

CHAPTER 6

CONCEPTUALIZATION AND IMPLEMENTATION OF STRESS PACKAGES

This chapter documents the conceptualization and implementation of the packages in the Ground-Water Flow (GWF) Process that simulate hydrologic stresses to a ground-water system. The stress packages add terms to the flow equation representing inflows or outflows. Mathematically, these are boundary conditions. The six packages documented are the Well (WEL) Package, Recharge (RCH) Package, General-Head Boundary (GHB) Package, River (RIV) Package, Drain (DRN) Package, and Evapotranspiration (EVT) Package. Input instructions are contained in Chapter 8. Programming is documented in Chapter 9.

In Chapter 2, the finite-difference flow equation for MODFLOW was developed (eq. 2–26):

$$\begin{aligned} & CV_{i,j,k-1/2} h_{i,j,k-1} + CC_{i-1/2,j,k} h_{i-1,j,k} + CR_{i,j-1/2,k} h_{i,j-1,k} \\ & + (-CV_{i,j,k-1/2} - CC_{i-1/2,j,k} - CR_{i,j-1/2,k} - CR_{i,j+1/2,k} - CC_{i+1/2,j,k} - CV_{i,j,k+1/2} + HCOF_{i,j,k}) h_{i,j,k} \quad (6-1) \\ & + CR_{i,j+1/2,k} h_{i,j+1,k} + CC_{i+1/2,j,k} h_{i+1,j,k} + CV_{i,j,k+1/2} h_{i,j,k+1} = RHS_{i,j,k} \end{aligned}$$

The flow equation was formulated so that inflows are added to the left side, with outflows represented as negative inflows. Stresses are incorporated to the flow equation by adding terms to HCOF and RHS. A stress term that is a coefficient of head, $h_{i,j,k}$, is added to HCOF. A stress term that is a constant is subtracted from RHS because RHS has been moved to the right side of the flow equation.

Well Package

The Well (WEL) Package is designed to simulate features such as wells that withdraw water from or add water to the aquifer at a constant rate during a stress period, where the rate is independent of both the cell area and the head in the cell. The discussion in this section is developed on the assumption that the features to be simulated are actually wells, either discharging or recharging; however, the package can be used to simulate any features for which the recharge or discharge can be directly specified.

The flow rate, Q , for a well is specified by the user as a fluid volume per unit time at which water is added to the aquifer. Negative values of Q are used to indicate well discharge (pumping), whereas positive values of Q indicate a recharging well. Four data values are required for each well—the row, column, and layer number of the cell in which the well is located, and the recharge rate, Q , of the well. The wells are redefined each stress period.

At each iteration, as the matrix equations are formulated, the value of Q for each well is subtracted from the RHS value (eq. 6–1) for the cell containing that well. Where more than one well falls within a single cell, the calculation is repeated for each well as the RHS term for that cell is assembled. Thus the user specifies the recharge associated with each individual well, and these are in effect summed within the program to obtain the total recharge to the cell.

The WEL Package does not directly accommodate wells that are open to more than one layer of the model. A well of this type, however, can be represented as a group of single-cell wells, each open to one of the layers tapped by the multilayer well, and each having an individual Q term specified for each stress period. If this approach is used, the recharge (negative for discharge) of the multilayer well must be divided or apportioned in some way among the individual layers, externally to the model program. A common method of doing this is to divide the well discharge (negative for pumping) in proportion to the layer transmissivities:

$$\frac{Q_n}{Q_w} = \frac{T_n}{\sum T} \quad (6-2)$$

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where

- Q_n is the recharge (negative for pumping) from layer n to a particular well in a given stress period (L^3T^{-1}),
- Q_w is the total recharge (negative for pumping) for the well in that stress period (L^3T^{-1}),
- T_n is the transmissivity of layer n (L^2T^{-1}), and
- $\sum T$ represents the sum of the transmissivities of all layers penetrated by the well (L^2T^{-1}).

IMPORTANT: Equation 6-2, or some other method of apportioning the recharge, must be implemented by the user externally to the program for each multilayer well, and for each stress period. This approach, in which a multilayer well is represented as a group of single layer wells, fails to take into account the interconnection between various layers provided by the well itself, and is thus an incomplete representation of the problem. The Multi-Node Well Package has been created to directly simulate multilayer wells (Halford and Hanson, 2002).

Recharge Package

The Recharge (RCH) Package is designed to simulate areally distributed recharge to the ground-water system. Most commonly, areal recharge occurs as a result of precipitation that percolates to the ground-water system. Recharge applied to the model is defined as

$$QR_{i,j} = I_{i,j} DELR_j DELC_i \quad (6-3)$$

where

- $QR_{i,j}$ is the recharge flow rate applied to the model at horizontal cell location (i,j) expressed as a fluid volume per unit time (L^3T^{-1}); and
- $I_{i,j}$ is the recharge flux (in units of length per time, LT^{-1}) applicable to the map area, $DELR_j DELC_i$, of the cell.

Values of recharge flux, $I_{i,j}$, are specified by the user at each stress period. These values of recharge flux are multiplied by horizontal cell areas, $DELR_j DELC_i$, to obtain values of $QR_{i,j}$. The recharge, $QR_{i,j}$, is applied to a single cell within the vertical column of cells located at (i,j) . The RCH Package does not allow for recharge to occur simultaneously at multiple depths in the same vertical column because natural recharge enters the ground-water system at the top. In the simplest situation, the top of the ground-water system will occur in model layer 1; however, the vertical position of the top of the system may vary with horizontal location and with time as the water-table rises and falls. Three options for specifying the cell in each vertical column of cells that receives the recharge have been implemented as described below.

The cell within each vertical column to which the recharge is applied is specified through the recharge option code, NRCHOP, and optional two-dimensional variable IRCH. The options include: (1) application of the recharge to model layer 1; (2) application of the recharge to any cell in the vertical column as specified by layer numbers contained in two-dimensional variable $IRCH_{i,j}$; and (3) application of the recharge to the uppermost variable-head cell in the vertical column, provided no constant-head cell is above the variable-head cell in the column. Under options 1 and 2, if a cell designated to receive recharge is no flow or constant head, then no recharge is added. Under the third option, if a constant-head cell is in a vertical column of cells and no variable-head cell is above, then no recharge is applied to this column because the assumption is that any recharge would be intercepted by the constant-head source. Recharge flux values that are read into the model must be expressed in units that are consistent with the length and time units used to represent all other model data.

In the formation of the matrix equations, the recharge flow rate, $QR_{i,j}$, associated with a given horizontal cell location (i,j) and vertical location, k , that is determined by the recharge option is subtracted from the value of $RHS_{i,j,k}$ (eq. 6-1). This is done at each iteration for all cells that receive recharge. Because recharge, as defined, is independent of aquifer head, nothing is added to the coefficient of head, $HCOF_{i,j,k}$.

Careful consideration should be given to the problem under study and to the other options employed in the simulation before deciding which of the three recharge options to use in a given situation. For example, figure 6-1 shows a situation in which a cross-section model has been used to simulate a hypothetical problem involving

recharge, seepage from a river, and seepage into a river (fig. 6-1A). The river is simulated by constant-head cells. Using the provision described in Chapter 5 for horizontal conductance formulation under water-table conditions, cells for which the computed head was lower than the bottom elevation were converted to no flow so that the uppermost variable-head cell in each vertical column contains the water table. This process yields the final distribution of variable-head, constant-head, and no-flow cells shown in figure 6-1B.

