Stage-Fall-Discharge Relations for Steady Flow in Prismatic Channels

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Laboratory studies of effect of variable backwater on stage-discharge relations at stream-gaging stations

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ABSTRACT

In a prismatic channel, the shape of the surface profile remains fixed as long as the discharge, the slope, and the boundary conditions remain fixed. This "Principle of rigidity of the profile" was previously applied to the observation, under laboratory conditions, of 41 M-1 type profiles for steady flow in prismatic channels. The same "Principle of rigidity of the profile" is now applied to the derivation, from these same profiles, of extensive data on relations among stage, fall, and discharge. These data are used to test the applicability of various methods of determining discharge under variable fall, under the particular conditions of boundary and slope which existed in the laboratory. Diagrams are presented to show that, using the uniform-flow ratings as a basis of comparisons, all the known curves of relation between stage, fall, and discharge become three-dimensional diagrams, rather than single curves. A new method of preparing such three-dimensional diagrams is proposed.

Other diagrams are presented to show that by introduction of an optimum amount of backwater and substitution of the resulting fixed-backwater rating in place of the uniform-flow rating as a basis of comparisons, the correlation between discharge ratios and fall ratios is so improved that a single curve of relation will, under all conditions that were tested, lead to a close approximation of the true discharge.

Throughout the report emphasis is placed upon the fact that the findings and conclusions are solely with respect to the particular conditions of boundary and slope which were used in the laboratory. The applicability of these conclusions to field conditions has not been extensively explored. The final section of the report presents suggestions for use in connection with field problems.

In the early part of the report, and incidental to the main theme, there is presented the theory of flow in prismatic channels. Data are presented to show the extent to which the laboratory observations are in agreement with this theory.

INTRODUCTION

PURPOSE

Gradually varied flow may be observed in almost any natural water course. From one cross section to another, even though the discharge is the same, variations may be expected in the velocities. Although usually due to changes in the size or shape of the cross sections of the stream channel, or a combination of these changes, the velocity variations may occur in perfectly uniform channels also. The changes often are so gradual as to escape casual observation, but the hydraulic problems involved are nevertheless problems in gradually varied flow. The problems are of particular importance on streams for which the discharge cannot be determined by reference to stage alone, but which also require the additional factor of fall as determined from an auxiliary gage. In such cases readings from the base gage and from the auxiliary gage determine two points on a surface profile for gradually varied flow. Thus a better understanding of this type of flow will contribute to better understanding of the stage-fall-discharge relations at gaging stations affected by variable fall.

Many hydraulic engineers have only meager knowledge of gradually varied flow. Even among those who work exclusively with flow of water in open channels the subject has been greatly neglected. Some have not realized the importance of the problems; others have been discouraged by the complex nature of the mathematics that forms the usual basis of approach. Furthermore, study of the problem by observations on natural water courses is handicapped by inability to evaluate the pertinent hydraulic factors. Study in the laboratory, where such factors may be controlled and evaluated, has until recently been considered impractical, because of the excessively long channel that was thought to be necessary.

Foremost among the reasons for the lack of experimental data on gradually varied flow profiles lies the incongruity between the length of laboratory channels and the length of the curves to be investigated. Except for comparatively small discharges or comparatively steep slopes, or a combination of both, the investigator is likely to find that the curve he wishes to investigate is several times the length of the longest channel available for the study. To develop a curve 1,000 feet long in a channel whose length is only 100 feet seems, at first glance, a bit irrational. Further study of the problem, however, indicates that, for prismatic channels, such studies can be made in a comparatively simple manner. A technique for this purpose was first suggested by Mononobe (1938) and later developed more fully by graduate students at the University of Illinois (Mitchell and Barron, 1946). After this work had been tested and reviewed, an agreement was reached between the University of Illinois and the United States Geological Survey to extend and enlarge the scope of the observations. The data obtained under this agreement have been published as Bulletin 381 of the University of Illinois Engineering Experiment Station (Lansford and Mitchell, 1949), hereafter referred to in this report as Bulletin 381.

In Bulletin 381 the objective was to obtain and publish laboratory observations. No attempt was made to apply the data to the particular use of the Geological Survey: the determination of stage-fall-discharge relationships. These relationships, however, were considered to be the ultimate objective of the Survey's interest in the data, and a study was begun at the earliest opportunity. The present report presents the results of that study.

SCOPE

This report is considered to be a companion volume of Bulletin 381. There is little, if any, duplication of material in the two reports. They deal with the same data, but with different objectives. Bulletin 381 presents laboratory observations with only such analyses as were essential to a proper presentation of the data; the present work applies these data to development of stage-fall-discharge techniques. Enough basic information has been included in the present work to make it self-sufficient for its intended purpose. For example, the uniform depth—discharge ratings for the several channels, which appeared as a part of Bulletin 381, have not been repeated here. In this report, however, for each of the several channels tables of depth *versus* k have been provided, and information has been provided for computing area for any depth in any channel. The factor k is defined as a function such that, when multiplied by area and the square root of the bed slope, the uniform discharge is obtained. Thus the uniform-depth-discharge ratings, although not tabulated in this report, may be obtained from information contained herein. Likewise, the detailed tables of profile observations which constitute almost 50 percent of Bulletin 381 have not been repeated here. Instead there has been condensed into a few pages a series of "smoothed profiles" differing from the original data only in the manner in which a completed rating table differs from the raw tabulation first read from a rating curve—that is, the data have been slightly adjusted to obtain smoothly changing values of the first and second differences. This smoothing has been essential to certain portions of the subsequent analysis and as presented herein may save other students many hours of tedious effort. On the other hand, profiles drawn through these smoothed data will differ only imperceptibly from profiles drawn from the original data of Bulletin 381. The smoothed profiles, tables 1 to 5 should not be confused with the computed profiles, tables 12 to 16, which form the basis of analysis in the final sections of the report.

Limitations of funds and of personnel have prevented the complete analysis of all the profiles presented in Bulletin 381. To provide the greatest range of information intensity of the analysis has been varied in the following order:

1. The six profiles of cross section 2—a rectangular channel lined with material of the same roughness throughout the entire depth range.

2. The seven profiles of cross section 5—a flood-plain channel lined identically with that of cross section 2.

3. The seven profiles of cross section 4—a rectangular channel lined with material of the same roughness throughout the entire depth range, this material being much rougher than for cross section 2.

4. The five profiles of cross section 3—a rectangular channel in which the lining of the upper part was identical with that of cross section 4, and in which the lower part was the same as that of cross section 2, except for the deterioration of lining material during the 2 years which had elapsed.

5. The four profiles of cross section 6—a flood-plain channel in which the lining of the lower part was identical to that of cross section 5, and in which the upper part was unlined.

In general, a given method of analysis or correlation was tried first on the profiles of item 1 above. In cases where further tests appeared desirable, they were continued on the profiles of item 2. If conclusions were still in doubt, tests were continued on the other profiles in the order indicated above.

Most of the results of such tests have been included in the following pages. For many of the techniques tested the results, as applied to these laboratory data, have proved unsatisfactory. It does not necessarily follow that these same techniques are unsatisfactory for other channels in which different conditions may be more favorable to the particular technique. Whether the results of a given test were good or poor, however, they have been included to the end that the reader may see for himself the nature of the results and thus make his own decision as to whether the method should be applied to his particular problem or analysis.

It cannot be too strongly emphasized that the tests contained herein are for steady flow in prismatic channels. Furthermore, the channel slope, although "mild" in the technical classification of fluid mechanics, is much steeper than generally encountered in field problems of gradually varied flow—this steepness of slope arises from practical limitations of the laboratory technique. Thus, the conclusions which may be drawn from the analysis of these data must not be blindly transferred to other problems on natural watercourses. Not only the probable flatter slopes, but also the irregularities of cross section of natural watercourses, and the phenomenon of changing discharge point toward untold complications. An ultimate objective to this study, of course, would be an answer to the question, "What change in the analysis of field problems is indicated by these investigations?" Much thought has been given to this question, and a number of suggestions have been made under the heading, "Application to field problems." (See pp. 142 to 146.)

ACKNOWLEDGMENTS

This report is an analysis of the data resulting from an investigation in the Fluid Mechanics and Hydraulics Laboratory at the University of Illinois as a part of the work of the Engineering Experiment Station (Lansford and Mitchell, 1949). In part it is similar to an earlier and less extensive analysis made in the department of Theoretical and Applied Mechanics of the same university (Mitchell and Barron, 1946). Except for the basic computations essential to the preparation of Bulletin 381 (Lansford and Mitchell, 1949), this analysis was made and the manuscript prepared by, or under the technical supervision of, William D. Mitchell, Hydraulic Engineer, working under the administrative direction of J. H. Morgan, District Engineer, Champaign, Ill. Other members of the Survey staff have assisted with the computations. Mr. John R. Stipp, Hydraulic Engineer, was in direct charge of computations and the preparation of tables. Mr. Salvatore A. Rocci, Hydraulic Engineer, assisted with the computations, and was in direct charge of the preparation of illustrations. Grateful acknowledgment also is made for the helpful advice and criticism of the Washington staff of the Survey, particularly W. S. Eisenlohr, Jr., of the Technical Coordination Branch, and C. H. Hardison, of the Surface Water Branch. The work was financed by Research and Development funds made available by C. G. Paulsen, Chief Hydraulic Engineer, through J. V. B. Wells, Chief, Surface Water Branch.

THEORY OF STEADY FLOW IN OPEN PRISMATIC CHANNELS

Steady flow is flow which is constant with respect to time; it is sufficiently accurate, for most channels, to say that, at a given point along the channel the discharge must be the same at all times within the period under observation. Such a condition rarely exists in the flow of natural streams, but often the condition is closely approached.

A prismatic channel is one for which all cross sections, perpendicular to the direction of general motion, are similar and parallel polygons which are equal in area. It should be noted that these characteristics apply to cross sections of the channel, but not necessarily to those portions of the cross sections which contain water.

There appears in the remaining articles of this section a presentation of the conventional concepts of flow under the limitations stated above. From many practical engineers, a purely mathematical discussion of this subject, including the varied-flow equation, will receive scant welcome. However, to avoid the necessity of involved or ambiguous statements during later sections of this report, it is imperative that certain mathematical relationships be recorded, and certain working formulas be explained. Most of these relations have been published many times before; they will not be new to workers familiar with varied-flow theory. For these, as well as for others willing to accept working formulas without worry over derivation, it is suggested that this section be disregarded and that a study of this report be resumed with the next section (p. 25).

TOTAL ENERGY OF A CROSS SECTION

COMPARISON FROM POINT TO POINT

At any point in a given flow, the total head (energy per unit weight of fluid) is composed of velocity head, pressure head, and elevation,

thus;

$$E = Z + \frac{p}{\gamma} + \frac{V^2}{2g},$$

in which E is total energy above the datum plane, Z is elevation above datum plane, p is intensity of pressure, V is velocity, all at the given point, γ is specific weight of the fluid, and g is the acceleration of gravity. Thus for point 1, figure 1,

$$E = Z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g}.$$

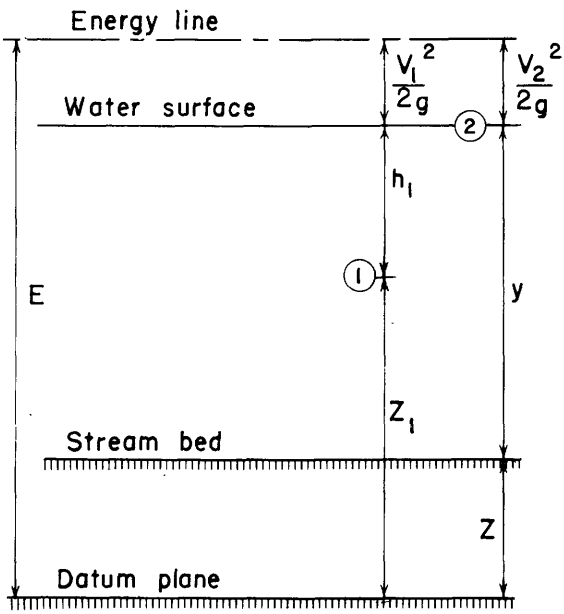


Figure 1. -Energy in a cross section.

Now if, in a given vertical plane, the pressure be hydrostatically distributed—that is, varies in ratio to lineal distance below the free surface—

then $$p_1 = \gamma h_1,$$

in which h_1 is the elevation of p_1 below the free surface. As the point p is allowed to approach a fixed free surface, Z increases by the amount that h decreases. Hence it follows that $Z_1 + p_1/\gamma$ is a constant, and may be replaced, for convience, by the values for p_2 in the free surface:

$$Z_1 + \frac{p_1}{\gamma} = Z_2 + \frac{p_2}{\gamma} = Z_2 + \frac{\gamma h_2}{\gamma} = Z_2 + 0 = Z + y,$$

in which Z is elevation of channel bed above datum plane, and y is total depth of flow.

Now if it be assumed that, in any cross section, the velocity is the same at all points, the velocity head $V^2/2g$ will be constant,

so that $$E = Z + y + \frac{V^2}{2g} = \text{a constant} \tag{1}$$

for all points in a given cross section.

Since this is the form of the basic equation of flow, both uniform and gradually varied, it will be apparent that two limitations have been imposed: (1) that pressure be hydrostatically distributed and (2) that the velocity be uniformly distributed throughout the cross section. The first limitation is observed by excluding from consideration all cases of rapidly varied flow—that is, cases in which the changes in flow occur in such abrupt manner as to disturb appreciably the normal distribution of pressure. The second limitation must be violated in most practical problems, but errors due to this cause may be compensated by use of a velocity-distribution coefficient.

VELOCITY-DISTRIBUTION COEFFICIENT

In equation (1), let it be considered that E is the energy per unit weight of fluid as applied to the cross section as a whole. E might be obtained by dividing the total flux of energy per unit time, E_t, by the mass discharge, γQ. However, to obtain the total flux of energy per unit time, it is necessary to obtain an integration, over the entire cross section, of the values at any point p. If V were truly a constant at all points in the cross section, this would be a simple algebraic operation, but because V is actually a variable, integration must be used, or, at least, the value of E should be computed for a very large number of points, each applied to its appropriate portion of the flow, and a summation obtained, that is,

$$E_t = C_1 \gamma Q \left\{ Z_1 + h_1 + \frac{V_1^2}{2g} \right\} + C_2 \gamma Q \left\{ Z_2 + h_2 + \frac{V_2^2}{2g} \right\} + \ldots . C_n \gamma Q \left\{ Z_n + h_n + \frac{V_n^2}{2g} \right\},$$

or, since $$Z_1 + h_1 = Z_2 + h_2 = Z_3 + h_3 = Z + y,$$

and since $$C_1 + C_2 + \ldots C_n = 1,$$

then, $$\frac{E_t}{\gamma Q} = Z + y + \left\{ C_1 \frac{V_1^2}{2g} + C_2 \frac{V_2^2}{2g} + \ldots\ldots C_n \frac{V_n^2}{2g} \right\}, \qquad \text{(a)}$$

in which C is the portion of the flow to which any V (with appropriate subscript) is applicable. Now if, in place of these varying values of V, there is used the average $V = Q/A$ (in which Q is total discharge and A is total area of the cross section),

$$E = \frac{E_t}{\gamma Q} = Z + y + \frac{V^2}{2g}. \qquad \text{(b)}$$

If V truly varies from point to point, it will be obvious that the result of equation (a) must differ from that for equation (b), for in any series of unequal numbers the average of their squares is greater than the square of their average. For example, with the numbers 3 and 5, the average of the squares is $(9 + 25)/2 = 17$, while the square of the average is 16. Thus if these values of 3 and 5 were used for V_1 and V_2 in computing E_t by equation (a), using $C_1 = C_2 = 0.5$, and then the value $V = 4$ was used to compute E_t by equation (b), it would be found necessary to apply to the latter computation a coefficient of 17/16 to harmonize the results. It will be noted that the greater the variation in the values of V, the larger will be the necessary coefficient. Thus, if $V_1 = 1$ and $V_2 = 7$, the average of the squares is 25, while the square of the average is 16, as before. It will also be noted that the required coefficient will vary with the relative values of C_1 and C_2. Using the principle of conservation of energy and the calculus, it can be demonstrated that

$$c_e = \frac{\Sigma a v^3}{A V^3},$$

in which v is the velocity at any point, and a is the portion of the cross section to which v applies, and A and V are values for the entire cross section.

Using the principle of conservation of momentum and the calculus, it can be demonstrated that

$$c_m = \frac{\Sigma a v^2}{A V^2},$$

in which a, v, A and V have the same significance as before.

There has been much heated discussion as to whether the c_e or c_m coefficient should be used. It might be pointed out that use of c_m weights the various v's in accordance with their appropriate area, whereas use of c_e weights them in accordance with their appropriate discharge. For some years, the use of c_m has been standard practice in the Geological Survey. Use of c_m as the velocity distribution coefficient will be continued in this report.

SPECIFIC ENERGY

Further consideration of the energy at a cross section provides a basis for approach to the problem of gradually varied flow. In a given cross section, let it be assumed that the discharge is to be held constant, but that means are provided for varying the depth at which flow will occur. This is readily imagined by assuming a channel which can be tilted, so as to allow the bed slope to vary. Under these circumstances different values of depth will result in different values of energy. The value of Z, being constant at a cross section, may now be disregarded, or in other words, the datum plane may be transferred to coincide with the bed of the channel. The energy with respect to this new datum is known as specific energy, or specific head, and denoted by the symbol H. Thus,

$$H = y + \frac{V^2}{2g}. \tag{2}$$

The manner in which H varies with y, depth, is illustrated by the accompanying figure 2, in which y has been plotted as ordinate and H as abscissa. Since y is

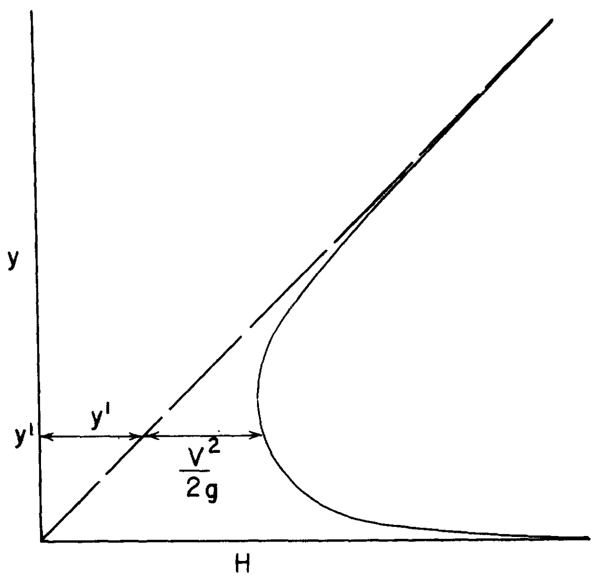


Figure 2. -Specific-energy diagram.

one of the factors of which H is composed, this portion of H may be represented by the 45° line through the origin of coordinates. That is, for any depth (y'), H is composed of an abcissa (y') plus an increment ($V^2/2g$) of velocity head. If y' is very great, the velocity will be very small, and the velocity head, involving the square of a very small number, will become negligible. Thus, at great depths, H approaches y asymptotically. If y' is very small, the velocity will be very great, and the velocity head, involving the square of a large number, will become extremely large. Thus, as y approaches zero, H approaches infinity. At some intermediate point, H must have a minimum value, which may be obtained by equating to zero the first derivative of equation (2):

$$\frac{dH}{dy} = \frac{dy}{dy} + \left\{\frac{d}{dy}\right\} \left\{\frac{V^2}{2g}\right\}.$$

But $$V = \frac{Q}{A}$$

so that $$\left\{\frac{d}{dy}\right\}\left\{\frac{V^2}{2g}\right\} = \left\{\frac{d}{dy}\right\} \left\{\frac{Q^2}{2gA^2}\right\} = \left\{\frac{Q^2}{2g}\right\}\left\{\frac{-2}{A^3}\right\} \left\{\frac{dA}{dy}\right\}$$

but $$dA = b(dy)$$

so $$\left\{\frac{d}{dy}\right\} \left\{\frac{V^2}{2g}\right\} = -\frac{Q^2b}{gA^3}. \qquad (3)$$

Therefore $$\frac{dH}{dy} = \frac{dy}{dy} - \frac{Q^2b}{gA^3},$$

which for minimum specific energy must equal zero, so that

$$\frac{dy}{dy} - \frac{Q^2b}{gA^3} = 0$$

or $$Q^2b = gA^3 \qquad (4)$$

whence $$\frac{Q^2b}{A^2} = gA$$

and $$\frac{V^2}{g} = \frac{A}{b}.$$

But A/b is the average depth,

whence $$\frac{V^2}{2g} = \frac{y_{av}}{2}; \qquad (5)$$

that is, H, the specific energy, is a minimum when the velocity head is equal to half the average depth. This value of the depth, y, for which H is a minimum, has great significance in open-channel flow, and is designated as y_c, the critical depth.

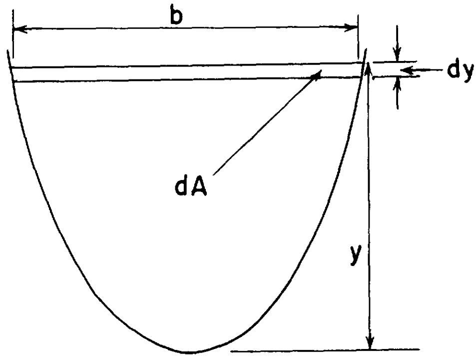


Figure 3. -Variable depth at a section.

UNIFORM FLOW

COMPARISON FROM SECTION TO SECTION

In the preceding section consideration has been given to the evaluation of E, the energy at a cross section. Attention is now invited to a comparison of the values of E for different cross sections. Referring to figure 4, a longitudinal section of a prismatic channel, with finite value of bed slope, S_0, it should first be mentioned that it is customary to measure depth as the vertical distance, y, rather than the distance y' normal to the stream bed. Obviously $y' = y \cos \beta$. Except for very steep slopes, however, $\cos \beta$ is so near unity as to make little difference whether y is measured in the vertical or normal direction. For example, the angle whose tangent is 0.003 (S_0 as used in the laboratory work) has for its cosine the value of 0.999996. Similarly x, the distance along the channel between any two cross sections, may be taken as the horizontal distance, rather than the distance parallel to the channel bottom.

Let figure 4 represent a reach of length x taken from a prismatic channel of infinite length, in which there is steady flow. At section 1, with depth y_1, and with respect to the horizontal datum plane,

$$E_1 = Z_1 + y_1 + \frac{V_1^2}{2g} = Z_1 + H_1$$

$$E_2 = Z_2 + y_2 + \frac{V_2^2}{2g} = Z_2 + H_2$$

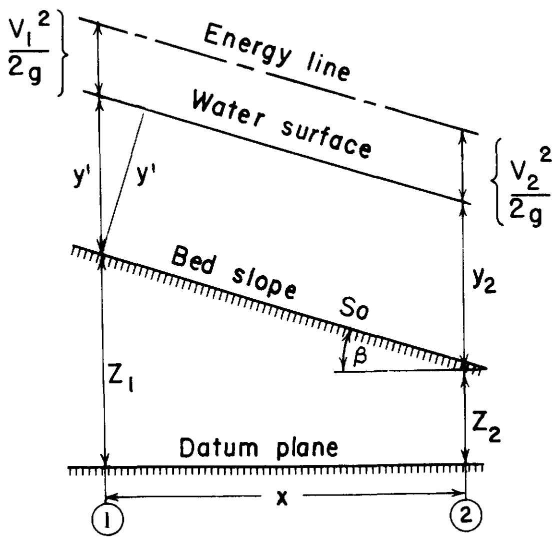


Figure 4. -Longitudinal section of uniform flow.

Since this channel is the same for all cross sections, not only between sections 1 and 2, but for any number of sections upstream and downstream, y_1 and y_2 must be equal. This follows from the fact that the controlling factors, whatever they may be, which lead to a depth y_1 at section 1, are identical to the factors which control the depth at section 2, and must therefore lead again to the depth y_1. The water surface must therefore be parallel to the stream bed. It follows that the velocities, and therefore the specific energy, H, must be the same at sections 1 and 2. Whence $E_1 = Z_1 + H_1$ and $E_2 = Z_2 + H_1$, so that $E_1 - E_2 = Z_1 - Z_2$. Dividing both members of this last equation by x discloses the loss per unit distance, or the rate of loss:

$$\frac{E_1 - E_2}{x} = \frac{Z_1 - Z_2}{x}$$

But from figure 4, $(Z_1 - Z_2)/x$ is the bed slope, S_0. Thus the slope of the energy line, as well as the slope of the water surface, is the same as the slope of the bed, S_0. This is a distinguishing characteristic of uniform flow. Flow is said to be uniform when all the elements which characterize the flow remain constant from one section to another.

UNIFORM DEPTH

Reference was made in the preceding article to "the controlling factors, whatever they may be, which lead to a depth y_1." These factors now will be examined. The movement of water from one cross section to another constitutes work, which can be accomplished only by expenditure of energy. In uniform flow, this energy cannot come from the water itself, for the specific energy remains the same at all cross sections. The energy must be derived from change in elevation, and all energy derived from this source must be expended for this purpose. Now the amount of energy needed for moving a given mass of water varies somewhat with the boundary conditions of the channel, but primarily with the velocity. If, in figure 4, the depth of flow were to drop below y_1, an increase in velocity would be required to pass the discharge through the decreased cross-sectional area. But this increased velocity would require the expenditure of greater energy, which is not available. Hence the water would not be entirely removed from the cross section, and the residual would immediately act to increase the depth. If the depth were to rise above y_1, a lower velocity would occur, and excess energy would be accumulated. This excess energy would immediately act to increase the velocity, and thus restore the original depth y_1. Thus it follows that, for a given discharge in a given prismatic channel, of very great length with a given bed slope, S_0, there is one, and only one, depth at which the flow will occur. This depth is known as the uniform depth, and designated by the symbol y_0. The discharge corresponding to this depth is known as the uniform discharge, Q_0.

COMPUTATIONS FOR UNIFORM FLOW

Problems in uniform flow frequently involve the determination of discharge for flow at a given depth. Here the objective is to determine an average velocity for a cross section such that the rate of change of the energy line, S (often called the friction slope), is equal to the rate of change of elevation, S_0. Defying rigorous mathematical analysis, the problem has led to empirical relationships, of which the Manning formula is perhaps the most popular:

$$V = \frac{1.486}{n} R^{2/3} S^{1/2}$$

in which n is a coefficient of roughness, dependent primarily upon the lining of the channel, R is the hydraulic radius (cross-sectional area divided by cross-sectional wetted perimeter), and S is the bed slope. The formula has objectionable features, among them being the fact that the two-thirds power of the hydraulic radius is, in some instances, an inadequate measure of boundary effects. In spite of its shortcomings, the formula is generally used for lack of a better one.

Fortunately, in the laboratory investigations the use of such a formula was unnecessary. With a given steady flow in the laboratory channel, the regulating equipment was manipulated until a uniform depth, y_0, was found to exist at all cross sections. The discharge, Q_0, was then measured, usually by current meter. Thus the values of y_0 versus Q_0, or the uniform stage-discharge relations, were established without need of a formula. This was a great advantage in preparing the computed profiles, tables 12 to 16, in that it permitted the computations to be based on laboratory observations, and kept them free of the complications which attend the use of formulas such as Manning's.

CLASSIFICATIONS OF FLOW

Means now may be described for the classification of types of flow and of the channels in which they occur. In discussion of specific energy, there was proposed the concept of a tilting channel, permitting a variation of the bed slope, but otherwise holding fixed all the physical features, including roughness. Attention is now invited to uniform flow in such a channel. Assuming a given constant discharge and remembering that uniform depth is dependent on the establishment of a velocity which will demand a rate of energy expenditure, S, exactly equal to S_0, the following facts become apparent: As the slope of the channel becomes very great the normal depth will become very small. Both the specific energy and the friction slope will be very great, but the specific energy is self-sustaining, and the friction slope is sustained by the high value of bed slope. As the lower end of the channel is raised, decreasing S_0, the value of normal depth, y_0, will increase. As long as the values of y are less than y_c (see p. 10), this increase in y_0 will be accompanied by a decrease in specific energy. Under these conditions, the channel is said to have a ***steep*** slope, and the state of the flow is said to be ***rapid***, or ***shooting***. As the slope of the channel is further decreased the condition is reached in which the normal depth will be exactly equal to the critical depth (each being equal to twice the velocity head) and the specific energy reaches a minimum value. Under these conditions, the channel is said to have ***critical*** slope, and the state of flow is said to be ***critical***. As the slope of the channel is still further decreased, the normal depth will continue to increase, but since y_0 is now greater than y_c, the increase in y_0 will be accompanied by an increase in specific energy. Under these conditions, the channel is said to have a ***mild*** slope, and the state of the flow is said to be ***tranquil***, or ***turbulent***. Other channel slopes, such as adverse, are recognized, but these have no application within the scope of this report. Another state of flow (laminar) may occasionally occur, but a different sort of criterion must be used for its proper description. This subject will be mentioned again in a later section of the report. (See p. 49.)

In the preceding paragraph, the assumptions are given as a constant (steady) discharge and uniform depth. This second assumption is the same as assuming a prismatic channel of very great length (see p. 13). In a much shorter channel, conditions of uniform flow will not necessarily exist, but rather the depth at successive sections may vary. Thus, if a channel of mild slope be obstructed by a dam, the section immediately upstream will have a depth greater than the uniform depth; if this channel ends abruptly as a free fall, the section immediately upstream will have a depth less than the uniform depth. There are other conditions of discharge, such as under a sluice gate, which may result in a condition of flow in which depth may be less than critical depth, as well as less than uniform depth. Such possible conditions give rise to further classification of flow: when the depth is greater than both the normal and critical, the flow is said to be *class 1*; when depth is greater than either of these values, but less than the other, the flow is said to be *class 2*; when depth is less than both values, flow is said to be *class 3*. In the measurement of river discharge, most of the problems which arise are in connection with class 1 flow in channels of mild slope -- or, as it is commonly expressed, the M-1 profiles. Although the laboratory data included a few observations on M-2 profiles (see p. 26), the present analysis is limited to M-1 conditions.

NONUNIFORM FLOW

GENERAL CONSIDERATIONS

Figure 5 represents a case of gradually varied flow. This example differs from uniform flow in several respects, the most obvious of which is that successive values of y are not equal but gradually increase from left to right. Since the chan-

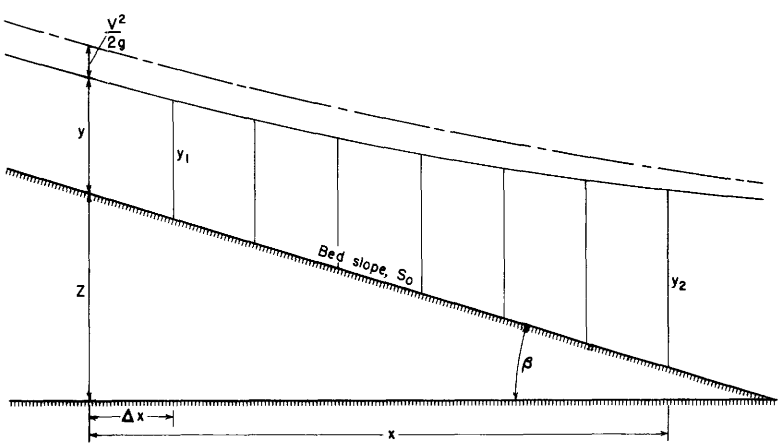


Figure 5. -Longitudinal section of a case of gradually varied flow.

nel cross section is the same at all points, and the discharge is constant, it follows that velocity (and velocity head) must decrease as y increases. As in uniform flow, the finite value of S_0 produces variation, from section to section, in the value of Z. Thus all the factors which enter into the evaluation of E become variable quantities.

To study the variations of these factors in the reach x between the point at which $y = y_1$ and the point at which $y = y_2$, recourse is made to a very short reach, Δx, such that the rates of variation within the reach are essentially the same as the rates of variation at any given point -- that is, the same as the slope of the various lines. This slope is obtained by taking the derivative, with respect to x, of equation (1), replacing d by the value Δ for a finite, but very short, reach:

$$\frac{\Delta E}{\Delta x} = \frac{\Delta Z}{\Delta x} + \frac{\Delta y}{\Delta x} + \frac{\Delta (V^2)}{\Delta x(2g)}. \tag{6}$$

GEOMETRY OF THE REACH, Δx

The interrelation of these variables is illustrated in figure 6, page 17, which is a plot of actual data from Bulletin 381, table 1, page 39 (Lansford and Mitchell, 1949). Data not given in table 1 have been computed by principles set forth under "Total energy of a cross section," with the assumption that c_m is unity. Datum plane has been taken as the horizontal plane which intersects the channel bed at station 0. Taking as Δx the reach from station 1212 to station 1182 and following the simple rule that change in any item is equal to the final value minus the initial value, the following observations become apparent:

(1) ΔE has the value 4.102 - 4.144 = -0.042, or the distance between points (1) and (2) in figure 6, the line through point (2) having been drawn parallel to the datum plane and through energy line at initial section.

(2) ΔZ has the value 3.546 - 3.636 = -0.090, or the distance between points (3) and (4), the line through point (4) having been drawn parallel to the datum plane and through bed elevation at initial section.

(3) Δy has the value of 0.523 - 0.466 = 0.057, or the distance between points (5) and (6), the line through point (6) having been drawn parallel to bed slope and through water surface at initial section.

(4) $\Delta(V^2/2g)$ has the value 4.060 - 4.069 = -0.009, or the distance between points (7) and (5), the line through point (7) having been drawn parallel to the energy line and through water surface at initial section.

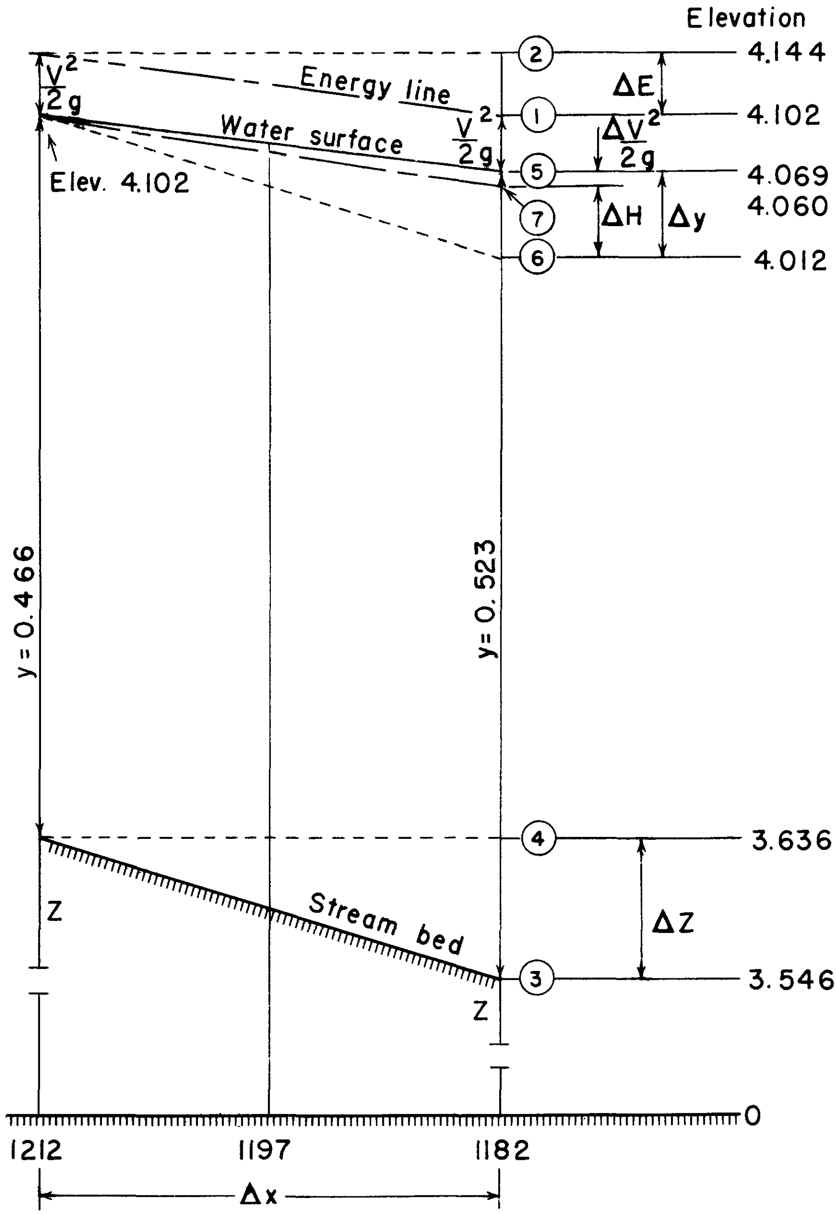


Figure 6.—Gradually varied flow in a very short reach.

(5) ΔH has the value of 4.060 - 4.012 = 0.048, or the distance between points (7) and (6), lines through these points having been drawn as already described; obviously ΔH represents the change of energy line with respect to the bed slope.

(6) Δx, of course, has the value 1182 - 1212 = -30, or the distance between $y = 0.523$ and $y = 0.466$.

Special attention is invited to the fact that the change in specific energy, ΔH, is equal to the algebraic sum of the change in depth, Δy, and the change in velocity head, $\Delta(V^2/2g)$.

COORDINATE SYSTEMS AND CONVENTIONS

Justification already has been made for the practice of measuring y and x in vertical and horizontal directions, rather than with respect to channel bed. It generally is convenient to establish the datum plane as was done in preceding article -- that is, the horizontal plane which intersects the channel bed at the point for which y has some predetermined value -- usually that for the downstream end of the reach of channel. (In the laboratory work, this value of y was invariably taken as 4.000 feet. For further discussion, see p. 35.)

In the use of equation (6) it is customary to adopt the convention that a sustaining slope -- that is, a channel which slopes downward in the direction of flow -- is a positive slope. With the convention as used in the preceding article, it will be noted that both dZ and dx are negative, so that dZ/dx (the sustaining slope) is positive in accordance with usual custom.

As a check upon the consistency of these evaluations, the numerical values of the preceding article are now substituted in equation (6):

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\frac{(V^2)}{(2g)} \tag{6}$$

$$\frac{-0.042}{-30} = \frac{-0.090}{-30} + \frac{+0.057}{-30} + \frac{-0.009}{-30}$$

$$0.0014 = 0.0030 - 0.0019 + 0.0003$$

$$0.0014 = 0.0014$$

In working with a finite reach of channel, great caution must be observed to keep the reach, Δx, sufficiently short to avoid the effects of curvature.

INTEGRATION OF THE VARIED-FLOW EQUATION

It has just been indicated that, under varied flow, computations for finite reaches are valid only if the reach, Δx, is very short. Thus for great accuracy, one is tempted to return to the infinitesimal reach, dx, and apply methods of integration to obtain values for a finite reach. This is the more desirable since, in most practical computations it is necessary to rearrange equation (6) into a form such that x may be computed from available data. In other words, it becomes desirable to combine as many as possible of the various factors, solve the resulting expression for dx, and integrate the final expression between appropriate limits of y. Unfortunately, the mathematics becomes extremely complicated for all except certain special cases.

Considering an infinitesimal reach:

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left\{\frac{V^2}{2g}\right\} \tag{6}$$

To appreciate the significance of this equation it is convenient to recall (see p. 9) that $y + V^2/2g = H$, so that equation (6) may be written

$$\frac{dE}{dx} = \frac{dZ}{dx} + \frac{dH}{dx} \tag{6a}$$

which usually is interpreted to indicate that the rate of change (with respect to x) of energy is equal to the rate of change of bed elevation plus the rate of change of specific energy. In theoretical mechanics it is customary to compute change as the final value of a quantity minus the initial value of the quantity. But, as flow always involves a loss of energy, such computations here result in undesirable negative values for dE. In this report, this situation is remedied by use of a dE which is the negative of the above, or initial value minus final value, and which, to avoid confusion, is termed loss rather than change.[1]

It is entirely appropriate that equation (6a) be interpreted to indicate that the rate of loss of energy is equal to the rate of loss of bed elevation plus the rate of loss of specific energy. It is not proper, however, to regard the rate of loss of energy as equal to the rate of loss of bed elevation plus the rate of change of specific energy. If, in equation (6) or (6a), one term is regarded as a loss, then all terms should be regarded as loss, or initial value minus final value.

[1]The author is indebted to Carl E. Kindsvater, Professor of Civil Engineering, Georgia Institute of Technology, for suggesting the use of this distinction between loss and change.

Considering figure 6 now to represent an infinitesimal reach, and using subscripts (1) and (2) to indicate the initial section and final section, respectively, $dE/dx = (E_1 - E_2)/dx = S$, and $dZ/dx = (Z_1 - Z_2)/dx = S_0$, in which both dE and dZ are regarded as loss. Making these substitutions in equation (6) gives

$$S = S_0 + \frac{dy}{dx} + \frac{d}{dx}\left[\frac{V^2}{2g}\right] \tag{7}$$

a form of expression which is common to all the methods which are subsequently discussed. The differences in the methods lie in the differences in treatment of the last term of the righthand member -- that is, the small, but sometimes significant effect of the change in kinetic energy. From equation (3), page 10,

$$\frac{d}{dy}\left[\frac{V^2}{2g}\right] = \frac{-Q^2b}{gA^3},$$

or

$$d\left[\frac{V^2}{2g}\right] = \frac{-Q^2b}{gA^3}\,dy$$

Making this substitution in equation (7),

$$S = S_0 + \frac{dy}{dx} - \frac{dy}{dx} \cdot \frac{Q^2b}{gA^3}$$

$$S - S_0 = \frac{dy}{dx}\left[1 - \frac{Q^2b}{gA^3}\right]$$

$$\frac{dx}{dy} = \frac{\left[1 - \dfrac{Q^2b}{gA^3}\right]}{S - S_0}$$

$$-dx = \frac{\left[1 - \dfrac{Q^2b}{gA^3}\right]}{S_0 - S}\,dy \tag{8}$$

In the interpretation of equation (8) it should be remembered that, as S and S_0 were expressed as rates of loss, dy also must be regarded as a loss, or $dy = y_1 - y_2$. Some authors have regarded dy as the change, $y_2 - y_1$, in which case the minus sign vanishes from the left member of the above equation. However such a mixture, in one equation, of the concepts of loss and change frequently has been a source of confusion, and of error in computations.

For a fixed discharge in a channel of known geometric characteristics, dx in formula (8) might be computed for two given values of y, provided S were known. Problems involved in computing S for uniform flow were discussed on page 13. In nonuniform flow it must be assumed that S is the same as for uniform flow under the given conditions of discharge and depth -- in other words, discharge through a given cross section is a unique function of depth and friction slope. An expression for such an S may be obtained by use of the fundamental equation $Q = AV$, and the substitution for V of one of the usual formulas for uniform flow. Using the Manning formula (see p. 13),

$$Q_0 = A \; \frac{1.486}{n} \; R^{2/3} \; S_0^{1/2}$$

Here, for a given depth of flow, y, the expression

$$A \; \frac{1.486}{n} \; R^{2/3}$$

is dependent only on the cross section. Bakhmeteff (1932) has suggested that, for simplicity, this expression may be replaced by a single symbol, K, called conveyance. Letting K_0 be that particular value of conveyance in which $S = S_0$, or the flow is uniform,

$$Q_0 = K_0 \; S_0^{1/2}$$

or,

$$S_0 = \frac{Q_0{}^2}{K_0{}^2} \tag{9}$$

Now if the flow be at the same depth, y, but be gradually varied instead of uniform, K must be the same as before, since it is dependent only on the cross section which is unchanged. However, for gradually varied flow, Q no longer is Q_0, nor is S equal to S_0, so the above equation becomes

$$S = \frac{Q^2}{K_0{}^2} \tag{10}$$

Similarily, the discharge Q_0, when flowing at a depth y other than y_0 will be related to a specific value of S and to a K dependent on the depth y, so that

$$S = \frac{Q_0{}^2}{K^2} \tag{11}$$

Dividing equation (11) by equation (9),

$$\frac{S}{S_0} = \frac{Q_0^2}{K^2} \cdot \frac{K_0^2}{Q_0^2} = \frac{K_0^2}{K^2}$$

Furthermore,

$$S_0 - S = S_0 \left[1 - \frac{S}{S_0}\right] = S_0 \left[1 - \frac{K_0^2}{K^2}\right]$$

and formula (8) becomes

$$- dx = \frac{1 - \dfrac{Q^2 b}{gA^3}}{S_0 \left[1 - \dfrac{K_0^2}{K^2}\right]} \cdot dy \qquad (12)$$

which, except for the sign of dx, is a common form of the differential equation for gradually varied flow.

DIRECT INTEGRATION

Bahkmeteff has observed that K, in general and between wide limits, is an exponential function of y. With that assumption, plus the ratio between S_0 and S_c, and making use of a table which he has computed, substitutions may be made in a rearrangement of equation (12) so that the distance, x, between two given and widely different values of y may be directly computed. In some instances, however, the ratio S_0/S_c may vary with y, so that it may be necessary to take comparatively small increments of y. Used with care, the method will yield results in satisfactory agreement with the results of other reliable methods.

Von Seggern (1950) made use of Bakhmeteff's observation that K was an exponential function of y, and added the thought that $M = A\sqrt{A/b}$ also is an exponential function of y, and that $(M_c/M)^2$ may be substituted in equation (12) in place of Q^2b/gA^3. With these assumptions, a table similar to Bakhmeteff's, and a second table to be used in connection with the new assumption, substitutions may be made in another rearrangement of equation (12) so that the distance, x, between two widely different values of y may be directly computed. In many instances the method will yield results which are in satisfactory agreement with those of other reliable methods.

A detailed comparison of the Bakhmeteff and Von Seggern methods will be found among the published discussions of Von Seggern's paper. A comparison between the results of these two methods and the laboratory observations will be found on page 55.

THE STEP METHOD

After noting the difficulties which attend any attempt at complete integration, one is inclined to look with greater favor upon the step method — the method in which average values at the ends of a reach are assumed to apply throughout the reach. This favorable attitude is the more pronounced in view of the extreme simplicity both of the required formula and of the method of its derivation. Figure 7

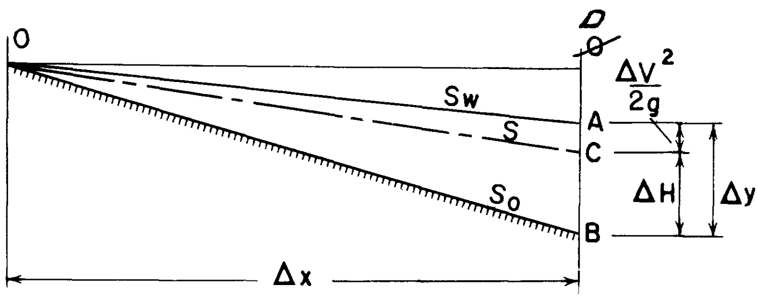


Figure 7. —Geometry of the step method.

reproduces certain pertinent features of figure 6. Point O is the water surface at the upstream end of the reach. Points A, B, and C are the points shown in figure 6 as 5, 6, and 7, respectively. Point D is the intersection of a line through A, B, and C with a horizontal line through point O.

$$(D\text{-}B) = (D\text{-}C) + (C\text{-}B). \tag{13}$$

It is obvious that

$$(D\text{-}B) = \Delta x \, S_0$$

and

$$(D\text{-}C) = \Delta x \, S$$

The significance of $(C\text{-}B)$ is not so apparent. It must first be noted that both $(D\text{-}B)$ and $(D\text{-}C)$ represent a loss, or initial value minus final value, and must be distinguished from change, which was defined (p. 16) as the final value minus the initial value. For consistency, as well as convenience, the final term in equation (13) should be viewed also as a loss. $(C\text{-}B)$, however, is not the loss in H between the initial and final sections; rather it is the gain in H or, as is generally called, the change in H. At either section, the value of H is represented by the elevation of the S line minus the elevation of the S_0 line. At the initial section, H, as depicted by figure 7, is zero; at the final section, H is represented by $(C\text{-}B)$. Hence the *loss* in H from the initial to the final section is

$$\Delta H = 0 - (C\text{-}B)$$

whence $$(C-B) = -\Delta H$$

Making these substitutions in equation (13),

$$\Delta x S_0 = \Delta x S - \Delta H$$

whence $$\Delta x = -\Delta H/(S_0 - S), \tag{14}$$

in which ΔH is defined as a loss, or initial value minus final value, consistent with the computation of S and of S_0.

For those who prefer an analytical approach, this same relation might have been derived directly from equation (8). In that equation, the numerator of the right-hand term might have been written: $dy - Q^2 b\, dy/gA^3$. As pointed out in the development of that equation,

$$-Q^2 b\, dy/gA^3 = d(V^2/2g)$$

Remembering that

$$dy + d(V^2/2g) = dH,$$

equation (8) becomes:

$$-dx = \frac{dy + d(V^2/2g)}{S_0 - S};\ dx = -dH/(S_0 - S)$$

the same as in the graphical development, except for the change between d and Δ, representing the difference between an infinitesimal and a finite reach. Since S varies from point to point throughout a finite reach, formula (14) should be used only for short reaches. Furthermore, S should be computed for each end of the reach, and the average value, $\bar{S}$, should be used as being more representative of the proper value of S. Thus the proper equation for step-method computation becomes:

$$\Delta x = -\Delta H/(S_0 - \bar{S}). \tag{15}$$

This formula has been used as the basis of extensive computations in the following section of this report. (See pp. 52-55.)

THE LABORATORY DATA

In the introduction to this report (see p. 3) reference was made to University of Illinois Engineering Experiment Station Bulletin 381 (Lansford and Mitchell, 1949) which presents the basic experimental data that form the foundation for the present report. In arranging with the university for the collection of the data, the primary objective of the Geological Survey was the acquisition of a reservoir of reliable data on which might be based detailed studies of stage-fall-discharge relations. For practical reasons, these detailed studies could not be immediately completed. Thus, Bulletin 381 was designed and published with the particular objective of preserving the base data and of making it immediately available to such parties as might care to make their own analysis. This report is designed to present an analysis of those data, but does not duplicate the tables of observations previously published.

In its field association with problems of stage-fall-discharge relations, the Geological Survey is almost always limited to cases of the M-1 type of profile: the profile in which the actual observed depths are greater than the normal depth and the normal depth is greater than the critical depth. In the design of the experiment, a basic consideration was that the resulting profiles should be of this type—that is, the channel should be one of mild slope. An exceedingly flat bed slope would perhaps more nearly have approached the slopes of natural streams, but it would have made the curves of such great length as to add greatly to the experimental work required and, for the length of channel available, might require greater accuracy than would be possible, even in the laboratory. A steeper slope would decrease the length of the curves, but would require that the channel roughness be greater to keep the depths greater than critical.

