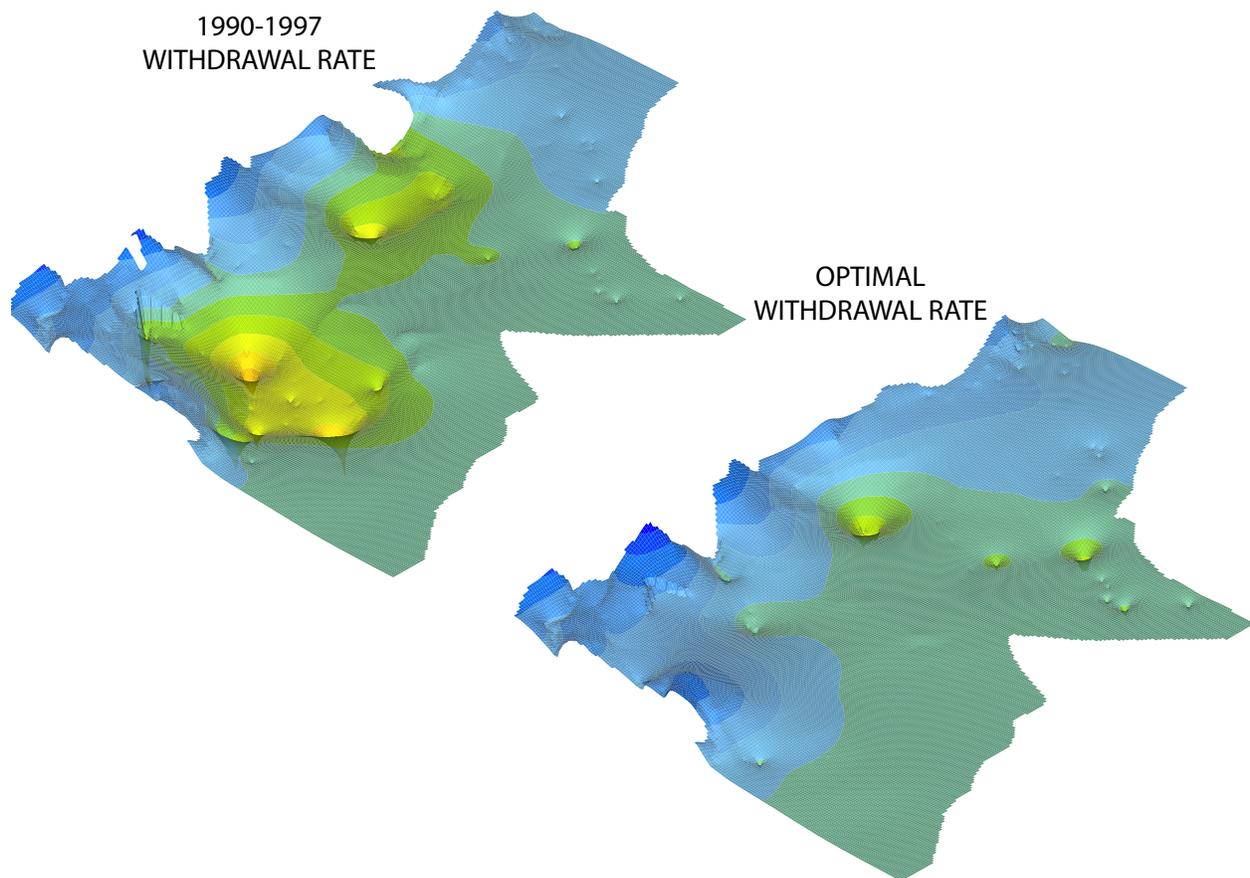


Prepared in cooperation with the
U.S. Army Corps of Engineers, Memphis District and the
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CONJUNCTIVE-USE OPTIMIZATION MODEL AND SUSTAINABLE-YIELD ESTIMATION FOR THE SPARTA AQUIFER OF SOUTHEASTERN ARKANSAS AND NORTH- CENTRAL LOUISIANA

Water-Resources Investigations Report 03-4231



U.S. Department of the Interior
U.S. Geological Survey

Cover: Oblique view of the resultant steady-state simulated hydraulic heads using baseline 1990-1997 withdrawal rate and the optimal withdrawal rate from scenario 3A.

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By Paul W. McKee, Brian R. Clark, and John B. Czarnecki

U.S. GEOLOGICAL SURVEY

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Little Rock, Arkansas
2004

U.S. DEPARTMENT OF THE INTERIOR

GALE A. NORTON, Secretary

U.S. GEOLOGICAL SURVEY

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CONVERSION FACTORS AND ABBREVIATIONS

	Multiply	By	To obtain
Length			
	inch (in.)	2.54	centimeter (cm)
	foot (ft)	0.3048	meter (m)
	mile (mi)	1.609	kilometer (km)
Area			
	square foot (ft ²)	0.09290	square meter (m ²)
	square mile (mi ²)	2.590	square kilometer (km ²)
Flow rate			
	cubic foot per second (ft ³ /s)	0.02832	cubic meter per second (m ³ /s)
	cubic foot per day (ft ³ /d)	0.02832	cubic meter per day (m ³ /d)
	gallon per minute (gal/min)	0.06308	liter per second (L/s)
	gallon per day (gal/d)	0.003785	cubic meter per day (m ³ /d)
	million gallons per day (Mgal/d)	0.04381	cubic meter per second (m ³ /s)
	billion gallons per day (Bgal/d)	43.81	cubic meter per second (m ³ /s)
Hydraulic conductivity			
	foot per day (ft/d)	0.3048	meter per day (m/d)

In this report, vertical coordinate information is referenced to the National Geodetic Vertical Datum of 1929 (NGVD of 1929). Horizontal coordinate information is referenced to North American Datum of 1927 (NAD 27).

Altitude, as used in this report, refers to distance above or below NGVD of 1929.

CONJUNCTIVE-USE OPTIMIZATION MODEL AND SUSTAINABLE-YIELD ESTIMATION FOR THE SPARTA AQUIFER OF SOUTHEASTERN ARKANSAS AND NORTH-CENTRAL LOUISIANA

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ABSTRACT

Conjunctive-use optimization modeling was done to assist water managers and planners by estimating the maximum amount of ground water that hypothetically could be withdrawn from wells within the Sparta aquifer indefinitely without violating hydraulic-head or stream-discharge constraints. The Sparta aquifer is largely a confined aquifer of regional importance that comprises a sequence of unconsolidated sand units that are contained within the Sparta Sand. In 2000, more than 35.4 million cubic feet per day (Mft³/d) of water were withdrawn from the aquifer by more than 900 wells, primarily for industry, municipal supply, and crop irrigation in Arkansas. Continued, heavy withdrawals from the aquifer have caused several large cones of depression, lowering hydraulic heads below the top of the Sparta Sand in parts of Union and Columbia Counties and several areas in north-central Louisiana. Problems related to overdraft in the Sparta aquifer can result in increased drilling and pumping costs, reduced well yields, and degraded water quality in areas of large drawdown.

A finite-difference ground-water flow model was developed for the Sparta aquifer using MODFLOW, primarily in eastern and southeastern Arkansas and north-central Louisiana. Observed aquifer conditions in 1997 supported by numerical simulations of ground-water flow show that continued pumping at withdrawal rates representative of 1990 - 1997 rates cannot be sustained indefinitely without causing hydraulic heads to drop substantially below the top of the Sparta Sand in

southern Arkansas and north-central Louisiana. Areas of ground-water levels below the top of the Sparta Sand have been designated as Critical Ground-Water Areas by the State of Arkansas. A steady-state conjunctive-use optimization model was developed to simulate optimized surface-water and ground-water withdrawals while maintaining hydraulic-head and streamflow constraints, thus determining the “sustainable yield” for the aquifer.

Initial attempts to estimate sustainable yield using simulated 1997 hydraulic heads as initial heads in Scenario 1 and 100 percent of the baseline 1990-1997 withdrawal rate as the lower specified limit in Scenario 2 led to infeasible results. Sustainable yield was estimated successfully for scenario 3 with three variations on the upper limit of withdrawal rates. Additionally, ground-water withdrawals in Union County were fixed at 35.6 percent of the baseline 1990-1997 withdrawal rate in Scenario 3. These fixed withdrawals are recognized by the Arkansas Soil and Water Conservation Commission to be sustainable as determined in a previous study. The optimized solutions maintained hydraulic heads at or above the top of the Sparta Sand (except in the outcrop areas where unconfined conditions occur) and streamflow within the outcrop areas was maintained at or above minimum levels. Scenario 3 used limits of 100, 150, and 200 percent of baseline 1990-1997 withdrawal rates for the upper specified limit on 1,119 withdrawal decision variables (managed wells) resulting in estimated sustainable yields ranging from 11.6 to 13.2 Mft³/d in Arkansas and 0.3 to 0.5 Mft³/d in Louisiana. Assuming the total

water demand is equal to the baseline 1990-1997 withdrawal rates, the sustainable yields estimated from the three scenarios only provide 52 to 59 percent of the total ground-water demand for Arkansas; the remainder is defined as unmet demand that could be obtained from large, sustainable surface-water withdrawals.

INTRODUCTION

The Sparta aquifer is a confined aquifer of regional importance within the Mississippi Embayment aquifer system. It consists of varying amounts of unconsolidated sand, interstratified with silt and clay lenses within the Sparta Sand of the Claiborne Group. It extends through eastern and southeastern regions of Arkansas, northern Louisiana, and portions of Texas, Mississippi, and Tennessee (fig. 1). The Sparta Sand ranges in total thickness from 100 to 1,000 feet (ft) and is stratigraphically positioned and confined between the overlying Cook Mountain Formation and the underlying Cane River Formation (Payne, 1968).

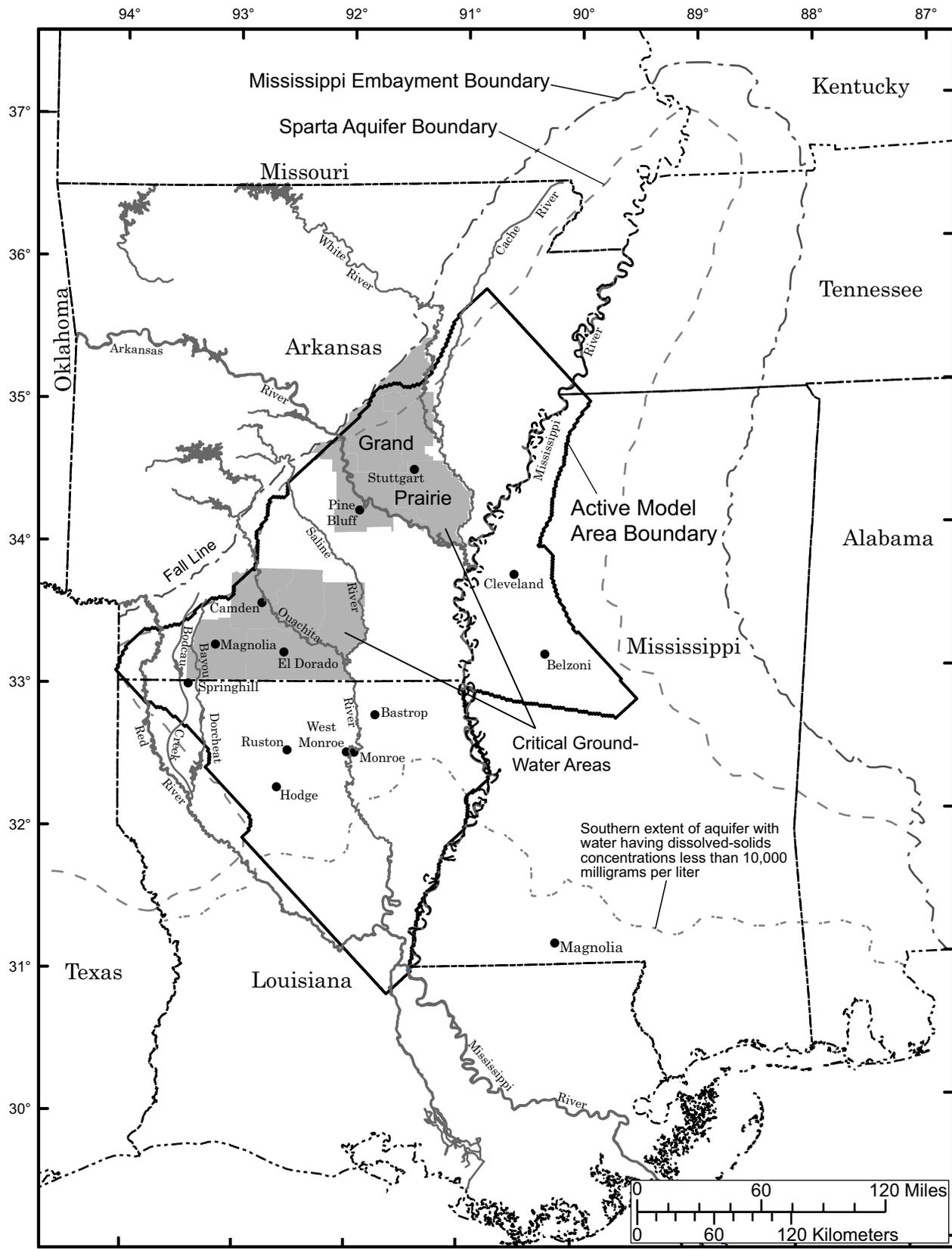
The Sparta aquifer of southeastern Arkansas and north-central Louisiana covers more than 32,500 mi² and is a major source of water for municipal, industrial, and agricultural uses. Since the early 1900's, the Sparta aquifer has provided abundant water of high quality primarily for industry and public supply. Total ground-water withdrawals from the Sparta aquifer in Arkansas have increased from 14.2 Mft³/d in 1965 (Halberg and Stephens, 1966) to an estimated 35.4 Mft³/d in 2000 (T.W. Holland, U.S. Geological Survey, written commun., 2002)—an increase of 150 percent. Through the years, the growing water demands of industries and municipalities have resulted in ground-water withdrawals from the Sparta aquifer that substantially exceed recharge to the aquifer. In addition, where ground-water level declines in the Mississippi River Valley alluvial aquifer are decreasing well yields, the Sparta aquifer is increasingly used for crop irrigation. These large withdrawals have caused considerable drawdown in the potentiometric surface resulting in the development of large cones of depression over 175 feet deep. Hydraulic heads (often used interchangeably with water-level altitude or potentiometric surface) are now below the top of the Sparta Sand in some parts of Union and Columbia Counties, Arkansas, and in several parishes of Louisiana (T.P. Schrader, U.S. Geological Survey, written commun., 2003).

