like that of eolian dunes, the current eroding their upstream faces and depositing the eroded material on the downstream faces. With any progressive change of conditions tending to increase the load, the dunes eventually disappear and the débris surface becomes smooth. The smooth phase is in turn succeeded by a second rhythmic phase, in which a system of hills travel upstream. These are called antidunes, and their movement is accomplished by erosion on the downstream face and deposition on the upstream face. Both rhythms of débris movement are initiated by rhythms of water movement.

Application of formulas.—While the principles discovered in the laboratory are necessarily involved in the work of rivers, the laboratory formulas are not immediately available for the discussion of river problems. Being both empiric and complex, they will not bear extensive extrapolation. Under some circumstances they may be used to compare the work of one stream with that of another stream of the same type, but they do not permit an estimate of a river's capacity to be based on the determined capacities of laboratory streams. The investigation made an advance in the direction of its primary goal, but the goal was not reached.

Load versus energy.—The energy of a stream is measured by the product of its discharge (mass per unit time), its slope, and the acceleration of gravity. In a stream without load the energy is expended in flow resistances, which are greater as velocity and viscosity are greater. Load, including that carried in suspension and that dragged along the bed, affects the energy in three ways. (1) It adds its mass to the mass of the water and increases the stock of energy pro rata. (2) Its transportation involves mechanical work, and that work is at the expense of the stream's energy. (3) Its presence restricts the mobility of the water, in effect increasing its viscosity, and thus consumes energy. For the finest elements of load the third factor is more important than the second; for coarser elements the second is the more important. For each element the second and third together exceed the first, so that the net result is a tax on the stream's energy. Each element of load, by drawing on the supply of energy, reduces velocity and thus reduces capacity for all parts of the load. This principle affords a condition by which total capacity is limited. Subject to this condition a stream's load at any time is determined by the supply of débris and the fineness of the available kinds.

Flume transportation.—In the experiments described above—experiments illustrating stream transportation—the load traversed a plastic bed composed of its own material. Other experiments were arranged in which the load traversed a rigid bed, the bottom of a flume. Capacities are notably larger for flume transportation than for stream transportation, and their laws of variation are different. Rolling is an important mode of progression. For rolled particles the capacity increases with coarseness, for leaping particles with fineness. Capacity increases with slope and usually with discharge also, but the rates of increase are less than in stream transportation. Capacity is reduced by roughness of bed.

Vertical velocity curve.—The vertical distribution of velocities in a current is controlled by conditions. The level of maximum velocity may have any position in the upper three-fourths of the current. In loaded streams its position is higher as the load is greater. In unloaded streams its position is higher as the slope is steeper, as the discharge is greater, and as the bed is rougher.

Pitot tube.—The constant of the Pitot velocity gage—the ratio between the head realized and the theoretic velocity head—is not the same in all parts of a conduit, being less near the water surface and greater near the bottom or side of the conduit.

NOTATION.

Certain letters are used continuously in the volume as symbols for quantities, a definition accompanying the first use. These are arranged alphabetically in the following list, with brief characterization and page reference. The list does not include letters used temporarily as symbols and defined in immediate connection with their use; and if the same letter has both temporary and continuous uses only the continuous use is here given.

Symbol	Definition	Page.
A	A constant of the Pitot-Darcy gage.	254
(A), (B), etc.	Grades of débris, separated by sieves	21
(A_1 G_4), etc.	Mixtures of débris grades, the letters and figures indicating components and proportions	169
a	A pressure constant (Lechalas)	193
α	Numerical constant in equation $C=b_2(1-\alpha R)R^m$	125
Accents (´ ` ^ ˇ)	An accent over the symbol of a variable indicates the nature of its influence on a function. Thus $A=f(\acute{B}, \grave{C}, \hat{D}, \check{E})$ states that A is an increasing function of B, a decreasing function of C, a maximum function of D, and a minimum function of E	96
b	A constant in equation (109)	191
b'	A constant depth in equation (21).	88
b_1	A constant capacity in equation (10)	64
b_2	A constant capacity in equation (54)	129
b_3	A constant in equation (64)	139
b_4	A constant in equation (75)	151
b_5	A constant in equation (91)	186
C	Capacity of a stream for traction of débris (in gm./sec.)	35
C_r	Readjusted value of capacity	141, 151
c	Constant coefficient in $y=cx^j$	99
c_1	Constant coefficient in $C=c_1S^{j_1}$	109
D	Mean diameter (in feet) of particles of débris	21
d	Depth of current (in feet)	33
d	Differential	97
E	Efficiency of stream for traction of débris; capacity per unit discharge per unit slope; C/QS	36
e	Base of Naperian logarithms	61
F	Linear fineness of débris; the reciprocal of D	21, 183
F_2	Bulk fineness of débris; the number of particles in a cubic foot.	21, 183
ft./sec	Feet per second; unit of velocity	34
ft.^{3}/sec	Cubic feet per second; unit of discharge	34
g	Acceleration of gravity	225
gm./sec	Grams per second; unit of load and of capacity for load	34
H	Range of fineness	96
H, H_1	Readings of comparator, Pitot-Darcy gage	254
I	Synthetic index of relative variation	99
I_1	I for capacity and slope	122
I_3	I for capacity and discharge	147
I_4	I for capacity and fineness	153
I_{ds}	I for capacity and depth, slope being constant	164
I_{dQ}	I for capacity and depth, discharge being constant	164
I_{dV}	I for capacity and depth, mean velocity being constant	164
I_e	I for efficiency and slope	122
I_R	I for capacity and slope, form ratio being constant	119
I_V	I for capacity and mean velocity.	157
I_{VS}	I for capacity and mean velocity, slope being constant	157
I_{VQ}	I for capacity and mean velocity, discharge being constant	157
I_{Vd}	I for capacity and mean velocity, depth being constant	157
I_w	I for capacity and slope, width being constant	119
i	Index of relative variation; exponent in $y=vx^i$	99
i_1	i for capacity and slope	99
i_2	i for capacity and form ratio	130
i_3	i for capacity and discharge	141
i_4	i for capacity and fineness	153
j	Exponent in $y=cx^j$	99
j_1	Exponent in $C=c_1S^{j_1}$	109
K, k	Constants of the Pitot-Darcy gage	254
κ	A constant discharge, corresponding to competent discharge	139
L	Length, as a dimension of units	139
L	Load; mass of débris transported through a cross section per unit time (in gm./sec.)	35
M	Mass, as a dimension of units	139

Symbol	Meaning	Page.
m	Exponent in $C=b_2(1-\alpha R)R^m$	129
m_1	Exponent; $=m+1$	128, 129
n	Exponent in $C=b_1(S-\sigma)^n$	61, 64, 96
n_1	Exponent in $d=\frac{b'}{S^{n_1}}$	88
o	Exponent in $C=b_3(Q-\kappa)^o$	139
p	Exponent in $C=b_4(F-\phi)^p$	151
p. e.	Probable error	89
π	Ratio of circumference of circle to diameter	21
ϕ	A constant linear fineness, corresponding to competent fineness.	151
Q	Discharge (in ft.³/sec.)	35
R	Form ratio; d/w	36
ρ	Value of R corresponding to maximum capacity	129
S	Slope, in per cent, of stream bed or water surface	34
s	Slope of stream bed or water surface; = fall per unit distance.	34
σ	A constant slope, corresponding to competent slope	64
Σ	Sum of	228
T	Time, as a dimension of units	139
U	Duty of water (in gm./sec.) for traction of débris; capacity per unit discharge; C/Q	36
V	Velocity of current (in ft./sec.)	163
V_b	Velocity of stream at contact with bed (Lechalas)	193
V_m	Mean velocity; discharge÷area of cross section; Q/dw	33, 94, 155
V_s	Velocity of stream at surface (Lechalas)	194
v	Variable coefficient in $y=vx^i$	99
v_1	Variable coefficient in $C=v_1S^{i_1}$	99
v_3	Variable coefficient in $C=v_3Q^{i_3}$	141
w	Width of stream channel (in feet)	67

THE TRANSPORTATION OF DÉBRIS BY RUNNING WATER.

By Grove Karl Gilbert.

CHAPTER I.—THE OBSERVATIONS.

INTRODUCTION.

GENERAL CLASSIFICATION.

Streams of water carry forward débris in various ways. The simplest is that in which the particles are slidden or rolled. Sliding rarely takes place except where the bed of the channel is smooth. Pure rolling, in which the particle is continuously in contact with the bed, is also of small relative importance. If the bed is uneven, the particle usually does not retain continuous contact but makes leaps, and the process is then called *saltation*, an expressive name introduced by McGee.[1] With swifter current leaps are extended, and if a particle thus freed from the bed be caught by an ascending portion of a swirling current its excursion may be indefinitely prolonged. Thus borne it is said to be suspended, and the process by which it is transported is called *suspension*. There is no sharp line between saltation and suspension, but the distinction is nevertheless important, for it serves to delimit two methods of hydraulic transportation which follow different laws. In suspension the efficient factor is the upward component of motion in parts of a complex current. In other transportation, including saltation, rolling, and sliding, the efficient factor is the motion parallel with the bed and close to it. This second division of current transportation is called by certain French engineers *entraînement* but has received no name in English. Being in need of a succinct title, I translate the French designation, which indicates a sweeping or dragging along, by the word *traction*, thus classifying hydraulic transportation as (1) hydraulic suspension and (2) hydraulic traction.

[1] McGee, W J, Geol. Soc. America Bull., vol. 19, p. 199, 1908.

The bed of a natural stream which carries a large load of débris is composed of loose grains identical in character with those transported. The material of the load is derived from and returned to the bed, and the surface of the bed is molded by the current. When débris is transported through artificial channels, such as flumes and pipes, the bed is usually rigid and unyielding. Trifling as this difference appears, it yet occasions a marked contrast in the quantitative laws of transportation, and in the laboratory the two kinds of transportation were the subjects of separate courses of experimentation. It is necessary, therefore, for present purposes, to base a second classification of hydraulic transportation on the nature of the bottom. As the bed is typically plastic in stream channels and typically rigid in flumes and other artificial channels, it is convenient to call the two classes *stream transportation* and *flume transportation.*

The second classification traverses the first and their combination gives four divisions—stream suspension, stream traction, flume suspension, and flume traction. This report treats of stream traction and flume traction. It contains the record and discussion of a series of experiments made in a specially equipped laboratory at the University of California, Berkeley, in the years 1907–1909.

STREAM TRACTION.

Previous to the Berkeley work little was known of the quantitative laws of stream traction. The quantity of material transported has sometimes been said to be proportional to the square of the slope, but I have failed to discover that the statement has a recorded basis in theory or observation. A statement more frequently encountered is to the effect that the quantity varies with the sixth power of the velocity; and the origin of this assertion is not in doubt. It is an erroneous version of a deductive law commonly attributed to Hopkins (1844) or Airy and Law (1885), although announced as early as 1823 by Leslie.[1] The law, as formulated by Hopkins, is that "the moving force of a current, estimated by the volume or weight of the masses of any proposed form which it is capable of moving, varies as the sixth power of the velocity"; and this law pertains not at all to the quantity of material moved, but to the maximum size of the grain or pebble or bowlder a given current is competent to move.

The subject of the competence of currents, or the relation of velocities to the size of particles they can move, has also been treated experimentally by several investigators, and some account of their work will be given in later chapters.

About the year 1883 Deacon made, in Manchester, England, a notable series of experiments in the field of stream traction. As a result of definite measurements of quantities of sand transported and of the velocities of the transporting currents, he announced[2] that the amount transported, instead of varying with the sixth power of the velocity, as had been supposed, actually varies with the fifth power.

In the field of flume traction the work has been somewhat more extensive, having as its special incentive the needs arising in ore mills for the conveyance of crushed rock; and a résumé of results will be found in the chapter on flume traction.

A still greater body of investigation has been conducted by German and French engineers with the use of laboratory models of river channels. The French work was done largely for the purpose of testing certain rules formulated by Fargue[3] for the improvement of navigable streams. The German experiments were and still are addressed to the broader subject of river engineering in general and include within their scope the scientific study of the ways in which rivers shape and reshape their channels. The quantitative laws of stream traction, which constitute the chief theme of the Berkeley work, thus fall within the province of the German investigators, but their study has not been taken up. There are three German laboratories, all well equipped, located severally at Dresden, Karlsruhe, and Berlin.[4]

The flow of a stream is a complex process, involving interactions which have thus far baffled mechanical analysis. Stream traction is not only a function of stream flow but itself adds a complication. Some realization of the complexity may be achieved by considering briefly certain of the conditions which modify the capacity of a stream to transport débris along its bed. Width is a factor; a broad channel carries more than a narrow one. Velocity is a factor; the quantity of débris carried varies greatly for small changes in the velocity along the bed. Bed velocity is affected by slope and also by depth, increasing with each factor; and depth is affected by discharge and also by slope. If there is diversity of velocity from place to place over the bed, more débris is carried than if the average velocity everywhere prevails, and the greater the diversity the greater the carrying power of the stream. Size of transported particles is a factor, a greater weight of fine débris being carried than of coarse. The density of débris is a factor, a low specific gravity being favorable. The shapes of particles affect traction, but the nature of this influence is not well understood. An important factor is found in form of channel, efficiency being affected by turns and curvature and also by the relation of depth to width. The friction between current and banks is a factor and therefore likewise the nature of the banks. So, too, is the viscosity of the water, a property varying with temperature and also with impurities, whether dissolved or suspended.

The enumeration might be extended, complexity might be further illustrated by pointing

[1] Sir John Leslie's analysis is to be found in his Elements of natural philosophy, and in the edition of 1829 occurs at pages 426–427 of volume 1. David Stevenson mentions 1823, which probably indicates the first edition. William Hopkins gives a different analysis with practically the same result, and does not mention Leslie. The passage quoted occurs in Cambridge Philos. Soc. Trans., vol. 8, p. 233, 1844, and is probably from his earliest discussion of the subject. Wilfred Airy's later but evidently independent analysis appears in Inst. Civil Eng. Proc., vol. 82, pp. 25–26, 1885, with expansion by Henry Law on pp. 29–30.

[2] Deacon, G. F., Inst. Civil Eng. Proc., vol. 98, pp. 93–96, 1894.

[3] Fargue, L., Annales des ponts et chaussées, 1894.

[4] The equipment and work of the laboratory of river engineering of the Technical High School of Dresden are discussed in the Zeitschrift für Bauwesen, vol. 50, pp. 343–360, 1900, and vol. 55, pp. 664–680, 1905; the equipment of the laboratory of the Technical High School "Fredericiana" of Karlsruhe in the same journal, vol. 53, pp. 103–136, 1903, and vol. 60, pp. 313–328, 1910; and the equipment and work of the Laboratory for River Improvement and Naval Architecture, Berlin, in vol. 56, pp. 123–151, 323–324, 1906.

out the influence of conditions on one another, and the difficulty of measuring the detrital loads of streams might be dwelt upon, but enough has been said to warrant the statement that an adequate analysis with quantitative relations can not be achieved by the mere observation of streams in their natural condition. It is necessary to supplement such observation by experiments in which the conditions are definitely controlled.

OUTLINE OF COURSE OF EXPERIMENTATION.

In the work of the Berkeley laboratory capacity for hydraulic traction was compared with discharge, with slope, depth, and width of current, and with fineness of débris; and minor attention was given to velocity and to curvature of channel. For the principal experiments a straight trough was used, the sides being vertical and parallel, the ends open, the bottom plane and horizontal. Through this a stream of water was run, the discharge being controlled and measured. Near the head of the trough sand was dropped into the water at a uniform rate, the sand grains being of approximately uniform size. At the beginning of an experiment the sand accumulated in the trough, being shaped by the current into a deposit with a gentle forward slope. The deposit gradually extended to the outfall end of the trough, and eventually accumulation ceased, the rate at which sand escaped at the outfall having become equal to the rate at which it was fed above. The slope was thus automatically adjusted and became just sufficient to enable the particular discharge to transport the particular quantity of the particular kind of sand. The slope was then measured. Measurement was made also of the depth of the current; and the mean velocity was computed from the discharge, width, and depth.

In a second experiment, with the same discharge, the sand was fed to the current at a different rate, and the resulting slope and depth were different. By a series of such experiments was developed a law of relation between the quantity of sand carried, or the load, and the slope necessary to carry it, this law pertaining to the particular discharge and the particular grade of sand. The same experiments showed also the relations of the velocity of the current to slope and load.

Another series of experiments, employing a greater or a less discharge, gave a parallel set of relations between slope, load, and velocity. By multiplying such series the relations between discharge and slope, discharge and load, and discharge and velocity were developed.

Then a third condition was varied, the width of channel; and finally the remaining condition under control, the size of the sand grains. Thus data were obtained for studying the quantitative relations between load, slope, discharge, width, and fineness, as well as the relations of depth and mean velocity to all others. In all, the range of conditions included six discharges, six widths of channel, and eight grades of sand and gravel, but not all the possible combinations of these were made. The actual number of combinations was 130, and under each of these were a series of measurements of load, slope, and depth. There were also limited series of experiments involving a greater number of discharges and a greater number of widths. The separate determinations of load and slope numbered nearly 1,200, and those of depth about 900.

SCOPE OF EXPERIMENTS.

Before proceeding to a fuller description of apparatus and experiments, let us consider to what extent the conditions of the laboratory were representative of the conditions which exist in the natural stream.

The sand used came from the beds of American and Sacramento rivers and was assumed to be representative of river sand in general. No attention was paid to the influence on traction of the form and density of grains. Each sample used was separated from the natural mixture by means of two sieves and was composed of grains which passed through a certain mesh and were arrested by a mesh slightly smaller. In the sand carried by a river near its bed the range of size is much wider. The limit of coarseness is found in those particles which the current is barely able to roll, the limit in fineness in those particles which the swirls of the current are not quite able to lift into suspension; and the limits vary from point to point of the channel bed. This difference in condition was not wholly ignored, but a short supplementary series of experiments was made with definite mixtures of sand of various sizes, as well as with a natural mixture.

The straight channel of the laboratory differs materially from the curved channels of nature. It gives comparatively uniform depths and velocities from side to side and from point to point in the direction of the flow, while in a curved channel the depths and velocities vary greatly both across and along the channel. This difference in condition also received some attention, a short series of experiments being made with crooked and curved channels.

The vertical sides of the troughs did not well represent the sloping banks of rivers, and no attempt was made to measure the qualification due to this difference. The cross section of the laboratory current was essentially a rectangle and the ability of the current to transport was found to be definitely related to the ratio between depth and width; but satisfactory connection was not made between this relation and the forms of cross section in rivers.

The thalweg of a river channel traverses an alternation of deeps and shoals, the deeps being characterized by a different system of velocities and by a different line of separation between the grades of débris carried severally by suspension and traction; but these contrasts were touched only in a qualitative way in the work of the laboratory.

Each experiment dealt with a slope in adjustment with a particular discharge and a particular grade of sand. In a natural stream the discharge is subject to variation, and its changes cause changes in the fineness of. the material carried along the bottom. Load and the local slopes are ever in process of adjustment to the temporary conditions of discharge and fineness, but the adjustment is never complete. The general or average slope is adjusted to an indeterminate discharge which is neither the smallest nor the greatest. For this phase of disparity allowance is not easily made.

One of the conditions affecting velocities is friction on bed and sides of channel. Friction on the bed depends partly on the roughness of the bed and partly on the consumption of energy by traction. Its laws are the same in laboratory and in river. Friction on the sides depends on the character of the channel wall and must be materially greater on river banks than on the smooth sides of the experiment trough. The magnitude of the difference was not determined. Velocities are affected by the viscosity of the water, variations in this factor being caused by differences in temperature and by impurities in solution and in suspension. The transportation of small particles is affected by adhesion, a property varying with the mineral character of the particles and with the impurities of the water. These factors were ignored but are probably negligible in comparison with the factors tested. It may be mentioned, however, that the water of the laboratory was practically free from suspended material, whereas that of rivers is usually highly charged at the time of most active traction.

These comparisons serve to show that the investigation treats of a group of important factors of the general problem of stream traction but by no means comprehends all. Its results constitute only a contribution to the subject.

ACCESSORY STUDIES.

Incidental and accessory to the main inquiry were a number of minor inquiries. One pertained to the Pitot tube, a second to other methods of measuring velocity near the bottom, a third to the relation between the mean velocity of a loadless stream and the roughness of its channel bed, and a fourth to the mechanical process of hydraulic traction.

FLUME TRACTION.

In the experiments on flume traction the bed of the channel was not composed of loose débris but was the unyielding bottom of the trough. The same apparatus was used, with appropriate modifications. In each experiment slope of channel was predetermined, the trough being placed with definite inclination. The bed of the channel was given a definite quality of roughness or smoothness, and the material of the load was of a particular fineness or of a definite mixture of sizes. With a definite discharge flowing through the trough, débris was fed to the current at a definite rate, and the rate was gradually increased until clogging occurred. The rate of feed just before clogging was then recorded as the maximum load under the particular conditions. The series of experiments used two widths of channel, five textures of channel bed, six discharges, and seven grades of sand and gravel, besides mixtures. There were nearly 300 determinations of load.

APPARATUS AND MATERIAL.

EXPERIMENT TROUGHS.

The trough in which most of the experiments were made was of wood, 31.5 feet long, with an inside width of 1.96 feet. The height of the sides was 1.8 feet at the head and 1 foot at the end, the change being made by a series of steps. Its proportions and general relations are illustrated by figure 1. The surfaces were planed and painted. At the head, where water entered, the trough was connected with a tank by a flexible joint, a groove of the under side of the trough bottom resting on a semicylindric member of the tank, so as to constitute a hinge, and the walls of the trough being connected with the sides of the tank by a sheet of flexible rubber. Here also was a gate by which the flow from the tank could be stopped. Close to the opposite end of the trough was a cross trough 11 feet long, 2.5 feet wide, and 3 feet deep, rigidly attached to the experiment trough and extending below it. A rectangular opening in the bottom of the experiment trough, an opening having the width of that trough, permitted sand in transportation by the current to sink into the cross trough, which contained boxes to receive it. The width of the experiment trough was varied by means of a longitudinal partition which was given various positions. Its width at the end was also varied by means of two oblique partitions, the "outfall contractor," which merged with the sides a few feet from the end and could be adjusted as desired. For certain experiments

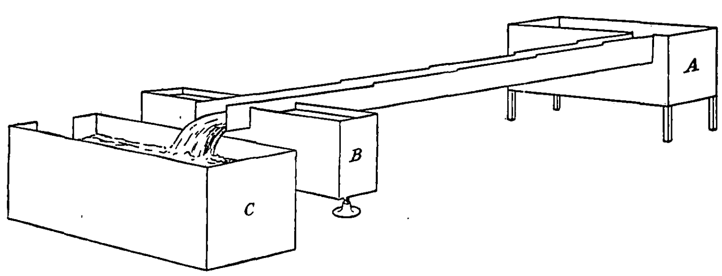


FIGURE 1.—Diagrammatic view of shorter experiment trough, showing relations to stilling tank (*A*), cross tank (*B*), and settling tank (*C*).

false bottoms were added, with surfaces specially prepared as to roughness. These will be specifically described in connection with the corresponding observations. A second trough, having the same function as the one just described was 150 feet long but similar in width and style. Its sides were higher and it was not hinged at the head but remained horizontal. By temporary arrangements of partitions curved and crooked channels 1 foot wide were constructed within this trough. The shorter trough was installed in the basement of the Mining Building of the University of California; the longer one on the campus near by. (See Pl. I, frontispiece.) The longer trough was remodeled for the experiments on flume traction.

A third trough 14 feet long and 0.67 foot wide had its wooden sides replaced for a space of 3.5 feet, at midlength, by plate glass, so that observation could be made from the side. It was provided with a sliding diaphragm, to be described in another place.

A fourth trough, of iron, was used only in the experiments on flume traction and will be described in connection with those experiments.

A few experiments were made also in a trough carrying the waste water of the 150-foot trough. This had a width of 0.915 foot.

WATER SUPPLY.

The water was taken from the municipal mains of Berkeley. As it was not practicable to draw freely on this source, a moderate supply was made to serve for a long series of experiments, being stored in a sump and pumped up as required. By repeated use it acquired a certain amount of fine detritus in suspension, but the quantity was not sufficient to obstruct the view of the experiments—or rather, when it was found obstructive, a fresh supply of clear water was substituted.

THE WATER CIRCUIT.

Starting from the storage tank or sump, the water was lifted by a power pump to a high trough 13 feet above. In passing through this trough it was first quieted by baffles and then regulated as to surface level by means of a spillway about 13 feet wide, the overflow returning directly to the sump. At the end it sank slowly through a vertical shaft, or leg, whence it issued in a jet through an aperture regulated by a measuring gate. After spending its force against a water cushion it passed through a stilling tank and then through the experiment trough. From that it fell a short distance to a settling tank, and thence returned to the sump.

In figure 2 the circuit is shown diagrammatically but without accuracy as to the arrangement and relative sizes of the parts of the apparatus.

DISCHARGE.

For the control and determination of the discharges used in the experiments, a measuring gate was provided. Near the lower end of the vertical leg of the high trough the water issued through a rectangular opening in a brass plate. The gate, also of brass, sliding along the plate, controlled the size of the opening, its motion being given by rack and pinion and its position shown by a suitable scale. The head was about 6 feet and was determinate. The gate and its calibration are described in Appendix B (pp. 257–259).

The head was regulated by means of the spillway in the high trough, and the amount of overflow on the spillway was controlled by a gate valve in the supply pipe just above the pump. As a check on the control, the position of the water surface was shown in an inclined glass tube outside of the high trough. The index tube being nearly horizontal, its meniscus had a magnified motion and the condition of the head could be seen at a glance. (See fig. 88, p. 257.)

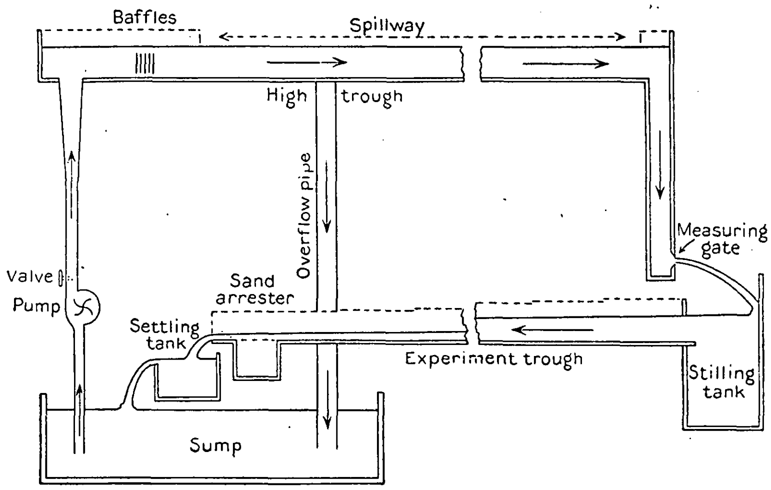


FIGURE 2.—Diagram of water circuit.

SAND FEED.

Above the experiment trough, and near its head, was a hopper-shaped box from which sand was delivered to the current in the trough. The box ended downward in an edge which stood transverse to the trough. Along this edge were a series of openings whose size was determined by a movable notched plate of brass. Water was supplied to the sand in the hopper, both at the top and near the bottom, the amount being regulated by valves. This water came from a small reservoir that was kept full by diverting part of the jet issuing from the measuring gate, and its use therefore added nothing to the measured discharge.

For some of the experiments débris was fed by hand, the quantity being regulated by means of a measuring box and a watch.

SAND ARRESTER.

The cross trough attached to the experiment trough and extending below it (see figs. 1 and 2) had along its bottom a track on which moved a platform car. This car carried two iron boxes to receive the sand. The boxes were rectangular and a little broader than the experiment trough. Openings protected by wire gauze permitted water to drain from them when they were lifted out.

SETTLING TANK.

The function of the settling tank was to catch sand which was carried past the cross trough. It was fitted with a system of partitions providing two alternative courses through which the stream could be turned, and with a hinged partition—the "deflector"—by which the diversion was made.

GAGE FOR DEPTH MEASUREMENT.

A frame resting on the experiment trough bore in vertical position a slender brass rod. This was raised and lowered by rack and pinion, and its relative height could be read on a scale. Depth of water was measured by reading the scale first with the rod end at the water surface and again with it at the débris surface.

LEVEL FOR SLOPE MEASUREMENT.

A surveyors' level stood a few yards from the trough, about equally distant from the ends, and was used, with a light rod, to measure relative heights for the determination of slopes of water surface and sand surface.

PITOT-DARCY GAGE.

A pressure-gaging apparatus of the Pitot-Darcy type, but of special pattern, was used to measure velocities of current. Its two apertures were directed severally upstream and downstream. Its external form was designed to give the least possible resistance to the current. The reading scale, with rubber-tube connection, had a fixed position, while the receiving member was moved from point to point. A fuller description is contained in Appendix A (pp. 251–256).

SAND AND GRAVEL.

The débris used in the experiments was obtained from three streams—Sacramento River 7 miles below the mouth of the American, American River 8 miles above its mouth, and Strawberry Creek in Berkeley. The débris from the creek was relatively coarse and was used only in the experiments on flume traction. The mean density of the river material was 2.69; that of the creek gravel 2.53. The forms of the grains of sand are shown in Plate II. To prepare the débris for use it was sorted into grades by a system of sieves, and in the laboratory records each grade was designated by the limiting sieve numbers. Thus the grains of the 40–50 grade passed through a sieve with 40 meshes to the inch and were caught by a sieve with 50 meshes to the inch. For the sake of brevity the grades are commonly indicated in this report by letters in parentheses—(A), (B), etc.—and the same notation is extended to mixtures of two or more sizes. Neither of these notations, however, is suited for the mathematical discussion of the laboratory data, and three others were devised. These are, first, the mean diameter of particles, designated by D; second, the reciprocal of the mean diameter, or the number of particles, side by side, in a row 1 foot long, designated by F; third, the number of particles necessary to occupy, without voids, the space of 1 cubic foot, designated by F_2. In the sense that the notation of D distinguishes by magnitudes, the notations of F and F_2 distinguish by minitudes. F is otherwise called *linear fineness*, and F_2 *bulk fineness*.

To determine the several constants for a grade of débris, a sample was weighed and its particles were counted. Then, N being the number of particles in the sample, W their weight, G their density, and W_0 the weight of a cubic foot of water,

$$F_2=\frac{NGW_0}{W}$$

Defining mean diameter as the diameter of a sphere having the volume of the average particle—

$$F=\sqrt[3]{\frac{\pi}{6}F_2}$$

$$D=\frac{1}{F}=\sqrt[3]{\frac{6F_2}{\pi}}$$

In the following table the grades of sand and gravel are characterized by the several notations.

TABLE 1.—*Grades of débris.*

Grade name.	Sieves used in separation (meshes to 1 inch).	D, mean diameter of particles (foot).	F, number of particles to linear foot.	F_2, number of particles to cubic foot.	Range of D or F.	Range of F_2.
(A)...	50–60	0.00100	1,002	1,910,000,000	1.13	1.44
(B)...	40–50	.00123	812	1,023,000,000	1.17	1.60
(C)...	30–40	.00166	602	417,000,000	1.44	2.99
(D)...	20–30	.00258	388	111,500,000	1.56	3.80
(E)...	10–20	.00561	178	10,770,000	1.95	7.41
(F)...	6– 8	.0104	95.9	1,685,000	1.40	2.74
(G)...	4– 6	.0162	61.8	451,000	1.43	2.92
(H)...	3– 4	.0230	43.4	156,000	1.36	2.51
(I)...	1– 2	.0547	[18.3]	11,900	[2.00]	[8.00]
(J)...	½– 1	.110	[9.1]	1,440	[2.00]	[8.00]
(K)...	⅓– ½	.200	[5.0]	239	[1.50]	[3.37]

The sixth column of the table contains an index of the range in diameter within each grade, and this is also the range in linear fineness. The index is the ratio between the diameters of the apertures of the two sieves by means of which the grade was separated. It would express accurately the ratio of the diameter of the largest particle to that of the smallest particle in the same grade if all the particles were spherical, or if all had precisely the same shape. But there are actual differences of form sufficient to modify materially the character of the separation. For the same mean diameter a prolate form will pass a sieve that will arrest a sphere, and an oblate form may be arrested where a sphere will pass. By reason of this qualifying condition, the actual range is somewhat greater than the tabulated estimate. The range for bulk fineness, given in the last column, is the cube of the range of linear fineness.

METHODS OF EXPERIMENTATION.

The methods here described are those used in the investigation of stream traction. Those used in studying flume traction are set forth in the chapter on that subject.

GENERAL PROCEDURE FOR A SINGLE EXPERIMENT.

The experiment trough stands horizontal. The width of channel has been fixed by the placing or the omission of the partition. The head gate is open. The openings in the hopper have been set to a particular size. The outfall contractor has been adjusted to a width previously found suitable for the conditions of the experiment. The two sand-catching boxes stand on the car, one of them being in position under the opening in the trough bottom.

The pump is started. The measuring gate is opened until its index reaches the point corresponding to the desired discharge. A valve associated with the pump is turned, if necessary, to adjust the water level in the high trough. When the flow in the experiment trough has become steady, or nearly steady, the sand feed is started by opening the valves which admit water to the hopper.

The "run" has now begun, and it is continued without change until the slope of the deposit constituting the channel bed has become stable. The car in the cross trough is now moved so as to bring the reserve sandbox into position, a stop watch is started, and the deflector in the settling tank is shifted. These changes begin the period of load measurement. During this period measurements are made of the depth of the current and, under certain conditions, of the slope of the water surface, and the character of the sand bed is recorded. The measuring period is terminated by again shifting the sand boxes and the deflector, and the watch is stopped. The head gate is now closed, sand feed and pump are stopped, and the discharge-measuring gate is closed. Next the slope of the channel bed is measured, and the sand caught during the period recorded by the watch is weighed.

DETAILS OF PROCEDURE.

In the following paragraphs some details and variants will be described, with comments on methods and apparatus. As the investigation had no precedents to follow, its methods were necessarily developed by a process of trial and and failure. Many defects of method were remedied as the work went on. A few that were recognized after much work had been done were tolerated to the end, because their correction would have made the body of results less homogeneous.

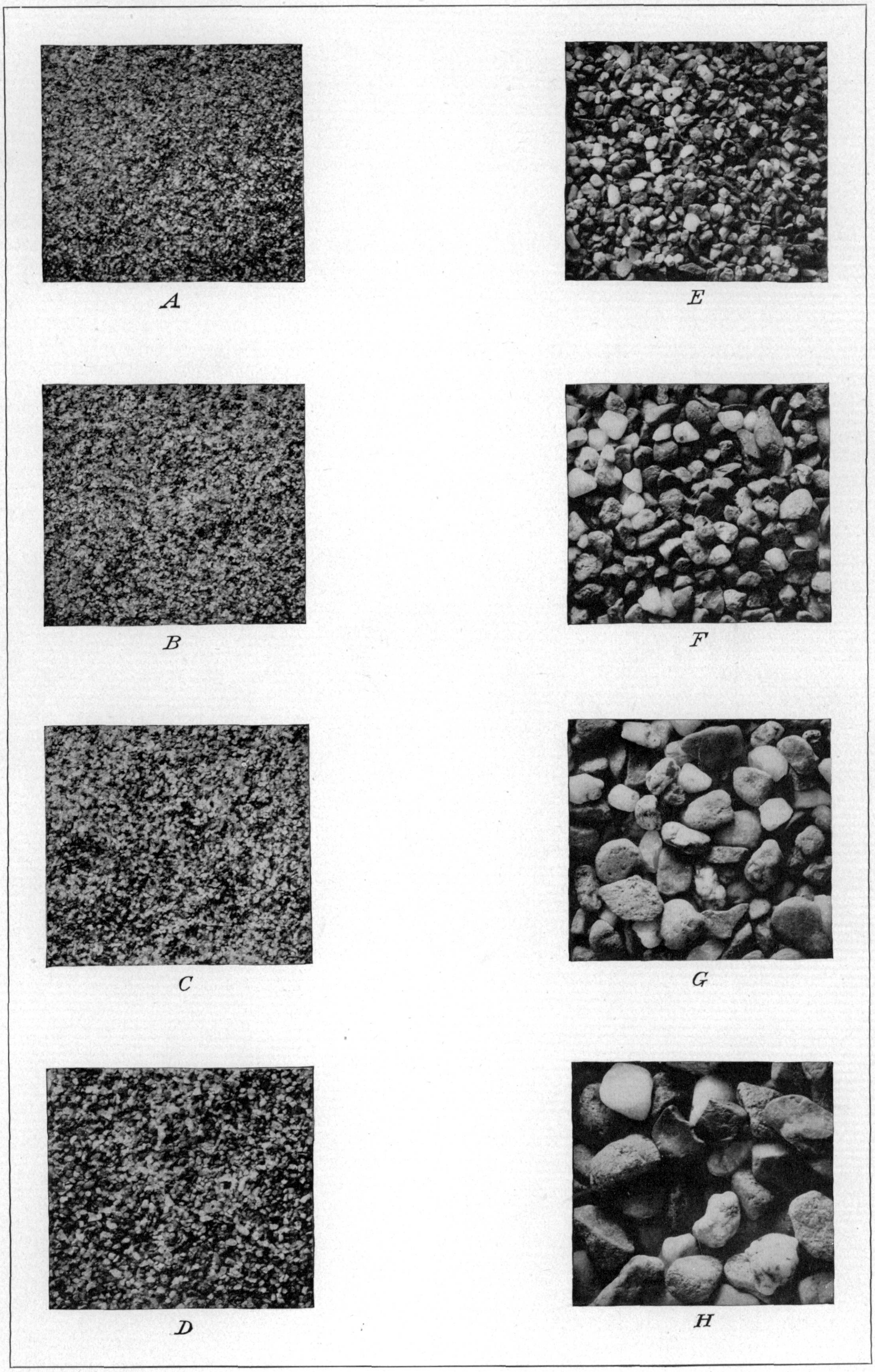
WIDTH OF CHANNEL.

The full width of the 31-foot trough and the 150-foot trough was 1.96 feet. By the use of partitions the channel was given widths approximately two-thirds, one-half, one-third, two-ninths, and one-ninth of a full width. The scale of width was, in feet, 0.23, 0.44, 0.66, 1.00, 1.32, 1.96. A few experiments employed also the widths 1.2, 1.4, 1.6, and 1.8 feet.

DISCHARGE.

As the discharge was determined by flow through an aperture of adjustable size, under an adjustable head, the precision of its measurement involved (1) the precision of setting the gate at the aperture, (2) the precision of calibrating the aperture for different positions of the gate, and (3) the precision of adjusting the head.

The rating (described in Appendix B) was effected by a volumetric method, believed to be adequate, the average probable error of its determinations being about 1 in 500.



DÉBRIS USED IN EXPERIMENTS

The gate was set by bringing an index mark opposite a graduation mark on a scale, the two marks being on brass plates in contact. The gate was controlled by rack and pinion, and considerable force was necessary to move it. The limit of error may have been 0.002 foot. The ordinary error is believed to have been less than 0.001 foot. An error of 0.001 foot in the setting would cause an error of 0.002 ft.^{3}/sec., or $\frac{1}{250}$ of the medium discharge.

The determination of head is subject to an accidental error and a systematic error. The accidental error pertains to the adjustment of water level in the high trough, by means of the valve at the pump, with observation of the tube index of water level. It was possible to give this adjustment a refinement comparable with that of the hook gage, but in practice that refinement was not attained, because a close watch was not kept on the index. It was found by experience that the fluctuations of level (occasioned by fluctuations of the electric current supplying power to the pump) were small, and they were usually neglected, a practical calibration of the valve at the pump being arranged so that it could receive the proper setting for each setting of the discharge measuring gate. The ordinary error of the adjustment of the head is estimated at 0.003 foot, which would occasion an error in the discharge of 1 in 4,000.

The remaining possibility of error is connected with the history of the apparatus. At the time of the calibration of the measuring gate the laboratory occupied temporary quarters. In its removal to permanent quarters there was a measurement and readjustment of the vertical distance constituting the head. Also, for the work with the long trough the measuring gate was transferred to a replica of the high trough, which may have differed in some particular affecting the constants. As the work of calibration was at no time repeated, there was no check on the errors which may have been thus introduced. In a general way, they are probably of the same order of magnitude as the errors of adjustment of water surface. It is believed that all other errors affecting discharge are small in comparison with that connected with the measuring gate.

The vertical width of the aperture by which discharge was regulated was 2 inches. The head, measured from the middle of the aperture, was 6.0 feet. The horizontal dimensions of the aperture, during experimental work, ranged from 0.1 inch to 6.0 inches, and the corresponding discharges are given in the following table:

TABLE 2.—*Gate readings and corresponding discharges.*

Gate opening (inches).	Discharge (ft.^{3}/sec.).	Gate opening (inches).	Discharge (ft.^{3}/sec.).
0.1	0.019	1.5	0.272
.2	.039	1.6	.290
.3	.058	1.7	.308
.4	.075	1.8	.327
.5	.093	1.9	.345
.6	.111	2.0	.363
.7	.128	2.5	.454
.8	.146	3.0	.545
.9	.164	3.5	.639
1.0	.182	4.0	.734
1.1	.200	4.5	.828
1.2	.218	5.0	.923
1.3	.237	5.5	1.021
1.4	.255	6.0	1.119

THE FEEDING OF SAND.

The fact that the hourglass has been used to measure time suggests that the flow of dry sand through an aperture may be uniform. Such a flow was not tested in the laboratory because the plan for experimentation required that sand should be used over and over, and it was not practicable to dry it. The hopper was a device intended to produce a uniform flow of wet sand. Moist sand will not flow through a small opening; but if enough water is present to more than fill the voids, adhesion is overcome and flow takes place, as in a quicksand. The freedom of the flow depends on the amount of water. It was found difficult to maintain a uniform condition in the hopper. Another difficulty arose from clogging of the openings, and this was occasioned by shreds of wood fiber and similar impurities in the sand. The second difficulty was largely obviated, after a time, by making the openings larger and fewer; but the hopper feeding was at best not sufficiently uniform to be used in measuring the load carried by the experimental stream.

For all experiments in which a large quantity of débris was carried, the material was fed to the current by hand and was measured in the feeding. A small box of known capacity was filled with the material and emptied into the current at regular intervals timed by a watch or clock. If the interval was long, the measured unit was dumped on a sloping table above the trough and gradually fed to the current by means of a scraper. Hand feeding had the defect of discontinuity, as well as irregularity in detail, but it had the advantage of measurement, and in certain experiments its measurement of load gave an important check on the measurement of débris delivered at the outfall end of the trough. With a perfect and stable adjustment of conditions the two should agree, and their disagreement served to show that the slope of the channel bed had not become perfectly adjusted, or else that its adjusted condition was subject to rhythmic oscillation.

In some of the later work the rate of feed was measured from time to time by intercepting the stream of sand falling from the hopper during a definite number of seconds and weighing the sample thus caught.

THE COLLECTION OF SAND.

In the original construction of the apparatus for arresting the sand the opening in the bottom of the trough was covered by a coarse wire screen, which lay flush with the trough bottom. This was intended to separate the current above from the still water below and prevent the formation of eddies, which might keep the sand from settling to the collecting box and might also check the current. It fulfilled its purpose and was altogether satisfactory for currents of moderate velocity, but with high velocities it interfered with the arrest of the sand, letting a considerable fraction pass on to the settling tank. It was accordingly removed, apparently without bad results. Eddies were formed, but the anticipated difficulties were not realized.

On the whole the apparatus for arresting sand was successful. It was only with the finer débris and at the highest velocities that the fraction of load escaping to the settling trough was too large to be neglected in the weighing.

DETERMINATION OF LOAD.

The sand collected, in sand box and settling tank, during the period recorded by the stop watch was weighed without drying, and the gross weight was afterward corrected by an allowance for the contained water. In order to determine the proper allowance a preliminary study had been made, and as a result of that study a definite procedure was adopted for bringing the wet sand to a particular "standard" condition. After the sand-collecting box had been lifted from the trough all water which would drain from it by gravity alone was allowed to escape. It was then removed to smaller boxes for weighing. These boxes were jarred by tapping, which caused the sand grains to readjust their contacts and settle together, excluding a part of the interstitial water, which appeared at the surface and was poured off. The sand was then weighed. It is of interest to note that in the condition thus adopted as a convenient standard sand occupies less space than when dry, moist, or supersaturated; its voids are at a minimum.

The period recorded by the stop watch was ordinarily about 10 minutes but was made less when the current was most heavily loaded, because of the limited capacity of the sand-collecting box, and was extended for the lightest loads. Its beginning was sharply defined by the shifting of the sand boxes, which could be made to coincide within a second with the starting of the watch. Its end was somewhat less definite, but the error in time is believed to be small in comparison with the whole period.

The load per second was computed by dividing the total load, namely, the corrected weight of sand, by the number of seconds in the stop-watch reading. Its error included (1) the error of timing, (2) the error of standardizing the sand and correcting for contained water, and (3) the error in weighing. There are no definite data bearing on its amount, and nothing better can be recorded than a general impression that the results are reliable within 2 per cent, that the precision is lower than that of the discharge measurement, and that the error in determination of load is notably less than the error, presently to be considered, in correlating load with slope.

When the rate of feed was regulated by the periodic contribution of a measureful of débris, the weighings of the unit, from time to time, showed inequalities from which precision could be estimated. A computation indicated the average probable error, for a run, as about 1 per cent. This depended chiefly on the standardization, and as that was less perfect for the débris as fed than for the débris as collected, the ordinary measurement of collected load is presumably affected by a smaller probable error.

DETERMINATION OF SLOPE.

The observations of slope were made with surveyors' level and rod. The rod, made for the purpose, without unnecessary length or weight, was graduated to hundredths of a foot and read by eye estimate to thousandths. It was held by an assistant while the observer and recorder stood at the telescope. The positions were determined by a graduation of the trough, which was marked at every foot. To measure the water slope, heights of the surface were taken at several points along the trough. To measure the sand slope, heights were taken at intervals of either 2 or 4 feet, the shorter interval being used with the shorter trough. The water slope could not be measured when the surface was rough. When the débris surface was rough, it was usually graded before measurement by scraping from crests into adjacent hollows.

The observed heights were plotted on section paper, with relatively large vertical scale, and a straight line was drawn through or among them. The line served the purpose of a preliminary determination of slope, and the plots were inspected for the detection of systematic errors. As a result of this inspection a portion of the profile was selected for the determination of slope, and from the observations on this portion the slope was computed by least-squares method.

CONTRACTOR.

As will be explained more fully in another connection, the slope measurements were affected (1) by systematic errors connected with the conditions under which the water entered and escaped from the trough, and (2) by accidental errors arising from rhythm. One of the measures used to diminish the systematic errors was the contraction of the current at the outfall end of the trough. The apparatus for this purpose consisted of two boards as wide as the depth of the trough and arranged as in figure 3. Their attachment to the sides was flexible, so that the degree of convergence and the width of aperture at the outfall could be modified at will. This apparatus will be called the outfall *contractor*. The theory and efficiency of the contractor will be considered in the discussion of the slope errors.

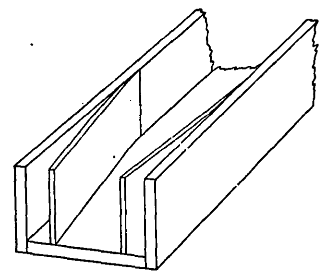


FIGURE 3.—The contractor.

MEASUREMENT OF DEPTH.

The depth of the current was measured at midwidth and near midlength of the trough. The determination was made by means of the gage already described (p. 21), during the period of time for which the load was measured. As the water surface was subject to rhythmic fluctuation, a series of observations of its position were made, and their mean was used. A series of observations of the position of the débris surface were sometimes made also, but usually only a single observation, and the reading obtained was subtracted from the mean of readings on the water surface. The observations of the débris surface were subject to an error which was regarded as more serious than that of the observations of water surface because, being essentially systematic, it could not be eliminated by repetition. The presence of the gage rod in the water modified the distribution of velocities, and this modification included an increase of the current's velocity a little below the end of the rod. As the bottom was approached by the rod, the current scoured a hollow in the bed immediately under it; and if the rod were lowered to actual contact, the reading would give an excessive estimate of depth. What was attempted was to lower the rod to a position as nearly as possible at the level of undisturbed parts of the bed surrounding the visible hollow. This was a matter of judgment, but not of confident judgment, because the actual bed was concealed by a cloud of saltatory débris particles. It is therefore recognized that the measurements of depth are uncertain.

Whenever the water profile as well as the débris profile was surveyed, an independent estimate of depth was obtained by subtracting one profile from the other. This mode of determination avoided the error incident to the gage work and was on the whole satisfactory, but unfortunately the number of experiments to which it could be applied was not large. During the greater part of the experimentation, the importance of the water profile was not recognized, and this particular use of it was essentially an afterthought.

The values from profiles being assumed to have relatively small errors, both systematic and accidental, it is possible to measure by their aid the precision of the values from gage readings. Of 118 depths which were measured by both methods, the gage gave the greater value for 36, the lesser for 78; and the average for gage values was 0.0045 ± 0.0007 foot less than the average for profile values. Independently of this apparent systematic error, the probable error of a single measurement with the gage was ± 0.007 foot.

MEASUREMENT OF VELOCITY.

The mean velocity of the current is computed by dividing the discharge by the area of the cross section, or the product of width and depth. Its precision depends on those of the determinations of discharge, width, and depth; and as the precision for discharge and width is relatively very high, the precision of mean velocity may be regarded as identical with that of depth.

The attempts to measure velocity close to the channel bed were not successful. This is much regretted, because it is believed that bed velocity is a prime factor in traction and that slope and discharge exert their influence chiefly through bed velocity. The mode of measurement to which most attention was given was that by the Pitot-Darcy gage, and special forms of that instrument were constructed for the purpose. The difficulty which seemed insuperable was essentially the same as that encountered in the measurement of depth. As the instrument approached the current-molded bed of débris, the bed retreated, with the formation of a hollow. In the presence of the instrument the normal velocity at the bed did not exist. Inseparable from this difficulty is a property of the instrument. When it is held close to the bottom or side of a channel its constant is not the same as in the free current. The system of flow lines and velocities with which the stream passes the obstructing object determines the instrument's constant, and when that system is modified by a neighboring object the constant changes. The nature of these difficulties is such that it was not thought worth while to experiment with other gages and meters which limit the freedom of the current.

Other devices tried were of one type. Small objects, such as currants or beans, only slightly denser than water, were placed in the current and watched. The lighter ones would not remain near the bottom. The heavier ones were visibly retarded when they touched the bed and were also retarded when close to the bottom by the cloud of saltatory sand, which has a slower average velocity than the water it suffuses.

MODES OF TRANSPORTATION.

MOVEMENT OF INDIVIDUAL PARTICLES.

ROLLING.

In stream traction sliding is a negligible factor. The roughness of the bed causes particles that retain contact to roll. When, as in most of the experiments, the grains are of nearly uniform size, each moving grain has to surmount obstacles with diameter like its own, and when it reaches the summit of an obstacle it usually possesses a velocity which causes it to leap. So rolling is chiefly the mere prelude to saltation. With mixed débris the same is true for the finer grains, but the coarser may roll continuously over a surface composed of the finer, and the coarsest of all, those close to the limit of competence, move solely by rolling.

The large particle, as it rolls over the bed of smaller particles, indents the bed, and its contact involves friction. The energy thus expended comes from the motion of the water, and its communication depends on differential motion between water and particle. Except under special conditions, to be mentioned later, the load travels less rapidly than the carrier, and it is also true that in a load of mixed débris the finer parts outstrip the coarser.

SALTATION.

In stream traction the dominant mode of particle movement is saltation. Because saltation grades into suspension it has often been explained in the same manner, by appeal to upward movement of filaments of current, but the recent studies have led me to entertain a different view. Before this view is presented an account will be given of certain observations which were made with the use of the trough having glass sides.

Through the trough was passed a current transporting sand of uniform grade, and the conditions were such that the sand bed and water surface were smooth. In the same water floated a few fine particles and thin flakes of mica, illustrating suspension, but there was no intergradation of the two processes. Viewed from the side, the saltation was seen to occupy at the bottom of the current a space with a definite upper limit, parallel to the sand bed. Within this space—the zone of saltation—the distribution of flying grains was systematic, the cloud being dense below and thin above, but not perceptibly varying from point to point along the bed. Viewed from above, the surface of the cloud seemed uniform and level, and it all appeared to be moving in the same direction. There was no suggestion of swirls in the current.

When, in looking from the side, attention was directed to the base of the zone, it was easy to watch grains that traveled half by rolling and half by skipping, and these moved quite slowly; but higher in the zone the motions were so rapid and diverse that all was a blur. To resolve this blur a sliding diaphragm was arranged. This consisted of a short board with a hole in it. The board hung

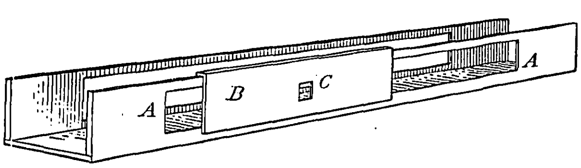


FIGURE 4.—Diagrammatic view of part of experiment trough with glass panels (*A*) and sliding screen (*B*). *C*, Hole in screen.

outside the wall of the trough, being supported by a cleat above in such manner that it could be slid along the trough. (See fig. 4.) The hole, about 2 inches square, gave a restricted view of the saltation zone. By sliding the board in the direction of the current and keeping the eyes opposite, a traveling field of view was obtained. Manifestly if the field traveled at the same rate as the current, any object moving with the current would appear at rest to the observer, because there would be no relative motion of observer and object; and if objects in the water were moving (horizontally) at different rates, those coinciding in rate with the field would be seen as if at rest, while the others would be seen as moving.

When the field was moved slowly the rolling grains ceased to be distinct but were replaced in distinctness by grains that seemed to bob up and down. These vibrated through a space of two or three diameters, as if repeatedly striking the bed and rebounding. In interpreting this appearance, allowance must be

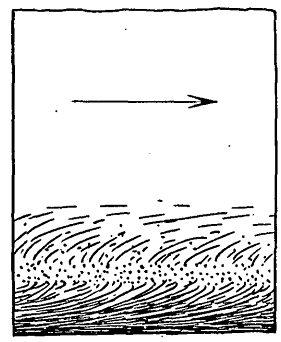


FIGURE 5.—Appearance of the zone of saltation, as viewed from the side with a moving field.

made for the fact that the grains were distinctly seen because they were moving horizontally about as fast as the diaphragm. Their paths were really low-arching curves, and only the vertical factor remained when the horizontal was abstracted. It is probable also that the appearance of rebounding was largely illusory, most of the grains either stopping at the end of the leap, or else leaping next time with a different velocity.

When the field was moved somewhat faster, the bobbing grains disappeared and there came into distinct vision a set of grains quite free from the bed and occupying a belt within the saltation zone. All the zone above and below them was blurred. In the middle of the belt vertical motion was to be discerned but was less conspicuous than in the lower zone. Where distinctness graded into blurring, lines of motion could be seen which were oblique and curved, the lines above the belt curving forward and those below backward, as shown in figure 5.

With progressively faster motion of the field the belt of distinct vision rose higher, until the top of the zone was reached, when all the lower part was blurred.

The systematic gradation of velocity and other features from the bed upward and the sensible uniformity of process over the whole width of channel are not consistent with the idea that the saltation zone is invaded by eddies of large dimensions, such as would be competent to sustain the grains by the upward components of their motions. If there were large ascending and descending strands of current the visible surface of the zone would be locally raised and depressed by them. We must, indeed, assume that the flow is turbulent, in the technical sense, because parallel or laminar flow is impossible with velocities competent for traction, but the eddies may be assumed to be of small dimensions in relation to the depth of the zone, and the lines of flow with which saltation is concerned may be assumed to be approximately parallel to the general direction of the current.

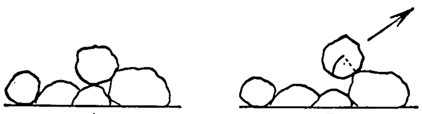


FIGURE 6.—The beginning of a leap, in saltation.

The explanation I would substitute for that of the uplift of grains by rising strands is that each grain is projected from the bed with an initial velocity which gives it a trajectory analogous to that of a cannon ball. The following fuller statement, though given with little qualification, should be understood as largely hypothetic.

In figure 6 the current is supposed to move from left to right above the grains of débris shown in outline. A grain which in *A* is at rest appears in *B* in an advanced position, having been rolled upward and forward about an undisturbed grain which lies in its way. (The moving grain is doubtless more likely to roll against two other grains than a single one, but the principle is the same.) In moving to its new position the center of gravity of the grain describes a curve convex upward. The grain continuously gains in velocity, and the acceleration also increases as the direction of motion comes to make a smaller angle with the direction of the current. At each instant the accelerative force due to the current and that of gravity are combined and have a resultant direction; and the combined or resultant accelerative force may be resolved into two parts, one of which coincides in direction with the motion of the center of gravity and the other with the line joining the center of gravity and the point of contact. The last-mentioned component presses the moving grain against the stationary grain. Opposed to it is the centrifugal force arising from the curvature of the grain's path; and the point is finally reached where the centrifugal force dominates and the grain is free. Under the ordinary conditions of saltation this point is not the crest of the obstruction, but is on the upstream side, so that the grain's direction of motion at the instant of separation is obliquely upward. Thus the free grain is initially moving upward as well as forward, and it has almost literally made a leap from the bed.

If the grain were at that instant released from all influences but gravity, its path before returning to the bed would be the arc of a quadric parabola with vertical axis. The actual deviation of its trajectory from the parabolic form is analogous to that observed in gunnery, for it arises from the resistance of a fluid; but the laws of resistance are not the same for air and water, and the frictional acceleration in one case is negative while in the other it is mainly positive. The trajectory in gunnery is shorter than the ideal parabolic arc; in saltation it is longer.

Figure 8 gives diagrammatically the trajectory of a saltatory grain. In figure 7 *AB* is a portion of the same trajectory. Let the space *AC* represent the instantaneous velocity of the grain, and let the line *AD* represent in direction and length the velocity of the water about the grain. Then, *C* and *D* being connected by a line, *CD* represents in direction and magnitude the relative velocity of water and grain, or the velocity of the water as referred to the grain. By reason of the mutual resistance of water and grain, this relative motion accelerates the grain, the acceleration being a function of the differential velocity, the size of the grain, and other conditions. On the line *CD*, showing the direction of the acceleration, let the space *CE* represent its amount. Then from *E* draw the vertical *EF*, represent-

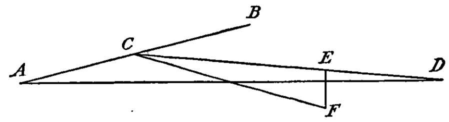


FIGURE 7.—Diagram of accelerations affecting a saltatory grain.

ing, to the same scale, the acceleration of the grain by gravity. Connect C and F; the line CF represents in magnitude and direction the resultant acceleration of the grain. These relations are independent of the particular directions of motion of the grain and the water. Let us now introduce the assumptions, believed to be practically true for the laboratory conditions, that the water in the region of saltation moves parallel to the bed and that its velocity increases notably with distance from the bed. In the ascending part of its path the grain encounters filaments of the current with higher and higher velocity. This tends to increase the relative velocity, but the grain is at the same time gaining in horizontal velocity and the gain tends to diminish the relative velocity. Unless the leap is short in relation to the size of the grain, the second of these tendencies is the greater, and at the highest point of its path the grain is moving nearly as fast as the

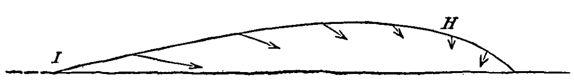


FIGURE 8.—Theoretic trajectory of a saltatory particle, the initial point being at I. Arrows indicate acceleration.

water. In the descending part of its path it encounters slower moving filaments of current, and at some point (H, fig. 8) its horizontal motion may equal that of the adjacent water. Then beyond H it passes through filaments moving still more slowly, and its acceleration from the reaction of the current becomes negative. The acceleration due to gravity is of course uniform and downward, and its combination with that due to the current yields a system of directions and magnitudes of the type indicated in figure 8 by short arrows. In the shorter and lower trajectories it is probable that the critical point H is not reached.

If the position of the grain before leaping (fig. 6) is such that only a relatively short roll suffices to free it, then its initial velocity is small and the angle of ascent at which it is freed is low. It has a short, flat trajectory, and its velocity at the highest point is moderate. If the original roll is longer there is time to acquire speed before the leap; the initial velocity is large and the angle of ascent is relatively high. It has a long and high trajectory and when at the crest has been accelerated to high velocity. If a grain at the end of a leap touches the bed at a favorable point it may leap again without coming to rest, and the impetus of the first flight will thus enhance the initial velocity of the second.

In the observations with the moving field the grains seen most distinctly were those which moved horizontally with the field and at the same time had little vertical motion. So each belt of distinctness contained grains at the tops of their trajectories and was practically made up of such grains. The grains producing the curved lines in figure 3 were ascending or descending obliquely, and their horizontal components of motion coincided with the motion of the field for an instant only.

In general the observations seem to show that the summit velocities of the leaping grains increase systematically with the height of the leap, and this generalization is in perfect accord with the hypothesis that the paths of grains are determined primarily by initial impulse.

Under the hypothesis the series of velocities observed by aid of the moving field are not velocities of current, for the initial velocities of grains, being caused by the current, require that the water outspeed all the grains at the bottom of the zone of saltation. At the top of the zone there must be at least a slight advantage with the current, provided the water velocities increase upward. That the water velocities do increase upward can hardly be doubted, for in sweeping along the sand the stream expends energy, and as its energy subsists in velocity, the expenditure involves retardation. Moreover, the grains of sand are at the same time most numerous and slowest near the bottom of the zone, so that their effect is there greatest.

In this connection it is to be observed that the width of the belt of distinct vision in the moving field (fig. 5) is greater for the upper part of the zone of saltation than for the lower. As distinct vision is limited to a certain (undetermined) range in horizontal velocities, this fact implies that the increase in horizontal speed of sand grains with distance from the bed is less rapid in the upper part of the zone than in the lower.

The preceding discussion is subject to two qualifications, the first of which is connected with the retardation of the current at the side of the trough. By reason of that retardation the zone of saltation is shallower near the side and does not include the longer and higher leaps. Figure 9 gives an ideal conception of the cross section of the zone and the distribution of flying grains within it. Observation from the side penetrates but a short distance into the cloud, the distance being least where the cloud is most dense. The practical limit of visual penetration may be assumed to take some such form as the line *AB*. Thus the

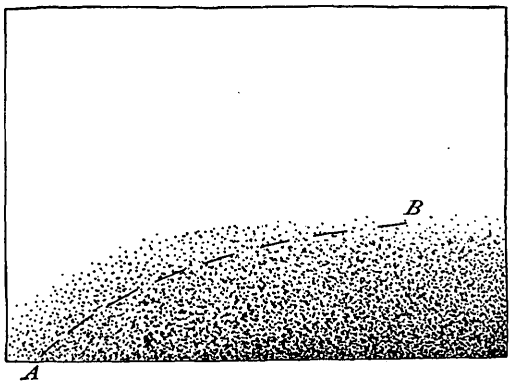


FIGURE 9.—Ideal transverse section of zone of saltation at side of experiment trough.

tract actually studied in the work with the moving field was somewhat superficial and was not in strictness a vertical section of the zone.

The second qualification is connected with turbulence. In steady flow the motion at each point of a stream is constant in velocity and direction. When the general velocity exceeds a certain minimum, which for the streams we have to consider is very small, the flow is not steady, but involves eddies or vortices, which as a rule move onward with the current. In consequence of these eddies the course of each particle of water is sinuous, and the sinuous courses interweave. The flow is then said to be turbulent. Usually there are both large and small eddies, the minute ones being multitudinous. As the axes of whirling movements have all attitudes, the directions of motion, as a rule, have upward or downward components, and the suspension of particles of débris is due to the upward components. Particles so small that they can not come to rest on the bottom are thereby lifted and relifted and kept in the body of the water. Under the conditions arranged for the study of saltation there appeared to be no large eddies, but the zone was unquestionably pervaded by small ones, excited by the roughness of the bed and by the differential motions of water and leaping grains. With increasing strength of current the texture of turbulence would enlarge and saltation pass into suspension. With a diversified débris, instead of the uniform material actually used, there would be phases of action in which the paths of small grains were made sinuous by turbulence, while those of larger grains remained simple in form. The trajectory of saltation, as described, may therefore be regarded as a simple type of path which combines in all proportions with the sinuous type of path characterizing suspension.

Through the entire zone of saltation motion is being communicated to particles of the load by the water, and there is a corresponding loss of motion by the water. That loss reduces all the stream's velocities but makes the greatest reduction in the lower part of the zone of saltation. The loss of velocity in the lower strands reduces their power to cause particles to leap. The greater the load the greater this reduction, and thus the quantity of load is automatically regulated.

COLLECTIVE MOVEMENT.

In the experiment used to illustrate saltation the collective movement of the sand was uniform, the conditions of the experiment having been adjusted to that end. But it is equally possible so to adjust them as to make the collective movement rhythmic. Uniformity is in fact an intermediate phase between two rhythmic phases, which are of contrasted types. These phases will be described.

In another experiment a bed of sand was first prepared with the surface level and smooth. Over this a deep stream of water was run with a current so gentle that the bed was not disturbed. The strength of current was gradually increased until a few grains of sand began to move and then was kept steady. Soon it was seen that the feeble traction did not affect the whole bed, but only certain tracts, and after a time a regular pattern developed and the bed exhibited a system of waves and hollows. As the waves grew the amount of transportation increased, showing that, under the given conditions, the undulating surface was better adapted to traction than the plane. With such waves and hollows are associated a special mode of transportation, which is illustrated in figure 10. A current reaching the bed at *A* follows the rising slope and crest of the wave to *C* and then shoots free, to reach the bed again at *D*. The space overleaped between *C* and *D* is occupied by one or more slow-moving eddies. From *A* to *C* there is traction, the material being derived from the slope between *A* and *B*. At *C* the débris, being abandoned by the current, is dumped, and it slides by gravity down the slope *CE*. So the upstream face of the wave is eroded and the downstream face built out, with the result that the wave, as a surface form, travels downstream. As this is precisely what takes place when a sand hill travels under the influence of the wind, the name of the eolian hill has been borrowed, and the waves are called *dunes*.[1] In one of the narrower troughs of the laboratory the dunes formed a single line. In a wider trough their arrangement sometimes suggested a double line, the crests of one being opposite the hollows of the other, but their arrangement continually changed. On the bed of a broad, shallow stream they are apt to have a subregular imbricated pattern.[2] In a deep stream a single dune may be nearly as broad as the channel. In the laboratory the forms were inconstant, but the type was about as broad as long, with the front edge convex downstream. In natural streams the dunes show great variety in outline, some being described as longest in the direction of the current and others as greatly extended in the transverse direction. They vary in size with the size of the stream, but especially with the depth, and are transformed and remodeled with increase and reduction of discharge. The horizontal dimensions of most laboratory examples may be conveniently described in

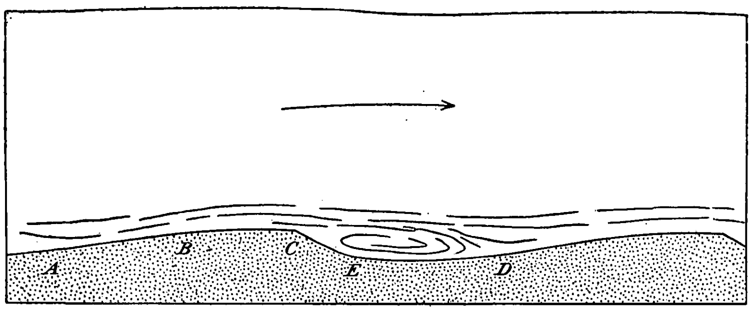


FIGURE 10.—Longitudinal section illustrating the dune mode of traction.

inches, but river examples may require scores or hundreds of feet.[3] The maximum height in the laboratory was probably 2 inches; for those in Sacramento River 2 feet has been reported, and for those in the Mississippi 22 feet.

In each series of laboratory experiments to determine the relation of load to slope the initial run was made with a small load, while for the succeeding runs the load was progressively increased. Enlargement of the load caused increase of slope and velocity, with decrease of depth, and these changes were accompanied by changes in the mode of transportation. In the earlier runs dunes were formed, and these marched slowly down the trough. Then, somewhat abruptly, the dunes ceased to appear, and for a number of runs the channel bed was without waves and approximately plane, although somewhat ruffled in the run immediately following the disappearance of dunes. Finally a third stage was reached in which the bed was characterized by waves of another type. These are called *antidunes*, because they are contrasted with dunes in their direction of movement; they travel against the current instead of with it. Their downstream slopes are eroded and their upstream slopes receive deposit. They travel much faster than the dunes, and their profiles are more symmetric. The water surface, which shows only slight undulation in connection with dunes, follows the profiles of anti-

[1] This is the name chiefly used by Swiss investigators (see De Candolle, Arch. sci. phys. et nat., vol. 9, p. 242, 1883, and Forel, idem, vol. 10, p. 43, 1884), and many observers compare the subaqueous feature to the eolian; but the specific title commonly used in the United States and Great Britain is *sand wave*, and some French engineers employ *grève*. In the present paper, dune is preferred to sand wave because there is occasion to distinguish two species of débris waves.

[2] The imbricated pattern is frequently seen beneath tidal waters, where ripple marks due to the reaction of wind waves are transformed into dunes when the tidal current sweeps across them. It is then usually to be ascribed to a difference in direction of the two actions. An elaborate account of its development in rivers is given by H. Blasius, who has recently investigated the whole subject of the rhythmic features of river beds. See Zeitschr. Bauwesen, vol. 60, pp. 465–472, 1910.

[3] Arthur Hider, who studied dunes in the lower Mississippi, reported a maximum length, crest to crest, of 750 feet, a maximum height of 22 feet, and a maximum progression of 81 feet in a day. See Mississippi River Comm. Rept., 1882, pp. 83–88 (=Chief Eng. U. S. A., Rept., 1883, pp. 2194–2199).

dunes closely and shares their transformations.[1] (See fig. 11.) Usually each antidune occupied the full width of the experiment trough; and in natural streams, so far as I have observed, they either reach from side to side of the channel or else form well-defined rows in the direction of the current. Not only is a row of antidunes a rhythm in itself, but it goes through a rhythmic fluctuation in activity, either oscillating about a mean condition or else developing paroxysmally on a plane stream bed and then slowly declining. Paroxysmal increase starts at the downstream end of a row and travels upstream, gaining in force for a time, and the climax is accompanied by a combing of wave crests. Where the débris is very coarse, as on the outwash plains of glaciers, a din of clashing bowlders is added to the roar of the water.[2]

Of the phases of process in the laboratory Mr. Murphy writes:

Their [the dunes'] form is continually changing as they move forward; they divide and again unite, the parts traveling at different rates, and new ones form on top of the older ones. The grains roll up the gentle slope, fall over the crest, are covered by other grains, and rest until the dune again passes over them and they are again uncovered. Thus the time during which they are in motion is small compared to the time during which they rest. As the velocity of the current increases, the rate of feeding being correspondingly increased, the size of the dunes and their rate of movement increase. Thus we find that when the discharge is 0.363 ft.^{3}/sec., load 11 gm./sec., and slope 0.32 per cent, the dunes are 7 to 9 inches long and one-half inch high and move at the rate of 0.56 foot a minute, but when the discharge is 0.734 ft.^{3}/sec. and the load 30 gm./sec., the dunes are 13 to 15 inches long, three-fourths of an inch high and move at the rate of 1.5 feet a minute. As the velocity of the current increases some of the grains leap as well as roll, and some, instead of dropping over the crest of a dune and resting, leap to the next dune. The dune grows less distinct in form and finally at a critical velocity it disappears, dune motion ceases, and the sand surface becomes comparatively even. This condition of even surface flow continues as the slope increases until at another critical velocity antidune movement begins. A profile of the sand surface for this kind of motion is shown in figure 11. For this experiment the trough width is 1.32 feet, discharge 0.734 ft.^{3}/sec., load 213 gm./sec. and the sand slope 1.23 per cent. These sand waves are from 2 to 3 feet in length from crest to crest, they extend the width of the trough, and some of them are 0.5 foot in height from crest to trough of the wave. They travel slowly upstream, some of the sand being scoured from the downstream face in the vicinity of Y (fig. 11) and deposited on the upstream face at X. Some of these waves remain for two minutes or longer, but most of them not longer than one minute. A whitecap forms on the surface of the water when the larger waves disappear. Sometimes two or more will disappear at once and leave the surface without waves for a distance of 10 feet or more. Only a portion of the sand transported takes part in the formation of these sand waves. The velocity in a wave trough is greater than near the crest. The sand grains flow nearly parallel to the bed as they pass through the trough, but at the crest they have an up and down motion as well as a forward motion. On the crest of the larger waves their forward motion is small compared with their vertical motion. * * *

There is a sand movement by rotation or whirls that aids transportation. These whirls have been observed during dune motion only for smaller sizes of sand. They are of short duration, lasting usually less than one minute, but in this time one of these may scour a hole 1 to 3 inches deep and 4 to 10 inches in length. They usually start near the side of the trough, the axis inclining downstream and toward the center, making an angle of 30° to 60° with the side and a small angle at the bottom. These whirls are 3 to 5 inches in diameter and the sand grains are thrown violently up as well as downstream by them. This movement aids transportation by its lifting action, some of the grains being carried in suspension for a short distance by it.

The change in the appearance of a loaded stream as the load is increased, the discharge remaining constant, is very striking. For no load the water surface is even and smooth. As fine sand is fed into the water at a slow rate, small sand dunes will form on the bottom and many little waves will form on the surface. As the rate of feeding is increased, the slope and velocity increasing, these waves become larger and fewer and have the shape of an inverted canoe. These canoes are side by side, the number depending on the trough width and size of waves. When the width was 1.0 foot two sets formed side by side; when

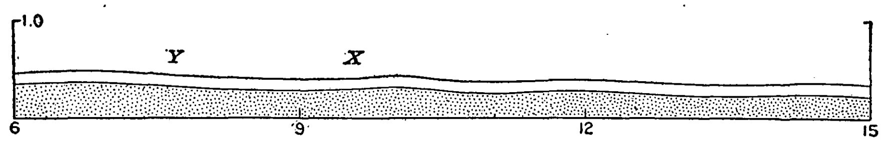


FIGURE 11.—Longitudinal section illustrating the antidune mode of traction. The numbers show distance in feet from the head of the experiment trough.

[1] Antidunes, though less common than dunes, are by no means rare under natural conditions. They are described by Vaughan Cornish (Geog. Jour. (London), vol. 13, p. 624, 1899; Scottish Geog. Mag., vol. 17, pp. 1-2, 1901), and are mentioned by John S. Owens (Geog. Jour., vol. 31, p. 424, 1908).

[2] The sequence of bed characters—dune, smooth, antidune—was observed by John S. Owens in studies with natural currents in 1907, and the characters were correlated with velocities. With depths of 3 to 6 inches and a bed of sand, he noted sand ripples [dunes] when the velocities, measured by floats, were from 0.85 to 2.5 ft./sec., and the appearance of antidunes at a velocity of about 3 ft./sec. (Geog. Jour. (London), vol. 31, pp. 416, 424, 1908). Sainjon and Partiot, studying the movement of débris in the Loire, had previously observed that whereas with low velocities the entire bottom load was transported through the progress of dunes, with higher velocities the débris was swept along from crest to crest and the dunes were reduced in height (Annales des ponts et chaussées, 5th ser., vol. 1, pp. 270-272, 1871).

the width was 1.32 feet three sets formed. As the critical velocity at which dune motion ceases is approached these waves begin to disappear, and when this velocity is reached the water surface is waveless. This waveless condition continues as the rate of feed increases until sand motion in antidunes begins, when large waves, the width of the trough and corresponding in length to the sand waves beneath them, are formed as illustrated in figure 11.

In order to show the magnitude of these surface waves, wave traces have been drawn. Some of these are given in figure 12. A sheet of galvanized iron 4 feet long and 1 foot high was divided into inch squares by lines. This plate was moistened and covered with fine dust. It was held vertical at a given place over the experiment trough and on signal was dropped into the trough and taken out again as quickly as possible. The dust was removed from that part of the plate in the water, leaving a well-defined outline of the wave. This wave trace was quickly sketched on paper by means of the lines marking the squares. Trace *A*, figure 12, is for zero feed; *B* is for a very small feed; *C* is for a larger feed, the surface being covered with the canoe-shaped waves; *D* shows one of the larger waves associated with antidunes.

The slopes at which the phases of traction change are lower for large streams than for small, and lower for fine débris than for coarse. The phase with smooth bed—which may conveniently be called the smooth phase—covers a greater range of conditions with mixed débris than with assorted.

The processes associated with dunes and antidunes were briefly studied in the glazed trough and with the moving field. Transportation by saltation follows the entire profile of the antidunes but traverses the dunes only from *A* to *C* of figure 10. The velocity of saltatory grains is greatest where erosion takes place, namely, along the upstream slopes of dunes and the downstream slopes of antidunes, and it may reasonably be inferred that the water velocities are greatest in those places. The eye detected no difference in water

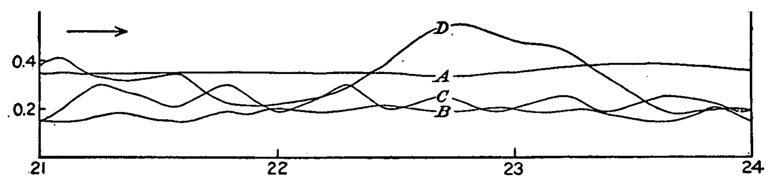


FIGURE 12.—Profiles of water surface, automatically recorded, showing undulations associated with the antidune mode of traction. Numbers show distance, in feet, from the head of experiment trough.

depth over the two slopes of the antidune, and if the depth is the same so also is the mean velocity; but the ratio of bed velocity to mean velocity is known to vary with conditions.

The cause of the changes in process has not been adequately investigated, but a few suggestions may be made. To assist in a search for controlling conditions, the factors connected with the two critical points—the change from dune phase to smooth and the change from smooth to antidune—were tabulated from the experiments with sand of a single grade (C); the positions of the critical points being estimated by Mr. Murphy at a time when the details of the experiments were freshly in mind. In Table 3 *w* is the width of trough, in feet; *Q* the discharge, in cubic feet per second; *S* the per cent of slope; *d* the depth of water in feet; V_m the mean velocity, in feet per second; *L* the load, in grams per second; and L_1 the load per unit width. The data in this table are taken from a preliminary reduction of the observations and are less accurate than the results of the final adjustment, which appear in Table 12 (p. 75). They suffice, however, for the present purpose.

TABLE 3.—*Data connected with changes in mode of transportation.*

w	*Q*	First critical point.					Second critical point.				
		S	*d*	V_m	*L*	L_1	*S*	*d*	V_m	*L*	L_1
0.66	0.093	1.00	0.076	1.86	13	20	1.86	0.058	2.44	44	67
.66	.182	.93	.115	2.41	28	46	1.72	.092	3.01	88	134
.66	.363	.82	.190	2.91	48	73	1.51	.165	3.37	145	221
.66	.545	.74	.272	3.05	70	106	1.34	.209	3.97	184	281
1.00	.182	1.15	.082	1.76	41	41	2.02	.056	3.25	111	111
1.00	.363	.89	.134	2.71	70	70	1.70	.109	3.33	201	201
1.00	.545	.81	.180	3.03	95	95	1.51	.142	3.75	244	244
1.00	.734	.73	.227	3.24	113	113	1.34	.194	3.78	301	301
1.32	.182	1.20	.068	2.03	33	25	2.15	.051	2.70	121	92
1.32	.363	1.00	.107	2.57	76	57	1.76	.099	2.77	210	159
1.32	.545	.87	.145	2.84	106	80	1.53	.111	3.71	251	190
1.32	.734	.75	.178	3.13	140	106	1.33	.149	3.73	293	222
1.96	.363	1.04	.081	2.29	67	34	1.91	.062	2.99	219	112
1.96	.734	.84	.133	2.82	138	71	1.48	.108	3.47	354	181
1.96	1.119	.74	.182	3.14	191	98	1.22	.150	3.81	418	213

The factors were then plotted in various combinations on logarithmic section paper, and certain approximate numerical relations were thus discovered. The first critical point is reached when $d=0.016\ V_m^{2.3}$, or when $d=0.0045\ L_1^{0.85}$. The second critical point is reached when $d=0.004\ V_m^{3.3}$, or when $d=0.0003\ L_1^{1.15}$. The coefficients and exponents are not well defined by the data, but the general indications are (1) that each change in phase occurs when the depth of water bears a certain numerical relation to a power of the mean velocity near the cube, and (2) that the changes occur when the depth bears a certain numerical relation to the amount of load carried in each unit of width of current.

At the bottom the stream is limited and restricted by the bed of débris; at the top by the water surface. To the space between these bounds, a space measured by the depth, the eddies or convolutions of the current are confined. Within the range of conditions covered by the experiments the normal mode of flow involves sinuosity of the filaments of current, and the tendency toward diversity of internal movement is strong in proportion as the velocity is high. A particular relation between depth and velocity corresponds to a sort of equilibrium between the factors of turbulence and restraint, in accordance with which the sinuosity of the lines of flow is reduced to a minimum and the water surface and channel bed are approximately plane. This gives the smooth phase of traction. When the depth is increased without increase of velocity, the reduction of restraint permits the development of internal diversity, and this carries with it diversity of the plastic bed, giving the dune phase of traction. When the velocity is increased without increase of depth, the restraint is overpowered, and a diversified but systematic arrangement of flow lines develops, which carries with it systematic diversity of both water surface and channel bed and gives the antidune phase.

There will be occasion to speak of these relations in another connection in Chapter XIV.

It may be noted, as a possible contribution toward an explanatory analysis, that gravity opposes the current on the upstream side of the antidune (fig. 11) and assists the current on the downstream side. It is where gravity accelerates that load is increased by erosion, and where gravity retards that load is reduced by deposition. In the case of the dune, however, erosion occurs where gravity is a retarding force. But here the descending current, which is accelerated by gravity and which is observed to have gained speed, is free from the bed and bears no load (fig. 10). In order to transport when it resumes contact with the bed it must take débris from the bed, and by so taking it erodes.

UNITS.

The system of units to which the laboratory data have been reduced and which will be employed in their discussion is hybrid in that it includes the foot and the gram. The foot is made the fundamental unit for length, area, and volume because it is the unit employed generally by English-speaking engineers. The gram is made the unit of mass, primarily because it is of convenient magnitude, but also because of the manifest advantage of introducing the metric system wherever no practical difficulties interfere. It happens that the two measures which are given in grams are of categories unfamiliar alike to the engineer and the general reader, so that the gram unit encounters no conflicting habit of thought. One measure is the mass of a grain of sand or a pebble, the other the mass of the débris carried by a stream in a second.

The unit of time, for the indication of rates, is one second. Velocity is given in distance per second, ft./sec.; discharge in volume per second, ft.^{3}/sec.; and load in mass per second, gm./sec.

In hydraulic and hydrodynamic treatises slope of streams is measured by the quotient of fall by distance, or the tangent of its angle, and the unit slope is taken as 45°. For practical purposes this unit is inconvenient because it transcends experience, and engineers commonly avoid it by speaking of slope in percentage or in fractions of 1 per cent, thus making 1 per cent the actual unit. For most purposes I find the smaller unit most convenient, but I have occasional use for the larger unit and shall accordingly use both. To avoid confusion the symbol S will be used with the smaller unit, and where discrimination is important it will be called *per cent slope*, while the symbol s will be used with the larger unit. $S=100\ s$.

TERMS.

Load.—The quantity of débris transported by a stream through any cross section in a unit of time is its *load* at that section. A part is carried in suspension and a part by traction, but as we are here concerned with traction only, the tractional load is to be understood when the word is used without specification. Load is measured in grams per second, gm./sec. For certain engineering purposes it is desirable to consider load as volume, not as the sum of the volumes of individual grains, but as the gross space occupied by the débris as a natural deposit. For the débris used in the experiments in stream traction, with the mixture of sizes ordinarily found in a river deposit, the weight of 1 cubic foot is about 50,000 grams, or 110 pounds. The symbol for load is L.

Capacity.—The maximum load a stream can carry is its capacity. It is measured in grams per second, gm./sec. As the work of the laboratory was largely to determine capacity by measuring maximum load, the two terms are to a large extent interchangeable in the discussion of laboratory data, but the distinction is nevertheless important. The symbol for capacity is C.

Capacity is a function of various conditions, such as slope and discharge, and the chief purpose of the laboratory investigation was to discover the relations of capacity to conditions. When a fully loaded stream undergoes some change of condition affecting its capacity, it becomes thereby overloaded or underloaded. If overloaded, it drops part of its load, making a deposit. If underloaded, it takes on more load, thereby eroding its bed. Through these reactions the profiles of stream beds are adjusted, so far as stream beds are composed of débris. If the bed is of rock in place, the underloaded stream can not obtain its complement of débris, but nevertheless it attacks the bed. By dragging débris over the rock it files or corrades the bed of its channel. It is a general fact that the loads of streams flowing on bedrock are less than their capacities.

Competence.—Under certain combinations of controlling factors capacity is zero or negative. If then some one factor be changed just enough to render capacity positive, that factor in its new condition is said to be *competent*, or else to be a measure of the stream's competence. For example, a stream at its low stage can not move the débris on its bed; with increase of discharge a velocity is acquired such that traction begins; and the discharge is then said to be competent. A stream flowing over a too gentle slope has no capacity, but coming to a steeper slope it is just able to move débris; the steeper slope is said to be competent. A current flowing over débris of various sizes transports the finer but can not move the coarser; the fineness of the débris it can barely move is the measure of its competence.

Discharge.—The quantity of water passing through any cross section of a stream in a unit of time is the *discharge* of the stream at that point. It is measured in cubic feet per second, ft.^{3}/sec. The symbol is Q.

Slope.—The inclination of the water surface in the direction of flow is known as the slope of the stream. It is the ratio which fall, or loss of head, bears to distance in the direction of flow. Per cent slope, as explained on page 34, is numerically 100 times as great, being the fall in a distance of 100 units. The terms slope and per cent slope are also applied in this report to the inclination of the bed of the channel.

Size of débris.—The relative magnitude of the débris particles making up the load may be considered from two opposed viewpoints. Thus we may say that the load varies inversely with the coarseness of the débris, or that it varies directly with the fineness. The second viewpoint is here preferred because it conduces to symmetry in formulation. Two very different measures of fineness have been considered, and on page 21 the material of the laboratory is listed under both. One defines fineness by the number of particles which, placed side by side, occupy the linear space of 1 foot. The other defines it by the number of particles required to fill the space of 1 cubic foot. *Linear fineness* is the more readily conceived, because it appeals to vision. *Bulk fineness* is the more easily determined. The symbol of linear fineness is F, of bulk fineness F_2.

Form ratio.—Of the variable factors which in combination produce the multifarious channel forms of natural streams, the laboratory dealt extensively with but a single one, the relation of depth to width. The relation is a simple ratio, and either of the two terms might be made the divisor. The width has been chosen because, as in the case of fineness its selection conduces to symmetry in formulation. The ratio $\frac{\text{depth of stream}}{\text{width of channel}}$ will be called *form ratio*. Its symbol is R.

Duty and efficiency.—Capacity varies with discharge, but is not proportional to it. The load which may be borne by a unit of discharge varies with the discharge and also with the other conditions. It is the capacity per unit discharge, or the quotient of capacity by discharge, and will be called the *duty*. The symbol is U.

Capacity varies also with slope but is not proportional to it. As the product of discharge by slope (by the acceleration of gravity) measures the stream's potential energy per unit time per unit distance, capacity also varies with the energy but is not proportional to it. The load which may be borne by a unit of discharge on a unit slope varies with all the conditions of transportation. It is the capacity per unit discharge and unit slope, or the quotient of capacity by the product of discharge and slope, and will be called the *efficiency*. It is a measure of the stream's potential work of transportation in relation to its potential energy. Its symbol is E.

The primary definition of efficiency in mechanics is the ratio of work done to energy expended; it implies that the work may be expressed in the same unit as energy. The ratio is always less than unity. But there is an important secondary use of the term, applied to cases in which the result accomplished can not be expressed in terms of energy. In such cases the ratio may have any magnitude, as it arises from the comparison of incongruous quantities. It does not measure economy of energy but relative accomplishment in respect to any condition selected for comparison. As capacity for transportation is not statable in units of energy, the use of the term efficiency in this connection falls under the second definition.

Symbols.—An index of symbols, with brief definitions and references to pages for fuller definition, may be found on page 13.

TABLE OF OBSERVATIONS ON STREAM TRACTION.

The observations on stream traction are presented in Table 4. As the original notes are voluminous, certain combinations and reductions were made before tabulation. The reduction of the slope observations involved discrimination, and the mode of reduction is therefore described below. Each horizontal line of the table contains the record of a single experiment.

The observations are arranged according to (1) fineness of débris, (2) width of channel, (3) discharge, (4) load and slope. The categories of fineness begin with single sizes, taken in order from fine to coarse, and follow with mixtures; they constitute subtables, each designated by a letter in parentheses. The arrangement by load and slope is approximate only. In a general way the sequences of load and slope are parallel, each increasing as the other increases, but the data are not perfectly harmonious, and where the two sequences differ the arrangement is somewhat irregular.

The first column, for all divisions of the table except (J), gives width of trough; the second, discharge. The third, fourth, and fifth pertain to load and give, respectively, the load as measured by débris fed at the head of the trough, the load as measured by débris caught at the outfall end, and the period, in minutes, during which débris was collected at the outfall end. The precision of measurement is probably somewhat higher where the period is relatively long.

The next three columns pertain to slope. The sixth contains the slope, in per cent, of the water surface; the seventh, the slope of the bed of débris as shaped by the current; and the eighth, the extreme distance between points at which were made observations used in computing the slope or slopes. Nearly all the experiments for which the recorded distance is 16 feet or less were made in the shorter trough, of which the gross length was 31.5 feet, the distance between the débris-feeding apparatus and the débris-arresting apparatus being 24.5 feet. The experiments for which the recorded distance exceeds 16 feet were made in the longer trough, the distance from the feeding station to the collecting station being at least 16 feet longer than the space covered by the observations used in the computation.

The ninth column shows the depth at a single point as measured by the gage (see p. 21); the tenth gives the mean depth as estimated from the records of water profile and bed profile (see p. 25).

In the eleventh column, headed "Character of bed," a generalization is given from notes on the mode of transportation, the condition of the water surface during the run, and the condition of the bed of débris after the withdrawal of the water. The original notes are somewhat varied in scope and nomenclature, and it seemed best to make the tabulated record more simple. The words *dunes, smooth,* and *antidunes* denote the three modes of traction described on pages 30–33. The word *transition* is used where the mode of traction was intermediate between one of the rhythmic modes and the smooth mode, and also where different modes of traction obtained in different parts of the trough.

In the final column the word *free* indicates that the contractor (p. 25) was not used, but the experiment trough retained its full width to the end.

The reduction of the slope observations was preceded by a careful study of them with reference to their systematic and accidental errors. As a result of that study certain criteria of exclusion were adopted, by means of which, it is thought, the influence of systematic errors was materially reduced. The criteria were applied through an inspection of the plotted profiles of water surface and débris bed. The observations excluded were those believed to be much affected either by the peculiar conditions near the head of the trough or by the peculiar conditions near the outfall end, so that the retained observations constituted a continuous series covering the middle.

The observations to be reduced were ascribed equal weights. They consisted of a series of level readings (h_0, h_1, h_2 . . . h_{n-1}, h_n), each giving the vertical distance of a point of the débris surface, or of the water surface, below a horizontal plane of reference, together with the horizontal distances of the same points from an initial point in the axis of the trough. The observed points being at equal intervals along the trough, and the zero of distances being made coincident with the first of the observed points, the horizontal distances may be represented by 0, l, $2l$, . . . $(n-1)l$. The following formula[1] was used to compute the slope, the number of terms in numerator or denominator being $\frac{n}{2}$ when n was an even number, and $\frac{n+1}{2}$ when n was odd.

$$S=\frac{100}{l}\,\frac{(h_n-h_o)n+(h_{n-1}-h_1)(n-2)+(h_{n-2}-h_2)(n-4)+\dots}{n^2+(n-2)^2+(n-4)^2+\dots} \quad (1)$$

The numerical operations were simple.

In division (J) of the table the arrangement is somewhat different, and a column is added at the left. The division records experiments made with débris prepared by mixing, in definite proportions, two or more of the grades of débris to which the preceding divisions pertain. The notation adopted to designate these mixtures is analogous to that for chemical compounds, subscript figures being used to indicate approximate proportions. This notation, together with the more precise indication of the proportions, is given in the left-hand column.

[1] Essentially a least-squares formula, although developed from apparently independent considerations.

TABLE 4 (A).—*Observations on load, slope, and depth, with débris having 25,200 particles to the gram, or grade (A).*

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
0.66	0.093	27	24	5	1.12	1.22	16	0.065	0.083	....	Contracted.
		67	74	5	....	2.14	16	....	....	Antidunes	Do.
	.182	19	18	10	....	.69	16	.126	....	Smooth	Contracted.
		93	96	4	....	1.59	14	.095	....	Antidunes	Do.
	.545	59	58	5	.56	.55	16	.272	.284	Smooth	Contracted.
		147	157	3	....	1.07	16	....	....	Antidunes	Do.
1.00	.182	10.5	9.4	10	.58	.53	16	.118	.136	Dunes	Contracted.
		151	155	3	....	2.04	16	....	....	Antidunes	Do.
	.363	20	19	8	....	.42	14	.175	....	Transition	Contracted.
		240	245	3	....	1.64	16	....	....	Antidunes	Do.
	.734	52	51	4	....	.44	14	.264	....	Smooth	Contracted.
		317	317	3	....	1.19	16	....	....	Antidunes	Do.
1.32	.182	12	12	10	.65	.63	16	.091	.106	Dunes	Contracted.
		17	18	10	.83	.75	16	.084	.091	....do	Do.
		18	17	10	....	.73	16	.075	....	....do	Do.
		23	21	8	.80	.79	16	.080	.083	....do	Do.
		37	38	6	....	.99	16	.063	.087	Transition	Do.
		50	51	5	1.11	1.10	16	.058	.079	Antidunes	Do.
		75	68	5	....	1.37	16	.061	....	[Antidunes]	Do.
		73	73	5	....	1.41	16	.060	....	Antidunes	Do.
		129	120	5	....	1.79	16	....	....	[Antidunes]	Do.
		120	135	3	....	1.86	16	....	....	....do	Do.
	.363	7.3	8.9	8	.37	.27	16	.191	.195	Dunes	Contracted.
		42	38	6	.60	.51	16	.124	.131	Transition	Do.
		45	41	5	....	.58	16	.115	....	....	Do.
		44	43	5	....	.67	16	.112	....	....	Do.
		65	74	4	....	.84	16	.112	....	....	Do.
		69	76	5	.77	.90	16	.130	....	Transition	Do.
		125	133	4	....	1.18	16	.115	....	Antidunes	Do.
		121	140	5	....	1.14	16	.106	....	....do	Do.
		....	177	4	....	1.43	16	.089	....	....do	Do.
		214	202	4	....	1.55	16	.097	....	[Antidunes]	Do.
		....	240	4	....	1.65	16	....	....	....do	Do.
	.734	38	38	5	.35	.35	10	....	.267	Dunes	Contracted.
		82	84	4	.53	.48	16	.205	.207	Smooth	Do.
		86	89	4	....	.51	16	.198	....	....do	Do.
		127	156	4	....	.73	16	.190	....	Antidunes	Do.
		151	154	3	....	.83	16	....	....	....do	Do.
		191	192	3	....	.81	16	....	....	[Antidunes]	Do.
		176	215	3	....	.97	16	....	....	Antidunes	Do.
		....	240	3	....	.95	16	....	....	....do	Do.
		....	327	3	....	1.13	16	....	....	....do	Do.
		....	365	3	....	1.24	16	....	....	[Antidunes]	Do.
1.96	.363	8.1	8.7	6	.50	.36	16	.179	.187	Dunes	Contracted.
		20	20	6	....	.41	16	.118	....	....	Do.
		19	21	6	.69	.55	16	.118	.129	....	Do.
		33	32	5	.67	.60	16	.103	.107	....	Do.
		58	60	5	.78	.78	16	.090	.097	....	Do.
		81	92	5	.97	.94	16	.082	.084	Antidunes	Do.
		128	127	5	....	1.10	16	.073	....	....do	Do.
		143	128	5	....	1.17	16	.077	....	....do	Do.
		128	144	4	....	1.18	16	.075	....	....do	Do.
		192	179	4	....	1.38	16	.072	....	[Antidunes]	Do.
		215	193	4	....	1.50	16	.076	....	....do	Do.
		227	218	4	....	1.59	16	.067	....	....do	Do.
		....	264	3	....	1.73	16	.069	....	Antidunes	Do.
		....	283	3	....	1.77	16	.070	....	[Antidunes]	Do.
	.734	7.7	8.3	10	.18	.18	16	.283	.293	....	Contracted.
		32	33	6	....	.36	12	.180	....	....	Do.
		74	83	4	....	.45	16	.156	....	....	Do.
		70	80	5	.58	.49	16	.156	.152	Smooth	Do.
		105	105	5	.79	.56	16	.137	.146	....do	Do.
		148	163	4	....	.79	16	.142	....	Transition	Do.
		....	167	4	....	.75	16	.135	....	....	Do.
		214	179	3	....	.94	16	.129	....	Antidunes	Do.
		221	231	3	....	.98	16	.135	....	[Antidunes]	Do.
		....	279	3	....	1.01	16	.130	....	Antidunes	Do.
	1.119	94	91	4	.30	.45	16	.220	.214	Smooth	Contracted.
		99	102	4	....	.39	16	.207	....	[Smooth]	Do.
		186	130	4	.33	.57	16	.194	.201	Smooth	Do.
		169	188	3	....	.59	16	.196	....	....do	Do.
		180	221	3	....	.63	16	.182	....	....do	Do.
		198	229	3	....	.60	16	.187	....	....	Do.
		221	258	4	....	.81	16	.165	....	Transition	Do.
		....	291	3	....	.79	16	.179	....	Antidunes	Do.
		....	341	3	....	.95	16	....	....	....do	Do.
		....	346	3	....	1.02	16	.176	....	....do	Do.

TABLE 4 (B).—*Observations on load, slope, and depth, with débris having 15,400 particles to the gram, or grade (B).*

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
0.23	0.093		4.0	9		0.85	16	0.192		Transition	Free.
			6.7	7		.94	16	.184		do	Do.
			8.1	7		1.01	16	.174		Smooth	Do.
			20	7		1.49	16	.146		do	Do.
			33	7		2.12	16			Antidunes	Do.
	.182		4.0	10		.73	16	.364		Dunes	Free.
			18	6		1.12	16	.284		Smooth	Do.
			31	5		1.55	16	.230		Transition	Do.
			32	7		1.55	16	.232		do	Do.
.44	.093		5.3	6		.73	16	.136		Dunes	Free.
			11	7		.90	16	.110		Smooth	Do.
			18	4		1.18	16	.093		Transition	Do.
			34	7		1.62	16	.070		Antidunes	Do.
			53	5		2.31	16	.072		do	Do.
			64	4		2.38	16	.068		do	Do.
	.182		8.5	7		.50	16	.194		Transition	Free.
			16	5		.75	16	.177		Smooth	Do.
			27	6		.98	16	.146		Transition	Do.
			73	8		1.66	16			Antidunes	Do.
			76	4		1.73	16	.139		do	Do.
.66	.093		5.1	8		.75	16	.089		Dunes	Free.
			9.2	6		.84	16	.080		do	Do.
			17	7		1.23	16	.060		Smooth	Do.
			15	9		1.32	12	.059		Transition	Do.
			22	5		1.41	16	.058		do	Do.
			28	5		1.47	16	.050		do	Do.
			34	5		1.63	16	.056		do	Do.
			41	5		2.01	16	.054		do	Do.
			53	5		2.09	16	.049		Antidunes	Do.
			57	5		2.17	16	.058		do	Do.
			93	7		2.96	16	.037		do	Do.
	.182	4.8	4.2	8	0.32	.35	16	.175	0.183	Dunes	Contracted.
		3.5	6.0	8	.37	.36	16	.175	.183	do	Do.
		14	12	7		.69	16				Do.
		15	16	6		.68	16	.137			Do.
		23	20	8		.66	16	.113			Free.
		27	21	4		.72	16	.110			Do.
		24	19	8		.81	16	.107		Smooth	Do.
		42	42	7		.98	16	.113			Do.
		47	39	6		1.06	16	.101			Do.
		41	44	9		1.18	16				Do.
		51	47	8		1.17	16	.112			Do.
			55	5		1.32	16	.107		Transition	Do.
		55	55	8		1.35	16				Do.
			55	5		1.36	16	.103		do	Do.
			67	7		1.51	16			Antidunes	Do.
		81	72	7		1.54	16			do	Do.
			88	4		1.51	16	.091		[Antidunes]	Do.
			105	5		2.00	16	.082		Antidunes	Do.
			112	6		1.88	16	.083		[Antidunes]	Do.
			119	6		2.05	16	.077		do	Do.
		166	126	4		2.24	16			Antidunes	Do.
			139	5		2.19	16	.115		[Antidunes]	Do.
			159	5		2.46	16	.096		do	Do.
			169	5		2.79	16	.082		do	Do.
			213	4		3.14	16			do	Do.
			236	3		3.06	16			do	Do.
			254	3		2.91	16			do	Do.
			268	4		2.93	16			do	Do.
			377	3		4.03	16			do	Do.
	.363	29	29	5		.51	16	.216	.221	Transition	Contracted.
		44	40	8		.68	16	.212		do	Free.
		52	39	6		.69	16	.205		do	Do.
		58	49	6		.75	16	.202		Smooth	Do.
		73	55	5		.80	16	.188		do	Do.
			77	4		.98	16	.178		Transition	Do.
			91	5		.99	16	.189		Antidunes	Do.
			104	5		1.14	16	.191		do	Do.
			148	4		1.54	16	.177		[Antidunes]	Do.
			152	5		1.43	16	.155		do	Do.
			164	4		1.63	16			do	Do.
			167	5		1.66	14	.158		do	Do.
			186	3		1.71	16			do	Do.
			226	4		1.81	16			do	Do.
	.545		5.8	6	.18	.21	16	.471	.472	Dunes	Contracted.
		16	16	9		.23	14	.367		do	Do.
		34	35	5	.47	.46	14	.291	.300	Transition	Do.
		64	54	8		.63	16	.292		do	Free.
		67	68	6		.74	16	.258		do	Do.
		79	81	6		.72	14	.260		Smooth	Do.
		108	104	5		.87	16	.220			Do.
		105	111	5		.89	16	.226		Transition	Do.
			132	5		1.00	16	.230		Antidunes	Do.
			169	5		1.32	16	.231		do	Do.
			242	3		1.81	16			[Antidunes]	Do.
			271	3		1.81	16			do	Do.
			304	3		1.79	16			do	Do.

TABLE 4 (B).—*Observations on load, slope, and depth, with débris having 13,400 particles to the gram, or grade (B)*—Con.

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
1.00	0.182	9.9	11	5		0.61	16	0.106		Dunes	Contracted.
		9.9	13	5		.61	16	.102			Do.
			28	6		.97	16	.073		Transition	Free.
			39	5		1.15	16	.081		do	Do.
			51	5		1.29	16	.073		Smooth	Do.
			98	4		1.87	16	.060		Antidunes	Do.
			116	4		2.14	16	.067		do	Do.
			157	3		2.43	16	.066		do	Do.
			182	4		2.63	16	.070		[Antidunes]	Do.
	.363	0	0		0.030	.044	60	.475		Dunes forming	Contracted.
		2.1	1.3	390	.18	.16	60	.344	.323	[Dunes]	Do.
		1.9	1.6	360	.17	.16	60	.321	.321	do	Do.
		6.4	6.2	60		.28	48			do	Do.
		6.8	6.8	44	.27	.31	44	.278	.271	do	Do.
		10.5	7.2	5		.23	16	.214	.244	do	Do.
		9.6	9.4	71		.33	40	.224		do	Do.
		8.7	9.7	6		.29	16	.208		do	Do.
		9.3	11	62		.31	44	.254		Dunes	Do.
		18	21	30	.42	.43	40	.188	.193		Do.
		25	25	15	.43	.49	36	.195	.176		Do.
		28	28	4		.54	16	.164			Do.
		37	49	8		.63	32			Transition	Do.
			46	4		.78	14	.154		Smooth	Free.
			55	9		.65	12	.150		do	Do.
			79	4		1.00	16	.120		Antidunes	Do.
			153	4		1.51	16	.107		do	Do.
			206	4		1.66	16	.129		do	Do.
			218	3		1.78	16	.112		do	Do.
			268	3		2.00	16			do	Do.
			305	3		2.14	16			[Antidunes]	Do.
			347	3		2.28	16			do	Do.
	.545		107	5		.79	12	.194		Smooth	Free.
			117	5		.83	16	.167		Transition	Do.
			167	4		1.16	16	.157		Antidunes	Do.
			199	3		1.31	16	.169		do	Do.
			219	3		1.29	16			[Antidunes]	Do.
			274	3		1.46	16			Antidunes	Do.
			306	3		1.63	16	.144		do	Do.
			315	3		1.65	16			[Antidunes]	Do.
			396	2		1.84	16			do	Do.
			504	2		2.10	16			do	Do.
	.734	0	0			.012	40	.735	.740	Dunes forming	Contracted.
		3.9	4.7	60	.14	.15	40	.497	.500		Do.
		4.7	4.9	90	.14	.18	36	.496	.497		Do.
		18	17	20	.25	.26	28	.332	.334		Do.
		37	38	10	.36	.38	24	.282	.267		Do.
		56	52	10	.41	.48	28	.257	.256		Do.
			54	8		.45	16	.257		Transition	Free.
		56	59	10		.50	28	.257			Contracted.
			60	7		.53	16	.262		Transition	Free.
			62	9		.44	16	.257		do	Do.
		84	80	5	.31	.54	28	.250	.241		Contracted.
			102	5		.68	16	.237		Transition	Free.
			188	5		.92	16	.207		Smooth	Do.
			244	3		1.16	16			Antidunes	Do.
			305	3		1.30	16			do	Do.
			370	2		1.46	16			[Antidunes]	Do.
			416	2		1.72	16			do	Do.
			433	2		1.56	16			do	Do.
1.32	.182	7.3	5.2	8	.40	.40	16	.129	.126	Dunes	Contracted.
		5.8	5.3	8		.45	16			[Dunes]	Do.
			14	6		.80	16	.081		Dunes	Free.
			17	6		.98	16	.061		do	Do.
			34	6		1.12	16	.063		Transition	Do.
			27	9		1.18	12	.057		do	Do.
			43	6		1.22	16	.056		Smooth	Do.
			56	6		1.41	16	.053		Transition	Do.
			68	6		1.66	16	.041		do	Do.
			81	5		1.71	16	.051		do	Do.
			80	5		1.77	16	.038		do	Do.
			93	7		1.88	16	.045		Antidunes	Do.
			115	4		2.11	16	.058		do	Do.
			127	4		2.33	16			do	Do.
			142	4		2.32	16	.037		[Antidunes]	Do.
			152	4		2.46	16	.039		do	Do.
	.363	8.3	5.2	5	.21	.25	16	.227	.220	Dunes	Contracted.
		8.7	7.9	9	.42	.26	16	.203	.197	[Dunes]	Do.
		17	17	9	.53	.43	16	.153	.163	do	Do.
		19	16	9	.64	.43	16	.147	.163	do	Do.
			22	6		.47	14	.131		Dunes	Free.
			22	6		.66	16	.128		[Dunes]	Do.
			25	4		.69	14	.124		Dunes	Do.
			38	6		.82	12	.109		do	Do.
			89	7		1.07	16	.098			Do.
			104	6		1.20	16	.099			Do.
			136	5		1.28	16	.080		Antidunes	Do.
			131	5		1.33	16	.072		do	Do.
			134	5		1.37	16	.098		[Antidunes]	Do.

TABLE 4 (B)—*Observations on load, slope, and depth, with débris having 13,400 particles to the gram, or grade (B)*—Con.

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
1.32	0.363		162	7		1.54	16	0.092		Antidunes	Free.
			173	5		1.63	16	.082		do	Do.
			257	3		1.90	16			[Antidunes]	Do.
			261	3		2.12	16			do	Do.
	.545	30	29	5	0.09	.47	12	.206	0.207	Dunes	Contracted.
		40	55	4	.57	.54	14	.162	.165	do	Do.
		50	54	4	.68	.61	14	.154	.152	do	Do.
		82	89	4		.76	16	.139			Do.
		82	92	5		.76	16	.141		Smooth	Do.
			105	5		.75	14	.148		do	Free.
			104	4		.84	16	.158		Transition	Do.
			147	4		.98	16	.141		Antidunes	Do.
			218	3		1.28	16	.106		do	Do.
			264	4		1.49	16			do	Do.
			278	3		1.49	16			[Antidunes]	Do.
			361	3		1.85	16			do	Do.
	.734	26	30	5	.33	.31	12	.289	.267		Contracted.
			39	7		.41	16	.226		Transition	Free.
			55	6		.50	16	.219		do	Do.
			56	6		.50	16	.195		do	Do.
			63	5		.53	16	.206		do	Do.
			90	5		.58	16	.199		Smooth	Do.
			129	6		.87	16	.174		do	Do.
			136	6		.67	16	.170		do	Do.
			134	5		.78	16	.172		do	Do.
			144	4		.85	16	.165		Transition	Do.
			167	4		.90	16	.173		do	Do.
			178	5		.96	16	.169		Antidunes	Do.
			182	5		.92	16	.166		do	Do.
			204	4		1.01	16			do	Do.
			255	3		1.10	16			[Antidunes]	Do.
			268	3		1.14	16			do	Do.
			280	3		1.17	16			do	Do.
			341	3		1.35	16			do	Do.
			342	3		1.45	16			do	Do.
			383	2		1.51	16			do	Do.
			485	2		1.65	16			do	Do.
			516	3		1.66	16			do	Do.
1.96	.363	8.1	7.5	8	.47	.31	16	.162		Dunes	Contracted.
			18	8		.65	16	.104			Free.
			25	12		.62	16	.110			Do.
			36	23		.59	14	.070			Do.
		47	40	12		.77	14	.087			Do.
			44	8		.92	16	.080		Smooth	Do.
			59	4		1.01	16	.059		Transition	Do.
		80	66	8		1.03	16	.068			Do.
			68	6		1.07	16	.078		Transition	Do.
			96	5		1.21	16	.062		do	Do.
			137	3		1.45	16	.063		Antidunes	Do.
			147	3		1.48	16	.044		do	Do.
			177	3		1.62	16	.050		[Antidunes]	Do.
			186	3		1.64	16			do	Do.
			214	3		1.81	16			do	Do.
			249	3		2.00	16	.060		do	Do.
	.545		53	23		.54	16	.132			Free.
		86	59	6		.70	16	.127		Smooth	Do.
		75	70	5		.69	16	.130			Do.
			72	18		.71	16	.126			Do.
		109	90	3		.78	16	.091			Do.
			89	5		.80	16	.111		Transition	Do.
			94	4		.98	16	.115		do	Do.
			106	4		.97	16			do	Do.
			144	4		1.12	16	.099		do	Do.
			217	4		1.39	16				Do.
			264	3		1.60	16	.095		Antidunes	Do.
			299	3		1.58	16			[Antidunes]	Do.
			307	3		1.54	16			Antidunes	Do.
			303	3		1.61	16			[Antidunes]	Do.
			356	3		1.73	16			Antidunes	Do.
	.734	7.7	7.6	6	.31	.23	10	.250	.257	Dunes	Contracted.
		38	40	5	.46	.27	16	.188	.196		Do.
		35	40	8		.44	12	.171			Free.
		101	66	8	.44	.53	14	.161			Do.
		89	88	4		.59	16				Do.
			98	9		.64	14	.139			Do.
			103	6		.56	14	.148			Do.
			186	3		.80	16			Antidunes	Do.
			185	3		1.05	16			do	Do.
		245	198	3		1.03	16			[Antidunes]	Do.
			239	4		1.05	16	.150		Antidunes	Do.
			332	3		1.40	16			[Antidunes]	Do.
			341	3		1.39	16	.146		Antidunes	Do.
			352	3		1.38	16			[Antidunes]	Do.
			348	3		1.45	16	.124		Antidunes	Do.
			469	2		1.64	16			[Antidunes]	Do.
			520	2		1.87	16			do	Do.

TABLE 4 (B).—*Observations on load, slope, and depth, with débris having 13,400 particles to the gram, or grade (B)*—Con.

Width.	Dis-charge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
1.96	1.119	6.5	6.4	10	0.21	0.19	16	0.411	0.406	Dunes	Contracted.
		14	13	10	.28	.18	16	.353	.375	[Dunes]	Do.
		29	32	11	.37	.32	16	.345	.337	Dunes	Do.
		53	59	4	...	.28	12	.229	...	Transition	Do.
		112	109	8	...	.59	16	.315	...	...	Free.
		...	110	6	...	.61	16	.211	...	Smooth	Do.
		126	129	5	...	.65	12	.188	...	[Smooth]	Do.
		...	191	4	...	.72	16	.188	...	Smooth	Do.
		...	230	4	...	.73	16	.184	...	...	Do.
		...	317	3	...	.91	16	.187	...	Transition	Do.
		...	310	2	...	1.00	16	.176	...	...	Do.
		...	356	2	...	.93	16	...	...	...	Do.
		...	355	2	...	.98	16	...	...	...	Do.
		...	406	3	...	1.13	16	...	...	...	Do.
		...	418	3	...	1.31	16	.145	...	Antidunes	Do.

