

# REGIONALIZATION OF HARMONIC-MEAN STREAMFLOWS IN KENTUCKY

By Gary R. Martin and Kevin J. Ruhl

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## CONVERSION FACTORS

<i>Multiply</i>	<i>By</i>	<i>To obtain</i>
inch (in.)	25.4	millimeter
foot (ft)	0.3048	meter
mile (mi)	1.609	kilometer
square mile (mi <sup>2</sup> )	2.590	square kilometer
cubic foot (ft <sup>3</sup> )	0.02832	cubic meter
cubic foot per second (ft <sup>3</sup> /s)	0.02832	cubic meter per second
cubic foot per second per square mile ((ft <sup>3</sup> /s)/mi <sup>2</sup> )	0.0109	cubic meter per second per square kilometer

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# REGIONALIZATION OF HARMONIC-MEAN STREAMFLOWS IN KENTUCKY

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## ABSTRACT

Harmonic-mean streamflow ( $Q_h$ ), defined as the reciprocal of the arithmetic mean of the reciprocal daily streamflow values, was determined for selected stream sites in Kentucky. Daily mean discharges for the available period of record through the 1989 water year at 230 continuous-record streamflow-gaging stations located in and adjacent to Kentucky were used in the analysis. Periods of record affected by regulation were identified and analyzed separately from periods of record unaffected by regulation. Record-extension procedures were applied to short-term stations to reduce time-sampling error and, thus, improve estimates of the long-term  $Q_h$ .

Techniques to estimate the  $Q_h$  at ungaged stream sites in Kentucky were developed. A regression model relating  $Q_h$  to total drainage area and streamflow-variability index was presented with example applications. The regression model has a standard error of estimate of 76 percent and a standard error of prediction of 78 percent.

## INTRODUCTION

Streamflow information pertaining to the quality and quantity of water is critical for the protection and wise management of surface-water resources in Kentucky. Harmonic-mean streamflow, the reciprocal of the arithmetic mean of the reciprocal daily streamflow values, is useful for assessing the availability of water for assimilation of certain types of wastes. Kentucky water-quality standards for protection of human health from exposure to selected toxic substances incorporate the  $Q_h$ . The U.S. Geological Survey (USGS), in cooperation with the Kentucky Natural Resources and Environmental Protection Cabinet (KNREPC), analyzed available data from the streamflow-gaging network, located in and adjacent to Kentucky, to estimate the long-term  $Q_h$  and to develop a regional relation to estimate  $Q_h$  values at ungaged stream sites in Kentucky.

Discharge of contaminants can interfere with surface-water uses. The health of aquatic communities and humans can be adversely affected by exposure to elevated concentrations of contaminants. Consumption of toxic substances in fish and water can be detrimental to human health, and any adverse effects on health can be cumulative. Concentrations of such toxic substances in streams, when discharged at a constant rate, are inversely proportional to streamflow. Procedures for defining pertinent streamflow characteristics in Kentucky are, therefore, needed to set contaminant-discharge limits.

Pursuant to the 1987 amendments to the Clean Water Act, States are required to establish concentration limits for toxic substances that interfere with surface-water use. The KNREPC has established allowable concentration limits for selected toxic substances for protection of human health based on numerical criteria and associated risk factors published in 'Quality Criteria for Water 1986' (U.S. Environmental Protection Agency, 1986a). The  $Q_h$  was adopted as the governing "design" flow for setting waste-load-discharge limits for selected cancer-linked substances. It was determined that  $Q_h$  can provide the most representative estimate of long-term average instream exposure concentrations of these substances (Kentucky Natural Resources and Environmental Protection Cabinet, 1990).

## **Purpose and Scope**

The purpose of this report is to provide (1)  $Q_h$  values at streamflow-gaging stations, considering periods of record affected by regulation and periods of record unaffected by regulation separately, and (2) a procedure for estimating the  $Q_h$  at ungaged stream sites unaffected by regulation or local diversions. This report presents  $Q_h$  values for 230 continuous-record streamflow-gaging stations in the study area. Procedures for estimating the  $Q_h$  at ungaged stream sites are described and illustrated with example computations.