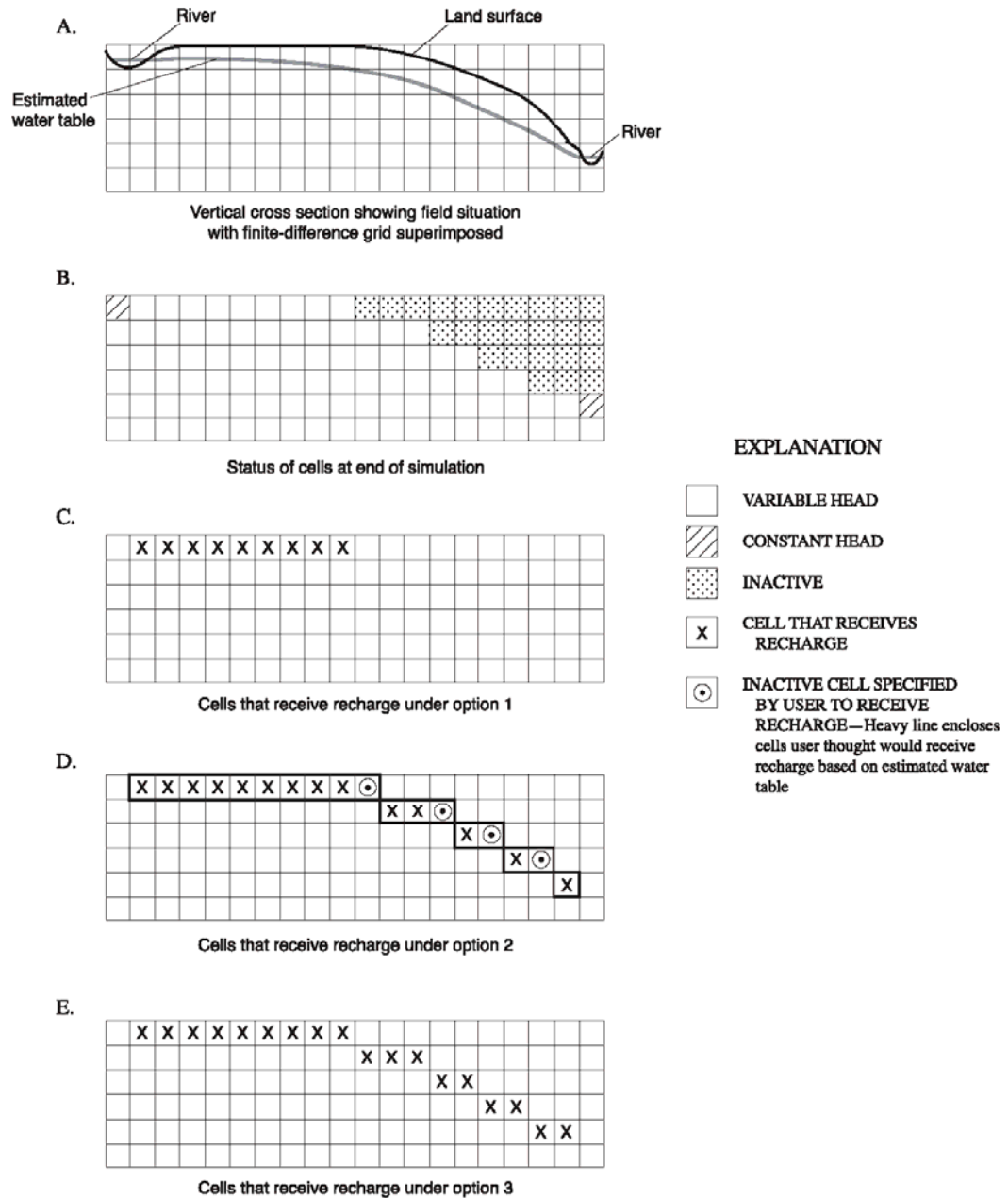


Figure 6-1. Hypothetical problem showing which cells receive recharge under the three options available in the Recharge Package. (From McDonald and Harbaugh, 1988.)

Figure 6-1C illustrates the recharge distribution to the model if option 1 above is utilized. Under this option, recharge is permitted only to the top layer of the model. Thus, once the water-table shape has been simulated by the

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use of no-flow cells in the top layer, recharge to the vertical columns beneath those cells is shut off. This clearly fails to simulate the given system.

Figure 6-1D illustrates the recharge distribution if option 2 is used, assuming that the user specifies recharge cells prior to the simulation on the basis of an estimated water-table position, which differs slightly from the computed water table obtained in the simulation process. Four of the cells that the user had designated as recharge cells have converted to a no-flow condition and thus receive no recharge.

Figure 6-1E illustrates the recharge distribution under option 3, which turns out to be the option best suited for this particular situation. Under this option, recharge enters the uppermost variable-head cell in each vertical column, except where constant-head cells have been used to represent the river. Thus, a continuous distribution of recharge to the water table is simulated.

For the typical situation of recharge from precipitation, option 3 is the easiest to use. The model user does not have to be concerned about determining which is the highest variable-head cell in a vertical column because the program automatically determines this throughout the simulation. Option 1, however, can be useful in situations where recharge should not pass through the no-flow cells in layer 1. For example, some cells may be designated no-flow because they are impermeable. Any recharge specified for those cells should not pass into layer 2. Alternately, option 3 could still be used in this situation by specifying that the recharge rate is zero at the impermeable cells. Similarly, option 2 may be useful when layers other than layer 1 have outcrop areas and when recharge to the specified layers should not penetrate through no-flow cells to a lower layer.

The RCH Package can be used to simulate recharge from sources other than precipitation—for example, artificial recharge. Discharge can also be simulated using the RCH Package by specifying negative values of the recharge flux. If the ability to apply recharge to more than one cell in a vertical column of cells is required, then the Well Package, which allows recharge or discharge to be specified at any model cell, can be used.

General-Head Boundary Package

The function of the General-Head Boundary (GHB) Package is to simulate flow into or out of a cell i,j,k , from an external source in proportion to the difference between the head in the cell and the head assigned to the external source. The constant of proportionality is called the boundary conductance. Thus a linear relation between flow into the cell and head in the cell is established:

$$QB_n = CB_n (HB_n - h_{i,j,k}), \quad (6-4)$$

where

- n is a boundary number,
- QB_n is the flow into cell i,j,k from the boundary (L^3T^{-1});
- CB_n is the boundary conductance (L^2T^{-1});
- HB_n is the head assigned to the external source (L); and
- $h_{i,j,k}$ is the head in cell i,j,k (L).

The relation between cell i,j,k and the external source is shown schematically in figure 6-2. The constant-head source is represented by the apparatus on the right in figure 6-2, which holds the source head at the level HB_n regardless of other factors; the link between the source and cell i,j,k is represented by the block of porous material having conductance CB_n . Note that figure 6-2 shows no mechanism to limit flow in either direction as $h_{i,j,k}$ rises or falls.

A graph of inflow from a general-head boundary and head in the cell containing the boundary as given by equation 6-4 is shown in figure 6-3. The GHB Package provides no limiting value of flow to bound the linear function in either direction; and as the difference between the head in the cell containing the boundary and the source head increases, flow into or out of the cell continues to increase without limit. Accordingly, the GHB Package must be used with care to insure that unrealistic flows into or out of the system do not develop during the course of simulation. The GHB Package is included in MODFLOW without a specific real-world conceptualization in mind. The River, Drain, and Evapotranspiration Packages documented in the following sections of this chapter incorporate

the same head-dependent form, except that they limit the flow to simulate the conceptualization of specific hydrologic features.

QB_n of equation 6-4 is defined as an inflow to the aquifer, and according to the flow convention used in MODFLOW, inflows are added to the left side of equation 6-1. In terms of the variables HCOF and RHS, the term $-CB_n$ is added to $HCOF_{i,j,k}$ and the term $CB_n HB_n$ is subtracted from $RHS_{i,j,k}$ as the matrix equations are assembled.

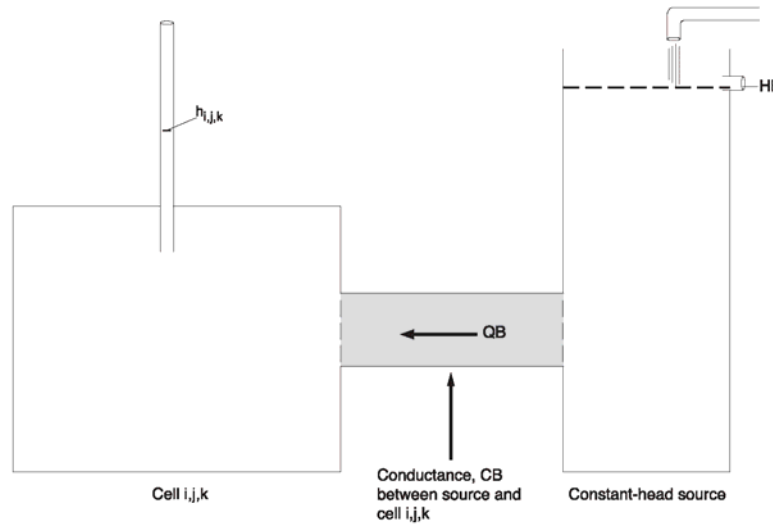


Figure 6-2. Schematic diagram illustrating principle of General-Head Boundary Package. (Modified from McDonald and Harbaugh, 1988.)