It appeared feasible to line the channel with a roughness material which would provide a value of n of about 0.02. The Manning formula indicated that, with this value of n, a slope of S_0 = 0.003 would provide normal depths well above the critical. With an effective channel length of about 135 feet, this bed slope provided a total change in bed elevation of about 0.40 foot, which appeared to be satisfactory from the standpoint of the probable accuracy with which the observations could be made. Later it was found feasible to vary the channel with respect to roughness, and with respect to shape of cross section. However, the amount of labor involved in varying the bed slope would have been so great as to make that change prohibitive. Thus, throughout the series of experimental observations, channel conditions were made to conform with conditions which might be met in the field, but only within the limitations imposed by practical laboratory requirements.

DESCRIPTION OF THE DATA

With respect to material which is useful in the present analysis, and in contrast with other material which is primarily informative, Bulletin 381 contains the following data:

1. Details of the cross sections for which profiles were observed. (See fig. 4, p. 14.)

2. Normal depth-discharge ratings for cross sections 2-6, inclusive. (See fig 6, pp. 18-19 and tables 42-46, pp. 81-85.)

3. Observed surface profiles: for the seven cross sections, there are a total of 41 profiles. (See figs. 8-12, pp. 27-31, and tables 1-41, pp. 39-80.)

4. Typical velocity-distribution diagrams for representative cross sections. (See figs. 13-15, pp. 32, 33 and fol. 34.)

For cross sections 5-7, the dimensions given in Bulletin 381 are nominal; a careful survey indicated that elevations for the break in side slopes should be at 1.011 instead of 1.00, and 1.207 instead of 1.200. A table of areas, based on these refined elevations, appears in this report as table 27. (See p. 154.)

A few words might be said concerning the observed surface profiles. For all cross sections, each profile was observed from the greatest depth which could be obtained (limited either by the depth of the laboratory channel, or in the case of low discharges, by the maximum depth which could be induced by extreme manipulation of the backwater gate) to that point near normal depth at which the water surface becomes so nearly parallel to the bed slope as to represent the practical limit of observation. For cross sections 1 to 4, inclusive, depths were observed at points along the channel which were 15 feet apart. For cross sections 5 to 7, inclusive, depths were observed at 10-foot intervals, except for very great depths of low discharge, for which 20-foot intervals were used. In cross section 3, y_n = 1.512 feet (see Bull. 381, table 22, p. 60) a portion of the M-2 curve, approximately 300 feet in length, was included. Likewise for cross section 5, y_n = 1.704 feet (see Bull. 381, table 35, p. 74) a portion of the M-2 curve, approximately 230 feet in length, has been included.

Owing to various causes, among which might be mentioned the personal error in observations and the effects of local disturbances in the channel, differences between depths for successive equal increments of length are sometimes somewhat irregular. In some phases of the analysis (such as substitutions in equation (19), p. 44) it has appeared advisable to smooth out these irregularities by applying small and compensating changes to the observed values. Such a table, designated as a smoothed profile, has been prepared for each of the observed profiles of cross sections 2-6, inclusive. These smoothed profiles, not published in Bulletin 381, are found as tables 1-5 in this report.

Table 1.--Smoothed profiles, cross section 2

y_0=0.391	y_0=0.612	y_0=0.812	y_0=1.121	y_0=1.489	y_0=1.840
------	------	------	------	3.7080	3.7025
------	------	------	------	3.6670	3.6638
------	------	------	------	3.6260	3.6252
------	------	------	------	3.5851	3.5867
------	------	------	3.5090	3.5442	3.5484
------	------	------	3.4668	3.5034	3.5103
------	------	------	3.4246	3.4627	3.4724
------	------	------	3.3824	3.4221	3.4347
------	------	------	3.3403	3.3816	3.3972
------	------	------	3.2982	3.3412	3.3599
------	------	3.2150	3.2561	3.3009	3.3228
------	------	3.1720	3.2140	3.2607	3.2859
------	------	3.1290	3.1720	3.2206	3.2492
------	------	3.0860	3.1300	3.1806	3.2128
------	------	3.0430	3.0880	3.1407	3.1767
------	3.0030	3.0000	3.0460	3.1009	3.1409
------	2.9590	2.9570	3.0040	3.0612	3.1054
------	2.9150	2.9140	2.9621	3.0216	3.0702
------	2.8710	2.8710	2.9202	2.9822	3.0353
------	2.8270	2.8280	2.8783	2.9430	3.0007
------	2.7830	2.7850	2.8364	2.9040	2.9664
------	2.7391	2.7420	2.7945	2.8652	2.9324
------	2.6952	2.6990	2.7526	2.8266	2.8987
------	2.6513	2.6561	2.7108	2.7882	2.8653
------	2.6074	2.6132	2.6690	2.7500	2.8322
------	2.5635	2.5703	2.6272	2.7120	2.7994
------	2.5196	2.5274	2.5854	2.6743	2.7669
------	2.4757	2.4845	2.5436	2.6369	2.7347
------	2.4318	2.4416	2.5018	2.5998	2.7028
------	2.3879	2.3987	2.4600	2.5630	2.6712
2.3280	2.3440	2.3558	2.4183	2.5266	2.6400
2.2830	2.3002	2.3129	2.3767	2.4906	2.6092
2.2380	2.2564	2.2700	2.3352	2.4550	2.5788
2.1930	2.2126	2.2271	2.2939	2.4198	2.5489
2.1480	2.1688	2.1842	2.2528	2.3850	2.5195
2.1030	2.1250	2.1413	2.2119	2.3507	2.4907
2.0580	2.0812	2.0984	2.1713	2.3169	2.4625
2.0131	2.0374	2.0556	2.1310	2.2836	2.4350
1.9682	1.9936	2.0128	2.0910	2.2508	2.4082
1.9233	1.9499	1.9701	2.0513	2.2185	2.3821
1.8784	1.9062	1.9275	2.0120	2.1867	2.3567
1.8335	1.8625	1.8850	1.9731	2.1554	2.3320
1.7886	1.8188	1.8427	1.9346	2.1246	2.3080
1.7438	1.7752	1.8006	1.8965	2.0943	2.2847
1.6990	1.7316	1.7587	1.8589	2.0645	2.2621

Table 1.--Smoothed profiles, cross section 2 --Continued

y_o=0.391	y_o=0.612	y_o=0.812	y_o=1.121	y_o=1.489	y_o=1.840
1.6542	1.6880	1.7170	1.8218	2.0352	2.2402
1.6094	1.6445	1.6755	1.7852	2.0064	2.2190
1.5646	1.6010	1.6343	1.7491	1.9781	2.1985
1.5199	1.5576	1.5934	1.7136	1.9503	2.1788
1.4752	1.5143	1.5528	1.6787	1.9230	2.1599
1.4305	1.4711	1.5125	1.6444	1.8962	2.1418
1.3859	1.4280	1.4725	1.6108	1.8699	2.1245
1.3413	1.3850	1.4329	1.5779	1.8441	2.1079
1.2967	1.3422	1.3937	1.5458	1.8188	2.0920
1.2522	1.2996	1.3549	1.5145	1.7940	2.0768
1.2077	1.2572	1.3166	1.4841	1.7697	2.0623
1.1633	1.2151	1.2788	1.4546	1.7459	2.0485
1.1189	1.1733	1.2415	1.4261	1.7226	2.0353
1.0746	1.1318	1.2048	1.3987	1.6999	2.0227
1.0303	1.0907	1.1689	1.3725	1.6783	2.0107
.9861	1.0501	1.1340	1.3476	1.6583	1.9992
.9421	1.0101	1.1003	1.3241	1.6403	1.9882
.8984	.9709	1.0680	1.3021	1.6243	1.9777
.8551	.9326	1.0373	1.2817	1.6103	1.9677
.8123	.8954	1.0084	1.2630	1.5981	1.9582
.7701	.8595	.9814	1.2461	1.5874	1.9491
.7287	.8252	.9564	1.2310	1.5779	1.9404
.6882	.7927	.9335	1.2177	1.5694	1.9321
.6487	.7623	.9128	1.2061	1.5618	1.9242
.6104	.7344	.8944	1.1960	1.5550	1.9167
.5736	.7095	.8784	1.1872	1.5489	1.9096
.5387	.6881	.8648	1.1795	1.5433	1.9029
.5062	.6702	.8535	1.1727	1.5382	1.8966
.4767	.6558	.8442	1.1667	1.5335	1.8907
.4512	.6444	.8366	1.1614	1.5291	1.8852
.4317	.6355	.8304	1.1567	1.5250	1.8801
.4182	.6287	.8254	1.1525	1.5212	1.8754
.4097	.6238	.8214	1.1488	1.5177	1.8711
.4047	.6202	.8182	1.1455	1.5144	1.8671
.4012	.6175	.8157	1.1426	1.5113	1.8634
.3985	.6155	.8139	1.1401	1.5084	1.8600
.3964	.6141	.8127	1.1379	------	------
.3948	.6132	.8120	1.1360	------	------
.3936	.6127	.8117	1.1344	------	------
.3927	.6124	.8116	1.1330	------	------
.3921	.6122	------	------	------	------
.3917	.6121	------	------	------	------
.3914	.6120	------	------	------	------
.3912	------	------	------	------	------
.3911	------	------	------	------	------

Table 2.--Smoothed profiles, cross section 3

y_o=0.404	y_o=0.606	y_o=0.773	y_o=1.071	y_o=1.512	y_o=0.404	y_o=0.606	y_o=0.773	y_o=1.071	y_o=1.512
------	------	------	------	3.6699	1.5796	1.6561	1.6830	1.8096	2.0905
------	------	------	------	3.6295	1.5349	1.6130	1.6418	1.7737	2.0628
------	------	------	------	3.5892	1.4902	1.5700	1.6008	1.7382	2.0355
------	------	------	------	3.5490	1.4455	1.5270	1.5601	1.7032	2.0086
------	------	------	3.4637	3.5089	1.4008	1.4840	1.5197	1.6687	1.9821
------	------	------	3.4209	3.4689	1.3561	1.4411	1.4796	1.6348	1.9560
------	------	------	3.3782	3.4290	1.3114	1.3982	1.4398	1.6014	1.9303
------	------	------	3.3356	3.3892	1.2667	1.3554	1.4003	1.5686	1.9051
------	------	------	3.2931	3.3495	1.2221	1.3127	1.3612	1.5364	1.8805
------	------	------	3.2507	3.3099	1.1775	1.2701	1.3225	1.5049	1.8566
------	------	3.1845	3.2084	3.2704	1.1330	1.2277	1.2842	1.4741	1.8336
------	------	3.1407	3.1662	3.2310	1.0886	1.1855	1.2463	1.4440	1.8117
------	------	3.0969	3.1241	3.1917	1.0444	1.1436	1.2089	1.4147	1.7911
------	------	3.0532	3.0821	3.1525	1.0004	1.1021	1.1720	1.3862	1.7718
------	3.0077	3.0095	3.0402	3.1134	.9566	1.0611	1.1357	1.3586	1.7536
------	2.9636	2.9659	2.9984	3.0744	.9130	1.0207	1.1001	1.3320	1.7363
------	2.9195	2.9223	2.9567	3.0356	.8698	.9811	1.0654	1.3065	1.7199
------	2.8755	2.8788	2.9151	2.9970	.8271	.9425	1.0318	1.2822	1.7044
2.7890	2.8315	2.8353	2.8736	2.9586	.7851	.9051	.9996	1.2592	1.6898
2.7442	2.7875	2.7919	2.8322	2.9204	.7439	.8691	.9692	1.2376	1.6761
2.6994	2.7436	2.7485	2.7909	2.8825	.7037	.8345	.9411	1.2174	1.6633
2.6546	2.6997	2.7052	2.7497	2.8450	.6647	.8017	.9157	1.1988	1.6514
2.6098	2.6558	2.6619	2.7087	2.8080	.6273	.7709	.8932	1.1819	1.6404
2.5650	2.6120	2.6187	2.6678	2.7716	.5918	.7423	.8735	1.1668	1.6303
2.5202	2.5682	2.5755	2.6270	2.7360	.5586	.7161	.8564	1.1535	1.6211
2.4754	2.5244	2.5324	2.5863	2.7015	.5280	.6926	.8417	1.1418	1.6128
2.4306	2.4807	2.4893	2.5457	2.6684	.5002	.6721	.8292	1.1316	1.6054
2.3858	2.4370	2.4463	2.5052	2.6363	.4755	.6551	.8187	1.1228	1.5989
2.3410	2.3933	2.4033	2.4649	2.6048	.4541	.6416	.8100	1.1153	1.5933
2.2962	2.3497	2.3604	2.4247	2.5736	.4362	.6316	.8029	1.1090	1.5886
2.2514	2.3061	2.3175	2.3847	2.5425	.4220	.6246	.7972	1.1038	1.5847
2.2066	2.2625	2.2747	2.3448	2.5114	.4117	.6196	.7927	1.0996	------
2.1618	2.2190	2.2319	2.3051	2.4804	.4055	.6161	.7892	1.0962	------
2.1170	2.1755	2.1892	2.2656	2.4494	.4034	.6136	.7865	1.0934	------
2.0722	2.1320	2.1465	2.2263	2.4185	.4026	.6116	.7845	------	------
2.0274	2.0886	2.1039	2.1872	2.3876	.4023	.6101	.7831	------	------
1.9826	2.0452	2.0613	2.1483	2.3568	.4021	.6090	------	------	------
1.9378	2.0018	2.0188	2.1096	2.3261	------	.6082	------	------	------
1.8930	1.9585	1.9764	2.0711	2.2955	------	.6076	------	------	------
1.8482	1.9152	1.9341	2.0328	2.2651	------	.6071	------	------	------
1.8034	1.8719	1.8919	1.9948	2.2350	------	.6067	------	------	------
1.7586	1.8287	1.8498	1.9571	2.2053	------	.6064	------	------	------
1.7138	1.7855	1.8078	1.9197	2.1760	------	.6062	------	------	------
1.6690	1.7423	1.7660	1.8826	2.1471	------	.6061	------	------	------
1.6243	1.6992	1.7244	1.8459	2.1186	------	------	------	------	------

Table 3.--Smoothed profiles, cross section 4

y_o=0.528	y_o=0.780	y_o=1.028	y_o=1.369	y_o=1.877	y_o=2.335
------	------	------	------	3.5384	3.6492
------	------	------	3.4696	3.5009	3.6142
------	------	------	3.4282	3.4636	3.5794
------	------	------	3.3869	3.4265	3.5448
------	------	------	3.3457	3.3896	3.5104
------	------	------	3.3046	3.3529	3.4762
------	------	3.2222	3.2636	3.3164	3.4422
------	------	3.1791	3.2227	3.2801	3.4084
------	------	3.1361	3.1819	3.2441	3.3748
------	------	3.0932	3.1412	3.2084	3.3414
------	3.0157	3.0504	3.1006	3.1730	3.3082
------	2.9720	3.0076	3.0601	3.1379	3.2752
------	2.9283	2.9649	3.0197	3.1031	3.2425
------	2.8846	2.9223	2.9794	3.0686	3.2101
------	2.8409	2.8798	2.9392	3.0344	3.1780
2.7950	2.7972	2.8374	2.8991	3.0005	3.1462
2.7501	2.7535	2.7950	2.8591	2.9669	3.1148
2.7052	2.7098	2.7527	2.8193	2.9336	3.0838
2.6603	2.6662	2.7105	2.7797	2.9006	3.0533
2.6154	2.6226	2.6684	2.7403	2.8680	3.0234
2.5705	2.5790	2.6264	2.7011	2.8358	2.9942
2.5256	2.5354	2.5845	2.6621	2.8040	2.9658
2.4807	2.4919	2.5427	2.6232	2.7726	2.9383
2.4358	2.4484	2.5010	2.5845	2.7416	2.9117
2.3909	2.4049	2.4594	2.5460	2.7110	2.8860
2.3460	2.3614	2.4179	2.5077	2.6808	2.8612
2.3011	2.3179	2.3765	2.4696	2.6510	2.8373
2.2562	2.2745	2.3352	2.4317	2.6216	2.8143
2.2113	2.2311	2.2940	2.3940	2.5926	2.7922
2.1665	2.1877	2.2529	2.3565	2.5641	2.7709
2.1217	2.1443	2.2119	2.3193	2.5361	2.7504
2.0769	2.1010	2.1710	2.2824	2.5086	2.7307
2.0321	2.0577	2.1302	2.2458	2.4817	2.7117
1.9874	2.0144	2.0896	2.2095	2.4554	2.6934
1.9427	1.9711	2.0492	2.1736	2.4297	2.6757
1.8980	1.9278	2.0090	2.1381	2.4047	2.6586
1.8534	1.8846	1.9690	2.1030	2.3804	2.6420
1.8088	1.8414	1.9293	2.0684	2.3568	2.6259
1.7643	1.7982	1.8899	2.0343	2.3340	2.6103
1.7199	1.7550	1.8508	2.0008	2.3120	2.5951
1.6756	1.7119	1.8121	1.9679	2.2908	2.5803
1.6314	1.6690	1.7738	1.9357	2.2703	2.5659
1.5873	1.6264	1.7360	1.9042	2.2505	2.5519
1.5433	1.5842	1.6987	1.8735	2.2314	2.5383
1.4994	1.5424	1.6620	1.8436	2.2129	2.5251

Table 3.--Smoothed profiles, cross section 4 --Continued

y_o=0.528	y_o=0.780	y_o=1.028	y_o=1.369	y_o=1.877	y_o=2.335
1.4556	1.5011	1.6259	1.8146	2.1950	2.5123
1.4119	1.4603	1.5905	1.7865	2.1777	2.4999
1.3683	1.4201	1.5558	1.7593	2.1610	2.4879
1.3248	1.3805	1.5218	1.7330	2.1449	2.4763
1.2814	1.3416	1.4886	1.7076	2.1294	2.4651
1.2381	1.3034	1.4562	1.6832	2.1145	2.4543
1.1949	1.2660	1.4247	1.6598	2.1002	2.4439
1.1519	1.2294	1.3941	1.6374	2.0865	2.4339
1.1091	1.1937	1.3645	1.6160	2.0734	2.4243
1.0666	1.1589	1.3359	1.5957	2.0608	2.4151
1.0245	1.1251	1.3084	1.5765	2.0487	2.4063
.9830	1.0923	1.2821	1.5584	2.0371	2.3979
.9422	1.0606	1.2571	1.5414	2.0260	2.3899
.9023	1.0301	1.2334	1.5255	2.0154	2.3823
.8635	1.0009	1.2111	1.5106	2.0053	2.3751
.8260	.9732	1.1903	1.4967	1.9957	2.3683
.7900	.9471	1.1711	1.4838	1.9866	2.3619
.7557	.9228	1.1535	1.4719	1.9780	2.3559
.7233	.9004	1.1376	1.4610	1.9699	2.3503
.6930	.8801	1.1234	1.4511	1.9623	------
.6650	.8620	1.1109	1.4422	1.9552	------
.6396	.8462	1.1000	1.4343	1.9486	------
.6171	.8328	1.0906	1.4272	1.9425	------
.5979	.8218	1.0825	1.4206	1.9369	------
.5817	.8130	1.0755	1.4144	1.9318	------
.5683	.8062	1.0694	1.4084	1.9272	------
.5575	.8011	1.0640	1.4026	------	------
.5490	.7972	1.0592	1.3970	------	------
.5425	.7940	1.0549	1.3916	------	------
.5377	.7912	1.0510	------	------	------
.5343	.7888	1.0475	------	------	------
.5320	.7868	1.0444	------	------	------
.5305	.7852	1.0416	------	------	------
.5295	.7840	1.0391	------	------	------
.5288	.7831	1.0369	------	------	------
.5283	.7825	1.0350	------	------	------
.5280	.7821	1.0334	------	------	------
------	.7818	1.0320	------	------	------
------	.7815	1.0308	------	------	------
------	.7813	1.0298	------	------	------
------	.7811	1.0290	------	------	------
------	.7810	1.0284	------	------	------
------	------	1.0280	------	------	------

Table 4.--Smoothed profiles, cross section 5

y_o=0.456	y_o=0.687	y_o=0.905	y_o=1.158	y_o=1.285	y_o=1.704	y_o=2.525
------	------	------	------	------	------	3.6740
------	------	------	------	------	------	3.6477
------	------	------	------	------	------	3.6217
------	------	------	------	------	------	3.5960
------	------	------	------	------	------	3.5706
------	------	------	------	------	------	3.5455
------	------	------	------	------	------	3.5207
------	------	------	------	------	------	3.4962
------	------	------	------	------	------	3.4720
------	------	------	------	------	------	3.4481
------	------	------	------	------	------	3.4245
------	------	------	------	------	------	3.4012
------	------	------	------	------	------	3.3782
------	------	------	------	------	------	3.3555
------	------	------	------	------	------	3.3331
------	------	------	------	------	3.1540	3.3110
------	------	------	------	------	3.1253	3.2892
------	------	------	------	------	3.0966	3.2677
------	------	------	------	------	3.0679	3.2465
------	------	------	------	------	3.0393	3.2256
------	------	------	------	------	3.0107	3.2050
------	------	------	------	------	2.9821	3.1847
------	------	------	------	------	2.9535	3.1647
------	------	------	------	------	2.9250	3.1450
------	------	------	------	------	2.8965	3.1256
------	------	------	------	------	2.8680	3.1065
------	------	------	------	------	2.8396	3.0877
------	------	------	------	------	2.8112	3.0692
------	------	------	------	2.7630	2.7828	3.0510
------	------	------	------	2.7335	2.7545	3.0331
------	------	------	------	2.7040	2.7262	3.0155
------	------	------	2.6800	2.6745	2.6980	2.9981
------	------	------	2.6503	2.6450	2.6698	2.9809
------	------	2.6035	2.6206	2.6155	2.6417	2.9639
------	2.5630	2.5736	2.5909	2.5860	2.6137	2.9471
------	2.5330	2.5437	2.5612	2.5565	2.5858	2.9305
------	2.5030	2.5138	2.5315	2.5270	2.5580	2.9141
2.4880	2.4730	2.4839	2.5018	2.4975	2.5303	2.8979
2.4580	2.4430	2.4540	2.4721	2.4680	2.5028	2.8819
2.4280	2.4130	2.4241	2.4424	2.4385	2.4755	2.8661
2.3980	2.3830	2.3943	2.4127	2.4090	2.4484	2.8505
2.3680	2.3530	2.3645	2.3830	2.3796	2.4215	2.8351
2.3380	2.3230	2.3347	2.3533	2.3502	2.3948	2.8200
2.3080	2.2930	2.3049	2.3236	2.3208	2.3683	2.8052
2.2780	2.2630	2.2751	2.2939	2.2914	2.3420	2.7907
2.2480	2.2330	2.2453	2.2642	2.2620	2.3159	2.7766
2.2180	2.2031	2.2155	2.2345	2.2326	2.2901	2.7629
2.1880	2.1732	2.1857	2.2048	2.2033	2.2646	2.7496
2.1580	2.1433	2.1559	2.1751	2.1740	2.2394	2.7368
2.1280	2.1134	2.1262	2.1454	2.1447	2.2145	2.7245
2.0980	2.0835	2.0965	2.1157	2.1154	2.1899	2.7127
2.0680	2.0537	2.0668	2.0860	2.0861	2.1656	2.7015
2.0380	2.0239	2.0371	2.0564	2.0569	2.1416	2.6909
2.0080	1.9941	2.0074	2.0268	2.0277	2.1180	2.6808
1.9780	1.9643	1.9777	1.9972	1.9985	2.0948	2.6712
1.9480	1.9346	1.9480	1.9677	1.9693	2.0720	2.6621
1.9180	1.9049	1.9184	1.9382	1.9402	2.0496	2.6535
1.8880	1.8752	1.8888	1.9088	1.9111	2.0277	2.6453
1.8581	1.8456	1.8592	1.8795	1.8821	2.0063	2.6375
1.8282	1.8160	1.8296	1.8503	1.8532	1.9855	2.6301
1.7983	1.7864	1.8001	1.8212	1.8245	1.9653	2.6231
1.7684	1.7569	1.7706	1.7922	1.7960	1.9458	2.6165
1.7385	1.7274	1.7411	1.7633	1.7678	1.9270	2.6103
1.7086	1.6980	1.7117	1.7345	1.7399	1.9090	2.6045
1.6787	1.6686	1.6823	1.7059	1.7123	1.8918	2.5991

Table 4.--Smoothed profiles, cross section 5 --Continued

y_o=0.456	y_o=0.687	y_o=0.905	y_o=1.158	y_o=1.285	y_o=1.704	y_o=2.525
1.6488	1.6393	1.6530	1.6775	1.6851	1.8754	2.5940
1.6189	1.6100	1.6238	1.6493	1.6583	1.8598	2.5892
1.5890	1.5808	1.5947	1.6213	1.6319	1.8450	2.5847
1.5592	1.5516	1.5657	1.5935	1.6060	1.8311	2.5805
1.5294	1.5225	1.5368	1.5660	1.5806	1.8181	2.5765
1.4996	1.4934	1.5081	1.5388	1.5557	1.8060	2.5727
1.4699	1.4644	1.4796	1.5119	1.5314	1.7948	2.5691
1.4403	1.4355	1.4513	1.4853	1.5077	1.7845	2.5657
1.4108	1.4067	1.4233	1.4591	1.4847	1.7751	1.5624
1.3814	1.3780	1.3956	1.4334	1.4625	1.7666	2.5592
1.3521	1.3495	1.3683	1.4083	1.4412	1.7589	2.5561
1.3229	1.3213	1.3415	1.3839	1.4209	1.7520	2.5531
1.2938	1.2935	1.3153	1.3603	1.4017	1.7459	2.5502
1.2649	1.2662	1.2898	1.3376	1.3838	1.7406	2.5474
1.2362	1.2395	1.2650	1.3159	1.3674	1.7361	2.5447
1.2077	1.2136	1.2410	1.2953	1.3528	1.7323	2.5421
1.1795	1.1886	1.2179	1.2759	1.3403	1.7292	2.5396
1.1516	1.1646	1.1958	1.2578	1.3302	1.7267	------
1.1241	1.1416	1.1748	1.2411	1.3225	1.7247	------
1.0970	1.1196	1.1550	1.2259	1.3168	1.7231	------
1.0704	1.0985	1.1365	1.2123	1.3127	1.7219	------
1.0443	1.0783	1.1194	1.2004	1.3098	1.7210	------
1.0183	1.0589	1.1037	1.1904	1.3077	------	------
.9924	1.0402	1.0894	1.1825	1.3061	------	------
.9666	1.0221	1.0764	1.1766	1.3049	------	------
.9409	1.0045	1.0646	1.1725	1.3040	------	------
.9154	.9874	1.0538	1.1699	1.3033	------	------
.8901	.9708	1.0438	1.1684	1.3028	------	------
.8651	.9546	1.0345	1.1676	1.3024	------	------
.8404	.9388	1.0258	1.1671	1.3021	------	------
.8161	.9234	1.0176	1.1668	1.3019	------	------
.7923	.9084	1.0099	1.1666	1.3018	------	------
.7691	.8938	1.0027	------	------	------	------
.7466	.8796	.9958	------	------	------	------
.7248	.8658	.9893	------	------	------	------
.7037	.8524	.9831	------	------	------	------
.6834	.8394	.9772	------	------	------	------
.6639	.8268	.9716	------	------	------	------
.6453	.8146	.9663	------	------	------	------
.6276	.8028	.9613	------	------	------	------
.6108	.7914	.9566	------	------	------	------
.5949	.7804	.9522	------	------	------	------
.5799	.7698	.9481	------	------	------	------
.5659	.7592	.9443	------	------	------	------
.5529	.7494	.9408	------	------	------	------
.5409	.7400	.9376	------	------	------	------
.5299	.7310	.9347	------	------	------	------
.5199	.7214	.9321	------	------	------	------
.5108	.7122	.9298	------	------	------	------
.5026	.7044	------	------	------	------	------
.4953	------	------	------	------	------	------
.4890	------	------	------	------	------	------
.4833	------	------	------	------	------	------
.4783	------	------	------	------	------	------
.4740	------	------	------	------	------	------
.4703	------	------	------	------	------	------
.4672	------	------	------	------	------	------
.4646	------	------	------	------	------	------
.4625	------	------	------	------	------	------
.4608	------	------	------	------	------	------
.4595	------	------	------	------	------	------
.4585	------	------	------	------	------	------
.4578	------	------	------	------	------	------
.4573	------	------	------	------	------	------
.4570	------	------	------	------	------	------
.4568	------	------	------	------	------	------
.4567	------	------	------	------	------	------

Table 5.--Smoothed profiles, cross section 6

y_o=0.471	y_o=0.905	y_o=1.294	y_o=2.431	y_o=0.471	y_o=0.905	y_o=1.294	y_o=2.431
------	------	------	3.6597	1.7983	1.8297	1.8288	2.4983
------	------	------	3.6331	1.7684	1.8001	1.8005	2.4914
------	------	------	3.6066	1.7385	1.7706	1.7725	2.4848
------	------	------	3.5801	1.7086	1.7411	1.7448	2.4785
------	------	------	3.5537	1.6787	1.7117	1.7174	2.4725
------	------	------	3.5273	1.6488	1.6823	1.6904	2.4668
------	------	------	3.5010	1.6190	1.6530	1.6638	2.4613
------	------	------	3.4747	1.5893	1.6238	1.6376	2.4560
------	------	------	3.4485	1.5597	1.5947	1.6119	2.4508
------	------	------	3.4223	1.5302	1.5657	1.5867	2.4457
------	------	------	3.3962	1.5008	1.5368	1.5621	2.4407
------	------	------	3.3702	1.4715	1.5081	1.5381	2.4358
------	------	------	3.3443	1.4423	1.4796	1.5148	------
------	------	------	3.3186	1.4132	1.4513	1.4922	------
------	------	------	3.2930	1.3842	1.4233	1.4704	------
------	------	------	3.2676	1.3553	1.3956	1.4495	------
------	------	------	3.2424	1.3265	1.3683	1.4296	------
------	------	------	3.2174	1.2978	1.3415	1.4108	------
------	------	------	3.1926	1.2692	1.3153	1.3932	------
------	------	------	3.1680	1.2408	1.2898	1.3769	------
------	------	------	3.1437	1.2126	1.2650	1.3620	------
------	------	------	3.1197	1.1847	1.2410	1.3486	------
------	------	------	3.0960	1.1571	1.2179	1.3368	------
------	------	------	3.0726	1.1299	1.1958	1.3267	------
------	------	------	3.0495	1.1031	1.1748	1.3184	------
------	------	------	3.0267	1.0768	1.1550	1.3119	------
------	------	------	3.0042	1.0510	1.1365	1.3072	------
------	------	------	2.9820	1.0257	1.1194	1.3039	------
------	------	2.7660	2.9602	1.0007	1.1037	1.3015	------
------	------	2.7364	2.9388	.9758	1.0894	1.2996	------
------	------	2.7068	2.9177	.9510	1.0764	1.2981	------
------	------	2.6772	2.8970	.9263	1.0646	1.2969	------
------	------	2.6476	2.8767	.9017	1.0538	1.2960	------
------	------	2.6180	2.8569	.8772	1.0438	1.2953	------
------	2.6070	2.5885	2.8375	.8528	1.0345	1.2948	------
------	2.5770	2.5590	2.8185	.8286	1.0258	1.2944	------
------	2.5470	2.5295	2.8000	.8046	1.0176	1.2941	------
2.4880	2.5170	2.5000	2.7819	.7809	1.0099	1.2939	------
2.4580	2.4870	2.4705	2.7642	.7576	1.0027	1.2938	------
2.4280	2.4570	2.4411	2.7470	.7349	.9958	------	------
2.3980	2.4270	2.4117	2.7303	.7129	.9893	------	------
2.3680	2.3970	2.3823	2.7140	.6917	.9831	------	------
2.3380	2.3670	2.3529	2.6982	.6715	.9772	------	------
2.3080	2.3370	2.3235	2.6829	.6525	.9716	------	------
2.2780	2.3070	2.2942	2.6681	.6349	.9663	------	------
2.2480	2.2770	2.2649	2.6538	.6188	.9613	------	------
2.2180	2.2470	2.2356	2.6400	.6041	.9566	------	------
2.1880	2.2171	2.2063	2.6267	.5907	.9522	------	------
2.1580	2.1872	2.1770	2.6139	.5785	.9481	------	------
2.1280	2.1573	2.1477	2.6016	.5674	.9443	------	------
2.0980	2.1274	2.1185	2.5898	.5573	.9408	------	------
2.0680	2.0975	2.0893	2.5785	.5481	.9376	------	------
2.0380	2.0676	2.0601	2.5677	.5397	.9347	------	------
2.0080	2.0378	2.0309	2.5574	.5319	.9321	------	------
1.9780	2.0080	2.0018	2.5476	.5245	.9298	------	------
1.9480	1.9782	1.9727	2.5383	.5174	------	------	------
1.9180	1.9484	1.9437	2.5295	.5105	------	------	------
1.8880	1.9187	1.9148	2.5211	.5037	------	------	------
1.8581	1.8890	1.8860	2.5131	.4970	------	------	------
1.8282	1.8593	1.8573	2.5055	.4904	------	------	------
				.4839	------	------	------

ANALYSIS OF THE DATA

PRELIMINARY CONSIDERATIONS

CONVENTIONS AND USE OF TERMS

The framework and general principles relating to conventions and the use of terms has been presented in connection with the theory of varied flow in prismatic channels (see pp. 15-18). A more detailed statement, moulded within that framework, is a prerequisite to the analysis of the data.

As already mentioned in consideration of the varied-flow equation (see p. 18) it is customary to adopt the convention that a downward slope (in the direction of flow) is positive. This is facilitated by assuming the channel stationing to increase in the upstream direction with the line of the bed slope passing through

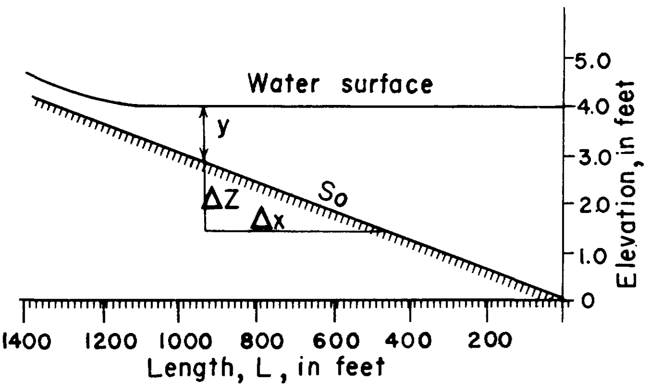


Figure 8. -Analytical orientation of profile.

the origin of coordinates (see fig. 8), and with the value of $y = y_{max}$ coincident with the vertical axis. (This y_{max} is the maximum value of y in which the investigator is interested. For example, if the profile were the result of an obstruction, such as a dam, in the channel, y_{max} would be the depth immediately upstream from the dam.) With respect to the laboratory data, consistency among the profiles was obtained by the invariable use of y_{max} as 4.000 feet. Now, if the horizontal axis of coordinates be taken as the datum plane, or level from which elevations are measured, it is apparent that at any point the elevation of the bed is S_0L, and the elevation of the water surface is

$$\text{Elev.} = S_0L + y, \tag{16}$$

L being the distance, in feet, measured from the point at which the channel bed intersects the datum plane, and also the distance from the point at which y is equal to y_{max}

Table 6.--Stationing for the profiles

Cross section	y_o	Bulletin 381		Smoothed profiles		Correction to Bulletin 381
		Observed y	Published Stationing	Smoothed y	Revised Stationing	
2	0.391	2.326	560	2.3280	560	0
2	.612	2.998	333	3.0030	333	0
2	.812	3.212	263	3.2150	262	-1
2	1.121	3.509	166	3.5090	164	-2
2	1.489	3.706	101	3.7080	99	-2
2	1.840	3.703	109	3.7025	108	-1
3	.404	2.790	404	2.7890	404	0
3	.606	3.008	332	3.0077	332	0
3	.773	3.183	277	3.1845	276	-1
3	1.071	3.464	185	3.4637	187	+2
3	1.512	3.670	125	3.6699	129	+4
4	.528	2.789	405	2.7950	404	-1
4	.780	3.018	332	3.0157	330	-2
4	1.028	3.222	265	3.2222	264	-1
4	1.369	3.475	185	3.4696	187	+2
4	1.419	3.512	175	------	176	+1
4	1.877	3.540	178	3.5384	181	+3
4	2.335	3.646	171	3.6492	180	+9
5	.456	2.490	503	2.4880	504	+1
5	.687	2.563	479	2.5630	478	-1
5	.905	2.604	465	2.6035	466	+1
5	1.158	2.680	439	2.6800	440	+1
5	1.285	2.763	413	2.7630	413	0
5	1.704	3.154	291	3.1540	287	-4
5	2.525	3.674	135	3.6740	133	-2
6	.471	2.487	503	2.4880	503	0
6	.905	2.607	463	2.6070	464	+1
6	1.294	2.766	412	2.7660	412	0
6	2.431	3.670	131	3.6597	129	-2

REVISIONS TO STATIONING

In Bulletin 381 (Lansford and Mitchell, 1949) it is stated (pp. 22, 25-26) that the stationing, or distance L, presented as a part of each profile table, is a result of computation rather than observation. The increments of distance, Δx, are of course a matter of observation, being the measured distance between the point gages (see p. 26). However since the value of y_{max} (4.000 feet) is beyond the depth which could be observed in the laboratory, the desired consistency of stationing among the various profiles could be obtained only by computing a stationing for some point on each profile. This was determined by matching the computed and observed data near the downstream end of the observed profiles. For each profile the computed data were plotted on the basis of depth versus L. Beginning with the greatest depth which was observed on two or more gages and extending stream-upward through 10 consecutive observed depths, the values of L corresponding to each observed depth were read from the curve—the few slightly greater depths which were observed on only a single gage were not used in this determination, since there was no verification of their accuracy. The average differences were computed between the stations so determined and the temporary arbitrary stations which had been assigned to the observed depths. This difference was the correction then applied throughout the length of the profile to convert temporary stationing to stationing as shown in the profile tables.

As pointed out in Bulletin 381, these values were not required during the progress of observations; except as a matter of convenience, the stationing could have been neglected entirely until the time of the analysis. It also has been pointed out that the values of stationing published in Bulletin 381 might later be slightly revised as a result of more detailed analysis.

On the basis of the more complete analysis described in the following pages, a slight revision has been made to the stationing of some of the profiles. Although the changes are small, never more than a few feet, in the interest of accuracy they have been incorporated in the profiles shown in this report. Table 6, page 36, indicates the corrections to be applied to various tables in Bulletin 381, and at the same time provides an index to the stationing which is applicable to the smoothed profiles presented as tables 1-5 in the preceding section of this report.

EVALUATION OF VELOCITY-DISTRIBUTION COEFFICIENTS

The necessity of a velocity-distribution coefficient has been discussed on page 7. This coefficient was determined by plotting a number of diagrams similar to those shown in Bulletin 381 as figures 13-15 (pp. 32-34). Each area between lines of equal velocity was planimetered, and computations were made as shown in right-hand section of table 7, page 38 of this report, under the subheading "Total cross section." These, with the two lines of computations at the foot of the table, are sufficient to determine the coefficient. The remainder of table 7, which is for the section shown as figure 15 in Bulletin 381, has been presented to show a summary of conditions in this cross section for subdivisions as shown in figure 9.

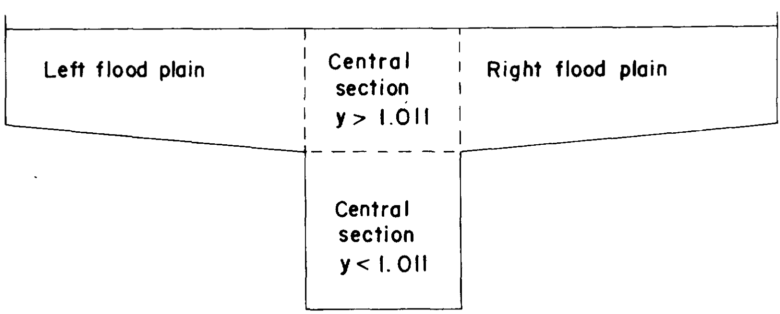


Figure 9. -Subdivisions of flow in cross section 5.

Table 7.--Computation of a velocity-distribution coefficient

	Left flood-plain section				Central section, y > 1.011				Right flood-plain section				Central section, y < 1.011				TOTAL CROSS SECTION			
Avg. vel.	Plani. diff.	Area	a.v	a.v²	Plani. diff.	Area	a.v	a.v²	Plani. diff.	Area	a.v	a.v²	Plani. diff.	Area	a.v	a.v²	Plani. diff.	Area	a.v	a.v²
2.12	276	0.0447	0.0948	0.2010	36	0.0058	0.0123	0.0261	918	0.1486	0.3150	0.6678	0	0	0	0	1,230	0.1991	0.4221	0.8949
2.05	1,245	.2015	.4131	.8469	956	.1548	.3173	.6505	1,209	.1957	.4012	.8225	0	0	0	0	3,410	.5520	1.1316	2.3198
1.95	1,118	.1810	.3530	.6884	822	.1331	.2595	.5060	797	.1290	.2516	.4906	0	0	0	0	2,737	.4431	.8640	1.6848
1.85	1,001	.1620	.2997	.5544	624	.1010	.1868	.3456	691	.1119	.2070	.3830	0	0	0	0	2,316	.3749	.6936	1.2832
1.75	749	.1213	.2123	.3715	1,138	.1842	.3224	.5642	566	.0916	.1603	.2805	0	0	0	0	2,453	.3971	.6949	1.2161
1.65	639	.1034	.1706	.2815	690	.1117	.1843	.3041	589	.0954	.1574	.2597	239	0.0387	0.0639	0.1054	2,157	.3492	.5762	.9507
1.55	620	.1004	.1556	.2412	348	.0563	.0873	.1353	623	.1009	.1564	.2424	1,452	.2351	.3644	.5648	3,043	.4926	.7635	1.1834
1.45	540	.0874	.1267	.1837	145	.0235	.0341	.0494	685	.1109	.1608	.2332	793	.1284	.1862	.2700	2,163	.3502	.5078	.7363
1.35	572	.0926	.1250	.1688	[illegible]	.0120	.0162	.0219	593	.0960	.1296	.1750	749	.1213	.1638	.2211	1,988	.3218	.4344	.5864
1.25	548	.0887	.1109	.1386	31	.0050	.0062	.0078	511	.0827	.1034	.1292	716	.1159	.1449	.1811	1,806	.2924	.3655	.4569
1.15	400	.0648	.0745	.0857	14	.0023	.0026	.0030	424	.0686	.0789	.0907	772	.1250	.1438	.1654	1,610	.2606	.2997	.3447
1.05	286	.0463	.0486	.0510	3	.0005	.0005	.0005	323	.0523	.0549	.0576	611	.0989	.1038	.1090	1,223	.1980	.2079	.2183
.75	497	.0805	.0604	.0453	0	0	0	0	660	.1068	.0801	.0601	838	.1357	.1018	.0764	1,995	.3230	.2422	.1816
Total	8,491	1.3746	2.2452	3.8580	4,881	.7902	1.4295	2.6144	8,589	1.3904	2.2566	3.8923	6,170	.9988	1.2726	1.6932	28,131	4.554	7.2034	12.0571

$V = Q/A = 7.2034/4.554 = 1.5818$

$c_m = \Sigma a.v^2/AV^2 = 12.0571/7.2034 \times 1.5818 = 1.058$

Note: Pitometer performance coefficient of 1.04 has not been applied to these velocities.

Table 8.--Summary of values of c_m

Range	Cross section				
	2	3	4	5	6
$0 < y \leq 1.5$		1.04			
$1.5 < y \leq 2.0$		1.05			
$2.0 < y \leq 2.5$	1.04	1.06	1.10	1.05	1.05
$2.5 < y \leq 3.0$		1.07			
$3.0 < y \leq 4.0$		1.08			

Table 8, page 38, summarizes the values of c_m for the various cross sections. It was found that appreciable variation occurred between one and another of the cross sections, but only in cross section 3 was there appreciable variation within the cross section itself. A more elaborate study might have disclosed further slight variations. However, it should be pointed out that the distribution coefficients were so near to unity as to have only a slight effect on the velocity head. (See fig. 16, p. 57 , for example, in which a profile has been plotted both with and without the correction for c_m). Because of the small magnitude of these effects, and also because the labor, both in observation and computation, is very great, a more elaborate determination of distribution coefficients appeared to be unjustifiable.

COMPUTATION OF HYDRAULIC RADIUS

Much of the analysis which follows is independent of hydraulic radius, so that the difficulties which arise from this troublesome factor are not everywhere a handicap to the work. However, in the evaluation of Manning's *n*, and in the attempts to correlate *S* and *V* with channel characteristics, there was definite need for this factor, and considerable attention was directed toward its proper computation. The results are presented here not only as a necessary prelude to discussion of Manning's *n* and the slope-velocity correlations, but also for information and guidance to those who may deal with other problems (such as slope-area determinations) wherein the geometry of the cross section approaches that of cross section 5.

For the rectangular section, the hydraulic radius, R, was computed in the conventional manner,

$$R = A/\text{w.p.}, \tag{17}$$

in which A is the cross-sectional area in square feet and *w.p.* is the wetted perimeter, in feet. These values, when used in a formula such as Manning's, appear to give quite satisfactory results, except for small values of y. A curve of R as abscissa against y as ordinate will generally have the characteristic shape of the solid line farthest to the right in figure 10, page 40 . The curve passes through the origin at an angle of 45° to the axes, assumes a gradually increasing slope with increasing y, and finally becomes asymptotic to the line $R = b/2$, in which b is the width of the channel.

As is generally realized, the use of equation (17) for a flood-plain cross section, such as cross section 5, leads to very unsatisfactory results. When computed by such a formula, R plotted against y becomes a curve as shown by the solid line farthest to the left in figure 10. Within the rectangular portion of the channel, the curve again passes through the origin at an angle of 45°, assumes a gradually increasing slope with increasing y, and would eventually become asymptotic to the line $R = b/2 = 0.50$ ft. At the depth $y = 1.01$, the value of R is 0.3344 foot. But as b is suddenly increased above this depth, a corresponding

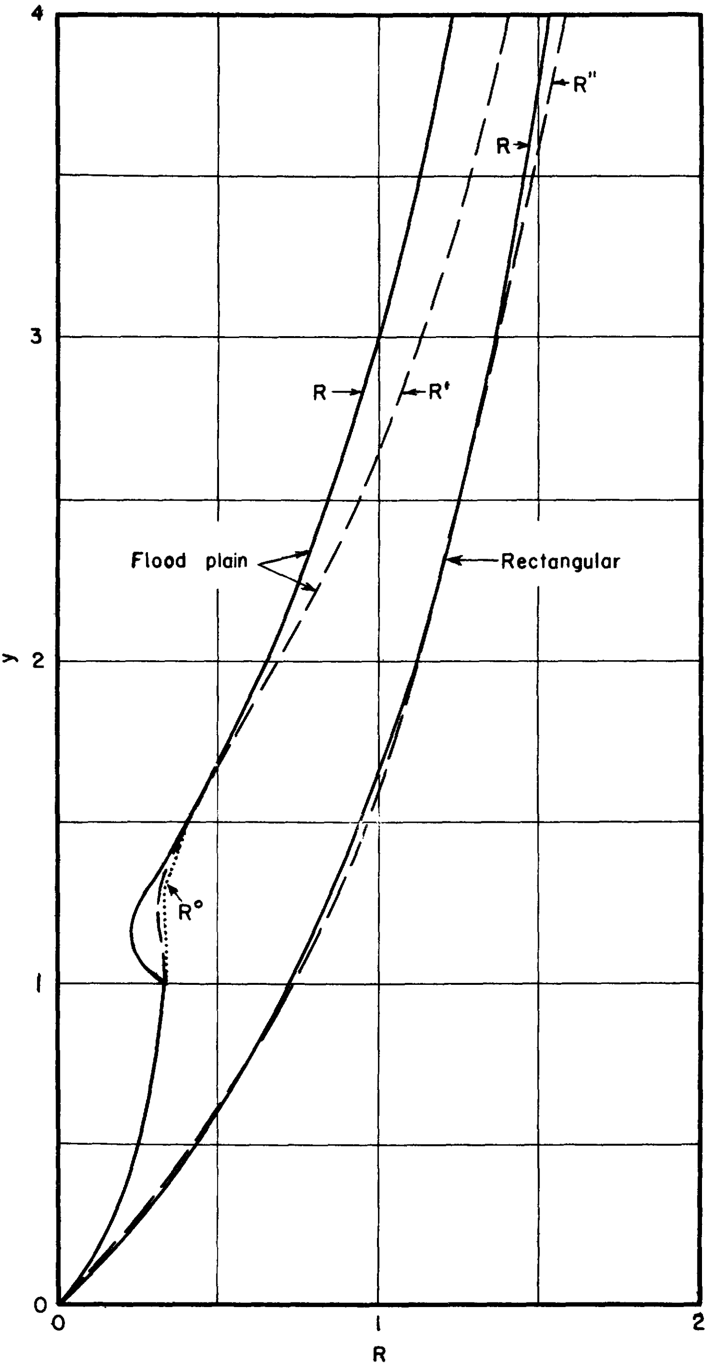


Figure 10.—Hydraulic radii.

break to the left occurs in the values of R, so that for $y = 1.20$, $R = 0.2268$ foot. After b again becomes a constant, the values of R begin once more to increase with y, but within practical limits of y remain substantially less than for the rectangular channel. The unsatisfactory feature of this relationship lies in the decrease of R with an increase of y. Velocity formulas conventionally assume that V varies directly (although perhaps exponentially) with R, and thus, in the instance given above, would indicate a very pronounced decrease in average velocity between $y = 1.01$ and $y = 1.20$. Assuming, as in the Manning formula, that velocity varies as the two-thirds power of R, the indicated ratio of mean velocities would be $(0.2268/0.3344)^{2/3} = 0.77$. The actual ratio, as taken from the data for cross section 5, is $1.925/2.069 = 0.93$. In other words, use of the Manning formula to compute the discharge for cross section 5 at depth $y = 1.20$ would give a value about 17 percent low if R were computed as area divided by wetted perimeter.

Methods have been proposed to overcome this difficulty. A common procedure is to impose mythical channel boundaries by extending upward the vertical sides of the small rectangular channel and then computing the discharge separately for the resulting subdivisions of the channel. But the question immediately arises as to procedure for computing the wetted perimeter. To include the mythical boundaries would result in an overall lowering of the value of boundary roughness, with no way of computing the new value. Experience has demonstrated that, for many problems, acceptable results are obtained by using the mythical boundaries only for subdividing the area, the hydraulic radius for each subdivision then being obtained by dividing each area by its actual wetted perimeter. For various values of y in cross section 5, such computations were made, and an overall value of hydraulic radius obtained by weighting the value for each subdivision by the ratio of its area to the total area. Results of these computations are shown, in figure 10, by the dotted line labelled $R°$.