Several Arkansas counties within the extent of the Sparta aquifer have been designated Critical Ground-Water Areas (CGWA) by the Arkansas Soil and Water Conservation Commission (ASWCC) because hydraulic heads in the Sparta aquifer are below the top of the Sparta Sand (fig. 1). This indicates that withdrawal rates have already exceeded those that can be sustained indefinitely. A recently calibrated flow model of the Sparta aquifer (McKee and Clark, 2003) simulates this condition and indicates these areas will expand if withdrawal rates equivalent to those representing the period from 1990 to 1997 continue.

To estimate withdrawal rates that could be sustained relative to the constraints of critical ground-water designation, the U.S. Geological Survey (USGS), in cooperation with the Memphis District of the U.S. Army Corps of Engineers (MCOE) and the ASWCC, applied conjunctive-use optimization modeling to the existing Sparta aquifer flow model during 2000-2002. Optimized withdrawal rates form the basis for estimating sustainable yield from the Sparta aquifer. Additionally, the conjunctive-use optimization model determines the maximum available withdrawals from major streams for supplementing ground water to meet the total water demand. The Sparta aquifer flow model was modified and recalibrated for the purpose of evaluating potential pumping scenarios and optimizing withdrawal rates to determine sustainable yield for the Sparta aquifer. Documentation of the development, calibration, and application of the Sparta aquifer flow model is given by McKee and Clark (2003).

Purpose and Scope

This report describes the application and evaluation of a conjunctive-use optimization model that was developed from a linear programming routine utilizing the calibrated Sparta aquifer flow model (McKee and Clark, 2003). The objective of the optimization modeling was to simulate maximum ground-water withdrawals from the Sparta aquifer and surface-water withdrawals from major streams in the Sparta outcrop/subcrop area while maintaining desirable hydraulic heads in the aquifer and streamflow in the outcrop. In essence, the objective was to estimate the "sustainable yield"—indefinite ground-water withdrawals without compromising the integrity of the Sparta aquifer in Arkansas and Louisiana or streamflow in seven streams. Although the numerical flow model includes western Mississippi for hydrologic boundary purposes, no



(Modified from Arthur and Taylor, 1990)

Figure 1. Extent of Sparta aquifer and location of model area with Arkansas Soil and Water Conservation Commission Critical Ground-Water Areas shaded.

hydraulic-head data from Mississippi were used for calibration; therefore, sustainable-yield estimations for Mississippi are not included in this report. The results of the optimization modeling can provide water managers and policy makers with information that can be used to assist in management of the ground-water resources of the Sparta aquifer.

Previous Studies

Optimization management of ground-water systems has been explored and successfully applied by many researchers. Reichard (1995) offers a thorough review of these studies discussing an optimization approach to ground-water/surface-water management. For the Sparta aquifer, sustainable yield was estimated in Union County using one control point of specified minimum acceptable hydraulic head and varying Union County pumpage within the ground-water model to achieve the target hydraulic head (Hays, 2000). Results from Hays (2000) determined that 28 percent of the 1997 withdrawal rate in Union County could be sustained while maintaining acceptable hydraulic heads above the top of the Sparta Sand Formation at one control location centered in the deepest cone of depression. This rate has since been recognized by ASWCC as a goal to limit pumping from the Sparta aquifer in Union County through conservation and alternate water supplies. The study described in this report, however, is the first to explore and describe conjunctive-use optimization management for the Sparta aquifer, although similar investigations have been conducted in the overlying Mississippi River Valley alluvial aquifer by Peralta and others (1985) for the Grand Prairie region and most recently by Czarnecki and others (2003) for the alluvial aquifer in northeastern Arkansas.

Study Area Description

The study area comprises terrain in southeastern Arkansas, north-central Louisiana, and northwestern Mississippi (fig. 1). Land-surface altitudes range from more than 500 ft along the western boundary and outcrop recharge zones to less than 100 ft along the Mississippi River. The principal drainages are the Mississippi, Arkansas, Saline, Ouachita, and Red Rivers, Bayou Dorcheat, and Bodcau Creek. Mean annual precipitation is approximately 50 inches (Freiwald,

1985). Water use from the Sparta aquifer is for municipal supply, manufacturing of forest, chemical, and other industrial products, agriculture, and aquaculture.

Hydrogeology of the Sparta Aquifer

The Sparta aquifer is composed of a sequence of unconsolidated sand, silt, and clay units within the Sparta Sand of the Mississippi Embayment aquifer system and extends from south Texas, northward into Louisiana, Arkansas, and Tennessee, and eastward into Mississippi (fig. 1). A detailed description of the hydrogeology of the Sparta aquifer is discussed in numerous reports including Payne (1968), Hosman and others (1968), and Petersen and others (1985). A large part of the Sparta aquifer is overlain by Quaternary-age alluvial and terrace deposits that form the Mississippi River Valley alluvial aquifer. The Sparta Sand ranges in total thickness from 100 to 1,000 ft and is stratigraphically positioned between the overlying Cook Mountain Formation and underlying Cane River Formation. These two formations are low-permeability, fine-grained, clay-rich units that confine flow within the more permeable Sparta Sand. The Sparta Sand outcrops on both the west and east sides of the Mississippi Embayment. North of 35 degrees latitude, the Sparta Sand becomes part of the Memphis Sand as the Cane River Formation undergoes a facies change where deposits change from marine clays to sand (Hosman and others, 1968).

The Sparta aquifer is unconfined in the outcrop area and becomes confined as it dips toward the central axis of the Mississippi Embayment (generally corresponding with the Mississippi River) and southward toward the Gulf of Mexico where it is deeply buried in the subsurface (Hosman and others, 1968). The total sand thickness within the aquifer generally ranges from 200 to 600 ft. Sources of recharge to the Sparta aquifer are infiltration of precipitation on the outcrop, leakage from streams in the outcrop, leakage from the alluvium where the Sparta subcrops, and leakage from adjacent aquifers through confining layers where the vertical hydraulic gradient is towards the Sparta aquifer. Most natural discharge is to streams in the outcrop and to adjacent units with lower potentiometric surfaces. Generalized ground-water flow in the Sparta aquifer is from the outcrop areas to the axis of the Mississippi Embayment except where heavy withdrawals in some areas have caused a reversal of flow towards the resultant cones of depression (Hosman and others, 1968).

The horizontal hydraulic conductivity of the Sparta aquifer generally ranges from 10 to 200 ft/d with an average of about 70 ft/d across the extent of the Mississippi Embayment. The average specific storage for the Sparta aquifer is about 6×10^{-7} /ft and storativity is 3×10^{-4} based on an average thickness of 500 ft (Hosman and others, 1968).

Sparta Aquifer Development and Associated Effects

The earliest known withdrawals from the Sparta aquifer in the study area began in 1898 in Pine Bluff, Arkansas, for industrial use. It was not until about 1920 that substantial withdrawals from the aquifer began for rice irrigation in the Grand Prairie area. Since that time, the aquifer has been heavily used for industrial and municipal supply, providing water of excellent quality. The principal areas of ground-water withdrawal in the study area are in Jefferson, Union, and Arkansas Counties in Arkansas, Ouachita Parish in Louisiana, and Bolivar County in Mississippi (all counties and parishes shown on figure 3). Currently, more than 1,760 wells are screened in the Sparta aquifer in Arkansas and Louisiana, and more than 35.4 Mft³/d was withdrawn from the aquifer in Arkansas in 2000 from more than 900 wells (T.W. Holland, U.S. Geological Survey, written commun., 2003). Individual wells completed in the Sparta aquifer generally yield 100 to 500 gal/min, with less common rates up to 1,200 gal/min. Continued ground-water withdrawal through time has caused the potentiometric surface to decline more than 175 ft in some areas in the aquifer (Joseph, 1998; T.P. Schrader, U.S. Geological Survey, written commun., 2003), which also has affected natural flow directions.

Acknowledgements

The conjunctive-use optimization model routine used in this report is an adaptation of enhancements made by Brian Wagner (U.S. Geological Survey) to MODMAN 3.0. Wesley Danskin (U.S. Geological Survey) provided guidance on the use of MODMAN and the large-scale optimization solver MINOS. Stream-flow constraints for streams used in the optimization model were compiled by Steve Loop (Arkansas Soil and Water Conservation Commission) for six streams

in Arkansas and by Paul Ensminger (U.S. Geological Survey) for one stream in Louisiana.

CONJUNCTIVE-USE OPTIMIZATION MODEL

A conjunctive-use optimization model, hereafter referred to as the “optimization model,” was developed for the Sparta aquifer using three modeling packages known as MODFLOW-96, MODMAN, and MINOS (Harbaugh and McDonald, 1996; Greenwald, 1998; Murtaugh and Sanders, 1998). The optimization model determined maximum total surface-water and ground-water withdrawals from the system while maintaining desirable hydraulic heads in the aquifer and stream-flows in the outcrop.

Ground-Water Flow Model

The Sparta aquifer ground-water flow model, hereafter referred to as the “ground-water model,” used in the optimization model was developed, calibrated, and documented by McKee and Clark (2003). Characteristics of the ground-water model are listed in table 1. The ground-water model simulates flow in the Sparta aquifer covering 38,220 mi² in southeastern Arkansas, north-central Louisiana, and western Mississippi (fig. 1). The ground-water model is a two-layer, transient, three dimensional (3D) numerical model with a uniform grid spacing of 1 mi² developed using MODFLOW-2000 (Harbaugh and others, 2000). The quasi-3D confining unit between the two layers represents a mixture of continuous and discontinuous clay lenses in much of south-central Arkansas and north-central Louisiana separating the upper and lower water-bearing zones of the Sparta aquifer. Calibration of the ground-water model involved a simulation period from 1898 to 1997 and used non-linear regression methods (Hill and others, 2000) to match a total of 795 hydraulic-head observations from 316 wells over four time periods—1970, 1985, 1990, and 1997. Field measurements were not available to compare with simulated fluxes in and out of the Sparta outcrop streams. Thorough analysis of the calibration indicated that the model could simulate observed conditions in the Sparta aquifer within acceptable error. The calibrated flow model has a root mean square error (RMSE) of 17.99 ft for all observations, and 18.93 ft in 1997 (McKee and Clark, 2003). Calibration statistics such as the mean, mean

absolute, and root-mean-square difference between observed and simulated hydraulic head for all observations are listed in table 1.

Table 1. Characteristics of the Sparta aquifer ground-water flow model

[RMSE, root mean square error; Mft³/d, million cubic feet per day]

Characteristic	Value
Area (square miles)	38,220
Model layers	2
Calibration period	1898 to 1997
Stress periods	28
Stress period length, range (years)	1 to 20
Cells with wells representing 1996 distribution of withdrawals	1,152
Baseline 1990-1997 withdrawals (Mft ³ /d)	32
Hydraulic conductivity (feet per day)	2.5 to 47.9
Specific yield	0.20
Specific storage (feet ⁻¹)	3.54x10 ⁻⁶ to 1.5x10 ⁻⁴
River cells (defines streams included in the model)	488
Hydraulic head observation periods	1990, 1985, 1990, 1997
Hydraulic head observations, all four periods	795
Range in observed hydraulic head values in 1997 (altitude, in feet relative to NGVD of 1929)	-224 to +306
Mean difference between observed and simulated hydraulic head, all four periods (feet)	0.36
Mean absolute difference between observed and simulated hydraulic head, all four periods (feet)	13.78
Root mean square error (RMSE) between observed and simulated hydraulic head, all four periods (feet)	17.99

Ground-water flow in the Sparta aquifer was simulated from 1898 through 1997. Steady-state conditions were simulated in the ground-water model for the year 1898 and represent aquifer predevelopment conditions. The ground-water model then simulates 100 years of system response to stress by dividing the model into 27 stress periods (28 total, including steady-state predevelopment) with each stress period representing a period of time for which ground-water withdrawals were relatively unchanged. The length of stress periods ranges from 1 to 20 years; therefore, seasonal

effects are not simulated. The most recent stress period (1990-1997) is 8 years.

Boundary conditions include no flow on the bottom, western edge, and a portion of the eastern edge of the model, as well as head-dependent flow boundaries on much of the top, northern, southern, and portions of the eastern edge of the model. Specified flow boundaries exist along the western edge of the model in what is known as the outcrop-subcrop area of the Sparta aquifer. In addition, 15 streams in Arkansas and Louisiana were simulated in the Sparta aquifer outcrop area using head-dependent flow boundaries (McKee and Clark, 2003).

Hydraulic properties of the ground-water system are represented in the Sparta aquifer flow model by the distribution of faults, hydraulic conductivity, storage, and vertical leakance parameters throughout the model domain. The final parameter estimates of the model are considered reasonable estimates for the type of material and conditions found in the Sparta aquifer. Horizontal hydraulic conductivity values ranged from 2.5 to 47.9 ft/d. Horizontal hydraulic conductivity of horizontal flow boundaries, representing faults, range from 5.0 x 10⁻⁶ to 1.0 ft/d. Specific yield values were fixed at 0.2. Specific storage values range from 3.54 x 10⁻⁶ to 1.5 x 10⁻⁴ft⁻¹. The hydraulic conductivity value used in the calculation of riverbed conductance representing leakance through overlying units, ranged from 3.1 x 10⁻⁵ to 9.3 x 10⁻³. Recharge ranges from 0.11 to 1.10 inches per year, with the highest recharge applied to the northern portion of the outcrop-subcrop area.