TABLE 4 (C).—*Observations on load, slope, and depth, with débris having 5,460 particles to the gram, or grade (C).*

Width.	Dis-charge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
0.44	0.093	...	3.2	7	...	0.64	16	0.161	...	Dunes	Free.
		...	5.3	5	...	.85	16	.141	...	Transition	Do.
		...	9.4	5	...	.94	16	.131	...	Smooth	Do.
		...	12	5	...	1.01	16	.122	...	do	Do.
		...	19	7	...	1.26	16	.117	...	do	Do.
		...	26	4	...	1.45	16	.111	...	do	Do.
		...	35	5	...	1.59	16	.111	...	do	Do.
		...	34	4	...	1.71	16	.100	...	do	Do.
		...	46	4	...	2.24	16	...	...	Antidunes	Do.
	.182	...	9.7	6	...	.61	16	.235	...	Dunes	Free.
		...	14	6	...	.70	16	.233	...	Transition	Do.
		...	38	6	...	1.33	16	.168	...	Smooth	Do.
		...	43	5	...	1.32	16	.176	...	do	Do.
		...	47	4	...	1.38	16	.172	...	do	Do.
		...	51	5	...	1.52	16	.153	...	do	Do.
		...	97	3	...	2.16	16	.138	...	Antidunes	Do.
.66	.093	...	3.5	10	...	.54	16	.108	...	Dunes	Free.
		...	8.6	13	...	.79	16	.084	...	[Dunes]	Do.
		...	10	7	...	.98	16	.077	...	Dunes	Do.
		...	15	6	...	.97	16	.078	...	do	Do.
		...	15	6	...	1.05	16	.075	...	do	Do.
		...	17	6	...	1.05	16	.077	...	do	Do.
		...	16	6	...	1.11	16	.073	...	Transition	Do.
		...	22	6	...	1.36	16	.066	...	Smooth	Do.
		...	35	5	...	1.56	16	.063	...	do	Do.
		...	48	4	...	1.98	16	.055	...	Antidunes	Do.
		...	61	4	...	2.38	16	.050	...	do	Do.
		...	74	3	...	2.52	16	.052	...	[Antidunes]	Do.
		...	100	3	...	3.02	16	...	...	Antidunes	Do.
		...	109	5	...	3.41	16	...	...	do	Do.
		...	142	3	...	3.73	16	...	...	do	Do.
		...	146	3	...	3.79	16	...	...	do	Do.
		...	156	3	...	4.04	16	...	...	do	Do.
	.182	2.0	2.3	420	0.29	.24	64	.242	0.238	[Dunes]	Contracted.
		...	9.7	11	...	.54	16	.136	...	do	Free.
		...	16	8	...	.64	16	.124	...	do	Do.
		...	18	7	...	.76	16	.132	...	Dunes	Do.
		...	18	7	...	.72	16	.127	...	do	Do.
		...	28	6	...	.94	16	.117	...	Transition	Do.
		...	39	5	...	1.16	16	.096	...	Smooth	Do.
		...	45	7	1.20	1.17	16	.099	.110	[Smooth]	Do.
		...	72	5	...	1.54	16	...	...	Smooth	Do.
		...	78	8	1.69	1.57	16	.089	.093	do	Do.
		...	105	4	...	1.95	16	...	...	Transition	Do.
		...	113	4	...	2.24	16	.062	...	Antidunes	Do.
		...	123	4	...	2.32	16	...	...	do	Do.
		...	133	4	...	2.33	16	...	...	do	Do.
		...	146	3	...	2.58	16	...	...	do	Do.
		...	155	4	...	2.54	16	...	...	do	Do.
		...	163	3	...	2.63	16	...	...	do	Do.
		...	190	4	...	2.82	16	...	...	[Antidunes]	Do.
		...	205	3	...	2.99	16	...	...	do	Do.
		...	220	3	...	3.05	16	...	...	do	Do.
		...	217	3	...	3.20	16	...	...	do	Do.
		...	246	3	...	3.45	16	...	...	do	Do.
	.363	...	22	5	...	.44	16	.241	...	Dunes	Free.
		...	32	5	...	.64	16	.222	...	Transition	Do.
		...	34	6	...	.70	16	.211	...	do	Do.
		38	37	4	...	.61	16	.213	...	[Transition]	Contracted.

TABLE 4 (C).—*Observations on load, slope, and depth, with débris having 5,460 particles to the gram, or grade (C)*—Con.

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
0.66	0.363	35	35	4		0.63	16	0.216		Transition	Contracted.
		37	40	4		.66	16	.212		[Transition]	Do.
			39	5		.67	16	.205		Transition	Free.
			54	5		.87	16	.186		Smooth	Do.
			78	6		.99	14	.174		do	Do.
			83	5		1.04	16	.167		do	Do.
		82	98	3		1.08	14	.168		Transition	Contracted.
		89	99	3		1.07	16	.173			Do.
		92	97	3		1.08	16	.164			Do.
			121	5		1.19	14	.174		Smooth	Free.
			121	4		1.23	14	.168		do	Do.
			134	4		1.41	16	.161		Transition	Do.
			161	3		1.71	16			Antidunes	Do.
			182	3		1.87	16			do	Do.
			183	3		1.91	16			do	Do.
			228	3		1.97	14			[Antidunes]	Do.
			241	3		2.04	16			do	Do.
			245	3		2.08	16	.165		do	Do.
			240	3		2.11	16			do	Do.
			281	3		2.29	16			do	Do.
			292	3		2.57	16			do	Do.
	.545	2.8	2.8	410	0.15	.20	56	.579	0.577	[Dunes]	Contracted.
			36	7		.54	16	.312		Dunes	Free.
		44	45	4		.51	16	.288			Contracted.
		46	48	4		.56	14	.284			Do.
		46	49	4		.57	16	.278		Transition	Do.
		45	47	4		.58	16	.288		[Transition]	Do.
			41	7		.60	16	.303			Free.
			47	8		.60	16	.308		Transition	Do.
			53	8		.57	16	.300		do	Do.
			76	4		.78	16	.255		do	Do.
			88	3		.82	16	.260		do	Do.
		111	117	3		.96	16	.223		do	Contracted.
		119	115	3		.98	16	.223		[Transition]	Do.
		117	121	3		.99	16	.228		do	Do.
			130	3		1.01	16	.242		Smooth	Free.
			151	5		1.12	16	.226		do	Do.
			152	4		1.12	16	.239		[Smooth]	Do.
			162	5		1.21	16	.222		Smooth	Do.
			196	4		1.53	16	.234		Antidunes	Do.
			239	4		1.56	16			[Antidunes]	Do.
			241	3		1.64	16			do	Do.
			252	3		1.69	14			Antidunes	Do.
			275	3		1.73	14			[Antidunes]	Do.
			346	3		2.04	16			do	Do.
	.734	67	61	4		.62	14	.327		[Transition]	Contracted.
		67	64	4		.62	14	.318		do	Do.
		67	67	4		.58	16	.320		do	Do.
		152	168	3		.99	14	.263		do	Do.
		164	163	3		1.04	16	.277		Transition	Do.
1.00	.182		7.6	9		.54	16	.117		Dunes	Free.
			11	12		.57	16	.116		do	Do.
			16	9		.73	16	.110		do	Do.
			17	5		.75	16	.107		do	Do.
			18	5		.74	16	.108		do	Do.
		28	26	10		.91	16			do	Do.
		39	41	8		1.10	16			do	Do.
		53	54	6		1.31	16			Transition	Do.
		53	56	11		1.32	16			do	Do.
		79	70	3		1.52	16			Smooth	Do.
			84	6		1.68	16	.065		do	Do.
			97	4		1.85	16	.062		do	Do.
		105	106	4		1.96	16			Transition	Do.
			114	4		2.07	16	.051		Antidunes	Do.
			128	4		2.24	16	.065		do	Do.
			131	4		2.32	16			do	Do.
			145	3		2.48	16			do	Do.
			166	3		2.66	16			[Antidunes]	Do.
	.363	1.9	1.9	94	.17	.19	48	.322	.326	Dunes	Contracted.
		6.5	5.1	41	.26	.24	48	.266	.275	[Dunes]	Do.
		19	18	10	.42	.42	48	.209	.221	do	Do.
			24	8		.44	16	.187		Dunes	Free.
			30	6		.52	16	.164		do	Do.
		42	39	7	.61	.65	40	.168	.170		Contracted.
			46	6		.73	16	.142		Transition	Free.
		46	48	4		.70	16	.143		[Transition]	Contracted.
		43	48	4		.73	16	.141		Transition	Do.
			58	6		.82	16	.144		do	Free.
			61	5		.83	16	.133		do	Do.
		85	83	8		.95	32	.135		Smooth	Contracted.
		75	81	3		.97	16			[Smooth]	Free.
		85	86	9		.93	32	.123		Smooth	Contracted.
		75	84	3		.96	16			do	Free.
			86	5		.97	16	.133		do	Do.
			104	5		1.09	16	.121		do	Do.
			104	5		1.09	16	.133		do	Do.
		140	124	3		1.25	16	.114		[Smooth]	Contracted.
		158	131	3		1.26	16	.114		do	Do.
			147	5		1.38	16	.115		Smooth	Free.

TABLE 4 (C).—*Observations on load, slope, and depth, with débris having 5,460 particles to the gram, or grade (C)*—Con.

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
1.00	0.363		166	5		1.50	16			Antidunes	Free.
			233	4		1.99	16			do.	Do.
			257	3		1.98	16			do.	Do.
			256	3		2.10	16			[Antidunes]	Do.
			255	4		2.11	16			do.	Do.
			349	3		2.48	16			do.	Do.
			362	3		2.65	16			do.	Do.
	.545		48	5		.51	16	0.211		Dunes	Free.
			57	5		.53	16	.206		[Dunes]	Do.
		60	68	4		.61	16	.193		Transition	Contracted.
		61	64	4		.62	16	.189		do.	Do.
			71	5		.62	16	.193		do.	Free.
			107	5		.83	16	.176			Do.
			107	5		.88	16	.175		Smooth	Do.
		146	150	3		1.02	14	.167		do.	Contracted.
			153	4		1.05	16	.173		do.	Free.
			165	4		1.09	16	.163		do.	Do.
			169	5		1.00	16	.160		do.	Do.
		158	173	3		1.12	14	.166		[Smooth]	Contracted.
		156	181	3		1.12	14	.160		do.	Do.
			205	4		1.28	16	.148		Smooth	Free.
			266	4		1.50	16	.136		Transition	Do.
			315	3		1.60	16			Antidunes	Do.
			292	4		1.70	16			do.	Do.
			296	4		1.75	16			do.	Do.
			281	3		1.87	16			[Antidunes]	Do.
			355	3		1.95	16			do.	Do.
			391	3		1.94	16			do.	Do.
			398	3		1.97	16			do.	Do.
			431	3		2.17	16			do.	Do.
	.734	1.9	1.6	60	0.11	.13	48	.581	0.579	[Dunes]	Contracted.
		2.6	2.1	170	.11	.16	32		.570	do.	Do.
		2.6	3.0	180	.16	.14	40		.537	do.	Do.
			13	12	.21	.21	56			Dunes	Free.
		13	15	20	.17	.27	40	.459	.467	[Dunes]	Contracted.
			14	12	.26	.33	76	.420	.413	do.	Free.
		42	39	8		.34	36	.322		do.	Do.
		74	84	4		.58	16	.232		Transition	Contracted.
		76	85	4		.58	14	.241		do.	Do.
		103	117	5		.57	36	.261		[Transition]	Do.
			77	4		.61	14	.249		Transition	Free.
			95	6		.59	16	.233		do.	Do.
			132	4		.81	16	.221		do.	Do.
		160	194	3		.99	16	.194			Contracted.
			181	6		1.01	16	.203		Smooth	Do.
			230	4		1.15	16	.187		do.	Do.
			267	4		1.29	16	.199		Transition	Do.
			308	4		1.43	16			Antidunes	Do.
			309	3		1.50	16			do.	Do.
			329	3		1.51	16			do.	Free.
			444	3		1.78	16			[Antidunes]	Do.
			501	3		2.07	16			do.	Do.
	1.119	111	118	3		.57	14	.312		Transition	Contracted.
		114	129	3		.66	14	.305		[Transition]	Do.
		325	348	2		1.17	16	.256		[Smooth]	Do.
		325	370	2		1.22	16	.253		Smooth	Do.
1.32	.182	11	9.6	10	.66	.66	16	.093	.099	Dunes	Contracted.
			7.8	9		.70	12	.100		do.	Free.
			13	6		.84	14	.090		do.	Do.
			21	5		.98	16	.076		do.	Do.
			30	6		1.14	16	.067		Transition	Do.
			43	5		1.24	16	.067		do.	Do.
			58	5		1.64	16	.055		Smooth	Do.
			99	5		2.08	16	.040		Transition	Do.
			120	5		2.23	16	.049		Antidunes	Do.
			126	4		2.10	16	.056		[Antidunes]	Do.
			130	4		2.29	16	.051		do.	Do.
			141	3		2.34	16	.045		Antidunes	Do.
	.363		8.8	6	.33	.32	16	.163	.174	Dunes	Contracted.
		7.8	10	6	.31	.35	16	.171	.170	do.	Do.
		26	26	6	.55	.56	16	.140	.178	[Dunes]	Do.
			27	6		.56	16	.127		Dunes	Free.
			38	5		.72	16	.123		do.	Do.
			51	5		.79	16	.122		do.	Do.
			60	5		.89	16	.120		Transition	Do.
			85	5		1.07	16	.106		Smooth	Do.
			111	6		1.23	14	.104		do.	Do.
			118	5		1.27	16	.100		do.	Do.
			130	4		1.32	16	.097		do.	Do.
			129	4		1.33	16	.085		do.	Do.
			154	4		1.46	16	.090		Transition	Do.
			158	4		1.44	16	.089		do.	Do.
			190	3		1.80	16			Antidunes	Do.
			237	3		1.92	16			[Antidunes]	Do.
			264	3		2.14	16			do.	Do.
			266	3		2.18	16			Antidunes	Do.
			300	3		2.30	16			[Antidunes]	Do.
			308	3		2.46	16			Antidunes	Do.

TABLE 4 (C).—*Observations on load, slope, and depth, with débris having 5,460 particles to the gram, or grade (C)*—Con.

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
1.32	0.545		36	6		0.56	12	0.186		Dunes	Free.
			55	6		.62	16	.172		do	Do.
			79	5		.69	16	.152		do	Do.
			96	5		.83	16	.152		Transition	Do.
			118	5		.85	16	.141		Smooth	Do.
			159	5		1.10	16	.132		do	Do.
			210	5		1.37	16	.118		Transition	Do.
			248	4		1.57	16			Antidunes	Do.
			256	3		1.58	16			[Antidunes]	Do.
			340	3		1.89	16			do	Do.
			362	3		1.95	16			do	Do.
			407	3		2.08	16			do	Do.
			394	3		2.20	16			Antidunes	Do.
	.734	4.3	4.7	120	0.18	.16	48	.409	0.416	[Dunes]	Contracted.
			14			a .20		.324		do	Do.
			29			a .35		.292		do	Do.
			53			a .51		.210		do	Do.
			58	6		.50	14	.214		Dunes	Free.
			74			a .55		.189			Contracted.
			105	5		.65	16	.186		Transition	Free.
			113			a .75		.117			Contracted.
			120			a .73		.183			Do.
			159	4		.87	16	.167		Smooth	Free.
			227	4		1.05	16	.163		[Smooth]	Do.
			252	4		1.15	16	.149		Smooth	
			309	3		1.37	16	.143		Transition	Do.
			368	3		1.62	16			Antidunes	Do.
			405	3		1.72	16			[Antidunes]	Do.
			432	3		1.82	16			do	Do.
			456	3		1.90	16			do	Do.
1.96	.363	11	9.5	10	.45	.35	16	.128	.141	[Dunes]	Contracted.
			16	6		.59	16	.106		Dunes	Free.
		25	26	10	.73	.71	16	.105	.113	do	Contracted.
			40	7		.90	16	.100		Transition	Free.
			60	4		.99	16	.089		do	Do.
			77	4		1.09	16	.080		do	Do.
			90	4		1.21	16	.078		Smooth	Do.
			124	4		1.41	16	.067		do	Do.
			163	4		1.63	16	.063		Transition	Do.
			175	4		1.67	16	.064		do	Do.
			203	4		1.85	16	.067		do	Do.
			227	4		1.93	16	.061		do	Do.
			227	4		2.03	16	.060		do	Do.
	.545		30	8		.56	16	.148		Dunes	Free.
			55	5		.70	16	.127		do	Do.
			79	5		.79	16	.113		do	Do.
			101	6		.92	16	.115		Transition	Do.
			138	5		1.07	16	.100		Smooth	Do.
			165	4		1.10	16	.098		do	Do.
			210	4		1.34	16	.094		Transition	Do.
			241	4		1.38	16	.094		do	Do.
			266	4		1.58	16	.080		do	Do.
			321	3		1.85	16	.071		Antidunes	Do.
			331	3		1.85	16			do	Do.
			355	3		1.96	16			do	Do.
			386	3		2.06	16			[Antidunes]	Do.
	.734	36	32	8	.42	.45	16	.205	.195	[Dunes]	Contracted.
		36	41	10	.54	.41	18	.194	.209	Dunes	Do.
			49	5		.37	16	.177		do	Free.
			55	5		.49	12	.171		do	Do.
		70	63	6	.53	.60	16	.149	.161	[Dunes]	Contracted.
		68	73	7	.46	.58	16	.165	.175	Dunes	Do.
			72	5		.61	16	.155		do	Free.
			87	4		.60	16	.155		Transition	Do.
		121	101	6	.67	.70	16	.120	.145	do	Do.
			96	5		.75	16	.147		Dunes	Do.
			166	3		.92	14	.139		Transition	Do.
			241	4		1.17	16	.117		Smooth	Do.
			259	4		1.22	16	.113		[Smooth]	Do.
			302	3		1.31	16	.106		Transition	Do.
			334	3		1.53	16	.105		Antidunes	Do.
			350	3		1.49	14			[Antidunes]	Do.
			362	3		1.45	16			do	Do.
			368	3		1.50	16			do	Do.
	1.119		90	5		.53	16	.222		Dunes	Free.
		117	112	6	.68	.61	16	.188	.205		Contracted.
		130	131	4	.60	.64	16	.184	.202	Smooth	Do.
		133	138	6	.65	.56	16	.180	.180	do	Do.
			140	5		.65	16	.196			Free.
		129	149	5	.79	.69	16	.186	.199	Smooth	Do.
		132	158	4	.79	.71	16	.189	.192	do	Do.
			215	5		.78	16	.178		do	Do.
			253	3		.78	16	.171		do	Do.
			331	3		.97	16	.166		do	Do.
			360	3		1.03	16	.152		do	Do.
			397	3		1.12	16			Transition	Do.

a Computed graphically from data in a notebook afterward lost.

TABLE 4 (D).—*Observations on load, slope, and depth, with débris having 1,460 particles to the gram, or grade (D).*

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
0.66	0.093	....	5.3	6	....	0.80	16	0.077	....	Dunes	Free.
		....	8.1	6	....	.94	16	.077	....	do	Do.
		....	13	5	....	1.19	16	.075	....	Transition	Do.
		....	20	4	....	1.39	16	.076	....	do	Do.
		....	21	5	....	1.68	16	.063	....	Smooth	Do.
		....	29	4	....	1.83	16	.058	....	do	Do.
		....	36	6	....	1.88	16	.061	....	do	Do.
		....	45	4	....	1.98	16	.063	....	do	Do.
		....	47	4	....	2.25	16	.056	....	do	Do.
		....	62	4	....	2.47	16	.056	....	do	Do.
	.182	6.4	5.8	7	....	.39	16	.166	....	Dunes	Contracted.
		....	14	6	....	.77	16	.147	....	do	Free.
		....	17	6	....	.82	16	.135	....	do	Do.
		....	23	6	....	.95	16	.123	....	Transition	Do.
		....	30	5	....	1.10	16	.105	....	do	Do.
		....	47	4	....	1.26	16	.108	....	Smooth	Do.
		....	53	4	....	1.40	16	.108	....	do	Do.
		....	83	8	....	1.95	16	.090	....	do	Do.
		....	108	9	....	2.10	16	.090	....	do	Do
		....	121	4	....	2.26	16	090	....	do	Do.
		....	127	4	....	2.28	16	.093	....	do	Do.
		....	134	4	....	2.39	16	.090	....	do	Do.
	.545	8.9	6.9	6	.30	.19	16	.460	0.464	Dunes	Contracted.
		8.9	9.5	5	.28	.20	16	.422	.425	[Dunes]	Do.
		18	17	5	.61	.51	14	.403	.382	Dunes	Do.
		32	27	5	.61	.57	16	.318	.322	....	Do.
		33	37	4	....	.64	16	.316	....	....	Do.
		....	64	5	....	.81	12	.267	....	Transition	Free.
		....	92	4	....	.96	16	.237	....	Smooth	Do.
		....	145	4	....	1.25	16	217	....	do	Do
		....	172	4	....	1.42	16	.211	....	do	Do.
		....	203	4	....	1.55	12	.202	....	do	Do.
		....	209	4	....	1.58	12	.209	....	do	Do.
		....	218	3	....	1.57	12	.191	....	do	Do.
		....	231	3	....	1.65	14	.177	....	do	Do.
		....	331	3	....	1.98	12	.196	....	do	Do.
		....	310	3	....	2.02	12	....	....	Transition	Do.
1.00	.182	....	9.5	7	....	.69	14	.110	....	Dunes	Free.
		....	16	5	....	.84	16	.100	....	do	Do.
		....	22	5	....	1.06	14	.094	....	Transition	Do.
		....	27	5	....	1.08	16	.099	....	do	Do.
		....	28	4	....	1.14	16	.088	....	do	Do.
		....	48	4	....	1.34	16	.086	....	do	Do.
		....	49	4	....	1.39	16	.082	....	Smooth	Do
		....	56	4	....	1.57	16	.071	....	do	Do
		....	80	4	....	1.83	16	.066	....	do	Do
		....	98	4	....	2.09	16	.070	....	do	Do
		....	109	4	....	2.33	14	.066	....	do	Do
		....	126	4	....	2.55	14	.062	....	do	Do
		....	152	3	....	2.75	16	.064	....	do	Do
	.363	0	0	....	.037	....	40	.418	....	Dunes	Contracted.
		1.8	2.3	81	.16	.18	28	.292	.289	[Dunes]	Do.
		1.8	2.0	66	.19	.18	24	.283	.286	do	Do.
		5.9	5.4	20	32	.25	24	.245	.252	Dunes	Do.
		12	12	22	.27	.35	20	.224	.224	[Dunes]	Do.
		12	12	26	.56	.43	20	.235	.229	do	Do.
		23	23	12	.53	.58	24	.209	.199	do	Do.
		....	28	6	....	.65	16	.170	....	Dunes	Free.
		....	35	5	....	.80	16	.164	....	Transition	Do.
		....	86	4	....	1.14	16	.136	....	Smooth	Do.
		....	112	4	....	1.32	16	.120	....	do	Do.
		....	130	4	....	1.44	14	.116	....	do	Do.
		....	141	3	....	1.49	16	.117	....	do	Do.
		....	170	3	....	1.65	14	.115	....	do	Do.
		....	181	4	....	1.75	16	.103	....	do	Do.
		....	229	4	1.85	1.94	14	.104	.099	do	Do.
		....	258	3	2.00	2.11	14	.101	.098	do	Do.
	.545	30	30	4	.53	.53	16	.236	.240	....	Contracted.
		32	31	4	....	.55	16	.241	....	....	Do.
		....	66	4	....	.78	16	.187	....	Transition	Free.
		....	143	4	....	1.11	16	.166	....	Smooth	Do.
		....	168	4	....	1.26	16	.162	....	do	Do.
		....	229	4	....	1.58	16	.136	....	do	Do.
		....	256	3	....	1.61	16	.136	....	do	Do.
		....	281	3	....	1.62	14	.140	....	do	Do.
		....	310	3	....	1.74	14	.136	....	do	Do.
		....	340	3	....	1.91	14	.136	....	do	Do.

TABLE 4 (D).—*Observations on load, slope, and depth, with débris having 1,460 particles to the gram, or grade (D)*—Con.

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
1.00	0.734	0	0	....	0.085	....	40	....	....	Dunes forming........	Contracted.
		5.8	6.5	29	.18	0.21	36	0.491	0.482	[Dunes]........	Do.
		19	12	18	.22	.32	24	.423	.411	.....do........	Do.
		20	21	15	.26	.37	36	.413	.410	.....do........	Do.
		49	56	4	....	.61	16	.285	....	.....do........	Do.
		....	66	4	....	.61	12	.250	....	Dunes........	Free.
		83	82	3	....	.75	16	.241	....	Smooth........	Contracted.
		....	108	4	....	.83	14	.220	....	Transition........	Free.
		....	170	3	....	.98	14	.206	....	Smooth........	Do.
		....	189	4	....	1.09	16	.198	....	.....do........	Do.
		....	193	4	....	1.19	16	.195	....	.....do........	Do.
		....	265	3	....	1.39	16	.189	....	.....do........	Do.
		....	293	3	....	1.46	16	.185	....	.....do........	Do.
		....	354	3	....	1.73	16	.175	....	.....do........	Do.
		....	377	3	....	1.76	14	.162	....	.....do........	Do.
1.32	.363	8.9	11	10	.50	.30	16	.137	.140	[Dunes]........	Contracted.
		10	11	10	.40	.34	16	.171	.171	.....do........	Do.
		25	23	8	.46	.67	16	.137	.154	Dunes........	Do.
		59	53	5	....	1.00	16	.124	....	[Dunes]........	Do.
		60	57	2	....	1.05	16	.129	....	Dunes........	Do.
		100	89	4	....	1.25	16	.107	....	....	Free.
		102	97	3	....	1.25	16	.111	....	Smooth........	Do.
		154	135	3	....	1.54	16	.090	....	.....do........	Do.
		166	136	3	....	1.57	14	.094	....	[Smooth]........	Do.
		169	161	3	....	1.67	14	.089	....	Smooth........	Do.
		195	236	3	....	1.89	16	.088	....	.....do........	Do.
		202	247	3	....	1.91	16	.087	....	.....do........	Do.
	.734	17	15	10	.28	.34	16	.360	.354	Dunes........	Contracted.
		22	25	10	.33	.37	16	.272	.304	.....do........	Do.
		48	43	10	.60	.52	14	.243	.227	[Dunes]........	Do.
		50	49	8	.51	.58	16	.224	.248	.....do........	Do.
		53	47	8	.64	.61	16	.233	.231	Dunes........	Do.
		51	60	5	....	.62	14	.202	....	....	Do.
		65	72	5	....	.55	16	.201	....	....	Do.
		68	59	5	.75	.70	16	.206	.228	....	Do.
		86	72	5	.66	.79	16	.184	.192	....	Do.
		90	83	5	....	.67	16	.181	....	....	Do.
		99	84	4	1.28	1.15	16	....	....	[Smooth]........	Do.
		172	170	3	1.00	1.00	12	.154	.164	Smooth........	Do.
		202	174	3	.97	.98	14	.160	.168	.....do........	Do.
		192	209	3	....	1.15	12	.156	....	.....do........	Do.
		192	200	3	....	1.17	12	.152	....	.....do........	Do.

TABLE 4 (E).—*Observations on load, slope, and depth, with débris having 142 particles to the gram, or grade (E).*

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
0.66	0.182	25	21	15		1.20	16	0.125		Dunes	Contracted.
		25	21	19		1.27	16	.117			Do.
		31	28	15		1.43	16	.115		Transition	Do.
		31	31	13		1.48	16	.115			Do.
		31	30	15		1.56	16	.114			Do.
	.363	38	31	9		1.04	16	.200		Transition (dunes to smooth).	Contracted.
		38	31	9		1.04	16				Do.
		48	42	9		1.27	16	.197		Transition	Do.
		48	46	10		1.26	16	.200			Do.
		48	47	10		1.29	16	.185			
	.734	48	47	6		1.03	16	.372		Dunes	Contracted.
		48	50	6		1.09	16	.387		....do	Do.
		96	95	4		1.43	16			....do	Do.
		96	97	4		1.46	16			....do	Do.
	1.119	50	44	6		.56	14	.562		Dunes	Contracted.
		91	80	3		1.23	16			[Dunes]	Do.
		91	85	3		1.31	16			....do	Do.
		101	94	3		1.11	16			Dunes	Do.
		101	87	3		1.24	16			[Dunes]	Do.
		101	94	3		1.31	16			....do	Do.
		101	95	3		1.29	16			....do	Do.
		193	167	3		1.38	16			....do	Do.
		193	184	3		1.28	16			Dunes	Do.
		193	186	3		1.41	16				Do.
1.00	.182	6.4	5.2	21		.59	16			Dunes	Contracted.(?)
		19	19	10		1.11	16	.094			Do.
		19	19	16		1.12	16	.087			Do.
		47	44	8		1.80	16	.081			Do.
		50	44	8		1.80	16	.077			Do.
		50	50	9		1.94	16	.093			Do.
		78	75	5		2.42	16				Do.
	.363				0.065		32	.357			Contracted.
			Tr.		.11		32	.314		Dunes forming	Do.
		2.1	2.9	61	.23	.22	48	.241	0.277	[Dunes]	Do.
		2.1	2.2	60		.25	48	.242		Dunes	Do.
		6.7	7.7	20	.46	.44	48	.218	.215		Do.
		6.6	7.9	20	.39	.44	32	.212	.208		Do.
		24	22	34	.80	.87	16	.161	.167	Transition	Free.
		50	53	9	1.24	1.28	16			....do	Contracted.
		95	104	5	1.92	1.83	16	.113		....do	Do.
		142	168	4	2.26	2.29	16	.110	.115	....do	Do.
	.734				.04		32	.618		[Dunes]	Contracted.
			Tr.		.07		32	.562		....do	Do.
		2.1	1.7	70		.19	48	.447		....do	Do.
		2.1	3.6	63	.17	.18	48	.442	.459	....do	Do.
		6.9	4.5	31	.31	.29	32	.411	.403	....do	Do.
		6.9	10	25	.31	.26	40	.403	.407	....do	Do.
		21	20	8	.43	.46	28	.364	.363	....do	Do.
		21	20	18		.48	16	.303		....do	Do.
		24	26	16	.61	.47	16	.301		Dunes	Do.
		41	44	10		.73	36	.324		[Dunes]	Do.
		41	47	6		.74	36	.326		....do	Do.
		47	45	8	.82	.83	16	.273		Dunes	Do.
		87	86	6		1.19	16	.236		Transition	Do.
		95	100	4	1.17	1.23	16	.223			Do.
		142	135	3	1.50	1.46	16	.187			Do.
		142	150	3	1.47	1.50	16	.189			Do.
		203	220	2	1.80	1.61	16			Smooth	Do.
	1.119	47	50	6		.52	16	.380			Contracted.
		47	50	8		.59	16	.406			Do.
		189	170	3		1.15	16				Do.
		189	190	3		1.18	16				Do.
1.32	.363	20	20	15		.67	16	.131		Dunes	Contracted.
		20	20	10		.73	16	.129		....do	Do.
		86	93	5		1.78	16	.097			Do.
	.734	33	33	9		.60	16	.239		Dunes	Contracted.
		86	85	4		1.10	16	.198			Do.
		86	86	3		1.12	16	.211			Do.
	1.119	50	46	6		.58	16	.324		Dunes	Contracted.
		50	50	4		.62	16	.317		....do	Do.
		172	163	3		1.13	16				
		172	171	3		1.22	16				

TABLE 4 (F).—*Observations on load, slope, and depth, with débris having 22.1 particles to the gram, or grade (F).*

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
0.66	0.182	11	11	17		1.29	16	0.118		Smooth	Contracted.
		11	12	23		1.31	16	.129		[Smooth]	Do.
		58	52	11		2.51	16	.102		Smooth	Do.
		58	53	8		2.50	16	.106		do	Do.
	.363	26	26	17		1.12	16	.207		Smooth	Contracted.
		26	26	12		1.13	16	.204		do	Do.
		71	80	6		1.89	16	.176			Do.
		71	71	10		1.96	16	.170			Do.
	.734	35	37	9		.97	16	.346		Dunes	Contracted.
		35	38	6		1.00	16	.340		[Dunes]	Do.
		106	103	3		1.68	16			Dunes	Do.
		106	108	3		1.75	16			do	Do.
1.00	.182	6.8	6.8	21		1.36	16	.090		Smooth	Free. (?)
		41	42	7		2.49	16	.078		do	Do.
		41	44	7		2.53	16	.080		do	Do.
	.363	10	9.5	18		.85	16	.160		Smooth	Free. (?)
		10	10	20		.91	16	.161		do	Do.
		51	47	14		1.65	16	.128		[Smooth]	Do.
		51	51	10		1.70	16	.130		do	Do.
		51	56	8		1.68	16	.140		Smooth	Do.
		120	120	3		2.47	16				Do.
	.734	26	25	14		.77	16	.268		Dunes	Contracted.
		26	25	12		.77	16	.265		do	Do.
		104	98	5		1.52	16	.217		do	Do.
		104	114	5		1.60	16	.211		do	Do.
	1.119	52	53	5		.80	16	.330		Dunes	Free. (?)
		52	56	5		.85	16	.343		do	Do.
		207	197	3		1.65	16			do	Do.
		207	209	3		1.67	16			do	Do.
1.32	.363	21	21	7		1.16	16	.116		Smooth	Contracted.
		21	21	21		1.21	16	.118		do	Do.
		70	68	9		2.05	16	.114		Transition	Do.
		70	72	5		2.07	16	.108		do	Do.
	.734	26	26	13		.83	16	.212		Dunes	Contracted.
		26	27	12		.85	16	.209		do	Do.
		26	27	12		.86	16	.215		[Dunes]	Do.
		104	102	3		1.48	16	.176			Do.
		104	106	4		1.58	16	.180			Do.
	1.119	58	50	12		.74	16	.275		Dunes	Contracted.
		58	50	9		.78	16	.288		do	Do.
		58	59	6		.84	16	.284		do	Do.
		212	188	2		1.55	16			[Dunes]	Do.
		212	208	2		1.60	16			Dunes	Do.

TABLE 4 (G).—*Observations on load, slope, and depth, with débris having 5.9 particles to the gram, or grade (G).*

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
0.66	0.363	10	11	15		1.11	16	0.198			Contracted.
		25	25	12		1.44	16	.186			Do.
		25	28	15		1.48	16	.192			Do.
		51	47	6		1.82	16	.175			Do.
		50	50	8		1.90	16	.175			Do.
		100	100	3		2.56	16	.160			Do.
		100	105	3		2.70	16	.158			Do.
	.734	10	8.5	14		.68	16	.373			Contracted.
		10	11	23		.70	16	.364			Do.
		25	26	11		.95	16	.342			Do.
		50	48	7		1.19	16	.322			Do.
		50	49	11		1.19	16	.324			Do.
		105	104	5		1.71	16	.292			Do.
		203	210	3		2.43	16	.261			Do.
		210	219	3		2.35	14	.264			Do.
	1.119	10	12	18		.62	16	.558			Contracted.
		50	49	6		.98	16	.460			Do.
		50	54	6		1.02	16	.451			Do.
		100	94	4		1.35	16	.414			Do.
		100	114	4		1.32	16	.411			Do.
		203	212	3		1.97	16	.374			Do.
		203	215	3		1.95	16	.378			Do.
		304	311	3		2.40	16	.354			Do.
		304	331	3		2.36	16				Do.

TABLE 4 (G).—*Observations on load, slope, and depth, with débris having 5.9 particles to the gram, or grade (G)*—Con.

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
1.00	0.363	10	10	20		1.27	16	0 143		Smooth	Contracted.
		20	21	15		1.48	16	.139		[Smooth].	Do.
		25	25	12		1.61	16	.136		Smooth	Do.
		25	28	18		1.62	16	.141		do	Do.
		34	34	12		1.76	16	.132		do	Do.
		51	50	9		2.09	16	.129		do	Do.
		102	97	5		2.74	16	.114			Do.
	.734	10	11	42		.78	16	.272		Smooth	Contracted.
		20	20	15		.86	16	.248		[Smooth].	Do.
		25	25	15		.95	16	.248		do	Do.
		25	26	11		.97	16	.251		Smooth	Do.
		50	49	10		1.27	16	.235		[Smooth].	Do.
		101	100	5		1.69	16	.214		Smooth	Do.
		201	189	4		2.30	16	.190			Do.
		201	212	3		2.37	16	.191			Do.
	1.119	10	10	20		.64	16	.389		Smooth	Contracted.
		25	24	15		.66	16	.357			Do.
		25	25	17		.67	16	.359			Do.
		50	53	6		.97	16	.324			Do.
		50	54	7		.90	16	.326			Do.
		101	100	4		1.22	16	.308			Do.
		102	103	4		1.31	16	.307			Do.
		204	214	3		1.78	16	.275			Do.
		306	297	3		2.04	14	.252			Do.
		306	310	3		2.21	14	.253			Do.
1.32	.363	25	24	12		1.90	16	.115			Contracted.
		25	26	9		1.97	16	.104			Do.
		51	49	10		2.25	16	.108			Do.
		51	47	10		2.34	16	.105			Do.
		98	95	3		3.02	16	.093			Do.
		98	123	4		3.10	16	.097			Do.
	.734	10	9	21		.82	16	.210			Contracted.
		10	12	18		.82	16	.210			Do.
		25	24	13		1.08	16	.194			Do.
		25	27	13		1.14	16	.200			Do.
		51	52	6		1.41	16	.193			Do.
		102	102	4		1.82	16	.171			Do.
		203	200	3		2.44	16	.163			Do.
	1.119	10	14	19		.71	16	.297			Contracted.
		51	46	4		.90	16	.270			Do.
		51	67	6		1.07	16	.261			Do.
		98	95	3		1.31	16	.248			Do.
		98	110	4		1.40	16	.245			Do.
		197	200	3		1.83	16	.230			Do.
		295	282	3		2.17	16	.201			Do.
		304	302	3		2.26	16	.202			Do.

TABLE 4 (H).—*Observations on load, slope, and depth, with débris having 2.0 particles to the gram, or grade (H).*

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
0.66	0.363	10	9.2	21		1.49	16	0.184			Contracted.
		10	12	16		1.58	16	.184			Do.
		21	17	18		1.80	16	.183			Do.
		21	22	14		1.84	16	.173			Do.
		52	51	5		2.43	16	.167			
		52	56	6		2.47	16	.173			
	.734	10	7.9	18		.90	16	.345			Contracted.
		10	11	17		.95	16	.348			Do.
		21	20	17		1.10	16	.333			Do.
		21	22	15		1.19	16	.344			Do.
		52	51	8		1.50	16	.320			Do.
		52	52	8		1.51	16	.310			Do.
		105	105	4		2.02	16	.289			Do.
		209	209	3		2.69	16	.250			Do.
		209	222	3		2.92	16	.253			Do.
	1.119	10	12	16		.74	16	.502			Contracted.
		10	10	20		.81	16	.510			Do.
		10	12	21		.89	16	.503			Do.
		26	26	15		1.03	16	.470			Do.
		52	53	13		1.26	16	.442			Do.
		52	54	10		1.28	16	.437			Do.
		52	53	9		1.33	16	.447			Do.
		105	98	3		1.65	14	.391			Do.
		105	106	6		1.63	16	.398			Do.
		105	113	3		1.62	16	.392			Do.
		209	209	3		2.31	16	.340			Do.
		209	209	3		2.38	16	.334			Do.

TABLE 4 (I).—*Special group of observations on load, slope, and depth, with débris of grade (C); for discussion of form ratio.*

Width.	Discharge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
1.00	0.734	74	84	4		0.58	16	0.232		Transition	Contracted.
		76	45	4		.58	14	.241		do	Do.
		160	194	3		.99	16	.194			Do.
	.923	96	103	3		.58	16	.270		Transition	Contracted.
		94	103	3		.62	16	.282		do	Do.
		254	280	2		1.09	16	.233			Do.
	1.119	111	118	3		.57	14	.312		Transition	Contracted.
		114	129	3		.66	14	.305			Do.
1.20	.734	66	74	3		.51	16	.222		Transition	Contracted.
		67	74	3		.60	16	.215		[Transition]	Do.
		193	218	3		1.05	16	.173		Transition	Do.
		199	223	3		1.07	16	.175		[Transition]	Do.
	.923	99	104	3		.65	14	.232		Transition	Contracted.
		249	266	3		1.06	16	.210		do	Do.
		246	274	3		1.01	16	.201		do	Do.
	1.021	105	107	3		.57	16	.250		[Transition]	Contracted.
		110	109	4		.54	16	.262		Transition	Do.
		304	325	3		1.20	16	.197		Smooth	Do.
	1.119	108	114	4	0.56	.62	12	.269	.277	[Transition]	Contracted.
		111	123	4		.61	16	.269		Transition	Do.
		322	292	3		1.00	16	.224		[Smooth]	Do.
		339	326	3		1.03	14	.231		Smooth	Do.
1.40	.734	76	74	4		.63	16	.199		Transition	Contracted.
		76	75	4		.59	14	.196		do	Do.
		220	215	3		1.04	16	.158		Smooth	Do.
		216	226	3		1.07	16	.151		do	Do.
	.923	72	79	4		.51	16	.235		[Transition]	Contracted.
		82	74	4		.57	16	.237		Transition	Do.
		271	279	3		1.03	16	.176		Smooth	Do.
		283	297	3		1.04	14	.175		[Smooth]	Do.
	1.021	82	97	4		.58	16	.248			Contracted.
		292	319	3		1.03	14	.181		[Smooth]	Do.
		289	323	3		1.08	16	.180		Smooth	Do.
	1.119	105	104	4		.57	16	.254			Contracted.
		123	130	4		.61	16	.231			Do.
		292	325	3		.97	14	.213		Smooth	Do.
		284	337	3		1.01	14	.215		[Smooth]	Do.
		292	312	3		1.07	16	.203		Smooth	Do.
1.60	.734	69	70	4		.60	12	.191			Contracted.
		265	236	3		1.14	16	.121		Smooth	Do.
		269	268	3		1.12	16	.122		do	Do.
	.923	67	82	4		.45	12	.233		Dunes	Contracted.
		69	69	5		.46	12	.202		do	Do.
		67	66	4		.50	12	.224		[Dunes]	Do.
		298	299	3		1.05	16	.161		Smooth	Do.
		289	280	3		1.07	16	.161		do	Do.
	1.021	67	66	4		.39	12	.243			Contracted.
		63	73	4		.47	12	.244			Do.
		66	80	4		.51	12	.232			Do.
		310	311	3		1.06	16	.169			Do.
	1.119	82	83	4		.49	16	.227		Transition	Contracted.
		96	91	4		.47	12	.234		[Transition]	Do.
		85	98	4		.55	12	.233		do	Do.
		336	332	3		.99	12	.188			Do.
1.80	.734	69	69	4		.61	16	.174		Dunes	Contracted.
		70	72	4		.60	16	.174		do	Do.
		211	204	3		1.04	16	.137		Smooth	Do.
		205	203	3		1.06	16	.130		do	Do.
	.923	70	75	4		.53	14	.206		Dunes	Contracted.
		67	74	4		.58	14	.204		do	Do.
		260	258	3		1.04	16	.157		[Smooth]	Do.
		260	273	3		1.04	16	.152		Smooth	Do.
	1.021	70	69	4		.51	16	.224		Dunes	Contracted.
		69	76	4		.52	14	.227		[Dunes]	Do.
		70	76	4		.57	16	.225		Dunes	Do.
		292	323	3		1.00	14	.155		Smooth	Do.
		296	308	3		1.01	16	.158		do	Do.
	1.119	64	66	4		.42	14	.240		Dunes	Contracted.
		75	74	4		.47	14	.243		do	Do.
		386	375	3		1.05	16	.160		Smooth	Do.
		363	385	3		1.05	12	.167		do	Do.

TABLE 4 (I).—*Special group of observations on load, slope, and depth, with débris of grade (C)*—Continued.

Width.	Dis-charge.	Load.			Slope.			Depth.		Character of bed.	Outfall.
		Feed.	Collection.	Period.	Water surface.	Bed.	Distance.	By gage.	By profiles.		
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*	*Per cent.*	*Per cent.*	*Feet.*	*Feet.*	*Feet.*		
1.96	0.734	85	83	4	....	0.61	16	0.147	....	[Transition]	Contracted.
		84	88	4	....	.69	16	.146	....	Transition	Do.
		234	236	3	....	1.05	16	.117	....	Smooth	Do.
		222	231	3	....	1.09	16	.123	....	....do	Do.
		234	250	3	....	1.12	16	.122	....	....	Do.
	.923	96	97	3	....	.62	16	.176	....	Transition	Contracted.
		246	279	3	....	1.02	16	.143	....	....	Do.
	1.021	84	85	4	....	.50	16	.194	....	[Transition]	Contracted.
		85	87	4	....	.57	14	.200	....	Transition	Do.
		94	104	3	....	.54	16	.193	....	[Transition]	Do.
		328	343	3	....	1.09	16	.141	....	....do	Do.
		316	352	3	....	1.09	16	.144	....	Transition	Do.
	1.119	82	82	4	....	.45	12	.214	....	Dunes	Contracted.
		85	84	4	....	.57	16	.222	....	Transition	Do.
		82	94	4	....	.52	16	.219	....	....do	Do.
		345	341	3	....	1.00	16	.150	....	Smooth	Do.
		328	363	3	....	1.00	16	.147	....	[Smooth]	Do.

TABLE 4 (J).—*Observations on load, slope, and depth, with débris of two or more grades mixed.*

Designation of mixture, component grades, and percentages by weight.	Width of trough.	Dis-charge.	Load.			Slope of bed.		Depth by gage.	Character of bed.
			Feed.	Collection.	Period.	Per cent.	Distance.		
	Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*		*Feet.*	*Feet.*	
(A_1C_1)=(A)50 : (C)50	1.00	0.363	42	43	8	0.58	16	....	Transition.
			90	91	4	.92	16	....	Do.
			128	140	4	1.23	16	....	Antidunes.
			171	169	3	1.47	16	....	Do.
(A_3G_1)=(A)75 : (G)25	1.00	.363	20	18	12	.51	16	....	Smooth.
			90	74	5	.93	16	....	Transition.
			80	89	4	.95	16	....	Do.
			175	164	3	1.38	16	....	Antidunes.
			175	176	3	1.36	16	....	Do.
(A_2G_1)=(A)67 : (G)33	1.00	.363	20	20	12	.53	16	....	Smooth.
			42	42	6	.62	16	....	Do.
			42	42	6	.64	16	....	Do.
			87	88	3	1.00	16	....	Antidunes.
			110	119	3	1.18	16	....	Do.
(A_1G_1)=(A)50 : (G)50	1.00	.363	22	22	8	.68	16	....	Smooth.
			46	43	6	1.01	16	....	Transition.
			91	93	4	1.42	16	....	Do.
(A_1G_2)=(A)33 : (G)67	1.00	.363	42	40	6	1.36	16	....	Smooth.
(A_1G_4)=(A)22 : (G)78	1.00	.363	22	21	20	1.30	16	....	
			43	42	7	1.79	16	....	
(B_4F_1)=(B)78 : (F)22	1.00	.363	42	43	6	.59	16	....	Smooth.
			84	92	3	.86	16	....	Do.
			113	105	3	.99	16	....	Do.
			113	125	3	1.05	16	....	Do.
			169	159	2	1.38	16	....	Transition.
(B_2F_1)=(B)64 : (F)36	1.00	.363	39	42	7	.57	16	....	Smooth.
			78	80	5	.85	16	....	Do.
			105	109	4	1.11	16	....	Do.
			157	157	2	1.49	16	....	Do.
(B_1F_1)=(B)47 : (F)53	1.00	.363	45	49	7	.73	16	....	Transition.
			90	95	4	1.14	16	....	Do.
			120	130	3	1.41	16	....	Smooth.
			181	162	2	1.61	16	....	Do.
(B_1F_2)=(B)31 : (F)69	1.00	.363	28	35	11	.86	16	....	Dunes.
			41	40	6	1.06	16	....	Do.
			81	82	4	1.56	16	....	Do.
(B_1F_4)=(B)18 : (F)82	1.00	.363	16	17	15	.82	16	....	Dunes.
			27	29	10	1.16	14	....	Do.
			40	43	6	1.46	16	....	Do.
(C_4E_1)=(C)79 : (E)21	1.00	.363	43	49	8	.70	16	....	Smooth.
			85	84	4	.93	16	....	Do.
			115	118	3	1.15	16	....	Do.
			171	171	2	1.52	16	....	Do.

TABLE 4 (J).—*Observations on load, slope, and depth, with débris of two or more grades mixed*—Continued.

Designation of mixture, component grades, and percentages by weight.	Width of trough.	Dis-charge.	Load.			Slope of bed.		Depth by gage.	Character of bed.
			Feed.	Collec-tion.	Period.	Per cent.	Dis-tance.		
	Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*		*Feet.*	*Feet.*	
(C_2E_1)=(C)65 : (E)35	1.00	0.182	78	81	3	1.77	16	....	Smooth.
			104	105	4	2.06	16	....	Do.
		.363	43	42	7	.69	16	....	Transition.
			85	85	4	1.03	16	....	Do.
			114	117	3	1.27	16	....	Smooth.
			155	170	3	1.57	16	....	Do.
			171	169	3	1.65	16	....	Do.
			155	170	4	1.60	16	....	Do.
			171	173	3	1.74	16	....	Do.
(C_1E_1)=(C)48 : (E)52	1.00	.182	26	28	12	.99	16	0.096	Transition.
			40	40	10	1.17	16	.090	Do.
			40	41	10	1.27	16	.091	Do.
			53	50	6	1.43	16	.081	Smooth.
			51	60	....	1.50	16	....	Do.
			53	52	6	1.51	16	.082	Do.
			51	61	....	1.55	16	....	Do.
			81	67	4	1.82	16	....	Do.
			77	78	....	1.95	16	....	Do.
			81	87	5	1.89	16	....	Do.
		.363	39	40	6	.74	16	.150	Transition.
			39	41	7	.76	16	.151	Do.
			78	78	5	1.07	16	....	Smooth.
			104	93	3	1.37	16	....	Do.
			104	115	3	1.40	16	....	Do.
			104	119	3	1.43	16	....	Do.
			157	158	3	1.85	16	....	Do.
(C_1E_2)=(C)31 : (E)69	1.00	.182	26	26	10	1.10	16	.093	Transition.
			39	39	8	1.37	16	.088	Do.
			52	57	6	1.64	16	.082	Smooth.
			78	76	3	2.22	16	....	Do.
		.363	39	39	8	.77	16	....	Dunes.
			39	39	9	.84	16	....	Do.
			78	81	3	1.25	16	....	Do.
			108	112	3	1.54	16	....	Transition.
			157	158	3	2.10	16	....	Do.
(C_1E_4)=(C)19 : (E)81	1.00	.182	20	20	14	1.02	16	....	Dunes.
			26	28	10	1.29	16	.085	Do.
			39	40	6	1.61	16	.079	Do.
			39	42	6	1.65	16	.084	Do.
			52	51	6	1.88	16	.078	Transition.
			52	54	12	1.93	16	.082	Do.
		.363	40	40	8	1.00	16	....	Dunes.
			53	56	7	1.14	16	....	Do.
			78	76	4	1.45	16	....	Transition.
			109	109	3	1.75	16	....	Do.
(C_4G_1)=(C)80 : (G)20	1.00	.363	31	32	11	.62	16	....	Dunes.
			93	92	5	1.07	16	....	Transition.
			140	140	4	1.30	16	....	Do.
			187	194	3	1.48	16	....	Smooth.
			187	195	3	1.62	16	....	Do.
(C_2G_1)=(C)67 : (G)33	1.00	.363	31	29	....	.59	16	....	Smooth.
			31	29	12	.63	16	....	Do.
			31	31	10	.62	16	....	Do.
			61	66	7	.90	16	....	Do.
			92	90	4	1.03	16	....	Do.
			92	95	5	1.08	16	....	Do.
			122	120	3	1.20	16	....	Do.
			183	180	2	1.65	16	....	Do.
(C_1G_1)=(C)50 : (G)50	1.00	.363	17	18	15	.71	16	....	Smooth.
			50	50	10	.99	16	....	Do.
			100	105	3	1.34	16	....	Do.
			133	128	3	1.62	16	....	Do.
			133	145	3	1.66	16	....	Do.
(C_1G_2)=(C)33 : (G)67	1.00	.363	16	15	16	.95	16	....	Transition.
			32	34	10	1.17	16	....	Smooth.
			32	36	10	1.27	16	....	Do.
			64	69	7	1.59	16	....	Transition.
(E_4G_1)=(E)80 : (G)20	1.00	.363	15	17	15	.72	16	....	Transition.
			31	31	10	1.00	16	....	Smooth.
			61	58	8	1.39	16	....	Transition.
(E_2G_1)=(E)67 : (G)33	1.00	.363	16	15	18	.76	16	....	Smooth.
			32	30	11	1.03	16	....	Do.
			63	59	6	1.45	16	....	Transition.
(E_1G_1)=(E)50 : (G)50	1.00	.363	16	18	18	.87	16	....	Smooth.
			31	34	10	1.12	16	....	Do.
			62	60	8	1.60	16	....	Transition.

TABLE 4 (J).—*Observations on load, slope, and depth, with débris of two or more grades mixed*—Continued.

Designation of mixture, component grades, and percentages by weight.	Width of trough.	Discharge.	Load.			Slope of bed.		Depth by gage.	Character of bed.
			Feed.	Collection.	Period.	Per cent.	Distance.		
	Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*		*Feet.*	*Feet.*	
(E_1G_2)=(E)33 : (G)67	1.00	0.363	16	15	14	0.95	16	....	Transition.
			23	23	12	1.18	16	....	Smooth.
			30	29	12	1.35	16	....	
			30	32	11	1.33	16	....	
			45	49	7	1.73	16	....	Transition.
			45	54	7	1.74	16	....	Do.
$(A_1C_1G_2)$=(A)25 : (C)25 : (G)50	1.00	.363	16	13	15	.77	16	....	Transition.
			16	17	10	.79	16	....	Do.
			16	14	15	.82	16	....	Smooth.
			32	33	10	.88	16	....	Do.
				88	5	1.32	16	....	Do.
			126	131	4	1.59	16	....	Transition.
			126	151	4	1.67	16	....	Do.
(CḊEFG)=(C)45 : (D)35 : (E)12 : (F)6 : (G)2.	1.00	.182	21	25	12	.82	14	....	Dunes.
			42	45	10	1.23	16	....	Transition.
			84	84	5	1.72	16	....	
		.363	67	69	8	.81	16	....	Smooth.
			67	69	6	.85	16	....	Do.
			84	80	5	.91	16	....	Do.
			84	83	5	.93	16	....	Do.
			84	83	5	.95	16	....	Do.
			113	120	5	1.18	16	....	
			113	118	5	1.19	16	....	
			169	172	4	1.53	16	....	
			169	173	4	1.54	16	....	
		.545	84	90	6	.65	16	....	Smooth.
			84	92	5	.67	16	....	Do.
			169	151	3	1.07	16	....	Do.
			169	177	3	1.13	16	....	Do.
			253	228	3	1.40	16	....	Do.
			253	242	3	1.37	16	....	

TABLE 4 (K).—*Observations on load, slope, and depth, with unassorted débris.* [a]

Width.	Discharge.	Load.			Slope of bed.		Depth by gage.	Character of bed.	Outfall.
		Feed.	Collection.	Period.	Per cent.	Distance.			
Feet.	*Ft.³/sec.*	*Gm./sec.*	*Gm./sec.*	*Minutes.*		*Feet.*	*Feet.*		
1.00	0.182	19	18	15	0.74	16	....	Dunes	Contracted.
		19	18	15	.79	16	....	.....do	Do.
		38	38	8	1.00	16	....	Smooth	Do.
		75	76	3	1.55	16	....	.....do	Do.
		145	147	2	2.28	16	....	.....do	Do.
	.363	22	20	10	.43	16	0.169	Transition	Contracted.
		22	27	12	.50	16	.176	.....do	Do.
		38	34	6	.59	16	.157	Smooth	Do.
		38	42	6	.67	16	.158	.....do	Do.
		77	74	5	.89	16	....	.....do	Do.
		154	150	3	1.31	16	....	.....do	Do.

[a] The very coarsest particles were removed by passing the sample through a 6-mesh sieve, and the very finest by passing it over a 60-mesh sieve. It retained the equivalents of grades (A), (B), (C), (D), (E), and (F).

CHAPTER II.—ADJUSTMENT OF OBSERVATIONS.

OBSERVATIONS ON CAPACITY AND SLOPE.

THE OBSERVATIONAL SERIES.

Each of the experiments in stream traction involved six quantities—(1) a fineness, or grade of débris, (2) a width of trough, (3) a discharge, (4) a slope, (5) a load, or capacity, and (6) a depth of current. The experiments were arranged in series, for each of which grade, width, and discharge were constant, while within each the magnitudes of slope, capacity, and depth were varied. There will be frequent occasion to mention these secondary units of the experimental work, and whenever the title *series* seems not sufficiently specific they will be called *observational series*. The number of such series recorded in Table 4 is 153.

The factors of grade, width, and discharge, which are related to an individual series as fixed conditions, or constants, do in fact assume the character of variables when series is compared with series; but their modes of determination and combination are not of such character that their numerical values can be checked and adjusted by means of recorded relations.

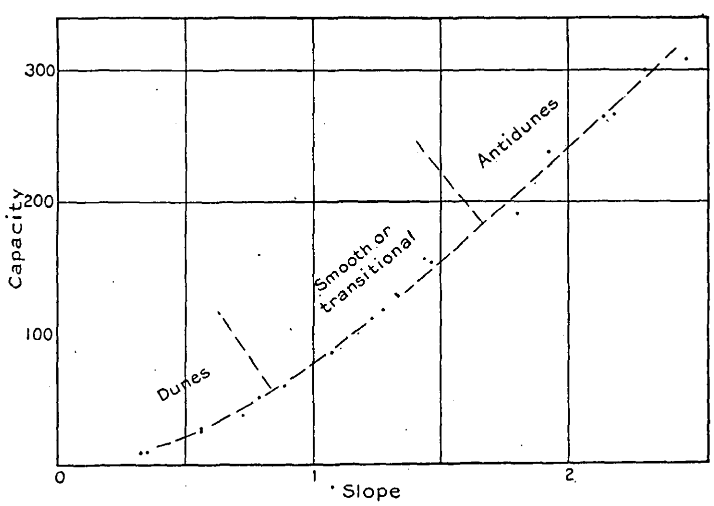


FIGURE 13.—Plot of a single series of observations on capacity and slope. Capacity in grams of débris per second. Slope in per cent. The modes of traction are indicated.

In each experiment the values of slope, load, and depth are mutually dependent; within each series they form a triple progression, the depth decreasing while slope and load increase; but the laws of these interdependent variations are partly masked by irregularities in the sequences. As a preliminary to the general discussion, the observational values were subjected to a process of adjustment, whereby the sequences were freed from irregularities. The irregularities are made manifest by the comparison of the sequences of two variables, and first consideration will be given to those of capacity and slope.

Figure 13 exhibits the relations of capacity to slope as observed in a single series of experiments (that for grade (C), with $w=1.32$ feet and $Q=0.363$ ft.^{3}/sec.). The ordinates indicate capacity, as measured by débris delivered at the lower end of the trough; the abscissas represent slope, as measured on the bed of the channel. The arrangement of the observational dots suggests that if the observations were harmonious the dots would fall in a line of simple curvature. Such a line would express the law connecting capacity and slope. The departures of the dots from such linear arrangement represent irregularities, or errors, in the experimental data. The adjustment proposed is the replacement of the imperfectly alined dots by a generalized or representative line, or the replacement of the inharmonious values of capacity and slope by a system of harmonious or adjusted values.

ERRORS.

As a first step in the treatment of the errors of the data they were studied with a view to the discrimination of the systematic and the accidental.

The three modes of traction—the dune, the smooth, the antidune—although intergrading, are mechanically different. It was surmised that they might differ in efficiency, so that the capacity-slope curve might show a step in passing from one to another; and it was also surmised that the law connecting capacity with slope might not be the same for the several modes. A suggestive observation had shown that on very low slopes—slopes so low that capacity is minute—the current changes an artificially smoothed bed of débris to a system of dunes, and that with the development of dunes the load is notably increased without any change in general slope. To test the surmises all the series were plotted on logarithmic section paper. Figure 14 shows a logarithmic plot of the same data which appear in figure 13; and it will be observed that the line suggested by the points has much less curvature in the logarithmic plot. Its approximation to a straight line makes the study of its local peculiarities comparatively easy. The examination of the plots, while not disproving the surmises, showed that whatever diverse influences may be exerted by the modes of traction, they are too small to be discriminated from the irregularities due to other causes.

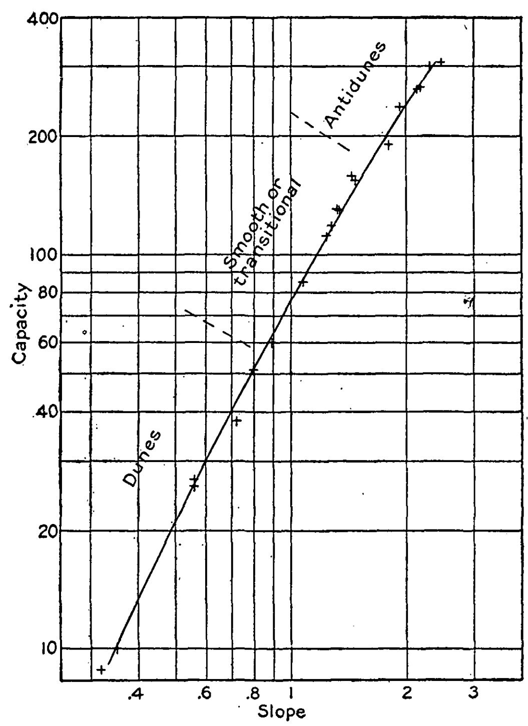


FIGURE 14.—Logarithmic plot of a series of observations on capacity and slope. Compare figure 13.

Other sources of systematic error are connected with the methods of experimentation.

INTAKE INFLUENCES.

As the water entered the experiment trough from the stilling tank it was accelerated, the gain in mean velocity being associated with a quick descent in the surface profile. Beyond this descent the profile usually rose somewhat, and there was commonly a moderate development of fixed waves. This development was modified and the waves were on the whole reduced by the addition of the débris. As it fell into the water the débris had no forward momentum, and it therefore tended to retard the current. But the débris also accumulated on the bottom, reducing the depth of the water at that point, and this reduction necessitated an increase in mean velocity. In the immediate neighborhood of the place where débris was fed the slope of the water was affected by an abnormality distinguishable from the intake abnormality proper, and the joint abnormality faded gradually downstream. The nature of these features varied with the discharge and load, with the gradual development of the adjusted slope, and also with the mode of feeding. During the greater part of the experimental work the feeding was either automatic and continuous or else manipulated by hand in such way as to make it nearly continuous, but for a minor part the feeding was intermittent, a measureful of débris being dumped into the water at regular intervals.

OUTFALL INFLUENCES.

In all the earlier work the trough had the same cross section at the lower end as elsewhere, and the water fell freely from its open end to the settling tank. As the resistance to its forward motion was less at the outfall than within the trough, the water flowed faster there. Its faster flow diminished the resistance just above, and thus the influence of outfall conditions extended indefinitely upstream. An expression of this influence was found in the water profile, which was usually convex in the lower part of the trough, the degree of convexity diminishing upstream. Its effect on the profile of the bed is not readily analyzed, because that profile is adjusted through the velocity of water at the bottom of the current, and the bed velocity is not simply related either to mean velocity or to depth.

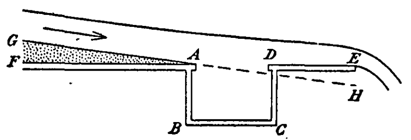


FIGURE 15.—Diagrammatic longitudinal section of outfall end of experiment trough, illustrating influence of sand arrester on water slope.

A second factor at the outfall end was the arrangement for separating the débris from the current. This included a well, *ABCD*, figure 15, which was sunk below the trough bed and into which the débris sank, while the current passed on to the outfall at *E*. In part of the work the space *AD* was entirely open; in another part a coarse screen was stretched across it. In either case the resistance of this part of the channel bed differed from the resistance along the débris slope and may have been greater or less. From the well to the outfall, *DE*, the frictional resistance was less than elsewhere. As the fixed part of the channel bed, *DE*, was horizontal and the débris portion, *GA*, was inclined, the profile of the bed changed at *A*. Projected forward, the slope *GA* passed below *E* to *H*, and when the débris slope, down which the transporting current flowed, was steep, the part *DE* was related to it somewhat as a dam. The tendency of the quasi-dam was to retard the current near the outfall and make the water profile concave, and in some of the experiments the profile actually became concave. Other outfall factors were recognized, but they are not here mentioned because they are believed to be of relatively small importance.

In the reduction of observations on slope an attempt was made to lessen the effect of intake and outfall influences by omitting from the calculations the profile data obtained near the ends of the trough. The data from a considerable number of experiments were finally discarded altogether and do not appear in the tables. To replace the discarded data experiments were afterward made with a modified apparatus.

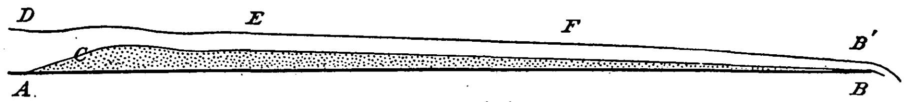


FIGURE 16.—Diagrammatic longitudinal section of débris bed and stream, in a long trough.

CHANGES IN APPARATUS.

As the terminal influences of all kinds diminish with distance from the trough ends, a manifest mode of avoidance is to employ a very long experimental trough and determine slopes from observations in the medial portion exclusively. A trough length of 150 feet was tried and proved moderately successful for very low slopes. For steeper slopes there were practical difficulties—the need either of a trough with adjustable slope or of a very deep trough and a correspondingly large stock of assorted débris—and these led to the consideration of other plans. The one adopted was to contract the channel at the outfall and thus increase the resistance to flow at that point, and with the contracted outfall to use a trough of moderate length. The theory of this plan may be illustrated by a diagram. In figure 16, *AB* represents in profile the bed of a long trough, *CB* the profile of débris, and *DB'* the water profile. In the tract *EF* the water and débris profiles are nearly parallel and depth and velocities are therefore practically uniform. From *F* to *B'* the water profile is notably convex because the resistance to flow afforded by the water itself steadily diminishes toward *B'*.

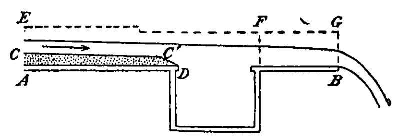


FIGURE 17.—Diagrammatic longitudinal section of outfall end of trough, illustrating influence of contractor.

The plan undertook to introduce at *F*, by contraction, a resistance equivalent to that afforded by the water beyond *F* and then dispense with the portion of the trough between *F* and *B*. In the longitudinal section, figure 17, *ADB* is the bed of the trough, with the well for catching débris, as already shown in figure

15; *EGB* is the profile of one side wall. The walls converge from *F* to *G* (see fig. 3), producing the contraction at the outfall. It was found that the bed of débris, instead of running to a feather edge at *D* (compare fig. 16), held its thickness to *C′* and ended in a steep incline.

The device of contraction accomplished its purpose of avoiding terminal difficulties, but it was found to aggravate certain other difficulties, next to be described.

RHYTHM.

Whenever the profile of a current was determined by a series of measurements applied to the water surface, that surface was found to be in a state of unrest. Its position in any vertical fluctuated upward and downward rhythmically. The amplitude of oscillation, which might be great or small, was not constant; that is, the rhythm was not simple, but compound. It consisted apparently of many rhythmic elements differing one from another in period and amplitude.

The rhythmic quality, thus easily appreciated by watching the play of the surface in relation to a fixed point, permeated every function of the current—the slope of its profile, both local and general, the slope of its bed, the quantity of débris transported, the mode of its transportation. The rhythm of the dune has already been described, but associated with the dunes were greater débris waves, also traveling downstream and each involving the volume of many dunes. In the bed of the long trough a series of them could be seen; in the shorter trough one or two might be made out, or the effect might be only an alternate temporary steepening and flattening of the general slope. The rhythm of the antidune was accompanied by rhythmic paroxysms and doubtless by other rhythms which escaped recognition because the steep slopes with which the antidune was associated were not studied in the long trough. Figure 18 shows a few channel-bed profiles in which rhythmic features appeal to the eye, but the greater number of such profiles merely show an irregularity in which periodicity is not conspicuous. It is probable that the currents were affected by numerous coexistent rhythms, which served to confuse one another and thus masked periodicity except when some one rhythm was stronger than the rest.

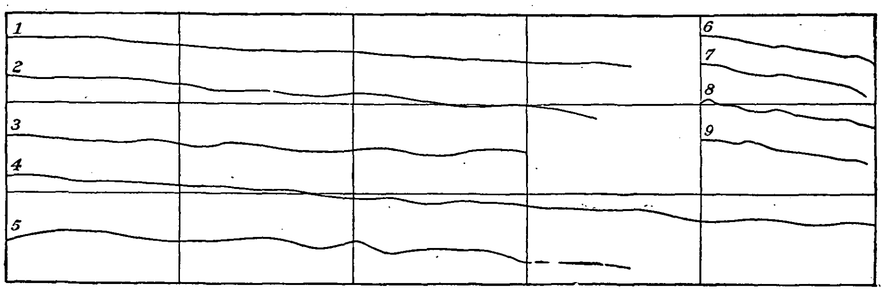


FIGURE 18.—Profiles of channel bed, illustrating tractional rhythms associated with dunes of greater magnitude. Scales: Each horizontal space =20 feet; each vertical space =0.4 foot. For Nos. 1 to 5 the average slope is 0.2 per cent, and the average load (of grade (C)) per foot of channel width is 7 gm./sec. For Nos. 6 to 9 the average slope is 0.6 per cent and the average load 37 gm./sec.

The condition of relatively smooth channel bed which intervened between the conditions characterized severally by dunes and antidunes was also a condition of relative uniformity in all the activities of the current, and when it prevailed the rhythmic variations were at a minimum.

The rhythms of the transporting stream manifestly constitute a group of phenomena worthy of systematic study, but the Berkeley laboratory, having a definite and different theme, treated them only as difficulties interfering with its work. It sought the capacity for load inhering in the average of all the diverse slopes presented by the rhythms, and it necessarily treated the deviations of slope measurements from that average as accidental errors.

The rhythms affected the determinations of loads as well as slopes. The variations of profile were effected by erosion and deposition, and a current which was eroding or depositing carried more load at one point than at another. As the loads were largely determined by weighing the débris delivered at the trough end in a limited time, the amount obtained would depend in part on the phase of slope variation near the point of delivery.

In the earlier experiments with contraction at outfall—as in other experiments—it was necessary to continue a run, with uniform discharge and uniform feed of débris, until the slope of the sand bed had been automatically adjusted to the conditions. The criteria adopted for recognition of a state of adjustment were two—that the water slope equal the bed slope and that the rate of delivery of débris equal the rate of feed. It was found impracticable to satisfy these tests, because both slopes and the rate of delivery fluctuated through a wide range, and an approximate adjustment, if attained, could not be made to continue. The state of affairs may be likened to the waving of a flag in the wind; at the outer margin the amplitude of the undulation is much greater than close to the staff. In the long trough the outfall end corresponded to the staff, giving a fixed position and element of uniformity to which the profiles conformed, and the rhythmic departures were greater with distance from the outfall. The shorter trough when combined with end contraction represented a segment of the long trough at a distance from the outfall and was correspondingly subject to great fluctuations. Despite these difficulties, the nature of which was not well understood at the time, a large number of experiments were made in this way.

The work with free outfall was affected chiefly by terminal influences, and as these produced systematic errors there was danger of false conclusions. The work with contracted outfall was affected by accidental errors of such magnitude as largely to mask the nature of the laws sought. Between these perils of Scylla and Charybdis a middle course was finally steered by using a moderate amount of contraction, whereby the recognized systematic errors were practically avoided without the introduction of insuperable rhythmic irregularities.

SLOPES OF DÉBRIS AND WATER SURFACE.

The slope of the bed of débris to which measurement was applied had been established by the stream as that appropriate to the stream's load of débris. It was caused by the load, in conjunction with the discharge and other conditions, and it accurately sufficed to give the stream capacity for that load. This was my point of view in arranging the experimental methods, and accordingly one of the principal measurements undertaken was that of the débris profile. But the slope more generally considered in hydraulic studies is that of the water surface. Head, the hydraulician's ordinary measure for the determination of power, is the vertical interval between two points of the water surface, and slope is the loss of head in a unit of distance. Under conditions of uniform flow the two profiles are parallel, but for various reasons our experimental currents ordinarily lacked so much of uniformity that the two slopes were appreciably different. I do not find it easy to decide which slope should be regarded as the true correlative of capacity for traction, but as all our laboratory data include the débris slope, while the determinations of water slope were relatively infrequent, the discussion of results has adhered almost exclusively to the former. If the water slope is the true correlative, then the use of the débris slope involves a systematic error.

THE LOGARITHMIC PLOTS.

When the data of an observational series are plotted on ordinary section paper, as in figure 13, and a representative line is drawn through or among them, that line is the graphic equivalent of

$$C=f\ (S) \qquad (2)$$

When they are plotted on logarithmic section paper, as in figure 14, and a representative line is drawn, that line is the graphic equivalent of

$$\log C=f_1\ (\log S) \qquad (3)$$

The second equation, or line, is the logarithmic form of the first.

As already mentioned, a logarithmic plot was made of each observational series. The plot included primarily the slopes of the débris bed and the determinations of load from the delivery of débris at the end of the trough, but it included also, with distinctive notation, such determinations as were available of water-surface slopes and of load based on the rate of feed at the head of the trough. The notation also classified observations with reference to the three modes of traction and distinguished experiments made with use of the contractor from those with free outfall.

A critical study of these plots led to several conclusions. (1) Judged by the internal evidence of regularity and irregularity, the observations of débris slope and water slope have about the same quality. (2) Similarly, the observations of load fed and load delivered have about the same quality. (3) As already stated, the law of sequence is not discovered to change in passing from one process of traction to another; the assumption of a continuous law is the best practicable. (4) Except for very low slopes, the results obtained with the use of the contractor do not differ widely from those without it. (5) There are differences between bodies of experimental data obtained at different stages of the work—differences of uncertain source but presumably connected with modes of manipulation—which make it desirable to treat such bodies separately whenever practicable. (6) The best representative line is not straight but curved, and its curvature is always in one direction.

If the representative line were straight, function (3) would have the form

$$\log C = \log A + n \log S \text{ --------- } (4)$$

in which A is a constant capacity and n a ratio; and function (2) would have the form

$$C = AS^n \text{ -------------- } (5)$$

On some of the plots the observational points are too irregular to afford trustworthy evidence of curvature. On most of them the indicated curvature is slight. From inspection of the data during the progress of the experiments it was thought that the true representative line would prove to be straight, in which case the accurate determination of two points on the line would suffice; and some of the experimental work was adjusted to that theory. The series giving data for but two points on the logarithmic plot of course furnished no evidence as to curvature of the representative line.

For all those cases in which the position of the best representative line could be inferred, with close approximation, from the arrangement of the observational points, a satisfactory adjustment could be made by simply drawing the line and then converting its series of positions into figures; but this procedure would afford no control for the curvature of representative lines in cases where the observational points were few or inharmonious. In order to make the stronger series support the weaker, the plan was adopted of (1) connecting the lines with a formula of interpolation, (2) correlating the constants of the formula with conditions of experimentation, and thus (3) giving deductive control to the lines of the weaker series.

SELECTION OF AN INTERPOLATION FORMULA.

The best interpolation formula is one which embodies the true theory of the relation to which the observations pertain. In the present case the true theory is not known, but there are certain conditions which a theory must satisfy, and these may be used as criteria in the selection of a form for empiric formulas of interpolation. Subject to these criteria, the form selected should serve to minimize the discrepancies between observed and adjusted values.

For the study of the character of the curve to represent best the logarithmic plot of observations, the data for débris of grade (G) were selected. The experiments furnishing those data were all performed by one method, the method using moderate contraction of the trough at outfall; and for that grade the apparent curvature of the logarithmic graph is greater than for most others. The data were first plotted (on logarithmic paper) in groups, each group containing the data for three graphs which pertain to the same width of channel but to different discharges. It was assumed that the three graphs, if correctly drawn, would constitute a system, the one for the medium discharge being intermediate in form and position between the other two; and in drawing them on this assumption the forms were mutually adjusted. Then a rearrangement was made which grouped together data agreeing as to discharge but differing as to width of channel, and further adjustment was made. Selection was finally made of the graph for $w=0.66$ foot and $Q=0.734$ ft.^{3}/sec., and this was drawn, through the selected positions, by the aid of a flexible ruler. Thus the curve in figure 19 is a graphic generalization not only from its particular series of observations but from several related series. It was assumed to be a typical or representative curve for the function $\log C=f_1(\log S)$; and the corresponding

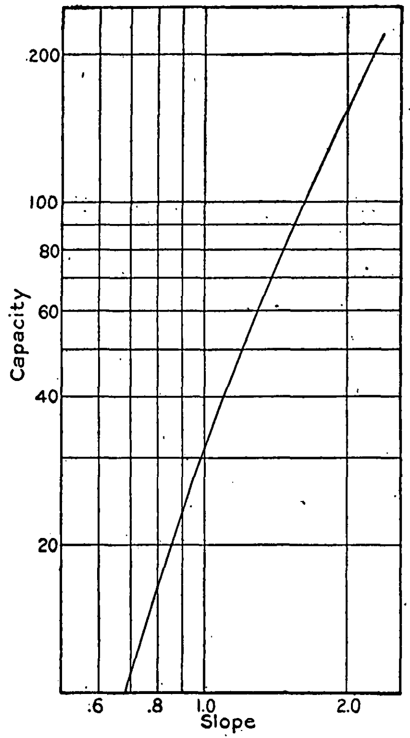


FIGURE 19.—Logarithmic graph of $C=f(S)$, for grade (G), $w=0.66$ foot, $Q=0.734$ ft.³/sec.

plot on ordinary section paper (the line *BD* in fig. 20) was assumed to represent typically the function $C=f(S)$. The coordinates of the line *BD* are given in Table 5.

TABLE 5.—*Values of capacity for traction, graphically generalized from data of Table 4 (G), for $w=0.66$ foot and $Q=0.734$ ft.³/sec.; corresponding to the curve of $\log C=f_1(\log S)$ in figure 19, and the curve BD in figure 20.*

Slope.	Capacity.	Slope.	Capacity.
Per cent.	*Gm./sec.*	*Per cent.*	*Gm./sec.*
0.8	16.3	1.8	124
1.0	31.2	2.0	153
1.2	49.9	2.2	185
1.4	72.0	2.4	220
1.6	97.0		

A number of tentative formulas were now compared with this empiric line, their parameters being computed so that they would fit, as nearly as practicable, the values of C in Table 5. Certain functions, including the simpler functions of the circular arc and the exponential function $C=e^{n(S+a)}$, could not be fitted, even approximately, to the data; but the following functions yielded curves closely resembling that of figure 20:

$$C=bS+cS^2+dS^3 \qquad (6)$$
$$C=bS+cS^2 \qquad (7)$$
$$C=a+bS+cS^2 \qquad (8)$$
$$C=-\lambda+bS^n \qquad (9)$$
$$C=b(S-\sigma)^n \qquad (10)$$
$$C=bS^{nS^m} \qquad (11)$$

Functions (6), (7), and (8) are special cases of the general formula of interpolation with integral exponents:

$$x=a+a_1y+a_2y^2 \ . \ . \ . \ +a_ny^n \qquad (12)$$

No. (11) is a somewhat involved power function suggested by results of a preliminary discussion of the laboratory data. Nos. (9) and (10) are special cases of the general parabolic function

$$(x+a)^n=b(y+a_1)^m \qquad (13)$$

and have the virtue of facilitating the graphic treatment of the material. Their logarithmic equivalents are, respectively,

$$\log (C+\lambda)=\log b+n \log S \qquad (14)$$
$$\log C=\log b+n \log (S-\sigma) \qquad (15)$$

and, as each of these is the equation of a straight line, the graphic derivation of the exponent, by means of logarithmic section paper, becomes a simple matter after the value of λ or σ has been determined.

The adjustment of equations (6) to (11) to the specific data in Table 5 gives them the following forms, (6a) being derived from (6), etc.:

$$C=-29.5S+67.59S^2-7.194S^3 \qquad (6a)$$
$$C=-12.86S+44.08S^2 \qquad (7a)$$
$$C=-19.25+16.94S+34.48S^2 \qquad (8a)$$
$$C=-10.0+41.2S^{1.98} \qquad (9a)$$
$$C=70.5(S-0.39)^{1.63} \qquad (10a)$$
$$C=31.2S^{2.68S^{0.21}} \qquad (11a)$$

When the curves corresponding to these equations are plotted for the region covered by the empiric line *BD*, they coincide very closely with that line. The greatest departure is in the curve for (7a), but its divergence is not sufficient to throw it out of apparent harmony with the series of points representing the original observations.

In order to exhibit further the properties of the formulas, their curves were extrapolated in both directions from the locus *BD*. Table 6 contains the numerical data used in plotting the extensions. Figure 20 gives the extensions of the curves for slopes greater than those of the experiments, and figure 21 the extensions for smaller slopes.

TABLE 6.—*Numerical data computed for the construction of curves in figures 20 and 21.*

Equation	(6a)	(7a)	(8a)	(9a)	(10a)	(11a)
Values of C corresponding to—	*Gm./sec.*	*Gm./sec.*	*Gm./sec.*	*Gm./sec.*	*Gm./sec.*	*Gm./sec.*
$S=0$	0	0	−19.25	−10.0	Imaginary.	0
$S=0.1$	....	−.845	....	−9.57	...do	.0014
$S=0.2$	....	−.810	....	−8.30	...do	.073
$S=0.3$	....	.108	−11.07	−6.20	...do	.490
$S=0.4$	....	1.908	−6.97	−3.28	.039	1.59
$S=0.5$	....	4.59	−1.96	.45	1.93	3.64
$S=0.6$	....	8.15	3.33	4.98	5.54	6.81
$S=0.7$	....	12.6	9.50	10.33	10.46	11.14
$S=0.8$	16.3	17.9	16.36	16.48	16.49	16.67
$S=3.0$	327	358	332	353	337	323
$S=4.0$	504	654	589	631	571	616
$S=5.0$	645	1,039	920	990	849	676
$S=6.0$	705	1,510	1,315	1,422	1,172	84[illegible]
$S=7.0$	645	2,070	1,741	....	1,531	998
$S=8.0$	409	2,718	2,313	2,519	1,923	1,140
$S=9.0$	−31	....	2,916	....	2,355	1,276
$S=10.0$	....	....	....	3,926	2,818	1,400
	Per cent.	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*
Values of S when $C=0$	0 .45 8.94	0 .29	−1.03 .54	−0.49	0.39	0
Values of S when C=maximum	6.04					
Values of S when C=minimum	.22					
Values of S corresponding to point of inflection	3.33	....	....	....	....	4.37

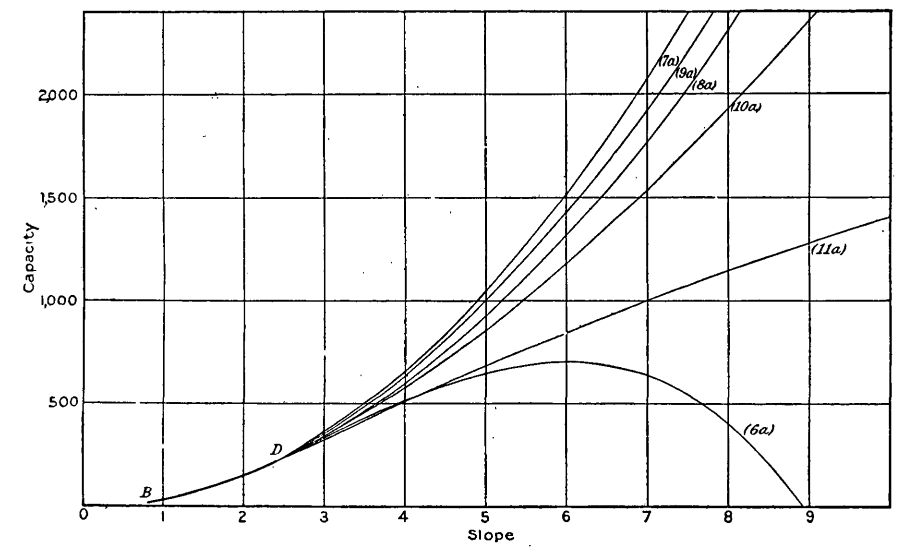


FIGURE 20.—Extrapolated curves of $C=f(S)$ for tentative equations of interpolation, and for slopes greater than 2.4 per cent.

The approximate range of this series of experiments is from a slope of 0.8 per cent to one of 2.4 per cent. The extrapolated curves pertain to slopes from 2.4 to 10 per cent and from 0.8 to 0 per cent. The prompt divergence of the lines as they leave the locus to which they were adjusted shows that they have widely different values for purposes of extrapolation, and therefore presumably for purposes of interpolation.

Attention being given first to the curves for higher slopes (fig. 20), it will be observed that four of them ascend with progressively increasing rate. The curve of formula (11a) ascends continuously, but its rate of ascent changes at the slope of 4.37 per cent from an increasing rate to a decreasing rate. The curve of formula (6a) exchanges its increasing rate of ascent for a decreasing rate at the slope of 3.33 per cent, attains a maximum at a slope of about 6 per cent, and crosses the line of zero capacity before reaching the slope of 9 per cent.

The general characteristics of stream traction do not admit of a maximum in the relation of capacity to slope. Capacity for traction is clearly an increasing function of the stream's velocity, and the velocity is clearly an increasing function of the slope. There is reason also to believe that capacity increases at an increasing rate up to the slope corresponding to infinite capacity. There are three forces concerned in traction—first, the force of the current, of which the direction is parallel to the slope; second, a component of gravity, when gravity is resolved in directions parallel and normal to the slope; third, the resistance of the bed, which is a function not only of the others, but inversely of the slope. Within the range of experimental slopes the component of gravity is negligible in comparison with the force of the current, and the influence of slope on the resistance is relatively unimportant; but as the angle of stability for loose material is approached the resistance diminishes rapidly, and at the slope of instability (65 to 70 per cent for river sand) gravity is competent to transport without the aid of current, and the stream's capacity is infinite. All these factors depend on slope, and as the increment to capacity verges on infinity in approaching the

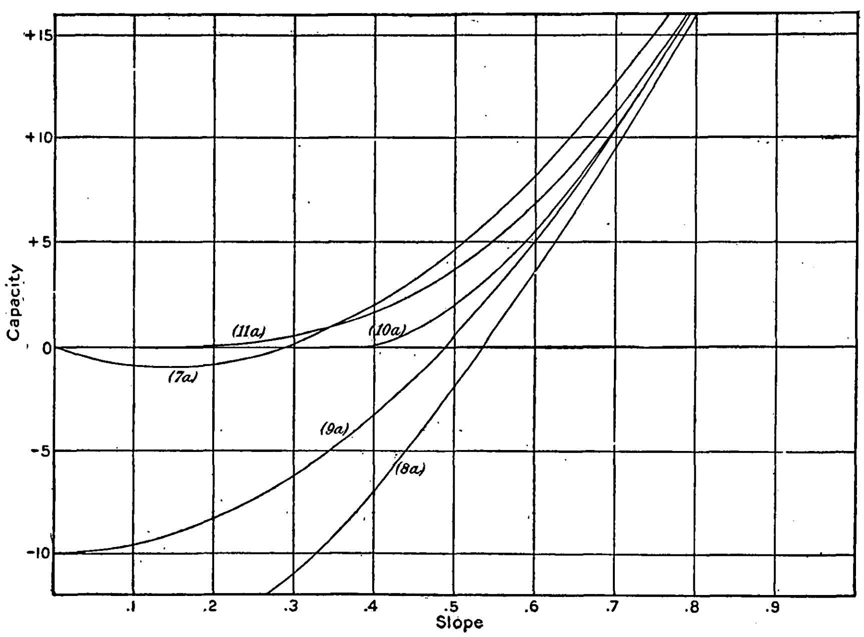


FIGURE 21.—Extrapolated curves of $C=f(S)$ for tentative equations of interpolation and for slopes less than 0.8 per cent.

slope which limits variation, it is highly probable that capacity grows continuously with slope.

This criterion suffices for the rejection not only of the specific formulas (6a) and (11a), but also of their types, (6) and (11). In formula (6a) the occurrence of the maximum value of C is determined by the negative coefficient of S^3; and it is true as a general fact that equations of the class indicated by (12) yield maxima whenever the coefficient of the highest power of the independent variable is negative. It is possible, or perhaps probable, that if each series of laboratory values were to be formulated under (7) or (8) the conditions for maxima would be found to occur. On the whole, the extrapolations for higher slopes tend to restrict choice to forms (9) and (10), with some reservation as to forms (7) and (8).

Figure 21 gives extrapolated curves for slopes less than 0.8 per cent and represents the same equations as figure 20, except that the curve for (6a) is omitted. It will be observed that it magnifies greatly the space between 0 and B in figure 20, the scale of slopes being 10 times and the scale of capacities 100 times as large. The implications of the functions for low slopes are specially important because extrapolation from laboratory conditions to those of natural streams will nearly always involve the passage from higher to lower slopes.

Curves (7a) and (11a) reach the origin of coordinates—that is, their equations indicate that at the zero of slope there is no capacity for traction. Formula (11a) gives small but finite capacities for very low slopes; but under formula (7a) finite capacities cease when the slope falls to 0.29 per cent, and for lower slopes there is indication of negative capacities. If the conditions of traction permitted, negative capacity might be interpreted as capacity for traction upstream; but as this is inadmissible, the negative values may be classed as surd results arising from the imperfection of the correlation between an abstract formula and a concrete problem. The curves of (8a) and (9a) also intersect the axis of slope at some distance from the origin, and their extensions indicate negative capacity. The curve of (10a) becomes tangent to the axis of slope at the point corresponding to a slope of 0.39 per cent and there ends, having no continuation below the axis. It is the real limb of a parabola of which all other parts are imaginary. It expresses to the eye the implication of formula (10a) that traction ceases when the slope is reduced to 0.39 per cent, and that its cessation is not abrupt but gradual; and also the implication of the general formula (10) that traction ceases when the slope is reduced to the value σ.

It is a matter of observation that when slope is gradually reduced, the current becoming feebler and the capacity gradually less, the zero of capacity is reached before the zero of slope. For each group of conditions (fineness, width, discharge) there is a particular slope corresponding to the zero of capacity. It is also a matter of observation that the change in capacity near the zero is gradual. Formulas (7a), (8a), (9a), and (10a) therefore accord with the data of observation in the fact that they connect the zero of capacity with a finite slope; formula (11a), which connects zero capacity with zero slope, is discordant. Also, formulas (10a) and (11a) accord with the data of observation in that they make the approach of capacity to its zero gradual; while formulas (7a), (8a), and (9a), which make the arrival of capacity at its zero abrupt, are in that respect discordant.

But one of the formulas (10a), shows qualitative agreement with both of the criteria applied through extrapolation to low slopes; and that formula is one of the two which respond best to the criterion applied through extrapolation to high slopes. That type of formula, or

$$C = b_1 \, (S - \sigma)^n \quad \text{.........} \quad (10)$$

was therefore selected for the reduction of the more or less irregular series of observational values of capacity to orderly series better suited for comparative study.

In rewriting the formula the coefficient is changed from b to b_1, because corresponding coefficients b_2, b_3, etc., are to be used in a series of formulas expressing the relations of capacity to various conditions. As slope is a ratio between lengths, $(S-\sigma)^n$ is of zero dimensions and b_1 is of the unit C; it is the value of capacity when $S-\sigma=1$.

The slope which is barely sufficient to initiate traction has been defined (p. 35) as the *competent slope.* To whatever extent σ represents the competent slope the formula has a rational basis. The local potential energy of a stream, or the energy available at any cross section in a unit of time, is simply proportional to the product of discharge by slope or, if the discharge be constant, is proportional to the slope. So long as the slope is less than that of competence the energy is expended on resistances at contact with wetted perimeter and air and on internal work occasioned by those resistances. When the slope exceeds the competent slope, part of the energy is used as before and part is used in traction. The change from competent slope to a steeper slope increases the available energy by an amount proportional to the increase of slope, and the increase of energy is associated with the added work of traction. Capacity for traction, beginning at competent slope, increases pari passu with the increase of the excess of slope above the competent slope, and there is manifest propriety in treating it as a function of the excess of slope rather than of the total slope. It is of course also a function of the total slope; but an adequate formula for its relation to the excess of slope may reasonably be supposed to be simpler than a formula for its relation to total slope. If σ represents competent slope, then the relation of capacity to $S-\sigma$ should be simpler than its relation to S.

Instructive information as to the relative simplicity of the two functions is obtained by comparing their logarithmic graphs. In figure 22 the curved line AB has been copied from the curve in figure 19. It is the graph of $\log C = f(\log S)$ for grade (G), width 0.66 foot, and discharge 0.734 ft.³/sec. If for S we substitute $S-0.3$, we modify the graph by moving each point of it to the left by an amount equal to $\log S - \log(S-0.3)$; and we produce the line CD, which is the graph of $\log C = f_{II}(\log(S-0.3))$. If in similar manner we derive the graph of $\log C = f_{III}(\log(S-0.6))$, the result is the line EF. CD curves in the same direction as AB but less strongly; EF curves in the opposite direction. It is evident that the three curves belong to a continuous series, and that somewhere between CD and EF a member of the series is straight or approximately straight. That straight line, GH, is the graph of $\log C = f_{IV}(\log(S-0.39))$; but as it is straight, its equation may be written

$$\log C = \log b_1 + n \log(S-0.39),$$

in which $\log b_1$ is the ordinate of the intersection of the line with the axis of $\log C$, and n is the trigonometric tangent measuring the inclination of the line to the axis of $\log S$. This is identical with equation (15) except that 0.39 appears in place of σ; and in fact

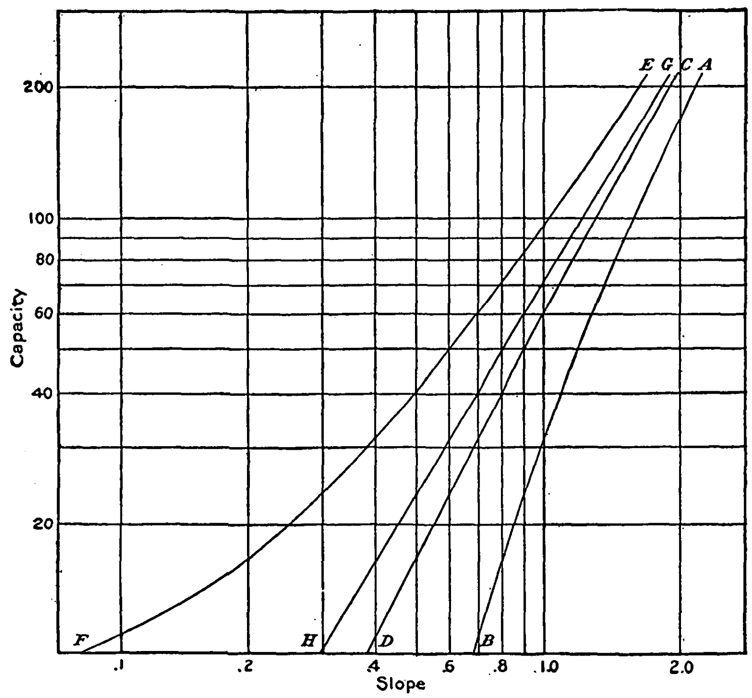


FIGURE 22.—The relation of σ in $C=f(S-\sigma)$ to the curvature of the logarithmic graph.

the value of σ in equation (10a) was computed graphically by means of the logarithmic plot.

The line AB, being the graph of $\log C = f_I(\log S)$, is also the logarithmic graph of $C=f(S)$. The line GH, being the graph of $\log C = f_{IV}(\log(S-\sigma))$ is also the logarithmic graph of $C=f_V(S-\sigma)$. Their relation in respect to simplicity is that of the curve to the straight line.

In view of these suggestions of harmony it is peculiarly pertinent to inquire whether σ is actually representative of competent slope; and it will be convenient to make that inquiry in connection with the determination of values of σ for the several series of observations on capacity and slope.

THE CONSTANT σ AND COMPETENT SLOPE.

In the experimental data for graded débris—Table 4, (A) to (H)— are 117 series of values of capacity and slope. After these had been plotted and inspected in a comparative way, it was decided to restrict the main discussions to 92 series only, the discarded series being all short as well as somewhat discrepant among themselves. Of the 92 series retained, only 30 afford information as to the corresponding values of σ; that is, only 30 of the logarithmic plots exhibit curvature so definitely that the approximate magnitudes of the constants necessary to eliminate it can be inferred. In some of the remaining series the observations are not so distributed as to bring out the curvature. For others the observational positions on the plots are too widely dispersed to give good indication of the character of the best representative line.

On the 30 logarithmic plots the curves approximately representing the observations were drawn, and the values of σ necessary to replace the curves by straight lines were computed graphically in the manner just indicated. These values are given, to the nearest tenth of 1 per cent of slope, in Table 7, where they are arranged with reference to the conditions of the experiments.

TABLE 7.—*Values of σ in $C=b_1(S-\sigma)^n$, estimated from logarithmic plots of observations.*

Grade	F	Range of F	w (feet).	Values of σ for discharge (ft.³/sec.) of—							
				0.093	0.182	0.363	0.545	0.734	0.923	1.021	1.119
(A)	25,200	1.77	0.66	—	—	...	—	...	...	...	...
			1.00	...	—	—	...	—	...	...	...
			1.32	...	0.2	0.1	...	—	...	...	...
			1.96	...	...	—	...	—	...	...	...
(B)	13,400	2.35	.23	0.7	.4	...	...	...	...	...	...
			.44	.5	.0	...	...	...	...	...	...
			.66	—	—	—	—	...	...	...	...
			1.00	...	—	.1	—	0.1	...	...	...
			1.32	...	—	—	—	—	...	...	...
			1.96	...	...	—	—	—	...	...	...
(C)	5,460	2.85	.44	.4	—	...	...	...	...	...	...
			.66	.2	.1	—	0.1	—	...	...	...
			1.00	...	.3	.1	—	.1	...	...	—
			1.32	...	—	.1	—	.1	...	...	...
			1.96	...	...	.3	—	—	...	...	—
(D)	1,460	3.58	.66	—	—	...	—	...	...	...	...
			1.00	...	—	—	—	—	...	...	...
			1.32	...	...	—	...	—	...	...	...
(E)	142	12.65	.66	...	—	—	...	—	...	...	—
			1.00	...	.0	.0	...	.0	...	...	—
			1.32	...	...	—	...	—	...	...	—
(F)	22.1	2.15	.66	...	—	—	...	—	...	...	...
			1.00	...	—	—	...	—	...	...	—
			1.32	...	...	—	...	—	...	...	—
(G)	5.9	2.92	.66	...	...	.6	...	.4	...	...	0.3
			1.00	...	...	.6	...	.3	...	...	.3
			1.32	...	...	—	...	.4	...	...	—
(H)	2.0	2.51	.66	...	...	—	...	.6	...	...	.4

NOTE.—The horizontal dashes indicate series of observations to which values of σ are to be assigned.

The same table shows the distribution of the experimental series which fail to give values of σ but to which it is proposed to assign values. In order to assign these values properly it is necessary to know the laws of variation of σ with reference to the conditions of experimentation. These are suggested in part by the roughly determined values of the table and are otherwise indicated by general considerations. The variations are connected with at least three conditions—discharge, width, and fineness—and are less surely connected with range of fineness.

Considering first the variation of σ with discharge and giving attention in the table to values of σ falling in the same horizontal line, we find by inspection that invariably the value for a larger discharge is either less than or equal to the value for the corresponding smaller discharge. The indication is that σ is a decreasing function of discharge. This relation might have been inferred from general considerations, on the theory that σ represents competent slope. Competent slope is the slope giving competent velocity along the bed, but bed velocity also varies directly with discharge. With large discharge less slope is necessary to induce competent velocity; with small discharge more slope. In other words, competent slope varies inversely with discharge. In a preliminary discussion of the traction data for débris of grades (B) and (C) it was found that capacity is more sensitive to changes in slope than to changes in discharge. The relation of capacity to slope and discharge jointly being tentatively represented by

$$C \propto S^n Q^m \text{..........} (16)$$

the values of n and m were computed for many different conditions, and it was found that on the average $\frac{m}{n}=0.34$. There is, however, considerable variation in the ratio, and it is relatively large when C is small. On plotting the values of the ratio in relation to C and graphically extrapolating, it was found that when $C=0$ the ratio is about 0.5. The condition of zero capacity is that of competent bed velocity and competent slope.

For any particular value of C in (16) the product of S^n by Q^m is constant, or

$$S^n \propto \frac{1}{Q^m}$$

whence

$$S \propto \frac{1}{Q^{\frac{m}{n}}}$$

But, as we have just seen, when C has the particular value $C=0$, $\frac{m}{n}=0.5$. Therefore competent slope varies inversely as $Q^{0.5}$. Its assumed representative, σ, is assumed to vary inversely with the square root of discharge.[1]

Turning now to the relation of σ to width, w, and examining Table 7, we see that σ is not exclusively either an increasing or a decreasing function of w. Where the smallest widths are concerned, as with $Q=0.093$ ft.³/sec. and $Q=0.182$ ft.³/sec., for grades (B) and (C), the function is decreasing. Where the greater widths are concerned it is for the most part increasing. There is a rational explanation for such double relationship in the case of competent slope.

Figure 23 represents two troughs in cross section. Each has a bed of débris, and they are assumed to be carrying the same total discharge. In the wider there is less discharge for each unit of width, and the tendency of the smaller discharge is to reduce velocity. Therefore to maintain a particular velocity—namely, the competent velocity—the tendency is to produce a relatively steep slope. So far as this factor is concerned, competent slope is relatively steep for a wider trough. But there is another factor; the velocity is influenced by the resistance of the sides of the trough. This resistance is greater where the water is deeper, because the surface of contact is broader; and the water is deeper in the narrower trough. The tendency of the resistance is to reduce velocities and therefore to make the slope for competent velocity steeper in the narrower trough.

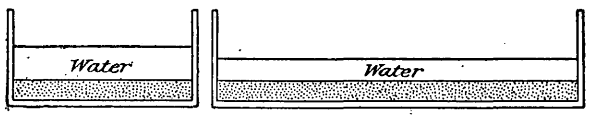


FIGURE 23.—Diagrammatic sections of laboratory troughs, illustrating relation of current depth to trough width.

In very wide troughs the influence of the sides is of minor importance, the influence of discharge per unit width dominates, and the competent slope varies directly with the width. In very narrow troughs the influence of the sides dominates, and the competent slope varies inversely with the width. For some intermediate width the two tendencies are balanced, and the competent slope has its minimum value. In figure 24 abscissas measured from O represent width of trough, and ordinates represent competent slope. The curve sketched, while not quantitative, has adequate basis for its broader features and

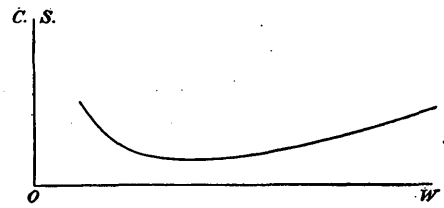


FIGURE 24.—Ideal curve of competent slope (*C. S.*) in relation to width of trough (*W*).

shows the general character of the relation of competent slope to width. From its minimum it ascends gradually on the side of greater width, and on the side of lesser width rises with relative rapidity toward a vertical asymptote near the axis of competent slope. These various characters are approximately paralleled by the variations of σ as shown in the table.

The relation of σ to fineness does not come out very clearly in Table 7, and a different

[1] The preliminary discussion on which is based $\sigma \propto \frac{1}{Q^{0.5}}$ did not have the advantage of the formula using σ. A rediscussion, to be found in Chapter V, yields $\sigma \propto \frac{1}{Q^{0.77}}$; but it was not practicable to give the adjustment the benefit of this later work without repeating the greater part of the computations of the paper. It is not believed that the advantage to the results would be commensurate with the labor involved.

arrangement is therefore given in Table 8, which omits the values of σ not affording comparison and adds three interpolated values. The general fact thus shown is that σ increases as fineness diminishes, but to this there are two exceptions. The more important exception is in grade (E), of which all values of σ are less than the corresponding values for either grade (C) or grade (G). The values for grades (B) and (C) appear to be about the same, although the two grades differ notably in fineness. The exceptions are associated with peculiarities of the grades with respect to range of fineness. In grade (E) the range for bulk fineness is about twice as great as in any of the other grades; and if this character indicates the cause of its abnormally small values of σ, then the abnormally high values in grade (B) are explained by its small range of fineness as compared to the range of grade (C).

TABLE 8.—*Values of σ, from Table 7, arranged to show variation in relation to fineness of débris.*

Grade		(B).	(C).	(E).	(G).	(H).
Fineness (F_2)		13,400	5,460	142	5.9	2.0
Range of fineness		2.35	2.85	12.65	2.92	2.51
Width.	Discharge.	Values of σ.				
0.44	0.093	0.5	0.4	....	....	....
.44	.182	.0	[.2]	....	....	....
.66	.363	....	[.1]	....	0.6	....
.66	.734	....	[.1]	....	.4	0.6
1.00	.182	....	.3	0.0	....	....
1.00	.363	.1	.1	.0	.6	....
1.00	.734	.1	.1	.0	.3	....
1.32	.734	....	.1	....	.4	....

The values of σ inferred from the logarithmic plots are not sufficiently precise to yield quantitative laws of the relations of σ to fineness and the range of fineness, and it is again necessary to seek instead the laws governing competent slope. Does competent slope vary inversely with fineness, and what is its law? Does competent slope vary inversely with range of fineness, and what is its law?

Inasmuch as fine débris is moved by a relatively slow current and as the force of the current is a direct function of slope, the competent slope for fine débris is less than for coarse. If we accept the thesis of Leslie and Hopkins (see p. 16) that competent bed velocity (V_{cb}) varies with the sixth root of the volume or mass of the débris particle, then, since bulk fineness is the reciprocal of volume,

$$V_{cb} \propto \frac{1}{F_2^{1/6}}$$

If we assume, from the Chezy formula, that mean velocity of current is proportional to the square root of slope and apply it to component mean velocity (V_{cm}) and competent slope (S_c), we have

$$V_{cm} \propto S_c^{0.5}$$

If we further assume that bed velocity is proportional to mean velocity, then, by combining the three proportions and reducing, we obtain

$$S_c \propto \frac{1}{F_2^{0.33}} \qquad (17)$$

As each of the three assumed laws is subject to important qualifications, the product of their combination must be regarded as but a rough approximation to the law connecting competent slope with fineness.

Further light on the law is afforded by some experiments made for the specific purpose of determining competent slope. In these experiments the discharge remained constant while the velocity was modified by changing the width of the outfall. The slope of the bed had been prepared in advance, and the slope of the water surface was measured for each width of outfall. At each stage of the experiment the movement of grains of débris along the bottom was noted by such phrases as "many," "several," "few," "very few," "none," the words being used in that order as a sort of scale. Competent slope was inferred from a comparison of these notes with the recorded slopes of the water surface. The results are given in Table 9.

In a closely related series of experiments, Table 10, a slope of débris was prepared in advance, a small discharge was passed over it, and the discharge was progressively increased, with notes on the movement of débris grains. These experiments gave competent discharge.

In each series the depths were measured, and from these the mean velocities (V_m) were computed.

The experimental determinations were indefinite for several reasons. In the first place, a stream with the critical bottom velocity does not transport débris and therefore does not establish its own slope of bed. The slopes artificially prepared were of necessity imperfectly adjusted. In the next place, the prepared slopes did not imitate the natural diversity of detail but were plane. When a current of competent velocity passed over one of them it immediately began to shape the bed into dunes, and as the modeling proceeded the activity of transportation increased. After the dunes were formed, a smaller general velocity, or a less discharge, or a lower general slope was competent. The experiments being made in sets, the first of a set gave a result from the plane bed and the others from a more or less diversified bed.

In the third place, the particles composing one of the experimental grades of débris were not of uniform mobility. Not only were they of diverse size, as indicated by the "range of fineness," but they were different in shape and in specific gravity, so that some were able to resist a considerably stronger current than others. The competent slope for the least mobile particles was materially steeper than that for the most mobile, and no mode of gaging average mobility was discovered.

TABLE 9.—*Experimental data on competent slope.*

Grade of débris.	Width of trough.	Width of contractor.	Discharge.	Slope of débris.	Slope of water.	Depth.	Mean velocity.	Notes on movement of débris.
	Feet.	*Feet.*	*Ft.³/sec.*	*Per ct.*	*Per ct.*	*Feet.*	*Ft./sec.*	
(B).....	1.00	0.30	0.363	0.048	0.01	0.490	0.74	A very few grains moving. Small dunes forming.
		.40	.363	.03	.06	.425	.85	Many grains moving. Dunes forming.
		.30	.363	.028	.03	.475	.76	A very few grains moving. Some dunes forming.
C).....	1.00	.30	.363	.10	.01	.451	.80	A very few grains moving in a few places.
		.40	.363	.10	.042	.391	.93	Many grains moving. Dunes forming.
		.30	.734	.10	.045	....	....	Dunes forming rapidly.
(D).....	1.00	.30	.363	.02	.21	.465	.78	No grains moving.
		.40	.363	.02	.34	.418	.87	An occasional grain moving.
		.30	.734	.02	.33	.743	.99	Very few, if any, grains moving.
		.40	.734	.02	.57	.658	1.12	Many grains moving. Dunes forming.
		.40	.363	.06	.03	.392	.93	A very few grains moving. No dunes forming.
		.40	.363	.10	.055	....	....	Very few grains moving.
		.45	.363	.10	.054	.373	.97	Some small grains moving.
		.50	.363	.10	.056	.357	1.02	Several small grains moving. No dunes forming.
		.55	.363	.10	....	....	....	Many grains moving.
(E).....	1.00	.40	.363	.05	.043	.387	.94	No transportation.
		.40	.734	.035	.025	.618	1.19	Do.
		.50	.363	.035	.057	.357	1.02	Do.
		.50	.734	.035	.069	.562	1.31	Some transportation in lower half of slope.
		.60	.363	.05	.086	.330	1.10	Very few grains moving in lower half of slope.
		.60	.363	.10	.096	.314	1.16	A few grains moving.
(G).....	1.00	.70	.363	.40	.22	.256	1.42	No grains moving.
		.87	.363	.40	.29	.238	1.52	Do.
		.87	.363	.50	.46	.225	1.61	A few grains moving at one place.
		.87	.734	.50	.46	.360	1.01	A few grains moving in places.
		.87	.363	.65	.70	.201	1.80	Very few, if any, grains moving.
		.87	.363	.80	.80	.184	1.98	An occasional grain moving.
		.87	.363	1.00	.93	.173	2.10	Several grains moving in center half of slope.

When an experiment was begun with a velocity well below competence, and the velocity was gradually increased, the first movement detected would be the saltation of some small or light particle, and then the number of particles moving would gradually grow with the quickening of current.

An attempt to correlate the "notes on movement of débris" in Table 10 for a discharge of 0.363 ft.³/sec. gave the following values of water slope for equivalent phases of movement:

Grade	(B)	(C)	(D)	(E)	(G)
Slope of water (per cent)	0.03	0.03	0.06	0.10	0.93

By assuming the power function $S_c = aF_2^n$, or $\log S_c = \log a + n \log F_2$, and plotting $\log S_c$ in relation to $\log F_2$, the value found for n is about -0.5, but it has a large uncertainty. The

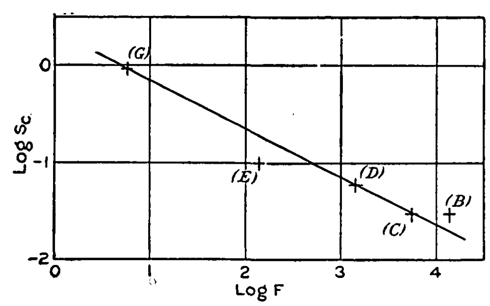


FIGURE 25.—Logarithmic plot of competent slope in relation to fineness of débris.

plot, figure 25, shows the competent slope for grade (E) smaller than would be indicated by its neighbors, but not so much smaller as is the value of σ for grade (E) in Table 7. It also makes the competent slope for grade (B) aberrant in the same sense as the corresponding value of σ. The data are too vague to give much value to such correspondences; but the broader resemblances aid in connecting σ with competent slope.