Within the range of depths required for these computations, this $R°$ curve appears quite satisfactory; the objectionable break in the R curve has been greatly improved, and above a depth of about 1.6 feet the R curve and the $R°$ curve are, for a considerable range, nearly coincident. Unfortunately, the method cannot be recommended for extremely large values of y. In the central section, as y increases the area increases proportionally, but the wetted perimeter is held constant. Hence the hydraulic radius for this section increases rapidly with depth and eventually becomes so great as to dominate the computation for weighting R and lead to weighted values of hydraulic radius much greater that the asymptotic limit, $R = b/2$. Thus, in principle at least, another method is desirable such that, for the central section, the area will be limited to values which are in keeping with the limitation upon wetted perimeter.

An examination of figure 15 (Bull. 381) suggests that the boundary effects of the small rectangular section are reflected upward and inward from reentering boundary angles (the breaks in cross section at which $y = 1.011$ feet). When reviewed in the light of a paper by Keulegan (1938), this fact suggests the use of a subdivision which is explained as follows: Bisectors, ci and fi, (see fig. 11) were drawn for the reentering angles, X and Y. These divided the cross section into

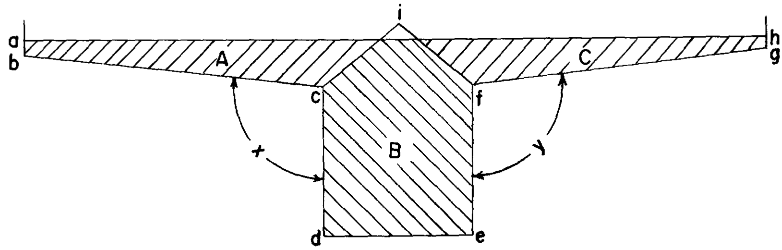


Figure 11. -Subdivision of flood-plain cross section for computing R'.

three areas, A, B, and C. Their hydraulic radii were taken as being A/abc, $B/cdef$, and C/fgh, respectively. The hydraulic radius of the entire cross section (hereafter designated as R') was then obtained by weighting the hydraulic radii just computed by area, as follows:

$$R' = \frac{A^2/abc + B^2/cdef + C^2/fgh}{A + B + C}. \tag{18}$$

Comparing the change in R' with change in y, it was found that there was nearly a constant value of R' between the values of $y = 1.01$ and $y = 1.20$, in which range the value of b increased from 1.00 to 5.00. Plot of this relation appears as the dashed line (R') farthest to the left in figure 10. It will be noted that, for shallow depths on the flood plain, values for R' are nearly the same as for $R°$. At depth of 4 feet, R' is appreciably greater than either R or $R°$, but for extremely great depths (not shown in figure 10) R', like R, will become asymptotic to $b/2$.

To determine whether this method of computing hydraulic radius is consistent, from a practical standpoint, with the conventional procedure (equation 17), a corresponding computation was made for the rectangular section. For this, bisectors were drawn for the salient angles of the cross section (the 90° angles between bottom and sides,) thus dividing this cross section into three parts. Computations then were completed for R'' by the method outlined above for R'. Results are shown by the dashed line farthest to the right in figure 10. Since there is negligible difference in the results, it appears that the method of computing R' as used in the flood-plain cross section is consistent with conventional procedure for computing hydraulic radius.

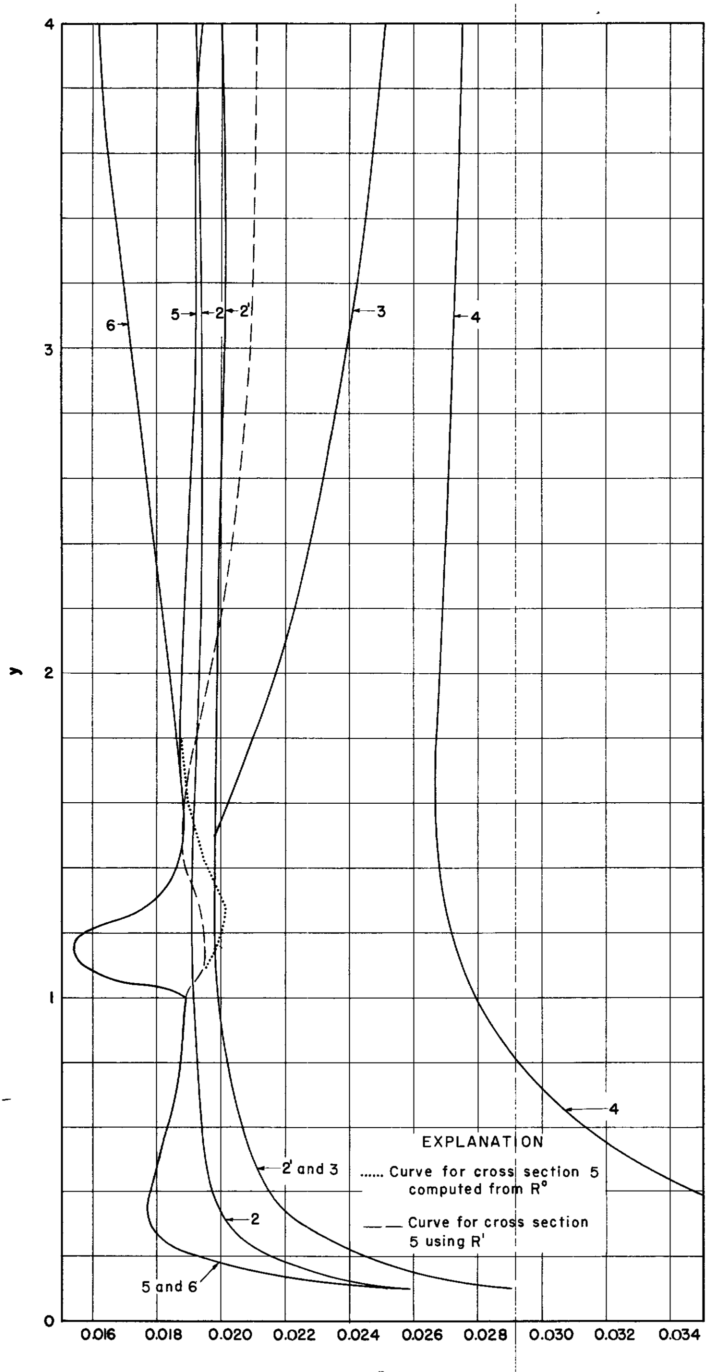


Figure 12.—Values of Manning's n for the various cross sections.

VALUES OF MANNING'S n

As was the case with hydraulic radius, most of the analysis which follows is independent of the value of Manning's n. However, a better understanding of the various channel ratings, as well as additional light on the applicability of the Manning formula, may be gained by an examination of the manner in which n varies between the several cross-sections, and between various depths within any cross section. This information is presented in figure 12, page 43. It will be noted that for low values of y, all the cross sections show an apparent increase in the value of n as y decreases. If n be considered as a characteristic of the channel lining, which for any cross section is uniform for depths of less than 1.5 feet, then the apparent increase of n with decrease of y can be accounted for only by the fact that the two-thirds power of the hydraulic radius inadequately reflects the effect of the shape of the channel. Particular attention is invited to the curves for the flood-plain cross sections 5 and 6. (For values of y less than 1.5 feet the two cross sections are identical.) The continuous lines indicate the variation in n when R is computed in the conventional manner of equation (17). The dotted and dashed lines indicate the corresponding variation, for cross section 5, based on R° and R', respectively. It will be noted that in the range of rapidly expanding width (between $y = 1.01$ and $y = 1.20$) and immediately above, the use of R°, or of R', results in much more stable values of n.

DIRECT METHOD OF ANALYSIS

In an earlier section of this publication, "Integration of the varied-flow equation by the step method," it was demonstrated both graphically and analytically that

$$\Delta x = -\Delta H/(S_0 - S). \tag{14}$$

With appropriate knowledge of the velocity-distribution coefficient, and of the uniform-flow stage-discharge relation, ΔH may be readily determined for any two values of y. Bearing in mind that S_0 is normally a known quantity, it will be obvious that the profile computations may be readily made if S can be properly evaluated. Thus it appeared most desirable that S should be computed for the laboratory data and correlations made between these values of S, the known discharges, and the channel characteristics. If a relation could be found which was applicable to all cross sections and all discharges, it would represent considerable progress in the evolution of a better formula for the computation of friction slope for gradually varied flow.

Introducing y and the velocity distribution coefficient into equation (14) and solving for S,

$$S = \frac{\Delta_x S_0 + (y_1 + c_m V_1{}^2/2g) - (y_2 + c_m V_2{}^2/2g)}{\Delta x}, \tag{19}$$

in which $\Delta x = x_1 - x_2$ is a positive quantity if the initial section (1) is upstream, or a negative quantity if the initial section is downstream from the final section (2). If y_2 and y_1 be taken sufficiently close together, the resulting value of S may be considered as applicable to the point at which $y = (y_2 + y_1)/2$, the average value of y. In computing S for the rectangular channel, Δx was taken as 15 feet; for the flood-plain channel Δx was taken as 10 feet. The values of y used in preliminary computations were those taken directly from the observed profiles. However, y had been observed only to thousands of a foot, and it was found that this was insufficient refinement to avoid "step breaks" in the computed values of S. Therefore the values of y as given in the observed profiles were smoothed and recomputed to ten-thousandths of a foot, giving the values shown in tables 1 to 5, pages 27 to 34. Using these tables and formula (19), values were computed and large-scale plots were made of y_{avg} as ordinate against S as abscissa for each of the smoothed profiles.

Tables 9 and 10, pages 46, 47, containing the basic data for the proposed correlation, were then prepared by reading from the large-scale plots the values of S corresponding to the indicated cross sections, depths, and discharges.

In the attempted correlations, each cross section was first considered separately. The first step was a plot, on logarithmic coordinates, of S as abscissa against V, the actual velocity under which the specific S was observed, as ordinate. A common symbol was used to identify common values of y, but different symbols for different values of y. Each plotted point was labelled by its proper discharge. There resulted a great dispersion of points; but it appeared that, at least for the higher values of S and V, parallel straight lines might be passed through those points for which y was a constant. If this were the case, all the points could be brought to a common curve by dividing V by some function of y.

In attempting to discover this unknown function of y, the straight lines were drawn as described above, and intercepts, i, read from a common ordinate. (The ordinate $S = 0.0004$ was used as a matter of convenience.) These intercepts then were plotted on logarithmic coordinates against various functions of y. Of the plots attempted, the one involving hydraulic radius was most satisfactory, giving a relation, for example, in cross section 4, of $R - 0.27 = .0002365\ i^2$. Unfortunately, the relation proved not to be constant for all cross sections, the relation for cross section 2, for example, being $R - 0.15 = .000141\ i^2$. The variation of the coefficient of i was to be expected and might have been correlated with variation between the channel linings, but the variation in the adjustment to R could not be logically interpreted. Thus, for lack of a common relation between R and i in the several cross sections, no further attempts were made to bring the data for all cross sections into a single plot.

The data for each individual cross section, however, now were recomputed. This was accomplished by dividing V by $(R + c)^{0.5}$, in which c was constant for a given cross section, and having, for example, the value of -0.15 for cross section 2. The resulting plot for cross section 2 is presented on page 48 as

Table 9.--Friction slope and its exponent, rectangular channels

y	Cross section 2			Cross section 3			Cross section 4		
	Q	S	b	Q	S	b	Q	S	b
1.0	3.90	0.000113	0.4815	3.77	0.000127	0.4869	3.79	0.000290	0.4943
1.0	7.99	.000583	.4913	7.37	.000541	.4917	7.62	.001253	.4981
1.0	12.38	.001536	.4971	10.90	.001240	.4941	11.64	.003000	.5000
1.0	16.98	.003000	.5000	16.30	.003000	.5000	-----	--------	------
1.5	3.90	.000037	.4863	3.77	.000033	.4809	3.79	.000093	.5017
1.5	7.99	.000191	.4958	7.37	.000198	.5034	7.62	.000326	.4930
1.5	12.38	.000487	.4992	10.90	.000441	.5050	12.19	.000926	.4994
1.5	20.13	.001283	.4988	18.10	.001198	.5046	19.10	.002224	.4975
1.5	30.22	.002932	.4998	29.55	.003000	.5000	21.85	.003000	.5000
1.5	30.53	.003000	.5000	-----	--------	------	-----	--------	------
2.0	3.90	.000012	.4725	3.77	.000023	.4938	3.79	.000025	.4771
2.0	7.99	.000113	.5102	7.37	.000132	.5153	7.62	.000142	.4918
2.0	12.38	.000223	.4994	10.90	.000232	.5033	12.19	.000396	.4963
2.0	20.13	.000566	.4966	18.10	.000607	.5002	19.10	.000920	.4919
2.0	30.22	.001381	.5022	29.80	.001485	.4923	29.91	.002460	.4977
2.0	40.39	.002444	.5016	40.28	.003000	.5000	32.58	.003000	.5000
2.0	45.18	.003000	.5000	-----	--------	------	-----	--------	------
2.5	7.99	.000089	.5291	3.77	.000015	.4965	3.79	.000013	.4755
2.5	12.38	.000176	.5202	7.37	.000094	.5226	7.62	.000112	.5111
2.5	20.13	.000307	.4958	10.90	.000156	.5082	12.19	.000259	.5063
2.5	30.22	.000784	.5039	18.10	.000384	.5019	19.10	.000537	.4956
2.5	40.39	.001344	.5012	29.80	.001086	.5054	29.91	.001340	.4963
2.5	60.83	.003000	.5000	51.37	.003000	.5000	39.98	.002299	.4927
2.5	-----	--------	------	-----	--------	------	43.67	.003000	.5000
3.0	7.99	.000076	.5456	7.37	.000068	.5260	7.62	.000092	.5254
3.0	12.38	.000155	.5402	10.90	.000107	.5092	12.19	.000174	.5098
3.0	20.13	.000258	.5146	18.10	.000261	.5028	19.10	.000362	.5003
3.0	30.22	.000487	.5042	29.80	.000547	.4860	29.91	.000848	.4971
3.0	40.39	.000881	.5054	62.80	.003000	.5000	39.98	.001223	.4809
3.0	77.45	.003000	.5000	-----	--------	------	55.10	.003000	.5000
3.5	20.13	.000222	.5296	29.80	.000405	.4891	29.91	.000580	.4976
3.5	30.22	.000362	.5109	74.51	.003000	.5000	39.98	.000833	.4820
3.5	40.39	.000594	.5060	-----	--------	------	66.81	.003000	.5000
3.5	94.85	.003000	.5000	-----	--------	------	-----	--------	------

Table 10.--Friction slope and its exponent, flood-plain channels

y	Cross section 5			Cross section 6		
	Q	S	b	Q	S	b
0.5	0.79	0.002378	0.5005	0.83	0.002453	0.4948
0.5	0.89	.003000	.5000	.89	.003000	.5000
0.8	.79	.000725	.4985	.83	.000727	.4918
0.8	1.32	.001974	.4964	1.59	.003000	.5000
0.8	1.59	.003000	.5000	-----	--------	------
1.0	.79	.000464	.5039	.83	.000563	.5104
1.0	1.32	.001376	.5091	1.84	.002380	.5003
1.0	1.84	.002379	.5003	2.07	.003000	.5000
1.0	2.07	.003000	.5000	-----	--------	------
1.1	.79	.000400	.5128	.83	.000452	.5145
1.1	1.32	.001138	.5160	1.84	.001807	.5013
1.1	1.84	.001811	.5014	2.39	.003000	.5000
1.1	2.39	.003000	.5000	-----	--------	------
1.2	.79	.000240	.5090	.83	.000277	.5118
1.2	1.32	.000680	.5113	1.84	.001125	.5004
1.2	1.84	.001118	.4999	3.01	.003000	.5000
1.2	2.71	.002228	.4929	-----	--------	------
1.2	3.01	.003000	.5000	-----	--------	------
1.5	.79	.000036	.5003	.83	.000073	.5323
1.5	1.32	.000122	.5111	1.84	.000200	.5018
1.5	1.84	.000200	.5018	3.98	.000945	.5030
1.5	2.71	.000452	.5048	7.24	.003000	.5000
1.5	3.87	.000878	.5017	-----	--------	------
1.5	7.24	.003000	.5000	-----	--------	------
2.0	1.32	.000025	.5208	1.84	.000030	.4999
2.0	1.84	.000044	.5169	3.98	.000132	.4966
2.0	2.71	.000061	.4945	18.40	.003000	.5000
2.0	3.87	.000124	.4938	-----	--------	------
2.0	11.30	.001157	.4985	-----	--------	------
2.0	18.01	.003000	.5000	-----	--------	------
2.5	1.84	.000011	.5007	3.98	.000065	.5199
2.5	2.71	.000033	.5164	30.63	.002498	.4960
2.5	3.87	.000066	.5166	32.77	.003000	.5000
2.5	11.30	.000374	.4944	-----	--------	------
2.5	30.61	.003000	.5000	-----	--------	------
3.0	11.30	.000192	.4998	30.63	.001065	.4955
3.0	31.28	.001503	.5015	49.86	.003000	.5000
3.0	44.61	.003000	.5000	-----	--------	------
3.5	31.28	.000740	.4926	30.63	.000545	.4950
3.5	59.71	.003000	.5000	69.21	.003000	.5000

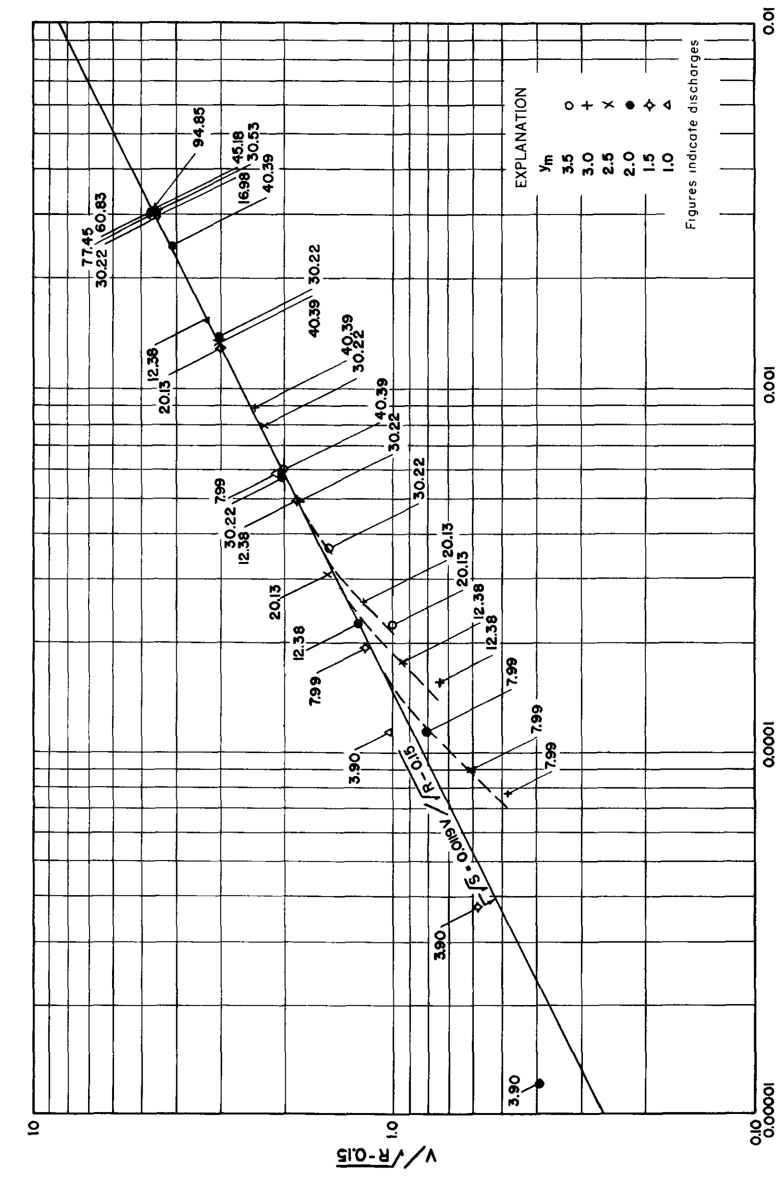


Figure 13—Friction slope in cross section 2.

figure 13. It will be noted that, for values of S greater than about 0.0003, all the points lie very close to the curve $S^{0.5} = 0.0119\,V/(R\text{-}0.15)^{0.5}$. However, for lower values of S the points depart from this curve, and it will be noted that points of equal discharge tend to define independent curves branching away from the parent relationship. Of these branches, all except that for the lowest discharge trend in the same direction. A positive reason is not evident for this apparent discordance of the curve for lowest discharge, but it seems appropriate to point out that for this curve the values of S were computed from differentials of y which were extremely small and may therefore be subject to very considerable errors. Neglecting this one curve, all the others indicate that as S decreases, the slope of the curve, or the appropriate exponent of S, sharply increases and in fact appears to approach a 45° line which would represent an S with an exponent of 1. Such a value for the exponent of S would, of course, be indicative of laminar flow. Thus the plot may be interpreted to indicate that, under the extreme back-water conditions for which these low values of S are appropriate, turbulent flow was not completely developed.

It was next assumed that the relationship indicated by figure 13 should have the more general form

$$S^b = f(V, y)$$

and efforts were made to evaluate b, and correlate these values. For uniform flow at any given depth, y, by equation (9),

$$K_0 = \frac{Q_0}{S_0^{0.5}} = \frac{V_0 A}{S_0^{0.5}}, \tag{20}$$

a value depending only on channel characteristics. For nonuniform flow at the same depth, these channel characteristics remain unchanged, unless, indeed, roughness is a function of velocity, which is contrary to usual assumption. Assuming the channel characteristics remain unchanged, then K remains unchanged and

$$Q = K_0 S^b, \text{ or } K_0 = \frac{Q}{S^b} = \frac{VA}{S^b}. \tag{21}$$

Combining equations (20) and (21) and solving for b,

$$b = \frac{\tfrac{1}{2}\log S_0 + \log V - \log V_0}{\log S}. \tag{22}$$

Values of b were computed by this formula for all the values of S which appear in tables 9 and 10, pages 46, 47. These values of b (see tables 9 and 10) then were plotted against promising characteristics of the flow, including the Froude Number, Reynolds Number, and various ratios between V, V_0, y, and y_0. The most

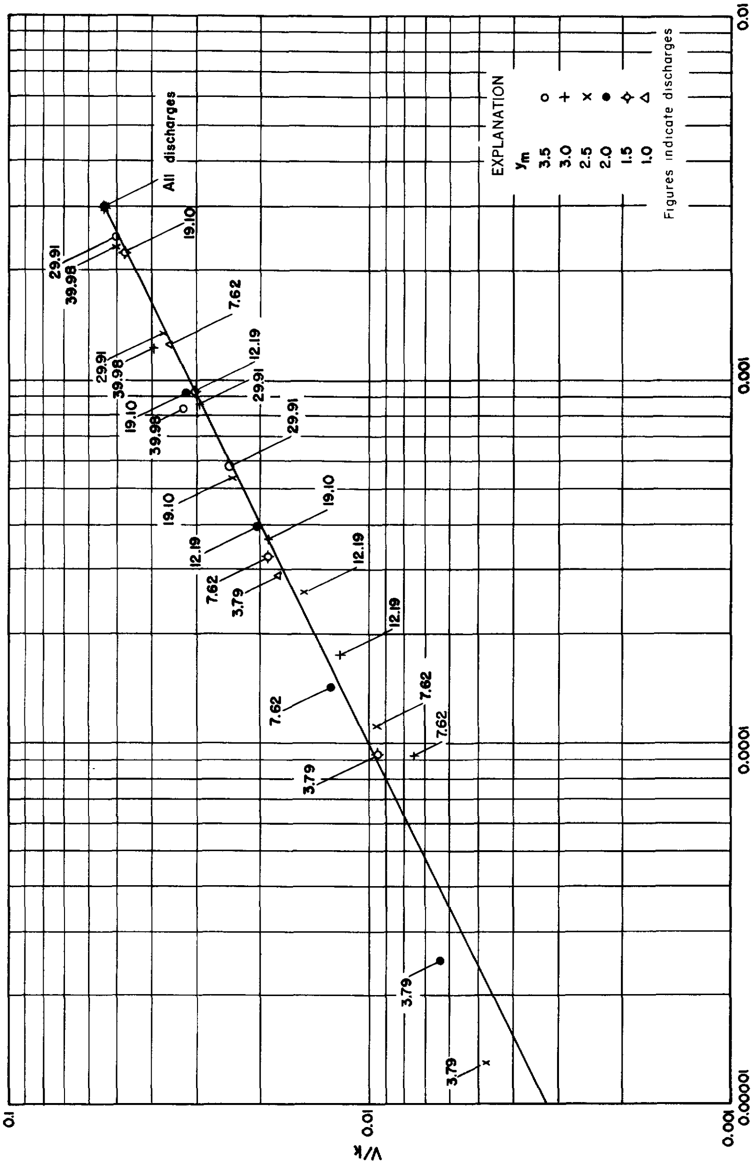


Figure 14. —Relationship between V/k and friction slope, cross section 4.

satisfactory plots were obtained with respect to the Froude Number, which was computed as $F = c_m V^2/gR$, in which R is the conventional hydraulic radius, and for which R', as described on page 42, was substituted when considering the flood-plain channels. These plots indicated a definite trend of increase in b with low values of F, but there was much scattering of the points, and the data for the lowest profiles were in contradiction to the general trend. Still the trend was sufficiently pronounced to suggest use of the relation in test computations of backwater profiles. Agreement between these computed profiles and observed profiles was not materially better than the agreement obtained from computations by less involved methods. Thus, because the improvements in results were not definite enough to justify the vast increase in the work of computation, and also because of the contradicting trend of the data for the profile of lowest discharge, no further use was made of the variable exponent of friction slope. Assuming, instead, that b is invariably 0.5, equation (21) reduces to

$$K_0 = VA/S^{0.5} \tag{23}$$

which, combined with equation (20) gives

$$S = V^2(S_0/V_0^2). \tag{24}$$

For a given channel, S_0 is constant and V_0 is a function of y. Thus it becomes convenient to introduce a factor $k = V_0/S_0^{0.5}$, so that

$$S = \left(\frac{V}{k}\right)^2 \tag{25}$$

The extent to which equation (25) is appropriate is illustrated by figure 14, page 50, in which, for cross section 4, values of V/k have been plotted as ordinates with values of S as abscissa. The curve has been drawn to conform with equation (25). As in figure 13, there are some notable departures of particular discharges from the slope of the curve as drawn, but since the trend of the lowest discharge is opposite to the trend of the higher discharges, the use of the curve as drawn, involving the exponent 2 in equation (25) seems most appropriate. This equation has been used for computation of S in the remainder of this report. (See p. 52.)

INDIRECT METHODS OF ANALYSIS

In the direct method of analysis, attempts were made to evaluate S, the slope of a point on the energy gradient. Because this slope changes from point to point along the channel, it was necessary to work with reaches of channel of such short length that the change could be neglected. This in turn involved the use of such small values of Δy that depths observed to thousandths of a foot, even when smoothed to ten-thousandths of a foot, were comparatively rough approximations. On the other hand, the values of y might be regarded as highly accurate with respect to a method of analysis which would permit the use of long reaches of channel. Because of curvature in both the line of the energy gradient and the line of the water surface, use can be made of long reaches only by indirect methods; that is, it becomes necessary (1) to assume that a particular method of computation is appropriate, (2) to compute the surface profile by this method, and finally (3) to compare the computed profile with the observed profile, thereby ascertaining whether or not the particular method is truly appropriate.

Of two indirect methods which produce equally satisfactory results, one may be the more desirable from the practical viewpoint. If only the total length of reach between two widely different values of y is desired, a method which completely integrates the varied-flow equation may be a great saving in labor. On the other hand, if it is desired to define accurately the shape of the profile between these two widely different values of y, many additional points on the profile may need to be computed solely for this purpose, so that the total number of steps may be greatly increased. For this purpose the step method, because of its simplicity, flexibility, and widespread acceptance, may be regarded as the more desirable.

THE STEP METHOD

The basic formula of the step method has been presented:

$$\Delta x = -\Delta H/(S_0 - \bar{S}) \tag{15}$$

$\bar{S}$ being the mean value of S at the two ends of the reach Δx, and S being computed by the method deemed most appropriate. It has been indicated in the preceding section, and it is now assumed, that the most appropriate method of computing S is

$$S = \left(\frac{V}{k}\right)^2. \tag{25}$$

Values of k appear in this report as tables 28-32, pages 152 to 154.

In the development of equation (15), it was specified (see page 24) that ΔH should be considered as the loss in H, or H at the initial section minus H at the final section. Thus all the terms in the right member of equation (15) represent loss in the reach Δx. Whether Δx itself should be regarded as a loss (initial value minus final value) or a change (final value minus initial value) is a question which may be viewed in the light of convenience, and the objective of the computations. In fact there are instances in which Δx is appropriately regarded merely as a length of reach. In the present study, it is both convenient and appropriate to regard Δx as a loss, so that, if L_1 be the value of the stationing for the initial section (1) and L_2 be the corresponding value for the final section

Table 11.--Sample computation of profile by step method

[Cross section 5: y_o = 0.456 ft; Q = 0.79 cfs]

y	A	V	$c_m V^2/2g$	H	ΔH	k	V/k	S	$\bar{S}$	$S_o - \bar{S}$	$\frac{\Delta H}{S_o - \bar{S}}$	L	Elev.
4.00	15.564	0.05076	0.00004	4.00004		88.225	0.000,575,35	0.000,000,3				0	4.00000
					0.39998				0.000,000,4	0.002,999,6	133.34		
3.60	13.564	.05824	.00006	3.60006		84.530	.000,688,99	.000,000,5				133.34	4.00002
					.39998				.000,000,6	.002,999,4	133.35		
3.20	11.564	.06832	.00008	3.20008		79.794	.000,856,20	.000,000,7				266.69	4.00007
					.39997				.000,001,0	.002,999,0	133.37		
2.80	9.564	.08260	.00011	2.80011		74.183	.001,113,5	.000,001,2				400.06	4.00018
					.19997				.000,001,4	.002,998,6	66.69		
2.60	8.564	.09225	.00014	2.60014		71.012	.001,299,1	.000,001,7				466.75	4.00025
					.19996				.000,002,0	.002,998,0	66.70		
2.40	7.564	.10444	.00018	2.40018		67.512	.001,547,0	.000,002,4				533.45	4.00035
					.19994				.000,003,0	.002,997,0	66.71		
2.20	6.564	.12035	.00024	2.20024		63.612	.001,891,9	.000,003,6				600.16	4.00048
					.19991				.000,004,7	.002,995,3	66.74		
2.00	5.564	.14198	.00033	2.00033		59.097	.002,402,5	.000,005,8				666.90	4.00070
					.19984				.000,008,1	.002,991,9	66.79		
1 80	4.564	.17309	.00049	1.80049		53.644	.003,226,6	.000,010,4				733.69	4.00107
					.19969				.000,016,4	.002,983,6	66.93		
1.60	3.564	.22166	.00080	1.60080		46.872	.004,729,0	.000,022,4				800.62	4.00186
					.09972				.000,029,0	.002,971,0	33.56		
1.50	3.064	.2578	.00108	1.50108		43.140	.005,975,9	.000,035,7				834.18	4.00254
					.09953				.000,048,4	.002,951,6	33.72		
1.40	2.564	.3081	.00155	1.40155		39.378	.007,824,2	.000,061,2				867.90	4.00370
					.09916				.000,087,4	.002,912,6	34.05		
1.30	2.064	.3828	.00239	1.30239		35.914	.010,659	.000,113,6				901.95	4.00585
					.03946				.000,129,8	.002,870,2	13.75		
1.26	1.864	.4238	.00293	1.26293		35.065	.012,086	.000,146,1				915.70	4.00710
					.03925				.000,165,6	.002,834,4	13.85		
1.22	1.664	.4748	.00368	1.22368		34.892	.013,608	.000,185,2				929.55	4.00865
					.03897				.000,207,0	.002,793,0	13.95		
1.18	1.470	.5374	.00471	1.18471		35.522	.015,129	.000,228,9				943.50	4.01050
					.03875				.000,253,0	.002,747,0	14.11		
1.14	1.308	.6040	.00596	1.14596		36.292	.016,643	.000,277,0				957.61	4.01283

Table 11.--Sample computation of profile by step method--Continued

y	A	V	$c_m V^2/2g$	H	ΔH	k	V/k	S	$\bar{S}$	$S_o - \bar{S}$	$\frac{\Delta H}{S_o - \bar{S}}$	L	Elev.
1.14	1.308	0.6040	0.00596	1.14596		36.292	0.016,643	0.000,277,0				957.61	4.01283
					0.03864				0.000,302,4	0.002,697,6	14.32		
1.10	1.180	.6695	.00732	1.10732		36.979	.018,105	.000,327,8				971.93	4.01579
					.03865				.000,352,2	.002,647,8	14.60		
1.06	1.084	.7288	.00867	1.06867		37.559	.019,404	.000,376,5				986.53	4.01959
					.01938				.000,387,0	.002,613,0	7.42		
1.04	1.047	.7545	.00929	1.04929		37.840	.019,939	.000,397,6				993.95	4.02185
					.01950				.000,407,1	.002,592,9	7.52		
1.02	1.020	.7745	.00979	1.02979		37.946	.020,411	.000,416,6				1,001.47	4.02441
					.01960				.000,426,8	.002,573.2	7.62		
1.00	1.000	.7900	.01019	1.01019		37.793	.020,903	.000,436,9				1,009.09	4.02727
					.09761				.000,498,0	.002,502,0	39.01		
.90	.900	.8778	.01258	.91258		37.123	.023,646	.000,559,1				1,048.10	4.04430
					.09666				.000,649,8	.002,350,2	41.13		
.80	.800	.9875	.01592	.81592		36.287	.027,214	.000,740,6				1,089.23	4.06769
					.09513				.000,884,0	.002,116.0	44.96		
.70	.700	1.1286	.02079	.72079		35.211	.032,052	.001,027,3				1,134.19	4.10257
					.09249				.001,260,0	.001,740,0	53.16		
.60	.600	1.3167	.02830	.62830		34.081	.038,634	.001,492,6				1,187.35	4.16205
					.03581				.001,628,7	.001,371,3	26.11		
.56	.560	1.4107	.03249	.59249		33.581	.042,009	.001,764,8				1,213.46	4.20038
					.03481				.001,941.8	.001,058,2	32.90		
.52	.520	1.5192	.03768	.55768		33.004	.046,031	.002,118,9				1,246.36	4.25908
					.01693				.002,241,3	.000,758,7	22.31		
.50	.500	1.5800	.04075	.54075		32.498	.048,618	.002,363,7				1,268.67	4.30601
					.00832				.002,418,6	.000,581,4	14.31		
.49	.490	1.6122	.04243	.53243		32.416	.049,735	.002,473,6				1,282.98	4.33894
					.00821				.002,532,5	.000,467,5	17.56		
.48	.480	1.6458	.04422	.52422		32.330	.050,906	.002,591,4				1,300.54	4.38162
					.00810				.002,688,0	.000,312,0	25.96		
.47	.470	1.6809	.04612	.51612		31.854	.052,769	.002,784,6				1,326.50	4,44950
					.00753				.002,855,0	.000,145,0	51.93		
.46056		1.7153	.04803	.50859		31.713	.054,088	.002,925,5				1,378.43	4.59585

(2), $L_1 - L_2 = \Delta x$, which, combined with equation (15), gives $L_2 = L_1 + [\Delta H/(S_0 - \bar{S})]$ in which ΔH still is defined as the loss, $H_1 - H_2$. This equation is appropriate to the computation of any and all types of profiles, and yields the same arrangement of the values for L irrespective of whether the initial section be chosen upstream or downstream from the final section.

Computations by the above formula are conveniently made in the form of table 11, pages 53 and 54. In the first column are listed values of y, beginning with y_{max} as described on page 35. Successive values of y are those corresponding to intervals needed for properly defining the profile, or for mitigating the effects of curvature, whichever may be smaller. Values of A in column 2 are the computed product of y and width b, or for the flood-plain cross sections, are taken from the table of areas (see p. 152, table 27). V is obtained by dividing the known discharge of the profile by A. To compute the velocity head, $c_m V^2/2g$, c_m is taken from table 8, on page 38. H is the sum of y and velocity head, and k is taken from tables 28 to 32. Column 3 divided by column 7 gives column 8, V/k, which is squared to obtain S, column 9. For each reach, the downstream is considered as the initial section, so that ΔH is the remainder of H on any line minus H on the following line. Values of S at each end of a reach are averaged to obtain $\bar{S}$; this subtracted from the constant value (0.003) for S_0 yields column 11 which, when divided into ΔH from column 6, gives column 12. Value of the stationing for the final end of the reach (L_2) is then obtained by adding column 12 to the value of the stationing for the initial end of the reach, the results being shown in the next-to-last column of the table. In the final column, elevation is the sum of y and S_0L. (See equation (16), p. 35 .)

DIRECT-INTEGRATION METHODS

Of the several methods of direct integration which have been proposed in recent years, those which appeared most adaptable to the laboratory data (namely the Bakhmeteff method and the Von Seggern method) have been discussed in the preceding section of this report. (See pp. 21, 22.) As a check upon the reliability and extent of agreement of various methods, selected representative profiles were computed not only by the step method, but also by the above methods of direct integration.

COMPARISON OF RESULTS

A comparison of results is most conveniently made by figures 15 and 16, pages 56, 57. Figure 15 presents the curves for the highest and lowest discharges observed for cross section 2. The curves represent the results of step-method computations and are identical with the curves presented as part of figure 8 in Bulletin 381, (Lansford and Mitchell, 1949). It will be noted that for the high discharge there is no appreciable difference between the results of the various methods. For the low discharge, the direct-integration methods produce eleva-

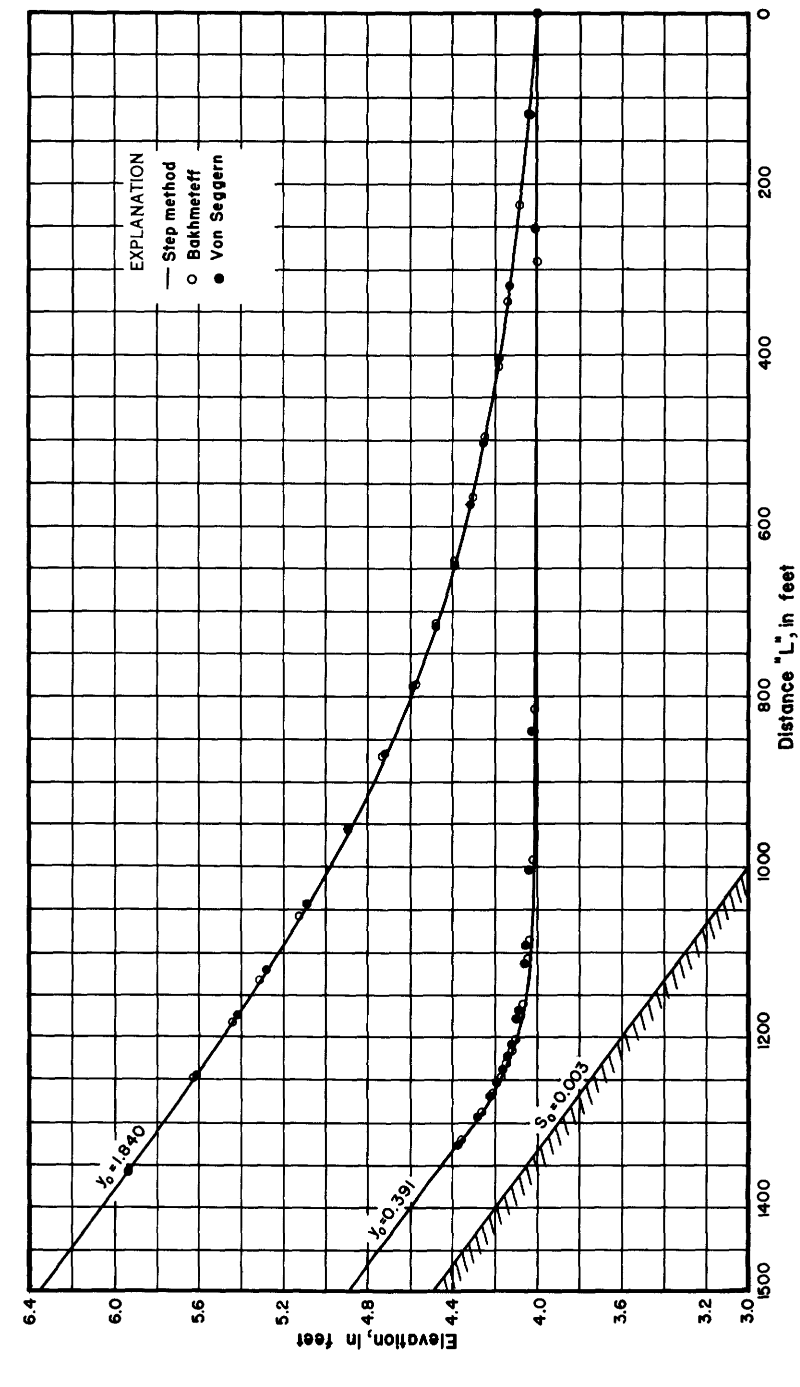


Figure 15.—Comparison of computation methods, cross section 2.

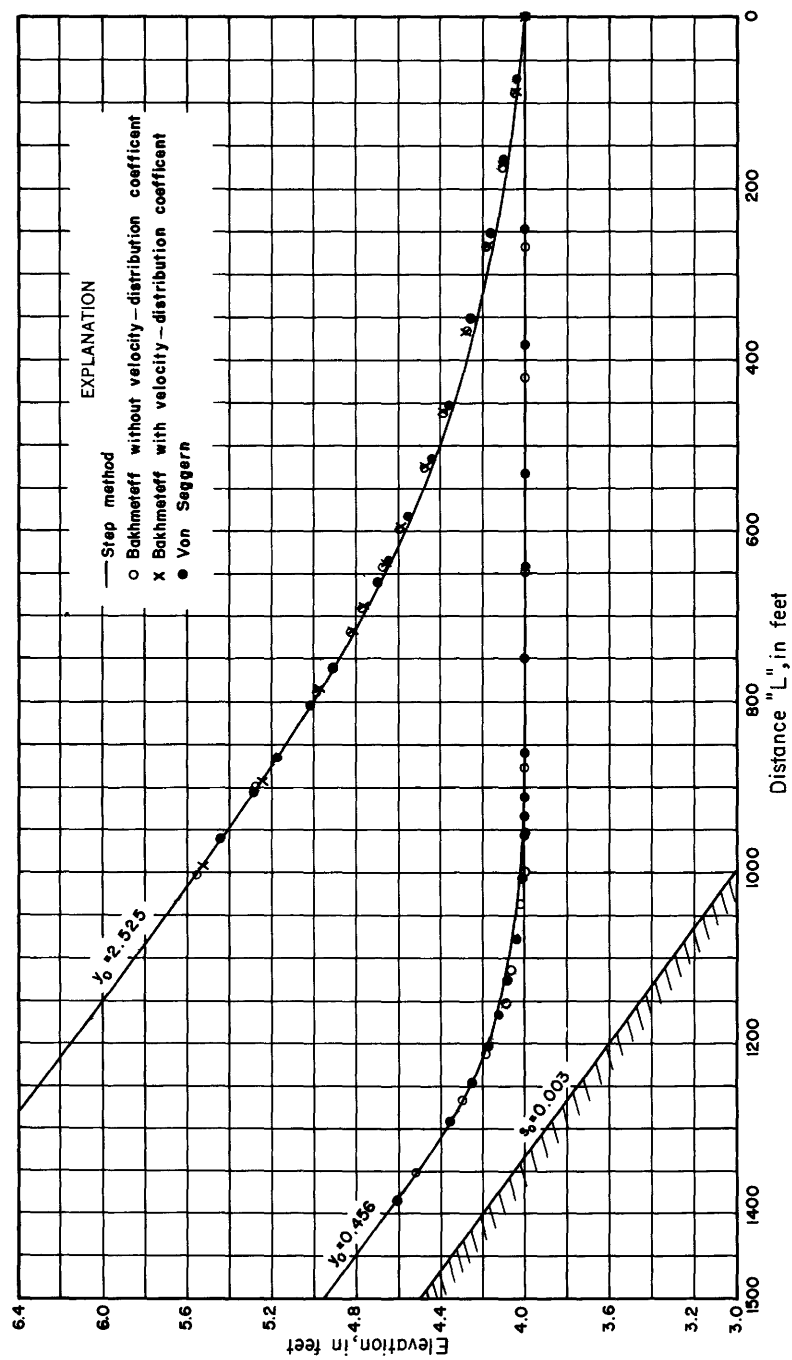


Figure 16. --Comparison of computation methods, cross section 5.

tions which, between stations 800 and 1300, are slightly higher than those for the step method. Comparison with the corresponding curve of Bulletin 381 indicates that the step-method results are in slightly better agreement with the observed data.

Figure 16 presents similar data for the highest and lowest discharges observed for cross section 5. Comparing this plot with figure 11 of Bulletin 381 and bearing in mind that the step-method results are identical in the two figures, it will be noted that, although the results of the direct integration methods differ only slightly from the results of the step method, these differences are generally in the opposite direction from the observed data.

Thus, while the work involved in step-method computations is slightly greater, the results appear to be slightly more consistent with observed data. Furthermore, in the following sections on stage-fall-discharge relations, it is necessary that the profiles be carefully defined throughout their entire length; thus a great number of subdivisions is required even by the direct-integration methods. For these reasons, the step method was adopted as the standard procedure for computing the profiles.

TABLES OF PROFILES BY THE STEP METHOD

Using the step method, profiles were computed for all the profiles which were observed in cross sections 2 to 6, inclusive. These are identical with the continuous lines of figures 8 to 12 in Bulletin 381. Summary of the computations appears as tables 12 to 16, inclusive, on pages 59 to 77. It will be noted that these computed profiles are based on the laboratory data in that the hydraulic properties of the cross sections under uniform-flow conditions were determined experimentally, the properties thus determined for a given depth being assumed to apply to conditions of gradually varied flow at the same depth. Furthermore, the computed profiles (except for the two lowest discharges for cross section 5) were plotted and carefully compared with their observed counterparts, thus verifying the validity of the computed profiles. The values of y_0 and Q given in the column headings are related to each other through the use of the stage-discharge relation for uniform flow as given in table 42 in Bulletin 381 or as can be determined without reference to that bulletin through use of tables 27 to 32 of this publication as described on page 3.