Optimization Model

For optimization-modeling described in this report, modifications were made to MODMAN 4.0 to (1) incorporate stream withdrawal cells as decision variables, (2) allow specification of streamflow constraints, and (3) account for streamflow water budgeting. Modifications to the MODMAN code were initially provided by Brian Wagner (U.S. Geological Survey) in a modification of MODMAN 3.0, and adapted by John Czarnecki (U.S. Geological Survey) to MODMAN 4.0. In addition, the ability to aggregate wells within a subarea of the model and to treat an aggregate-well pumping rate as a single decision variable was added to MODMAN 4.0. However, that ability was not utilized; instead, 1,152 ground-water-withdrawal decision variables and 7 surface-water-withdrawal decision variables were specified.

The optimization modeling process (fig. 2) begins with the calibration and adaptation of a MODFLOW-based ground-water flow model to be compatible with the optimization modeling software (MODMAN 4.0). Adaptation entailed the conversion of the flow model from MODFLOW 2000 (Hill and others, 2000) to MODFLOW 96 (McDonald and Harbaugh, 1988; Harbaugh and McDonald, 1996), and verifying that the results were the same. Steady-state conditions were selected (rather than transient conditions) because the maximized withdrawals are intended to represent sustainable yield of the system (a rate that can be maintained indefinitely).

A management problem is formulated to maximize a parameter, such as water production from ground water and surface water, within selected constraints, such as maintaining hydraulic heads in the

aquifer above a minimum altitude or maintaining a minimum amount of streamflow. An input objective function is prepared from the component of the management problem for a conjunctive-use version of MODMAN 4.0 to generate response coefficients for each specified withdrawal cell in the model. The response-coefficient matrix consists of changes in hydraulic head or streamflow at each constraint location that occur in response to pumping at a single well or river cell at a unit rate (Greenwald, 1998; Ahlfeld and Mulligan, 2000).

After all the response coefficients are calculated, they are combined to form a data-input set along with hydraulic-head and streamflow constraints, and are formulated as a linear program in mathematical programming system (MPS) format. The linear optimization program is run under MINOS. If a feasible solution

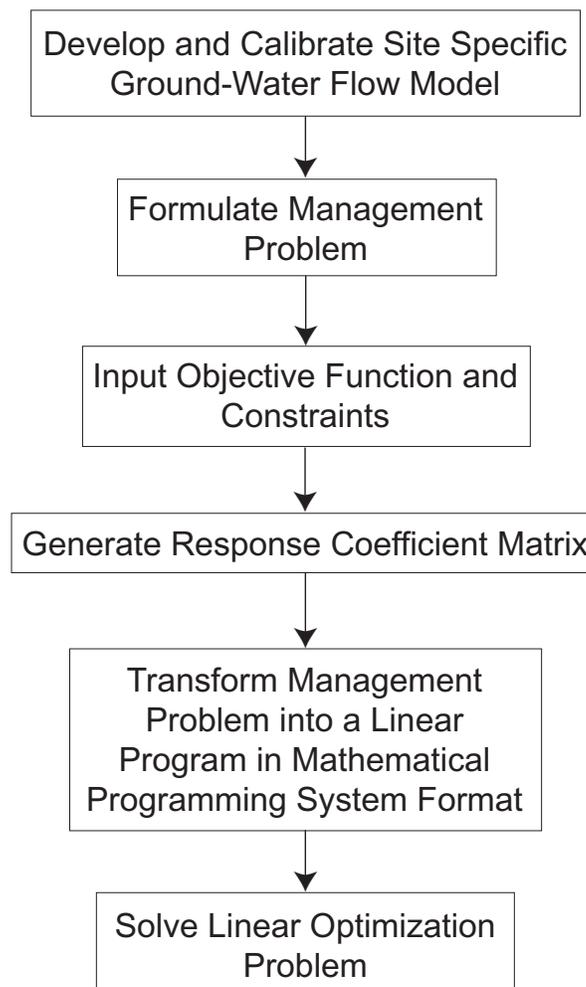


Figure 2. Flow chart of optimization modeling process (modified from Greenwald, 1998)

exists, MINOS will provide estimates of optimal (maximum) values of ground-water and surface-water withdrawals. MINOS also identifies points in the model where hydraulic-head or streamflow constraints have been reached.

Optimal ground-water withdrawal rates calculated using the optimization model were evaluated by applying them in the flow model, to compare the resulting simulated hydraulic head and streamflow against the specified hydraulic-head and streamflow constraints. In a strictly linear model, such as one for a confined aquifer, ground-water flow is a function of hydraulic head through only the hydraulic-gradient term in Darcy's law:

$$Q = -K \frac{dh}{dl} A \quad (1)$$

where Q is ground-water flow (in cubic feet per day); K is the hydraulic conductivity (in feet per day); $\frac{dh}{dl}$ is the hydraulic gradient, dimensionless; h is hydraulic head, (in feet); l is a distance over which the gradient is measured (in feet); and A is the cross-sectional area through which flow occurs (in square feet).

For unconfined conditions, A also is a function of hydraulic head. If changes in hydraulic head are small relative to the total saturated thickness, then A will remain about the same. However, if substantial change in saturated thickness occurs, A can change appreciably, because

$$A = bw \quad (2)$$

where b is the saturated thickness (in feet), which varies with hydraulic head; and w is the width through which flow occurs (in feet).

Problem Formulation

The optimization model was formulated as a linear programming problem with the objective of maximizing water production from wells and from streams subject to: (1) maintaining streamflow at or above minimum specified rates; (2) maintaining ground-water levels at or above specified levels; and (3) limiting

ground-water withdrawals to a maximum of 200 percent of the baseline 1990-1997 withdrawal rate. Steady-state conditions were selected (rather than transient conditions) because the maximized withdrawals are intended to represent sustainable yield of the system (a rate that can be maintained indefinitely). In this model, the decision variables (a term used in optimization modeling to identify variables that can be part of a management scheme) are the withdrawal rates at 1,152 model cells corresponding to well locations and at 7 model cells corresponding to stream reaches on the out-crop/subcrop area of the Sparta aquifer.

Constraints (specified limits) on hydraulic head and streamflow are designated at specific cells called control points within the ground-water model grid (fig. 3). These control points and the constraints assigned must be satisfied for a feasible solution of the objective function.

Objective Function

The objective of the optimization model is to maximize water production from ground-water and surface-water sources. The objective function of the optimization model has the form:

$$\text{maximize } z = \sum q_{well} + \sum q_{river} \quad (3)$$

where z is the total managed water withdrawal, in million cubic feet per day;

$\sum q_{well}$ is the sum of ground-water withdrawal rates from all managed wells, in million cubic feet per day; and

$\sum q_{river}$ is the sum of surface-water withdrawal rates from all managed stream reaches, in million cubic feet per day.

Streamflow Constraints

Streamflow is regulated in Arkansas by ASWCC for purposes of maintaining water quality, navigation, and species habitat. Streamflow constraints are specified as the minimum amount of flow required at individual river model cells. The equation governing the relation between streamflow constraints and flow into and out of a stream is

$$q_{head}^R + \sum q_{overland}^R \pm \sum q_{ground\ water}^R - \sum q_{diversions}^R - \sum q_{river}^R \geq q_{minimum}^R \quad (4)$$

where q_{head}^R is the flow rate into the head of stream reach R , in cubic feet per day;
 $\sum q_{overland}^R$ is the sum of all overland and tributary flow to stream reach R , in cubic feet per day;
 $\sum q_{ground\ water}^R$ is the net sum of all groundwater flow to or from stream reach R , in cubic feet per day;
 $\sum q_{diversions}^R$ is the sum of all surface-water diversions from stream reach R , in cubic feet per day;
 $\sum q_{river}^R$ is the sum of all potential withdrawals, not including diversions, from stream reach R , in cubic feet per day; and
 $q_{minimum}^R$ is the minimum permissible surface-water flow rate for stream reach R , in cubic feet per day.

Optimal surface-water withdrawals were calculated at seven points along seven streams within the Sparta aquifer outcrop/subcrop area for which streamflow constraints were specified. These streams include the Arkansas River, Cache River, Bayou Dorcheat, Ouachita River, Red River, Saline River, and White River (fig. 3), all of which have mean annual flows greater than 1,000 ft³/s within the Sparta outcrop/sub-

crop area. Streamflow constraint data were determined using the median August mean daily flow values for the period of record for each stream (Steve Loop, Arkansas Soil and Water Conservation Commission, written commun., 2002; Paul Ensminger, U.S. Geological Survey, written commun., 2002). All streamflow data were based on historical streamflow data obtained from USGS streamflow gaging stations located on nearby reaches of these seven streams. Streamflow constraints were specified in the most downstream point of each stream segment (seven total). Flow into the most upstream point of each stream was specified based on mean annual flow. Overland flow was distributed equally at river model cells within a stream reach based on the difference in long-term mean streamflow between neighboring streamflow-gaging stations for a specific stream reach; or if such data were unavailable, areal estimates of runoff based on drainage areas were used (Elton Porter, U.S. Geological Survey, written commun., 2001). Surface-water diversion rates that occurred in 2000 (T.W. Holland, U.S. Geological Survey, written commun., 2002) were subtracted from specified overland flow at the appropriate river cells. Overland flow, streamflow constraints, and optimum withdrawal rates for each stream are listed in table 2.

Table 2. Flow and optimized surface-water withdrawals for major streams within the Sparta aquifer outcrop area specified within the conjunctive-use optimization model for all three scenarios (3A-3C)

[Mft³/d, million cubic feet per day; August mean is the median August mean daily flow for period of record]

Stream name	Number of ground-water model cells	Flow in stream entering outcrop/subcrop area (Mft ³ /d)	Net overland flow per river cell (Mft ³ /d)	Flow in stream leaving outcrop/subcrop area (Mft ³ /d)	Streamflow constraint (Mft ³ /d)	Rationale for value of constraint	Period of record	Optimized surface-water withdrawal (Mft ³ /d)
Arkansas	13	3,751	-0.006	3,751	1,492	August mean	1970-1997	2,259
Bayou Dorcheat	26	67	1.2	98	2	August mean	1929-1932, 1937-1979	96
Cache	50	109	0.2	119	36	August mean	1988-2000	83
Ouachita	39	340	9.6	713	171	August mean	1929-1997	542
Red	55	1,700	2.4	1,832	473	August mean	1945-1997	1,359
Saline	14	74	0.7	85	16	August mean	1941-1999	69
White	11	2,316	4.6	2,366	1,378	August mean	1981-1995	988

Hydraulic-Head Constraints

Equation 3 is computed such that the following constraints are maintained:

$$h_c \geq h_{\text{minimum}} \quad (5)$$

where h_c is the hydraulic-head altitude at constraint location c ; and

h_{minimum} is the hydraulic-head altitude at the top of the Sparta Sand.

To accommodate the ASWCC Critical Ground-Water Area criterion that hydraulic heads within the Sparta aquifer should remain above the top of the Sparta Sand, hydraulic-head constraints were specified at 2,549 model cells (fig. 3).

A uniform distribution of model cells with an approximate 5-mile spacing was selected as the control points for which hydraulic heads are maintained at or above the minimum specified head during the optimization process. Specification of hydraulic-head constraints at every model cell is numerically difficult and was found to be unneeded for obtaining an acceptable solution for ground-water withdrawals that yield hydraulic-head values at or above the specified constraint. In the simulation/optimization model of Czarnecki and others (2003), the differences between simulated values of hydraulic head and associated hydraulic-head constraints were compared to differences at all model cells and resulted in nearly identical distribution for the two data sets. Based on the results of Czarnecki and others (2003), the assumption was made that hydraulic-head control points specified at an approximate 5-mile spacing provided an adequate distribution of control points for the optimization model.

The Sparta aquifer is not confined everywhere in the model, such as in the outcrop/subcrop area or in a small area in northeastern Louisiana (fig. 3). For this reason, hydraulic-head constraints were set at 50 percent of predevelopment saturated thickness in these areas to prevent an unrealistic optimized solution or hydraulic heads from dropping below the bottom of the Sparta Sand. Everywhere else, hydraulic-head constraints were set at the top of the Sparta Sand.

Specified Limits on Ground-Water Withdrawals

The spatial distribution of the withdrawal decision variable (managed wells) was specified as those wells “that represent withdrawal rates” of the Sparta aquifer flow model from 1990-1997. The withdrawal

rates from these wells were based on 1995 and 1996 water-use data from Louisiana and Arkansas that were adjusted to adequately represent the time period from 1990-1997 in the last stress period of the Sparta aquifer flow model (ground-water model stress period 28; McKee and Clark, 2003), hereafter referred to as “baseline 1990-1997 withdrawal rates.” For optimization, 1,152 one-square mile cells were used to represent managed wells from more than 1,760 individual wells and aggregated water-use data in 1995-1996. All managed wells were assigned to layer 2 of the ground-water model; 398 in Arkansas, 725 in Louisiana, and 29 in Mississippi. The baseline 1990-1997 withdrawal rates for all managed wells total 32 Mft³/d.

Test simulations using the baseline 1990-1997 withdrawal rates applied to steady-state conditions yielded areas with dry cells. Therefore, ground-water withdrawal limits were specified at each well in the model as a multiple of the amount pumped in the baseline 1990-1997 withdrawal rate, such that

$$0 \leq q_{\text{well}i} \leq M q_{\text{well}i(1990-1997)} \quad (6)$$

where $q_{\text{well}i}$ is the optimal ground-water withdrawal for well i , in cubic feet per day;

M is a multiplier between 1 and 2; and

$q_{\text{well}i(1990-1997)}$ is the total amount withdrawn using the baseline 1990-1997 withdrawal rate from well i , in cubic feet per day.

This equation holds true for all withdrawals except those in Union County, which were held at 28 percent of the Hays (2000) 1997 withdrawal rate, discussed further in the Scenario 3 section.

Specified Limits on Surface-Water Withdrawals

No limits were imposed on optimized withdrawals from streams such that the range in optimal withdrawal was between zero and the maximum amount of water available at the most downstream point of each stream segment where constraints were specified. This specification permitted analysis of where water could be produced and the maximum amount available.