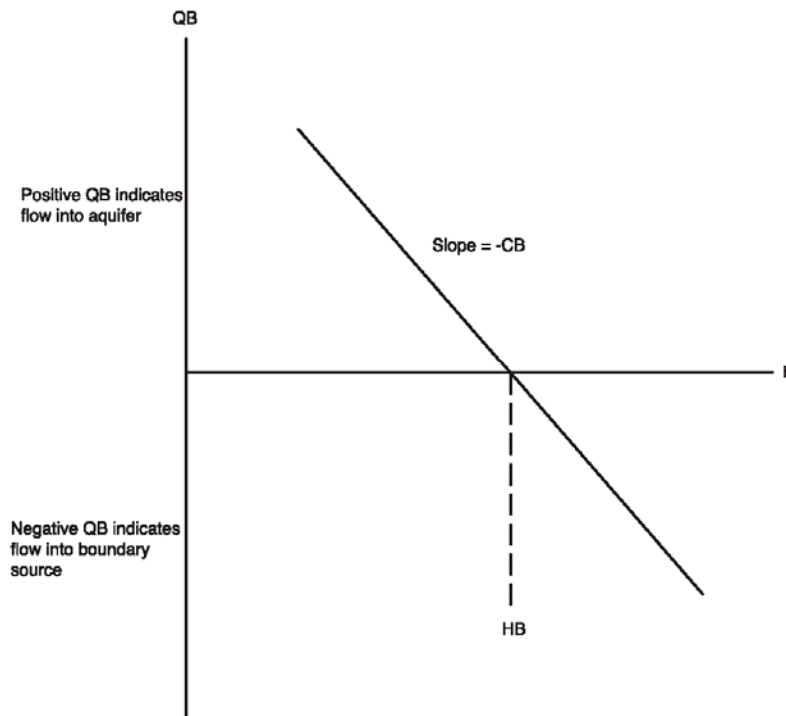


Figure 6-3. Plot of flow, QB , from a general-head boundary source into a cell as a function of head, h , in the cell where HB is the source head. (Modified from McDonald and Harbaugh, 1988.)

River Package

Rivers and streams contribute water to or drain water from the ground-water system, depending on the head gradient between the river and the ground-water regime. The purpose of the River (RIV) Package is to simulate the effects of flow between surface-water features and ground-water systems. To accomplish this purpose, terms representing seepage to or from the surface features must be added to the ground-water flow equation for each cell affected by the seepage.

Figure 6-4 shows two rivers divided into reaches so that each reach is completely contained in a single cell. River-aquifer seepage is simulated between each reach and the model cell that contains that reach. RIV does not simulate surface-water flow in the river – only the river/aquifer seepage. [Other packages that also simulate surface-water flow have been developed for MODFLOW (for example, Prudic, 1989, and Prudic, Konikow, and Banta, 2004).] Accordingly, the order of numbering for reaches has no impact on calculations in the RIV Package. River seepage is independently simulated for each river reach.

The cross section in figure 6-5 shows a single cell containing a river reach. In the conceptualized system (fig. 6-5A), the open water of a river is separated from the ground-water system by a layer of low permeability riverbed material. Figure 6-5B shows an idealization of this system in which the river-aquifer interconnection is represented as a simple conductance through which one-dimensional flow occurs.

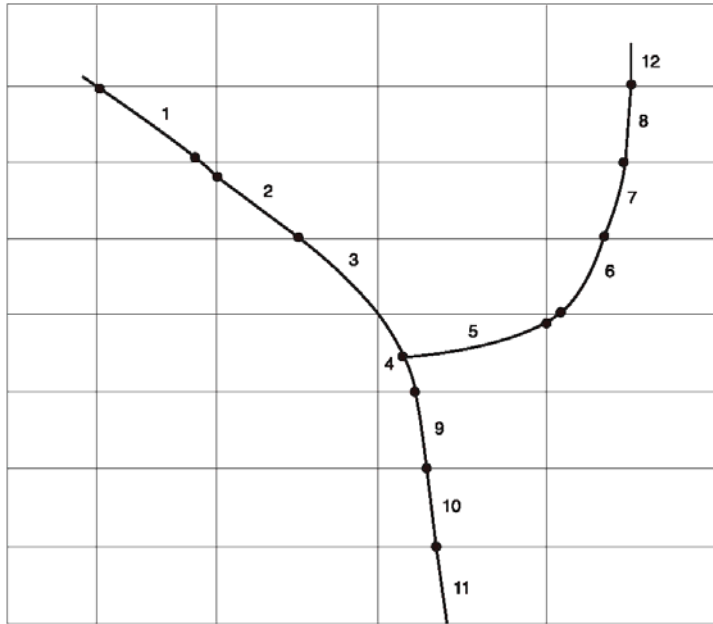


Figure 6-4. Discretization of two rivers into reaches. Some small reaches are ignored. (Modified from McDonald and Harbaugh, 1988.)

The assumption is made that measurable head losses between the river and the aquifer are limited to those across the riverbed layer itself—that is, that no substantial head loss occurs between the bottom of the riverbed layer and the point represented by the underlying model node. Further, an assumption is made that the underlying model cell remains fully saturated—that is, the water level does not drop below the bottom of the riverbed layer. Under these assumptions, flow between the river and the ground-water system for reach n is given by

$$QRIV_n = CRIV_n (HRIV_n - h_{i,j,k}) \quad (6-5)$$

where

$QRIV_n$ is the flow between the river and the aquifer, taken as positive if it is directed into the aquifer (L^3T^{-1});

$HRIV_n$ is the water level (stage) in the river (L);

$CRIV_n$ is the hydraulic conductance of the river-aquifer interconnection (L^2T^{-1}); and

$h_{i,j,k}$ is the head at the node in the cell underlying the river reach (L).

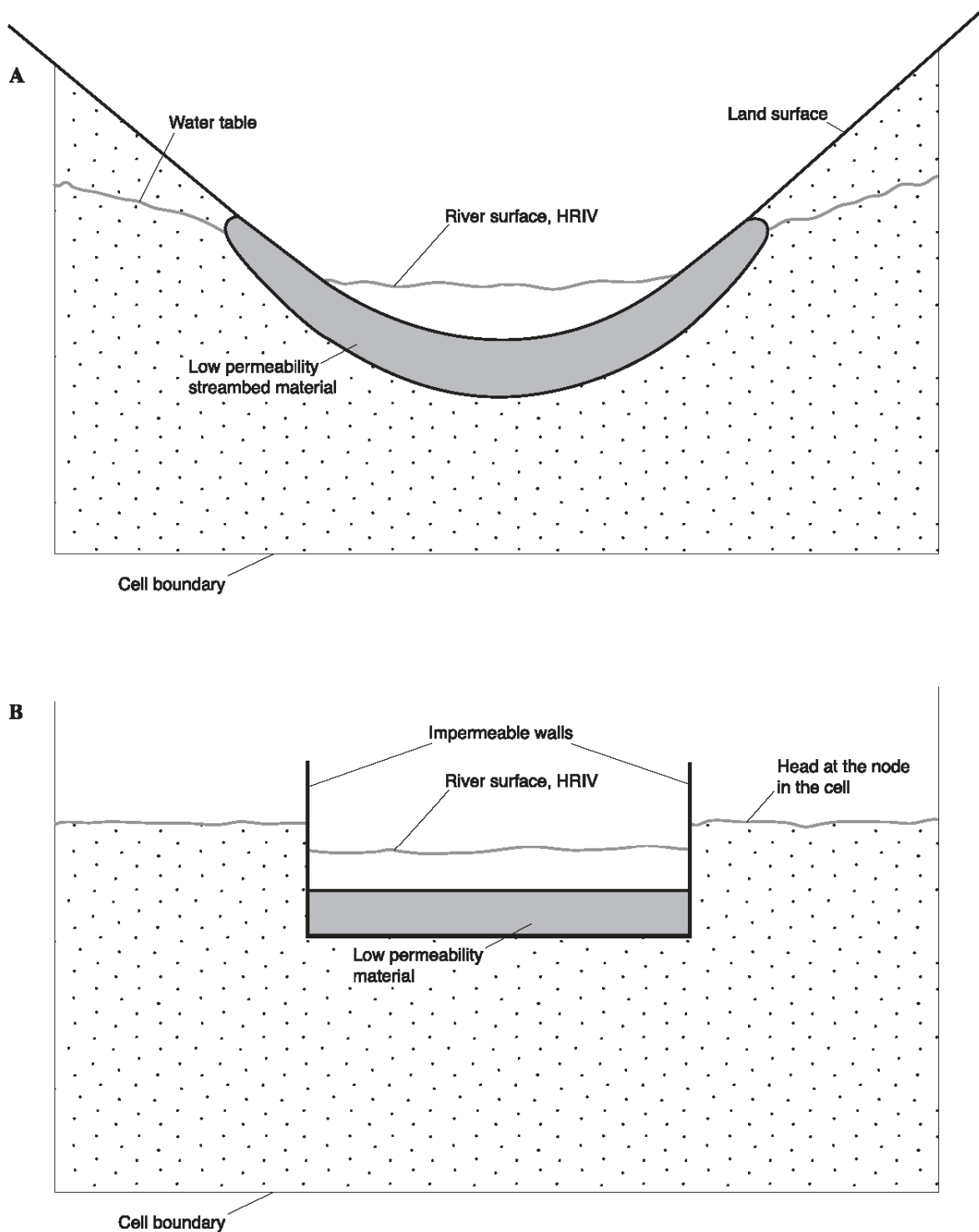


Figure 6-5. (A) Cross section of an aquifer containing a river and (B) conceptual representation of river-aquifer interconnection in a simulation. (From McDonald and Harbaugh, 1988.)