Table 12.--Summary of step-method computation of profiles, cross section 2

y	Q = 3.90, y_o = 0.391		Q = 7.99, y_o = 0.612		Q = 12.38, y_o = 0.812		Q = 20.13, y_o = 1.121		Q = 30.22, y_o = 1.489		Q = 40.39, y_o = 1.840	
	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.
4.00	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000
3.60	133.47	4.00041	133.91	4.00173	134.73	4.00419	137.10	4.01130	142.24	4.02672	150.46	4.05138
3.40	------	-------	------	-------	------	-------	206.07	4.01821	214.43	4.04329	227.95	4.08385
3.20	266.99	4.00097	268.01	4.00403	269.91	4.00973	275.43	4.02629	287.61	4.06283	307.63	4.12289
3.00	------	-------	335.15	4.00545	337.73	4.01319	345.27	4.03581	362.08	4.08624	390.29	4.17087
2.90	------	-------	------	-------	------	-------	------	-------	------	-------	433.06	4.19918
2.80	400.57	4.00171	402.38	4.00714	405.77	4.01731	415.74	4.04722	438.27	4.11481	477.08	4.23124
2.70	------	-------	------	-------	------	-------	------	-------	477.18	4.13154	522.65	4.26795
2.60	467.39	4.00217	469.72	4.00916	474.09	4.02227	487.03	4.06109	516.79	4.15037	570.19	4.31057
2.50	------	-------	------	-------	------	-------	------	-------	557.24	4.17172	620.26	4.36078
2.40	534.25	4.00275	537.21	4.01163	542.77	4.02831	559.42	4.07826	598.72	4.19616	673.73	4.42119
2.36	------	-------	------	-------	------	-------	------	-------	------	-------	696.33	4.44899
2.32	------	-------	------	-------	------	-------	------	-------	------	-------	719.81	4.47943
2.30	------	-------	------	-------	------	-------	596.18	4.08854	641.49	4.22447	------	-------
2.28	------	-------	------	-------	------	-------	------	-------	------	-------	744.32	4.51296
2.24	------	-------	------	-------	------	-------	------	-------	------	-------	770.07	4.55021
2.20	601.16	4.00348	604.89	4.01467	611.96	4.03588	633.38	4.10014	685.90	4.25770	797.30	4.59190
2.16	------	-------	------	-------	------	-------	------	-------	------	-------	826.38	4.63914
2.12	------	-------	------	-------	------	-------	------	-------	------	-------	857.79	4.69337
2.10	------	-------	------	-------	646.79	4.04037	671.15	4.11345	732.49	4.29747	------	-------
2.08	------	-------	------	-------	------	-------	------	-------	------	-------	892.28	4.75684
2.04	------	-------	------	-------	------	-------	------	-------	------	-------	930.98	4.83294
2.02	------	-------	------	-------	------	-------	------	-------	------	-------	952.46	4.87738
2.00	668.13	4.00439	672.84	4.01852	681.82	4.04546	709.62	4.12886	782.08	4.34624	975.80	4.92740
1.98	------	-------	------	-------	------	-------	------	-------	------	-------	1,001.49	4.98447
1.96	------	-------	------	-------	------	-------	------	-------	803.01	4.36903	1,030.24	5.05072
1.94	------	-------	------	-------	------	-------	------	-------	------	-------	1,063.21	5.12963
1.92	------	-------	------	-------	------	-------	------	-------	824.74	4.39422	1,102.32	5.22696
1.91	------	-------	------	-------	------	-------	------	-------	------	-------	1,125.23	5.28569
1.90	------	-------	706.95	4.02085	717.11	4.05133	748.99	4.14697	------	-------	1,151.20	5.35360

Table 12.--Summary of step-method computation of profiles, cross section 2--Continued

y	Q = 3.90, y_o = 0.391		Q = 7.99, y_o = 0.612		Q = 12.38, y_o = 0.812		Q = 20.13, y_o = 1.121		Q = 30.22, y_o = 1.489		Q = 40.39, y_o = 1.840	
	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.
1.89	------	-------	------	-------	------	-------	------	-------	------	-------	1,181.45	5.43435
1.88	------	-------	------	-------	------	-------	------	-------	847.43	4.42229	1,217.73	5.53319
1.87	------	-------	------	-------	------	-------	------	-------	------	-------	1,263.49	5.66047
1.86	------	-------	------	-------	------	-------	------	-------	------	-------	1,326.69	5.84007
1.8584	------	-------	------	-------	------	-------	------	-------	------	-------	1,339.83	5.87789
1.84	------	-------	------	-------	------	-------	------	-------	871.31	4.45393	--------	-------
1.80	735.19	4.00557	741.18	4.02354	752.71	4.05813	789.52	4.16856	896.80	4.49040	--------	-------
1.76	------	-------	------	-------	------	-------	------	-------	924.19	4.53257	--------	-------
1.72	------	-------	------	-------	------	-------	------	-------	953.89	4.58167	--------	-------
1.70	------	-------	775.54	4.02662	788.70	4.06610	831.63	4.19489	------	-------	--------	-------
1.68	------	-------	------	-------	------	-------	------	-------	987.07	4.64121	--------	-------
1.66	------	-------	------	-------	------	-------	849.02	4.20706	1,005.41	4.67623	--------	-------
1.64	------	-------	------	-------	------	-------	------	-------	1,025.31	4.71593	--------	-------
1.62	------	-------	------	-------	------	-------	866.82	4.22046	1,047.21	4.76163	--------	-------
1.60	802.38	4.00714	810.09	4.03027	825.20	4.07560	------	-------	1,071.84	4.81552	--------	-------
1.58	------	-------	------	-------	------	-------	885.10	4.23530	1,100.20	4.88060	--------	-------
1.56	------	-------	------	-------	------	-------	------	-------	1,134.12	4.96236	--------	-------
1.55	------	-------	------	-------	------	-------	------	-------	1,154.24	5.01272	--------	-------
1.54	------	-------	------	-------	------	-------	903.94	4.25182	1,177.38	5.07214	--------	-------
1.53	------	-------	------	-------	------	-------	------	-------	1,205.09	5.14527	--------	-------
1.52	------	-------	------	-------	------	-------	------	-------	1,239.74	5.23922	--------	-------
1.51	------	-------	------	-------	------	-------	------	-------	1,286.49	5.36947	--------	-------
1.50389	------	-------	------	-------	------	-------	------	-------	1,327.04	5.48501	--------	-------
1.50	836.04	4.00812	844.87	4.03461	862.37	4.08711	923.48	4.27044	--------	-------	--------	-------
1.46	------	-------	------	-------	------	-------	943.86	4.29158	--------	-------	--------	-------
1.42	------	-------	------	-------	------	-------	965.31	4.31593	--------	-------	--------	-------
1.40	869.77	4.00931	879.96	4.03988	900.46	4.10138	------	-------	--------	-------	--------	-------
1.38	------	-------	------	-------	------	-------	988.14	4.34442	--------	-------	--------	-------
1.36	------	-------	------	-------	916.03	4.10809	------	-------	--------	-------	--------	-------
1.34	------	-------	------	-------	------	-------	1,012.84	4.37852	--------	-------	--------	-------

1.32	--------	-------	--------	-------	931.84	4.11552	--------	-------
1.30	903.59	4.01077	915.46	4.04638	--------	-------	1,040.21	4.42063
1.28	--------	-------	--------	-------	947.94	4.12382	1,055.27	4.44581
1.26	--------	-------	--------	-------	--------	-------	1,071.59	4.47477
1.24	--------	-------	--------	-------	964.38	4.13314	1,089.54	4.50862
1.22	--------	-------	--------	-------	--------	-------	1,109.78	4.54934
1.20	937.52	4.01256	951.56	4.05468	981.26	4.14378	1,133.34	4.60002
1.18	--------	-------	--------	-------	--------	-------	1,162.10	4.66630
1.17	--------	-------	--------	-------	--------	-------	1,179.66	4.70898
1.16	--------	-------	--------	-------	998.65	4.15595	1,200.52	4.76156
1.15	--------	-------	--------	-------	--------	-------	1,226.69	4.83007
1.14	--------	-------	--------	-------	--------	-------	1,262.82	4.92846
1.13221	--------	-------	--------	-------	--------	-------	1,306.07	5.05042
1.12	--------	-------	--------	-------	1,016.72	4.17016	--------	-------
1.10	971.62	4.01486	988.55	4.06565	--------	-------	--------	-------
1.08	--------	-------	--------	-------	1,035.66	4.18698	--------	-------
1.06	--------	-------	1,003.67	4.07101	--------	-------	--------	-------
1.04	--------	-------	--------	-------	1,055.76	4.20728	--------	-------
1.02	--------	-------	1,019.06	4.07718	--------	-------	--------	-------
1.00	1,005.97	4.01791	--------	-------	1,077.52	4.23256	--------	-------
.98	--------	-------	1,034.78	4.08434	1,089.26	4.24778	--------	-------
.96	--------	-------	--------	-------	1,101.75	4.26525	--------	-------
.94	--------	-------	1,050.91	4.09273	1,115.24	4.28572	--------	-------
.92	--------	-------	--------	-------	1,130.07	4.31021	--------	-------
.90	1,040.71	4.02213	1,067.59	4.10277	1,146.83	4.34049	--------	-------
.88	--------	-------	--------	-------	1,166.51	4.37953	--------	-------
.87	--------	-------	--------	-------	1,178.00	4.40400	--------	-------
.86	--------	-------	1,085.00	4.11500	1,191.12	4.43336	--------	-------
.85	--------	-------	--------	-------	1,206.71	4.47013	--------	-------
.84	--------	-------	--------	-------	1,226.07	4.51821	--------	-------

Table 12.--Summary of step-method computation of profiles, cross section 2--Continued

y	Q = 3.90, y_o = 0.391		Q = 7.99, y_o = 0.612		Q = 12.38, y_o = 0.812	
	L	Elev.	L	Elev.	L	Elev.
.83	--------	-------	--------	-------	1,252.53	4.58759
.82012	--------	-------	--------	-------	1,296.91	4.71085
.82	--------	-------	1,103.40	4.13020	--------	-------
.80	1,076.10	4.02830	--------	-------	--------	-------
.78	--------	-------	1,123.31	4.14993	--------	-------
.76	1,090.51	4.03153	1,134.06	4.16218	--------	-------
.74	--------	-------	1,145.55	4.17665	--------	-------
.72	1,105.14	4.03542	1,158.05	4.19415	--------	-------
.70	--------	-------	1,171.96	4.21588	--------	-------
.68	1,120.06	4.04018	1,188.08	4.24424	--------	-------
.66	--------	-------	1,207.92	4.28376	--------	-------
.65	--------	-------	1,220.36	4.31108	--------	-------
.64	1,135.39	4.04617	1,235.81	4.34743	--------	-------
.63	--------	-------	1,256.80	4.40040	--------	-------
.62	--------	-------	1,291.98	4.49594	--------	-------
.61812	--------	-------	1,303.91	4.52985	--------	-------
.60	1,151.29	4.05387	--------	-------	--------	-------
.56	1,168.04	4.06412	--------	-------	--------	-------
.52	1,186.17	4.07851	--------	-------	--------	-------
.50	1,196.03	4.08809	--------	-------	--------	-------
.48	1,206.75	4.10025	--------	-------	--------	-------
.46	1,218.80	4.11640	--------	-------	--------	-------
.44	1,233.11	4.13933	--------	-------	--------	-------
.43	1,241.73	4.15519	--------	-------	--------	-------
.42	1,252.19	4.17657	--------	-------	--------	-------
.41	1,266.02	4.20806	--------	-------	--------	-------
.40	1,287.88	4.26364	--------	-------	--------	-------
.39491	1,310.51	4.32644	--------	-------	--------	-------

Table 13.--Summary of step-method computations of profiles, cross section 3

y	Q = 3.77 y_o = 0.404		Q = 7.37 y_o = 0.606		Q = 10.90 y_o = 0.773		Q = 18.10 y_o = 1.071		Q = 29.80 y_o = 1.512	
	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.
4.00	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000
3.60	133.57	4.00071	134.26	4.00278	135.39	4.00617	139.16	4.01748	150.70	4.05210
3.40	------	-------	------	-------	------	-------	209.30	4.02790	227.89	4.08367
3.20	267.21	4.00163	268.76	4.00628	271.34	4.01402	279.91	4.03973	306.76	4.12028
3.00	------	-------	336.14	4.00842	339.49	4.01847	351.19	4.05357	387.90	4.16370
2.80	400.94	4.00282	403.62	4.01086	407.96	4.02388	423.13	4.06939	471.92	4.21576
2.70	------	-------	------	-------	------	-------	------	-------	515.18	4.24554
2.60	467.85	4.00355	471.22	4.01366	476.71	4.03013	495.99	4.08797	559.55	4.27865
2.50	------	-------	------	-------	------	-------	------	-------	605.69	4.31707
2.40	534.80	4.00440	539.00	4.01700	546.87	4.04061	570.17	4.11051	653.05	4.35915
2.30	------	-------	------	-------	------	-------	607.74	4.12322	702.42	4.40726
2.20	601.80	4.00540	606.96	4.02088	616.46	4.04938	645.76	4.13728	754.36	4.46308
2.10	------	-------	------	-------	651.46	4.05438	684.31	4.15293	809.69	4.52907
2.06	------	-------	------	-------	------	-------	------	-------	833.02	4.55906
2.02	------	------	------	------	------	-------	------	-------	857.19	4.59157
2.00	668.87	4.00661	675.21	4.02563	686.70	4.06010	723.69	4.17107	------	-------
1.98	------	-------	------	-------	------	-------	------	-------	883.37	4.63011
1.94	------	-------	------	-------	------	-------	------	-------	909.71	4.66913
1.90	------	-------	709.44	4.02832	722.08	4.06624	763.63	4.19089	937.44	4.71232
1.86	------	-------	------	-------	------	-------	------	-------	966.84	4.76052
1.82	------	-------	------	-------	------	-------	------	-------	998.34	4.81502
1.80	736.02	4.00806	743.76	4.03128	757.90	4.07370	804.50	4.21350	------	-------
1.78	------	-------	------	-------	------	-------	------	-------	1,032.51	4.87753
1.74	------	-------	------	-------	------	-------	------	-------	1,070.19	4.95057
1.70	------	-------	778.20	4.03460	793.57	4.08071	846.52	4.23956	1,112.77	5.03831
1.68	------	-------	------	--------	------	-------	------	-------	1,136.60	5.08980
1.66	------	-------	------	-------	------	-------	------	-------	1,162.62	5.14786
1.64	------	-------	------	-------	------	-------	------	-------	1,191.53	5.21459
1.62	------	-------	------	-------	------	-------	------	-------	1,224.22	5.29266
1.60	803.28	4.00984	812.76	4.03828	829.78	4.08934	890.01	4.27003	1,262.30	5.38690

Table 13.--Summary of step-method computations of profiles, cross section 3--Continued

y	Q = 3.77 y_o = 0.404		Q = 7.37 y_o = 0.606		Q = 10.90 y_o = 0.773		Q = 18.10 y_o = 1.071		Q = 29.80 y_o = 1.512	
	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.
1.58	------	-------	------	-------	------	-------	------	-------	1,308.50	5.50550
1.57	------	-------	------	-------	------	-------	------	-------	1,336.38	5.57914
1.56	------	-------	------	-------	------	-------	907.91	4.28373	1,368.97	5.66691
1.55	------	-------	------	-------	------	-------	------	-------	1,408.23	5.77469
1.54	------	-------	------	-------	------	-------	------	-------	1,459.39	5.91817
1.53	------	-------	------	-------	------	-------	------	-------	1,531.39	6.12417
1.52712	------	-------	------	-------	------	-------	------	-------	1,559.67	6.20613
1.52	------	-------	------	-------	------	-------	926.15	4.29845	--------	-------
1.50	836.97	4.01091	847.53	4.04259	867.51	4.10253	------	-------	--------	-------
1.48	------	-------	------	-------	------	-------	945.35	4.31605	--------	-------
1.44	------	-------	------	-------	------	-------	964.71	4.33413	--------	-------
1.40	870.71	4.01213	882.50	4.04750	904.77	4.11431	984.93	4.35479	--------	-------
1.36	------	-------	------	-------	------	-------	1,006.24	4.37872	--------	-------
1.32	------	-------	------	-------	------	-------	1,028.99	4.40697	--------	-------
1.30	904.54	4.01362	917.87	4.05361	943.10	4.12930	--------	-------	--------	-------
1.28	------	-------	------	-------	------	-------	1,053.68	4.44104	--------	-------
1.26	------	-------	------	-------	958.82	4.13646	--------	-------	--------	-------
1.24	------	-------	------	-------	------	-------	1,081.17	4.48351	--------	-------
1.22	------	-------	------	-------	974.84	4.14452	1,096.42	4.50926	--------	-------
1.20	938.50	4.01550	953.80	4.06140	------	-------	1,113.05	4.53915	--------	-------
1.18	------	-------	------	-------	991.22	4.15366	1,131.54	4.57462	--------	-------
1.16	------	-------	968.37	4.06511	------	-------	1,152.58	4.61774	--------	-------
1.14	------	-------	------	-------	1,008.05	4.16415	1,177.59	4.67277	--------	-------
1.13	------	-------	------	-------	--------	-------	1,192.40	4.70720	--------	-------
1.12	------	-------	983.10	4.06930	--------	-------	1,209.42	4.74826	--------	-------
1.11	------	-------	------	-------	--------	-------	1,229.70	4.79910	--------	-------
1.10	972.64	4.01792	------	-------	1,025.46	4.17638	1,255.30	4.86590	--------	-------
1.09	------	-------	------	-------	--------	-------	1,290.27	4.96081	--------	-------
1.08171	------	-------	------	-------	--------	-------	1,335.20	5.08731	--------	-------
1.08	------	-------	998.02	4.07406	--------	-------	--------	-------	--------	-------

1.06	--------	-------	--------	-------	1,043.59	4.19077
1.04	--------	-------	1,013.16	4.07948	--------	-------
1.02	--------	-------	--------	-------	1,062.69	4.20807
1.00	1,007.05	4.02115	1,028.58	4.08574	--------	-------
.98	--------	-------	--------	-------	1,083.13	4.22939
.96	--------	-------	1,044.35	4.09305	--------	-------
.94	--------	-------	--------	-------	1,105.51	4.25652
.92	--------	-------	1,060.57	4.10171	1,117.73	4.27319
.90	1,041.89	4.02567	--------	-------	1,130.95	4.29285
.88	--------	-------	1,077.39	4.11217	1,145.49	4.31647
.86	--------	-------	--------	-------	1,161.88	4.34564
.84	--------	-------	1,095.01	4.12503	1,181.09	4.38327
.83	--------	-------	--------	-------	1,192.33	4.40699
.82	--------	-------	--------	-------	1,205.23	4.43569
.81	--------	-------	--------	-------	1,220.61	4.47183
.80	1,077.44	4.03232	1,113.78	4.14134	1,240.12	4.52036
.79	--------	-------	--------	-------	1,267.42	4.59226
.78073	--------	-------	--------	-------	1,309.47	4.70914
.76	1,091.95	4.03585	1,134.29	4.16287	--------	-------
.74	--------	-------	1,145.49	4.17647	--------	-------
.72	1,106.72	4.04016	1,157.62	4.19286	--------	-------
.70	--------	-------	1,171.04	4.21312	--------	-------
.68	1,121.83	4.04549	1,186.59	4.23977	--------	-------
.66	--------	-------	1,205.40	4.27620	--------	-------
.65	--------	-------	1,216.67	4.30001	--------	-------
.64	1,137.43	4.05229	1,230.20	4.33060	--------	-------
.63	--------	-------	1,247.46	4.37238	--------	-------
.62	--------	-------	1,272.39	4.43717	--------	-------
.61206	--------	-------	1,308.04	4.53618	--------	-------
.60	1,153.72	4.06116	--------	-------	--------	-------
.56	1,171.08	4.07324	--------	-------	--------	-------
.54	1,180.35	4.08105	--------	-------	--------	-------
.52	1,190.19	4.09057	--------	-------	--------	-------
.50	1,200.85	4.10255	--------	-------	--------	-------
.48	1,212.72	4.11816	--------	-------	--------	-------

Table 13.--Summary of step-method computations of profiles, cross section 3 --Continued

y	$Q = 3.77$, $y_o = 0.404$	
	L	Elev.
.46	1,226.57	4.13971
.45	1,234.71	4.15413
.44	1,244.25	4.17275
.43	1,256.03	4.19809
.42	1,272.12	4.23636
.41	1,300.29	4.31087
.40804	1,311.21	4.34167

Table 14.--Summary of step-method computation of profiles, cross section 4

y	Q = 3.79 y_o = 0.528		Q = 7.62 y_o = 0.780		Q = 12.19 y_o = 1.028		Q = 19.10 y_o = 1.369		Q = 20.15 y_o = 1.419		Q = 29.91 y_o = 1.877		Q = 39.98 y_o = 2.355	
	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.
4.00	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000
3.80	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	88.57	4.06571
3.60	133.64	4.00092	134.58	4.00374	136.59	4.00977	141.68	4.02504	142.70	4.02810	156.21	4.06863	181.43	4.14429
3.40	------	-------	------	-------	205.23	4.01569	213.46	4.04038	215.11	4.04533	237.27	4.11181	280.18	4.24054
3.30	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	332.48	4.29744
3.20	267.38	4.00214	269.56	4.00868	274.19	4.02257	286.10	4.05830	288.50	4.06550	321.22	4.16366	387.33	4.36199
3.10	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	445.41	4.43623
3.00	334.30	4.00290	337.25	4.01175	343.54	4.03062	359.85	4.07955	363.16	4.08948	409.10	4.22730	507.64	4.52292
2.90	------	-------	------	-------	------	-------	------	-------	------	-------	454.96	4.26488	575.40	4.62620
2.80	401.26	4.00378	405.12	4.01536	413.39	4.04017	435.05	4.10515	439.49	4.11847	502.51	4.30753	650.90	4.75270
2.76	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	684.05	4.81215
2.72	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	719.39	4.87817
2.70	------	-------	------	-------	------	-------	------	-------	------	-------	552.15	4.35645	------	-------
2.68	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	757.37	4.95211
2.64	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	798.69	5.03607
2.60	468.28	4.00484	472.89	4.01867	483.91	4.05173	512.22	4.13666	518.09	4.15427	604.45	4.41335	844.29	5.13287
2.56	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	895.55	5.24665
2.52	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	954.70	5.38410
2.50	------	-------	------	-------	------	-------	551.75	4.15525	558.50	4.17550	660.23	4.48069	988.41	5.46523
2.48	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	1,025.79	5.55737
2.46	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	1,067.93	5.66379
2.44	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	1,116.39	5.78917
2.42	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	1,173.83	5.94149
2.41	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	1,207.46	6.03238
2.40	535.37	4.00611	541.66	4.02498	555.31	4.06593	592.10	4.17630	599.87	4.19961	720.75	4.56225	1,245.56	6.13668
2.39	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	1,289.57	6.25871
2.38	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	1,341.98	6.40534
2.37	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	1,406.64	6.58992
2.36	------	-------	------	-------	------	-------	------	-------	------	-------	746.70	4.60010	1,491.66	6.83498
2.35835	------	-------	------	-------	------	-------	------	-------	------	-------	------	-------	1,508.91	6.88508

Table 14.--Summary of step-method computation of profiles, cross section 4--Continued

y	Q = 3.79, y_o = 0.528		Q = 7.62, y_o = 0.780		Q = 12.19, y_o = 1.028		Q = 19.10, y_o = 1.369		Q = 20.15, y_o = 1.419		Q = 29.91, y_o = 1.877	
	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.
2.32	------	-------	------	-------	------	-------	------	-------	------	-------	773.90	4.64170
2.30	------	-------	------	-------	591.43	4.07429	633.45	4.20035	642.43	4.22729	------	-------
2.28	------	-------	------	-------	------	-------	------	-------	------	-------	802.60	4.68780
2.24	------	-------	------	-------	------	-------	------	-------	------	-------	833.15	4.73945
2.20	602.56	4.00768	610.52	4.03156	627.92	4.08376	676.07	4.22821	686.50	4.25950	865.95	4.79785
2.16	------	-------	------	-------	------	-------	------	-------	------	-------	901.61	4.86483
2.12	------	-------	------	-------	------	-------	------	-------	------	-------	941.02	4.94306
2.10	------	-------	645.15	4.03545	664.85	4.09455	720.30	4.26090	732.53	4.29759	------	-------
2.08	------	-------	------	-------	------	-------	------	-------	------	-------	985.58	5.03674
2.06	------	-------	------	-------	------	-------	------	-------	------	-------	1,010.47	5.09141
2.04	------	-------	------	-------	------	-------	------	-------	------	-------	1,037.56	5.15268
2.02	------	-------	------	-------	------	-------	------	-------	------	-------	1,067.49	5.22247
2.00	669.89	4.00967	679.96	4.03988	702.32	4.10696	766.66	4.29998	779.48	4.33844	1,101.11	5.30333
1.98	------	-------	------	-------	------	-------	------	-------	------	-------	1,139.73	5.39919
1.96	------	-------	------	-------	------	-------	------	-------	799.88	4.35964	1,185.48	5.51644
1.94	------	-------	------	-------	------	-------	------	-------	------	-------	1,242.25	5.66675
1.92	------	-------	------	-------	------	-------	------	-------	820.95	4.38285	1,318.54	5.87562
1.90	------	-------	714.99	4.04497	740.47	4.12141	815.93	4.34779	------	-------	1,437.86	6.21358
1.89577	------	-------	------	-------	------	-------	------	-------	------	-------	1,476.67	6.32578
1.88	------	-------	------	-------	------	-------	------	-------	842.81	4.40843	--------	-------
1.86	------	-------	------	-------	------	-------	836.69	4.37007	------	-------	--------	-------
1.84	------	-------	------	-------	------	-------	------	-------	865.60	4.43680	--------	-------
1.82	------	-------	------	-------	------	-------	858.24	4.39472	------	-------	--------	-------
1.80	737.43	4.01229	750.30	4.05090	779.50	4.13850	------	-------	889.52	4.46856	--------	-------
1.78	------	-------	------	-------	------	-------	880.71	4.42213	------	-------	--------	-------
1.76	------	-------	------	-------	------	-------	------	-------	914.83	4.50449	--------	-------
1.74	------	-------	------	-------	------	-------	904.31	4.45293	------	-------	--------	-------
1.72	------	-------	------	-------	------	-------	------	-------	941.88	4.54564	--------	-------
1.70	771.30	4.01390	785.96	4.05788	819.69	4.15907	929.29	4.48787	------	-------	--------	-------
1.68	------	-------	------	-------	------	-------	------	-------	971.17	4.59351	--------	-------

1.66	------	-------	------	-------	------	-------	956.04	4.52812	--------	-------
1.64	------	-------	------	-------	------	-------	------	-------	1,003.49	4.65047
1.62	------	-------	------	-------	------	-------	985.05	4.57515	--------	-------
1.60	805.27	4.01581	822.06	4.06618	861.47	4.18441	------	-------	1,040.04	4.72012
1.58	------	-------	------	-------	------	-------	1,017.15	4.63145	1,060.52	4.76156
1.56	------	-------	------	-------	878.75	4.19625	1,034.75	4.66425	1,082.98	4.80894
1.54	------	-------	------	-------	------	-------	1,053.69	4.70107	1,108.04	4.86412
1.52	------	-------	------	-------	896.46	4.20938	1,074.22	4.74266	1,136.67	4.93001
1.50	839.37	4.01811	858.77	4.07631	------	-------	1,097.01	4.79103	1,170.46	5.01138
1.49	------	-------	------	-------	------	-------	--------	-------	1,190.14	5.06042
1.48	------	-------	------	-------	914.66	4.22398	1,122.65	4.84795	1,212.38	5.11714
1.47	------	-------	------	-------	------	-------	--------	-------	1,238.13	5.18439
1.46	------	-------	------	-------	------	-------	1,152.31	4.91693	1,268.86	5.26658
1.45	------	-------	------	-------	------	-------	--------	-------	1,307.48	5.37244
1.44	------	-------	------	-------	933.46	4.24038	1,187.91	5.00373	1,359.82	5.51946
1.43319	------	-------	------	-------	------	-------	--------	-------	1,411.50	5.66769
1.43	------	-------	------	-------	------	-------	1,209.13	5.05739	--------	-------
1.42	------	-------	------	-------	------	-------	1,233.67	5.12101	--------	-------
1.41	------	-------	------	-------	------	-------	1,262.92	5.19876	--------	-------
1.40	873.64	4.02092	896.31	4.08893	952.99	4.25897	1,299.59	5.29877	--------	-------
1.39	------	-------	------	-------	------	-------	1,349.47	5.43841	--------	-------
1.38269	------	-------	------	-------	------	-------	1,402.79	5.59106	--------	-------
1.36	------	-------	------	-------	973.43	4.28029	--------	-------	--------	-------
1.32	------	-------	------	-------	995.04	4.30512	--------	-------	--------	-------
1.30	908.14	4.02442	935.06	4.10518	------	-------	--------	-------	--------	-------
1.28	------	-------	------	-------	1,018.18	4.33454	--------	-------	--------	-------
1.26	------	-------	951.00	4.11300	--------	-------	--------	-------	--------	-------
1.24	------	-------	------	-------	1,043.39	4.37017	--------	-------	--------	-------
1.22	------	-------	967.29	4.12187	1,057.03	4.39109	--------	-------	--------	-------
1.20	942.97	4.02891	------	-------	1,071.58	4.41474	--------	-------	--------	-------

Table 14.--Summary of step-method computation of profiles, cross section 4--Continued

y	Q = 3.79, y_o = 0.528		Q = 7.62, y_o = 0.780		Q = 12.19, y_o = 1.028	
	L	Elev.	L	Elev.	L	Elev.
1.18	--------	-------	984.00	4.13200	1,087.26	4.44178
1.16	--------	-------	--------	-------	1,104.40	4.47320
1.14	--------	-------	1,001.23	4.14369	1,123.49	4.51047
1.12	--------	-------	--------	-------	1,145.34	4.55602
1.10	978.28	4.03484	1,019.13	4.15739	1,171.37	4.61411
1.09	--------	-------	--------	-------	1,186.69	4.65007
1.08	--------	-------	--------	-------	1,204.23	4.69269
1.07	--------	-------	--------	-------	1,225.04	4.74512
1.06	--------	-------	1,037.89	4.17367	1,250.81	4.81243
1.05	--------	-------	--------	-------	1,284.96	4.90488
1.04	--------	-------	--------	-------	1,337.79	5.05337
1.03828	--------	-------	--------	-------	1,351.45	5.09263
1.02	--------	-------	1,057.79	4.19337	--------	-------
1.00	1,014.36	4.04308	--------	-------	--------	-------
.98	--------	-------	1,079.31	4.21793	--------	-------
.96	1,029.09	4.04727	--------	-------	--------	-------
.94	--------	-------	1,103.19	4.24957	--------	-------
.92	1,044.06	4.05218	1,116.41	4.26923	--------	-------
.90	--------	-------	1,130.89	4.29267	--------	-------
.88	1,059.35	4.05805	1,147.12	4.32136	--------	-------
.86	--------	-------	1,165.83	4.35749	--------	-------
.84	1,075.06	4.06518	1,188.51	4.40553	--------	-------
.83	--------	-------	1,202.20	4.43660	--------	-------
.82	--------	-------	1,218.39	4.47517	--------	-------
.81	--------	-------	1,234.47	4.51341	--------	-------
.80	1,091.32	4.07396	1,261.74	4.58522	--------	-------
.79	--------	-------	1,306.01	4.70803	--------	-------
.7878	--------	-------	1,322.07	4.75401	--------	-------
.76	1,108.33	4.08499	--------	-------	--------	-------
.72	1.126.43	4.09929	--------	-------	--------	-------

.68	1,146.20	4.11860
.66	1,156.99	4.13097
.64	1,168.71	4.14613
.62	1,181.71	4.16513
.60	1,196.61	4.18983
.58	1,214.74	4.22422
.57	1,225.92	4.24776
.56	1,239.38	4.27814
.55	1,256.81	4.32043
.54	1,282.79	4.38837
.53328	1,329.32	4.52124

Table 15.--Summary of step method computation of profiles, cross section 5

y	Q = 0.34 y_o = 0.350		Q = 0.56 y_o = 0.350		Q = 0.79 y_o = 0.456		Q = 1.32 y_o = 0.687		Q = 1.84 y_o = 0.905		Q = 2.71 y_o = 1.158		Q = 3.87 y_o = 1.285		Q = 11.30 y_o = 1.704		Q = 31.28 y_o = 2.525	
	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.
4.00	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000	0	4.00000
3.80	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	78.37	4.03511
3.60	133.34	4.00002	133.34	4.00002	133.34	4.00002	133.35	4.00005	133.41	4.00023	133.49	4.00047	133.65	4.00095	133.69	4.00107	159.63	4.07889
3.40	------	------	------	------	------	------	------	------	------	------	200.26	4.00078	200.53	4.00159	202.20	4.00660	245.17	4.13551
3.30	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	290.19	4.17057
3.20	266.68	4.00004	266.68	4.00004	266.69	4.00007	266.74	4.00022	266.86	4.00058	267.05	4.00115	267.46	4.00238	271.17	4.01351	337.37	4.21211
3.10	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	387.44	4.26232
3.00	------	------	------	------	------	------	------	------	333.60	4.00080	333.88	4.00164	334.46	4.00338	340.78	4.02234	441.59	4.32477
2.96	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	464.77	4.35431
2.92	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	489.09	4.38727
2.88	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	514.79	4.42437
2.84	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	542.23	4.46669
2.80	400.02	4.00006	400.03	4.00009	400.06	4.00018	400.16	4.00048	400.36	4.00108	400.75	4.00225	401.55	4.00465	411.32	4.03396	571.86	4.51558
2.78	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	587.71	4.54313
2.76	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	604.40	4.57320
2.74	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	622.07	4.60621
2.72	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	640.94	4.64282
2.70	------	------	------	------	------	------	------	------	------	------	------	------	------	------	447.06	4.04118	661.27	4.68381
2.69	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	672.07	4.70621
2.68	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	683.40	4.73020
2.67	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	695.26	4.75578
2.66	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	707.77	4.78331
2.65	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	721.06	4.81318
2.64	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	735.22	4.84566
2.63	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	750.40	4.88120
2.62	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	766.83	4.92049
2.61	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	784.78	4.96434
2.60	466.69	4.00007	466.71	4.00013	466.75	4.00025	466.89	4.00067	467.15	4.00145	467.69	4.00307	468.78	4.00634	483.24	4.04972	804.69	5.01407
2.59	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	827.05	5.07115
2.58	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	852.60	5.13780
2.57	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	882.75	5.21825
2.56	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	919.69	5.31907
2.55025	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	966.25	5.44900
2.50	------	------	------	------	------	------	------	------	------	------	501.20	4.00360	------	------	519.98	4.05994	------	------
2.40	533.37	4.00011	533.39	4.00017	533.45	4.00035	533.65	4.00095	533.99	4.00197	534.74	4.00422	536.24	4.00872	557.47	4.07241	------	------

2.30	------	------	------	------	------	------	------	------	567.44	4.00232	568.32	4.00496	570.08	4.01024	595.98	4.08794
2.20	600.05	4.00015	600.08	4.00024	600.16	4.00048	600.45	4.00135	600.91	4.00273	601.95	4.00585	604.04	4.01212	635.95	4.10785
2.16	------	------	------	------	------	------	------	------	------	------	------	------	------	------	652.47	4.11741
2.12	------	------	------	------	------	------	------	------	------	------	------	------	------	------	669.40	4.12820
2.10	------	------	------	------	------	------	------	------	634.41	4.00323	635.66	4.00698	638.15	4.01445	------	------
2.08	------	------	------	------	------	------	------	------	------	------	------	------	------	------	686.83	4.14049
2.04	------	------	------	------	------	------	------	------	------	------	------	------	------	------	704.90	4.15470
2.00	666.73	4.00019	666.79	4.00037	666.90	4.00070	667.32	4.00196	667.96	4.00388	669.47	4.00841	672.48	4.01744	723.76	4.17128
1.96	------	------	------	------	------	------	------	------	------	------	------	------	------	------	743.67	4.19101
1.92	------	------	------	------	------	------	------	------	------	------	------	------	------	------	765.00	4.21500
1.90	------	------	------	------	------	------	------	------	701.58	4.00474	703.43	4.01029	707.13	4.02139	776.38	4.22914
1.88	------	------	------	------	------	------	------	------	------	------	------	------	------	------	788.35	4.24505
1.86	------	------	------	------	------	------	------	------	------	------	------	------	------	------	801.08	4.26324
1.84	------	------	------	------	------	------	------	------	------	------	------	------	------	------	814.78	4.28434
1.82	------	------	------	------	------	------	------	------	------	------	------	------	------	------	829.79	4.30937
1.80	733.42	4.00026	733.52	4.00056	733.69	4.00107	734.34	4.00302	735.30	4.00590	737.61	4.01283	742.26	4.02678	846.60	4.33980
1.79	------	------	------	------	------	------	------	------	------	------	------	------	------	------	855.92	4.35776
1.78	------	------	------	------	------	------	------	------	------	------	------	------	------	------	866.03	4.37809
1.77	------	------	------	------	------	------	------	------	------	------	------	------	------	------	877.18	4.40154
1.76	------	------	------	------	------	------	------	------	------	------	------	------	756.50	4.02950	889.69	4.42907
1.75	------	------	------	------	------	------	------	------	------	------	------	------	------	------	904.13	4.46239
1.74	------	------	------	------	------	------	------	------	------	------	------	------	------	------	921.42	4.50426
1.73	------	------	------	------	------	------	------	------	------	------	------	------	------	------	943.44	4.56032
1.72104	------	------	------	------	------	------	------	------	------	------	------	------	------	------	974.46	4.64442
1.72	------	------	------	------	------	------	------	------	------	------	------	------	770.89	4.03267	------	------
1.70	------	------	------	------	------	------	------	------	769.17	4.00751	772.16	4.01648	------	------	------	------
1.68	------	------	------	------	------	------	------	------	------	------	------	------	785.48	4.03644	------	------
1.66	------	------	------	------	------	------	------	------	------	------	786.12	4.01836	------	------	------	------
1.64	------	------	------	------	------	------	------	------	------	------	------	------	800.32	4.04096	------	------
1.62	------	------	------	------	------	------	------	------	------	------	800.20	4.02060	------	------	------	------
1.60	800.13	4.00039	800.32	4.00096	800.62	4.00186	801.74	4.00522	803.32	4.00996	------	------	815.49	4.04647	------	------
1.58	------	------	------	------	------	------	------	------	------	------	814.44	4.02332	------	------	------	------
1.56	------	------	------	------	------	------	------	------	------	------	------	------	831.11	4.05333	------	------
1.54	------	------	------	------	------	------	------	------	------	------	828.88	4.02664	------	------	------	------
1.52	------	------	------	------	------	------	------	------	------	------	------	------	847.35	4.06205	------	------
1.50	833.51	4.00053	833.77	4.00131	834.18	4.00254	835.73	4.00719	837.96	4.01388	843.58	4.03074	------	------	------	------
1.48	------	------	------	------	------	------	------	------	------	------	------	------	864.48	4.07344	------	------
1.46	------	------	------	------	------	------	------	------	------	------	858.65	4.03595	------	------	------	------
1.44	------	------	------	------	------	------	------	------	------	------	------	------	882.96	4.08888	------	------
1.42	------	------	------	------	------	------	------	------	------	------	874.24	4.04272	892.95	4.09885	------	------

Table 15.--Summary of step method computation of profiles, cross section 5--Continued

y	Q = 0.34 y_o = 0.250		Q = 0.56 y_o = 0.350		Q = 0.79 y_o = 0.456		Q = 1.32 y_o = 0.687		Q = 1.84 y_o = 0.905		Q = 2.71 y_o = 1.158		Q = 3.87 y_o = 1.285	
	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.	L	Elev.
1.40	866.91	4.00073	867.30	4.00190	867.90	4.00370	870.18	4.01054	873.58	4.02074	------	------	903.68	4.11104
1.38	------	------	------	------	------	------	------	------	------	------	890.59	4.05177	915.45	4.12635
1.36	------	------	------	------	------	------	------	------	------	------	899.15	4.05745	928.77	4.14631
1.34	------	------	------	------	------	------	------	------	------	------	908.07	4.06421	944.65	4.17395
1.33	------	------	------	------	------	------	------	------	------	------	------	------	954.20	4.19260
1.32	------	------	------	------	------	------	------	------	------	------	917.46	4.07238	965.52	4.21656
1.31	------	------	------	------	------	------	------	------	------	------	------	------	979.89	4.24967
1.30	900.37	4.00111	900.99	4.00297	901.95	4.00585	905.61	4.01683	911.36	4.03408	927.47	4.08241	1,000.37	4.30111
1.29785	------	------	------	------	------	------	------	------	------	------	------	------	1,008.19	4.32242
1.28	------	------	------	------	------	------	------	------	------	------	938.32	4.09496	------	------
1.26	913.78	4.00134	914.53	4.00359	915.70	4.00710	------	------	927.51	4.04253	950.31	4.11093	------	------
1.24	------	------	------	------	------	------	------	------	------	------	963.96	4.13188	------	------
1.23	------	------	------	------	------	------	------	------	------	------	971.68	4.14504	------	------
1.22	927.20	4.00160	928.11	4.00433	929.55	4.00865	------	------	944.62	4.05386	980.21	4.16063	------	------
1.21	------	------	------	------	------	------	------	------	------	------	989.81	4.17943	------	------
1.20	------	------	------	------	------	------	942.84	4.02852	------	------	1,000.95	4.20285	------	------
1.19	------	------	------	------	------	------	------	------	------	------	1,014.60	4.23380	------	------
1.18	940.64	4.00192	941.74	4.00522	943.50	4.01050	------	------	962.94	4.06882	1,032.67	4.27801	------	------
1.17	------	------	------	------	------	------	------	------	------	------	1,060.38	4.35114	------	------
1.16958	------	------	------	------	------	------	------	------	------	------	1,061.91	4.35531	------	------
1.14	954.11	4.00233	955.45	4.00635	957.61	4.01283	------	------	983.38	4.09014	------	------	------	------
1.10	967.61	4.00283	969.25	4.00775	971.93	4.01579	983.30	4.04990	1,007.36	4.12208	------	------	------	------
1.06	981.15	4.00345	983.18	4.00954	986.53	4.01959	1,001.26	4.06378	1,037.32	4.17196	------	------	------	------
1.04	987.94	4.00382	990.20	4.01060	993.95	4.02185	1,010.77	4.07231	1,055.78	4.20734	------	------	------	------
1.02	994.75	4.00425	997.27	4.01181	1,001.47	4.02441	1,020.76	4.08228	1,077.64	4.25292	------	------	------	------
1.01	------	------	------	------	------	------	1,025.96	4.08788	1,090.28	4.28084	------	------	------	------
1.00	1,001.57	4.00471	1,004.38	4.01314	1,009.09	4.02727	1,031.23	4.09369	1,103.94	4.31182	------	------	------	------
.99	------	------	------	------	------	------	------	------	1,118.58	4.34574	------	------	------	------
.98	------	------	------	------	------	------	------	------	1,134.75	4.38425	------	------	------	------
.97	------	------	------	------	------	------	------	------	1,152.89	4.42867	------	------	------	------
.96	------	------	------	------	------	------	1,053.12	4.11936	1,173.68	4.48104	------	------	------	------
.95	------	------	------	------	------	------	------	------	1,198.20	4.54460	------	------	------	------
.94	------	------	------	------	------	------	------	------	1,227.05	4.62115	------	------	------	------
.93	------	------	------	------	------	------	------	------	1,264.67	4.72401	------	------	------	------
.92	------	------	------	------	------	------	1,076.59	4.14977	1,319.66	4.87898	------	------	------	------
.91405	------	------	------	------	------	------	------	------	1,373.18	5.03359	------	------	------	------

.90	1,035.81	4.00743	1,040.31	4.02093	1,048.10	4.04430	--------	-------
.88	--------	-------	--------	-------	--------	-------	1,102.28	4.18684
.84	--------	-------	--------	-------	--------	-------	1,131.18	4.23354
.80	1,070.32	4.01096	1,077.09	4.03127	1,089.23	4.06769	1,165.30	4.29590
.78	--------	-------	--------	-------	--------	-------	1,185.77	4.33731
.76	--------	-------	--------	-------	--------	-------	1,210.06	4.39018
.74	--------	-------	--------	-------	--------	-------	1,239.90	4.45970
.73	--------	-------	--------	-------	--------	-------	1,258.48	4.50544
.72	--------	-------	--------	-------	--------	-------	1,281.46	4.56438
.71	--------	-------	--------	-------	--------	-------	1,312.26	4.64678
.70	1,105.26	4.01578	1,115.26	4.04578	1,134.19	4.10257	1,360.59	4.78177
.69387	--------	-------	--------	-------	--------	-------	1,407.72	4.91703
.60	1,140.90	4.02270	1,155.91	4.06773	1,187.35	4.16205	--------	-------
.56	1,155.44	4.02632	1,173.28	4.07984	1,213.46	4.20038	--------	-------
.52	1,170.23	4.03069	1,191.75	4.09525	1,246.36	4.25908	--------	-------
.50	1,177.75	4.03325	1,201.60	4.10480	1,268.67	4.30601	--------	-------
.49	--------	-------	--------	-------	1,282.98	4.33894	--------	-------
.48	1,185.37	4.03611	1,212.00	4.11600	1,300.54	4.38162	--------	-------
.47	1,189.23	4.03769	1,217.48	4.12244	1,326.50	4.44950	--------	-------
.46056	--------	-------	--------	-------	1,378.43	4.59585	--------	-------
.46	1,193.12	4.03936	1,223.22	4.12966	--------	-------	--------	-------
.45	1,197.05	4.04115	1,229.19	4.13757	--------	-------	--------	-------
.44	1,201.01	4.04303	1,235.40	4.14620	--------	-------	--------	-------
.43	1,205.01	4.04503	1,241.94	4.15582	--------	-------	--------	-------
.42	1,209.08	4.04724	1,249.12	4.16736	--------	-------	--------	-------
.41	1,213.21	4.04963	1,257.05	4.18115	--------	-------	--------	-------
.40	1,217.41	4.05223	1,265.83	4.19749	--------	-------	--------	-------
.39	1,221.67	4.05501	1,275.77	4.21731	--------	-------	--------	-------
.38	1,226.01	4.05803	1,287.47	4.24241	--------	-------	--------	-------
.37	1,230.47	4.06141	1,303.33	4.27999	--------	-------	--------	-------
.36	1,235.07	4.06521	1,330.08	4.35024	--------	-------	--------	-------
.35350	--------	-------	1,365.50	4.45000	--------	-------	--------	-------
.35	1,239.81	4.06943	--------	-------	--------	-------	--------	-------
.34	1,244.72	4.07416	--------	-------	--------	-------	--------	-------
.33	1,249.82	4.07946	--------	-------	--------	-------	--------	-------
.32	1,255.26	4.08578	--------	-------	--------	-------	--------	-------
.31	1,261.15	4.09345	--------	-------	--------	-------	--------	-------
.30	1,267.53	4.10259	--------	-------	--------	-------	--------	-------
.29	1,274.60	4.11380	--------	-------	--------	-------	--------	-------
.28	1,282.72	4.12816	--------	-------	--------	-------	--------	-------
.27	1,293.38	4.15014	--------	-------	--------	-------	--------	-------
.26	1,310.67	4.19201	--------	-------	--------	-------	--------	-------
.25250	1,338.46	4.26788	--------	-------	--------	-------	--------	-------

Table 16. --Summary of step method computations of profiles, cross section 6

y	Q = 0.83 y_o = 0.471		Q = 1.84 y_o = 0.905		Q = 3.98 y_o = 1.294		Q = 30.63 y_o = 2.431	
	L	Elev.	L	Elev.	L	Elev.	L	Elev.
4.00	0	4.00000	0	4.00000	0	4.00000	0	4.00000
3.80	------	-------	------	-------	------	-------	73.08	4.01924
3.60	133.35	4.00005	133.38	4.00014	133.55	4.00065	147.89	4.04367
3.40	------	-------	------	-------	------	-------	225.26	4.07578
3.30	------	-------	------	-------	------	-------	265.28	4.09584
3.20	266.70	4.00010	266.79	4.00037	267.23	4.00169	306.56	4.11968
3.10	------	-------	------	-------	------	-------	349.51	4.14853
3.00	------	-------	------	-------	334.14	4.00242	394.72	4.18416
2.90	------	-------	------	-------	------	-------	443.17	4.22951
2.86	------	-------	------	-------	------	-------	463.77	4.25131
2.82	------	-------	------	-------	------	-------	485.28	4.27584
2.80	400.06	4.00018	400.26	4.00078	401.14	4.00342	------	-------
2.78	------	-------	------	-------	------	-------	507.93	4.30379
2.74	------	-------	------	-------	------	-------	531.98	4.33594
2.70	------	-------	------	-------	------	-------	557.85	4.37355
2.68	------	-------	------	-------	------	-------	571.65	4.39495
2.66	------	-------	------	-------	------	-------	586.14	4.41842
2.64	------	-------	------	-------	------	-------	601.47	4.44441
2.62	------	-------	------	-------	------	-------	617.82	4.47346
2.60	466.75	4.00025	467.03	4.00109	468.28	4.00484	635.40	4.50620
2.58	------	-------	------	-------	------	-------	654.52	4.54356
2.57	------	-------	------	-------	------	-------	664.79	4.56437
2.56	------	-------	------	-------	------	-------	675.64	4.58692
2.55	------	-------	------	-------	------	-------	687.13	4.61139
2.54	------	-------	------	-------	------	-------	699.40	4.63820
2.53	------	-------	------	-------	------	-------	712.60	4.66780
2.52	------	-------	------	-------	------	-------	726.97	4.70091
2.51	------	-------	------	-------	------	-------	742.72	4.73816
2.50	------	-------	------	-------	------	-------	759.28	4.77784
2.49	------	-------	------	-------	------	-------	777.88	4.82364
2.48	------	-------	------	-------	------	-------	800.96	4.88288
2.47	------	-------	------	-------	------	-------	828.70	4.95610
2.46	------	-------	------	-------	------	-------	863.77	5.05131
2.45531	------	-------	------	-------	------	-------	884.65	5.10926
2.40	533.44	4.00032	533.84	4.00152	535.63	4.00689	------	-------
2.30	------	-------	------	-------	569.42	4.00826	------	-------
2.20	600.15	4.00045	600.73	4.00219	603.33	4.00999	------	-------
2.10	------	-------	------	-------	637.41	4.01223	------	-------
2.00	666.89	4.00067	667.76	4.00328	671.72	4.01516	------	-------
1.90	------	-------	------	-------	706.38	4.01914	------	-------
1.80	733.69	4.00107	735.10	4.00530	741.57	4.02471	------	-------
1.76	------	-------	------	-------	755.85	4.02755	------	-------
1.72	------	-------	------	-------	770.30	4.03090	------	-------
1.70	------	-------	768.97	4.00691	------	-------	------	-------
1.68	------	-------	------	-------	784.96	4.03488	------	-------
1.64	------	-------	------	-------	799.90	4.03970	------	-------
1.60	800.65	4.00195	803.12	4.00936	815.20	4.04560	------	-------
1.56	------	-------	------	-------	830.99	4.05297	------	-------
1.52	------	-------	------	-------	847.45	4.06235	------	-------
1.50	834.23	4.00269	837.76	4.01328	------	-------	------	-------

Table 16.--Summary of step method computation of profiles, cross section 6 --Continued

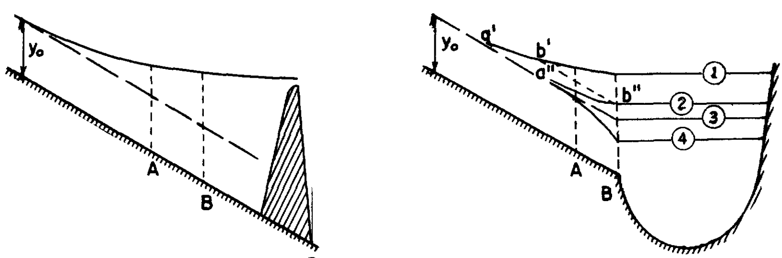
y	Q = 0.83 y_o = 0.471		Q = 1.84 y_o = 0.905		Q = 3.98 y_o = 1.294	
	L	Elev.	L	Elev.	L	Elev.
1.48	------	-------	------	-------	864.89	4.07467
1.46	------	-------	------	-------	874.12	4.08236
1.44	------	-------	------	-------	883.82	4.09146
1.42	------	-------	------	-------	894.15	4.10245
1.40	868.00	4.00400	873.38	4.02014	905.34	4.11602
1.38	------	-------	------	-------	917.76	4.13328
1.36	------	-------	------	-------	932.10	4.15630
1.35	------	-------	------	-------	940.35	4.17105
1.34	------	-------	------	-------	949.77	4.18931
1.33	------	-------	------	-------	960.87	4.21261
1.32	------	-------	------	-------	974.74	4.24422
1.31	------	-------	------	-------	994.12	4.29236
1.30694	------	-------	------	-------	1,002.51	4.31447
1.30	902.12	4.00636	911.16	4.03348	--------	-------
1.26	915.91	4.00773	927.31	4.04193	--------	-------
1.22	929.82	4.00946	944.42	4.05326	--------	-------
1.18	943.84	4.01152	962.74	4.06822	--------	-------
1.14	958.04	4.01412	983.18	4.08954	--------	-------
1.10	972.48	4.01744	1,007.16	4.12148	--------	-------
1.06	987.22	4.02166	1,037.10	4.17130	--------	-------
1.04	994.73	4.02419	1,055.69	4.20707	--------	-------
1.02	1,002.36	4.02708	1,077.55	4.25265	--------	-------
1.01	--------	-------	1,090.19	4.28057	--------	-------
1.00	1,010.09	4.03027	1,103.85	4.31155	--------	-------
.99	--------	-------	1,118.49	4.34547	--------	-------
.98	--------	-------	1,134.66	4.38398	--------	-------
.97	--------	-------	1,152.80	4.42840	--------	-------
.96	--------	-------	1,173.59	4.48077	--------	-------
.95	--------	-------	1,198.11	4.54433	--------	-------
.94	--------	-------	1,226.93	4.62079	--------	-------
.93	--------	-------	1,264.51	4.72353	--------	-------
.92	--------	-------	1,319.50	4.87850	--------	-------
.91405	--------	-------	1,373.02	5.03311	--------	-------
.90	1,049.83	4.04949	--------	-------	--------	-------
.80	1,092.02	4.07606	--------	-------	--------	-------
.70	1,138.76	4.11628	--------	-------	--------	-------
.60	1,195.75	4.18725	--------	-------	--------	-------
.56	1,225.18	4.23554	--------	-------	--------	-------
.52	1,265.19	4.31557	--------	-------	--------	-------
.51	1,279.52	4.34856	--------	-------	--------	-------
.50	1,297.96	4.39388	--------	-------	--------	-------
.49	1,322.65	4.45795	--------	-------	--------	-------
.48	1,361.95	4.56585	--------	-------	--------	-------
.47571	1,394.33	4.65870	--------	-------	--------	-------

STAGE-FALL-DISCHARGE RELATIONS FOR THE LABORATORY CHANNELS

DETERMINATION OF DISCHARGE UNDER BACKWATER

In the operation of most gaging stations, discharge is determined from stage, or elevation of the water surface. This is accomplished by use of rating curves, developed from an appropriate number of discharge measurements, for each of which both the discharge (the dependent variable) and the stage (the independent variable) have been observed. The curves presented as figure 6, pages 18 to 19, in Bulletin 381 (Lansford and Mitchell, 1949) become rating curves for the laboratory channels if the scale of depth, y, be considered as a scale of elevation.