TABLE 10.—*Experimental data on competent discharge.*

Grade of débris.	Width of trough.	Width of contractor.	Discharge.	Slope of débris.	Depth.	Mean velocity.	Notes on movement of débris.
	Feet.	*Feet.*	*Ft.³/sec.*	*Per ct.*	*Feet.*	*Ft./sec.*	
(E)....	1.32	0.90	0.039	2.00	....	....	Many grains moving. Dunes quickly formed.
			.019	1.95	0.015	0.96	Some grains moving; many moving as small channels are formed or surface becomes rough.
			.019	1.00	.014	1.03	No grains moving.
			.039	1.00	.026	1.14	Many grains moving.
			.039	1.00	.024	1.23	Several grains moving. Dunes forming.
			.019	1.00	.018	.80	No grains moving.
	1.00	.72	.010	2.00	.015	.67	A few grains moving in a few places. Dunes forming.
			.019	1.94	.025	.76	Many grains moving. Dunes forming.
		.92	.019	1.00	....	....	A few grains moving in a few places.
			.029	1.00	....	....	Several grains moving in several places.
			.039	.50	.053	.74	No grains moving.
			.058	.50	.060	.96	A few grains moving in a few places.
			.075	.50	.065	1.15	Several grains moving in several places.
			.093	.50	.070	1.33	Transportation.
	.66	.46	.010	2.00	.022	.69	A few grains moving in a few places.
			.019	2.00	.028	1.03	Several grains moving in nearly all parts of trough.
			.010	1.13	....	....	No motion.
			.019	1.13	.042	.69	A few grains moving in a few places.
			.039	1.13	.050	1.19	Many grains moving. Surface becoming rough.
(F).....	.66	.46	.039	2.51	.043	1.36	An occasional grain moving. Several grains moving after a time.
			.058	1.05	....	....	No grains moving.
			.075	1.05	.071	1.59	A few grains moving in a few places.
			.093	1.05	.092	1.52	Several grains moving in some places.
			.111	1.05	....	....	Many grains moving.
	1.00	(?)	.146	1.00	.086	1.70	Few grains moving in some places.
			.164	1.00	.093	1.77	Several grains moving.
(G)....	.66	.45	.146	1.10	.119	1.84	No grains moving.
			.164	1.10	.126	1.95	Very few grains moving.
			.182	1.10	.139	1.96	A few grains moving near middle of trough.
			.200	1.10	.143	2.10	A few grains moving.
			.218	1.10	.153	2.13	Several grains moving in lower half of trough.
			.164	1.58	.112	2.20	Some grains moving in lower half of trough.
			.182	1.58	.104	2.63	Several grains moving in lower half of trough, but very few in upper half.
			.200	1.58	.104	2.88	Many grains moving in lower half, but few in upper half.
			.146	2.05	.106	2.07	Several grains moving in middle of trough; many in lower part. Cutting of grade.
			.111	2.05	.088	1.89	A few grains moving.
			.093	2.05	.088	1.58	Very few grains moving.
			.075	2.05	.070	1.61	No grains moving.
			.058	2.50	.052	1.61	Do.
			.075	2.50	.054	2.08	A few grains moving.
			.093	2.50	.072	1.95	Many grains moving.
	1.00	.70	.182	1.00	.124	1.47	No grains moving.
			.218	1.00	.141	1.55	No (or very few) grains moving.
			.255	1.00	.153	1.67	Very few moving.
			.290	1.00	.159	1.84	Many grains moving.
			.182	1.45	.089	2.04	Very few small grains moving.
			.218	1.45	.098	2.23	Several grains moving.
			.255	1.45	.105	2.43	Many grains moving.
			.182	1.90	.090	2.02	Several grains moving.
			.146	1.90	.080	1.83	Very few grains moving.
			.111	1.90	.059	1.88	No grains moving.
			.093	2.52	.061	1.52	Do.
			.111	2.52	.067	1.66	A very few grains moving in part of trough.
			.128	2.52	.074	1.73	A few grains moving, except near head of trough.
			.146	2.52	....	....	Many grains moving. Grade cutting in places.
	1.32	.92	.272	1.05	.115	1.79	No grains moving.
			.327	1.05	.125	1.99	Very few grains moving.
			.345	1.05	.128	2.04	A few grains moving.
			.363	1.05	.130	2.11	Several grains moving.
			.218	1.40	.075	2.21	Very few grains moving near center of trough.
			.237	1.40	.079	2.27	None moving except at middle of trough.
			.255	1.40	.084	2.27	Very few moving except at middle of trough.
			.272	1.40	.090	2.28	A few grains moving.
			.290	1.40	.099	2.22	Several moving in middle and lower parts; very few in upper.
			.218	2.00	.079	2.09	A few grains moving.
			.237	2.00	.090	1.99	Several grains moving in middle part of trough.
			.255	2.00	.097	1.98	Several grains moving.
			.146	2.50	.055	2.01	Very few grains moving.
			.164	2.50	.061	2.04	Several grains moving.
			.182	2.50	.069	1.99	Do.
(H)....	.66	.46	.363	1.10	.212	2.57	Occasionally a grain moving.
			.454	1.10	.260	2.62	A few grains moving in lower half of trough.
			.545	1.10	.399	2.74	Several grains moving in lower two-thirds of trough, and occasionally a grain in upper third.
			.639	1.10	.329	2.91	Several grains moving in middle third of trough; a few in upper and lower parts.
		.45	.272	1.30	.163	2.49	Occasionally one grain moving.
			.308	1.30	.171	2.70	Some grains moving below middle; 1 to 3 grains in a cross section.
			.345	1.30	.184	2.82	2 to 5 grains moving in a cross section.
			.182	2.05	.132	2.05	No grains moving.
			.218	2.05	.137	2.39	A few grains moving in lower three-fourths of trough.
			.255	2.05	.141	2.72	Many grains moving in lower three-fourths of trough.

In treating the data of Table 9, the assumption was made that the note "several grains moving" corresponds to competent slope; and averages were found of the values of mean velocity and depth.[1] These are:

Grade	(E)	(F)	(G)	(H)
Mean velocity (ft./sec.)	1. 10	1. 52	2. 14	2. 83
Depth (foot)	0. 020	0. 092	0. 108	0. 218

When the logarithms of these numbers are plotted the positions fall well in line, and the representative line gives (V_{cm} indicating the mean velocity corresponding to competent bed velocity)

$$V_{cm} = \frac{3.2}{F_2^{0.22}} \text{ .... } (18)$$

Assuming again that bed velocity is proportional to mean velocity, and again assuming the validity of the Chezy formula, we obtain from (18)

$$S_c \propto \frac{1}{F_2^{0.44}} \text{ .... } (19)$$

The two values of the exponent of F_2 derived from the experiments, namely, -0.44 and -0.50, are both larger than the deductive value, -0.33, of equation (17), but the disparity is quite natural in view of the indefiniteness of the data and the uncertainties of the assumptions. Collectively the values indicate an order of magnitude.

The influence of range of fineness on competent slope appears to be of the same nature as its influence on σ, though much less pronounced, but the determinations of competent slope are too indefinite to give the greatest value to the comparison. It is significant, however, that while the logarithmic plots for grade (E) and width 1.00 and for three different discharges (Table 7) all yield values of σ less than 0.05 per cent, the experiments on competent slope (Table 9) record for one of the discharges "no grains moving" with a slope of 0.21 per cent, and for another "very few, if any, grains moving" with a slope of 0.33 per cent. The values of σ in this case fall far below those for the most mobile components of the débris grade which has the largest range of fineness. The general facts appear to be that σ varies decreasingly with range of fineness, and that competent slope is subject to a variation of the same kind, which may or may not be of the same magnitude.

The cause of this variation is not surely known, but a plausible suggestion in regard to it may be made. In the experiments with mixtures of two or more grades it was found that before the slope had been established, especially when low velocities were used, the current tended to sort the débris, building deposits with the coarser part and delivering the finer material at the end of the trough. In experiments with a single grade the same tendency doubtless existed. It was in fact observed in connection with dunes and antidunes, which sometimes showed a shading in color due to partial sorting with respect to density, the heavier particles being dark, the lighter pale. As the differences of size within a grade were not such as to appeal strongly to the eye, considerable sorting with respect to size might take place without attracting attention. With the ordinary routine of the experiments, which began in each series with low slopes and velocities and gradually increased them, the influences of such sorting may have been systematic, and thus may have modified that relation of values which finds expression in the constant σ. The result of such influence would be more pronounced for grade (E) than for grades with smaller range of fineness. It is easy to see also that an allied influence may have affected to some extent the interpretation of the experiments on competent slope.

The variations of σ are paralleled in so many ways by the variations of competent slope as to leave little doubt that the one is in some way representative of the other. It can not be said that the constant σ, arbitrarily introduced to rectify curves and thereby facilitate interpolation, is the equivalent of the slope of competence—if for no other reason than that the competent slope for a grade of débris made of unequal grains eludes precise definition—but it may well be a complex function of the competent slopes of all the different sorts of grains contained in one of the laboratory grades.

For the practical purpose of obtaining values of σ for use in formulas of interpolation, the preceding discussion yields a large body of pertinent information. Sigma varies inversely

[1] A few series of observations on competent velocity have been made by others. They pertain chiefly to flume traction and are cited in Chapter XII. Login, whose results are given in Chapter VII, used the methods of stream traction but omitted to measure the sizes of materials transported.

with a power of discharge, approximately the 0.5 power. It varies inversely with a somewhat smaller power of the bulk fineness of débris. It varies inversely with the range of fineness of the grades of débris, this variation serving to qualify the preceding. It varies with width of channel, the variation including a minimum. Without attempting to give definite symbolic expression to these laws of variation, they were applied to the practical problem, and by a series of adjustments the skeleton of values of σ in Table 8 was developed into a system covering the whole range of experimental conditions. That system is presented in Table 11.

TABLE 11.—*Values of σ, in per cent of slope, as adjusted for use in interpolation equations of the form $C=b_1(S-\sigma)^n$.*

Grade.	Width (feet).	Values of σ for discharge ($ft.^3/sec.$) of—							
		0.093	0.182	0.363	0.545	0.734	0.923	1.021	1.119
(A).....	0.66	0.40	0.20	......	0.04	......	......	......	......
	1.00	......	.12	0.07	......	0.05	......	......	......
	1.32	......	.17	.10	......	.07	......	......	......
	1.96	......	......	.17	......	.10	......	......	......
(B).....	.23	.70	.60	......	......	......	......	......	......
	.44	.50	.10	......	......	......	......	......	......
	.66	.30	.10	.08	.07	......	......	......	......
	1.00	......	.12	.08	.06	.06	......	......	......
	1.32	......	.17	.12	.10	.08	......	......	......
	1.96	......	......	.18	.14	.12	......	......	0.10
(C).....	.44	.40	.20	......	......	......	......	......	......
	.66	.16	.11	.08	.06	.04	......	......	......
	1.00	......	.15	.11	.09	.07	......	......	.06
	1.32	......	.22	.16	.13	.11	......	......	......
	1.96	......	......	.24	.20	.17	......	......	.14
(D).....	.66	.19	.14	......	.08	......	......	......	......
	1.00	......	.17	.12	.12	10	0.08	......	......
	1.32	......	......	.16	......	.11	......	......	......
(E).....	.66	......	......	.09	......	.06	......	......	......
	1.00	......	.08	.06	......	.04	......	......	.03
	1.32	......	......	.04	......	.03	......	......	.02
(F).....	.66	......	.33	.21	......	.17	......	......	......
	1.00	......	.44	.31	......	.22	......	......	.17
	1.32	......	......	.39	......	.28	......	......	.23
(G).....	.66	......	......	.50	......	.36	......	......	.27
	1.00	......	......	.58	......	.41	......	......	.33
	1.32	......	......	.71	......	.50	......	......	.40
(H).....	.66	......	......	.80	......	.56	......	......	.48

INTERPOLATION.

The values of σ having been assigned, the data of Table 4 were once more plotted on logarithmic paper, the ordinates again representing load or capacity, and the abscissas representing $S-\sigma$, or observed slope less the constant slope σ. As in the preliminary plotting for inspection, the primary data were (1) the estimates of load from the quantity of débris delivered at the end of the trough and (2) the associated slopes of the bed of débris, and accessory data were added with distinctive notation. The secondary data were used chiefly to indicate the relative precision of the primary data, the primary having greater weight when agreeing more closely with the secondary. The illustrative plot, figure 26, shows only the primary data.

The next step was to draw through and among the observational points the best representative straight line. It is a property of the logarithmic plot that its distances represent ratios, and the scale of ratios is everywhere the same. Similar errors of observational positions are shown by similar distances in all parts of the plot, provided the errors are considered as fractional parts of the plotted quantities. Each observational point should be given the same influence in determining the

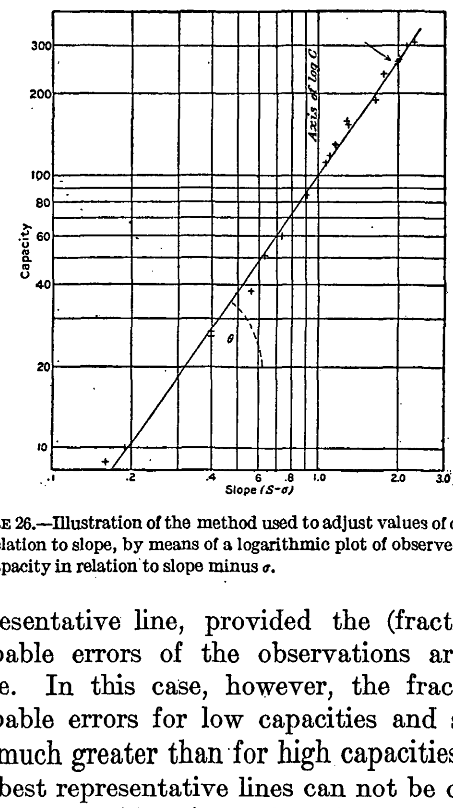


FIGURE 26.—Illustration of the method used to adjust values of capacity, in relation to slope, by means of a logarithmic plot of observed values of capacity in relation to slope minus σ.

representative line, provided the (fractional) probable errors of the observations are the same. In this case, however, the fractional probable errors for low capacities and slopes are much greater than for high capacities, and the best representative lines can not be drawn without consideration of weights. Adjustment by the least-squares method was considered and experiments were tried, but the labor entailed by the necessity of using weights was not thought to be warranted by the quality of the data. The following simpler and less rigorous method was employed:

A group of observational points corresponding to the highest slopes was selected by inspection of the plot, and its center of gravity was computed by a graphic method. In the case shown by figure 26, the group includes six points and the center of gravity is the point indicated by an arrow. The representative line was made to pass through this point and was otherwise adjusted to position by eye estimate, with consideration of all the observational points and their supposed weights. By this method the observations of greatest weight were enabled to fix one point on the line and were also consulted, along with observations of less weight, as to its direction or attitude. The method obviously left much to personal judgment, but had a rigorous method been attempted it would have been difficult or impossible to avoid the use of nonrigorous judgment in the assignment of weights.

The line when drawn is a generalized expression, for a single series of observations, of the relation of capacity to slope less σ. It is the graphic equivalent, or graph, of a specific equation of the form

$$\log C = \log b_1 + n \log (S-\sigma) \quad (20)$$

which is the logarithmic equivalent of

$$C = b_1 (S-\sigma)^n \quad (10)$$

The inclination of the line ($\tan \theta$) was next measured, or computed, giving the numerical value of n; and its point of intersection with the axis of $\log C$ was determined, giving the value of b_1. The values of n, b_1, and σ for the 92 series are shown in Table 15.

The graph was used, instead of the equivalent equation, in computing values of C corresponding to systems of values of S. The results are given in Table 12 and constitute the data for further generalizations. To compute values of C for a particular observational series, the value of σ for the series was first subtracted from each value of S in the adopted system, then each remainder $(S-\sigma)$ was applied as an argument to the graph and the corresponding value of C read off. The range of values thus computed and tabulated was either limited by the range of observational values, or else included a moderate extrapolation, which never exceeded 10 per cent of the observational range.

PRECISION.

If the position of the straight line in each of the logarithmic plots (fig. 26) had been determined by rigorous methods, it would be possible to compute by rigorous methods the probable errors of the quantities implied by its position. As only approximate methods were employed in placing them, only approximate measures of precision are attainable, and an elaborate treatment would be unprofitable. The precision of the attitude of the line, corresponding to the quantity n, has not been estimated; but computation has been made of the precision of its position in the direction of the axis of $\log C$; and also of the precision of the observations, on the assumption that the distances of the observational points from the straight line represent errors of observation. The precision of the position of the line involves the precision of the adjusted values of capacity, and also the precision of the coefficient of the equation of adjustment. The method of computation, given below, is also the method employed in various other computations of precision, the results of which appear in later chapters.

Each logarithmic plot, corresponding to an observational series, was treated separately. The distance of each plotted observational point from the representative straight line, in a direction parallel to the axis of $\log C$, was measured. This distance, interpreted by the scale of the section paper, gives the logarithm of the ratio between an observed capacity and the corresponding adjusted capacity. By using a strip of the section paper as measuring scale, it was possible to read the ratio directly. It was also possible, without computation, and as a simple matter of reading, to subtract unity from the ratio and multiply the remainder by 100, thus recording directly the residual, or observational error, as a per cent of the quantity measured. The facility of this operation determined the estimation of all errors in percentage.

From the residuals thus obtained, probable errors were computed by the following approximate formulas, in which m is the number of the residuals and [v] is the sum of the residuals, irrespective of sign:

Probable error of an observation =

$$0.845 \frac{[v]}{\sqrt{m(m-1)}}$$

Probable error of adjusted capacities =

$$0.845 \frac{[v]}{m\sqrt{m-1}}$$

Table 12 contains the computed probable errors for all those series, 82 in number, in which the number of observations is not less than 4. For a smaller number of observations it was possible so to frame adjusting equations as to leave no residuals; and although this was not done, the propriety of applying the computation to such cases was not evident.

The arithmetical mean of the 82 values of probable error for adjusted capacities is ± 2.50 per cent. The corresponding mean for observed capacities is ± 8.80 per cent. Another measure of the precision of the observations, collectively, is the arithmetical mean, irrespective of sign, of all the residuals (966 in number) involved in the 82 series. That mean is 11.5 per cent.

While these estimates of precision are computed specifically for capacities or loads, the sources of the errors are not restricted to the observations of loads but include also the observations of slope and discharge. With use of the same diagrams it would be possible to consider slope as a function of capacity and compute the probable errors of slope determinations. The relations are such that, within any series, the probable error of slope, considered as a percentage, is less than the probable error of capacity.

The residuals within a series are, as a rule, relatively great for the lower slopes; but this is true only when the residuals are considered as fractional parts of the capacity values to which they pertain. If the residuals be measured in the unit of capacity, then they are relatively great for the higher slopes. The gradation of precision in relation to slope was discussed with some care, and an elaborate system of weights was prepared for the adjusted values of capacity. While these weights were of service in connection with various combinations afterward made, it has not seemed necessary to include them in the printed tables. It is to be understood, however, that the probable errors associated with each series in Table 12 apply to the values of capacity as a group, the probable errors (in per cent) of the smaller values being relatively great and those of the larger values relatively small.

The residuals represent chiefly the errors of observation, but they include also errors introduced in the process of adjustment. To whatever extent the formula of adjustment misrepresents the actual relation between the variables, to whatever extent the assigned values of σ are inaccurate, and to whatever extent the representative lines of the plot are misplaced, factors of error are introduced which tend to increase the residuals. The estimates of probable error of observations are therefore larger than they would be if the methods of adjustment were perfect.

The observations giving large percentage residuals were, as a class, treated in the graphic adjustments as of relatively low weight; but in the computations of probable error they were treated as of equal weight. The estimates of probable error, and their average values, are larger than they would be if computed with regard to weights. The influence of this factor can not be definitely evaluated, but ratios brought out in the discussion of residuals for the assignment of weights indicate that it has some importance. It is thought that the average probable error of the adjusted values may be as low as ± 2.0 per cent, and that of the observations as low as ± 7.0 per cent.

On the other hand, the computations take account only of the discrepancies revealed by comparing observations of the same series and do not cover such discrepancies as exist between one series and another. The estimates of probable error are smaller than they would be if both classes of discrepancies were included. This matter receives further consideration in Chapters V and VI.

DUTY.

The duty of water traction, as defined by the units adopted for this paper, is the capacity in grams per second for each cubic foot per second of discharge, and its formula is $U=\frac{C}{Q}$. The duty corresponding to each adjusted capacity has been computed, and the values appear in Table 12.

It is sometimes desirable to treat duty as the ratio which the mass of the load, or capacity, bears to the mass of the carrier. To obtain the value of duty as a ratio of masses the corresponding value of U should be divided by 28,350, the number of grams in a cubic foot of water.

It is sometimes desirable to treat duty as a volume of débris, including voids—that is, as the space occupied by débris as a deposit, either before or after transportation. In such cases the American engineer commonly uses the cubic yard as a unit of volume and 24 hours as a unit of time. To obtain the approximate duty in cubic yards of débris per 24 hours, divide the duty in gm./sec. by 14. The exact value of the appropriate divisor depends on the specific gravity of the débris particles and on the percentage of voids in the débris aggregate.

EFFICIENCY.

The measure of efficiency, as defined on page 36, is $E=\frac{C}{QS}$. The computed efficiencies are tabulated in Table 12, along with the adjusted capacities and duties.

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency.*

[Gr., grade of débris; w, width of experiment trough in feet; Q, discharge in cubic feet per second; S, slope of channel bed, in per cent; C, capacity for stream traction, in grams per second; U, duty of water for traction, $=\frac{C}{Q}$; E, efficiency of current, $=\frac{C}{QS}$.]

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	w	Q	S	C	U	E	Adjusted data.	Observation.
(A)	0.66	0.093	1.2	23.0	247	206		
			1.4	32.3	347	248		
			1.6	42.6	458	286		
			1.8	53.5	576	320		
			2.0	65.5	704	352		
			2.2	78.0	840	382		
(A)	.66	.182	.7	18.7	103	147		
			.8	25.0	137	171		
			.9	32.0	176	196		
			1.0	39.5	217	217		
			1.2	56.3	309	257		
			1.4	75.0	412	294		
			1.6	96.0	528	330		
			1.8	119	654	363		
(A)	.66	.545	.5	49.8	91.4	183		
			.6	66.0	121	202		
			.7	83.0	152	217		
			.8	101	185	231		
			.9	120	220	244		
			1.0	140	257	257		
			1.2	184	358	282		
(A)	1.00	.182	.5	8.2	45.1	90.2		
			.6	12.5	68.6	114		
			.7	17.6	95.7	137		
			.8	23.5	129	161		
			.9	30.0	165	183		
			1.0	37.5	206	206		
			1.2	54.5	300	250		
			1.4	74.0	407	291		
			1.6	97.0	534	334		
			1.8	122	670	372		
			2.0	149	819	410		
			2.2	179	984	447		
(A)	1.00	.363	.4	17.0	46.8	117		
			.5	26.9	74.1	148		
			.6	38.2	105	175		
			.7	51.8	143	204		
			.8	66	182	228		
			.9	83	229	254		
			1.0	100	276	276		
			1.2	140	386	322		
			1.4	185	510	364		
			1.6	235	648	405		
			1.8	290	799	443		
(A)	1.00	.734	.4	42.5	58.0	145		
			.5	65	88.6	177		
			.6	91	124	207		
			.7	121	165	236		
			.8	154	210	262		
			.9	191	260	289		
			1.0	231	315	315		
			1.2	320	436	363		
(A)	1.32	.182	.5	7.0	38.5	77	1.6	5.2
			.6	11.2	61.6	103		
			.7	16.3	89.6	127		
			.8	22.3	122	152		
			.9	29.1	160	178		
			1.0	36.8	202	202		
			1.2	54.3	298	248		
			1.4	74.8	411	294		
			1.6	98	538	336		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(A)	1.32	0.182	1.8	124	681	378	1.6	5.2
			2.0	152	835	418		
			2.2	183	1,010	459		
(A)	1.32	.363	.2	3.6	9.9	49.6	2.3	7.6
			.3	10.5	28.9	96.3		
			.4	19.5	53.7	134		
			.5	30.2	83.2	166		
			.6	42.6	117	195		
			.7	56.0	154	220		
			.8	71	196	245		
			.9	87	240	267		
			1.0	104	287	287		
			1.2	142	391	326		
			1.4	183	504	360		
			1.6	227	626	391		
			1.8	276	760	422		
			2.0	328	904	452		
(A)	1.32	.734	.3	27.3	37.2	124	2.3	7.2
			.4	48.2	65.6	164		
			.5	73.5	100	200		
			.6	103	140	233		
			.7	135	184	263		
			.8	170	232	290		
			.9	219	285	317		
			1.0	250	341	341		
			1.2	344	469	391		
			1.4	445	606	433		
(A)	1.96	.363	.4	12.6	34.7	86.8	2.6	9.4
			.5	22.4	61.7	123		
			.6	34.0	93.7	156		
			.7	47.3	130	186		
			.8	72.2	171	214		
			.9	79	218	242		
			1.9	96	264	265		
			1.2	136	375	312		
			1.4	178	490	350		
			1.6	227	625	390		
			1.8	278	766	425		
			2.0	335	923	462		
(A)	1.96	.734	.2	11.2	15.3	76.5	3.4	10.9
			.3	29.5	40.2	134		
			.4	52	70.8	177		
			.5	78	106	212		
			.6	105	143	238		
			.7	136	185	264		
			.8	169	230	288		
			.9	204	278	309		
			1.0	240	327	327		
			1.2	320	436	363		
(A)	1.96	1.119	.4	100	89.4	224	3.9	12.3
			.5	138	123	246		
			.6	180	161	268		
			.7	222	198	283		
			.8	266	238	298		
			.9	311	278	309		
			1.0	359	321	321		
(B)	.23	.093	.8	2.8	30.1	37.5	1.2	2.6
			.9	5.3	57.0	63.3		
			1.0	7.8	83.9	83.9		
			1.2	12.5	134	112		
			1.4	17.1	184	131		
			1.6	21.6	232	145		
			1.8	26.1	281	156		
			2.0	30.5	328	164		
			2.2	39.4	375	170		
(B)	.23	.182	.7	3.3	18.1	25.9		
			.8	6.7	36.8	46.0		
			.9	10.0	55.0	61.1		
			1.0	13.3	73.1	73.1		
			1.2	20.0	110	91.5		
			1.4	26.6	146	104		
			1.6	33.2	182	114		
			1.8	39.9	219	122		
(B)	.44	.093	.7	4.6	49.5		2.0	4.9
			.8	7.4	79.6			
			.9	10.2	110			
			1.0	13.2	142			
			1.2	19.5	210			
			1.4	26.2	282			
			1.6	33.0	355			
			1.8	40.0	430			
			2.0	47.2	508			
			2.2	54.7	588			
			2.4	62.0	667			
			2.6	70	753			

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(B)	0.44	0.182	0.5	7.7	42.3	84.6	0.9	2.1
			.6	11.0	60.5	101		
			.7	14.9	81.9	117		
			.8	19.0	104	130		
			.9	23.7	130	144		
			1.0	28.7	158	158		
			1.2	40.0	220	193		
			1.4	52.7	290	207		
			1.6	66	363	229		
			1.8	81	445	247		
(B)	.66	.093	.7	4.3	46.2	66.0	2.9	9.7
			.8	6.2	66.7	83.4		
			.9	8.4	90.3	100		
			1.0	10.8	116	116		
			1.2	16.3	175	146		
			1.4	22.9	247	176		
			1.6	30.0	323	202		
			1.8	38.0	409	227		
			2.0	46.5	500	250		
			2.2	56.0	602	274		
			2.4	66.0	710	296		
			2.6	76.5	823	316		
			2.8	88.0	946	338		
			3.0	100	1,080	360		
(B)	.66	.182	.3	2.8	15.4	51.3	1.8	9.7
			.4	5.5	30.2	75.5		
			.5	8.9	48.9	97.9		
			.6	12.8	70.4	117		
			.7	17.3	95.1	136		
			.8	22.2	122	152		
			.9	27.6	152	169		
			1.0	33.5	184	184		
			1.2	46.7	257	214		
			1.4	61.4	338	241		
			1.6	78	429	268		
			1.8	96	528	293		
			2.0	113	621	310		
			2.2	134	736	335		
			2.4	156	858	357		
			2.6	179	984	378		
			2.8	202	1,110	397		
			3.0	228	1,250	413		
			3.2	255	1,400	438		
			3.4	281	1,540	453		
			3.6	310	1,700	472		
			3.8	340	1,870	492		
			4.0	371	2,040	510		
			4.2	402	2,210	526		
(B)	.66	.363	.5	26.0	71.6	143	1.4	5.4
			.6	35.2	97	162		
			.7	45.6	126	180		
			.8	56.9	157	196		
			.9	68.3	188	209		
			1.0	81	223	223		
			1.2	107	295	246		
			1.4	137	378	270		
			1.6	168	463	289		
			1.8	200	551	306		
			2.0	234	644	322		
(B)	.66	.545	.2	7.9	14.5	72.5	3.6	13.0
			.3	17.4	31.9	106		
			.4	28.7	52.6	132		
			.5	41.3	75.8	152		
			.6	55.0	101	168		
			.7	70	128	183		
			.8	86	158	198		
			.9	102	187	208		
			1.0	120	220	220		
			1.2	158	290	242		
			1.4	198	363	259		
			1.6	240	440	275		
			1.8	284	521	289		
			2.0	330	606	303		
(B)	1.00	.182	.5	7.3	40.1	80.2	1.1	3.4
			.6	10.8	59.4	99.0		
			.7	15.0	82.4	118		
			.8	19.5	107	134		
			.9	24.6	135	150		
			1.0	30.1	165	165		
			1.2	42.8	235	179		
			1.4	57	313	223		
			1.6	73	401	259		
			1.8	90	495	275		
			2.0	108	594	297		
			2.2	129	709	322		
			2.4	150	824	343		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(B)	1.00	0.182	2.6	173	954	366	1.1	3.4
			2.8	197	1,080	386		
			3.0	224	1,230	410		
(B)	1.00	.363	.2	3.7	10.2	51.0	3.6	16.5
			.3	9.4	25.9	86.3		
			.4	16.6	45.8	114		
			.5	25.4	70	140		
			.6	35.3	97.2	162		
			.7	46.2	127	181		
			.8	58.5	161	201		
			.9	71	196	218		
			1.0	85	234	234		
			1.2	116	320	267		
			1.4	149	411	293		
			1.6	186	512	320		
			1.8	225	620	344		
			2.0	266	732	366		
			2.2	310	854	388		
			2.4	356	981	419		
(B)	1.00	.545	.8	99	182	228	1.9	6.0
			.9	120	220	244		
			1.0	143	262	262		
			1.2	193	354	295		
			1.4	247	453	323		
			1.6	305	560	350		
			1.8	368	675	375		
			2.0	435	798	399		
			2.2	505	927	421		
(B)	1.00	.734	.2	11.0	15.0	75	2.8	11.4
			.3	25.0	34.1	114		
			.4	42.8	58.3	146		
			.5	63	86	172		
			.6	86	117	195		
			.7	110	150	214		
			.8	138	188	235		
			.9	168	229	254		
			1.0	199	271	271		
			1.2	268	365	304		
			1.4	344	469	335		
			1.6	422	575	359		
			1.8	507	691	384		
(B)	1.32	.182	.4	3.7	20.3	50.8	1.6	5.4
			.5	6.6	36.3	72.6		
			.6	10.1	55.5	92.5		
			.7	14.1	77.5	111		
			.8	18.7	103	129		
			.9	23.8	131	146		
			1.0	29.3	161	161		
			1.2	42.0	231	176		
			1.4	55.8	307	219		
			1.6	71	390	244		
			1.8	88	484	269		
			2.0	105	577	288		
			2.2	124	682	310		
			2.4	144	791	330		
			2.6	166	912	351		
(B)	1.32	.363	.3	6.9	....	....	2.4	10.9
			.4	13.6	37.5	93.8		
			.5	21.8	60.1	120		
			.6	31.2	86.0	143		
			.7	41.9	115	164		
			.8	53.1	146	182		
			.9	65.5	180	200		
			1.0	79	218	218		
			1.2	109	300	250		
			1.4	141	389	278		
			1.6	177	488	305		
			1.8	215	592	329		
			2.0	255	702	351		
			2.2	297	818	372		
(B)	1.32	.545	.5	39.8	73	146	4.1	18.3
			.6	56	103	172		
			.7	75	138	197		
			.8	95	174	218		
			.9	116	213	237		
			1.0	140	257	257		
			1.2	191	351	292		
			1.4	248	455	325		
			1.6	310	569	355		
			1.8	375	688	382		
(B)	1.32	.734	.3	21.7	29.6	98.7	2.1	9.7
			.4	39.0	53.1	133		
			.5	59.5	81.0	162		
			.6	83	113	188		
			.7	110	150	214		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(B)	1.32	0.734	0.8	139	189	236	2.1	9.7
			.9	170	232	258		
			1.0	204	278	278		
			1.2	279	380	317		
			1.4	361	492	351		
			1.6	450	613	383		
			1.8	545	742	412		
(B)	1.96	.363	.4	7.7	21.2	50.3	3.9	15.5
			.5	14.3	39.4	76.8		
			.6	22.4	61.7	103		
			.7	32.0	88.2	126		
			.8	43.0	118	148		
			.9	54.8	151	168		
			1.0	67.6	186	186		
			1.2	97	267	222		
			1.4	130	358	256		
			1.6	167	460	287		
			1.8	208	573	318		
			2.0	253	697	348		
			2.2	300	826	375		
(B)	1.96	.545	.5	29.8	54.7	109	2.9	11.3
			.6	44.2	81.1	135		
			.7	60.5	111	159		
			.8	79	145	181		
			.9	98	180	200		
			1.0	120	220	220		
			1.2	168	308	257		
			1.4	221	406	290		
			1.6	282	517	323		
			1.8	345	633	352		
(B)	1.96	.734	.3	18.6	25.3	84.3	4.1	17.1
			.4	35.8	48.8	122		
			.5	56.0	76.3	155		
			.6	78.5	107	178		
			.7	103	140	200		
			.8	130	177	221		
			.9	158	215	239		
			1.0	190	259	259		
			1.2	257	350	292		
			1.4	330	450	322		
			1.6	407	554	346		
			1.8	488	665	369		
			2.0	575	783	392		
(B)	1.96	1.119	.2	13.4	12.0	60	5.4	24.1
			.3	36.1	32.3	108		
			.4	65.0	58.1	145		
			.5	98	87.5	175		
			.6	135	121	202		
			.7	175	156	223		
			.8	218	195	241		
			.9	263	235	261		
			1.0	313	280	280		
			1.2	417	373	311		
(C)	.44	.093	.6	2.3	24.7	41.2	3.3	10.1
			.7	4.1	44.1	63.0		
			.8	6.1	65.6	82.0		
			.9	8.4	90.4	100		
			1.0	10.9	117	117		
			1.2	16.4	176	147		
			1.4	22.6	243	174		
			1.6	29.4	316	197		
			1.8	36.6	394	219		
			2.0	44.2	475	238		
			2.2	52.3	562	255		
(C)	.44	.182	.6	8.7	47.8	79.7	2.3	6.1
			.7	12.1	66.5	95		
			.8	15.9	87.4	109		
			.9	20.1	110	122		
			1.0	24.7	136	136		
			1.2	34.3	188	157		
			1.4	45.3	249	178		
			1.6	57.2	314	196		
			1.8	70	385	214		
			2.0	84	462	231		
			2.2	98	538	244		
(C)	.66	.093	.5	3.3	35.5	71.0	1.7	6.8
			.6	5.0	53.8	89.7		
			.7	6.9	74.2	106		
			.8	9.0	96.8	121		
			.9	11.3	121	134		
			1.0	13.8	148	148		
			1.2	19.4	209	174		
			1.4	25.7	276	197		
			1.6	32.4	348	217		
			1.8	39.7	427	237		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(C)	0.66	0.093	2.0	47.5	511	256	1.7	6.8
			2.2	56	602	274		
			2.4	65	699	291		
			2.6	74	796	306		
			2.8	84	904	323		
			3	95	1,020	340		
			3.2	105	1,130	353		
			3.4	117	1,260	371		
			3.6	128	1,380	384		
			3.8	140	1,510	398		
			4	153	1,650	412		
			4.2	166	1,780	424		
(C)	.66	.182	.3	3	16.5	55	1.2	5.8
			.4	5.7	31.3	75.8		
			.5	9	49.5	99		
			.6	12.9	70.9	118		
			.7	17.1	94	134		
			.8	21.8	120	150		
			.9	26.7	147	163		
			1	32.1	176	176		
			1.2	44.1	242	202		
			1.4	57	313	223		
			1.6	71	390	244		
			1.8	87	478	265		
			2	102	560	280		
			2.2	120	659	299		
			2.4	138	758	316		
			2.6	157	863	332		
			2.8	177	892	347		
			3	198	1,090	363		
			3.2	219	1,200	375		
			3.4	241	1,320	388		
			3.6	266	1,460	406		
			3.8	289	1,590	419		
(C)	.66	.363	.4	16.5	45.5	114	1.8	9.0
			.5	23.3	64.2	128		
			.6	31.9	87.9	146		
			.7	41.2	113	161		
			.8	51.1	141	176		
			.9	62	171	190		
			1	73	201	201		
			1.2	98	270	225		
			1.4	124	342	244		
			1.6	152	419	262		
			1.8	183	514	280		
			2	215	597	296		
			2.2	248	683	310		
			2.4	283	780	325		
			2.6	320	882	339		
(C)	.66	.545	.2	6.6	12.1	60.5	2.1	10.1
			.3	14.8	27.2	90.7		
			.4	24.8	45.5	114		
			.5	36.5	67.0	134		
			.6	49.2	90.2	150		
			.7	63.3	116	166		
			.8	78.5	144	180		
			.9	95	174	193		
			1	112	205	205		
			1.2	150	275	229		
			1.4	190	349	249		
			1.6	233	427	267		
			1.8	280	514	285		
			2	328	602	301		
			2.2	380	697	317		
(C)	.66	.734	.5	51	69.5	139	3.8	8.5
			.6	68	93	154		
			.7	87	118	169		
			.8	107	146	182		
			.9	129	176	196		
			1	152	207	207		
(C)	1.00	.182	.4	4.5	24.7	61.8	1	4.4
			.5	7.6	41.8	83.6		
			.6	11.4	62.6	104		
			.7	15.6	85.8	123		
			.8	20.4	112	140		
			.9	25.6	141	157		
			1	31.2	171	171		
			1.2	44	242	202		
			1.4	57.7	317	226		
			1.6	72	396	247		
			1.8	90	495	275		
			2	106	582	291		
			2.2	125	686	312		
			2.4	145	797	332		
			2.6	166	912	351		
			2.8	189	1,040	372		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(C)	1.00	0.363	0.2	3.3	9.1	45.5	1.2	6.2
			.3	9.6	26.4	88		
			.4	17.5	48.2	120		
			.5	26.6	73.3	147		
			.6	36.6	101	168		
			.7	47.5	131	187		
			.8	59	163	204		
			.9	72	198	220		
			1	85	234	234		
			1.2	113	311	259		
			1.4	143	394	281		
			1.6	175	482	301		
			1.8	210	579	322		
			2	245	675	338		
			2.2	281	774	352		
			2.4	320	882	367		
			2.6	360	992	381		
			2.8	401	1,100	393		
(C)	1.00	.545	.5	48.2	88.4	177	1.1	5.5
			.6	64.9	119	198		
			.7	82	150	214		
			.8	100	183	229		
			.9	119	218	242		
			1.0	140	257	257		
			1.2	183	336	280		
			1.4	228	418	299		
			1.6	275	504	315		
			1.8	326	598	332		
			2.0	377	692	346		
			2.2	430	789	359		
(C)	1.00	.734	.2	12.5	17.0	85	7.8	37.7
			.3	27.3	37.2	124		
			.4	44.2	60.2	150		
			.5	63.2	86.2	172		
			.6	84	114	190		
			.7	106	144	206		
			.8	130	177	221		
			.9	155	211	234		
			1.0	180	245	245		
			1.2	235	320	267		
			1.4	294	401	286		
			1.6	352	480	300		
			1.8	416	567	315		
			2.0	483	658	329		
			2.2	550	750	341		
(C)	1.00	1.119	.5	96	85.8	172	2.3	5.1
			.6	128	114	190		
			.7	162	145	207		
			.8	198	177	221		
			.9	236	211	234		
			1.0	276	247	247		
			1.2	361	323	269		
			1 4	451	403	288		
(C)	1.32	.182	.6	5.6	30.8	51.3	2.4	8.4
			.7	8.7	47.8	68.3		
			.8	12.3	67.6	84.5		
			.9	16.6	91.2	101		
			1.0	21.4	118	118		
			1.2	32.7	180	150		
			1.4	46.5	256	183		
			1.6	62	341	213		
			1.8	80	440	244		
			2.0	99	544	272		
			2.2	120	659	299		
			2.4	144	791	329		
(C)	1.32	.363	.3	6.4	17.6	58.7	1.2	5.5
			.4	13.6	37.5	93.8		
			.5	22.1	60.9	122		
			.6	31.9	87.9	146		
			.7	42.2	116	166		
			.8	53.8	148	185		
			.9	66	182	202		
			1.0	79	218	218		
			1.2	105	289	241		
			1.4	135	372	266		
			1.6	166	458	286		
			1.8	199	548	304		
			2.0	233	642	321		
			2.2	270	744	338		
			2.4	307	846	352		
(C)	1.32	.545	.6	56.3	103	172	1.8	8.5
			.7	73.3	134	191		
			.8	91	167	209		
			.9	109	200	222		
			1.0	129	237	237		
			1.2	171	314	262		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(C)	1.32	0.345	1.4	215	395	282	1.8	8.3
			1.6	263	483	302		
			1.8	312	573	318		
			2.0	362	664	332		
			2.2	415	762	346		
			2.4	470	862	359		
(C)	1.32	.734	.2	9.3	12.7	63.5	2.2	9.0
			.3	25.0	34.1	114		
			.4	43.2	58.9	147		
			.5	63.8	86.9	174		
			.6	86	117	195		
			.7	109	148	211		
			.8	133	181	226		
			.9	160	218	242		
			1.0	187	255	255		
			1.2	243	331	276		
			1.4	304	414	296		
			1.6	367	500	312		
			1.8	432	589	327		
			2.0	500	681	340		
			2.2	570	777	353		
(C)	1.96	.363	.5	10.5	28.9	57.8	3.5	15.5
			.6	17.8	49.0	81.7		
			.7	26.5	73.0	104		
			.8	36.6	101	126		
			.9	47.9	132	147		
			1.0	60.0	165	165		
			1.2	88	242	202		
			1.4	119	328	234		
			1.6	154	424	265		
			1.8	193	532	295		
			2.0	235	648	324		
			2.2	280	771	350		
(C)	1.96	.545	.5	25.7	47.2	94.4	2.0	7.0
			.6	39.3	72.1	120		
			.7	55.0	101	144		
			.8	72	132	165		
			.9	91	167	186		
			1.0	111	204	204		
			1.2	156	286	238		
			1.4	205	376	268		
			1.6	259	475	297		
			1.8	316	580	322		
			2.0	375	688	344		
			2.2	440	808	367		
(C)	1.96	.734	.4	34.0	46.3	116	2.8	13.2
			.5	55	74.9	150		
			.6	78	106	177		
			.7	103	140	200		
			.8	131	178	222		
			.9	160	218	242		
			1.0	190	259	259		
			1.2	255	347	289		
			1.4	323	440	314		
			1.6	394	537	336		
(C)	1.96	1.119	.5	88	78.7	157	2.2	8.9
			.6	130	116	193		
			.7	177	158	226		
			.8	228	204	255		
			.9	283	253	281		
			1.0	343	307	307		
			1.2	480	429	358		
(D)	.66	.093	.6	2.7	29.0	48.4	2.6	8.1
			.7	3.9	41.9	59.9		
			.8	5.4	58.1	72.5		
			.9	7.1	76.4	84.9		
			1.0	9.1	97.9	97.9		
			1.2	13.5	145	121		
			1.4	18.6	200	143		
			1.6	24.7	266	166		
			1.8	31.4	338	188		
			2.0	38.5	414	207		
			2.2	46.3	498	226		
			2.4	55.2	594	248		
			2.6	64.3	692	266		
(D)	.66	.182	.4	4.7	25.8	64.5	4.2	15.2
			.5	7.7	42.3	84.6		
			.6	11.3	62.1	104		
			.7	15.3	84.1	120		
			.8	19.7	108	135		
			.9	24.6	135	150		
			1.0	29.8	164	164		
			1.2	41.0	225	188		
			1.4	53.4	293	209		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(D)	0.66	0.182	1.6	67	368	230	4.2	15.2
			1.8	82	450	250		
			2.0	98	538	269		
			2.2	114	626	284		
			2.4	132	726	302		
			2.6	150	824	317		
(D)	.66	.545	.2	4.7	8.5	42.5	3.4	15.3
			.3	11.7	21.5	71.6		
			.4	20.7	38.0	95		
			.5	31.2	57.2	114		
			.6	43.1	79.1	132		
			.7	56.2	103	147		
			.8	70.2	129	161		
			.9	86	158	176		
			1.0	101	185	185		
			1.2	137	251	209		
			1.4	175	321	229		
			1.6	217	398	249		
			1.8	261	479	266		
			2.0	308	568	284		
			2.2	357	655	298		
(D)	1.00	.182	.7	11.3	62.1	88.7	2.5	9.1
			.8	15.1	83	104		
			.9	19.2	106	118		
			1.0	24.0	132	132		
			1.2	34.5	190	158		
			1.4	46.4	255	182		
			1.6	59.8	329	206		
			1.8	74	407	226		
			2.0	90	495	248		
			2.2	106	582	264		
			2.4	124	682	284		
			2.6	144	792	304		
			2.8	164	902	324		
(D)	1.00	.363	.3	6.9	19.0	63	3.3	13.2
			.4	13.2	36.4	91.0		
			.5	20.9	57.6	115		
			.6	29.6	81.6	136		
			.7	39.0	107	153		
			.8	49.6	137	171		
			.9	61	168	187		
			1.0	73	201	201		
			1.2	99	273	228		
			1.4	127	350	250		
			1.6	158	435	272		
			1.8	190	524	291		
			2.0	225	620	310		
			2.2	261	819	327		
			2.4	299	824	343		
(D)	1.00	.545	.5	27.6	50.6	101	1.2	3.9
			.6	40.2	73.8	123		
			.7	55.0	101	144		
			.8	71	130	162		
			.9	89	163	181		
			1.0	108	198	198		
			1.2	152	279	232		
			1.4	201	369	264		
			1.6	257	472	295		
			1.8	317	582	323		
			2.0	382	700	350		
(D)	1.00	.734	.2	6.9	9.4	47.0	2.5	11.5
			.3	17.5	23.8	79.3		
			.4	31.0	42.2	106		
			.5	47.0	64.0	128		
			.6	65.0	88.6	148		
			.7	85	116	166		
			.8	106	144	180		
			.9	129	176	196		
			1.0	153	208	208		
			1.2	208	282	236		
			1.4	268	365	361		
			1.6	332	452	283		
			1.8	388	542	302		
(D)	1.32	.363	.5	11.9	32.8	65.6	7.0	24.3
			.6	18.8	50.9	84.8		
			.7	27.0	74.4	106		
			.8	36.6	101	139		
			.9	47.2	130	144		
			1.0	59.2	163	163		
			1.2	87	240	200		
			1.4	118	325	232		
			1.6	154	424	265		
			1.8	194	534	297		
			2.0	237	653	326		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(D)	1.32	0.734	0.3	10.2	13.9	46.3	5.3	21.4
			.4	20.9	28.5	71.2		
			.5	34.0	46.3	92.6		
			.6	49.2	67.0	112		
			.7	67	91.3	130		
			.8	87	118	148		
			.9	108	147	163		
			1.0	131	178	178		
			1.2	185	252	210		
(E)	.66	.363	1.0	24.8	68.3	68.3	0.9	2.1
			1.2	36.2	99.7	83.1		
			1.4	49.4	136	97.2		
(E)	.66	.734	1.0	40.0	54.5	54.5		
			1.2	59.0	80.4	67.0		
			1.4	82	112	80		
			1.6	108	147	92		
(E)	1.00	.182	.4	2.4	13.2	33.0	1.9	5.0
			.5	3.8	20.9	41.8		
			.6	5.5	30.4	50.7		
			.7	7.4	40.6	58.0		
			.8	9.6	52.7	65.0		
			.9	12.0	66.0	73.3		
			1.0	14.8	81.4	81.4		
			1.2	20.7	114	95		
			1.4	27.6	152	108		
			1.6	35.3	194	121		
			1.8	43.5	239	133		
			2.0	52.5	288	144		
			2.2	62.4	343	156		
			2.4	73	401	167		
			2.6	84	461	177		
			2.8	96	527	188		
(E)	1.00	.363	.2	1.3	3.6	18.0	5.1	14.6
			.3	3.3	9.1	30.3		
			.4	6.0	16.5	41.2		
			.5	9.2	25.3	50.6		
			.6	13.1	36.1	60.1		
			.7	17.5	48.2	68.9		
			.8	22.5	62.0	77.5		
			.9	28.0	77.2	85.8		
			1.0	33.8	93.1	93.1		
			1.2	47.2	130	108		
			1.4	62.0	171	122		
			1.6	78.5	216	135		
			1.8	96	264	147		
			2.0	115	317	158		
			2.2	137	378	172		
			2.4	159	438	182		
(E)	1.00	.734	.2	4.2	5.7	28.5	5.0	19.5
			.3	9.1	12.4	41.3		
			.4	15.2	20.7	51.8		
			.5	22.4	30.5	61.0		
			.6	30.8	42.0	70.0		
			.7	40.0	54.5	77.9		
			.8	49.8	67.8	84.8		
			.9	60.5	72.4	91.6		
			1.0	72.0	98.1	98.1		
			1.2	98	134	112		
			1.4	126	172	123		
			1.6	156	213	133		
			1.8	188	256	142		
			2.0	222	302	151		
(E)	1.00	1.119	.5	44	39.3	76.6	3.2	6.4
			.6	60	54	89		
			.7	77	69	98		
			.8	97	87	108		
			.9	117	105	117		
			1.0	138	123	123		
			1.2	188	168	140		
			1.4	242	216	154		
(E)	1.32	.363	.6	15.3	42.2	70.3		
			.7	20.3	55.9	79.9		
			.8	25.4	70.0	87.5		
			.9	30.7	84.6	94.0		
			1.0	36.3	100	100		
			1.2	49.3	136	113		
			1.4	63.5	175	125		
			1.6	78.8	217	136		
			1.8	95	262	145		
			2.0	112	309	154		
(E)	1.32	.734	.5	24.7	33.7	67.4		
			.6	33.0	45.0	75.0		
			.7	42.0	57.2	81.7		
			.8	51.7	70.4	88.0		
			.9	62	84.5	93.9		
			1.0	73	99.5	99.5		
			1.2	97	132	110		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(E)	1.32	1.119	0.6	49.4	44.2	73.7	1.8	3.6
			.7	65.0	58.1	83.0		
			.8	82.5	73.7	92.1		
			.9	102	91.1	101		
			1.0	123	110	110		
			1.2	171	153	128		
(F)	.66	.182	1.2	9.3	51.0	42.5	.8	1.7
			1.4	13.7	75.3	53.8		
			1.6	18.9	104	65.0		
			1.8	24.7	136	75.6		
			2.0	31.4	172	86.0		
			2.2	38.7	213	96.8		
			2.4	46.8	257	107		
			2.6	55.7	306	118		
(F)	.66	.363	1.0	20.5	56.5	56.5	2.3	4.6
			1.2	29.9	82.4	68.7		
			1.4	41.0	113	80.8		
			1.6	53.2	147	91.9		
			1.8	67.0	185	103		
			2.0	81.5	225	112		
(F)	.66	.734	.8	31.6	43.0	47.8	.4	.8
			1.0	39.0	53.2	53.2		
			1.2	55.3	75.3	62.8		
			1.4	73.0	99.5	71.0		
			1.6	93.5	127	79.4		
			1.8	115	157	87.2		
(F)	1.00	.182	1.2	4.2	23.1	17.6		
			1.4	7.3	40.1	28.6		
			1.6	11.3	62.1	38.8		
			1.8	16.4	90.1	50.0		
			2.0	22.5	124	62.0		
			2.2	29.8	164	74.6		
			2.4	38.2	210	87.5		
			2.6	47.7	262	101		
(F)	1.00	.363	.8	7.4	20.4	25.5	2.0	4.8
			.9	10.5	28.9	32.1		
			1.0	14.0	38.6	38.6		
			1.2	22.8	62.8	52.3		
			1.4	33.4	92.0	65.7		
			1.6	46.0	127	79.4		
			1.8	60	165	91.7		
			2.0	76	209	104		
			2.2	94	259	118		
			2.4	112	309	129		
			2.6	133	367	141		
(F)	1.00	.734	.7	20.0	27.3	39.0	.5	1.0
			.8	27.3	37.2	46.5		
			.9	35.1	47.8	53.1		
			1.0	42.8	58.3	58.3		
			1.2	63	85.8	71.5		
			1.4	85	116	82.9		
			1.6	109	148	92.6		
			1.8	136	185	103		
(F)	1.00	1.119	.7	37.5	33.5	47.9	.9	1.8
			.8	51.0	45.6	57.0		
			.9	64.8	57.9	64.3		
			1.0	79.5	71.0	71.0		
			1.2	112	100	83.3		
			1.4	150	134	95.8		
			1.6	191	171	107		
			1.8	236	211	117		
(F)	1.32	.363	1.2	21.4	59.0	48.2	1.2	2.4
			1.4	31.0	85.4	61.0		
			1.6	41.5	114	71.3		
			1.8	53.0	146	81.1		
			2.0	66.0	182	91.0		
			2.2	80	220	100		
(F)	1.32	.734	.8	22.9	29.6	37.0	3.2	7.2
			.9	31.0	40.1	44.6		
			1.0	40.2	52.0	52.0		
			1.2	61.6	79.7	66.4		
			1.4	87.0	119	85.0		
			1.6	115	149	93.2		
(F)	1.32	1.119	.8	43.3	38.7	48.4	3.2	7.2
			.9	56.0	50.0	91.1		
			1.0	70.0	62.6	62.6		
			1.2	101	90.2	75.2		
			1.4	138	123	87.9		
			1.6	178	159	99.4		
			1.8	221	197	109		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*.	*C*	*U*	*E*	Adjusted data.	Observation.
(G)	0.66	0.363	1.0	8.5	23.4	23.4	2.2	5.8
			1.2	14.8	40.8	34.0		
			1.4	23.0	63.4	45.3		
			1.6	32.7	90.1	56.3		
			1.8	43.7	120	66.7		
			2.0	56.0	154	77.0		
(G)	.66	.734	.7	10.3	13.3	19.0	1.5	4.3
			.8	16.0	20.7	25.9		
			.9	22.8	29.5	32.8		
			1.0	30.4	39.3	39.3		
			1.2	48.3	62.5	52.1		
			1.4	69	89.2	63.7		
			1.6	93	127	79.4		
			1.8	120	155	86.2		
			2.0	149	193	96.5		
			2.2	181	234	106		
			2.4	216	280	117		
			2.6	253	327	126		
(G)	.66	1.119	.6	11.9	10.6	17.7	1.2	3.6
			.7	19.0	17.0	24.3		
			.8	27.7	24.7	30.9		
			.9	37.6	33.6	37.3		
			1.0	49.0	43.8	43.8		
			1.2	75.0	67.0	55.8		
			1.4	107	95.6	68.3		
			1.6	143	128	80.0		
			1.8	182	163	90.6		
			2.0	227	203	102		
			2.2	276	247	112		
			2.4	325	290	121		
			2.6	382	341	131		
(G)	1.00	.363	1.2	9.7	26.7	22.2	1.9	5.0
			1.4	16.3	44.9	35.0		
			1.6	24.4	67.2	42.0		
			1.8	34.1	94.0	52.2		
			2.0	45.2	125	62.5		
			2.2	57	157	71.4		
			2.4	71	196	81.7		
			2.6	86	237	91.2		
			2.8	102	281	100		
			3.0	120	331	110		
(G)	1.00	.734	.7	8.0	10.9	15.6	1.9	5.5
			.8	13.3	18.1	22.6		
			.9	19.5	26.6	29.6		
			1.0	26.9	36.6	36.6		
			1.2	44	60.0	50.0		
			1.4	65	88.6	63.3		
			1.6	89	121	75.6		
			1.8	115	157	87.8		
			2.0	145	198	99.0		
			2.2	177	241	110		
			2.4	212	289	120		
			2.6	249	339	130		
(G)	1.00	1.119	.6	13.5	12.1	20.1	4.3	13.6
			.7	22.7	20.3	29.0		
			.8	33.6	30.0	37.5		
			.9	46.0	41.1	45.6		
			1.0	61.0	54.4	54.4		
			1.2	92	82.2	68.5		
			1.4	128	114	81.5		
			1.6	171	153	95.6		
			1.8	217	194	108		
			2.0	268	239	120		
			2.2	321	287	130		
			2.4	380	340	142		
(G)	1.32	.363	1.8	18.1	49.5	27.7	2.4	4.8
			2.0	28.3	78.0	39.0		
			2.2	39.9	101	45.9		
			2.4	53.8	148	61.7		
			2.6	69.8	192	73.9		
			2.8	88.0	242	86.5		
			3.0	109	300	100		
			3.2	134	369	115		
(G)	1.32	.734	.8	7.9	10.8	13.5	2.2	5.8
			.9	12.8	17.4	19.3		
			1.0	18.9	25.8	25.8		
			1.2	33.8	46.1	38.4		
			1.4	52.0	70.8	50.6		
			1.6	74.0	101	63.2		
			1.8	98.0	134	75.4		
			2.0	125	170	85.0		
			2.2	155	211	95.9		
			2.4	187	255	106		
			2.6	222	303	117		
			2.8	260	354	126		

TABLE 12.—*Adjusted values of capacity, based on data of Table 4, with corresponding values of duty and efficiency*—Con.

Conditions.				Adjusted data.			Probable error (per cent) of—	
Gr.	*w*	*Q*	*S*	*C*	*U*	*E*	Adjusted data.	Observation.
(G)	1.32	1.119	0.7	17.9	16.0	22.9	3.1	8.7
			.8	28.0	25.0	31.2		
			.9	39.5	35.3	38.1		
			1.0	52.2	46.6	46.6		
			1.2	81	72.4	60.3		
			1.4	115	103	73.6		
			1.6	153	137	85.6		
			1.8	194	173	96.2		
			2.0	238	213	106		
			2.2	285	255	116		
			2.4	335	299	125		
			2.6	387	346	133		
(H)	.66	.363	1.2	3.1	8.5	7.1	1.0	3.5
			1.4	6.9	19.0	13.6		
			1.6	12.4	34.2	21.4		
			1.8	19.4	53.5	29.7		
			2.0	28.0	77.1	38.6		
			2.2	38.1	105	47.8		
			2.4	49.8	137	57.1		
			2.6	63	174	66.9		
			2.8	78	215	76.8		
(H)	.66	.734	.8	4.5	6.8	8.5	1.6	4.9
			.9	8.2	12.4	13.8		
			1.0	12.8	19.4	19.4		
			1.2	24.6	37.3	31.1		
			1.4	39.2	59.4	42.4		
			1.6	57.0	86.4	54.0		
			1.8	77.0	117	65.0		
			2.0	100	152	76.0		
			2.2	125	189	85.9		
			2.4	151	229	95.6		
			2.6	181	274	105		
			2.8	213	323	115		
			3.0	246	373	126		
			3.2	282	427	133		
(H)	.66	1.119	.7	6.0	5.4	7.7		11.2
			.8	11.1	9.9	12.4	3.2	
			.9	17.6	15.7	17.4		
			1.0	25.2	22.5	22.5		
			1.2	43.5	38.8	32.3		
			1.4	65.3	58.3	41.6		
			1.6	91.0	81.3	50.8		
			1.8	120	107	59.5		
			2.0	152	136	68.0		
			2.2	186	166	75.5		
			2.4	223	199	83.0		
			2.6	263	235	90.4		

OBSERVATIONS ON DEPTH.

MODE OF ADJUSTMENT.

To adjust the observations of depth it is necessary to deal with their relations to another variable, and either slope or capacity might be used. The selection of the particular variable for comparison was a matter of convenience only, because adjustment to either one would bring the depth values into orderly relation to the adjusted values of the other also; but the question of convenience was not unimportant. A fairly thorough preliminary study was therefore made, in which the depth measurements for many series were plotted in relation, severally, to measurements of capacity and measurements of slope.

In figure 27 the horizontal scale is that of slope, the vertical of depth. The round dots represent observations made with grade (C), width 1.32 feet, and discharge 0.734 ft.³/sec. Despite irregularities, the grouping suggests as

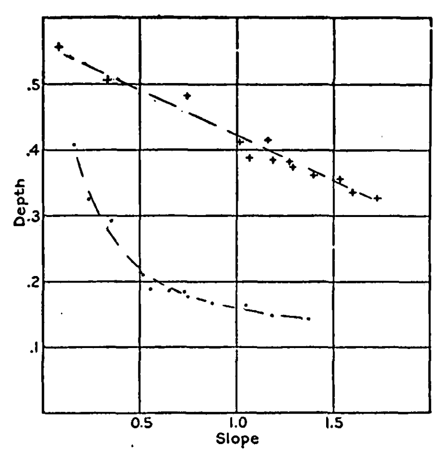


FIGURE 27.—Observations of depth of current, in relation to slope, plotted as dots. The crosses show logarithms of the same depths and slopes.

the representative line some such curve as that drawn. On taking account of the physical conditions, it is evident that as the slope is flattened, the current is slowed and the depth increased, and that zero slope gives infinite depth. The theoretic curve, therefore, has an asymptote in the vertical line corresponding to zero slope. Similarly the depth is reduced by increase in slope but remains finite for very high slopes. The theoretic curve has as asymptote a horizontal line corresponding, exactly or approximately, with the (horizontal) line of zero depth. These asymptotes relate the curve $d=f(S)$ to the hyperbola. In the same figure, but with use of a different scale, are a series of crosses which show the same observations as they appear when plotted on logarithmic section paper. They are the plot of observations on $\log d=f_{\text{I}} (\log S)$; and their arrangement suggests that the representative line may be straight.

Plots were made to show the relations of depth observations to associated capacity, and these also suggest the hyperbola and the straight line. If, however, the locus of $d=f_{\text{II}}(C)$ is a hyperbola it differs materially from that of $d=f(S)$, for as depth increases and current slackens, capacity becomes zero when current reaches the value of competence, and depth is not then infinite. So the line of zero capacity is not an asymptote to the curve.

It is to be observed also that the representative lines for $\log d=f_{\text{I}} (\log S)$ and $\log d=f_{\text{IV}} (\log C)$ can not both be straight, for if they were there could be derived from them a straight line representing $\log C=f_{\text{V}} (\log S)$, and it has already been found that that line is curved. It is, indeed, probable that neither of the logarithmic plots involving depth is straight; yet there are cogent practical reasons for assuming one or the other to be so. One reason is the very great convenience of the straight-line function, and another that the relatively small range of the depth data renders impracticable such a discussion of the curvature of logarithmic loci as was made in the case of capacity versus slope. Accordingly, the most orderly plots of $\log d=f_{\text{I}} (\log S)$ and $\log d=f_{\text{IV}} (\log C)$ were compared with special reference to curvature. For the function of $\log C$ the plots were found to indicate curvature in one direction only, while for the function of $\log S$ they indicated slight curvatures in both directions, with the straight line as an approximate mean. The function $\log d=f_{\text{I}} (\log S)$ was accordingly selected for the adjustment of the depth observations; and the representative line on the logarithmic plot was assumed to be straight. In accordance with that assumption, the adopted formula of interpolation was

$$d=\frac{b'}{S^{n_1}} \qquad (21)$$

with its logarithmic equivalent,

$$\log d=\log b'-n_1 \log S \qquad (22)$$

The coefficient b' is a depth, the depth corresponding to a slope of 1 per cent.

The data were all plotted on logarithmic section paper. The notation was made to distinguish depth measurements made at a single point by means of the gage (see p. 25) from those based on full profiles of water surface and bed of débris. The former were used exclusively in the drawing of the representative lines, but not because they were regarded as of higher authority. It was thought best not to combine data which in certain cases were known to be incongruous; and the gage observations covered the whole range of the work, while the profiles did not. The measurements by profile were used in criticising the measurements by gage, and they determined the acceptance or rejection of certain gage measurements. It was noted that in some series of observations the depth measurements by the two methods were in close accord, while in others there was a large systematic difference; and certain series were rejected because of such large differences.

The plots were made to distinguish also the observations associated with different modes of traction—the dune, smooth, and antidune modes. The observations with the smooth mode were assumed to be best, as a class; and these, together with the observations connected with the transitional phases of traction, were used to fix an initial point of each representative line. The direction of the line was then adjusted by eye estimate to make it representative of all the points of the particular series. In this adjustment consideration was given to the conditions affecting the measurements of both depths and slopes.

The lines were first drawn for those observational series which appeared from the plots to be most harmonious, and for these there was discovered a tendency toward parallelism, within each group, of lines associated with a particular sand grade and channel width but differing as to discharge. For numerous other groups, involving very irregular observational positions, such parallelism was assumed; and under that assumption the better series of a group were made to control the directions of the lines for the poorer. For three groups the directions were interpolated by use of the directions found in affiliated groups.

For each observational series the direction of the representative line gave the value of n_1 in equations (21) and (22), and the intersection of the line with the axis of log d gave the value of b'.

TABLE 13.—*Values of* n_1 *in* $d=\frac{b'}{S^{n_1}}$.

Grade of débris.	Value of n_1 for trough having width (in feet) of—					
	0.23	0.44	0.66	1.00	1.32	1.96
(A)	....	0.44	....	....	0.50	0.60
(B)	0.49	[.44]	0.40	0.34	.70	.55
(C)	....	.44	.56	.53	.48-.58	.48
(D)	....	....	.40	....	.40-.59	....
(E)	....	....	[.35]	.34	....	....
(F)	....	....	.30	.26	[.26]	....
(G)	....	....	.28	.30	.26	....
(H)	....	....	.34	....	....	....

In Table 13 the determinations of n_1 are assembled, being arranged vertically with respect to grade of débris and horizontally with respect to width of trough. Collectively they indicate that, for constant discharge, the depth varies inversely with the slope and less rapidly than the slope. Comparatively they indicate considerable range in the rate of variation. There is some suggestion of system in the distribution of values of n_1, and it is especially probable that to the coarser débris belong the smaller values, but irregularity is, on the whole, more in evidence than system. The irregularity is to be ascribed largely to the imperfection of the data, and imperfection is believed to inhere especially in the measurements of depths by means of the gage. (See p. 25.)

Table 14 contains series of adjusted or interpolated values of depth. These were computed for slopes with the constant interval of 0.2 per cent, the range of slope within each series being approximately the same as the range through which the depth observations were made. The range is usually less than the range of slopes for which capacities were computed, the difference being occasioned by the fact that satisfactory measurements of depth could not be made during vigorous antidune traction. The computations were graphic, the values of adjusted depth being read directly from the logarithmic plot.

The number of series represented in the table is 73, being one-fourth less than the number for which capacities and slopes were tabulated. For 18 series the depth measurements were not reduced, the chief reasons for rejections being either (1) that the number of observations was very small, (2) that the observations included none made in connection with the smooth phase of traction, and (3) that the measurements with gage differed systematically by a large amount from corresponding measurements by the method of profiles.

TABLE 14.—*Adjusted values of depth of current* (d), *with values of mean velocity* (V_m) *and form ratio* $\left(R=\frac{d}{w}\right)$.

[n_1, Exponent in adjusting formula (21); p. e., probable error, in per cent, of adjusted values.]

Grade..	(A)			(A)			(A)			(A)			(A)		
w......	1.32			1.32			1.96			1.96			1.96		
Q......	.363			.734			.363			.734			1.119		
n_1......	.50			.50			.60			.60			.60		
p. e....	1.5			1.8			0.7			1.7			3.4		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.2	0.218	1.25	0.165	....	....	....	....	....	....	0.255	1.47	0.130	0.232	3.35	0.118
.4	.154	1.78	.117	0.227	2.45	0.172	0.136	1.36	0.170	.167	2.24	.085	.182	4.25	.093
.6	.126	2.09	.096	.185	3.00	.141	.107	1.74	.155	.131	2.88	.067	.153	5.05	.078
.8	.109	2.53	.083	.160	3.45	.122	.090	2.05	.046	.110	3.41	.056	....	....	....
1.0	.098	2.83	.074	....	....	....	.079	2.35	.040	....	....	....	....	....	....
1.2	.090	3.10	.068	....	....	....	.071	2.62	.036	....	....	....	....	....	....
1.4	....	....	....	....	....	....	.064	2.87	.033	....	....	....	....	....	....

TABLE 14.—*Adjusted values of depth of current (d), with values of mean velocity (V_m) and form ratio* $\left(R=\frac{d}{w}\right)$—Con.

Grade	(B)			(B)			(B)			(B)			(B)		
w	.23			.23			.44			.44			.66		
Q	.093			.182			.093			.182			.093		
n_1	.49			.49			[.44]			[.44]			.55		
p. e.	3.3			...			5.5			2.6			2.3		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
.6	...	...	...	...	...	...	0.128	1.73	0.294	0.220	1.90	0.502	...	...	...
.8	0.200	2.02	0.880	0.327	2.43	1.43	.113	1.96	.259	.184	2.27	.421	...	...	...
1.0	.179	2.17	.780	.292	2.70	1.28	.102	2.08	.235	.163	2.56	.371	0.079	1.80	0.118
1.2	.163	2.49	.715	.266	2.96	1.17	.094	2.36	.216	.146	2.81	.336	.069	2.03	.105
1.4	.150	2.68	.660	.247	3.02	1.08	.088	2.53	.202	.135	3.04	.310	.063	2.25	.095
1.6	.141	2.86	.620	.231	3.42	1.01	.083	2.67	.190	...	...	...	.057	2.45	.087
1.8	.133	3.04	.583	.218	3.61	.96	.079	2.82	.182	...	...	...	.053	2.65	.081
2.0	...	...	...	...	...	...	.075	2.95	.173	...	...	...	.050	2.82	.076
2.2	...	...	...	...	...	...	.072	3.08	.167	...	...	...	.047	3.00	.072
2.4	...	...	...	...	...	...	.069	3.20	.160	...	...	...	.045	3.15	.068
2.6	...	...	...	...	...	...	.067	3.32	.154	...	...	...	...	...	...

Grade	(B)			(B)			(B)			(B)			(B)		
w	.66			.66			.66			1.00			1.00		
Q	.182			.363			.545			.182			.363		
n_1	.40			.40			.40			.34			.34		
p. e.	2.1			2.1			2.0			3.3			2.5		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.2	0.208	1.33	0.317	...	...	...	0.427	1.97	0.650	...	...	...	0.239	1.53	0.239
.4	.176	1.76	.240	0.246	2.22	0.373	.323	2.60	.490	...	...	...	.188	1.94	.188
.6	.133	2.08	.203	.210	2.63	.318	.274	3.04	.418	0.101	1.81	0.101	.164	2.24	.164
.8	.118	2.34	.181	.187	2.95	.283	.245	3.40	.370	.091	2.00	.091	.148	2.46	.148
1.0	.108	2.56	.165	.171	3.23	.259	.223	3.70	.340	.084	2.15	.084	.137	2.66	.137
1.2	.100	2.75	.153	.158	3.49	.240	.207	3.97	.315	.079	2.39	.079	.128	2.83	.128
1.4	.095	2.94	.144	.148	3.71	.226	.194	4.24	.296	.075	2.40	.075	.122	2.98	.122
1.6	.090	3.10	.137	...	...	...	...	...	...	.072	2.51	.072	.117	3.12	.117
1.8	.085	3.25	.130	...	...	...	...	...	...	.069	2.62	.069	.112	3.25	.112
2.0	.082	3.38	.125	...	...	...	...	...	...	.066	2.71	.066	.108	3.36	.108
2.2	.079	3.50	.120	...	...	...	...	...	...	.064	2.80	.064	.105	3.48	.105
2.4	...	...	...	...	...	...	...	...	...	.062	2.92	.062	...	...	...
2.6	...	...	...	...	...	...	...	...	...	.061	3.00	.061	...	...	...

Grade	(B)			(B)			(B)			(B)			(B)		
w	1.00			1.32			1.32			1.32			1.96		
Q	.734			.182			.545			.734			.363		
n_1	.34			.70			.70			.70			.55		
p. e.	1.4			2.5			2.4			2.6			3.3		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.2	0.347	2.10	0.347	0.200	0.68	0.153	...	...	...	...	...	...	...	...	...
.4	.275	2.68	.275	.124	1.12	.094	0.233	1.76	0.176	0.256	2.10	0.194	0.117	1.58	0.059
.6	.239	3.10	.239	.094	1.48	.071	.175	2.35	.133	.193	2.83	.146	.094	1.98	.047
.8	.217	3.41	.217	.077	1.82	.057	.143	2.88	.108	.158	3.48	.120	.080	2.33	.041
1.0	.200	3.67	.200	.066	2.13	.050	.123	3.38	.093	...	...	...	.071	2.65	.036
1.2	...	...	...	.058	2.42	.046	.108	3.82	.082	...	...	...	.064	2.93	.033
1.4	...	...	...	.052	2.70	.039	.097	4.27	.073	...	...	...	.059	3.20	.030
1.6	...	...	...	.047	2.96	.036	...	...	...	...	...	...	.054	3.43	.028
1.8	...	...	...	.043	3.20	.033	...	...	...	...	...	...	.051	3.67	.026
2.0	...	...	...	.040	3.45	.031	...	...	...	...	...	...	.048	3.88	.025
2.2	...	...	...	.038	3.70	.029	...	...	...	...	...	...	...	...	...
2.4	...	...	...	.036	3.92	.027	...	...	...	...	...	...	...	...	...

Grade	(B)			(B)			(B)			(C)			(C)		
w	1.96			1.96			1.96			.44			.44		
Q	.545			.734			1.119			.093			.182		
n_1	.55			.55			.55			.44			.44		
p. e.	2.8			5.4			3.8			0.6			0.7		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.2	...	...	...	0.268	1.40	0.136	0.400	1.45	0.203	...	...	...	...	...	...
.4	0.168	1.63	0.086	.183	2.05	.094	.271	2.13	.138	...	...	...	...	...	...
.6	.135	2.04	.069	.147	2.55	.075	.217	2.66	.110	0.163	1.28	0.370	0.240	1.73	0.546
.8	.115	2.38	.059	.126	2.98	.064	.185	3.11	.094	.143	1.46	.328	.213	1.96	.480
1.0	.102	2.70	.052	.112	3.35	.057	.163	3.51	.083	.130	1.62	.298	.192	2.16	.436
1.2	.092	2.30	.047	...	...	...	.147	3.89	.075	.120	1.75	.275	.177	2.34	.400
1.4	.085	2.26	.043	...	...	...	...	...	...	.112	1.87	.256	.165	2.50	.375
1.6	...	...	...	...	...	...	...	...	...	.106	2.00	.242	.157	2.66	.355
1.8	...	...	...	...	...	...	...	...	...	.101	2.10	.230	...	...	...

TABLE 14.—*Adjusted values of depth of current (d), with values of mean velocity (V_m) and form ratio* $\left(R=\frac{d}{w}\right)$—Con.

Grade..	(C)			(C)			(C)			(C)			(C)		
w......	.66			.66			.66			.66			.66		
Q......	.093			.182			.363			.545			.734		
n_1......	.56			.56			.56			.56			.56		
p. e....	0.8			1.8			1.4			1.3			1.9		
S	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*
0.2	...	...	...	0.270	1.03	0.410				0.562	1.43	0.850	...	...	...
.4	0.132	1.08	0.199	.183	1.52	.277	0.288	1.88	0.438	.381	2.13	.580	...	...	...
.6	.105	1.35	.160	.146	1.89	.222	.230	2.38	.350	.304	2.68	.461	0.340	3.26	0.515
.8	.090	1.58	.135	.125	2.21	.188	.196	2.80	.298	.260	3.16	.392	.290	3.82	.440
1.0	.079	1.79	.119	.110	2.50	.167	.173	3.20	.264	.228	3.60	.346	.255	4.34	.386
1.2	.071	1.97	.107	.100	2.77	.150	.157	3.53	.238	.206	4.00	.312	.230	4.80	.349
1.4	.065	2.14	.099	.092	3.00	.138	...	...	...	...	...	...	...	...	...
1.6	.061	2.30	.092	.085	3.23	.128	...	...	...	...	...	...	...	...	...
1.8	.057	2.45	.086	.080	3.44	.120	...	...	...	...	...	...	...	...	...
2.0	.054	2.60	.082	.075	3.64	.113	...	...	...	...	...	...	...	...	...
2.2	.051	2.72	.077	.071	3.83	.107	...	...	...	...	...	...	...	...	...
2.4	.048	2.86	.074	...	...	...	...	...	...	...	...	...	...	...	...
2.6	.046	3.00	.071	...	...	...	...	...	...	...	...	...	...	...	...

Grade..	(C)			(C)			(C)			(C)			(C)		
w......	1.00			1.00			1.00			1.00			1.32		
Q......	.182			.363			.545			.734			.182		
n_1......	.53			.53			.53			.53			.58		
p. e....	1.9			0.9			1.5			2.2			1.7		
S	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*
0.2	...	...	...	0.307	1.18	0.307	...	...	...	0.450	1.66	0.450	...	...	...
.4	0.139	1.30	0.139	.212	1.72	.212	...	...	...	.310	2.39	.310	...	...	...
.6	.113	1.62	.113	.171	2.14	.171	...	...	...	.250	2.96	.250	0.100	1.38	0.075
.8	.098	1.88	.098	.147	2.49	.147	0.190	2.86	0.190	.214	3.45	.214	.084	1.65	.064
1.0	.086	2.13	.086	.130	2.80	.130	.170	3.22	.170	.190	3.88	.190	.074	1.88	.056
1.2	.078	2.34	.078	.118	3.08	.118	.153	3.54	.153	.172	4.26	.172	.067	2.09	.050
1.4	.072	2.55	.072	.108	3.34	.108	.141	3.84	.141	...	...	...	.061	2.29	.046
1.6	.067	2.74	.067	.102	3.60	.102	.132	4.14	.132	...	...	...	.056	2.47	.043
1.8	.063	2.90	.063	...	...	...	.123	4.52	.123	...	...	...	.053	2.73	.040
2.0	.059	3.08	.059	...	...	...	...	...	...	...	...	...	.050	2.81	.037
2.2	.056	3.24	.056	...	...	...	...	...	...	...	...	...	.047	2.96	.035
2.4	.054	3.38	.054	...	...	...	...	...	...	...	...	...	.045	3.12	.034
2.6	...	...	...	...	...	...	...	...	...	...	...	...	.043	3.27	.032

Grade..	(C)			(C)			(C)			(C)			(C)		
w......	1.32			1.32			1.32			1.96			1.96		
Q......	.363			.545			.734			.363			.545		
n_1......	.48			.48			.48			.48			.48		
p. e....	2.3			.8			1.1			1.2			1.1		
S	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*
0.2	...	...	...	...	...	...	0.344	1.63	0.261	...	...	...	...	...	...
.4	0.168	1.66	0.128	...	...	...	.246	2.27	.187	0.132	1.18	0.068	0.166	1.66	0.085
.6	.138	2.00	.105	0.174	2.36	0.132	.203	2.76	.153	.108	1.44	.056	.136	2.02	.069
.8	.120	2.28	.092	.152	2.71	.115	.177	3.17	.134	.095	1.66	.048	.118	2.34	.060
1.0	.108	2.54	.082	.136	3.02	.104	.158	3.50	.120	.085	1.85	.043	.106	2.61	.054
1.2	.099	2.76	.075	.125	3.30	.095	.145	3.84	.110	.078	2.02	.040	.097	2.86	.049
1.4	.092	2.96	.070	.116	3.54	.088	.135	4.14	.102	.072	2.18	.037	.090	3.10	.046
1.6	.086	3.15	.065	.108	3.78	.083	.124	4.40	.096	.068	2.32	.035	.084	3.30	.043
1.8	...	...	...	...	...	...	...	...	...	.064	2.46	.033	.079	3.50	.040
2.0	...	...	...	...	...	...	...	...	...	.061	2.59	.031	.075	3.69	.038
2.2	...	...	...	...	...	...	...	...	...	.058	2.71	.030	...	...	...

Grade..	(C)			(C)			(D)			(D)			(D)		
w......	1.96			1.96			0.66			0.66			0.66		
Q......	.734			1.119			.093			.182			.545		
n_1......	.48			.48			.40			.40			.40		
p. e....	1.0			1.0			1.3			1.1			1.5		
S	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*	*d*	V_m	*R*
0.2	...	...	...	...	...	...	...	...	...	...	...	...	0.461	1.81	0.640
.4	0.194	1.96	0.099	0.240	2.40	0.122	...	...	...	0.172	1.61	0.258	.349	2.38	.531
.6	.159	2.38	.081	.197	2.92	.100	...	...	...	.143	1.92	.217	.296	2.80	.450
.8	.138	2.73	.070	.172	3.36	.088	0.087	1.63	0.132	.127	2.17	.192	.265	3.14	.403
1.0	.124	3.04	.063	.153	3.73	.078	.080	1.78	.132	.115	2.40	.174	.242	3.42	.369
1.2	.113	3.31	.058	.140	4.08	.072	.074	1.91	.112	.107	2.59	.161	.224	3.69	.342
1.4	.105	3.56	.053	...	...	...	.069	2.02	.105	.100	2.77	.151	.212	3.91	.322
1.6	.099	3.80	.050	...	...	...	.066	2.13	.100	.094	2.92	.143	.200	4.13	.305
1.8	...	...	...	...	...	...	.063	2.23	.095	.090	3.07	.135	.191	4.33	.291
2.0	...	...	...	...	...	...	.060	2.33	.091	.086	3.21	.130	.183	4.50	.280
2.2	...	...	...	...	...	...	.058	2.42	.088	.082	3.36	.125	...	...	...
2.4	...	...	...	...	...	...	.056	2.50	.085	.079	3.48	.120	...	...	...
2.6	...	...	...	...	...	...	.054	2.58	.082	.077	3.60	.116	...	...	...

TABLE 14.—*Adjusted values of depth of current (d), with values of mean velocity (V_m) and form ratio* $\left(R=\frac{d}{w}\right)$—Con.

Grade	(D)			(D)			(D)			(D)			(D)		
w	1.00			1.00			1.00			1.00			1.32		
Q	.182			.363			.545			.734			.363		
n_1	.42			.42			.42			.42			.40		
p. e.	.8			1.8			.9			1.5			2.1		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.2				0.278	1.32	0.278									
.4				.207	1.77	.207	0.252	2.16	0.252				0.163	1.68	0.133
.6	0.116	1.57	0.116	.174	2.10	.174	.213	2.56	.213	0.260	2.87	0.260	.138	1.98	.106
.8	.103	1.76	.103	.154	2.36	.154	.188	2.89	.188	.231	3.21	.231	.124	2.22	.094
1.0	.094	1.93	.094	.140	2.60	.140	.172	3.19	.172	.210	3.51	.210	.113	2.43	.086
1.2	.087	2.09	.087	.130	2.81	.130	.158	3.43	.158	.195	3.80	.195	.105	2.62	.080
1.4	.081	2.22	.081	.122	3.00	.122	.149	3.67	.149	.183	4.04	.183	.099	2.78	.075
1.6	.077	2.34	.077	.115	3.18	.115	.141	3.88	.141	.173	4.28	.173	.094	2.93	.071
1.8	.073	2.47	.073	.108	3.33	.108	.134	4.08	.134	.164	4.49	.164	.090	3.07	.068
2.0	.070	2.57	.070	.104	3.48	.104	.128	4.27	.128	.157	4.69	.157	.086	3.20	.065
2.2	.067	2.67	.067	.101	3.63	.101	.123	4.44	.123						
2.4	.065	2.77	.065	.097	3.77	.097									
2.6	.063	2.87	.063												
2.8	.061	2.95	.061												
3.0	.059	3.04	.059												

Grade	(D)			(E)			(E)			(E)			(E)		
w	1.32			.66			.66			1.00			1.00		
Q	.734			.363			.734			.182			.363		
n_1	.59			[.35]			[.35]			.34			.34		
p. e.	1.7			1.6						2.6			1.6		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.2	0.423	1.30	0.323										0.252	1.44	0.252
.4	.281	1.97	.214										.198	1.83	.198
.6	.222	2.50	.169										.173	2.10	.173
.8	.187	2.97	.143										.157	2.32	.157
1.0	.164	3.38	.125	0.129	2.14	0.195	0.206	2.66	0.313	0.097	1.88	0.097	.146	2.50	.146
1.2	.147	3.78	.112	.122	2.28	.183	.194	2.83	.293	.091	2.02	.091	.137	2.66	.137
1.4				.115	2.40	.173	.183	3.00	.285	.087	2.13	.087	.129	2.80	.129
1.6				.109	2.52	.166	.175	3.14	.265	.083	2.22	.083	.123	2.94	.123
1.8										.080	2.31	.080	.118	3.05	.118
2.0										.077	2.39	.077	.114	3.16	.114
2.2													.112	3.27	.112
2.4													.108	3.37	.108

Grade	(E)			(F)			(F)			(F)			(F)		
w	1.00			.66			.66			1.00			1.00		
Q	.734			.182			.363			.182			.363		
n_1	.34			.30			.30			.26			.26		
p. e.	2.6			1.5			0.2						1.2		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.2	0.395	1.85	0.395												
.4	.312	2.34	.312												
.6	.272	2.70	.272												
.8	.246	2.98	.246				0.227	2.43	0.345				0.162	2.24	0.162
1.0	.228	3.23	.228	0.134	2.05	0.205	.213	2.59	.321	0.099	1.83	0.099	.153	2.38	.153
1.2	.214	3.44	.214	.127	2.16	.194	.201	2.74	.303	.095	1.92	.095	.145	2.50	.145
1.4	.203	3.63	.203	.122	2.28	.185	.192	2.86	.290	.091	2.00	.091	.139	2.60	.139
1.6	.194	3.80	.194	.117	2.36	.178	.185	2.98	.280	.088	2.07	.088	.135	2.69	.135
1.8				.113	2.46	.172	.178	3.08	.270	.085	2.14	.085	.131	2.78	.131
2.0				.108	2.52	.167	.173	3.18	.262	.083	2.20	.083	.127	2.85	.127
2.2				.106	2.60	.162	.167	3.28	.254	.081	2.26	.081			
2.4				.103	2.67	.157				.079	2.32	.079			
2.6				.101	2.74	.153				.078	2.37	.078			

TABLE 14.—*Adjusted values of depth of current* (d), *with values of mean velocity* (V_m), *and form ratio* $\left(R=\frac{d}{w}\right)$—Con.

Grade	(F)			(G)			(G)			(G)			(G)		
w	1.32			.66			.66			.66			1.00		
Q	.363			.363			.734			1.119			.363		
n_1	[.26]			[.26]			[.26]			[.26]			.30		
p. e.	2.1			0.2			0.2			0.4			0.4		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.6	......	......	......	......	......	......	0.389	2.86	0.588	0.465	3.68	0.705	......	......	......
.8	......	......	......	0.222	2.49	0.336	.358	3.10	.540	.436	3.92	.660	......	......	......
1.0	0.127	2.17	0.096	.218	2.65	.315	.338	3.30	.508	.413	4.12	.626	0.158	2.30	0.158
1.2	.121	2.27	.092	.198	2.80	.300	.320	3.48	.481	.396	4.30	.600	.149	2.42	.149
1.4	.116	2.36	.088	.189	2.91	.287	.306	3.63	.460	.382	4.44	.580	.143	2.54	.143
1.6	.112	2.44	.085	.183	3.02	.277	.294	3.78	.443	.371	4.58	.561	.137	2.65	.137
1.8	.108	2.52	.083	.176	3.13	.268	.285	3.90	.430	.361	4.70	.548	.132	2.74	.132
2.0	.106	2.59	.081	.172	3.23	.260	.276	4.02	.418	.352	4.80	.532	.128	2.83	.128
2.2	.103	2.65	.079	.167	3.30	.253	.270	4.13	.405	.346	4.90	.522	.124	2.93	.124
2.4	......	......	......	.163	3.39	.247	.263	4.22	.395	.338	5.00	.512	.121	3.00	.121
2.6	......	......	......	.158	3.46	.242	.257	4.32	.387	.332	5.10	.502	.118	3.07	.118
2.8	......	......	......	......	......	......	.252	4.41	.378	.327	5.19	.494	.116	3.14	.116
3.0	......	......	......	......	......	......	......	......	......	......	......	......	.113	3.22	.113

Grade	(G)			(G)			(G)			(G)		
w	1.00			1.00			1.32			1.32		
Q	.734			1.119			.363			.734		
n_1	.30			.30			.26			.26		
p. e.	0.6			0.5			1.1			1.2		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.6	0.287	2.57	0.287	0.376	3.00	0.376	......	......	......	......	......	......
.8	.363	2.80	.263	.344	3.27	.344	......	......	......	0.216	2.60	0.164
1.0	.246	2.99	.246	.322	3.50	.322	......	......	......	.203	2.75	.154
1.2	.232	3.16	.232	.302	3.70	.302	......	......	......	.194	2.89	.147
1.4	.222	3.30	.222	.290	3.86	.290	......	......	......	.186	3.00	.141
1.6	.213	3.43	.213	.279	4.04	.279	0.114	2.41	0.087	.180	3.12	.136
1.8	.206	3.56	.206	.269	4.18	.269	.112	2.48	.084	.174	3.22	.132
2.0	.199	3.68	.199	.260	4.31	.260	.108	2.55	.082	.169	3.30	.128
2.2	.194	3.78	.194	.253	4.43	.253	.105	2.62	.080	.165	3.39	.125
2.4	.188	3.87	.188	.247	4.55	.247	.103	2.67	.078	.162	3.47	.122
2.6	.184	3.96	.184	.241	4.61	.241	.101	2.74	.077	.158	3.54	.120
2.8	.180	4.05	.180	......	......	......	.099	2.78	.075	......	3.61	.118
3.0	......	......	......	......	......	......	.098	2.83	.074	......	......	......
3.2	......	......	......	......	......	......	.096	2.87	.073	......	......	......

Grade	(G)			(H)			(H)			(H)		
w	1.32			.66			.66			.66		
Q	1.119			.363			.734			1.119		
n_1	.26			.34			.34			.34		
p. e.	0.7			1.7			0.9			0.5		
S	d	V_m	R	d	V_m	R	d	V_m	R	d	V_m	R
0.6	0.303	2.80	0.230	......	......	......	......	......	......	0.560	2.83	0.850
.8	.281	3.03	.213	......	......	......	0.382	2.93	0.580	.507	3.12	.770
1.0	.264	3.22	.200	0.220	2.52	0.336	.354	3.16	.538	.470	3.36	.713
1.2	.251	3.38	.190	.207	2.68	.316	.333	3.36	.504	.441	3.59	.670
1.4	.241	3.52	.183	.197	2.82	.300	.316	3.53	.479	.420	3.78	.636
1.6	.232	3.66	.176	.188	2.96	.286	.302	3.70	.458	.400	3.94	.608
1.8	.226	3.78	.170	.181	3.07	.276	.290	3.84	.440	.385	4.10	.584
2.0	.219	3.88	.165	.174	3.18	.267	.280	4.00	.425	.372	4.25	.564
2.2	.214	3.98	.162	.168	3.29	.258	.271	4.13	.411	.360	4.40	.548
2.4	.208	4.07	.157	.164	3.39	.250	.263	4.25	.399	.349	4.50	.530
2.6	.203	4.17	.154	.159	3.48	.243	.256	4.36	.388	.341	4.64	.518
2.8	......	......	......	......	......	......	.250	4.46	.379	......	......	......
3.0	......	......	......	......	......	......	.244	4.57	.370	......	......	......

PRECISION.

Probable errors were computed in the manner described on page 73. The largest error found for a series of adjusted values of depth is ± 5.5 per cent; and the arithmetical mean of the 66 determinations is ± 1.70 per cent. The mean of 66 determinations of the probable error of an observation of depth is ± 5.47 per cent; and the arithmetical mean of the (710) residuals, or differences (irrespective of sign) between observed and adjusted depths, is 6.65 per cent.

The residuals of any series, computed as fractional parts of the quantity measured, are greater for small depths than for large. Computed in fractions of a foot, they do not vary notably with depth.

The average value of the depths to which these measures of precision pertain is 0.176 foot. Using this factor to convert the preceding average percentage errors into (approximately) equivalent linear errors, we have, for the average of probable errors of adjusted depths ± 0.003 foot; for the average of probable errors of observed depths ± 0.010 foot, and for the average residual 0.011 foot.

The errors thus estimated include (1) the strictly accidental errors of observation, (2) the more or less systematic influences exerted on the measurements by the diverse modes of traction, and (3) errors occasioned by the assumptions underlying the method of reduction; but they do not include such systematic errors as are shown by comparing gage measurements with profile measurements. (See p. 26.) In connection with the descriptions of methods of measurement it is stated that the probable error of a single measurement by gage was computed from the comparison of values by two methods (in all cases where both are used), and on the assumption that the measures by the profile method are relatively accurate. This computation gave a value smaller than ± 0.010 foot, namely, ± 0.007 foot. The groups of measurements to which the two estimates of precision apply are not the same, although they overlap. One group includes all gage measurements of 66 series, their number being 710; the other group includes all those gage measurements of the 92 series which were checked by profile measurements, their number being 118.

By comparing these results with those reported on page 74, it is seen that the computed probable errors for depth are smaller than those for capacity. It is nevertheless believed that the measurements of load were more precise than those of depth. The discrepancy is accounted for by considering, first, that the estimates of precision, instead of applying simply to the measurements of load and depth, apply to capacity for load as a function of slope and to depth as a function of slope; and, second, that the relation of load to slope is subject to continual rhythmic variation, while the relation of depth to slope is little influenced by that variation.

MEAN VELOCITY.

As the discharge, Q, equals the product of the sectional area of the current, wd, by the mean velocity, V_m, we have

$$V_m = \frac{Q}{wd} \quad \text{(23)}$$

For each observational series, Q and w are constant and

$$V_m \propto \frac{1}{d} \quad \text{(24)}$$

Substituting for d its value in the interpolation equation (21), and remembering that b' is a constant for each series, we obtain

$$V_m \propto S^{n_1} \quad \text{(25)}$$

By means of (23) a value of mean velocity was computed for each adjusted value of d, and these values are given in Table 14. They involve all the assumptions of the formula for the reduction of the depth observations and have the same fractional measures of precision.

FORM RATIO.

The adjusted values of d were used also for the computation of the form ratio, R, which is the quotient of the depth of the current by its width; and a value of R is tabulated with each value of d. Within each observational series the form ratios are proportional to the depths, and they have the same measures of precision.

GRAPHIC COMPUTATION.

All these computations were made by graphic methods. For each observational series a plot was made on logarithmic section paper. By way of illustration the plot for grade (B), width 1.96 feet and discharge 1.119 ft.³/sec. is reproduced in figure 28, the finer lines of the logarithmic net being omitted. The vertical lines represent values of slope; the horizontal lines, capacity, depth, mean velocity, and form ratio. For the convenience of having the graphs close together their scales are made to differ, the ratio of one to another being 10, 100, or 1,000. In the particular instance shown in figure 28 the same line represents $C=100$, $d=0.1$, $V_m=1.0$, and $R=0.1$. With use of this notation were plotted the equations for C, d, V_m, and R, as functions of S. In their logarithmic forms these are

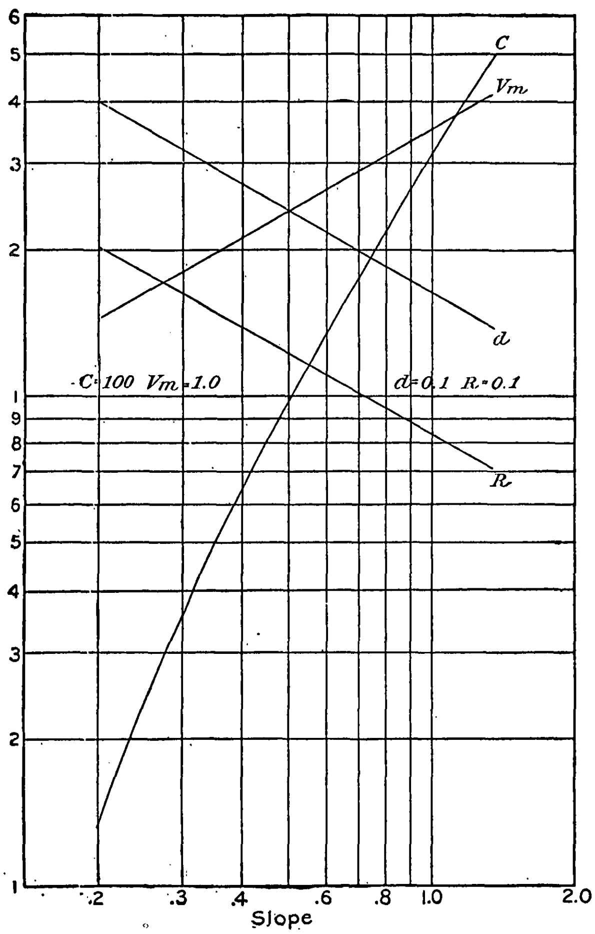


Figure 28.—Logarithmic computation sheet, combining relations of capacity, mean velocity, form ratio, and slope.

$$\log d = \log b' - n_1 \log S$$

$$\log V_m = \log \frac{Q}{w} - \log d = \log \frac{Q}{w} - \log b' + n_1 \log S$$

$$\log R = \log d - \log w = \log \frac{b'}{w} - n_1 \log S$$

$$\log C = \log b_1 + n \log (S - \sigma)$$

Their loci are the straight lines marked d, V_m, and R and the curve marked C.

After the preparation of these constructions the values of d, V_m, and R, corresponding to the selected series of values of S, were read from the sheets, affording the data of Table 14. The sheets had also many other uses, for in recording the relations of four interdependent variables to slope they also recorded their relations to one another. The points of the four loci which lie in the same vertical represent corresponding values of the several variables, and this property made it possible to read from the plot the value of a variable corresponding to a particular value of one of the others. For example, if it is desired to learn the capacity corresponding to a mean velocity of 2 ft./sec., the intersection of the velocity line with the line representing 2 on the scale of velocities is first found. From that intersection a vertical is followed or drawn to the capacity line and the position of the second intersection is read on the capacity scale.

A large part of the numerical data cited in the following discussions were either taken directly from the computation sheets or based upon them.

The accuracy of the computations by logarithmic graph may be characterized by saying that it is slightly below that by slide rule. The theoretic accuracy is the same, but tests of the logarithmic paper employed showed it to be a less perfect instrument than the slide rule, which was used for a large body of routine computations.

CHAPTER III.—RELATION OF CAPACITY TO SLOPE.

INTRODUCTION.

A series of chapters, beginning with this one, are given to the discussion of the observational data. The discussions make use of the adjusted values of capacity for stream traction, slope of stream bed, and depth of current, with their derivatives, contained in Tables 12 and 14. Associated with those adjusted values are certain grades of transported material, or degrees of fineness, certain widths of channel, and certain discharges of the transporting stream. The leading subjects of discussion are the relations of capacity to slope, discharge, fineness, and form ratio, but consideration is also given to the relations of capacity to depth and velocity, and to the relations which duty, efficiency, and depth bear to various conditions. The discussion is essentially empiric, its course being guided in small degree only by theoretic considerations.

The treatment of the relation of capacity to slope first views it as conditioned by channels of fixed width, and then as subject to the relatively ideal condition of fixed form ratio.

IN CHANNELS OF FIXED WIDTH.

THE CONDITIONS.

In each observational series the width of channel was constant, and so also were the discharge and the grade of débris constituting the load. As the load was changed, the slope responded; velocity responded to change of slope; and with variation of velocity went variation of depth. The ratio of depth to width, or the form ratio, was therefore a variable; so that the stream which dragged a large load down a steep slope differed in form, and to that extent in type, from the stream which moved a small load along a gentle slope. In a few cases it is possible so to combine data from different series as to discover the relation of capacity to slope for streams which have similar cross sections; and these will be examined in another place; but the principal discussion relates to streams with constant width and variable form ratio.

THE SIGMA FORMULA.[1]

Those properties of the formula

$$C=b_1(S-\sigma)^n \quad \text{..........} \quad (10)$$

which determined its adoption for the reduction of the observational data to a more orderly system led also to the consideration of its availability as an empiric formula for the general relation of the stream's capacity for traction to the slope of its bed. With a view to this second use, the specific values of b_1, σ, and n derived for the purpose of the reduction—values which are recorded in Table 15—were arranged and combined in various ways, in order to discover, if possible, definite relations to the several conditions in accordance with which the experiments were varied. It has already been noted (p. 71) that the critical slope, σ, varies inversely with fineness of débris, with discharge, and (probably) with range of fineness within a grade, and that it varies inversely with width of trough when that width is relatively small, but directly with width when width is relatively large.

$$\sigma=f\,(\grave{F}, \grave{H}, \check{w}, \grave{Q}) \quad \text{........} \quad (26)$$

written to express these relations in symbols, introduces H to designate range of fineness and distinguishes trend of function by means of accents. As the notation by accents will be frequently used, its definition may be made explicit. Where the function is direct, or *increasing*, its value increasing with the increase of the independent variable, the symbol of the variable is given the acute accent (′). Where the function is inverse, or *decreasing*, its value decreasing with increase of the variable, the grave accent (\`) is used. For a *maximum* function, first increasing and then decreasing (^) is used, and for a *minimum* function (˘).

The discussion of the values of b_1 showed (1) that they vary directly and in marked degree, but irregularly, with F_2, (2) that they vary

[1] Since these lines were penned I have discovered that this title duplicates one in the field of higher mathematics. Nevertheless I retain it because of its mnemonic convenience. The two fields of application are so distinct that serious confusion will not be occasioned.

directly and approximately in simple ratio, with Q, and (3) that they vary increasingly, if at all, but very slightly with w, while (4) a relation to H could not be disentangled from the relation to F_2. The discussion of the values of n showed (1) that they vary inversely and irregularly with F_2, (2) that they vary inversely and more regularly with Q, and (3) that the variation in respect to w is direct for small widths and inverse for large widths, while (4) the relation to H is not separable.

The data on the three parameters are summarized in equation (27), which is an expansion of equation (10). In all probability f_{I} and f_{II} are as complex as f, but no factors are introduced of which the influence was not definitely shown by the discussion.

$$C = f_{\text{I}}(\acute{F}_2, \acute{Q})\,[S - f(\grave{F}_2, \grave{H}, \breve{w}, \grave{Q})]^{f_{\text{II}}(\grave{F}, \hat{w}, \grave{Q})} \quad \text{(27)}$$

This equation is subject to a qualification connected with the assignment of values to σ. It will be recalled that that assignment was somewhat arbitrary, and also that the values of σ entered into the computation of the values of b_1 and n. A systematic error in the values of σ would therefore cause systematic errors in the other parameters and might vitiate conclusions as to the laws of their variation. A search was made for evidence of such errors, the search making use of the principle (easily demonstrated) that a positive error in σ would cause a positive error in b_1 and a negative error in n. While the result of the search was negative, it is not to be supposed that the values of σ have high precision. To their errors, in combination with the obscure influences of the varying range of fineness and with the errors of observation, are to be ascribed the irregularities of the constants of the adjusting equations.

While the algebraic relations are such that minor errors in the values of σ might have important influence on values of b_1 and n, their influence on the interpolated values of C would be small.

The uncertainties affecting the several elements of equation (27) are so great that no attempt will be made to develop from it a definite and quantitative expression for the relation of capacity to slope. For this reason the preceding paragraphs have attempted to present only the general tenor of the discussion and have purposely omitted details. The results, despite important qualifications, show clearly that any general expression of the law connecting capacity and slope which might be based on formula (10) would be highly complex.

With reference to the main subject of this chapter, the following section is of the nature of a digression, its purpose being to define a method and terminology used in several of the succeeding chapters.

THE POWER FUNCTION AND THE INDEX OF RELATIVE VARIATION.

One of the algebraic forms to which the title *power function* is applied is

$$y = ax^n \quad \text{(28)}$$

If the coefficient be suppressed, leaving

$$y \propto x^n$$

this is the exact equivalent of the familar "y varies as the nth power of x." This mode of comparing the rate of variation of one thing with the rate of variation of another is extensively employed, and it so commends itself by its simplicity that its use is constantly extended into fields where its applicability is approximate only. Having occasion to make much use of certain variants of this function, I find it important to obtain a clear conception of its properties and shall therefore give the matter somewhat elementary attention—with due apology to the mathematical reader.

If we consider x and y merely as numbers, the rate of variation with y with respect to x is the ratio of the differential increment of y to that of x. That ratio is

$$\frac{dy}{dx} = anx^{n-1} \quad \text{(29)}$$

If we consider x and y (and also the constant a) as powers of a common base, equation (28) becomes

$$\log y = \log a + n \log x \quad \text{(30)}$$

The rate of variation of $\log y$ with respect to $\log x$ is, differentiating,

$$\frac{d \log y}{d \log x} = n \quad \text{(31)}$$

Now, in saying that one quantity varies as a certain power of another, or in using such a function as (28), the index of variation, or that by which is indicated the comparative rate of variation, is the exponent *n;* and the value of this exponent, in terms of the variables, is found not in (29) but in (31). The expression "y varies as the nth power of x" is equivalent to "the rate of variation of y, considered as a power, is n times the rate of variation of x, considered as a power."

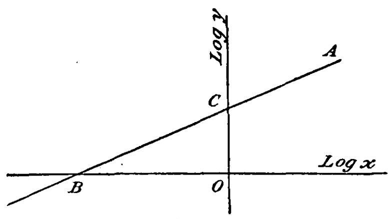


FIGURE 29.—Logarithmic locus of the power function.

An equivalent result would be attained if attention were given to the quality of the growth of x and y. Considering their growth as occurring by additive increments, equation (29) gives the ratio between their rates of increase. Considering their growth as a matter of multiplication by ratios, the additive increments are increments to logarithms, and equation (31) gives the ratio between rates of increase.

Equation (30), the logarithmic equivalent of (28), is the equation of a straight line. Representing it by AB in figure 29, its inclination $\frac{CO}{BO}$ equals n, and its intersection with the axis of log y gives $CO = \log a$. These familiar properties enhance the utility of function (28) by enabling the investigator to discuss its constants on logarithmic section paper.

In many, probably a large majority, of the physical problems to which the power function is applied, it is found that the exponent, n, does not have a constant value through the entire range of observed values of x and y. The locus of $\log y = f(\log x)$ is then not a straight line but a curve, which we may represent by AB in figure 30. At any point of the curve C, its minute element, not distinguishable from a straight line, has an inclination, $\frac{CE}{DE}$, which is homologous with n in (31) and (28) and which we may call n_2. The value of n_2 varies from point to point of the curve, so that if we try to express the relation of y to x in the form of equation (28) we must regard the exponent as a variable.

The element at C may also be thought of as part of the tangent line CD, of which the equation is

$$\log y = \overline{FO} + n_2 \log x$$

whence

$$y = \log^{-1} \overline{FO} x^{n_2}$$

On comparing this with (28), it is seen that $\log^{-1} \overline{FO}$ corresponds to a. Let us replace it by a_1, giving

$$y = a_1 x^{n_2} \text{ - - - - - - - - - - - - - (32)}$$

It is evident that for any other point of the curve, as C', the tangent intersects the axis of log y at a point different from F, and this corresponds to a different value of a_1. In other words, if we would express in an equation of the type of (28) the same relation between two variables that is expressed by the logarithmic locus in figure 30, we must make the coefficient as well as the exponent variable.

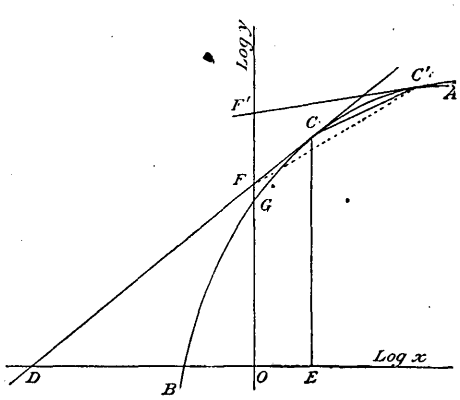


FIGURE 30.—Locus of $\log y = f(\log x)$, illustrating the nature of the index of relative variation.

The values of a_1 and n_2 are evidently functions of the independent variable, x.

It is possible to give to the relation shown by the logarithmic locus an algebraic expression which is identical in form with (28) and in which the coefficient is a constant. That is to say, it is possible to segregate the variability of parameters in the exponent; but when that is done the exponent no longer corresponds to the expression "varies as the nth power of," and the expression thus lacks the essential quality of (28). To illustrate, let us assume that the position of the point F is fixed, so as to give a constant value to a_1. The values of n_2 then correspond to the inclinations of lines FC, FC'', etc., drawn to points on the curve; but these lines are not tangents and their inclinations are not those of the corresponding elements of the curve.

In order to satisfy the condition, from equation (28), that $y=a$ when $x=1$ and $\log x=0$, the fixed point must be at the intersection, G, of the curve with the axis of $\log y$. The lines connecting it with other points of the curve are therefore chords.

In the discussions of our laboratory data use will be made of both these variants of the power function; and they will be distinguished, from one another as well as from (28), by the following notation. In

$$y=vx^i \quad \text{(33)}$$

the coefficient and exponent are both variable and are functions of x. The symbol v is chosen for the coefficient to signalize its variability. The exponent, i, denotes the instantaneous ratio of the variation of y to the variation of x, when those variations are viewed as ratios. It is the first differential coefficient of $\log y$ with respect to $\log x$, and it will be spoken of as the *index of relative variation.*

In

$$y=cx^j \quad \text{(34)}$$

the coefficient is constant and the exponent variable.

Whenever, in the investigation of the natural law connecting two variables, pairs of simultaneous values of the variables are known by observation, it is possible to plot a curve representing empirically $\log y=f(\log x)$—such a curve as $ACGB$ in figure 30. The directions of the elements of that curve, or the values of i, are essentially facts of observation. They depend exclusively on the phenomena and are independent of the units in which observational data are expressed. It is different with the values of j, for those depend on the position of the point, G, in which the curve intersects the axis of $\log y$, and therefore on the position of the axis. The position of the axis corresponds to $\log x=0$ or $x=1$ and is thus dependent on the magnitude of the unit by which the independent variable is measured.

In other chapters of this report much attention is given to the values of i, and the discussion of the variations of such values is used as a mode of treating empirically the relations between the various factors of the general problem of traction.

THE SYNTHETIC INDEX.

Recurring to figure 30, let us give attention to a restricted portion of the curve, for example, the part between C and C'. The value of i corresponding to the point C is the inclination of the line CF; the value of i corresponding to C' is the inclination of $C'F'$. Between the two are a continuous series of other values. The inclination of the chord connecting C and C', conconsidered as a ratio or exponent, is intermediate between the extreme values of i. If the sequence of values follows a definite law, the value given by the chord equals some sort of a mean derived from the others; and, in any case, it is in a sense representative of the group. It may be called a *synthetic index* of relative variation between the indicated limits.

If the coordinates of C be $\log x'$ and $\log y'$, and the coordinates of C' be $\log x''$ and $\log y''$, then, representing the synthetic index by I,

$$I=\frac{\log y''-\log y'}{\log x''-\log x'} \quad \text{(35)}$$

As the direction of the chord depends on the positions of C and C' upon the curve, so the value of I depends on the limits between which it is computed. As the direction of the chord gives no information concerning the direction of any part of the curve, so the value of I can not be used to determine any particular value of i. It is used in the following pages for the comparison of different functions for which the data span approximately the same range of conditions.

APPLICATION TO THE SIGMA FUNCTION.

Let us now represent the relation of capacity to slope by an equation of form (33),

$$C=v_1\,S^{i_1} \quad \text{(36)}$$

and develop the value of i_1 from equation (10). The logarithmic equivalent of (10) (p. 64) is

$$\log C = \log b_1 + n \log (S - \sigma) \quad \text{(15)}$$

Differentiating, and dividing both members by $d \log S$,

$$\frac{d \log C}{d \log S} = \frac{n\, d \log (S-\sigma)}{d \log S} \quad \text{(37)}$$

The first member of this equation is the value of i_1, and i_1 may be substituted for it. Also

$$d \log (S-\sigma) = \frac{dS}{S-\sigma}$$

and

$$d \log S = \frac{dS}{S}$$

Introducing these values in (37) and reducing, we have

$$i_1 = \frac{nS}{S-\sigma} \quad \text{(38)}$$

The values of n and σ being known for each observational series, it is possible, by means of equation (38), to compute i_1 for any value of S. The values of i_1 have been computed for series of values of S having the uniform interval of 0.2 per cent (and below 1.0 per cent the interval of 0.1 per cent) and having such range in each observational series as to correspond with the range of the observations. These values are recorded in Table 15. It is proposed to discuss their relations to various conditions, beginning with slope.

TABLE 15.—*Values of i_1, the index of relative variation for capacity in relation to slope.*

Conditions of experimentation												
Grade	(A)											
w	0.66			1.00			1.32			1.96		
Q	0.093	0.182	0.545	0.182	0.363	0.734	0.182	0.363	0.734	0.363	0.734	1.119
Parameters of adjusting equation												
σ	0.40	0.20	0.04	0.12	0.07	0.05	0.17	0.10	0.07	0.17	0.10	0.08
b_1	32.2	56.2	150.3	47.4	113.3	252.3	50.1	122.8	282.4	128.9	280.1	397.0
n	1.50	1.60	1.42	1.81	1.70	1.70	1.80	1.53	1.59	1.58	1.40	1.21
S	Values of i_1.											
0.2								3.05			2.79	
.3								2.29	2.08		2.09	
.4					2.07	1.94		2.04	1.93	2.74	1.86	
.5			1.54	2.39	1.98	1.89	2.72	1.91	1.85	2.39	1.74	1.63
.6			1.52	2.27	1.92	1.85	2.51	1.83	1.80	2.20	1.68	1.57
.7		2.24	1.50	2.19	1.89	1.83	2.38	1.78	1.77	2.08	1.63	1.53
.8		2.12	1.49	2.13	1.87	1.81	2.28	1.75	1.74	2.00	1.60	1.50
.9		2.05	1.48	2.09	1.85	1.80	2.22	1.72	1.72	1.94	1.57	1.48
1.0		1.99	1.48	2.06	1.83	1.79	2.17	1.70	1.71	1.90	1.55	1.47
1.2	2.25	1.92	1.47	2.02	1.81	1.78	2.10	1.67	1.69	1.84	1.52	1.44
1.4	2.10	1.86		1.99	1.79		2.05	1.65	1.68	1.80		1.43
1.6	2.00	1.82		1.96	1.78		1.02	1.63		1.76		1.42
1.8	1.93	1.79		1.95	1.77		1.99	1.62		1.74		1.41
2.0	1.88			1.93			1.97	1.61		1.72		1.40
2.2	1.84			1.92			1.95					1.39

TABLE 15.—*Values of i_1, the index of relative variation for capacity in relation to slope*—Continued.