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Figure 6-6 shows an isolated view of the idealized riverbed conductance of figure 6-5B, as it crosses an individual cell. The length (L_n) of the conductance block is the length of the river as it crosses the node; the width (W_n) is the river width; the distance of flow is taken as the thickness (M_n) of the riverbed layer; and K_n is the hydraulic conductivity of the riverbed material. The conductance can then be computed as

$$CRIV_n = \frac{K_n L_n W_n}{M_n} \quad (6-6)$$

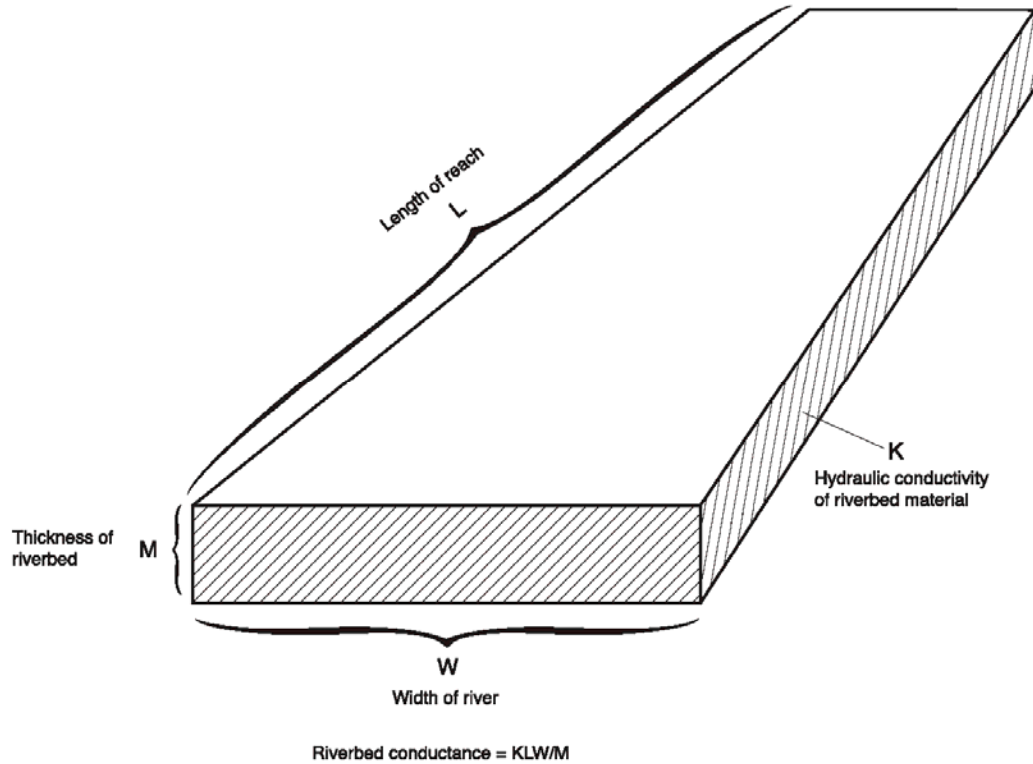


Figure 6-6. Idealization of riverbed conductance in an individual cell.
(From McDonald and Harbaugh, 1988.)

Equation 6-5 normally provides an acceptable approximation of river-aquifer interaction over a certain range of aquifer head values. In most cases, however, if water levels in the aquifer fall below a certain point, seepage from the river ceases to depend on head in the aquifer. This can be visualized by returning to the concept of a discrete riverbed layer. Figure 6-7A shows the situation described by equation 6-5; the water level in the aquifer is above the bottom of the riverbed layer, and flow through that layer is proportional to the head difference between the river and the aquifer. In figure 6-7B, the water level in the aquifer has fallen below the bottom of the riverbed layer, leaving an unsaturated interval beneath that layer; if it is assumed that the riverbed layer itself remains saturated, the head at its base will simply be the elevation at that point. If this elevation is designated $RBOT_n$, the flow through the riverbed layer is given by

$$QRIV_n = CRIV_n (HRIV_n - RBOT_n) \quad (6-7)$$

where $QRIV_n$, $CRIV_n$, and $HRIV_n$ are as defined for equations 6-5 and 6-6. Obviously, further declines in head below $RBOT_n$ produce no increase in flow through the riverbed layer; the flow simply retains the constant value given by equation 6-7, as long as head remains below $RBOT_n$. MODFLOW uses these concepts in simulating river-aquifer interaction—that is, flow between a river and a node i,j,k is simulated according to the equation set

$$Q_{RIV_n} = CRIV_n (HRIV_n - h_{i,j,k}), \quad h_{i,j,k} > RBOT_n \quad (6-8A)$$

$$Q_{RIV_n} = CRIV_n (HRIV_n - RBOT_n), \quad h_{i,j,k} \leq RBOT_n \quad (6-8B)$$

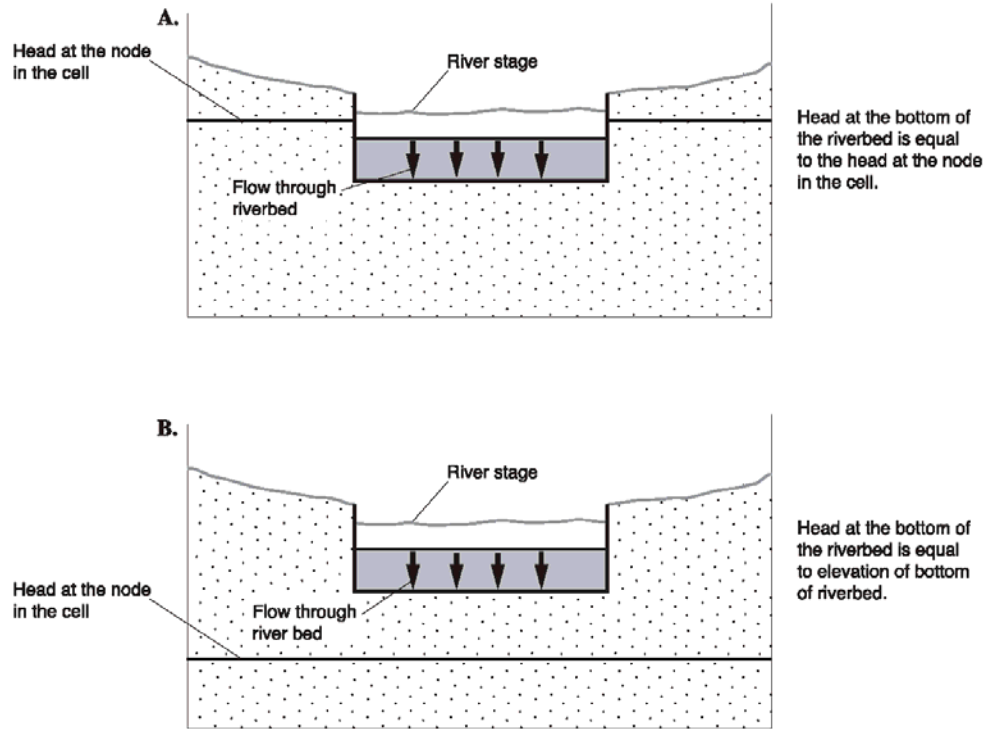


Figure 6-7. Cross sections showing the relation between head at the bottom of the riverbed layer and head in the cell. Head in the cell is equal to the water-table elevation. (Modified from McDonald and Harbaugh, 1988.)

Figure 6-8 shows a graph of flow from a river reach as a function of the head, h , in the cell containing the reach, as calculated using equation 6-8. Flow is zero when h is equal to the water level in the river, $HRIV$. For values of h higher than $HRIV$, flow is into the river, represented as a negative inflow to the aquifer. For values of h lower than $HRIV$, flow is positive into the aquifer. This positive flow increases linearly as h decreases, until h reaches $RBOT$; thereafter the flow remains constant. Thus, the RIV Package is mathematically identical to the GHB Package as long as h is greater than $RBOT$.