## **Physiography and Geology**

The physiography of the State reflects the lithology of the surface rocks and is largely defined by the Cincinnati Arch (fig. 1). The axis of the Cincinnati Arch trends northward from south-central Kentucky to just south of the Inner and Outer Bluegrass boundary where it divides into two branches. The branches are approximately parallel but are separated by approximately 25 mi at the Ohio River (McFarland, 1950). Lithologic units dip from the arch, a structural high, so that geologic features are generally symmetrical on each side of the axis of the arch.

East and west from the Cincinnati Arch, progressively younger rocks are exposed at the surface. The oldest exposed rocks are part of the Jessamine Dome and the areas adjacent to it. The location of this area corresponds approximately to the Inner Bluegrass region (fig. 1). These rocks consist of limestone, shale, and sandstone of Ordovician age. Narrow bands of shales and limestones of Silurian and Devonian age surround this area and correspond to the Outer Bluegrass region. An expansive area of limestone of Mississippian age (Mississippian Plateaus Region) is exposed starting at the Ohio River in northeastern Kentucky and extending southwest to the State boundary and northwest in a crescent-shaped area surrounding the Western Kentucky Coal Field. The eastern boundary of this area is the Cumberland Escarpment (fig. 1). Sandstones, shales, siltstones, and coals of Pennsylvanian age in eastern and northwestern Kentucky--the youngest rocks in Kentucky--compose the Eastern and Western Kentucky Coal Fields. Alluvial deposits of Cretaceous and Tertiary age occur in extreme western Kentucky in the Mississippi Embayment.

Much of the Mississippian Plateau is characterized by karst features such as sinkholes, caves, springs, and gaining and losing streams. Most well-developed karst features are located in a band originating in west-central Kentucky and extending to south-central Kentucky, southeast to the State boundary, east along the boundary, and then northeast and north (areas shown in black in fig. 2). Less well-developed karst features are in central and south-central Kentucky. The streams in karst areas commonly have sustained base flow during dry-weather periods.

## **Climate**

Annual precipitation in Kentucky averages about 47 inches (Conner, 1982). The distribution of precipitation varies areally, year-to-year, and seasonally. The mean annual precipitation in Kentucky ranges areally from about 41 to 53 inches. Rainfall generally decreases to the north, reflecting the increase in distance from the source of precipitation, which is primarily the subtropical Atlantic Ocean and Gulf of Mexico. Considerable year-to-year variation in precipitation has occurred in Kentucky. During the period 1951-80, annual precipitation at reporting stations ranged from 14.5 to 78.6 in. (Conner, 1982).

Seasonally, the greatest amount of precipitation occurs during late winter and early spring in all areas except in north-central Kentucky, which receives the largest monthly precipitation in July. Winter precipitation is associated with frontal activity; however, in summer, convective thunderstorms produce most