Conditions of experimentation												
Grade	(B)											
w	0.23		0.44		0.66				1.00			
Q	0.093	0.182	0.093	0.182	0.093	0.182	0.363	0.545	0.182	0.393	0.545	0.734
Parameters of adjusting equation: σ	0.70	0.60	0.50	0.10	0.30	0.10	0.08	0.07	0.12	0.08	0.06	0.06
b_1	23.8	33.2	29.7	34 3	19.4	39.8	91.4	132.8	37.6	97.5	157.2	219.3
n	0.93	0.99	1.15	1.63	1.64	1.64	1.45	1.38	1.69	1.54	1.53	1.52
S	Values of i_1.											
0.2								2.13		2.57		2.17
.3						2.46		1.80		2.10		1.89
.4						2.19		1.68		1.93		1.78
.5				2.08		2.05	1.73	1.61	2.22	1.84		1.72
.6				1.96		1.97	1.67	1.57	2.11	1.78		1.69
.7		6.96	4.04	1.90	2.87	1.92	1.64	1.54	2.04	1.74		1.66
.8	7.41	3.98	3.08	1.86	2.63	1.88	1.61	1.51	1.98	1.71	1.65	1.64
.9	4.17	2.98	2.60	1.83	2.46	1.85	1.59	1.50	1.95	1.69	1.63	1.62
1.0	3.10	2.49	2.31	1.81	2.34	1.82	1.58	1.49	1.92	1.68	1.62	1.61
1.2	2.22	1.99	1.98	1.78	2.19	1.79	1.55	1.47	1.88	1.65	1.61	1.60
1.4	1.85	1.74	1.80	1.76	2.09	1.77	1.54	1.46	1.85	1.63	1.59	1.58
1.6	1.65	1.59	1.68	1.74	2.02	1.75	1.53	1.45	1.83	1.62	1.58	1.58
1.8	1.52	1.49	1.60	1.73	1.97	1.74	1.52	1.44	1.81	1.61	1.58	1.57
2.0	1.43		1.54		1.93	1.73	1.51	1.43	1.80	1.60	1.57	
2.2	1.36		1.50		1.90	1.72			1.79	1.60	1.57	
2.4			1.46		1.88	1.71			1.78	1.59		
2.6			1.43		1.86	1.71			1.77			
2.8					1.84	1.70			1.76			
3.0					1.82	1.70			1.76			
3.2						1.69						
3.4						1.69						
3.6						1.69						
3.8						1.69						
4.0						1.68						
4.2						1.68						

Conditions of experimentation												
Grade	(B)								(C)			
w	1.32				1.96				0.44		0.66	
Q	0.182	0.363	0.545	0.734	0.363	0.545	0.734	1.119	0.093	0.182	0.093	0.182
Parameters of adjusting equation: σ	0.17	0.12	0.10	0.08	0.18	0.14	0.12	0.10	0.40	0.20	0.16	0.11
b_1	39.5	96.5	165.4	233.5	93.9	153.5	228.2	363.3	22.6	34.4	18.2	38.6
n	1.61	1.54	1.55	1.57	1.64	1.60	1.46	1.43	1.43	1.50	1.58	1.54
S	Values of i_1.											
0.2								2.86				
.3		2.56		2.14			2.44	2.15				2.43
.4	2.80	2.20		1.96	2.99		2.09	1.91				2.12
.5	2.44	2.02	1.94	1.87	2.57	2.23	1.93	1.79			2.33	1.97
.6	2.25	1.92	1.86	1.81	2.35	2.09	1.83	1.73	4.29	2.22	2.16	1.88
.7	2.13	1.85	1.81	1.78	2.21	2.01	1.77	1.68	3.39	2.10	2.05	1.83
.8	2.05	1.81	1.78	1.75	2.12	1.94	1.72	1.64	2.86	2.00	1.98	1.79
.9	1.99	1.77	1.75	1.72	2.06	1.90	1.69	1.62	2.57	1.93	1.92	1.75
1.0	1.94	1.75	1.73	1.71	2.01	1.86	1.66	1.60	2.39	1.88	1.87	1.73
1.2	1.88	1.71	1.70	1.68	1.94	1.82	1.63	1.57	2.15	1.81	1.82	1.70
1.4	1.84	1.68	1.67	1.67	1.89	1.78	1.60		2.00	1.76	1.79	1.67
1.6	1.80	1.66	1.66	1.65	1.85	1.76	1.58		1.91	1.72	1.76	1.65
1.8	1.78	1.64	1.65	1.64	1.83	1.74	1.57		1.84	1.69	1.74	1.64
2.0	1.76	1.63			1.81		1.56		1.79	1.67	1.72	1.63
2.2	1.75	1.62			1.79				1.75	1.65	1.70	1.62
2.4	1.74										1.69	1.61
2.6	1.73										1.69	1.61
2.8											1.68	1.60
3.0											1.67	1.60
3.2											1.66	1.59
3.4											1.66	1.59
3.6											1.65	1.59
3.8											1.65	1.59
4.0											1.65	
4.2											1.64	

TABLE 15.—*Values of i_1, the index of relative variation for capacity in relation to slope*—Continued.

Conditions of experimentation												
Grade	(C)											
w	0.66			1.00					1.32			
Q	0.363	0.545	0.734	0.182	0.363	0.545	0.734	1.119	0.182	0.363	0.545	0.734
Parameters of adjusting equation: σ	0.08	0.06	0.04	0.15	0.11	0.09	0.07	0.06	0.22	0.16	0.13	0.11
b_1	82.9	122.8	162.1	40.4	100.1	159.2	199.2	301.6	33.7	99.9	156.1	218.5
n	1.46	1.48	1.50	1.59	1.41	1.33	1.35	1.39	1.85	1.40	1.35	1.30
S	Values of i_1.											
0.2	......	2.12	......	......	3.14	......	2.18	......	......	......	......	2.90
.3	......	1.86	......	......	2.23	......	1.76	......	......	3.00	......	2.12
.4	1.83	1.75	......	2.54	1.95	......	1.64	......	......	2.33	......	1.80
.5	1.74	1.69	1.63	2.27	1.81	1.63	1.57	1.58	......	2.06	......	1.67
.6	1.68	1.65	1.61	2.12	1.73	1.57	1.53	1.55	2.92	1.91	1.72	1.60
.7	1.65	1.62	1.59	2.02	1.68	1.53	1.50	1.52	2.70	1.82	1.65	1.55
.8	1.62	1.60	1.58	1.95	1.64	1.50	1.48	1.50	2.55	1.75	1.61	1.51
.9	1.60	1.59	1.57	1.90	1.61	1.48	1.46	1.49	2.45	1.70	1.57	1.49
1.0	1.59	1.58	1.57	1.87	1.59	1.47	1.45	1.48	2.37	1.67	1.55	1.47
1.2	1.56	1.56	......	1.81	1.55	1.44	1.43	1.46	2.27	1.61	1.51	1.44
1.4	1.55	1.55	......	1.78	1.53	1.43	1.42	1.45	2.20	1.58	1.49	1.42
1.6	1.54	1.54	......	1.75	1.51	1.42	1.41	......	2.14	1.55	1.47	1.40
1.8	1.53	1.54	......	1.73	1.50	1.41	1.41	......	2.11	1.54	1.45	1.39
2.0	1.52	1.53	......	1.71	1.49	1.40	1.40	......	2.08	1.52	1.44	1.38
2.2	1.52	1.53	......	1.70	1.49	1.39	1.39	......	2.06	1.51	1.43	1.37
2.4	1.51	......	......	1.69	1.48	......	......	......	2.04	1.50	1.42	......
2.6	1.51	......	......	1.68	1.47	......	......	......	......	......	......	......
2.8	......	......	......	1.68	1.47	......	......	......	......	......	......	......
3.0	......	......	......	......	......	......	......	......	......	......	......	......
3.2	......	......	......	......	......	......	......	......	......	......	......	......
3.4	......	......	......	......	......	......	......	......	......	......	......	......
3.6	......	......	......	......	......	......	......	......	......	......	......	......
3.8	......	......	......	......	......	......	......	......	......	......	......	......

Conditions of experimentation											
Grade	(C)				(D)						
w	1.96				0.66			1.00			
Q	0.363	0.545	0.734	1.119	0.093	0.182	0.545	0.182	0.363	0.545	0.734
Parameters of adjusting equation: σ	0.24	0.20	0.17	0.14	0.19	0.14	0.08	0.17	0.12	0.10	0.08
b_1	93.4	155.7	245.1	438.3	13.2	37.2	115.2	32.8	87.9	129.4	174.6
n	1.62	1.50	1.34	1.59	1.80	1.55	1.51	1.67	1.49	1.68	1.52
S	Values of i_1.										
0.2	......	......	......	......	......	......	2.52	......	......	......	2.54
.3	......	......	......	......	......	......	2.06	......	2.48	......	2.12
.4	......	......	2.33	......	......	2.38	1.89	......	2.12	......	1.90
.5	3.12	2.49	2.03	2.21	......	2.15	1.80	......	1.95	2.11	1.81
.6	2.71	2.24	1.87	2.07	2.64	2.02	1.74	......	1.86	2.04	1.75
.7	2.47	2.09	1.77	1.99	2.47	1.93	1.71	2.20	1.80	1.97	1.72
.8	2.32	1.99	1.70	1.93	2.36	1.88	1.68	2.12	1.75	1.93	1.69
.9	2.21	1.92	1.65	1.88	2.28	1.83	1.66	2.06	1.71	1.90	1.67
1.0	2.14	1.87	1.62	1.85	2.22	1.80	1.64	2.01	1.69	1.87	1.65
1.2	2.03	1.79	1.56	1.80	2.14	1.75	1.62	1.94	1.65	1.84	1.63
1.4	1.96	1.74	1.53	......	2.08	1.72	1.60	1.90	1.63	1.81	1.61
1.6	1.91	1.71	1.50	......	2.04	1.70	1.59	1.86	1.61	1.80	1.60
1.8	1.87	1.68	......	......	2.01	1.68	1.58	1.84	1.59	1.79	1.59
2.0	1.84	1.66	......	......	1.99	1.66	1.57	1.82	1.58	1.78	......
2.2	1.82	1.65	......	......	1.97	1.65	1.57	1.81	1.57	......	......
2.4	......	......	......	......	1.96	1.64	......	1.80	1.56	......	......
2.6	......	......	......	......	1.94	1.63	......	1.79	......	......	......
2.8	......	......	......	......	......	......	......	1.78	......	......	......

TABLE 15.—*Values of i_1, the index of relative variation for capacity in relation to slope*—Continued.

Conditions of experimentation		(D)		(E)								
	Grade..	(D)		(E)								
	w	1.32		0.66		1.00				1.32		
	Q	0.363	0.734	0.363	0.734	0.182	0.363	0.734	1.119	0.363	0.734	1.119
Parameters of adjusting equation	σ	0.16	0.11	0.16	0.11	0.08	0.06	0.04	0.03	0.04	0.03	0.02
	b_1	80.1	160.0	33.8	50.2	17.0	37.3	76.5	145.9	38.8	76.1	126.1
	n	1.77	1.65	1.93	1.73	1.72	1.70	1.58	1.58	1.57	1.50	1.77
S		Values of i_1.										
0.2							2.43	1.98				
.3			2.60				2.13	1.83				
.4			2.27			2.16	2.00	1.76				
.5		2.60	2.11			2.06	1.93	1.72	1.69		1.59	
.6		2.41	2.02			1.99	1.89	1.69	1.67	1.68	1.57	1.83
.7		2.29	1.95			1.95	1.86	1.68	1.65	1.67	1.56	1.82
.8		2.21	1.91			1.92	1.84	1.67	1.64	1.66	1.55	1.82
.9		2.15	1.88			1.90	1.82	1.66	1.64	1.65	1.55	1.81
1.0		2.11	1.85	2.09	2.17	1.88	1.81	1.65	1.63	1.64	1.54	1.81
1.2		2.04	1.81	2.03	2.13	1.85	1.79	1.64	1.62	1.63	1.54	1.80
1.4		2.00		1.99	2.10	1.83	1.78	1.63	1.62	1.62		
1.6		1.97			2.08	1.82	1.77	1.62	1.61	1.61		
1.8		1.94				1.81	1.76	1.62	1.61	1.61		
2.0		1.92				1.80	1.76	1.61	1.61	1.60		
2.2						1.79	1.75		1.61			
2.4						1.79	1.75		1.60			
2.6						1.78						
2.8						1.78						

Conditions of experimentation		(F)										(G)
	Grade..	(F)										(G)
	w	0.66			1.00				1.32			0.66
	Q	0.182	0.363	0.734	0.182	0.363	0.734	1.119	0.363	0.734	1.119	0.363
Parameters of adjusting equation	σ	0.33	0.21	0.17	0.44	0.31	0.22	0.17	0.39	0.28	0.23	0.50
	b_1	12.02	30.5	52.2	7.96	28.2	65.0	106.9	30.3	71.1	126.4	27.5
	n	1.86	1.69	1.61	2.31	1.88	1.61	1.60	1.63	1.73	1.49	1.74
S		Values of i_1.										
0.2												
.3												
.4												
.5												
.6												
.7							2.35	2.08				
.8						3.07	2.22	2.00		2.65	2.09	
.9				1.98		2.87	2.13	1.94		2.50	2.00	
1.0			2.14	1.94		2.72	2.06	1.90		2.40	1.94	3.48
1.2		2.57	2.04	1.88	3.65	2.54	1.98	1.83	2.42	2.25	1.85	2.98
1.4		2.44	1.99	1.83	3.38	2.42	1.91	1.79	2.27	2.15	1.78	2.71
1.6		2.35	1.94	1.80	3.20	2.33	1.87	1.76	2.16	2.09	1.74	2.53
1.8		2.28	1.91	1.78	3.07	2.27	1.84	1.74	2.08		1.71	2.41
2.0		2.23	1.89		2.97	2.23			2.03			2.32
2.2		2.19			2.90	2.19			1.99			2.25
2.4		2.16			1.84	2.16						2.20
2.6		2.14			1.79	2.14						2.16
2.8												2.12
3.0												2.08

TABLE 15.—*Values of i_1, the index of relative variation for capacity in relation to slope*—Continued.

Conditions of experimentation												
	Grade..	(G)								(H)		
	w......	0.66		1.00			1.32			0.66		
	Q......	0.734	1.119	0.363	0.734	1.119	0.363	0.734	1.119	0.363	0.734	1.119
Parameters of adjusted equation	σ.......	0.36	0.28	0.58	0.41	0.33	0.71	0.50	0.40	0.80	0.56	0.48
	b_1......	64.5	85.0	23.6	65.7	115.5	15.39	62.3	114.5	19.38	52.9	75.1
	n.......	1.69	1.78	1.85	1.70	1.63	2.37	1.71	1.54	2.01	1.72	1.67
S		Values of i_1.										
0.2		....	....	....	....	....	....	....	....	....	....	....
.3		....	....	....	....	....	....	....	....	....	....	....
.4		....	....	....	....	....	....	....	....	....	....	....
.5		....	....	....	....	....	....	....	....	....	....	....
.6		....	3.23	....	....	3.63	....	....	....	....	....	....
.7		3.49	2.89	....	4.11	3.09	....	....	3.61	....	....	5.31
.8		3.08	2.68	....	3.49	2.78	....	4.58	3.09	....	5.75	4.17
.9		2.82	2.54	....	3.12	2.58	....	3.86	2.78	....	4.56	3.58
1.0		2.65	2.43	....	2.88	2.44	....	3.43	2.57	....	3.92	3.21
1.2		2.42	2.29	3.57	2.58	2.25	....	2.94	2.32	6.03	3.23	2.78
1.4		2.28	2.20	3.15	2.40	2.14	....	2.64	2.16	4.69	2.88	2.54
1.6		2.19	2.14	2.90	2.29	2.06	....	2.49	2.06	4.02	2.65	2.38
1.8		2.12	2.09	2.71	2.20	2.00	3.91	2.38	1.99	3.62	2.50	2.27
2.0		2.07	2.05	2.60	2.14	1.96	3.67	2.29	1.93	3.35	2.40	2.20
2.2		2.03	2.02	2.51	2.09	1.92	3.50	2.22	1.89	3.16	2.32	2.14
2.4		1.99	2.00	2.43	2.05	1.89	3.36	2.17	1.85	3.01	2.25	2.09
2.6		1.96	1.98	2.38	2.02	....	3.26	2.12	1.83	2.90	2.20	2.05
2.8		....	....	2.33	....	....	3.18	2.08	....	2.81	2.16	....
3.0		....	....	2.29	....	....	3.11	....	....	....	2.12	....
3.2		....	....	....	....	....	3.05	....	....	....	2.09	....
3.4		....	....	....	....	....	3.00	....	....	....	....	....

VARIATION OF THE INDEX.

Each column of the table contains a set of values of i_1 which pertain to the same grade, fineness, width, and discharge and of which the changes are related to slope only. In figure 31 a number of these sets are plotted in relation to slope. The curves have a strong family likeness, arising from the fact that the data were all adjusted by the sigma formula; but the likeness would not altogether disappear if the assumptions of that formula were abandoned. The general relations of the index to slope are as follows:

(1) It varies decreasingly with slope.

(2) Its rate of change is greater for low slopes than for high.

The upper group of curves all pertain to grade (C) and width 1.00 foot, but represent different discharges. They show (3) that the rate of change for similar slopes is greater for small discharges than for large.

The second group of curves all pertain to grade (C) and discharge 0.363 ft.^{3}/sec., but represent different widths. They show (4) that the rate of change for similar slopes is greater for broad channels than for narrow, or, as the depth varies inversely with the width, that the rate of change is greater for shallow streams than for deep.

The third group of curves all pertain to width 1.00 foot and discharge 0.363 ft.^{3}/sec., but represent different grades of débris. They show (5) that the rate of change is greater for coarse débris than for fine.

In the third group the curves for grades (A), (B), (C), (D), and (E) lie close together, while those for the coarser grades (F) and (G) are well separated. This is probably connected with the fact that the range of fineness gradually increases from (A) to (E) and then drops abruptly from (E) to (F). The influence of increasing range approximately neutralizes that of decreasing fineness, and the inference is (6) that the rate of change in the index is greater for small range than for large.

Consider now the variations of the index in relation to width. In figure 32 (p. 106) the ordinates, as before, represent values of i_1 and the abscissas represent width of channel. The points fixed by the data are shown by the dots.

(7) The upper group of curves all pertain to grade (C) and discharge 0.182 ft.^{3}/sec., but represent different slopes. Their common char-

acter is a distinct minimum. From the neighborhood of width 0.66 foot there is increase of i_1 in the direction of greater width, and also in the direction of less width.

(8) The position of the minimum is apparently the same for low slopes as for high.

(9) The minimum is most strongly marked in case of the gentler slopes.

The second group of curves all pertain to grade (C) and a slope of 1.0 per cent but differ in respect to discharge. Each of them shows a minimum, except the curve for discharge

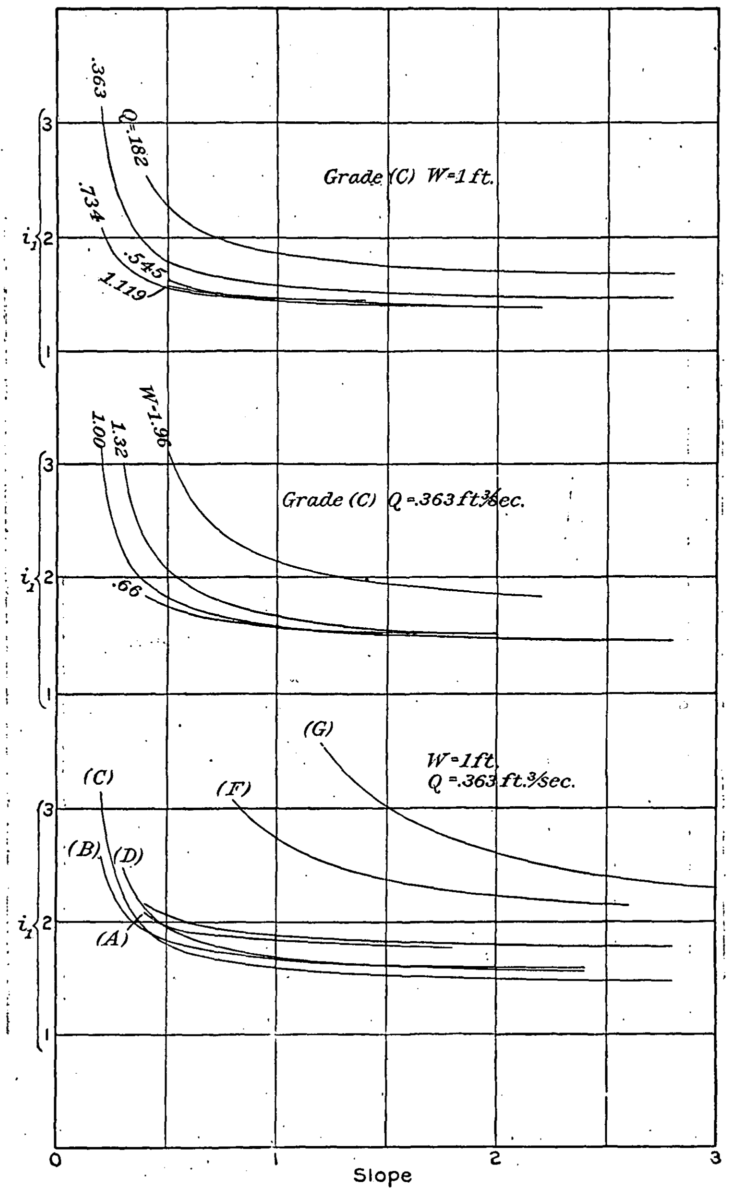


FIGURE 31.—Variations of i_1 in relation to slope.

0.093 ft.³/sec., which has but two fixed points. They show also that—

(10) The position of the minimum is related to discharge. For large discharges it is associated with relatively large widths, for smaller discharges with smaller widths.

(11) The minimum is more pronounced, or the associated rates of change in the index are higher in case of small discharge than of large.

Various analogies, which appear in another part of this paper, render it probable that all the preceding inferences are of a general character; but those in regard to width are not sustained by all the data.

The curves of the third group are based on observations with grade (B) and are drawn, like those of the first group, to contrast the relations of the index to width for different slopes. With the gentler slopes they give indications of a minimum, but not with the steeper. The character of the discrepancy is such as to suggest that the values of the index computed for width 0.23 foot vary too rapidly with slope; and this result might be brought about by assigning too large a value to σ. A critical review of the data, however, failed to find warrant for any material change in that constant.

It is believed that a group of discrepancies which this instance illustrates are connected with the history of the experimental work. The first grade to be investigated was (B), and the methods of manipulation were subject to various minor changes, which were not always recorded; but the range of conditions was large. Grade (C) was next taken up, and again the range of conditions was large. Other grades followed, with less elaborate range of conditions; but the work on grade (G) was somewhat expanded, in order to learn the influence of coarser débris on various factors. The work on (G) also had the advantage of the fullest

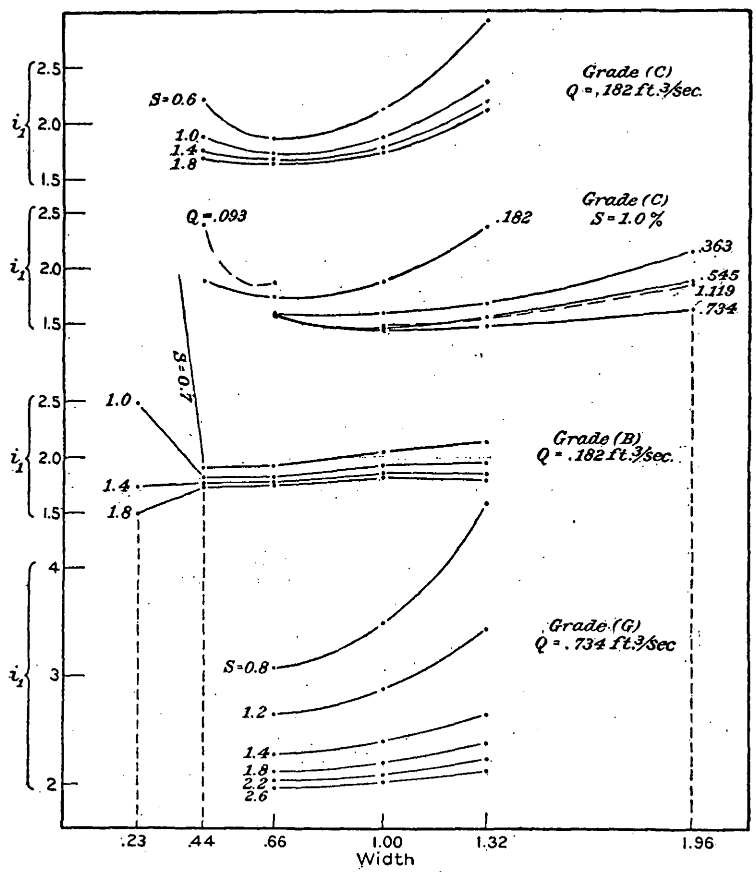


FIGURE 32.—Variations of i_1 in relation to width of channel.

development of experimental method as well as that of uniformity of method. Because of this history it is believed that the results for grades (C) and (G) are of higher authority than those of other grades; and the belief is strengthened by the general symmetry and internal consistency of the (C) and (G) results. The inferences, given in preceding paragraphs, from data of grades (C) and (G) are therefore accepted, and the discordance of data for grade (B), while not specifically explained, is ascribed in a general way to unrecorded differences in laboratory methods.

The curves of the fourth group of figure 32 all pertain to grade (G) and discharge 0.734 ft.³/sec. but differ in respect to slope. Compared with the first and second groups they are seen to be consistent with the inference as to a minimum value of i_1, but the minimum falls outside the range of widths for which data were obtained. With grade (C) and discharge 0.734 ft.³/sec. the minimum falls between widths 1 foot and 1.32 feet, but nearer to the former. With grade (G) and the same discharge it apparently falls with some width less than 0.66 foot. This indicates that—

(12) The position of the minimum is related to fineness. For the finer débris it is associated with relatively great width; for the coarser, with smaller width.

The curves of the fourth group support the ninth inference, that the minimum is most strongly marked for the gentlest slope.

In the study of the data many other comparisons of the influence of width were made, but they are not here illustrated. Their chief service was in indicating the comparative value of different divisions of the body of data. The general fact brought out—and one emphasized in various other ways—is that the measures of

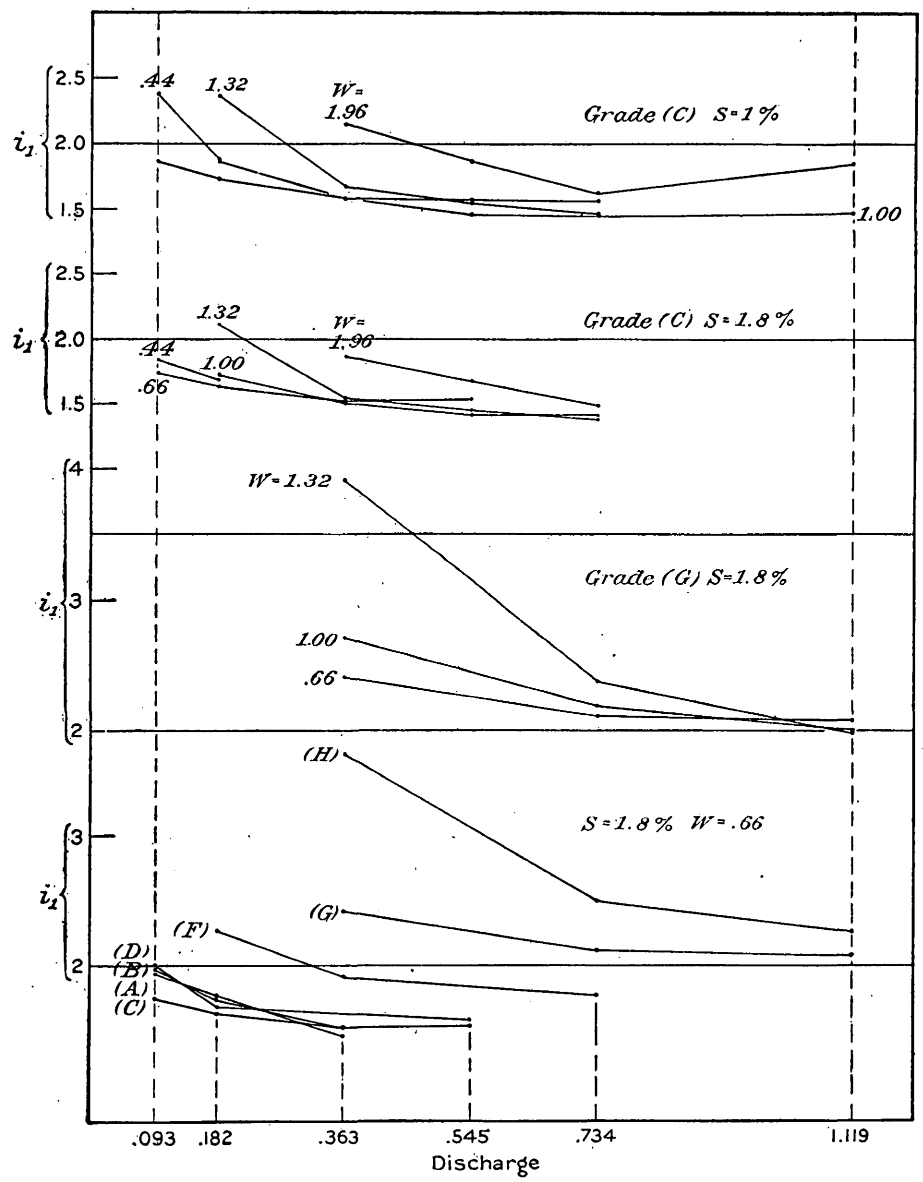


FIGURE 33.—Variations of i_1 in relation to discharge.

precision derived from discrepancies of observations within a single series by no means cover the whole field. The discrepancies discovered when properties of different series are compared are quite as important and must be given consideration in connection with the broader generalizations.

Let us now consider the relations of the varying value of i_1 to discharge. These are illustrated by figure 33. The curves of the upper group all pertain to grade (C) and slope 1.0 per cent but differ in respect to width of channel. Those of the second group pertain to grade (C) and slope 1.8 per cent; those of the third group to grade (G) and slope 1.8 per cent. The general fact is that—

(13) As discharge increases the value of i_1 diminishes. There are three exceptions, of which

two do not exceed the computed probable errors of the data, and the third is connected with a value of i_1 to which the lowest weight is ascribed.

(14) The rate of change in the index is greater for small discharges than for large.

(15) For the same discharges the rate of change in the index is greater for wide channels than for narrow, and is therefore greater for shallow streams than for deep.

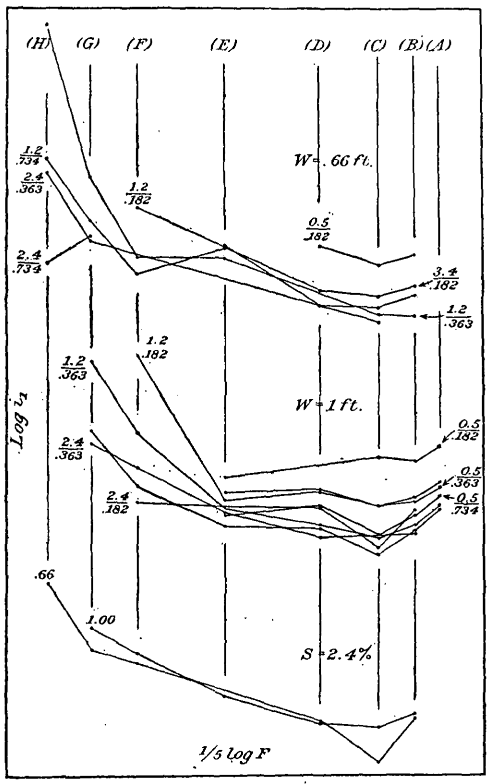


FIGURE 34.—Variations of i_1 in relation to fineness of débris.

The curves of the lowest group all pertain to width 0.66 foot and slope 1.8 per cent, but differ as to grade of débris. They indicate that—

(16) The rate of change in the index is greater for coarse débris than for fine. The peculiarities of spacing, as in a previous instance, may show the influence of the factor of range in fineness within the several grades.

To consider now the relations of the values of i_1 to the fineness of débris, the comparison is made with linear fineness—instead of bulk fineness, as in discussing σ, and it is found convenient to plot the logarithms of the quantities instead of the quantities themselves. In figure 34 the curves of the upper group are derived from experiments conducted with a trough width of 0.66 foot, and each one pertains to a particular combination of slope and discharge. Those of the second group are derived from experiments with a trough width of 1 foot.

(17) In the main they show decrease of i_1 with increase of fineness, but the finer grades give the opposite indication. The data are not sufficiently harmonious to determine whether the law of change is continuous or involves a reversal. If it is continuous, i_1 is an inverse function of F.

In view of the fact that the double variation of i_1 in relation to width is a complicating factor and of the further fact that that variation is less pronounced with high slopes than with low, two curves (the lowest group of fig. 34) were constructed from data pertaining to the highest practicable slope, 2.4 per cent. Each curve belongs to a particular width of trough, and each is a composite with respect to discharge. Their indication is practically the same as that of the other groups.[1]

The character of the material has not seemed to warrant a quantitative discussion of the variations of the index of variation, and a summary of the qualitative discussion is necessarily limited to generalities. The index of relative variation or the sensitiveness of capacity for traction to change of slope is a decreasing function of the slope, the discharge, the fineness of débris, and the range of fineness and is a minimum function of width of channel.

In symbols,

$$i_1 = f(\check{S}, \check{Q}, \check{F}, \check{H}, \breve{w}) \quad \text{........} \quad (39)$$

If we assume tentatively that the function replacing i_1 in the exponent is the product of functions of the individual conditions—that is, if we write

$$i_1 = (f_{\text{I}}(S))\ (f_{\text{II}}(Q))\ (f_{\text{III}}(F))\ (f_{\text{IV}}(H))\ (f_{\text{V}}w))$$

then we must also recognize that in $f_{\text{I}}(S)$, f_{I} is itself a function of Q, F, H, and w, that f_{II} is a function of F and w, and that f_{V} is a function of

[1] In the data on flume traction the relation of capacity to fineness exhibits peculiarities quite analogous to those here found in the relation of i_1 to fineness. The capacity is larger for very fine and very coarse débris than for intermediate grades. A tentative explanation (see Chapter XII) connects the larger capacity for fine débris with a transition in process from traction to suspension.

S, Q, and F. Parallel complexities would also arise if attempt were made to formulate the relations by means of such an expression as

$$i_1 = f_{\text{I}}(S) + f_{\text{II}}(Q) + f_{\text{III}}(F) + f_{\text{IV}}(H) + f_{\text{V}}(w)$$

The sensitiveness of capacity to slope appears to be a function of the conditions jointly rather than severally.

The development of complexity within complexity suggests that the actual nature of the relation is too involved for disentanglement by empiric methods, but that conclusion does not necessarily follow. Just as a highly complex mathematical expression may be the exact equivalent of a fairly simple expression of a different type, so a physical law may defy formulation when approached in a certain way yet yield readily when the best method of approach has been discovered.

FORMULATION WITH CONSTANT COEFFICIENT.

For the relation of capacity to slope the formula equivalent to (34) is

$$C = c_1 S^{j_1} \quad \text{.............} (40)$$

in which the constant coefficient c_1 is the value of capacity when $S=1$.

From equation (10) (p. 96),

$$c_1 = b_1 (1-\sigma)^n \quad \text{...........} (41)$$

Also as (10) and (40) give equivalent expressions for C, we may equate their second members:

$$c_1 S^{j_1} = b_1 (S-\sigma)^n$$

Substituting the value of c_1 from (41),

$$b_1 (1-\sigma)^n S^{j_1} = b_1 (S-\sigma)^n$$

whence

$$S^{j_1} = \left(\frac{S-\sigma}{1-\sigma}\right)^n$$

and

$$j_1 = n \frac{\log (S-\sigma) - \log (1-\sigma)}{\log S} \quad \text{......} (42)$$

By means of this formula the values of j_1 in Table 16 were computed. These values have been subjected to systematic comparison with the associated conditions (S, Q, w, etc.) in the same manner as were the values of the exponent i_1; but the results of the comparison need not be given in detail because they are parallel to those for i_1. They are in fact identical so far as verbal statement is concerned and differ only in quantitative ways. It is true in the main, and almost without exception, that the variations of j_1 are less rapid than those of i_1; and their average range is about half as great.

TABLE 16.—*Values of j_1, in $C=c_1S^{j_1}$, the coefficient c_1 being a constant.*

Conditions of experimentation: Grade	(A)											
w	0.66			1.00			1.32			1.96		
Q	0.093	0.182	0.545	0.182	0.363	0.734	0.182	0.363	0.734	0.363	0.734	0.119
c_1	15	39.4	142	37.6	100	231	36.5	104	252	96.1	242	359
S	Values of j_1.											
0.2								2.09			1.91	
.4					1.93	1.85		1.86	1.80	2.21	1.68	1.39
.6			1.50	2.15	1.88	1.82	2.32	1.76	1.75	2.03	1.61	1.35
.8		2.06	1.49	2.10	1.85	1.80	2.23	1.72	1.73	1.96	1.57	1.33
1.0		1.99	1.48	2.06	1.83	1.79	2.17	1.70	1.71	1.90	1.55	1.31
1.2	2.36	1.95	1.47	2.04	1.82	1.78	2.13	1.68	1.70	1.87	1.54	
1.4	2.27	1.93		2.02	1.81		2.10	1.67	1.68	1.84		
1.6	2.20	1.91		2.01	1.80		2.08	1.66		1.82		
1.8	2.16	1.88		2.00	1.80		2.06	1.65		1.81		
2.0	2.12			1.99			2.05	1.65		1.80		
2.2	2.09			1.99			2.04					

TABLE 16.—*Values of j_1, in $C=c_1S^{j_1}$; the coefficient c_1 being a constant*—Continued.

Conditions of experimentation												
Grade	(B)											
w	0.23		0.44		0.66				1.00			
Q	0.093	0.182	0.093	0.182	0.093	0.182	0.363	0.545	0.182	0.363	0.545	0.734
c_1	7.8	13.4	13.3	28.9								
S	Values of j_1.											
0.2								1.69		1.95		1.79
.4						1.96		1.57		1.78		1.68
.6				1.88		1.89	1.63	1.53	2.00	1.73		1.64
.8	4.57	3.09	2.63	1.84	2.47	1.85	1.60	1.50	1.95	1.70	1.64	1.62
1.0	3.10	2.49	2.31	1.81	2.34	1.82	1.58	1.49	1.92	1.68	1.62	1.61
1.2	2.60	2.21	2.16	1.79	2.26	1.81	1.56	1.48	1.90	1.66	1.61	1.61
1.4	2.34	2.05	2.02	1.78	2.20	1.80	1.55	1.47	1.88	1.65	1.61	1.60
1.6	2.18	1.94	1.94	1.77	2.16	1.79	1.54	1.46	1.87	1.64	1.60	1.59
1.8	2.05	1.86	1.88	1.77	2.13	1.78	1.54	1.46	1.86	1.64	1.60	1.59
2.0	1.96		1.83		2.10	1.77	1.53	1.45	1.85	1.64	1.59	
2.2	1.89		1.79		2.08	1.77			1.84	1.63	1.59	
2.4			1.76		2.06	1.76			1.84	1.63		
2.6			1.73		2.04	1.76			1.83			
2.8					2.03	1.75			1.83			
3.0					2.02	1.75			1.82			
3.2						1.75						
3.4						1.75						
3.6						1.74						
3.8						1.74						
4.0						1.74						
4.2						1.74						

Conditions of experimentation												
Grade	(B)								(C)			
w	1.32				1.96				0.44		0.66	
Q	0.182	0.363	0.545	0.734	0.363	0.545	0.734	1.119	0.093	0.182	0.093	0.182
c_1	29.2	79.3	140	205	67.8	120	189	312	10.9	24.7	13.8	32.3
S	Values of j_1.											
0.2								1.96				
.4	2.26	1.92		1.81	2.36		1.82	1.69				1.88
.6	2.08	1.82	1.79	1.76	2.16	1.98	1.74	1.65	3.08	2.05	2.00	1.80
.8	2.00	1.78	1.75	1.73	2.06	1.91	1.70	1.62	2.60	1.95	1.93	1.76
1.0	1.94	1.75	1.73	1.71	2.01	1.87	1.66	1.60	2.39	1.88	1.88	1.73
1.2	1.92	1.72	1.71	1.69	1.97	1.84	1.64	1.58	2.26	1.83	1.85	1.71
1.4	1.89	1.71	1.70	1.68	1.94	1.82	1.63		2.17	1.80	1.83	1.70
1.6	1.87	1.70	1.69	1.67	1.92	1.81	1.62		2.11	1.77	1.81	1.69
1.8	1.85	1.69	1.68	1.67	1.90	1.80	1.61		2.06	1.76	1.80	1.68
2.0	1.84	1.68			1.89		1.61		2.03	1.75	1.79	1.67
2.2	1.83	1.68			1.88				1.99	1.74	1.78	1.67
2.4	1.82										1.77	1.66
2.6	1.82										1.76	1.66
2.8											1.76	1.66
3.0											1.75	1.65
3.2											1.75	1.65
3.4											1.75	1.65
3.6											1.74	1.64
3.8											1.74	1.64
4.0											1.74	
4.2											1.73	

TABLE 16.—*Values of j_1, in $C=c_1S^{j_1}$, the coefficient c_1 being a constant*—Continued.

Conditions of experimentation												
Grade..	(C)											
w......	0.66			1.00					1.32			
Q......	0.363	0.545	0.734	0.182	0.363	0.545	0.734	1.119	0.182	0.363	0.545	0.734
c_1	73.4	112	153	31.2	84.9	140	181	277	21.3	78.4	130	184
S	Values of j_1.											
0.2	....	1.76	....	....	2.03	....	1.65	....	....	....	....	1.86
.4	1.68	1.65	....	2.12	1.73	....	1.53	1.55	....	1.92	....	1.60
.6	1.63	1.61	1.58	1.98	1.66	1.51	1.49	1.51	2.61	1.78	1.62	1.54
.8	1.61	1.59	1.58	1.91	1.62	1.49	1.47	1.49	2.46	1.71	1.58	1.49
1.0	1.59	1.58	1.57	1.87	1.59	1.47	1.45	1.48	2.37	1.67	1.55	1.47
1.2	1.58	1.57	....	1.84	1.57	1.46	1.44	1.47	2.32	1.64	1.53	1.45
1.4	1.57	1.56	....	1.82	1.56	1.45	1.43	1.47	2.28	1.62	1.51	1.44
1.6	1.56	1.56	....	1.80	1.55	1.44	1.43	....	2.25	1.60	1.50	1.43
1.8	1.55	1.55	....	1.79	1.54	1.43	1.42	....	2.23	1.59	1.49	1.42
2.0	1.55	1.55	....	1.78	1.53	1.42	1.42	....	2.21	1.58	1.49	1.42
2.2	1.54	1.55	....	1.77	1.53	1.42	1.42	....	2.19	1.57	1.48	1.41
2.4	1.54	....	....	1.76	1.52	....	....	....	2.17	1.56	1.47	....
2.6	1.54	....	....	1.76	1.52	....	....	....	....	....	....	....
2.8	....	....	....	1.75	1.51	....	....	....	....	....	....	....

Conditions of experimentation											
Grade..	(C)				(D)						
w......	1.96				0.66			1.00			
Q......	0.363	0.545	0.734	1.119	0.093	0.182	0.545	0.182	0.363	0.545	0.734
c_1	59.8	111	191	345	9.1	29.5	102	24.0	72.5	108	154
S	Values of j_1.										
0.2	....	....	....	....	....	....	1.91	....	....	....	1.92
.4	....	....	1.88	....	....	2.02	1.74	....	1.84	....	1.75
.6	2.38	2.03	1.73	1.95	2.41	1.90	1.69	....	1.77	1.94	1.70
.8	2.22	1.94	1.66	1.89	2.29	1.84	1.66	2.06	1.72	1.90	1.67
1.0	2.14	1.87	1.62	1.85	2.22	1.80	1.64	2.01	1.69	1.87	1.65
1.2	2.08	1.83	1.60	1.82	2.18	1.78	1.63	1.98	1.67	1.85	1.64
1.4	2.04	1.80	1.58	....	2.15	1.76	1.62	1.95	1.65	1.84	1.63
1.6	2.01	1.78	1.56	....	2.12	1.74	1.61	1.93	1.64	1.84	1.63
1.8	1.99	1.76	....	....	2.10	1.73	1.61	1.91	1.63	1.83	1.62
2.0	1.97	1.75	....	....	2.09	1.72	1.60	1.90	1.63	1.82	....
2.2	1.95	1.74	....	....	2.08	1.71	1.60	1.89	1.62	1.82	....
2.4	....	....	....	....	2.07	1.71	....	1.88	1.61	....	....
2.6	....	....	....	....	2.06	1.70	....	1.87	....	....	....
2.8	....	....	....	....	....	....	....	1.87	....	....	....

Conditions of experimentation											
Grade..	(D)		(E)								
w......	1.32		0.66		1.00				1.32		
Q......	0.373	0.734	0.363	0.734	0.182	0.363	0.734	1.119	0.363	0.734	1.119
c_1	58.9	132	24.8	40.1	14.7	33.6	71.5	139	36.4	72.6	122
S	Values of j_1.										
0.2	....	....	....	....	....	2.03	1.76	....	....	....	....
.4	....	2.02	....	....	1.99	1.89	1.70	....	....	....	....
.6	2.25	1.93	....	....	1.94	1.85	1.68	1.65	1.66	1.56	1.82
.8	2.16	1.88	....	....	1.98	1.82	1.66	1.64	1.65	1.55	1.81
1.0	2.11	1.85	2.09	2.17	1.88	1.81	1.65	1.63	1.64	1.54	1.81
1.2	2.07	1.83	2.06	2.15	1.86	1.80	1.64	1.63	1.63	1.54	1.80
1.4	2.05	....	2.03	2.13	1.85	1.79	1.64	1.62	1.62	....	....
1.6	2.03	....	....	2.12	1.85	1.79	1.63	1.62	1.62	....	....
1.8	2.02	....	....	....	1.84	1.78	1.63	1.62	1.62	....	....
2.0	2.01	....	....	....	1.84	1.78	1.63	1.62	1.62	....	....
2.2	....	....	....	....	1.83	1.77	....	1.62	....	....	....
2.4	....	....	....	....	1.83	1.77	....	1.62	....	....	....
2.6	....	....	....	....	1.82	....	....	....	....	....	....
2.8	....	....	....	....	1.82	....	....	....	....	....	....

TABLE 16.—*Values of j_1, in $C=c_1S^{j_1}$, the coefficient c_1, being a constant*—Continued.

Conditions of experimentation											
Grade	(F)										(G)
w	0.66			1.00				1.32			0.66
Q	0.182	0.363	0.734	0.182	0.363	0.734	1.119	0.363	0.734	1.119	0.363
c_1	5.7	20.4	38.7	2.1	13.9	43.6	79.6	13.5	40.3	86.5	8.3
S	Values of j_1.										
0.2											
.4											
.6											
.8					2.85	2.14	1.95		2.52	2.01	
1.0		2.14	1.94		2.73	2.06	1.90		2.40	1.94	3.48
1.2	2.67	2.08	1.91	3.87	2.65	2.02	1.87	2.54	2.32	1.88	3.21
1.4	2.60	2.05	1.88	3.71	2.58	1.98	1.84	2.45	2.27	1.85	3.04
1.6	1.53	2.03	1.86	3.59	2.52	1.95	1.82	2.38	2.22	1.83	2.91
1.8	2.47	2.01	1.85	3.50	2.48	1.94	1.81	2.33		1.80	2.82
2.0	2.43	1.99		3.43	2.45			2.29			2.76
2.2	2.41			3.36	2.42			2.26			2.70
2.4	2.39			3.31	2.39						2.65
2.6	2.38			3.27	2.37						2.61
2.8											2.58
3.0											2.55

Conditions of experimentation											
Grade	(G)								(H)		
w	0.66		1.00			1.32			0.66		
Q	0.734	1.119	0.363	0.734	1.119	0.363	0.734	1.119	0.363	0.734	1.119
c_1	30.3	48.6	4.8	26.8	60.1	0.82	19.0	52.0	0.76	12.8	25.2
S	Values of j_1.										
0.2											
.4											
.6		2.77			2.91						
.8	2.85	2.55		3.15	2.60		3.93	2.81		4.68	3.63
1.0	2.65	2.43		2.88	2.44		3.43	2.57		3.92	3.21
1.2	2.53	2.36	3.94	2.72	2.34		3.22	2.44	7.65	3.54	2.98
1.4	2.45	2.31	3.67	2.61	2.27		3.00	2.35	6.57	3.31	2.83
1.6	2.38	2.27	3.48	2.63	2.22		2.88	2.27	5.93	3.15	2.75
1.8	2.34	2.24	3.35	2.48	2.18	5.34	2.79	2.23	5.50	3.04	2.64
2.0	2.30	2.21	3.25	2.43	2.15	5.10	2.72	2.19	5.20	2.95	2.58
2.2	2.27	2.19	3.16	2.39	2.12	4.92	2.66	2.15	4.96	2.88	2.53
2.4	2.24	2.17	3.10	2.36	2.10	4.77	2.61	2.12	4.77	2.82	2.49
2.6	2.22	2.16	3.04	2.34		4.65	2.57	2.10	4.63	2.76	2.45
2.8			2.99			4.55	2.54		4.49	2.72	
3.0			2.94			4.45				2.69	
3.2						4.38				2.66	
3.4						4.32					

The greatest contrast between the rates of variation of the two exponents is found when relations to slope are considered, the least when relations to fineness are considered.

The variations of the two exponents with respect to slope are illustrated by figure 35. In the example from which the curves in this figure were computed the range of variation, within the experimental limits, is considerably above the average, but the example is otherwise typical. The rate of change is everywhere smaller for j_1 than for i_1, and the total change, or range of variation, is therefore smaller. This relation prevails throughout the range of slopes covered by the experiments but would not be found to hold for slopes far outside of that range. The average range of j_1, for the investigated cases, falls between one-half and one-third of the average range of i_1.

It thus appears that a modification of the plan of formulation which dispenses with variation in the coefficient and thereby concentrates all expression of variation in the other parameter, the exponent, tends also to diminish diversity in that parameter.

EFFECT OF CHANGING THE UNIT OF SLOPE.

The values of j_1 are functions not only of the conditions of experimentation and of the constant slope σ but also of the unit used for the measurement of slope. (See p. 99.) In the term log $(1-\sigma)$, which enters into the value of j_1 (equation 42), 1 is the unit of slope; and the relative magnitude of 1 and σ changes as the unit changes. Other terms of the formula are also (implicitly) functions of the unit, but the various influences are not compensatory, and the resultant is of such nature that the values of j_1 vary inversely with the magnitude of the unit. It will be recalled that in the notation of this paper the symbol S pertains to the unit $\frac{\text{height}}{\text{distance}} \div 100$, and the symbol s to the unit $\frac{\text{height}}{\text{distance}}$. The curve marked j_1 in figure 35 represents values of the exponent computed with use of the smaller unit, and the curve marked k represents a coordinate system of values computed with use of the larger unit. The latter curve, produced, would intersect the curve of i_1 at a point corresponding to $s=1$ or $S=100$.

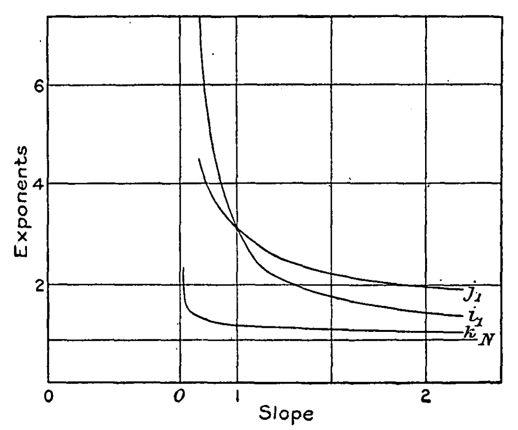


FIGURE 35.—Variations of exponents i_1, j_1, and k, in relation to slope. The scale of slope is in per cent.

It would be possible, by employing the larger unit in the notation for slope, to construct a table equivalent to Table 16 in which the values of j_1 would all be smaller and would have in each series less range. For many of the series they would approach closely the associated values of n. This reduction of exponents would be accompanied by an enormous increase in the values of coefficients, each value of c_1 in the table being magnified by the factor 100^n. The increase would result from the fact that c_1 is the capacity corresponding to unit slope, while the unit slope in that case would be 45°. Considered as capacities, the values of c_1 would be in a sense fictitious, because the laws of traction with which we are dealing do not apply (see p. 63) to so high a slope as 45°.

To recur for a moment to the general account of the index of relative variation, it will be recalled that the index was shown to be independent of the units of the observational quantities. In this particular instance the value of the index, $n\frac{S}{S-\sigma}$, has two factors, of which the first is an abstract number and the second is a ratio between slopes and is independent of the unit of slope. The ordinates of the curve marked i_1 in figure 35 are therefore independent of the slope unit, and the curve is a fact of observation, plus the assumptions of the formula of adjustment.

PRECISION.

Because the values of i_1 were computed from values of n and σ, their precisions are involved with those of n and σ, but the relation is not simple. The precision of n depends partly on the harmony of the observations of load and slope and partly on the precision of σ. It is not feasible to measure the precision of σ for individual series of observations, and the precisions of individual values of n and i_1 are therefore indeterminate. All that has been attempted is to derive a rough estimate of average precision from average values of the quantities involved. The estimated average probable error of values of n is 3.9 per cent, and the corresponding estimate for values of i_1 is 4.6 per cent.

The precision of i_1 varies with slope within each series represented by a column in Table 15, being relatively high for the steeper slopes. The probable error, if measured in the same unit with i_1, is much larger for gentle slopes than for steep; if measured in percentage, it is somewhat larger for the gentler slopes. Measured in percentage, its value for the steeper slopes approximates the corresponding probable error of n.

EVIDENCE FROM EXPERIMENTS WITH MIXED DÉBRIS.

The observations on capacity and slope when the débris transported consisted of a mixture of two or more grades were reduced in the same general manner as those for single grades. It was not thought advisable to make any adjustment of the values of σ, but each logarithmic plot was treated independently. For about one-third of the mixtures the best value appeared to be zero. For another it fell between the values which had been assigned to the finer and coarser components of the mixture. For the remaining third it exceeded the values associated with the components. When the load was a mixture of many grades combined in approximately natural proportions, and also when it consisted of unsorted débris from a river bed, the values of σ were small and for the most part zero.

These features were scrutinized with special interest because the properties of grade (E) had suggested that great range of fineness might determine very low values of σ. On the whole, the data from mixtures favor that view, but their support is by no means unanimous.

TABLE 17.—*Values of i_1 for mixtures of two or more grades of débris.*

Conditions of experimentation: Grade	(A_1C_1)	(A_3G_1)	(A_2G_1)	(A_1G_1)	(A_1G_2)	(A_1G_4)	(B_4F_1)	(B_2F_1)	(B_1F_1)	(B_1F_2)	(B_1F_4)	(C_4E_1)
w	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Q	0.363	0.363	0.363	0.363	0.363	0.363	0.363	0.363	0.363	0.363	0.363	0.363
Parameters of interpolation equation: σ	0.25	0.30	0.20	0	0	0	0.25	0.25	0.50	0.50	0.50	0.30
n	1.06	1.38	1.53	1.98	2.09	2.21	1.08	0.96	1.51	1.59	1.60	1.10
S	Values of i_1.											
0.4												
.6	1.82	2.76	2.30	1.98			1.85	1.64				2.20
.8	1.55	2.21	2.05	1.98			1.57	1.39	4.02	4.23	4.27	1.76
1.0	1.42	1.97	1.92	1.98	2.09	2.21	1.44	1.28	3.02	3.17	3.20	1.57
1.2	1.34	1.84	1.84	1.98	2.09	2.21	1.36	1.21	2.59	2.72	2.75	1.47
1.4	1.30	1.76	1.79	1.98	2.09	2.21	1.31	1.17	2.35	2.47	2.49	1.40
1.6	1.26	1.70		1.98	2.09	2.21		1.14	2.19	2.31	2.33	1.36
1.8						2.21			2.09			1.32
2.0						2.21						

Conditions of experimentation: Grade	(C_2E_1)	(C_2E_1)	(C_1E_1)	(C_1E_1)	(C_1E_2)	(C_1E_2)	(C_1E_4)	(C_1E_4)	(C_4G_1)	(C_2G_1)	(C_1G_1)
w	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Q	0.182	0.363	0.182	0.363	0.182	0.363	0.182	0.363	0.363	0.363	0.363
Parameters of interpolation equation: σ	0	0.30	0	0.40	0.20	0.30	0	0.30	0	0.30	0.50
n	1.78	1.13	1.60	1.12	1.40	1.25	1.55	1.33	1.97	1.24	1.13
S	Values of i_1.										
0.4											
.6		2.26							1.97	2.49	
.8		1.81		2.23		2.02	1.55		1.97	1.99	3.01
1.0		1.62	1.60	1.86	1.75	1.79	1.55	1.90	1.97	1.78	2.26
1.2		1.51	1.60	1.68	1.68	1.67	1.55	1.77	1.97	1.66	1.93
1.4		1.44	1.60	1.56	1.63	1.60	1.55	1.69	1.97	1.58	1.76
1.6	1.78	1.39	1.60	1.49	1.60	1.54	1.55	1.63	1.97	1.53	1.64
1.8	1.78	1.36	1.60	1.44	1.57	1.51	1.55	1.59	1.97	1.49	1.56
2.0	1.78		1.60	1.40	1.55	1.48	1.55	1.56			
2.2	1.78		1.60		1.54	1.45	1.55				

Conditions of experimentation: Grade	(C_1G_2)	(E_4G_1)	(E_2G_1)	(E_1G_1)	(E_1G_2)	($A_1C_1G_2$)	(CDEFG)			Natural.	
w	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Q	0.363	0.363	0.363	0.363	0.363	0.363	1.82	0.363	0.545	0.182	0.363
Parameters of interpolation equation: σ	0.60	0	0.30	0.40	0	0.48	0	0	0	0.30	0
n	1.47	1.87	1.90	1.28	2.03	1.67	1.63	1.48	1.22	1.43	1.84
S	Values of i_1.										
0.4											1.84
.6		1.87							1.22		1.84
.8		1.87	3.04	2.56		4.19	1.63	1.48	1.22	2.29	1.84
1.0	3.68	1.87	2.71	2.13	2.03	3.22	1.63	1.48	1.22	2.04	1.84
1.2	2.94	1.87	2.53	1.92	2.03	2.79	1.63	1.48	1.22	1.90	1.84
1.4	2.58	1.87	2.42	1.79	2.03	2.55	1.63	1.48	1.22	1.82	1.84
1.6	2.36	1.87	2.34	1.70	2.03	2.39	1.63	1.48	1.22	1.76	1.84
1.8	2.21	1.87		1.64	2.03	2.28	1.63	1.48	1.22	1.72	
2.0					2.03					1.68	
2.4										1.65	
2.6										1.63	
2.8										1.62	

The adjusted values of capacity may be found in Table 12, together with a variety of other data. The values of i_1 are contained in Table 17, having been computed for the range of slopes covered by the observations. In order to compare these with the indexes of relative variation for the single grades from which the mixtures were made, Table 18 has been prepared, containing indexes corresponding to the uniform slope of 1.2 per cent. An analysis of this table shows that 3 mixtures are more sensitive to variation of slope than are components, 13 are less sensitive, and 18 show sensitiveness of intermediate rank. A general average shows the mixtures 11 per cent less sensitive than the (means of) components, but the contrast is much more pronounced for natural river débris and for the most complex artificial combination than for the simpler mixtures.

TABLE 18.—*Comparison of values of i_1 for mixtures and their components.*

[S=1.2; w=1.00.]

Component grades.		Q	Value of i_1 when ratio of finer to coarser is—						
Finer.	Coarser.		1:0	4:1	2:1	1:1	1:2	1:4	0:1
(A)	(C)	0.363	1.81	...	...	1.34	...	...	1.55
(A)	(G)	.363	1.81	1.84	1.84	[a]1.98	[a]2.09	[a]2.21	3.57
(B)	(F)	.363	1.65	1.36	1.21	2.59	2.72	2.75	2.54
(C)	(E)	.182	1.81	...	[a]1.78	[a]1.60	1.68	[a]1.55	1.85
(C)	(E)	.363	1.55	1.47	1.51	1.68	1.67	1.77	1.79
(C)	(G)	.363	1.55	[a]1.97	1.66	1.93	2.94	...	3.57
(E)	(G)	.363	1.79	[a]1.87	2.53	1.92	[a]2.03	...	3.57
(A), (C)	(G)	.363	1.34	...	...	2.79	...	...	3.57
Mixture of (C), (D), (E), (F), and (G)		.182	1.81	...	...	[a]1.63	...	...	...
		.363	1.55	...	...	[a]1.48	...	...	3.57
		.545	1.44	...	...	[a]1.22	...	...	...
Natural combination ranging in fineness from (A) to (F)		.182	2.02	...	...	1.90	...	...	3.65
		.363	1.81	...	...	[a]1.84	...	...	2.54

[a] For these, $\sigma=0$ and $i_1=n$.

The general tenor of the evidence from mixtures is to show that in passing from the laboratory conditions (with graded débris) to natural conditions it will be proper to reduce the estimates of the sensitiveness of capacity to changes of slope.

RELATION OF INDEX TO MODE OF TRACTION.

One of the possible ways of bridging the chasm between laboratory conditions and river conditions is through the consideration of modes of traction. River discharges are enormously greater than the experimental, river slopes are relatively minute, and river channel sections are very dissimilar in form and proportions, but the three modes of traction have the same sequence in rivers as in the laboratory troughs. This consideration has led to the separation of certain data connected with a single mode of traction—the one characterized by smooth surfaces of water and débris. Table 19 contains values of the index of relative variation, i_1, associated with smooth traction and grouped with reference to various conditions. In the upper division of the table all the values pertain to the same grade of débris (C) and are arranged according to discharges and channel widths; in the lower division all pertain to the same width of channel and are arranged according to discharges and grades of débris.

TABLE 19.—*Values of i_1 associated with the smooth mode of traction.*

I. Values pertaining to the same grade of débris, (C).

Width (feet).	Value of i_1 for discharge (ft.³/sec.) of—					
	0.093	0.182	0.363	0.545	0.734	1.119
0.44	2.15	1.88	...	...	...	...
.66	1.79	1.70	1.59	1.59	1.58	...
1.00	...	1.81	1.59	1.48	1.48	...
1.32	...	2.20	1.61	1.55	1.51	...
1.96	...	...	1.96	1.79	1.62	1.93

II. Values pertaining to the same width of channel, 0.66 feet.

Grade.	Value of i_1 for discharge (ft.³/sec.) of—					
	0.093	0.182	0.363	0.545	0.734	1.119
(B)	2.09	1.82	1.61	1.54	...	...
(C)	1.79	1.70	1.59	1.59	1.58	...
(D)	2.01	1.72	...	1.64	...	...
(E)	...	1.99	2.13	...	...	...
(G)	...	...	2.41	...	2.19	2.20
(H)	...	...	3.35	...	2.65	2.54

Inspection of the table shows that under this special condition the index (1) varies inversely with discharge, (2) varies inversely with fineness, and (3) varies both inversely and directly with width. In all these respects the variations are similar to those noted when the constant condition is slope, but the rates of variation are not the same. To illustrate the differences in rate I introduce Table 20, in which data of Table 19 are compared, in parallel columns, with similar data (from Table 15) conditioned by constant slope. The same data are also presented graphically and with some generalization in figure 36. On giving attention to the first division of the table and to the upper diagram, it will be seen that the variation of the index with discharge is somewhat less when the constant factor is mode of traction than when it is slope. The second division and second diagram show that the variation with fineness is also less for constant mode of traction. The third division and diagram indicate that the variation with width of channel is of the same order of magnitude in the two cases, but that for constant mode of traction the minimum value of the index is associated with greater width.

The comparison does not indicate that the condition of uniform mode of traction is

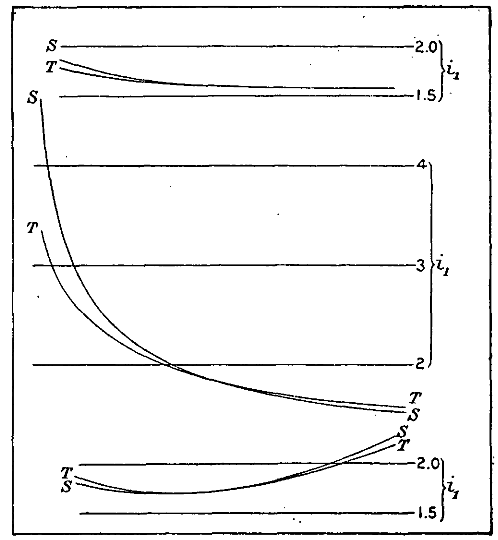


FIGURE 36.—Variations of i_1 under condition of uniform mode of traction (T, T, T), and under condition of uniform slope (S, S, S). Vertical distances represent i_1, with a common scale but different zeros. Horizontal distances represent, for the upper pair of curves, discharge; for the second pair, logarithms of fineness; for the lower pair, width of channel; the values increasing from left to right.

greatly to be preferred to that of uniform slope as a basis for the extension of laboratory generalizations to large natural streams.

TABLE 20.—*Relations of i_1 to discharge, débris grade, and channel width, when conditioned (1) by a constant mode of traction and (2) by a constant slope.*

Grade, (C) *w,* 0.66 foot.			*w,* 0.66 foot. *Q,* 0.363 ft.³/sec.			*Grade,* (C) *Q,* 0.182 ft.³/sec.		
Q	Values of i_1 with—		Grade.	Values of i_1 with—		*w*	Values of i_1 with—	
	Smooth traction.	Constant slope 1.0 per cent.		Smooth traction.	Constant slope 1.4 per cent.		Smooth traction.	Constant slope 1.2 per cent.
0.093	1.79	1.87	(B)	1.61	1.54	0.44	1.88	1.81
.182	1.70	1.73	(C)	1.59	1.55	.66	1.70	1.70
.363	1.59	1.59	(E)	2.13	1.99	1.00	1.81	1.81
.545	1.59	1.58	(G)	2.41	2.71	1.32	2.20	2.27
.734	1.58	1.57	(H)	3.35	4.69	....	....	....

IN CHANNELS OF SIMILAR SECTION.

THE CONDITIONS.

It will be shown in the following chapter that one of the important conditions affecting capacity is the relation of stream depth to stream width, or the form ratio R. The matter has, in fact, already received some attention in connection with the variation of σ. Now, in each observational series the width is constant while the depth varies, so that the form ratio is a variable. Its variations accompany and are inseparable from those of slope; and the variation of capacity (within an observational series), which up to this point has been treated as if it were purely a function of slope, is in reality a function of slope and form ratio jointly. To separate the two factors and thereby discover the relation of capacity to slope for streams of similar section, it is necessary to bring together data obtained by use of troughs with different widths, selecting points of two or more adjusted series which are characterized by the same ratio of depth to width. In every such comparison the capacity and slope associated with the narrower trough are relatively large, while for the wider trough they are relatively small. The opportunities for comparison are not abundant, because in the main the observational series which are of like conditions except as to width do not overlap in respect to form ratio.

SIGMA AND THE INDEX.

In the records of the main body of experiments 24 cases of overlap are found, all associated with the finer grades of débris, from (A) to (D). In each of these the comparison includes two widths only, no instance occurring in which it can be extended to three. There is, however, a special group of experimental series, planned in part for this particular purpose, in which the trough-width interval is so small that triple overlaps occur. The special experiments were made with débris of grade (C); the trough widths were 1.0, 1.2, 1.4, 1.6, 1.8, and 1.96 feet; and the experiments yield nine triple overlaps. In six of these the extent of overlap is such that numerical comparisons have been made for more than one value of R.

With the aid of the computation sheets described on page 95, a table was compiled in which adjusted capacities and slopes were arranged with respect to form ratio, and in this table, which has not been printed, the matter of overlaps was canvassed. Table 21 contains the data involved in the triple overlaps.

TABLE 21.—*Selected data, for grade (C), showing the relation of capacity to slope when the form ratio is constant.*

R	Q=0.734			Q=0.923			Q=1.021			Q=1.119		
	w	S	C	w	S	C	w	S	C	w	S	C
0.07	1.60	1.38	354									
	1.80	1.17	262									
	1.96	.75	118									
.08	1.60	1.08	226									
	1.80	.89	151									
	1.96	.49	48									
.09				1.60	1.29	413						
				1.80	.94	210						
				1.96	.58	88						
.10	1.40	1.33	346				1.60	1.19	405			
	1.60	.72	102				1.80	.82	195			
	1.80	.55	57				1.96	.50	75			
.11	1.40	1.06	224	1.40	1.44	522				1.60	1.07	380
	1.60	.60	71	1.60	.83	185				1.80	.74	193
	1.80	.45	35	1.80	.60	83				1.96	.50	88
.12				1.40	1.17	357				1.60	.87	261
				1.60	.69	130				1.80	.62	134
				1.80	.50	56				1.96	.41	55
.13	1.20	1.43	382	1.40	.96	243	1.40	1.16	380			
	1.40	.72	103	1.60	.58	94	1.60	.65	135			
	1.60	.44	37	1.80	.42	36	1.80	.47	58			
.14										1.40	1.20	470
										1.60	.60	130
										1.80	.45	66
.15										1.40	.99	331
										1.60	.51	95
										1.80	.39	47
.16										1.40	.83	236
										1.60	.44	71
										1.80	.34	33

To illustrate the use made of such data, the case of $Q=1.119$ ft.^{3}/sec. and $R=0.15$ is selected. In that example the values of slope and capacity are given for the trough widths 1.4, 1.6, and 1.8 feet. These values come from three adjusting equations, which are, for the widths severally, $C=386\ (S-0.08)^{1.72}$, $C=400\ (S-0.10)^{1.60}$, and $C=430\ (S-0.12)^{1.67}$. The graphs of the equations are shown in figure 37. On each graph is a dot indicating the point which corresponds to the tabulated values of C and S, and for each of these points the ratio R is 0.15. The curves represent the relations of C to S under the condition of uniform width. The three dots are points on an undrawn curve to express the relation of C to S under the condition of constant form ratio.

It is convenient, in discussing this undrawn curve, to assume that its equation involves σ and is otherwise of the same type as the equations used in discussing the data for constant width. In this case, moreover, the assumption is countenanced by the fact that traction is limited by the competent slope. By making the assumption, it is possible to compute all the parameters of the curve from the coordinates of the three known points and write its equation:

$$C = 469\ (S - 0.23)^{1.25} \qquad (43)$$

In the present connection the most significant of the parameters is σ; and the values of σ have been computed for each of the cases of Table 21. They are assembled in Table 22.

TABLE 22.—*Values of σ corresponding to data in Table 21.*

R	Value of σ when Q is—			
	0.734	0.923	1.021	1.119
0.07	0	....	....	....
.08	0	....	....	....
.09	....	−.44	+.04	....
.10	+.20	....	....	....
.11	+.20	+.35	....	+.05
.12	....	+.33	....	+.11
.13	+.14	+.31	+.13	....
.14	....	....	....	+.21
.15	....	....	....	+.24
.16	....	....	....	+.24

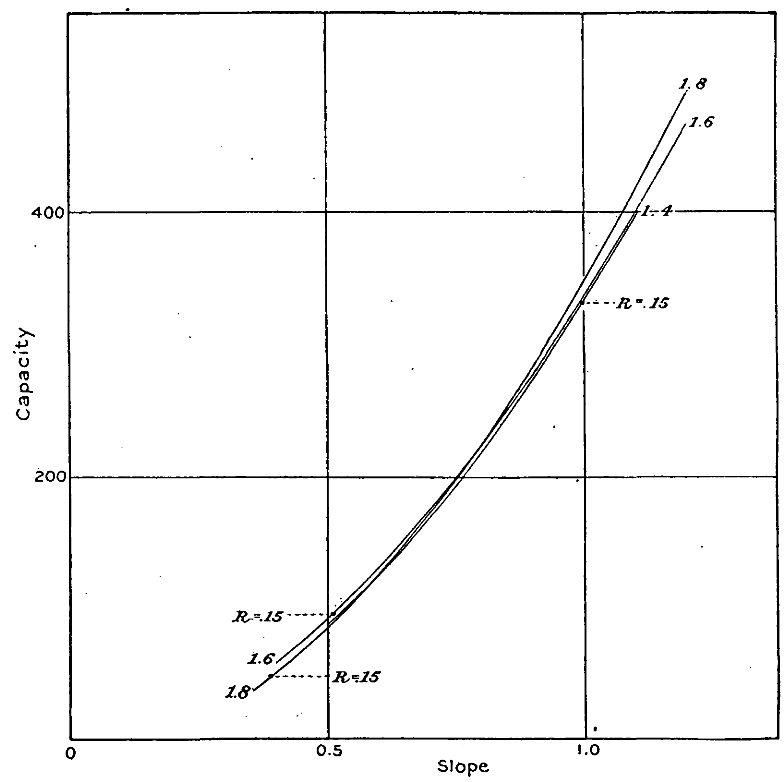


FIGURE 37.—Curves of $C=f(S)$ for trough widths of 1.4, 1.6, and 1.8 feet, showing points which agree as to form ratio. Grade (C). Q=1.119 ft.³/sec. R=0.15.

One value of σ has a negative sign. As this yields positive capacity for negative slope, thus transcending the physical conditions of traction, and as the value is numerically large, an error of importance is betrayed. The error appears not to be one of computation and must be ascribed either to the observational data or to assumptions made in the work of reduction and adjustment. In either case its existence serves to qualify the values of σ as of low precision.

In view of that qualification, the study of the values of σ in detail appears unprofitable and only their mean will be considered. In deriving the general mean a subsidiary mean was first found for each case of overlap and these were afterward combined, this course being followed because when several values of σ are derived from the same overlap of series they are not independent. The general mean thus derived is +0.08, and this may be compared with a mean similarly obtained from the associated equations under condition of constant width, namely, +0.12. So far as this moderate difference in magnitude has significance, it indicates that the index of relative variation, i_1, has the smaller range under the condition of constant form ratio; for the index's range in magnitude is a direct function of σ.

THE SYNTHETIC INDEX.

Another mode of viewing the data of Table 21 ignores the middle term of the triple overlap and considers only the first and last. From the values of capacity and slope in these two terms a value of the synthetic index can be computed by equation (35). It is possible in this way to broaden the range of data by including the cases of simple overlap; and it is possible to make many pertinent comparisons by computing the corresponding values of the synthetic index for observational series in which the constant condition is width instead of form ratio. The indexes with form ratio constant and with width constant may be symbolized severally by I_R and I_w.

TABLE 23.—*Values of synthetic index under condition that R is constant, (I_R); with coordinate values, under condition that w is constant, (I_w).*

Grade.	Q.	Width.		R.	I_R.	I_w. Channel narrow.	I_w. Channel wide.
(A)	0.363	1.32	1.96	.06	2.01	1.74	1.99
				.07	2.23	1.80	2.17
	.734	1.32	1.96	.10	1.77	1.82	1.75
				.12	1.90		
(B)	.182	1.00	1.32	.06	1.95	1.87	1.92
				.07	2.09	2.00	2.02
				.08	2.22	2.06	2.16
				.09	2.49		
	.363	.66	1.00	.20	1.69		1.75
	.545	1.32	1.96	.07	2.00	1.75	1.96
				.08	2.11		
	.734	1.00	1.32	.20	1.80	1.69	1.82
	.734	1.32	1.96	.12	2.05		
(C)	.182	.44	.66	.30	1.71		1.88
	.182	.66	1.00	.10	1.74	1.69	1.79
				.12	1.88	1.75	1.92
				.14	2.02	1.82	2.04
	.182	1.00	1.32	.05	2.11	1.70	
				.06	2.36	1.77	2.26
				.07	2.67	1.87	2.43
	.363	.66	1.00	.20	1.53	1.62	1.64
				.30	1.96		2.07
		1.00	1.32	.10	1.72	1.58	1.66
				.12	1.87	1.65	1.88
	.363	1.32	1.96	.06	2.22	1.72	2.24
	.545	1.00	1.32	.12	1.56	1.44	1.52
				.14	1.64	1.48	
	.734	.66	1.00	.40	1.46	1.20	1.60
	.734	1.00	1.32	.16	1.47	1.48	
				.18	1.54	1.52	
				.20	1.59	1.73	
	.734	1.20	1.60	.13	2.01	1.91	
	.734	1.32	1.96	.09	1.60		
	.734	1.40	1.80	.10	2.06	2.00	2.01
				.11	2.17		
	.734	1.60	1.96	.07	1.81	1.92	1.95
				.08	1.96	1.98	2.07
	.923	1.40	1.80	.11	2.10		
				.12	2.18	1.91	2.08
				.13	2.30		
	.923	1.60	1.96	.09	1.93	1.86	1.98
	1.021	1.40	1.80	.13	2.08	1.99	
	1.021	1.60	1.96	.10	1.95	1.86	1.99
	1.119	1.40	1.80	.14	2.00		
				.15	2.10	1.49	2.13
				.16	2.20		
	1.119	1.60	1.96	.11	1.93	1.87	2.07
				.12	2.07	1.94	2.13
(D)	.363	1.00	1.32	.10	2.02	1.67	
	.734	1.00	1.32	.16	1.86		
				.18	1.94	1.66	
				.20	2.11	1.72	1.97
Adjusted means					1.88	1.72	1.89

Table 23 contains values of I_R computed for the 9 cases of overlap shown in Table 21 and for 24 cases of simple overlap previously mentioned. Where the overlap is extensive, values of I_R were computed for two or more values of R. Each of these values is based on data of two width-constant series, pertaining to different trough widths, and whenever the extent of such an associated width-constant series permitted, a value of I_w was computed from its data for the same range of slope as that covered by the value of I_R. The table thus comprises data for the coordinate values of the indexes under conditions of constant form ratio and constant width.

The most important generalization from the data in Table 23 is contained in the adjusted means which appear at the bottom. In the derivation of these means due account was taken of the fact that the several individual values from the same overlap are not independent and also of the fact that the coordinate data for constant width are incomplete. The mean for constant form ratio, 1.88, is practically the same as that one for constant width which is associated with the wider channels, 1.89, and is notably greater than the mean associated with the narrower channels, 1.72. On the whole it is indicated that the sensitiveness of capacity to slope is the greater for traction conditioned by constant form ratio, in the proportion of 1.88 to 1.805, or as 1.04 to 1.

When the values of I_R and I_w are arranged according to the associated values of R, the following relations are brought out:

R	Mean I_R.	Mean I_w.		$\frac{I_R}{I_w}$
		Narrower trough.	Wider trough.	
0.05 to 0.08	2.14	1.86	2.10	1.08
.09 to .12	1.93	1.81	1.94	1.03
.14 to .40	1.80	1.63	1.86	1.03

SUMMARY.

Only a small fraction of the observational data are available for the discussion of the capacity-slope relation under the condition of constant form ratio, and the discussion is therefore limited to a comparison of its features under that condition with corresponding features under the condition of constant channel width. The results of such comparison may be summarized as follows: The sensitiveness of tractional capacity to variation of slope is in general greater under the condition of constant form ratio, but the difference is of moderate amount. The difference is somewhat less (at least within the limits of available data) for broad and shallow streams than for streams that are narrow and deep. The range of sensitiveness, or its variation with variation of slope, appears to be somewhat less under the condition of constant form ratio. The generalizations in regard to traction by currents of variable depth but invariable width may be extended, with only moderate qualification, to the case of currents which retain geometric similarity of section while slope is varied.

REVIEW.

With increase of the slope of descent goes increase of a stream's energy (per unit time, per unit distance). With the increase of energy goes increase of capacity for the transportation of débris along the channel bed. The increase of energy is strictly proportional to the increase of slope, but the increase of capacity follows a different law. The law is not simple, but one feature persists through all its manifestations: The capacity for traction increases more rapidly than the slope. The difference in rapidity, or the magnitude of the difference between the rates of change for capacity and slope, is itself a variable, depending on a variety of conditions. The study of the relation of capacity to slope is here treated as a study of the influence of conditions on the magnitude of the difference between the two rates of change.

The magnitude of that difference is indicated by a quantity, of the nature of an exponent, called the index of relative variation (of capacity, as compared to slope), and designated by i_1. The index may be defined as the first differential coefficient of log C with respect to log S. It is illustrated by saying that the capacity varies, instantaneously, as the i_1 power of the slope.

For the greater part of the field covered by the experiments the index falls between 1.4 and 3.0, but under some conditions it is considerably higher. It varies with slope, being higher for low slopes and lower for high. It varies with discharge, being relatively high for small discharges. It varies with fineness, being relatively high for coarse débris. Briefly, it varies inversely with slope, discharge, and fineness. It varies also with width of channel, decreasingly for relatively narrow channels and increasingly for relatively broad channels; so that, for any particular combination of slope, discharge, and fineness there is a width characterized by a minimum value of the index.

It is furthermore true that no one of these variations is itself constant in rate, the rate of each having its own law of variation. Thus the complexity of the relation of capacity to slope is such as to be characterized by variations of the rates of variation of rates of variation; and there are even vistas of higher orders of variability. The law connecting capacity with slope may be susceptible of much more compact expression, but such formulation must probably await the development of a mechanical theory of stream traction.

Formulation founded on the index of relative variation, while bringing out clearly certain general features of the law, is not able to afford a complete quantitative statement. It may be likened to a map in definite hachures as contrasted with one in definite contours. As the hachure tells the direction and rate of slope but omits the absolute altitude, so the index tells the relative change under given conditions but omits the absolute capacity. To remedy this defect, the experiment was tried of substituting for the equation $C=v_1S^{i_1}$, in which v_1 is variable, the equation $C=c_1S^{j_1}$, in which c_1 is constant for each series of observations. Formulation by means of c_1 and j_1 is more nearly analogous to the contour map, but the variability of the exponent j_1 is no less formidable than that of i_1 while the definition and derivation of j_1 are less simple and its significance is less clear.

Further utilizing the analogy of the map, we may think of the capacity-slope relation as an undulating topography, in which the vertical element is $\frac{C}{S}$ or $\frac{f(C)}{f_1(S)}$ and the horizontal elements are qualifying conditions. Formulation is a mode of representing this topography, the hills and valleys of which do not depend on the mode but are real. Two modes have been tried, each with limitations, but the ideal mode is not known. The contour map or the relief model would serve admirably if the qualifying conditions were two only, but as they number at least four, a graphic or plastic expression is possible only in space of n dimensions.

By reason of the complexity of the relation of capacity to slope and because of the lack of a mechanical theory of flow and traction, the laboratory data do not warrant inferences as to the quantitative relations of capacity to slope for rivers.

Of various attempts to evade the complexity, two are thought worthy of record. In each series of experiments the mode of traction changes with increase of slope, first from dune to smooth, then from smooth to antidune, but the critical slopes are not the same for different discharges or widths or degrees of fineness. It appeared possible to gain in simplicity by treating separately the data associated with a single mode of traction, and data for the smooth mode were accordingly segregated and discussed. Greater simplicity was not found, but the range of variation is somewhat smaller for the single mode of traction.

The second attempt was connected with the form of cross section of the current. Within a single series of experiments the width was constant and the depth varied, so that the capacity was conditioned not only by slope but by form ratio, $R=\frac{d}{w}$. By comparing one observational series with another it is possible to obtain data conditioned by difference in slope, but without difference in form ratio. The discussion of such data developed only moderate modification of the results previously obtained and no reduction in complexity.

DUTY AND EFFICIENCY.

For the purposes of this paper duty has been defined as the ratio of capacity to discharge: $U=\frac{C}{Q}$. Combining this with $C=f(S)$, the most general expression for the relation of capacity to slope, we have $U=\frac{f(S)}{Q}$. Under no form discovered for $f(S)$ is this expression reducible to simpler terms. For each value of discharge, duty is simply proportional to capacity; and the entire discussion of this chapter applies to duty as well as capacity. The parameters, n, i_1, j_1, I_w, and σ may be transferred, without modification, to formulas for duty.

Efficiency has been defined as the ratio of capacity to the product of discharge by slope:

$$E=\frac{C}{QS} \qquad (44)$$

The combination of this with

$$C=v_1S^{i_1} \qquad (36)$$

yields

$$E=v_1\frac{S^{i_1}}{QS}=\frac{v_1}{Q}S^{i_1-1} \qquad (45)$$

The transformation of the exponent is important. While capacity varies as the i_1 power of slope, efficiency varies as the i_1-1 power. The index of relative variation is 1 less for efficiency than for capacity. Table 15 could therefore be adapted to efficiency by diminishing all its values by unity. As the lowest known values of i_1 are greater than unity—the lowest in Table 15 is 1.31—it follows that efficiency is an increasing function of slope under all tested conditions.

The combination of (44) with

$$C=e_1S^{i_1} \quad \text{.............} (40)$$

yields

$$E=\frac{e_1}{Q}S^{i_1-1} \quad \text{............} (46)$$

showing that in passing from the field of capacity to that of efficiency the exponent associated with a constant coefficient also is reduced by unity.

Following the form of equation (35), we have

$$I_1=\frac{\log C''-\log C'}{\log S''-\log S'} \quad \text{........} (35a)$$

where C' and C'' are specific values of capacity corresponding severally to the slopes S' and S''. Designating by I_e the synthetic index of efficiency in relation to slope, and by E' and E'' the efficiencies corresponding to C' and C'', we have

$$I_e=\frac{\log E''-\log E'}{\log S''-\log S'} \quad \text{.......} (35b)$$

As $C=ESQ$, $\log C'=\log E'+\log S'+\log Q$, and $\log C''=\log E''+\log S''+\log Q$. Substituting these values in (35a) and reducing, we have

$$I_1=\frac{\log E''-\log E'}{\log S''-\log S'}+1$$

Subtracting the members of this expression from those of (35b) and transposing, we have

$$I_e=I_1-1 \quad \text{.............} (47)$$

That is, the synthetic index of relative variation of efficiency with reference to slope is less by unity than the corresponding index for capacity.

It is evident that at competent slope, when capacity is zero, efficiency also is zero. Like capacity, it increases with increase of slope. Under the assumption that its law of increase is of the same type, its value varying with a power of $(S-\sigma)$, two expressions have been derived, but neither has been found reducible to simple form. The algebraic work being omitted, they are

$$E=v_{\text{II}}(S-\sigma)^{n-\frac{S-\sigma}{S}}$$

$$E=\frac{b_1}{Q}(S-\sigma)^{n-\frac{\log S-2}{\log (S-\sigma)-2}}$$

In the first of these expressions the coefficient is variable, being a function of S; so that the exponent may be regarded as the index of relative variation for efficiency in relation to $(S-\sigma)$. As $\frac{S-\sigma}{S}$ falls to zero when S falls to the limiting value σ, and as it approximates unity when S is indefinitely large, the values of the exponent lie between n and $n-1$ for all practical cases. In the second expression the coefficient is constant, with respect to slope, but the exponent is transcendental and intractable.

Thus it appears that the derived expression for efficiency as a function of $S-\sigma$ is not simply related to the coordinate expression for capacity and is not available for practical purposes; but it does not necessarily follow that the actual relation of efficiency to slope can not be formulated for practical purposes by an equation of the sigma type. All that is really shown is that if capacity and efficiency are both formulated in that way, the results are not consistent. Formula (10) was adopted for the capacity-slope relation, not because it expresses a demonstrated law of relation, but because it so far simulates the real law of relation as to be available for the marshaling of the observational data. It seems quite possible that had the data been first translated from terms of capacity into terms of efficiency, the type of formula would have been found equally available.

By way of testing the matter a few comparative computations were made, observational series being selected for the purpose from those which in the adjustment gave small probable errors. From the original data in Table 4 values of efficiency were computed, and these were plotted on logarithmic paper in relation to $S-\sigma$, the values of σ being those employed in the adjusting equations. In four of the nine cases treated the locus indicated was a straight line; and the drawing of the line gave values of B and n_{II} in $E=B\ (S-\sigma_1)^{n_{II}}$. In each of the remaining five cases the indicated locus was a curve, and the curvature was such as to indicate a larger constant in place of σ. This larger constant, σ_1, was determined graphically, and the other parameters were computed as before. The results are given in Table 23a, together with comparative data from Table 15.

TABLE 23a.—*Comparison of parameters in the associated functions of capacity and efficiency,* $C=b_1(S-\sigma)^n$ *and* $E=B\ (S-\sigma_1)^{n_{II}}$.

Grade.	w	Q	b_1	B	σ	σ_1	$\sigma_1-\sigma$	n	n_{II}	$n-n_{II}$
(B)	0.66	0.182	39.8	196	0.10	0.10	0	1.64	0.67	0.97
(B)	1.00	.363	97.5	268	.08	.18	.10	1.54	.56	.98
(C)	.66	.182	38.6	187	.11	.11	0	1.54	.63	.91
(C)	.66	.545	123	193	.06	.06	0	1.48	.56	.92
(C)	1.00	.363	100	280	.11	.31	.20	1.48	.37	1.11
(C)	1.32	.363	100	240	.16	.22	.06	1.40	.54	.86
(G)	.66	.734	64.5	78	.36	.56	.20	1.69	.74	.91
(G)	1.00	.734	65.7	63	.41	.41	0	1.70	.96	.74
(H)	.66	.734	52.9	58.6	.56	.76	.20	1.72	.80	.92
Means							.09			.92

The proximate inferences from these plots and comparisons are, first, that efficiency may be formulated, with sufficient accuracy for practical purposes, as proportional to a power of $S-\sigma_1$; second, that, when it is thus formulated, the approximate values of σ_1 are in general larger than the values of σ obtained in the formulation of capacity; and, third, that the values of the exponent are smaller than the equivalent values for capacity, the differences usually being somewhat less than unity.

The field of these inferences was also traversed by a mathematical inquiry, of which the results are more definite. If the relation of efficiency to slope be formulated by

$$E=B\ (S-\sigma_1)^{n_{II}} \qquad (47a)$$

the exponent n_{II} is always less than n, but never so small as $n-1$. For the range of conditions covered by the experiments, it is little greater than $n-1$. The value of σ_1 is always greater than the corresponding value of σ, the difference being usually small. The difference is greater when the value of the exponent is relatively small. Equation (47a) is incompatible with the corresponding equation for capacity, (10). If the locus of $E=f(S)$ be separately plotted by means of the two equations, the resulting curves are not coincident, but they intersect at three points and lie close together elsewhere (in the practical field) unless the difference between σ and σ_1 is large.

On the whole, it appears entirely feasible to formulate efficiency by means of equation (47a).

CHAPTER IV.—RELATION OF CAPACITY TO FORM RATIO.

INTRODUCTION.

Details of channel form in a natural stream are highly diversified. In connection with the bendings to right and left the current is thrown to one side and the other, with the result that the cross section is not, for the most part, symmetric about a medial axis but shows greater depth on the side of the swifter flow. In the straight channels of the laboratory there was little departure from bilateral symmetry and the cross section was approximately rectangular. For this reason those relations of traction to form of cross section which are found to exist in the laboratory can not, in general, be inferred of natural streams. Nevertheless there is probably an approximate correspondence between the two types when the tractional properties of a broad, shallow channel are compared with those of a narrow, deep channel; and to that extent the discussion of form ratio ($R=\frac{d}{w}$) is pertinent to the problems of natural streams.

In connection with the study of the laboratory data the form ratio is a factor of great importance, for not only is capacity for traction directly conditioned by it, but it affects every law of relation between capacity and another condition.

In the discussion of capacity in relation to slope the effects which might have been referred to form ratio were treated instead as due to width, while small account was taken of the coordinate influence of depth. For many purposes the choice of viewpoint is indifferent, but when large and small channels are to be compared there is decided advantage in taking account of form ratio. The form ratios of laboratory channels and river channels, for example, are of the same order of magnitude, but the widths are not.

SELECTION OF A FORMULA.

MAXIMUM.

When identical discharges are passed through troughs of different width and are loaded with débris of the same grade, and the loads are adjusted so as to establish the same slope, it is usually found, not only that the capacity varies with the width, but that some intermediate width determines a greater capacity than do the extreme widths. That is, the curve of capacity in relation to width exhibits a maximum. The form ratio varies inversely with the width; and the same maximum appears when the capacity is compared with form ratio. The curves in figure 38, introduced to illustrate this fact, show data from Tables 12 and 14 for grade (C), with $Q=0.363$ ft.³/sec. and $S=1.0$ per cent. In the upper curve capacities are compared with widths; in the lower one the same capacities are compared with form ratios.

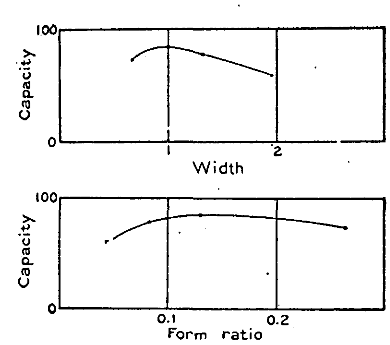


FIGURE 38.—Illustration of the relation of capacity to width of channel and to form ratio, when slope and discharge are constant.

The formula for the discussion of such relations must be one affording a maximum. It must also satisfy various physical conditions, as will presently appear.

The explanation of the maximum, so far as its main elements are concerned, is not difficult. The phenomenon was in fact anticipated in the planning of the experiments, and certain courses of experimentation were arranged with special regard to the discovery of the form ratio of highest efficiency.

Conceive a stream of constant discharge and flowing down a constant slope but of variable width. The field of traction is determined by the width, and the evident tendency of this factor is to make the capacity increase as the width increases. The rate of traction for each unit of width is determined by the bed velocity in that unit, and the bed velocity is intimately associated with the mean velocity. Velocity varies directly with depth, and, inasmuch as increase of width causes (in a stream of constant discharge) decrease of depth, the tendency of this factor is to make capacity decrease as width increases. Velocity is also affected by lateral resistance, the retarding influence of the side walls of the channel. The retardation is greater as the wall surface is greater, therefore as the depth is greater, and therefore as the width is less. As capacity varies inversely with the retardation, and as the retardation varies inversely with width, it follows that the tendency of this factor is to make capacity increase as width increases. Thus the influence of width on capacity is threefold: Its increase (1) enlarges capacity by broadening the field of traction, (2) reduces capacity by reducing depth, and (3) enlarges capacity by reducing the field of side-wall resistance. Now, without inquiring as to the laws which affect the several factors, it is evident that when the width is greatly increased a condition is inevitably reached in which the depth is so small that the velocity is no longer competent and capacity is nil. It is equally evident that when the width is gradually and greatly reduced the field of traction must become so narrow that the capacity is very small, and eventually the current must be so retarded by side-wall friction that its bed velocity is no longer competent and capacity is nil. For all widths between these limits capacity exists, and somewhere between them it attains a maximum.

The forms of algebraic function which afford a maximum are many; but no general examination of them is necessary, because the physical conditions of the problem serve to indicate the appropriate type. As just observed, the variation of form ratio (when discharge and slope are constant) involves simultaneous variations of width and depth. To develop an expression for the relation of capacity to form ratio, it is convenient first to determine separately the relations of capacity to width and to depth, and then to combine the two functions.

CAPACITY AND WIDTH.

To consider separately the response of capacity for traction to variation of width it is necessary to relinquish, for the time being, the assumption of constant discharge and variable depth, and substitute for it the assumption of constant depth and slope, with variable discharge. That is, we are to conceive a stream of constant slope, of which the width is progressively increased or diminished and of which the discharge is varied in such way as to maintain a constant depth. Figure 39 represents the cross section of such a stream, whether natural or of the laboratory type.

Near the sides the current is retarded by side friction. Also, the freedom of its internal movements is restricted by the sides, just as it is everywhere restricted by the upper surface and the bed. These lateral influences diminish with distance from the sides and finally cease to be perceptible. We may thus recognize, in a broad stream, two lateral portions, *AB*, in which capacity is affected by the sides, and a medial portion, *AA*, in which capacity is not thus affected. In the medial portion total capacity is strictly proportional to the distance $\overline{AA}$; or, in other words, the capacity per unit

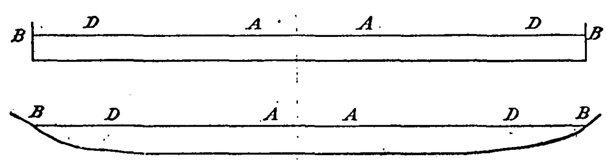


FIGURE 39.—Cross sections of stream channels; to illustrate the relation of capacity to width.

distance, C_1, is uniform. In a lateral portion the capacity per unit distance diminishes as the side is approached. Whatever the law of diminution, the total capacity of a lateral portion is equivalent to the capacity per unit distance in the medial portion, multiplied by some distance $\overline{AD}$, less than $\overline{AB}$. Therefore the total capacity for the whole stream is

$$C = C_1\,(\overline{AA} + 2\overline{AD}) = C_1\,(w - 2\overline{DB}) \quad \text{.......(48)}$$

It is evident that for a shallow stream the distances $\overline{AB}$ and $\overline{DB}$ are less than for a deep stream; and while the assumption may not be strictly accurate, it must be approximately true that $\overline{DB}$ is proportional to the depth. Making that assumption and introducing the numerical constant α, I replace $2\overline{DB}$ by αd, and write

$$C = C_1\,(w - \alpha d)$$

As we are here concerned only with the law of variation of C, we may conveniently replace this by the proportion

$$C \propto w - \alpha d.$$

Substituting for d its equivalent Rw,

$$C \propto w\,(1-\alpha R).$$

As d is by postulate constant, and as $w=\frac{d}{R}$, $w \propto \frac{1}{R}$. We may therefore substitute $\frac{1}{R}$ for w in the proportion above, obtaining

$$C \propto \frac{1-\alpha R}{R} \quad \text{............(49)}$$

This expression gives the relation of capacity to form ratio, so far as that relation depends on variation of width. Eventually it is to be complemented by an expression similarly dependent on variation of depth.

The preceding analysis involves the assumption that the stream is so broad, in relation to its depth, that its medial portion is unaffected by lateral influences. The resulting proportion, (49), is not necessarily applicable to narrower streams. It is quite conceivable that when the channel is so narrow that the reaction of the sides affects all parts of the current the variation of capacity follows a different law. The analytic consideration of the case of narrower channels has not been attempted, but some information has been obtained from the experiments. The following examination of experimental data is directed toward this question and also toward that of the magnitude of the constant α.

TABLE 24.—*Relation of capacity for traction to width of channel, when slope and depth are constant.*

Grade.	Slope (per cent).	Depth (feet).	Value of C when width (feet) is—					
			0.23	0.44	0.66	1.00	1.32	1.96
(B)	0.8	0.08	....	....	8.4	[17.4]	20.4	43
		.10	....	....	15	26.6	35.4	66
		.12	....	8.6	21.0	38.0	57.5	95
		.15	....	15.2	36.7	60.0	112	145
		.20	2.8	32.0	62.5	113	....	....
	1.2	.08	....	....	29	50	88	142
		.10	....	22.1	46	74	156	211
		.12	....	32.0	65	103	....	290
(C)	.8	.10	....	....	12.5	22.0	25.0	42
		.12	....	....	19.2	34	....	76
		.14	....	[5.3]	27	51	78	135
		.20	....	....	52	111	....	....
	1.2	.10	....	....	45	80	109	171
		.12	....	....	[64]	115	151	[300]
		.14	....	20.0	81	156	221	480
			Geometric means.					
(B) and (C).	0.8 and 1.2	.08	....	....	15.6	29.4	42.4	78.0
		.10	....	....	29.6	50.7	75.2	115
		.12	....	....	36.1	62.5	....	158
		.14 } .15 }	....	11.7	43.1	78.2	124	211

The assumptions of the present section include constant slope and constant depth, with discharge and capacity adjusted to variation of width. The experiments involve constant width and constant discharge, with automatic adjustment of slope and depth to variation of load. In order to check the analysis by means of the laboratory data it is necessary to employ some method of interpolation. Two methods were tried, but only one need be described.

Attention being first restricted to a particular grade of débris and a particular slope of channel, the computation sheets (p. 95) for the different discharges were entered with a particular depth as argument, and the associated values of capacity and slope were taken out. These values were plotted on logarithmic section paper as a series of points. Through these points was drawn a curve—the locus of $\log C = f(\log S)$, under the condition that d is constant. By means of this curve values of C were interpolated, corresponding to selected values of S. The process was then repeated with other depths, other widths, and other grades; and in this way were obtained sets of values of capacity in relation to width, under the condition of constant depth and slope. Such interpolated values of capacity are presented in Table 24. It was found that the data for grades (B) and (C) only are full enough to serve the present purpose.

The tabulated capacities are also plotted, in relation to width, in the upper and second divisions of figure 40. If the data were precise, and if equation (48) were strictly accurate, the

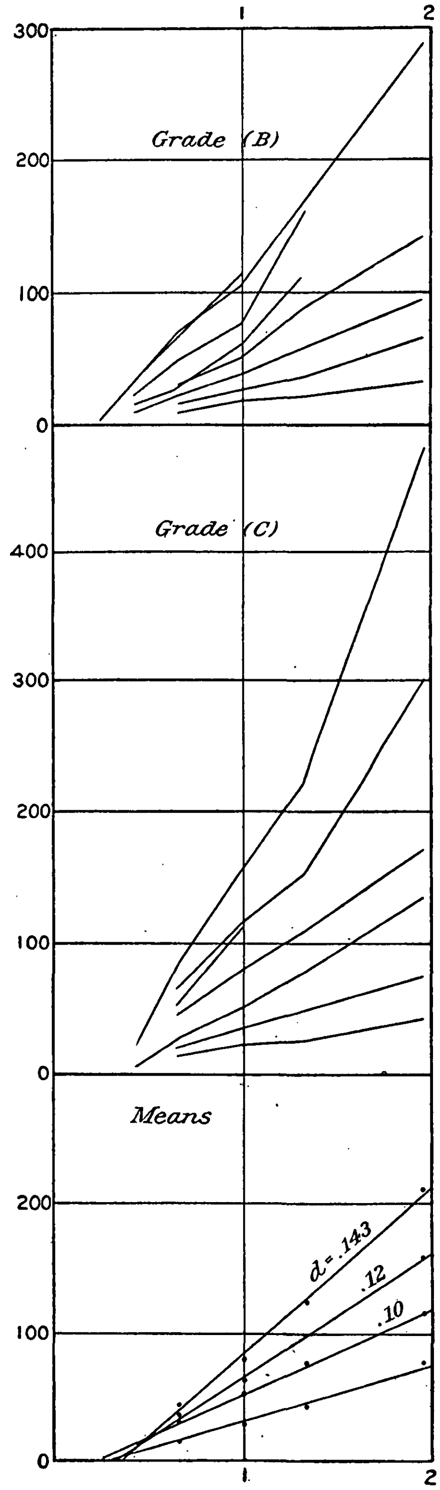


FIGURE 40.—Capacity for traction in relation to width of channel, when depth and slope are constant. Scale of capacities, vertical; widths, horizontal.

oblique lines of the figure would all be straight and would all intersect the line of zero capacity somewhere to the right of the origin. The irregularities of the lines are of such distribution as to indicate that they are occasioned chiefly by the imperfection of the data, and so far as may be judged by their inspection the formula is substantially correct.

As the individual lines do not well indicate the points of intersection with the horizontal axis, a set of composites were prepared, each combining the data for a particular depth, without distinction as to grade of débris or slope of channel. In the computations for these a few interpolations were first made, and then the capacities were combined by taking their geometric means. The numerical results appear at the bottom of Table 24, and these are represented by dots in the lower division of figure 40. The indications of the dots were then generalized by drawing straight lines among them, and the intersections of these lines with the line of zero capacity gave points corresponding to D in figure 39. More strictly, the distance of each intersection from the origin gave an estimate of the quantity $2\ \overline{DB}$ in equation (48). As each estimate is associated with a particular depth, and as $\alpha = \frac{2\ \overline{DB}}{d}$, the intersections give also values of the constant α in (49).

d	$2\ \overline{DB}$	α
0.08	0.30	3.7
.10	.24	2.4
.12	.325	2.7
.143	.36	2.5

As the plotted dots are so irregular as to admit of much latitude in the drawing of the lines, these values of the constant are far from precise. No inference may be drawn from their differences, and collectively they serve only to indicate an order of magnitude. For fine sand, with slopes of about 1 per cent, the constant α has a probable magnitude of 2 or 3.

CAPACITY AND DEPTH.

To consider separately the response of capacity to variation of depth, the assumption of constant discharge and variable width must again be laid aside, and there must be substituted for it the assumption of constant width and slope, with variable discharge. That is, we are to conceive a stream of constant width, of which the discharge and load are simultaneously varied in such way as to maintain the slope unchanged, and the changes in depth are to be compared with the changes in load or capacity. To make the results strictly coordinate with those for the control of capacity by width, we should deal only with channels so broad and shallow that the lateral portions (fig. 39) are small in comparison with the medial portions, but this is not practicable. It is possible, however, to minimize the influence of lateral retardation by selecting groups of data in which the form ratio is small.

Table 25 contains data pertaining to débris of grade (C), a channel width of 0.66 foot, and a slope of 1 per cent. The values of capacity are taken from Table 12 and the values of depth from Table 14. It appears by inspection that the capacity increases with the depth. On plotting the pairs of values on logarithmic section paper, it is found that they may be represented approximately by a straight line. The examination of many such plots showed that the most accurate representative line has a gentle curvature, but for the present purposes it suffices to assume that the line is straight. That is to say, it is found to be approximately true, and the assumption is made, that C varies with some power of d, or

$$C \propto d^{m_1} \quad \text{..............} (50)$$

In the particular case $m_1 = 1.90$, and other values of m_1 are shown in Table 26.

TABLE 25.—*Values of capacity and depth for currents transporting débris of grade (C), when the width is 0.66 foot and the slope 1.0 per cent.*

Discharge (ft.³/sec.)	0.093	0.182	0.363	0.545	0.734
Capacity (gm./sec.)	13.8	32.1	73	112	152
Depth (foot)	.079	.110	.173	.228	.255

TABLE 26.—*Values of m_1 in $C \propto d^{m_1}$, when slope is constant.*

Grade.	Slope. (per cent).	Value of m_1 when width (feet) is— 0.66	1.00	1.32	1.96
(C)	0.6	2.20	2.50	3.10	3.41
	1.0	1.90	2.10	2.32	2.97
(G)	1.0	2.67	2.96	3.95	....
	1.6	2.05	3.43	2.83	....
	2.4	1.80	2.26	2.48	....

Comparison of the tabulated values of m_1 shows that they have considerable range and that their variations in magnitude are definitely related to several conditions. The sensitiveness of capacity to variation of depth (when width and slope are constant) is greater as the slope is less, is greater as the fineness is less, and, with one exception, is greater as the width is greater:

$$m_1 = f\,(\grave{S},\ \grave{F},\ \acute{w}) \quad \text{..........} (51)$$

As w is by postulate constant, and as $d = w\,R$, it follows that $d \propto R$. R may therefore be substituted for d in proportion (50), giving

$$C \propto R^{m_1} \quad \text{..............} (52)$$

This is an expression for the relation of capacity to form ratio, so far as that relation depends on variation of depth. It is the complement of proportion (49) on page 126.

CAPACITY AND FORM RATIO.

We now return to the assumption of constant discharge. Having obtained an approximate expression for the law of capacity's variation in response to change of width and a coordinate expression for variation on response to change of depth, we next inquire how these may be combined into an expression for the response of capacity to simultaneous changes of width and depth, when those changes are of such character as to leave discharge constant.

If the functions (49) and (52) be assumed to be independent, their simple and direct combination gives

$$C \propto \frac{1 - \alpha R}{R} R^{m_1} \quad \text{..........} (53)$$

This formula, as may readily be shown, satisfies an important condition by providing for a maximum value of C, but the assumption involved in its construction is not strictly warranted. The law embodied in $\frac{1-\alpha R}{R}$ is based on the constancy of d and may not be valid when d is made variable, while the law embodied in R^{m_1} may not be valid when w is made variable.

Viewing the matter from another side, we may say that the difficulty would not exist if the values of α and m_1 depended only on fineness, slope, and discharge and were independent of R; but they are in fact functions of R. (We have already seen that m_1 is an increasing function of w, and the variation of α with d

will appear in another connection.) An inquiry as to the nature of the functions—an inquiry here unrecorded except as to its result—served to determine that the variations of α and m_1, as functions of R, affect capacity in opposite senses, so that their influences are at least partly compensatory. It appeared also that each influence is relatively great when R is large and relatively small when R is small, so that their laws of distribution include a compensatory factor.

Despite the existence of this difficulty, which is palliated rather than cured by features of compensation, the function in (53) has been adopted as the best practicable formula for the relation of capacity to form ratio. The logical defect in the combination of its two factors may mean that its accuracy is inferior to that of the factors; but as the factors are confessedly only approximate, the defect is not inconsistent with the possession of even superior accuracy by the combination. In the treatment of so intricate a subject by methods which are dominantly empiric altogether adequate formulation is not to be hoped for; but the modicum of physical foundation afforded to formula (53) is believed to give it advantage over a purely mathematical expedient.

A slight transformation gives it more convenient form. Moving R from the denominator to the numerator and making $m_1 - 1 = m$, we have

$$C \propto (1 - \alpha R)\ R^m$$

It is convenient also to change from a proportion to an equation; introducing a coefficient, b_2,

$$C = b_2\ (1 - \alpha R)\ R^m \quad \text{--------(54)}$$

As R is a ratio between lengths, and α (p. 125) is also a numerical quantity without dimensions, b_2 is of the unit of C. It is the value of capacity when $(1 - \alpha R)\ R^m = 1$.

Let us now consider the properties of the formula. For the sake of giving a visible illustration, it has been applied to the example already used in figure 38. As the equation has three parameters, its constants require for their determination three pairs of values of R and C. The example furnishes four pairs, and these, by approximate adjustment, give

$$C = 346\ (1 - 2.18\ R)\ R^{0.52} \quad \text{------(55)}$$

The corresponding curve is shown in figure 41.

This conforms to the physical conditions by indicating two values of R for which capacity is nil, and an intermediate value for which capacity is at maximum. On referring to (54), it is evident that $C=0$ when $R=0$, and also when $1 - \alpha R = 0$, or $R = \frac{1}{\alpha}$.

In ascribing finite capacity to all small values of R the formula is inaccurate, for when the form ratio is gradually reduced the velocity must always fall below competence before the ratio reaches zero. The true function might be represented by some such curve as the broken line D in the figure. It has not seemed advisable to complicate the formula by a modification which might remedy this defect.

In the region of the larger values of R the formula is subject to a qualification already mentioned on page 125. The larger values are associated with narrow channels, in which the

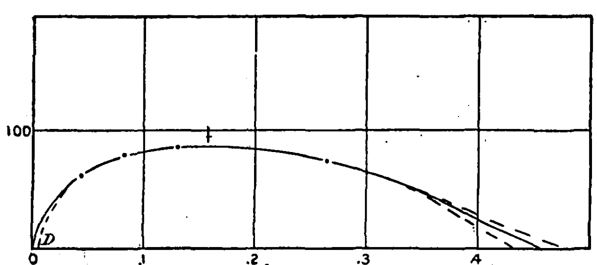


FIGURE 41.—Plot of equation (55). Capacity, vertical; form ratio, horizontal.

influences of the side walls or banks are dominant. So far as the laboratory data give indication, the formula is applicable to these values; but the adjustment might be less satisfactory with channel walls of a different character.

The region of the maximum value of capacity, which constitutes the chief field for the application of the formula, is little affected by the qualifications which have been mentioned.

Differentiating equation (54) and equating the first differential coefficient with zero, we have

$$b_2\ m\ R^{m-1} - b_2\ \alpha\ (m+1)\ R^m = 0$$

whence $R = \frac{1}{\alpha} \frac{m}{m+1}$, which is the condition giving C its maximum value. Designating this value of R by ρ, we have

$$\rho = \frac{1}{\alpha} \frac{m}{m+1} \quad \text{------------(56)}$$

$$\alpha = \frac{1}{\rho} \frac{m}{m+1} \quad \text{------------(57)}$$

Substituting from (57) into (54), we have

$$C=b_2\left(1-\frac{R}{\rho}\frac{m}{m+1}\right)R^m \text{.....(58)}$$

Equations (54) and (58) are alternative expressions of the same relation, the one involving α without ρ, the other ρ without α. Each has its field of superior convenience, alike for computations and discussion.

To obtain the maximum capacity, ρ is substituted for R in (58). The equation reduces to

$$C_{max}=\frac{b_2}{m+1}\rho^m \text{..........(59)}$$

DISCUSSION OF EXPERIMENTAL DATA.

SCOPE AND METHOD OF DISCUSSION.

The equations adopted for the formulation of $C=f(R)$, namely, equations (54) and (58), involve four constants. b_2 is a quantity of the unit of capacity; ρ is a ratio, the form ratio corresponding to maximum capacity; α is a ratio connected with side-wall resistance; and m is an exponent. There is a mutual dependence between α and ρ; but b_2, α, and m, grouped together in (54), are independent; and so are b_2, ρ, and m, grouped in (58). In the following discussion of the relation of capacity to form ratio, equations of the form of (54) and (58) are derived from groups of experimental data; and these are compared in such way as to show the control of their constants by the conditions of slope, discharge, and fineness.

A "group" of experimental data, for this purpose, includes values of capacity and form ratio from at least three observational series, all pertaining to the same fineness, slope, and discharge, but to different widths. Three pairs of observational values suffice to determine the three independent parameters; with a greater number the problem is usually one of adjustment, to determine the most probable values of the parameters.

The computation of the constants by algebraic methods is tedious, and a graphic method was substituted. When an equation of the form of (58) is plotted on logarithmic section paper the shape and size of the curve are determined wholly by the exponent m: its position measured in the direction of the axis of log R is determined by the value of ρ; and its position with respect to the axis of log C by the value of b_2. A graphic process based on these properties gave solutions of sufficient approximation for the purposes of the discussion.

SENSITIVENESS AND THE INDEX OF RELATIVE VARIATION.

The sensitiveness of capacity to variation of form ratio is indicated graphically by the inclination of the logarithmic locus $C=f(R)$. As that locus is a curve, the sensitiveness varies with R. The form of the curve, as mentioned in the last paragraph, is determined by the exponent m, and the steepness (see fig. 42) of its legs varies directly with m. The exponent is thus a general index of sensitiveness.

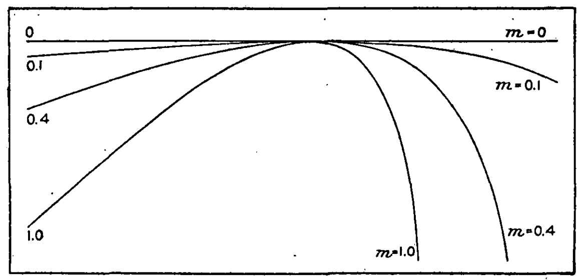


FIGURE 42.—Logarithmic plots of $C=b_2\left(1-\frac{R}{\rho}\frac{m}{m+1}\right)R^m$, corresponding to the same value of b_2 and ρ, but different values of m.