The conceptualization of river-aquifer interaction used here assumes that this interaction is independent of the location of the river reach within the cell, and that the level of water in the river is uniform over the reach and constant over each stress period. The latter assumption implies that conditions of flow in the river do not vary substantially during the stress period—for example, the river does not go dry or overflow its banks, or such events are of such short duration as to have no effect on river-aquifer interaction.

If the assumption is satisfied that all substantial head loss occurs across a discrete, rectangular riverbed layer, the application of equations 6-6 and 6-8 is straightforward. The idealized riverbed conductance can be further generalized to include a variable width and a meandering river segment. In this case, the length would be the total length of the river reach in the cell, and the width would be the average width along the length. If reliable field measurements of river seepage and associated head difference are available, they may be used to calculate an effective conductance. Otherwise, a conductance value must be chosen more or less arbitrarily and adjusted during model calibration.

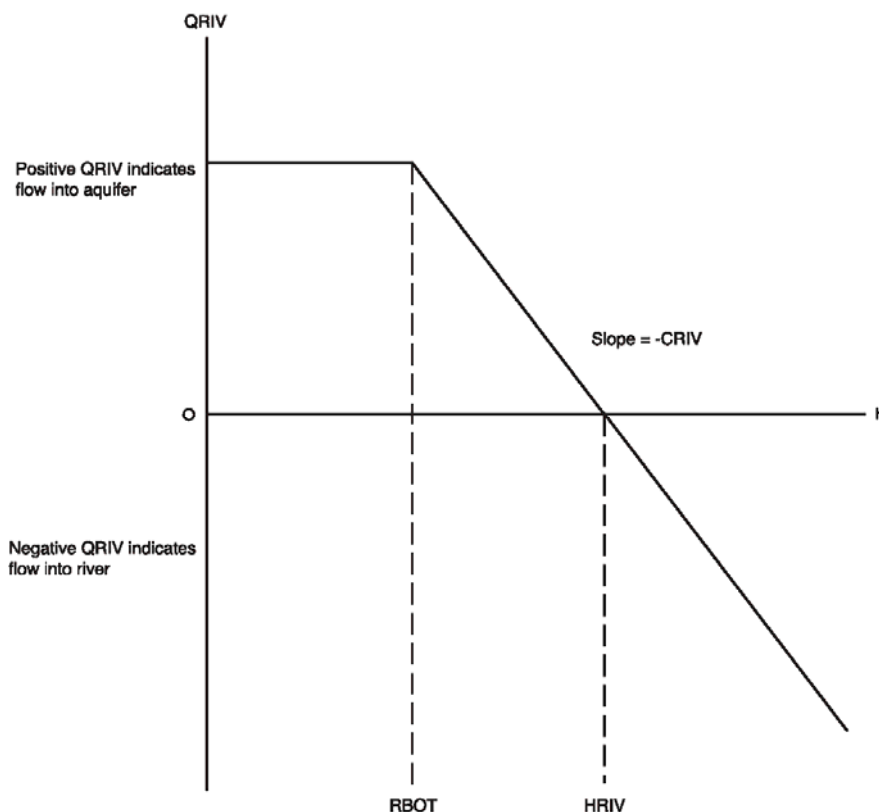


Figure 6-8. Plot of flow, $QRIV$, from a river into a cell as a function of head, h , in the cell where $RBOT$ is the elevation of the bottom of the riverbed and $HRIV$ is the head in the river. (From McDonald and Harbaugh, 1988.)

The flow between the river and aquifer is in general a three-dimensional process, and its representation through a single conductance term and riverbed elevation is only approximate in situations in which the riverbed is much different from the idealized confining layer. For example, the bed may be discontinuous, multiple zones of low conductivity may be below the river, or perhaps no discrete riverbed layer can be identified. The user should recognize that formulation of a single conductance term to account for the resulting three-dimensional flow process is inherently an empirical exercise, and that adjustment during calibration is almost always required. Certain rules can be formulated, however, to guide the initial choice of conductance in these situations. For example, the area through which flow occurs should still be viewed as the product of the total length of the river reach in the cell and the average width; the assumed distance of flow should not exceed the vertical interval between the riverbed and node i,j,k ; and, if distinct layers can be recognized within this interval, these layers should normally be treated as conductances in series in formulating an equivalent conductance.

The application of equation 6-8 is the most difficult in situations where a discrete riverbed does not exist. In this case, equation 6-8 should be applied as an approximation. The following is a simple justification for applying the general relation of equation 6-8 in cases of no distinct riverbed. Although a strictly linear relation between head and seepage may not exist, the seepage into the river will increase as aquifer head increases above the river stage. Likewise, when the river is losing, seepage from the river will increase as head drops below the river stage. Thus, choosing a value for $CRIV$ that will approximate the seepage as represented by equation 6-8A should be possible, although the value of $CRIV$ should probably vary with aquifer head. The existence of a depth below the river for which a further decrease in water level will not cause additional seepage also seems reasonable, and thus equation 6-8B would apply. This situation is conceptualized in figure 6-9, where the water table has fallen substantially below the river and only a narrow saturated connection exists between the river and the regional water table. When the water table drops, as shown in figure 6-9, a region in the saturated column will form in which the head gradient above the

water table is nearly 1. Further lowering the water table will not increase the gradient, and therefore the seepage should not increase. A break in saturation also will occur if the water table lowers enough, but leakage will not substantially increase once the head drops low enough to cause the region of unit gradient.

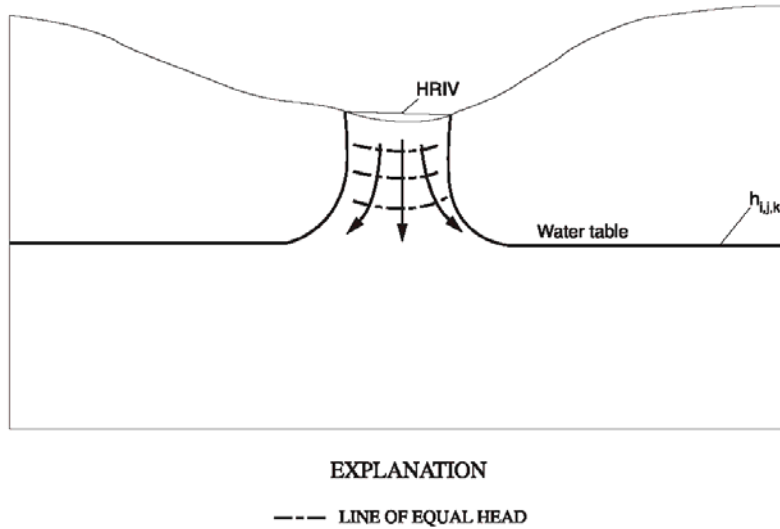


Figure 6-9. Limiting seepage from a river at unit hydraulic gradient. (Modified from McDonald and Harbaugh, 1988.)

Unfortunately, the above argument that equation 6-8 can be qualitatively applied in the situation of no distinct riverbed does not provide much guidance on how to determine appropriate values for $CRIV_n$ and $RBOT_n$. One approach is to assume that the maximum seepage from the stream is the seepage in the aquifer in a column of water in which unity head gradient occurs. Darcy's law can be applied to compute the vertical flow, Q_{max} , through the water column. If the water column is assumed to have the same horizontal dimensions as the stream channel, L_n and W_n , then the result is:

$$Q_{max} = K_{aq} L_n W_n dh/dl ,$$

where

K_{aq} is aquifer hydraulic conductivity (LT^{-1}), and
 dh/dl is the head gradient through the water column (dimensionless).

The head gradient is 1, so $Q_{max} = K_{aq} L_n W_n$.

Equation 6-5 can then be used to compute the conductance that will produce Q_{max} when aquifer head has a specified value of $h_{i,j,k}$. If $h_{i,j,k}$ is replaced by $RBOT_n$, then $CRIV_n$ is:

$$CRIV_n = \frac{K_{aq} L_n W_n}{HRIV_n - RBOT_n} \quad (6-9)$$

In this situation, rather than being a distinct riverbed-bottom elevation, $RBOT_n$ is the value of aquifer head at which the seepage from the river will equal Q_{max} . Seepage will not increase if the aquifer head goes lower. Regardless of the approach used to determine these values initially, some evaluation or adjustment is essential during the calibration process.

Data describing each river reach are specified by the user for each stress period. Input consists of six entries for each river reach: the layer, row, and column of the cell containing the reach, and the three values needed to calculate seepage—river level or stage ($HRIV_n$), the conductance of the river-aquifer interconnection ($CRIV_n$), and the "elevation of the bottom of the riverbed," or level at which the limiting value of river seepage is attained ($RBOT_n$).