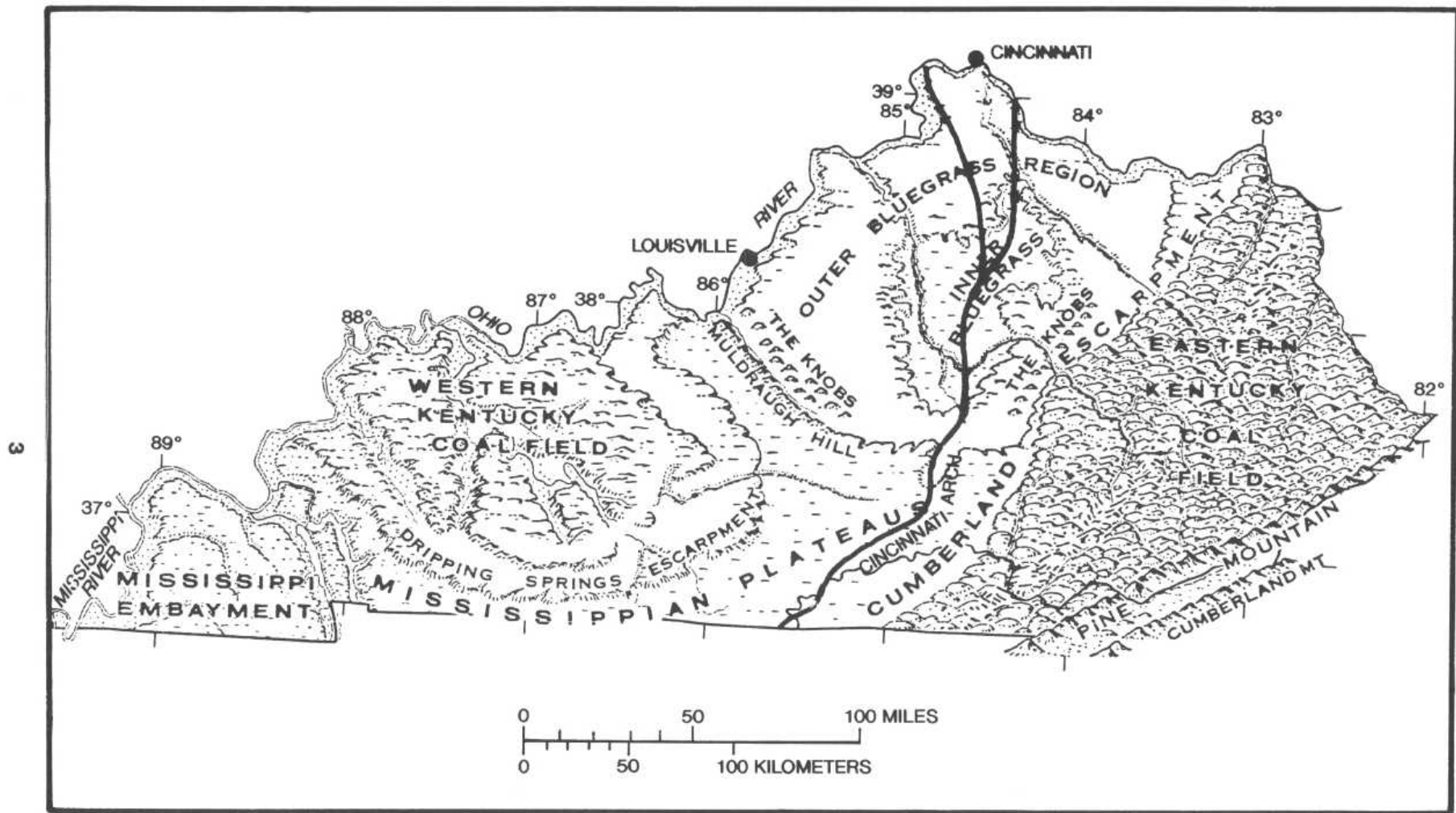


Figure 1.--Physiographic regions in Kentucky [From Kentucky Geological Survey, 1980].