SUSTAINABLE-YIELD ESTIMATION

Sustainable yield was estimated by using the optimization model. In this section, sustainable yield

and unmet demand are defined and the development and results of scenarios are described.

Determination of Sustainable Yield and Unmet Demand

The optimization model provides estimates of sustainable yield from both ground water and surface water. The primary concern of State planners and water-resource managers for the Sparta aquifer is sustainability—the development and use of ground water for an indefinite time without causing unacceptable environmental, economic, or social consequences (Alley and others, 1999). The term “sustainable yield” is used to define that quantity of water that can be withdrawn indefinitely by reaching a system equilibrium (steady state) without compromising the integrity of the aquifer or with respect to agreed upon criteria. In Arkansas, the ASWCC has established critical criteria for a confined aquifer, such as the Sparta aquifer. Criteria include (1) hydraulic head in wells must be above the top of the aquifer formation and (2) the rate of decline of hydraulic head in wells must not be more than 1 foot per year over a 5-year period. If these criteria are violated, the ASWCC may designate an area to be a Critical Ground-Water Area.

Unmet demand is defined as the difference between the optimized ground-water withdrawal rate (sustainable yield) and a desired withdrawal rate (demand that is not met by ground water). For example, if the desired withdrawal rate is 9.8 Mft³/d, but the optimized ground-water withdrawal rate is calculated to be only 6.7 Mft³/d, the unmet demand is 3.1 Mft³/d (the difference between these two values). Note that determining the optimized withdrawal rate is an independent process that is not affected by the desired withdrawal rate. Therefore, unmet demand will vary as the desired withdrawal rate changes.

The development of an optimization model scenario to achieve feasible optimized solutions of estimated sustainable yield from the Sparta aquifer was a step-wise process involving several scenario formulations. The general process involved formulating a scenario by assigning initial conditions, upper and lower specified limits on withdrawals from managed wells, and hydraulic-head and streamflow constraints at control points. An infeasible solution occurs when optimization is not possible for the given scenario formulation without violating constraints. Despite the difficulties of investigating the cause of infeasible solutions, much is

learned about the system behavior to direct and guide scenario development. For the purpose of discussion, the progression of scenario development is separated and described in three scenarios of which the results of scenario 1 lead to scenario 2 which lead to scenario 3.

For all scenarios, MODMAN was used to estimate optimized ground-water withdrawal rates from all 1,152 managed wells in the optimization model. The lower and upper specified limits on withdrawals from these managed wells were a percentage from baseline 1990-1997 withdrawal rates (100 percent). The distribution and rates of withdrawal for the baseline 1990-1997 wells managed in the optimization model are shown in figure 4. For feasible scenario solutions, the sustainable yield was estimated for the Sparta aquifer in Arkansas and Louisiana independently (table 3). Sustainable yield estimations for ground water are listed in table 4 by county and parish for each feasible scenario, as well as examples of unmet demand calculated using hypothetical ground-water demand rates. Optimized withdrawals from streams were the same through each feasible scenario (3A-3C).

Total optimized withdrawals of surface water from all streams for which low-flow constraints were specified were about 5,300 and 100 Mft³/d in Arkansas and Louisiana, respectively. The sustainable yield from surface water is much larger than from ground water, and these large sustainable yields represent a potential source of water that could supplement ground water and meet total water demand.

The following sections describe three scenarios that were tested with the optimization model. Scenario 1 looks at the viability of using 1997 simulated hydraulic heads as initial conditions in the optimization model. Scenario 2 tests the feasibility of stressing all pumping wells at the continued withdrawal rate of 1990-1997. As described in the following sections, scenarios 1 and 2 are infeasible. Scenario 3 provides estimates of sustainable yield using variations on withdrawals throughout the model area while recognizing predefined sustainable yield in Union County from Hays (2000). To accomplish this, withdrawals in Union County were treated as managed wells for ease of post-processing. Union County managed wells were not allowed to decrease below 28 percent of the Hays (2000) 1997 withdrawal rate, discussed further in the Scenario 3 section. In all cases, withdrawals in Union County were optimized at the lower specified limit of 28 percent of the Hays (2000) 1997 withdrawal rate.

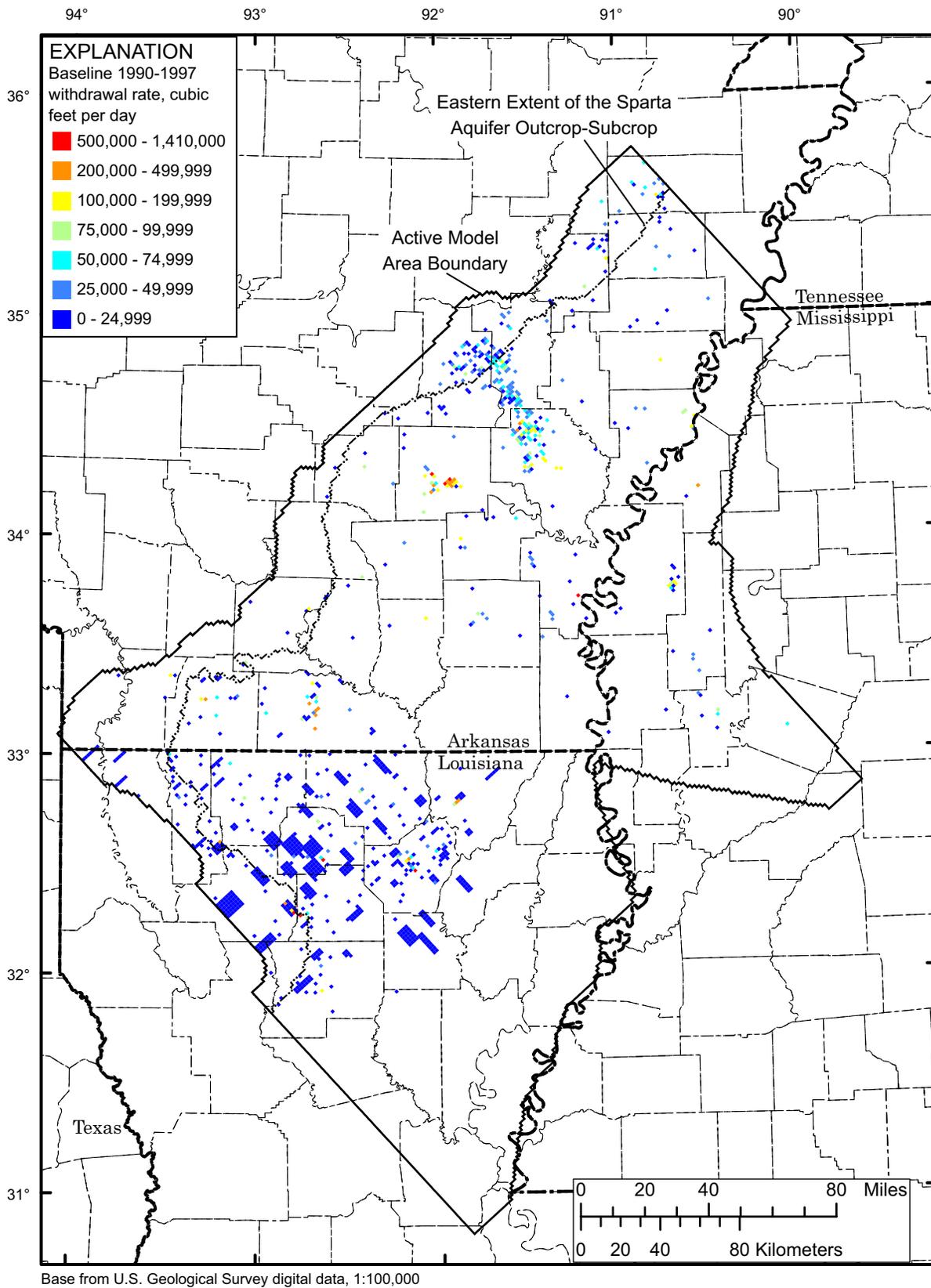


Figure 4. Baseline 1990-1997 ground-water withdrawal rates used in conjunctive-use optimization model.

Table 3. Summary of optimization scenarios and associated constraints including sustainable yield estimates and total 1997 withdrawals for Arkansas and Louisiana

[Mft³/d, million cubic feet per day; Sum of sustainable yield and unmet demand may not equal baseline withdrawal rate because of rounding; NA, not applicable]

Study area (managed wells ¹)	Scenario	Number of managed wells with non-zero flow	Baseline 1990-1997 withdrawal rate (Mft ³ /d)	Lower specified limit withdrawal rate ²	Upper specified limit withdrawal rate ²	Sustainable ground-water yield (Mft ³ /d)	Unmet demand ³ (Mft ³ /d)	Optimized withdrawal from streams ⁴ (Mft ³ /d)
Arkansas (398)	1	NA	22.4	0	100	infeasible	NA	NA
	2	NA	22.4	100	100	infeasible	NA	NA
	3A	193	22.4	0	100	11.6	10.9	5,300
	3B	146	22.4	0	150	12.7	9.8	5,300
	3C	135	22.4	0	200	13.2	9.2	5,300
Louisiana (725)	1	NA	8.2	0	100	infeasible	NA	NA
	2	NA	8.2	100	100	infeasible	NA	NA
	3A	110	8.2	0	100	0.3	7.9	96
	3B	73	8.2	0	150	0.4	7.8	96
	3C	69	8.2	0	200	0.5	7.7	96

¹Does not include 29 wells in Mississippi.

²Withdrawal rate in percent of baseline 1990-1997 withdrawals; 22.4, 8.2, and 1.3 Mft³/d for Arkansas, Louisiana, and Mississippi, respectively.

³Unmet demand assuming a desired demand equal to baseline 1990-1997 withdrawals.

⁴Not considered in unmet demand calculation; available streamflow to be considered as source of water for areas where current demand is unmet.

Table 4. Estimated sustainable yield and unmet demand by county and parish for each scenario in Arkansas and Louisiana

[All values are in million cubic feet per day. Negative values indicate no unmet demand]

County name	Sustainable yield based on an upper withdrawal limit of:			Unmet demand based on a demand of the baseline rate and a sustainable yield from:			Unmet demand based on sustainable yields from scenarios 3a, 3b, and 3c, respectively, and a demand of:			
	Baseline 1990-1997 withdrawal rate	100 percent of baseline rate (scenario 3a)	150 percent of baseline rate (scenario 3b)	200 percent of baseline rate (scenario 3c)	Scenario 3a	Scenario 3b	Scenario 3c	100 percent of baseline rate (scenario 3a)	150 percent of baseline rate (scenario 3b)	200 percent of baseline rate (scenario 3c)
Arkansas	4.73	0.84	0.02	0.02	3.89	4.72	4.71	3.89	7.08	9.44
Bradley	0.12	0.00	0.00	0.00	0.12	0.12	0.12	0.12	0.18	0.24
Calhoun	0.05	0.03	0.00	0.00	0.02	0.05	0.05	0.02	0.07	0.10
Chicot	0.03	0.03	0.05	0.06	0.00	-0.02	-0.03	0.00	0.00	0.00
Cleveland	0.04	0.04	0.05	0.07	0.00	-0.01	-0.03	0.00	0.01	0.01
Columbia	0.74	0.00	0.00	0.00	0.74	0.74	0.74	0.74	1.10	1.47
Cross	0.15	0.15	0.22	0.29	0.00	-0.07	-0.15	0.00	0.00	0.00
Dallas	0.08	0.08	0.00	0.00	0.00	0.08	0.08	0.00	0.13	0.17
Desha	1.04	1.04	1.56	2.08	0.00	-0.52	-1.04	0.00	0.00	0.00
Drew	0.22	0.22	0.08	0.07	0.00	0.14	0.15	0.00	0.25	0.37
Grant	0.15	0.15	0.22	0.29	0.00	-0.07	-0.15	0.00	0.00	0.00
Jackson	0.10	0.10	0.15	0.20	0.00	-0.05	-0.10	0.00	0.00	0.00
Jefferson	5.58	5.58	6.80	6.27	0.00	-1.22	-0.69	0.00	1.57	4.89
Lafayette	0.15	0.00	0.00	0.00	0.15	0.15	0.15	0.15	0.22	0.30
Lee	0.16	0.16	0.00	0.00	0.00	0.16	0.16	0.00	0.24	0.32
Lincoln	0.25	0.25	0.37	0.14	0.00	-0.12	0.10	0.00	0.00	0.35
Lonoke	1.79	0.13	0.02	0.03	1.65	1.76	1.76	1.65	2.66	3.54
Miller	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Monroe	0.05	0.00	0.00	0.00	0.05	0.05	0.05	0.05	0.08	0.11
Ouachita	0.28	0.16	0.00	0.00	0.12	0.28	0.28	0.12	0.43	0.57
Phillips	0.57	0.57	0.70	0.94	0.01	-0.13	-0.36	0.01	0.16	0.21
Poinsett	0.60	0.60	0.77	0.80	0.00	-0.17	-0.20	0.00	0.13	0.40
Prairie	2.82	0.14	0.11	0.15	2.67	2.70	2.66	2.67	4.11	5.48

Table 4. Estimated sustainable yield and unmet demand by county and parish for each scenario in Arkansas and Louisiana--Continued

[All values are in million cubic feet per day. Negative values indicate no unmet demand]