The use of these curves assumes there is a fixed and invariable relation between stage and discharge. Such an assumption obviously is true for uniform flow (see pp. 11-13). It also is true for certain conditions of nonuniform flow, as, for example, those in which an M-1 surface profile is produced by nonuniformity of the channel, so that for a given stage and discharge the profile always will be the same. The rating curve for a channel upstream from a fixed spillway (see fig. 17 (left), below) is an example of such conditions. If a gage be located at any point such as *A* or *B*, the stage will be greater than for uniform flow; but as long as the discharge is constant, the stage at *A* or *B* will be constant and the simple stage-discharge relation effective. In discussions on stream gaging, it is quite common to designate this as the "normal" stage-discharge relation. It will be obvious that, although this relation may be fixed and invariable, it nevertheless may differ materially from the uniform-stage-discharge relation described on pages 11-14 , and which also is referred to by many writers as the "normal" stage-discharge relation. To avoid confusion between the two, the relationship determined by the conditions of figure 17 (left) will be designated as the fixed-backwater stage-discharge relation.



Fixed source of nonuniformity   Variable source of nonuniformity

Figure 17.—Variations of water surface under fixed discharge.

But the surface curve between gages A and B may result from other causes than a fixed spillway. Let it be supposed that the channel discharges into a reservoir, as in figure 17 (right), and that the elevation of the water surface in this reservoir is subject to variation throughout wide limits. Assuming that the discharge is the same as for figure 17 (left), and elevation 1 of the reservoir is such that the stage at B is the same as before, then the profile upstream from B will be the same as before. If the reservoir surface is lowered to elevation 2, then the point b' on the first profile will be transferred to b'' on the second profile. In fact, the profile between gages A and B will be identical in form with the curve $a'b'$. The stage will be reduced at both gages A and B, although the discharge remains constant. Hence, under such variable backwater, the stage-discharge relation at each point will be variable, rather than fixed. It will be noted, however, that the reduction in elevation at B will be greater than the corresponding change at A, so that the fall in the reach A-B will be increased. If the reservoir surface is further lowered to elevation 3 so that the depth at B is equal to y_0, the condition of uniform flow will be established. The depth at A also will be y_0, the water-surface slope will be parallel to the bed slope, and the uniform stage-discharge relation will be effective. This is the limiting condition of the problem as now considered. Lowering the reservoir surface to elevation 4, so the depth at B is less than y_0, will result in an M-2 surface profile in the reach A-B—a situation beyond the scope of the present study.

It was noted above that the variation in stage at points A and B was accompanied by a variation in fall, so that stage and fall may now be used as two independent variables for the determination of discharge. Reliable values of discharge could be more easily obtained by the use of stage and slope of the water surface, both at gage A. Unfortunately, the slope of the water surface at a point is not susceptible to observation under field conditions. It can only be approximated by use of the fall in a reach which, of course, may be transformed to the average slope in the reach by dividing by the length, in feet, between the gages. An examination of the difficulties which arise from this approximation, and of the methods for mitigating these difficulties represents the objective of the present study.

Referring again to figure 17 (right), it will be noted that the shifting of the profile a'-b' to the position a''-b'' is an application of the principle of rigidity, described in Bulletin 381 (p. 17) as follows:

> In a prismatic channel the shape of the backwater profile remains fixed as long as the discharge, the boundary conditions and the slope are fixed. Any change in the depth at which the water is flowing does not produce a change in the shape of the curve; it merely transfers that particular portion of the curve to a different part of the channel. As the depth of water is increased, the curve is displaced, rigidly, in the upstream direction, and another portion of the curve, characterized by a slope which is more nearly horizontal, is brought into place; as the depth of water is decreased the curve is displaced, rigidly, in the downstream direction and another portion of the curve, characterized by a slope which is more nearly that of the channel bed, is brought into place. In neither case has any portion of the curve been altered; only a different part of the same curve has been brought under observation.

Thus, means are now at hand for determination of an unlimited amount of data for studies of stage-fall-discharge relations in the laboratory channels. It is necessary only to decide what length of reach is to be investigated and to consider two gages, separated always by this length of reach, to be moved at will to any location on the channel. Thus, if it be desired to study a reach of 100 feet in length, the upper gage may be considered to be located at station 100 and the lower gage at station 0. From any of the profile tables (tables 1-5 or 12-16), the discharge and the elevation at the two stations may be determined. (Values in the profile tables are given at such intervals that straight line interpolation may be used when necessary.) Subtracting the elevation obtained on the lower gage from that obtained on the upper gage (tables 12-16), provides the value of the fall, and complete data are available which are equivalent to a field observation of stage, fall, and discharge. If, now, another set of data are desired under backwater conditions only slightly less severe, it may be considered that the upper gage is located at, say, station 130, and the lower gage at station 30. From the profile tables another pair of stages may be read, which result in slightly lower stage at the base gage and slightly more fall in the reach.

The technique just described can be applied either to the smoothed profiles, tables 1 to 5, or to the computed profiles, tables 12 to 16. Experience has demonstrated, however, that in the studies which are to follow, it is more desirable to use tables 12 to 16. Under great backwater effects the observed fall becomes extremely small, in many cases less than 0.0005 foot. (See, for example, the F_m column of table 24, p. 95 .) Tables 1 to 5, it will be recalled, were prepared from data observed only to thousandths of a foot, so that not infrequently falls computed from these tables will be in error by more than 0.0001 foot, thus leading to possible errors of more than 20 percent in the computation of fall ratios. Such errors lead to considerable scatter in subsequent plottings and thus tend to obscure the conclusions. The scatter from this cause is practically eliminated by the use of tables 12 to 16, in which elevations are computed to five decimal places. From the fact that the computed profiles are essentially the same as the smoothed profiles (differing largely only in degree of refinement) it follows that studies based on tables 12 to 16 will lead to essentially the same results as those based on tables 1 to 5. Thus, to prevent conclusions being obscured by scatter due to insufficient refinement in fall, the studies which follow have been based on the computed profiles, table 12 to 16.

In order that readers who wish to make further studies may have convenient access to data in usable form, selected simultaneous values of stage, fall, and discharge for a 400-foot reach of the computed profiles are presented as tables 33 to 37, pages 155 to 159. Appropriate values of y for each profile are listed, together with the discharge, in the tables. The following example will illustrate the computation of the associated falls. The fall, F_m, in cross section 2, as shown in table 33 for a depth of 1.60 feet and a discharge of 3.90 cfs, was computed from table 12, page 59, using a value for y_0 of 0.391 foot (Q of 3.90 cfs). The station and elevation for y of 1.60 feet are found to be 802.38 and 4.00714, respectively. Subtracting 400 feet, the length of the reach, from 802.38 gives 402.38, the station of the downstream end of the reach. From column 2 of the table using 402.38 and straight-line interpolation, the elevation of this station is found to be 4.00172. Subtracting this water-surface elevation from that at the upstream end of the reach gives 0.00542, the listed fall.

In working with field observations, the data which fail to support a particular analysis, or method of procedure, sometimes are viewed with suspicion. Their recording may have been affected by wind, or ice, or poor measuring conditions, or by errors in reading a gage. But for the data which may now be computed from tables 12 to 16, these excuses cannot be made. Values of discharge are the result of many observations which have been crosschecked and correlated. Values of stage are computed to the fifth decimal place. The analysis of these data by any given method becomes a test, not of the data, but of the method, as it applies to the particular set of channel conditions that existed in the laboratory. If the method gives acceptable results under the severe conditions of this test, it may be considered as generally applicable to other stage-fall-discharge problems. If it does not give acceptable results, it must be regarded as a method suitable only for conditions which are less severe than those used herein. With these thoughts in mind, the data have been computed and plotted by the several methods which follow.

RESUME OF CURRENT METHODS

THE STAGE-FALL-DISCHARGE DIAGRAM

This, perhaps the oldest of all the methods of analysis, is illustrated by figures 18 and 19, pages 82 and 83. The first of these is for cross section 2 (rectangular); the other is for cross section 5 (flood plain). Each has been prepared for a reach 400 feet in length. Since the bed slope is 0.003, the no-backwater fall (or fall for uniform flow) is 1.20 feet. The right-hand curves, which represent the uniform stage-discharge relations of tables 42 and 45, Bulletin 381 (Lansford and Mitchell, 1949), have been so labeled. Other solid lines represent the positions which would be occupied by the stage-discharge curves if the fall were fixed at the values shown. Preparation of these diagrams is quite simple. For example, in plotting figure 19, the needed values of stage, fall, and discharge are abstracted from table 33, page 155, until enough are obtained to permit the drawing of lines of equal fall.

In practice, it sometimes is necessary to develop such diagrams from comparatively few points. To obtain a systematic spacing of the lines, it is sometimes assumed that lines of equal fall should vary with the square-root relation. Curves developed on this assumption and based on the no-backwater stage-

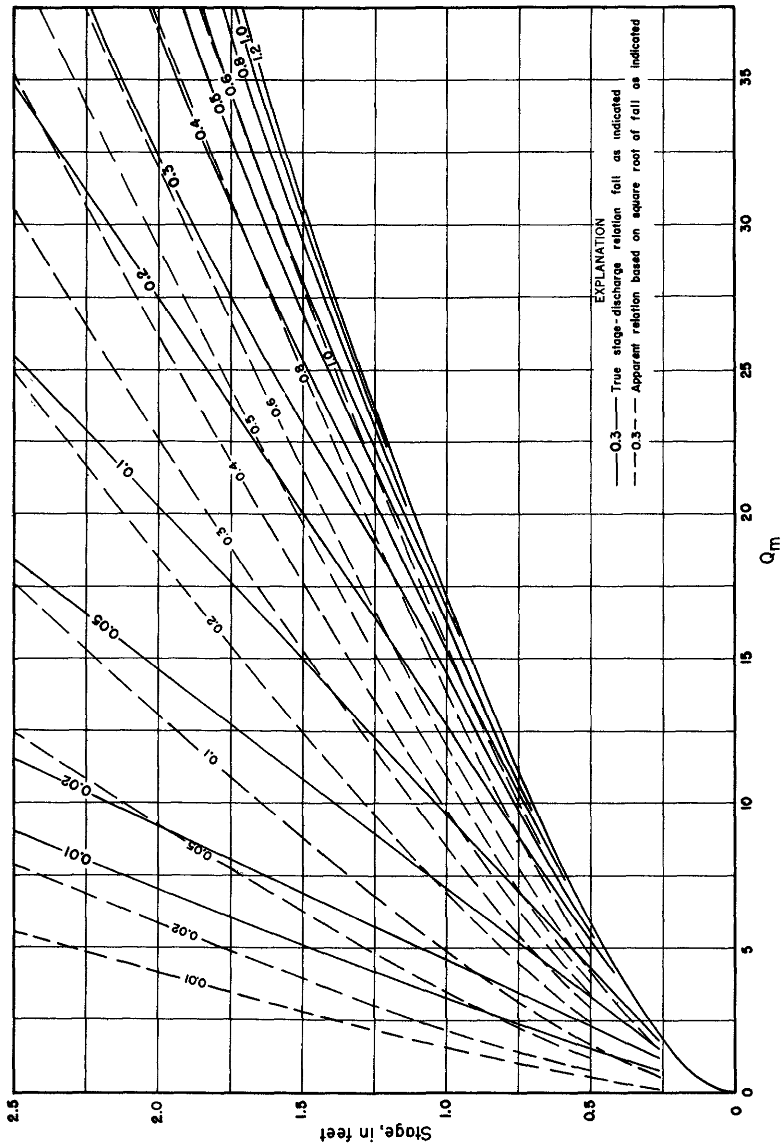


Figure 18. —Stage-fall-discharge diagram, cross section 2, 400-foot reach.

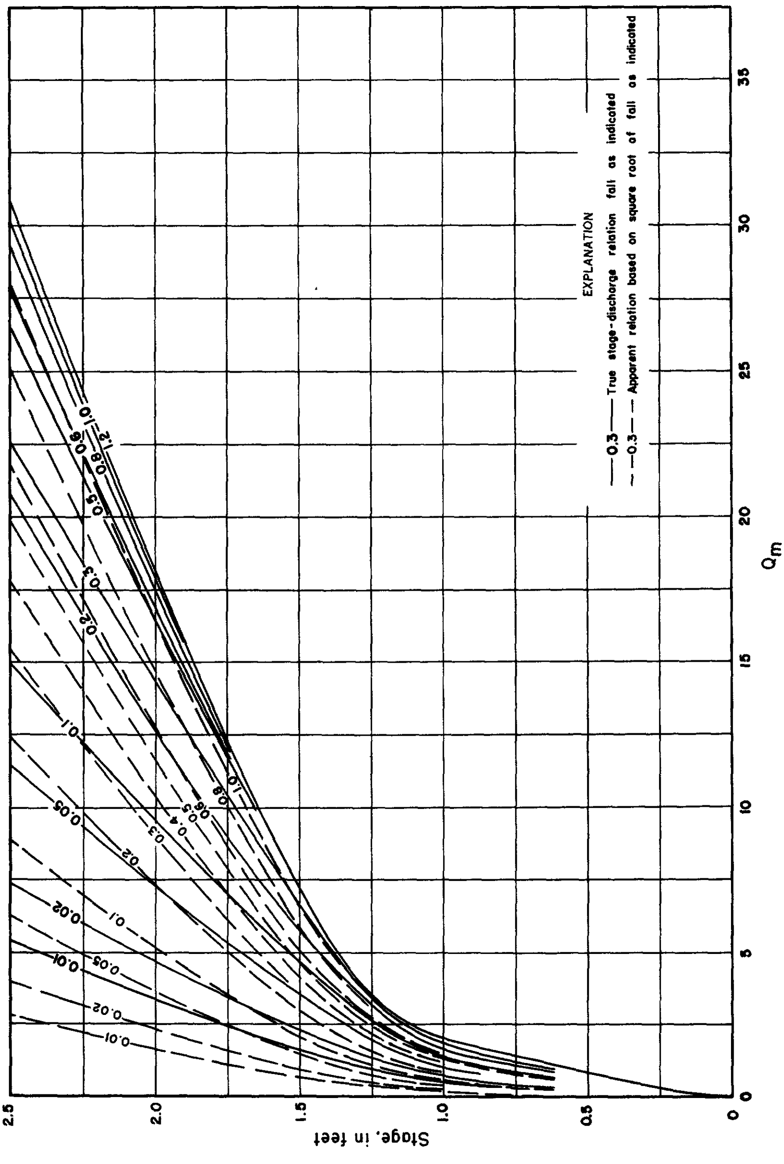


Figure 19.—Stage-fall-discharge diagram, cross section 5, 400-foot reach.

discharge relation appear as dashed lines in figures 18 and 19. The percentage differences are enormous and must lead immediately to the conclusion that, in this case at least, the true discharge does not vary from the no-backwater stage-discharge relation in accordance with the square-root-of-the-fall relation. In practice, of course, many of the discharge measurements, if not all, might have been obtained under backwater, and the curves would have been adjusted to fit these measurements. Such adjustments, properly applied, would tend to bring the diagram to the position of the solid lines, and with sufficient measurements the diagram might become an adequate expression of the stage-fall-discharge relations. Since, however, there is such sharp departure from the square-root-of-the-fall relation, caution must be used in the extrapolation of diagrams based on meager data.

THE BOYER METHOD

In U. S. Geological Survey Water-Supply Paper 888 (Corbett and others, 1943) it is pointed out (p. 132),
"if at a given stage two discharges Q_1 and Q_2 occur at different times with the corresponding energy slopes S_{e1} and S_{e2} their relation may be expressed by the equation [5]:

$$Q_1/Q_2 = S_{e1}{}^p/S_{e2}{}^p"$$

It then is explained that, for reasons as stated above, the fall in a reach is not a direct measure of the slope of the energy line, but
"the slope S_e as expressed in equation (5) may be considered as a function of the fall in a reach, so that the equation of relation becomes

$$Q_1/Q_2 = \text{function } F_1/F_2 \tag{6}$$

in which the falls are the difference in the heights of the water surface at the ends of the reach. If the velocity-head increment $\alpha V^2/2g$ is large enough to warrant consideration, it should be added to each observed fall. "

Now if Q_m and F_m be observed for a given backwater condition, and if Q_0 and F_0 be observed for conditions of no backwater, a pair of ratios may be obtained (Q_m/Q_0 and F_m/F_0). If the backwater condition be permitted to change, and another observation made of Q_m and F_m, a second pair of ratios becomes available. This procedure may be repeated to obtain as many pairs of ratios as are desired. Points may then be plotted, using the discharge ratios as ordinate and the fall ratios as abscissa; and the nature of the function as referred to above may be examined. Such a plot is commonly spoken of as a "Boyer diagram" in recognition of the development work of Mr. M. C. Boyer, at the time a hydraulic engineer of the U. S. Geological Survey.

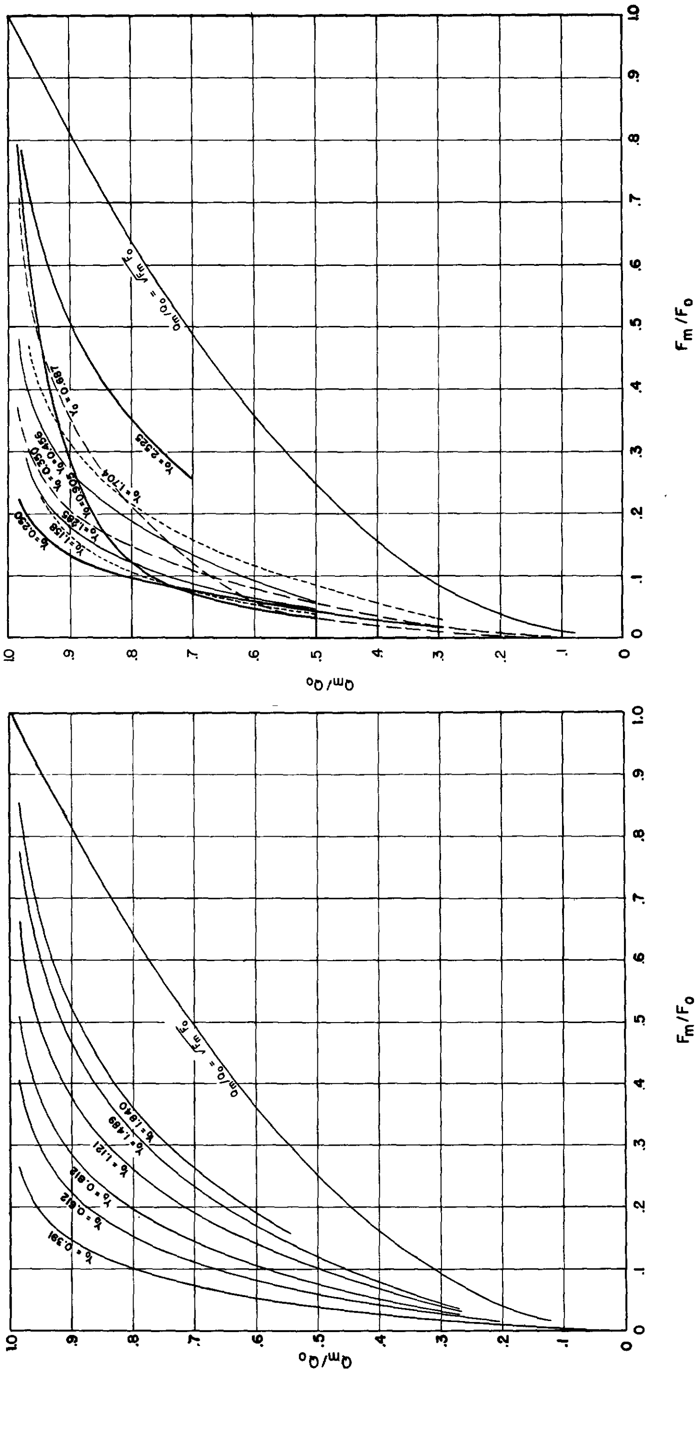


Figure 20. —Boyer diagram (all profiles) cross sections 2 (left) and 5 (right).

Figure 20 (left) is a Boyer diagram developed for the laboratory data, cross section 2, using a length of reach of 400 feet. It will be noted that the points plot, not as a single curve, but as a family of curves: a separate curve for each value of y_0 (or Q_m). It will also be noted that as the discharge (represented by value of y_0) increases, the curve of relation approaches but does not reach the position which would be represented by the relation $Q_m/Q_0 = \sqrt{F_m/F_0}$.

Figure 20 (right) presents a similar study of data for cross section 3. Again it is obvious that there is a curve for each value of y_0(or Q_m). But here the pattern of the curves is much more complex than for the rectangular channel; in fact, because of the braiding of the curves a general statement is extremely difficult. For the higher values of the discharge ratio, however, it may be noted that, beginning with the lowest discharge, the succeeding curves of low discharge, as plotted, lie progressively toward the right (toward the square-root-of-the-fall position). Thus, reading from left to right, we find y_0 values of 0.250, 0.350, 0.436, and 0.687. The condition of near-bankfull discharge at uniform depth, represented by the curve $y_0 = 0.903$, cuts erratically across the pattern, resembling the curves for high discharge at the upper end, but resembling the curves for low discharge at the lower end. The curve for the next higher value of y_0 (1.138, with flow on the flood plain) is quite similar to the curve for $y_0 = 0.250$, and succeedingly higher discharge curves (y_0 values of 1.283, 1.704 and 2.323) fall progressively further toward the right. Thus, for y_0 values which are entirely within the small rectangular channel, the pattern is similar to that for cross section 2, and for y_0 values which are above the flood plain another pattern somewhat similar to that for cross section 2 is superimposed upon the first. The entire pattern, however, is much too complicated to be developed from meager data such as might be collected in the field.

It is to be expected that the variation of the function, as shown in the first Boyer diagram, is due in substantial part to neglect of the velocity-head increment. To what extent this is true is shown by figure 21. Since the curves for discharges of 3.90 and 40.39 second-feet ($y_0 = 0.391$ and 1.840 ft) form an envelope for all the other curves developed for cross section 2, only these two have been shown in figure 21 (left). The curves shown in continuous lines show the range of variation of the function, with correction for velocity-head increment but still for a reach of 400 feet. Although the curves more nearly approach the square-root-of-the-fall relation, the variation still is quite large. Further improvement may now be obtained by shortening the length of reach, as indicated by the plotting of the curves shown in dashed lines further to the right on figure 21 (left). The length of reach has been taken as 120 feet, and correction has been made for velocity-head increment. It will be noted that these curves show less variation, and are still closer to the position represented by the square-root-of-the-fall relation. In fact, this relation appears to be the position which all these curves, when corrected for change in velocity head, approach as a limit as the reach is made shorter. This is due to the fact that, as the reach becomes shorter, the variation of fall in the reach becomes more representative of variation in slope at the base gage. That the square-root relation applies to slope at the base gage is demonstrated by the curve farthest to the right in this

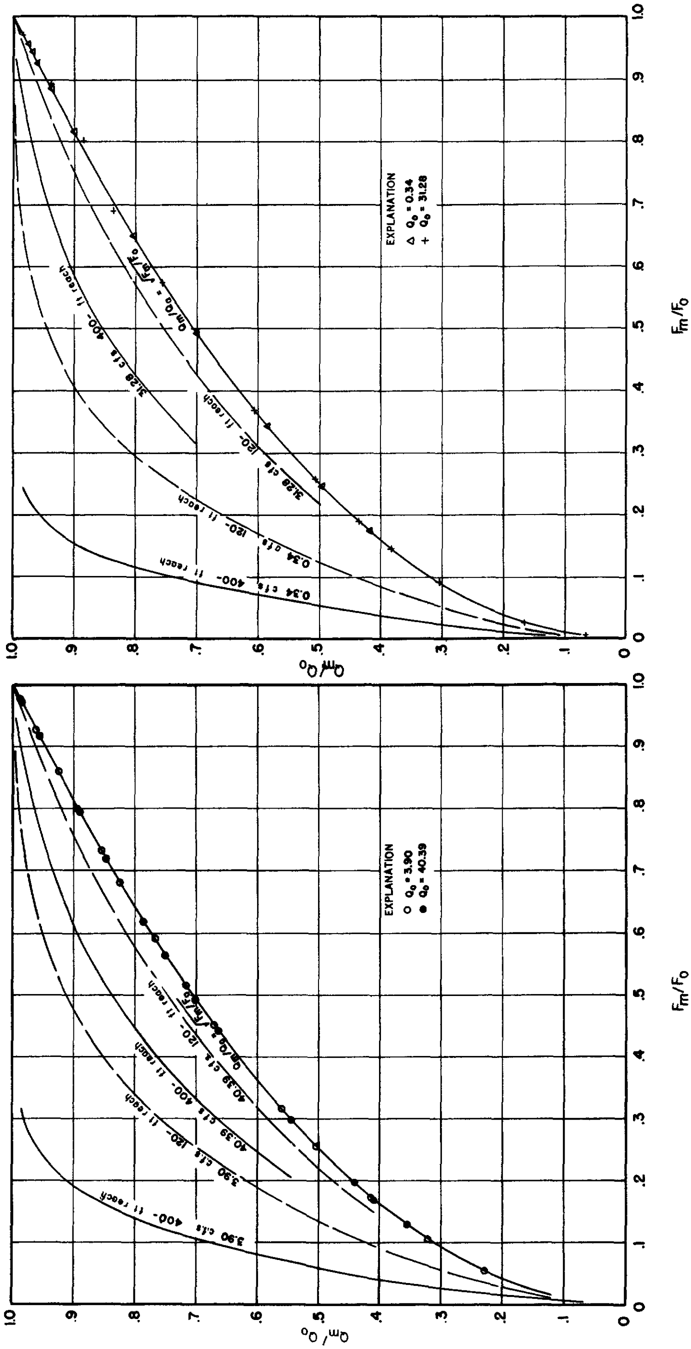


Figure 21.—Boyer diagram for various-length reaches, cross sections 2 (left) and 5 (right), with velocity-head correction.

figure. This curve has been drawn so that Q_m/Q_0 is equal to the square root of F_m/F_0, and data for the 3.90 and the 40.39 second-foot discharges, for a very short reach, and corrected for change in velocity head, have been plotted thereon. The only differences between the curve and the plotted points appear to be those which arise from lack of refinement of the computations. Figure 21 (right) presents similar studies of cross section 5 data.

The computations from which these plots were made are summarized in the tables which follow. Table 17, page 90, presents the data for the upper enveloping curve for cross section 2, 400-foot reach, while table 18, page 90, presents data for the lower enveloping curve for the same cross section and reach. Tables 19 and 20, page 91, are corresponding data for the same cross section, 120 foot reach. In these tables columns 5 and 6 give the discharge ratios and fall ratios used in plotting the enveloping curves of figure 20 (left). (The adjustments for velocity-head increment that were used in plotting figure 21 have not been shown in tables 17 to 20.) The data are derived in the following manner: For a given observed depth, y_m, and profile, Q_m, the values of fall for a 400-foot reach may be found in table 33, page 155. The computation of this fall was as follows: For $y_m = 2.00$, $y_0 = 0.391$ (Q_m = 3.90), columns 2 and 3 of table 12 give L = 668.13 and elev. = 4.00439. The L is entered in column 2 of table 17. Entering the same columns of table 12 with L = 668.13 - 400.00 = 268.13, you find, by interpolation, a water-surface elevation of 4.00098 feet. The difference between the two elevations, or 0.00341 feet is F_m, the observed fall for a reach of 400 feet as given in table 33. This is entered in column 3 of table 17. The uniform discharge, Q_0, for $y = 2.00$ ft, is found from table 42 of Bulletin 381 to be 45.18 cfs, which is entered in column 4 of table 17. Values of F_0, y_0 and Q_m are constant throughout table 17, and are as given in the table heading. Columns 5 to 10, inclusive, are computed as indicated at the top of the column. Computation of column 11, the final column, will be explained in a later section (see page 92). For the Boyer diagrams, only the results of columns 5 and 6 are required. The other columns will be used in connection with other types of diagrams.

To obtain all the curves of figure 20 (left), it was necessary to compute tables similar to table 17 for each of the profiles. For the four intermediate profiles (those between the highest and lowest discharges) the only values that were used were those for discharge ratio and fall ratio. These have been condensed into a single table, and presented as table 21, page 93.

A similar set of computations, parallel in all respects to those just described, was required for the plotting of figure 20 (right), the Boyer diagram for cross section 5. These are presented as tables 22 to 25, pages 94-95, which are comparable in form and content to tables 17 to 20, and as table 26, page 96-97, which is comparable to table 21.

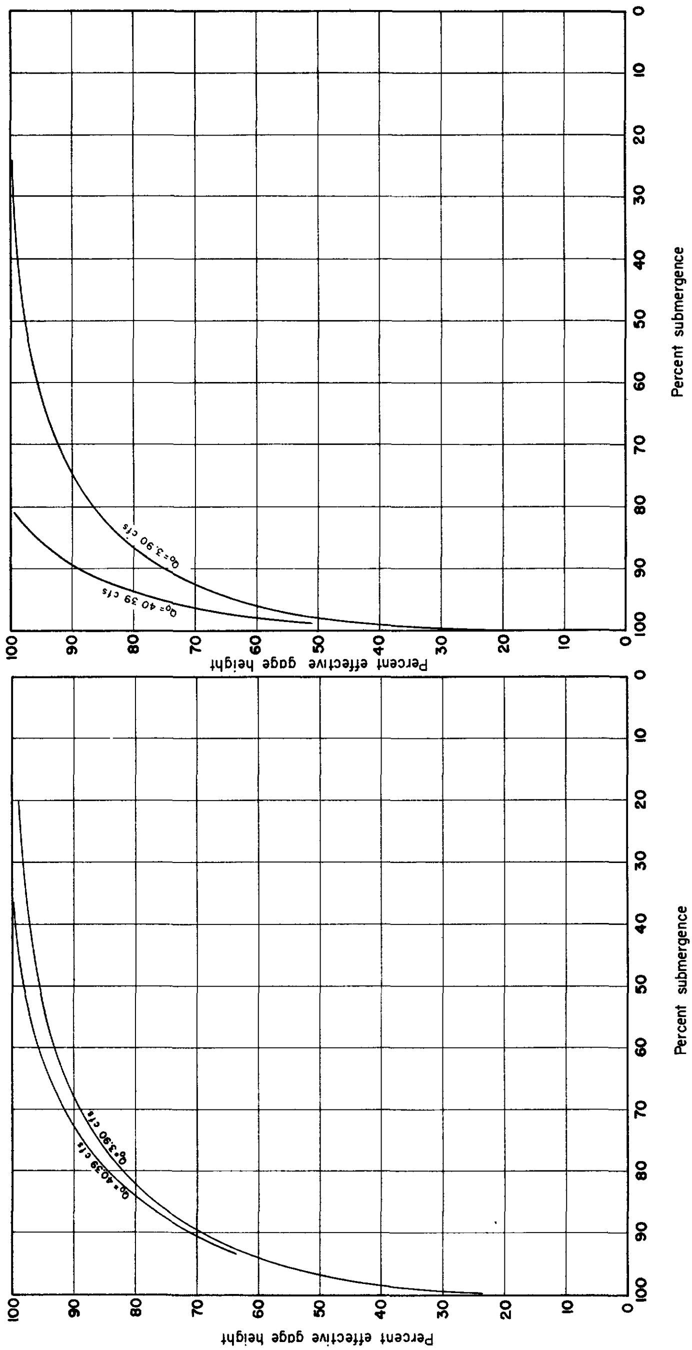


Figure 22. —Submergence diagrams, cross section 2, 400-foot reach (left) and 120-foot reach (right).

Table 17.—Computation of stage-fall-discharge ratios, cross section 2, 400-foot reach; $y_o = 0.391$ ft

[F_o = 1.200 ft; Q_m = 3.90 cfs]

y_m	L	F_m	Q_o	Q_m/Q_o	F_m/F_o	F_m/y_o	F_m/y_m	y_m/F_o	y_o/y_m	Percent submergence
2.50000	500.82	0.00215	60.83	0.0641	0.0018	0.0055	0.0009	2.0833	0.1564	99.91
2.00000	668.13	.00341	45.18	.0863	.0028	.0087	.0017	1.6667	.1955	99.83
1.80000	735.19	.00422	39.20	.0995	.0035	.0108	.0023	1.5000	.2172	99.77
1.60000	802.38	.00542	33.38	.1168	.0045	.0139	.0034	1.3333	.2444	99.66
1.40000	869.77	.00712	27.72	.1407	.0059	.0182	.0051	1.1667	.2793	99.49
1.20000	937.52	.00977	22.23	.1754	.0081	.0250	.0081	1.0000	.3258	99.19
1.10000	971.62	.01170	19.57	.1993	.0098	.0299	.0106	.9167	.3555	98.94
1.00000	1,005.97	.01436	16.98	.2297	.0120	.0367	.0144	.8333	.3910	98.56
.90000	1,040.71	.01811	14.48	.2693	.0151	.0463	.0201	.7500	.4344	97.99
.80000	1,076.10	.02377	12.09	.3226	.0198	.0608	.0297	.6667	.4888	97.03
.70000	1,112.60	.03263	9.84	.3963	.0272	.0835	.0466	.5833	.5586	95.34
.60000	1,151.29	.04792	7.74	.5039	.0399	.1226	.0799	.5000	.6517	92.01
.50000	1,196.03	.08110	5.81	.6713	.0676	.2074	.1622	.4167	.7820	83.78
.46000	1,218.80	.10878	5.08	.7677	.0906	.2782	.2365	.3833	.8500	76.35
.42000	1,252.19	.16788	4.38	.8904	.1399	.4294	.3997	.3500	.9310	60.03
.41000	1,266.02	.19888	4.21	.9264	.1657	.5086	.4851	.3417	.9537	51.49
.40000	1,287.88	.25355	4.05	.9630	.2113	.6485	.6339	.3333	.9775	36.61
.39491	1,310.51	.31530	3.96	.9848	.2628	.8064	.7984	.3291	.9901	20.16
.391	limit	1.200	3.90	1.0000	1.0000	3.0691	3.0691	.3258	1.0000	------

Table 18.—Computation of stage-fall-discharge ratios, cross section 2, 400-foot reach; $y_o = 1.840$ ft

[F_o = 1.200 ft; Q_m = 40.39 cfs]

y_m	L	F_m	Q_o	Q_m/Q_o	F_m/F_o	F_m/y_o	F_m/y_m	y_m/F_o	y_o/y_m	Percent submergence
2.90000	433.06	0.18789	74.06	0.5454	0.1566	0.1021	0.0648	2.4167	0.6345	93.52
2.50000	620.26	.28015	60.83	.6640	.2335	.1523	.1121	2.0833	.7360	88.79
2.28000	744.32	.36877	53.82	.7505	.3073	.2004	.1617	1.9000	.8070	83.83
2.20000	797.30	.41639	51.32	.7870	.3470	.2263	.1893	1.8333	.8364	81.07
2.08000	892.28	.51335	47.62	.8482	.4278	.2790	.2468	1.7333	.8846	75.32
2.00000	975.80	.61120	45.18	.8940	.5093	.3322	.3056	1.6667	.9200	69.44
1.98000	1,001.49	.64251	44.58	.9060	.5354	.3492	.3245	1.6500	.9293	67.55
1.91000	1,125.23	.79885	42.47	.9510	.6657	.4342	.4182	1.5917	.9634	58.18
1.90000	1,151.20	.83069	42.17	.9578	.6922	.4515	.4372	1.5833	.9684	56.28
1.89000	1,181.45	.86672	41.87	.9647	.7223	.4710	.4586	1.5750	.9735	54.14
1.88000	1,217.73	.90810	41.58	.9714	.7568	.4935	.4830	1.5667	.9787	51.70
1.87000	1,263.49	.95661	41.28	.9784	.7972	.5199	.5116	1.5583	.9840	48.84
1.86000	1,326.69	1.01557	40.98	.9856	.8463	.5519	.5460	1.5500	.9892	45.40
1.85840	1,339.83	1.02664	40.93	.9868	.8555	.5580	.5524	1.5487	.9901	44.76
1.840	limit	1.200	40.39	1.0000	1.0000	.6522	.6522	1.5333	1.0000	34.78

Table 19.—Computation of stage-fall discharge ratios, cross section 2, 120-foot reach; $y_o = 0.391$ ft

[F_o = 0.360 ft; Q_m = 3.90 cfs]

y_m	L	F_m	Q_o	Q_m/Q_o	F_m/F_o	F_m/y_o	F_m/y_m	y_m/F_o	y_o/y_m	Percent submergence
3.60000	133.47	0.00037	98.42	0.0396	0.0010	0.0009	0.0001	10.0000	0.1086	99.99
2.50000	500.82	.00086	60.83	.0641	.0024	.0022	.0003	6.9444	.1564	99.97
2.00000	668.13	.00149	45.18	.0863	.0041	.0038	.0007	5.5556	.1955	99.93
1.80000	735.19	.00190	39.20	.0995	.0053	.0049	.0011	5.0000	.2172	99.89
1.60000	802.38	.00250	33.38	.1168	.0069	.0064	.0016	4.4444	.2444	99.84
1.40000	869.77	.00340	27.72	.1407	.0094	.0087	.0024	3.8889	.2793	99.76
1.20000	937.52	.00498	22.23	.1754	.0138	.0127	.0042	3.3333	.3258	99.58
1.10000	971.62	.00619	19.57	.1993	.0172	.0158	.0056	3.0556	.3555	99.44
1.00000	1,005.97	.00790	16.98	.2297	.0219	.0202	.0079	2.7778	.3910	99.21
.90000	1,040.71	.01046	14.48	.2693	.0291	.0268	.0116	2.5000	.4344	98.84
.80000	1,076.10	.01449	12.09	.3226	.0402	.0371	.0181	2.2222	.4888	98.19
.70000	1,112.60	.02108	9.84	.3963	.0586	.0539	.0301	1.9444	.5586	96.99
.60000	1,151.29	.03288	7.74	.5039	.0913	.0841	.0548	1.6667	.6517	94.52
.56000	1,168.04	.04071	6.95	.5612	.1131	.1041	.0727	1.5556	.6982	92.73
.50000	1,196.03	.05980	5.81	.6713	.1661	.1529	.1196	1.3889	.7820	88.04
.46000	1,218.80	.08267	5.08	.7677	.2296	.2114	.1797	1.2778	.8500	82.03
.43000	1,241.73	.11436	4.56	.8553	.3177	.2925	.2660	1.1944	.9093	73.40
.42000	1,252.19	.13165	4.38	.8904	.3657	.3367	.3135	1.1667	.9310	68.65
.41000	1,266.02	.15674	4.21	.9264	.4354	.4009	.3823	1.1389	.9537	61.77
.40000	1,287.88	.19971	4.05	.9630	.5547	.5108	.4993	1.1111	.9775	50.07
.39491	1,310.51	.24371	3.96	.9848	.6770	.6233	.6172	1.0970	.9901	38.29
.391	limit	.360	3.90	1.0000	1.0000	.9207	.9207	1.0861	1.0000	7.93

Table 20.—Computation of stage-fall-discharge ratios, cross section 2, 120-foot reach; $y_o = 1.840$ ft

[F_o = 0.360 ft; Q_m = 40.39 cfs]

y_m	L	F_m	Q_o	Q_m/Q_o	F_m/F_o	F_m/y_o	F_m/y_m	y_m/F_o	y_o/y_m	Percent submergence
3.60000	150.46	0.04098	98.42	0.4104	0.1138	0.0223	0.0114	10.0000	0.5111	98.86
3.40000	227.95	.04699	91.31	.4423	.1305	.0255	.0138	9.4444	.5412	98.62
2.90000	433.06	.07314	4.06	.5454	.2032	.0398	.0252	8.0556	.6345	97.48
2.50000	620.26	.11087	70.83	.6640	.3080	.0603	.0444	6.9444	.7360	95.57
2.40000	673.73	.12538	57.62	.7010	.3483	.0681	.0522	6.6667	.7667	94.78
2.28000	744.32	.14759	53.82	.7505	.4100	.0802	.0647	6.3333	.8070	93.53
2.20000	797.30	.16632	51.32	.7870	.4620	.0904	.0756	6.1111	.8364	92.44
2.08000	892.28	.20325	47.62	.8482	.5646	.1105	.0977	5.7778	.8846	90.23
2.00000	975.80	.23747	45.18	.8940	.6596	.1291	.1187	5.5556	.9200	88.13
1.98000	1,001.49	.24749	44.58	.9060	.6875	.1345	.1250	5.5000	.9293	87.50
1.91000	1,125.23	.29260	42.47	.9510	.8128	.1590	.1532	5.3056	.9634	84.68
1.90000	1,151.20	.30058	42.17	.9578	.8349	.1634	.1582	5.2778	.9684	84.18
1.89000	1,181.45	.30893	41.87	.9647	.8581	.1679	.1635	5.2500	.9735	83.65
1.88000	1,217.73	.31765	41.58	.9714	.8824	.1726	.1690	5.2222	.9787	83.10
1.87000	1,263.49	.32703	41.28	.9784	.9084	.1777	.1749	5.1944	.9840	82.51
1.86000	1,326.69	.33696	40.98	.9856	.9360	.1831	.1812	5.1667	.9892	81.88
1.85840	1,339.83	.33886	40.93	.9868	.9413	.1842	.1823	5.1622	.9901	81.77
1.840	limit	.360	40.39	1.0000	1.0000	.1957	.1957	5.1111	1.0000	80.43

STAGE-RATIO AND SUBMERGENCE METHODS

The stage-ratio method is described in Water-Supply Paper 888 (Corbett and others, 1943) as having been used with some success at gaging stations in the Ohio River and Tennessee River basins. In this method, the ratio of the gage heights (to the same datum) at each end of the reach for each discharge measurement is determined and designated as the gage ratio. At the base gage, a second ratio is determined by dividing the effective gage height by observed gage height. This is designated as the stage ratio (the no-backwater gage height, y_0, divided by the observed gage height, y_m). Curves are developed of effective gage height plotted against discharge, and of stage ratio plotted against gage ratio. In use, the latter curve is entered with the gage ratio, and the stage ratio is found. With this and the known observed gage height, the effective gage height may be computed. Thereafter the discharge may be determined from the effective gage height-discharge curve. In this method, it appears that the datum used will have considerable bearing on the sensitivity, at least, of the results. Obviously, if mean sea-level datum, for example, is used for all the gages, all the ratios will be much nearer to unity and the sensitivity of the curves will be less than if the datum plane were near the channel bed.

In any event, the stage-ratio method appears to be in fact merely a less convenient version of the submergence method which has at times been used at several stations in Illinois. In this method, the ratio obtained by dividing the effective gage height by observed gage height (y_0/y_m) and multiplying by 100 is designated as "percent effective gage height" and is plotted against "percent submergence" to give a curve of relation. The submergence is obtained from readings of the gages at each end of the reach by following formula:

$$\text{Submergence} = \frac{\text{Elev. of WS at lower gage - Elev. bed at upper gage}}{\text{Elev. of WS at upper gage - Elev. bed at upper gage}}$$

This formula has been applied to compute column 11, the final column, of tables 17 to 20 and 22 to 25. Taking again as an example line 2 of table 17, it has been pointed out (page 88) that the water-surface elevation at the lower gage (station 268.13) is 4.00098 ft, and at the upper gage (station 668.13) 4.00439 ft. The elevation of the bed at the upper gage is S_0L = 0.003 x 668.13 ft = 2.00439 ft. Hence,

$$\text{submergence} = \frac{4.00098 - 2.00439}{4.00439 - 2.00439} = \frac{1.99659}{2.00000} = 0.9983$$

or 99.83 percent, the value which has been entered in column 11 of table 17.