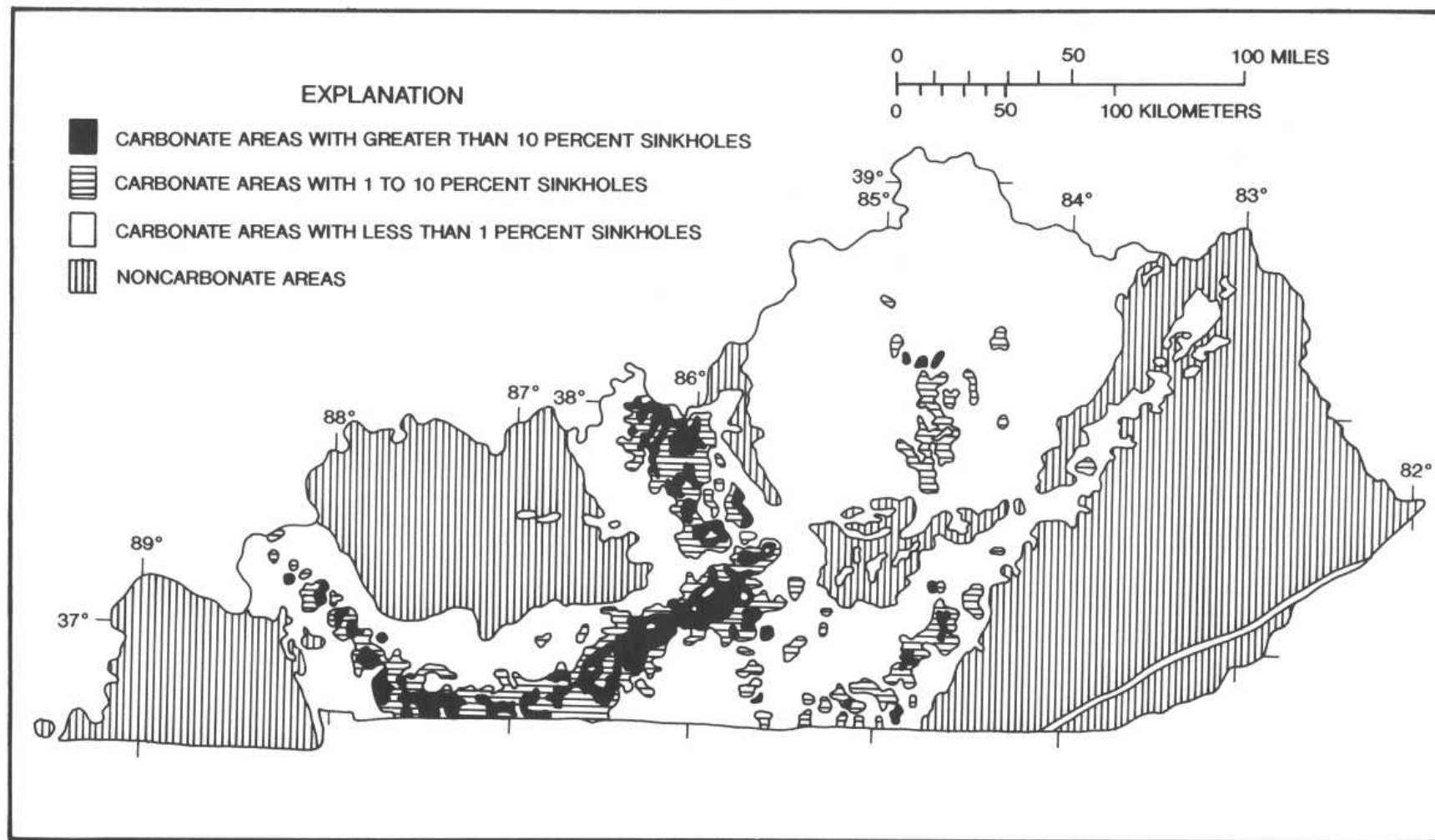


Figure 2.--Generalized carbonate areas and surficial karst development in Kentucky  
[From Crawford and Webster, 1986].

of the precipitation. Large amounts of rainfall in Kentucky have been associated with tropical cyclones originating in the Gulf of Mexico. Kentucky's dry season occurs during the fall, and October is the driest month. The Bermuda High, which normally resides off the coast of the southeastern United States during summer, moves inland in the fall. In October, the normal position of the Bermuda High is over Kentucky and Tennessee. The High suppresses convective activity and inhibits the movement of fronts (Conner, 1982). As a result, streamflow depends primarily on the discharge of ground water during late summer and early fall.

## THEORETICAL BASIS FOR, AND COMPUTATION OF, HARMONIC-MEAN STREAMFLOW

The  $Q_h$  is one of several streamflow statistics that have been adopted by Federal and State water-quality-management agencies as design flows for use in waste-load allocation. Design flows are stream discharges that are used to set waste-load-discharge limits on the basis of permissible instream concentrations of contaminants.