The logarithmic equivalent of (54) is

$$\log C=\log b_2+\log(1-\alpha R)+m\log R$$

Differentiating, we have

$$d\log C=d\log(1-\alpha R)+m\,d\log R$$

Dividing by $d\log R$, we have

$$\frac{d\log C}{d\log R}=\frac{d\log(1-\alpha R)}{d\log R}+m$$

Making substitutions from

$$\frac{d\log C}{d\log R}=i_2$$

$$d\log(1-\alpha R)=\frac{-\alpha dR}{1-\alpha R}$$

and

$$d\log R=\frac{dR}{R}$$

and reducing, we have

$$i_2=m-\frac{\alpha R}{1-\alpha R} \text{.........(60)}$$

It is evident that $\frac{\alpha R}{1-\alpha R}$ increases with increase of R; therefore i_2 decreases with increase of R. Also i_2 is positive when $m > \frac{\alpha R}{1-\alpha R}$ and negative when $m < \frac{\alpha R}{1-\alpha R}$. In passing from positive to negative, i_2 passes through zero when $m = \frac{\alpha R}{1-\alpha R}$ or when $R = \frac{1}{\alpha}\frac{m}{m+1} = \rho$ (cf. equation 56). When $R=0$, the limiting condition for a broad, shallow stream, $\frac{\alpha R}{1-\alpha R} = 0$ and $i_2 = m$. As this condition may only be approached but not reached, the positive values of the index are all less than m. When $\alpha R = 1$, or $R = \frac{1}{\alpha}$, the condition limiting traction for a narrow, deep stream, $\frac{\alpha R}{1-\alpha R} = +\infty$ and $i_2 = -\infty$. The negative values of the index therefore range to infinity.

The progressive changes of the index are conveniently illustrated by the characters of the curves in figure 42. Consider, for example, the curve corresponding to $m=1.0$. As this curve is the logarithmic locus of $C=f(R)$, its inclination to the horizontal represents i_2. At the extreme left, corresponding to a very small value of R (a shallow, wide stream), its inclination is a little less than 45°; or the value of i_2 is a little less than 1.0, the value of m. As the curve ascends i_2 is positive. The inclination then diminishes gradually and becomes zero at the apex of the curve, which corresponds to $R=\rho$. As the curve begins its descent the forward inclination corresponds to negative values of i_2, and these values increase numerically, becoming very large as the curve approaches verticality.

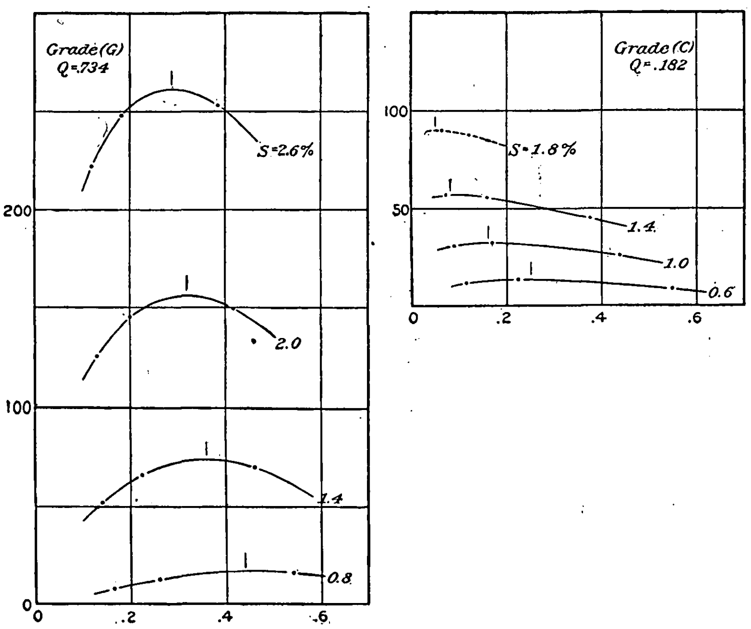


FIGURE 43.—Relation of capacity, C, to form ratio, R. The variation of the function $C=b_2(1-\alpha R)R^m$ with slope. Scale of C vertical; scale of R horizontal.

CONTROL OF CONSTANTS BY SLOPE.

To illustrate the way in which the index of sensitiveness and other constants defining the relation of capacity to form ratio are influenced by the condition slope, two sets of computations were made. The first set used data pertaining to débris of grade (C) and to a discharge of 0.182 ft.^{3}/sec.; the second, data for grade (G) and 0.734 ft.^{3}/sec. The observational data and the computed constants appear numerically in Table 27 and graphically in figure 43. The dots in the figure represent points fixed by the observations, and through these have been drawn curves conforming to formulas (54) and (58). Short vertical lines note the positions of the maxima, where $R=\rho$.

On referring to the table, it is seen that as the slope increases the values of m, ρ, and α decrease, while the value of b_2 increases. With steeper slopes of channel, capacity is relatively less sensitive to changes in the form of cross section; with steeper slopes, the form of section giving highest capacity for traction is relatively shallow and broad; with steeper slopes, the reduction of capacity through retardation of the current by the channel sides is relatively small; and the general effect of steepened slopes is increased capacity.

Each set of values of a constant exhibits an orderly progression, marred only by minor irregularities due to the inaccuracies of graphic computation. The results recorded in Tables 28 and 29 are much less orderly. It is to be noted that the remarkable orderliness in the present case is not due to high precision of the data but to the fact that the values of C and R assembled in Table 27 had previously been adjusted, as described in Chapter II, in a manner which made them orderly with respect to slope.

TABLE 27.—*Observed and computed quantities illustrating the influence of slope on the relation of capacity to form ratio.*

Grade and discharge (ft.³/sec.).	Observational data.				Constants of equations.				i_2
	Slope (per cent).	Width (feet).	Form ratio.	Capacity (gm./sec.)	m	ρ	α	b_2	
(C) 0.182	0.6	0.44	0.546	8.7					−1.55
		.66	.222	12.9	0.44	0.25	1.22	34.4	+ .07
		1.00	.113	11.4					+ .28
	1.0	.44	.436	24.7					− .65
		.66	.167	32.1	.20	.158	1.05	55.5	− .01
		1.00	.086	31.2					+ .10
	1.4	.44	.375	45.3					− .46
		.66	.138	57.0	.08	.079	.94	75.2	− .07
		1.00	.072	57.7					+ .01
	1.8	.66	.120	87	[.05	.05	.95	110]	− .08
		1.00	.063	90					− .01
(G) 0.734	.8	.66	.540	16.0					−1.42
		1.00	263	13.3	1.55	.44	1.38	170	+1.07
		1.32	.164	7.9					+1.26
	1.4	.66	.460	69					− .50
		1.00	.222	65	.80	.36	1.23	296	+ .42
		1.32	.141	52					+ .59
	2.0	.66	.418	149					− .32
		1.00	.199	145	.55	.32	1.11	451	+ .27
		1.32	.128	125					+ .41
	2.6	.66	.387	253					− .26
		1.00	.184	249	.45	.29	1.07	662	+ .20
		1.32	.120	222					+ .30

CONTROL OF CONSTANTS BY DISCHARGE.

Table 28 and figure 44 exhibit quantities and curves illustrating the influence of discharge on the relation of capacity to form ratio. The data employed are those of grade (C) with a channel slope of 1 per cent, and of grade (G) with a channel slope of 2 per cent. Of the seven curves in the figure, four were determined by three observational points. As three is the minimum number of given points for the determination of a curve of this type, each curve passes through all the points. In each of the other cases four points are given by the observations, and it was not found possible to pass one of the curves through all the points. If the formulas are correct, the incompatibility indicates errors of the data. Errors of similar magnitude inferably affect the three-point groups of data, and this inference qualifies the determination of the curves and their equations. Of the same general tenor is the fact that the four curves under grade (C), pertaining to discharges which increase in arithmetic progression, do not exhibit an orderly progression as to shape.

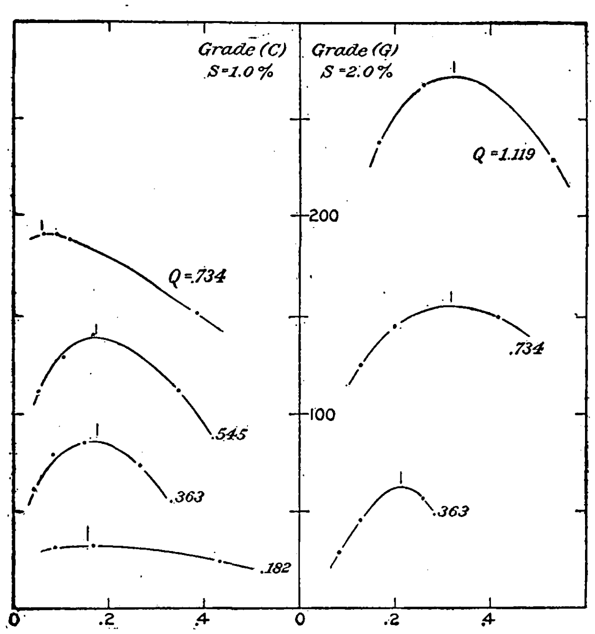


FIGURE 44.—Relation of capacity, C, to form ratio, R. The variation of the function $C=b_2(1-\alpha R)R^m$ with discharge. Scale of C vertical; scale of R horizontal.

TABLE 28.—*Observed and computed quantities illustrating the influence of discharge on the relation of capacity to form ratio.*

Grade and slope (per cent).	Observational data.				Constants of equations.				i_2
	Discharge (ft.³/sec.).	Width (feet).	Form ratio.	Capacity (gm./sec.).	m	ρ	α	b_2	
(C) 1.0	0.182	0.44	0.436	24.7					−0.65
		.66	.167	32.1	0.20	0.158	1.05	55.5	−.01
		1.00	.086	31.2					+.10
	.363	.66	.264	73					−.84
		1.00	.130	85					+.13
		1.32	.082	79	.52	.157	2.18	340	+.30
		1.96	.043	60					+.42
	.545	.66	.346	112					−.91
		1.00	.170	140	.40	.174	1.64	392	+.01
		1.32	.104	129					+.19
		1.96	.054	111					+.30
	.734	.66	.386	152					−.38
		1.00	.190	180	.05	.061	.78	229	−.12
		1.32	.120	187					−.05
		1.96	.063	190					−.00
(G) 2.0	.363	.66	.260	56					−1.15
		1.00	.128	45.2	1.52	.215	2.80	1,611	+.96
		1.32	.082	28.3					+1.22
	.734	.66	.418	149					−.32
		1.00	.199	145	.55	.320	1.11	451	+.27
		1.32	.128	125					+.41
	1.119	.66	.532	227					−.96
		1.00	.260	268	.58	.322	1.14	.830	+.16
		1.32	.165	238					+.35

The same incongruities of course characterize the tabulated numerical results. Increase of discharge is accompanied, in the main, by diminution of m and α, but the indications are inconclusive as to ρ and b_2.

CONTROL OF CONSTANTS BY FINENESS.

With reference to the influence of fineness of débris, the comparative data are restricted to three grades. Table 29 and figure 45 record the data and results for grades (B), (C), and (G), with use of a single slope and a single discharge. The curves for grades (B) and (C), which fall close together, are so different in form as to intersect; and the difference shown by the forms is otherwise expressed by incongruities among the values of constants. With the exception of b_2, the constants for grade (B) are intermediate between those of the other grades, whereas the intermediate grade is (C). If we ignore this incongruity and compare the constants for grades (B) and (C) with those for the coarse grade (G), we find that m, ρ, and α are all relatively small for the fine grades, or vary inversely with fineness, while b_2 varies directly with fineness. The variations of m and ρ are more pronounced than that of α.

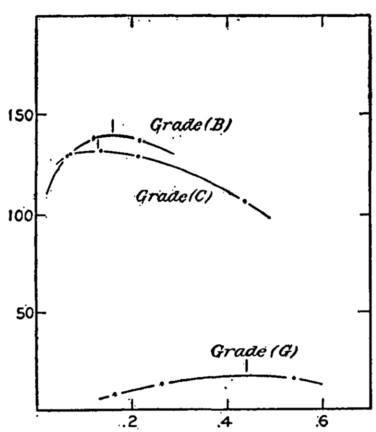


FIGURE 45.—Relation of capacity, C, to form ratio, R. The variation of the function $C = b_2 (1 - \alpha R) R^m$ with fineness of débris. Scale of C vertical; scale of R horizontal.

TABLE 29.—*Observed and computed quantities illustrating the influence of the fineness of débris on the relation of capacity to form ratio.*

Slope (per cent) and discharge (ft.³/sec.).	Observational data.				Constants of equations.				i_2
	Grade and fineness (F_2).	Width (feet).	Form ratio.	Capacity (gm./sec.).	m	ρ	α	b_2	
0.8	(B)	1.00	0.217	138					−0.09
0.734	13,400	1.32	.120	139	0.20	0.16	1.04	242	+.06
		1.96	.064	130					+.13
	(C)	.66	.440	107					−.48
	5,460	1.00	.214	130	.12	.126	.85	191	−.10
		1.32	.134	133					−.01
		1.96	.070	131					+.06
	(G)	.66	.540	16					−1.42
	5.9	1.00	.263	13.3	1.55	.44	1.38	170	+1.07
		1.32	.164	7.9					+1.26

SPECIAL GROUP OF OBSERVATIONS.

The special group of observations recorded in Table 4 (I), page 51, were arranged largely for the purpose of defining ρ, the optimum form ratio. They differed from the main body of observations in that the interval between the discharges employed and the interval between the widths employed were both smaller; and they were restricted to a single grade of débris. They had the advantage of an experimental method believed to be the best developed in the laboratory; and in view of this advantage their series were constituted of fewer individual observations than those of the main body of experiments. The results have not been satisfactory, and attempts at formulation in the present connection have developed marked incongruities. In figure 46 three curves are given, and the corresponding numerical data appear in Table 30. Each curve is based on five observational points, but they are so irregularly placed that their control is feeble.

On comparing the two cases having the same discharge, it is seen that the greater slope is associated with the smaller values of m, ρ, and α and with the greater value of b_2, the difference being most strongly marked for m and ρ. On comparing the two cases having the same slope, it is seen that the greater discharge is associated with the smaller values of m and ρ and with the larger value of b_2, while the values of α are nearly equal.

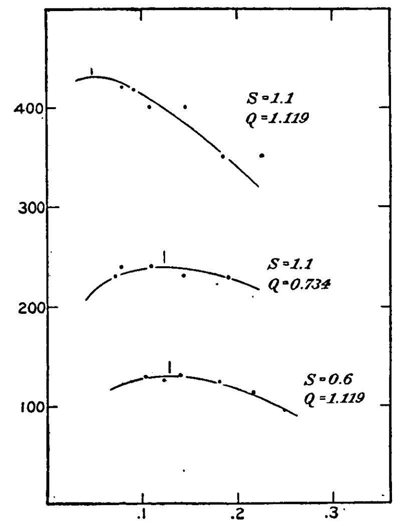


FIGURE 46.—Relation of capacity, C, to form ratio, R. Variation of the function $C=b_2(1-\alpha R)R^m$ with slope and discharge. Data from special group of experiments with débris of grade (C). Scale of C vertical; scale of R horizontal.

TABLE 30.—*Observed and computed quantities illustrating the influence of slope and discharge on the relation of capacity to form ratio.*

Grade.	Discharge (ft.³/sec.).	Observational data.				Constants of equations.				i_2
		Slope (per cent).	Width (feet).	Form ratio.	C	m	ρ	α	b_2	
(C)	0.734	1.1	1.0	0.190	230					−0.25
			1.2	.143	240					− .07
			1.4	.108	240	0.3	0.123	1.88	586	+ .05
			1.6	.079	231					+ .12
			1.8	.072	230					+ .14
	1.119	.6	1.2	.216	114					− .72
			1.4	.180	125					− .35
			1.6	.140	131	.5	.129	2.58	546	− .06
			1.8	.122	125					+ .05
			1.96	.102	130					+ .15
	1.119	1.1	1.2	.185	350					− .34
			1.4	.145	400					− .28
			1.6	.108	400	.1	.048	1.90	643	− .16
			1.8	.091	418					− .11
			1.96	.078	420					− .07

SUMMARY AS TO CONTROL BY CONDITIONS.

The treatment of the observational data by means of a formula specially designed to show the relation of capacity for traction to the proportions of the cross section develops incongruities. These are of such distribution as to indicate that they are due in chief part to the observations and their methods of adjustment. Discrepancies which manifestly pertain to the data are so large that it is not practicable to determine whether the imperfections of the formula are important.

The exponent m varies inversely with slope, with discharge, and with fineness. Thus all the conditions which tend to increase capacity tend also to make capacity less sensitive to changes in form ratio.

The optimum form ratio, ρ, varies inversely with slope and with fineness. As to its variation with discharge the evidence is not unanimous; either it varies inversely under some circumstances and directly under others, or else its proper variation is inverse and data of contrary import are erroneous. The latter view is thought more probable, because in many other connections the controls of slope and discharge follow parallel lines.

The constant α, which represents the resistance of side walls or banks to the flow of the stream, is also a decreasing function of slope, discharge, and fineness.

The constant b_2, which is of the unit of capacity, varies directly with slope and inversely with fineness, and the evidence as to its variation with discharge is conflicting. As b^2 is the value of capacity when

$$(1-\alpha R)\ R^m=1, \text{ or } \left(1-\frac{R}{\rho}\,\frac{m}{m+1}\right) R^m=1$$

it corresponds to a rather complicated relation between R, m, and α, or R, m, and ρ; and this relation makes the interpretation of the lack of order among its tabulated values a difficult matter.

If b_2 is left out of the account, it is possible to generalize by saying that the constants of equations (54) and (58) vary decreasingly with the conditions which affect capacity increasingly.

$$m=f(\grave{S},\ \grave{Q},\ \grave{F}) \qquad (61)$$

$$\rho=f_{\text{I}}(\grave{S},\ \grave{Q},\ \grave{F}) \qquad (62)$$

$$\alpha=f_{\text{II}}(\grave{S},\ \grave{Q},\ \grave{F}) \qquad (63)$$

THE OPTIMUM FORM RATIO.

The ratio of depth to width which gives to a stream its greatest capacity for traction is of importance to the engineer whenever he has occasion to control the movement of débris. The title *optimum ratio* is especially appropriate when his desire is to promote that movement.

The range of values for the ratio, under laboratory conditions, is from 1:2 to 1:20. One effect of this wide range, when taken in connection with the variety of conditions by which the ratio is controlled, is to complicate the formulation of practical rules; but this difficulty is not insuperable. It is qualified to an important degree by the consideration that capacity, in the region of its maximum, changes very slowly with change of form ratio, so that an approximate determination of the ratio has practical value.

The values of the ratio given in Table 31 are appropriate to the conditions of the Berkeley laboratory—that is, they pertain to troughs a few inches or a few feet wide, with smooth vertical sides. It is important to note also that they apply only to transportation of débris over a bed of débris, and not to flume traction, which has a different law. (See p. 213.)

TABLE 31.—*Estimated ratios of depth of current to width of trough, to enable a given discharge, on a given slope, to transport its maximum load.*

Material transported.	Slope (per cent).	Ratio for discharge (ft.^{3}/sec.) of—			
		0.25	0.50	0.75	1.00
Fine sand	0.5	1:4	1:6	1:8	1:9
	1.0	1:6	1:9	1:12	1:15
	2.0	1:10	1:16	1:20	....
Coarse sand or fine gravel	1.0	....	....	1:25	1:3
	2.0	....	1:25	1:3	1:5
	3.0	1:2	1:3	1:4	1:7

No way has been found to extend the quantitative results to rivers. It can hardly be questioned that the optimum ratio for rivers varies inversely with slope, discharge, and fineness of débris, but its absolute amount can not be inferred from the experimental results. River slopes are relatively very small and river discharges are relatively very large, and the two differences affect the ratio in opposite ways. To compute the joint result we should have definite and precise information as to the laws of dependence, but our actual knowledge is qualitative and vague.

In this connection it is of interest to record a single observation on river efficiency. Where Yuba river passes from the Sierra Nevada to the broad Sacramento Valley its habit is rather abruptly changed. In the Narrows it is narrow and deep; a few miles downstream it has become wide and shallow. Its bed is of gravel, with slopes regulated by the river itself when in flood, and the same material composes the load it carries.

In the Narrows the form ratio during high flood is 0.06 and the slope is 0.10 per cent. Two miles downstream the form ratio is 0.008 and the slope is 0.34 per cent. Thus the energy necessary to transport the load where the form ratio is 0.008 is more than three times that which suffices where the form ratio is

0.06; and it is evident that the larger ratio is much more efficient than the smaller. The data do not serve to define the optimum form ratio, but merely show that it is much greater than 0.008. In this instance the slopes and fineness are of the same order of magnitude as those realized in the laboratory, but the discharges are of a higher order.

When water without detrital load is conveyed by an open rectangular conduit, the form ratio of highest efficiency is that which yields the highest mean velocity. It is approximately 1:2. This corresponds to the maximum value of the optimum ratio for traction, and the correspondence might have been expected on theoretic grounds. The two factors which, in ultimate analysis, determine capacity for traction are velocity of current along the bed and width of bed. When discharge and slope are such as barely to afford competence with the most favorable form ratio, that ratio is one giving the highest velocity, namely, 1:2. The other factor, width of bed, is evidently favored by lower values of R; and therefore, as the conditions recede from the limit of competence, the optimum form ratio becomes smaller. This line of reasoning might, in fact, have been used to show a priori—what has actually been shown by the experiments—that the value of ρ varies inversely with slope, discharge, and fineness.

SUMMARY.

Capacity for traction varies with the depth of the current, being approximately (though not precisely) proportional to a power of the depth. Capacity varies also with the width of the current, being approximately proportional to the width less a constant width. This constant width is equivalent to the product of the depth by a numerical constant. When the discharge is constant, any change of width causes a change of depth and also a change of form ratio, $R=\frac{d}{w}$. A formula for the variation of capacity in relation to form ratio, when the discharge is constant, is based on the above-mentioned properties and takes the form

$$C=b_2\ (1-\alpha R)\ R^m \qquad (54)$$

in which α is a numerical constant; or

$$C=b_2\left(1-\frac{R}{\rho}\,\frac{m}{m+1}\right)R^m \qquad (58)$$

in which ρ is the optimum form ratio, or the form ratio giving the highest capacity.

That capacity should have a maximum value corresponding to some particular value of form ratio is made to appear from theoretic considerations, and the fact of a maximum is shown by the experimental data. The same data show that the optimum form ratio has different values under different conditions, its values becoming smaller as slope, or discharge, or fineness increases.

The sensitiveness of capacity to the control of form ratio is indicated in the formulas by the exponent of R, and that also varies with conditions. It becomes smaller as slope, or discharge, or fineness increases.

It is believed that all the generalizations from the laboratory results may be applied to natural streams, but only in a qualitative way; the disparity of conditions is so great that the numerical results can not be thus applied.

CHAPTER V.—RELATION OF CAPACITY TO DISCHARGE.

FORMULATION AND REDUCTION.

As a condition controlling the capacity of a stream for the traction of débris, discharge is the coordinate of slope. Each of the two factors is proportional to the potential energy of the stream, on which traction depends; and the control of each is exercised through the control of velocity. Their fundamental difference in relation to traction is connected with depth of current. When velocity is augmented by increase of slope, the depth is reduced; when it is augmented by increase of discharge, the depth is increased. Notwithstanding this difference, the relations of capacity to discharge parallel those of capacity to slope to a remarkable extent. Thanks to this parallelism, the discussions of the present chapter may be based in considerable part on those which have preceded.

The data for the comparison of capacity with discharge are contained in Table 12. In each division of that table assigned to a grade of débris are a number of subdivisions pertaining severally to particular widths of channel. Each column of such a subdivision pertains to a particular discharge and contains a series of adjusted capacities corresponding to an orderly series of slopes. The values of capacity connected with the same slope and comprised in the same subdivision constitute a group illustrating the relation of capacity to discharge. Table 32 contains in its upper part a number of such groups, selected and arranged for the present purpose. So far as practicable they pertain to the same slope, but it was not possible to secure absolute uniformity in this respect and at the same time make the representation include data of all the grades of débris. For grades (A) to (E) the slope is 1.0 per cent; for grades (F) to (H), 1.2 per cent.

TABLE 32.—*Data on the relation of capacity to discharge, with readjusted values of capacity, C_r, and values of the index of relative variation, i_3.*

Conditions: Grade / Slope (per cent) / Width (feet)		(A) 1.0 .66	(A) 1.0 1.00	(A) 1.0 1.32	(A) 1.0 1.96	(B) 1.0 .23	(B) 1.0 .44	(B) 1.0 .66	(B) 1.0 1.00	(B) 1.0 1.32	(B) 1.0 1.96
	Q	C	C	C	C	C	C	C	C	C	C
Data	0.093	[14.8]	....	....	....	7.8	13.2	10.8	....	....	....
	.182	39.5	37.5	36.8	....	13.3	28.7	33.5	30.1	29.3	....
	.363	....	100	104	96	....	....	81	85	79	67.6
	.545	140	....	....	....	....	....	120	143	140	120
	.734	....	231	250	240	....	....	....	199	204	190
	1.119	....	....	....	359	....	....	....	....	....	313
Adjusting equations	κ	.028	.048	.067	.105	.002	.017	.033	.057	.080	.125
	o	1.08	1.05	1.09	1.00	.79	1.00	1.13	1.12	1.06	1.08
	b_3	290	370	389	367	49.5	170	270	313	313	312
	Q	C_r	C_r	C_r	C_r	C_r	C_r	C_r	C_r	C_r	C_r
Readjusted capacities	0.093	15.1	....	....	....	7.8	13.2	11.1	....	....	....
	.182	38.5	37.5	37.0	....	13.3	28.7	31.6	30.3	28.5	....
	.363	89	99	103	96	....	....	77	83	83	67.6
	.545	142	167	174	159	....	....	127	140	140	123
	.734	....	231	250	234	....	....	....	201	201	185
	1.119	....	....	....	370	....	....	....	....	....	311
	Q	i_3	i_3	i_3	i_3	i_3	i_3	i_3	i_3	i_3	i_3
Index of relative variation	0.093	1.54	....	....	....	0.81	1.21	1.75	....	....	....
	.182	1.28	1.43	1.72	....	.80	1.10	1.38	1.63	1.89	....
	.363	1.17	1.21	1.33	1.41	....	....	1.24	1.33	1.36	1.64
	.545	1.14	1.15	1.24	1.24	....	....	1.20	1.25	1.24	1.40
	.734	....	1.12	1.20	1.17	....	....	....	1.21	1.19	1.30
	1.119	....	....	....	1.10	....	....	....	....	....	1.21
Probable error (per cent)	C	....	....	....	....		....	4.8	1.4	2.4	1.4
	C_r	....	....	....	....	....	....	2.4	0.7	1.2	0.7

TABLE 32.—*Data on the relation of capacity to discharge, with readjusted values of capacity, C_r, and values of the index of relative variation, i_3*—Continued.

Conditions	Grade	(C)	(C)	(C)	(C)	(C)	(D)	(D)	(D)	(E)
	Slope (per cent)	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	Width (feet)	.44	.66	1.00	1.32	1.96	.66	1.00	1.32	.66
	Q	C	C	C	C	C	C	C	C	C
Data	0.093	10.9	13.8	...	...		9.1	...	...	...
	.182	24.7	32.1	31.2	21.4	...	29.8	24.0	...	...
	.363	...	73	85	79	60	...	73	59.2	24.8
	.545	...	112	140	129	111	101	108	...	...
	.734	...	152	180	187	190		153	131	40
	1.119	...	...	276	...	343	...	...	...	...
Adjusting equations	κ	.020	.041	.071	.099	.156	.059	.102	.143	.108
	o	1.02	.94	1.00	1.05	1.14	.90	.88	.81	...
	b_3	157	213	285	305	354	195	224	201	...
	Q	C_r	C_r	C_r	C_r	C_r	C_r	C_r	C_r	C_r
Readjusted capacities	0.093	10.9	13.2	...	...	...	9.2	...	...	...
	.182	24.7	33.9	33.0	22.0	...	29.6	24.3	...	...
	.363	...	74	84	75	59	67	69	59.2	...
	.545	...	112	135	130	120	102	110	97	...
	.734	...	151	187	190	190	...	151	131	...
	1.119	...	...	299	...	340	...	...	...	...
	Q	i_3	i_3	i_3	i_3	i_3	i_3	i_3	i_3	i_3
Index of relative variation	0.093	1.30	1.68	...	...	...	2.46	...	...	...
	.182	1.14	1.21	1.64	2.30	...	1.33	2.00	...	...
	.363	...	1.06	1.24	1.44	2.00	1.07	1.22	1.33	...
	.545	...	1.02	1.15	1.28	1.60	1.01	1.08	1.10	...
	.734	...	1.00	1.11	1.21	1.45	...	1.02	1.01	...
	1.119	...	...	1.07	...	1.32	...	...	...	...
Probable error (per cent)	C	...	2.2	3.6	2.7	2.9	...	2.4	...	...
	C_r	...	1.0	1.6	1.3	1.5	...	1.2	...	...
Conditions	Grade	(E)	(E)	(F)	(F)	(F)	(G)	(G)	(G)	(H)
	Slope (per cent)	1.0	1.0	1.2	1.2	1.2	1.2	1.2	1.2	1.2
	Width (feet)	1.00	1.32	.66	1.00	1.32	.66	1.00	1.32	.66
	Q	C	C	C	C	C	C	C	C	C
Data	0.093	...	...	...	...	...	...	...	...	...
	.182	14.8	...	9.3	4.2	...	...	...	...	...
	.363	33.8	36.3	29.9	22.8	21.4	14.8	9.7	...	3.1
	.545	...	...	...	...	...	...	...	...	...
	.734	72	73	55.3	63	61.6	48.3	44.0	33.8	24.6
	1.119	138	123	...	112	101	75	92	81	43.5
Adjusting equations	κ	.188	.263	.139	.242	.339	.199	.345	.433	.233
	o	...	...	...	...	...	.94	...	...	1.24
	b_3	...	...	...	...	...	.83	...	...	.54
	Q	C_r	C_r	C_r	C_r	C_r	C_r	C_r	C_r	C_r
Readjusted capacities	0.093	...	...	...	...	...	...	...	...	...
	.182	...	...	...	...	...	...	...	...	...
	.363	...	...	...	...	...	15.1	...	...	4.2
	.545	...	...	...	...	...	...	...	...	...
	.734	...	...	...	...	...	46	...	...	22.5
	1.119	...	...	...	...	...	76	...	...	46
	Q	i_3	i_3	i_3	i_3	i_3	i_3	i_3	i_3	i_3
Index of relative variation	0.093	...	...	...	...	...	...	...	...	...
	.182	...	...	...	...	...	...	...	...	...
	.363	...	...	...	...	...	2.04	...	...	3.46
	.545	...	...	...	...	...	...	...	...	...
	.734	...	...	...	...	...	1.29	...	...	1.82
	1.119	...	...	...	...	...	1.22	...	...	1.57
Probable error (per cent)	C	...	...	...	...	...	...	...	...	...
	C_r	...	...	...	...	...	...	...	...	...

The same data are plotted in figure 47, where horizontal distances represent logarithms of discharge and vertical distances logarithms of capacity. For each of the above-mentioned "groups" the plotted points are connected by a series of straight lines, and each of the broken lines thus produced is the rough logarithmic graph of an equation $C=f(Q)$. The graphs are arranged according to grades of débris, and secondarily according to widths of channel, their order from left to right corresponding to the sequence from narrower to broader channels. In effecting this arrangement graphs were moved bodily up or down but not to the right or left.

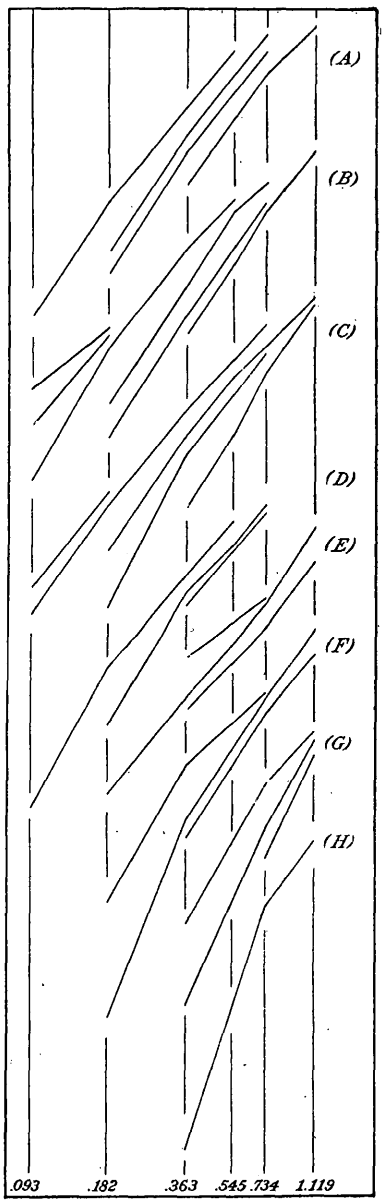


FIGURE 47.—Logarithmic plots of the relation of capacity to discharge. The horizontal scale is that of log Q, the vertical of log C. The zeros of log C for the different plots are not the same.

It appears by inspection that the graphs bend toward the right as they ascend. To this rule there are a few exceptions, but the only strongly marked exceptions are connected with grade (E), the data for which have previously been recognized as anomalous. As the inclination of each line indicates the sensitiveness of capacity to the control of discharge, the general bending to the right, or the reduction of inclination in passing from lower to higher discharges shows that sensitiveness diminishes as discharge increases. This feature is similar to one observed in studying the relation of capacity to slope, and as that feature was found to be connected with competence, the resemblance leads at once to the suggestion that here also is a connection with competence.

If a very small discharge be made to flow over a sloping bed of débris and the discharge be gradually increased, transportation of débris will commence when the competent discharge is reached and will increase with further increase of discharge. It is a plausible hypothesis that the capacity is more simply related to the excess of discharge above the competent quantity than to the total discharge. Following the procedure in the case of capacity and slope we may assume that capacity is proportional to a power of the excess of discharge above a constant discharge, the constant discharge being closely related to competent discharge. In the following formula, constructed on the plan of equation (10),

$$C=b_3\,(Q-\kappa)^o \quad \text{..........} \quad (64)$$

κ is a constant discharge and b_3 is a constant numerically equal to the value of capacity when the discharge equals $\kappa+1$, although it is not strictly a capacity. The dimensions of capacity are $M^{+1}\,T^{-1}$, of discharge $L^{+3}\,T^{-1}$, and of $(Q-\kappa)^o$ $L^{+3o}\,T^{-o}$; and these values give to b_3 the dimensions $L^{-3o}\,M^{+1}\,T^{o-1}$.

As a preliminary to the adjustment of the observational data of Table 32 by this formula, values of κ were graphically computed by the method previously employed in connection with σ. The computations were applied to all groups of values of capacity in the table, except such as comprise less than three capacities and except also the aberrant data of grade (E). In Table 33 the results are arranged with reference to grade of débris and channel width.

TABLE 33.—*Values of the discharge constant, κ, computed from the discharges and capacities of Table 32.*

Grade.	Value of κ when width of channel (in feet) is—			
	0.66	1.00	1.32	1.96
(A)	....	0.070	0.075	0.215
(B)	0.056	.095	.035	.000
(C)	.025	.100	.030	.060
(D)	.060	.130	....	....
(F)	.149	.130	.180	....
(G)	.280	.160	....	....
(H)	.326	....	....	....

The tabulated values of κ show great irregularity. It is not difficult, however, to recognize a tendency to grow larger in passing from finer débris to coarser; and there is also, though it is ill defined, a tendency to increase with increasing width. The cause of the first-mentioned tendency is readily understood, because relatively coarse débris requires a relatively swift current to move it; and the reality of the second also finds support when the conditions affecting competent discharge are considered.

Postulating, initially, a channel of some particular width, containing a stream of which the discharge is barely competent, let us assume that the width is increased. In spreading to the new width the stream loses depth and velocity, and its velocity is no longer competent. That the velocity may again become competent the discharge must be increased. Thus it is in general true that competent discharge increases with increase of width. In the cross section of a broad laboratory current, figure 48, a medial portion, *AA*, is unaffected

FIGURE 48.—Ideal cross section of a stream in the experiment trough, illustrating the relation of competent discharge to width.

by side-wall resistance and has competent velocity. Two lateral portions, *AB*, have less than competent velocity. Let us imagine these lateral portions replaced by narrower divisions, *AD*, in which velocities are the same as in *AA*. The effective width for the maintenance of competent velocity is then $\overline{DD} = w - 2\,\overline{BD}$. If now width and discharge be increased or diminished, with maintenance of competent velocity, the quantity $2\,\overline{BD}$ is unaffected, so that it may be regarded as a constant. Velocities being, by hypothesis, uniform through the whole space *DD*, the discharge is proportional to the width of that space.

$$Q_c \propto w - \text{a constant} \quad \text{(65)}$$

This result assumes a channel so wide that a medial portion is unaffected by side-wall resistance. It may not be true for narrower channels. If it were to be refined, the constant would be found to be a function of depth (compare p. 129); but in applying its principle to the values of κ no allowance was made for that factor.

On the theory that κ is closely related to competent discharge and has similar properties, and with the assumption that (65) may be applied to narrow channels as well as broad, the principle of (65) was used in adjusting the tabulated values of κ in relation to width of channel. The formula assumed was

$$\kappa \propto w - 0.2$$

The value of the constant was arbitrarily fixed, after several trials, no criterion for selection being discovered except the harmony of results.

With the aid of this formula it was possible to combine the data of Table 33 in such way as to afford a better view of the relation of κ to grade, or fineness; and this was done. By multiplication or division, each value was reduced to its equivalent for a channel width of 1 foot, and means were taken. These means appear in the second line of Table 34. They were compared with the mean diameters of particles for the several grades, the comparison being made on logarithmic section paper. In figure 49, showing this plot, the numbers indicate weights. While the plotted points do not fall well into line, they leave no question that the value of κ rises as the débris becomes coarser. On the assumption that the function is a power function, a straight line was drawn to represent it; and by this line the mean values of κ were adjusted. Values for the other trough widths were then computed. These appear in Table 32.

TABLE 34.—*Adjustment of system of values of κ for a channel width of 1 foot and slopes of 1.0 and 1.2 per cent.*

Grade	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
Mean diameter of particles (foot)	0.00100	0.00123	0.00166	0.00258	0.00561	0.01040	0.01620	0.02300
Mean κ computed from Table 33	.074	.078	.047	.117	....	.188	.367	.925
Adjusted κ	.048	.057	.071	.102	.188	.242	.345	.406

In making these adjustments due account was taken of the fact that not all the data used pertain to the same slope. Allowance for slope difference was made in plotting the values of κ to adjust for fineness, and again in applying the results of that adjustment. The adjustments for slope are based on the following consideration.

By use of the data in Table 12, values of κ pertaining to the same grade and width but differing as to slope were compared. In one case κ appeared to increase with increase of slope, but in most cases it appeared to decrease.

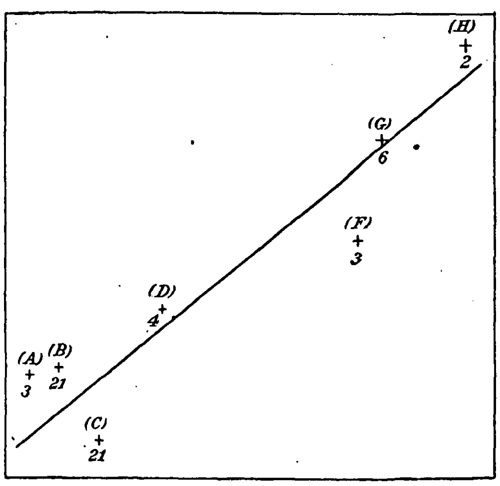


FIGURE 49.—Logarithmic plot of $\kappa = f(D)$.

Little weight, however, was given to such determinations, because it was evident that they were controlled by the values of σ, which had been somewhat arbitrarily assigned in the first adjustment of the observations. A more satisfactory indication was obtained by reasoning based on the relative sensitiveness of capacity to the several controls of slope and discharge. The index of relative variation of capacity with respect to slope being i_1, we may designate the corresponding index with respect to discharge by i_3. As capacity varies simultaneously with S^{i_1} and Q^{i_3} it is evident that for any particular value of capacity S^{i_1} and Q^{i_3} vary inversely. The proportion $Q^{i_3} \propto \frac{1}{S^{i_1}}$ yields

$$Q \propto \frac{1}{S^{\frac{i_1}{i_3}}}$$

From a discussion of the relations of the two indexes, to be found on a later page of the present chapter, it appears that the average ratio of i_1 to i_3 is 1.36 but that the ratio varies somewhat with conditions. For the condition $C=0$ it is assumed to be 1.3; or $\kappa \propto \frac{1}{S^{1.3}}$.

While considerable uncertainty attaches to the numerical relations, there can be no question that κ varies directly with width and inversely with fineness and slope. As any change in width affects the form ratio in the opposite sense, we may write

$$\kappa = f(\grave{S}, \acute{F}, \grave{R}) \qquad (66)$$

A system of values of κ having been thus arranged, logarithmic plots were made of the capacities of Table 32, in relation to $Q-\kappa$, and straight lines were drawn representing equations of the type of (64). In cases where the plotted points did not fall well in line, recourse was had to weights based on the probable errors recorded in Table 12. The lines thus drawn gave, by their inclinations and intersections, the values of the other parameters, o and b_3, as well as the adjusted values of capacity, C_r, all of which are presented in Table 32. The plots for data of grades (E) and (F) and for two of the three groups under grade (G) exhibited so great irregularities that it was decided not to use them.

The similarity of equation (64) to equation (10) makes it unnecessary to repeat the reasoning by which (38) was educed, and the equation for the index of relative variation for capacity in relation to discharge may be written directly:

$$i_3 = \frac{oQ}{Q-\kappa} \qquad (67)$$

By its use the values of i_3 in Table 32 were computed.

MEASURES OF PRECISION AND THEIR INTERPRETATION.

In 16 of the columns of Table 32 the values of capacity before and after adjustment constitute data suitable for the computation of probable errors. As the number of residuals is in each case small, the particular values of probable errors (recorded at the bottom of the table) are not themselves of high precision; but the averages have greater claim to attention. It should be added that the residuals were treated as of equal weight, despite the fact that in the adjustment which determined them the relative weights of the observational data were recognized.

The probable errors of the upper line, marked "C," pertain to the adjusted values of capacity taken from Table 12, which are here treated as observations. The probable errors of the lower line, marked "C_r," pertain to the groups of readjusted values of capacity.

The average probable error (or, strictly, the mean of the nine tabulated errors) of the readjusted capacities is ± 1.3 per cent. The corresponding average for the capacities before readjustment is ± 2.64 per cent.

It is of interest to compare the last figure with the previously computed average probable error of the adjusted values of capacity in Table 12, namely, ± 2.50 per cent. The earlier estimate applies to 66 series of values, the later to 36 values taken from 36 of the series. The 36 values may be regarded as properly representative of the series from which they come. Of the unrepresented series, a portion escaped because the values they furnished to Table 32 fell in groups of less than four, and such a group did not afford suitable data for computation of probable error. The others were omitted because the groups they constituted involved incongruities so great that they were rejected in the readjustment. The omission of incongruous groups evidently had the effect of lowering the estimate of average probable error, and to give validity to the comparison due allowance should be made for that effect. The discarded values were accordingly treated so far as necessary to compute their probable errors, and it was found that by the inclusion of these the estimate of average probable error was raised from 2.64 to 3.2 per cent. The revised estimate is believed to be properly comparable with the earlier estimate of 2.50 per cent. It will be recalled that the original observations were characterized by accidental errors, ascribed chiefly to rhythm, and by systematic errors, ascribed chiefly to methods of observation. The nature of the first adjustment was such that its computed probable errors were little affected by the errors of the second class. The adjustment was conditioned by a formula involving the assumption that capacity varies as a power of $(S-\sigma)$, and also by various assumptions involved in the arrangement of a system of values for σ. Whatever errors were introduced in connection with these assumptions tended to increase the estimates of probable error; but they may also be supposed to have aggravated somewhat the errors of the class not covered by the computations of probable error.

The errors falling outside the estimates of probable error were largely of such nature as to affect an observational series in its entirety, and it was expected that they would be revealed in the failure of groups of quantities taken from different series to exhibit an orderly sequence. Abundant evidence of their existence has been encountered in various discussions, including the control by conditions of the sensitiveness of capacity to slope, of the sensitiveness of capacity to form ratio, of the value of the optimum form ratio, and of the value of the constant κ; but the present discussion is the only one affording an estimate of their magnitude.

Assuming that the estimate of 2.5 per cent represents the influence of a restricted class of errors, and that the estimate of 3.2 per cent represents the joint influence of that class and a second class, the independent influence of the second class is represented by

$$\sqrt{3.2^2-2.5^2}=\pm\ 2.0 \text{ per cent}$$

The indication is that the two classes of errors are of nearly equal importance.[1]

These results are qualified by the fact that the estimate of 3.2 per cent includes not only the errors of the adjusted values of capacity in

[1] The considerations making it probable that ± 2.5 per cent is an overestimate for the average error of the first class (p. 74) do not apply to the estimate of ± 3.2 for the combined error. If the average error for the first class is as low as ± 2.0 per cent, the computed average for the second class becomes ± 2.5 per cent.

Table 12, but also whatever errors affect the method of adjustment in connection with the discussion of the control of capacity by discharge. That method includes the assumption that capacity varies as a power of $(Q-\kappa)$ and also certain assumptions as to the control of κ by various conditions. If the errors involved in those assumptions could be eliminated or discriminated, the general estimate for the values in Table 12 would be somewhat reduced.

As the chief result of this discussion, the general precision of the main body of material contributed by the Berkeley experiments is characterized by an average probable error slightly in excess of 3 per cent.

This estimate applies specifically to the adjusted capacities of Table 12 as those capacities are related to slope and discharge, and it can not be extended to derivatives of those capacities without qualification. It is believed that the precision of the readjusted capacities of Table 32 is higher, and also that of the values of I_1 in Table 23, but that the values of i_1, j_1, and i_3, in Tables 15, 16, and 32, rank lower.

In the adjustment of the observations on capacity in relation to slope, and also in the adjustment of observations on depth and slope, many cases were treated in which the observations were either not sufficiently numerous or not sufficiently harmonious to afford good control of the parameters of the adjusting equations. In such cases the parameters were estimated in groups, with orderly sequences of values. A similar method was employed also in readjustments in relation to discharge. While this procedure appeared, and still appears, to be the best practicable, it can hardly fail to introduce a certain amount of error. The terms of the adjusted sequences are inevitably associated with different form ratios; and the laws connecting form ratio with capacity are so different from the other laws of the system as to determine sequences less simple than those actually used. The ideal adjustment would take simultaneous cognizance of the complicated interrelations of capacity, slope, discharge, and form ratio; but such comprehensive treatment can not be attempted with profit until we have a better theoretic knowledge of the physical reactions.

CONTROL OF RELATIVE VARIATION BY CONDITIONS.

The sensitiveness of capacity to changes of discharge is indicated by i_3, the index of relative variation. Inspection of the values of that index recorded in Table 32 shows that they vary inversely with discharge and directly with width, and both these tendencies are also to be inferred from the lines of figure 47.

The table further indicates that the rate of change in the index in response to change of discharge is greater for small discharges than for large. This feature might also have been inferred from the plotted lines, but its systematic expression in the table is a product of the formula, $i_3=\frac{oQ}{Q-\kappa}$, by which the values were computed.

The relation of the index to width of channel is not similarly dominated by the formula, although somewhat influenced by the assignment of values of κ. The variation of the index with width is in general more pronounced for low discharges than for high, but in the data for high discharges occur two exceptions to the general law. These exceptions are ascribed to irregularities in the data. As the width for any particular discharge varies inversely with the form ratio, it follows that the index is a decreasing function of form ratio.

The relation of the index to slope is not shown by the table. It was the subject of a special inquiry, including 32 comparisons. In each comparison a value of the index for a particular slope was contrasted with the value for a slope twice as great, the other conditions being the same. In 25 instances the greater index was associated with the smaller slope; in 7 instances with the larger. The mean of the indexes computed for smaller slopes was 1.51; the mean for larger slopes 1.17. The general law appears to be that the index varies inversely with slope. The seven instances of opposite tenor are all associated with large discharges; and their occurrence is ascribed to a systematic error connected with the assignment of values to κ.

Table 35 compares values of the index with fineness. Despite irregularities, it is evident that the values tend to increase in passing from finer to coarser grades—that is, their variation in respect to fineness is inverse.

TABLE 35.—*Values of the exponent i_3 arranged to show variation in relation to fineness of débris.*

Grade.	Values of i_3.				
	$Q=0.363$			$Q=0.182$	
	$w=0.66$	$w=1.00$	$w=1.32$	$w=0.66$	$w=1.00$
(A)	1.17	1.21	1.33	1.28	1.43
(B)	1.24	1.33	1.36	1.38	1.63
(C)	1.06	1.24	1.44	1.21	1.64
(D)	1.05	1.25	1.33	1.30	2.02
(G)	2.04				
(H)	3.46	....	....	....	....

In summary, the sensitiveness of capacity to variation of discharge is greater as slope, discharge, form ratio, and fineness are less.

$$i_3=f\,(\grave{S},\ \grave{Q},\ \grave{F},\ \grave{R})\ \text{..........}\ (68)$$

DUTY AND EFFICIENCY.

The variation of capacity with discharge is indicated in general terms by an equation of the type of (33):

$$C=v_3Q^{i_3}\ \text{..............}\ (69)$$

Duty being the quotient of capacity by discharge, this gives

$$U=\frac{v_3Q^{i_3}}{Q}=v_3Q^{i_3-1}\ \text{..........}\ (70)$$

and, as efficiency is the quotient of capacity by discharge and slope,

$$E=\frac{v_3}{S}\ Q^{i_3-1}\ \text{............}\ (71)$$

That is, the index of relative variation for both duty and efficiency, in relation to discharge, is less by unity than the corresponding index for capacity. Therefore the values of the index in Table 32 need only to be reduced by unity to apply to duty and efficiency.

Under ordinary conditions the index for duty and efficiency falls between unity and zero; or, in other words, duty and efficiency increase with increase of discharge, but their increase is less rapid than that of discharge. Exceptionally the increase is much more rapid, the exceptions being associated with discharges little above the limit of competence. On the other hand, there appear to be conditions under which the index falls below zero, so that duty and efficiency diminish with increase of discharge. The diminution indicated by the figures in the column (of Table 32) for grade (D) and width 0.66 foot is only of the order of magnitude of the probable error; but a pronounced diminution would be inferred from the values of the index for grade (B) and width 0.23 foot. As the results from the last-mentioned group of observations stand by themselves in various respects, some reservation is felt in regard to them, and there is at least room for doubt whether the diminution is actually demonstrated.

With respect to all conditions the variations of the index for duty and efficiency follow the same laws as the index for capacity; but, as a consequence of the uniform reduction by unity, the rates of variation are higher. If, for example, in passing from a smaller to a larger discharge, the index for capacity falls from 1.40 to 1.20, a reduction of one-seventh, the index for efficiency falls from 0.40 to 0.20, a reduction of one-half.

Lines of reasoning strictly parallel to those employed in the last section of Chapter III yield the following conclusions:

The synthetic index of relative variation for the duty of water in relation to discharge is less by unity than the corresponding synthetic index for capacity in relation to discharge.

The synthetic index of relative variation for efficiency in relation to discharge is less by unity than the corresponding synthetic index for capacity in relation to discharge.

If the duty of water, or if efficiency, be assumed to vary as some power of $Q-\kappa$, the exponent of that power (expressing the instantaneous rate of variation) equals $o-\frac{Q-\kappa}{Q}$. As discharge increases from κ toward infinity, the exponent diminishes from o toward $o-1$.

If the relation of efficiency to discharge (and similarly for the relation of duty to discharge) be expressed by

$$E=B_1(Q-\kappa_1)^{o_{II}}\ \text{..........}\ (71a)$$

the value of o_{II} is always less than the corresponding value of o, the difference approaching but not exceeding unity. The value of κ_1 is always greater than the corresponding value of κ, usually much greater. It was found by trial that, within the range of conditions realized in the laboratory, the difference between values of efficiency computed directly by means of (71a) and values computed indirectly by means of (64) is not large, its order of magnitude being that of the probable errors.

The control of duty and efficiency by discharge is further considered in the following section.

COMPARISON OF THE CONTROLS OF DISCHARGE AND SLOPE.

CONTROLS OF CAPACITY.

We are now in position to compare the influences exerted by slope and discharge, severally, on capacity. The general fact brought out by the comparison is that capacity is more sensitive to changes of slope than to changes of discharge, but the difference in sensitiveness is not the same for all conditions.

TABLE 36.—*Comparison of the index of relative variation, i_1, for capacity and slope, with the index, i_3, for capacity and discharge.*

[For grades (A) to (D) the data are for S=1.0; for grades (G) and (H) for S=1.2.]

Grade.	Q	w=0.44		w=0.66		w=1.00		w=1.32		w=1.96	
		i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3
(A)	0.182			1.99	1.28	2.06	1.43	2.17	1.72		
	.363					1.83	1.21	1.70	1.33	1.90	1.41
	.545			1.48	1.14						
	.734					1.79	1.12	1.71	1.20	1.55	1.17
	1.119									1.31	1.10
(B)	.093	2.31	1.21	2.34	1.75						
	.182	1.81	1.10	1.82	1.38	1.92	1.63	1.94	1.89		
	.363			1.58	1.24	1.67	1.33	1.75	1.36	2.01	1.64
	.545			1.49	1.20	1.62	1.25	1.73	1.24	1.87	1.40
	.734					1.61	1.21	1.71	1.19	1.66	1.30
	1.119									1.60	1.21
(C)	.093	2.39	1.30	1.88	1.68						
	.182	1.88	1.14	1.73	1.21	1.87	1.64	2.37	2.30		
	.363			1.59	1.06	1.59	1.24	1.67	1.44	2.14	2.00
	.545			1.58	1.02	1.47	1.15	1.55	1.28	1.87	1.60
	.734			1.57	1.00	1.45	1.11	1.47	1.21	1.62	1.45
	1.119					1.48	1.07			1.85	1.32
(D)	.093			2.22	2.46						
	.182			1.80	2.33	2.01	2.00				
	.363					1.69	1.22	2.11	1.33		
	.545			1.64	1.01	1.87	1.08				
	.734					1.65	1.02	1.85	1.01		
(G)	.363			2.98	2.04						
	.734			2.42	1.29						
	1.119			2.29	1.22						
(H)	.363			6.03	3.46						
	.734			3.23	1.82						
	1.119			2.78	1.57						

In Table 36 values of the index of relative variation are brought together from Tables 15 and 32. The selection includes all such as correspond in respect to débris, trough width, slope, and discharge, with the exception of those of trough width 0.23 foot, which appear to be anomalous. There are 64 pairs of values.

Of the 64 comparisons, 62 show capacity as more sensitive to slope, 2 as more sensitive to discharge. The two exceptional cases are from experiments with débris of grade (D) and with channel width 0.66 foot; and the data from those experiments were reexamined in search for an explanation of what seems an anomaly. No explanation was found, and, as the observations are supported by the estimates of precision, it remains probable that there are real exceptions to the general rule.

The means of the 64 values of i_1 and i_3 are, severally, 1.93 and 1.42; and the ratio of the first to the second is 1.36. On the average, the sensitiveness of capacity to slope is one-third greater than the sensitiveness of capacity to discharge.

To ascertain the variation of the ratio $\frac{i_1}{i_3}$ with discharge, the values in Table 36 were specially grouped for the taking of partial averages. The first group gave comparative ratios for discharges of 0.093 and 0.182 ft.^{3}/sec., by means of four sets of index values, each set agreeing as to all conditions other than discharge. The second group gave comparative ratios for discharges of 0.182, 0.363, and 0.734 ft.³/sec., by means of five sets of index values; and two other groups made other comparisons, as shown in Table 37. The upper division of the table gives mean values of i_1 and i_3; the lower division, their ratios. It will be observed that within each group the values of the indexes decrease as discharge increases, while the values of the ratio, as a rule, increase with the increase of discharge. To the first rule there are no exceptions; the exceptions to the second are not so important as to leave the principle in doubt.

TABLE 37.—*Variations of the indexes i_1 and i_3, and their ratio, in relation to discharge.*

Group.	Sets.	Q=0.093		Q=0.182		Q=0.363		Q=0.545		Q=0.734		Q=1.119	
		i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3
1	4	2.23	1.48	1.81	1.21								
2	5			2.02	1.74	1.73	1.30			1.65	1.16		
3	8					1.76	1.41	1.69	1.26	1.59	1.19		
4	7					2.67	1.97			1.97	1.32	1.85	1.22
		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3	
1		1.50		1.50									
2				1.16		1.33				1.42			
3						1.25		1.35		1.34			
4						1.35				1.49		1.52	

The variations of the indexes with discharge have already been illustrated in another way (pp. 107, 143). The new fact brought out is that the superiority of slope over discharge, as a factor controlling capacity, is more pronounced for large discharges than for small, under like conditions of slope, width, and fineness.

TABLE 38.—*Variations of the indexes i_1 and i_3, and their ratio, in relation to width of channel.*

Group.	Sets.	w=0.44		w=0.66		w=1.00		w=1.32		w=1.96	
		i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3
1	4	2.10	1.19	1.94	1.50						
2	8			1.67	1.17	1.71	1.35	1.83	1.55		
3	8					1.63	1.20	1.66	1.28	1.83	1.50
		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3	
1		1.77		1.29							
2				1.40		1.27		1.18			
3						1.36		1.30		1.22	

A different grouping of the index values, but similar in principle, gave the means and ratios of Table 38, which is related to channel width just as Table 37 is related to discharge. With a single exception, the mean values of indexes increase with width; thus illustrating general facts previously noted on pages 104 and 143. Without exception, the ratios of i_1 to i_3 decrease with increase of width. The new fact brought out is that the superiority of slope over discharge, as a factor controlling capacity, is more pronounced for narrow channels than for wide, under like conditions of slope, discharge, and fineness.

As form ratio varies inversely with width, it follows that the superiority of slope is more pronounced, under like conditions, when form ratio is large than when it is small.

A third grouping of the index values, making a similar comparison of their averages and ratios with fineness of débris, is reported in Table 39 (p. 147). The mean values of indexes increase on the whole in passing from finer to coarse grades, but there is much irregularity. The same irregularity was encountered when these relations were examined in other connections. (See pp. 108 and 143.) The ratios decrease on the whole from finer to coarser, and there is but one discordance among the sequences.

To compare the variations of the indexes with changes in slope, the 32 pairs of values of

i_3 mentioned on page 143 were contrasted with corresponding values of i_1. The results are summarized in Table 40, below, in which each of the three groups indicates that the ratio of the indexes increases with increase of slope.

TABLE 39.—*Variations of the indexes i_1 and i_3, and their ratio, in relation to fineness of débris.*

Group.	Sets.	(A)		(B)		(C)		(D)		(G)		(H)	
		i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3
1	11	1.77	1.28	1.74	1.39	1.76	1.45	....	....	....	....	....	....
2	8	....	....	1.79	1.38	1.65	1.32	1.87	1.54	....	....	....	....
3	3	....	....	....	....	....	....	....	....	2.70	1.66	4.63	2.64
		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3	
1	....	1.38		1.25		1.21		....		....		....	
2	....	....		1.30		1.26		1.21		....		....	
3	....	....		....		....		....		1.69		1.76	

TABLE 40.—*Variations of the indexes i_1 and i_3, and their ratio, in relation to slope.*

Group.	Sets.	$S=0.5$		$S=1.0$		$S=1.2$		$S=2.0$		$S=2.4$	
		i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3	i_1	i_3
1	18	1.90	1.56	1.68	1.23	....	....	....	....	....	....
2	10	....	....	1.87	1.45	....	....	1.66	1.12	....	....
3	4	....	....	....	....	2.82	1.47	....	....	2.13	1.08
		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3		i_1/i_3	
1	....	1.22		1.35		....		....		....	
2	....	....		1.29		....		1.49		....	
3	....	....		....		1.91		....		1.98	

To sum the results of the preceding paragraphs: The sensitiveness of capacity for traction to changes of slope, as measured by the exponent i_1, is in general greater than its sensitiveness to changes of discharge, measured by i_3. The superiority of the control by slope persists through nearly (or perhaps quite) the entire range of conditions realized in the laboratory. If the superiority is measured by the ratio $\frac{i_1}{i_3}$, its average value (based on 64 comparisons) is 1.36, and it increases with increase of slope, discharge, form ratio, and fineness of débris.

$$\frac{i_1}{i_3}=f(\acute{S}, \acute{Q}, \acute{F}, \acute{R}) \quad \text{..........(72)}$$

Another mode of comparing the controls by slope and discharge is by means of the synthetic index of relative variation (p. 99), and in some respects this mode is more satisfactory than the one given above. The synthetic index, I_3, of the relative variation of capacity and discharge was computed for 21 cases, in each of which the greater discharge was approximately double the lesser. The synthetic index I_1, of the relative variation of capacity and slope, was computed for 21 pairs of cases, the greater slope being double the lesser. Each value of I_3 was joined to two discharges and a slope and associated with a pair of values of I_1. Each of the two values of I_1 was joined to one discharge and two slopes, the slopes being so chosen, whenever possible, that their mean coincided with the slope of the associated I_3. The mean of the 42 values of I_1 is 1.92; that of 21 values of I_3 is 1.42; and the ratio of these means is 1.35. The result is practically identical with that obtained by the discussion of values of i_1 and i_3.

CONTROLS OF DUTY.

The index of relative variation of the duty of water in relation to slope (p. 121) is i_1, the same as the index for capacity and slope. The corresponding index for the duty of water in relation to discharge (p. 144) is i_3-1. The ratio of these indexes, $\frac{i_1}{i_3-1}$, is evidently greater than the ratio $\frac{i_1}{i_3}$, which has just been considered—that is, the superiority of the control by slopes, as compared with the control by discharge, is more strongly marked in the case of duty than in the case of capacity.

For general averages, $\frac{i_1}{i_3}=\frac{1.93}{1.42}=1.36$, and $\frac{i_1}{i_3-1}=\frac{1.93}{0.42}=4.3$.

In Table 41 the values of $\frac{i_1}{i_3-1}$ correspond to those of $\frac{i_1}{i_3}$ in Tables 37 to 40—that is, they are based on the partial means of those tables and are arranged under the same groups, the purpose being to show how the dominance of control by slope, as expressed by a ratio, varies with certain conditions.

TABLE 41.—*Variations of the ratio $\frac{i_1}{i_3-1}$ in relation to discharge, width of channel, fineness of débris, and slope.*

	Values of $\frac{i_1}{i_3-1}$ for group—			
	1	2	3	4
$Q=$ 0.093	4.6			
.182	8.6	2.7		
.363		5.8	4.3	2.8
.545			6.5	
.734		10.3	8.4	6.1
1.119				8.4
$w=$ 0.44	11.2			
.66	3.9	9.8		
1.00		4.9	8.1	
1.32		3.3	5.9	
1.96			3.7	
Grade (A)	6.3			
(B)	4.5	8.0		
(C)	3.9	5.1		
(D)		3.5		
(G)			4.9	
(H)			3.1	
$S=$ 0.5	3.4			
1.0	7.3	4.2		
1.2			6.0	
2.0		13.8		
2.4			26.6	

In comparing this table with the tables of $\frac{i_1}{i_3}$, the most conspicuous feature noted is that the variation of $\frac{i_1}{i_3-1}$ with all conditions is much more pronounced than the variation of $\frac{i_1}{i_3}$. Associated with this is the fact that the exceptions or apparent reversals observed in Tables 37 and 39 are not repeated in Table 41.

In verbal generalization of the tabulated results it is to be borne in mind (1) that the alphabetic order in which the grades are arranged is the order from fine to coarse, and (2) that variation with respect to form ratio is the inverse of variation with respect to width of channel. The general fact is that the dominance of control by slope, as compared with control by discharge—a dominance always pronounced—is notably increased by increase of slope, discharge, fineness, or form ratio.

$$\frac{i_1}{i_3-1}=f\,(\acute{S},\ \acute{Q},\ \acute{F},\ \acute{R}) \quad \text{.......(73)}$$

CONTROLS OF EFFICIENCY.

The index of relative variation of efficiency in relation to slope (p. 144) is i_1-1; and the corresponding index for efficiency in relation to discharge is i_3-1. The ratio of these indexes $\frac{i_1-1}{i_3-1}$ is greater than $\frac{i_1}{i_3}$, the corresponding ratio in the case of capacity, with exception of those doubtful cases in which $i_1<i_3$. That is, the superiority of the control by slope, as compared to the control by discharge, is more strongly marked in the case of efficiency than in the case of capacity. It is also true that $\frac{i_1-1}{i_3-1}<\frac{i_1}{i_3-1}$; or that the superiority of control by slope is less strongly marked for efficiency than for duty. For the general averages $i_1=1.93$ and $i_3=1.42$ the computed values of the ratios expressing superiority of control by slope are, as to capacity 1.36, as to efficiency 2.21, as to duty 4.3.

TABLE 42.—*Variations of the ratio $\frac{i_1-1}{i_3-1}$ in relation to discharge, width of channel, fineness of débris, and slope.*

	Values of $\frac{i_1-1}{i_3-1}$ for group—			
	1	2	3	4
$Q=$ 0.093	2.6			
.182	3.9	1.4		
.363		2.4	1.8	1.7
.545			2.7	
.734		4.1	3.1	3.0
1.119				3.9
$w=$ 0.44	5.5			
.66	1.9	3.9		
1.00		2.0	3.1	
1.32		1.5	2.4	
1.96			1.7	
Grade (A)	2.7			
(B)	1.9	3.6		
(C)	1.7	2.0		
(D)		1.6		
(G)			3.0	
(H)			2.4	
$S=$ .05	1.6			
1.0	2.9	1.9		
1.2			3.9	
2.0		5.5		
2.4			14.1	

In Table 42 the values of $\frac{i_1-1}{i_3-1}$ correspond to those of $\frac{i_1}{i_3-1}$ in Table 41 and are similarly derived from data of Tables 27 to 40. The purpose of the table is to show how the dominance of control by slope, as expressed by a ratio, varies with certain conditions.

A comparison of tabulated values for the several ratios shows that the ratios associated with efficiency vary with conditions more rapidly than those associated with capacity, but less rapidly than those associated with duty. In Table 42, just as in Table 41, there are no exceptions as to the direction of the trend of variation.

Bearing in mind that the alphabetic order in which the grades are arranged is the order from fine to coarse, and that variation with respect to form ratio is the inverse of variation with respect to width, we see that the general fact shown by the table is that the dominance of control by slope—a dominance always pronounced—is notably increased by increase of slope, discharge, fineness, or form ratio.

$$\frac{i_1-1}{i_3-1}=f(\acute{S}, \acute{Q}, \acute{F}, \acute{R}) \quad (74)$$

SUMMARY.

With débris of a particular size and a channel bed of a particular slope, there is a particular discharge which is barely competent to cause transportation. With increase of discharge above this barely competent discharge, there is a proportional addition to the stream's potential energy. The relation of capacity to discharge is formulated on the assumption that the capacity is proportional to some power of the added energy, and therefore to the same power of the added discharge. As each grade of débris is somewhat heterogeneous as to the size of its grains, this assumed principle can not be applied strictly; the practical assumption is that capacity varies with a power of the difference between the discharge and a constant discharge, the constant being so chosen as best to harmonize the data.

By means of such formulation the data were readjusted and the rate of variation of capacity with discharge, or the index of relative variation, i_3, has been computed for a variety of conditions. It is found to be greater as the slope of channel, the discharge, the fineness of débris, and the form ratio are less. The average of the values computed for laboratory conditions is 1.42 and the ordinary range is from 1.00 to 2.00.

The rate at which the efficiency of the stream and the duty of the stream's water vary with discharge is denoted by an index which is less by unity than that for capacity. Its average is 0.42 and its ordinary range is from 0 to 1.00.

It has previously been shown that the corresponding indexes showing the relation of capacity to slope are larger. In other words, capacity is more sensitive to changes of slope than to changes of discharge. If relative sensitiveness to the two controls be expressed by a ratio, the average value of that ratio is 1.36. The ratio varies with conditions, being relatively large when slope, discharge, fineness, and form ratio are relatively small.

The primary adjustment of observations of capacity, described in Chapter II, was an adjustment with respect to slope. The probable errors computed from differences between adjusted and unadjusted values were influenced by only a portion of the observational errors. In readjusting values of capacity with respect to discharge, another division of the observational errors was encountered and its importance was estimated. The probable errors computed from the results of the two adjustments are believed to represent with sufficient approximation the order of precision of the adjusted values of capacity, which constitute the main body of data for the discussions of the report. The order of precision is expressed by saying that the average probable error of the adjusted values is a little more than 3 per cent.

CHAPTER VI.—RELATION OF CAPACITY TO FINENESS OF DÉBRIS.

FORMULATION.

To study the laws affecting the control of capacity for traction by fineness of débris, capacities should be compared which are subject to like conditions in all other respects. For this purpose data from Table 12 were arranged as in Table 43, where the capacities in each horizontal line are conditioned by the same slope, discharge, and width of channel. All the data of that table pertain to a slope of 1.0 per cent; but similar tables were constructed for slopes of 0.5, 0.7, 1.4, and 2.0 per cent.

The same data were also plotted on logarithmic paper; and, after a preliminary examination, five sets were selected for special investigation. The plots of these appear in figure 50, where ordinates are logarithms of capacity and abscissas are logarithms of linear fineness. It is to be noted that the zero of log C is not the same for the different graphs. The graphs were moved up or down, so as to avoid confusion through intersection.

The first law illustrated by the plots is that capacity increases as fineness increases; the second, that it increases more rapidly for small fineness than for great fineness. Despite irregularities of the data it is evident that the locus of log $C=f$ (log F) is a curve, and that the function has a general resemblance to those found in comparing capacity with slope and discharge.

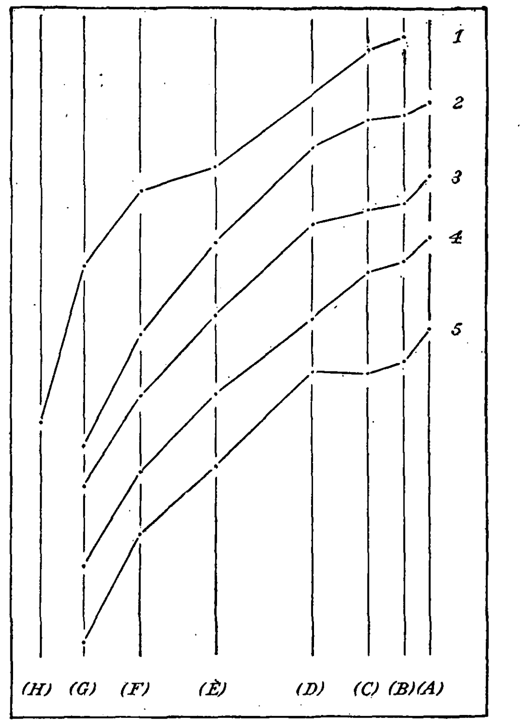


FIGURE 50.—Logarithmic plots of capacity for traction in relation to fineness of débris; corresponding to data in Table 44.

TABLE 43.—*Values of capacity for traction, arranged to illustrate the relation of capacity to grades of débris.*

Conditions.			Value of C for grade—							
S	w	Q	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
1.0	0.66	0.093		10.8	13.8	9.1				
		.182	39.5	33.5	32.1	29.8				
		.363		81	73		24.8	20.5	8.5	
		.545	140	120	112	101				
		.734			152		40	39	30.4	12.8
		1.119							49	25.2
	1.00	.093								
		.182	37.5	30.1	31.2	24.0	14.8			
		.363	100	85	85	73	33.8	14.0		
		.545		143	140	108				
		.734	231	199	180	153	72	42.8	26.9	
		1.119			276		138	79.5	61	
	1.32	.093								
		.182	36.8	29.3	21.4					
		.363	104	79	79	59.2	36.3			
		.545		140	129					
		.734	250	204	187	131	73	40.2	18.9	
		1.119					123	70	52.2	
	1.96	.093								
		.182								
		.363	96	67.6	60.0					
		.545		120	111					
		.734	240	190	190					
		1.119	359	313	343					

For any velocity, as determined by slope, discharge, and width, there is a competent fineness, marking the limit between traction and no traction; and to this extent, at least, the relation of fineness to traction is analogous to the relations of slope and discharge. It is not easy to carry the analogy further, because slope and discharge are conditions of active force, and fineness is a condition of reactive force, or resistance; but an experiment in formulation reveals a parallelism quite as striking as that between the capacity-slope and capacity-discharge functions.

Assuming

$$C = b_4 \, (F - \phi)^p \text{ ....} \quad (75)$$

in which ϕ is a constant fineness and b_4 a constant of the numerical value of capacity when $F = \phi + 1$,[1] the five sets of data in Table 44 were treated graphically for the determination of ϕ and p. The methods were such as have already been described (pp. 65 and 139). The formulas were then used to compute readjusted values of capacity, C_r, and values of the index of relative variation, i_4, and probable errors were also computed.

TABLE 44.—*Numerical data connected with the plots in figure 50, and illustrating the relation of capacity for traction to grades of débris.*

No. of plot in figure 50		1				2				3			
Conditions of experiments	w	0.66				1.00				1.00			
	Q	0.363				0.734				0.363			
	S	1.4				0.7				1.4			
Parameters of equations	ϕ	41				55				49			
	p	0.62				0.60				0.58			
	b_4	4.5				2.4				3.7			
Average probable errors	C	6.8				7.6				4.9			
	Cr	2.8				2.9				1.9			
		C	C_r	i_4	i_1	C	C_r	i_4	i_1	C	C_r	i_4	i_1
Data and computed results	Grade (A)	......	......	......	......	121	140	0.63	1.83	185	196	0.61	1.79
	Grade (B)	137	145	0.65	1.54	110	123	.64	1.66	149	173	.62	1.63
	Grade (C)	124	123	.67	1.55	106	102	.66	1.50	143	143	.63	1.53
	Grade (D)	......	......	.69	......	85	76	.70	1.72	127	108	.66	1.63
	Grade (E)	49.4	59	.81	2.10	40	42	.87	1.68	62	62	.80	1.78
	Grade (F)	41	37	1.08	1.99	20	21.7	1.42	2.35	33.4	34.4	1.18	2.42
	Grade (G)	23	22	1.82	2.71	8	7.4	5.45	4.11	16.3	16.3	2.80	3.15
	Grade (H)	6.9	7.1	13.50	4.69	......	......	......	......	......	......	......	......

No. of plot in figure 50		4				5			
Conditions of experiments	w	1.32				1.32			
	Q	0.734				0.363			
	S	1.0				2.0			
Parameters of equations	ϕ	48				55			
	p	0.61				0.50			
	b_4	3.8				11.4			
Average probable errors	C	1.7				6.2			
	Cr	0.6				2.4			
		C	C_r	i_4	i_1	C	C_r	i_4	i_1
Data and computed results	Grade (A)	250	250	0.64	1.71	328	319	0.53	1.97
	Grade (B)	204	219	.65	1.71	255	285	.54	1.63
	Grade (C)	187	180	.66	1.47	233	242	.55	1.52
	Grade (D)	131	133	.70	1.85	237	190	.58	1.92
	Grade (E)	73	74	.84	1.54	112	115	.72	1.60
	Grade (F)	40.2	40.2	1.22	2.40	66	67	1.19	2.03
	Grade (G)	18.9	19	5.20	3.43	28.3	27.4	4.55	3.67
	Grade (H)	......	......	......	......	......	......	......	......

PRECISION.

The average probable error of the readjusted capacities was found to be ±2.1 per cent. This error is to be ascribed in part to discordance of the data among themselves, and in part to discordance of the formula with the data; but the distribution of the residuals is not such as to imply important discordance of the formula.

[1] The dimensions of capacity are $M^{+1} T^{-1}$, and of fineness L^{-3}. The constant b_4, being equal to $C / (F-\phi)^p$, has dimensions $L^{+3p} M^{+1} T^{-1}$.

There are, however, important discordances among the data. Considered as errors, the discordances constitute a group which were not detected in the adjustments of capacities to slopes and to discharges, but which escaped those tests because related peculiarly to the grades of débris. From the residuals of the present readjustment the average probable error of capacities before readjustment is estimated at ±5.4 per cent, whereas the average probable error of the body of once-adjusted capacities from which these were selected was estimated at 2.5 per cent. On the assumption that the estimate for the whole body of values applies to the selected group, the share of error associated with the grades is estimated as $\sqrt{5.4^2-2.5^2}=\pm 4.8$ per cent.

Inspection of the logarithmic plots suggested that part of the discordance of the data is systematic. To bring out the systematic element the original values of capacity in Table 44 were divided by the readjusted values, and means were taken of the quotients. The means are listed below, and in figure 51 they are plotted logarithmically in relation to fineness. The plotted points conspicuously out of line are those for grades (B) and (D), the capacities determined for grade (B) being relatively too small and those for grade (D) too large. The same result was obtained from a canvass of a wide range of data.

Ratios of original values of capacity to adjusted values.

Grade	(A)	(B)	(C)	(D)	(E)	(F)	(G)
Linear fineness	1,002	812	602	388	178	95.9	43.4
Mean ratio	0.96	0.89	1.01	1.13	0.98	0.97	1.02

It is surmised that these errors arise in part from variations of experimental method, and this suspicion attaches especially to grade (B), which was the first to be treated in the laboratory. It attaches much less to grade (D), for

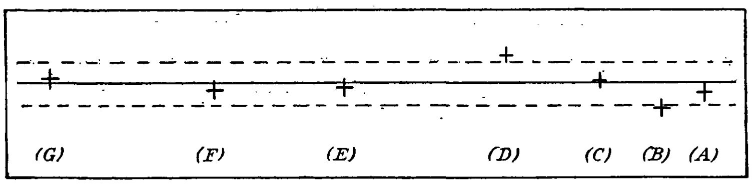


FIGURE 51.—Average departures of original values of capacity from the system of values readjusted in relation to fineness of débris. The horizontal line represents the readjusted system. The broken lines, above and below, correspond to departures of 10 per cent.

which the experimental method was about the same as for grades (A), (E), and (F). So far as the quality of the experimental work is gaged by the probable error, that on grade (D) would appear to be considerably below the average.

It is surmised that systematic error may also be connected with properties of the grades other than that of fineness. Range of fineness has already been appealed to as an explanation of apparent anomalies as to competent slope. From the experiments with mixtures (Chapter IX) we know that great range within a grade would tend to increase capacity for traction. As the range is small for grade (A), regularly increases to grade (E), and is again small from (F) to (H), the influence of this factor tends to enhance the capacity of grade (E); but the effect of that influence is not apparent in the diagram.

The separation of grades, being effected by sieves, was a gaging of grains by certain dimensions. The grains may have differed also as to shapes and densities, and each of these properties would affect capacity. The débris first obtained for the laboratory had been washed from the bed of Sacramento River when a flood broke the levee below the mouth of American River. This furnished material for the finer grades. Other lots of débris were taken from the bed of American River, and these furnished material chiefly for the coarser grades. So far as the separations from the different lots coincided, they were used indiscriminately. It is possible that grade (D) was composed chiefly of the coarser particles of a fine alluvium, while grade (E) was composed chiefly of the finer particles of a relatively coarse alluvium. As any natural body of tractional débris is the result of a sorting process in which tractionable particles are separated from the immovable on one side and from the suspendible on the other, it is likely to include among its coarser grains many which are tractionable only because of low density or favorable shape, and among its finer grains many which escape suspension because of high density or unfavorable shape.

Thus it appears possible that the superior mobility of grade (D) was determined by properties other than size. Unfortunately the record is not of such character that the value of this suggestion can now be tested.

A third suggestion pertains to the gaging of fineness. The method of gaging included a weighing and involved certain assumptions as to homogeneity in average density and in shape which may not have been fully warranted.

These various suggestions, while not susceptible of test at the present time, are sufficiently plausible to show the possibility of definite causes for the discordances discovered by the comparison of data from different grades. In my judgment it is proper to ascribe the greater discordances to such causes, and to view them as abnormalities with respect to the law connecting capacity with fineness.

In view of the magnitude of the abnormalties or discordances, it does not appear profitable to extend the readjustment of data to other and shorter sets. The five sets in Table 44 were selected because they included great range in fineness, and because they were qualified to yield fairly definite values of the constant ϕ.

VARIATIONS OF THE CONSTANT ϕ.

The laws which control the variation of ϕ have not been developed from the observations, but their general character may be inferred deductively by considering the relations of competent fineness to various conditions—it being assumed that ϕ is intimately related to competent fineness. Postulate a current of which the velocity is determined by a particular slope, a particular discharge, and a particular width. For this current a certain fineness is competent. Increase of slope or discharge increases the velocity and makes a lower fineness competent. Decrease of width, which corresponds to increase of form ratio, increases velocity and makes a lower fineness competent. Thus competent fineness, and therefore ϕ, varies inversely with the slope, discharge, and form ratio.

$$\phi=f\,(\grave{S},\ \grave{Q},\ \grave{R}) \text{ .... } (76)$$

INDEX OF RELATIVE VARIATION.

Framing an equation of the type of (33)—

$$C=v_4F^{i_4} \text{ .... } (77)$$

in which i_4 is the index of relative variation for capacity and fineness. The formula for the index (cf. pp. 100 and 141) is

$$i_4=\frac{pF}{F-\phi} \text{ .... } (78)$$

With this formula, values of i_4 were computed from the data in Table 44, and they are given in the lower part of that table.

By inspection it appears that the index increases as fineness diminishes, its growth being at first slow but becoming rapid as competent fineness is approached. Because of the discordances of the data it is not easy to derive a body of values of the index for discussion in relation to other conditions, but it is relatively easy to obtain comparative values of the synthetic index, I_4, and the variations of these values may be assumed to show the same trends as the variations of i_4. Values of I_4 were computed between corresponding data of grades (C) and (G) by the formula

$$I_4=\frac{\log C_{\rm I}-\log C_{\rm II}}{\log F_{\rm I}-\log F_{\rm II}}$$

in which $C_{\rm I}$ and $C_{\rm II}$ are specific capacities corresponding to the finenesses $F_{\rm I}$ and $F_{\rm II}$; and the results are given in Table 45.

From these results it is inferred (1) that the index varies inversely with the slope, (2) that it varies inversely with discharge, and (3) that it varies directly with width, and therefore inversely with form ratio. The response is in general of a very pronounced character, but to this there is exception in one of the comparisons with width. It is possible that the index is a maximum function of width and a minimum function of form ratio. With some reservation on this point, we may generalize:

$$i_4=f\,(\grave{S},\ \grave{Q},\ \acute{F},\ \grave{R}) \text{ .... } (79)$$

If equation (79) be compared with equations (39) and (68), it will be seen that the variation of the capacity-fineness index observes the same laws of trend as the variations of the capacity-slope index and the capacity-discharge index. In view of this general parallelism of variation, it is thought that the relative magnitudes of average i_4, average i_1, and average i_3 may be adequately discussed by means of a moderate number of comparisons. Accordingly only those values of i_4 computed from the five equations of Table 44 are used. The corresponding values of i_3 are not available, but those of i_1

are given in Table 15. They have been copied into Table 45, so as to be conveniently compared.

Inspection shows that i_4 is in general much smaller than i_1 but that it becomes greater as the limit of competence is approached. As to the first of these generalizations there can no question, but the second is not equally satisfactory. In the vicinity of competence the value of i_4 is highly sensitive to the influence of ϕ; and in the same region i_1 is highly sensitive to σ. The features of the table might be produced by slight overestimates of ϕ or by slight underestimates of σ. In view of this consideration it is probably best to leave the higher values of the index out of the account and base a computation of averages wholly on the lower values. Including only the 28 values of each index associated with grades (A) to (F), the means are, for i_4 0.77, for i_1 1.79; and the ratio of the second to the first is 2.4; that is, the sensitiveness of capacity to control by slope is estimated to be 2.4 times as great as its sensitiveness to control by fineness. The ratio of sensitiveness for slope and discharge i_1/i_3 having been estimated at 1.36, it follows that the ratio for discharge and fineness is $\frac{2.4}{1.36}=1.8$.

$$\text{Mean } i_1 : \text{mean } i_3 : \text{mean } i_4 :: 2.4 : 1.8 : 1.0.$$

It is to be understood that these estimates are of the most general character. The ratios doubtless vary notably with conditions.

The property with which capacity has been compared in this chapter is linear fineness, F, defined as the reciprocal of diameter, or as the number of grains to the linear foot. Bulk fineness, F_2, defined as the reciprocal of volume, is proportional to the cube of linear fineness. It follows that the index of relative variation when capacity is compared with bulk fineness is one-third the corresponding index, i_4, for capacity and linear fineness; and the same factor applies to synthetic indexes. If bulk fineness were substituted for linear fineness in equations of the form of (75), the values of ϕ would be quite different and the values of p would be uniformly one-third as great.

TABLE 45.—*Values of I_4, the synthetic index of relative variation for capacity and fineness, compared with slope, discharge, and width of channel.*

Fixed conditions: Coarser grade	(G)		(G)		(G)		(G)		(G)		(G)	
Finer grade	(C)		(C)		(C)		(C)		(C)		(C)	
Slope (per cent)					1.8		2.0		2.0		2.0	
Discharge (ft.³/sec.)	0.363		0.734						0.363		0.734	
Width (feet)	1.32		1.32		1.32		1.32					
Variables compared	S	I_4	S	I_4	Q	I_4	Q	I_4	w	I_4	w	I_4
	1.8	1.05	1.0	0.99	0.363	1.05	0.363	0.92	0.66	0.59		
	2.4	.76	2.0	.61	.734	.65	.734	.61	1.00	.74	1.00	0.57
									1.32	.92	1.32	.61

DUTY AND EFFICIENCY.

The relations of duty and efficiency to capacity involve discharge and slope but are independent of fineness. Fineness, therefore, has exactly the same control of duty and efficiency that it has of capacity, and the conclusions of this chapter apply without qualification to duty and efficiency.

SUMMARY.

Capacity for traction is greater for fine débris than for coarse—that is, capacity increases with fineness. The law of increase admits of formulation in a manner strictly analogous to that employed in comparing capacity with slope and discharge—that is, it is found that capacity varies approximately with a power of the fineness less a constant fineness. The value of the constant fineness varies with conditions, being greater as slope, discharge, and form ratio are greater. The rate at which capacity varies with change of fineness, or the index of relative variation, is not the same for all conditions, being greater as slope, discharge and form ratio are less. Under similar conditions the rate is less than the corresponding rate for capacity and slope, the average ratio between them being as 1 to 2.4.

The arrangement of capacities in accordance with the assumed law of increase develops discrepancies which are believed to be of the nature of systematic errors. The largest of these have a magnitude of about 10 per cent. They are tentatively ascribed to peculiarities of the débris used in experiments and to imperfectly developed laboratory methods.

CHAPTER VII.—RELATION OF CAPACITY TO VELOCITY.

PRELIMINARY CONSIDERATIONS.

The work of stream traction is accomplished by the movement of water along the bed of the channel. For that reason the system of water movements and water velocities near the bed is intimately related to the load or capacity for load. In certain parts of this paper and in the writings of some other investigators use is made of the term "bed velocity," or its equivalent, but the term has no satisfactory definition. The difficulties which are encountered in this connection have to do also with the vertical velocity curve.

In all the streams with which we are here concerned the flow is eddying or turbulent. At any point the direction of motion and the velocity are constantly changing. If a mean be taken of the instantaneous forward components of velocity—the components parallel to the axis of the stream—it gives for the point a mean velocity coordinate with the mean velocity for the cross section obtained by dividing the discharge by the sectional area. It will be observed that the mean at a point is a mean with respect to time, while the sectional mean is primarily a mean with respect to space. The mean at a point, as thus defined, being called V_p, the vertical velocity curve may be defined as the curve obtained by plotting the values of V_p for any vertical of the current in their relation to depth. As commonly drawn by hydraulic engineers, it terminates downward at some distance from the origin of velocities, OD—say at B in figure 52—connoting a finite velocity for the water in actual contact with the bed. This implication contravenes a theorem of hydrodynamics that the velocity at contact with the wall of a conduit is either zero or indefinitely small. The theorem is believed to have been established experimentally by the work of J. L. M. Poiseuille [1] and is generally accepted. In the direct study of the velocities of streams instrumental observation is not carried from surface to bed, but ceases at some point, C, and the drawing of the curve below that point is a matter of inference. The inference accordant with the hydrodynamic principle is that the curve changes its course below C and reaches the origin at or near O.[2] This inference accords also with our observations in connection with the study of saltation (see p. 29); and those observations suggest likewise that the curve is materially modified by the resistances to the current involved in the work of saltation.

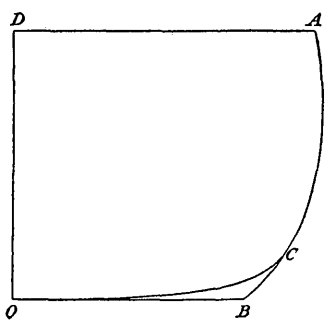


FIGURE 52.—Vertical velocity curve, drawn to illustrate its theoretic character near the stream's bed. OD is the origin of velocities.

It thus appears that in the region with which traction studies are specially concerned the range of V_p is great. The work of traction depends on a system of velocities and not on a single one, and there is no individual value of V_p with special claim to the title "bed velocity." It would be possible to define bed velocity as the value of V_p at some particular distance from the bed or at a distance constituting some particular fraction of the depth of current; but such a definition would be hard to apply.

However smooth a stream bed of débris may be in its general aspect, it is never smooth as regards details. Figure 53 gives an ideal profile, the intersection of a bed by a vertical plane. Not only are there salients and reentrants, but some of the reentrants communicate with the voids within the mass of débris. In many of

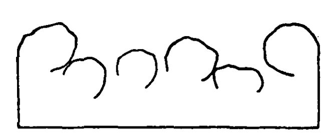


FIGURE 53.—Ideal profile of a stream bed composed of débris grains.

[1] See Lamb's Hydrodynamics, 3d ed., p. 544, 1906.

[2] See Cunningham, Allan, Hydraulic experiments at Roorkee, p. 46, 1875, and Inst. Civil Eng. Proc., vol. 71, p. 23, 1882, where he discusses the horizontal velocity curve; and Von Wagner, idem, p. 90.

the reentrants are doubtless stationary eddies, with reversed currents where the value of V_p is negative. It appears equally difficult to give definition to the bed as a datum from which to measure upward, and to select and define a locus for bed velocity. There is reason to suspect also that the problem as thus stated is unduly simplified by the assumption that the bed is a stable entity, clearly separate from the zone of saltation above. It did indeed so appear when the process of saltation was studied through the glass wall of the observation trough, but what was witnessed was the phase of the process at the edge of the channel bed, where the current was retarded by the resistance of the channel wall. At a distance from that wall, in the region where the cloud of saltatory particles effectually precludes visual observation, the passage from stability to mobility may be less definite. I am led to this suggestion by the observations, quoted by McMath,[1] of a civil engineer who descended in a diving bell to the bottom of the Mississippi at a point where the depth was 65 feet and the bottom of sand. Stepping to the bed, he sank into it about 3 feet, and then thrusting his arm into the yielding mass, could feel its flowing motion to a depth of 2 feet, the velocity diminished downward. In interpreting these phenomena, allowance must be made for the fact that the presence of the diving bell created an abnormal condition and if it rested on the bed put a stop to saltation. The flow of the sand is then to be ascribed to the difference in water pressure on the two sides of the bell. But the fact of the flow seems to indicate an antecedent state of mobility, a layer of the bed being supersaturated so as to have the properties of quicksand. If such a layer exists, then the transition from the bed to the saltation zone is not abrupt but gradual.

The difficulties in attempting to define bed velocity are supplemented by those which affect the measurement of velocities near the bed while traction is in progress (p. 26), and together they have served to prevent the use of bed velocity as a factor for quantitative comparison with capacity. This result has been regretted because the forces which accomplish traction are applied directly through the velocities of water near the bed, and it was admitted only after the failure of repeated attempts to obtain serviceable estimates of bed velocity.

In the present chapter observed or interpolated capacities are compared with mean velocities of the stream, mean velocity being computed as the quotient of measured discharge by measured sectional area. The measurements of discharge and width being relatively simple and accurate, the determinations of mean velocity have the same degree of precision as the measurements of depth. (See p. 26.)

In comparing capacity with mean velocity, it is convenient always to treat fineness of débris and width of channel as constants, but it is also advantageous to recognize three separate points of view as to the status of discharge, slope, and depth.

First, we may treat discharge as constant, in which case slope and depth vary, along with velocity and capacity. Each of the observational series (Tables 4, 12, and 14) conforms to this viewpoint. When discharge is constant, the increase of power necessary to increase velocity is given by increase of slope, and the increase of velocity causes the unchanged discharge to occupy less space. As velocity and capacity increase, slope increases and depth decreases.

Second, we may treat slope as constant. With slope constant, the increase of power necessary to increase velocity is given by increase of discharge, but the rate at which discharge is increased is greater than the rate of increase given to velocity, and the increased discharge therefore requires more space. As velocity and capacity increase, both discharge and depth also increase.

Third, we may treat depth as constant. To increase velocity by increasing slope will, as we have seen, reduce depth. To increase velocity by increasing discharge will, as we have seen, increase depth. To increase velocity without changing depth, it is necessary to enlarge both slope and discharge. No experiments were conducted with fixed depths, but the data for this comparison are readily obtained by interpolation.

It is proposed to examine the relation of capacity to velocity from each of these viewpoints, developing the results so far as necessary to give a basis for a comparison of the viewpoints.

[1] McMath, R. E., Van Nostrand's Mag., vol. 20, p. 227, 1879.

A preliminary remark applies to all. For each grade of débris and width of channel, and for each specific assumption of a constant discharge, slope, or depth, there is necessarily a competent mean velocity, below which no traction takes place. The conception of such a competent velocity has underlain all the discussions of competent slope, competent discharge, competent fineness, and competent form ratio. A broad analogy therefore points to the propriety of formulating the capacity-velocity relation as other relations of capacity have been formulated. And the inference from analogy finds support in logarithmic plots of $C=f(V_m)$ under each of the three above-mentioned conditions. It may fairly be assumed, therefore, that the index of relative variation for capacity and velocity itself varies with velocity, being relatively small for high velocities, being relatively large for low velocities, and becoming indefinitely large as competent velocity is approached.

For the purposes of this chapter, however, it has seemed best to employ a simpler method, using only the synthetic index of relative variation—characterized by the symbol I. Calling the synthetic index for the variation of capacity with respect to mean velocity I_V, we may conveniently distinguish by I_{VQ}, I_{VS}, and I_{Vd} the values associated severally with the special cases of constant discharge, constant slope, and constant depth.

The computations of the index are made chiefly by the formula

$$I_V=\frac{\log C'-\log C''}{\log V_m'-\log V_m''} \quad \text{-------} (80)$$

in which C' and C'' are specific capacities, and V_m' and V_m'' are the corresponding mean velocities. Graphically, I_V is the inclination of a line connecting two points of which the coordinates are, for the first, log C' and log V_m', and for the second, log C'' and log V_m''. Where the available data serve to place more than two points on the logarithmic plot of $C=f(V_m)$, definite suggestion may thereby be made that the line connecting the extreme points does not constitute the most probable location of the chord theoretically corresponding to I_V; and in such cases a line is drawn with regard to all the data, and its inclination is measured on the plot.

The subject of competent velocity, which is of interest independently of the formulation of capacity and velocity, will be considered at the end of the chapter.

THE SYNTHETIC INDEX WHEN DISCHARGE IS CONSTANT.

In Table 14 are 73 series of values of V_m, each value corresponding to a stated value of S. The coordinate series in Table 12 contain values of C corresponding to the same values of S. From each pair of series were taken the highest and lowest values of V_m and the corresponding values of C, and from these four quantities was computed a value of I_{VQ}. The 73 values of the index are shown in Table 46, where the arrangement is such as to exhibit the variation of the values with respect to discharge.

TABLE 46.—*Values of I_{VQ}, the synthetic index of relative variation for capacity in relation to mean velocity, when discharge is constant.*

Grade.	w	Value of I_{VQ} when discharge (in ft.³/sec.) is—					
		0.093	0.182	0.363	0.545	0.734	1.119
(A)	1.32	....	....	4.05	....	3.62	....
	1.96	....	....	3.55	....	3.22	2.38
(B)	.23	5.45	4.50	....	....	....	....
	.44	4.32	4.45	....	....	....	....
	.66	5.76	4.61	2.93	4.22	....	....
	1.00	....	5.68	5.37	....	5.15	....
	1.32	....	2.92	....	2.48	2.51	....
	1.96	....	....	4.59	3.43	3.38	3.49
(C)	.44	6.31	4.30	....	....	....	....
	.66	3.36	3.29	2.82	3.04	2.92	....
	1.00	....	3.63	3.56	2.52	3.11	....
	1.32	....	2.54	3.90	3.27	3.69	....
	1.96	....	....	4.36	3.74	3.69	3.89
(D)	.66	5.38	4.30	....	4.59	....	....
	1.00	....	4.16	4.13	4.34	3.94	....
	1.32	....	....	5.34	....	3.34	....
(E)	.66	....	....	5.98	....	6.00	....
	1.00	....	5.10	5.35	....	3.43	....
(F)	.66	....	7.50	6.67	....	....	....
	1.00	....	11.46	9.63	....	....	....
	1.32	....	....	8.41	....	....	....
(G)	.66	....	....	9.22	....	8.30	10.60
	1.00	....	....	8.80	....	8.49	8.05
	1.32	....	....	13.60	....	10.67	8.26
(H)	.66	....	....	12.57	....	8.97	8.49

The values which lie in any horizontal line agree as to all conditions except discharge. On comparing the columns for discharges 0.093 and 0.182 ft.³/sec., it is seen there are six lines carrying values in both columns. The means of these values are 4.80 and 4.21, the greater mean belonging with the smaller discharge. In the column for discharge 0.182 are nine values coordinate with values in the column for discharge 0.363, and the means for the two groups of nine are 5.31 and 5.04. Again the greater mean is associated with the smaller discharge, and the same relation is found by other combinations. The partial means resulting from these reductions are arranged in the upper division of Table 47. The general fact exhibited is that the index, other conditions being the same, varies inversely with discharge.

The same method was employed to discover the nature of the control of the index, first, by grade of débris and, second, by width of channel; and the partial means are grouped in the middle and lower divisions of Table 47. Of the eight pairs of means connected with grade, seven agree in testifying that the index varies inversely with fineness. The exceptional testimony comes from the comparison of grades (B) and (C); and although it is emphatic, it serves rather to illustrate the general discordance of data from the experiments with grade (B) than to qualify the general law as to the index.

TABLE 47.—*Partial means based on Table 46, illustrating the control of I_{VQ}, by discharge, fineness, and width.*

Number of values.	Means of I_{VQ} for discharge (ft.³/sec.) of—					
	0.093	0.182	0.363	0.545	0.734	1.119
6	4.80	4.21				
9		5.31	5.04			
7			3.90	3.51		
16			6.58		5.47	
8					5.81	5.64

Number of values.	Means of I_{VQ} for grade—							
	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
4	3.45	3.49						
6		4.01	3.79					
9			3.34	4.39				
5			3.21		5.17			
5			3.44			8.73		
4					5.19		8.71	
3						8.24	10.54	
3							9.39	10.01

Number of values.	Means of I_{VQ} for width (feet) of—					
	0.23	0.44	0.66	1.00	1.32	1.96
2	4.97	4.38				
4		4.84	3.80			
15			5.59	5.84		
12				5.56	5.87	
7					3.36	3.62

Of the five pairs of means connected with width, three show increase of the index with increase of width and two give the opposite indication. The contrasts are not strong in any case, and the nature of the law is not clear. It may be that the normal variation with width is so slight as to be masked by accidental errors of the data; or it may be that the index, like the constant σ, is a minimum function of form ratio.

By dividing the slope interval covered by a computed value of the index and computing separately the indexes for the two subintervals, it was found that the index associated with the higher slopes has a smaller value than that for the lower slopes—that is, the index varies inversely with slopes.

To compare the control of capacity by mean velocity with its control by slope the 73 values of I_w (the synthetic index of relative variation of capacity and slope, under condition of constant width) corresponding to the tabulated values of I_{VQ} were computed. Each value of I_{VQ} is greater than the corresponding value of I_w, the ratio ranging from 1.4 to 4.0. The mean of 73 values of I_{VQ} is 5.33; the mean I_w is 2.05; and the ratio of the means is 2.60. It is in general true that the greater the indexes the greater the ratio between them.

In Table 48 means of the two indexes are shown for the several grades of débris. Each index varies with fineness and so, too, does the ratio of indexes.

TABLE 48.—*Synthetic indexes, comparing the control of capacity by mean velocity with the control by slope, and comparing both controls with grades of débris.*

Grade.	Number of separate determinations.	Mean I_{VQ}	Mean I_w	$\frac{\text{Mean } I_{VQ}}{\text{Mean } I_w}$	$\frac{\text{Mean } I_w}{\text{Mean } I_{VQ}}$
(A)	5	3.62	1.87	1.93	0.52
(B)	18	4.35	1.94	2.24	.45
(C)	19	3.57	1.85	1.93	.52
(D)	9	4.50	1.87	2.41	.42
(E)	5	5.17	1.83	2.83	.35
(F)	5	8.73	2.27	3.84	.26
(G)	9	9.56	2.60	3.68	.27
(H)	3	10.01	3.20	3.12	.32
	73	5.33	2.05	2.60	.38

MEAN VELOCITY VERSUS SLOPE.

The comparison of I_{VQ} with I_w affords, incidentally, an estimate of the relative variation of mean velocity and slope. The rate of variation of capacity with mean velocity being I_{VQ}, the rate of variation of mean velocity with capacity is $1/I_{VQ}$; and the rate of variation of capacity with slope being I_w, the rate of variation of mean velocity with slope is

$1/I_{VQ} \times I_w = I_w/I_{VQ}$. The values of I_w/I_{VQ} listed in Table 48 are therefore estimates of the relative variation of mean velocity in relation to slope, and they have the same quality as the corresponding values of I_w and I_{VQ}. They are subject to the limiting conditions of constant discharge, fineness, and width, and they are averages of variability for practically the entire range of conditions realized in the laboratory, with the exception of those in the immediate neighborhood of competence. For this range of conditions mean velocity varies, on the average, as the 0.38 power of the slope.

This result is comparable with the generalization embodied in the Chezy formula, which makes mean velocity vary with the 0.5 power of the slope. The two are not inconsistent because they pertain severally to loaded and loadless streams. In a loaded stream increase of slope augments load and thus develops rapidly a factor of resistance from which the loadless stream is free. Velocity, being limited by resistances, develops less rapidly when the conditions are such that the resistances develop more rapidly.

THE SYNTHETIC INDEX WHEN SLOPE IS CONSTANT.

For a selected slope, values of V_m may be found in Table 14 which agree as to width of channel and grade of débris and differ only as to the discharges with which they are associated; and in Table 12 may be found the corresponding values of capacity. Such pairs of values, when occurring in series of two to five, constitute data for the computation of values of I_{VS}. To cover the entire range of tabulated data without needless repetition choice was made of slopes 0.6, 1.0, 1.4, and 2.0 per cent. The data associated with these slopes gave 66 values of the index, the values being essentially independent except in a few instances. They are shown in Table 49. From them were derived the sets of partial means arranged in Table 50, the method of derivation being that described in connection with Table 47.

The means show that I_{VS} varies inversely with slope, the variation being of moderate amount. They leave little question that it varies inversely with fineness, though the evidence is somewhat conflicting. They indicate also a direct variation with trough width, the opposite tendency being indicated only by the means for the greatest widths. The plots of the data, not here reproduced, show that the index varies inversely with discharge.

TABLE 49.—*Values of I_{VS}, the synthetic index of relative variation for capacity in relation to mean velocity, when slope is constant.*

Grade.	S	Value of I_{VS} for width (feet) of—					
		0.23	0.44	0.66	1.00	1.32	1.96
(A)	.06	....	....	....	....	2.18	1.87
(B)	.6	....	....	3.82	4.60	3.24	5.15
	1.0	2.42	2.60	4.39	4.10	3.40	4.62
	1.4	3.30	....	3.80	4.45	3.24	....
	2.0	....	....	4.95	4.17	....	....
(C)	.6	....	4.38	3.20	3.31	4.20	2.80
	1.0	....	2.83	2.98	3.01	3.48	2.45
	1.4	....	2.89	2.34	3.35	3.10	2.03
	2.0	....	....	1.59	....	....	1.33
(D)	.6	....	....	3.55	2.55	4.10	....
	1.0	....	....	3.40	2.49	....	....
	2.0	....	....	2.72	2.85	....	....
(E)	.6	....	....	....	3.38	....	....
	1.0	....	....	3.68	3.06	2.39	....
	1.4	....	....	2.29	2.85	....	....
	2.0	....	....	....	2.79	....	....
(F)	1.0	....	....	2.21	2.94	....	....
	1.4	....	....	4.86	5.80	....	....
	2.0	....	....	4.92	4.72	....	....
	1.0	....	....	3.97	4.62	6.50	....
	1.4	....	....	3.63	4.92	2.06	....
	2.0	....	....	3.50	3.10	5.05	....
	1.4	....	....	9.78	....	....	....
	2.0	....	....	5.84	....	....	....

TABLE 50.—*Partial means based on Table 49, illustrating the control of I_{VS} by slope, fineness, and width.*

Number of values.	Means of I_{SV} for slope (per cent) of—			
	0.6	1.0	1.4	2.0
12	3.68	3.35	....	....
16	....	3.63	3.43	....
11	....	....	4.23	3.81

Number of values.	Means of I_{VS} for grade—							
	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
2	2.02	4.19	......	......	......	......	......	......
13	......	3.81	2.87	......	......	......	......	......
6	......	......	30.5	3.13	......	......	......	......
4	......	......	......	2.82	3.23	......	......	......
5	......	......	......	......	2.93	4.11	4.05	......
2	......	......	......	......	......	......	3.56	7.81

Number of values.	Means of I_{VS} for width (feet) of—					
	0.23	0.44	0.66	1.00	1.32	1.96
1	2.42	2.60	....	....	....	....
4	....	3.17	3.23	3.47	3.54	2.97
18	....	....	3.57	3.72	....	....
11	....	....	....	3.46	3.71	....
6	....	....	....	....	3.27	3.15

THE SYNTHETIC INDEX WHEN DEPTH IS CONSTANT.

The capacities and mean velocities corresponding to particular depths were derived from the computation sheets described on page 95. The selected depths were 0.10, 0.14, 0.20, and 0.28 foot, and the data available for these depths afforded 32 values of the synthetic index, I_{Vd}. These are recorded in Table 51, and partial means derived from them are arranged in Table 52 so as to show variation in relation to fineness of débris and width of channel.

TABLE 51.—*Values of I_{Vd}, the synthetic index of relative variation for capacity in relation to mean velocity, when depth of current is constant.*

Width (feet).	Depth (feet).	Value of I_{Vd} for grade—					
		(A)	(B)	(C)	(D)	(E)	(G)
0.44	0.14	....	....	3.73	....	....	....
.66	.10	....	....	2.59	....	....	....
	.14	....	4.12	3.16	....	....	....
	.20	....	3.51	3.39	....	....	....
	.28	....	....	2.62	....	....	....
1.00	.14	....	....	3.59	3.17	....	....
	.20	....	4.02	2.80	3.51	4.46	....
	.28	....	....	3.44	....	....	7.86
1.32	.10	....	3.05	3.86	....	....	....
	.14	3.23	2.17	3.10	3.52	....	....
	.20	3.65	2.03	....	....	....	7.22
1.96	.10	....	3.92	3.02	....	....	....
	.14	2.36	3.41	3.60	....	....	....
	.20	2.23	4.50	4.27	....	....	....

TABLE 52.—*Partial means based on Table 51, illustrating the control of I_{Vd}, by fineness and width.*

Number of values.	Means of I_{Vd} for grade—					
	(A)	(B)	(C)	(D)	(E)	(G)
4	2.87	3.03	....	....	....	....
8	....	3.59	3.40	....	....	....
3	....	....	3.16	3.40	....	....
1	....	....	3.44	....	4.46	....
2	....	3.04	....	....	....	7.54

Number of values.	Means of I_{Vd} for width (feet) of—				
	0.44	0.66	1.00	1.32	1.96
1	3.73	....	3.28	....	....
4	....	3.17	3.46	....	....
3	....	....	3.71	2.88	....
7	....	....	....	3.00	3.09

With a single exception in each group of means, the indication is that the index varies inversely with fineness and directly with width. Under the condition of constant depth each change in mean velocity is accompanied by changes of both slope and discharge, and the influences of the two can not be examined separately. The features of the logarithmic plots, not here reproduced, show that the index varies inversely with slope and discharge, considered together.

THE THREE INDEXES.

In bringing together the results outlined in the preceding paragraphs we may replace width of channel by form ratio, bearing in mind that the two factors are so related that their variations are in opposite senses. So far as qualitative statement is concerned, the three synthetic indexes are identical in properties. The sensitiveness of capacity for traction to the control of mean velocity of current varies inversely with slope of channel, discharge, fineness of débris, and form ratio.

$$I_5 = f(\grave{S}, \grave{Q}, \grave{F}, \grave{R}) \quad \text{........} \quad (81)$$

As to the first three conditions the generalization is unqualified, but it is possible that the function as to form ratio is of the minimum class instead of inverse.

The three indexes are not of the same magnitude. In comparing them I_{Vd} was made the standard, partly because the values computed for it are fewer. To compare with each of its 32 values, that value of I_{Vs} which most nearly represents the same group of conditions was selected, and also a pair of values of I_{VQ} which collectively represent nearly the same conditions. Means were then derived for each index for each of the six grades to which they pertain, and also general means—all of which are shown in Table 53. In the same table are the ratios between general means and also between the corresponding grade means. For the general means I_{VQ} is 7 per cent greater than I_{Vd}, and I_{VQ} is the greater for all the partial means but one. For all the partial means but one I_{Vs} is smaller than I_{Vd}, and for the general means it is 9 per cent smaller. Capacity is most sensitive to the mean velocity conditioned by constant discharge and least sensitive to the mean velocity conditioned by constant slope.

These results have a theoretic connection with the fact that capacity is more sensitive to changes of slope than to those of discharge. When discharge is constant the changes of velocity are caused by changes of slope, and the changes in capacity are those due to the changes of slope. When slope is constant the changes of velocity are caused by changes of discharge, and the changes in capacity are those due to changes of discharge. When depth is constant the changes of velocity are caused by concurrent changes of slope and discharge, and the changes in capacity are intermediate between those caused by slope alone and by discharge alone.

TABLE 53.—*Comparison of synthetic indexes of relative variation for capacity and mean velocity, under the several conditions of constant discharge, constant depth, and constant slope.*

Grade.	Number of comparisons.	Means.			Ratios.	
		I_{VQ}	I_{Vd}	I_{VS}	$\frac{I_{VQ}}{I_{Vd}}$	$\frac{I_{VS}}{I_{Vd}}$
(A)	4	3.34	2.87	1.97	1.16	0.69
(B)	9	3.55	3.41	4.19	1.04	1.23
(C)	13	3.60	3.55	2.96	1.01	.83
(D)	3	4.14	3.40	2.63	1.22	.77
(E)	1	4.39	4.46	3.06	.99	.69
(G)	2	8.84	7.54	3.86	1.17	.51
General means....		3.98	3.68	3.21	1.07	.91

It is profitable to consider the same facts also in relation to depth of current. Postulate an initial status, with a particular discharge and slope, determining a certain velocity, depth, and capacity. First, increase the slope until the velocity is doubled. The capacity is increased, let us say (borrowing mean I_{VQ} from the table) to 15.8 times its initial amount. At the same time the depth is reduced one-half. Second, after returning to the initial status, increase slope and discharge by such amounts as to double the velocity without changing the depth. The capacity grows (mean I_{Vd}) to 12.8 times its original amount. Third, starting from the initial condition as before, increase the discharge until velocity is doubled. The capacity grows (mean I_{VS}) to 9.2 times its original amount; and the depth is at the same time increased, being more than doubled. Thus, for the same (doubled) mean velocity, the capacity is greater as the depth is smaller. Mean velocity is more efficient for traction as depth is less.

Now, the primary direct cause of stream traction is bed velocity. A concurrent cause theoretically exists in the component of gravity parallel to the slope, acting directly on the load, but for all ordinary stream slopes this factor is negligible. Slope and discharge are (essentially) indirect causes and are causes only in so far as they occasion bed velocity. They also determine mean velocity, and, from one point of view, mean velocity may be said to control capacity by controlling bed velocity.

Let us assume, for the moment, that bed velocity determines capacity irrespectively of depth. Then the variations of capacity above described imply corresponding variations of bed velocity, and, as the mean velocity does not change, we may infer that the ratio of bed velocity to mean velocity is a function—a decreasing function—of depth.

This proposition is, to say the least, worthy of consideration, but it fails of demonstration because the assumption which paved the way for it is not valid. It is not true that the relation of capacity to bed velocity is independent of depth. In the first place, change of depth, when not accompanied by change of mean velocity, causes change in the mode of traction. Within the range of the above hypothetic conditions may occur both the dune rhythm and the antidune rhythm; and at least one of these has an influence on capacity. Moreover, these rhythms involve diversity of velocity from point to point along the bed, so that "bed velocity" has not a simple definition.

In the second place, the load, or the work of traction, reacts on the vertical distribution of velocities. In figure 54 the line ABC is assumed as the vertical velocity curve of a stream flowing in a straight conduit and bearing no load.

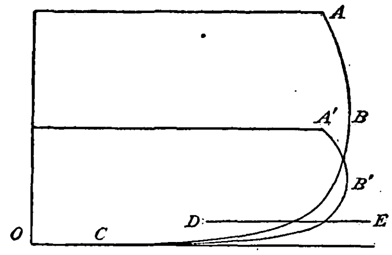


FIGURE 54.—Ideal curves of velocity in relation to depth, illustrating their relation to the zone of saltation. O=zero of velocity (horizontal) and of distance from the bottom (vertical).

$A'B'C$, identical except as to vertical dimensions, is assumed as the curve corresponding to the same mean velocity in a current one-half as deep. Introducing, now, the condition of traction, we may represent the upper limit of the zone of saltation by the line DE. The potential velocities within the zone are evidently quite different for the two depths of current, and they give advantage, for traction, to the shallower current. The work of saltation tends to retard the lower filaments of current and through these the higher filaments, reducing the mean velocity and increasing the depth. If mean velocity and depth be restored by suitable changes in slope and discharge, a new and different vertical velocity curve will be obtained for each depth. While I am not able to deduce the exact nature of the changes, it seems clear that those portions of the new curves within the zone of saltation will be contrasted in some such way as are the potential curves of the drawing, and that for purposes of traction the advantage will still be with the shallow current. Provided the portions of the velocity curves within the zone of saltation show steeper gradient for the current of less depth, tractional capacity will for that current bear a higher ratio to mean velocity.