County name	Baseline 1990-1997 withdrawal rate	Sustainable yield based on an upper withdrawal limit of:			Unmet demand based on a demand of the baseline rate and a sustainable yield from:			Unmet demand based on sustainable yields from scenarios 3a, 3b, and 3c, respectively, and a demand of:		
		100 percent of baseline rate (scenario 3a)	150 percent of baseline rate (scenario 3b)	200 percent of baseline rate (scenario 3c)	Scenario 3a	Scenario 3b	Scenario 3c	100 percent of baseline rate (scenario 3a)	150 percent of baseline rate (scenario 3b)	200 percent of baseline rate (scenario 3c)
Pulaski	0.02	0.02	0.03	0.04	0.00	-0.01	-0.02	0.00	0.00	0.00
Saline	0.02	0.02	0.03	0.05	0.00	-0.01	-0.02	0.00	0.00	0.00
St. Francis	0.07	0.06	0.08	0.11	0.01	-0.01	-0.04	0.01	0.02	0.03
Union	2.20	0.78	0.78	0.78	1.42	1.42	1.42	1.42	2.52	3.62
Woodruff	0.43	0.40	0.60	0.80	0.02	-0.17	-0.37	0.02	0.04	0.05
Louisiana										
Bienville	1.17	0.00	0.00	0.00	1.17	1.17	1.17	1.17	1.76	2.35
Bossier	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.00	0.01	0.01
Caddo	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Caldwell	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01
Claiborne	0.29	0.00	0.00	0.00	0.29	0.29	0.29	0.29	0.44	0.59
Jackson	0.84	0.00	0.00	0.00	0.84	0.84	0.84	0.84	1.26	1.68
La Salle	0.01	0.01	0.02	0.02	0.00	-0.01	-0.01	0.00	0.00	0.00
Lincoln	0.98	0.00	0.00	0.00	0.98	0.98	0.98	0.98	1.47	1.96
Morehouse	0.75	0.03	0.04	0.05	0.72	0.71	0.70	0.72	1.09	1.45
Natchitoches	0.01	0.01	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Ouachita	2.70	0.00	0.00	0.00	2.70	2.70	2.70	2.70	4.05	5.40
Richland	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Union	0.59	0.00	0.00	0.00	0.59	0.59	0.59	0.59	0.88	1.18
Webster	0.60	0.01	0.02	0.03	0.58	0.58	0.57	0.58	0.87	1.17
Winn	0.26	0.23	0.28	0.37	0.03	-0.02	-0.11	0.03	0.11	0.15

Scenario 1

The first scenario formulated for the optimization model attempted to use 1997 simulated hydraulic heads from the ground-water model as the initial heads. However, continued heavy withdrawals from the Sparta aquifer caused hydraulic heads at the end of 1997 to be below the top of the Sparta Sand in many areas within the aquifer (fig. 5). Therefore, in the optimization model, some initial heads were below the constraint; thus, the solution was infeasible from the outset. To resolve this discrepancy, steady-state simulations of predevelopment conditions were used for initial conditions in scenarios 2 and 3 because hydraulic heads were above the top of the Sparta Sand prior to aquifer development. However, an area of approximately 100 mi² in northeastern Webster and western Claiborne Parishes in Louisiana (fig. 3) had predevelopment heads below the top of the Sparta Sand. Lithologic data were unavailable to dispute or confirm this condition. Unconfined conditions are most likely the result of cell geometry misrepresenting the actual aquifer geometry. To resolve this problem in scenarios 2 and 3, this area was treated in similar fashion as the outcrop/subcrop areas with regard to assigning hydraulic-head constraints at 50 percent of the predevelopment saturated thickness.

Scenario 2

The second scenario transitioned from the first, after resolving issues with initial heads, by attempting to find an optimized solution using 100 percent of the baseline 1990-1997 withdrawal rates as the lower specified limit on withdrawals from managed wells. A feasible solution for this scenario would indicate this ground-water demand could be sustainable indefinitely. Unfortunately, this solution is infeasible, because the scenario 1 discussion already indicated observed and simulated 1997 hydraulic heads below the top of the Sparta Sand as a result of continued, heavy withdrawals. The demand of baseline 1990-1997 withdrawal rates cannot be met; therefore, the lower specified limit on withdrawals from managed wells must be lowered to something less than 100 percent, which will create an unmet demand.

Scenario 3

In previous work by Hays (2000), sustainable yield estimates were calculated for the Sparta aquifer in Union County. In these estimates, simulated withdrawals were varied iteratively to achieve the specified minimum hydraulic head at the top of the Sparta Sand at one constraint location centered in the deepest cone of depression formed beneath the city of El Dorado. Hays's study resulted in an estimated sustainable yield of 28 percent of the 1997 withdrawal rate in Union County. This result, which was recognized by ASWCC, was used as a goal to reduce withdrawals from the Sparta aquifer in Union County. Since that time, local industry, the city of El Dorado, and Union County have been initiating conservation measures to reduce withdrawals from the Sparta aquifer through water reuse and alternative water sources for industry. Therefore, in all subsets of scenario 3, withdrawals in Union County are not allowed to decrease below 28 percent of the Hays (2000) 1997 withdrawal rate, which is equivalent to approximately 35.6 percent of the baseline 1990-1997 withdrawal rate specified for Union County in the current optimization model.

Scenario 3 transitioned from scenario 2 by allowing withdrawals from managed wells other than those in Union County to vary between 0 (lower specified limit) and some percent of the baseline 1990-1997 withdrawals (upper specified limit). Upper specified limits of 100, 150, and 200 percent were used in three subsets (A-C) that make up scenario 3 to find optimized solutions and determine any unmet demand assuming a desired demand equal to baseline withdrawals. Lower specified limits of 0 percent were used for all three scenarios for wells other than those in Union County.

All subsets of scenario 3 initially resulted in infeasible solutions. However, only three constraint locations in the southwest portion of the model (fig. 6A) caused the optimization to be infeasible. The head constraint (as discussed previously in the Hydraulic-Heads Constraint section) is set equal to the Sparta top in areas where the aquifer is confined and to 50 percent of the original saturated thickness where the aquifer is unconfined (outcrop area). The three constraint locations that caused the infeasible solutions are within or close to the defined Sparta Sand outcrop area (fig. 6). Because of uncertainty in the boundary between confined and unconfined in that transition area, and because there is some inherent uncertainty in the interpolated top of the Sparta Sand, it was reasonable to adjust the constraint at each of the three infeasible

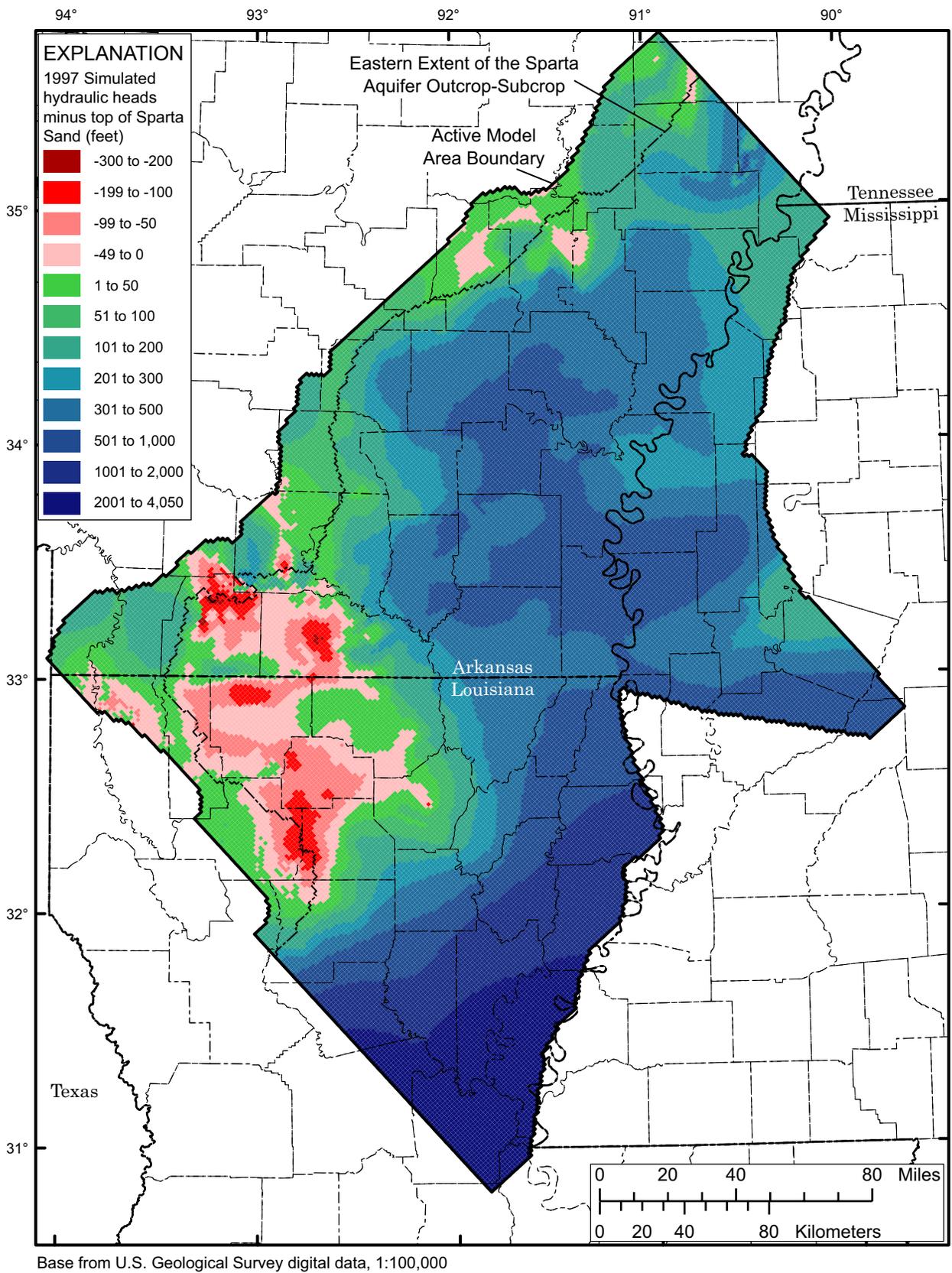


Figure 5. Difference between Sparta aquifer 1997 simulated heads and the top of the Sparta Sand.

locations. A relaxation of 10 feet in the constraint at these three locations resulted in feasible optimization solutions.

For scenario 3A, lower and upper specified limits on withdrawals from managed wells other than those in Union County were set at zero and 100 percent of baseline 1990-1997 withdrawal rates, respectively. These specified limits on managed wells, coupled with the hydraulic-head constraint over much of the Sparta aquifer results in an optimized solution. However, withdrawals from many managed wells were optimized at the lower specified limit of zero in the Grand Prairie area and Columbia County, and in much of the already stressed areas in north-central Louisiana (fig. 6A). Managed wells with non-zero flow totaled 193 out of 398 possible wells in Arkansas and 110 out of 725 wells in Louisiana (table 3). Other than Union County, withdrawals from these managed wells with non-zero flow were optimized at the upper specified limit (100 percent of the baseline 1990-1997 withdrawal rate). The distribution of optimal withdrawals was restricted when the hydraulic head at any point in the optimization model reached the minimum-head constraint. This occurred in two locations in scenario 3A—the first near the border of Lonoke and Prairie Counties in Arkansas and the second in Claiborne Parish in Louisiana (fig. 6A). Sustainable yield for scenario 3A was calculated to be 11.6 Mft³/d in Arkansas and 0.3 Mft³/d in Louisiana. Using baseline 1990-1997 withdrawal rates as the desired demand, unmet demand was calculated to be 10.9 Mft³/d and 7.9 Mft³/d in Arkansas and Louisiana, respectively (table 3).

For scenario 3B, the withdrawals from managed wells other than those in Union County were allowed to vary between zero and 150 percent of baseline 1990-1997 withdrawal rates. This scenario produced an optimized solution that was similar to the scenario 3A solution because most withdrawals from managed wells in the stressed areas of the Grand Prairie area and north-central Louisiana were optimized at the lower specified limit of zero. Managed wells with non-zero flow as compared to Scenario 3A were fewer in Arkansas with 146 out of 398 possible wells and fewer in Louisiana with 73 out of 725 possible wells (table 3). As in scenario 3A, withdrawals from these managed wells with non-zero flow were optimized at the upper specified limit. Hydraulic-head constraints were reached at the same two locations as for scenario 3A, along with an additional point located in the northwestern corner of the model area (fig. 6B). Sustainable yield for scenario

3B was calculated to be 12.7 Mft³/d in Arkansas and 0.4 Mft³/d in Louisiana. Using baseline 1990-1997 withdrawal rates as the desired demand, unmet demand was calculated to be 9.8 Mft³/d and 7.8 Mft³/d in Arkansas and Louisiana, respectively (table 3), which could be obtained from the optimized surface-water withdrawals.

For scenario 3C, withdrawals from managed wells other than those in Union County were allowed to vary between zero and 200 percent of the baseline 1990-1997 withdrawal rate. This scenario produced an optimized solution that was similar to the scenario 3B solution. Managed wells with non-zero flow as compared to scenario 3B were fewer in Arkansas with 135 out of 398 possible wells and fewer in Louisiana with 69 out of 725 possible wells (table 3). As with both scenarios 3A and 3B, withdrawals from these managed wells with non-zero flow were optimized at the upper specified limit. Hydraulic-head constraints also were reached at the same three locations in the model as that of scenario 3B (figs. 6B, 6C). Sustainable yield for scenario 3C was calculated to be 13.2 Mft³/d in Arkansas and 0.5 Mft³/d in Louisiana. Using baseline 1990-1997 withdrawal rates as the desired demand, unmet demand was calculated to be 9.2 Mft³/d and 7.7 Mft³/d in Arkansas and Louisiana, respectively (table 3).

Analysis of Scenario Sustainable Yield Estimation

Sustainable yield is a function of managed well distributions and lower and upper specified limits on withdrawals from managed wells. However, in each of the scenarios, the managed well distributions are fixed. For the scenarios with feasible solutions, sustainable-yield estimations increase as the upper specified limit on withdrawals from managed wells increase (fig. 7). However, continuing to increase withdrawals at managed well locations beyond 200 percent of the baseline rate may be unrealistic.