Two general types of design flows are now in use--hydrologically based and biologically based (U.S. Environmental Protection Agency, 1986b). Hydrologically based design flows are derived from standard extreme-value statistical analysis of daily mean streamflow data that include only a single extreme value per year (lowest or highest). In a given year, however, several occurrences of streamflow may be more severe than the extreme value in other years during the period of record, and these occurrences are excluded in extreme-value analysis. Extreme-value analysis does, however, permit assignment of annual exceedence (or nonexceedence) probabilities. The 7-day, 10-year low flow ( $7Q_{10}$ ), computed using the lowest arithmetic mean of discharge over 7 consecutive days during each year of record, is an example of a commonly used hydrologically based design flow. Biologically based design flows, in contrast, are intended to be closely associated with observed biological, toxicological, and ecological effects of contaminants. To that end, all of the annual streamflow record is considered to account for the cumulative adverse effects of multiple within-year extreme values on organisms. Water-quality criteria for protection of aquatic life and human health can be implemented using biologically based design flows.

Currently (1993), Kentucky uses hydrologically based design flows for protection of aquatic life and human health from toxic substances that are not linked to cancer. Biologically based design flows are used for purposes of protecting human health from cancer-linked substances. Water-quality standards for purposes of protecting human health from cancer-linked substances were established based on the health risks of long-term (lifetime) exposures through (1) consumption of water and (2) consumption of water and fish tissue.

Biologically based design flows are computed using the harmonic mean of a streamflow time series, typically daily mean streamflow. Estimates of long-term (lifetime) average exposure concentrations of contaminants in streams can be made using  $Q_h$  computed from the long-term, period-of-record daily mean streamflows. The average exposure concentration,  $C_{avg}$ , during  $N$  days can be computed (L.A. Rossman, U.S. Environmental Protection Agency, written commun., 1988; Rossman, 1990b) as

$$C_{avg} = \frac{\sum_{t=1}^N C_t}{N} = \frac{\sum_{t=1}^N \frac{W}{Q_t}}{N} = \frac{W \sum_{t=1}^N \frac{1}{Q_t}}{N} = \frac{W}{N} \cdot \frac{1}{\frac{\sum_{t=1}^N \frac{1}{Q_t}}{N}} = \frac{W}{Q_h} \quad (1)$$

where

- $C_t$  is the average exposure concentration for a given day (mass/volume);
- $W$  is constant instream contaminant loading rate (mass/time);
- $Q_t$  is the average streamflow for a given day (volume/time); and
- $Q_h$  is harmonic-mean streamflow (volume/time).

Given an allowable long-term average exposure concentration for a contaminant specified by a water-quality standard, together with the computed  $Q_h$  at a site, the allowable instream contaminant loading rate ( $W$ ) can be determined from equation 1.

Thus,  $Q_h$  is the reciprocal of the arithmetic mean of the reciprocal of the daily mean streamflows, or

$$Q_h = \frac{N}{\sum_{t=1}^N \frac{1}{Q_t}} = \frac{N}{\frac{1}{Q_1} + \frac{1}{Q_2} + \dots + \frac{1}{Q_N}} \quad (2)$$

Low streamflows within the period of record analyzed are weighted more heavily than moderate or high streamflows. For example, the harmonic mean of the integers from 1 through 10 (i.e.,  $Q_t = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ ) is 3.4, compared to the arithmetic mean of 5.5. Thus,  $Q_h$  is biased toward the low end of the range of streamflows during a given period of record, and therefore,  $Q_h$  may correlate with other low-flow statistics.

A value of zero for daily mean streamflow cannot be handled directly in the above formulation because division by zero yields an undefined value. To adjust for zero flows, the harmonic mean of the nonzero daily mean streamflows was multiplied by the ratio of the number of nonzero daily mean streamflows to the total number of daily mean streamflows in the period of record analyzed. This proportional adjustment for zero flows is consistent with current U.S. Environmental Protection Agency design-flow methodology (Rossman, 1990a).