RELATIVE SENSITIVENESS TO CONTROLS.

The synthetic index of relative variation for capacity in relation to mean velocity, when the limiting condition is constant discharge, namely, I_{VQ}, has been estimated (p. 158) as 2.60 times the corresponding index, I_w for capacity in relation to slope. The same index has been estimated (Table 53) as 1.07 times I_{Vd} and 1.19 times I_{VS}. Combination of ratios indicates that I_{Vd} is 2.43 times and I_{VS} 2.18 times as great as I_w. While these figures have an appearance of exactitude, their order of precision is really low. They are built on averages of individual values of indexes, which among themselves are highly diversified. At best they represent the average of values covered by the range of experiments in the laboratory, but in part they are based on values covering much narrower ranges. Moreover, it was not possible, except in the case of I_{VQ} and I_w, to derive the compared series of values of the index from data representing exactly the same conditions. For these reasons the numerical results should be accepted only as indicating an order of sequence and an order of magnitude. The quantitative response of capacity to the change of mean velocity is much larger than its response to change of slope, probably more than twice as large. Minor differences depend on the conditions under which mean velocity varies. The response is greatest when velocities are subject to the condition of constant discharge, less when the restrictive condition is constant depth, and least when it is constant slope.

COMPETENT VELOCITY.

The demonstrations by Leslie, Hopkins, Airy, and Law of the proposition that the diameter of the largest particle a current can move is proportional to the square of the velocity involve the principle that the pressure of a current is proportional to the square of its velocity, and also the assumption that the forward pressures on different parts of the particle are the same, so that the total pressure is proportional to the sectional area of the particle.[1] Under that assumption the total pressure may be conceived as applied to the center of gravity of the particle, a consideration of importance when the motion given to the particle is of the nature of rolling or overturning. These assumptions are not strictly true, because in the immediate vicinity of the channel bed the velocity increases with distance from the bed. Moreover, as we have seen (p. 29), the rate of increase is a diminishing rate. As a consequence of the inequality of velocity and its mode of distribution (1) the average pressure on the upstream face of a large particle is greater than the corresponding average pressure on a small particle, (2) the point of application of the total pressure (the point which determines the lever arm in overturning and rolling) is always above the center of gravity, and (3) the point of application may be differently related to the center of gravity in particles of different sizes. The general effect of these qualifying circumstances is to reduce the difficulty of moving large particles, and thus to make the rate at which competent diameter of particle increases with velocity (at any particular level) somewhat greater than that of the square of the velocity.

On the other hand, it is to be observed that in stream traction the roughness of the channel bed is defined by the coarseness of the load, and the system of velocities near the bed is a function of several things, one of which is the roughness. It is by no means impossible that the vertical velocity curve of a stream flowing over a bed of coarse débris is an enlarged replica of the curve of a shallower stream flowing over a bed of finer débris, in which case the law of Leslie might hold despite the qualifications mentioned above. The problem is too

[1] For references see footnote on page 16.

complicated, for the present at least, for full deductive treatment, and there are no adequate experimental data.

Most of the earlier experiments on competence pertained to flume traction, but it is probable that in those of T. Login (1857) the conditions were such as to give stream traction. The currents he employed were shallow, and he measured velocities by means of floats which occupied half the depth of the water.[1] As dimensions of transported particles are not included in his report of observations, a numerical formula can not be based on it.

Experimental data on competent velocity for stream traction.

[By T. Login.]

	Velocity in ft./sec.
Brick clay, mixed with water and then allowed to settle	0.25
Fresh-water sand	.67
Sea sand	1.10
Rounded pebbles, size of peas	2.00

John S. Owens, experimenting on the transporting power of sea currents, made use of small streams flowing from one tide pool to another. Measuring velocities by means of floats, he tested the ability of currents to move pebbles, 0.5 inch to 6 inches in diameter, over a channel bed of sand.[2] His results are formulated in

$$D_1 = \frac{45\ V^2}{W-64}$$

where D_1 is diameter of pebble in inches, V velocity in ft./sec., and W the weight in pounds of a cubic foot of the material of the pebble. This is equivalent to

$$D = \frac{0.059}{G-1}\ V^2 \quad \text{(82)}$$

where D is the diameter in feet and G the density of the material.[3]

Of the Berkeley experiments on competence, recorded in Tables 9 and 10, a single series bears on the point under consideration, but its bearing is less direct than could be desired. By selecting from Table 9 data corresponding to a trough width of 1 foot and a discharge of 0.363 ft.³/sec. I was able to make a logarithmic plot of diameters of particles of grades (B), (C), (D), (E), (G) in relation to competent mean velocities, and this plot gave the following equation:

$$D = 0.0025\ V_m^{2.7} = \frac{0.0042}{G-1}\ V_m^{2.7} \quad \text{(83)}$$

The exponent 2.7 is connected with the fact that V_m in this case is mean velocity with discharge constant. It is easy to infer from the data assembled in Table 53 that if either depth or slope had been the constant condition in the experiments a smaller exponent would be indicated. So the possibility remains that the Leslie law is true of mean velocities provided the depths increase along with the velocities, and the experimental data manifestly do not apply to bed velocities.

In order to compare the Berkeley observations with Login's, the diameters of his materials have been computed by equation (83). The diameter found for brick clay, 0.00006 foot, is much too large; but this result is readily accounted for by the fact that adhesion is an important factor in the resistance of clay to the action of the current. If the computed diameters for his "fresh-water sand," 0.00085 foot, and "sea sand," 0.0032 foot, are correct, those materials correspond severally to our grades (A) and (D), a very fine sand and a coarse sand. The diameter found for "pebbles, size of peas," is 0.016 foot, or one-fifth of an inch.

The coefficient obtained by Owens, 0.059 in equation (82), is 14 times as large as our coefficient, 0.0042 in equation (83), a contrast which accords in a general way with the difference between the classes of phenomena observed. The pebbles he tested were rolled over a bed of relatively fine material, which gave them a smooth pathway with little resistance, while the grains to be moved in our experiments rested among similar grains and were less readily dislodged.

[1] Royal Soc. Edinburgh Proc., vol. 3, p. 475, 1857.
[2] Geog. Jour., vol. 31, pp. 415–420, 1908.
[3] See also experiments by T. E. Blackwell, cited in Chapter XII.

CHAPTER VIII.—RELATION OF CAPACITY TO DEPTH.

INTRODUCTION.

As a condition controlling capacity for traction, depth has several distinct aspects; and the nature of its control depends altogether on the character of associated conditions. Three aspects will here be considered. They all assume that size of débris and width of channel are constant, and they are severally characterized by the limiting conditions of constant discharge, constant slope, and constant mean velocity. In examining the nature of the controls, and in comparing them with one another and with other controls, use will be made of the synthetic index; and the method of discussion will be similar to that of the preceding chapter. The symbols for the synthetic index, under the three limiting conditions, will be severally

$$I_{dQ},\ I_{ds},\ \text{and}\ I_{dV}.$$

WHEN DISCHARGE IS CONSTANT.

A stream of constant discharge, flowing in a channel of constant width, can change its depth only by changing its mean velocity, and depth and velocity vary in opposite senses. Whatever the ratio by which the mean velocity is increased or diminished, the depth diminishes or increases in the same ratio. It follows that the law of change for capacity in relation to mean velocity is the inverse of the law of change for capacity in relation to depth. So far as these laws are expressed by values of the synthetic indexes,

$$I_{dQ} = -I_{VQ}.$$

Independent computations of I_{dQ} are therefore unnecessary, as the values of I_{VQ} in Tables 46, 47, and 48 need only change of sign to become the corresponding values of I_{dQ}. The following summary statement of the general features of the control of capacity by depth is but a condensation and adaptation of the statement on pages 157–158.

Under the condition of constant discharge capacity varies inversely with depth. Its rate of variation is more than twice the rate at which it varies with slope of channel. The rate responds to changes in discharge, slope of channel, fineness of débris, and form ratio, diminishing as those factors increase (with possible exception as to form ratio).

Depth, when subject to the condition of constant discharge, varies inversely with slope. Estimates of the average rate of variation are contained in the last column of Table 48.

WHEN SLOPE IS CONSTANT.

A stream flowing down a constant slope, in a channel of constant width, and transporting débris of a particular grade changes its depth when the discharge is changed. The depth is greater as the discharge is greater. The capacity for traction also is greater as the discharge is greater. Therefore the capacity varies in the same sense as the depth.

Table 54 contains 68 values of I_{ds}, computed from data of Tables 12 and 14. The method of derivation was identical with that already described for Table 49, with the exception that values of depth were used instead of values of mean velocity.

TABLE 54.—*Values of I_{ds}, the synthetic index of relative variation for capacity in relation to depth of current, when slope is constant.*

Grade.	S	Value of I_{ds} for width (feet) of—					
		0.23	0.44	0.66	1.00	1.32	1.96
(A)	0.6					2.31	2.28
(B)	.6			2.02	2.40	2.93	2.14
	1.0	1.07	2.17	2.05	2.18	2.52	1.85
	1.4	.89		1.78	1.97	2.36	1.41
	2.0			1.59	1.83		
(C)	.6		3.44	2.27	2.51	2.56	3.29
	1.0		2.12	1.98	2.22	2.85	2.13
	1.4		1.80	2.34	2.04	2.36	2.65
	2.0			2.27	2.22		
(D)	.6			1.84	1.97	2.03	
	1.0			2.18	2.30	2.12	
	1.4			2.01	2.16		
	2.0			1.87	2.75		
(E)	.6				1.90		
	1.0			1.02	1.70		
	1.4			1.09	1.79		
	2.0				2.00		
(F)	1.4			2.40	3.59		
	2.0			2.03	2.87		
(G)	1.0			2.74	3.02	3.88	
	1.4			2.18	2.91	3.07	
	2.0			1.96	2.51	3.01	
(H)	1.0			2.37			
	1.4			2.95			
	2.0			2.22			

The derivation of means in Table 55 followed the precedents of Tables 47 and 50. An examination of logarithmic plots shows that I_{ds} varies inversely, but only slightly, with discharge. The means in Table 55 show that it varies inversely with slope and fineness, but they give no clear indication of its essential relation to width.

TABLE 55.—*Partial means based on Table 54, illustrating the control of I_{ds}, by slope, fineness, and width.*

Number of values.	Means of I_{ds} for slope (per cent) of—			
	0.6	1.0	1.4	2.0
14	2.66	2.13		
18		2.12	2.10	
13			2.40	2.24

Number of values.	Means of I_{ds} for grade—							
	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
2	2.29	2.53						
15		2.08	2.39					
10		2.13	2.33	2.12				
6			2.22	2.23	1.55			
4			2.22	2.20		2.72		
8			2.28				2.78	
3		1.81	2.20	2.02			2.29	2.51

Number of values.	Means of I_{ds} for width (feet) of—					
	0.23	0.44	0.66	1.00	1.32	1.96
2	0.98		1.91			
4		2.38	2.16			
19			1.98	2.36		
11				2.37	2.70	
6					2.59	2.24

For each of the 68 values of I_{ds}, the corresponding value of the synthetic index of the relation of capacity to discharge, I_3, was computed, the relations being such that the paired values represent exactly the same conditions. It was found that, without exception, the values of I_{ds} are greater than the companion values of I_3. Table 56 contains a series of comparative means and their ratios, the ratio of general means being 1.62.

TABLE 56.—*Synthetic indexes, comparing the control of capacity by depth with the control by discharge, and comparing both controls with grades of débris.*

Grade.	Number of separate determinations.	Mean I_{ds}.	Mean I_3.	Mean I_{ds} / Mean I_3	Mean I_3 / Mean I_{ds}
(A)	2	2.29	1.37	1.67	0.60
(B)	17	1.95	1.30	1.50	.67
(C)	17	2.41	1.36	1.77	.56
(D)	10	2.12	1.26	1.68	.59
(E)	6	1.58	0.99	1.59	.63
(F)	4	2.72	1.81	1.50	.67
(G)	9	2.81	1.76	1.59	.63
(H)	3	2.51	1.70	1.48	.68
	68	2.25	1.39	1.62	.62

DEPTH VERSUS DISCHARGE.

The comparison of I_{ds} with I_3 affords, incidentally, an estimate of the relative variation of depth and discharge. The rate of variation of capacity with depth being I_{ds}, the rate of variation of depth with capacity is $1/I_{ds}$; and, the rate of variation of capacity with discharge being I_3, the rate of variation of depth with discharge is $1/I_{ds} + I_3 = I_3/I_{ds}$. The values of I_3/I_{ds} in Table 56 are therefore estimates of the variation of depth in relation to discharge, under the limiting condition of constant slope. They are of the quality of the synthetic index and are based on the general range of conditions realized in the laboratory, except those in the neighborhood of competence. For this range of conditions depth varies, on the average, with the 0.62 power of discharge.

WHEN VELOCITY IS CONSTANT.

By interpolation from the data recorded in Tables 12 and 14, values of capacity and depth may be found corresponding to selected values of mean velocity. Such values were derived for mean velocities of 2, 3, and 4 ft./sec., and from them were computed the 42 values of I_{dv} in Table 57.

TABLE 57.—*Values of I_{dv}, the synthetic index of relative variation for capacity in relation to depth of current, when mean velocity is constant.*

Grade.	V_m	Value of I_{dv} for width (feet) of—					
		0.23	0.44	0.66	1.00	1.32	1.96
(A)	3					−0.32	−0.75
	4						− .69
(B)	2			+0.09	−0.26	+ .25	+ .32
	3	−0.26	−0.48	− .14	−1.02	+ .18	+ .19
	4			− .34		− .37	
(C)	2		− .75	0	+ .21	+ .22	
	3			− .57	− .36	− .32	
	4			+ .09	− .25	− .35	
(D)	2			− .34	− .27	+ .08	−1.50
	3			− .22	− .75	− .92	− .44
	4				−1.50		
(E)	2				−1.03		
	3				− .93		
(G)	3			−1.08	−1.35		
	4			−2.69	−1.19	−3.17	
(H)	3			− .75			

Most of these values are negative, but nine are positive. The mean of the 42 values, combined with regard to sign, is −0.54. A positive value, considered by itself, indicates that capacity varies directly with depth, and a negative value that the variation is inverse. At least two interpretations may be considered—(1) that the actual variation is usually inverse but under some conditions is direct; (2) that the law of inverse variation is general and that the apparent exceptions are due entirely to the imperfection of the data. The second interpretation accords the better with the analogies afforded by other branches of the subject and is provisionally accepted.

The interpretation directs attention to the general discordance of the computed values of I_{dV}, and a suggestion may be made as to the cause of that discordance. First, the measurement of depth was the most difficult of the direct measurements performed in the laboratory, and it was peculiarly subject to possibilities of systematic error. Second, the range of slopes through which depth could be observed was much less than in the case of capacity, and this made the work of adjustment less satisfactory. Third, in the computation of this particular index the depth data enter twice, and they enter in such way that their errors cumulate instead of canceling.

In Table 58 the relations of I_{dV} to various conditions are shown by the comparison of corresponding means. I_{dV} is seen to vary directly with mean velocity, and it may fairly be inferred to vary inversely with fineness, but the data as to width of channel are contradictory and inconclusive.

The control of the index by mean velocity affords information as to its relation to slope and discharge. Mean velocity varies directly with both slope and discharge, and it does not change without corresponding change in at least one of these factors. It follows that the index, which varies directly with mean velocity, also varies directly with at least one of the factors slope and discharge. In the groups of data from which values of the index were computed, increase of velocity was associated either with increase of slope or with increase of both slope and discharge but in no case with decrease of slope or discharge. It seems, therefore, proper to infer that I_{dV} varies in magnitude directly with both slope and discharge. In this respect it stands as an exception among the indexes of relative variation connected with traction. All other species of I and also i vary inversely with slope and discharge. As a check on this exceptional result, certain probable errors have been computed. In the upper division of Table 58 −0.19, the mean of 12 values of I_{dV} with a velocity of 2 ft./sec., is compared with −0.41, the corresponding mean for a velocity of 3 ft./sec. The difference between these means, −0.22, has a probable error of ±0.13. A similar difference, appearing in the comparison of indexes for velocities of 3 and 4 ft./sec., is −0.24±0.18. As the two differences give testimony of the same tenor, their joint evidence is stronger than that of either separately, so that the discussion of the residuals leaves the presumption in favor of the conclusion that this particular index constitutes a real exception in its relation to slope and discharge.

TABLE 58.—*Partial means based on Table 57, illustrating the control of I_{dV}, by mean velocity, fineness and width.*

Number of values.	Mean of I_{dV} for mean velocity (ft./sec.) of— 2	3	4
12	−0.19	−0.41	
9		− .57	−0.81

Number of values.	Mean of I_{dV} for grade— (A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)
2	−0.54	+0.18						
10		− .11	−0.32					
7			− .15	−0.57				
2				− .51	−0.98			
5			− .29				−1.90	
1							−1.08	−0.75

Number of values.	Mean of I_{dV} for width (feet) of— 0.23	0.44	0.66	1.00	1.32	1.96
1	−0.26	−0.48				
2		− .61	−0.07			
9			− .64	−0.58		
8				− .49	−0.61	
5						−0.44

THE THREE CONDITIONS COMPARED.

When depth is increased without change of slope (or width or grade of débris), its increase is effected by increase of discharge, with the result that capacity is increased, so that capacity is an increasing function of depth. When depth is increased without change of discharge, its increase is effected by reducing slope, with the result that capacity is reduced, so that capacity is a decreasing function of depth. When depth is increased without change of velocity, its increase requires increase of discharge accompanied by diminution of slope; and as these changes have opposite influences on capacity, it is not evident a priori whether capacity will be enlarged or reduced. The experimental data show that it is slightly reduced, so that capacity is a decreasing function of depth.

When depth is reduced without change of slope, and the reduction is continued progressively, a stage is eventually reached in which the velocity is no longer competent for traction. It is probable, therefore, that, under this condition, an approximate formula for $C=f(d)$ might involve a depth constant and be similar to the formula (64) used for $C=f(Q)$.

Reduction of depth without change of discharge involves increase of velocity, and it is evident that competence does not lie in that direction. But increase of depth involves reduction of velocity and leads eventually to a competent velocity. The limiting depth corresponding to competence is therefore a great depth instead of a small one. As mean velocity now varies inversely with depth, a coordinate formula might take the form

$$C=B\left(\frac{1}{d}-\frac{1}{\delta}\right)^n$$

B being used as a general constant and δ as a constant depth.

When depth is reduced without change of mean velocity, the efficiency of the mean velocity is enhanced and competence is not approached. When depth is increased, the efficiency of the unchanged mean velocity is diminished and a (large) competent depth may, under some conditions, be realized.

To show the numerical relations of the indexes by which these three capacity-depth functions are severally characterized, certain means are assembled in Table 59. It was not found possible to procure values of the different indexes representing closely the same conditions, and what was done was to derive for each index the mean of all determinations made for each particular grade of débris.

The arrangement by grades points again to the fact that all the indexes vary inversely with fineness of débris, but the rate of variation is in fact somewhat greater than these series of values suggest. The observations on the coarser grades were made with steeper average slopes and larger average discharges than those on the finer grades, and the effect of high slopes and large discharges (except in case of I_{dv}) is to reduce the indexes of relative variation.

TABLE 59.—*Comparison of synthetic indexes of relative variation for capacity and depth, under the several conditions of constant discharge, constant slope, and constant mean velocity.*

Grade.	Number of separate determinations.			Means.			$\frac{I_{dQ}}{I_{ds}}$
	I_{dQ}	I_{ds}	I_{dV}	I_{dQ}	I_{ds}	I_{dV}	
(A)	5	2	3	− 3.62	2.29	−0.59	−1.57
(B)	18	17	12	− 4.24	1.95	− .15	−2.17
(C)	19	17	12	− 3.58	2.41	− .33	−1.49
(D)	9	10	7	− 4.39	2.12	− .56	−2.07
(E)	5	6	2	− 5.17	1.58	− .98	−3.29
(F)	5	4	......	− 8.73	2.72	....	−3.21
(G)	9	9	5	− 9.45	2,81	−1.30	−3.36
(H)	3	3	1	−10.01	2.51	− .75	−3.99
	73	68	42	− 5.28	2.25	− .56	−2.34

From the general means at the bottom of the table it appears that, for the range of laboratory conditions, capacity is 2.34 times as sensitive to the control of depth when the limiting condition is constant discharge as when the limiting condition is constant slope, and about nine times as sensitive as when the limiting condition is constant mean velocity.

One of the results of the discussion is to emphasize the importance, when considering the relation of tractional work to depth, of sharply discriminating the conditions under which depth is regarded as a variable.

So far as the variations of the capacity-depth indexes admit of formulation in the symbols used for other indexes,

$$I_{dQ}=f\,(\grave{S}\,\grave{Q}\,\dot{F}) \qquad (84)$$

$$I_{ds}=f_{I}\,(\grave{S}\,\grave{Q}\,\grave{F}) \qquad (84a)$$

$$I_{dv}=f_{II}\,(\acute{S}\,\acute{Q}\,\grave{F}) \qquad (84b)$$

COMPARISON OF CONTROLS BY SLOPE, DISCHARGE, MEAN VELOCITY, AND DEPTH.

In Chapter V the general sensitiveness of capacity to slope is compared with that of capacity to discharge by means of coordinate values of the exponent i. In Chapter VII the sensitiveness to slope is compared with sensitiveness to mean velocity (with discharge constant) by means of coordinate values of I. The two methods of comparison are so far related that their results should be fairly harmonious. In the present chapter the same measure of sensitiveness to mean velocity, converted by change of sign to a measure of sensitiveness to depth (with discharge constant), has been compared with a similar measure for depth with slope constant. Finally, the measure last mentioned has been compared with a coordinate measure of the sensitiveness of capacity to discharge, thus completing a circle and affording opportunity for a checking of estimates in respect to consistency. The ratios resulting from the four comparisons are as follows:

$$\frac{\text{index of control by slope}}{\text{index of control by discharge}} = 1.36$$

$$\frac{\text{index of control by velocity (discharge constant)}}{\text{index of control by slope}} = 2.60$$

$$\frac{\text{index of control by depth (slope constant)}}{\text{index of control by discharge}} = -1.62$$

$$\frac{\text{index of control by velocity (discharge constant)}}{\text{index of control by depth (slope constant)}} = -2.34$$

The combination, by multiplication, of the first and second equations gives

$$\frac{\text{index of control by velocity (discharge constant)}}{\text{index of control by discharge}} = 3.51$$

The combination of the third and fourth gives an equation with identical first member, but the second member is 3.79. The two results differ by 8 per cent.

CHAPTER IX.—EXPERIMENTS WITH MIXED GRADES.

ADJUSTMENT AND NOTATION.

The tractional load of a natural stream includes particles with great range in size. The grades of débris used in the laboratory had narrowly limited ranges. Although the reasons for this limitation were believed to be adequate, the possibility was recognized that the laws discovered by the use of artificial grades might not apply without modification to natural grades; and in view of this possibility a series of experiments were arranged to bridge over the interval between the artificial and the natural in this particular respect. The same apparatus and the same general methods being used, observations were made first on mixtures of two grades; then on mixtures of three or more, and finally on a natural combination of sizes.

When work on mixtures was begun, need was soon found for a modification of the experimental procedure. During the automatic process of adjusting the slope to the load the current acted unequally on the components of the mixture, carrying forward an undue share of the finer part and depositing an undue share of the coarser. To escape the difficulties introduced by this partial re-sorting, a run was interrupted after it had gone far enough to indicate the approximate slope, the débris was taken from arrester and experiment trough and was throughly remixed, and then the approximate slope was artificially constructed in the trough, after which the run was continued.

The observational data are contained in divisions (J) and (K) of Table 4. The values of capacity in relation to slope were adjusted in the same manner as with individual grades, except that each value of σ for an adjusting equation was derived from the data of the particular observational series, without influence from related series. The adjusted capacities are recorded in Table 60, and with them are fractional capacities computed for the several grades composing the mixtures. Table 17 contains the constants of the adjusting equations and also series of values of the index of relative variation.

In each set of experiments with mixture of two grades the proportions of the components were varied. The usual series of proportions was approximately 4 : 1, 2 : 1, 1 : 1, 1 : 2, and 1 : 4. To denote these mixtures, a notation has been adopted similar to one employed in chemistry; for example, a mixture of grades (B) and (F) in the proportion 4 : 1 is designated (B_4F_1).

The proportions of some of the mixtures are told accurately by the subscript figures; those of others only approximately. The theoretic or standard mode of apportionment was by weight of dry material, but when this was not convenient the material was weighed or measured in moist or saturated condition, and the actual ratio for dry weight was afterward learned by computation. The actual proportions are given in percentages in Table 4 (J).

TABLE 60.—*Adjusted values of capacity in relation to slope, for mixtures of two or more grades of débris and for an unsorted natural alluvium, based on data of Table 4 (J) and (K).*

Grade	(A_1C_1)			(A_8G_1)			(A_2G_1)			(A_1G_1)		
Q	0.363			0.363			0.363			0.363		
Component	(A)	(C)	Total.	(A)	(G)	Total.	(A)	(G)	Total.	(A)	(G)	Total.
S	Value of *C*.											
0.5	16	16	32	13	4	17	13	7	20	8	9	17
.6	23	23	46	22	7	29	21	10	31	11	12	23
.7	30	30	60	32	11	43	29	15	44	14	15	29
.8	37	37	74	44	15	59	39	19	58	18	19	37
.9	44	44	88	57	19	76	49	24	73	22	23	45
1.0	51	51	102	71	23	94	60	30	90	......	......	......
1.2	65	66	131	99	33	132	85	42	127	32	33	65
1.4	80	81	161	131	44	175	113	56	169	44	45	89
1.6	95	96	191	165	55	220	......	......	......	57	58	115
1.8	......	......	......	......	......	......	......	......	......	......	......	......
2.0	......	......	......	......	......	......	......	......	......	......	......	......
Probable error (per cent)	......	......	......	......	......	2.8	......	......	4.6	......	......	......

TABLE 60.—*Adjusted values of capacity, in relation to slope, for mixtures of two or more grades of débris and for an unsorted natural alluvium, based on data of Table 4 (J) and (K)*—Continued.

Grade	(A_1G_2)			(A_1G_4)			(B_4F_1)			(B_2F_1)		
Q	0.363			0.363			0.363			0.363		
Component	(A)	(G)	Total.	(A)	(G)	Total.	(B)	(F)	Total.	(B)	(F)	Total.
S	Value of C.											
0.5							26	7	33	22	12	34
.6							37	10	47	29	17	46
.7							48	14	62	38	21	59
.8							60	17	77	47	25	72
.9							72	20	92	54	30	84
1.0	7	14	21	3	9	12	83	23	106	62	35	97
1.2	10	21	31	4	14	18	107	30	137	78	43	121
1.4	14	28	42	5	20	25	133	37	170	93	52	145
1.6	19	37	56	7	26	33				109	61	170
1.8				9	34	43						
2.0				12	42	54						
Probable error (per cent)									2.1			1.1

Grade	(B_1F_1)			(B_1F_2)			(B_1F_4)			(C_4E_1)		
Q	0.363			0.363			0.363			0.363		
Component	(B)	(F)	Total.	(B)	(F)	Total.	(B)	(F)	Total.	(C)	(E)	Total.
S	Value of C.											
0.5												
.6										29	8	37
.7	22	24	46				2	11	13	40	11	51
.8	26	30	56	9	19	28	3	13	16	51	14	65
.9	31	35	66	11	23	34	4	15	19	62	17	79
1.0	37	41	78	12	28	40	5	18	23	74	20	94
1.2	48	55	103	17	37	54	6	25	31	97	26	123
1.4	61	68	129	21	48	69	7	32	39	122	32	154
1.6	75	83	158	26	59	85	9	40	49	146	39	185
1.8	89	100	189							171	46	217
Probable error (per cent)			0.1									0.7

Grade	(C_2E_1)			(C_2E_1)			(C_1E_1)			(C_1E_1)		
Q	0.182			0.363			0.182			0.363		
Component	(C)	(E)	Total.	(C)	(E)	Total.	(C)	(E)	Total.	(C)	(E)	Total.
S	Value of C.											
0.5												
.6				20	11	31						
.7				28	15	43				14	15	29
.8				36	19	55				19	21	40
.9				44	24	68	12	12	24	25	26	51
1.0				53	28	81	14	15	29	30	32	62
1.2				70	37	107	19	20	39	42	44	86
1.4				88	47	135	24	26	50	53	57	110
1.6	44	24	68	106	57	163	29	32	61	65	71	136
1.8	55	29	84	124	67	191	36	38	74	78	84	162
2.0	66	35	101				42	46	88	90	97	187
2.2	77	42	119				53	58	111			
Probable error (per cent)						0.7			1.8			2.3

TABLE 60.—*Adjusted values of capacity, in relation to slope, for mixtures of two or more grades of débris and for an unsorted natural alluvium, based on data of Table 4 (J) and (K)*—Continued.

Grade	(C_1E_2)			(C_1E_2)			(C_1E_4)			(C_1E_4)		
Q	0.182			0.363			0.182			0.363		
Component	(C)	(E)	Total.	(C)	(E)	Total.	(C)	(E)	Total.	(C)	(E)	Total.
S	Values of *C*.											
0.5												
.6												
.7				8	17	25						
.8				11	23	34						
.9				13	29	42	3	13	16	6	28	34
1.0	7	15	22	16	35	51	4	15	19	8	33	41
1.2	9	21	30	22	48	70	5	20	25	11	46	57
1.4	12	27	39	28	63	91	6	26	32	14	61	75
1.6	15	34	49	34	77	111	8	32	40	18	76	94
1.8	18	41	59	42	92	134	9	39	48	21	92	113
2.0	22	48	70	48	108	156	11	45	56	25	109	134
2.2	25	55	80	56	124	180	12	53	65			
Probable error (per cent)			2.2			5.6			0.1			1.6

Grade	(C_4G_1)			(C_2G_1)			(C_1G_1)			(C_1G_2)		
Q	0.363			0.363			0.363			0.363		
Component	(C)	(G)	Total.	(C)	(G)	Total.	(C)	(G)	Total.	(C)	(G)	Total.
S	Values of *C*.											
0.5	17	4	21	11	6	17						
.6	24	6	30	19	9	28						
.7	32	8	40	27	14	41	10	10	20			
.8	42	10	52	36	18	54	15	16	31			
.9	54	13	67	45	23	68	21	22	43	4	8	12
1.0	65	16	81	55	27	82	27	28	55	6	12	18
1.2	93	23	116	74	37	111	40	40	80	11	22	33
1.4	126	32	158	96	48	144	53	53	106	17	33	50
1.6	164	41	205	119	59	178	67	67	134	23	47	70
1.8	208	52	260	141	71	212	80	81	161	31	61	92
2.0												
Probable error (per cent)			1.9			1.2			4.4			2.2

Grade	(E_4G_1)			(E_2G_1)			(E_1G_1)			(E_1G_2)		
Q	0.363			0.363			0.363			0.363		
Component	(E)	(G)	Total.	(E)	(G)	Total.	(E)	(G)	Total.	(E)	(G)	Total.
S	Value of *C*.											
0.6	10	2	12									
.7	13	3	16	6	3	9						
.8	17	4	21	9	5	14	8	8	16			
.9	21	5	26	13	7	20	10	10	20	4	9	13
1.0	25	6	31	18	9	27	13	13	26	6	11	17
1.2	35	9	44	29	15	44	18	19	37	8	16	24
1.4	47	12	59	43	22	65	24	25	49	11	22	33
1.6	60	15	75	59	30	89	31	31	62	14	29	43
1.8	75	19	94				37	38	75	18	37	55
2.0										23	45	68
Probable error (per cent)												1.4

TABLE 60.—*Adjusted values of capacity, in relation to slope, for mixtures of two or more grades of débris and for an unsorted natural alluvium, based on data of Table 4 (J) and (K)*—Continued.

Grade	($A_1C_1G_2$)				($C_{45}D_{35}E_{12}F_6G_2$)											
Q	0.363				0.182						0.363					
Component	(A)	(C)	(G)	Total.	(C)	(D)	(E)	(F)	(G)	Total.	(C)	(D)	(E)	(F)	(G)	Total.
S	Value of C.															
0.6																
.7	2	2	5	9	8.2	6.6	2.3	1.1	0.3	18.9	24.2	18.9	6.5	3.2	1.1	54
.8	4	4	9	17	10.6	8.2	2.8	1.4	.5	23.5	29.2	22.8	7.8	3.9	1.3	65
.9	7	7	13	27	12.8	10.0	3.4	1.7	.6	28.4	35.0	27.3	9.4	4.7	1.6	78
1.0	9	9	20	38	15.2	11.8	4.1	2.0	.7	33.9	40.9	31.8	10.9	5.4	1.8	91
1.2	16	16	33	65	20.3	16.0	5.5	2.7	.9	45.8	53.5	41.6	14.3	7.1	2.4	119
1.4	24	24	50	98	26.6	20.6	7.1	3.5	1.2	59	67.5	52.5	18.0	9.0	3.0	150
1.6	34	34	69	137	32.8	25.6	8.8	4.4	1.5	73	81.9	63.7	21.8	10.9	3.6	182
1.8	45	45	91	181	39.6	30.8	10.6	5.3	1.8	88	95.4	74.2	25.4	12.7	4.2	212
2.0					47.3	36.8	12.6	6.3	2.1	105						
Probable error (per cent)				4.0												0.4

Grade	($C_{45}D_{35}E_{12}F_6G_2$).						Natural.	
Q	0.545						0.182	0.363
Component	(C)	(D)	(E)	(F)	(G)	Total.		
S	Values of C.							
0.4								17
.5								26
.6	36.4	28.4	9.7	4.8	1.6	81		36
.7	44.1	34.3	11.8	5.9	2.0	98	15	47
.8	50.4	39.9	13.7	6.8	2.3	114	20	60
.9	59.4	46.2	15.8	7.9	2.6	132	26	75
1.0	68.0	52.8	18.1	9.0	3.0	151	33	91
1.2	85.0	66.1	22.7	11.3	3.8	189	47	126
1.4	103	80.2	27.5	13.7	4.6	229	63	168
1.6	121.5	94.5	32.4	16.2	5.4	270	80	213
1.8							98	
2.0							117	
2.2							138	
2.4							159	
2.6							181	
Probable error (per cent)						0.4	2.6	0.8

MIXTURES OF TWO GRADES.

The relation borne by the traction of mixtures to the traction of separate grades is most clearly shown by the records from varied combinations of two grades only. In all experiments with mixtures the same width of channel, 1 foot, was used; and from the adjusted capacities for mixtures of two it is possible to select a full set associated with the same discharge, 0.363 ft.³/sec., and the same slope, 1.4 per cent. These capacities are arranged for comparative examination in Table 61, and to them are added, from Table 12, the corresponding capacities for the separate grades.

TABLE 61.—*Capacities for traction, with varied mixtures of two grades.*

Grade.	Capacities.			Grade.	Capacities.			Grade.	Capacities.			Grade.	Capacities.			Grade.	Capacities.		
	Total.	(A)	(G)		Total.	(C)	(G)		Total.	(B)	(F)		Total.	(C)	(E)		Total.	(E)	(G)
(A)	185	185	0	(C)	143	143	0	(B)	149	149	0	(C)	143	143	0	(E)	62	62	0
(A_3G_1)	175	131	44	(C_4G_1)	158	126	32	(B_4F_1)	170	133	37	(C_4E_1)	154	122	32	(E_4G_1)	59	47	12
(A_2G_1)	169	113	56	(C_2G_1)	144	96	48	(B_2F_1)	145	93	52	(C_2E_1)	135	88	47	(E_2G_1)	65	43	22
(A_1G_1)	89	44	45	(C_1G_1)	106	53	53	(B_1F_1)	129	61	68	(C_1E_1)	110	53	57	(E_1G_1)	49	24	25
(A_1G_2)	42	14	28	(C_1G_2)	50	17	33	(B_1F_2)	69	21	48	(C_1E_2)	91	28	63	(E_1G_2)	33	11	22
(A_1G_4)	25	5	20	(G)	16	0	16	(B_1F_4)	39	7	32	(C_1E_4)	75	14	61	(G)	16	0	16
(G)	16	0	16					(F)	33	0	33	(E)	62	0	62				

Each vertical column of capacities shows the tractional power of the current, first for the finer component alone, then for mixtures with progressively increasing shares of the coarser component, and finally for the coarser alone. The order of the different groups is that of the contrast in fineness between the finer and coarser components. If the linear fineness of the finer be divided, in each case, by the fineness of the coarser, the ratios obtained are

(AG)	(CG)	(BF)	(CE)	(EG)
16.2	9.7	8.5	3.4	2.9

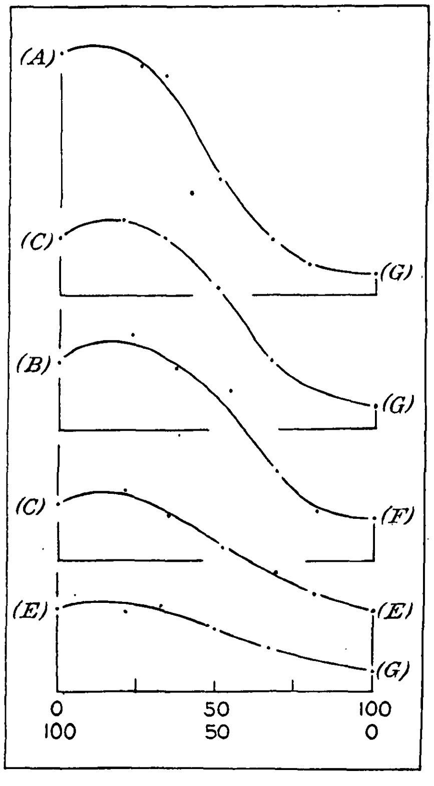


FIGURE 55.—Tractional capacity for mixed débris, in relation to proportions of component grades.

The data of the table are plotted, with a different arrangement, in figures 55 and 56. In figure 55 the total capacities are plotted in relation to the proportions of fine and coarse in the mixtures. The vertical scale being of capacity, the horizontal, if read from right to left, is of percentage of the finer component, or if read from left to right, is of percentage of the coarser. Figure 56 (p. 174) shows, in parallel columns, the capacities for the component grades, those for the coarser components being at the left.

Attention may first be directed to the total capacity curves of figure 55. If the capacities for mixtures were related in the simplest manner to the proportions of components, the curve of capacities would be a straight line joining the points (at the extreme left and extreme right) given by the capacities for the components. The (EG) curve—that placed lowest in the diagram—lying wholly above such a hypothetic line, would suggest that all mixtures give an advantage in traction, but this suggestion is not supported by the other curves. The upper curve, for example, would lie as much below as above a straight line joining its extremities. The capacity for the mixture appears always to exceed what may be called the pro rata estimate when the finer component has the higher proportion, but it may fall below that estimate when the coarser component predominates.

The curves of figure 56 show the capacities apportioned to the components of the mixtures. Each one represents that portion of the total load which consists of the material of one component. In each curve of the left-hand column the ordinate at the right represents the load of the coarser material when by itself, and the successive ordinates toward the left show how the load is modified by the admixture, in the material fed to the current, of gradually increasing percentages of the finer débris. There are no observations with very small percentages of the finer material, but in each case the second fixed point of the curve shows an increase of load. The addition of the finer débris not only increases the total capacity but increases the capacity for the coarser débris. The amount of the latter increase appears to be greater as the contrast in fineness of components is greater, and in the extreme case the capacity for the coarser is multiplied by 3.5. The capacity of the current for débris of grade (G) is 16 gm./sec.; but when that débris is mixed with twice its weight of grade (A), which is 16.2 times as fine, the capacity for the mixture is so large that one-third of it, apportioned to grade (G), is 56 gm./sec. In three of the five suites of experiments the highest capacity recorded for the coarser débris corresponds to the mixture of 1:1. In the others it corresponds approximately to 2:1 and 1:2. If the position of the maximum is related to contrast in fineness, it is associated with a larger ratio of the finer component when the contrast is great.

The corresponding curves for the finer component (at the right in fig. 56) show that capacity for finer débris is influenced by the admixture of coarser, but not in the same way. Each addition of the coarser reduces capacity, the rate of reduction being at first gradual, then more rapid, and afterward gradual.

The general effect of the addition of fine material being to increase capacity, and the effect of adding coarse material to reduce capacity, let us now inquire the effect of diversifying the material of the load by exchanging part of it for a finer grade and part for a coarser. Probably no general answer to this question may be derived from our data, but a partial answer is possible if we assume that the initial grade is separated from each of the substituted components by the same contrast of fineness, expressed as a ratio. To take a concrete example, from Table 61, the capacity for grade (A) is 185 gm./sec., for grade (G) 16 gm./sec., and for (A_1G_1), their equal mixture, 89 gm./sec. When for half of (A) there is substituted an equal amount of (G), which is 16.2 times coarser, the capacity is changed in the ratio $\frac{89}{185}=0.48$. When for half of (G) there is substituted an equal amount of (A), which is 16.2 times finer, the capacity is changed in the ratio $\frac{89}{16}=5.56$. The geometric mean of these ratios, 1.64, may plausibly stand for the effect of substituting an equal mixture of (A) and (G) for a grade symmetrically intermediate between (A) and (G). The method is easily criticized, and its assumptions will certainly not bear close scrutiny, but it nevertheless yields a sort of composite which is of use in showing that the general effect of diversifying a stream's tractional load is to enlarge capacity. Corresponding composite ratios have been obtained from the other examples of Table 61 and are given below.

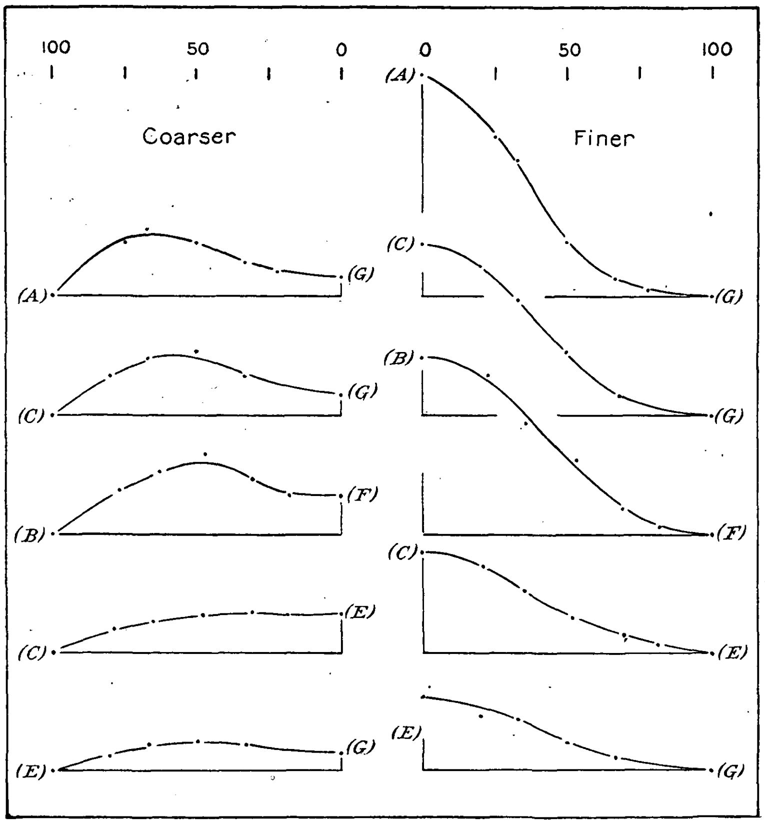


FIGURE 56.—Tractional capacities of components of mixed grades, in relation to the percentages of the components in the mixtures. Curves at left show capacities for coarser component, at right for finer. Ratios at left show percentages of finer component, at right of coarser.

Grade.	Ratio of fineness.	Ratio of capacity change attributed to mixture.
(AG)	16.2	1.64
(CG)	9.7	2.22
(BF)	8.5	1.72
(CE)	3.4	1.17
(EG)	2.9	1.56

The composite ratios have been arranged in the order of the ratio of fineness, but the comparison shows no correspondence. They prove equally inharmonious when compared with the fineness of the finer component, the fineness of the coarser component, or the fineness of the mixture. Their irregularities must be ascribed to observational errors and to causes not at present to be discriminated from observational errors. It is of interest, however, to note that the large measure of capacity change associated with the (CG) combination might be inferred also from a comparison which involves a different viewpoint and also some practically independent data. If we think of grades (A), (C), and (E) as modifiers of capacity for grade (G), we may compare their efficiencies by means of the following quantities, taken from Table 61:

Grade.	Capacity.	Grade.	Capacity.
(A_1G_1)	89	(A_1G_2)	42
(C_1G_1)	106	(C_1G_2)	50
(E_1G_1)	49	(E_1G_2)	33

The superiority of grade (C) as a modifier for (G) is thus brought out without making use of the capacities for the uncombined grades (A), (C), (E), and (G); and the result from mixtures of 1:1 is supported by that from mixtures of 1:2.

CONTROL BY SLOPE AND DISCHARGE.

The preceding comparisons are conditioned by a discharge of 0.363 ft.³/sec., a slope of 1.4 per cent, and a channel width of 1 foot. With a different set of conditions a different set of quantitative relations would be found, and the qualitative also would doubtless be modified. The observations on mixtures included no other width, and there was but a single set of experiments using a different discharge, but the range in slope was coordinate with that for the separate grades.

Figure 57 shows the capacity-slope curves for the (AG) set of experiments, figure 58 for the (BF) set, and figure 59 for the (CE) set. In figure 57 the curves for mixtures form a graded series between those for the component grades, and there is almost perfect harmony of form and attitude. As the points representing capacities associated with a slope of 1.4 per cent all lie in the same vertical line, and as similar points for another slope lie in some other vertical line, it is evident by inspection that inferences from data of any other available slope would be practically identical with those from the slope of 1.4 per cent. It is also evident

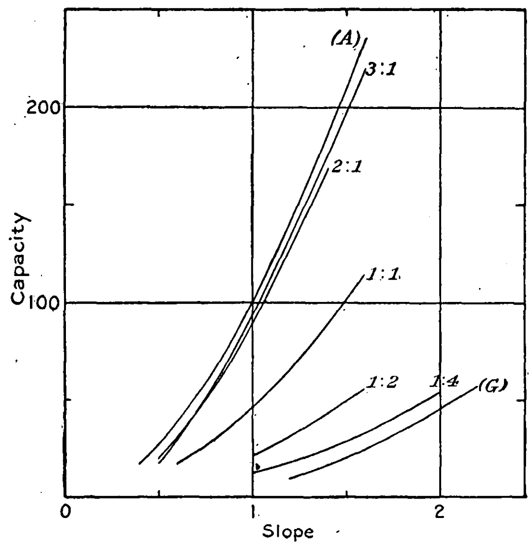


FIGURE 57.—Curves of capacity in relation to slope for grade (A), grade (G), and mixtures of those grades. The ratios of components in the mixtures are indicated.

that indexes of relative variation, i_1 or I_1, for the mixtures constitute, with those for the components, an orderly system. The same remarks apply also to the (CE) groups of

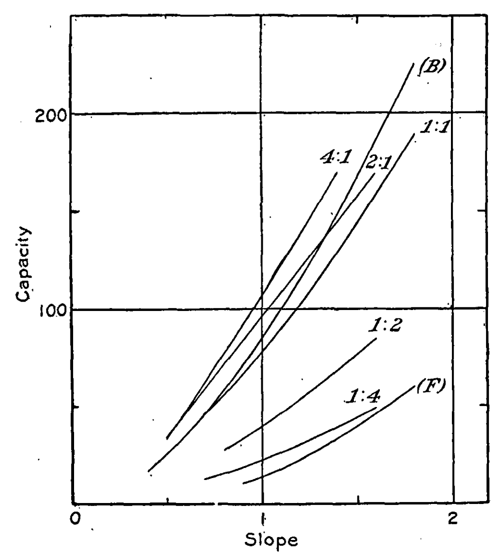


FIGURE 58.—Curves of capacity in relation to slope for grade (B), grade (F), and mixtures of those grades. The ratios of the components in the mixtures are indicated.

curves, but they do not apply to the (BF) group in figure 58. The attitudes of the curves for mixtures are there out of harmony with the attitudes of the (B) and (F) curves. The curves for the mixtures seem to belong to a different system, intersecting or tending to intersect the curves for the components. Evidently the indexes of relative variation are inharmonious, and evidently the inferences drawn from data for the slope of 1.4 per cent would not be duplicated by a discussion of data from a slope of 1.0 per cent or 1.6 per cent.

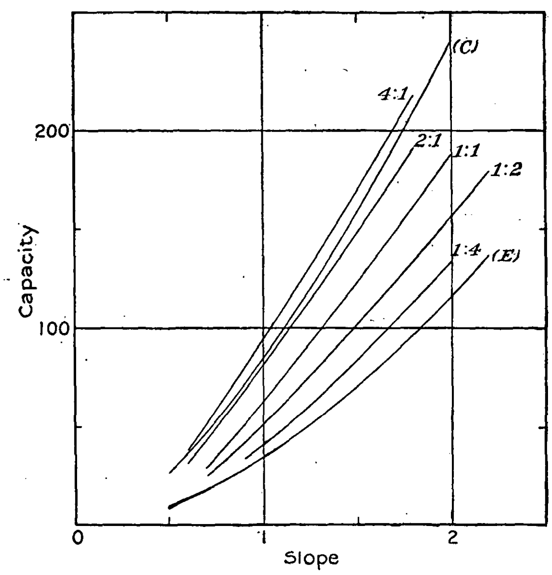


FIGURE 59.—Curves of capacity in relation to slope for grade (C), grade (E), and mixtures of those grades. The ratios of the components in the mixtures are indicated.

Graphic comparison has been extended to the remaining data of Table 60, so far as dual mixtures are concerned, but no marked discordance has been discovered outside of the (BF) group. Although the cause of the exceptional discordance has not been found, I believe that it should be ascribed to some exceptional though unknown circumstance and not be permitted to nullify the otherwise harmonious testimony. The tenor of that testimony is that the relation of capacity to slope is substantially the same for mixtures as for simpler grades of débris. There is, however, a noteworthy qualification to this statement, in that the values of σ have a smaller average for mixtures than for component grades.

The single comparison possible between results obtained with different discharges indicates that with mixtures, just as with their components, capacity increases with discharge in more than simple ratio.

MIXTURES OF MORE THAN TWO GRADES.

Experiments were made with a mixture of three grades, and with a mixture of five. The former was observed with discharges of 0.182 and 0.363 ft.³/sec., the latter with discharges of 0.182, 0.363, and 0.545 ft.³/sec. (See Tables 4 (J) and 60.)

Brief consideration only will be given to the data from the mixture of three, $(A_1C_1G_2)$, because the points on which they bear are more fully covered by the data from the mixture of five. Grades (A) and (C) differ from one another in fineness much less than either differs from grade (G). The triple mixture may therefore be thought of as half coarse and half fine, with the distinction that the fine half is made up of two grades. By comparing data from it with data from the closely related mixtures (A_1G_1) and (C_1G_1), in which the fine portion is of a single grade, we may throw light on the question whether the advantage to traction which is obtained by substituting two grades for one may be augmented by further diversification. With discharge 0.363 ft.³/sec. and slope 1.4 per cent the capacities for the three mixtures are

(A_1G_1)	$(A_1C_1G_2)$	(C_1G_1)
89	98	106

As the capacity for the triple mixture has a value midway between those for the two dual mixtures, no advantage is indicated for the greater diversification.

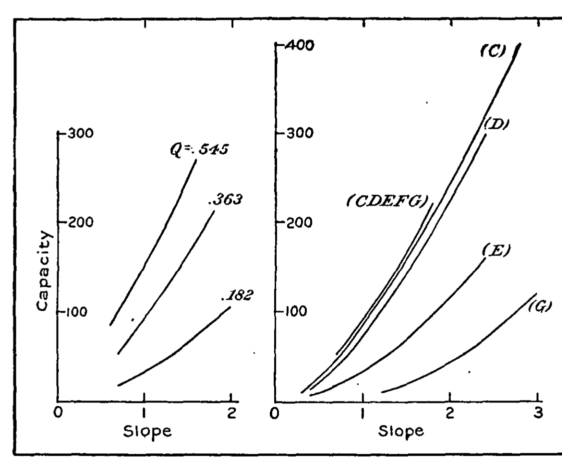


FIGURE 60.—Curves of capacity in relation to slope for a mixture of five grades, (CDEFG). Comparison of mixture curves for three discharges, and of mixture curve with curves for component grades.

In the mixture of five grades the proportions were so arranged as to approximate a natural combination. The components and their percentages are shown by writing $(C_{45}D_{35}E_{12}F_6G_2)$. The curves of capacity in relation to slope are shown, for the three discharges, in the left-hand diagram of figure 60. In the right-hand diagram the curve for discharge 0.363 ft.³/sec. is repeated, and with it are placed the corresponding curves for four of the component grades—that for grade (F) being omitted because it is nearly coincident with the one for grade (E).

The curve for the mixture is not only of the same type with the others, but runs nearly parallel with those nearest. For all slopes the capacities obtained for the mixture are greater than for any component. The fact that the mixture gives a greater capacity than does the fine grade (C) alone shows that the addition to (C) of 35 per cent of (D) and 20 per cent of still coarser grades works an advantage instead of a detriment.

In figure 58 it is seen that the addition of 20 per cent of coarser débris to grade (B) increases capacity, and in figure 59 that a similar increase accompanies the addition of 20 per cent of coarser débris to grade (C). So far as the case of the more complex mixture is comparable to these, there is no indication that a mixture of great complexity has advantage for traction over a mixture of two components only. As to this point the inference from data of the five-part mixture is supported by that already drawn from a datum of the $(A_1C_1G_2)$ mixture, and it is

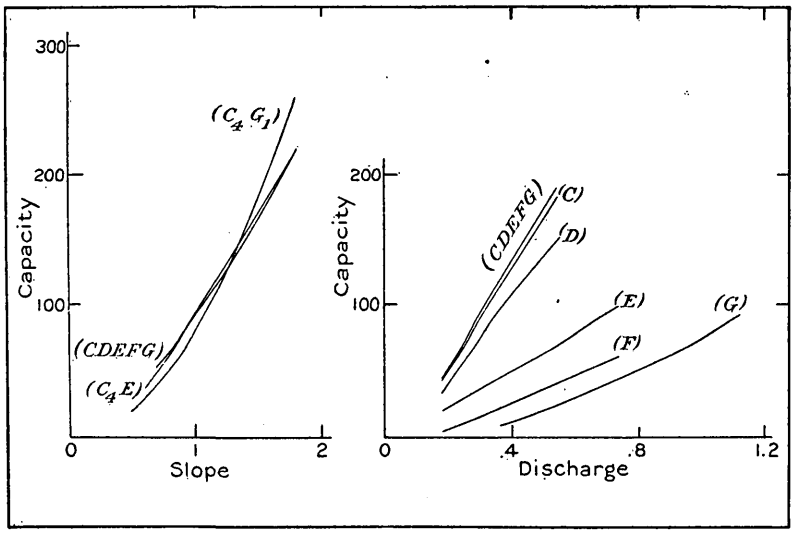


FIGURE 61.—Capacity-slope curves for related mixtures and capacity-discharge curves for mixture and component grades.

further supported by the facts brought together in the left-hand diagram of figure 61, which shows the capacity-slope curves of mixtures (C_4E_1) and (C_4G_1) along with that for the five-part mixture.

The right-hand diagram of figure 61 is a group of plots of capacity as a function of discharge. These plots pertain to the complex mixture and its five components, and all are conditioned by a slope of 1.2 per cent. They show that the tractional superiority of the mixture is not confined to the use of a particular discharge, and they indicate also that the capacity-discharge relation is essentially the same for the mixture as for separate grades. The locus of $C=f(Q)$ for the mixture is approximately a straight line, and if produced it intersects the axis of Q to the right of the origin. It might be expressed by an equation in the form of (64) with an exponent near unity.

A NATURAL GRADE.

Two series of experiments were made with an alluvium in its natural condition, except that the very finest constituents had been removed by passing it over a 60-mesh sieve. The observations are recorded in Table 4 (K), and the adjusted capacities in Table 60. In figure 62 the capacity-slope curves are plotted, and each is accompanied, for comparison, by the corresponding curves for grades (A) and (C).

The approximate mechanical analysis of this material, stated in terms of the separated grades of the laboratory series, is as follows:

	Per cent.		Per cent.
(A)	6	(D)	42
(B)	13	(E)	10
(C)	27	Coarser than (E)	2

The fact that the capacities (fig. 62) are greater than those for grade (C), notwithstanding the dominance of a component corresponding to the coarser grade (D), testifies again to the advantage for traction of a mixture as compared to a grade of narrow range in fineness.

It will be observed also that each of the curves from the natural grade resembles closely its neighbors from artificial grades. So far as their evidence goes the type is the same for both, and the tendency of their evidence is to show that the laws connecting capacity with slope, as developed by the study of sorted débris, apply also to unsorted stream alluvium.

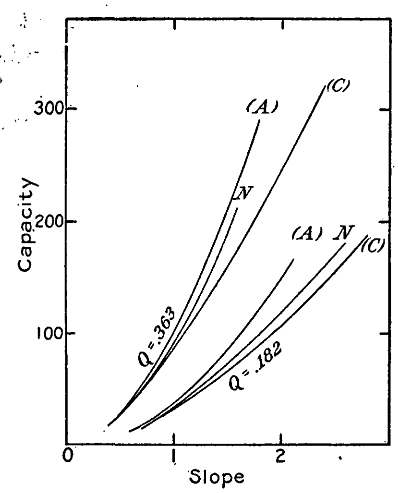


FIGURE 62.—Capacity-slope curves for a natural grade of débris, compared with curves for sieve-separated grades.

Table 63 (p. 180) gives computed finenesses for various mixtures and for this natural grade of débris. The fineness of the natural grade is nearly identical with that of grade (C), and the two thus afford a direct comparison between the capacities of a natural grade and a narrowly limited grade. For the same discharge and slope their tabulated capacities are respectively 168 and 143 gm./sec., the ratio of advantage to the natural grade being 1.17. The computed fineness of grade (CDEFG) does not correspond to that of any simple grade, but a comparison made by means of interpolation gives 150 and 131 gm./sec. as corresponding capacities for the mixture and a simple grade of the same mean fineness, and the ratio of the first to the second is 1.15. These ratios are smaller than those estimated from data for binary mixtures (p. 174), but are coordinate in value. The question of relative authority will be considered later.

CAUSES OF SUPERIOR MOBILITY OF MIXTURES.

When a finer grade of débris is added to a coarser the finer grains occupy interspaces among the coarser and thereby make the surface of the stream bed smoother. This quality of smoothness appealed to the eye during the progress of the experiments. One of the coarser grains, resting on a surface composed of its fellows, may sink so far into a hollow as not to be easily dislodged by the current, but when such hollows are partly filled by the smaller grains its position is higher and it can withstand less force of current. In other words, the larger particles are moved more readily on the smoother bed, and this fact also was a matter of direct visual observation. The promotion of mobility applies not only to the starting of the grain but to its continuance in motion. It encounters less resistance as it rolls or skips along the bed, and it is less apt to be arrested. When a single large particle travels along a bed composed wholly of grains much smaller it rarely leaps, but rolls instead, and it must in general be true that the larger particles in mixtures roll more and skip less than their smaller companions.

The admixture of finer débris thus changes the mode of traction for the coarser, and it is believed that the enhanced capacity is due mainly to this change. Capacity for the coarser is increased because the new condition reduces its resistance to the force of the current.

The fact that under some conditions the capacity for fine material is slightly increased by the addition of coarser is not so easily explained. The coarser grains do not make the bed smoother but rougher. The rougher bed retards the current. Even while rolling the larger grains are holding back the water, and the larger grains reduce the area of bed on which the traction of the smaller takes place. In these ways the presence of the coarse material tends to reduce the capacity of the current for the fine, and these factors certainly seem adequate to explain the general fact that capacity for the finer débris is reduced by admixture of the coarser.

Two factors may be named with the opposite tendency. The first is the impact of the coarser particles. In rolling and leaping they disturb the finer, tending thus to dislodge them from their resting places and either start them forward or else give them new positions from which they may be more easily swept. The second is the production of diversity in the current. Every obstruction diversifies the current. The deflection necessary to pass it both constitutes and causes diversity of direction, and diversity of direction necessitates diversity of velocity. If a pebble is placed on the sandy bed of a small stream, the transformation of the adjacent parts of the bed by the diversified current is obvious. The building up of the bed in the lee of the pebble testifies to lowered velocity, and the scouring at the side to heightened velocity. In the same way each coarser grain of a heterogeneous stream load diversifies the current about it and gives to such of the filaments as are accelerated greater power for the traction of finer grains. With reference to the transportation of the finer débris, the coarser grains have the function of obstructions whether they are partly embedded or lie on the surface or are rolled along.

If the factors concerned in the traction of the finer components of mixtures have been correctly stated, it is not difficult to understand that under most conditions the net result of their influences will be a reduction of capacity, and also that special conditions may determine an increase.

VOIDS.

The packing together of larger and smaller grains which tends toward a smooth stream bed tends also toward the reduction of interstitial spaces within the bedded débris, thus reducing the percentage of voids. It was suggested in the laboratory that the percentage of voids, used inversely, might serve as a sort of index of mobility, and estimates of the voids were accordingly made for most of the materials employed in the experiments with mixed grades. Partly for this purpose, and partly to obtain the factors needed to correct the weighings of load for interstitial water, a series of special weighings were made.

A vessel holding 535 cubic centimeters was filled with saturated débris and weighed. Afterward the same débris was weighed in a dry condition. The computation was made by the formula

$$\text{Percentage of voids} = \frac{W' - W}{535}$$

in which W' is the weight of saturated débris in grams and W the weight of dry débris.

Table 62 contains the estimated voids for the binary mixtures. In each series the percentages are smaller for the mixtures than for the component grades; and when the percentages were plotted in relation to the proportions of component grades (after the manner of the capacities in figs. 55 and 56) each series was found to indicate a minimum. The positions of the minima correspond to mixtures with 30 to 40 per cent of the finer grade of débris. In comparing voids with capacities, the minima of the void curves are to be considered in relation to the maxima of the capacity curves.

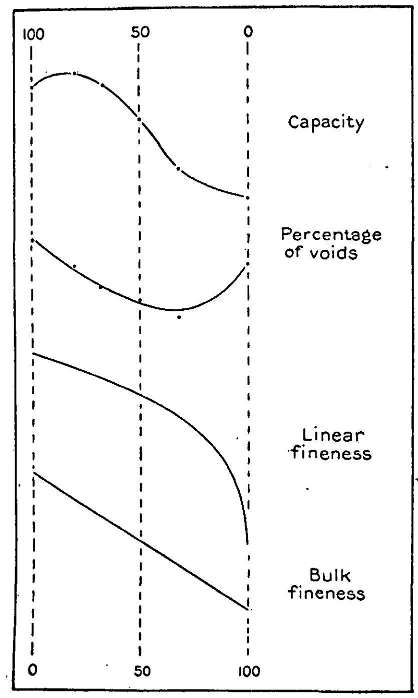


FIGURE 63.—Curves showing the relations of various quantities to the proportions of fine and coarse components in a mixture of two grades of débris, (C) and (G). The horizontal scale, when read from left to right, shows the percentage of the coarser component in the mixture.

Those maxima, however, are associated with mixtures having 60 to 90 per cent of the finer grade; and the attempt at correlation therefore fails. A single void curve is reproduced in figure 63, together with the corresponding capacity curve.

TABLE 62.—*Percentages of voids in certain mixed grades of débris, compared with the percentages in the component grades.*

Percentage of finer grade in mixture.	Percentage of voids in grade—					
	(AC)	(AG)	(BF)	(CE)	(CG)	(EG)
0	44	38	37	40	38	38
20	....	28	31	33	....	....
33	....	24	29	32	26	29
50	40	29	29	33	30	28
67	....	31	34	35	33	34
80	....	37	39	37	38	36
100	44	44	46	44	44	40

FINENESS.

A similar attempt was made to correlate the capacity curves of figures 55 and 56 with fineness. In computing the fineness of a mixture, the finenesses of its components were used as data, and the combination was made with bulk finenesses (p. 21) as follows: Denoting the proportions of components by a, b, c, etc., their bulk finenesses by F_2', F_2'', F_2''', etc., and the fineness of the mixture by F_2^m,

$$F_2^m = \frac{aF_2' + bF_2'' + cF_2''' + \text{etc.}}{a+b+c+\text{etc.}} \quad \text{....} \quad (85)$$

Linear fineness, F, was then computed by formula (88).

The results are listed in Table 63, together with the corresponding capacities for traction when the discharge is 0.363 ft.³/sec. and the slope is 1.4 per cent.

TABLE 63.—*Finenesses of mixed grades and their components.*

[Computed from data in Tables 1, 4 (J), and 4 (K).]

Grade.	$F_2/10^6$	F	C	Grade.	$F_2/10^6$	F	C
(A)	1,910	1,002	185	(C)	417	602	143
(A_1C_1)	1,163	848	161	(C_4G_1)	334	557	158
(C)	417	602	143	(C_2G_1)	278	526	144
				(C_1G_1)	209	478	106
				(C_1G_2)	139.3	418	50
(A)	1,910	1,002	185	(G)	0.451	61.8	16
(A_3G_1)	1,432	908	175				
(A_2G_1)	1,273	873	169				
(A_1G_1)	955	794	89	(E)	10.77	178	62
(A_1G_2)	637	678	42	(E_4G_1)	8.71	166	59
(A_1G_4)	420	604	25	(E_2G_1)	7.33	157	65
(G)	0.451	61.8	16	(E_1G_1)	5.61	143	49
				(E_1G_2)	3.89	127	33
				(G)	0.451	61.8	16
(B)	1,023	812	149				
(B_4F_1)	798	747	170				
(B_2F_1)	656	700	145	(A)	1,910	1,002	185
(B_1F_1)	482	632	129	($A_1C_1G_2$)	582	673	98
(B_1F_2)	318	538	69	(C)	417	602	143
(B_1F_4)	186	459	39	(G)	0.451	61.8	16
(F)	1.685	95.9	33				
				(C)	417	602	143
(C)	417	602	143	(CDEFG)	228	492	150
(C_4E_1)	332	557	154	(D)	111.5	388	127
(C_2E_1)	275	524	135	(E)	10.77	178	62
(C_1E_1)	206	476	110	(F)	1.685	95.9	33
(C_1E_2)	137	406	91	(G)	0.451	61.8	16
(C_1E_4)	88	358	75				
(E)	10.77	178	62				
				Natural.	419	603	168

The comparisons of capacity with fineness for series of mixtures of two grades are illustrated by figure 63, where the horizontal scale is that of the proportions of fine and coarse in the mixture. The capacity curve is identical with the second in figure 55, and the other curves pertain to the same series of (C G) mixtures. The curves for capacity and linear fineness are strongly discordant, capacity changing most rapidly with mixtures approximately in the ratio of 1:1, and fineness changing most rapidly when the proportion of the finer grade is minute. The graph of bulk fineness is a straight line and betrays no sympathy with the sigmoid curve of capacity, though somewhat less discordant than the curve of linear fineness.

It is quite evident that the peculiar relation of capacity to the proportions of a binary mixture is not to be either accounted for or formulated as a relation of fineness, and we have just seen that it can not be formulated in terms of the percentage of voids. The elimination of those two associated factors leaves it—so far as our recognized alternatives are concerned—to be ascribed wholly to modifications of the texture of the channel bed and the consequent modifications of the mode of transportation, and to these factors it is not practicable to give numerical expression.

RELATION OF CAPACITY TO FINENESS, FOR NATURAL GRADES.

There is another way of comparing the capacities pertaining to mixed grades with the fineness of the grades, which largely avoids the influence of changing mode of traction and which throws a side light on the relation of capacity to fineness in the case of natural grades. Instead of comparing the data for different mixtures of the same two simple grades, it compares data from similar mixtures of different pairs of simple grades.

Figure 64 has been compiled from data in Table 63. Its upper group of five dots represents the logarithms of capacity in relation to the logarithms of linear fineness, for combinations of fine and coarse in the ratio of 4:1 (one ratio of 3:1 being included). The line drawn among them gives, by its inclination, an estimate of I_4, the synthetic index of capacity in relation to fineness, for a range in fineness from 166 to 908, the value being 0.70. The next group of dots corresponds to mixtures of two fine to one coarse and gives 0.57 as a value of the index. The next group, distinguished by crosses, corresponds to mixtures of one part fine with one of coarse. It includes six points, but one of these stands far from the line suggested by the others. The line, as drawn, represents an index value of 0.62. In the fourth group, distinguished by X's and corresponding to mixtures of one fine to two coarse, the points are so irregularly placed that no line can be drawn; and a fifth group, not reproduced, is equally irregular. Fineness seems to control capacity when the finer component of the mixture is the more important, but other factors mask its influence when the coarser component dominates.

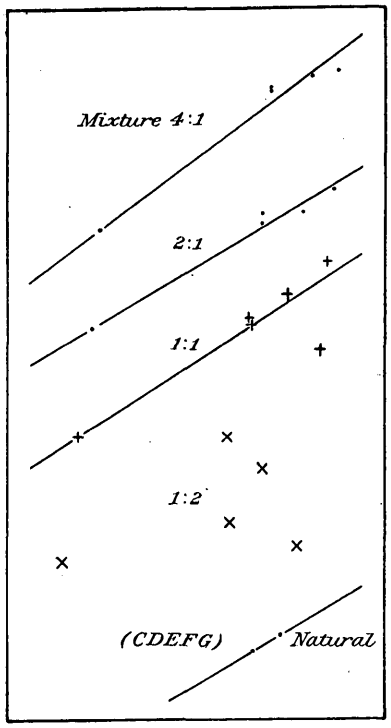


FIGURE 64.—Logarithmic plots of capacity in relation to linear fineness, for related mixtures of débris.

In preparing the mixture ($C_{45}D_{35}E_{12}F_6G_2$) the endeavor was made so to apportion the components as to approximate a natural grade. To whatever extent that effort succeeded, the data obtained with use of the mixture are comparable with those afterward obtained with a natural grade. The capacities and finenesses of the complex mixture and the natural grade are represented by two plotted points near the bottom of figure 64, and the line drawn through them gives 0.57 as a value of the index. Unfortunately the range in fineness covered by the two is small—from 492 to 603—so that their determination of I_4 has little weight; its close agreement with other values may be largely accidental.

In Table 45 mean values of I_4 based on work with the sieve-separated grades are so arranged as to show their control by slope, discharge, and trough width. On making the indicated allowances for differences in condition, and comparing those values with the values indicated by figure 64, it appears that the latter are somewhat smaller but that the differences are not great. The index values from data of the experiments with mixtures range from 0.57 to 0.70. Coordinate values from the experiments with single, sieve-separated grades would range from 0.70 to 0.90.

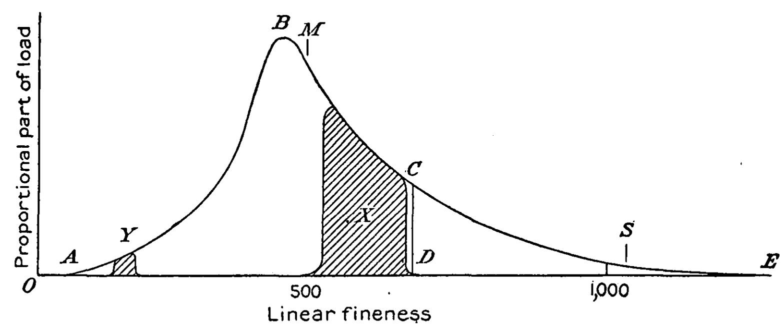


FIGURE 65.—Curve illustrating the range and distribution of finenesses in natural and artificial grades of débris.

In considering the bearings of these estimates there is advantage in giving graphic expression to the conditions they severally represent. When the results of mechanical analyses of sands and similar materials are presented graphically, the usual practice is to plot integrated quantities, or proportions, of ingredients on a scale of fineness or coarseness; but for the present purpose it is convenient to use the quantities without integration. In figure 65 the horizontal scale is that of linear fineness, and the ordinates are relative quantities of components of different finenesses. The curve *ABC* represents the composition of the débris constituting the tractional load of a river and is based on a sample of alluvium from a river bed. The area *ABCD*, between the curve and the axis of fineness, represents the total load and corresponds to the 100 per cent of integrative diagrams. The range in fineness is limited at *A* by competent fineness for traction, coarser material on the river bed not being moved by the particular discharge to which the diagram corresponds. It is limited in the opposite direction by competent fineness for suspension, the area CED representing suspendible material entangled with the bed load. The mean fineness of the tractional load is marked at M.

In sorting débris for the laboratory experiments, the material of the river's load was divided by sieves, and this partition might be represented in the diagram by a series of vertical lines—slightly flexed to take account of the influence on the separation of irregularity in shape of particles. The shaded areas X and Y may represent the constitution as to fineness of two of the sieve-separated grades.

The experiments show that for a narrowly limited grade, such as X or Y, which has the same mean fineness as the river alluvium, $ABCD$, the capacity for traction is much less than for the alluvium. They indicate also, though less decisively, that for a nearly equal mixture of a fine grade with a coarse, both being narrowly limited, the capacity is nearly the same as for the unsorted alluvium, provided the mean fineness is the same. The essential property appears to be abundance of both coarse and fine, and not multiplicity of grades of fineness.

Passing now from capacity to capacity's rate of variation—in respect to fineness—or to the valuation of i_4, we find a certain parallelism. For the narrowly limited grades the sensitiveness of capacity to fineness, as measured by the index i_4, extends upward from about 0.60; while values of the synthetic index, I_4, range from 0.70 to 1.00 or more. As to these values the data are not abundant. For mixtures of two narrow grades we have a few estimates of I_4, of which the largest is 0.70; and for grades similar to $ABCD$ a single weak estimate of 0.57. If we conclude that the sensitiveness of capacity to fineness is less for natural grades than for the narrowly limited grades, we must base the inference almost wholly on the data from the mixtures of two grades, connecting the latter with natural grades by aid of the analogy outlined above. This I am willing to do, but at the same time I would record my recognition of the weakness of the evidence and reasoning. It is estimated that, on the average, the capacity of streams for natural grades of débris varies with the 0.60 to 0.75 power of linear fineness. This is equivalent to saying that capacity varies with the 0.20 to 0.25 power of bulk fineness, or with the fifth or fourth root of bulk fineness.

While the curve in figure 65 is based on the mechanical analysis of material which constituted the tractional load of a river current, there is no reason to believe that its form presents a dominant type. Inspection of other analyses, in fact, suggests that such curves exhibit much variety and may sometimes even present two maxima. The load which a natural current carries is determined not only by the two limits of competence, but by the character of the material within its reach. Neighboring affluents of a river may bring to it strongly contrasted grades of débris, or their tribute may at one time be much finer than at another. Moreover, a river is not a simple current, but a complex of currents, which vary in competence and in the character of their loads. It is true that, the channel being considered as a whole, its load at one point is essentially the same as just above or just below, but the mode of movement involves a continual remodeling of the bed and a sorting and re-sorting of the material. The load at any particular point and time is conditioned by many factors of the complex. For this reason a representative sample of a river's load is not easy to define or to collect.

In view of this complexity it is difficult to apply even a simple formula to problems in river engineering, and refinement in formulation would be of little avail. For the same reason it is not practicable to derive a formula directly from river data, and the product of the laboratory is the best available, despite the artificial simplicity of its conditions.

DEFINITION AND MEASUREMENT OF MEAN FINENESS.

The term "mean fineness," as here used, is not free from the possibility of misapprehension. As the fineness of débris is a property depending on the size of component particles, it is not unnatural to think of fineness as a property of the particles—and there is, for that matter, a fineness of particles. To obtain the mean fineness of particles, one would first determine the finenesses of the individual particles, and then the mean of those finenesses. The basal unit would be the particle. In deriving the mean fineness of a body of débris the basal unit is some unit by which quantity of débris is measured. It may be a unit of weight or a unit of volume. In this report a body of débris is conceived to be composed of equal volume units, each of which has a determined or determinable fineness, and its mean fineness is the mean of the finenesses of the volume units. In dealing with bulk fineness the mean computed is the arithmetical mean of the finenesses of units. In dealing with linear fineness the mean computed is the cube root of the arithmetical mean of the cubes of the finenesses of units.

The intricacy of the definition of mean linear fineness arises from the relation of linear fineness to bulk fineness. The fundamental concept is that of bulk fineness, and the definition of linear fineness rests upon it. Linear fineness is essentially a derivative of bulk fineness, and mean linear fineness is an exactly similar derivative of mean bulk fineness. To pass from an assemblage of linear finenesses to their mean, it is necessary to pass through bulk fineness, and that passage involves cubes and cube root.

Bulk fineness is defined as the number of particles in a unit volume (1 cubic foot), it being assumed there are no voids. It is the reciprocal of the volume of the particle—which might be called bulk coarseness. There are two practical modes of measuring it. If the specific gravity of the débris be known (or assumed), measurement includes a weighing and a counting. Then, W being the weight, N the number of particles, G the specific gravity, W_0 the weight of a cubic foot of water, and F_2 the bulk fineness,

$$F_2=\frac{W_0GN}{W} \quad \text{------------} (86)$$

If the specific gravity be not known, measurement includes two weighings and a counting. Then, W being the weight in air and W_1 the weight in water,

$$F_2=\frac{W_0N}{W_1-W} \quad \text{------------} (87)$$

This procedure determines bulk fineness when all particles have the same volume; when they are of different volumes it determines mean bulk fineness. As a matter of fact, all our measurements in the laboratory were of mean fineness. It is not possible by any method of sorting with which I am acquainted to separate from a natural alluvium a grade which is really uniform in fineness.

When the mean fineness of a sample of débris is desired, there is no need to separate it into grades, because the process for measuring the mean fineness of the whole is identical with that for measuring the fineness of a grade, When bodies of débris of known finenesses are mingled, the mean fineness of the mixture is computed by a formula (85), which sums the finenesses by unit volumes (or weights) and then divides by the number of unit volumes.

Linear fineness is defined as the reciprocal of the mean diameter of the particles of the débris. Like bulk fineness, it is treated as a property of the body of débris and not as a property of the particle. Mean diameter is defined as the diameter of a sphere having the same volume as the particle. Defined thus, linear fineness is a function of volume of particle, and as bulk fineness is also a function of that volume, the two have a fixed relation:

$$F=\left(\frac{\pi}{6}\right)^{\frac{1}{3}}F_2^{\frac{1}{3}} \quad \text{------------} (88)$$

Substituting in (88) from (86) and (87), we have

$$F=\left(\frac{\pi}{6}\right)^{\frac{1}{3}}\left(\frac{W_0GN}{W}\right)^{\frac{1}{3}} \quad \text{---------} (89)$$

$$F=\left(\frac{\pi}{6}\right)^{\frac{1}{3}}\left(\frac{W_0N}{W_1-W}\right)^{\frac{1}{3}} \quad \text{---------} (90)$$

The computations of fineness for this report used (89) or (90); or, what is equivalent, they first determined bulk fineness by (86) or (87), and then derived linear fineness by (88). The computations of mean linear fineness applied (88) to mean bulk fineness.

It would have been possible to formulate fineness in such a way that the definition of linear fineness would be direct and comparatively simple, but any such formulation would encounter complexity in some of its parts, provided it established a logical relation between linear fineness and bulk fineness. Its adoption would also involve the sacrifice of simplicity in the measurement of fineness. Any system requiring the direct measurement of diameters would be inferior for practical purposes to the one here used.

The subject of scales of fineness has been elaborated because nearly all the results as to the control of capacity by fineness would be quite different if a different scale were used. If, for example, mean linear fineness had been defined as the arithmetical mean of linear finenesses, the curve for linear fineness in figure 63 would be a straight line, while the line for bulk fineness would be a curve similar to that shown for linear fineness but turned through 180°.

When the fineness of the tractional load of a stream is to be determined by means of a sample of the débris constituting its bed, account must be taken of another factor. Omitting considerations affecting the selection of a sample, which belong to Chapter XIII, let us assume that the sample in hand is representative of the stream's tractional load. In addition to the débris which was carried along the bed, it inevitably includes finer material which was carried in suspension. Suspended particles are arrested along with the coarser and form part of every stream deposit. Once lodged in the interstices of coarser particles, they are sheltered from the current and are not again disturbed so long as the coarser material remains. If the deposition of the coarser débris is very rapid the amount of entangled finer stuff may be small, but when deposition is slow the interstices act continuously as traps and catch suspendible débris until they are filled. The latter is the usual condition, and the tractional sample therefore ordinarily contains a considerable percentage of suspensional material. To separate the two it is necessary to draw an arbitrary line, for the graduation in fineness is complete. As regards interstitial space, the tractional part of the sample is comparable with the more complex mixtures of the laboratory, and its voids may be estimated as 25 per cent of the whole space. The suspensional débris packed in these voids may be assumed itself to include 25 per cent of voids, so that the net volume of its particles is three-fourths that of the containing voids, or 18.75 per cent of the whole space. The net volume of the tractional particles being 75 per cent of the whole space, the two divisions of the sample bear the relation, by net volume or by weight, of 75 to 18.75, or of 4 to 1. This gives a practical rule for separation. The sample should be divided, with aid of sieves and scales, into a coarser four-fifths and a finer one-fifth, and only the coarser part should be used in estimating mean fineness. In figure 65 the entrapped suspensional material is represented by the triangular area *CDE*.

SUMMARY.

The purpose of the experiments with mixtures was to bring the results from work with separate grades into proper relation with phenomena of unsorted natural material. The indications given by these experiments are in part direct and in part conditioned by the principle adopted in framing a scale of fineness. The adopted principle makes the conception of bulk fineness fundamental and that of linear fineness derivative.

The capacities for traction observed in the experiments with narrowly limited grades are less than for equivalent grades with greater diversity in fineness. A study of data from mixtures of two narrow grades indicates that the ratio of advantage for diversified débris is from 1.17 to 2.22, the mean of five estimates, from different groups of data, being 1.66. Two comparisons of results from highly diversified grades, with results from nearly homogeneous grades of the same fineness, give as estimates of the ratio of advantage 1.15 and 1.17. The larger estimates were made by an indirect method but are independent of the scale of fineness. The smaller estimates were made by a direct method but involve the theory of the scale of fineness. In combining the two groups of estimates, greater weight is assigned to the smaller, not because they are of recognized higher authority, but because the same scale of fineness will almost necessarily be used in applying the results of the investigation to practical questions. The compromise value of 1.2 is adopted, as a correction to be applied to values of capacity in Table 12 in estimating capacities for diversified grades of like fineness.

The advantage of diversification appears to arise largely from the fact that the finer particles, by filling spaces between the coarser, make a smoother road for the travel of the coarser, and it is not proved that a highly diversified débris gives higher capacity than one containing only two sizes of particles.

It is especially notable that when fine material is added to a previously homogeneous coarse material not only is the total capacity increased, but the capacity for the coarser part of the load is increased, and it may even be enlarged several fold. The general effect of adding coarse to fine is to reduce the stream's capacity for the fine, but under some conditions there is a slight increase.

The general relations of capacity to slope, discharge, and fineness (and presumably to form ratio also) are the same for natural and other complex grades of débris as for the sieve-sorted grades of the laboratory, but some of the constants are not quite the same.

The sensitiveness of capacity to slope is on the average the same for both classes of débris grades, but the variation of sensitiveness in relation to slope, as determined by the constant σ, is somewhat less for natural grades.

As to the relation of capacity to discharge comparison was limited to a single example, and that suggested no modification of the constants derived from work with laboratory grades.

The sensitiveness of capacity to fineness is somewhat less for natural grades than for the the laboratory grades. No values of the constant ϕ were obtained for complex grades, and comparisons of sensitiveness were made only by means of the synthetic index of relative variation. The average value of that index, for natural grades of débris transported under laboratory conditions, is estimated at 0.20 to 0.25 for bulk fineness.

CHAPTER X.—REVIEW OF CONTROLS OF CAPACITY.

INTRODUCTION.

In the preceding seven chapters the relations of capacity for stream traction to a variety of factors have been examined one at a time. It is now proposed to bring together some of the discovered elements of control. The experimental data thus far considered pertain to straight channels, and the factors of control connected with bending channels have not received attention. Those factors must be included when the attempt is made to bring laboratory results into relation with river phenomena, but as they constitute a category by themselves it is convenient to leave them out of the account in correlating the results from straight-channel work.

The immediate determinants of capacity are (1) the velocities of the current adjacent to the channel bed, (2) the widths of channel bed through which those velocities are effective in moving débris, and (3) the mobility of the débris constituting the bed and the load. It was not found practicable to measure bed velocity, but measurement was applied to its two chief determinants, slope and discharge, and also to its intimate associate, mean velocity, and these have been discussed separately. Width has entered into the discussion chiefly as an associate of depth in the determination of form ratio. By reason of these and other interrelations the six controls of capacity which have been discussed—slope, discharge, fineness, depth, mean velocity, and form ratio—are not independent, and not all should appear in a general equation. Slope, discharge, and fineness being accepted as of primary importance, it is feasible to add but one of the others, and choice has been made of form ratio.

FORMULATION BASED ON COMPETENCE.

The functions used in discussing the relations of capacity to slope, discharge, and fineness are similar, and each involves a conception of competence. Competence enters also the theory of the relation of capacity to form ratio, but it enters in a different way. It is convenient to omit at first the form-ratio function and consider together the three which are similar. They are:

$$C=b_1(S-\sigma)^n \text{ .... } (10)$$

$$C=b_3(Q-\kappa)^o \text{ .... } (64)$$

$$C=b_4(F-\phi)^p \text{ .... } (75)$$

Each of these equations expresses the law of variation of capacity with respect to one condition when the other two conditions are constant, and in that sense they are independent; but there is a mutual dependence of parameters which is of so complete a character that they are essentially simultaneous. The dependence of parameters is more readily stated by means of a specific instance than in general terms. In equation (10) b_1, σ, and n are constant so long as Q and F hold the same values; they do not vary with variation of S. But when the values of Q and F are changed those of b_1, σ, and n are modified. Through this control of its parameters the equation involves the relation of capacity to discharge and fineness.

The coefficient b_1 is the value of capacity when $(S-\sigma)=1$; b_3 when $(Q-\kappa)=1$; b_4 when $(F-\phi)=1$. Replacing them by b_5, as the numerical value of capacity when $(S-\sigma)=1$, $(Q-\kappa)=1$, and $(F-\phi)=1$, we may combine the three equations into

$$C=b_5\ (S-\sigma)^n\ (Q-\kappa)^o\ (F-\phi)^p \text{ .... } (91)$$

The constant b_5 is not of the same unit with either b_1, b_3, or b_4. Its dimensions, derived from those of the variables of (91), are $L^{3p-3o}M^{+1}T^{o-1}$.

From the experimental data have been computed 92 values of n, 20 values of o, and 5 values of p. (See Tables 15, 32, and 44.) All these are positive. The following statistical summary gives a general idea of their relative magnitudes. Its figures are not based on the same range of observational data; but the ranges for o and p correspond approximately with the middle part of the range for n.

Exponent.	Number of determinations.	Mean value.	Range of values.
n	92	1.59	0.93–2.37
o	20	1.02	.81–1.24
p	5	.58	.50–.62

It will be recalled that while the forms of the equations involving σ, κ, and ϕ were based on the conception of competence, it was not found possible to correlate those parameters strictly with competent slope, discharge, and fineness. The correlations were obstructed by phenomena of dune rhythms and of diversified fineness and could not be completed, but the forms of equation were found to be well adapted to the combined expression of observational data above the region of competence. Their relation to competence is not absolute but intimate, and it is so intimate that certain properties of the parameters may properly be inferred from the physical theory of competence.

When the swiftest velocity on the bed is barely able to move débris, there is a threefold condition of competence. For the particular discharge and fineness, the slope is competent; for the particular slope and fineness, the discharge is competent; for the particular slope and discharge, the fineness is competent. The conditions of competence for the three factors controlling capacity are thus not only similar but simultaneous and coincident. Neither factor can sink alone to the limiting level of competence, but the three arrive together. This is an important principle and lies at the foundation of the systematic interdependence of parameters and variables.

In equation (91) the quantities $(S-\sigma)$, $(Q-\kappa)$, and $(F-\phi)$ become zero simultaneously. When $S=\sigma$, then also $Q=\kappa$, and $F=\phi$; and vice versa. ...... (92)

As capacity varies directly with $(S-\sigma)$, $(Q-\kappa)$, and $(F-\phi)$, it is also true that each of these varies directly with capacity. Any change of condition which affects capacity affects those three quantities in the same sense. For example, suppose discharge to be increased. This not only increases $(Q-\kappa)$ and thereby increases capacity, but it also increases $(S-\sigma)$ and $(F-\phi)$. One mode of expressing this fact is to say that capacity measures the remoteness of each controlling factor from the initial status of competence, and all recede or approach together.

Let us now make a more definite assumption, that discharge is increased while slope and fineness remain the same. The resulting increase of $(S-\sigma)$, as S is unchanged, implies a diminution of σ; and the increase of $(F-\phi)$ implies a diminution of ϕ. That is, σ and ϕ vary inversely with discharge. Parallel reasoning shows that σ and κ vary inversely with fineness, and that κ and ϕ vary inversely with slope.

These relations are here developed deductively from the theory of competence. They have been developed inductively from the observational data; for equations (26), (66), and (77) include

$$\left.\begin{array}{l} \sigma=f(\grave{Q}, \grave{F}) \\ \kappa=f_{I}(\grave{S}, \grave{F}) \\ \phi=f_{II}(\grave{S}, \grave{Q}) \end{array}\right\} \text{............} (93)$$

No way has been found in which to study the exponents deductively. The only evidences of order discovered by comparison of observational data pertain to n, which has been found (equation 27) to vary inversely with discharge and fineness. The question whether o and p follow similar trends could not be answered by the adjusted data because of the cumulative effect of accidental errors. There is, however, considerable force in analogic reasoning, based not only on equations (93), but on other elements of symmetry in the relations of capacity to the several factors—elements to be noted later. The state of the evidence may be expressed by

$$\left.\begin{array}{l} n=f_{III}(\grave{Q}, \grave{F}) \\ [o=f_{IV}(\grave{S}, \grave{F})] \\ [p=f_{V}(\grave{S}, \grave{Q})] \end{array}\right\} \text{............} (94)$$

It is convenient to have a name for the group of constants designated by Greek letters, and as they define the conditions of competence, they may be called *competence constants*.

The exponent n and the associated competence constant σ, as they vary with Q and F

and do not vary with S, are controlled by Q and F. As they both vary inversely with Q and with F, it follows that they vary directly one with the other. It is evidently true in general that each exponent varies directly with the associated competence constant.

$$\left.\begin{array}{l} n=f_{VI}(\acute{\sigma}) \\ o=f_{VII}(\acute{\kappa}) \\ p=f_{VIII}(\acute{\phi}) \end{array}\right\} \text{-----------} (95)$$

With σ constant, the variation of $S-\sigma$ is determined by variation of S. Considered as additive, their variations are identical; but if we regard the changes as ratios, the changes in $(S-\sigma)$ are proportional to $\frac{1}{S-\sigma}$ and those in S to $\frac{1}{S}$. The ratio between these fractions, which is $\frac{S}{S-\sigma}$, is a measure of the sensitiveness of $(S-\sigma)$ to changes in S. It is evident that as S increases the sensitiveness diminishes. As capacity varies with a power of $(S-\sigma)$, the sensitiveness of capacity to slope becomes less as the slope increases.

Any change in S causes, according to (92) and (93), a change of opposite character in κ and o. When S is increased, κ and o are reduced. As the sensitiveness of $(Q-\kappa)$ to change in Q is measured by $\frac{Q}{Q-\kappa}$, it is evident that the reduction of κ lessens the sensitiveness. This has the effect also of lessening the sensitiveness of capacity to discharge; and that sensitiveness is further lessened by the reduction of o. Parity of reasoning shows that increase of slope lessens the sensitiveness of capacity to fineness, so that the effect of increasing slope is to reduce the sensitiveness of capacity to all three of its controlling factors. It is evident also that a similar result would be reached if the analysis began by assuming an increase of discharge or fineness.

It is a general principle that any change in one of the control factors, slope, discharge, and fineness, causing capacity to increase, has the effect also of making capacity less sensitive to changes in each and all of the control factors; and the inverse proposition is of course equally true. The statement being phrased to include both, the sensitiveness of capacity to the three controlling conditions varies inversely with capacity.

The term "sensitiveness," as used in the preceding paragraphs, is equivalent to the more specific "index to relative variation," for which the symbol i has been used; and by reference to various studies of the control of the index by conditions it may be seen that the entire scope of the general principle just stated has been covered by essentially inductive generalizations. From equations (39), (68), and (79),

$$\left.\begin{array}{l} i_1=f_{IX}(\grave{S},\ \grave{Q},\ \grave{F}) \\ i_3=f_{X}(\grave{S},\ \grave{Q},\ \grave{F}) \\ i_4=f_{XI}(\grave{S},\ \grave{Q},\ \grave{F}) \end{array}\right\} \text{-----------} (96)$$

This checking of deductive by inductive results helps to establish the second and third equations of (94), which were inferred from analogies.

Very little is known of the nature of the functions in (93) to (96), beyond the fact that those of (95) are increasing and the others decreasing. Deductive reasoning has not been successfully applied, and induction has escaped the entanglement of accidental errors in only a single instance and to a limited extent. The exceptional instance is that represented by the first equation of group (96). The symbols being translated into words, that equation reads: The index of relative variation for capacity in relation to slope varies inversely with slope, with discharge, and with fineness. There are in fact three distinct propositions, and each of these might be expressed by a separate equation. As to the first proposition, that the index varies inversely with slope, it was found, inductively, that the rate at which it varies with slope is itself a decreasing function of slope and also of discharge and fineness; and knowledge of similar character was gained as to the second and third propositions (pp. 104–108). Representing by di_{1S}, di_{1Q}, and di_{1F} the rates of variation of the index in relation to slope, discharge, and fineness, severally, we have

$$\left.\begin{array}{l} di_{1S}=f_1(\grave{S},\ \grave{Q},\ \grave{F}) \\ di_{1Q}=f_2(\grave{Q},\ \grave{F}) \\ di_{1F}=f_3(\grave{F}) \end{array}\right\} \text{-----------} (97)$$

These fragmentary determinations are all of one tenor, and in view of the remarkable symmetries already discovered among the elements of equation (91), they render probable the general proposition:

The rate at which capacity varies inversely with each of the three controlling conditions, slope, discharge, and fineness, itself varies inversely with each of the conditions. ...... (98)

Returning to (92) and (93), we may indicate certain corollaries.

Starting from the status of competence, let us assume that slope is increased, with destruction of the status, and that the status is restored by reducing discharge. In the restored status σ is greater than in the original, κ is less, and ϕ is unchanged. It is evident that the nature of the result does not depend on the particular assumptions, and that we may pass to the general proposition:

When capacity is zero, the competence constants are so related that a change in any one of them involves a change of contrary sign in some other. ...... (99)

Starting from a status characterized by a particular value of capacity, we may first break it by increasing slope and then restore it by decreasing discharge (fineness remaining unchanged). The first change reduces κ and ϕ; the second increases σ and ϕ. It does not appear whether the net result for ϕ involves change in its value, but if so the change is probably small in relation to the increase in σ and the decrease in κ. It is evident that the nature of the result does not depend on the particular assumption, and that we may pass to two general propositions, each of which includes (99) as a special case:

Under the condition that capacity is constant, the competence constants are so related that a change in any one of them involves a change of contrary sign in some other. ...... (100)

Under the condition that capacity is constant, the values of slope, discharge and fineness are so related that a change in any one of them involves a change of contrary sign in some other. ...... (101)

It follows also that

Under the condition that capacity is constant, the value of each controlling condition (S, Q, or F) is so related to the corresponding competence constant (σ, κ, or ϕ) that the two vary in same sense. ...... (102)

Propositions (100) and (102) are deduced from equations (93). By parity of reasoning equations (94) yield (103) and (104), but these two propositions share whatever uncertainty attaches to (94).

Under the condition that capacity is constant, the exponents n, o, p are so related that a change in any one of them involves a change of contrary sign in some other. ...... (103)

Under the condition that capacity is constant, the value of each controlling condition (S, Q, or F) is so related to the corresponding exponent (n, o, or p) that the two vary in the same sense. ...... (104)

As capacity can not be increased under (91) without increasing S, Q, or F, and as the increase of one of these involves under (93) the decrease of two competence constants, without any change of the third, it follows that the competence constants, collectively, vary in-inversely with capacity. The same reasoning, if applied to (91) and (94), yields a similar conclusion as to the exponents. To combine the two in a single statement:

The competence constants σ, κ, and ϕ, taken as a group, and the exponents n, o, and p, taken as a group, vary inversely with capacity. ...... (105)

I find it not easy to bring into combination the laws of internal relation between parameters of a group and the laws which connect the groups with capacity; but if these laws be regarded as conditions, it is possible to frame more comprehensive theorems of tentative character. Equations (106) are of this class and are thought worthy of examination, although the data at hand do not suffice for their testing.

$$\left.\begin{aligned} f_1(\acute{\sigma})+f_2(\acute{\kappa})+f_3(\acute{\phi})-f_4(\grave{C})=0 \\ f_5(\acute{n})+f_6(\acute{o})+f_7(\acute{p})-f_8(\grave{C})=0 \end{aligned}\right\} \text{----} \quad (106)$$

The equation under discussion,

$$C=b_5(S-\sigma)^n(Q-\kappa)^o(F-\phi)^p \text{------} \quad (91)$$

is an expression of relation between capacity and three of its controls, namely, slope (S), discharge (Q), and fineness (F). It involves seven parameters, of which six are functions of the independent variables, S, Q, and F. It is thus a bare framework, and the completion of the structure calls for the replacement of the six parameters by their values in terms of the variables. The laws contained in the equations and propositions numbered (92) to (98), with their corollaries, (99) to (105), are contributions toward the completion of the structure, but they are largely of the nature of restrictions. They impose conditions to be satisfied by the perfected equation.

Some of the conditions are already embodied in the form of (91), and with reference to such conditions it is important that the origin of that form be not overlooked. The form assumes that the three competence constants are the values of the corresponding variables when capacity is zero, whereas their identification with those values is by no means complete. The definition and recognition of the status of competence are so obstructed by the complicating conditions of nonhomogeneous débris and dune rhythm that no more can be asserted than an indefinitely representative relation. For most purposes, however, we are little concerned with conditions in the immediate neighborhood of the competence limit, so that this qualification is of small practical moment. Outside of the neighborhood of competence the support of the form is empiric; it has served well as a scheme for the marshaling of the observations. The support is qualified, in turn, by the fact that the observations are not of such harmony and precision as to discriminate nicely among formulas of adjustment. In view of these qualifications, the possibility has been recognized that some of the laws above enumerated might emanate from the form of the equation and have no other basis; and in view of this possibility the foundations of each conclusion have been scrutinized. I believe that all the inferred laws, from (92) to (105), are essentially inductive.

It is easy to understand that any constructive effort which should hang all supplementary conditions on the framework of (91) would result in a formula so unwieldly as to be useless. It is a matter of faith with me that if our data were so precise as to substitute definite quantitative relations for the fascicle of trends and indefinite parallelisms they have actually furnished, some way would be found leading from complexity to simplicity. I am not without hope that the presentation here made may suggest to the mechanist, familiar with the aspects of solved problems of similar difficulty, a rational theory under which the data may advantageously be recombined.

In an effort to discover unities among the complexities of the capacity relations, equation (91) was given the following form:

$$\frac{C}{b_5}=\sigma^n\kappa^o\phi^p\left(\frac{S}{\sigma}-1\right)^n\left(\frac{Q}{\kappa}-1\right)^o\left(\frac{F}{\phi}-1\right)^p \text{--} \quad (108)$$

The three factors making the second division of the second member, being independent of the units of measurement, seemed well adapted to the expression of comprehensive harmonies, if such exist.

The following negations were demonstrated:

The quantities $\frac{S}{\sigma}$, $\frac{Q}{\kappa}$, and $\frac{F}{\phi}$ are not equal, nor are the ratios between them constant.

The quantities $\frac{S}{\sigma}-1$, $\frac{Q}{\kappa}-1$, and $\frac{F}{\phi}-1$ are not equal, nor are the ratios between them constant.

The quantities $\left(\frac{S}{\sigma}-1\right)^n$, $\left(\frac{Q}{\kappa}-1\right)^o$, and $\left(\frac{F}{\phi}-1\right)^p$ are not equal, nor are the ratios between them constant.

It was also found that the symmetric factors in equation (91), namely, $(S-\sigma)^n$, $(Q-\kappa)^o$, and $(F-\phi)^p$, are not equal, nor are the ratios between them constant.

THE FORM-RATIO FACTOR.

In its relation to form ratio capacity has two zeros, one corresponding to a high ratio, the other to a low. Each of these corresponds also to a competent bed velocity, so that into a perfect formula competence would enter twice. The formula adopted, however, ignores the element of competence, chiefly because its recognition, which would add a complication, was not seen to be of advantage for the expression of the control of capacity in the more important regions outside the vicinity of competence. The accepted formula is

$$C=b_2\left(1-\frac{m}{m+1}\ \frac{R}{\rho}\right)R^m \quad (58)$$

The quantity ρ is that value of R which corresponds to the maximum value of C—the maximum standing between the two zeros—and b_2 is a capacity constant. The function as a whole qualifies capacity by means of a numerical factor and may be combined with (91) by multiplication of the factors:

$$C=b(S-\sigma)^n(Q-\kappa)^o(F-\phi)^p\left(1-\frac{m}{m+1}\ \frac{R}{\rho}\right)R^m \quad (109)$$

The coefficient b, replacing b_5 and b_2, is a quantity of the same unit with b_5 (see p. 186), but numerically independent.

The function now added is of distinct type from the others, for instead of advancing by a continuous law from zero to infinity it first rises to a finite maximum and then returns to zero. The first three factors are harmonious; the fourth discordant. At every stage in the investigation the discussion of the laboratory data has been hampered by this discordance. In order to treat adequately the relation of capacity to either slope, discharge, or fineness, it was necessary to isolate that relation by equalizing other conditions, and slope or discharge or fineness could readily be equalized; but the form-ratio factor was intractable. By means of interpolation it was possible to assemble varied data characterized by the same form ratio, but that did not meet the difficulty. It was necessary to take account of the relation of the particular ratio to the optimum ratio, ρ; and the value of ρ varies with all other conditions.

The sensitiveness of capacity to form ratio, as measured by the index of relative variation, is less than its sensitiveness to other conditions. The average of 48 values tabulated in Chapter IV is 0.24, while similar averages for fineness, discharge, and slope are three, five, and seven times as great. The distribution of sensitiveness, in relation to values of the independent variables, is illustrated by figure 66, where four curves are plotted, each representing a particular instance, selected as typical. The vertical scale is the same for all; and the ordinates represent values of the index of relative variation. The horizontal scale is that of slope for the curve SS, of discharge for the curve QQ, of fineness for the curve FF, and of form ratio for the curve $R\rho R$. The vertical cc represents the competence constants and is an asymptote to three of the curves. The horizontal line mm gives the value of the exponent m corresponding to the form-ratio index. For values of R greater than ρ the index is negative, but its curve is drawn above the zero line to represent sensitiveness, which is not affected by sign.

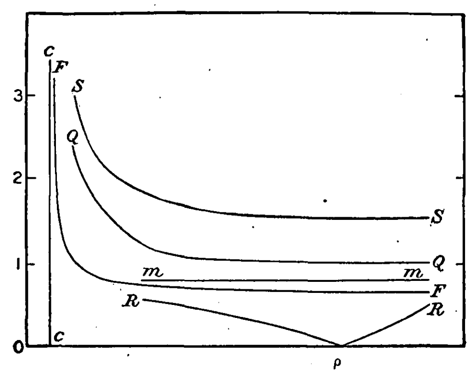


FIGURE 66.—Typical curves illustrating the distribution of the sensitiveness of capacity for traction to various controlling conditions. Ordinates represent values of the index of relative variation: abscissas, to four different scales, represent values of slope, discharge, linear fineness, and form ratio.

The two parameters of the form-ratio factor have laws of variation similar to those of the other parameters; each varies inversely with values of all independent variables except its own.

$$\rho=f\,(\grave{S},\ \grave{Q},\ \grave{F}) \quad (61)$$

$$m=f_1\,(\grave{S},\ \grave{Q},\ \grave{F}) \quad (62)$$

It follows that each varies directly with each of the other parameters (σ, κ, ϕ, n, o, and p).

The relations of the parameters to form ratio are less simple. In all cases the evidence from the observational data is conflicting, and as the several cases have come up for consideration the trend of evidence has seemed now in one direction and now in another. Impressions as to those trends are recorded in the preceding chapters, but when assembled they fail to indicate any general principle. Recourse is therefore had to theoretic considerations alone, and the conclusions reached can claim no direct support from the tabulated values of exponents and competence constants.

The conclusion (105) that the competence constants collectively vary inversely with capacity is based on the fact that individually they vary inversely with things which promote capacity, namely, slope, discharge, and fineness. Let us now assume that those three conditions remain constant and consider the effect of varying form ratio. Initially let the form ratio be so small that the bed velocity is competent; σ, κ, and ϕ are severally equal to the competent values of S, Q, and F. Now change width and depth so as to increase the form ratio and capacity becomes finite. That capacity may be finite, σ, κ, and ϕ must be less than S, Q, and F, and as the latter have not changed the competence constants have been reduced by the increase of form ratio. By parity of reasoning it can be shown that if the initial form ratio be so large as to make the bed velocity competent a reduction of form ratio will cause a reduction of σ, κ, and ϕ. Somewhere between the two form ratios of competence lies ρ, the form ratio of maximum capacity, and between the same limits lie minimum values of the competence constants.

The greater the capacity induced by adjustment of form ratio, the greater the reduction of slope, for example, necessary to reduce capacity to zero, and as this reduction varies directly with the depression of σ below the initial value of S, it follows that the minimum value of σ (and similarly of κ and ϕ) coincides with the maximum of capacity.

The conclusion that the competence constants vary inversely with capacity is therefore true for the case in which changes in capacity are caused by changes in form ratio. It can be shown also that the exponents, n, o, and p, follow the same law.

The extension of this principle to the domain of form ratio gives assurance that the conclusions embodied in equations and propositions (99) to (108), conclusions which were reached from phenomena of slope, discharge, and fineness, are not vitiated by the traversing phenomena of form ratio.

The function in the form-ratio factor of (109) being characterized by a maximum, the variations of parameters with respect to form ratio are characterized by a minimum. This law may be so combined with those of (93), (94) (61), and (62) as to yield the following system of equations for the trends of changes in parameters consequent on changes in the four independent variables of equation (109):

$$\left.\begin{array}{ll} \sigma = f_1 (\grave{Q}, \grave{F}, \breve{R}) & n = f_5 (\grave{Q}, \grave{F}, \breve{R}) \\ \kappa = f_2 (\grave{S}, \grave{F}, \breve{R}) & o = f_6 (\grave{S}, \grave{F}, \breve{R}) \\ \phi = f_3 (\grave{S}, \grave{Q}, \breve{R}) & p = f_7 (\grave{S}, \grave{Q}, \breve{R}) \\ \rho = f_4 (\grave{S}, \grave{Q}, \grave{F}) & m = f_8 (\grave{S}, \grave{Q}, \grave{F}) \end{array}\right\} \text{-- (110)}$$

In the development of the form-ratio factor of equations (58) and (109), detailed in Chapter IV, the factor first appeared as $(1 - \alpha R)R^m$, the quantity α being a numerical coefficient introduced to represent the resistance to the current occasioned by the sides of the channel. It was afterward shown that $\alpha = \frac{m}{m+1} \frac{1}{\rho}$, and that form of coefficient was substituted. These relations show that either α or m varies in value with the character and amount of the resistance by the channel sides; and, in point of fact, both do. Nor is that control restricted to the parameters of form ratio. Lateral resistance affects also, and in comparable degree, the parameters of slope, discharge, fineness, and capacity. The sides of the laboratory channels were vertical and were of wood, planed and painted. Had they been smoother or rougher, or had they been inclined, the whole system of values given by the experiments would have been different. There is no reason, however, to question that they would have yielded the same qualitative results.

DUTY AND EFFICIENCY.

The discussions of duty and efficiency in Chapters III and V give reason for the belief that the variations of either quantity in relation to controlling conditions may advantageously be expressed by an equation identical in form with (109). Such an equation would not be interconvertible with (109), nor would an equation for duty be the exact equivalent of one for efficiency. By the aid of reasonable assumptions the parameters of either equation might be derived from the parameters of another, but the results of computations by the several equations would not be strictly compatible. These discordances may be demonstrated as properties of the algebraic forms.

They are also connected with the fact that the algebraic forms imperfectly represent the physical phenomena. It is possible that the physical facts are closely represented by some one of the equations, in which case the incompatibilities of the other two equations are wholly of the nature of errors; but if this be so, our data do not enable us to indicate the truer equation.

The exponent in the slope factor is greater for the capacity and duty equations than for the efficiency equation, the difference being a large fraction of unity. The exponent in the discharge factor is greater for the capacity equation than for those of duty and efficiency, the difference being a large fraction of unity. The competence constant in the slope factor is smaller in the capacity and duty equations than in that for efficiency, the difference being a small fraction of the constant. The competence constant in the discharge factor is smaller for the capacity equation than for the others, the difference being relatively large. The interrelations of parameters and the relations of parameters to independent variables are qualitatively the same for the equations of duty and efficiency as for the equation of capacity.

THE FORMULA OF LECHALAS.

THE FORMULA.

The only earlier serious attempt to formulate the transportation of débris, so far as I am informed, is that of C. Léchalas, who wrote in 1871, under the title "Note sur les rivières à fond de sable."[1] His discussion makes use of Dubuat's experiments on competent velocity (1786), Darcy and Bazin's formulas for velocities in conduits and rivers (1878), and observational data accumulated by the French engineering corps in connection with projects for the improvement of navigable rivers. In the following abstract of the more elementary part of his discussion the symbolic notation and the terminology of the present paper are to some extent substituted for those of the original.

Postulate a stream of fixed width and constant discharge, traversing a bed of uniform sand, unlimited in quantity but without accessions. So long as the velocity along the bed exceeds a certain value the current transports sand. Below that limit the sand is undisturbed.

The discharge, Q, and width, w, being known, the mean depth, d, the slope, s, the mean velocity, V_m, and the bed velocity, V_b, are given by the following three equations, of which (1) and (3) are from Darcy and Bazin. The constants are in meters.

$$\frac{ds}{V_m}=0.00028+\frac{0.00033}{d} \qquad (1)$$

$$Q=wd\,V_m \qquad (2)$$

$$V_b=V_m-10\sqrt{ds} \qquad (3)$$

The sand travels (1) by rolling, (2) by suspension. A particle of water impinging on the bottom gives motion to a sand grain, the motion having a direction which depends on the impact and on the positions of adjoining particles, solid and liquid. The grain is projected free from the bottom or is rolled along it, the particular result depending on the inclination and force of the impact and on various conditions which affect the resistance. Suspension corresponds especially to impacts associated with high velocities. Suspension is rare below a certain critical velocity for each density and size of sand grain. Transportation is slow at low stages of a variable stream, rapid and by suspension at high stages. The grains describe trajectories analogous to those of the water particles, but shorter; and there are frequent returns to the bottom, as well as restings between excursions. Larger grains are lifted less high, or are rolled only, or remain at rest. Small grains afford a better hold (prise) in relation to their weight. The smallest of all are carried in the body of the current.

The amount by which the pressure on the upstream face of a grain immersed in a current exceeds the pressure on the downstream face is proportional to the square of the velocity. Represent it by aV_b^2, the coefficient a depending on size, form, and position. For the sand of the Loire, the resistance developed equals $a\,0.25^2$, as that sand is immobile when $V_b<0.25$. "The difference is equal to the product of the mass of the grains by their velocity, projected on the same axis as V_b—that is to say, on the axis of the stream. This product, being pro-

[1] Annales des ponts et chaussées, Mém. et doc., 5th ser., vol. 1, pp. 381-431, 1871.

portional to the discharge [load] of sand, C, may be represented by the expression hC. We have then:

$$C=\frac{a}{h}\left(V_b^2-0.25^2\right)=m\left(V_b^2-0.06\right)\text{- - -}(4)$$

The value of the m will be sought from observation, which will correct in a measure for the introduction of V_b into the equation without allowance for the speed of the grains, etc. Subtraction of 0.06 ceases when the sands are prevented by suspension from rubbing on the bottom; therefore the formula becomes $C=mV_b^2$ for velocities above a certain limit. (It is readily understood that m, like a, is only approximately constant.)"

The rate at which dunes advance has been measured, in the French rivers, in relation to the velocities of the associated currents. It rises with V_b until the critical velocity is reached, and then drops as the change is made from rolling to suspension. The advance of dunes, depending on the fall of grains into the eddy (fig. 10) when they have been rolled to the crest, is affected by the introduction of suspension because then only a part of the traveling grains are received by the eddy.

Observations made on the Loire give as the limiting bed velocity for transportation [competent velocity for transportation], $V_{bc}=0.25$ meters per second. According to an engineer who has discussed those observations [H. L. Partiot?], the corresponding surface velocity, V_s, is equal to $\sqrt{0.11}$; and the formula for the rate of advance of the dunes,

$$\text{Rate of advance}=0.00013\ (V_s^2-0.11)\ \text{-}(5)$$

is good for all values of V_s up to 1.016. One might base on this a formula for load in relation to surface velocity, but the formula would be incomplete unless developed so as to take account of the depth; and it is best for the present to adhere to equation (4), which connects load with bed velocity.

Lechalas, however, for a temporary purpose, uses formulas of Darcy and Bazin to connect V_b with V_s, under certain assumptions as to depth, and with their aid computes for the Loire the critical bed velocity, V_{bcc}, at which suspension of the sands begins. $V_{bcc}=0.55$ meters per second. This is the velocity corresponding to $V_s=1.016$, the surface velocity which limits the applicability of formula (5).

The following table contains the observational data on dunes of the Loire and compares the observed rates of dune advance with rates computed by formula (5).