Figure 22 (left), page 89, is a submergence relation curve developed from the data for cross section 2, using a length of reach of 400 feet. It will be noted that the points again develop a family of curves, although for this reach the variation appears to be somewhat smaller than in the Boyer diagram for the

Table 21.--Fall ratios and discharge ratios, cross section 2, 400 foot reach
[for profiles not shown in tables 17 and 18]

y_m	y_o = 0.612		y_o = 0.812		y_o = 1.121		y_o = 1.489	
	F_m/F_o	Q_m/Q_o	F_m/F_o	Q_m/Q_o	F_m/F_o	Q_m/Q_o	F_m/F_o	Q_m/Q_o
2.80000	------	------	------	------	------	------	0.0897	0.4274
2.50000	0.0075	0.1313	0.0183	0.2035	0.0496	0.3309	.1180	.4968
2.00000	.0120	.1768	.0293	.2740	.0816	.4456	.2104	.6689
1.80000	.0150	.2038	.0367	.3158	.1047	.5135	.2913	.7709
1.68000	------	------	------	------	------	------	.3766	.8467
1.60000	.0191	.2394	.0474	.3709	.1408	.6031	.4736	.9053
1.56000	------	------	------	------	------	------	.5527	.9373
1.54000	------	------	------	------	------	------	.6088	.9542
1.53000	------	------	------	------	------	------	.6449	.9630
1.52000	------	------	------	------	------	------	.6887	.9717
1.51000	------	------	------	------	------	------	.7448	.9805
1.50389	------	------	------	------	------	------	.7775	.9863
1.489	------	------	------	------	------	------	1.0000	1.0000
1.40000	.0253	.2882	.0640	.4466	.2059	.7262	------	------
1.30000	------	------	------	------	.2651	.8068	------	------
1.24000	------	------	------	------	.3232	.8632	------	------
1.20000	.0353	.3594	.0927	.5569	.3835	.9055	------	------
1.17000	------	------	------	------	.4547	.9393	------	------
1.15000	------	------	------	------	.5319	.9632	------	------
1.14000	------	------	------	------	.5925	.9758	------	------
1.13221	------	------	------	------	.6638	.9855	------	------
1.121	------	------	------	------	1.0000	1.0000	------	------
1.10000	.0431	.4083	.1174	.6326	------	------	------	------
1.00000	.0540	.4706	.1564	.7291	------	------	------	------
.94000	------	------	.1956	.7997	------	------	------	------
.90000	.0704	.5518	.2362	.8550	------	------	------	------
.87000	------	------	.2836	.8997	------	------	------	------
.85000	------	------	.3328	.9322	------	------	------	------
.84000	------	------	.3686	.9487	------	------	------	------
.83000	------	------	.4196	.9664	------	------	------	------
.82012	------	------	.5090	.9848	------	------	------	------
.812	------	------	1.0000	1.0000	------	------	------	------
.80000	.0989	.6609	------	------	------	------	------	------
.74000	.1273	.7453	------	------	------	------	------	------
.70000	.1580	.8120	------	------	------	------	------	------
.68000	.1802	.8491	------	------	------	------	------	------
.66000	.2114	.8898	------	------	------	------	------	------
.65000	.2329	.9111	------	------	------	------	------	------
.64000	.2616	.9334	------	------	------	------	------	------
.63000	.3033	.9557	------	------	------	------	------	------
.62000	.3782	.9804	------	------	------	------	------	------
.61812	.4046	.9850	------	------	------	------	------	------
.612	1.0000	1.0000	------	------	------	------	------	------

Table 22.--Computation of stage-fall-discharge ratios, cross section 5, 400-foot reach; $y_o = 0.250$ ft

[F_o = 1.200 ft; Q_m = 0.34 cfs]

y_m	L	F_m	Q_o	Q_m/Q_o	F_m/F_o	F_m/y_o	F_m/y_m	y_m/F_o	y_o/y_m	Percent submergence
2.50000	500.03	0.00008	30.61	0.0111	0.0001	0.0003	0	2.0833	0.1000	100.00
2.00000	666.73	.00015	18.01	.0189	.0001	.0006	.0001	1.6667	.1250	99.99
1.80000	733.42	.00021	13.41	.0254	.0002	.0008	.0001	1.5000	.1389	99.99
1.60000	800.13	.00033	9.15	.0372	.0003	.0013	.0002	1.3333	.1562	99.98
1.40000	866.91	.00066	5.53	.0615	.0006	.0026	.0005	1.1667	.1786	99.95
1.20000	933.92	.00165	3.01	.1130	.0014	.0066	.0014	1.0000	.2083	99.86
1.10000	967.61	.00270	2.39	.1423	.0022	.0108	.0025	.9167	.2273	99.75
1.00000	1,001.57	.00456	2.07	.1643	.0038	.0182	.0046	.8333	.2500	99.54
.90000	1,035.81	.00726	1.83	.1858	.0060	.0290	.0081	.7500	.2778	99.19
.80000	1,070.32	.01077	1.59	.2138	.0090	.0431	.0135	.6667	.3125	98.65
.70000	1,105.26	.01555	1.35	.2519	.0130	.0622	.0222	.5833	.3571	97.78
.60000	1,140.90	.02243	1.12	.3036	.0187	.0897	.0374	.5000	.4167	96.26
.50000	1,177.75	.03291	.89	.3820	.0274	.1316	.0658	.4167	.5000	93.42
.40000	1,217.41	.05177	.67	.5075	.0431	.2071	.1294	.3333	.6250	87.06
.30000	1,267.53	.10185	.45	.7556	.0849	.4074	.3395	.2500	.8333	66.05
.28000	1,282.72	.12725	.41	.8293	.1060	.5090	.4545	.2333	.8929	54.55
.27000	1,293.38	.14911	.38	.8947	.1243	.5964	.5523	.2250	.9259	44.77
.26000	1,310.67	.19072	.36	.9444	.1589	.7629	.7335	.2167	.9615	26.65
.25250	1,338.46	.26601	.35	.9714	.2217	1.0640	1.0535	.2104	.9901	------
.250	limit	1.200	.34	1.0000	1.0000	4.8000	4.8000	.2083	1.0000	------

Table 23.--Computation of stage-fall-discharge ratios, cross section 5, 400-foot reach; $y_o = 2.525$ ft

[F_o = 1.200 ft; Q_m = 31.28 cfs]

y_m	L	F_m	Q_o	Q_m/Q_o	F_m/F_o	F_m/y_o	F_m/y_m	y_m/F_o	y_o/y_m	Percent submergence
3.00000	441.59	0.30614	44.61	0.7012	0.2551	0.1212	0.1020	2.5000	0.8417	89.80
2.88000	514.79	.36964	41.14	.7603	.3080	.1464	.1283	2.4000	.8767	87.17
2.80000	571.86	.42859	38.86	.8049	.3572	.1697	.1531	2.3333	.9018	84.69
2.72000	640.94	.51011	36.62	.8542	.4251	.2020	.1875	2.2667	.9283	81.25
2.65000	721.06	.61543	34.68	.9020	.5129	.2437	.2322	2.2083	.9528	76.78
2.61000	784.78	.70469	33.58	.9315	.5872	.2791	.2700	2.1750	.9674	73.00
2.59000	827.05	.76315	33.04	.9467	.6360	.3022	.2947	2.1583	.9749	70.53
2.57000	882.75	.83957	32.50	.9625	.6996	.3325	.3267	2.1417	.9825	67.33
2.56000	919.69	.88714	32.23	.9705	.7393	.3513	.3465	2.1333	.9863	65.35
2.55025	966.25	.94268	31.97	.9784	.7856	.3733	.3696	2.1252	.9901	63.04
2.525	limit	1.200	31.28	1.0000	1.0000	.4752	.4752	2.1042	1.0000	52.48

Table 24.—Computation of stage-fall-discharge ratios, cross section 5, 120-foot reach; $y_o = 0.250$ ft

[F_o = 0.360 ft; Q_m = 0.34 cfs]

y_m	L	F_m	Q_o	Q_m/Q_o	F_m/F_o	F_m/y_o	F_m/y_m	y_m/F_o	y_o/y_m	Percent submergence
2.80000	400.02	0.00002	38.86	0.0087	0.0001	0.0001	0	7.7778	0.0893	100.00
2.50000	500.03	.00004	30.61	.0111	.0001	.0002	0	6.9444	.1000	100.00
2.00000	666.73	.00007	18.01	.0189	.0002	.0003	0	5.5556	.1250	100.00
1.80000	733.42	.00010	13.41	.0254	.0003	.0004	.0001	5.0000	.1389	99.99
1.60000	800.13	.00019	9.15	.0372	.0005	.0008	.0001	4.4444	.1562	99.99
1.40000	866.91	.00044	5.53	.0615	.0012	.0018	.0003	3.8889	.1786	99.97
1.20000	933.92	.00131	3.01	.1130	.0036	.0052	.0011	3.3333	.2083	99.89
1.10000	967.61	.00222	2.39	.1423	.0062	.0089	.0020	3.0556	.2273	99.80
1.00000	1,001.57	.00381	2.07	.1643	.0106	.0152	.0038	2.7778	.2500	99.62
.90000	1,035.81	.00605	1.83	.1858	.0168	.0242	.0067	2.5000	.2778	99.33
.80000	1,070.32	.00875	1.59	.2138	.0243	.0350	.0109	2.2222	.3125	98.91
.70000	1,105.26	.01211	1.35	.2519	.0336	.0484	.0173	1.9444	.3571	98.27
.60000	1,140.90	.01646	1.12	.3036	.0457	.0658	.0274	1.6667	.4167	97.26
.50000	1,177.75	.02358	.89	.3820	.0655	.0943	.0472	1.3889	.5000	95.28
.45000	1,197.05	.02926	.78	.4359	.0813	.1170	.0650	1.2500	.5556	93.50
.40000	1,217.41	.03753	.67	.5075	.1042	.1501	.0938	1.1111	.6250	90.62
.35000	1,239.81	.05082	.56	.6071	.1412	.2033	.1452	.9722	.7143	85.48
.30000	1,267.53	.07824	.45	.7556	.2173	.3130	.2608	.8333	.8333	73.92
.28000	1,282.72	.09969	.41	.8293	.2769	.3988	.3560	.7778	.8929	64.40
.27000	1,293.38	.11838	.38	.8947	.3288	.4735	.4384	.7500	.9259	56.16
.26000	1,310.67	.15370	.36	.9444	.4269	.6148	.5912	.7222	.9615	40.88
.25250	1,338.46	.21496	.35	.9714	.5971	.8598	.8513	.7014	.9901	14.87
.250	limit	.360	.34	1.0000	1.0000	1.4400	1.4400	.6944	1.0000	------

Table 25.—Computation of stage-fall-discharge ratios, cross section 5, 120-foot reach; $y_o = 2.525$ ft

[F_o = 0.360 ft; Q_m = 31.28 cfs]

y_m	L	F_m	Q_o	Q_m/Q_o	F_m/F_o	F_m/y_o	F_m/y_m	y_m/F_o	y_o/y_m	Percent submergence
3.60000	159.63	0.06114	62.80	0.4981	0.1698	0.0242	0.0170	10.0000	0.7014	98.30
3.30000	290.19	.08469	53.57	.5839	.2352	.0335	.0257	9.1667	.7652	97.43
3.00000	441.59	.12655	44.61	.7012	.3515	.0501	.0422	8.3333	.8417	95.78
2.88000	514.79	.15357	41.14	.7603	.4266	.0608	.0533	8.0000	.8767	94.67
2.80000	571.86	.17772	38.86	.8049	.4937	.0704	.0635	7.7778	.9018	93.65
2.72000	640.94	.20896	36.62	.8542	.5804	.0828	.0768	7.5556	.9283	92.32
2.65000	721.06	.24600	34.68	.9020	.6833	.0974	.0928	7.3611	.9528	90.72
2.61000	784.78	.27325	33.58	.9315	.7590	.1082	.1047	7.2500	.9674	89.53
2.60000	804.69	.28109	33.31	.9391	.7808	.1113	.1081	7.2222	.9712	89.19
2.59000	827.05	.28942	33.04	.9467	.8039	.1146	.1117	7.1944	.9749	88.83
2.58000	852.60	.29815	32.77	.9545	.8282	.1181	.1156	7.1667	.9787	88.44
2.57000	882.75	.30752	32.50	.9625	.8542	.1218	.1197	7.1389	.9825	88.03
2.56000	919.69	.31749	32.23	.9705	.8819	.1257	.1240	7.1111	.9863	87.60
2.55025	966.25	.32776	31.97	.9784	.9104	.1298	.1285	7.0840	.9901	87.15
2.525	limit	.360	31.28	1.0000	1.0000	.1426	.1426	7.0139	1.0000	85.74

Table 26.--Fall ratios and discharge ratios, cross section 5, 400-foot reach [for profiles not shown in tables 22 and 23]

y_m	y_o = 0.350		y_o = 0.456		y_o = 0.687		y_o = 0.905	
	F_m/F_o	Q_m/Q_o	F_m/F_o	Q_m/Q_o	F_m/F_o	Q_m/Q_o	F_m/F_o	Q_m/Q_o
2.50000	0.0001	0.0183	0.0002	0.0258	0.0006	0.0431	0.0013	0.0601
2.00000	.0003	.0311	.0005	.0439	.0014	.0733	.0028	.1022
1.80000	.0004	.0418	.0008	.0589	.0022	.0984	.0042	.1372
1.60000	.0007	.0612	.0014	.0863	.0040	.1443	.0074	.2011
1.40000	.0015	.1013	.0029	.1429	.0082	.2387	.0160	.3327
1.30000	------	------	------	------	------	------	.0269	.4532
1.20000	.0038	.1860	.0077	.2625	.0229	.4385	.0493	.6113
1.10000	.0063	.2343	.0128	.3305	.0405	.5523	.0994	.7699
1.05000	------	------	------	------	------	------	.1552	.8364
1.00000	.0107	.2705	.0223	.3816	.0767	.6377	.2558	.8889
.96000	------	------	------	------	------	------	.3943	.9340
.94000	------	------	------	------	------	------	.5071	.9534
.93000	------	------	------	------	------	------	.5875	.9684
.92000	------	------	------	------	------	------	.7005	.9787
.91405	------	------	------	------	------	------	.7951	.9892
.905	------	------	------	------	------	------	1.0000	1.0000
.90000	.0172	.3060	.0364	.4317	.1383	.7213	------	------
.80000	.0257	.3522	.0557	.4969	.2432	.8302	------	------
.75000	------	------	------	------	.3486	.8980	------	------
.73000	------	------	------	------	.4134	.9296	------	------
.71000	------	------	------	------	.5232	.9635	------	------
.70000	.0377	.4148	.0846	.5852	.6199	.9778	------	------
.69387	------	------	------	------	.7062	.9851	------	------
.687	------	------	------	------	1.0000	1.0000	------	------
.60000	.0559	.5000	.1336	.7054	------	------	------	------
.50000	.0865	.6292	.2519	.8876	------	------	------	------
.48000	------	------	.3132	.9294	------	------	------	------
.47000	------	------	.3677	.9634	------	------	------	------
.46056	------	------	.4820	.9875	------	------	------	------
.456	------	------	1.0000	1.0000	------	------	------	------
.44000	.1207	.7368	------	------	------	------	------	------
.40000	.1630	.8358	------	------	------	------	------	------
.38000	.1999	.8889	------	------	------	------	------	------
.37000	.2308	.9333	------	------	------	------	------	------
.36000	.2881	.9655	------	------	------	------	------	------
.35350	.3689	.9825	------	------	------	------	------	------
.350	1.0000	1.0000	------	------	------	------	------	------

Table 26.--Fall ratios and discharge ratios, cross section 5, 400-foot reach--Continued

y_m	y_o = 1.158		y_o = 1.285		y_o = 1.704	
	F_m/F_o	Q_m/Q_o	F_m/F_o	Q_m/Q_o	F_m/F_o	Q_m/Q_o
2.80000	------	------	------	------	0.0282	0.2908
2.50000	0.0027	0.0885	0.0057	0.1264	.0492	.3692
2.20000	------	------	------	------	.0816	.4941
2.00000	.0060	.1505	.0125	.2149	.1259	.6274
1.90000	------	------	------	------	.1674	.7211
1.84000	------	------	------	------	.2081	.7897
1.80000	.0093	.2021	.0194	.2886	.2489	.8427
1.76000	------	------	------	------	.3146	.9018
1.75000	------	------	------	------	.3390	.9180
1.74000	------	------	------	------	.3699	.9347
1.73000	------	------	------	------	.4105	.9520
1.72104	------	------	------	------	.4710	.9683
1.704	------	------	------	------	1.0000	1.0000
1.60000	.0164	.2962	.0346	.4230	------	------
1.52000	------	------	.0469	.5092	------	------
1.44000	------	------	.0684	.6252	------	------
1.40000	.0366	.4901	.0862	.6998	------	------
1.36000	------	------	.1149	.7882	------	------
1.34000	------	------	.1374	.8395	------	------
1.32000	.0571	.6259	.1721	.8938	------	------
1.31000	------	------	.1991	.9236	------	------
1.30000	------	------	.2410	.9532	------	------
1.29785	------	------	.2583	.9603	------	------
1.285	------	------	1.0000	1.0000	------	------
1.26000	.0886	.7570	------	------	------	------
1.22000	.1295	.8522	------	------	------	------
1.20000	.1642	.9003	------	------	------	------
1.19000	.1896	.9249	------	------	------	------
1.18000	.2259	.9476	------	------	------	------
1.17000	.2859	.9713	------	------	------	------
1.16958	.2893	.9713	------	------	------	------
1.158	1.0000	1.0000	------	------	------	------

same conditions (fig. 20, left). No effort has been made to correct this diagram for change in velocity head.

Any hope that this method of correlation might be improved by shortening the reach is shattered by the plotting of figure 22 (right), which is a similar computation for a reach of 120 feet. It will be noted that as the length of reach becomes shorter the variation in the curves of relation becomes greater.

Results of this technique as applied to data for cross section 5 are presented as figure 23 (400-ft and 120-ft reaches), on page 99. It will be noted that the family of curves for this cross section has a much greater spread than for cross section 2.

MISCELLANEOUS METHODS

POSSIBLE COMBINATIONS OF DIMENSIONLESS RATIOS

To the end that no meritorious method might go unexamined, a survey has been made of the possible uses of other combinations of factors for the curve of relation. The elements which may be used in various combinations are as follows:

Q_m. This is the actual discharge: the discharge which is measured, or would be obtained by a discharge measurement.

Q_0. This is the uniform-flow discharge: the discharge which, for a given gage reading, would exist if no backwater were present.

y_m. This is the actual depth.

y_0. This is the so-called uniform depth: the depth which, for a given discharge, would exist if no backwater were present.

F_m. This is the actual fall in the reach, as indicated by the difference in reading of two gages set to same datum.

F_0. This is the so-called uniform fall: the fall which under conditions of no backwater will exist for a given gage reading or a given discharge, or both. In the laboratory channels it is independent of such variation and is equal to 0.003 times the length of the reach.

$c_m V_0^2/2g$. The velocity head for discharge Q_m flowing at depth y_0.

$c_m V_m^2/2g$. The velocity head for discharge Q_m flowing at depth y_m.

F'. The velocity head increment, or velocity head at the upper gage minus the velocity head at the lower gage. It is a correction to be added to the fall.

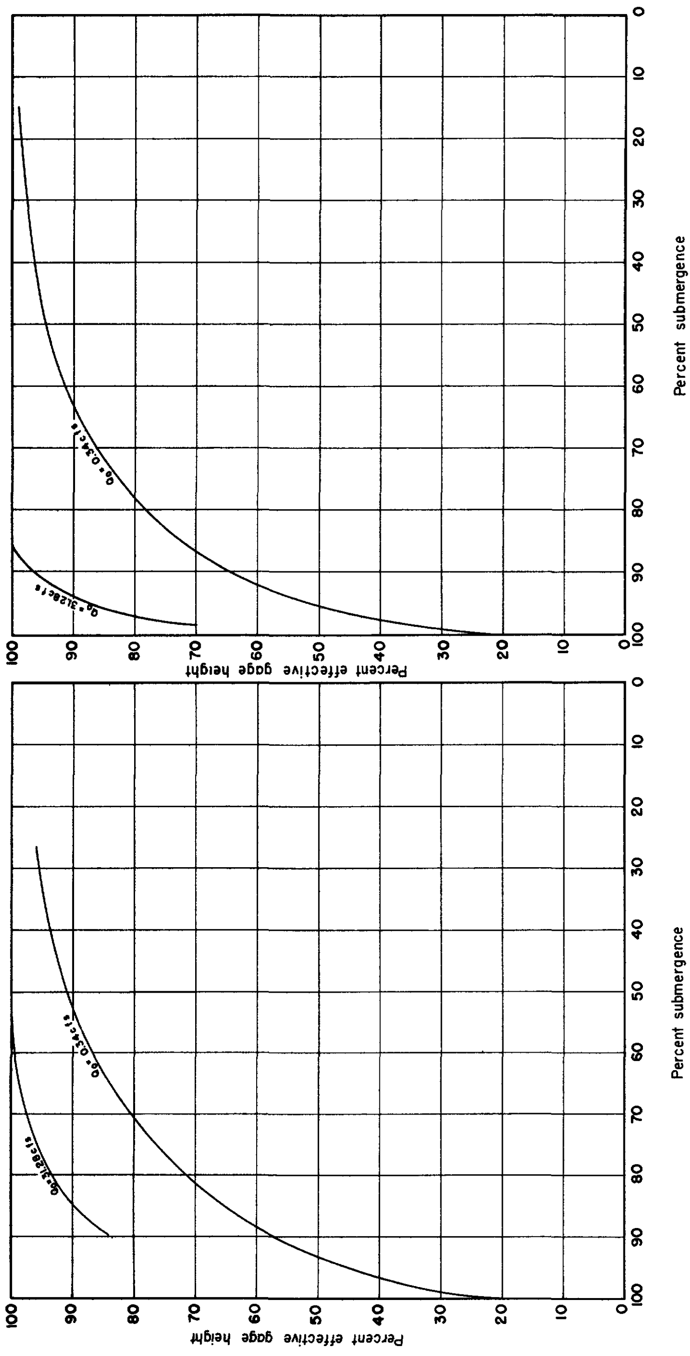


Figure 23. —Submergence diagrams, cross section 5, 400-foot reach (left) and 120-foot reach (right).

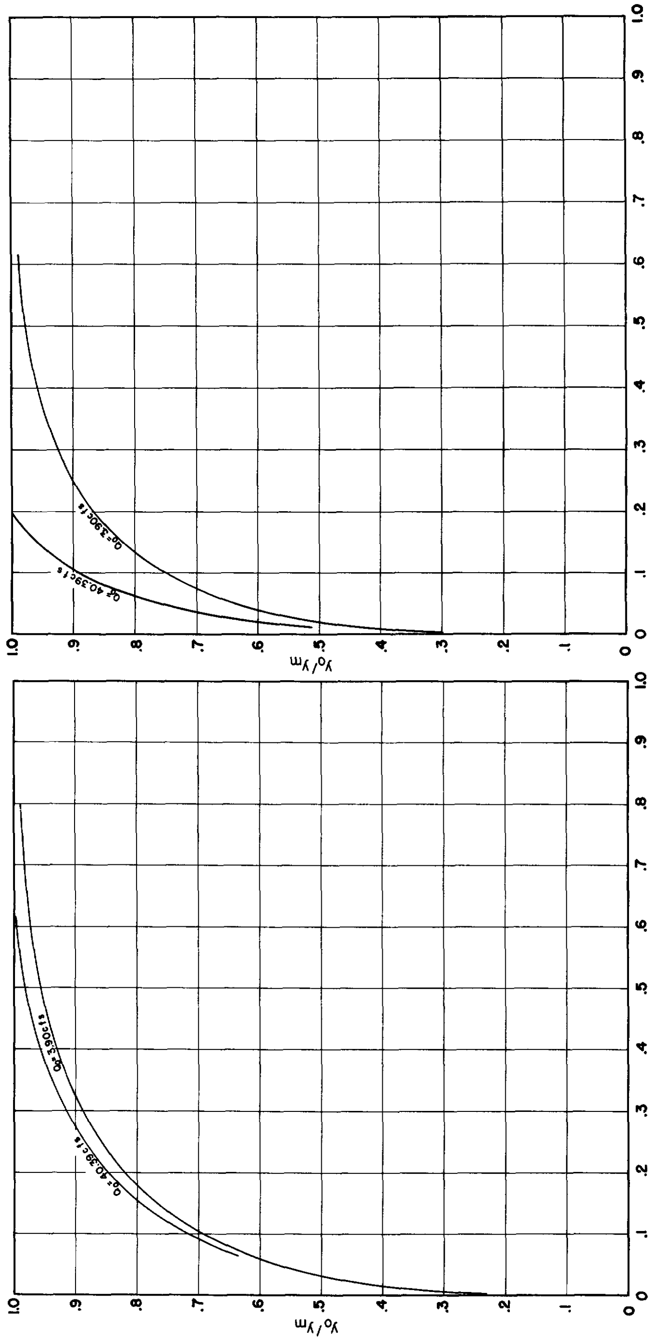


Figure 24.—$y_0/\ y_m$ versus $F_m\ /\ y_m$ cross section 2, 400-foot reach (left) and 120-foot reach (right).

Two other elements (Q_r, the "rating" discharge and F_r, the "rating" fall), both of which imply the presence of appreciable backwater, will be added later, in connection with fixed-backwater ratings. Expressions might also be written for specific head and its variation within the reach. They might also be written for the energy line and its variation within the reach but this, however, is pointless at this time. In fact, there is no merit, here, in writing expressions for velocity head and its variation in the reach except to take cognizance of the fact that they exist. In a search for a practical method of correcting for backwater effects, any method which involves use of the velocity head is to be avoided. If the velocity head is included, solutions may be made only by trial--that is, a discharge must be assumed in order that the velocity head may be evaluated, and the velocity head must be evaluated before the discharge may be computed. Hence, for practical purposes, consideration may be limited now to the first six of the above elements.

For practical use, it appears that these elements should be combined in the form of dimensionless ratios. Since only the first two have the dimension of L^3T^{-1}, these must be used together, while the remaining four (for each of which the dimension is L) may be used in any combination. Thus the following combinations become possible:

$$\frac{Q_m}{Q_0} \quad \frac{y_m}{y_0} \quad \frac{y_m}{F_m} \quad \frac{y_m}{F_0} \quad \frac{y_0}{F_m} \quad \frac{y_0}{F_0} \quad \frac{F_m}{F_0}$$

The reciprocals of these ratios may, of course, be used. Such use, however, will not result in the development of any new relations, but merely in a change in shape of the curves. In this study the above expressions have been inverted, if need be, to obtain the range of values which tends to lie below, rather than above, unity. The following arrangements have been found to be most usable:

$$\frac{Q_m}{Q_0} \quad \frac{y_0}{y_m} \quad \frac{F_m}{y_m} \quad \frac{y_m}{F_0} \quad \frac{F_m}{y_0} \quad \frac{F_m}{F_0}$$

It will be noted that y_0/F_0 has now been deleted. For any discharge this value is a constant, regardless of the magnitude of other ratios. Hence it is not suitable for use in these studies.

After values are computed for the six ratios listed above, curves of relation may be obtained by plotting any one of the ratios as ordinate against any other as abscissa. This would provide 15 methods of plotting. Practical considerations, however, lead to the immediate elimination of nearly half the number. In order that the plot may have practical use, it is necessary that in one ratio both numerator and denominator be known and that in the second ratio one of the values be known and the other be either Q_m or y_0. The values which are known (assuming that a curve or relation between uniform gage height and uniform discharge has been developed) are Q_0, y_m, F_m and F_0. Of the above ratios, the only ones containing a known value and either Q_m or y_0 are

$$\frac{Q_m}{Q_0} \quad \frac{y_0}{y_m} \quad \frac{F_m}{y_0};$$

the only ones made up of two known values are

$$\frac{F_m}{F_0} \quad \frac{F_m}{y_m} \quad \frac{y_m}{F_0}.$$

Thus the number of combinations that can be considered in light of the above analysis is limited to nine, as follows

$$(1) \quad \frac{Q_m}{Q_0} \text{ against } \frac{F_m}{F_0}$$

$$(2) \quad \frac{y_0}{y_m} \text{ against } \frac{F_m}{F_0}$$

$$(3) \quad \frac{F_m}{y_0} \text{ against } \frac{F_m}{F_0}$$

$$(4) \quad \frac{Q_m}{Q_0} \text{ against } \frac{F_m}{y_m}$$

$$(5) \quad \frac{y_0}{y_m} \text{ against } \frac{F_m}{y_m}$$

$$(6) \quad \frac{F_m}{y_0} \text{ against } \frac{F_m}{y_m}$$

$$(7) \quad \frac{Q_m}{Q_0} \text{ against } \frac{y_m}{F_0}$$

$$(8) \quad \frac{y_0}{y_m} \text{ against } \frac{y_m}{F_0}$$

$$(9) \quad \frac{F_m}{y_0} \text{ against } \frac{y_m}{F_0}$$

Data for the plotting of these relations, for the profiles of highest and lowest discharge in both cross section 2 and cross section 5, are contained in tables 17 to 20 and 22 to 25, pages 90 to 91 and 94 to 95, derivation of which has been previously explained.

RESULTS OF VARIOUS RATIO COMBINATIONS

The first of the above combinations already has been discussed at length. This is the plot presented in figure 20 as the Boyer diagram.

A plot of the second combination, y_0/y_m against F_m/F_0, has been made, although the results are not shown herein. It may be demonstrated that if there is a simple exponential relation between y_0 and Q_0 there will also be a corresponding relation between (Q_m/Q_0) and (y_0/y_m). In agreement with this principle, the plots of y_0/y_m against F_m/F_0 were found to be much like the Boyer diagrams. There was a somewhat wider variation between the curves, and a greater departure from the square-root-of-the-fall relation. There appeared to be no advantage over the Boyer diagram.

A study of combination (3) above, F_m/y_0 against F_m/F_0, reveals a limitation in the practical use of the relations which was not covered in the preceding discussion. This combination is lacking in one of the essential requirements, in that neither y_m nor Q_0 appears in either of the ratios. Only the fall, and not the depth, is taken into account. The relations are insufficient to give a determinate result.

The fourth relation, Q_m/Q_0 against F_m/y_m, develops also into a family of curves. It offers so little promise that the plot has been omitted from this report. The curves are practically superimposed for low values of Q_m/Q_0; but for high values of this ratio the curves disperse rapidly, as the ratios of F_m/y_m approach their limits of F_0/y_0.

Somewhat the same criticism is applicable to the fifth plot, of y_0/y_m against F_m/y_m. In the limit, as F_m/y_m approaches F_0/y_0, a range of dispersion occurs which is dependent only on the range of y_0. However, for a 400-foot reach of cross section 2, the maximum error found in determining discharge by use of a single mean curve to represent this relation was near 2 percent. Figure 24 (left), page 100, presents for cross section 2 the enveloping positions of the true curves (of highest and lowest discharges used in these computations). Figure 25 (left), page 104, shows similar curves for cross section 5. Unfortunately, the comparatively accurate results which might be obtained in the first instance appear not to hold under other circumstances. This is further evident from figures 24 (right) and 25 (right), prepared for reaches of 120 feet. It will be noted that the range of variation becomes considerably greater

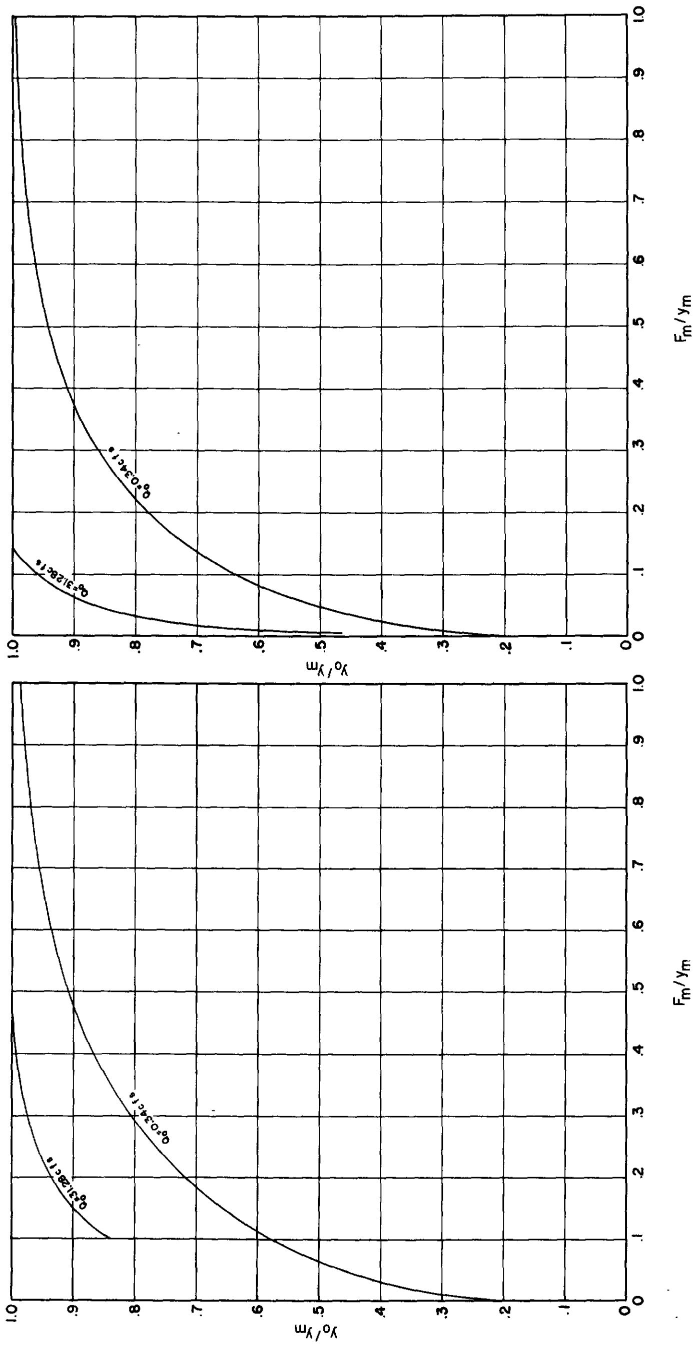


Figure 25.—y_0/y_m versus F_m/y_m cross section 5, 400-foot reach (left) and 120-foot reach (right).

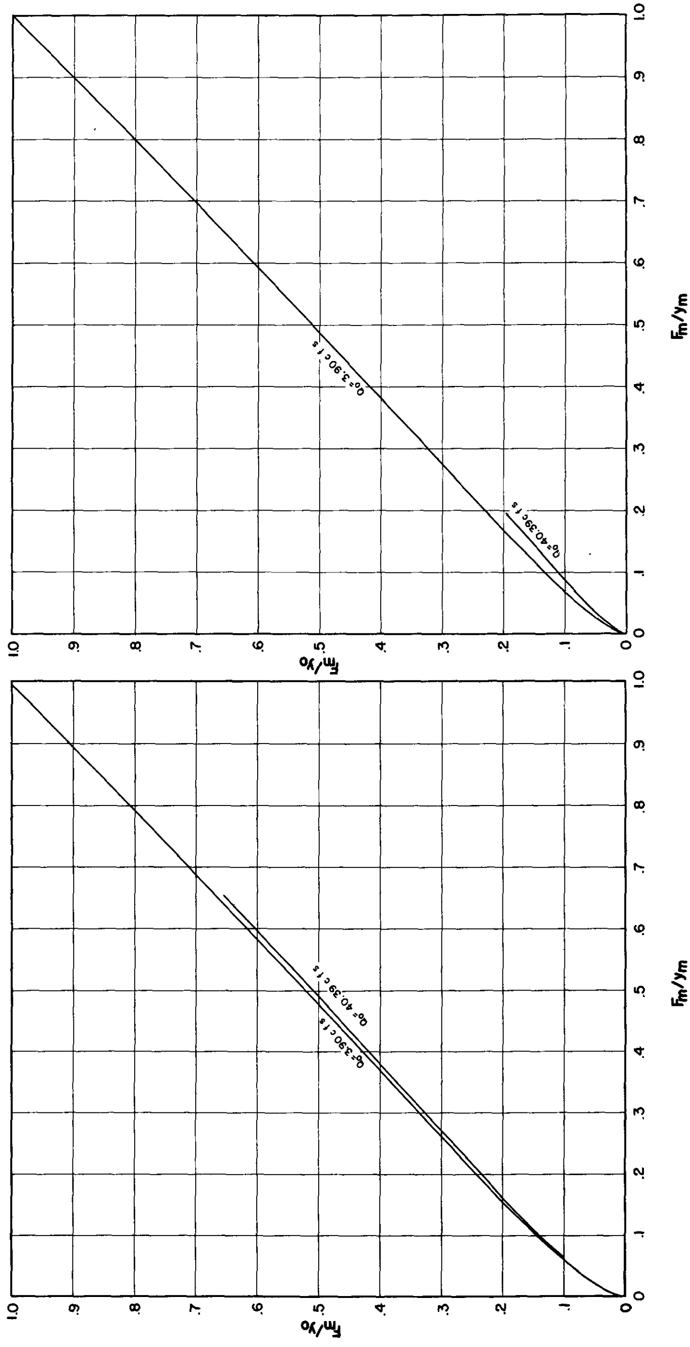


Figure 26.—F_m / y_0 versus F_m / y_m cross section 2, 400-foot reach (left) and 120-foot reach (right).

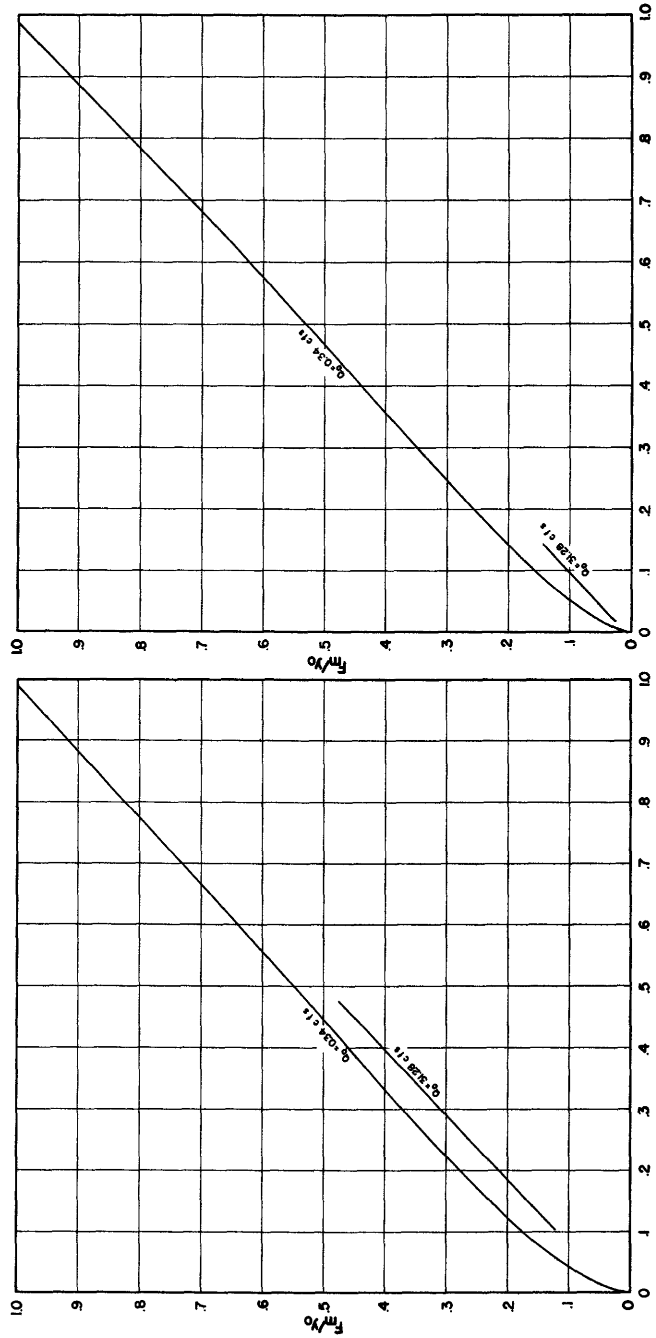


Figure 27.—F_m / y_0 versus F_m / y_m cross section 5, 400-foot reach (left) and 120-foot reach (right).

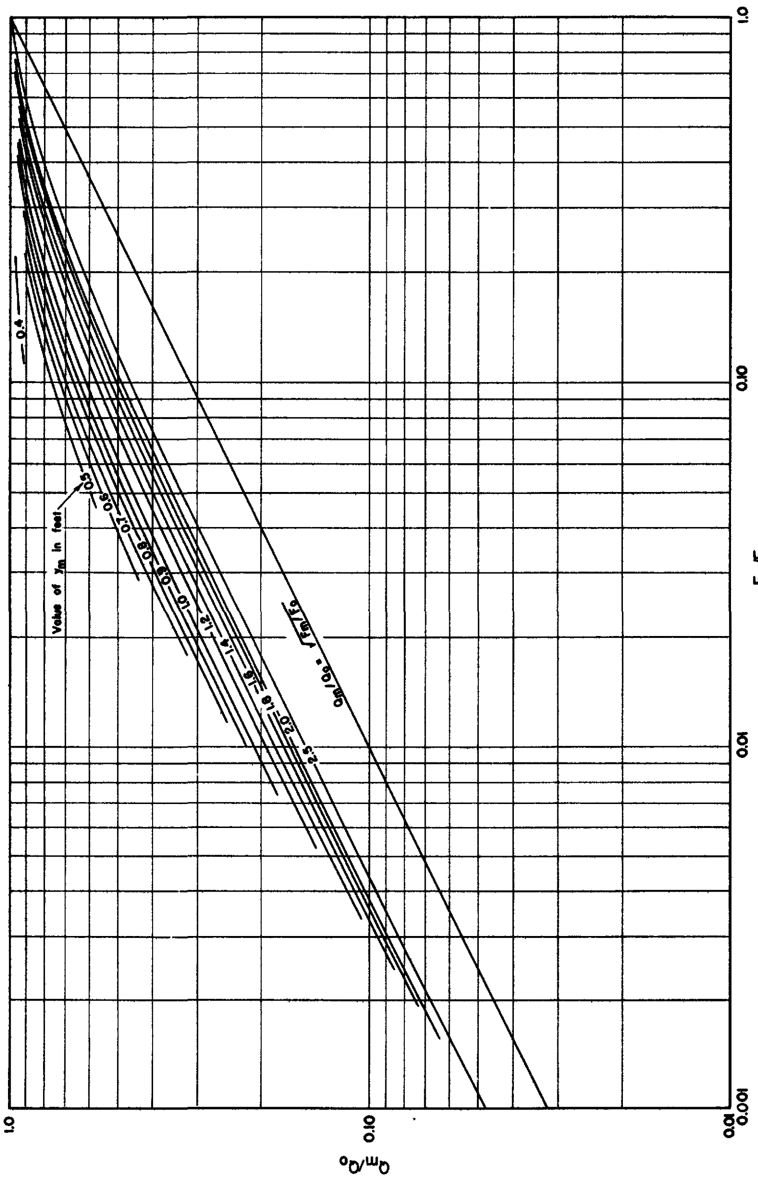


Figure 28.—Three-dimensional Boyer diagram, cross section 2.

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The sixth relation, F_m/y_0 against F_m/y_m, appears at first glance to offer a very acceptable plot. It has been presented for the 400-foot reach in cross section 2 as figure 26 (left), page 105. Closer inspection discloses that it also develops family characteristics. It also will be apparent that, as a condition of no backwater is approached, both of the ratios approach the limit of F_0/y_0. Each member of the family thus terminates on a straight line through the origin at 45° slope. Even for extreme backwater conditions, the variation from such line is small. Since this is true, the lines must be drawn to a high degree of accuracy to attain satisfactory results in determination of discharge. Furthermore, as the length of the reach decreases, the range of variation becomes considerably greater. Figure 26 (right) shows the enveloping positions for the 120-foot reach in cross section 2. Corresponding curves for cross section 5 are shown as figure 27, page 106.

The three remaining relations – Q_m/Q_0 against y_m/F_0, y_0/y_m against y_m/F_0, and F_m/y_0 against y_m/F_0 – offer so little promise that no plots have been shown. In each case one of the factors is y_m/F_0. Since F_0 is a constant for these conditions of uniform prismatic channel, the results are essentially the same as obtained by plotting directly against the observed depth.

It will be noted that, throughout these discussions, for any given length of reach F_0 is constant. This is in keeping with the fundamental concept of the uniform-flow rating for the prismatic channels. In the event that a fixed-backwater rating (see p. 78) is considered, a variable F_r will replace the constant F_0; and the relations discussed above should be reconsidered.

PROPOSED METHODS

RECAPITULATION

A comprehensive investigation of the possible combinations of stage, fall, and discharge in a prismatic channel fails to reveal any two-dimensional solution, based on the uniform-flow rating, which is universally acceptable. Any of the three-dimensional curves of relation which have been presented as figures 18 to 27 will give accurate results to the extent to which the curves are accurately drawn and accurately read. It will be noted, however, that all except figures 18 and 19 require trial solutions. That is, when entering the diagram with the known ratio, it is necessary to assume a discharge before a value of the second ratio may be obtained. If the discharge as computed from this second ratio does not agree with that which was assumed, a second assumption must be made. Such solutions, at best, are a tedious procedure.

This difficulty obviously would not arise if, for the family of curves presented for each of the methods, a single curve could be substituted with acceptable accuracy. In this respect it should be noted that, for the Boyer method, as the length of reach is decreased, the family of curves more nearly approaches a single curve and also approaches the curve expressing the square-root-of-the-fall relation. For the other three methods illustrated in figures 22 to 27 as the reach is *lengthened* each family more nearly approaches a single curve. Thus for those instances in which a single curve is used to represent the family group, it appears that accuracy should improve with the Boyer method as the reach is made shorter. It appears possible that accuracy should improve with the other methods as the reach is made longer. This latter appearance, however, may be an illusion, since the convergence of the family may merely represent a loss of sensitivity. In any event, the fact remains that the use of a single curve to represent the family is only an approximation of the true relationship.

THREE-DIMENSIONAL BOYER DIAGRAM

The true relationships may be shown, and the difficulties of trial solutions still may be avoided, by a diagram such as figures 28 to 34, page 107 and pages 110 to 115. The values of the fall ratio as abscissa have been plotted against values of the discharge ratio as ordinate, but the third variable has been taken as observed stage. No assumptions are required. The diagram may be entered with a known fall ratio and known observed stage, and the value of Q_m/Q_0 directly determined.

Let it be required, for example, to determine the discharge for depth of 2.00 feet in cross section 2, when the fall in 400 feet is 0.105 feet. The fall ratio, F_m/F_0, is 0.105/1.200, or 0.0875. Entering figure 28 with this value, and proceeding to the line $y_m = 2.00$, we find Q_m/Q_0 equal to 0.460. From figure 6, page 19, in Bulletin 381 (Lansford and Mitchell, 1949) or from table 42, page 81 in that bulletin, Q_0 is equal to 45.18 cfs. Hence Q_m must equal 45.18 cfs × 0.460, or 20.8 cfs.

Although the diagram may be prepared on any type of cross-section sheet, the use of logarithmic coordinates is strongly recommended. Two advantages will be obvious, thus: (1) In those ranges for which backwater effects are high and the values of the ratios become very small, there is sufficient expansion to permit accurate use. (2) Throughout most of their range, the lines of equal depth will be systematically spaced, nearly parallel to one another, and nearly parallel to a line representing the relation $Q_m/Q_0 = (F_m/F_0)^{0.5}$. All curves must, of course, converge to pass through the point 1, 1. Thus when meager data are available, the preparation of the diagram will be expedited by keeping the curves in this form.

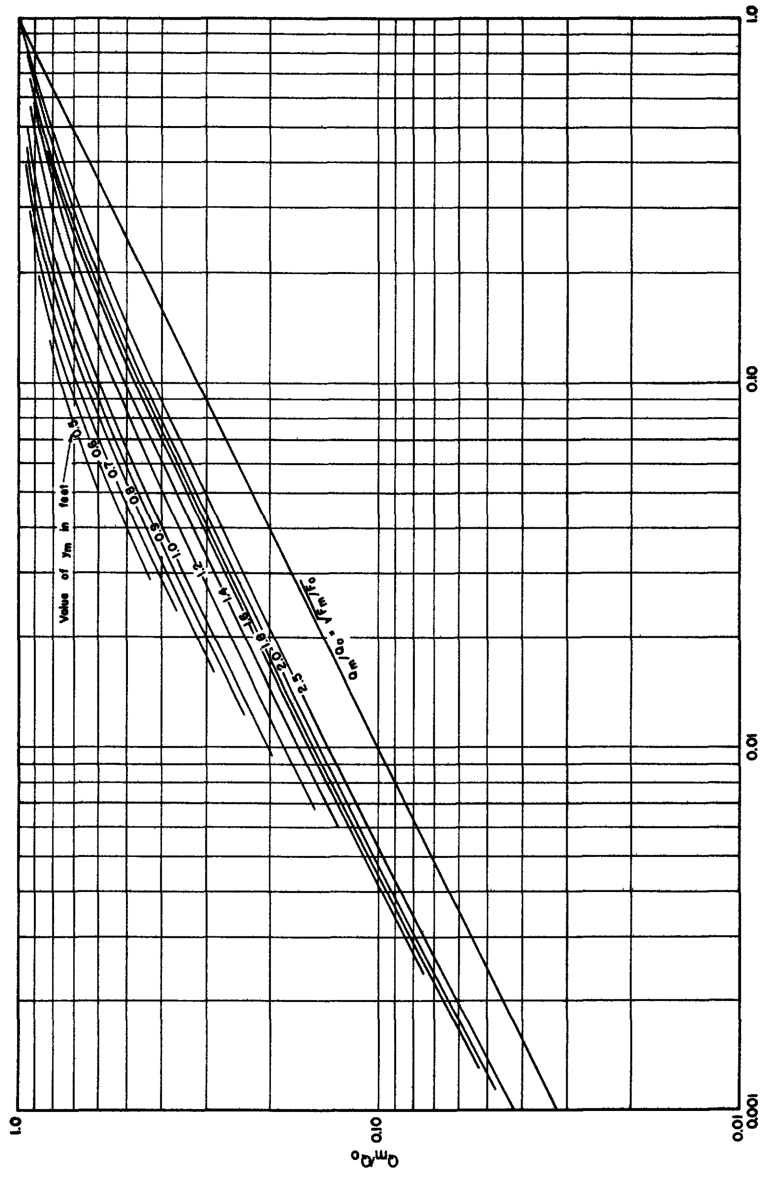


Figure 29.—Three-dimensional Boyer diagram, cross section 3.

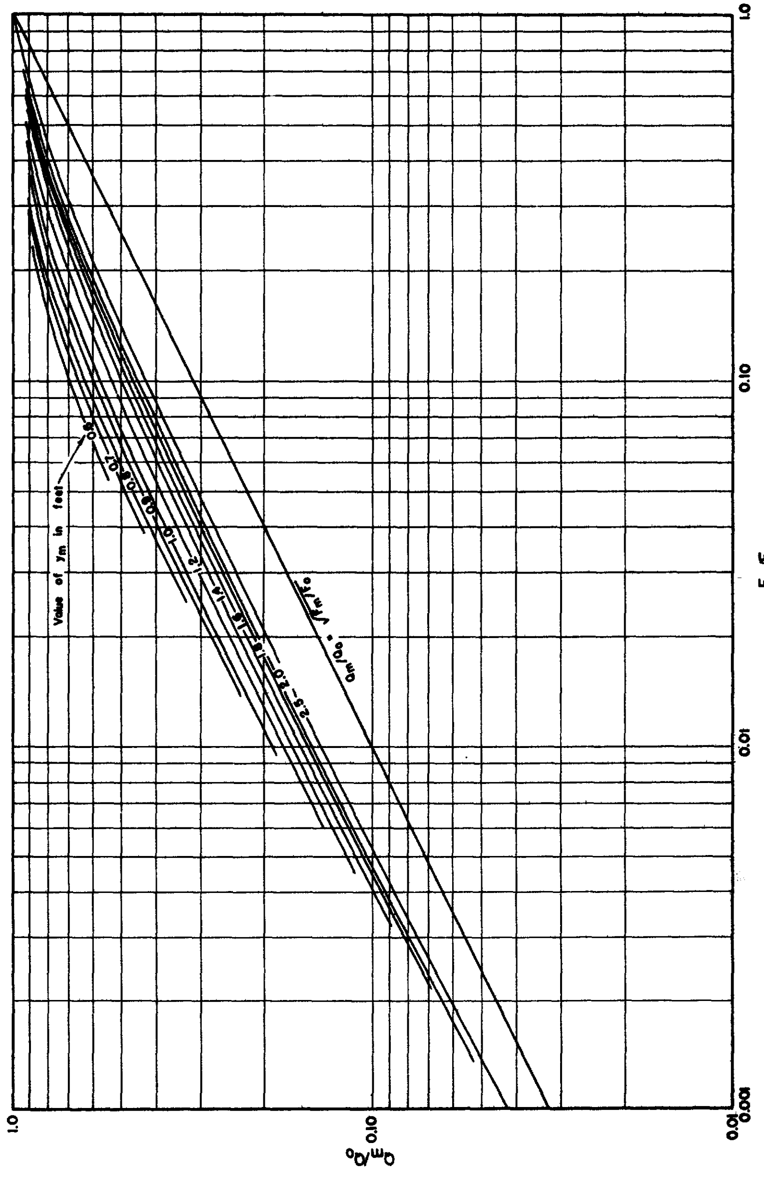


Figure 30. —Three-dimensional Boyer diagram, cross section 4.

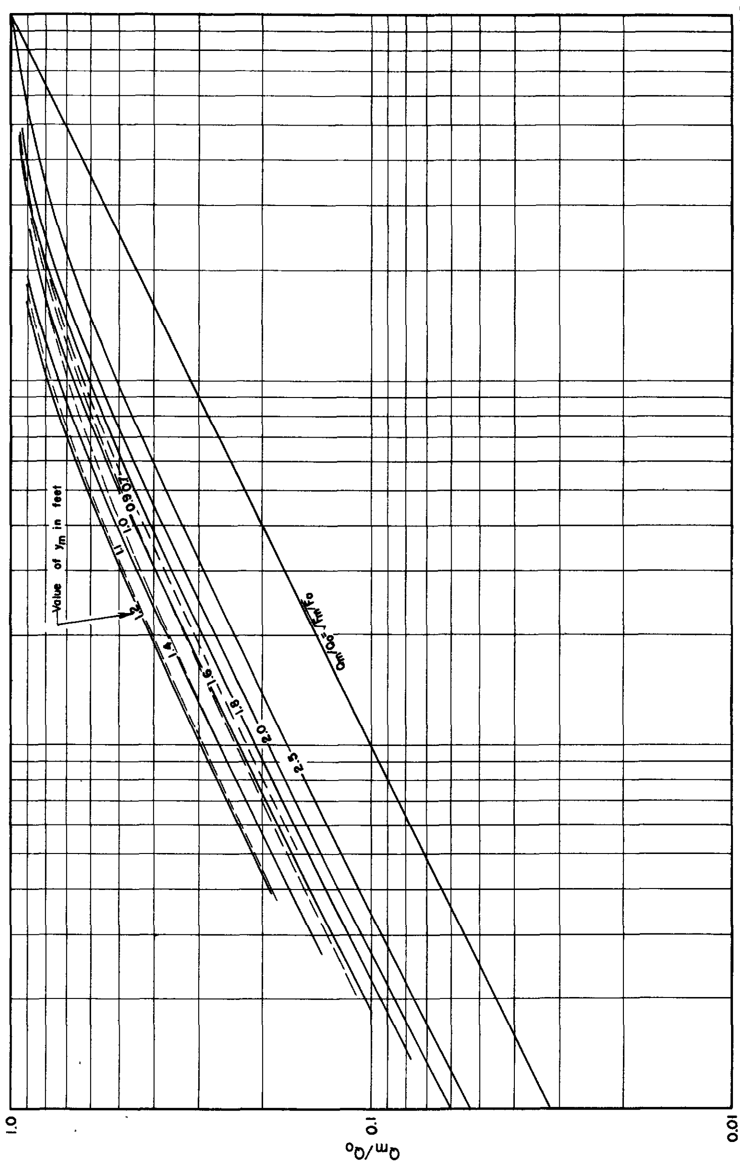


Figure 31. --Three-dimensional Boyer diagram, cross section 5 (y_m 2.5 through 0.7).