The above proportional adjustment for zeros can sometimes yield illogical results. For example, revising the above numerical example by changing the one to a zero (i.e.,  $Q_t = 0, 2, 3, 4, 5, 6, 7, 8, 9, 10$ ), the computed  $Q_h$  using the proportional adjustment,  $(9/10)(9/(1/2+1/3+ \dots +1/10))$ , is 4.2. This result is larger than when the one is included instead of the zero.

Alternate methods of treating zero flows were explored. One alternate method is the substitution of some small value, such as 0.01 or 0.001, for zero. Values of  $Q_h$  computed using such substitutions can differ significantly from values computed using the proportional adjustment discussed above (even for stations with few days of zero flow in the streamflow record). For example, substituting 0.01 for zero in the previous example (i.e.,  $Q_t = 0.01, 2, 3, 4, 5, 6, 7, 8, 9, 10$ ) yields a harmonic mean of approximately 0.1, a value significantly lower than  $Q_h$  computed using the proportional adjustment. Substituting 0.001 for zero (i.e.,  $Q_t = 0.001, 2, 3, 4, 5, 6, 7, 8, 9, 10$ ) yields a harmonic mean of approximately 0.01, an order of magnitude lower than when 0.01 is substituted for zero. Thus, the results are sensitive to the number substituted for zero, and the results may be significantly affected by one or a few observations of zero flow in the streamflow record. Research concerning alternate methods for treating zero flows is needed, given the sensitivity of the results to the treatment applied. This research is beyond the scope of this investigation.

Streamflow statistics are subject to error associated with the particular time period sampled (time-sampling error). Gaged record may occur during either an abnormally wet or dry period, thus making it unrepresentative of long-term average climatic conditions. Time-sampling error decreases as record length increases.

Record-extension (augmentation) techniques may be used to reduce time-sampling error. Record extension is achieved by relating concurrent streamflows (and streamflow statistics) at a short-term and a nearby long-term (index) station that is hydrologically similar. The  $Q_h$  at the index station and the relation between the concurrent streamflows at both stations may be used to provide an estimate of the long-term  $Q_h$  at the short-term station. A mathematical record-extension technique, Maintenance of Variance Extension Type 1 (MOVE.1) as described by Hirsch (1982), was used in this study. The estimate was computed using log-transformed values of the concurrent nonzero daily mean streamflows as

$$Q_{h(s)} = M_s + \left(\frac{S_s}{S_l}\right) \times (Q_{h(l)} - M_l), \quad (3)$$

where

- $Q_{h(s)}$  is the estimated long-term  $Q_h$  for the short-term station;
- $Q_{h(l)}$  is  $Q_h$  for the long-term station;
- $M_s, M_l$  are the mean of the daily mean streamflows for the concurrent period at the short- and long-term stations, respectively; and
- $S_s, S_l$  are the standard deviations of the daily mean streamflows for the concurrent period at the short- and long-term stations, respectively.

For example, this method was used to provide an improved estimate of long-term  $Q_h$  at the South Fork Licking River at Hayes, Kentucky (station 03253000) by relating concurrent flows at the Licking River at Catawba, Kentucky (station 03253500) during the 1929-31 water years. A graphical representation of the MOVE.1 line relating the concurrent nonzero flows is shown in figure 3. A  $Q_h$  of 184 ft<sup>3</sup>/s at the station at Catawba corresponds to an estimated long-term  $Q_h$  of 21.4 ft<sup>3</sup>/s at the station at Hayes. Without using the MOVE.1 record-extension procedure and instead computing directly from the streamflow data for 1929-31, the short-term  $Q_h$  at the station at Hayes would be 3.57 ft<sup>3</sup>/s. Nearly an order of magnitude less than the estimated long-term  $Q_h$  at the station at Hayes, the short-term  $Q_h$  reflects the severe drought conditions that occurred during the 1929-31 period. This difference is unusually large, and it illustrates the need for record-extension procedures to reduce potentially large time-sampling errors at short-term stations.

## DETERMINATION OF HARMONIC-MEAN STREAMFLOWS AT STREAMFLOW-GAGING STATIONS IN THE STUDY AREA

Available data through the 1989 water year from continuous-record streamflow-gaging stations were used to determine the values of  $Q_h$ . The streamflow data were first compiled and reviewed to define appropriate periods of record for analysis. Separate computations were made for regulated and unregulated periods of record at the stations affected by regulation.