The source of water for the increased sustainable yields with increased ground-water withdrawals is leakage from surface water in the outcrop/subcrop area and through the overlying confining unit. The amount of water required by simulations with maximum allowable ground-water withdrawals greater than 200 percent of the 1990-1997 baseline rate is considered unrealistic because the wells physically would not be able to pump that much water, and the distribution of pumping wells would need to be changed to satisfy

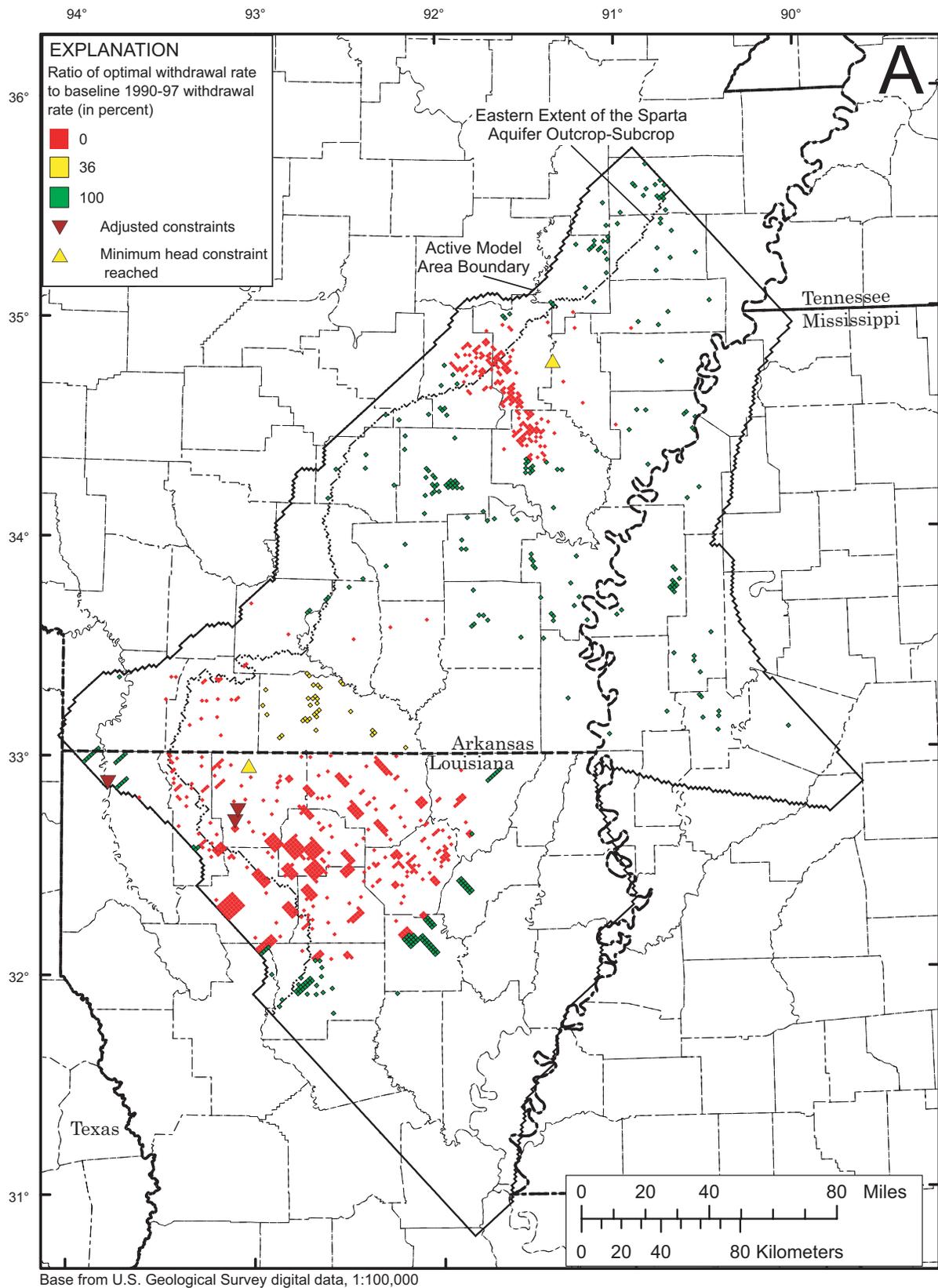


Figure 6. Ratio of optimal ground-water withdrawal to baseline 1990-1997 withdrawals (in percent) for (A) scenario 3A, (B) scenario 3B, and (C) scenario 3C including locations where hydraulic-head constraints are reached.

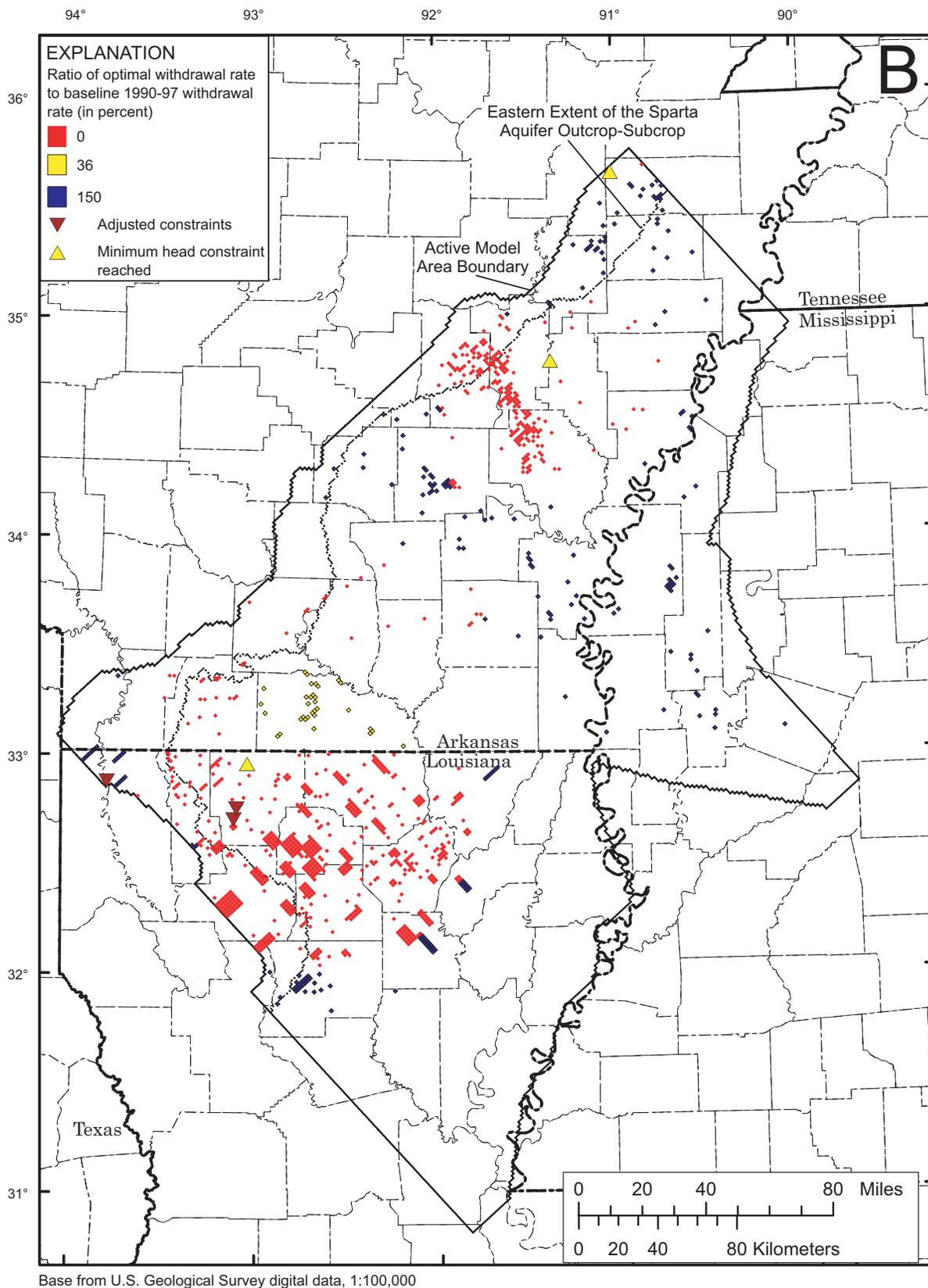


Figure 6. Ratio of optimal ground-water withdrawal to baseline 1990-1997 withdrawals (in percent) for (A) scenario 3A, (B) scenario 3B, and (C) scenario 3C including locations where hydraulic-head constraints are reached.

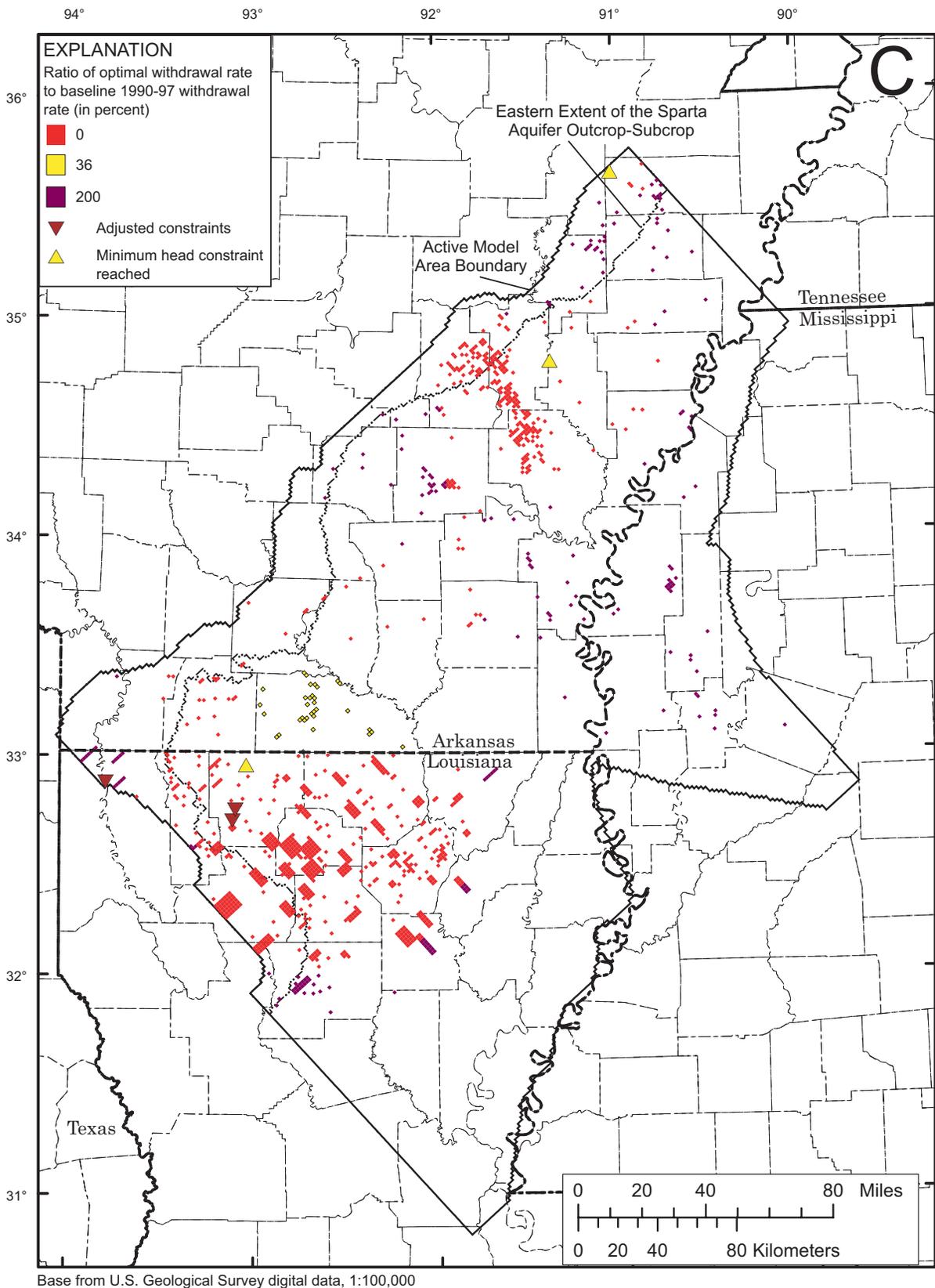


Figure 6. Ratio of optimal ground-water withdrawal to baseline 1990-1997 withdrawals (in percent) for (A) scenario 3A, (B) scenario 3B, and (C) scenario 3C including locations where hydraulic-head constraints are reached.

model constraints for managed wells with no flow. In addition, the distribution of wells likely would not be suitable for ground-water users.

Sustainable yield may be increased by strategic placement of managed wells in the model (other than the present fixed well locations) and optimizing withdrawal rates. This strategy could be used to determine the location of future wells to be drilled into the Sparta aquifer. Results of the optimization modeling indicate that substantial increases in maximum allowable ground-water withdrawals produce small incremental increases in sustainable yield (fig. 7). Assuming the total water demand is equal to the baseline 1990-1997 withdrawal rates, the sustainable yield estimated from scenario 3A-C only provides 52 to 59 percent of the total demand for Arkansas; the remainder is unmet demand that could be obtained from surface water.

Infeasible Scenarios

Several other scenarios were investigated. In many, the optimization problem was formulated with the lower specified limit on withdrawals from managed wells set at something greater than zero. For these scenarios, there were no feasible solutions. This means that some areas of the Sparta aquifer cannot be pumped and maintain hydraulic head above the top of the Sparta Sand while other areas are allowed to pump at rates equal to or greater than baseline 1990-1997 withdrawal rates.