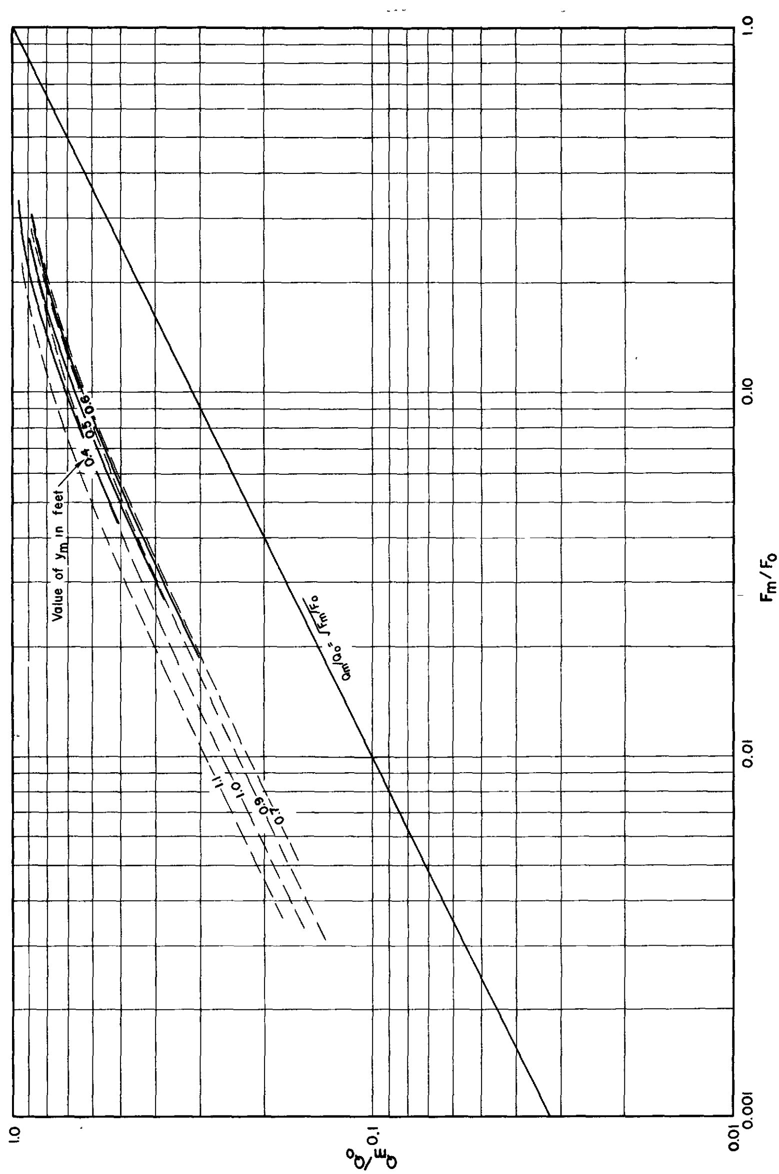


Figure 32.—Three-dimensional Boyer diagram, cross section 5 (y_m 1.1 through 0.4).

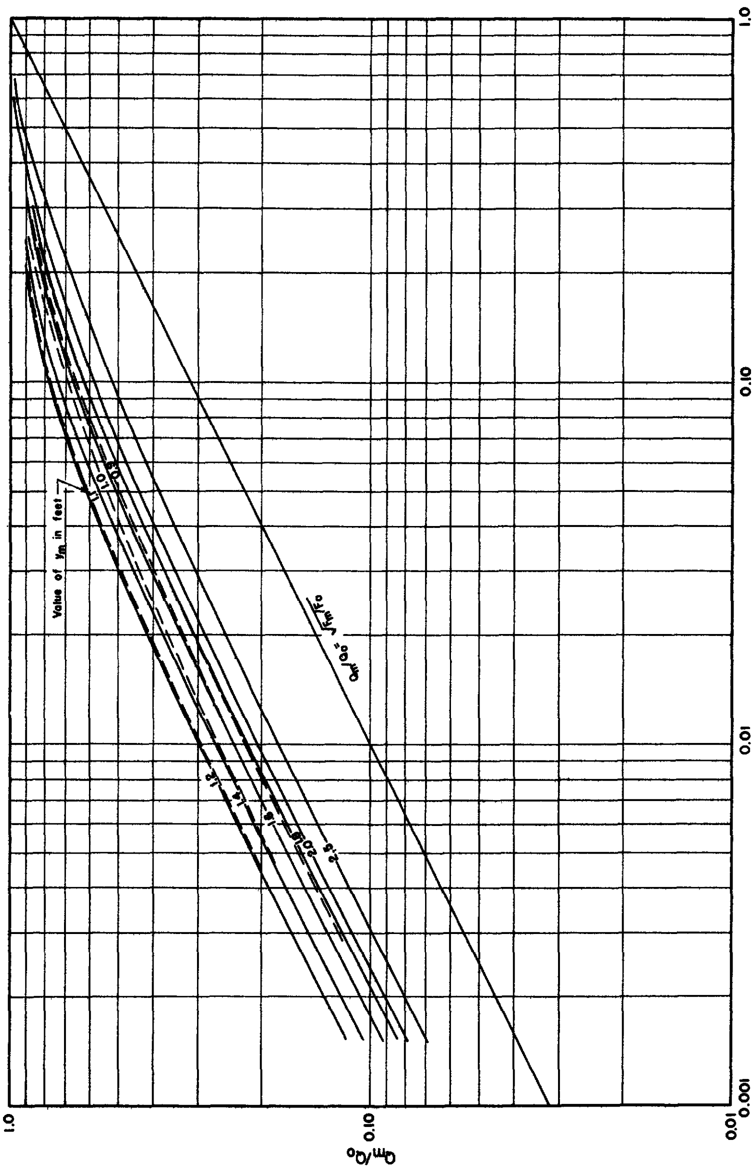


Figure 33.—Three-dimensional Boyer diagram, cross section 6 (y_m 2.5 through 0.9).

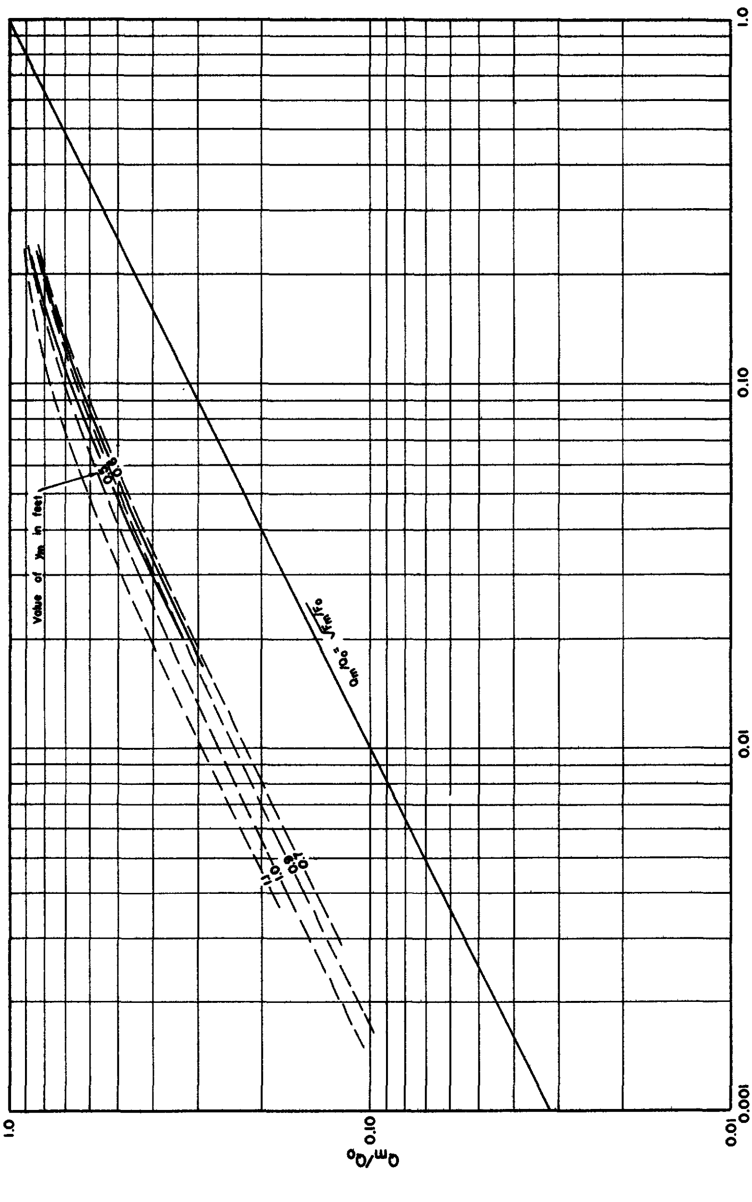
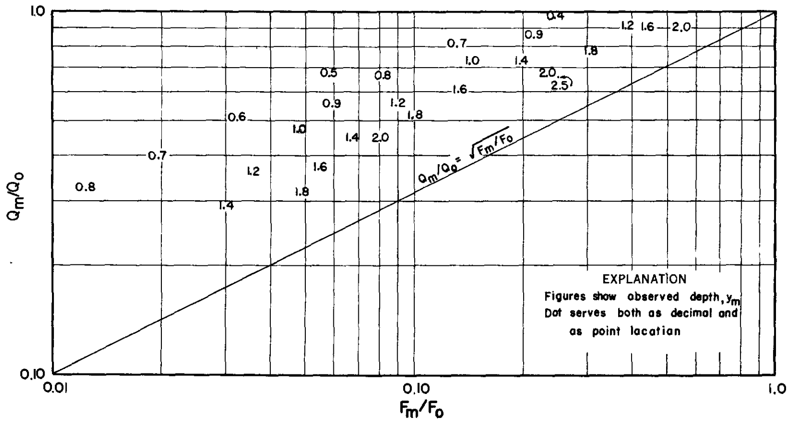


Figure 34. --Three-dimensional Boyer diagram, cross section 6 (y_m 1.1 through 0.5).



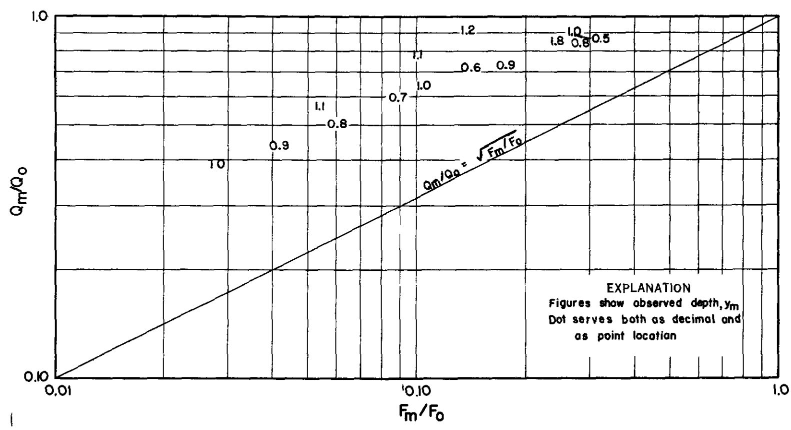


Figure 35.—Three-dimensional Boyer plot, developed from smoothed profiles, cross sections 2 (above) and 5 (below).

It will be noted that for channels such as those represented by cross sections 2, 3 and 4 the systematic spacing of the lines of equal depth is quite simple. Unfortunately, for channels such as those represented by cross sections 5 and 6, the spacing, although systematic, is subject to reversals in trend, so that certain sections of the diagrams are superimposed. (See figs. 31-34.) Beginning with the greatest value of y_m (2.5), it will be noted that, for any given fall ratio, the values of discharge ratio ***increase*** with decreasing values of y_m down to observed depth of 1.2 feet—the point below which, at the upper end of the reach, water no longer covers the entire width of the flood plain. As y_m is decreased below this value, the values of discharge ratio for any given fall ratio progressively ***decrease*** to the point $y_m = 0.7$. Here there occurs a second reversal in trend, so that as y_m is further decreased, values of the discharge ratio again increase in a manner similar to that for the upper portion of the channel.

A completely generalized statement as to the circumstances under which discharge ratio decreases with decreasing values of y_m is not warranted by available data. There appears to be a relation between this phenomenon and the fact that the water covers the flood plain throughout only a portion of the reach.

It will be recalled that figures 18 to 34 have been plotted from computations such as those presented in tables 17 to 26. These computations, in turn, were based on tables 12 to 16, which are tables of computed rather than observed profiles. An attempt was made to use the smoothed profiles based on the observed data, but the data available were found to be too meager to allow the complete development of the families of curves. Illustrations of the results to be obtained by use of the smoothed observations (tables 1 to 5) are to be found in figure 35, page 116. Although families of curves could not be accurately drawn from these data, the plotted points verify the fact that the same relations exist here as in figures 18 to 34.

The third variable, observed depth, might also be used with other combinations of depth and fall ratios such as heretofore discussed. Its use with Q_m/Q_0 and F_m/F_0, however, has the following definite advantages: (1) The terminal points of every curve are known, since each curve must start from the origin and end at the point 1,1. (2) The spacing and shape of the curves, as discussed above, are systematic. (3) In the event the method is used in those cases in which profile curvature and velocity-head increment are truly negligible, the plot should tend to resolve into a single curve. And conversely, (4) the development for any channel of a plot such as shown in the preceding figures will afford strong presumption that the effects of profile curvature and velocity-head increment, alone or in combination, are indeed factors in the stage-fall-discharge relation.

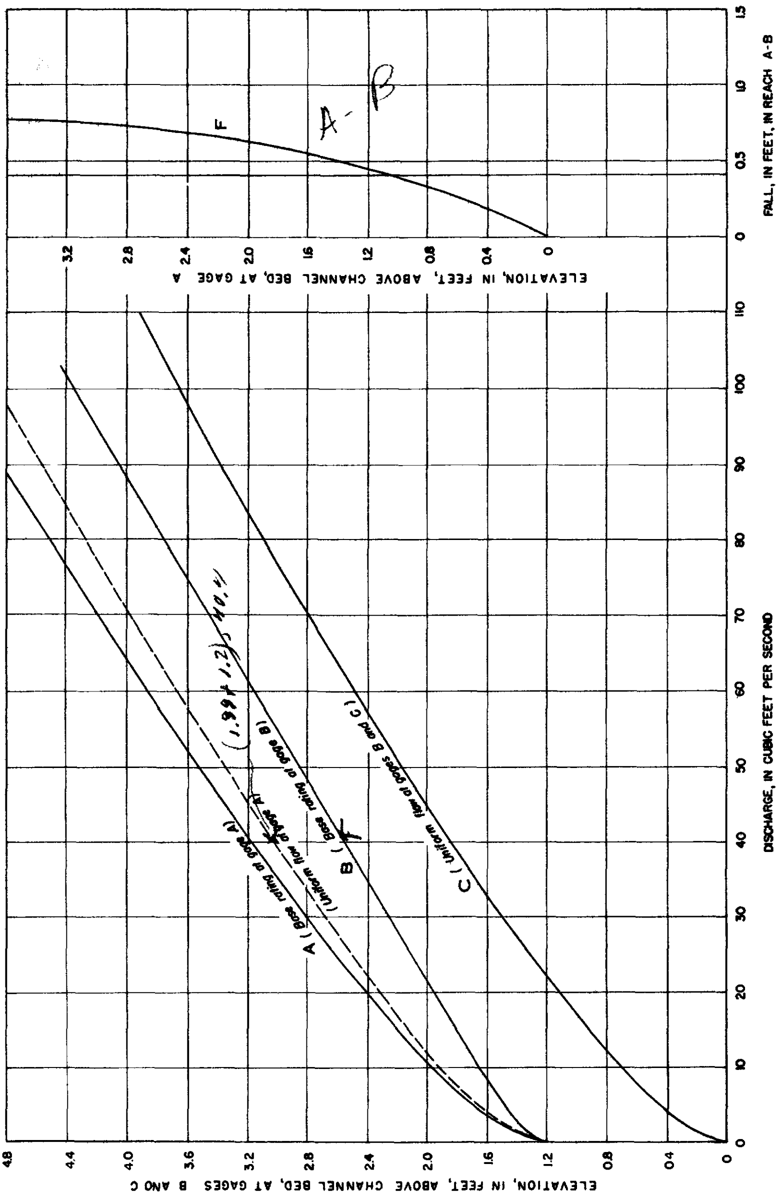


Figure 36.—Stage, discharge, and fall for a fixed-backwater condition, cross section 2.

FIXED-BACKWATER RATINGS

As explained at the beginning of this section (see page 78), there are two types of channel conditions for which there is a fixed and invariable relation between stage and discharge and for which fall is an unnecessary element in the rating: the first, uniform flow; the second, a condition of nonuniform flow induced by some fixed nonuniformity of the channel.

The second condition is illustrated by figure 17, page 78, in which it is necessary for all flow to pass over the crest of a fixed spillway which rises an appreciable distance above the channel bed. In varying degree this second condition commonly is encountered in the operation of gaging stations. Occasionally the station may be upstream from and in the backwater of a high dam which, even for highest discharges, is not submerged. More often it may be upstream from a dam of lesser height, which, as the discharge increases, will become at least partially submerged. Still more often the station may be upstream from a natural outcropping of ledge rock, or a group of boulders or other obstructing material, which, at low discharges, is said to "control" the stage-discharge relation. To the extent that any of these channel obstructions fulfill the function of a control--that is, determine the relation between stage and discharge--the steady-flow stage-discharge relation will be a simple two-dimensional curve, unencumbered by association with fall. When, with increasing discharge, these channel obstructions become submerged, and lose their effectiveness as a control, fall may or may not become an essential element of the steady-flow rating. The circumstances under which fall will become an essential element will be discussed subsequently. But first it is desirable to consider the simple case--that in which a dam is placed in an otherwise uniform channel and built to such height that it does not become submerged, even by the maximum discharge.

Let it be supposed that figure 17 (left) (page ~~90~~ 78) represents a longitudinal section of the channel for cross section 2 and that, at any point in the channel, there be erected a dam of large spillway capacity whose crest is 1.2 feet above the channel bed. (Downstream from the dam, the channel will be assumed to have the same cross section as upstream from the dam.) Let a gage be installed just downstream from the dam, another just upstream from the dam (taking care to avoid drawdown effects), and a third 400 feet upstream from the second. Let the gage farthest upstream be designated as *A*, the second one as *B*, and the one immediately downstream from the dam as *C*. For simplicity, let it be assumed that the longitudinal distance between *B* and *C* is negligible, so that the bed elevations at *B* and *C* may be considered to be the same. Finally, let it be assumed that, by some appropriate means such as current-meter measurements, a rating is developed for gage *B*, and is as shown by curve *B* of figure 36, page 118. (The fact that curve *B* has additional significance, as discussed on pages 130 and 134, does not preclude its use in the following explanation of basic concepts.)

Obviously, for any given rate of steady flow, an M-1 surface profile, or backwater curve, will now exist in the reach *A-B*, and for any of the rates of discharge treated in this report, the curve will be some segment of the profiles presented in table 12, pages 59 to 62. The particular segment can be determined by a simple computation, and hinges upon the fact that, for any discharge, the value of the depth y is known at gage *B*. For example, for the discharge of 40.39 cfs, the depth at *B*, as taken from figure 36, is 2.563 feet. Interpolating in table 12, the corresponding value of L is found to be 588.70. Thus the segment of profile which exists in the reach *A-B* is that shown between stations 588.70 and 988.70 of figure 15, page 56. The depth at gage *A* is computed by working with the next-to-last column and the first column of table 12 and interpolating a value of y equal to 1.990 for $L = 988.70$. A point ($Q = 40.39$, $y = 1.990$) may now be plotted on figure 36, the ordinate being with respect to scale of elevation above channel bed at upstream gage. This is a point on the two-dimensional steady-flow stage-discharge rating curve for gage *A*, under the fixed-backwater condition described above.

From the other backwater curves given in table 12, five additional points for defining the fixed-backwater rating at gage *A* may be determined. Another valuable point is obtained by noting that when the discharge becomes zero the water surface will be a horizontal line upstream from the crest of the dam and will intersect the channel bed at elevation zero on the upstream gage. From these 7 points, plotted logarithmically, curve *A* of figure 36 is drawn to represent the steady-flow stage-discharge relation for gage *A* under the fixed-backwater condition created by the dam previously described.

The dashed line in figure 36 is the uniform flow rating at gage *A*. If the imaginary dam be lowered, curve *B* would approach curve *C* as a limit and curve *A* would approach the Q_0 curve at gage *A* and the *F* curve (see below) would approach $F_0 = 1.20$ as a limit. Thus any curve *B* to the left of curve *C* will give a curve lying to the left of the Q_0 curve at gage *A*.

Let it be repeated that there are two types of channel conditions for which fall is superfluous to the steady-flow rating. In the first of these, uniform flow, fall is unnecessary because it is a constant; it always is the same as the fall in the channel bed. In the second type, nonuniform flow induced by some fixed nonuniformity of the channel, the fall may vary with discharge, but for a given discharge the fall always is the same, since the nonuniformity of the channel always is the same. As the discharge here is a unique function of stage, it follows that, for any given stage, the fall always is the same. This leads to consideration of the stage-fall curve which, although not needed under conditions described above, can now be easily determined. And it is well to become familiar with it now, for it becomes a very important element in ratings yet to be described.

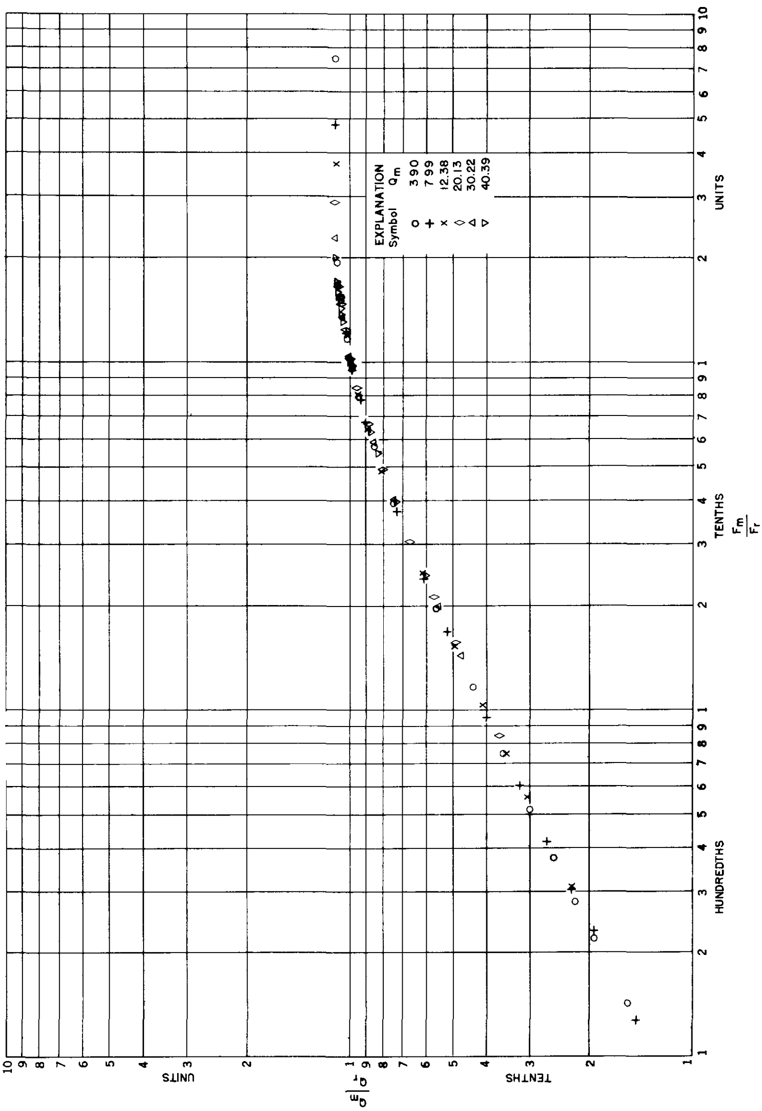


Figure 37.—Boyer diagram, for cross section 2, to accompany figure 36.

The fall for any stage at gage *A* may be obtained by entering figure 36 with the desired stage, moving horizontally to curve *A*, then noting the vertical intercept between curves *A* and *B*. A curve based on values so obtained is shown as curve *F* at the left of figure 36, where fall in the reach *A-B* is plotted as abscissa against the same ordinate as used for the rating at gage *A*.

Let it be remembered that the present discussion has dealt thus far with only one type of channel condition: that in which a dam is placed in an otherwise uniform channel and built to such height that it does not become submerged, even by the maximum discharge. That this assumption is fulfilled is evidenced by the fact that, in figure 36, curve *C*, the steady-uniform-flow rating for gage *C*, does not intersect curve *B*.

For next consideration, let the above conditions be supplemented by one additional assumption: although the dam never is submerged as the result of high discharge, it may become submerged as a result of some phenomenon farther downstream, such as backwater from a downstream confluence. As such submergence becomes appreciable, the rating for gage *B* begins to change, leading to the displacement, in the upstream direction, of the surface profile and thus to an increase in the stage at *A*, and a decrease in fall in the reach *A-B*. The steady-flow stage-discharge relation at *A* is no longer a simple two-dimensional relation but must be modified by consideration of the fall. For any given stage at *A*, and dependent upon the extent of submergence, the discharge and the fall in the reach *A-B* may be equal to or less than shown by figure 36. Thus it becomes convenient to allow the curves for gage *A* to become the base rating, or base for comparison of other conditions of fall and discharge. In other words, values from these curves may be used as the denominators in computing discharge ratios and fall ratios similar to those described on pages 84 to 118.

Values from this base rating cannot now be designated Q_0 and F_0, as they no longer are for conditions of uniform flow. They do represent conditions which may normally occur, and so might be designated Q_n and F_n -- a practice which was followed by Corbett, in U. S. Geological Survey Water Supply Paper 888. It has been suggested by Eisenlohr (personal communication, 1946) that a more appropriate subscript is *r*, thus indicating these are values from the base rating, whatever it may be. This suggestion is in keeping with concepts to be developed in succeeding pages and therefore is adopted here. Computations for Q_m/Q_r and F_m/F_r may be made in the same manner, and based on the same principles, as those for Q_m/Q_0 and F_m/F_0. (See pp. 79 to 81.)

Under the last assumption--that the rating for the dam may be affected by submergence as a result of some downstream phenomenon--only a portion of each profile may be used for computing discharge ratios and fall ratios. For a given discharge no portion of a profile may be used which provides, at the upstream end, a depth less than the corresponding ordinate of curve *A*, figure 36. Thus the

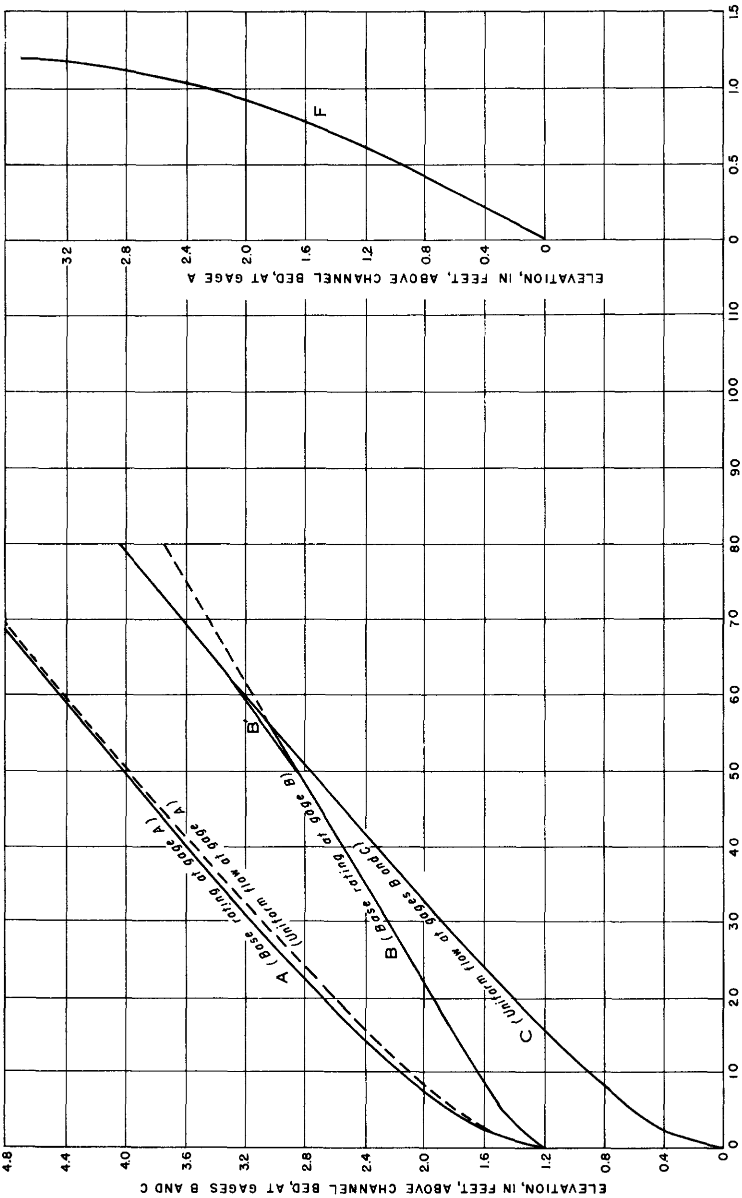


Figure 38.—Stage, discharge, and fall for a fixed-backwater condition, cross section 4.

maximum value of fall is F_r, and the maximum value of discharge is Q_r; the maximum possible values of Q_m/Q_r and F_m/F_r are unity. This limitation arises from the fact that all discharge must pass over the crest of the dam. If, for a given discharge, the depth of *A* were to become less than shown by figure 36, then the depth at *B* also would be less than shown by curve *B* (the profile having been displaced in the downstream direction), and there would be insufficient head to pass the given discharge over the dam. But let another assumption be made regarding the dam: that it be provided with sluice gages so that any discharge, however great, may be passed through the dam with a stage at *B* not appreciably greater than the stage, for the corresponding discharge, at gage *C*. Now with any given discharge, as the sluice gates are opened the stage at *B* will begin to fall, and the backwater profile will be displaced, rigidly, in the downstream direction, leading to a stage at *A* which is less than shown by curve *A*. For these conditions both Q_m/Q_r and F_m/F_r will become greater than unity. Under these circumstances the remaining portions of the profiles (those portions for which at the upstream end of the reach the depth is less than shown by curve *A*) may be used to compute values of the discharge ratio and the fall ratio. However, the ratio Q_m/Q_r cannot exceed the ratio Q_0/Q_r for as described above with curve *B* lowered to curve *C*, curve *A* can do no more than to lower to the Q_0 position at gage *A*.

The ratios which may evolve from these last two assumptions – (1) the drowning of the dam by some downstream phenomenon, so that backwater effects in the reach *A-B* are greater than usual, and (2) the opening of sluice gates, so that backwater effects in the reach *A-B* are less than usual – have been computed for cross section 2, and plotted as the Boyer diagram, figure 37, page 121. Here, at last, is a diagram for which all the data lie within very reasonable limits of a single curve. It will be noted that the points define a curve which passes through the point 1, 1. Values less than unity represent conditions described under assumption 1 above—the existence in reach *A-B* of backwater effects greater than those envisioned in the preparation of the curves of figure 36—whereas values greater than unity represent conditions described under assumption 2 above, or the existence in reach *A-B* of backwater effects which are less than those accounted for in the curves of figure 36.

Attention now is invited to a similar treatment for cross section 4. Let a dam, identical to that which was used for cross section 2, be placed in the channel, and let gages *A*, *B*, and *C* be located as before. (See page 119.) The curves which result from these assumptions are presented as figure 38, page 123. Curve *C* is first drawn. It is the steady-uniform-flow stage-discharge relation curve. It is applicable at gage *C* because channel characteristics below *C* are the same as in the reach *A-B*. Curve *B* is next drawn. As this is the rating for the dam, and the dam is identical with that used in cross section 2, curve *B* should be the same as curve *B* of figure 36. But a new factor has been injected! Curve *B*,

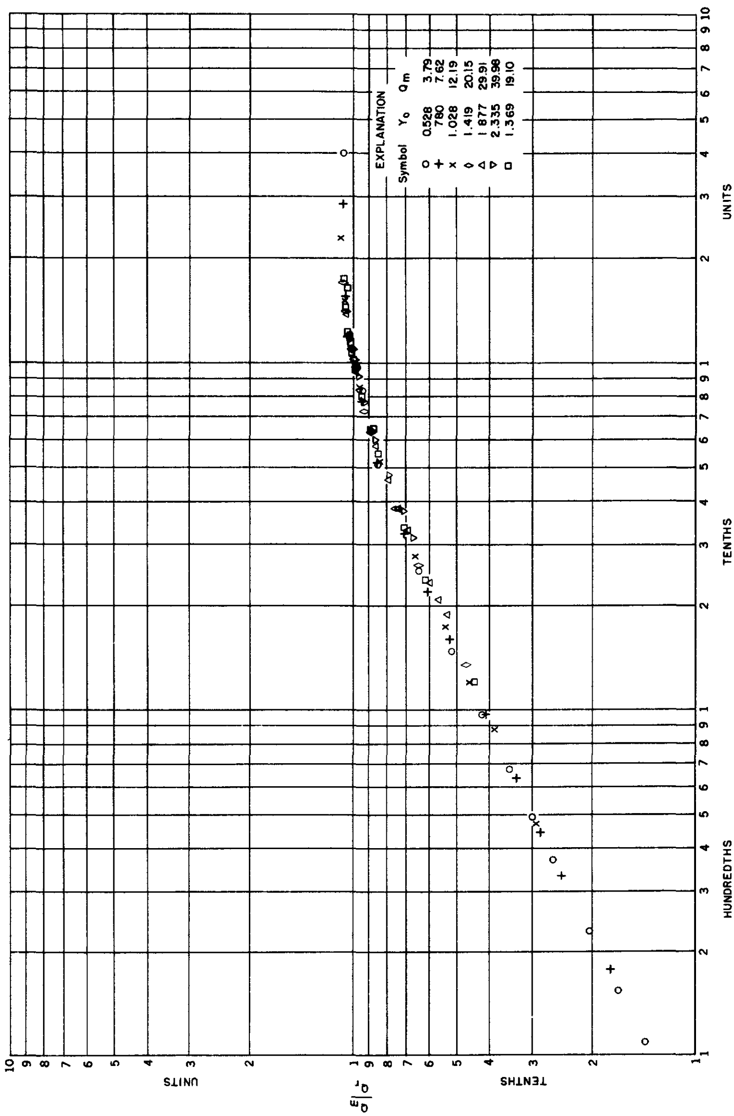


Figure 39.—Boyer diagram, for cross section 4, to accompany figure 38.

drawn as before, is found to cross curve *C* at a stage of about 3 feet. The explanation, which is obvious, is the same as that found at many stream-gaging stations: because of the increased roughness of cross section 4 over cross section 2, the channel capacity is much less, and above a stage of about 3 feet on gages *B* and *C* the discharge capacity of the channel is less than that of the dam. In other words, the dam is now subject to submergence at high discharge, even though only uniform channel conditions exist downstream from the dam; the dam becomes drowned by fixed channel control. The obvious procedure, well known to experienced rating-curve analysts, is to draw the transition curve *B'* between curves *B* and *C*. The curve *B-B'-C* thus becomes the rating for gage *B*.

With the technique described for cross section 2 (page 120) and each of the known profiles for cross section 4 points on curve *A* are computed. With logarithmic plotting and knowledge concerning the point of zero flow, curve *A*, the Q_r curve, is drawn. Fall in the reach *A-B*, or the F_r curve, is then determined as before, and is shown as curve *F*, at the left of figure 38.

Because the dam, at high discharges, becomes drowned by fixed channel control, a particular requirement for the F_r curve must now be met. It will be noted that the transition curve *B'* merges with curve *C* at a discharge of about 67 cfs. Now curve *C* is the uniform-flow curve so that, at gage *B*, uniform flow occurs for discharges greater than about 67 cfs. As there is an unobstructed uniform channel between gages *A* and *B*, it follows that for discharges greater than about 67 cfs uniform flow must occur throughout the reach *A-B*, and the fall in the reach must be equal to that of the channel bed, namely, 1.200 feet. It will be noted that the fall curve of figure 38 meets this requirement. If the rating curves were to be extended above gage height 3.5 feet, uniform flow would prevail under conditions of no backwater, so that for backwater conditions no satisfactory curve of Q_m/Q_r against F_m/F_r could be drawn; for above 3.5 feet Q_r would equal Q_0 and F_r would equal F_0, and the ratios would plot much the same as in figure 30. If backwater is to be expected at high stages, curve *B* should be drawn somewhat to the left of curve *C* even at high stages. The final position of curve *B*, and therefore curve *F*, should be determined by trial and error. The position of curve *B* and curve *F* that results in the least scatter of points in the Boyer diagram (fig. 40) is the best position for any one gage.

It will be obvious that, in figure 38, as in figure 36, the Q_r curve (curve *A*) and the F_r curve (curve *F*) are drawn for fixed-backwater conditions as dictated by the positions of curves *B* and *C*. If the rating at gage *B* were always as indicated by curve *B-B'-C*, the rating at gage *A* would be the simple two-dimensional relation of curve *A*. But now, as in cross section 2, let us assume that the stage-discharge relation at gage *B* may be influenced by (1) some phenomenon further downstream, such as backwater from a downstream confluence; and (2) by the operation of sluice gates in the dam. Under the first condition backwater effects in the reach *A-B* will be greater than, and under the second condition less than,

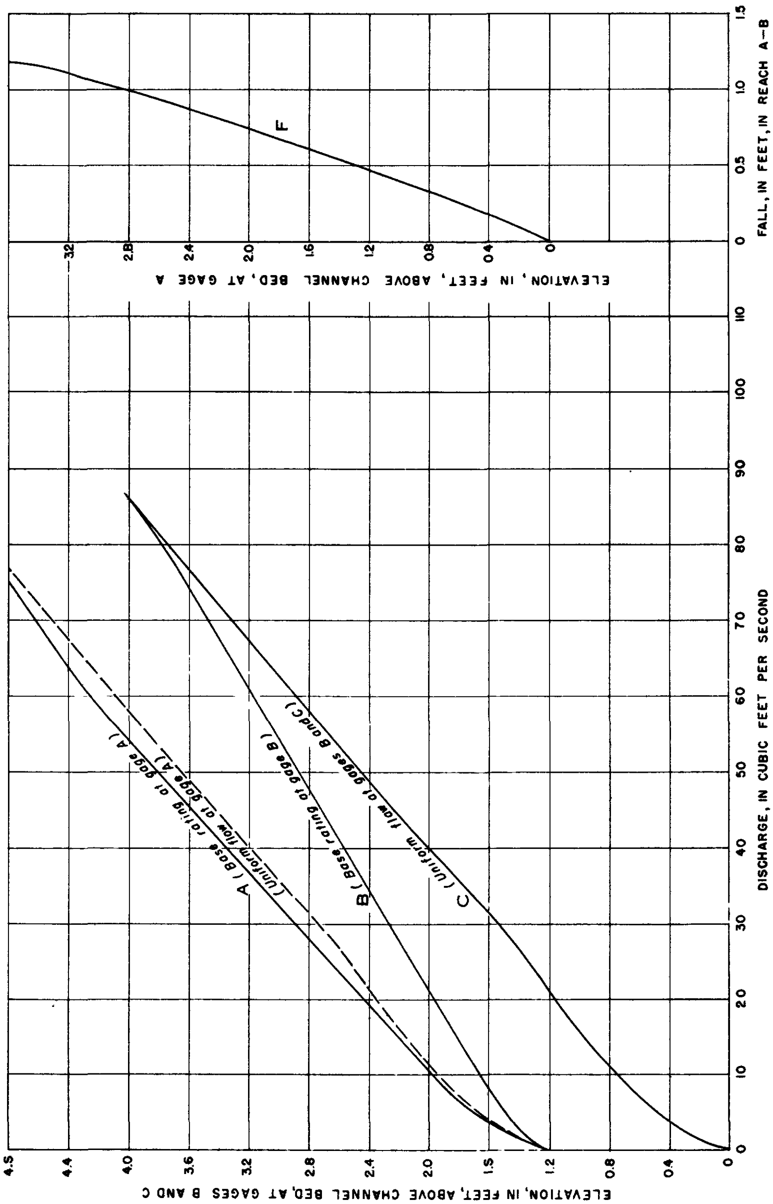


Figure 40.—Stage, discharge, and fall for a fixed-backwater condition, cross section 3.

the fixed-backwater conditions described by figure 38. (For high stages when uniform flow prevails at *B* and *C* opening of sluice gates between gages *B* and *C* could not lower the gage height at *B*.) Again use may be made of the backwater profiles for computing ratios of Q_m/Q_r and the corresponding values of F_m/F_r. Such ratios are presented as the Boyer diagram, figure 39, page 125. Again it will be noted that all the data fall within reasonable limits of a single curve. Given the depth at the upstream end of the reach, and the fall in the reach, computations based on factors from figures 38 and 39 will yield a close approximation to the true discharge.

Let attention now be directed to a similar treatment for cross section 3. Let a dam be erected in a channel of this cross section. Let it have characteristics identical with those described for other cross sections. Let gages *A*, *B*, and *C* be installed at locations as previously described.

The resulting curves for fixed-backwater conditions are shown in figure 40, page 127. Again curve *C*, the steady-uniform-flow stage-discharge relation curve, is drawn first. Curve *B* is next drawn, identical to curve *B* of figure 36, except that, as explained on page 126, it cannot cross curve *C*. As curve *B* approaches curve *C*, the former is bent slightly upward, the two curves gradually merging into one; the dam is completely drowned by fixed-channel control above the point of the merger. Here it occurs beyond the range of well-defined data but is assumed to be at stage of about 3.9 feet on gages *B* and *C*, so that uniform flow must exist for discharges beyond about 83 cfs. With the technique described for cross section 2 (page 120), each of the known profiles for cross section 3, knowledge concerning the point of zero flow and the discharge beyond which uniform flow must exist, and plotting logarithmically, curve *A*, the Q_r curve, is drawn. Fall in the reach *A-B*, or the F_r curve, is then determined as for the cross sections already described; it appears at the left of figure 40 as curve *F*.

Using these curves of Q_r and F_r for fixed-backwater conditions, and adding the assumptions that the stage-discharge relation at gage *B* may be influenced by (1) some phenomenon further downstream, such as backwater from a downstream confluence, and (2) by the operation of sluice gates in the dam, use is made of the backwater profiles for computing ratios of Q_m/Q_r and F_m/F_r. These ratios are plotted as the Boyer diagram, figure 41, page 129. It is apparent that the plotted points show more dispersion than was the case of the points of figure 37 and 39, but a mean curve would lie within about 4 percent of all the data. In other words, use of figures 40 and 41 will provide results within about 4 percent of the true discharges for cross section 3.

Consider now a similar reach of cross section 5, with the same obstruction as before, and gages *A*, *B*, and *C* located in the same positions. In figure 42, page 131, curve *C* again presents the steady-uniform-flow stage-discharge relation. Curve *B*, drawn as before, now intersects curve *C* at a discharge of about 11 cfs, indicating that the dam is drowned by fixed channel control at this very low

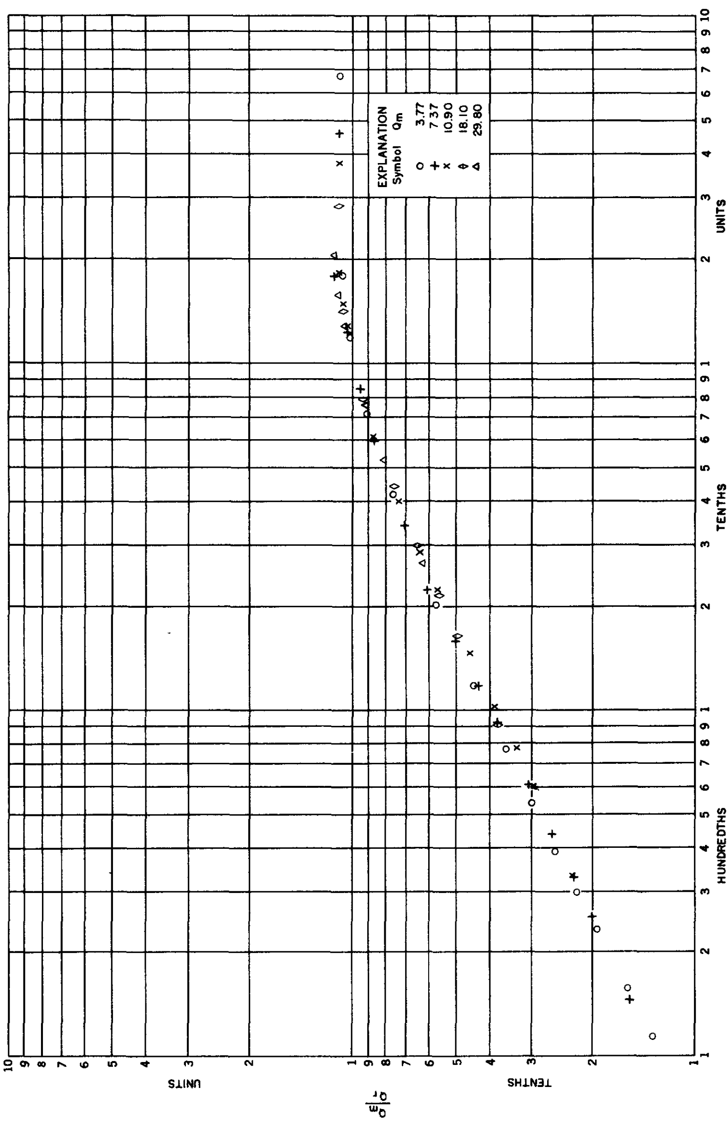


Figure 41. —Boyer diagram, for cross section 3, to accompany figure 40.

value, and that uniform flow would exist in reach A-B for higher discharges (or for a stage greater than about 1.7 feet at the upper end of the reach). In view of the results to be expected when Q_r equals Q_0, it becomes necessary to shift curve B to the position B' to obtain even reasonably satisfactory approach to a single curve of Q_m/Q_r against F_m/F_r. After such a shift, points on the Q_r curve (curve A) and the F_r curve (curve F) will appear as shown by the appropriate curves of figure 42. Values of Q_m/Q_r against F_m/F_r, computed as for the preceding cross sections, now plot as shown in the Boyer diagram, figure 43, page 132. It will be noted that dispersion in this plot is somewhat greater than in similar plots for other cross sections, but use of a single mean curve will result in errors of no more than about 6 percent. The shift of curve B to the position B' is the result of trial and error computations. Further discussion as to the propriety of trial and error solutions is given later in this report.

It is obvious that curve B' of figure 42 cannot reasonably be assumed to be a rating for fixed-backwater conditions at gage B unless those conditions are fixed by something other than the dam which previously has been described. Consider now the significance of the dam. Although it was assumed to be soundly built and carefully rated, and the rating was used in development of the curves of figures 36, 38, and 40 it was repeatedly subjected to degradation; first it was drowned by backwater from points farther downstream, and then it was riddled with sluice gates. Only under certain conditions was it allowed to control the stage-discharge relation, even at gage B; at other times, for a given discharge, the stage at B was sometimes higher, sometimes lower, than the dam alone would dictate. Except for the very important fact that it has provided the concepts which led to the curves of figures 36 to 41, the dam just as appropriately might have been a fisherman's net, or never have existed at all. But even though it were agreed that the dam did not exist at all, figures 36 to 43 still might be used for reliable computations of discharge under all combinations of stage and fall defined by the base data. (For stages higher than those defined by base data, additional adjustments to curve A, and hence to curves B and F, might be necessary to obtain highly reliable results.)

The purpose of the dam, which may now be considered to have been imaginary, was to give substance to the concept, in a uniform channel, of a base rating which differs from the uniform-flow rating. If a channel is indeed uniform, any rating other than the uniform-flow rating must be a very elusive phenomenon. It can be said only that, for a given discharge, there will be a given depth at the base gage, provided the fall has a properly related value. The assumption of the imaginary dam serves as a basis for correlation of the factors. The important fact to be derived is this: In the channels which have been studied, there may be found a stage-discharge relation (the Q_r curve) which, taken in conjunction with its associated fall relation (the F_r curve) will give close approximations to the true discharge, under all possible combinations of stage and fall, by the application of a single-curve relation Q_m/Q_r vs. F_m/F_r.

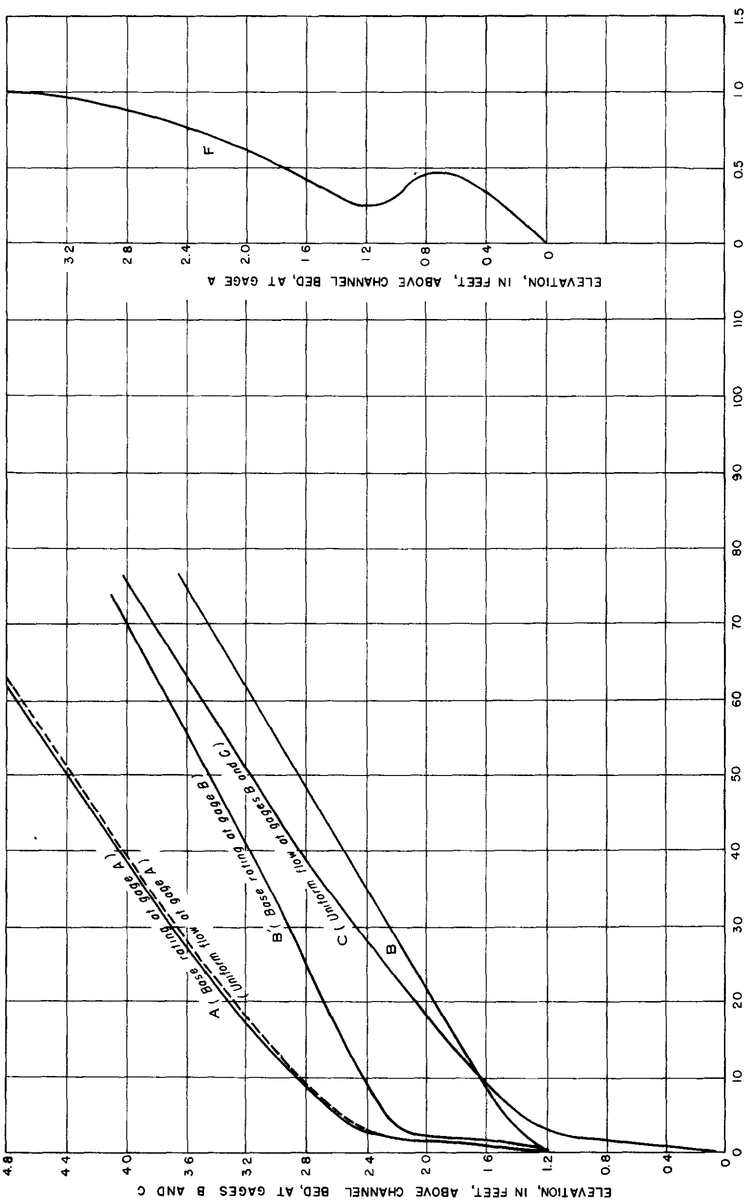


Figure 42. --Stage, discharge, and fall for a fixed-backwater condition, cross section 5.