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At the start of each iteration, terms representing river seepage are added to the flow equation for each cell containing a river reach. The choice of which river seepage equation to use, equation 6-8A or equation 6-8B, is made by comparing the most recent value of head at the cell to the value of $RBOT_n$ for the reach. Because this process is done at the start of each iteration, the most current value of head ($h_{i,j,k}$) is the value from the previous iteration. Thus, the check for which river seepage equation to use lags behind the seepage calculations by one iteration. If equation 6-8A is selected, then the term $-CRIV_n$ is added to the term $HCOF_{i,j,k}$ and the term $CRIV_n * HRIV_n$ is subtracted from $RHS_{i,j,k}$. If equation 6-8B is selected, then the term $CRIV_n(HRIV_n - RBOT_n)$ is subtracted from $RHS_{i,j,k}$.

Drain Package

The Drain (DRN) Package is designed to simulate the effects of features such as agricultural drains, which remove water from the aquifer at a rate proportional to the difference between the head in the aquifer and some fixed head or elevation, called the drain elevation, so long as the head in the aquifer is above that elevation. If, however, the aquifer head falls below the drain elevation, then the drain has no effect on the aquifer. The constant of proportionality is called the drain conductance. A mathematical statement of this situation is:

$$\begin{aligned} Q_{out} &= CD(h_{i,j,k} - HD), & h_{i,j,k} > HD \\ Q_{out} &= 0, & h_{i,j,k} \leq HD \end{aligned} \quad (6-10)$$

where

- Q_{out} is the flow from the aquifer into the drain (L^3T^{-1}),
- CD is the drain conductance (L^2T^{-1}),
- HD is the drain elevation (L), and
- $h_{i,j,k}$ is the head in the cell containing the drain (L).

Equation 6-10 is rewritten in terms of flow from the drain into the aquifer, QD , which is the flow convention used throughout this report:

$$QD_n = CD_n(HD_n - h_{i,j,k}), \quad h_{i,j,k} > HD_n \quad (6-11A)$$

$$QD_n = 0, \quad h_{i,j,k} \leq HD_n \quad (6-11B)$$

where the subscript n is a drain number.

Thus, from the perspective of inflow to the model, drain flow is either negative or zero. For purposes of simulation, an assumption is made that each model drain represents the part of a physical drain that overlies a single model cell. Figure 6-10 shows a graph of flow from a drain and head in the cell containing the drain as defined by equation 6-11; the function is similar to that for flow between a surface river and the aquifer (fig. 6-8) except that flow into the aquifer is excluded. With proper selection of coefficients, the River Package could in fact be used to perform the functions of the Drain Package.

Many physical conceptualizations can be approximated by equation 6-11. Figure 6-11A shows one conceptualization—a three-dimensional view of a buried drain tile (pipe containing slots) as used for agricultural drains. To justify representation by equation 6-11, the slope of the pipe is assumed to be great enough that once water enters the slots, it is carried away without filling the pipe. Accordingly, the drain runs only partially full, and the head representing the drain, HD_n , is the average elevation of the slots.

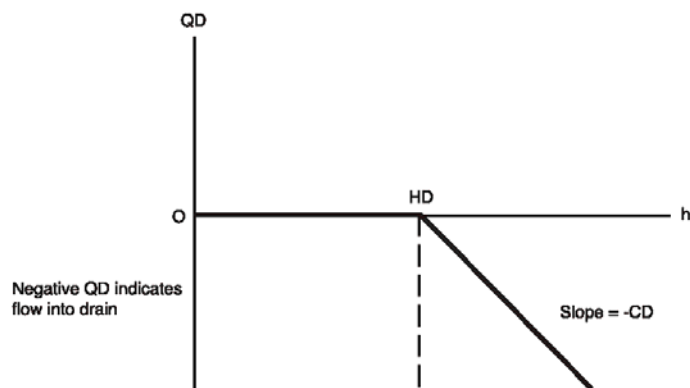


Figure 6-10. Plot of flow, QD , into a drain as a function of head, h , in a cell where elevation of the drain is HD and the conductance is CD . (Modified from McDonald and Harbaugh, 1988.)

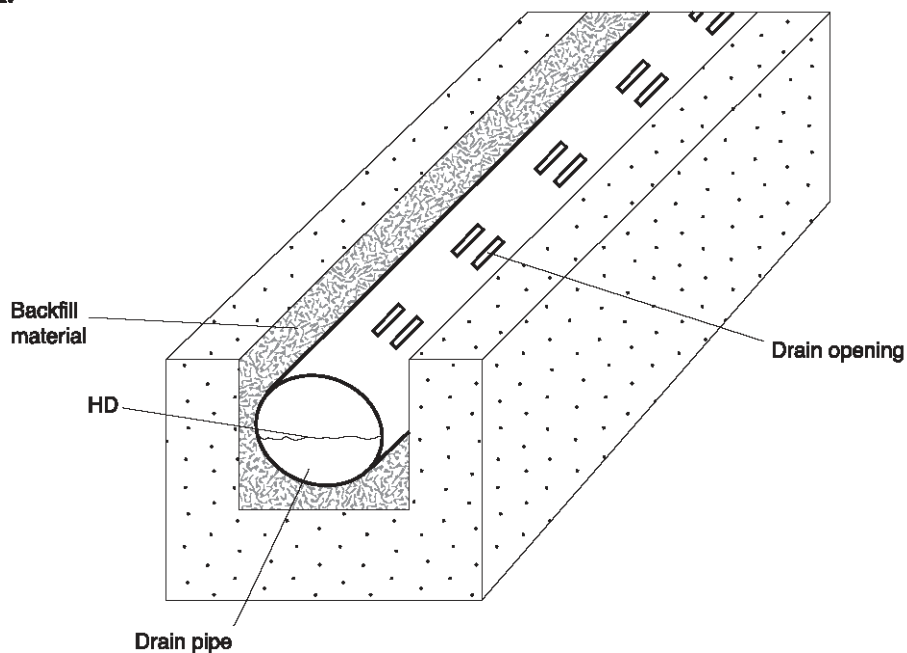
Figure 6-11B shows a cross section of cell i,j,k that is traversed by the drain. The head computed by the model for cell i,j,k ($h_{i,j,k}$) is actually an average value for the cell, and is normally assumed to prevail at some distance from the drain itself. HD_n prevails only locally at the drain pipe—it does not characterize the cell as a whole. Between the drain and the area in which head $h_{i,j,k}$ prevails, a radial or semiradial flow pattern exists in the vertical plane, normally characterized by progressively steeper head gradients as the drain is approached. The head loss within this converging flow pattern forms one part of the head difference $h_{i,j,k} - HD_n$. An additional component of head loss may occur in the immediate vicinity of the drain if the hydraulic conductivity in that region differs from the average value used for cell i,j,k owing to the presence of foreign material around the drain pipe. Finally, head losses occur through the wall of a drain pipe, depending upon the number and size of the openings in the pipe, and the degree to which those openings may be blocked by chemical precipitates, plant roots, or other obstructions.

The three processes discussed above—convergent flow toward the drain, flow through material of differing conductivity in the vicinity of the drain, and flow through the wall of the drain—each generate head losses that may be assumed proportional to the discharge, QD , into the drain from cell i,j,k . Because these head losses occur in series, the total head loss $HD_n - h_{i,j,k}$ also may be taken as proportional to QD_n . This has been done in the method of simulation embodied in the Drain Package. That is, the drain function is assumed to be described by equation 6-11. The coefficient CD_n of equation 6-11 is a lumped (or equivalent) conductance describing all of the head loss between the drain and the region of cell i,j,k in which the head $h_{i,j,k}$ can be assumed to prevail. Thus, CD_n depends on the characteristics of the convergent flow pattern toward the drain, as well as on the characteristics of the drain itself and its immediate environment.