Other scenarios were formulated whereby all hydraulic-head constraints were relaxed to the Sparta Sand bottom in Louisiana because currently there is no designated CGWA, as there is in Arkansas. In these scenario formulations, estimated sustainable yield appears to be much larger. However, non-linear model behavior, resulting from large changes in saturated

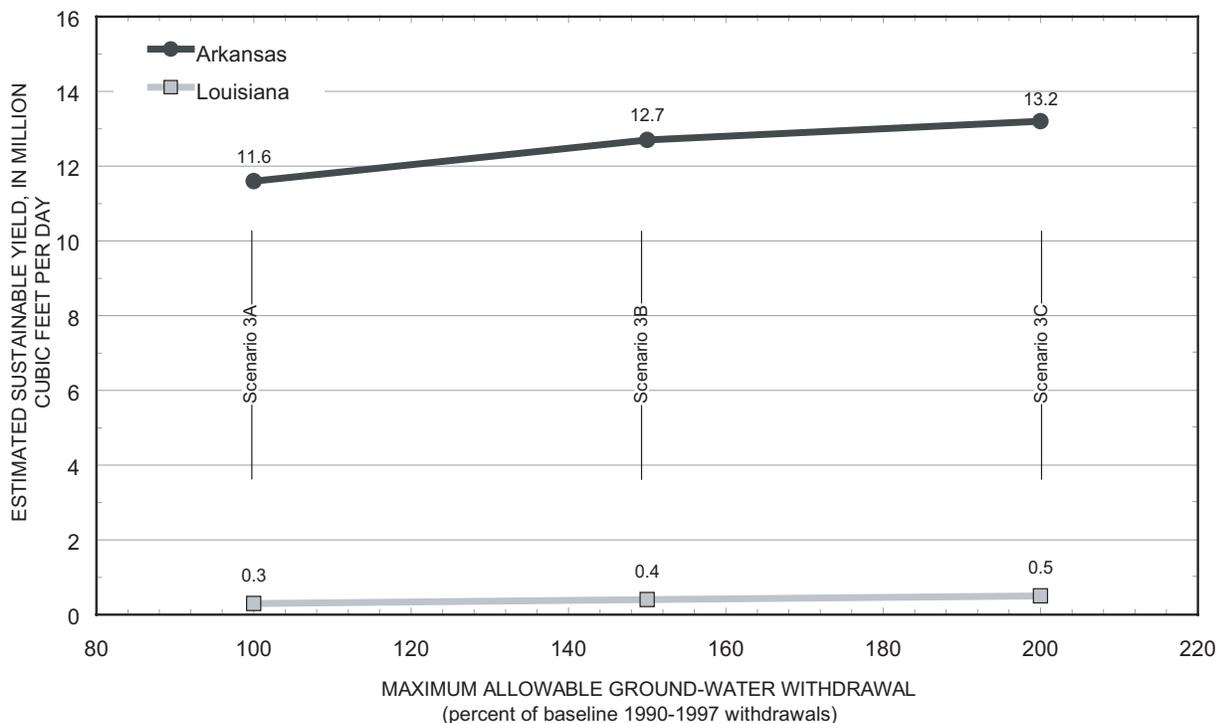


Figure 7. Graph showing scenario results of estimated sustainable yield as a function of the maximum allowable ground-water withdrawal.

thickness and transmissivity, was too large to render viable solutions with these scenario formulations. Non-linear optimization would be required to adequately determine the sustainable yield. These techniques are more complicated than linear or nearly linear boundary conditions (Ahlfeld and Mulligan, 2000). However, these types of scenarios could explore the effect of non-uniform water-management strategies between adjacent states. Therefore, two reasons emerged for continuing to maintain hydraulic-head constraints at the top of the Sparta Sand in Louisiana: (1) to maintain consistency with Arkansas criteria in lieu of any established criteria in Louisiana; and (2) to minimize any non-linear effects that could cause optimization problems as the Sparta aquifer becomes unconfined.

Verification of Optimized Withdrawals

Where unconfined conditions occur, large changes in saturated thickness and transmissivity can cause non-linear model response. Optimized withdrawal rates must be checked for errors associated with non-linear response of the ground-water model. Verification was done using the optimized withdrawals from each of the three scenarios as input to the Sparta aquifer flow model (MODFLOW-2000 version) to evaluate whether simulated steady-state hydraulic heads were within the specified constraints as computed by the optimization model (fig. 8). With the exception of the Sparta aquifer outcrop areas where hydraulic-head constraints were specified as the aquifer bottom, the optimized withdrawal rates produced hydraulic heads at or above the top of the Sparta Sand in most of the model area. In areas near the outcrop, hydraulic heads were below the top of the Sparta Sand by 10 to 50 ft, but maintained greater than 50 percent saturated thickness, as did all hydraulic heads in the outcrop area.

Model Assumptions and Limitations

To effectively utilize flow model results, an understanding of model assumptions and limitations is essential. The accuracy of ground-water models is limited by simplification of the conceptual model of a complex flow system, by space and time discretization effects, and by assumptions made in the formulation of the governing flow equations. Models also are limited by cell size, number of layers, boundary conditions, accuracy and availability of hydraulic property values,

accuracy of calibration, accuracy of ground-water withdrawal estimates, historical data for simulations, and parameter sensitivity. Models are limited by the availability of data and by the interpolations and extrapolations that are inherent in using data in a model. Although a model is calibrated, the calibration parameter values may not be unique in yielding the specific simulations of hydraulic head.

No additional managed wells other than those that were used to represent the baseline 1990-1997 withdrawal rates were assumed to be available for pumping in the optimization scenarios; only existing wells were used. Therefore, additional wells strategically located might have resulted in more yield. Also, the optimization scenarios focused on adjusting maximum allowable withdrawal rates based on the percentage of baseline 1990-1997 withdrawal rates. Some areas represented in the model likely could pump at rates much higher (>200 percent) than baseline 1990-1997 withdrawal rates. Both of these changes might increase yield. Another method of formulating the scenarios could be based on withdrawal rate magnitudes rather than percentages of a base rate.

Nonlinear effects are negligible except in the outcrop areas of the Sparta Sand. Because hydraulic-head constraints are set at the top of the Sparta Sand, the ground-water system must maintain confined conditions, which prevents the nonlinearity associated with unconfined systems. When the Sparta aquifer is confined, the flow equation is linear, but becomes nonlinear for unconfined conditions and, in this case, the linear solution from the optimization model begins to deviate from the non-linear solution. The system is unconfined in the outcrop-subcrop areas where hydraulic-head constraints were set to 50 percent saturated thickness of the aquifer, and some nonlinearity effects cause the estimated hydraulic heads from the optimized solutions to differ from simulated hydraulic heads from a flow ground-water flow model run using optimized withdrawal rates.

Hydraulic-head constraints near topographic high points in the top of the Sparta Sand cause the optimization model to reduce withdrawals over large areas to maintain hydraulic heads above the topographic high head constraint. The reduced withdrawals result in hydraulic heads more than 100 feet above the top of the Sparta Sand (fig. 8A-C) over much of the Sparta aquifer. Future alternative scenario runs might include discrete assignment of the hydraulic-head constraints that accommodate the highs and lows inherent in the top of

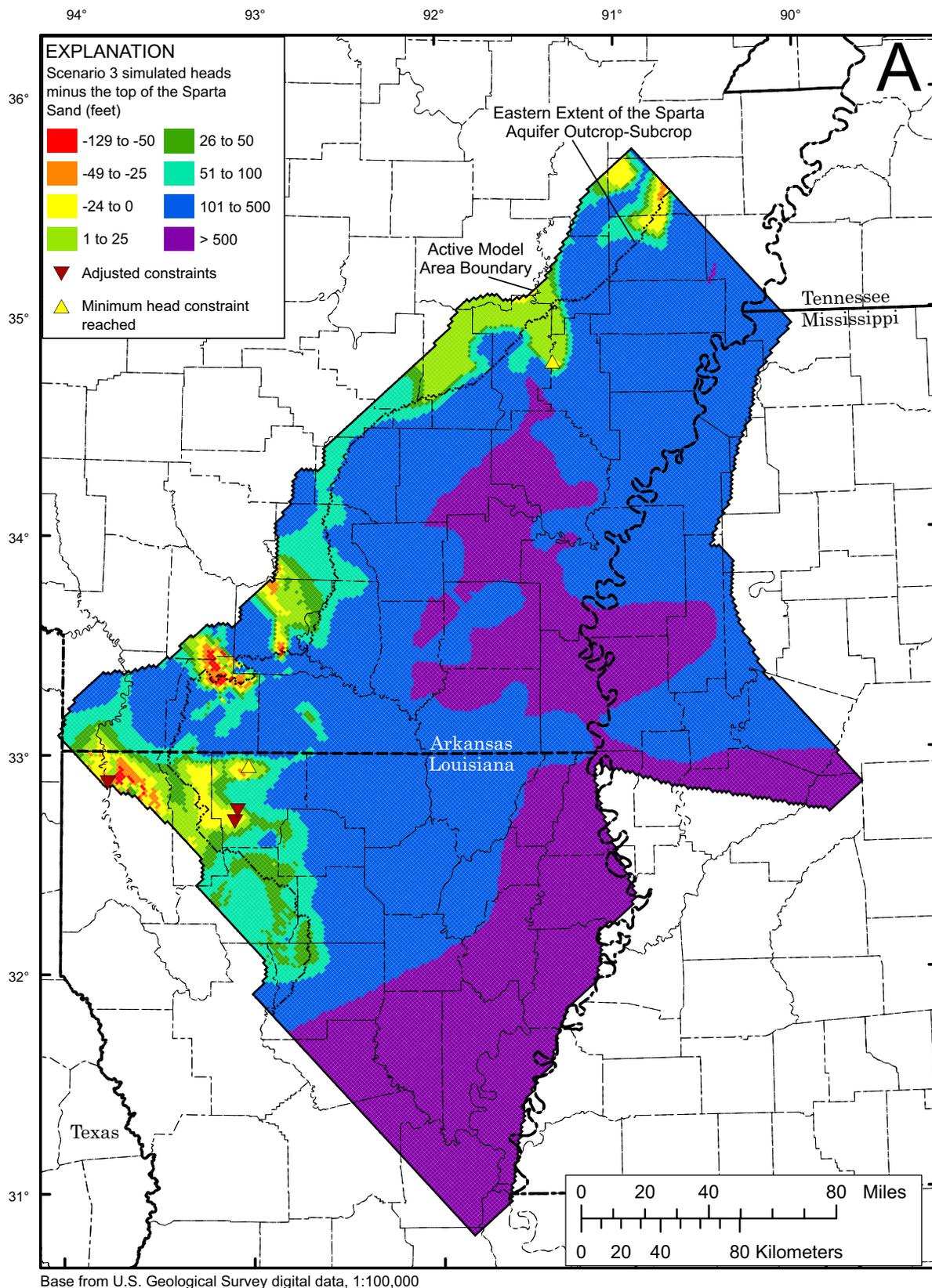


Figure 8. Difference between steady-state simulated hydraulic heads and the top of the Sparta Sand for (A) scenario 3A, (B) scenario 3B, and (C) scenario 3C using optimized withdrawals in the Sparta aquifer flow model for verification.

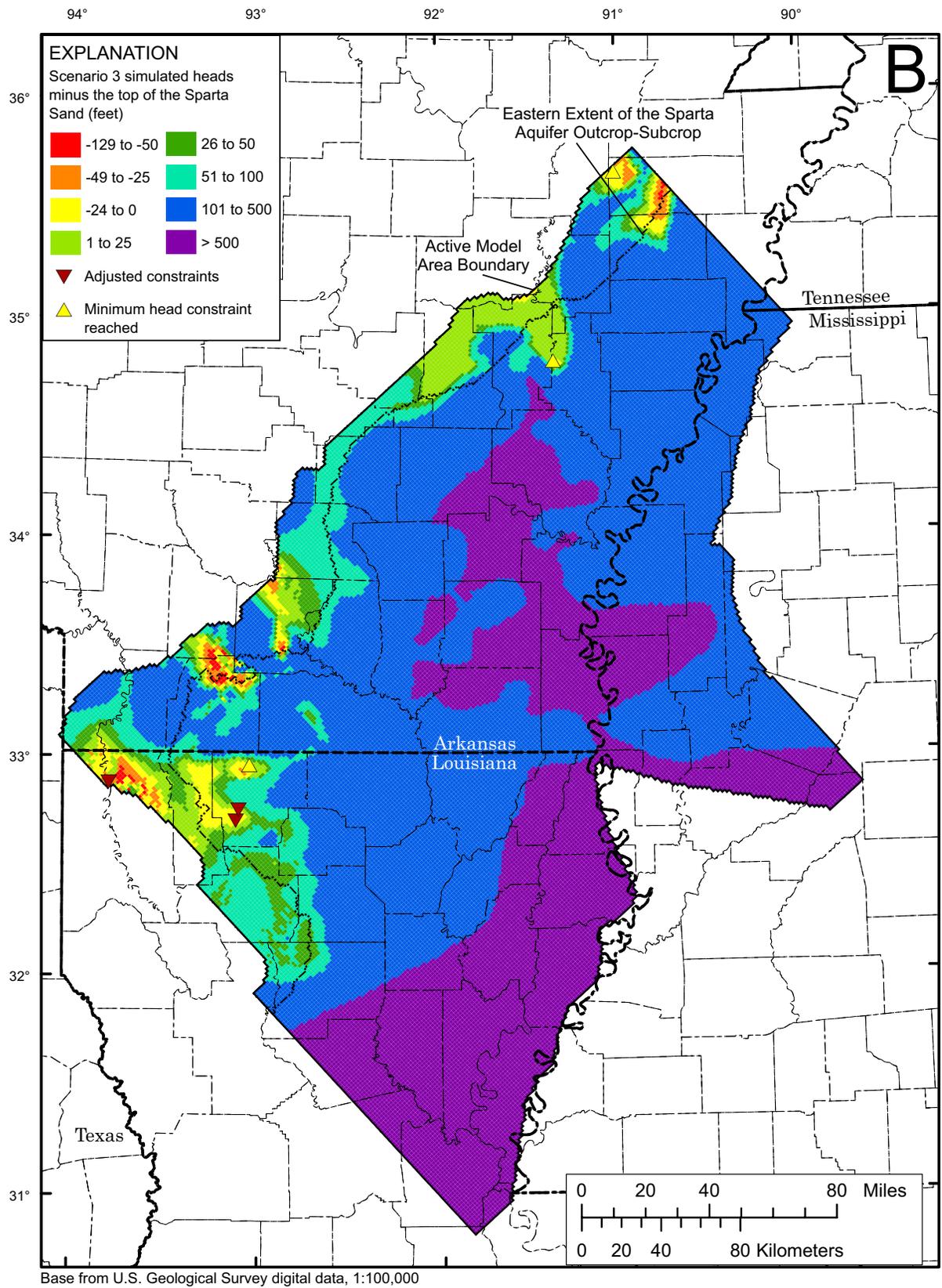


Figure 8. Difference between steady-state simulated hydraulic heads and the top of the Sparta Sand for (A) scenario 3A, (B) scenario 3B, and (C) scenario 3C using optimized withdrawals in the Sparta aquifer flow model for verification.