TABLE 63a.—*Data on subaqueous dunes of the Loire.*

Surface velocity.	Height of dunes.	Rate of dune advance.	
		Observed.	Computed.
Met./sec.	*Meters.*	*Met./sec.*$\times 10^{-5}$	*Met./sec.*$\times 10^{-6}$
0.58	0.900	3.0	3.0
.64	.300	3.3	3.9
.73	.300	5.1	5.5
.75	.782	6.3	5.9
.81	.967	6.7	7.1
.81	.967	7.5	7.1
.83	.760	7.6	7.5
1.00	.953	10.5	11.6
1.016	.920	12.4	12.0
1.016	.580	12.0	12.0
1.03	.487	6.2	12.35
1.05	.612	7.0	12.9
1.11	1.198	5.8	14.6
1.13	.650	8.7	15.2
1.33	.950	5.6	21.6

In later passages Lechalas recognizes the variations of velocity in passing from one vertical to another of the same stream section and makes (4) the formula for a division, one unit wide, of the cross section. Thus modified, it is applied in a variety of ways to practical engineering problems of the Loire.

DISCUSSION.

Lechalas's classification of transportation processes differs from that adopted for our work in that he makes saltation, at least verbally, a part of suspension. I am led, however, by a study of the more detailed descriptions of his colleague Partiot, to believe that the line practically drawn between rolling and suspension differs in small measure only from the line we have drawn between traction and suspension.

The lower critical velocity of Lechalas is the exact equivalent of our velocity competent for traction, and his upper critical velocity corresponds approximately to our velocity competent for suspension. The two attempts at formulation likewise agree in giving prominence to the factor of competence. They differ in the mode of using that factor, and they are actuated by different preconceptions.

In the first sentence of the passage (pp. 193–194) which has been inclosed in quotation marks to indicate its literal translation, Lechalas appears to equate the velocity of a particle of the load with the difference between the forward pressure of the current and the resistance given by the particle. Hooker[1] suggests that the acceleration of the particle instead of its velocity is intended; but with or without such emendation the author's reasoning is obscure to me, for I see no necessary physical relation between the number or mass of débris particles moved and the pressure of the current. The load may be defined as the product of the mass of particles by their average speed; and their speed, being produced by the pressure of the current, may be simply related to it, but any relation of the mass to the pressure is necessarily indirect and presumably involved.

Whatever the strength or weakness of the postulates on which the formula is based, the manner in which it incorporates the principle of competence gives it a rough resemblance to those we have developed, while the characterization of its constant m gives to it a large empiric factor; and it is in order to inquire whether, as an empiric formula, it finds support in the Berkeley data. As the Berkeley observations do not include bed velocities, the most direct comparison is impracticable; but an indirect relation may readily be established.

The difficulty we have found in defining bed velocity may be avoided, for the purpose of the present comparison, by accepting the definition used by Lechalas in

$$V_b = V_m - 10\sqrt{ds} \text{ ---------- } (3)$$

and by assuming depth to be constant. According to the Chezy formula this assumption makes V_m approximately proportional to $\sqrt{s}$, so that $10\sqrt{ds}$, in (3), is proportional to V_m. It follows that V_b is proportional to V_m, and this permits us to substitute V_m for V_b in equation (4) by changing the constants:

$$C = K(V_m^2 - k) \text{ ---------- } (6)$$

This expression implies that capacity for traction varies with mean velocity at a rate which diminishes as mean velocity increases but is never so low as that of the second power of mean velocity. The corresponding data from our experiments, namely, the data for capacity in relation to mean velocity under the condition of constant depth, are in accord with this, except that they indicate a limiting index of relative variation somewhat less than 2. In Table 51 the values of the synthetic index, I_{Vd}, range from 2.03 to 7.86; and a value of 2.03 for the synthetic index implies smaller values of the instantaneous index. This discrepancy is not important, and the formula of Lechalas, regarded as empirical, is probably adequate for the discussion of a body of observations on capacity and velocity. It could not, however, be used in connection with the Berkeley data unless both K and k (or m and 0.06 in equation (4)) were permitted to vary with conditions.

[1] Hooker, E. H., Am. Soc. Civil Eng. Trans., vol. 36, p. 256, 1896.

CHAPTER XI.—EXPERIMENTS WITH CROOKED CHANNELS.

EXPERIMENTS.

In order to study the influence which bends in the channel exert on capacity for traction, a short series of experiments were made with channels having angular bends and others with channels having curved bends. Each of these channels had a width of 1 foot and was shaped by means of partitions within a trough 1.96 feet wide. (See fig. 67.) Above and below the bends were straight reaches of the same width. All the experiments were made with débris of grade (C) and with a discharge of 0.363 ft.^{3}/sec. The loads were measured. In some experiments the head lost in the region of the bends was measured by means of level readings on the water surface above and below.

After each experiment the profile of the bed was determined by levelings at intervals of 1 foot, and in several cases the region of the bends was covered by such levelings and sketches as to make it possible to construct a contour map of the bed.

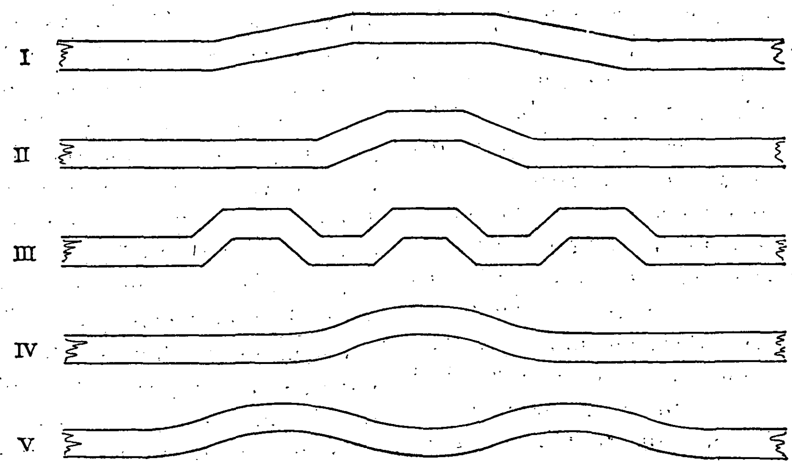


FIGURE 67.—Plans of troughs used in experiments to show the influence of bends on traction.

SLOPE DETERMINATIONS.

The profiles of the channel beds as shaped by the current were plotted. Through that part of the profile corresponding to the straight channel above the bends was drawn a straight line representing the mean slope for that region, and a similar straight line was drawn below the region of the bends. From a point on the first line near the position of the first bend to a point on the second line corresponding to a position several feet below the last bend a straight line was drawn, and this was assumed to represent the mean slope of the channel in the region affected most by the bends. The profiles above the bends showed evidence of the rhythms commonly observed in the straight-channel experiments. The profiles below the bends showed steeper undulations, which were ascribed to the influence of the strong agitation of the water in passing the bends. In estimating the slopes in the region of the bends the distance used was the length of the medial line of the channel.

The observations for head were made at points 2 feet and 4 feet above the first bend and 3 feet and 5 feet below the last bend, and the slopes were computed for the distances, on the medial line, between the points.

Check estimates of slope for straight channels were obtained by interpolation from Table 12, the determinations of load being used as arguments.

The data are assembled in Table 64, where the stronger and weaker determinations of the slope of the débris surface are severally indicated by the letters *a* and *b*. The measurements of water slope are thought to be coordinate in value with those of débris slope. The slopes computed from load measurements are probably of less weight.

TABLE 64.—*Comparison of slopes required for straight and crooked channels, respectively, under identical conditions of discharge, fineness, width, and load.*

[Débris of grade (C); discharge, 0.363 ft.³/sec.; width, 1 foot.]

Shape of crooked channel. (See fig. 67.)	Load.	Slope in straight channel.		Slope in crooked channel.	
		Computed from load by Table 12.	Profile of débris above first bend.	Average slope of débris.	Average slope of water surface.
	Gm./sec.	*Per cent.*	*Per cent.*	*Per cent.*	*Per cent.*
I.......	19	0.42	0.30 *b*	0.45 *b*	0.46
	75	.92	.89 *b*	.89 *a*	.93
	56	.77	.74 *b*	.80 *a*	.78
II......	60	.80	.70 *a*	.81 *b*	.72
III.....	61	.82	.94 *b*	1.15 *a*	....
IV......	60	.80	.76 *a*	.80 *a*	....
	60	.80	.85 *a*	.85 *b*	....
V.......	63	.83	.72 *b*	.83 *b*	....
	63	.83	.62 *b*	.80 *b*	....
	66	.85	.87 *b*	.81 *a*	....
	64	.84	.87 *a*	.77 *a*	....
	64	.84	.86 *a*	.80 *a*	....

NOTE.—Values marked *a* are given greater weight than those marked *b*.

FORMS AND SLOPES.

The combinations of bends in the experiment channels are shown in figure 67. In three of the channels the bends were angular; in two curved. In channel I the angle of deflection was 10.5°; there was a single group of four bends, returning the course to its original direction; and the short reaches were approximately 5 feet long. In channel II the arrangement was the same, with reaches of about 2.5 feet and deflection angles of 21.5°. In channel III were three groups of four deflections each, the angles being of 40.9° and the length of reach about 1.4 feet. Channel IV had the same proportions as No. II, with the substitution of curves for angles. The radius of curvature for the medial line was 6.55 feet. Channel V contained two groups of curves, each similar to the group in No. IV.

From the data (Table 64) connected with channels I, II, and III, it appears that with angular bends a greater slope is necessary to transport the load than when the channel is straight—that is, the capacity is reduced by angular bends. The reduction is greatest when the angle of deflection is greatest, and it is so small for an angle of 10° as to leave doubt whether it might not disappear altogether with a somewhat smaller angle.

The single group of curves (IV) appears to reduce capacity slightly (increase of slope for same load): but the double group (V) gives slopes indicating an increase of capacity.

As the bends of alluvial streams are curved, the curved experiment channels may be assumed to represent them better than do the angular channels, and it is possible that meandering channels have a greater capacity for traction than straight channels of the same length. There are, however, certain elements of incompleteness in the representation which make definite inference hazardous. The course of a stream which shapes its own channel through an alluvial plain is made up of bends and reaches. In passing from reach to bend there is a gradual increase of curvature until the radius of curvature, for the medial line, is between twice and three times the width of the channel, and the change from bend to reach is also gradual. The forms are automatically adjusted to the system of accelerations and velocities within the current. The angular change of direction in the bend may be one of a few degrees only, but in meandering streams it is commonly from 90° to 180°. In the artificial channels all curves were circular arcs with a radius of 6.55 times the channel width; there was no graduation in the radial acceleration due to deflection; the change from right-hand deflection to left-hand deflection was abrupt, without the intervention of a reach, and the changes of direction were through angles of 21.5° and 43°. That such differences are competent to affect transportation to a material extent is indicated by the relations of deeps and shoals (crossings) to bends. Fargue,[1] from a discussion of an artificially adjusted portion of the Garonne, reached the conclusion that the distance downstream from the apex of a curve to the deepest point of the associated deep and the distance downstream from a point of inflection to the associated crossing are each normally one-fourth the length of a stream unit—defined as the portion between two points of inflection. In our experiments (see fig. 68) each of these distances is one-half the length of the stream unit instead of one-fourth.

[1] Fargue, L., La forme du lit des rivières à fond mobile, Paris, 1908.

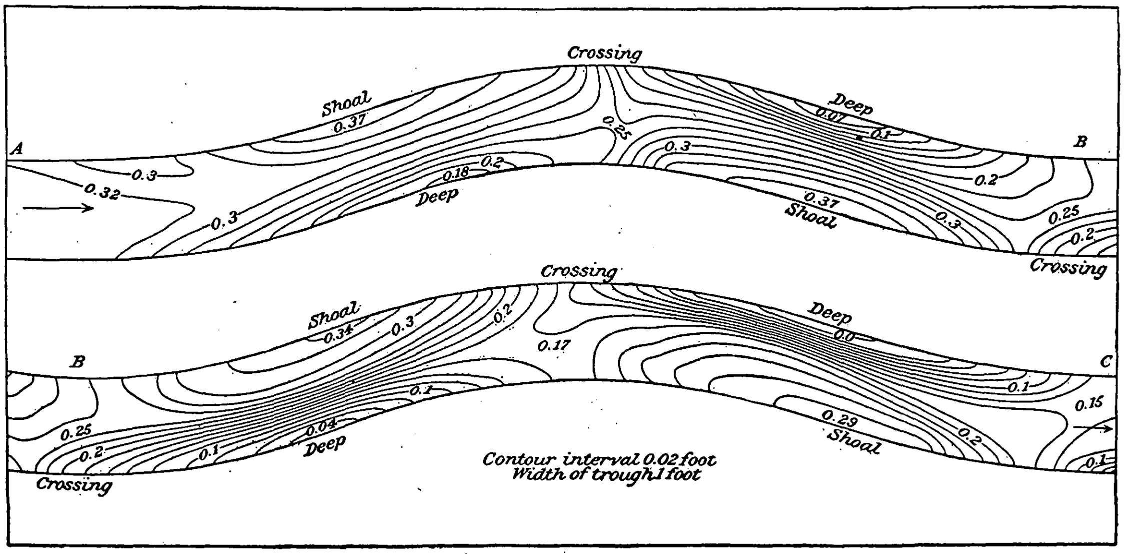


FIGURE 68.—Contoured plot of a stream bed, as shaped by a current.

In view of these facts, the apparent results of the experiments with crooked channels must be received with caution, and probably nothing more should be claimed for them than a general indication that the capacity of a moderately bent channel does not differ greatly from that of a straight channel.

FEATURES CAUSED BY CURVATURE.

Incidentally the experiments illustrated several consequences of curvature in addition to the influence on slope and capacity. At each turn the swiftest part of the current was thrown to the outer or concave side of the channel, and the slower parts moved toward the opposite side, the transfers giving to the current as a whole a twisting motion. The action on the débris became exceptionally strong near the outer side and exceptionally weak near the inner. A result of the strong action was that part of the load was thrown upward, so as to be temporarily suspended, and a result of the diversity of velocity was the maintenance of deep places near the outer wall and of shoals near the inner. Associated with the twisting motion were many whirls or eddies; and the general obliquity of motion had the effect of reducing the mean velocity in the direction of the general flow. The reduction of mean velocity was recorded in an increase of mean depth, which amounted, in the average of all examples, to 7 per cent and ranged from 2 to 14 per cent.

CHAPTER XII.—FLUME TRACTION.

THE OBSERVATIONS.

SCOPE.

That which distinguishes flume traction from stream traction is the fixity of the channel bed. In stream traction the shapes of the bed are adjusted to the rhythms of the mode of transportation, and its texture is that of the débris in transit. In flume traction the bed is unresponsive, but its texture, being independently determined, has an important influence on the mode of transportation. The experiments were arranged to determine the influences of different textures of bed on mode of traction and capacity for traction and were otherwise varied in respect to slope, width, discharge, and the character of débris transported.

GRADES OF DÉBRIS.

The material transported in the experiments included most of the grades already described (see Table 1 and Plate II) and also several mixtures not previously mentioned. In order conveniently to show the relations of the mixtures to their components, all the grades of débris used in the flume experiments are listed below in Table 65, the data of Table 1 being repeated so far as necessary. The elements of the table are defined at page 21. The material of the coarse grades (I) and (J) differs from that of the finer, being about 2 per cent less dense. Its particles also are somewhat less thoroughly rounded, their journey from the parent rock bed having been short.

TABLE 65.—*Grades of débris.*

Grade name.	Sieves used in separation (meshes to 1 inch).	*D.* Mean diameter of particles (foot).	*F.* Number of particles to linear foot.	F_2. Number of particles to cubic foot.
(B)........	40-50	0.00123	812	1,023,000,000
(C)........	30-40	.00166	602	417,000,000
(E)........	10-20	.00561	178	10,770,000
(G)........	4-6	.0162	61.8	451,000
(H)........	3-4	.0230	43.4	156,000
(I)........	1-2	.0547	18.3	11,900
(J)........	½-1	.110	9.1	1,440
(E_1G_1).....	....	.00698	143.2	5,610,000
($E_2H_1I_2$)...	....	.00706	141.7	5,430,000
($E_3H_2I_3J_2$)..	....	.00836	119.6	3,266,000

APPARATUS AND METHODS.

The experiment trough, a modification of that represented in the frontispiece, was 60 feet long and 1.91 feet wide, with vertical sides. The sides and bottom were of wood, planed and painted. For a portion of the experiments the bottom was covered by a false bottom, specially prepared to present a definite character of roughness, the sides remaining smooth. The trough was so arranged that it could be given various determinate slopes up to 3 per cent, and by means of an inclined false bottom a slope of 4.5 per cent was made. By means of a partition the width was reduced, for the greater part of the work, to 1.00 foot.

The width of trough at the outfall end was regulated by a contractor, as described on page 25. The débris was delivered at the outfall to a settling tank, which had two divisions; and a deflecting apparatus was so arranged that the delivery could be instantaneously diverted from one division to the other.

Above the trough near its head was a sloping platform on which measured units of débris were dumped at regular intervals, determined by a watch, and from which the débris was fed to the current by hand, with the aid of a scraper. The rate of feed was modified by changing the interval between dumpings, and by successive trials it was adjusted to the capacity of the current.

In accelerating the débris, as it fell into the water, the current was retarded, so that close to the feeding station it was slower than elsewhere. When the load was approximately adjusted to the general capacity of the current it constituted an overload in this particular tract, with the result that a portion was deposited. The load would then traverse an upper division of its course on a bed of débris, while in the lower and principal division it was in direct contact with the bottom of the trough. A tendency of the stream of débris to clog near the upper end of the trough, although moving freely beyond, was the ordinary criterion of the proper adjustment of the feed, and when this condition existed the load delivered at the outfall was assumed to represent the capacity of the stream. The outfall was then directed for a measured interval of time to a reserved division of the settling tank, and the débris thus separately received was weighed.

Besides the deposit connected with the feeding of débris there were transitory deposits of a rhythmic character, as described later.

Five characters of channel bed were used, namely, a planed and painted wood surface; a rough-sawn, unplaned wood surface; a surface of wood blocks, with grain vertical; a pavement of sand grains, set in cement; and a pavement of pebbles. (See Pl. III.)

PROCESSES OF FLUME TRACTION.

MOVEMENT OF INDIVIDUAL PARTICLES.

Flume traction differs from stream traction in its extensive substitution of rolling for saltation and in the important place it gives to sliding. The relative importance of these modes of particle movement is determined (1) by the texture of the bed surface in relation to the size of the particle, (2) by the velocity of the water in relation to the size of the particle, and (3) by the shape of the particle.

In stream traction the order of roughness of the bed is given by the fineness or coarseness of the material of the load, and this fact determines saltation as the dominant process. In flume traction the bed may be much smoother. On a smooth stream bed any particle with a broad facet is apt to slide, and a well-rounded particle to roll. Rolling is determined (rather than sliding) not only by the fact that the propulsive force of the current and the resistance given by the bed constitute a couple, but also by the fact that the current applies a greater force to the upper part of the particle than to the lower. The less smooth the bed surface, the greater its resistance and the more effective the couple in causing the particle to roll. With any particular texture of bed, the sizes of particles may determine their modes of progress, the largest sliding, those of smaller size rolling, and the smallest leaping. Increase of velocity tends to increase saltation at the expense of rolling and to increase rolling at the expense of sliding.

A particle rolled slowly is in continuous contact with the bed. A round particle may roll rapidly on a smooth bed without parting from it. Roughness of the bed causes changes of direction in the vertical plane, and such changes combined with high velocity cause leaps. If the particle is not round its rolling involves rise and fall of the center of mass, and such changes combined with high velocity cause leaps. Shape of particle may thus be a determinant between saltation and rolling, as well as between rolling and sliding.

A flattish particle, which may either slide or roll, travels faster when rolling. This is due partly to the fact that when it rolls it rolls on edge and thus projects farther into the current, and partly to the fact that the resistance at contact with the bed is greater for sliding than for rolling. It is also true that rolling particles as a class outstrip sliding particles as a class, the difference in speed being marked.

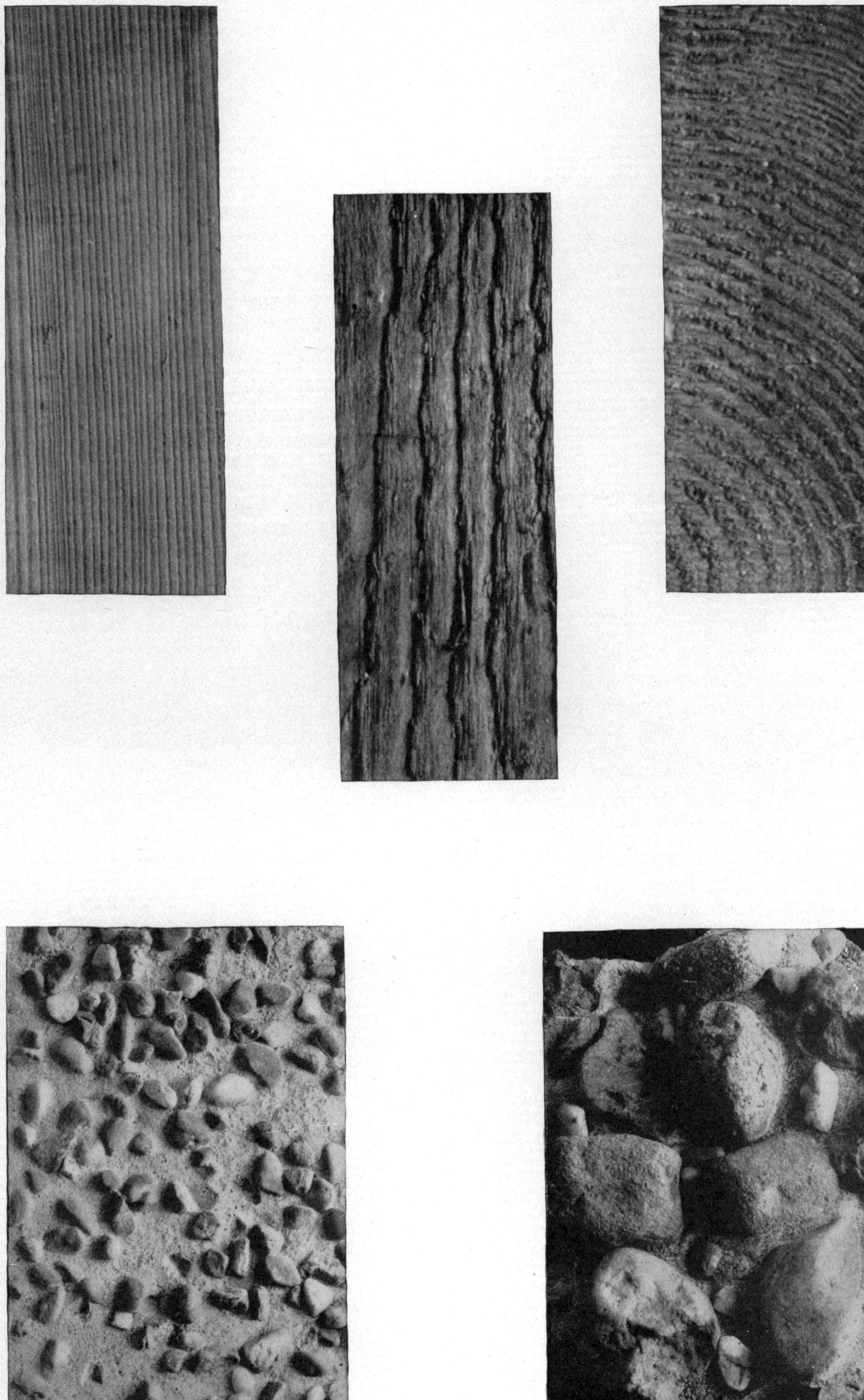
For particles of similar size those which dominantly roll outstrip those which dominantly leap. This is part of a more general fact that the better-rounded particles travel faster than the more angular. In the traction of mixed débris, where rolling is characteristic of larger particles and saltation of smaller, the larger travel faster than the smaller. There are thus two important ways in which rolling gives greater speed than saltation. It was not learned whether a particle which alternately rolls and leaps travels faster in one way than in the other.

A suspended particle, having the same speed as the water, outstrips all others. It is therefore possible that as saltation approaches the borderland of suspension its speed exceeds that of rolling.

When samples of different grades are fed to the same current in succession, it is found that the coarser travel the faster, whatever the mode of progression (except suspension). In the following record of experiments the speeds of grade (J) constitute an apparent exception, but their slowness is ascribed to the fact that the particles of that grade were relatively angular.

TABLE 66.—*Relative speeds of coarse and fine débris in flume traction.*

Depth of water.	Mean velocity of water.	Average speed of débris particles.		
		Grade (E).	Grade (H).	Grade (J).
Foot.	*Ft./sec.*	*Ft./sec.*	*Ft./sec.*	*Ft./sec.*
0.127	4.35	3.2	3.9	3.8
.182	3.04	2.3	2.4	....
.110	2.52	1.85	2.0	2.0
.182	1.52	.70	.85	....



ROUGH SURFACES USED IN EXPERIMENTS ON FLUME TRACTION.

It may be observed in passing that the recorded speed of the particles is on the average 75 per cent of the mean velocity of the water, the ratio being greater as the velocity is greater and as the depth is less. A higher ratio would of course be found if the speed of particles were to be compared with that of the lower part of the current. The percentage might be less than 75 if the stream were fully loaded.

In watching the traction of mixed débris it was observed, as already mentioned, that the larger particles traveled faster than the smaller. As it is difficult in such an observation to avoid giving attention largely to the more active particles, the observation applies especially to those which roll, but there is probably a similar contrast between the speeds of less active particles also. In an experiment with glass balls of different sizes it was found that the larger were rolled by the current somewhat faster than the smaller.

In attempting to understand the more rapid propulsion of the larger particles it is natural to compare the phenomenon with the more familiar fact that in the absence of water a large stone or ball will descend an incline with greater speed than a small one, but there is an important difference between the two cases. The object descending a dry incline is impelled by gravity, acting directly, and part of the resistance comes from the fluid in which it is immersed, whereas the rolling débris pebble is impelled chiefly by the moving fluid which surrounds it. The advantage in speed accruing to the larger pebble in flume traction is probably connected with the fact that the velocities of the current increase from the bottom upward. If the velocities were all the same the pressures applied to similar pebbles of different diameter would be proportional to their sectional areas, or to the squares of their diameters; but because of the gradation of velocities the increase of pressure in consequence of increase of diameter is more rapid than the increase of the square of the diameter. The chief resistance to the forward movement of the pebble is engendered at its contact with the bed and is of the nature of rolling resistance. For uniform speed, the rolling resistance of a wheel is proportional to its downward pressure divided by its diameter; and since in the case of the pebble the pressure is proportional to the cube of the diameter, the rolling resistance is proportional to the square. Increase in size of the rolling pebble is thus accompanied by increase of both propulsive force and rolling resistance, the increment to propulsive force being somewhat the larger; and the equality of force and resistance is restored by an increase of speed, which has the effect of reducing the propulsive force and increasing the resistance.[1] The reduction of propulsive force is connected with the fact that that force is determined at each instant by the velocities of the water, not as referred to the fixed bed, but as referred to the moving pebble.

The analysis of forces might be further developed, but the foregoing brief outline serves to indicate a mechanical principle underlying the observed fact that a current rolls large particles more swiftly than small. The principle is of fundamental importance in accounting for certain contrasts between the laws of flume traction and those of stream traction.

The discrimination of traction and suspension, usually easy in the experiments with stream transportation, was difficult in the flume work. The zone of saltation grew deeper with progressive increase of slope until it occupied the whole depth of the stream. Further increase of slope, with increase of load, made the cloud of particles denser, but there seemed no way of telling when the condition became that of a flowing mixture of water and sand.

COLLECTIVE MOVEMENT.

To the general fact that in flume traction the particles of the load either roll or slide in continuous contact with the fixed bed or else skip from point to point along it, there are two noteworthy exceptions.

When the transported débris includes particles which are small compared to the projections constituting the roughness of the bed, some of the débris finds lodgment among the projections. The roughness of the bed is thus diminished and the process of transportation is modified. The bed comes to be constituted in part of the fixed summits of projections and in part of mobile débris, and the process becomes a blending of flume traction proper and stream traction. When such conditions existed in the experiments it was found that the capacity was essentially that due to stream traction.

[1] The formula given by W. J. M. Rankine, in his "Applied mechanics" and elsewhere, for the rolling resistance of a wheel is $R=\frac{Q}{r}\left\{a+b(v-3.28)\right\}$ where R is resistance, Q gross load, r radius of wheel, v velocity in ft./sec., and a and b constants. The experimental values of a and b are such as to give velocity only a moderate influence on the resistance.

The second exception is a phenomenon of rhythm. It is probable that the rate of flume traction is always affected by rhythm, just as is that of stream traction, and the rhythm in load implies a rhythm in efficiency of current. Under certain conditions, of which the most important is small slope, the rhythm is manifested by the making of a local deposit from the load, a patch of débris appearing on the bed of the trough. Such a patch travels slowly downstream, being succeeded after an interval by another. With suitable variation of conditions the patches become more numerous, are regularly spaced, and occupy a greater share of the bed surface. They may even exceed the interspaces in area. When wide apart they are usually shaped in gentle slopes, but as their ranks close they assume the profiles of dunes, with steep frontal faces.

The advance of the débris patches is like that of typical dunes, in that the upstream slopes are eroded while the downstream slopes are aggraded; and in this way their travel is a factor in débris transportation. It does not, however, become the dominant factor, as in the dune phase of stream traction. There is always a large share of the load which passes over the deposits without being arrested and continues its journey across the intervening bare spaces.

In this rhythmic process there is a combination of elements belonging distinctively to flume traction and to stream traction, and it is possible that the process constitutes a transition from one system to the other; but, so far as developed by the experiments, it appears as a phase, a rhythmic phase, of flume traction. The slopes with which it was associated were much flatter than those necessary to carry the same load by the method of stream traction.

TABLE OF OBSERVATIONS.

In the following table the data are arranged primarily by character of bed surface and subordinately by width of channel, discharge, grade of débris, and slope of channel bed. The characters of bed surface are illustrated in Plate III. Load is given in grams per second, width in feet, discharge in cubic feet per second, and slope in percentage.

TABLE 67.—*Observations on flume traction, showing the relation of load to slope and other conditions.*

a. Over a surface of wood, planed and painted.

w	Q	S	Value of L for grade—								
			(C)	(E)	(G)	(H)	(I)	(J)	(E_1G_1)	($E_2H_1I_2$)	($E_3H_2I_3J_2$)
1.00	0.363	0.32	9.1	6.1	6.1						
		.50							28		
		.75	58	50	55						
		1.00							95	138	117
		1.28	154	107	126						
		1.50					248		166		
		1.93	294	183	199	243					
		2.00	294	202		268	334		271	397	406
		2.50				453			342		
		2.77		318	360	384					
		3.00	566	366	386	438	636	625	484	783	793
		3.41		444	479	524					
		3.93									1,092
		4.01								1,025	
		4.03	850					1,095	622		
		4.50		660	685	801					
	.734	.32	22.8	20.5	18.1						
		.50							68		
		.75	120	98	110						
		1.00							179	315	335
		1.28	261	212	219						
		1.50							297		
		1.93		372	348	449					
		2.00	561	381		474	660	778	427	909	949
		2.50				899	1,248	570			
		2.77		590	580	721					
		3.00	1,075	640	726	814	1,170	1,595	790	1,557	1,730
		3.41		771	765	976					
		3.93									2,360
		4.01								1,980	
		4.03						2,480	1,278		
		4.50		1,081	1,127	1,468					
1.91	.363	.75	49	41.6	58	30					
		1.28	110	103	117	137					
		1.93	230	193	222	255					
		2.77		324	392	463					
		3.44		439	471	569					
	.736	.75	134	114	147	115					
		1.28	282	222	248	325					
		1.93	432	368	430	501					
		2.77		641	725	799					
		3.44		857	846	1,028					

TABLE 67.—*Observations on flume traction, showing the relation of load to slope and other conditions*—Continued.

b. Over a surface of wood, rough-sawn, unplaned, and unpainted.

w	Q	S	Value of L for grade—					
			(E)	(G)	(H)	(I)	(J)	($E_2H_1I_2$)
1.00	0.363	1.07						186
		1.88	133	141	178			
		2.11						355
		2.20				304		
		2.88	247	284	342			
		3.09				515		609
		3.81	358	390	526			
		4.15				813	930	881
	.734	1.07						302
		1.88	269	313	343			
		2.11						646
		2.20				632		
		2.88	461	513	545			
		3.09				982	1,558	1,550
		3.81	673	720	772			
		4.15				1,605	2,140	1,665

c. Over a surface of rectangular wood blocks, with grain vertical.

w	Q	S	Value of L for grade—					
			(E)	(H)	(I)	(J)	($E_2H_1I_2$)	($E_3H_2I_3J_2$)
1.00	0.734	2.00	315	337	561	750	620	789
		3.00	580	693	962	1,397	1,089	1,360
		4.00	911	1,026	1,497	2,060	1,660	2,020

d. Over a pavement of sand grains, grade (G), set in cement, the débris being also of grade (G).

w	Q	S	L
1.00	0.363	1.00	12.1
		2.00	71
		3.00	175
		4.00	317
	.734	1.00	61.2
		2.00	214
		3.00	413
		4.00	656

e. Over a pavement of pebbles, grades (H) and (I), set in cement.

w	Q	S	Value of L for grade—							
			(E)	(G)	(H)	(I)	(J)	(E_1G_1)	($E_2H_1I_2$)	($E_3H_2I_3J_2$)
1.00	0.734	2.00	225	144	125			190	228	
		3.00	425	316	273	196		483	636	605
		4.00	630	531	463	401	471	714	992	1,059

ADJUSTMENT OF OBSERVATIONS.

FORMULATION.

In flume traction, as in stream traction, there is a finite slope—competent slope—corresponding to the zero capacity. An inspection of the observational data by plotting served to show that they could advantageously be adjusted by means of the formula based on the theory of competent slope:

$$C=b_1(S-\sigma)^n \qquad (10)$$

Forty-two observational series were found to give information as to the value of σ. Of these, 26 indicated positive values, three negative, and the remainder values so small as to be of uncertain sign. The mean of the 42 values is +0.29 per cent of slope. Eleven could be compared directly with values adopted in the adjustments of stream traction data, the mean of the eleven values being, for flume traction on a smooth surface +0.14 per cent, and for stream traction +0.28 per cent. This difference is consonant with observed differences in competent slope for the two modes of traction. With the aid of this information, and with use of considerations connected with modifications of the mode of flume traction by rough surfaces, but here omitted, a scheme of values of σ for the observational series of Table 67 was made out. The adjustments were then made, by the graphic methods described in Chapter II; and their results appear in Table 68. The same table records the parameters of the adjusting equations, and also the probable errors.

No adjusted values are given for grade (I) in the first division of the table, for the reason that the data, although apparently based on good observations, are strongly discordant. The observations were retained in the record because an aberrant fact, if established, may prove peculiarly valuable; but in this case the interpretation has not been discovered.

TABLE 68.—*Values of capacity for flume traction, adjusted in relation to slope of channel.*

a. Traction over a surface of wood, planed and painted.

w	*Q*	*S*	Value of *C* for grade—								
			(C)	(E)	(G)	(H)	(I)	(J)	(E_1G_1)	($E_2H_1I_2$)	($E_3H_2I_3J_2$)
1.00	0.363	0.5	28	25.5	25	....	....	....	29	....	....
		1.0	93	74	87	....	....	....	93	138	117
		1.5	186	134	158	....	....	....	170	266	254
		2.0	300	205	234	254	....	....	255	410	408
		2.5	435	283	315	350	....	....	350	568	580
		3.0	590	366	398	451	....	625	450	732	763
		3.5	760	453	486	558	....	860	558	910	955
		4.0	945	553	575	666	....	1,085	666	1,085	1,155
		4.5	....	654	665	780	....	....	....	....	....
Probable error (per cent)			2.9	3.7	0.8	1.1	....	....	1.3	1.8	1.2
Parameters of adjusting equations.		σ	0.04	0.06	0.25	0.35	....	1.80	0.15	0.25	0.40
		n	1.63	1.40	1.16	1.22	....	.96	1.30	1.29	1.28
		b_1	100	80	122	138	....	520	114	199	223

w	*Q*	*S*	Value of *C* for grade—								
			(C)	(E)	(G)	(H)	(I)	(J)	(E_1G_1)	($E_2H_1I_2$)	($E_3H_2I_3J_2$)
1.00	0.734	0.5	46	54	50	....	....	....	65	....	....
		1.0	177	145	151	....	....	....	179	336	335
		1.5	345	258	268	338	....	....	310	592	635
		2.0	548	382	393	470	....	790	453	870	970
		2.5	790	518	527	645	653	1,220	610	1,160	1,310
		3.0	1,050	665	668	830	910	1,630	770	1,465	1,675
		3.5	1,360	820	815	1,027	1,170	2,030	940	1,790	2,060
		4.0	1,700	985	965	1,235	1,440	2,430	1,108	2,120	2,450
		4.5	....	1,150	1,110	1,450	....	....	....	....	....
Probable error (per cent)			2.1	1.8	1.7	0.7	....	0.7	1.7	2.7	0.7
Parameters of adjusting equations.		σ	0.03	0.04	0.16	0.22	0.65	1.20	0.10	0.16	0.28
		n	1.60	1.35	1.20	1.29	1.01	.90	1.24	1.20	1.21
		b_1	185	154	187	221	470	965	202	415	500

w	*Q*	*S*	Value of *C* for grade—			
			(C)	(E)	(G)	(H)
1.91	0.363	1.0	77	69	85	75
		1.5	145	130	160	171
		2.0	224	200	241	275
		2.5	....	281	328	381
		3.0	....	370	415	491
		3.5	....	464	507	602
Probable error (per cent			....	0.5	3.0	1.5
Parameters of adjusting equations		σ	0.07	0.10	0.30	0.55
		n	1.44	1.42	1.16	1.09
		b_1	86	80	130	182

w	*Q*	*S*	Value of *C* for grade—			
			(C)	(E)	(G)	(H)
1.91	0.734	1.0	197	158	185	204
		1.5	324	281	320	378
		2.0	459	415	460	550
		2.5	....	560	607	730
		3.0	....	720	755	915
		3.5	....	880	910	1,095
Probable error (per cent)			....	1.4	3.3	2.4
Parameters of adjusting equations		σ	0.05	0.08	0.20	0.35
		n	1.16	1.30	1.12	1.06
		b_1	210	176	238	322

TABLE 68.—*Values of capacity for flume traction, adjusted in relation to slope of channel*—Continued.

b. Traction over a surface of wood, rough-sawn.

w	Q	S	Value of C for grade—					
			(E)	(G)	(H)	(I)	(J)	($E_2H_1I_2$)
1.00	0.363	1.5	97	....	120	....	....	229
		2.0	145	160	195	249	....	341
		2.5	200	220	279	374	....	460
		3.0	259	285	366	500	....	582
		3.5	331	355	460	630	....	710
		4.0	385	427	560	755	895	840
Probable error (per cent)....			....	....	....	....	....	4.6
Parameters of adjusting equations.		σ...	0.10	0.30	0.40	1.00	....	0.30
		n...	1.34	1.12	1.28	1.01	....	1.16
		b_1..	62	82	106	248	....	184

w	Q	S	Value of C for grade—					
			(E)	(G)	(H)	(I)	(J)	($E_2H_1I_2$)
1.00	0.734	1.5	....	....	....	....	....	450
		2.0	290	331	354	538	....	660
		2.5	390	434	462	752	....	885
		3.0	495	540	570	970	1,490	1,115
		3.5	600	648	680	1,100	1,800	1,360
		4.0	712	758	786	1,325	2,100	1,615
Probable error (per cent)....			....	....	....	....	....	2.3
Parameters of adjusting equations.		σ...	0.08	0.20	0.30	0.65	1.20	0.20
		n...	1.25	1.11	1.02	1.06	.76	1.18
		b_1..	128	173	207	390	950	325

c. Traction over wood blocks with grain vertical.

w	Q	S	Value of C for grade—					
			(E)	(H)	(I)	(J)	($E_2H_1I_2$)	($E_3H_2I_3J_2$)
1.00	0.734	2.0	313	354	540	730	613	780
		2.5	442	507	770	1,090	860	1,075
		3.0	583	673	1,008	1,415	1,110	1,390
		3.5	740	850	1,245	1,740	1,375	1,700
		4.0	900	1,026	1,500	2,040	1,660	2,010
Parameters of adjusting equations.		σ...	0.10	0.45	0.75	1.20	0.50	0.60
		n...	1.46	1.48	1.05	.81	1.16	1.06
		b_1..	123	201	430	880	385	545

d. Traction over pavement of sand grains, grade (G).

w	S	Value of C.	
		Q=0.363	Q=0.734
1.00	1.0	11	61
	1.5	36	131
	2.0	72	215
	2.5	119	310
	3.0	175	413
	3.5	240	525
	4.0	311	650
Parameters of adjusting equations.	σ...	0.50	0.30
	n...	1.70	1.72
	b_1..	36	101
Probable error (per cent)..............		1.1	0.1

TABLE 68.—*Values of capacity for flume traction, adjusted in relation to slope of channel*—Continued.

e. Traction over pavement of pebbles, grades (H) and (I).

w	Q	S	Value of C for grade— (E)	(G)	(H)	(I)	(E_1G_1)	($E_2H_1I_2$)	($E_3H_2I_3J_2$)
1.00	0.734	2.0	220	143	122		200	250	
		2.5	313	220	192		325	420	
		3.0	416	310	272	209	460	605	605
		3.5	529	412	363	290	600	800	830
		4.0	650	522	460	393	740	1,015	1,060
Parameters of adjusting equations.		σ	0.10	0.40	0.55	1.00	1.00	1.00	1.20
		n	1.50	1.60	1.52	1.65	1.19	1.26	1.28
		b_1	84	67	68	65	200	250	280

PRECISION.

Probable errors were computed from the residuals of 25 series, the residuals being measured on the plots as percentages. The greatest probable error computed for a series of adjusted values of capacity is ±4.6 per cent, and their average is ±1.3 per cent. The average of the 25 determinations of the probable error of an observation is ±3.8 per cent. The residuals number 139, and their average value, which also is a measure of the precision of the observations, is ±4.3 per cent.

A comparison of these measures with those obtained from the data for stream traction shows that the flume-traction data are decidedly the more harmonious. The average residual is more than twice as great for stream traction as for flume. Part of this difference may be due to the fact that the experiments with flume traction came last and had the benefit of previous experience, but it is to be ascribed chiefly to the fact that in flume traction the slope is constant, while in stream traction it is subject to rhythmic fluctuations.

DISCUSSION.

CAPACITY AND CHANNEL BED.

Data illustrating the influence of the character of the channel bed on the quantity of débris which a stream can transport have been assembled in Table 69. They are taken chiefly from the preceding table, but a few items are from Table 72 and Table 12. A single item, marked as interpolated, is based on a combination of data from Tables 68 and 72. They pertain to all the simple grades and mixed grades with which experiments were made in flume traction; and stream traction is represented, so far as possible, by coordinate data. Comparisons are made for slopes of 2 and 3 per cent and discharges of 0.363 and 0.734 ft.³/sec.

The greatest capacity is in each case associated with the smoothest of the tested channel beds, the surface being that of a plank, planed and painted, with the grain running parallel to the current.

TABLE 69.—*Comparison of capacities for flume traction associated with different characters of channel bed.*

[Width of trough, 1 foot.]

Q	S	Character of channel bed.	Value of C. Simple grades. (B)	(C)	(E)	(G)	(H)	(I)	(J)	Mixtures. (E_1G_1)	($E_2H_1I_2$)	($E_3H_2I_3J_2$)
0.363	2.0	Planed wood	388	300	205	234	254	364			410	
		Sawn wood			145	160	195	249			341	
		Sand pavement				72						
		Débris	266	245	115	45						
.363	3.0	Planed wood			366	398	451	600			732	
		Sawn wood			259	285	366	500			582	
		Sand pavement				175						
		Débris				120						
.734	2.0	Planed wood		548	382	393	470	711	790	453	870	970
		Sawn wood			290	331	354	533			660	
		Wood block			313		354	540	730		613	780
		Sand pavement				215						
		Gravel pavement			220	143	122			200	250	
		Débris		483	222	145						
.734	3.0	Planed wood			665	668	830	[1,060]	1,630	770	1,465	1,675
		Sawn wood			495	540	570	970	1,490		1,150	
		Wood block			583		673	1,008	1,415		1,110	1,390
		Sand pavement				413						
		Gravel pavement			416	310	272	209		460	605	605

Next in order are two varieties of unplaned wooden surface; the first being that of boards or planks, parallel to the current, retaining the roughness left by the saw; the second a pavement made by sawing planks of Oregon pine into short equal blocks and setting them on edge. Both these surfaces, as well as those described below, are illustrated in Plate III. These two varieties proved to have approximately the same properties in respect to traction, and the capacities associated with them are 23 per cent less than those for planed lumber. The range in ratio is not large for the different experiments, and the value given may be taken as a constant representing the difference in efficiency between new unplaned and planed wooden flumes. The difference tends, however, to diminish with wear, the unplaned lumber becoming smoother and the planed rougher.

The next grade of roughness was given by coarse sand—débris of grade (G)—set in cement, so as to constitute a pavement resembling sandpaper. The only material run over this was débris of the same grade, the special purpose being to compare flume traction with stream traction—the condition of fixed bed with that of mobile bed—when the degree of roughness is the same. The experiments gave the streams 50 per cent greater capacity when sweeping the débris over the fixed bed than when moving it at the same slope by the method of stream traction.

The sand pavement gives capacities half as great, on the average, as the surface of planed lumber, but the contrast is stronger for the smaller discharge and lower slope and less marked for the larger discharge and steeper slope.

The roughest surface used, a pavement of pebbles prepared by setting in cement a mixture of grades (H) and (I), gave still lower capacities. These range from 20 to 62 per cent of the corresponding capacities given by planed lumber. The obstructing influence of the rough bottom is most strongly manifested when the material transported has a coarseness corresponding to the texture of the pavement. For finer material its roughness is mitigated by the lodgment of débris, which has the effect of establishing a pavement of the finer material.

The word "débris" in the table indicates a channel bed composed of loose débris, the débris in transit, and the associated process is that of stream traction. The available data afford comparison only for the four finer grades, (B), (C), (E), and (G), the grades which would be designated sand. Each comparison, with an apparent exception to be considered immediately, shows stream traction to be less efficient than flume traction. When stream traction is compared with flume traction over a smooth surface, the observed ratio of efficiency ranges from 19 to 88 per cent, the smaller ratios being associated with the coarser grades of débris.

The exception occurs when capacity over a bed of débris is compared with capacity over a pavement of pebbles, the two capacities being found to be the same. The cases which afford this comparison are for grades (E) and (G), and these fine materials, by filling the hollows of the pavement, create a condition of bed in which stream traction dominates. The comparison is really between normal stream traction and stream traction modified by the appearance of crests of fixed pebbles in the channel bed. In harmony with this interpretation is the fact that capacities for stream traction and for traction over the gravel pavement, when compared severally with capacity for traction over smooth wood, both show contrasts which increase with coarseness of the load.

The important general facts brought out by the comparisons are (1) that with a given discharge, channel width, and slope, the process of flume traction is able to transport more débris than that of stream traction, and (2) that a stream's capacity for flume traction varies inversely with the roughness of the flume bed.

The first of these principles serves to explain certain phenomena of clogging. When there is fed to a flume a load greater than its stream is able to transport, a portion is deposited. This changes the character of the bed in such a way as to substitute stream traction for flume traction. Stream traction, being less efficient, can carry still less load, and a larger fraction is deposited. If the conditions remain unchanged the bed is built up until its slope becomes that necessary to carry the entire load by stream traction. Unless the trough is deep or short, overflow results.

When clogging is initiated by a temporary overloading, the stream loses power to carry its normal tractional load, and deposition continues unless the load is reduced considerably below the normal. In practical operations the first step toward the abatement of a clog is to stop all feeding of load above the deposit, that the stream may be able to take on load and thereby reduce the deposit.

CAPACITY AND SLOPE.

The rate at which capacity for flume traction is increased by increase of slope is contained implicitly in the values of n and σ assembled in Table 68; for n gives the rate of variation of capacity with $S-\sigma$, and the instantaneous rate of variation of capacity with slope is given by $i_1=\frac{nS}{S-\sigma}$. All the tabulated values of n except two are greater than unity; and in each of the cases where n is less than unity all values of i_1 computed for the range of slopes covered by the experiments are greater than unity. The general fact is thus indicated that, within the practical range of conditions, capacity increases with slope in more than simple ratio. Efficiency also increases as slope increases.

The sensitiveness of capacity to changes in slope varies with changes of condition; and this variation might be illustrated, as in treating of stream traction, by the tabulation and discussion of values of the index of relative variation, i_1. It will suffice, however, in this case to make comparisons by means of the synthetic index, I_1. Table 70 contains values of that index computed between the limits of $S=2.0$ and $S=3.5$. They represent 42 of the 51 series of values given in Table 68, the other nine series not having sufficient range for the computation. The arithmetical mean of the 42 values of I_1 is 1.46, and the range is from 1.08 to 2.08.

TABLE 70.—*Values of I_1 for flume traction, computed between the limits $S=2.0$ and $S=3.5$.*

w	Q	Character of channel bed.	Value of I_1.								
			Simple grades.						Mixtures.		
			(C)	(E)	(G)	(H)	(I)	(J)	(E_1G_1)	$(E_2H_1I_2)$	$(E_3H_2I_3J_2)$
1.00	0.363	Planed wood	1.66	1.44	1.30	1.41			1.40	1.42	1.52
		Sawn wood		1.43	1.42	1.54	1.66			1.32	
1.00	.734	Planed wood	1.66	1.37	1.30	1.40		1.69	1.30	1.29	1.35
		Sawn wood		1.30	1.20	1.17	1.27			1.08	
		Wood block		1.54		1.56	1.49	1.55		1.44	1.39
		Sand pavement			1.60						
		Gravel pavement		1.56	1.89	1.95			1.96	2.08	
1.91	.363	Planed wood		1.50	1.32	1.40					
1.91	.734	do		1.34	1.22	1.23					

Inspection of these data shows, first, that the values are always greater for $Q=0.363$ than for $Q=0.734$. The experiments deal with no other discharges, but it is probably true in general (as in case of stream traction) that increase of discharge is accompanied by decrease of the sensitiveness of capacity to slope.

If the index varies in a systematic way with fineness of débris, its increase is connected with decrease of fineness, but the finest débris of the table, (C), carries large values of the index. The apparent conflict of evidence has its parallel in the fuller data for stream traction (see p. 108 and fig. 34), and it is possible that the sensitiveness increases in two directions from a minimum value. Its variation might in that case be connected with the law relating capacity to fineness, as brought out in a later section.

The relation of the index to roughness of bed does not follow a simple law. Its values are in general least for the bed of rough lumber and progressively greater for planed lumber, wood blocks, and gravel pavement.

The greater sensitiveness of capacity to slope when the channel bed is a coarse pavement may be connected with the fact that the mode of transportation over such a bed is approximately stream traction; and this suggests that in flume traction the sensitiveness may be less than in stream traction. Direct comparison can not be made with use of the values of I_1 in Table 70, because the slopes used in stream traction experiments have less range; but special computations were made, so far as the data were found to overlap. The results are contained in Table 71 and indicate that the sensitiveness is greater for stream traction than for flume traction over a smooth bed, in case of grades (E) and (G), but less in case of grade (C).

TABLE 71.—*Comparison of values of I_1 for flume traction over a bed of planed wood, with corresponding values for stream traction.*

Width of trough (feet).	Grade of débris.	Discharge (ft.³/sec.).	Limiting values of slope (per cent).	I_1 for flume traction.	I_1 for stream traction.
1.00	(C)	0.363	0.5–2.0	1.71	1.60
		.734	.5–2.0	1.79	1.46
	(E)	.363	.5–2.0	1.50	1.99
		.734	.5–2.0	1.41	1.65
	(G)	.363	2.0–3.0	2.27	2.55
		.734	1.0–2.0	1.38	2.42

Various qualifications and doubts being omitted, the preceding paragraphs may be generalized by saying that the sensitiveness of capacity to slope is somewhat less in flume traction than in stream traction. It varies in both directions from a mean value expressed by the exponent 1.5, being greater as the slope is less, as the discharge is less, as the fineness is less, and as the channel bed is rougher. Efficiency for flume traction increases with slope.

CAPACITY AND DISCHARGE.

Special series of experiments were made to determine the variation of capacity with discharge. In each series the conditions of slope, width, and grade of débris were kept constant and the discharge was varied. The observations are given in Table 72, and the same table contains the adjusted values of capacity, together with the parameters of the adjusting equations. Inspection of logarithmic plots showed the propriety of adjusting by means of the formula used with the data for stream traction,

$$C = b_3 (Q - \kappa)^o \qquad (64)$$

and the computations were graphic.

TABLE 72.—*Observations and adjusted data illustrating the relation of capacity for flume traction to discharge, for a rectangular flume of planed wood 1 foot wide.*

[L, observed load; C, adjusted value of capacity; Q, discharge.]

Slope		2 per cent.					3 per cent.			
Grade of débris		(B)	(E)	(H)	(I)	($E_2H_1I_2$)	(E)	(H)	(I)	($E_2H_1I_2$)
	Q	L	L	L	L	L	L	L	L	L
Data of observation	0.039	......	......	......	......	......	28.0	25.0	......	......
	.093	54.5	......	......	......	......	79	128	50.6	87
	.182	151	96	136	110	144	169	228	272	325
	.363	388	202	268	385	397	366	446	567	718
	.545	619	322	417	528	617	580	635	922	1,147
	.734	......	446	546	738	907	785	914	1,170	1,550
Parameters of adjusting equations	κ	0.002	0.007	0.040	0.140	0.060	0.004	0.025	0.080	0.040
	o	1.34	1.09	.88	.68	1.05	1.10	.88	.79	.88
	b_3	1,430	630	760	1,000	1,345	1,125	1,175	1,650	2,220
Probable error (per cent)	......	0.9	0.2	0.4	0.9	0.5	0.3	1.0	0.7	1.4
	Q	C	C	C	C	C	C	C	C	C
Adjusted data	0.039	......	......	......	......	......	28	27	......	......
	.093	58	......	......	......	......	79	108	53	123
	.182	145	95	134	116	146	169	230	270	325
	.363	371	205	279	364	384	364	450	600	730
	.545	640	322	415	548	627	572	670	880	1,130
	.734	......	445	549	711	890	795	870	1,170	1,550

Several values of capacity in Table 72 agree as to conditions with values in Table 68, and these values would be identical if the experiments were homogeneous. A comparison shows that the values given by the experiments comparing capacity and discharge are in general the greater, the average difference being 6 per cent. This is evidently of the nature of systematic error and is probably connected with some change in apparatus or in detail of experimental method which occurred between the making of the two groups of experimental series.

The sensitiveness of capacity to changes of discharge varies with conditions. It is greater as the discharge is less, as the slope is less, and as the channel bed is rougher. It is relatively great for the coarsest and finest of the débris used and less for intermediate grades. Under similar conditions it is less for flume traction than for stream traction. Expressed as an exponent, I_3, its average value for the range of the experiments recorded in Table 72 is 1.26. Values of I_3 were also obtained from data in Table 68 by comparing the capacities for $Q=0.363$ ft.³/sec. with those for $Q=0.734$ ft.³/sec.; and the mean of such values is 0.97. These mean values are not necessarily inconsistent, for the synthetic index varies with the range in discharge for which it is computed and is lower as the discharges are higher; but a study of individual values shows that under identical conditions the data of Table 68 give the lower estimates of sensitiveness. The data as a whole indicate that, for the range of conditions realized in the experiments, the average value of the exponent expressing sensitiveness to discharge is 1.2. There can be no question that for the larger discharges used it falls below unity.

As the sensitiveness of the duty of water, and also the efficiency, to discharge is expressed by an exponent which is less by unity than the corresponding exponent for the sensitiveness of capacity to discharge, it follows that duty and efficiency vary little with discharge. In general they gain slightly with increase of discharge, but they lose when the discharge or slope is relatively large. This accords with a result obtained by G. A. Overstrom,[1] who found from experiments with launders that duty rose with increase of depth to a limited extent only.

CAPACITY AND FINENESS.

In Tables 68 and 72 the values of capacity standing in any horizontal line constitute a series illustrating the variations which are related to grades of débris, and if those in the columns for mixtures be excepted they illustrate the relations of capacity to fineness. Table 73 contains a selection of data from those tables, together with a single line taken from Table 69.

TABLE 73.—*Values of capacity for flume traction, illustrating the control of capacity by fineness of débris.*

Character of channel bed.	w	Q	S	Value of C for grade—						
				(B)	(C)	(E)	(G)	(H)	(I)	(J)
Planed wood	1.91	0.734	2.0		459	415	460	550		
	1.00	.363	2.0	388		202		268	383	
	1.00	.363	3.0		590	366	398	451		625
	1.00	.734	2.5		790	518	527	645	653	1,220
	1.00	.734	3.0		1,050	665	668	830	910	1,630
Sawn wood	1.00	.734	3.0			495	540	570	970	1,490
Wood block	1.00	.734	3.0			583		673	1,008	1,415
Gravel pavement	1.00	.734				416	310	272	200	
Débris (stream traction)	1.00	.363	2.0	266	245	115	45			

As the tables are examined, one of the features arresting attention is that in most of the series the smallest value of capacity does not appear at one end of the line but at some intermediate point. The occurrence of a minimum is in fact characteristic of all tested varieties of flume traction except that in which the bed is a pavement of pebbles. To give the feature graphic expression the data of the last five lines of Table 73 are plotted in figure 69, where the horizontal scale is that of linear fineness, F. The plotted points are far from regular, but the general character of the representative curves is unmistakable, and freehand lines have been drawn. On another sheet, not reproduced, the same data were plotted in relation to mean diameter of particles—the reciprocal of F—with similar result, except that the lower two curves became concave upward.

These curves illustrate the most important difference between the laws of flume traction and those of stream traction. In stream traction capacity increases continuously as fineness increases. In flume traction capacity increases with fineness when the grades of débris compared are relatively fine but increases with coarseness when the grades are relatively coarse. So far as these experiments show, the minimum of capacity corresponds to a coarse sand, but its position on the scale of fineness may be assumed to vary with slope, discharge, and roughness of bed.

The curve for flume traction over a bed paved with gravel shows no minimum but is of the same type as the curve for stream traction. This fact is confirmatory of an inference

[1] Quoted by R. H. Richards in Ore dressing.

already drawn (p. 207), that the transportation of fine débris over a fixed bed of coarser débris particles is essentially of the nature of stream traction. It may fairly be inferred that if we were able to extend this curve into the region of débris coarser than the gravel of the bed, a minimum would be developed.

If the curves were to be traced toward the right, by means of additional experiments with

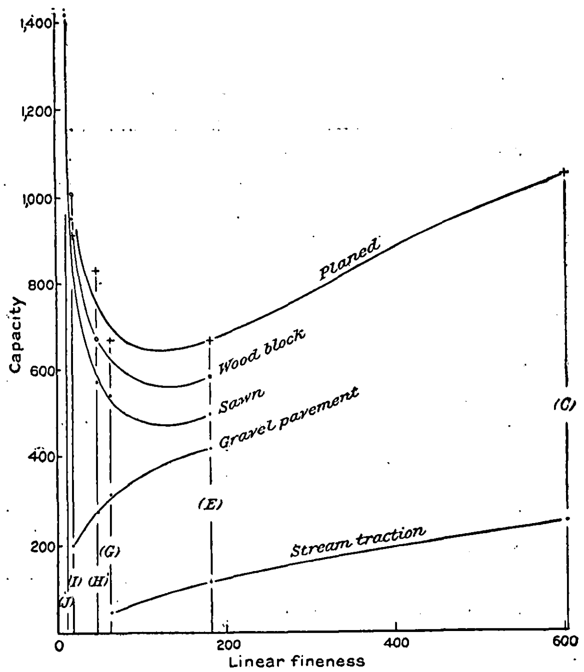


FIGURE 69.—Curves illustrating the relation of capacity for flume traction to fineness of débris. Data from bed of planed wood are recorded by crosses; from wood-block pavement by circles; other data by dots.

finer débris, there can be little doubt that they would be found to continue their ascent; but eventually, *as curves of traction,* they would come to an end with the passage of the process of transportation from traction into suspension. In the opposite direction they may be conceived to attain a maximum and then drop suddenly to the base line; for despite the law of increase of capacity with coarseness, there must be a degree of coarseness for which the force of the current is not competent, and when that is reached the ordinate of capacity becomes zero. The position of this limit, which I have in earlier pages called competent fineness, evidently depends on slope and discharge, as determining the force of the current, and on the degree of rounding of the débris.

The double ascent of the curve of flume traction is susceptible of plausible explanation, by means of considerations connected with the process of rolling. The process of rolling involves a question of space. Each rolling pebble occupies an area of the channel bed somewhat larger than its sectional area, even if the pebbles are arranged in the closest possible order. If we conceive the channel bed to be occupied by rolling pebbles of a particular size, separated by spaces which bear a definite ratio to the diameters of the pebbles; and again conceive it to be occupied by rolling pebbles of a larger size, with the same ratio between interspace and diameter; it is evident that the total volumes or masses of pebbles in the two cases will be proportional to the diameters. If the larger pebbles have twice the diameter of the smaller, then a given area of bed will contain twice as much rolling load of the larger pebbles as of the smaller. It is also true, as stated on an earlier page, that the rolling speed is somewhat greater for larger pebbles than for smaller. The tendency of these two factors is the same, to make the load greater for large particles than for small, when the process of transportation is rolling. The analysis is doubtless too simple—the degree of crowding on the bed, for example, may not be the same for different sizes, and the degree of crowding may affect the speed of rolling—but qualifying factors can hardly impair the qualitative inference that the rolling load increases with coarseness.

It is a matter of observation that, under similar conditions determining force of current, the dominant process in flume traction is for coarse débris rolling and for fine débris saltation. When the process is rolling, as just shown, capacity increases with coarseness of débris. When it is saltation, as illustrated by the body of experiments on stream traction, capacity increases with fineness. With the passage from saltation to suspension the effect is even heightened, and it is probable that in a number of the recorded experiments the process was largely that of suspension. Thus the double ascent of the capacity-fineness curve is determined by the distinctive properties of two (or three) modes of propulsion.

If the preceding explanation is well founded, the nature of the law connecting capacity with the degree of comminution of the débris in any particular case depends on those conditions which determine the dominant process of conveyance. If the channel bed is smooth, and if slope and discharge are so adjusted as to give a moderate velocity, the progression of sand may be by rolling, and in that case the capacity for different sands will vary inversely with their fineness. But if over the same smooth bed the current runs swiftly, sand will be made to travel by saltation, or by saltation and suspension, and then the capacity for different sands will vary directly with their fineness. The particular velocity with which the function reverses will depend on the quality of the bed, being lower if the bed is somewhat rough, because roughness changes rolling to saltation. The critical velocity will be higher for gravel than for sand, because higher velocity is needed to make coarser débris leap.

The experiments which have been made were not sufficiently varied to afford test for these inferences, and as there is no present opportunity for continuance of laboratory work the inferences must be regarded as largely hypothetic.

TABLE 74.—*Capacities for flume traction of mixed grades and their component simple grades.*

Character of bed.	Q	Mixed grade.	Capacity for mixture (gm./sec.).	Capacity for single grade (gm./sec.).				
				(E)	(G)	(H)	(I)	(J)
Planed wood	0.363	(E_1G_1)	450	225 366	225 398			
Do	.734	(E_1G_1)	770	385 665	385 668			
Do	.734	$(E_2H_1I_2)$	1,465	586 665		293 830	586 910	
Do	.734	$(E_3H_2I_3J_2)$	1,675	502 665		335 830	502 910	335 1,630
Sawn wood	.363	$(E_2H_1I_2)$	582	233 259		116 366	233 500	
Do	.734	$(E_2H_1I_2)$	1,115	446 495		223 570	446 970	
Wood block	.734	$(E_2H_1I_2)$	1,110	444 583		222 673	444 1,008	
Do	.734	$(E_3H_2I_3J_2)$	1,390	417 583		278 673	417 1,008	278 1,415
Gravel pavement	.734	(E_1G_1)	460	230 416	230 310			
Do	.734	$(E_2H_1I_2)$	605	242 416		121 272	242 209	
Do	.734	$(E_3H_2I_3J_2)$	605	181 416		121 272	181 209	121

MIXTURES.

Three mixtures of simple grades of débris were treated in the laboratory. The components of one, (E_1G_1), were a medium sand and a coarse sand. The others, $(E_2H_1I_2)$ and $(E_3H_2I_3J_2)$, combined medium sand with fine gravel and coarser gravel. All three were tested in relation to slope on the smooth channel bed and on the gravel pavement—one on the rough-sawn bed and two on the bed of wooden blocks. One entered into the experiments on capacity in relation to discharge. The results are contained in Tables 67, 68, and 72. The data from those tables which pertain to a channel width of 1 foot and a channel slope of 3 per cent are assembled in Table 74. The table contains also the capacity quota for each constituent, computed from the capacity for the mixture; and beneath each of these quotas is printed the capacity for the constituent when the entire load is composed of it.

A general fact brought out by this table is that the current can transport more of a mixture than it can of any one of the constituent grades. The table records a single exception, the capacity for the mixture being 1,390 gm./sec., while that for its coarsest constituent grade is 1,415 grams.

Another general fact shown is that the capacity for each component as part of a mixture is less than the capacity for the same component if transported separately. To this also there is a single apparent exception, but as it occurs among the data for traction over a gravel pavement it illustrates a feature of stream traction rather than flume traction.

The two principles may be illustrated together by saying that if a stream is carrying its full load of a grade narrowly limited in range of fineness, and a different grade of débris is added, the total load is thereby increased, but this increase is accompanied by a diminution of the quantity carried of the first-mentioned grade. In contrast with this is the law found for stream traction—that the load of the initially transported grade is increased by the moderate addition of other débris, provided the added débris is relatively fine.

The difference between the two cases is thought to be connected with rolling. In flume traction over a smooth bed the path for rolling particles is roughened by the presence of smaller particles. In stream traction the pathway for larger particles is smoothed by the presence of smaller particles and rolling is promoted.

In stream traction the capacity for a mixture is determined chiefly by the capacity for its finer components, and as mean fineness also depends chiefly on the fineness of the finer components, mean fineness is a serviceable gage of capacity. In flume traction the relation is quite different. Because of the double ascent of the curve of capacity and fineness, it may readily occur that the capacity for a mixture is most nearly related to that for the coarsest component—in fact, that is true of the three mixtures tested in our experiments—and when that is the case there is no parallelism between capacity and mean fineness.

CAPACITY AND FORM RATIO.

The data bearing on the relations of capacity to the depth and width of current, and their ratio, are meager. Most of the experiments were conducted with a single trough width, 1 foot. The only other width used was 1.91 feet, and its use was associated with but four grades of débris and a single character of channel bed—the smoothest.

Depths of current were not in general measured during the passage of loads, because the surfaces of load-bearing currents were usually so rough as to make good determinations impossible. Good measurements were made of unloaded streams, and the results are here tabulated. Attempts to measure depths of loaded streams yielded one result thought worthy of record. With a discharge of 0.734 ft.³/sec., a width of 1 foot, a slope of 4 per cent, and a full load of débris of grade ($E_3H_2I_3J_2$), the depth was 9 per cent greater than for the corresponding unloaded stream.

Table 76 compares the capacities found for a trough width of 1.91 feet with corresponding capacities for a width of 1 foot. By aid of Table 75 it brings capacities into relation also with depths.

TABLE 75.—*Depths and form ratios of unloaded streams, in troughs of wood, planed and painted.*

Q	S	w=1.91		w=1.00	
		d	R	d	R
0.363	1	0.076	0.040	0.120	0.120
	2	.062	.032	.096	.096
	3	.056	.029	.082	.082
.734	1	.119	.062	.194	.194
	2	.098	.051	.154	.154
	3	.086	.045	.136	.136

TABLE 76.—*Capacities for flume traction in troughs of different widths.*

Q	S	Grade (C).		Grade (E).		Grade (G).		Grade (H).	
		w=1.91	w=1.00	w=1.91	w=1.00	w=1.91	w=1.00	w=1.91	w=1.00
		Values of C.							
0.363	1.0	77	93	69	74	85	87	...	...
	2.0	224	300	200	205	241	234	275	254
	3.0	...	...	370	366	415	398	491	451
.734	1.0	197	177	158	145	185	151	...	...
	2.0	459	548	415	382	460	393	550	470
	3.0	...	...	720	665	755	668	915	830

Q	S	Grade (C).	Grade (E).	Grade (G).	Grade (H).
		$\frac{C_{1.91}}{C_{1.00}}$			
.363	1.0	0.83	0.93	0.98	...
	2.0	.75	.98	1.03	1.08
	3.0	...	1.01	1.04	1.08
.734	1.0	1.11	1.09	1.22	...
	2.0	.89	1.09	1.17	1.17
	3.0	...	1.08	1.13	1.10

Simple combinations of the quantities, into which it is not necessary to enter, show (1) that with constant depth, capacity increases with width and more rapidly than width, and (2) that with constant width, capacity increases with depth and more rapidly than depth. The rate of increase with depth, if expressed as an exponent, may be as low as 1.2 or as high as 2.0. From these premises, a line of reasoning parallel to that of Chapter IV shows that for flume traction, as for stream traction, the function $C=f(R)$ increases to a maximum and then decreases. The value of R corresponding to maximum capacity—the optimum form ratio—can not be determined from observations involving but two channel widths; but its limits can in some cases be indicated. For example, when the capacities conditioned by the same slope and discharge are approximately the same for the two trough widths, it may be inferred that the optimum falls between the values of R associated with the two capacities. Thus for grades (E), coarse sand, with a slope of 2 per cent, the capacities are about the same for the two widths, while the form ratios are 0.032 and 0.096, or one-thirtieth and one-tenth; and it is inferred that the optimum form ratio falls between those fractions.

From inferences of this sort, used in combination with the general principles of flume traction, a number of tentative conclusions have been drawn. They are of so hypothetic a character that the reasoning connected with them is not thought worthy of record, but the conclusions themselves may perhaps be of some service, until replaced by others of more secure foundation. They are: The ratio of depth to width giving a current the highest efficiency for flume traction (1) is greater for gentle slopes than for steep, (2) is greater for small discharge than for large, (3) is greater for fine débris than for coarse, (4) is greater for rough than for smooth channel beds, and (5) is in general less than for stream traction. The first and second propositions apply also to stream traction. The third is the reverse of the relation determined for stream traction, and its applicability may be limited to conditions under which rolling is the dominant mode of transit. By aid of the fifth and fourth Table 31 may be roughly applied to practical problems of flume traction.

TROUGH OF SEMICIRCULAR CROSS SECTION.

A few experiments were made with a trough of galvanized sheet iron, 1 foot wide, having a semicircular section. It was given a slope of 1 per cent, and in it were tested four grades of débris. The observational data appear in Table 77. The object of the experiments was to determine whether a channel with curved perimeter is more efficient or less efficient than one with rectangular section; the capacities obtained are compared, in the table, with those determined for a flat-bottomed trough of the same width. As the discharge used did not fill the semicylindric trough, the width of water surface was less than 1 foot. The width of channel bed occupied by the load ranged from one-fifth to one-half of the width of water surface. The medial depth of water was greater than in the rectangular channel, and the mean velocity was higher. The higher velocity is a factor favorable to the development of capacity; the narrower field of traction is an unfavorable factor. The resultant of the two was unfavorable, the capacities for the semicylindric trough being only half as great as for the rectangular. The result is qualified by the fact that the troughs compared were not of the same material, but the disparity of capacities is too great to be ascribed to that factor.

TABLE 77.—*Data on flume traction in a semicylindric iron trough of 0.5 foot radius; with comparative data for a rectangular wooden trough 1 foot wide.*

Slope (per cent).	Discharge (ft.³/sec.).	Grade of débris.	Capacity (gm./sec.).	
			In semicylindric trough.	In rectangular trough.
1.00	0.363	(C)	43	93
		(E)	33	74
		(F)	37	
	.734	(E)	64	145
		(F)	66	
		(G)	74	151
Depth of unloaded stream (foot)		$Q=0.363$	0.181	0.120
		$Q=0.734$	.257	.194
Width of water surface, unloaded stream (foot)		$Q=0.363$	.770	1.00
		$Q=0.734$	.874	1.00
Width of be occupied by oad (foot)			.17 to .37	1.00
Mean velocity of unloaded stream (ft./sec.)		$Q=0.363$	3.85	3.02
		$Q=0.734$	4.60	3.78

The fact that the doubling of the discharge did not double the observed load indicates that the duty of water diminishes as discharge increases, but the data are too few to give confidence to the inference. The three capacities with a discharge of 0.363 ft.^{3}/sec. indicate a minimum in the curve of $C=f(F)$, thus supporting the generalizations already made from the results with rectangular troughs.

The low efficiency here found for an open channel having a circular arc for its perimeter suggests that the cylindric form commonly given to closed conduits for the hydraulic conveyance of débris may not be the most efficient.

A second suggestion is connected with the fact that the semicylindric trough, while it narrows the field of traction, at the same time gives a high velocity to the water. It thus concentrates the available force and energy on the narrow field. Though the result is not favorable to capacity, it may be favorable to competence. When but a small load is to be transported, the practical problem may be one of competent velocity; and such a trough appears well adapted to the production of competent velocity with economy of discharge and slope.

SUMMARY.

In the transportation of débris in flumes much of the movement is usually by rolling and sliding. This is especially true if the current is gentle or the débris coarse. With a very swift current or with fine débris the particles travel by a series of leaps, and with the finest débris the load is suspended. When the conditions are such that the principal movement is by rolling and sliding, the capacity of the current increases with the coarseness of the débris transported, this law holding good up to the limit of coarseness at which the current is barely competent to start the particles. When the conditions are such that the principal movement is by saltation, the capacity of the current increases with the fineness of the débris, the law holding good up to and probably beyond the critical fineness at which the current is competent to carry the débris in suspension.

Under all conditions the capacity is increased by steepening the slope, and the increase of capacity is more rapid than the increase of slope. The capacity may vary with a power of the slope as low as the 1.2 power, or with one higher than the second. A general average for the experimental determinations is the 1.5 power.

Under all conditions the capacity is increased by enlarging the discharge. It may be increased in the same ratio, in a higher ratio, or in a somewhat lower ratio.

The highest capacity is associated with the smoothest channel bed. Progressive increase of roughness reduces capacity progressively until the texture of the bed becomes coarser than the débris of the load. The mode of transportation then passes from flume traction to stream traction. Under like conditions of slope, discharge, and character of débris, flume traction gives higher capacities than stream traction.

Rectangular or box flumes have higher capacity than semicylindric flumes of similar width. Up to a limit, which varies with conditions, the capacity is enlarged by increasing the width of channel at the expense of depth of current. The ratio of depth to width which gives highest efficiency has not been well covered by the experiments, but it is believed to be rarely greater than 1:10 and often as small as 1:30. For large operations the determination of width will usually represent a compromise between efficiency and the cost of construction and maintenance.

As most of the experiments were made with sorted débris, each grade being narrowly limited as to range in the size of its particles, and as most practical work is with aggregations having great range in size, the loads and capacities here reported need qualification. By experiments with mixtures of the laboratory grades it was found that the load carried of a mixture is greater than the load of any one of its important components taken separately. It is in general true that the capacities for complex natural grades of débris are greater than the tabulated capacities for the laboratory grades they most nearly resemble.

COMPETENCE.

The experiments in flume traction were practically limited in their range by phenomena of competence, and these limitations were of use in determining values of σ, κ, and ϕ; but no effort was made to observe competence directly and precisely. There are, however, a few observations by others, which may properly be assembled here, although it is not practicable to use them as checks on our work. Our indefinite data pertain to slope and discharge, while the observations of others pertain to velocity.

The experiments of Dubuat[1] (1783) have been assumed, both by him and by others, to pertain to stream traction, but his account of apparatus and methods makes it probable that what he really investigated was chiefly competence for flume traction. He used a trough of plank, with the grain lengthwise, and measured the velocity of the current by observing the speed of balls slightly heavier than water as they were swept along the bottom. In a current of a particular velocity he placed successively various kinds of débris and noted their behavior, then changed the velocity and repeated. His results are as follows:

	Competent bed velocity (ft./sec.).	
Potter's clay	Between 0.27 and	0.35
Coarse angular sand	0.7	1.1
River gravel:		
Size of anise seed	0.35	0.53
Size of peas	0.62	0.7
Size of common beans	1.1	1.55
Rounded pebbles, 1 inch in diameter	2.1	3.2
Angular flints, size of hen's eggs	3.2	4.0

J. W. Bazalgette, in discussing the flushing of sewers and therefore presumably considering flume traction rather than stream traction, quotes the following results of experiments by Robison:[2]

	Competent bed velocity (ft./sec.).
Fine sand	0.5
Sand coarse as linseed	.67
Fine gravel	1.0
Round pebbles, 1 inch in diameter	2.0
Angular stones, size of eggs	3.0

In 1857 T. E. Blackwell[3] conducted elaborate experiments to determine the velocities necessary to move various materials in sewers. His channel was of rough-sawn plank, 60 feet long and 4 feet wide, with level bottom. Velocities were measured by a tachometer, but the relation of the velocity measurements to the bed is not stated. The tests were applied to natural débris of various kinds and also to types of artificial objects likely to enter sewers. The objects were treated singly and in aggregates, with the general result that an aggregation requires higher velocity to move it than does a single object. It is evident that the experiments on single objects pertain to flume traction and some of those on aggregations to stream traction. From his tabulated results the subjoined data are selected as representing or illustrating the velocities competent for natural débris, the column of mean diameter being added by me. He infers from the experiments that (1) for objects of the same character competent velocity increases with the mass; (2) for objects of the same size and form it increases with the specific gravity; (3) for objects of different form it is greater in proportion as they depart from the form of a sphere; and (4) for objects in motion the rate of travel increases with the velocity of the current.

TABLE 78.—*Observations by Blackwell on velocity competent for traction.*

Material.	Volume.	Mean diameter.	Competent velocity.
Single objects (illustrating flume traction):	*Cubic inches.*	*Feet.*	*Ft./sec.*
Brickbat (roughly cuboid)	18.5	0.27	2.75–3.00
Do	12.98	.24	2.25–2.50
Do	13.6	.25	2.00–2.25
Do	7.33	.20	2.00–2.25
Do	4.76	.17	2.25–2.50
Do	2.59	.14	1.75–2.00
Flint (subangular)	10.37	.22	3.00–3.25
Do	6.05	.19	2.00–2.25
Do	4.11	.16	2.25–2.50
Do	1.95	.13	2.50–2.75
Bowlder (well rounded)		.20	1.50–1.75
Do		.16	2.25–2.50
Bowlder (fairly rounded)		.13	2.50–2.75
Aggregations (illustrating stream traction):			
Gravel		.042	2.25–2.50
Do		.021	1.25–1.50
Sand			0 –1.00

WORK OF OVERSTROM AND BLUE.

Certain experimental work on the capacity of currents for flume traction has for its specific purpose the determination of dimensions for launders, the flumes in which pulverized ore is conveyed. R. H. Richards's "Ore dressing"[4] contains an abstract of results obtained by G. A. Overstrom, accompanied by the statement that the experimental data are extensive but as yet unpublished. The troughs employed were flat-bottomed and probably of wood. For each slope, width of flume, and grade of transported material he found (1) that the duty of water varies with the discharge and that some particular discharge is associated with a maximum duty, so as to be the most economical; (2) that

[1] Dubuat-Nançay, L. G., Principes d'hydraulique, vol. 1, p. 100; vol. 2, pp. 57, 79, 95, Paris, 1786.
[2] Inst. Civil Eng. Proc., vol. 24, pp. 289–290, 1865.
[3] Accounts and papers [London], Sess. 2, 1857; Metropolitan drainage, vol. 36, Appendix IV, pp. 167–170, Pls. 1–5.
[4] Vol. 3, pp. 1592–1594, 1909.

the most economical discharge is sensibly proportional to the width, so that for each slope and grade of material there is a particular discharge per unit of width giving a maximum duty; and (3) that the most economical discharge is greater for low slopes than for high.

The first of these results is in fair accord with our own. Five of the nine values of the exponent o in Table 72 are less than unity. For the corresponding series the values of i_3 range both below and above unity and the corresponding values of the variable exponent for duty in relation to discharge, i_3-1, range below and above zero. The value zero evidently corresponds to a maximum value of duty. His third result is in strict accordance with ours; his second can not be compared without fuller details.

A diagram exhibiting his determinations for the traction of crushed quartz sized by 40-mesh and 150-mesh sieves shows for different discharges per unit width the variation of duty with slope. For the larger discharges duty varies as the first power of slope; for the smallest discharge with the second power. This corresponds to a variation of capacity with the second to third power of slope. Our most available data for comparison are those of grade (C), the capacity for which varies with the 1.66 power of slope (Table 70). As grade (C) was separated by 30-mesh and 40-mesh sieves, it is considerably coarser than the crushed quartz, and, being stream worn, it is less angular. The marked difference in the observed laws of variation is evidently susceptible of more than one interpretation, but it is thought to be connected with difference in fineness, as more fully stated on a following page.

F. K. Blue [1] made a series of experiments in which the trough was of sheet iron, 50 feet long, 5 inches deep, and 4 inches wide, the bottom being semicylindric with 2-inch radius. It was so mounted that it could be set to any slope up to 12 per cent. Two materials were used as load, the first a beach sand of 60-mesh average fineness, the other a sharp quartz sand of about 80-mesh fineness, containing about 10 per cent of slime from a stamp mill. With each material the discharge and load were varied; and for each combination of discharge and load the slope was adjusted to competence, and mean velocity was determined by means of a measurement of depth. Discharge and load were not measured directly but in certain combinations. Instead of discharge, the total volume of water and load was measured. This quantity was used chiefly in the computation of mean velocity, for which purpose it is better fitted than is discharge alone. Load was measured as a volume, the volume of the transported material as collected in a settling tank, and is reported only through a ratio, q, which is the quotient of the volume of load by the volume of discharge plus load. This is essentially a duty but differs materially from duty $\left(U=\frac{C}{Q}\right)$ as defined in the present report. Representing by W the weight in grams of a cubic foot of débris, including voids, and by v the percentage of voids, it follows from the definitions that

$$q=\frac{U}{W+U(1-v)},\ \text{and}\ U=\frac{qW}{1-q(1-v)}$$

From the discussion of his data Blue finds (1) that q varies as the square of the slope and (2) that it varies as the sixth power of the mean velocity. He does not specifically consider the relation of q to discharge, but examination of his tabulated data shows that q is but slightly sensitive to variations of discharge plus load.

As Blue's coarsest material, the beach sand, has approximately the fineness of our grade (A), while the finest we treated in flume traction is of grade (C), the most definite comparison of results can not be made, but there is nevertheless interest in such comparison as is possible. Computing values of U from his data for beach sand, and plotting them in relation to slope, I obtained $U \propto S^{2.02}$ This gives for capacity and slope, $C \propto S^{3.02}$; and the exponent 3.02 may be compared with values of I_1 in Table 70, for the smoothest kind of trough bed. The exponent for grade (C) is 1.66, and the exponent has a minimum value of 1.30 for grade (G). In accordance with the generalization (p. 208) that the sensitiveness of capacity to slope increases from a minimum toward both coarse grades and fine, we should expect for grade (A) an index of sensitiveness materially greater than 1.66. The data furnished by Blue thus tend to support the generalization, and additional support

[1] Eng. and Min. Jour., vol. 84, pp. 536–539, 1907.

is given by Overstrom's data, above cited. Such an inference is qualified as to Blue's data by the possibility that the exponent is affected in material degree by the form of the cross section of the trough.

Blue's data on velocity are of such character as to warrant a fuller discussion than he gives. His comparison with duty (q) is made without regard to the accessory conditions of discharge, depth, and slope, but we have seen that in stream traction these conditions materially affect the control of capacity and duty by mean velocity. It is not practicable so to arrange his data as to obtain results for the condition that discharge, depth, or slope is constant, but moderate approximations to such conditions may be obtained by grouping. I have divided his observations on the traction of beach sand into three groups—first, with reference to discharge (+load); second, with reference to depth; and, third, with reference to slope—and for each group have computed n on the assumption that capacity varies with the nth power of mean velocity. The resulting values are given in Table 79, and with them are placed the average of the corresponding exponents for stream traction, derived under the several conditions of constant discharge, constant depth, and constant slope (Table 53).

It appears (1) that in flume traction capacity is much more sensitive to variation of mean velocity than in stream traction; and (2) that in flume traction, as in stream traction, the order of sensitiveness as related to conditions is highest for constant discharge, intermediate for constant depth, and lowest for constant slope. In flume traction the sensitiveness appears to vary directly with discharge and depth and inversely with slope, while in stream traction it was found to vary inversely with discharge and slope, the variation with depth being indeterminate.

TABLE 79.—*Value of n in $C \propto V_m{}^n$, based on Blue's experiments on flume traction of beach sand.*

	Value of n under condition that small range is given to—		
	Q	d	S
The values of Q, d, or S being relatively—			
Small	5.7	5.7	3.6
Intermediate	6.2	5.0	5.0
Large	7.5	7.0	3.3
Mean	6.5	5.9	4.0
Comparative values of I_V from Table 53	3.98	3.68	3.21

CHAPTER XIII.—APPLICATION TO NATURAL STREAMS.

INTRODUCTION.

The flow of a river is a complex phenomenon. The transportation of débris by it involves intricate reactions. The quantity of débris transported depends on a variety of conditions, and these conditions interact one on another. Direct observation of what takes place at the base of the current is so difficult that the body of information thus obtained is small. In the work of the Berkeley laboratory the attempt was made to study the influence of each condition separately, and to that end all the conditions were subjected to control. This involved the substitution of the artificial for the natural; and while the principles discovered are such as must enter into the work of natural streams, their combinations there are different from the combinations of the laboratory. It is the province of the present chapter to consider the differences between the laboratory streams and natural streams, and in view of those differences the applicability of the laboratory results to problems connected with natural streams.

FEATURES DISTINGUISHING NATURAL STREAMS.

KINDS OF STREAMS.

Classification necessarily involves a purpose, or point of view, and there are in general as many scientific, or natural, or otherwise commendable classifications as there are functions to be subserved. The classification of streams here given has no other purpose than to afford a terminology convenient to the subject of débris transportation.

When the débris supplied to a stream is less than its capacity the stream erodes its bed, and if the condition is other than temporary the current reaches bedrock. The dragging of the load over the rock wears, or abrades, or corrades it. When the supply of débris equals or exceeds the capacity of the stream bedrock is not reached by the current, but the stream bed is constituted wholly of débris. Some streams with beds of débris have channel walls of rock, which rigidly limit their width and otherwise restrain their development. Most streams with beds of débris have one or both banks of previously deposited débris or alluvium, and these streams are able to shift their courses by eroding their banks. The several conditions thus outlined will be indicated by speaking of streams as *corrading*, or *rock-walled*, or *alluvial*. In strictness, these terms apply to local phases of stream habit rather than to entire streams. Most rivers and many creeks are corrading streams in parts of their courses and alluvial in other parts.

Whenever and wherever a stream's capacity is overtaxed by the supply of débris brought from points above a deposit is made, building up the bed. If the supply is less than the capacity, and if the bed is of débris, erosion results. Through these processes streams adjust their profiles to their supplies of débris. The process of adjustment is called gradation; a stream which builds up its bed is said to aggrade and one which reduces it is said to degrade.

An alluvial stream is usually an aggrading stream also; and when that is the case it is bordered by an alluvial plain, called a flood plain, over which the water spreads in time of flood.

If the general slope descended by an alluvial stream is relatively steep, its course is relatively direct and the bends to right and left are of small angular amount. If the general slope is relatively gentle, the stream winds in an intricate manner; part of its course may be in directions opposite to the general course, and some of its curves may swing through 180° or more. This distinction is embodied in the terms *direct alluvial* stream and *meandering* stream. The particular magnitude of general slope by which the two classes are separated is greater for small streams than for large. Because fineness is one of the conditions determining the general slope of an alluvial plain, and because the gentler slopes go with the finer alluvium, it is true in the main that meandering streams are associated with fine alluvium.

FEATURES CONNECTED WITH CURVATURE OF CHANNEL.

As nearly all the laboratory experiments were performed with straight channels, and as all natural channels are more or less curved, the features resulting from curvature constitute differences of which account must be taken in applying laboratory results. Some of these differences have been mentioned in connection with the short series of experiments with curved and bent channels, but a fuller account is desirable.

In a straight channel the current is swifter near the middle than near the sides and is swifter above mid-depth than below. On arriving at a bend the whole stream resists change of course, but the resistance is more effective for the swifter parts of the stream than for the slower. The upper central part is deflected least and projects itself against the outer bank. In so doing it displaces the slow-flowing water previously near that bank, and that water descends obliquely. The descending water displaces in turn the slow-flowing lower water, which is crowded toward the inner bank, while the water previously near that bank moves toward the middle as an upper layer. One general result is a twisting movement, the upper parts of the current tending toward the outer bank and the lower toward the inner.[1] Another result is that the swiftest current is no longer medial, but is near the outer or concave bank. Connected with these two is a gradation of velocities across the bottom, the greater velocities being near the outer bank. The bed velocities near the outer bank are not only much greater than those near the inner bank, but they are greater than any bed velocities in a relatively straight part of the stream. They have therefore greater capacity for traction, and by increasing the tractional load they erode until an equilibrium is attained. On the other hand, the currents which, crossing the bed obliquely, approach the inner bank are slackening currents, and they deposit what they can no longer carry.

It results that the cross section on a curve is asymmetric, the greatest depth being near the outer bank. As the winding stream changes the direction of its curvature from one side to the other, the twisting system of current filaments is reversed, and with it the system of depths, but the process of change includes a phase with more equable distribution of velocities, and this phase produces a shoal separating the two deeps. The shoal does not cross the channel in a direction at right angles to its sides but is somewhat oblique in position, tending to run from the inner bank of one curve to the inner bank of the other. In meandering streams it is usually narrow and is appropriately called a bar. In direct alluvial streams, where bends are apt to be separated by long, nearly straight reaches, it is usually broad and may for a distance occupy the entire width of the channel. In navigated rivers the locality of the bar is usually called a crossing, being the place where the thalweg, the line of strongest current, and the route of travel cross from side to side; and the name is often applied also to the bar itself.

The twisting current attacks the outer bank, being swifter at contact with that bank than in any other part of the wetted perimeter. If the bank consists of alluvium there is erosion, the amount being determined in part by resistances arising from roots, or adhesion of alluvial particles, or incipient cementation; and the eroded material, so far as it joins the bed load, helps to satisfy the bed current and limit downward erosion. In alluvial streams the erosion from concave banks offsets the deposition under convex banks, so that the channel may gradually shift its position without change of sectional area.

The sectional area may be either greater or less at a curve than on a reach, but the differences are normally[1] of small amount. Therefore the mean velocity does not vary greatly. The current in a curved channel, as compared to that in a straight channel, is characterized by diversity. Its bed velocities are both higher and lower, and the same is true of velocities along the banks. This diversity is favorable to traction, because capacity for traction varies with a high power of velocity; but the advantage to traction is partly offset by the fact that increase of velocity affects a smaller portion of the wetted perimeter than

[1] The system of movements here described has been observed by many students of rivers. They were demonstrated by the aid of a model channel by J. Thomson, in connection with an explanation which differs somewhat from that of the present text. See Roy. Soc. London Proc., pp. 5-8, 1876, and 356-357, 1877; also Inst. Mech. Eng. Proc., pp. 456-460, 1879.

[1] That is to say, they are of small amount when the system of depths is adjusted to the discharge, as explained on a later page.

is affected by reduction of velocity. There is also diversity in the directions followed by elements of current, and this diversity includes not only the twisting movement but various minor eddies and swirls. Diversified movements, by including upward movements, promote suspension, and in conjunction with diversified velocities they modify the partition of the load between traction and suspension. On the whole, suspension claims more in a diversified current, but it is also true that the line of separation between suspension and traction shifts to and fro in such a current. Much débris which is suspended in the swift water under the concave bank joins the bed load in passing the shoal between deeps, and the suspended load is still more restricted in passing the shoal of the convex bank. Deposition on the latter shoal includes both tractional and suspensional materials.

FEATURES CONNECTED WITH DIVERSITY OF DISCHARGE.

All streams vary in volume from season to season and from year to year. In a stream fed by springs the changes may be slight. At the opposite extreme are creeks and even rivers which exist only during storms. In most large streams the discharge at flood stage is many times greater than at low stage. Usually flood stages continue only for brief periods and in the aggregate occupy but a small fraction of the year.

It is broadly true that streams give shape to their own channels, and among alluvial streams there are few exceptions. It is broadly true also that the shapes of channels, including cross sections and plans, are the same for large streams as for small. But the large stream requires and develops a larger channel—broader, deeper, and winding in larger curves. Through variation of discharge the same stream is alternately large and small, so that its needs are different at different stages. At each stage it tends to fit its channel to the needs of the particular discharge. The formative forces residing in the current are so much stronger with large discharge than with small that the greater features of channel are adjusted to large discharge, and this despite the fact that floods are of brief duration. The feebler forces of smaller discharges modify the flood-made forms but do not succeed in completing their work of adjustment before it is interrupted by another flood. The deeps of high stage are pools at low stage and have currents too feeble for traction. As the reduced stream passes from pool to pool it crosses the shoal formed at high stage with quickened current. The velocities are still diversified, but the greater and smaller velocities have exchanged places. The slope of water surface is more diversified than at high stage, being lower at the pools and higher between them. Traction is restricted to the shoals, and the loads are small. The load at each shoal is obtained from the shoal itself and is deposited in the next pool, and in this way shallow channels are developed from pool to pool.

In contrasting the features of high and low stages, it has been convenient to use the terms as if high stage and low stage were specific and definite phases of stream activity, thereby ignoring the actual diversity in fluctuations of discharge. Floods are of all magnitudes, and each flood presents not only a maximum discharge but a continuous series of changing discharges. At each instant the stream contains a system of currents of which the details depend not only on the discharge but on the shapes of channel created by the work of previous discharges. So long as the discharge continues, its currents are eroding and depositing in such way as to remodel the channel for its own needs, and so long as the work of remodeling continues the loads and capacities at different cross sections are different.

With the changes in the values and distribution of velocities go changes in those values of competent fineness which on one side limit traction and on the other separate traction from suspension. With maximum discharge all the coarser grades of débris within the domain of the stream are in transit along the path of highest activity, and that path includes the deeps and the intervening shoals. With lessening discharge the coarsest material stops, but it stops chiefly in the deeps, because the change in bed velocity is there greatest. At the same time the coarsest of the suspended load escapes from the body of the stream and joins the bed load. By this double change the mean fineness of the tractional load is increased, and so also is the mean fineness of the suspended load. With continued reduction of discharge the tractional load in the deeps becomes gradually finer and at last ceases to move, while the graduated deposit caused by its arrest receives a final contribution from the suspended load. The tractional load on the shoals changes less in mean fineness and may cease to change altogether when the supply from the deep is cut off. It is then derived wholly from the subjacent bed and is greatly reduced in quantity. Soon the derivation becomes selective, the finer part being carried on while the coarser remains, with the result that the shallow channels on the bars come to be paved with particles which the enfeebled currents can not move.

If the section of the alluvium underlying a shoal be afterwards exposed, it is seen to be in the main heterogeneous but veneered at the top by a layer of its coarser particles. The typical section of a deposit in a deep shows the coarsest débris below and the finest at top, with a gradual change.

With the return of large discharge the modeling work of smaller discharges is rapidly obliterated, and the débris deposited in the pools rejoins the tractional and suspended loads.

SECTIONS OF CHANNEL.

Rock-walled channels result from the aggradation of corraded channels. Often they are recurrent temporary conditions of corraded channels. Their widths have been developed in connection with the work of corrasion and are less than the widths of alluvial streams. In the fact that their sides are immobile they resemble the laboratory channels, and their types of cross section are illustrated by the experiments with crooked channels. The channels of all alluvial streams are strongly asymmetric at the bends, and in the meandering streams the bends constitute the greater part of the course. Departure from symmetry is less pronounced in the reaches of direct alluvial streams, but even there a close approximation to symmetry is exceptional.

Alluvial streams tend to broaden their channels by eroding one or both banks. The influence of vegetation opposes this tendency. Often the erosion of the bank exposes roots, and some trees extend rootlets into the water. At low stages the bared parts of the flood channel are occupied by young plants. In these ways vegetation creates obstacles which retard the current at its contact with the bank and thus oppose erosion. If the current is strong erosion is merely retarded, not prevented; if the current is weak deposition may be induced. As a meandering stream encroaches on its concave bank, the convex bank encroaches on the stream, and channel width is maintained. A large stream is less affected than a small stream by the opposition of vegetation and maintains a channel of relatively small form ratio.

Some streams aggrade so rapidly that vegetation does not secure a foothold. By erosion of its banks such a stream broadens its channel and reduces its depth until the slackened current clogs itself by deposition of its load. The built-up bed becomes higher than the adjacent alluvial plain, and the stream takes a new course. Before the assumption of the new course the banks are overtopped by shallow distributaries which deposit their loads on the banks, thus building them up, until the stream is made to flow on a sort of elevated conduit; and when the main body of water at last leaves this pathway, it is apt to start its new course with a steepened slope and scour for itself a relatively narrow channel.

The building up of the bank by deposition from overflow is more pronounced in the presence of vegetation. The ridge thus created is called a *natural levee.* Its crest separates the channel from the flood plain and delimits at flood stage two provinces in which the conditions affecting transportation are strongly contrasted. In both provinces the general slope of the water surface is the same, but the broad sheet covering the plain has so little depth that its currents are sluggish. Between the banks are the normal channel depths and currents, and transportation is active, alike by traction and suspension. Beyond them transportation is effected almost wholly by suspension, and the coarser particles of the suspended load are deposited. As the flood subsides the lateral sheets of water are returned to the main channel by a draining process which involves the making and maintenance of small channels within the plain.

When the channel of a river is fully adjusted to the discharge the same load is transported through each section. All sections are then equally adapted to transportation, though in different ways. The most symmetric has a wide space at the bottom devoted to traction.

The least symmetric has a relatively narrow tractional space, but traction is there relatively active. The partition of the load between traction and suspension is not the same for the two sections, the tractional load having the greater range in the symmetric section and the suspended load in the asymmetric.

There is reason to believe that the sectional area is about the same in different parts of an adjusted channel. At low stages, when form is least adjusted to discharge, the sectional area is much larger for the asymmetric sections. At higher stages the contrast is less, and the greater area may be associated with either type of section. It is also true, if attention be restricted to the channel proper and the expansions over flood plains be excluded, that the variations in width from point to point of an adjusted channel are not of large amount. If it were strictly true that both sectional areas and widths are equal in different parts of an adjusted channel, it would follow (1) that mean depths are equal, and (2) that form ratios are equal, provided form ratio be defined as the ratio of mean depth to width. Such a generalization, while crude and doubtless subject to important qualification, nevertheless warrants the selection of mean depth rather than maximum depth as the quantity to be used in applying the conception of form ratio to rivers.

Assuming the generalization as an approximation to the actual fact and connecting with it the fact that all sections of an adjusted river are equally efficient for transportation, we are able to make a general application of the laboratory results on optimum form ratio to rivers. The ratio of mean depth to width in alluvial rivers, as a class, is very much smaller than in the laboratory examples by means of which the optimum ratio was discussed in Chapter IV. It is so much smaller that the range of form ratio for alluvial rivers overlaps but slightly the range observed in the laboratory. This disparity indicates, though without demonstrating, that the form ratios of the rivers are less than the optimum, and that their tractional capacities would be greater if they were narrower and deeper. As the optimum ratio is the one which enables a stream to transport its load with the least expenditure of head, it is probable that the slopes of most alluvial rivers can be lessened by artificially reducing their widths.

THE SUSPENDED LOAD.

In speaking above of the transfer of load from traction to suspension no consideration was given to capacity for suspension. Certain students of rivers, comparing discharges or velocities with the percentage of suspended material and finding a rough correspondence, have inferred that suspended load is a function of velocity; others, giving attention to conspicuous examples of noncorrespondence, have inferred that the suspended load depends only on the supply of suitable material. There is a measure of truth in both views, and their divergence is largely to be explained by considerations connected with competence.

The subject is illustrated by observations on the suspended load of Yuba River. Its water was sampled during flood stages, in 1879 [1] at Marysville, a load of 0.35 per cent, by weight, being found when the discharge was estimated at 26,000 ft.^{3}/sec., and a load of 0.42 per cent when the discharge was 18,000 ft.^{3}/sec. In 1906, at a time when the discharge was estimated at 33,000 ft.^{3}/sec., samples were taken at the same place and the load was found to be only 0.065 per cent. At the time of the earlier samplings hydraulic mining was in full operation in the basin of the Yuba, and the suspended load consisted chiefly of clay components of the auriferous gravels, artificially fed to the stream. In 1906 there was little hydraulic mining and the suspended load consisted of material washed from the surface of the land by rain.

At low stages in 1906 the water at Marysville was clear, but in 1879 a sampling when the discharge was 510 ft.^{3}/sec. gave a load of 0.86 per cent. The fact that the river's load in 1879 constituted a higher percentage at low stage than during flood is explained by the consideration that the turbid tributaries from the mines were less diluted by other tributaries at low stage than at high.

If we assume, first, that 0.065 per cent, observed at flood stage in 1906, represents the normal tribute from other sources than mining, and, second, that the mines contributed the same total amounts at low and high stages in 1879 (discharges being 510 and 18,000 ft.^{3}/sec.), and if we base on these assumptions a compu-

[1] Manson, Marsden, Report of determinations of sediment held in suspension, etc.: California State Engineer Report, 1880, Appendix B.

tation of the low stage percentage of load in 1879, we obtain an estimate of 12.6 per cent, which is 14 times greater than the observed load, 0.86 per cent. The discrepancy is altogether too great to be accounted for by errors in the explicit assumptions, and its explanation involves the factor of competence. The stronger currents of flood stages were competent to suspend heavier particles than were the feeble currents of low stages, and so a part of the load which at high stages reached the mouth of the river at Marysville was at low stages arrested on the way, being deposited in the low-water pools.

In these cases it is evident that the range of available fineness is determined, through velocity, by discharge, and that the load of débris *within the range of adequate fineness* is determined by supply. The load appears to bear no relation to capacity, and if the term capacity be used in the broad sense of a stream's ability to suspend material of unspecified fineness, then it is undoubtedly true, not merely of the Yuba but of all rivers, that the suspended load is less than the capacity and depends for its quantity on supply. If, however, capacity be considered with reference to particular degrees of fineness, the case is somewhat different, for a stream may carry a full load of that material for which it is barely competent and at the same time have less than a full load of finer material, and the matter is further complicated by interdependencies in virtue of which each element of load tends to limit the capacity for all other elements of load.

To show the basis for these statements and also to explain certain mutual relations between traction and suspension, it is necessary to give somewhat elementary consideration to the subject of capacity for suspension.

As already mentioned and implied in various connections, the process of suspension depends on the diversity in direction of the strands of the current. If the lines of flow were parallel to the stream bed, as is sometimes assumed for the sake of simplifying mathematical discussions, there would be no suspension.[1] In the sinuous and swirling movements which characterize the flow of streams strands of current are continually passing upward and downward and are as continually dividing and blending. Particles of débris too light to resist the lower elements of the current are swept upward and are retained in the body of the stream through a process analogous to the stirring of the domestic pot. While thus incorporated they are impelled downward by gravity, and all but the

[1] There is a theory originating with Dupuit (Études sur le mouvement des eaux, 1848) that suspension is due, or might be due in the ideal case of parallel flow lines, to reactions between solid particles and contiguous threads of current having different velocities. Under the postulate that the solid tends to move faster than the liquid, it is shown that the path of least resistance trends obliquely toward the swifter of adjacent threads of current, and therefore obliquely upward. As this theory retains place in current hydraulic literature, the fact that it is ignored in the text of the present paper may call for explanation. I do not accept the postulate and am of opinion also that the reasoning based on it ignores an essential factor. As a full statement and discussion of Dupuit's analysis would occupy much space, I will content myself with a statement of my own view. A good abstract of his theory, by E. H. Hooker, may be found in Am. Soc. Civil Eng. Trans., vol. 36, pp. 246–247, 320–322.

In various discussions of the subject the velocities are treated as "absolute"—that is, they are referred to the fixed walls and bed of the channel. As the only possible reactions between the solid particles and contiguous water are through relative velocities, it is better to focus attention on those by referring them to the center of the particle. Let us assume that the solid particle A, figure 70, is immersed in a current

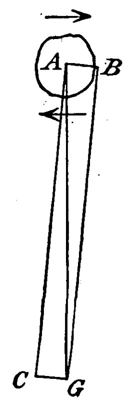


FIGURE 70.—Diagram of forces.

of which the parallel rectilinear filaments increase gradually in "absolute" velocity from below upward, and let us assume that at some instant it moves with the velocity and direction of the filament which is at the same level. Barring extraneous forces, it will continue indefinitely in the same direction and with the same velocity. The filament above moves, with reference to the particle, in a direction indicated by the arrow; the filament below moves in the opposite direction. Their relative velocities are the same, except for a possible difference of the second order of magnitude. The two filaments tend to draw the upper and lower parts of the particle in opposite directions, and the result is rotation. This is the only result dependent on the fact that the particle is solid. Now introduce the factor of density. The particle is denser than water. It is also part of a stream which is flowing, and the impulse it receives from gravity is greater than it would receive if it had the density of water. The component of gravity in the direction of flow, AB, acting on the excess of mass, draws the particle in the direction of flow. This component is proportional to the slope of the stream, which is a small fraction. At the same time the component of gravity normal to the direction of flow, AC, also draws the particle, which is equally free to move through the water in that direction. Its actual acceleration has the direction of their resultant, AG, which is vertically downward.

Dupuit's postulate was suggested and supported by the observed fact that a body floating down a stream moves faster than the visible current. Bérard demonstrated experimentally that the differential motion is due to the propulsion of the body by strands of current below the surface. See Annales des ponts et chaussées, 6th ser., vol. 12, pp. 830–835, 1886.

very finest actually move downward with reference to the surrounding water. From time to time they may touch the stream bed, but only to be lifted again by the next adequate rush of water.

Three ways are known in which the velocities of a stream are affected by the suspended load. In the first place, the load adds its mass to the mass of the stream, and as the stream's energy is proportional to the product of mass by slope, and the stream's velocity has its source in this energy, the addition tends to increase velocity. Second, the suspended particles are continually impelled downward by gravity. Also, as the strands of current containing them have curved courses, the particles are subject to tangential force, and because of their higher density this force is greater than the force simultaneously developed by the containing water, so that they are impelled through the water. Motion through the water, caused by these two forces, involves work; and this work is a direct consequence of suspension. The energy expended is potential energy, or energy of position, given to the particles by the flowing water, and its source is the energy of flow. So the work is the measure of the work of the stream in suspending the particles. It may therefore be regarded as a tax on the stream's energy, resulting in reduction of velocity. Third, the imperfect liquid constituted by the combination of water and débris is more viscous and therefore flows more slowly than the water alone. The solid particles do not partake of the internal shearing involved in the differential movements of the current, and by their rigidity they restrain the shearing of water in their immediate vicinity. Moreover, each particle is surrounded, through molecular forces, by a sphere or shell of influence which still further interferes with the freedom of water movement.

The relative importance of these factors varies with conditions, and no simple statement is possible because the influence of each factor follows a law peculiar to itself. The most important conditions affecting the influence of the mass of the load are discharge and slope, while for the work of suspension and for viscosity the important condition is the degree of comminution of the load.

The mass which the load contributes to the stream is equivalent, in relation to potential energy, to an increase of discharge, and its product by the stream's slope is proportional to potential energy in the same sense in which the stream's energy is proportional to the product of discharge and slope. In a series of experiments with loadless streams of water flowing in straight troughs, the mean velocity was found to vary approximately with the 0.25 power of the discharge and the 0.3 power of the slope. Under the particular conditions of these experiments

$$V_m = Q^{0.25}\, S^{0.3} \times \text{constant}.$$

Differentiating with reference to Q, we have

$$dV_m = dQ \times \frac{0.25\, S^{0.3}}{Q^{0.75}} \times \text{constant}.$$

The increment to Q being interpreted for present purposes as the suspended load, we see that the corresponding increment to mean velocity has a magnitude which varies directly but slowly with the slope and inversely but more rapidly with the discharge.

Each particle in suspension is drawn downward through the surrounding water by gravity. It is impelled through the water in an ever-changing direction by tangential force. The average speed of the resultant motions, referred to the surrounding water, is greater than the constant rate of descent the particle would acquire if sinking in still water. Therefore the work of suspension, measured by all the motions through the water, is greater than the work of simple subsidence, a quantity as to which much is known. The measure of a particle's work of subsidence per unit time is the product of its mass, less the mass of an equal volume of water, by its fall in unit time by the acceleration of gravity. If we call the mass of the particle M and its velocity of subsidence V_s, and assume its density to be 2.7, the measure of the work of subsidence is

$$\frac{1.7}{2.7} M V_s g = 0.63\, M V_s g$$

The coordinate measure of work for the stream's flow is the product of its mass by its fall in unit time by the acceleration of gravity, and the contribution which the particle, considered as a part of the stream, makes to the work of flow is therefore measured by the product of its mass by the fall of the stream in unit time by the acceleration of gravity. As the fall of the stream in unit time is equal to the mean velocity multiplied by the slope, the measure of the particle's work of flow is $Ms V_m g$. This is a measure of the particle's contribution to the stream's energy, while 0.63 $MV_s g$ is a coordinate measure of that factor of its draft on the stream's energy which depends on the direct action of gravity. The other factor of draft, the factor depending on tangential forces, varies with the violence of vertical movements and is of comparable importance only in the case of torrents. If we leave it out of account, the result of the comparison is that when 0.63 of the rate at which the particle is pulled through the water by gravity is greater than the rate at which it falls by reason of the general descent of the stream, its tax on the stream's energy is greater than its contribution thereto. Any allowance for the neglected factor would be equivalent to increasing the fraction 0.63.

To illustrate this relation by a concrete example: Mississippi River between Cairo and its mouth has, at flood stage, such velocity and slope that any suspended particle of silt which would sink in still water faster than half an inch a minute retards the current more through the work of suspension than it accelerates the current through the addition of its mass to the mass of the stream.

The velocity of subsidence has been elaborately studied. So far as river problems are concerned, it depends chiefly on the size of the suspended particles. For particles below a certain magnitude, which is controlled in part by impurities in solution, the velocity is zero. Between two critical diameters, which for quartz sand are about 0.02 and 0.5 millimeter [1] (0.00007 and 0.00016 foot) the velocity varies with the square of the diameter. Below the lower diameter the variation is more rapid, and above the upper it is less rapid, becoming for large particles as the square root of the diameter.

It follows that the consumption of energy involved in the suspension of the suspended load is an increasing function of the size of the particles into which the load is divided; in other words, it is a decreasing function of the degree of comminution. On the other hand, the contribution which it makes to energy by adding its mass to that of the water is independent of the degree of comminution.

The viscosity factor is not easily compared in a quantitative way with those just considered, but something may be said of the laws by which it is related to comminution. Attending first to that part which depends on interference with internal shearing of the water, let us conceive of a particle with center at C, figure 71, surrounded by water which is subjected to uniformly distributed shearing along planes parallel to $A_1AA_2B_2BB_1$, the direction of shearing being parallel to the line ACB. Conceive a right cylindroid figure tangent to the particle and parallel to AB, its bases being A_1HA_2G and B_1EB_2F. Motions being referred to C as origin, the cylindroid body of water would assume after a time, but for the presence of the particle, the form of the oblique cylindroid with bases $A_1H_1A_2G_1$ and $B_1E_1B_2F_1$.

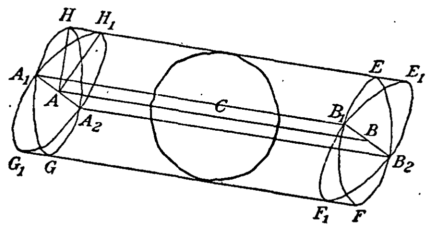


FIGURE 71.—Interference by suspended particle with freedom of shearing.

Because of the obstruction by the rigid particle the simple shearing motions thus indicated are replaced by other motions, components of which are normal to the shearing planes. It is assumed that the sum of the transverse elements of motion measures the action occasioned by the presence of the particle. The actual movements caused in the water doubtless affect regions within and without the cylindroid, but their nature need not be considered. The necessary transverse movements are equivalent to the transfer of a lunate wedge of water, $HA_1H_1A_2$ to a symmetric position on the opposite side of the plane $A_1A_2B_2B_1$ and a similar transfer of the wedge $FB_1F_1B_2$. Linear dimensions of the first-named wedge, in the direction of rectangular coordinates, are A_1A_2, HA, and HH_1. A_1A_2 equals a diameter of the particle. HA equals a semidiameter. As the angle HAH_1, being given by the general amount of shearing, is independent of the size of the particle, HH_1

[1] Richards, R. H., Textbook on ore dressing, pp. 262-268, 1909.

equals $AH \times \tan HAH_1$ and is proportional to a diameter of the particle. On the assumption that the diameters of the particle are equal it follows that the volume of the wedge is proportional to that of the particle, or to D^3. The distance of the center of gravity of the wedge from the plane $A_1A_2B_2B_1$, being a linear dimension of the wedge, is proportional to D, and the mean distance of transfer, which is double that distance, is also proportional to D. The quantity of motion normal to the shearing planes, occasioned by the particle, is measured by the wedge of water times the mean distance of transfer, and, the mass of the wedge being proportional to its volume, the quantity of motion is proportional to $D^3 \times D = D^4$. The quantity of motion occasioned by the entire suspended load, all its particles being assumed to have the same size, is proportional to ND^4, where N is the number of particles; and if the mass of the total load remain unchanged while its degree of comminution is varied, it is evident that N varies inversely with D^3. Therefore the quantity of internal motion occasioned by a particular mass of suspended matter of uniform grain is proportional (since $D^4/D^3 = D$) to the diameter of its particles. The fundamental assumption of the analysis is that this motion measures a resistance to the freedom of the water which is coordinate with viscosity and which may for practical purposes be considered as an addition to the resistance arising from the viscosity of the water.