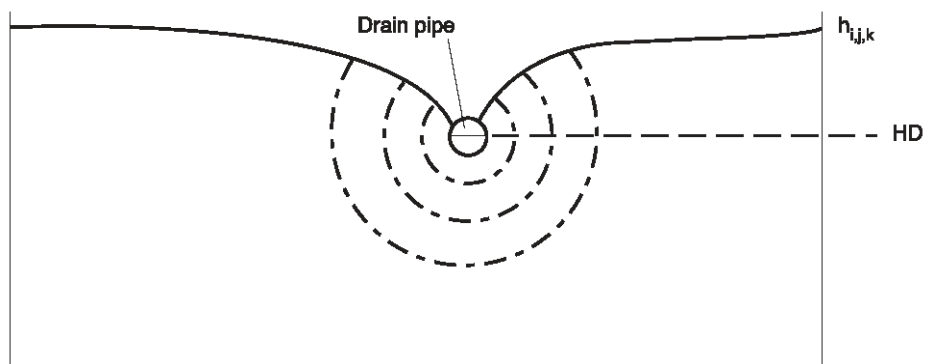
One could attempt to calculate values for CD_n by developing approximate equations for conductance for the three flow processes, and then calculate the equivalent series conductance. The conductance for each process would be based on the formulation of a one-dimensional flow equation. The formulations vary substantially depending on the specific drain system being simulated, so no general formulation for calculating CD_n is presented here. Also, in most situations a specific formulation would require detailed information that is not usually available, such as detailed head distribution around the drain, aquifer hydraulic conductivity near the drain, distribution of the fill material, hydraulic conductivity of fill material, number and size of the drain pipe openings, the amount of clogging materials, and the hydraulic conductivity of the clogging materials. In practice, calculating CD_n from measured values of QD_n and $HD_n - h_{i,j,k}$ using equation 6-11A is more common. If $HD_n - h_{i,j,k}$ is not accurately known, CD_n is usually adjusted during model calibration in order to match measured values of QD_n to model calculated values.

Figure 6-12 shows two other real-world conceptualizations that can sometimes be simulated using equation 6-11. Figure 6-12A shows an open channel, which could be natural or man made. If the channel carries water even when aquifer head is below the channel bed, then equation 6-11 presumably would not always apply because water could seep into the aquifer. In this case, the channel would be better simulated using the RIV Package as described earlier. But if the channel is dry unless ground water is seeping into it, and there is no chance for this water to seep back into the aquifer, then the Drain Package could provide a reasonable representation. For example, the head waters of a river might be acceptably simulated with DRN whereas the lower sections of a river might be simulated using RIV.

A.



B.



EXPLANATION

--- LINE OF EQUAL HEAD

Figure 6-11. (A) Factors affecting head loss in the immediate vicinity around a buried drain pipe in a backfilled ditch, and (B) cross section through cell i,j,k illustrating head loss in convergent flow into drain. (Modified from McDonald and Harbaugh, 1988.)

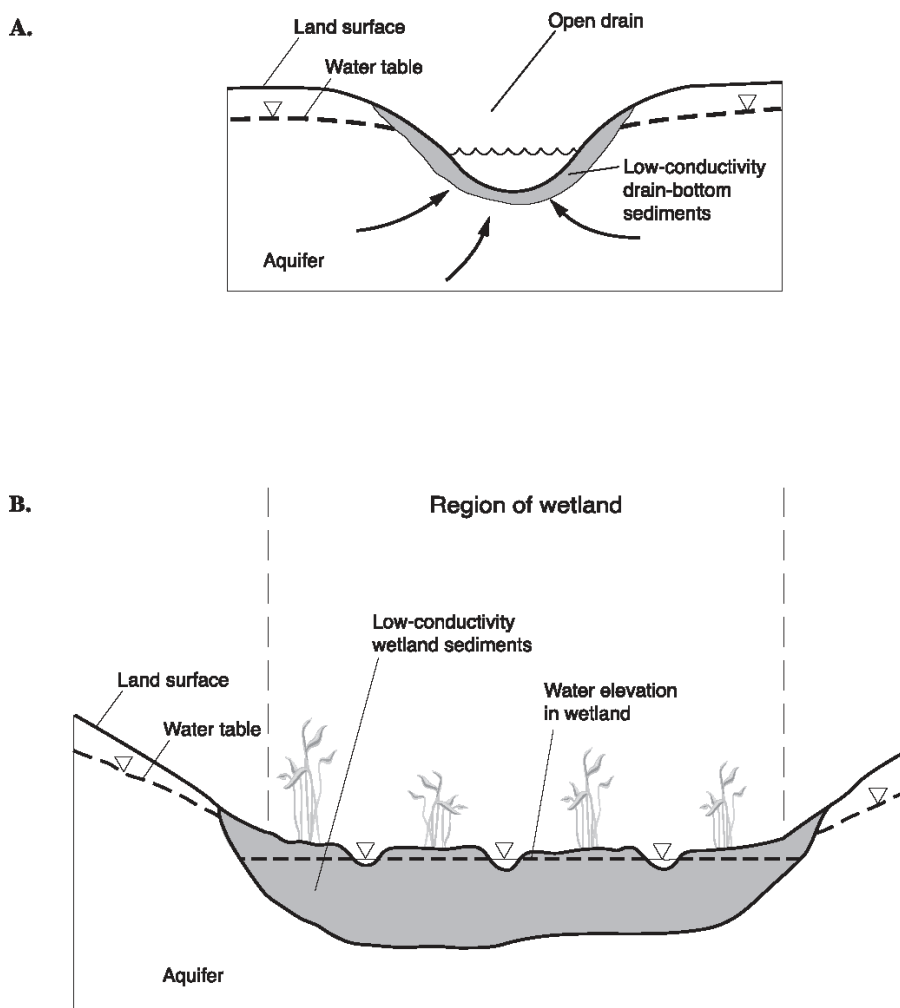


Figure 6-12. Alternate conceptualization of drains: (A) open drainage channel and (B) wetland.

Figure 6-12B shows a wetland, which can sometimes be conceptualized as functioning according to equation 6-11. If the wetland is wet primarily from ground-water seepage, then it will presumably dry up if the water table drops below the wetland. When wet, the conceptualized wetland water elevation is assumed to be fairly constant because excess ground-water seepage drains to a nearby river through a network of shallow channels or over the nearly flat surface. Seepage into the wetland is limited by low conductivity sediments in the wetland. Under these conditions, equation 6-11 would be a reasonable approximation. Although one physical wetland might cover many cells, the part covered by each cell would be viewed as a separate drain in MODFLOW. The drain conductance for each drain would depend on the area of the cell covered by the wetland and the properties of the lower conductivity sediments in the wetland.

According to the sign convention in MODFLOW, QD_n in equation 6-11 is defined as a flow into cell i,j,k and must be added to the left side of equation 6-1 for each cell affected by a drain, provided the head $h_{i,j,k}$ is above the drain elevation. This is accomplished in the Drain Package by testing to determine whether head exceeds drain elevation, and if so, by adding the term $-CD_n$ to $HCOF_{i,j,k}$ (equation 6-1) and subtracting the term CD_nHD_n from $RHS_{i,j,k}$, as the matrix equations are assembled.

Evapotranspiration Package

The Evapotranspiration (ET) Package simulates the effects of plant transpiration and direct evaporation in removing water from the saturated ground-water regime. The approach is based on the following assumptions: (1) when the water table is at or above a specified elevation, termed the "ET surface" in this report, evapotranspiration loss from the water table occurs at a maximum rate specified by the user; (2) when the depth of the water table below the ET surface elevation exceeds a specified interval, termed the "extinction depth" or "cutoff depth" in this report, evapotranspiration from the water table ceases; and (3) between these limits, evapotranspiration from the water table varies linearly with water-table elevation.

This can be expressed in equation form as

$$RET = EVTR, \quad h_{i,j,k} > SURF \quad (6-12A)$$

$$RET = EVTR \frac{h_{i,j,k} - (SURF - EXDP)}{EXDP}, \quad (SURF - EXDP) \leq h_{i,j,k} \leq SURF \quad (6-12B)$$

$$RET = 0, \quad h_{i,j,k} < SURF - EXDP \quad (6-12C)$$

where

- RET is the rate of loss per unit surface area of water table due to evapotranspiration, in units of volume of water per unit area per unit time (LT^{-1});
- $h_{i,j,k}$ is the head, or water-table elevation in the cell from which the evapotranspiration occurs (L);
- EVTR is the maximum possible value of RET (LT^{-1});
- SURF is the ET surface elevation, or the water-table elevation at which this maximum value of evapotranspiration loss occurs (L); and
- EXDP is the cutoff or extinction depth (L), such that when the distance between $h_{i,j,k}$ and SURF exceeds EXDP, evapotranspiration ceases.

In implementing the finite-difference approach, the volumetric rate of evapotranspiration loss from a given cell is required. This is given as the product of the loss rate per unit area, and the horizontal surface area, $DEL R_j DEL C_i$ of the cell from which the loss occurs:

$$QET = RET(DEL R_j DEL C_i) \quad (6-13)$$

where QET is the evapotranspiration loss, in volume of water per unit time (L^3T^{-1}), through the area $DEL R_j DEL C_i$.