### Streamflow-Data Compilation

Daily mean streamflows for the available period of record were retrieved from the USGS National Water Data Storage and Retrieval System (Hutchison and others, 1975). The data were checked and verified by comparing computed yearly and monthly summary statistics of the daily mean streamflows to published values

DAILY MEAN DISCHARGE FOR SOUTH FORK LICKING RIVER AT HAYES,  
KENTUCKY, IN CUBIC FEET PER SECOND

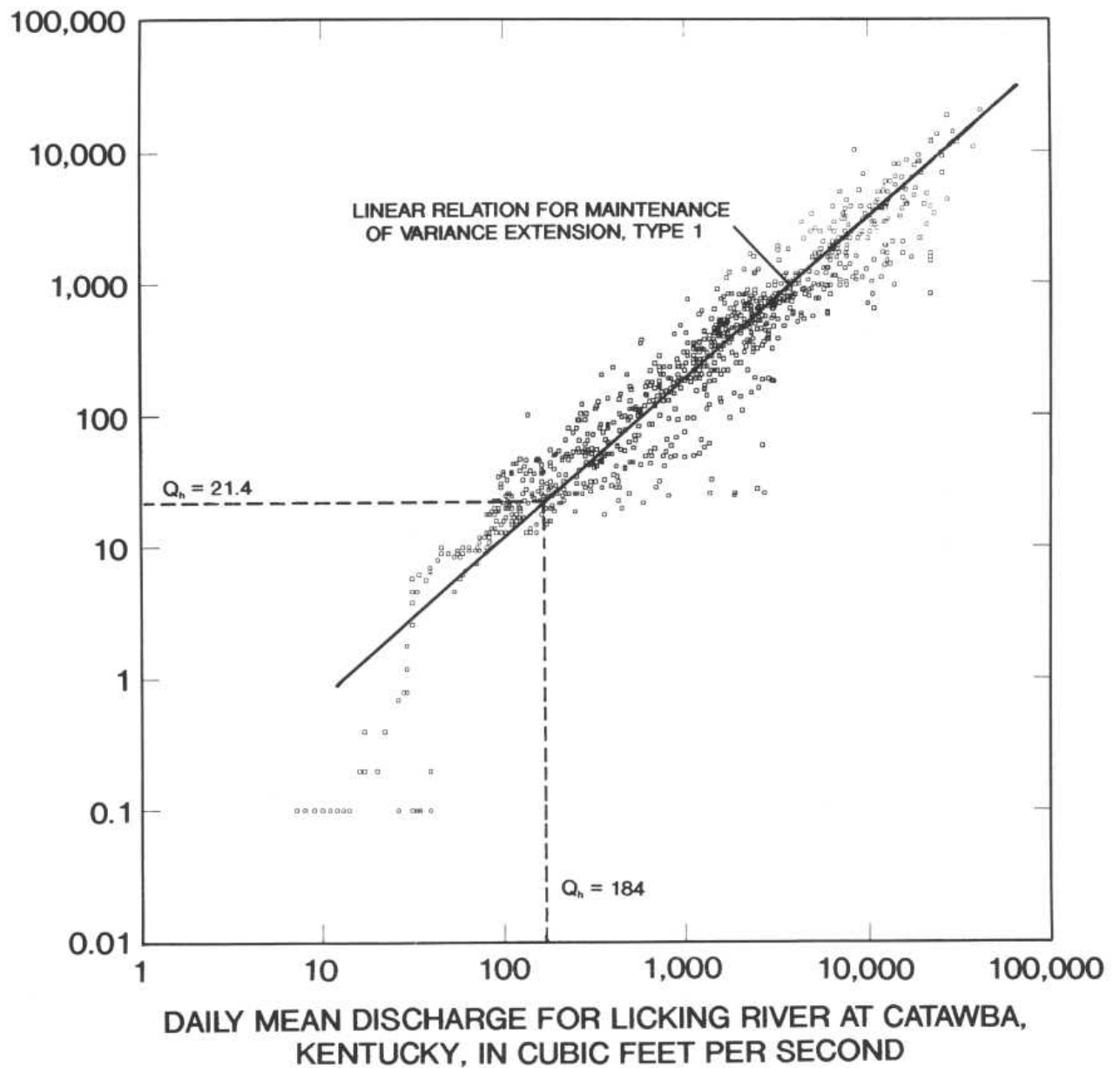


Figure 3.--Relation between concurrent daily mean flows for South Fork Licking River at Hayes, Kentucky and Licking River at Catawba, Kentucky, 1929-31 water years.

(U.S. Geological Survey, 1958a, 1958b, 1964a, 1964b, 1962-65, 1966-75, and 1976-90). Many of the stations in Kentucky are affected by regulation and (or) local diversions. Regulation by multipurpose or flood-control reservoirs reduces peak flows and generally augments low flows. Local diversions--localized transfers of water such as water supply withdrawals or wastewater discharges--artificially increase or decrease streamflows within a reach. Local diversions are common near municipalities and in urbanized areas. The extent of alterations in natural streamflows caused by regulation and local diversions is variable and difficult to quantify accurately. Periods of streamflow record affected by regulation and local diversions were considered separately from periods unaffected by regulation. Therefore, each continuous-record gaging station was screened to identify any periods of record affected by regulation (Melcher and Ruhl, 1984; Ruhl and Martin, 1991).

The retrievals included a total of 230 continuous-record streamflow-gaging stations (pl. 1), of which 54 also have regulated periods of record. Included with the stations located in Kentucky were several unregulated stations located nearby in adjacent States. The streamflow data in bordering States were retrieved to provide additional information for use in the regionalization of  $Q_h$  values. The period of record at the streamflow-gaging stations ranged from 1 to 78 years.