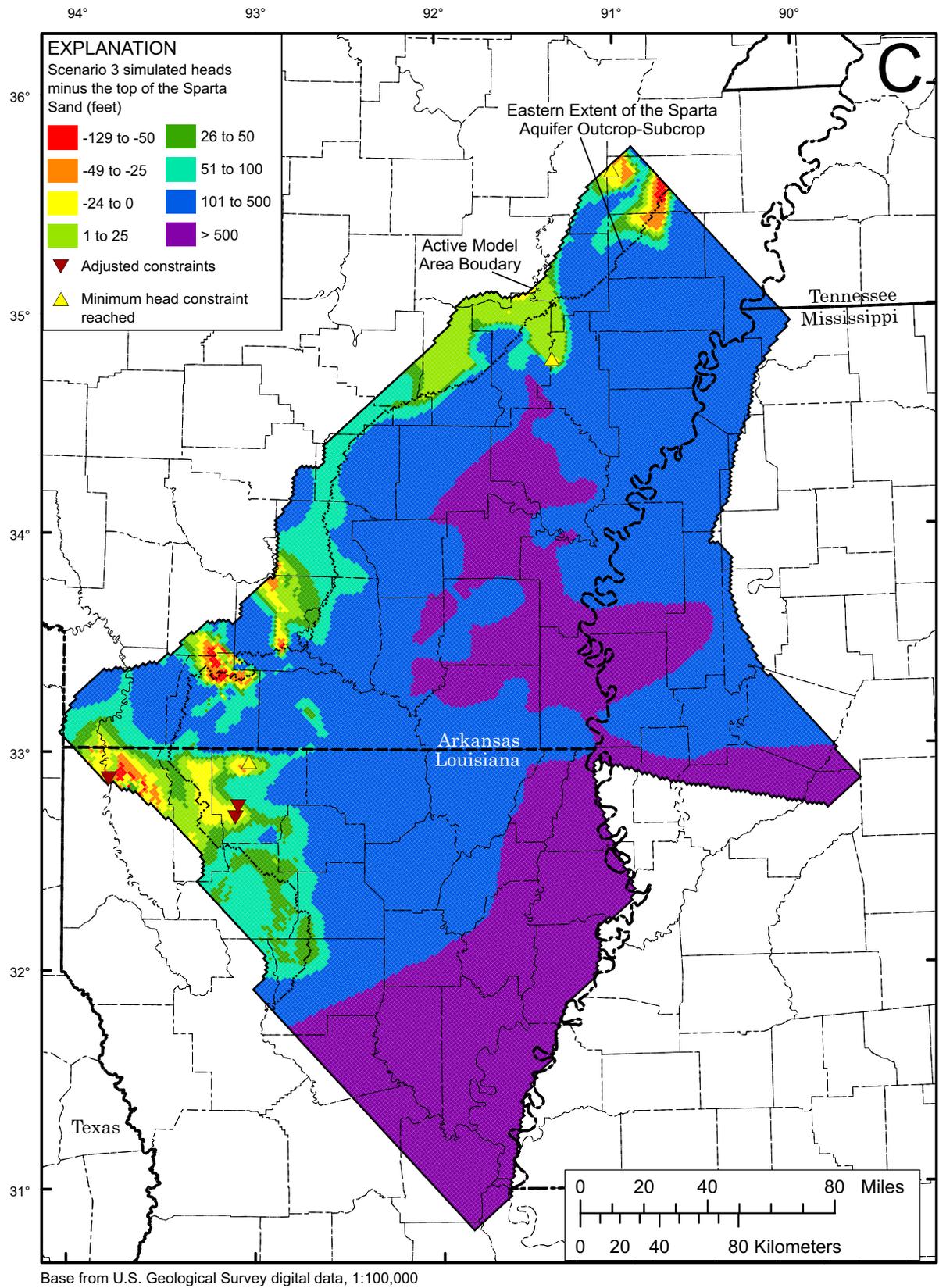


Figure 8. Difference between steady-state simulated hydraulic heads and the top of the Sparta Sand for (A) scenario 3A, (B) scenario 3B, and (C) scenario 3C using optimized withdrawals in the Sparta aquifer flow model for verification.

the Sparta Sand. These scenarios also may result in higher sustainable yields for the system because of relaxed constraints in areas where the hydraulic heads drop below the top of the Sparta Sand early in the simulation.

SUMMARY

Conjunctive-use optimization modeling was done to assist water managers and planners by estimating the maximum amount of ground water that hypothetically could be withdrawn indefinitely from wells within the Sparta aquifer without violating hydraulic-head or streamflow constraints. The Sparta aquifer is largely a confined aquifer of regional importance that comprises a sequence of unconsolidated sand units that are contained within the Sparta Sand. In 2000, more than 35.4 Mft³/d of water were withdrawn from the aquifer in Arkansas by more than 900 wells, primarily for industry, municipal supply, and crop irrigation. Several large cones of depression have formed in the potentiometric surface in the Grand Prairie area and near the cities of Pine Bluff and El Dorado in Arkansas and Monroe in Louisiana. Hydraulic heads in the Sparta aquifer are now below the top of the Sparta Sand in parts of Union and Columbia Counties and several areas in north-central Louisiana. Problems related to overdraft in the Sparta aquifer can result in increased drilling and pumping costs, reduced well yields, and degraded water quality in areas of large drawdown.

A finite-difference ground-water flow model was developed for the Sparta aquifer using MODFLOW, in eastern and southeastern Arkansas and north-central Louisiana. Observed aquifer conditions in 1997 supported by model simulations indicate that continued pumping at withdrawal rates representative of 1990-1997 rates cannot be sustained indefinitely and maintain hydraulic heads above the top of the Sparta Sand; a criteria used to designate Arkansas's Critical Ground-Water Areas. This report describes a steady-state conjunctive-use optimization model that was developed to simulate optimized surface-water and ground-water withdrawals while maintaining hydraulic-head and streamflow constraints, thus determining the "sustainable yield" for the aquifer.

Attempts to estimate sustainable yield initially resulted in infeasible solutions. However, continued scenario development produced a scenario with feasible solutions. A feasible solution returns estimated sustainable yield—optimized ground-water withdrawals

that can be maintained indefinitely without lowering hydraulic heads below the top of the Sparta Sand (except in the outcrop areas where unconfined conditions occur) or decreasing streamflows below minimum levels.

Scenarios 1 and 2 resulted in infeasible solutions, but were important findings in the progression of scenario development. From scenario 1, it was determined that steady-state simulations of predevelopment hydraulic heads should be used as the initial conditions for the optimization model to ensure that all hydraulic heads start out above the top of the Sparta Sand. For this reason, the solution was infeasible using the 1997 simulated hydraulic heads because hydraulic heads in many areas already were below the top of the Sparta Sand.

Scenario 2 transitioned from scenario 1 by attempting to find an optimized solution using 100 percent of the baseline 1990-1997 withdrawals as the lower specified limit on withdrawals from managed wells. This infeasible solution proves that the baseline 1990-1997 withdrawals cannot be sustained indefinitely.

For scenarios 3A-C, withdrawals from managed wells varied between zero and rates equal to 100, 150, and 200 percent of the baseline 1990-1997 withdrawal rates except for wells in Union County, which were held at 35.6 percent of the baseline 1990-1997 withdrawal rate. Estimated sustainable yields for scenario 3 ranged from 11.6 to 13.2 Mft³/d in Arkansas and 0.3 to 0.5 Mft³/d in Louisiana. Assuming the total water demand is equal to the baseline 1990-1997 withdrawal rates, the sustainable yield estimated from scenario 3A-C only provides 52 to 59 percent of the total demand for Arkansas; the remainder is unmet demand that could be obtained from surface water.

Sustainable yield is a function of managed well distributions and lower and upper specified limits on withdrawals from managed wells. For this report, 1,152 managed wells were used to represent the distribution of existing wells in the Sparta aquifer. In addition, lower and upper specified limits on withdrawals from managed wells were set as a percentage of the baseline 1990-1997 withdrawal rates. Hydraulic-head constraints have the largest effect on estimations of sustainable yield. Therefore, with no established criteria for maintaining hydraulic heads above the top of the Sparta Sand in Louisiana, decreasing the hydraulic-head constraint in Louisiana would increase the amount of estimated sustainable yield for both Arkan-

sas and Louisiana. However, non-linear model behavior was too great to render a viable solution for quantifying this increase. Sustainable yield also may be increased by strategic placement of managed wells in the model and optimizing withdrawal rates. This strategy could be used to determine the location of future wells to be drilled into the Sparta aquifer.

The results of this optimization model must be viewed within the limitations of the quality of data available for the Sparta aquifer system and representation of the system with a numerical simulation model. Results of model application scenarios demonstrate the usefulness of coupling numerical simulation and optimization for regional-scale evaluation of water-resource management strategies.

SELECTED REFERENCES

- Alley, W.M., Reilly, T.E., and Franke, O.L., 1999, Sustainability of ground-water resources: U.S. Geological Survey Circular 1186, 79 p.
- Ahlfeld, D.P. and Mulligan, A.E., 2000, Optimal management of flow in groundwater systems: San Diego, Calif., Academic Press, 185 p.
- Arthur, J.K., and Taylor, R.E., 1990, Definition of geohydrologic framework and preliminary simulation of ground-water flow in the Mississippi embayment aquifer system, Gulf Coastal Plain, United States: U.S. Geological Survey Water-Resources Investigations Report 86-4364, 97 p.
- Czarnecki, J.B., Clark, B.R., and Reed, T.B., 2003, Conjunctive-use optimization model of the Mississippi River Valley alluvial aquifer of northeastern Arkansas: U.S. Geological Survey Water-Resources Investigations Report 03-4230, 29 p.
- Danskin, W.R., and Freckleton, J.R., 1989, Ground-water flow modeling and optimization techniques applied to high-ground-water problems in San Bernardino, California: U.S. Geological Survey Open-File Report 89-75, 14 p.
- Freiwald, D.A., 1985, Average annual precipitation and runoff for Arkansas, 1951-80: U.S. Geological Survey Water-Resources Investigations Report 84-4363, 1 sheet.
- Greenwald, R. M., 1998, Documentation and user's guide: MODMAN, An optimization module for MODFLOW version 4.0: Freehold, New Jersey, HSI GeoTrans, 112 p.
- Halberg, H.N., and Stephens, J.W., 1966, Use of water in Arkansas, 1965: Arkansas Geological Commission Water Resources Summary 5, 12 p.
- Harbaugh, A.W., and McDonald, M.G., 1996, User's documentation for MODFLOW-96, an update to the U.S. Geological Survey modular finite-difference ground-water flow model: U.S. Geological Survey Open-File Report 96-485, 56 p.
- Harbaugh, A.W., Banta, E.R., Hill, M.C., McDonald, M.G., 2000, MODFLOW-2000, The U.S. Geological Survey Modular Ground-Water Model-User Guide to Modularization Concepts and the Ground-Water Flow Process: U.S. Geological Survey Open-File Report 00-92, 121p.
- Hays, P.D., 2000, Sustainable-Yield estimation for the Sparta aquifer in Union County, Arkansas: U.S. Geological Survey Water-Resources Investigations Report 99-4274, 17 p.
- Hill, M.C., Banta E.R., Harbaugh, A.W., and Anderman, E.R., 2000, MODFLOW-2000, The U.S. Geological Survey modular ground-water model user guide to the observation, sensitivity, and parameter-estimation processes and three post-processing programs: U.S. Geological Survey Open-File Report 00-184, 209 p.
- Hosman, R.L., Long, A.T., Lambert, T.W., and others, 1968, Tertiary aquifers in the Mississippi embayment, *with discussions of quality of the water*, by Jeffery, H.G.: U.S. Geological Survey Professional Paper 448-D, 29 p.
- Joseph, R.L., 1998, Potentiometric surface of the Sparta aquifer in eastern and south-central Arkansas and north-central Louisiana, and the Memphis aquifer in east-central Arkansas, October 1996-July 1997: U.S. Geological Survey Water-Resources Investigations Report 97-4282, 19 p.
- McKee, P.W., and Clark, B.R., 2003, Development and calibration of a ground-water flow model for the Sparta aquifer of southeastern Arkansas and north-central Louisiana and simulated response to withdrawals, 1998-2027: U.S. Geological Survey Water-Resources Investigations Report 03-4132, 71 p.
- McDonald, M.G., and Harbaugh, A.W., 1988, A modular three-dimensional finite-difference ground-water flow model: U.S. Geological Survey Techniques of Water-Resources Investigations, book 6, chap. A1, 586 p.
- Murtaugh, B.A., and Saunders, M.A., 1998, MINOS 5.5 user's guide: Systems Optimization Laboratory Technical Report SOL 83-20R, Stanford, California, Stanford University, 150 p.
- Payne, J.N., 1968, Hydrologic significance of the lithofacies of the Sparta Sand in Arkansas, Louisiana, Mississippi, and Texas: U.S. Geological Survey Professional Paper 569-A, 17 p.
- Petersen, J.C., Broom, M.E., and Bush, W.V., 1985, Geohydrologic units of the Gulf Coastal Plain in Arkansas: U.S. Geological Survey Water-Resources Investigations Report 85-4116, 20 p.

- Peralta, R.C., Yazdanian, Amin, Killian, P.J., and Shulstad, R.N., 1985, Future Quaternary groundwater accessibility in the Grand Prairie--1993: University of Arkansas, Agricultural Experiment Station Bulletin no. 877, 37 p.
- Reichard, E.G., 1995, Groundwater-surface water management with stochastic surface water supplies: A simulation optimization approach: *Water Resources Research*, v. 31, no. 11, p. 2845-2865, November 1995.