If the maximum value of QET (corresponding to EVTR) is designated QETM, equation 6-12 can be expressed in terms of volumetric discharge as

$$QET = QETM \quad h_{i,j,k} > SURF \quad (6-14A)$$

$$QET = QETM \frac{h_{i,j,k} - (SURF - EXDP)}{EXDP} \quad (SURF - EXDP) \leq h_{i,j,k} \leq SURF \quad (6-14B)$$

$$QET = 0 \quad h_{i,j,k} < SURF - EXDP \quad (6-14C)$$

Figure 6-13 shows a graph of evapotranspiration loss, QET, from a cell and head in the cell based on equation 6-14. Comparison of the ET function with the river or drain functions shows that the three are mathematically similar, except that the linear part of the ET function is bounded at both ends by constant values, rather than only at one end. Note that QET (eq. 6-14 and figure 6-13) represents outflow from the ground-water system, which is counter to the convention in MODFLOW of representing stresses as inflow. QET was developed in terms of outflow because it can be conceptually confusing to view evapotranspiration as a negative quantity.

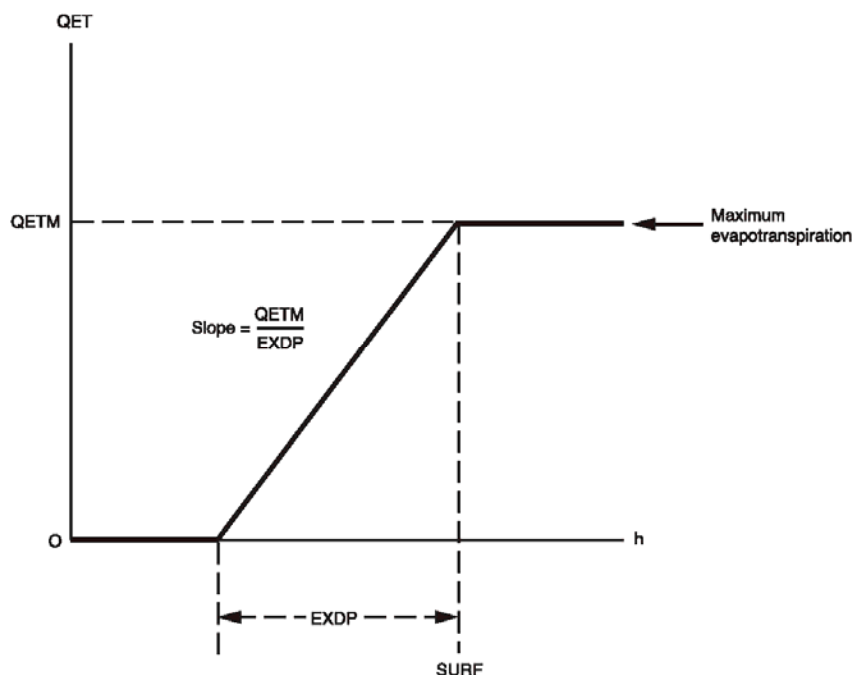


Figure 6-13. Plot of volumetric evapotranspiration, QET, as a function of head, h , in a cell where EXDP is the cutoff depth and SURF is the ET surface elevation. (Modified from McDonald and Harbaugh, 1988.)

Evapotranspiration is conceptualized as an areal phenomenon like recharge simulated by the Recharge Package. Evapotranspiration can be drawn from only one cell in each vertical column beneath the map area of each horizontal grid location. As with the Recharge Package, the user designates the layer for each horizontal location using one of three options. Under the first option, evapotranspiration is always drawn from the uppermost layer of the model; under the second option, the user specifies the cell within the vertical column at i,j from which the evapotranspiration is to be taken. In using either of these options, the computed evapotranspiration has no influence on the simulation if the designated cell is either a no-flow cell or a constant-head cell. Under the third option, evapotranspiration is taken from the uppermost variable-head cell in each vertical column, provided no constant-head cell is above it. A constant-head cell above a variable-head cell prevents evapotranspiration from occurring.

For each horizontal cell location, (i,j) , and for each stress period (unless an option is exercised to use prior values) the ET package reads values of EVTR (maximum evapotranspiration loss per unit area per unit time), with dimensions LT^{-1} . These rates are immediately multiplied by cell areas, $(DEL R_j)(DEL C_i)$, to obtain the maximum volumetric rate of evapotranspiration from each cell. Likewise, values of SURF (the ET surface elevation or water-table elevation at which evapotranspiration is maximum) and EXDP (the cutoff depth or extinction depth) are read for each horizontal cell location for each stress period. If the second option is used, the evapotranspiration layer is read for each horizontal location in the grid.

Equation 6-14 is rewritten in terms of inflow to a model cell as is the convention in MODFLOW. Equation 6-14C is further rearranged algebraically for convenient implementation in the code. Also, i,j subscripts are added to indicate the variables that have a different value for each horizontal cell location:

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$$QETin_{i,j,k} = -QETM_{i,j} \quad \text{for } h_{i,j,k} > SURF_{i,j} \quad (6-15A)$$

$$QETin_{i,j,k} = \frac{-QETM_{i,j}}{EXDP_{i,j}} h_{i,j,k} - QETM_{i,j} + \frac{QETM_{i,j} SURF_{i,j}}{EXDP_{i,j}} \quad \text{for } (SURF_{i,j} - EXDP_{i,j}) \leq h_{i,j,k} \leq SURF_{i,j} \quad (6-15B)$$

$$QETin_{i,j,k} = 0 \quad \text{for } h_{i,j,k} < SURF_{i,j} - EXDP_{i,j} \quad (6-15C)$$

In terms of the expressions HCOF and RHS of equation 6-1, evapotranspiration is added to the flow equation as follows:

1. if $h_{i,j,k} > SURF_{i,j}$, $QETM_{i,j}$ is added to $RHS_{i,j,k}$;
2. if $(SURF_{i,j} - EXDP_{i,j}) \leq h_{i,j,k} \leq SURF_{i,j}$, $\frac{-QETM_{i,j}}{EXDP_{i,j}}$ is added to $HCOF_{i,j,k}$, and $-QETM_{i,j} + \frac{QETM_{i,j} SURF_{i,j}}{EXDP_{i,j}}$ is subtracted from $RHS_{i,j,k}$; and
3. if $h_{i,j,k} < SURF_{i,j} - EXDP_{i,j}$, no changes are made in the terms $HCOF_{i,j,k}$ or $RHS_{i,j,k}$.

The value of SURF, the water-table elevation at which evapotranspiration is maximum, is sometimes assumed to be the land-surface elevation; however, the maximum evapotranspiration usually occurs at some depth below land surface because plants can generally withdraw the maximum amount from ground water when a fraction of the root zone is in contact with the water table. The cutoff or extinction depth, EXDP, is frequently assumed to be the distance from SURF down to the bottom of the deepest roots. Considerable variation can be introduced by climatic factors and plant type.

Summary of Stress Packages

Figure 6-14 schematically shows flow into a model cell containing a stress, Qin , and head in the cell, h , for all six stress packages documented in this chapter. These plots illustrate qualitatively the functional differences among the stress packages. Notice that the flows by the Well and Recharge Packages are independent of head in the model cell, whereas the flows calculated for the remaining packages are dependent on head in the model cell. The well inflow is negative, illustrating the typical situation of a pumping well, but any positive or negative value can be applied. Similarly, the recharge inflow is shown as positive, but negative values can be used as desired. The plot for evapotranspiration has been reversed to show inflow to the model rather than outflow as in figure 6-13.

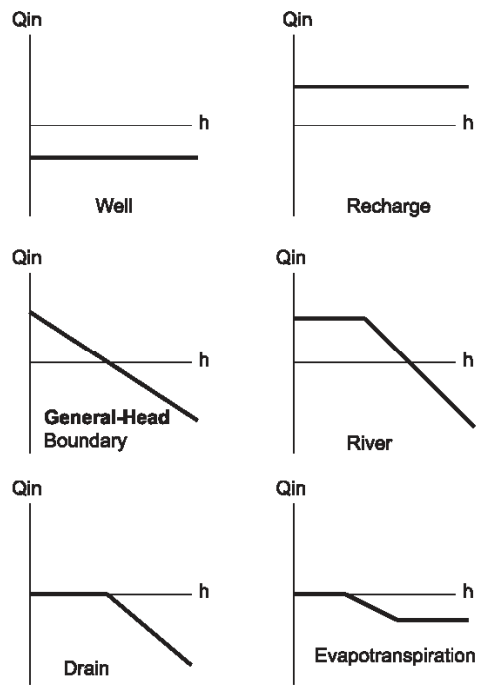


Figure 6-14. Comparison of stresses.