### **Unregulated Streamflow-Gaging Stations**

Daily mean streamflow at unregulated stations and for unregulated periods at subsequently regulated stations were used to compute values of  $Q_h$ . To compare station values of  $Q_h$ , these values were standardized to basin drainage area (reported in units of  $(\text{ft}^3/\text{s})/\text{mi}^2$ ) at each unregulated station. Where nearby hydrologically similar index stations were available and suitable relations were obtained, the MOVE.1 record extension techniques were applied.

Record-extension adjustments to  $Q_h$  values were attempted at all unregulated stations with less than 10 years of record. The accuracy of results depend on the availability of a well-correlated index station in a hydrologically similar setting. Adjusted  $Q_h$  values that were not consistent with other nearby long-term stations were not used. The correlation coefficient ( $r$ ) for the concurrent flows, though not used in the MOVE.1 calculation, is a measure of the strength of the linear relation; and  $r$  exceeded 0.80 for each station where the record extensions were used. Extensions were used at all the unregulated stations not affected by local diversion that had less than 6 years of record.

The extensions, while reducing time-sampling error at the station, generally improved consistency in  $Q_h$  and drainage-area-standardized  $Q_h$  values along stream reaches and among neighboring streams. A generalized depiction of the values of  $Q_h$  standardized to drainage area for unregulated stations not affected by local diversions is shown in figure 4. The largest values of drainage-area-standardized  $Q_h$  occur in karst areas (fig. 2) and also near Kentucky's eastern border with Virginia and West Virginia (an area roughly straddling the boundary between the Appalachian Plateaus and Valley and Ridge physiographic provinces). Sustained base flows are characteristic of streams in these areas (Ruhl and Martin, 1991; Hayes, 1991; and Friel and others, 1988) that have relatively high drainage-area-standardized  $Q_h$ .

Values of  $Q_h$ , drainage-area-standardized  $Q_h$ , total drainage area, and periods of record for stations not affected by regulation are listed in table 1 (back of report). The stations affected by local diversions and stations where the MOVE.1 record extensions were applied are identified by footnote in table 1.