That portion of the viscosity factor which depends on the molecular influence of the particle outside its boundary is still less susceptible of quantitative estimate but may yet be discussed with reference to the diameter of the suspended particle. In ignorance of the exact nature of the influence and also of the law by which it diminishes with distance outward from the boundary, I assume arbitrarily that at all distances from the particle less than l the freedom of water molecules is restricted, and that the amount of restriction is measured by the volume of the space in which the restriction is experienced. The imperfection of this assumption will of course affect any deduction from it. That volume is $\frac{1}{6}\pi\,(D+2l)^3 - \frac{1}{6}\,\pi\,D^3$. For the entire suspended load, assumed to consist of N equal particles, the total volume is N times as great. For a load of invariable weight but variable comminution, $N \propto \frac{1}{D^3}$, and the total volume is proportional to

$$\frac{\frac{1}{6}\pi\left((D+2l)^3 - D^3\right)}{D^3}$$

or, introducing a constant, K, and reducing, we have

$$\text{Volume} = K\left(\left(\frac{D+2l}{D}\right)^3 - 1\right)$$

Evidently the influence of this factor varies inversely with D. When D is very large as compared to $2l$, it approaches zero; when D is small as compared to $2l$, it is relatively very great. It is most sensitive to the control by D when the particles are very small.

The two divisions of the viscosity factor vary in their influence on velocity with the comminution of the load but in opposite ways, the influence of the first being greater as the particles are larger and that of the second as they are smaller. The laws of variation are such that their combination exhibits a minimum—that is, for some particular size of particle the influence on velocity is less than for particles either larger or smaller.

Another mode of treating the viscosity factor assumes that, so far as the viscosity effect is concerned, the molecular influence is equivalent to an enlargement of each particle to the extent of l on all sides. Then, reasoning as before with reference to interference with shearing, we obtain

$$\text{Resistance} \propto \frac{(D+2l)^4}{D^3}$$

This expression is not only simple but has the advantage of giving definite indication of the position of the minimum. The resistance is least when $D = 6l$.

We may now bring together the qualitative results of analysis, and write

$$V_{ms} = V_m + Wf_1(\acute{S}, \grave{Q}) - f_2(\acute{D}) - f_3(\check{D})$$

This may be read: The mean velocity (V_{ms}) of a stream carrying a load of suspended débris of diameter D equals the mean velocity (V_m) of the same stream when without load, plus a factor due to the weight (W) of the load and varying directly with the slope and inversely with the discharge, minus a factor due to the work of suspension and varying directly with the diameter of particles, minus a factor due to viscosity and varying inversely with the diameter if the particles are minute and directly with the diameter if they are larger.

It was convenient in the discussion to regard the whole suspended load as of one grade, but the result may be applied to any individual particle. For the actual stream with diversified load the equation might be written

$$V_{ms} = V_m + Wf_1(\acute{S}, \grave{Q}) - \Sigma f_2(\acute{D}) - \Sigma f_3(\grave{D}) \quad (111)$$

An investigation of the influence of suspended matter on viscosity has recently been made by Eugene C. Brigham and T. C. Durham.[1] The materials used by them were powders so fine that their tendency to settle in the water did not interfere with the conduct of the experiments. The work of suspension was therefore so small a factor as to be negligible. Various mixtures of water and powder were allowed to flow through a vertical capillary tube, impelled by their own weight, and the time of transmission for a determinate volume was noted. The time for a mixture containing 100 parts of water to 2.15 parts, by weight, of clay was found to be 15 per cent greater than the time for clear water; or the average velocity, within a slender tube, was reduced 15 per cent by the addition of the clay. If we assume the flow lines to have been parallel, as was probable, the theoretic increment of velocity due to the weight added by the clay was 1.6 per cent; if the flow lines were sinuous, the theoretic increment was less. The loss of velocity due to increased viscosity was therefore somewhat greater than 15 per cent and may have been as much as 16.6 per cent. Had the experiment been so arranged as to involve sinuous or turbulent flow, it would apply more cogently to the phenomena of rivers, but without that adjustment it serves to show that when the suspended material is exceedingly fine the loss of velocity through added viscosity is very much greater than the gain of velocity because of added weight.

The size of the particles was not reported, but the fact that the rate of settling was inappreciable suggests that their size may have fallen below that corresponding to a minimum influence on viscosity. Some light is thrown on this point by the results obtained with other fine powders. Interpolation from the reported data gives the following comparative estimates of the loss in velocity from the suspension of the same volumes of three substances: Infusorial earth, 3 per cent; graphite, 12 per cent; clay, 15 per cent. Of these the infusorial earth was coarsest, remaining in suspension largely because differing little in density from the water, while the graphite was a commercial variety said to be permanently suspended. An independent determination by L. J. Briggs and Arthur Campbell[2] gave a loss in velocity of 7 per cent, the material being a clay which "would not remain in a state of permanent suspension." The comparative data render it probable that the clay causing a retardation of 15 per cent was so finely divided as to give great effect to the molecular forces of the shell of influence.

The data from infusorial earth are useful in correlating the various factors which modify the stream's velocity. Assuming that the diatom tests composing the earth sample were similar to those figured in geologic textbooks, I have estimated the velocity of subsidence of particles having the same size but as dense as ordinary river silt and find that it corresponds to the critical velocity computed for the Mississippi at flood stage. That is, a suspended silt so fine as to have a large viscosity effect, so that a charge of 2.15 per cent reduces velocity by 3 per cent, is at the same time able, through its work of suspension, to consume all the energy it contributes to the current through its addition of mass. A finer silt would retard more by increasing viscosity, and a coarser silt would retard more through the work of suspension.

The available data are not fully demonstrative, but they render it highly probable that, under all conditions, streams are retarded by their suspended loads. If that be true, there is a capacity for suspension coordinate with capacity for traction. For each grade of suspended débris, and with any particular slope and discharge, it is possible by increasing the load so to retard the current that it is

[1] Am. Chem. Jour., vol. 46, pp. 278–297, 1911.

[2] Unpublished; communicated in letter.

barely competent to suspend débris of that grade, and the stream is then loaded to its full capacity.

In a natural stream the suspended particles are of many sizes, and all tend to reduce velocity. Some of the particles are near the limit of competence, and any increase of load will so reduce velocity that these can no longer be upheld. As to these coarsest particles the stream is loaded to full capacity, but not as to finer material. The addition of fine material will cause the arrest of some of the coarser, but will increase the total load. An exactly parallel statement may be made as to the tractional load.

To recur to the laboratory determinations of retardation by suspended material, it is of interest to note that the loads tested in the experiments, while greater than those ordinarily found in rivers, are representative of flood conditions in the more turbid streams. The load of 2.15 per cent, which caused retardations of 3 to 15 per cent, is equaled by ordinary floods of the Colorado of the West and is exceeded by ordinary floods of the Rio Grande and the Pecos. For the Rio Grande there are several records in the neighborhood of 10 per cent,[1] and small streams in arid lands are liable to receive similar loads as a result of violent local storms. On the other hand, the Mississippi near its mouth carries an average load of only 0.07 per cent, with a recorded maximum of 0.8 per cent.[2]

An attempt was made to measure the retardation of the current by tractional load. It was assumed that the bed resistance of a loaded stream has two parts, one due to the texture of the bed, the other to the work of traction, and that the reduction of velocity by traction could be measured by comparing speeds of loaded and unloaded streams on beds of the same texture and slope. A series of experiments were accordingly performed with unloaded streams flowing over beds composed of fixed grains of débris, or débris pavements, and it was thought that their mean velocities would be materially higher than those observed with streams otherwise similar but bearing loads. When the comparison was made, however, it was found that in 11 out of 16 experiments the observed velocities were higher with loaded than with unloaded streams; and the average of the 16 results was of the same tenor, ascribing a slight excess of velocity to the loaded streams. As the estimates of mean velocity had been based on observations of depth, and as the observations of depth were difficult in the case of loaded streams, it was thought possible that the result was affected by a systematic error in those depth measurements which had been made by means of the gage (p. 25). Examples were therefore sought in which the depth had been obtained by comparing the average height of the water surface during the run with the average height of the débris bed after the water had been drawn off. Thirteen instances were found in which such better-conditioned measures could be compared with measures of unloaded streams, and in 11 of the 13 comparisons the unloaded streams gave the higher mean velocities. The average of the 13 velocities found for loaded streams was 10 per cent less than the corresponding average for unloaded streams. The best of the available data, therefore, give evidence of the consumption of stream energy by traction, but the evidence is not so consistent as to free the matter from doubt. In Table 80 V_{ml} and V_{mu} represent the mean velocities of loaded and unloaded streams, and the various data are arranged according to the magnitude of the ratio of these velocities. This arrangement brings out the apparent fact that the reduction of mean velocity by traction is greater for small loads than for large, for gentle slopes than for steep, for low velocities than for high, and for large depths than for small. While it is entirely possible that such a result—a result opposed to my preconceptions—has been occasioned by a systematic error of observation, I am disposed to regard it rather as the expression of some physical law which has escaped my analysis. It is possibly connected with a fact brought out in the following chapter in the discussion of vertical velocity curves—the fact that addition of load has a pronounced influence on the distribution of velocities, increasing the contrast between velocities near the bed and the mean velocity.

If the variations of the load effect be ignored and attention given only to the means of quantities compiled in the table, it appears that 10

[1] U. S. Geol. Survey Water-Supply Paper 274, pp. 102–104, 1911.

[2] The average is on authority of Humphreys and Abbot (Physics and hydraulics of the Mississippi); the maximum is from an observation reported by J. A. Seddon (Rept. Chief Eng. U. S. A., 1887, p. 3094)—"231.9 grams per cubic foot."

per cent average reduction of velocity corresponds to an average load of 148.5 gm./sec. per cubic foot of discharge, or 0.52 per cent, by weight, of tractional load.

TABLE 80.—*Comparison of mean velocities of streams with and without tractional load.*

[Width, 1.00 foot.]

Grade of débris.	Discharge.	Slope.	Load.	Duty.	Depth of unloaded stream.	V_{ml}	V_{mu}	$\frac{V_{ml}}{V_{mu}}$
	Ft.³/sec.	*Per cent.*	*Gm./sec.*	*Gm./sec.*	*Foot.*	*Ft./sec.*	*Ft./sec.*	
(D)	0.734	0.37	21	28.6	0.410	1.79	2.43	0.74
(D)	.734	.32	12	16.4	.411	1.79	2.35	.76
(D)	.363	.43	12	33.1	.229	1.97	1.97	.80
(B)	.363	.43	21	58.0	.193	1.88	2.29	.82
(D)	.363	.58	23	63.4	.199	1.82	2.15	.85
(B)	.363	.54	25	69.0	.176	2.03	2.39	.85
(D)	.363	.35	12	33.1	.224	1.62	1.87	.87
(D)	.545	.53	30	55.0	.240	2.27	2.46	.92
(B)	.734	.38	38	51.7	.267	2.75	2.92	.94
(B)	.734	.48	52	70.8	.256	2.87	3.03	.95
(B)	.734	.54	80	109.0	.241	3.04	3.10	.98
(D)	.363	2.11	258	711.0	.098	3.70	3.37	1.10
(D)	.363	1.94	229	632.0	.099	3.66	3.30	1.11
	....	....	....	148.5	....	....	....	.90

THE TWO LOADS.

The reaction of tractional load on velocity affects primarily and chiefly the zone of saltation, but there is also a general retardation of the stream. The reaction of suspended load reduces all velocities, including those of the tractional zone. Thus the magnitude of each load affects capacity for the other load, and it also affects the conditions of competence for suspension and competence for traction.

As all parts of the load influence velocity, so all parts influence the general slope of an alluvial stream, which is automatically adjusted so as to give to the discharge the ability to transport all the material, coarse and fine, which is supplied. The adjustment is actually made through the coarser material, for the permanent deposits of the stream bed are from the tractional load.

Along with the adjustment of slope goes an automatic partition of the varied load into suspensional and tractional. If the lower Mississippi, for example, were to be supplied for the future with only that part of its load which is now carried in suspension, it would so reduce its slope that the slackened current would drop a portion of that load and thereafter move it by traction. If, on the other hand, the river were to be deprived of the fine débris now carried in suspension, it would so quicken its current as to lift into suspension a portion of the débris now carried by traction and would adjust its slope in such way as to maintain the partition of load.

An exception to the general law of automatic partition is found when the load has only small range in fineness, and this was illustrated by the artificial conditions of the laboratory; but it is not known that natural streams illustrate the exceptional case.

This phase of river adjustment is well illustrated by Yuba River, which I studied in 1904–1908. Where it issues from the mountains it carried a heavy load of coarse débris with which it was building up its bed at the edge of the Sacramento Valley. A dam thrown across it in the region of deposition arrested the tractional load for a time and gave an opportunity, by the aid of measurements, to estimate its amount. There were also samplings of the water and measurements of suspended load. The tractional load consisted mainly of gravel, with coarse sand and many bowlders, and the suspended load during flood included sand and finer débris. Fifteen miles below, near the mouth of the river, the tractional load consisted of sand, with rare small pebbles, and only clay and silt were in suspension. Here, too, the load suspended at flood stage was estimated from a sampling of the water. A computation based on the various data indicated that at flood stages the suspended load was approximately equal to the tractional load at each of the two localities.[1] While the data for this estimate were imperfect in many ways, they were nevertheless better than any other with which I am acquainted.

[1] A somewhat fuller statement may be found in Geol. Soc. America Bull., vol. 18, pp. 657-658, 1898.

Information as to suspended loads is fairly abundant, but there are no satisfactory data as to the complementary tractional loads.

The character of the partition of load depends in each particular instance on the relative proportions of the various grades in the débris with which the stream is supplied. It varies from point to point along the course of the same stream. In the case of Yuba River the variation was due to the fact that exceptional conditions created by hydraulic mining had thrown the bed profile out of adjustment; and many other streams are dealing with new and man-made conditions; but natural streams also have strongly contrasted load conditions in different parts of their courses. These arise, first, from the diversity of detritus furnished by tributaries, and, second, from the gradual comminution of the load as it is borne along.

The partition of load is greatly modified by variation of discharge and by temporary conditions of débris supply. A number of changes due to varying discharge have already been mentioned. The changes in velocity affect the grade of fineness marking the plane of partition. With reduction of discharge from a flood stage, the conditions of partition come to differ in pools and over shoals, and usually suspension ceases altogether, while traction is still continued on the shoals. With flood discharge, also, the partition is probably not quite the same for deep and shoal, a portion of the load traveling by suspension through the deeps but by traction over the shoals.

When a flood is occasioned by heavy rains the fine particles of soil are washed to the stream, with the result (1) that the suspended load is relatively large and (2) that the limiting grade is relatively fine. When an equal flood is caused by snow melting, without rain, the suspended load is smaller and the limiting grade coarser; in extreme cases there may be no suspension whatever. While a river is at low stage and without load, a local shower may wash to it a temporary supply of débris of which only the finer part will be immediately transported, giving suspension without traction.

The partition of load also varies greatly from point to point in the same cross section, competence for suspension and traction both responding to the control of velocity.

Despite these variable factors, it is quite possible that in the entire load of a stream large enough to be called a river there is a fairly definite ratio between suspension and traction. Knowledge of that ratio would be of practical importance to the engineer and geologist, because the measurement of the tractional load is always difficult, while measurement of the suspended load is merely a matter of routine and patience. The estimate afforded by the Yuba is a rough approximation and is qualified by the fact that the stream had not an established regimen but was engaged in adjusting its slope to new conditions of load.

Of other estimates of tractional load, the one most often quoted is that of Humphreys and Abbot. They found that the bar across one of the mouths of the Mississippi increased its width at the top at the rate of 338 feet a year and that the material added was similar to that of the tractional load. Assuming that the deposit had an outward extent to the depth of 100 feet and a transverse extent equal to the widths of all the mouths, they made a computation from which the tractional load was estimated at 11 per cent of the suspended load.[1]

An elaborate study of the loads of the Rhone was made by Adolphe Guérard,[2] who measured the suspended material by sampling the water for two years and computed the total output from the soundings of the sea bed, the survey of which had been repeated after an interval of 31 years. He found the suspended load less than one-fourth of the whole. Various partial estimates have been based on the march of subaqueous dunes. Where the volume of a dune and its rate of progression are known, their product determines definitely a part of the load, and if the data cover a year they yield at once a minimum estimate for the annual load. Some large dunes in various streams have been reported to persist from year to year, with progressive change of position, but Arthur Hider, who kept two tracts of the Mississippi channel under observation for about a year, found that the dunes were repeatedly readjusted in respect to size as the river stage changed, so that none could be identified through long periods, while the epochs of readjustment were characterized by general deposition or general scour, which could not be accounted for as a result of the

[1] Hydraulics of the Mississippi, p. 149.
[2] Inst. Civil Eng. Proc., vol. 82, pp. 308-310, 1885.

dune transformations.[1] A similar remodeling was observed by Partiot in the Loire.[2] Hider estimated the whole movement in dunes as 14,800 cubic yards in 24 hours, which corresponds to 2.2 per cent of the average suspended load reported by Humphreys and Abbot; but his judgment was that the entire movement of débris along the bed was at least ten times as great as the movement in dunes.

As a generalization from extensive observations of dunes in the river Loire, Sainjon formulates their rate of advance when the material is sand as a function of surface velocity, V_s. In his equation [3]

$$\text{Advance} = 0.00013\ (V_s^2 - 0.11)$$

the units are metric. Substituting feet for meters, we have

$$\text{Advance} = 0.00004\ (V_s^2 - 1.18)$$

The associated average height of dune crests [4] is 0.77 meter, or 2.54 feet. As the mean height of a dune is approximately one-half the height of its crest, we may multiply the rate of advance by 2.54/2 and obtain

$$0.00005\ (V_s^2 - 1.18)$$

as an expression for the Loire's load of sand carried in dunes, for each foot of channel width occupied by dunes. In a later discussion by Lechalas (see p. 193) it is shown that Sainjon's formula does not apply to surface velocities above 3.3 ft./sec., the rate of dune advance being then checked because part of the sand escapes into the body of the current and is not added to the downstream faces of the dunes; but the observations on dunes constitute the quantitative basis of the formulation of débris transportation by Lechalas, of which an account is given in Chapter X.

In this connection mention may be made of moving sand bodies of a different order of magnitude and probably of a different type. They are greater than the dunes of the same stream and are coordinate in size with the bars separating deeps but are distinguished from the bars by their migration downstream. In some of the "regularized" streams of Europe, where the main channel is artificially restricted to curves of large radius, they are developed in systematic alternation at the two sides, and the thalweg winds between them.[5] In the Mississippi they sometimes appear in the reaches. Their progress downstream is accomplished by deposition on forward slopes and erosion of rear slopes, but the forward slopes are not steep, like those of dunes, and their material is not wholly derived from the tractional load. Blasius [6] regards them as essentially dunes, correlating them specifically with dunes of reticulated pattern. My own view, not necessarily inconsistent with his, connects them with the fixed bars separating the deeps of a meandering stream. A free stream does not tolerate a straight channel. If a straight channel of moderate width be given to a stream, the current swings rhythmically to right and left, and if the banks yield it develops meanders. The meanders then migrate, according to laws of their own, and the bars are fixed in relation to the meanders. If the banks do not yield, the system of shoals and deeps established by the swinging current migrates slowly downstream. It is evident that the migration of these shoals is one of the factors—and may be an important factor—in the work of transportation; and also that every measurement of the migration of a shoal is a partial measurement of load.

Pilots of Mississippi steamboats observe that the bars at crossings are built up by floods, and such changes have been measured by engineers. The generalization has sometimes been made that deposition is a specific function of floods, but a more satisfactory interpretation is given by McMath,[7] who maintains that the rising river scours from the deeps to deposit on the shoals, and the falling river scours from the shoals to deposit in the deeps. The transfers are the joint work of traction and suspension. As such changes of the stream bed are measurable they afford quantitative data as to load, and it was from their observation that Hider, as previously quoted, inferred that the dune movement in the Mississippi includes but a small fraction of the tractional load.

[1] Mississippi River Comm. Rept. for 1882, pp. 80–88. Observations by W. H. Powless, made at a different place and in another year, are of the same tenor. See idem for 1881, pp. 66–120.

[2] Annales des ponts et chaussées, Mém., 5th ser., vol. 1, p. 270, 1871.

[3] Quoted, with some of the data, by Partiot, idem, pp. 271–273. The coefficient is there erroneously given as 0.0013.

[4] Given by Lechalas in the same volume, pp. 387–388.

[5] Engels, H., Zeitschr. Bauwesen, vol. 55, pp. 664–680, 1905.

[6] Idem, vol. 60, pp. 465–472, 1910.

[7] Mississippi River Comm. Rept. for 1881, p. 252.

The dune movement, the migrations of greater bars, and the transfers of débris from deep to shoal and shoal to deep are all competent to give information as to tractional load, but the estimates they give are minimum estimates, to be supplemented by estimates of the material which at flood stages is swept steadily along without contributing to any of the temporary deposits in such way as to be accessible to measurement.

AVAILABILITY OF LABORATORY RESULTS.

THE SLOPE FACTOR.

We are now ready to inquire whether, in view of the diversities and complexities affecting traction by natural streams, the formula for tractional capacity derived under the comparatively simple conditions of the laboratory is of practical value in connection with natural streams. The four factors of the formula may first be considered separately.

The general slope of a stream is the quotient of fall by distance, the distance being taken along the stream's course. It is best measured at high stage, because the chief work of grading the channel is accomplished by floods.

With reference to variations in capacity at a single locality, slope does not enter, the variations being referred to discharge; but account must be taken of slope in comparing different divisions of the same stream and in comparing one stream with another.

In all such cases the stream's slope is as definite a quantity and is susceptible of as precise measurement as is the slope of the laboratory channel. It differs as to its representative character. The laboratory slope is connected with a single discharge and a single grade of débris of determinable fineness. The slope of the natural stream does not represent the adjusting work of a determinable discharge but is a compromise product of the work of many discharges, and it is usually true that the velocities associated with these discharges have determined equally diverse mean finenesses of débris. The work of the natural stream, moreover, has been characterized by greater diversity, from point to point, of the bed velocities, and its system of velocities has been regulated in part by suspended load.

These difficulties would prove insuperable if attempt were made to infer the capacity of a natural stream from that of a laboratory stream, but they are not necessarily important in transferring a law of variation from a group of laboratory streams to an equally harmonious group of natural streams. If the diversification of discharges and finenesses is of the same type for the examples of natural streams between which comparison is made, it may well be that the slopes are comparable, one with another, in the same sense in which they are comparable in laboratory work, and that their relations to capacity should follow the same law.

THE DISCHARGE FACTOR.

Discharge differences must be considered when the tractional capacities of different streams are to be compared, and also in comparing the capacities of the same stream at different times.

In making comparison between different streams it is important that the discharges used be coordinate—that is, that they represent equivalent phases of stream work. If coordination be not secured, allowance must be made in one stream or the other for the variation of capacity with stage. In case the problem is such that the choice of phase is optional, preference should be given to flood phases, because the general slope and the details of channel shape are approximately adjusted to such phases. The greatest known discharge is probably less representative of the channel conditions than is the mean of annual maxima of discharge. It is believed that with use of discharges that are both representative and coordinate the discharge factor of the empiric formula may be applied. The result of such application will be the more satisfactory in proportion as the streams compared are allied in type and will be relatively unsatisfactory for streams in different climatic provinces or for comparison of a direct alluvial stream with one which meanders.

It is to be observed that in all studies of alluvial streams the discharge of which account should be taken in connection with traction is the discharge flowing in the channel proper. That which passes the banks ceases to contribute of its energy to the work of traction, and the portion of load diverted with it is not tractional.

The case of variation of discharge in the same stream is complicated by simultaneous variations of fineness and competence. In the experiments with sieve-separated grades of débris fineness and competence were constants with reference to discharge; but in a natural stream, where the tractional load may have great range in fineness, the mean fineness of the load varies with discharge, and the reason of its variation is that competence varies with discharge. The two competences which limit the range in fineness move up and down as discharge changes, and the mean fineness moves with them. Therefore the response of capacity to discharge can not be considered by itself.

For convenience in analyzing the conditions, let us assume first a discharge which is adjusted to the details of channel form, to the deeps and shoals. If, now, the discharge be increased, and with it the whole system of velocities, transportation will be everywhere stimulated, part of the tractional load will join the suspended, and the scouring of the deeps will bring into the tractional load a greater proportion of the coarser elements of the load. The mean fineness of that load will be reduced, and the capacity, while enlarged by increase of discharge, will be somewhat reduced by loss of fineness. The increase with discharge will be less than if the fineness were constant.

If, on the other hand, the discharge be reduced, some of the coarser material comes to rest, while finer débris is added from the suspended load. So the reduction of capacity from diminished discharge is qualified by the effect of increased fineness. But before the change in discharge has gone far the deeps become pockets for the reception of deposits, and traction is restricted to the intervening shoals, where it causes erosion. The erosion is selective, leaving an ever-increasing assemblage of residuary coarse material, which tends to protect the finer. The current on the shoals no longer obtains a full supply of the material for which it is competent, and the load and capacity part company. Or we may say that as the erosion of the shoals progresses the mean fineness of the accessible débris is reduced until a grade is reached for which the current is not competent. In either case the decadence of traction follows a law which is not well represented by the discharge term of the laboratory formula.

The above analysis postulates a wide range and somewhat equable distribution of fineness in the débris of the stream bed, a condition not always found. It might not apply, for example, to a stream which drains a district of friable sandstone and is therefore supplied with nothing coarser than sand. Nor would it apply well to a stream supplied with very coarse and very fine débris but not well supplied with intermediate grades.

In most alluvial streams, and probably in all meandering streams, the work of traction which is accomplished on the shoals at low stage and midstage is almost negligible in comparison with the high-stage traction. Not only is the rate of traction slow, but the field of traction is restricted. If a single formula will not fit both low and high stages, the one adjusted to high-stage variations will have the greater practical value.

Yet another consideration enters here, and one of peculiar importance. When discharge is reduced, and the competence of the current for traction is thereby changed, the coarse material eliminated from the tractional range ceases to be transported; but when discharge is increased, and the competence of the current for suspension is thereby changed, the fine material eliminated from the tractional range continues to be transported. It is, in fact, transported more rapidly, so that a greater amount passes a given section each second. For most or all practical purposes the change in mode of transportation is of no moment, and those purposes would be served by a formula which should include the material shifted and ignore the change in mode. In the system of reactions set up by change of discharge the two modes of transportation are so interwoven, in fact, that the practical discrimination of the suspended and tractional loads is impossible. Even in the laboratory experiments devised specially for the study of traction a certain amount of interplay was tolerated, for temporary suspension appeared over the crests of some of the antidunes and also in the bends of the crooked channels.

If the purely tractional point of view is to be exchanged for another, what shall be substituted? One natural suggestion is to include in a single view the entire load, suspended and tractional; another to include along with the tractional only that part of the suspended load which for part of the time is tractional also. That which would be included in one view and excluded from the other is the finer part of the suspended load, the part that does not sink to the bottom so long as the current is sufficiently active for traction. Being purely suspensional, its quantity is peculiarly a function of supply and is connected with discharge only through the association of discharge with rain. Wherever discharge is largely a matter of tribute from snowbanks, the suspended load is conspicuously independent of discharge.

If we exclude from view the purely suspensional material, a natural criterion for inclusion is the finest débris which low-stage discharge moves by traction, and as low-stage traction is limited to the bars, or interpool shoals, it is the finest tractional débris of those shoals. If we consider a gradual increase of discharge from least to greatest, we have at first no traction. Then for a particular discharge, which may be called the competent discharge, traction begins on the shoals, only the finest of the débris being moved. Gradually coarser and coarser material is included, the range in fineness and the load increasing together; but in this phase of action the load is not necessarily the equivalent of the capacity, for it may be limited by the supply of débris of requisite fineness. After a time another critical discharge is attained, which initiates the loaning of débris from traction to suspension, and thereafter a constantly increasing share of the traveling débris is suspended. As the suspended particles travel faster than the saltatory, and as capacity is the ability of the stream, measured in grams per second, to move débris past a sectional plane, the transfer from traction to suspension is an important factor in the enhancement of capacity.

The relation of capacity to discharge, contemplated from this viewpoint, has two elements in common with the discharge factor of the laboratory formula. It includes a competent discharge, corresponding to the zero of capacity, and it associates continuous increase of capacity with continuous increase of discharge. It differs, however, in important ways, and the possibility of expressing it by a definite formula is not evident. In the pool and rapid phase of activity the supply of débris suitable for traction is usually limited, and in many streams it is exhausted during each recurrence of the phase. In the phases of greater discharge, when traction occurs in the deeps as well as on the shoals, the sequence of capacities depends not only on discharge but on the relative proportions of débris of different grades of fineness in the material of the load. It is probable that for most streams the load-discharge function is discontinuous at the limit of the pool and rapid phase.

Because of this presumable discontinuity, because the tractional work while the pools exist accomplishes only a local transfer of débris, and because the work performed is usually of negligible amount in comparison with the work of larger discharges, it is probably better to ignore altogether the pool and rapid phase in any attempt at general formulation. If that be left out of account and if the general features of the laboratory formula be retained, the constant κ becomes the discharge which initiates traction in the deeps, and thus initiates *through* transportation of bottom load. If we accept that as a starting point, the material so fine as to be suspended by that discharge may be classed as purely suspensional, and other material suspended by larger discharges may be grouped with the tractional load. For the tractional load thus enhanced, or the *amplitractional* load, as it may conveniently be called, the rate of variation with discharge is evidently higher than the rates found for simple grades in the laboratory, and it may be much higher, for the débris diverted from traction to suspension, instead of lagging behind the lowest and slowest threads of the current, now speeds with the current's mean velocity.

It is possible that a practical formula for the fluctuations of an alluvial river's load may follow these lines, taking the form

$$C_a = b_3(Q - \kappa_1)^o \qquad (112)$$

where C_a is the capacity for amplitractional load, and κ_1 is the smallest discharge competent to establish a continuous train of traction through deeps and shoals; but the suggestion as to form has no better basis than analogy, and no data are known tending to determine the magnitude of the important parameter ***o***.

THE FINENESS FACTOR.

When the work of two natural streams is compared and the streams are of the same type, it is believed that the fineness factor of the laboratory formula is applicable. It is true that fineness enters in a relatively complex way into the determination of the loads of natural streams, but for the comparison indicated the elements of influence are severally represented by the experiments, and their totals should follow a law of the same type. For small discharges this inference is subject to certain qualifications, which will appear from what follows.

When the work of the same stream is compared under different discharges, a difference in fineness is developed under the laws of competence. With larger discharge the mean fineness is less than with small discharge, and the difference in fineness conspires with the difference in discharge to determine capacity. For reasons explained in the last section, however, capacity can not always be considered synonymous with load when the discharge is small.

THE FORM-RATIO FACTOR.

In the reaches of a direct alluvial stream there is approximate uniformity of depth at high stage, and the conditions involving form ratio are essentially like those realized in the laboratory. To such cases the principles developed in the laboratory studies should be applicable.

It is true in a general way, as already mentioned on page 223, that at a high stage of a natural stream the sectional area is about the same for the reaches as for the bends, and so too is the width. It follows that the mean depth is about the same, although the maximum depth may be very different. The high-stage capacity is also the same at every section, after the channel form has been adjusted to the discharge. If these generalizations are correct, the principle involved in the form-ratio factor of the laboratory formula is applicable to curving streams, provided form ratio is interpreted as the ratio of mean depth to width, and not as the ratio of maximum depth to width.

In the analysis of conditions determining the relation of capacity to form ratio (Chapter IV) an important rôle was ascribed to the resistance of the banks; and the quantity of that resistance was represented in one of the parameters of the formula, α. The optimum form ratio, ρ, was found to vary inversely with α and, therefore, to vary inversely with the resistance of the banks. The resistance afforded by river banks is greater than that given by the smooth walls of laboratory channels, and this element tends to make the optimum form ratio relatively small for rivers. Its influence, however, is overshadowed by those of slope and discharge. As the optimim ratio varies inversely with slope, and as most rivers have lower slopes than the experimental streams, the general tendency of the slope element is to make the ratio large for rivers. As the optimum ratio varies inversely with discharge, and as the discharges of natural streams are relatively large, the tendency of this element is to make the ratio small for natural streams. The rates of variation being unknown, the net result of the three influences can not be inferred deductively. The data from Yuba River, cited in Chapter IV (p. 135), show that for one case of a natural stream the optimum ratio is decidedly larger than that established by the stream in its alluvial phase and is of the order of magnitude of the determinations made in the laboratory.

THE FOUR FACTORS COLLECTIVELY.

The results of the preceding discussions admit of a certain amount of generalization. When different streams of the same type are compared, and especially when the type is alluvial, the law of their relative capacities at high stage may be expressed by the laboratory formula (109). The ability of that formula to express the variation of capacity with discharge in the same stream is problematic.

It has not been shown that the system of numerical parameters determined for laboratory conditions can be used in extending the application of the formula to natural streams. If the formula were rational, the result of an adequate mathematical treatment of the physical principles involved, the constants measured in the laboratory would be of universal application (with moderate qualification for the conditions imposed by the curvature of natural channels); but the constants of an empiric formula afford no basis for extensive extrapolation.

THE HYPOTHESIS OF SIMILAR STREAMS.

When the Berkeley experiments were planned it was assumed that the relations of capacity to various conditions would be found to be simple, and that the laboratory streams were representative of natural streams except as to the characters associated with bending channels. Because of the discovered complexity of the laws affecting capacity it is now apparent that the laboratory formulas can not be applied to streams in general. It is, however, probable that among the great variety of natural streams there is a more or less restricted group which is in such respect similar to the laboratory group that the empiric results of the laboratory—or at least the results embodied in exponents—may properly be applied to it.

The criteria of similarity between large and small have been discussed to some extent by others in connection with the investigation of hydraulic problems by means of models. William Froude inferred from theoretic considerations that if the speed of a ship and the speed of its miniature model "are proportional to the square roots of the dimensions, their resistances at those speeds will be as the cubes of their dimensions,"[1] and he afterward verified this result by experiments. T. A. Hearson, in projecting a model river for the investigation of various hydraulic problems, discussed separately the resistance to flow by the wetted perimeter, the influence of varying sectional area, and the influence of bends. He concluded that if the linear dimensions were kept in the same proportion, so that the river channel and its model were similar in the geometric sense, the velocities would be related as the square roots of the linear dimensions, and the discharges as the 2.5 powers of the linear dimensions. It would be necessary that the roughnesses of the channel surfaces have the same differences as the linear dimensions, and that the movable débris of the bed also follow the laws of linear dimensions.[2] His deductions were not tested by the construction and use of a model, but they derive a large measure of support from the verification of Froude's analogous theorem. So far as I am aware, all the models actually constructed to represent rivers and tidal basins have been given an exaggerated vertical scale.[3] O. Reynolds [4] made a series of models of tidal basins in which the scales of depth and of tidal amplitude were greater than the scale of length, the ratios ranging from 31:1 to 105:1. No adjustment was made as to size of débris, the requirements of his investigation being met by any material fine enough to be moved by the currents. A tidal oscillation was communicated to water resting on a level bed of sand, with the result that the bed was gradually molded into shapes more or less characteristic of estuaries. From general considerations a "law of kinetic similarity" was deduced:

$$\frac{p\sqrt{h}}{L} = \text{constant}$$

in which p is the tidal period, h the depth of water (proportional to the amplitude of the tide), and L the length of the estuary. Under this law the results were generally consistent, but there was found to be a limit to the range of suitable conditions, and this limit was formulated by

$$h^3 e = \text{constant}$$

in which e is the exaggeration of the vertical scale.

Eger, Dix, and Seifert,[5] making a model of a portion of Weser River for the purpose of studying the effect of projected improvements, adopted 1:100 as the scale of horizontal dimensions and depths, and 1:6.7 as the scale of mean diameters of débris particles composing the channel bed. It was then a matter, first of theory and computation but finally of trial, to select scales for discharge and slope. The main condition to be satisfied was that for discharges corresponding to high and low stages the depths of water should be properly related, according to the scale of linear dimensions. For the scale of discharges 1:40,000 was finally adopted, and for slopes 650:1. The resulting ratio of velocities was 1:4; and this ratio, combined with the ratio of débris sizes, was found to give a time ratio (for the accomplishment of similar changes in the bed of the stream) of 1:360. The scale of velocities being only 1:4 while the scale of distances was 1:100, there was an exaggeration of velocities in the ratio of 25:1.[6]

The quantities of débris moved being in the ratio of $1:100^3$, the distances moved in the

[1] These words are quoted from Inst. Naval Arch. Trans., vol. 15, p. 151, 1874. I have not seen Froude's original discussion of the subject.

[2] Inst. Civil Eng. Proc., vol. 146, pp. 216-222, 1900-1901.

[3] See Fargue, L., La forme du lit des rivières à fond mobile, pp. 57, 128, 1908. Fargue recommended for a model river a vertical scale of 1:100 and a horizontal scale of 1:20, from which he deduced a discharge ratio of 1:3,200 and a velocity ratio of 1:16.

[4] British Assoc. Adv. Sci. Repts. 1887, pp. 555-562; 1889, pp. 328-343; 1890, pp. 512-534; 1891, pp. 386-404.

[5] Zeitschr. Bauwesen, vol. 56, pp. 323-344, 1906.

[6] So stated by the authors. An allowance for the general principle that velocities are proportional to the square root of the hydraulic mean depth, and therefore to the square root of linear dimensions, would indicate 1:10 as the normal ratio of velocities, and give 2.5:1 as the exaggeration.

ratio of 1:100, and the times consumed in the ratio of 1:360, the ratio of loads (per second) was $1:\frac{100^4}{360}$ or 1:280,000. The results were satisfactory; it was found that the successive forms given to the river bed by variations of discharge were repeated in the model.

The exaggerations of the vertical scale by Reynolds and of the slope by Eger, Dix, and Seifert had the important effect of shortening the time necessary to produce the desired remodeling of the mobile bed. The absolute proportionality adopted by Froude and recommended by Hearson would have entailed a prohibitive consumption of time and might have added a serious complication in connection with the use of very fine débris.[1]

The similarity controlled by Reynold's law was a relation between the wave periods and dimensions of tidal basins and is not closely related to similarity in the control of tractional load. The similarities obtained in constructing the model of the Weser are more in point, because they involve an average rate of movement of débris; but they throw no light on the laws of variation of débris movement, which is the important matter in bridging the interval between our experiment streams and natural streams. After attempting to use various suggestions which came from the adjustments of the Weser model, I have returned to the principles of geometric similarity employed by Froude and Hearson.

Let us assume, as possible or plausible, that the principles developed in the laboratory, together with all parameters which are of the nature of ratios, are independent of the scale of operations and may be applied to streams of far greater magnitude, provided all linear factors are magnified in equal degree. If width and depth are enlarged in the same ratio, the form ratio is unchanged. If longitudinal distance and loss of head are enlarged in the same ratio, the slope is unchanged. If the dimensions of the transported particles are enlarged in the same ratio as the linear elements of channel, the linear coarseness is increased, or the linear fineness is reduced in that ratio.

The natural streams which may be considered as similar to the experimental streams constitute a class of moderate size. The form ratio for rivers ranges lower than for experimental streams, but there is some overlap. The smaller of the form ratios of the laboratory are representative of a considerable number of rivers. The slopes of rivers range lower than for laboratory streams, but here again there is overlap, and the natural streams which are similar in respect to slope are in general such as have coarse débris, so that there may be correspondence in that regard also. The similar natural streams to which hypothesis extends the laboratory results are those of large form ratio and steep slope, carrying coarse débris.

The primary difference between a large stream and a small one being one of discharge, and our general inquiry being directed to the valuation of capacity for traction, let us seek an expression for the relation of capacity to discharge when similar streams of different size are compared.

The laboratory data determine control of capacity by slope, discharge, fineness, and form ratio. In similar streams slope and form ratio are constant, and we need consider here only discharge and fineness. As we are comparing the laboratory streams as a group with similar natural streams, also taken as a group, it is advisable to employ a mode of formulation which lends itself to the use of averages, and the most convenient is that of the synthetic index. In

$$C \propto Q^{I_{3a}} F^{I_{4a}}$$

I_{3a} and I_{4a} are average values of the synthetic index and may be estimated, from data in Tables 32 and 43, as 1.32 and 0.77. Designating elements of the larger and smaller streams severally by subscripts $_{\prime\prime}$ and $_{\prime}$, we have, from the above,

$$\left.\begin{aligned} C_{\prime\prime} &\propto Q_{\prime\prime}^{1.32}\ F_{\prime\prime}^{0.77} \\ C_{\prime} &\propto Q_{\prime}^{1.32}\ F_{\prime}^{0.77} \\ \frac{C_{\prime\prime}}{C_{\prime}} &= \left(\frac{Q_{\prime\prime}}{Q_{\prime}}\right)^{1.32}\left(\frac{F_{\prime\prime}}{F_{\prime}}\right)^{0.77} \end{aligned}\right\} \quad (113)$$

If we designate by L the ratio between a linear dimension of the larger stream and the corresponding dimension of the smaller.

$$\frac{d_{\prime\prime}}{d_{\prime}} = \frac{w_{\prime\prime}}{w_{\prime}} = L, \text{ and } \frac{F_{\prime\prime}}{F_{\prime}} = \frac{1}{L}$$

Calling mean velocity V, bearing in mind that the hydraulic mean radius is a linear dimension of channel, and recalling that the Chezy formula

[1] Such considerations as these affected the selection of materials for the Berkeley experiments and prevented the employment of very gentle slopes.

makes V vary as the square root of the hydraulic radius, we have

$$\frac{V_{\prime\prime}}{V_{\prime}}=L^{0.5}$$

Then, since discharge is the product of width, depth, and velocity,

$$\frac{Q_{\prime\prime}}{Q_{\prime}}=\frac{w_{\prime\prime}}{w_{\prime}}\times\frac{d_{\prime\prime}}{d_{\prime}}\times\frac{V_{\prime\prime}}{V_{\prime}}=L\times L\times L^{0.5}=L^{2.5} \quad \text{(114)}$$

whence

$$\left(\frac{Q_{\prime\prime}}{Q_{\prime}}\right)^{-0.40}=\frac{1}{L}=\frac{F_{\prime\prime}}{F_{\prime}}$$

and

$$\left(\frac{F_{\prime\prime}}{F_{\prime}}\right)^{0.77}=\left(\frac{Q_{\prime\prime}}{Q_{\prime}}\right)^{-0.31}$$

Substituting this value of the fineness factor in (113), and reducing, we have

$$\frac{C_{\prime\prime}}{C_{\prime}}=\left(\frac{Q_{\prime\prime}}{Q_{\prime}}\right)^{1.01} \quad \text{(115)}$$

The result indicates that

$$\frac{C_{\prime\prime}}{C_{\prime}}=\frac{Q_{\prime\prime}}{Q_{\prime}}$$

whence

$$\frac{C_{\prime\prime}}{Q_{\prime\prime}}=\frac{C_{\prime}}{Q_{\prime}}$$

or, since $\frac{C}{Q}=U$, the tractional duty of water,

$$U_{\prime\prime}=U_{\prime}$$

That is, *for similar streams the tractional duty of water is the same.*

As the exponents connecting capacity with discharge, capacity with fineness, and mean velocity with hydraulic radius are all averages of low precision, the result is far from being so secure as might be inferred from equation (115). Its best support is really found in the plausibility of its conclusion. Our experience with a variety of physical laws makes it easy for us to believe that with suitable parity of conditions a unit of discharge will accomplish the same work as part of a large stream that it will accomplish as part of a small stream; and so the conclusion is plausible. The fact that an attempt to test the hypothesis of similar streams by combining it with experimental data has led to a plausible result is a fact favorable to the hypothesis.

Let us now assume the hypothetic law to be a real law and draw such inferences as may be warranted. The range of slopes which the experiments treat with confidence is from 0.5 to 3.0 per cent. Direct application is limited to streams having slopes within that range. By postulate,

$$\frac{D_{\prime\prime}}{D_{\prime}}=\frac{F_{\prime}}{F_{\prime\prime}}=L$$

Substituting in equation (114), we have

$$\frac{Q_{\prime\prime}}{Q_{\prime}}=\left(\frac{D_{\prime\prime}}{D_{\prime}}\right)^{2.5}$$

whence

$$\frac{Q_{\prime\prime}}{D_{\prime\prime}^{2.5}}=\frac{Q_{\prime}}{D_{\prime}^{2.5}}$$

That is to say, for similar streams, the ratio $\frac{Q}{D^{2.5}}$ may be regarded as constant. This relation affords a criterion for the discrimination of those natural streams which are similar to the laboratory streams, provided they are also similar in slope and form ratio. The following limiting values for $\frac{Q}{D^{2.5}}$ for different slopes are all estimated on the assumption of a form ratio of 0.05:

Slope (per cent).	Limiting values of $\frac{Q}{D^{2.5}}$
0.5	3,000,000–40,000,000
1.0	2,000,000–30,000,000
2.0	1,500,000–10,000,000
3.0	500,000– 4,000,000

The form ratio 0.05 is considerably below the average of the ratios developed in the laboratory, and it is also much above the average for alluvial rivers at flood stage. Any allowance which might be made for this discrepancy would have the effect of increasing the estimate of limiting values of the ratios of Q to $D^{2.5}$. Subject to this qualification the ratios indicate the types of natural streams which are "similar" to the laboratory streams and to which various laboratory results may be applied. The streams are in general either small creeks or else rivers transporting very coarse débris. As the slopes are determined by flood discharges, such discharges should be used in the classification.

For the streams thus classified as similar to laboratory streams the duty of water is of the same order of magnitude, and so are the rates of variation of duty with the several conditions of slope, discharge, and fineness. The rates of variation apply especially to comparisons of one stream with another. For the estimation of variation with discharge in the same stream something should be added to the laboratory rate to allow for the varying assistance which suspension gives to the work of traction (p. 234). For estimation of optimum form ratio something should be deducted from the laboratory indication to allow for the greater resistance of the channel walls.

SUMMARY.

Natural streams of alluvial type differ from the streams used in the laboratory in ways connected with the bendings of their courses and with variations of discharge. The differences affect forms of cross section, the distribution of velocities within the section, and the partition of the load between suspension and traction. The two portions of load are carried at the expense of the stream's energy, each reduces the velocity, and the reduction of velocity determines the limit of carrying power. The whole burden of the stream includes not only two divisions distinguished by mode of transportation but as many minor divisions as there are grades of débris, and the load carried of each grade reduces the capacity for all the grades.

These and other complexities make it difficult to apply the laboratory results to natural streams. It is probable that the forms of the laboratory formulas are applicable, with limitations, to the comparison of one stream with another, but the availability of the exponents is problematic. There are special difficulties in attempting to use the formulas for the comparison of capacities of the same stream at different stages, and in such comparisons the tractional load can not be considered by itself, because much material which is swept along the bed at lower stages is lifted by flood velocities into the body of the current.

It is thought that the laboratory formulas may be applied to natural streams which are geometrically similar to the laboratory streams—that is, to streams having the same slopes and form ratios and carrying débris of proportionate size. The class of streams to which the formulas apply by reason of similarity is necessarily restricted, being characterized in the main by high slopes and coarse débris. It can include few large streams.

CONCLUSION.

It was a primary purpose of the Berkeley investigation to determine for rivers the relation which the load swept along the bed bears to the more important factors of control. As a means to that end it was proposed to study the mode of propulsion and learn empirically the laws connecting its output with each factor of control taken separately. The review of results in the present chapter shows that the primary purpose was not accomplished. In the direction of the secondary purpose much more was achieved, and a body of definite information is contributed to the general subject of stream work. A valuable outcome is the knowledge that the output in tractional load is related to the controlling conditions in a highly complex manner, the law of control for each condition being qualified by all other conditions.

With the aid of the Berkeley experience it would be possible to avoid certain errors of method and arrange experiments which should yield more accurate measurements of the same general class—and it is natural that the experimenter should feel the desire to do his work over in a better way—but I am by no means sure that adequate advantage would reward a continuance of work on the same or closely related lines. The complex interactions could be given better numerical definition, but it may well be doubted whether their empiric definition would lead to their explanation. It is possible that the chasm between the laboratory and the river may be bridged only by an adequate theory, the work of the hydromechanist. It is possible also that it may be practically bridged by experiments which are more synthetic than ours, such experiments as may be made in the model rivers of certain German laboratories. (See p. 16.)

The practical applications for results from experiments in stream traction belong almost wholly to the field of river engineering. For the transportation of detritus and related materials by artificial currents, stream traction will rarely be used, because flume traction is more efficient. Our results in flume traction have therefore an immediate practical application, and as they were limited in range the advantage of extending them can hardly be questioned.

The report on the Berkeley investigation properly closes with this chapter, but there are several by-products which seem worthy of record. Some of them are presented in the following chapter, and others are contained in appendixes.

CHAPTER XIV.—PROBLEMS ASSOCIATED WITH RHYTHM.

RHYTHM IN STREAM TRANSPORTATION.

This chapter is concerned with certain problems upon which the Berkeley investigation touched, but which were not seriously attacked.

The low precision of the observations on stream traction, a precision characterized by an average error of about 11 per cent and an average probable error after adjustment between 2 and 3 per cent, had for its chief cause the failure to eliminate from the experiments the influence of rhythm. The slope of the water surface, the slope of the channel bed, and the load of débris transported were all subject without intermission to rhythmic fluctuations. If materially better observations of the same sort are to be made, this difficulty must be successfully dealt with, and the first step toward mastering it is to understand it. The removal of a difficulty, however, is neither the sole nor the most important result to be expected from the study of tractional rhythms. Underlying them are physical principles which are of importance in the dynamics of rivers, and their study constitutes one of the available lines of approach to the broader subject.

For their empiric study the general plan of the Berkeley apparatus is well adapted, but our experience indicates that certain details should be modified. The use of a long trough is advisable, with contraction at the outfall, and with delivery of the load to a settling tank beyond the outfall. The appliances and methods should be such as to secure uniformity in discharge, in character of débris, and in rate of feed.

The apparatus for regulating discharge, described on pages 20 and 257, was one of the most satisfactory parts of the Berkeley equipment. Its most important feature, as affecting precision, was the delivery of the water through an aperture under a considerable head.

Uniformity of débris can hardly be secured without the employment of an artificial, narrowly limited grade, and the available means of sorting is the sieve. It is to be observed, however, that after a grade has been separated by sieves it is still subject to sorting by current, the current recognizing differences of form and density which the sieves ignore. When débris that has once been handled by the stream is to be used a second time, remixing may be advisable.

None of the devices we employed to feed débris to the current achieved uniformity. Those which depended on the flow of wet débris through an aperture failed because the proportion of water could not be kept constant. The others depended on handwork and experienced the irregularity usual to handwork. An apparatus planned near the end of the experimental work, but never tried, is of such promise that its essential features are here described.

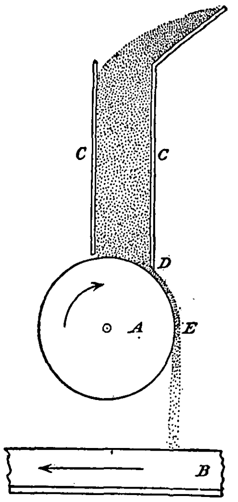


FIGURE 72.—Suggested apparatus for automatic feed of débris.

A drum, A in figure 72, is turned slowly by power, its rate being regulated by clockwork. Its position is above the trough containing the experiment stream, B. The surface of the drum is uniformly roughened. Above it is a vertical rectangular shaft, filled with moist débris. The shaft does not touch the drum. The width of the separating space at D is controlled by some suitable device. As the drum turns, a débris layer of uniform thickness is carried with it, and this falls into the stream. To prevent irregularities due to adhesion, a detaching device is placed at some point *E*, the device possibly consisting of an open-rank comb of elastic wire. The shaft should be smooth and of uniform section, so that the particles of débris near the drum may be brought into actual contact with one another by pressure of the débris above. That the débris may not be caused to flow by excess of moisture, it should be thoroughly drained before use. In a rough construction designed to test the practicability of the apparatus the drum surface was roughened by covering with a wire screen, and this was found to secure the delivery of the débris.

Uniformity of feed having been provided, the rhythms of transportation may be observed as oscillations in the débris delivered at the outfall. Rhymths of slope may be studied, at least initially, by observing changes in the profile of the water surface.

In the record of the Berkeley observations it is not practicable fully to discriminate rhythmic inequalities from those occasioned by irregularities of débris feeding, but there is reason to believe that several rhythms of different period coexist. The shortest rhythms are those connected with the dunes and antidunes, and these are evidently associated with rhythms of the flow of water.

RHYTHM IN THE FLOW OF WATER.

Reynolds,[1] treating of the flow of water through tubes, distinguishes two modes of flow as *direct* and *sinuous*. They are otherwise called steady and turbulent. In direct flow the filaments of current have simple lines, which are straight and parallel if the walls of the conduit are straight and parallel. In sinuous flow the filaments of current are neither simple nor parallel and may be intricately convoluted. The flow in tubes is direct for low velocities and sinuous for high, the critical velocity varying inversely with the diameter and the roughness of the tube and directly with the viscosity of the water. It would follow from his generalizations that the flow of such streams as were used in our experiments would be sinuous, and its actual sinuosity was a matter of observation. To a large extent the curvature of the flow lines was shown by the motions of minute suspended particles, and when this evidence was lacking it was still possible to infer diversity of current from continual changes in the configuration of the water surface.

It is probable that all the diversities of flow were rhythmic, for that is the nature of inequalities of motion developed by the interaction of constant forces; but in many cases the rhythms were so numerous and so related in period and other characters that their combination gave the impression of irregularity. In many other cases, however, some one rhythm was dominant, and these cases appear especially worthy of study.

The cycle of movements constituting a rhythm unit may be definitely related to space, or to time, or to both space and time. The pulsations of the water surface which obstructed our observations of water profile were manifestations of time factors. The regular sequence of dune crests was a manifestation of a space factor. The two phenomena coexisted. In the space interval from dune crest to dune crest certain elements of motion were constant. There was a large stationary vortex in the hollow between the crests (fig. 10, p. 31), and there might be stationary elements in the configuration of the water surface. At the same time the flow lines of the water at all points swayed in direction and were affected by variations of velocity, and these changes were rhythmic with respect to time. Doubtless many of the fluctuations belonged to or were associated with vortices which traveled with the general current. It is conceivable, or even probable, that the stationary features and the traveling features were coordinated, so that within the rhythmic space unit corresponding to the dune interval there was a cycle of variations of motion which was rhythmic with respect to time.

The rhythmic dunes marched slowly down the trough and with them marched the intercrest vortices and associated motions. To that extent the features of the bed controlled the features of the current. Nevertheless the dunes were not essential to the existence of a water rhythm characterized by a definite space interval. There was positive evidence that the dune

[1] Reynolds, Osborne, An experimental investigation of the circumstances which determine whether the motion of water shall be direct or sinuous and of the law of resistance in parallel channels: Roy. Soc. Philos. Trans., London, vol. 174, pp. 935-982, 1883. Also, The two manners of motion of water: Roy. Inst. Great Britain Proc., 1884.

interval was determined by a preexistent water rhythm. Not only was the dune interval a function of depth and velocity of current,[1] but the creation of dunes by water rhythm was repeatedly observed. In certain experiments a slow current, moving over a bed of débris artificially smoothed and leveled, was gradually quickened until transportation began. The movement of particles did not begin at the same time all along the bed but was initiated in a series of spots separated by uniform intervals, and the first result of the transportation was the creation of a system of dunes.

In certain experiments on flume traction a slow current, moving over a smooth channel bed of wood, swept along a small quantity of sand. With increase of the load of sand local deposits were induced, which took the form of thin straggling patches, similar to one another in outline and separated by approximately equal bare spaces. These moved slowly downstream, the mode of progress being similar to that of dunes, and with further increase of load they acquired the typical profile of dunes.

In both groups of experiments it was evident that the primary rhythm pertained to the water and was independent of the work of transportation. In the second group, where sand swept steadily forward over the whole area of the bed, it was evident that the water rhythm did not involve reversed currents along the bed and therefore did not include such stationary vortices as accompany dunes.

In the experiments on stream traction the development of dunes was conditioned by the three factors of velocity, depth, and load, besides an undetermined influence from fineness of débris.[2] It is probable that load and fineness enter only as factors of resistance, so that the essential conditions are velocity, depth, and bed resistance. Within certain rather wide ranges of value for these controlling factors the bed is molded into dunes. When the limit is exceeded by increase of velocity or resistance, or by decrease of depth, the dunes disappear and the bed becomes smooth and plane. At the same time the oscillations and other disturbances of the water surface are reduced; but as they do not altogether disappear it is to be inferred that the flow is still characterized by internal diversity.

With still further increase of velocity or resistance, or with further reduction of depth, another critical point is passed, and the process of traction becomes again rhythmic, but in an antithetic way. The bed is molded into antidunes, which travel upstream (pp. 31–34), and the water surface also is molded into waves, which copy the forms of the antidunes and move with them. The internal movement of the water is again characterized by a dominant rhythm, but the type of rhythm is different from that associated with dunes. The rhythm is also less stable, and its intensity exhibits a cycle of change. With low intensity the waves are nearly equal in height and length, but sooner or later inequalities develop and the higher waves overtake the lower and absorb them. This process increases the wave length in the upstream part of the trough, and the influence of the change is in some way communicated to the downstream end, where the waves are first formed, with the result that larger and larger waves develop. Finally a master wave, with curling crest, rushes through the trough from end to end, and this has the effect of wiping out the irregularities and restoring the status of low intensity. Various phases of the cycle are illustrated by the profiles in figure 12 (p. 33).

When combinations of velocity, resistance, and depth similar to those causing antidunes are made for a stream flowing through a rigid straight channel, without movable débris, the water develops surface waves, and these travel

[1] Measurements of dune interval in the laboratory were too few to demonstrate the factors of control, but comparison of laboratory data with data from other sources leaves little question that control is exercised by depth, velocity, and fineness. In the laboratory, where depths were a matter of inches, the dune interval rarely exceeded 2 feet. In Mississippi River depths measured in scores of feet are associated with intervals measured in hundreds of feet. In tidal estuaries, where dunes of a special type are exposed at low tide, the depths of the formative currents are intermediate between those of the laboratory and those of the Mississippi, while the intervals are measured in feet or tens of feet. For data of the Mississippi see Johnson, J. B., Rept. Chief Eng. U. S. A., 1879, pp. 1963–1967, and Eng. News, 1885, pp. 68–71; Powless, W. H., Mississippi River Comm. Rept. for 1881, pp. 66–120; and especially Hider, Arthur, Mississippi River Comm. Rept. for 1882, pp. 83–88. For data on the dunes of tidal estuaries see Cornish, Vaughan, Geog. Jour., vol. 18, pp. 170–202, 1901. Hider finds that the dune interval is greater at high stages of the river than at low, the depth and velocity both decreasing with the change from high to low. Cornish's observations show that the interval varies directly with depth of current under conditions which make it probable that the velocity varies inversely with depth.

[2] Eger, Dix, and Seifert (Zeitschr. Bauwesen, vol. 56, pp. 325–328) found that under certain conditions dunes were developed in a sand of uniform grade but not in a finer sand composed of several grades. In our experiments the smooth phase of transportation had greater range with mixed grades than with single grades.

downstream faster than the current. They are initially rhythmic, but their period is unstable because their velocity of propagation varies with their size. A wave with slight advantage in size will overtake the one in front of it, and then the two will unite, making a wave with still higher velocity. Thus the system tends, for a time at least, toward reduction of the number of waves and increase of the wave interval. As the waves grow by composition, their fronts steepen and the culminating phase is that of a "roll wave," or bore.[1] It may be noted, in passing, that the pulsations frequently observed in the overfall of a dam are probably rhythms of this type.

It may be assumed, at least tentatively, that all the dominant rhythms observed in straight conduits are initiated at the intake. Those associated with low velocity and an immobile bed appear to be stationary, but with a mobile bed they develop dunes, and they then travel downstream with the dunes. Those associated with high velocity and an immobile bed develop waves of translation which travel downstream. With a mobile bed antidunes are developed, and surface waves travel with them upstream. It is possible that the antidune waves coexist with the waves of translation, and that the cycle of intensity in the antidune phenomena results from the interactions of the two systems.

If there is warrant for the various correlations above indicated, the phenomena developed by our experiments in connection with modes of débris transportation afford a basis of classification for a considerable body of water rhythms. What we have called the smooth phase of traction marks a critical phase of water flow separating two types which are characterized by dominant rhythms, and the two dominant rhythms are in some way antithetic. The nature of their antithesis is not known to me, nor is the character of either rhythm; but it appears that the experimental and analytic study of the rhythms constitutes a field of research which is at the same time promising and important. I assume that the definite rhythms of water by which, when débris is present, the dunes and antidunes are caused are susceptible of analytic treatment, and I believe that the experiment trough affords the means of preliminary delineation and ultimate verification.

[1] See Cornish, Vaughan, Progressive waves in rivers: Geog. Jour. (London), vol. 29, pp. 23-31, 1907.

THE VERTICAL VELOCITY CURVE.

No systematic study was made of the distribution of velocities within the streams of the laboratory, but incidentally, in connection with several different minor inquiries, the vertical distribution of velocities in the medial plane was recorded; and it happens that some of these records serve to illustrate the dependence of that distribution—the vertical velocity curve—on certain conditions. The observations were made with the Pitot-Darcy gage.

The first to be reported were made in connection with a comparison of the free outfall (fig. 1, p. 19) and the contracted outfall (fig. 3, p. 25). The trough used with free outfall had smooth sides and bed, was 1 foot wide, and was set level. The stream of water, 0.734 ft.³/sec., carried no débris. Its surface slope gradually increased toward the outfall, and the mean velocity increased with the fall of the surface. Velocities were measured at three points, respectively, 5.5 feet, 2.3 feet, and 0.8 foot from the outfall. Their curves, given in figure 73, show that the acceleration as well as the velocity increased as the outfall was approached, and that there was a coordinate modification of the shape of the curve. In the first curve it is evident, despite a discordance among the determined points, that the level of maximum velocity is somewhat above mid-depth; in the second the maximum is near mid-depth; and in the third it is below. The acceleration, for which an integrated expression appears in the horizontal space between curves,

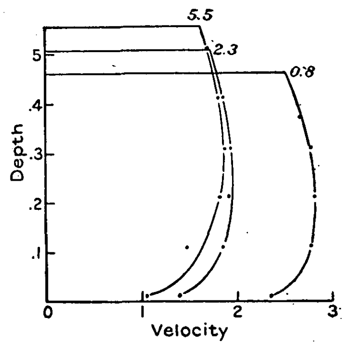


FIGURE 73.—Modification of vertical velocity curve by approach to outfall.

seems to have affected the lower part of the current more than the upper.

The trough used with contracted outfall was 0.92 foot wide and was contracted, in the manner shown in figure 3 (p. 25) to 0.30 foot. The oblique walls producing the contraction

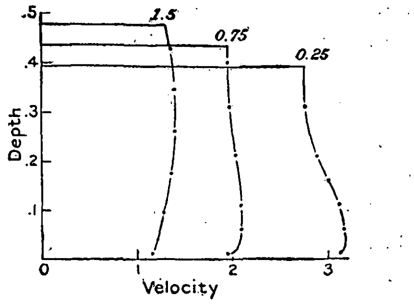


FIGURE 74.—Modification of vertical velocity curve by approach to contracted outfall.

were 3 feet long. The first velocity station was 1.5 feet from the outfall, at a point where the width was 0.64 foot; the second and third were at 0.75 and 0.25 foot from the outfall, corresponding to widths of 0.47 and 0.35 foot. The velocity curves, given in figure 74, show the descent of the plane of maximum velocity from a position slightly above middepth to

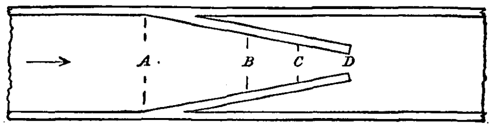


FIGURE 75.—Plan of experiment trough with local contraction. The letters show stations at which velocities were observed, and are repeated in figures 76 and 77. Scale, 1 inch=2 feet.

near the bottom. The acceleration, in this case connected with contraction of channel, as well as with the release at outfall from the channel resistance, is a function of depth.

In the third arrangement a trough 0.92 foot wide, with a slope of 0.58 per cent, was contracted at one point in the manner indicated in figure 75. This gave to the stream such a

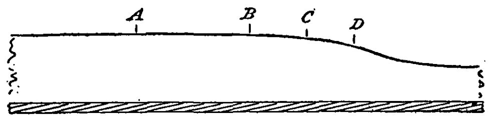


FIGURE 76.—Profile of water surface in trough shown in figure 75. Scale 1 inch=2 feet.

profile as is sketched in figure 76. Velocities were measured at four points, the distances from the point of extreme contraction being 2.0 feet, 1.0 foot, 0.5 foot, and 0, and the corresponding widths of current 0.92, 0.55, 0.37, and 0.19 foot. The velocity curves are plotted in figure 77. The level of maximum velocity is at or near the surface at the point of initial contraction, *A*, and then drops quickly, being below 0.9 depth in the narrowest strait, *D*. In this case also the acceleration appears to increase in regular manner with distance from the surface.

To avoid the peculiarities observed near the outfall, as well as those appropriate to intake

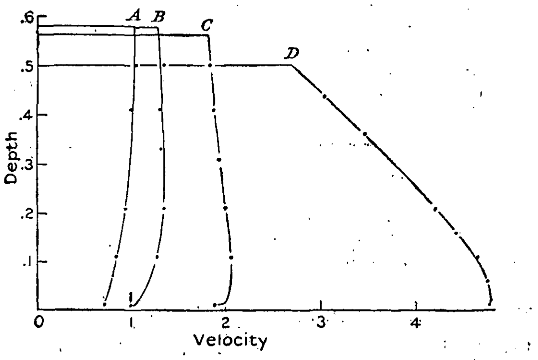


FIGURE 77.—Modification of vertical velocity curve by local contraction of channel. The letters indicate positions of velocity stations in trough. (See fig. 75.)

conditions, all other determinations of the curve were made near midlength of the trough, the ordinary distance from the outfall being 17 feet and the least distance 13 feet.

In figure 78 are three curves illustrating the influence of slope of channel. The trough was 1.96 feet wide and its sides and bed were smooth; the discharge was 1.119 ft.^{3}/sec. Curve *A* shows velocities 17 feet from the out-

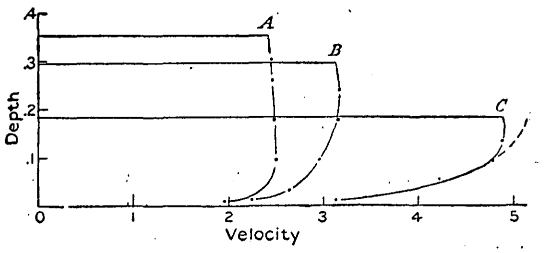


FIGURE 78.—Modification of vertical velocity curve when mean velocity is increased by change of slope.

fall, with the trough level; *B* corresponds to a slope of 0.26 per cent and *C* to 0.56 per cent. With the trough level, maximum velocity occurs at about 0.75 depth, and with both inclinations of trough its position is indicated at from 0.2 depth to 0.3 depth. As curve *C* differs in type from all others obtained, and as it may be made harmonious by rejecting a single observation, it is thought probable that the dotted line better expresses the fact. Its substitution throws the maximum to the surface of the water and makes a consistent series of the three curves.

In another series of observations the trough remained horizontal while the discharge was

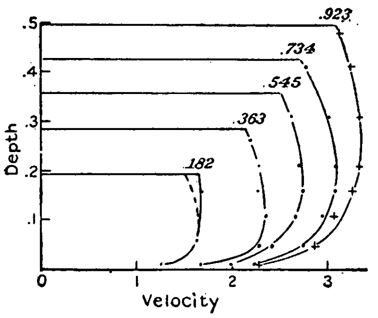


FIGURE 79.—Modification of vertical velocity curve when mean velocity is increased by change of discharge. The corresponding discharges are indicated in ft.³/sec.

varied. The width was 0.66 foot, and the discharges 0.182, 0.363, 0.545, 0.734, and 0.923 ft.³/sec. The resulting curves (fig. 79), with exception of that for the smallest discharge, form a consistent series, the level of maximum velocity rising slowly with increase of discharge. In this case also the discordant curve may be brought into harmony by the rejection of a

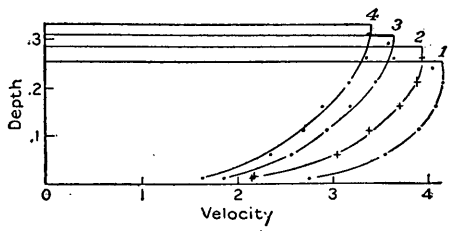


FIGURE 80.—Modification of vertical velocity curve by changes in the roughness of the channel bed. Curve 1, paraffin; 2, roughness of débris grade (A); 3, roughness of grade (D); 4, roughness of grade (F).

single observation, and a substitute curve is suggested in the diagram by a broken line.[1]

In another series the texture of the channel bed was made to vary. Constant features were the discharge, 0.734 ft.³/sec.; the width, 0.92 foot; the slope, 0.58 per cent; and the texture of the channel sides, which were planed and unpainted. The varieties of bed texture were (1) paraffin, coating a smooth board; (2) a pavement of débris of grade (A); and (3 and 4) similar pavements with grades (D) and (F). The observed curves, given in figure 80, indicate that the resistance occasioned by a rough bed retards the whole current but retards the lower parts in greater degree than the upper, so that the level of maximum velocity is raised. The amount of retardation is greater as the texture of the bed is coarser.

In figure 81 are two curves illustrating the influence of load on the vertical distribution of velocity. One curve was observed in a stream without load, flowing over a smooth and horizontal bed, the other in the same stream when carrying a load of 17 gm./sec. on a self-adjusted

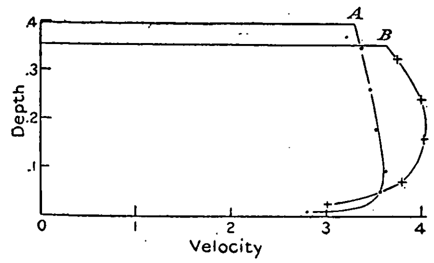


FIGURE 81.—Modification of vertical velocity curve by addition of load to stream, with corresponding increase of slope. *A*, without load; *B* with load.

slope of 0.64 per cent. To judge from the data shown in figure 78 (p. 245), the effect of slope alone would be to double or nearly double the mean velocity, but the actual increase was only 13 per cent. The acceleration due to slope was almost wholly neutralized by the retardation due to roughness of bed and to the work of traction. The retardation had also the effect of raising the level of maximum velocity.

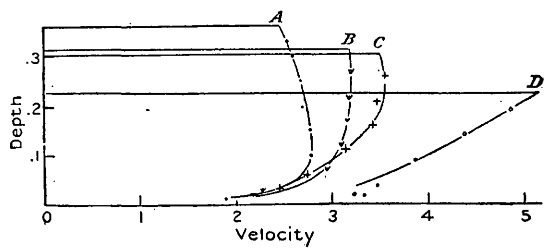


FIGURE 82.—Modification of vertical velocity curve by addition and progressive increase of load.

Similar contrasts are shown in figure 82, where the curve for an unloaded stream is compared with curves for the same stream when transporting three different loads. The constant factors in this case are: Width of channel,

[1] In this instance, in the one before mentioned, and also in the case of three curves in figure 80 the aberrant point records a measurement which was made very near the surface of the water. In such positions the constant of the Pitot-Darcy gage has a special value, and it is on the whole probable that the apparent errors of observation are occasioned by an error of the rating formula. The matter is discussed in Appendix A.

0.66 foot; discharge, 0.545 ft.^{3}/sec.; grade of débris, (C). The variables are as follows:

Load..........gm./sec..	0	38	53	194
Slope of bed...per cent..	0	0.38	0.56	1.14
Depth at 17 feet from outfall..............feet..	0.353	0.313	0.303	0.226
Mean velocity...ft./sec..	2.57	2.64	2.73	3.66
Level of maximum velocity; measured from the surface as a fraction of depth..................	0.7	0.1	0	0
Letter indicating curve in figure 82...............	*A*	*B*	*C*	*D*

With the slope of 0.38 per cent the sand moved in dunes; with 0.56 per cent the phase of traction was transitional between the dune and the smooth; with 1.14 it was transitional between the smooth and the antidune. The conspicuous change associated with the addition of load is the raising of the level of maximum velocity, and this is correlated also with increase of slope, increase of mean velocity, decrease of depth, and modification of the mode of traction.

The plotted points for velocities near the channel bed are irregular when the observations were made above a bed of loose débris, and little use has been made of them in drawing the curves. As previously mentioned, the presence of the gage caused a deflection of the lines of flow and the formation of a hollow in the bed. Not only was it impossible to observe with accuracy the relation of the instrument to the normal position of the bed, but the velocity observed was higher than that normally associated with the depth at which the instrument was placed. (See Appendix A.)

In most of the groups of curves the variations of form are associated with simultaneous variations of so many conditions that the nature of the control is not evident. For satisfactory interpretation a fuller series of observations seems to be required, but certain inferences may be drawn from those before us.

Many of the peculiarities of form are connected with the position of the level of maximum velocity. The movements of the maximum in relation to depth of current are of two kinds. It rises with increase of depth when that increase is caused by increase of discharge (fig. 79). It falls with increase of depth when that increase is independent of discharge (figs. 78, 81, and 82). Apparently depth, considered by itself, is not a factor of control. The maximum rises with increase of mean velocity when that increase is due to increase of discharge (fig. 79), or of slope (fig. 78), but falls with increase of mean velocity when the increase is due to lessened resistance of the channel bed (figs. 73, 80, 81, and 82). Apparently mean velocity, considered by itself, is not a factor of control. If we give attention to the three factors on which depth and mean velocity chiefly depend—namely, discharge, slope, and bed resistance—a more consistent relation is found. Variations of discharge affect only the group of curves in figure 79, and there the maximum rises with increase of discharge. Slope affects the groups in figures 78, 81, and 82, and in each case the maximum rises with increase of slope. Bed resistance affects the groups in figures 80, 81, and 82, and in each case the maximum rises with increase of resistance.

The lowering of the level of maximum velocity as the point of outfall is approached (fig. 73) is a harmonious feature, but in that case there is substituted for progressive reduction of bed resistance an abrupt cessation of all channel resistance. The resulting acceleration is propagated upstream, and its amount has a vertical distribution connected with pressure. In the case of contracted outfall (fig. 74), there is added an effect of convergence, which still further illustrates the graduation of acceleration in relation to pressure. The influence of contraction is important in other connections, but need not be further discussed in this place.

The influence of outfall may extend to a considerable number of the curves here figured. In most of the experiments made without contraction at outfall there was progressive decrease of depth and increase of mean velocity, from some point near the head of the trough to its end. This was most marked when the bed of the trough was horizontal (figs. 73, 78*A*, 79, 81*A*, and 82*A*). Reasoning from the observed fact that acceleration increases with depth, I think it probable that under such conditions the level of maximum velocity lies lower than it would with a uniform mean velocity.

Returning to the consideration of discharge, slope, and resistance, we may note that the variable resistance with which variations of the curve have been definitely correlated is bed resistance. In all the experiments the channel sides had the same texture, so that the side resistance was approximately proportional to the depth. The level of maximum velocity thus has the same relation to side resistance as to depth; it sometimes rises and sometimes falls when side resistance is increased. While a probability exists that side resistance influences the position of the maximum, the nature of its influence is not shown by the observations under consideration.

Discharge and slope, or the energy factors which they help to measure, urge the water forward, and their influence is applied to the whole stream. Bed resistance holds the stream back but is applied to its base only. The *obvious* tendency of these forces is to make the upper part of the stream move faster than the lower and produce a velocity curve with maximum at the water surface. This tendency is opposed by some other factor, unknown, which tends to depress the level of the maximum. Whatever that other factor may be, it loses in relative importance when discharge, slope, or bed resistance is increased.

A noteworthy feature of the curves is a tendency to change in character near the bed. The observations are not so precise nor so full as to afford a distinct characterization of the change, but there can be little question of its existence. It would appear that the peculiar conditions near the bed give great local importance to some factor of velocity control which is elsewhere of minor importance. Attention may also be directed to the fact that none of the curves resembles an ordinary parabola with horizontal axis. Had these been the vertical velocity curves to which mathematical formulas were first fitted, the equation of the parabola would not have been used.

The statement, above, that the factor tending to depress the level of maximum velocity below the water surface is unknown is perhaps rash, for several theories as to its nature have been advanced with confidence. To put the matter more cautiously—it has seemed to me that each theory of which I have read was effectually disposed of by the discussion which it aroused. However that may be, there is certainly room for another suggestion, and this I proceed to offer.

Reynolds[1] arranged an experiment in which a liquid was made to flow over another denser liquid, the two being immiscible. Below a certain velocity the surface of contact was smooth, but above the critical velocity the surface was occupied by a system of equal waves, which moved in the direction of flow but more slowly. With miscible liquids, or with two bodies of identical liquid, the waves are replaced by vortices. Under some conditions the vortices are as regularly spaced as the waves, but usually they are less regular, and various complications arise. The development of such vortices may readily be watched on a river surface wherever adjacent parts move with quite different velocities or move in opposite directions. Such vortices have vertical axes, and their direction of rotation is determined by the differential motion of the adjacent currents. If we conceive the water of a vortex as a body between parallel and opposed currents, then its direction of rotation is due to a mechanical couple contributed by the currents.

Transferring attention to the longitudinal vertical section of a stream, we find a couple of

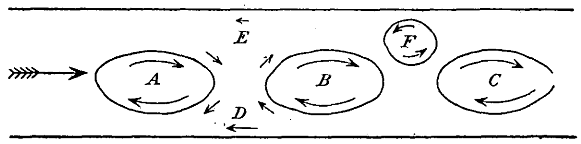


FIGURE 83.—Ideal longitudinal section of a stream, illustrating hypothesis to account for the subsurface position of the level of maximum velocity.

which one element is the general forward movement of the current and the other is the bed resistance. These tend to produce and maintain vortices with horizontal axes transverse to the channel and with forward rotation—that is, with the rotation of a wheel rolling forward in the direction of flow. To visualize these features, figure 83 gives an ideal section of a stream, with flow from left to right, and the ovals *A*, *B*, *C* represent a system of forward-rolling vortices. The arrows within an oval show direction of rotation, and it is important to recognize that the motions they indicate are referred to the center of the vortex, or to the vortex as a whole, and not to the fixed bed of the channel. With reference to the bed all parts of the vortex are moving toward the right, the lower part merely moving slower than the upper.

The tendency of vortices toward circular forms leaves certain tracts of the section unoccupied by the system of vortices. Consider the tract *D*, bounded below by the bed and above

[1] Roy. Soc. London Philos. Trans., vol. 174, pp. 943–944, 1883.

by parts of vortices *A* and *B*, and give attention to the motions by which the water in the tract is surrounded and influenced, taking care to refer each motion to the middle of the tract itself. Thus referred, the motion of the adjacent part of the rear vortex *A* is to the left and downward, that of the forward vortex *B* is to the left and upward, and that of the bed is to the left. The motions are indicated by arrows. The influences of the vortices tend to make the water about *D* rotate backward, while the influence of the bed tends to make it rotate forward. The result is not a priori evident, but may be assumed to be something different from simple rotation. Its possibilities will again be referred to.

Now consider the tract *E*, bounded below by parts of *A* and *B* and above by the water surface, and give attention to the motions by which its water is affected. Motions being referred as before to the tract itself, that of vortex *A* is to the right and downward, and that of vortex *B* is to the right and upward. Above is the motion of the air, which, in the absence of wind, is to the left, as indicated by an arrow. The three influences to which the tract of water is subject all tend to give a backward rotation, as indicated in a similar position at *F*. The vortex *F* is secondary to the *A*, *B*, *C* system and rotates in the opposite direction. If its motions be referred to the fixed stream bed, it is evident that the water in its upper part moves in the direction of the current less rapidly than the water in its lower part. The existence of such a vortex therefore tends to reduce the average velocity at the surface of the current and increase it at some lower level.

Abandoning now the specific and ideal case, we may state the hypothesis in general terms. Among the important causes of vortical motion in a river or other stream is the mechanical couple occasioned by the general forward motion of the water in conjunction with the resistance of the bed. This tends to form vortices with horizontal axes and forward roll; and the tendency is probably strongest in the lower part of the stream. In a space adjacent to two forward-rolling vortices there exists a tendency toward the development of a secondary, backward-rolling vortex, but this tendency is apt to be nullified by other and adverse influences except in the upper part of the stream. The free surface of the water does not oppose the development of such reversed vortices. Wherever reversed vortices abound the velocity at the surface (averaged with respect to time) is less than at some level below the surface.

The hypothesis as stated has no stationary element, but the phenomena of incipient dunes show that in certain cases repetitive motions are associated with stationary space divisions. A supplementary suggestion assigns these repetitive motions, or their initial phases, to the triangular space *D* in the ideal diagram, figure 83. The forces tending toward rotation in that space are antagonistic; and the suggestion is (1) that they produce some sort of alternating movement with regular periodicity, and (2) that the time interval of this movement, in combination with the forward movement of the major vortices *A*, *B*, *C*, yields a stationary space interval.

An investigation based on this line of suggestion and designed to test it should lead also to an explanation of the observed changes in the method by which traction is accomplished. The dune method is associated with a depth which is large in relation to the mean velocity and with a moderate bed resistance. It is replaced by other methods in consequence of (1) a reduction of depth, or (2) an increase of velocity, or (3) an increase of resistance. Reduction of depth diminishes the space for development of the hypothetic vortices. Increase of velocity or of resistance tends to enlarge their pattern. In either case the coercion of the water surface restricts the freedom of vortical movements and imposes conditions tending to modify their system.

THE MOVING FIELD.

The competent investigator is resourceful in the creation of new apparatus and methods as the need for them arises. While fully conscious of this fact, and of the further fact that no device is of assured value till it has been tried, I yet can not forbear to mention, for the benefit of others, a method which the Berkeley experience leads me to think valuable for the study of the internal details of a current of water.

In a current limited by the sides of a narrow, straight trough transverse movements are largely suppressed, so that most of the action can be learned from observation of what takes place in vertical planes parallel to the axis. The movements in a vertical plane may be exhibited by giving to the trough a glass side and by giving to the water in that plane exclusive illumination. When one of our laboratory currents, being illuminated from above, was viewed from the side, small particles in suspension were seen to be conspicuous, so conspicuous, in fact, as to resemble the motes in a sunbeam. Some of the particles were shreds of wood fiber worn from the trough, and these could be more easily followed by the eye because of their distinctive forms. Impressed by this phenomenon, I am confident that water movements in a vertical plane can be effectively revealed by giving to the water a suitable amount of suitable suspended material and by giving to the selected plane a brilliant and exclusive illumination.

With the aid of such simple arrangements much may be seen, but measurement will be difficult; and vortical movements may not be easily discriminated from those which are merely sinuous. It is believed, however, that both these results may be achieved by aid of the moving field. In a very simple form this device was employed by us in the study of processes of traction (p. 27), and despite the crudity of the apparatus it was found to be highly efficient. An experiment trough having in its side a glass panel, *AA* in figure 4, bore a sliding screen, *B*, in which was an opening, *C*. Moving débris was watched through the opening at the same time that the screen and opening were moved in the direction of the current. To the field of view was thus given a horizontal motion, and that motion was, in effect, subtracted from the motions of the objects observed. The apparent motions were motions in relation to the moving field, and not to the fixed trough. No provision was made for determining the velocity of the field, nor for measuring the apparent motions of the objects viewed; but even without these devices for quantitative work, the possibilities of the method of observation were sufficiently evident. With the necessary supplementary devices, the moving field promises to measure the horizontal and vertical components of motion in any part of the vertical section, and it thus makes possible the complete delineation of velocities and directions of details of current, so far as those details may be exhibited in longitudinal vertical sections. By giving to the field a suitable velocity it should be possible to see a traveling vortex which rotates in a vertical plane, just as vortices of horizontal rotation are seen on the surface of a stream.

A notable defect in the Berkeley arrangement was the requirement that the observer move his head in unison with the moving field. This interfered with steadiness and also limited narrowly the space covered by an observation. It could be remedied by substituting for the slide a car which should carry both observer and peephole at a determined rate.

Another suggested arrangement places the eye of the observer at a fixed telescope and

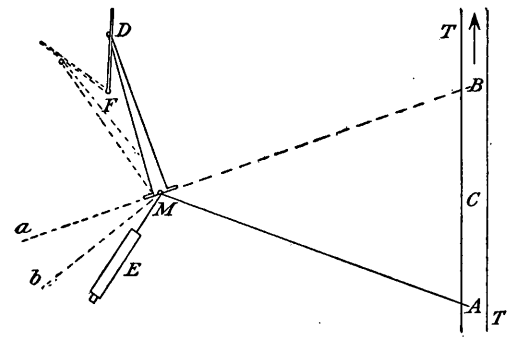


FIGURE 84.—Diagrammatic plan of suggested moving-field apparatus.

moves the field by means of a rotating mirror. This is illustrated by figure 84, where the trough *T* is shown in plan. The telescope *E* views the trough through the mirror, which is pivoted on a vertical axis at *M*. As the plane of the mirror rotates from *a* to *b* the field commanded by the telescope moves from *A* to *B*. It is evident that if the angular velocity of the mirror is constant the linear motion of the field along the trough will be relatively fast at *A* and *B* and relatively slow at *C*. The error thus arising may be avoided by some device of the nature of linkage. It will be corrected with sufficient approximation if the axis of the mirror be controlled by a rigidly attached arm *MD*, which is in turn controlled by an arm about one-third as long, the two having sliding contact at *D* and the short arm revolving uniformly about a vertical axis at *F*.

APPENDIX A.—THE PITOT-DARCY GAGE.

SCOPE OF APPENDIX.

Many measurements of velocity were made with the Pitot-Darcy gage, but as only a few have been finally utilized in the preparation of the report, a discussion of the instrument seemed not appropriate to the main text. Certain phases of our experience, however, are thought worthy of record because they have practical bearing on the utility and the use of such gages, and these are the subject of the appendix.

FORM OF INSTRUMENT.

Darcy developed the Pitot tube by adding a second tube, differently related to the current, and by connecting the two above with a chamber from which the air was partly exhausted. The water columns in the two tubes were thus lifted from the vicinity of the water surface to a convenient position, where their difference in height could readily be measured. In the gages constructed for our use the aperture of one tube was directed upstream and that of the other downstream. The tubes were borings in a single piece of brass, which was shaped on the outside in smooth contours, designed to interfere the least possible with the movement of the water. The form first given was afterward modified, and figure 85 shows the third and last design, with which most of the work was done. The openings had a diameter of 0.1 inch. At the opposite or upper end of the brass piece were stopcocks, and above these connection was made with rubber tubes, which led to the complementary part of the apparatus, where the difference in height of the two water columns was observed. It is convenient to call the member exposed to the current the *receiver*, and the complementary member the *comparator*.

In the comparator were two glass tubes, straight and parallel, with internal diameters of about 0.8 inch. At the top they were connected by an arch, and at the summit of the arch was a branch tube, with a pet cock, used in regulating the amounts of air and water. At the bottom they communicated with the rubber tubes through a brass piece, in which were two stopcocks, connected by gearing so as to open and close together. These parts were mounted on a board which also carried a scale of inches and decimals. A sliding index was arranged so that it could be set by the meniscus of a water column and its position then read on the scale; and there was a fixed mirror behind the tubes to aid in avoiding error from parallax. The board was supported in an inclined position, the slope given to the tubes and scale being that of 2½ horizontal to 1 vertical. This had the effect of making the movements of the columns 2.69 times as great for the same change of pressure as they would be if the tubes were vertical.

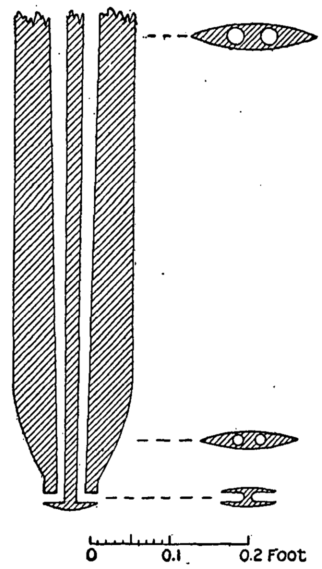


FIGURE 85.—Longitudinal section of lower end of receiver of Pitot-Darcy gage No. 3, with transverse sections at three points.

In preparing for observation, the internal air pressure was so adjusted that the columns stood near the middle of the scale. The receiver was then held, by a suitable frame, in the selected part of the current, and the stopcocks were opened. In the glass tube connected with the receiver opening facing upstream the column rose; in the other tube it fell. When the full effect of the current had been realized, the stopcocks at the bottom of the comparator were closed, and the heights of the columns were then read.

The use of the gage to measure velocities close to the bed of débris proved impracticable because the presence of the receiver modified the movement of the water and thereby modified the shape of the bed. (See pp. 26, 155.) This effect could have been reduced by using a different form of receiver. Darcy and Bazin[1] bent the tubes at the bottom in such a way that one or both openings met the water at some distance upstream from the vertical part of the tubes, and it is probable that the adoption of their design would have diminished the difficulty, although it could not have removed it. With such a design, however, the practicable forms for the second opening relate it to the piezometer, and the advantage of the downstream opening is lost.

That part of the design of the comparator which consists in the inclination of tubes and scale is not to be recommended. It refines by magnifying the reading, but it introduces possibilities of error in other ways. If the tubes are not straight or are not equally inclined, an error is occasioned which does not enter if they and the scale are vertical. Evidence of such error was found in the fact that the still-water or zero-velocity readings of the two columns were not always identical, but no ready means of correction was discovered.

Another source of error was detected in inequalities of sectional area of the glass tubes of the comparator. To show the nature of this error, let us assume that the pressure of the current at the upstream opening of the receiver is exactly equal to the negative pressure, or suction, at the downstream opening. If the glass tubes are of uniform and equal bore, one column moves upward just as much as the other moves downward, and the volume of air above the columns is unchanged. Now, assume that the tube containing the rising column has the greater diameter. It is evident that equal movement of the two columns will displace more air in the one tube than it will provide space for in the other, and the pressure of the confined air will be thereby increased. The effect of the increased pressure will be to lower both columns. There is also a secondary effect of small amount connected with the fact that the pressure of the confined air plus the head of water between the tops of the columns and the surface of the stream, on the one hand, and the atmospheric pressure, on the other, are in equilibrium, but into this we need not here enter.

As a means for the discussion of these errors the tubes were calibrated, by Prof. J. N. Le Conte, in the following manner: The tubes being closed at the bottom, a weighed quantity of water was introduced into one and the height of its column was read. By repeated additions of water and repeated readings the volumes of divisions of each tube were thus measured, the divisions being approximately 1.5 inches in length. In similar manner the volume was measured of the space above the straight tubes to the pet cock. The average sectional area of one tube was found to be 2.5 per cent greater than that of the other. The sectional area in the larger tube was found to vary through a range of at least 2.6 per cent, and the range for the smaller tube was 4.5 per cent.

A table of corrections to readings was computed from the data of calibration, and this table was practically applied. In comparing the rise of one column with the associated fall of the other the greatest correction applied amounted to a little less than 1 per cent. In the determination of velocity the largest correction applicable for this reason was about 0.3 per cent. These corrections, however, pertain only to the discussions of the instrument in the following pages. In the ordinary work of the gage they were not applied, because it was found that the errors of this class were practically eliminated when the same methods were employed in the preparation and in the use of rating formulas. The matter is mentioned here chiefly because errors from unequal tube caliber, which may sometimes prove important, appear not to have been allowed for in the discussions of Pitot-Darcy gages.

RATING METHODS.

The first and second gages were rated by the method of floats; the third was twice rated by the running-water method and several times by the still-water method. The floats used were

[1] Bazin, F. A., Recherches expérimentales sur l'écoulement de l'eau dans les canaux découverts: Acad. sci. Paris Mém. math. et phys., vol. 19, p. 49, Pl. IV, 1865. This memoir was published also as part of "Recherches hydrauliques," by H. Darcy and F. A. Bazin.

vertical cylinders of wood so adjusted that the submerged depth was twice the distance below the water surface of the apertures of the receiver. Some of the still-water ratings were made at the Geological Survey's rating station at Los Angeles, where a car running on a track at the side of a reservoir drew the receiver through the water of the reservoir. The others were made later in the long trough of the Berkeley laboratory, the car in this case running above the water. In the application of the running-water method a measured discharge was passed through a rectangular trough and a survey of velocities throughout a cross section was made by means of the gage. By using in this survey the rating formula obtained by the still-water method, and then comparing the mean velocity thus computed with that computed from the discharge and sectional area, a correction is obtained which may be applied to the still-water rating formula. Only a single comparison of this sort is practically available in connection with our instruments, and the terms of this are:

	Ft./sec.
Mean velocity by discharge and area..............	1.98
Mean velocity by gage (with still-water rating).	2.08±0.07

The resulting correction to the still-water ratings is −5 per cent, but this determination has small value because of the large probable error of one of the compared determinations.

The fact that the apparent correction is small is in accord with a property of the gage independently observed. By reason of the sinuosity of flow lines in a stream, the directions of motion are not parallel to the axis of a straight channel. Therefore, a current meter which records the velocity in the direction of flow instead of the component of velocity parallel to the channel axis yields an overestimate of mean velocity. Some Pitot-Darcy gages have been found to overestimate velocity when placed obliquely to the direction of flow, and for such the correction would be large. It was found, however, that the Berkeley gage when placed somewhat obliquely to the current gave a lower reading than when facing it squarely, and through this property it tended automatically to correct its readings for obliquity of current. If the correction were perfect, the still-water rating and running-water rating should be the same.

RATING FORMULA.

In the still-water ratings velocities independently determined were compared with resulting changes in the water columns of the comparator. Starting from the same level, one column rose with increase of speed and the other fell. Except for the influence of modifying conditions the changes should be equal, the positive velocity head being of the same amount as the negative velocity head. To test this point various sets of observations were plotted on section paper, the readings of the rising column being taken as abscissas and the readings of the falling column as ordinates. In most cases the plotted points fell well into line, and there was no question that the line represented by them was straight. That is, the true ratio of the negative pressure at the downstream opening of the receiver to the positive pressure at the upstream opening was constant, under the conditions of this particular series of trials. The value of the ratio was found to vary with conditions, and the several values found are so near unity as to confirm the theoretic belief that unity is the normal value.

In considering the variations of value it is first to be noted that the gage with which the observations were made, No. 3, being symmetric, could have either opening turned upstream, and it was in fact used both ways, with record of its position. But its symmetry was only approximate and therefore the two positions gave different results. Used in one way it will be called No. 3a, and in the other way No. 3b. It also happened that between the date of the Los Angeles ratings and that of the Berkeley ratings the receiver was accidently marred at one of its openings, and though its form was afterward restored as nearly as possible, some difference remained which affected its constants. The values of the pressure ratio are accordingly arranged in four groups in Table 81. Two of these groups also are subdivided with reference to the position of the receiver in relation to the perimeter of the current.

The ratio was notably larger after the accident than before, and the change was greater for 3a than for 3b. The greatest value of the ratio was given by trials in which the receiver ran close to the side of the reservoir, which in that case was a plank trough. The interpretation of these results will be considered in another connection.

TABLE 81.—*Ratio of the suction at one opening of the Pitot-Darcy gage to the pressure at the other.*

Place of rating.	Conditions.	Ratio, suction to pressure.	
		Gage 3a.	Gage 3b.
Los Angeles....	Deep water................	0.993±0.015	1.005±0.005
Berkeley.......	Shallow, mid-depth.......	1.038± .007	1.010± .002
Do.........	Shallow, near bed..........	....	1.016± .003
Do.........	Shallow, near surface......	....	1.014± .005
Do.........	Shallow, near side.........	1.052± .004	....

Let h represent the vertical space through which the column of water is raised by pressure from velocity V on the upstream opening of the receiver, and h_1 the simultaneous depression of the column connected with the downstream opening. Then, each being assumed to equal the velocity head,

$$h=h_1=\frac{V^2}{2g} \text{------------} (116)$$

$$h+h_1=\frac{2}{2g}V^2 \text{----------} (117)$$

In practice the full velocity head is not realized in instruments of the Pitot-Darcy type, and the coefficient determined by rating is less than $\frac{2}{2g}$. It is a common experience also in practical application of hydraulic formulas to find that qualification is advisable in other respects. I therefore substituted tentatively for (117) the formula

$$h+h_1=k\,V^u \text{-----------} (118)$$

and sought empiric values of u and k. The readings of the comparator corresponding to h and h_1 may be called H and H_1. The zero of the comparator scale being at its lower end, the difference between the readings corresponds to the sum of the spaces h and h_1. The readings are in inches, while the unit used for h, h_1, and g is the foot. Moreover, from the inclination of the comparator, the space between the two columns, as read on the scale, is 2.69 times the vertical space $h+h_1$. Therefore, the product of 12 by 2.69 being 32.28,

$$H-H_1=32.28\,(h+h_1) \text{------} (119)$$

Substituting in (118), we have

$$H-H_1=32.28\,k\,V^u \text{--------} (120)$$

or, making $K=32.38\,k$,

$$H-H_1=KV^u \text{----------} (121)$$

The observed quantities being H, H_1, and V, it was possible to plot on logarithmic section paper any series of values of $H-H_1$ in relation to the associated values of V, and thus compute graphically the corresponding values of u and K. The values were computed for all series of observations represented in Table 81, and they are given in Table 82. The mean of the seven values of u is 2.00, but their range is notable. The deviations from the normal may be ascribed in part to accidental errors. In the case of the third value, 1.90, and of the seventh, 1.94, the plotted positions are so scattered as to admit of considerable latitude in the drawing of the equation lines, but the control is much stronger for the values 2.09 and 2.12, and these could not be greatly reduced without violence to the facts of observation. It seems clear that the exponent is not wholly free from the influence of special conditions.

TABLE 82.—*Values of K and u in $H-H_1=KV^u$.*

Place of rating.	Conditions.	Gage.	K	u
Los Angeles........	Deep water.............	3a	0.70	2.00
Do............	....do..................	3b	.62	1.93
Berkeley...........	Shallow, mid-depth.....	3a	.72	1.90
Do............	....do..................	3b	.52	2.09
Do............	Shallow, near bed.......	3b	.70	1.91
Do............	Shallow, near surface....	3b	.46	2.12
Do............	Shallow, near side.......	3a	.66	1.94

For the practical purpose of rating the instrument, however, there is no advantage in departing from the normal exponent, and that was employed in the preparation of rating tables. The formula used for inferring velocities from readings is

$$V=A\sqrt{H-H_1} \text{---------} (122)$$

in which

$$A=\frac{1}{\sqrt{K}}=\frac{1}{\sqrt{32.28k}}$$

The values of A, graphically computed, are given in Table 83, and these values were used in the computations of velocities.

TABLE 83.—*Values of A in $V=A\sqrt{H-H_1}$ and values of gage efficiency.*

Place of rating.	Conditions.	Values of A.		Efficiency of gage.	
		Gage 3a.	Gage 3b.	3a.	3b.
Los Angeles	Deep water	1.22±0.02	1.26±0.01	0.67	0.63
Berkeley	Shallow, mid-depth	1.29± .04	1.30± .02	.60	.59
Do	Shallow, near bed		1.24± .01		.65
Do	Shallow, near surface		1.37± .03		.53
Do	Shallow, near side	1.26± .03		.63	

An inspection of the values of A with due attention to their probable errors serves to show that their differences are not to be regarded as wholly accidental, but must be ascribed in part to the variation of the instrumental constant with conditions. That the nature of the variations may be appreciated, the conditions will be more fully described. As already stated, gages 3a and 3b are the same symmetric instrument, but with opposite faces turned toward the current, while the instrument was modified to some minute extent by an injury and repair occurring between the Los Angeles and Berkeley ratings. Practically there were four gages, but so far as the mechanician could make them the four gages were identical in form. The reservoir at the Los Angeles rating station was broad and deep. The course followed by the receiver of the gage was several feet from the bank, at least 6 inches below the surface, and at least 1 foot above the bottom. The plank reservoir used in Berkeley was 1.96 feet wide, and the depth of water was 0.44 foot. In the ratings tabulated as at "mid-depth" the opening of the receiver was central in the cross section of the water. In the "near bottom" rating the center of the opening (its diameter being 0.01 foot) was 0.02 foot from the bottom; in the "near surface" rating the center was 0.03 foot below the surface of the water; and in the "near side" rating the center was 0.25 foot above the bottom and varied from 0.01 to 0.11 foot in its distance from the side, the course and the trough side not being quite parallel. The several relations of the opening to the water section are shown in figure 86.

There is nothing novel in the variation of the instrumental constant with minute differences in the shape of the receiver. This has been observed by all critical users of such instruments, and the custom is well established of giving a separate rating to each receiver. So far as I am aware the variation of the constant with the relation of the receiver to the walls of the conduit and to the water surface has not previously been recorded, and this is a matter of considerable moment, for various elaborate

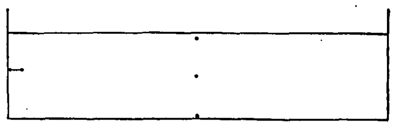


FIGURE 86.—Cross section of prism of water in trough, showing positions given to gage opening in various ratings.

discussions of the distribution of velocities within a cross section have assumed the constancy of the instrumental constant for all positions. As clearly indicated by the tabulated results, the constant A increases when the free surface of the water is approached and diminishes when a rigid wall of the conduit is approached.

In the application of these results (and of cognate results with which it was thought best not to burden these pages) to our velocity determinations a graphic table was constructed to show the relation of the instrumental constant to the position of receiver in the conduit; and this table (fig. 87) supplied the constant to be used with each individual observation. This procedure had an important influence on the interpretation of the running-water rating, making the apparent correction for that rating less than it would be if the mid-depth constant were used exclusively. Its influence on the vertical velocity curves (figs. 74 to 82) was to give them less curvature near the water surface, greater curvature in approaching the channel bed, and (sometimes) a somewhat lower level of maximum velocity.

In equation (117) $\frac{2g}{2}V^2$ equals twice the theoretic velocity head. If in equation (118) the exponent u be replaced by its mean value 2, we have kV^2 as an expression for the double head actually produced by the gage. The ratio of k to $\frac{2}{2g}$ measures the efficiency of the instrument in realizing the theoretic head. Since $A=\frac{1}{\sqrt{32.28\,k}}$, $k=\frac{1}{32.28\,A^2}$, and the measure of efficiency is

$$\frac{1}{32.28\,A^2}\div\frac{2}{2g}=\frac{32.16}{32.28}\frac{1}{A^2}=\frac{0.996}{A^2}$$

The values of the measure have been computed and are given in Table 83. The deep-water and mid-depth ratings being assumed to represent normal conditions, the mean of the corresponding efficiencies, namely, 0.62, may stand in a general way for the fraction of the theoretic head which is realized by this particular type of the Pitot-Darcy gage.

As the relation between velocity and observed head is that between agent and effect, it is evident that values of the efficiency ratio rather than those of the constant A should be compared in any attempt to explain the phenomena of variation. Restating from this viewpoint the results of comparative ratings, we have: The response of the head to changes of velocity is lessened when the receiver of the gage is brought near the free water surface and is increased when the receiver is brought near a rigid part of the stream's perimeter. It may be surmised that the differences in head are connected with the facility with which the flow lines of the water are diverted in passing around the instrument, regarded as an obstruction. In midcurrent the diversion is resisted by the inertia of surrounding water. Near the surface the resistance of water is partly replaced by resistance of more mobile air. Near the conduit the resistance of water is partly replaced by the resistance of an immobile solid.

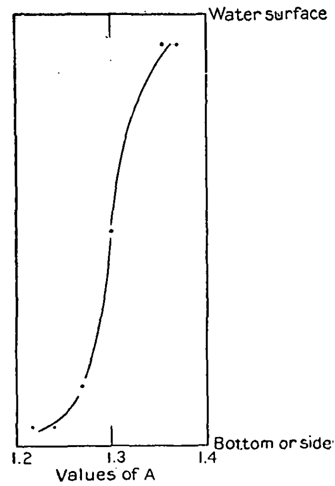


FIGURE 87.—Graphic table for interpolating values of A, in $V=A\sqrt{H-H_1}$, for observations made with gage 3b in different parts of a stream.

APPENDIX B.—THE DISCHARGE-MEASURING GATE AND ITS RATING.

THE GATE.

The gate for measurement of discharge is mentioned at page 20, and its general relation to other apparatus is shown in figure 2. In a section on the measurement of discharge, page 22, it is briefly described, and the history of its use is outlined. Figure 88 is designed to exhibit its relations more fully and shows the arrangement of the general apparatus of the laboratory at the time of its calibration.

The gate controlled an opening in the side of a vertical wooden shaft, of which the internal horizontal section measured 3 feet by 0.5 foot. At the top the shaft communicated freely with a long, narrow tank, to which water was continuously delivered by a pump. The surface level of water in the tank was regulated by means of an overflow weir and a valve above the pump and was observed in a glass tube or gage, *AB*, in figure 88, outside of the tank. This tube was given a slope of 1 in 10, so that

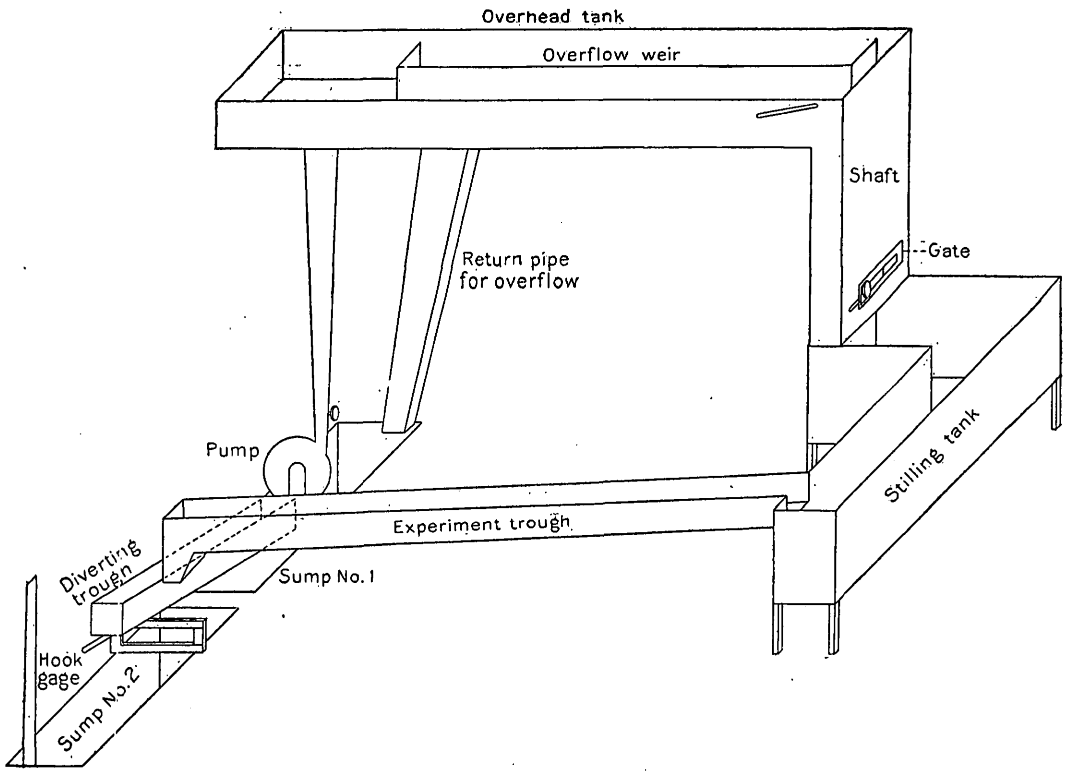


FIGURE 88.—Arrangement of apparatus connected with the rating of the discharge-measuring gate.

the movements of its water column afforded a magnified indication of changes in the water level within the tank.

Details of the gate are represented to scale in figure 89. A brass plate, *PP*, attached by screws to the outer face of the shaft, was pierced by an opening of 10.5 by 2 inches, the longer dimension being horizontal. About three sides the edges of the opening were beveled outward, leaving a 45° edge at the inner face of the plate. A sliding plate, *G*, rested against the inner face of the fixed plate. This was the gate proper, its function being to close either the whole or a definite portion of the opening and thus regulate the width of the issuing water jet. It overlapped by half an inch the margins of the opening, above and below, and was guided by two brass pieces, which appear in sections *b* and *c*. To the end adjacent to the water jet was given a chisel edge. The brass guides and the adjacent parts of the wooden shaft wall were shaped to a 45° bevel, which, if produced, would reach the edge of the opening in the fixed plate.

Movements of the slide were controlled from the outside. A brass rod, *O*, firmly attached to it and running parallel to its axis (fig. 89*d*), rested in a frame at the left of the opening (fig. 89*a*, *c*), where its motion was controlled by rack and pinion, the rack being cut on the upper side of the rod. At the left of the frame and pinion the rod slid along a brass scale graduated to inches and tenths; and the gate was set for any desired width of aperture by bringing an engraved index mark on the rod opposite the proper mark on the scale.

The operation of the gate was found to be satisfactory.

The head under which the jet issued, measured from the middle of the opening to the water surface in the upper tank, was 6.0 feet, plus or minus a small fraction observed by means of the gage above mentioned.

PLAN OF RATING.

The method of rating was volumetric and empiric. The gate having been set at a particular graduation, and the discharge having been continued until the rate of flow through the stilling tank and experiment trough had become steady, the outflow of the trough was diverted for a measured time into a special

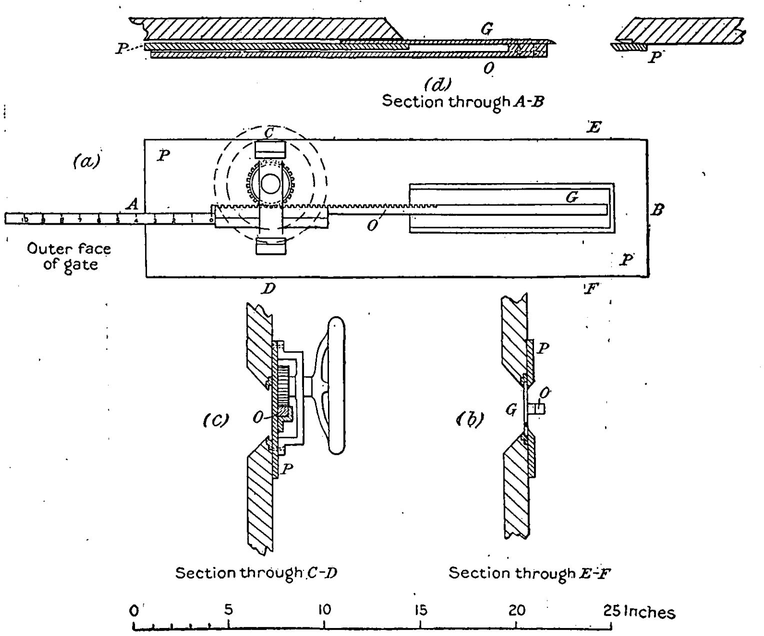


FIGURE 89.—Elevation and sections of gate for the measurement of discharge.

reservoir where its volume was measured. The special reservoir is shown in figure 88 as "sump No. 2," and the diverting apparatus also is indicated. The "diverting trough" was pivoted at the remote end and could be turned quickly by hand. In one position it received the discharge from the experiment trough and delivered it to sump No. 1, containing the supply for the pump; in another it permitted the water to fall into sump No. 2, arranged for measurement. The work of rating was performed by J. A. Burgess, and a full report upon it constituted his graduation thesis in engineering at the University of California. The details of apparatus and method were arranged by him, and the following account of the rating is essentially an abridgment of his report.

CALIBRATION OF THE MEASURING RESERVOIR.

Sump No. 2, constructed of concrete, was approximately rectangular but was not quite regular in form. No attempt was made to base computations of volume on its linear dimensions, but a scale of volumes was graduated directly. A hook gage was first installed, the hook being attached to a vertical rod, to which slow motion could be given. An index borne by the rod near its upper end followed a smooth surface which had been prepared to receive a graduation but initially was unmarked. The index was so arranged that by pressure it could be made to indent the prepared surface, thereby producing a mark of graduation. Starting with just enough water in the sump to permit the use of the hook, a line of graduation was marked. Then a cubic foot of water was added, the hook was readjusted, and a second line was marked. By repeating this process a scale was given to the gage, the unit of the scale being 1 cubic foot. The added units of water were measured in a wooden bottle made in the form of a cube with the opening at one corner; and the volume of the bottle was so adjusted that it contained 62.3 pounds of water. The capacity of the reservoir was about 38 cubic feet.

In the use of the scale thus provided, fractions of a cubic foot were read by means of a small free scale of equal parts applied obliquely to the space between two lines of graduation.

THE OBSERVATIONS.

The record of an observation included (1) the width of gate aperture, (2) the time interval, by stop watch, during which the water was delivered to sump No. 2, (3) two readings of the hook-gage scale, and their difference, giving the volume of water received by the sump, and (4) two gage readings of water level in the upper tank, one just before and one just after the period of volume measurement.

The quotient of the volume of water by the time in seconds gave the discharge for the indicated width of gate opening. The readings of the high-tank water level gave data for a correction to the head, resulting in a small correction to the computed discharges.

It was found that repeated observations with the narrower gate openings gave results nearly identical, while the results for wider openings showed more variation. The observations with wide openings were accordingly multiplied to increase the precision of the averages. The variation was ascribed to pulsations of the flowing water originating in the stilling tank. The general precision of the accepted values of discharge, listed in Table 2 (p. 23), is indicated by an average probable error of ±0.2 per cent. The largest computed probable error is ±0.4 per cent, being that of the discharge for a gate opening of 6 inches.

The general formula for discharge through a rectangular orifice when, as in the present case, the vertical dimension of the orifice is small in relation to the head, is

$$Q = cbd\sqrt{2gh}$$

in which h is the head, measured from the middle of the orifice, bd the area of the orifice, and c the constant of discharge. The observations give the following values of c for different settings of the gate:

Width of gate opening.	Value of c.
Inches. 1	0.704
2	.703
3	.667
4	.677
5	.677
6	.684
Mean.....	.686

For a "standard" orifice 2 inches square and a head of 6 feet Hamilton Smith's tables[1] give 0.604 as the value of c. The inner surface about the standard orifice is vertical and plane, whereas the surfaces about the orifice of our gate were oblique. The oblique guiding surfaces served to increase the velocity at the orifice and thus enlarge the constant of discharge. The variation of the constant with width of opening is probably connected with the fact that the beveled surfaces were not symmetrically arranged about the opening. On three sides they made an angle of 45° with the vertical plane of the orifice, but the edge of the slide, constituting the fourth side, was beveled at a smaller angle. The constants of the gate were thus affected by the conditions of its setting and a new rating would be necessary with a different setting.

[1] Hydraulics, p. 58.

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