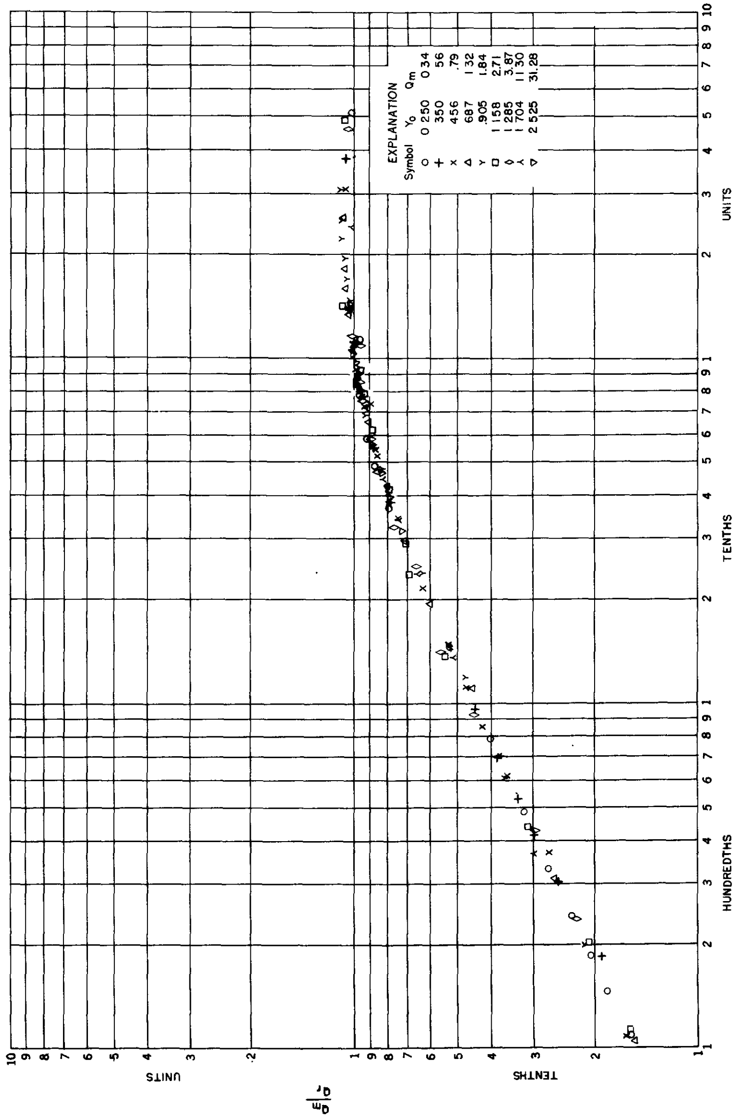


Figure 43.—Boyer diagram, for cross section 5, to accompany figure 42.

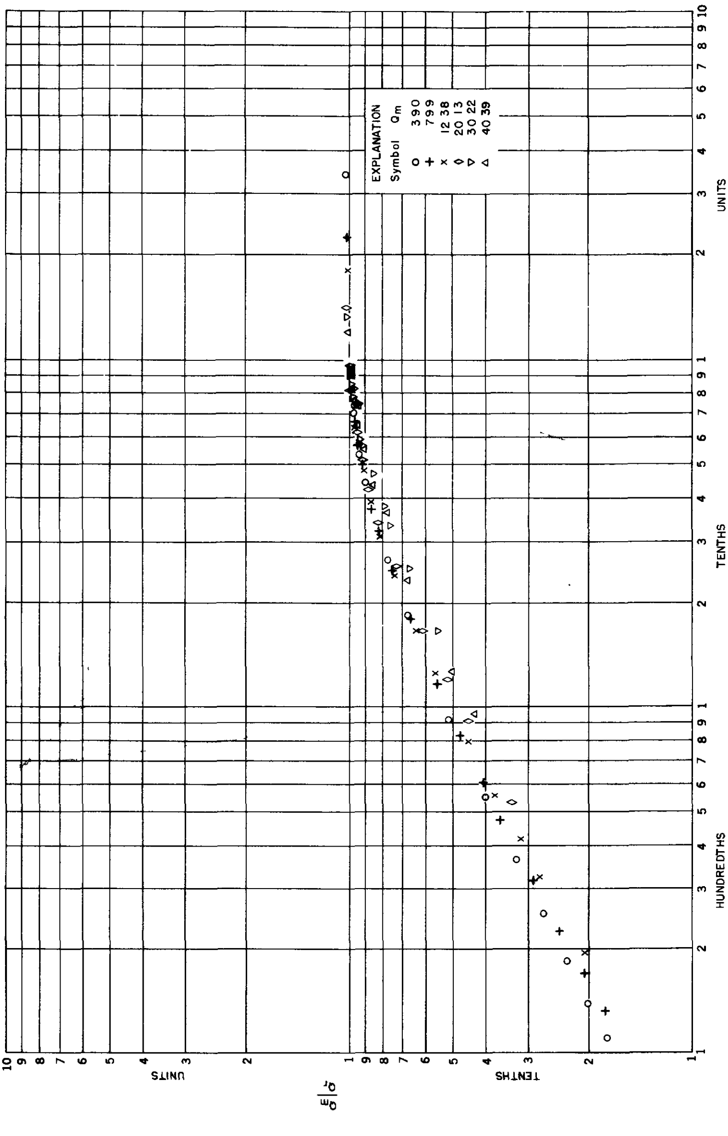


Figure 44. —Boyer diagram for the rating Q_r equals 0.99 Q_0 cross section 2.

It should not be assumed that just any stage-discharge relation may be used for the Q_r curve, even though the associated values of F_r are properly determined. For a given reach of a given channel there is an optimum position of the Q_r curve, (and the associated position of the F_r curve) which will give a minimum of dispersion of computed values of Q_m/Q_r when plotted against computed values of F_m/F_r. Many trials were required before the curves of figures 36, 38, 40 and 42 were evolved, and even these (particularly the curves for cross sections 3 and 4) may be subject to further improvement.

It has been suggested by Eisenlohr (personal communication) that the position of the Q_r curve might be obtained by multiplying the values from the Q_0 curve by some constant slightly less than unity, such as 0.99, 0.95, or even 0.90. Experience indicates that, for some channels (particularly a 400-foot reach of cross section 2), such technique is quite adequate, provided a particular optimum value of the constant is chosen. In fact, curve *B* (fig. 36) actually was drawn in such manner that the Q_r curve (curve *A*) would be in agreement with the relation $Q_r = 0.90\ Q_0$. It has been found, however, that a change in the shape of the cross section, or the length of the reach, or even the channel roughness, may result in a change in the optimum relation between Q_r and Q_0. For cross section 4, the relation $Q_r = 0.95\ Q_0$ was found to give better results than $Q_r = 0.90\ Q_0$, but neither was as satisfactory as the Q_r curve which has been shown as part of figure 38 and which results from the assumed position of curve *B*. For cross section 5, also, no coefficient of Q_0 could be found which provided a satisfactory position of the Q_r curve.

To further illustrate the variations which may occur as a result of a change in the position of the Q_r curve, it was assumed that, for a 400-foot reach of cross section 2, Q_r was equal to 0.99 Q_0. The associated F_r curve was computed from the backwater profiles by the same principles that were used for computing the F_r curves of figures 36, 38, 40, and 42. Ratios of Q_m/Q_r and F_m/F_r were then computed and were plotted as the Boyer diagram, figure 44, page 133. Comparison of this plot with the Boyer diagram of figure 37 gives evidence of the fact that, in a 400-foot reach of cross section 2, $Q_r = 0.99\ Q_0$ places the Q_r curve further from the optimum position than does $Q_r = 0.90\ Q_0$.

It appears that, for cross section 2, the relation $Q_r = 0.90\ Q_0$ is adequate because of the fact that, by happy coincidence, it also fulfills another and apparently more important requirement for the position of the Q_r curve. The nature of this other requirement can be described only by a more analytical approach to the whole problem, and will therefore be reserved for discussion in the following subsection.

ANALYTICAL APPROACH TO FIXED-BACKWATER RATINGS

The preceding article, presenting fixed-backwater ratings, has been designed to provide, both to the layman and to those who are constantly engaged in the analysis of slope-affected stream-flow stations, a simple picture and a simple solution of a very complex problem. In fact, in our present state of knowledge of the problem, it may represent the best picture and the best solution which can now be provided. However, as an approach to the question "Why are these things true?" some worth-while bits of information may be provided from an analytical viewpoint.

In an earlier paragraph (p. 86) it was explained that, for a very short reach, if the correction were made for change in velocity head, the relation

$$Q_m/Q_0 = \sqrt{F_m/F_0}$$

became an adequate correlation throughout the entire range of discharge and of backwater effect. In other words, if the slope of the energy line, S, at the base gage could be used in place of the fall, F, in a finite reach, discharge would vary with the square-root relation of the slopes. But as field investigations are necessarily based on fall in a finite reach, it is impractical to determine S. If a method could be evolved by which this observed fall in a finite reach could be corrected to become representative of the energy slope at the base gage, much of the problem would be solved.

It becomes appropriate, therefore, to look at the relation between these two factors. It necessarily is complex, depending at least on the cross section of the channel (including roughness), the length of reach, the discharge, and the amount of backwater. But, for any gaging station, the channel cross section and the length of reach are fixed; thus it is appropriate that they be assigned fixed values for this study. Furthermore, for a given discharge, the amount of backwater is a unique function of the stage at the base gage. Thus, for a 400-foot reach of cross section 2, the ratio, t, between effective fall and observed fall ($400S/F_m$) will appear as in figure 45, page 136. In these computations S was computed from the relation, $S = S_0(Q/Q_0)^2$ (see p. 21) and F_m was computed through use of table 12. It will be noted that "t" is the ratio of effective fall to measured fall, if effective fall is considered to be the fall that is the product of the length of reach and the slope of the energy line at the base gage and therefore the fall which divided by the length of reach would give the correct slope of the energy line at the base gage.

Attention should first be directed to the major family of curves, drawn in heavy lines. It will be noted that there is a different curve for each discharge and that each curve intercepts the y axis at the point $y = y_0$. It also will be noted that, for each of these curves, t increases rapidly with a slight increase in depth (introduction of a small amount of backwater), but soon reaches a maximum value beyond which, as depth increases, t decreases. Most notable of all is the fact

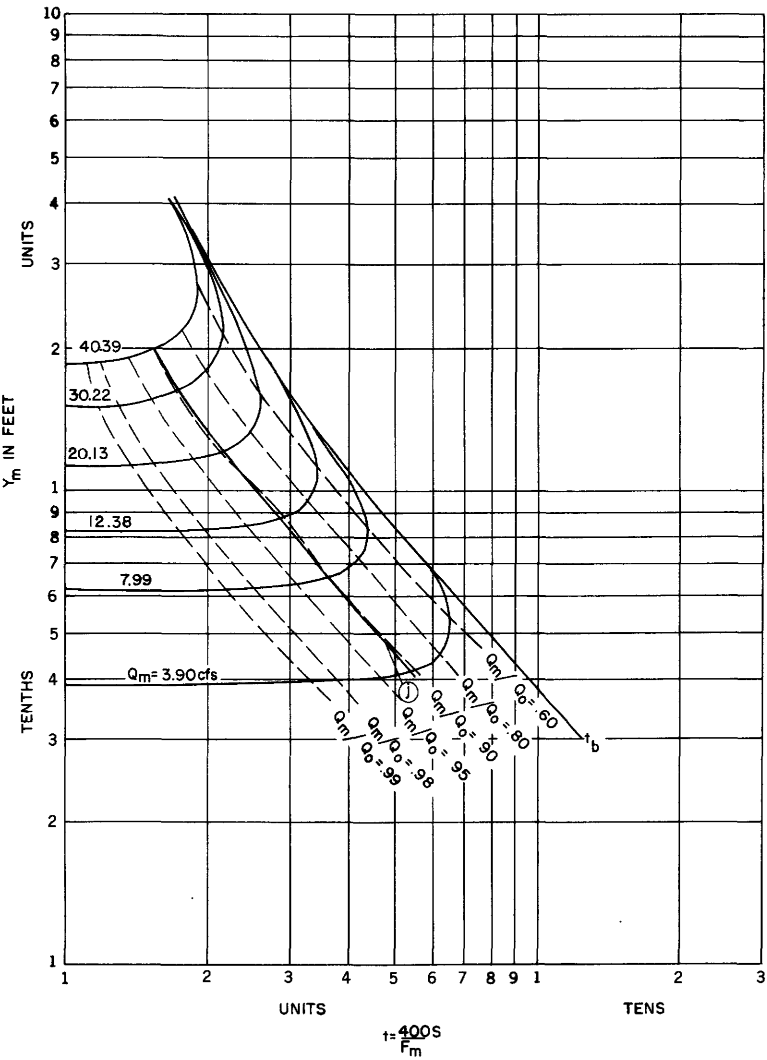


Figure 45.—Relation between observed fall and effective fall, 400-foot reach, cross section 2.

that, as depth increases, all the curves tend to merge into a single bounding curve, which has been labelled t_b. For very shallow depths the position of this curve has been estimated.

Attention now is invited to the second family of curves, drawn in light dashed line. Each of these lines indicates the variation between t and y, not for a given discharge, but for a given value of the discharge ratio, Q_m/Q_0. (For example, to compute the point common to the curves $Q_m/Q_0 = 0.80$ and $Q_m = 3.90$ cfs: $Q_0 = 3.90/0.80$, or 4.875 cfs, and, interpolating from table 42, Bulletin 381, $y_0 = 0.448$ foot. Interpolating from table 12, page 59, by the method described on page 81, $F_m = 0.12221$ foot. From formula (24), page 51, S varies as the square of Q_m/Q_0, so $S = 0.8^2 \times 0.003 = 0.00192$; $t = 400S/F_m = 6.284$.) It will be obvious that the line for $Q_m/Q_0 = 1.00$ will be coincident with the y axis. As this Q_m/Q_0 ratio is made smaller, the Q_m/Q_0 curves move rapidly to the right, so that for $Q_m/Q_0 = 0.60$, the curve is quite near the position of the t_b curve. For practical purposes, it may be considered that, for values of Q_m/Q_0 of about 0.4 or less, all the Q_m/Q_0 curves will be coincident with the t_b curve.

By definition, t is the coefficient to be applied to the observed fall to obtain effective fall at the base gage. After this correction has been applied, the relation $Q_m/Q_0 = \sqrt{F_m/F_0}$ will be valid. For those cases, then, in which Q_m/Q_0 does not exceed about 0.4, (remembering that here $t = t_b$),

$$Q_m/Q_0 = \sqrt{t_b\ F_m/F_0}\,. \tag{26}$$

Consider now the case in which Q_m/Q_0 is much larger, say 0.90. Now t is much smaller than t_b. Assume a condition such that, even though the proper Q_m/Q_0 to use, may vary slightly with depth, t will be proportional to t_b, or $t = f_b\ t_b$, f_b being a constant less than unity, say 0.60. Then

$$Q_m/Q_0 = \sqrt{f_b\ t_b\ F_m/F_0}\,. \tag{27}$$

It must be remembered that, for any observed depth, y_m, Q_0 is the discharge which would flow at y_m provided F_m were equal to F_0, and that Q_m is a variable discharge depending upon both y_m and F_m. For any particular y_m, let it be assumed that Q_m has a specific value, Q_r, for which the associated value of F_m will be the specific value F_r. Then, in place of equation (27), it may be written that

$$Q_r/Q_0 = \sqrt{f_b\ t_b\ F_r/F_0}\,. \tag{28}$$

If, now, in addition to f_b being a constant, it were possible at the same time that Q_r/Q_0 were also a constant, such as j, then

$$F_r = j^2 F_0/(f_b\, t_b). \tag{29}$$

Another look at figure 45 will now be profitable. Comparing the family of curves of Q_m/Q_0 with the t_b curve, it will be noted that at neither limit of the family of curves is there a proportional relation to (meaning, on log-log paper, equidistant from) t_b. For high values of Q_m/Q_0 these curves diverge from t_b with decreasing depth, while for $Q_m/Q_0 = 0.60$ the curve converges on t_b as depth decreases. Between these limits, one curve will be found which, more nearly than any other, is proportional to t_b. If one such curve be truly proportional, then both of the above requirements – that the discharge ratio, j, be constant and that f_b be constant – are fulfilled, and F_r will vary only with t_b.

In the development of figure 36 conditions were chosen such that curve A actually conforms to a constant ratio of 0.90 for Q_r/Q_0. And since Q_r is merely a specific case of Q_m, it follows that the rating of figure 36 is in conformance with the curve of figure 45 which is labelled $Q_m/Q_0 = 0.90$. On this figure has been added in heavy continuous line the curve $t = 0.59\ t_b$. It will be noted that the two curves are in close agreement throughout the range of well-defined data. Since the lines are so near together, for practical purposes they may be regarded as the same, hereafter referred to as the j-line. It now is possible to make some numerical substitutions in equation (29), using $j = 0.90$, $f_b = 0.59$, and remembering that, in this case, $F_0 = 0.003 \times 400 = 1.20$ ft. Hence $F_r = 1.65/t_b$. Using this equation and values of t_b from figure 45, it will be found that, up to the higher limit of well-defined data, computed values of F_r are in very good agreement with values from curve F (the F_r curve) of figure 36.

It should be further noted that, dividing equation (26) by equation (28),

$$Q_m/Q_r = (1/\sqrt{f_b})\ (\sqrt{F_m/F_r}). \tag{30}$$

As the validity of equation (26) was limited to discharge ratios of about 0.4or less, a similar limitation must be imposed upon equation (30). Using $f_b = 0.59$, equation (30) becomes:

$$Q_m/Q_r = 1.3\sqrt{F_m/F_r}\ ,$$

which is, indeed, the equation of the lower end of the curve (discharge ratios less than about 0.4) of figure 37.

The above limitation on equation (30) may be removed by substitution in place of the constant f_b another factor, f, which will vary primarily with the discharge ratio. The nature of this parameter, f, may be examined by referring again to figure 45. In general, for any given value of y_m, f is the value of t from the j-line divided by the value of t from the appropriate Q_m/Q_0 curve. Thus f_b is merely a special case of f, being appropriate for all values of Q_m/Q_0 less than about 0.4, as these all lie on a common t-curve (t_b) which is proportional to the j-line. But as values of Q_m/Q_0 increase to 0.6 and above, the value of f increases, so that, by the time Q_m/Q_0 becomes equal to Q_r/Q_0, the value of f is unity. Thus when Q_m and Q_r are equal, equation (30) becomes $Q_m/Q_r = \sqrt{F_m/F_r}$, which merely means that the curve of relation between discharge ratio and fall ratio, such as figure 37, will pass through the point 1,1. As Q_m/Q_0 becomes greater than Q_r/Q_0 the value of f becomes greater than unity, reaching as a limit the value of t from the j-line when Q_m/Q_0 is equal to unity.

To illustrate by example, let Q_m = 3.90 cfs and F_m = 1.20 feet. These are conditions of uniform flow, so Q_m/Q_0 will be equal to unity, and y_m (see table 42, Bull. 381, Lansford and Mitchell, 1949) will be 0.391 ft. From figure 36, Q_r = 3.50 cfs, and F_r = 0.165 foot. From figure 45, f = 5.8. Required, to check Q_m by substitution in equation (30), using $f = 5.8$ in place of f_b.

$$Q_m/3.50 = (1/\sqrt{5.8})\ (\sqrt{1.20/0.165})$$

$$Q_m = 3.50 \times 0.415 \times 2.70 = 3.50 \times 1.12 = 3.92 \text{ cfs},$$

which is well within acceptable limits of the known value of 3.90 cfs and demonstrates the applicability of the value of f. But at this high limit of Q_m/Q_r (that is; $Q_m/Q_0 = 1$), it is obvious that f will vary also with y_m, because of the great divergence between the j-line and the line $Q_m/Q_0 = 1$. To illustrate further, let Q_m = 40.39 cfs and F_m = 1.20 feet. Again these are conditions of uniform flow, so that $Q_m/Q_0 = 1$, and y_m (see table 42, Bull. 381, Lansford and Mitchell, 1949) will be 1.840 feet. From figure 37, Q_r = 36.3 cfs, and F_r = 0.60 foot. From figure 45, f = 1.62. Checking Q_m by substitution in equation (30),

$$Q_m/36.3 = (1/\sqrt{1.62})\ (\sqrt{1.20/0.60})$$

$$Q_m = 36.3 \times 0.79 \times 1.41 = 36.3 \times 1.11 = 40.4 \text{ cfs},$$

which agrees with the known value of 40.39 cfs and again demonstrates the applicability of the value of f. But here f has the value of 1.62, compared to 5.8 for the first example. Obviously f here has become largely a function of y_m. In fact, there is considerable evidence that, when $Q_m/Q_0 = 1$, f is inversely proportional to F_r, which is, of course, a function of y_m. It is for this reason that $(1/\sqrt{f})(\sqrt{F_m/F_r})$ remains fairly constant under uniform-flow conditions, as evidenced by the value of 1.12 in the first example and 1.11 in the second. The curve in figure 37 could be drawn to make Q_m/Q_r equal 1.11 for all values of F_m/F_r greater than 2, for we could be assured that under these conditions Q_m should very nearly equal Q_0.

The variation of f with y_m might seriously cloud the whole picture except for one saving circumstance: the value of f does not vary appreciably with depth except when the Q_m/Q_0 ratio is very near to unity. In other words, the two important factors affecting f operate, largely, one at a time, rather than simultaneously.

When the amount of backwater is great (small values of Q_m/Q_r), f is dominated by the discharge ratio; when the amount of backwater is small (values of Q_m/Q_r greater than unity and approaching the reciprocal of Q_r/Q_0), f is dominated by y_m. In some instances the two factors affecting f may operate simultaneously, and with some lack of harmony.

The foregoing observations are of course insufficient for the complete preparation of a rating analysis and are particularly handicapped by lack of a convenient method of computing t_b. They do however provide an insight into the problem of using observed fall in a reach as a measure of the friction slope at a gage and thus provide a firm basis for the development of stage-fall-discharge relations under field conditions. The following are the conclusions based on these analyses:

(1) There is an optimum position of the Q_r curve. The ideal position is that in which, for a plot similar to figure 45, the j-line will be for all values of y_m a constant ratio to the t_b line and at the same time will have a constant value, j, of Q_m/Q_0. If both these requirements can be entirely satisfied, $Q_r = jQ_0$. But in certain channels, such as cross section 5, both requirements cannot be met simultaneously. In such channels some dispersion must be expected in the plots of Q_m/Q_r against F_m/F_r, as described under (4) below. Experience indicates that such dispersion will be minimized by holding to the requirement that the j-line maintain a constant ratio to the t_b curve and selecting that ratio such that j varies as little as possible from a constant value of Q_m/Q_0. The dispersion in the plotting of the values of Q_m/Q_r against F_m/F_r, as evidenced by figures 39, 41, and 43, is due largely to the fact that for cross sections 3, 4, and 5 the two requirements mentioned above cannot be simultaneously satisfied.

(2) A forthright application of the principle that discharge varies as the square root of the friction slope would indicate that a correction factor should be applied to observed fall to transform it to friction slope (at the base gage). This, however, would lead to a complicated system of corrections. Instead, it is possible to approximate these corrections closely by combining most of them into a single parameter, the rating fall, having the form

$$F_r = j^2 F_0/(f_b\, t_b). \tag{29}$$

If the optimum position of the Q_r curve has been otherwise determined, values of F_r may perhaps be more conveniently computed from other data such as backwater profiles; but results should be substantially the same as from equation (29).

(3) If the Q curve has been located in the optimum position, and if the F_r curve has been drawn to agree, reasonable conformance to the relation

$$Q_m/Q_r = (1/\sqrt{f})(\sqrt{F_m/F_r}) \qquad (31)$$

may be expected, in which f is a further correction factor needed in the adjustment of F_m to effective fall. As f is largely a function of Q_m/Q_r it can be incorporated in the Q_m/Q_r against F_m/F_r curve. For low values of the discharge ratio, f will have the constant value f_b (t from the j-line divided by t from the t_b curve); when Q_m is equal to Q_r, f will be equal to unity; as values of Q_m/Q_r approach $Q_m/Q_0 = 1$ as a limit ($Q_m/Q_r = 1.11$ for the example shown in figure 45), f will vary inversely with F_r so as to make $Q_m/Q_r = (1/\sqrt{f})(\sqrt{F_m/F_r})$ a constant for all stages. In the range of Q_m/Q_r from about 0.60 to the maximum value of Q_m/Q_r, f will not be entirely a function of Q_m/Q_r but will be affected somewhat by variation in stage. The slight dispersion evidenced by figure 37 is largely the result of being unable to correct for this effect.

(4) The above paragraphs have considered the nearly ideal conditions, as in cross section 2, which makes possible the establishment of a j-line—that is, a curve, Q_m/Q_0 plotted against y_m, which will be in substantial agreement with another curve, $f_b\ t_b$ plotted against y_m. Analyses of other cross sections reveal that, in many cases, a j-line cannot be established, but that the $f_b\ t_b$ curve, regardless of the value assigned to f_b, will cross several of the curves of Q_m/Q_0. As the reasons for this departure from ideal conditions lie in the relation of various combinations of channel shape and channel roughness future exploration along theoretical lines would be likely to have little practical value. Rather, as has been indicated earlier, trial and error solution may be the most practical approach. The analytical approach can be used to explain why certain procedures should be followed, but the final position of the three curves of relation, y against Q_r; y against F_r; and Q_m/Q_r against F_m/F_r can best be determined by a series of trial computations each one of which should result in less scatter from a mean curve of Q_m/Q_r against F_m/F_r until the best solution is reached. In these successive trials parts of the Q_r curve, the F_r curve, or the Q_m/Q_r curve should be relocated if such relocation will result in a more consistent solution.

APPLICATION TO FIELD PROBLEMS

From those who are concerned with the analysis of rating curves for natural watercourses affected by variable fall questions may appropriately arise. What is here that will be helpful to us? Warning was given on page 4 that the discussions were to be confined to steady flow in prismatic channels, and that the irregularities of natural watercourses and the phenomenon of changing discharge point toward further complications. Furthermore, the amount of data provided for these analyses is very large, adequately covering the entire range of all combinations of stage and discharge; in contrast, field data often are meager in quantity and limited in range of conditions. How, then, can the foregoing analysis be useful to us in drawing our rating curves?

Let it be pointed out that the value of laboratory work lies largely in the establishment of general principles, rather than in the solution of specific problems. In complex and specialized situations laboratory results cannot take the place of engineering judgment, but familiarity with laboratory results should be expected to improve the quality of the engineering judgment that must be used. Such familiarity should be of assistance in identifying the significant factors of a complex problem and in providing qualitative evaluation of their effect. It is from this viewpoint that the following suggestions are made relative to analysis of field ratings.

In the first place, few channels in nature are uniform channels. Some may so nearly approach this condition that the techniques for uniform channels, as summarized in following paragraphs, will need to be employed. But many actually do have dams which are real, and which are only occasionally drowned by backwater or even affected by operation of gates. These are the simple cases. Here, without doubt, the Q_r and F_r curves should be developed from field data observed at times when the dam is fully effective; and here, if the dam is only subject to drowning by backwater from a downstream confluence, all values of Q_m/Q_r and F_m/F_r will be less than unity. On the other hand, if the dam is not subject to such drowning, but only to manipulation of gates, and if the Q_r and F_r curves have been developed for the particular case of the dam being fully effective (that is, all gates closed), then all values of Q_m/Q_r and of F_m/F_r will be greater than unity.

At the other extreme are those cases in which the only control appears to be a long reach of channel of comparatively uniform cross section and slope, plus backwater effects induced by some downstream confluence. Here the analyst may have a choice between two or more procedures. It may be that he will wish to try more than one of the possible techniques, finally settling on the one which appears best to fit his data and still offers a reasonable basis for any necessary extrapolation outside the range of conditions covered by the data. The possible techniques may be divided into two general categories, or simple modifications thereof: uniform-flow ratings and fixed-backwater ratings.

The uniform-flow rating, or any modification thereof, involves the assumption that the associated fall is a constant—that is, regardless of stage, the rating discharge will occur only when the fall has some value which is the same for all stages. Such analyses are sometimes referred to as "constant-fall ratings." Of these, the well-known unit-fall rating, expressed by the relation $Q_m/Q_r = \sqrt{F_m}$, F_r being assumed to equal unity, is the simplest case. For stations that always operate under a high amount of backwater, such a rating may be quite appropriate. If discharge measurements cover the entire range of flow conditions, and if such measurements closely conform to a unit-fall rating, there is no need to use more complicated techniques.

Care must be exercised, however, that the unit-fall analysis is not used where the variation in backwater is very great. Here this method may deteriorate into a stage-fall-discharge diagram such as discussed on pages 81 to 82, and illustrated by figures 18 and 19. Under such conditions of great variation in backwater, use of the constant-fall rating should be associated with three-dimensional plots of the Boyer diagram, as discussed on pages 109 to 117 and illustrated by figures 32 to 34. For Boyer diagrams in which the points scatter (as the data scatter in figure 35 (above), page 116), the points may be labelled with their observed gage height, and an examination may be made to determine whether a family of curves of equal gage height, similar to those in figures 32 to 34, can justifiably be drawn. If so, further improvement in the plot might be obtained by computing backwater profiles for desired discharges in the slope reach and abstracting needed data on stage, fall, and discharge from the computations. In development of the rating, such computed data should of course be given less weight than is given to reliable field observations. For favorable results along these lines, it appears that the field problem should exhibit the following features:

1. For a given gage height at the base (upstream) gage, there should be a wide variation of discharge. In other words, the station should be such that high gage height may result at one time from high discharge, at another time primarily from high backwater effects.

2. The channel gradient should be comparatively constant throughout the slope reach and "mild" by the technical classification of fluid mechanics. Within this limitation, the steeper the gradient, the more pronounced will be the family tendencies of the curves of relation.

3. The channel cross section should be as uniform as possible within the slope reach—that is, the stage-discharge relation at the base gage, when free of backwater, should be completely channel controlled. If, without backwater, the stage-discharge relation is materially affected by a section control, the rating fall should not be considered to be constant.

4. The reach should be free of backwater effects for such periods as to permit the development by usual stream-gaging methods of the two-dimensional no-backwater relation between stage and discharge.

For stations which tend to meet the above four requirements and for which a large number of measurements under a wide range of backwater are available, it seems appropriate to plot diagrams similar to figure 28 directly from observed data.

As an alternative to the procedure of the preceding paragraph the analyst may wish to undertake the second of the general catagory of techniques—namely, the development of a fixed-backwater rating. This involves the assumption that the associated fall is a variable—that is, for any given stage the rating discharge will occur only when the fall has some value which is a unique function of the observed stage, but not necessarily the same for different stages. In this case it would be desirable, from the analytical viewpoint, to place the Q_r curve in such position that the associated F_r curve could be expressed as $F_r = j^2F_0/f_bt_b$. (See pp. 137, 140.) From the practical viewpoint, however, it cannot be hoped that data will be available from which to evaluate all the factors in the above relation. Furthermore, in many cases it may be impossible to develop the no-backwater rating from discharge measurements because of the persistence of backwater conditions. As a substitute, the following steps are suggested:

1. Study the profiles of the floods of highest discharge. If these are found to be nearly parallel (having the same fall in the reach between the two gages), it is likely that this fall may be used as F_0, the fall that the maximum value of F_r cannot exceed. If these profiles are found not to be nearly parallel, study should be made of causes of backwater. Those profiles which appear to be least affected by backwater should be given greatest weight in estimating the maximum value of F_r.

2. Plot discharge measurements for which the fall is near to that found in step 1. Avoid use of measurements made under rapidly changing discharge. Using the plotted measurements as a guide, and assuming a constant fall curve (that is, the rating fall is the same for all stages), draw a tentative stage-discharge curve, which will be an approximate Q_0 curve. To obtain sufficient definition of this curve in the absence of suitable discharge measurements it may be desirable to evaluate the stage-conveyance curve from channel surveys.

3. Plot the remainder of the discharge measurements and make an auxiliary plot of stage against fall for each measurement. Working with the two plots together, draw the Q_r curve approximately 10 percent to the left of the tentative discharge curve at low stages, but nearer to the tentative curve at highest stages. Draw the F_r curve to agree (that is, if the Q_r curve passes through a discharge measurement, the F_r curve

should pass through the corresponding plotted fall; if the Q_r curve passes to one side of a discharge measurement, the F_r curve should pass to the same side of the corresponding plotted fall.) The F_r curve may be expected to approach zero fall at low stages, since at low stages nearly all of the uniform flow discharge (Q_0) can be obtained with a small observed fall. Obviously, when the discharge is zero, the fall must be zero; or, in event there is no water in a part of the slope reach, the concept of fall becomes meaningless.

4. Compute and plot Q_m/Q_r against F_m/F_r. Draw a curve among the points, causing it to pass through the point 1,1. If the curve extends above this value, it should rapidly become nearly parallel to the F_m/F_r axis. For low values of Q_m/Q_r (generally less than 0.50) the curve should approach one whose equation is $Q_m/Q_r = c\sqrt{F_m/F_r}$. Present experience indicates that for low ratios the value of c will be greater than unity and will depend upon the length of reach, the roughness of the channel, and perhaps the geometry of the cross section.

5. Assuming the F_r curve and the curve of relation between ratios to be correct, compute and plot the values of Q_r corresponding to each discharge measurement. The Q_r curve may now be redrawn to give better agreement with the plotted values of Q_r. Percentage variation between the adjusted value of each discharge measurement and the corresponding value from the Q_r curve should now be computed. If these variations are found to be within acceptable limits, the analysis may be considered as closed, and the necessary tables may be prepared.

If variation between adjusted measurements and the Q_r curve is great, further improvement sometimes may be obtained by assuming that the new Q_r curve is correct and by repeating appropriate portions of the above procedure, beginning with step 3. It may sometimes be helpful to compute the value of F_r that would make the scattering measurements plot on the Q_r curve and then adjust the F_r curve to average these values, keeping a smooth F_r curve. Then repeat steps 4 and 5. This is the procedure referred to earlier in this report as the trial-and-error method.

It is possible that with some modifications the procedure for developing fixed backwater ratings in uniform channels as outlined above may be equally applicable to reaches in which a section control is effective at low stages or even to reaches that include nonuniform-channel configurations. Laboratory verification of this presumption would be difficult in view of the many possible combinations of conditions that would need to be investigated. Perhaps the best verification would be through application to actual field conditions.

It will be noted that only in a few instances do these suggested techniques for solution of field problems differ materially from methods that long have been in use in many Geological Survey offices. Items 1 and 2, immediately above, are somewhat more far-reaching than heretofore proposed, but should result in a better understanding of the basic hydraulics of the particular slope reach, an approximation to the no-backwater rating, and a firmer foundation on which to base the analysis. The most outstanding innovation is the suggestion, in connection with uniform-flow (or constant-fall) ratings, of the three-dimensional aspect of the Boyer diagram, y_m being the third factor; but, if a constant-fall curve is assumed, the validity of the three-dimensional Boyer diagram is adequately documented. Beyond these points the conclusions of this report are largely confined to a clarification of the various types of slope-affected ratings, the conditions under which they may be expected to be appropriate, and to a verification by laboratory study of the fact that many of the methods of analysis now in use by the Geological Survey are, indeed, very appropriate.

SYMBOLS

(Except as otherwise explained in the text, numbers used as subscripts refer to particular points in the channel—for example, E_1 refers to total energy per pound of fluid at point 1, with respect to an established datum plane.)

A	Cross-sectional area of the fluid. (ft^2)
b	Width of channel at free surface of fluid. (ft)
b, as superscript	Variable exponent of the friction slope. (dimensionless)
b, as subscript	See immediately following prefixed symbol without subscript.
c	An empirical constant applied to hydraulic radius. (ft) Also used as a miscellaneous constant.
c, as subscript	Indicates prefixed symbol applies specifically to conditions of critical flow.
c_e	Velocity-distribution coefficient, based on principle of energy. (dimensionless)
c_m	Velocity-distribution coefficient, based on principle of momentum. (dimensionless)
d, as prefix	Indicates mathematical derivative of accompanying symbol.
E	Energy per pound of fluid, with respect to an established datum plane. (ft)
E_t	Total flux of energy per unit of time. (ft-lbs/sec)
F	Fall; difference in elevation of water surface between the two ends of a reach of channel. (ft)
F	Froude number; $c_m V^2/gR$. (dimensionless)
f	The ratio of t for any value of j (see below) to the value of t for any Q_m/Q_0, y_m being held constant. (dimensionless)
f_b	The value of f when t is equal to t_b (see below; dimensionless)
g	Acceleration of gravity; used as 32.16 ft/sec^2.
H	Specific energy; energy per pound of fluid with respect to bottom line of cross section as datum plane. (ft)
h	Elevation of water surface with respect to a given point in the fluid. (ft)
j	The ratio of the rating discharge to the uniform-flow discharge; Q_r/Q_0. (dimensionless)
K	Conveyance of a cross section. (cfs)
k	Conveyance divided by area of the cross section. (ft/sec)

L	Distance along the channel from point at which channel bed intersects datum plane. (ft)
M	The M-function; $M = A\sqrt{A/b}$. (ft$^{2\frac{1}{2}}$)
m, as subscript	Except when used with c, indicates prefixed symbol applies specifically to an actual (measured) condition.
n	Coefficient of roughness in the Manning formula. (ft$^{1/6}$?)
o, as subscript	Except when used with S, indicates prefixed symbol applies specifically to conditions of uniform flow.
p	Pressure intensity per unit area. (lbs/sq ft)
Q	Discharge. (cfs)
R	Hydraulic radius; A/w.p. (ft)
R'	Weighted hydraulic radius, as used for flood-plain cross sections. (See p. 42.)
R''	Weighted hydraulic radius, as computed for rectangular cross-sections. (See p. 42.)
r, as subscript	Indicates prefixed symbol applies specifically to a particular set of stage-fall-discharge conditions other than uniform flow that have been adopted as the base rating.
S	Slope of energy grade line; friction slope. (dimensionless)
$\bar{S}$	Mean of values of S for the two ends of the reach Δx. (dimensionless)
S_0	Slope of the channel bed. (dimensionless)
t	The ratio of effective fall at the base gage to observed fall in the slope reach. (dimensionless)
t_b	The maximum value of t for a given value of y_m. (dimensionless)
V	Mean velocity in a cross section; Q/A. (ft/sec)
w.p.	Wetted perimeter of a cross section. (ft)
x	Distance along the channel between any two cross sections. (ft)
y	Depth of fluid; perpendicular distance between stream bed and water surface; for this report, may be considered as vertical distance between stream bed and water surface. (ft)

Z	Elevation with respect to an established datum plane. (ft)
γ	Specific weight. (lbs/ft^3)
Δ	Indicates change in a reach of channel of such short length that under conditions of nonuniform flow the rates of variation within the reach are essentially the same as the respective rates of variation at any particular point within the reach.

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SUPPLEMENTARY TABLES

CHARACTERISTICS OF THE LABORATORY CHANNELS

AREAS OF FLOOD-PLAIN CROSS SECTION

Table 27.--Area, in square feet, of flood-plain cross sections

[Note: for depths less than 1.00, area is equal to depth]

y	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09	y
1.0	1.000	1.010	1.020	1.032	1.047	1.064	1.084	1.105	1.127	1.152	1.0
1.1	1.180	1.209	1.240	1.273	1.308	1.346	1.386	1.427	1.470	1.516	1.1
1.2	1.564	1.614	1.664	1.714	1.764	1.814	1.864	1.914	1.964	2.014	1.2
1.3	2.064	2.114	2.164	2.214	2.264	2.314	2.364	2.414	2.464	2.514	1.3
1.4	2.564	2.614	2.664	2.714	2.764	2.814	2.864	2.914	2.964	3.014	1.4
1.5	3.064	3.114	3.164	3.214	3.264	3.314	3.364	3.414	3.464	3.514	1.5
1.6	3.564	3.614	3.664	3.714	3.764	3.814	3.864	3.914	3.964	4.014	1.6
1.7	4.064	4.114	4.164	4.214	4.264	4.314	4.364	4.414	4.464	4.514	1.7
1.8	4.564	4.614	4.664	4.714	4.764	4.814	4.864	4.914	4.964	5.014	1.8
1.9	5.064	5.114	5.164	5.214	5.264	5.314	5.364	5.414	5.464	5.514	1.9
2.0	5.564	5.614	5.664	5.714	5.764	5.814	5.864	5.914	5.964	6.014	2.0
2.1	6.064	6.114	6.164	6.214	6.264	6.314	6.364	6.414	6.464	6.514	2.1
2.2	6.564	6.614	6.664	6.714	6.764	6.814	6.864	6.914	6.964	7.014	2.2
2.3	7.064	7.114	7.164	7.214	7.264	7.314	7.364	7.414	7.464	7.514	2.3
2.4	7.564	7.614	7.664	7.714	7.764	7.814	7.864	7.914	7.964	8.014	2.4
2.5	8.064	8.114	8.164	8.214	8.264	8.314	8.364	8.414	8.464	8.514	2.5
2.6	8.564	8.614	8.664	8.714	8.764	8.814	8.864	8.914	8.964	9.014	2.6
2.7	9.064	9.114	9.164	9.214	9.264	9.314	9.364	9.414	9.464	9.514	2.7
2.8	9.564	9.614	9.664	9.714	9.764	9.814	9.864	9.914	9.964	10.014	2.8
2.9	10.064	10.114	10.164	10.214	10.264	10.314	10.364	10.414	10.464	10.514	2.9
3.0	10.564	10.614	10.664	10.714	10.764	10.814	10.864	10.914	10.964	11.014	3.0
3.1	11.064	11.114	11.164	11.214	11.264	11.314	11.364	11.414	11.464	11.514	3.1
3.2	11.564	11.614	11.664	11.714	11.764	11.814	11.864	11.914	11.964	12.014	3.2
3.3	12.064	12.114	12.164	12.214	12.264	12.314	12.364	12.414	12.464	12.514	3.3
3.4	12.564	12.614	12.664	12.714	12.764	12.814	12.864	12.914	12.964	13.014	3.4
3.5	13.064	13.114	13.164	13.214	13.264	13.314	13.364	13.414	13.464	13.514	3.5
3.6	13.564	13.614	13.664	13.714	13.764	13.814	13.864	13.914	13.964	14.014	3.6
3.7	14.064	14.114	14.164	14.214	14.264	14.314	14.364	14.414	14.464	14.514	3.7
3.8	14.564	14.614	14.664	14.714	14.764	14.814	14.864	14.914	14.964	15.014	3.8
3.9	15.064	15.114	15.164	15.214	15.264	15.314	15.364	15.414	15.464	15.514	3.9

VALUES OF $k = V_o/\sqrt{S_o}$

Table 28.--Values of k, in feet per second, cross section 2

y	k	y	k	y	k	y	k
4.00	103.154	1.89	80.893	1.32	70.539	0.83	56.355
3.60	99.828	1.88	80.760	1.30	70.081	.82012	55.966
3.40	98.064	1.87	80.606	1.28	69.636	.82	55.975
3.20	96.217	1.86	80.451	1.26	69.146	.80	55.183
3.00	94.268	1.85840	80.422	1.24	68.672	.78	54.398
2.90	93.252	1.84	80.154	1.22	68.151	.76	53.668
2.80	92.200	1.80	79.522	1.20	67.644	.74	52.897
2.70	91.112	1.76	78.901	1.18	67.151	.72	52.136
2.60	89.994	1.72	78.251	1.17	66.881	.70	51.329
2.50	88.848	1.70	77.906	1.16	66.640	.68	50.529
2.40	87.667	1.68	77.572	1.15	66.362	.66	49.682
2.36	87.186	1.66	77.231	1.14	66.079	.65	49.268
2.32	86.692	1.64	76.904	1.13221	65.889	.64	48.839
2.30	86.445	1.62	76.546	1.12	65.531	.63	48.455
2.28	86.195	1.60	76.179	1.10	64.963	.62	47.999
2.24	85.697	1.58	75.827	1.08	64.408	.61812	47.909
2.20	85.180	1.56	75.463	1.06	63.832	.60	47.104
2.16	84.660	1.55	75.292	1.04	63.269	.56	45.317
2.12	84.139	1.54	75.093	1.02	62.648	.52	43.396
2.10	83.862	1.53	74.892	1.00	62.002	.50	42.430
2.08	83.597	1.52	74.711	.98	61.405	.48	41.384
2.04	83.053	1.51	74.529	.96	60.783	.46	40.325
2.02	82.774	1.50389	74.395	.94	60.133	.44	39.253
2.00	82.487	1.50	74.320	.92	59.455	.43	38.722
1.98	82.213	1.46	73.555	.90	58.749	.42	38.079
1.96	81.936	1.42	72.721	.88	58.091	.41	37.495
1.94	81.631	1.40	72.299	.87	57.752	.40	36.971
1.92	81.340	1.38	71.892	.86	57.405	.39491	36.615
1.91	81.193	1.36	71.445	.85	57.049	-------	------
1.90	81.043	1.34	71.014	.84	56.728	-------	------

Table 29.--Values of k, in feet per second, cross section 3

y	k	y	k	y	k	y	k
4.00	78.908	1.66	72.480	1.14	63.676	0.76	50.976
3.60	77.979	1.64	72.407	1.13	63.401	.74	50.182
3.40	77.486	1.62	72.354	1.12	63.118	.72	49.346
3.20	76.966	1.60	72.277	1.11	62.831	.70	48.512
3.00	76.438	1.58	72.221	1.10	62.541	.68	47.685
2.80	75.885	1.57	72.170	1.09	62.276	.66	46.805
2.70	75.598	1.56	72.140	1.08171	62.044	.65	46.346
2.60	75.319	1.55	72.111	1.08	61.973	.64	45.928
2.50	75.031	1.54	72.082	1.06	61.387	.63	45.499
2.40	74.749	1.53	72.051	1.04	60.812	.62	45.054
2.30	74.459	1.52712	71.978	1.02	60.178	.61206	44.689
2.20	74.158	1.52	71.996	1.00	59.519	.60	44.123
2.10	73.848	1.50	71.934	.98	58.908	.56	42.253
2.06	73.722	1.48	71.549	.96	58.272	.54	41.249
2.02	73.608	1.44	70.773	.94	57.608	.52	40.307
2.00	73.541	1.40	69.926	.92	56.916	.50	39.290
1.98	73.492	1.36	69.082	.90	56.193	.48	38.264
1.94	73.369	1.32	68.188	.88	55.519	.46	37.229
1.90	73.241	1.30	67.720	.86	54.814	.45	36.677
1.86	73.128	1.28	67.268	.84	54.120	.44	36.100
1.82	72.989	1.26	66.798	.83	53.717	.43	35.580
1.80	72.927	1.24	66.285	.82	53.348	.42	35.036
1.78	72.865	1.22	65.787	.81	52.925	.41	34.466
1.74	72.736	1.20	65.270	.80	52.536	.40804	34.368
1.70	72.601	1.18	64.768	.79	52.138	-------	------
1.68	72.529	1.16	64.248	.78073	51.869	-------	------

Table 30.--Values of k, in feet per second, cross section 4

y	k	y	k	y	k	y	k
4.00	71.879	2.24	61.732	1.54	53.825	1.05	43.887
3.80	71.060	2.20	61.378	1.52	53.500	1.04	43.608
3.60	70.169	2.16	61.027	1.50	53.189	1.03828	43.573
3.40	69.217	2.12	60.646	1.49	53.032	1.02	43.066
3.30	68.714	2.10	60.458	1.48	52.872	1.00	42.503
3.20	68.180	2.08	60.268	1.47	52.711	0.98	41.956
3.10	67.635	2.06	60.072	1.46	52.547	.96	41.384
3.00	67.065	2.04	59.892	1.45	52.380	.94	40.827
2.90	66.470	2.02	59.689	1.44	52.211	.92	40.206
2.80	65.843	2.00	59.483	1.43319	52.103	.90	39.558
2.76	65.581	1.98	59.291	1.43	52.039	.88	38.921
2.72	65.323	1.96	59.075	1.42	51.866	.86	38.299
2.70	65.186	1.94	58.875	1.41	51.690	.84	37.645
2.68	65.058	1.92	58.652	1.40	51.511	.83	37.351
2.64	64.786	1.90	58.444	1.39	51.331	.82	37.004
2.60	64.505	1.89577	58.400	1.38269	51.207	.81	36.696
2.56	64.230	1.88	58.230	1.36	50.772	.80	36.332
2.52	63.930	1.86	58.011	1.32	50.014	.79	36.007
2.50	63.784	1.84	57.768	1.30	49.604	.78780	35.921
2.48	63.636	1.82	57.540	1.28	49.209	.76	34.978
2.46	63.485	1.80	57.308	1.26	48.802	.72	33.574
2.44	63.348	1.78	57.069	1.24	48.382	.68	32.166
2.42	63.193	1.76	56.826	1.22	47.948	.66	31.425
2.41	63.105	1.74	56.576	1.20	47.500	.64	30.638
2.40	63.034	1.72	56.322	1.18	47.068	.62	29.918
2.39	62.961	1.70	56.061	1.16	46.587	.60	29.152
2.38	62.873	1.68	55.795	1.14	46.124	.58	28.330
2.37	62.800	1.66	55.543	1.12	45.611	.56	27.581
2.36	62.725	1.64	55.261	1.10	45.112	.55	27.154
2.35835	62.707	1.62	54.997	1.09	44.889	.54	26.778
2.32	62.406	1.60	54.703	1.08	44.628	.53328	26.499
2.30	62.234	1.58	54.425	1.07	44.364	-------	------
2.28	62.075	1.56	54.117	1.06	44.128	-------	------

Table 31.--Values of k, in feet per second, cross section 5

y	k	y	k	y	k	y	k
4.00	88.225	2.10	61.451	1.34	37.176	0.74	35.775
3.80	86.485	2.08	61.011	1.33	36.862	.73	35.514
3.60	84.530	2.04	60.087	1.32	36.531	.72	35.500
3.40	82.292	2.00	59.097	1.31	36.186	.71	35.229
3.30	81.072	1.96	58.101	1.30	35.914	.70	35.211
3.20	79.794	1.92	57.064	1.29785	35.839	.69387	35.259
3.10	78.465	1.90	56.496	1.28	35.418	.60	34.081
3.00	77.097	1.88	55.979	1.26	35.065	.56	33.581
2.96	76.542	1.86	55.440	1.24	34.879	.52	33.004
2.92	75.965	1.84	54.842	1.23	34.831	.50	32.498
2.88	75.383	1.82	54.256	1.22	34.892	.49	32.416
2.84	74.795	1.80	53.644	1.21	34.954	.48	32.330
2.80	74.183	1.79	53.348	1.20	35.138	.47	31.854
2.78	73.886	1.78	53.047	1.19	35.286	.46056	31.713
2.76	73.583	1.77	52.736	1.18	35.522	.46	31.751
2.74	73.274	1.76	52.421	1.17	35.697	.45	31.646
2.72	72.958	1.75	52.097	1.16958	35.746	.44	31.536
2.70	72.635	1.74	51.767	1.14	36.292	.43	31.419
2.69	72.471	1.73	51.427	1.10	36.979	.42	30.864
2.68	72.325	1.72104	51.106	1.06	37.559	.41	30.725
2.67	72.177	1.72	51.081	1.04	37.840	.40	30.581
2.66	72.007	1.70	50.405	1.02	37.946	.39	30.430
2.65	71.836	1.68	49.742	1.01	37.780	.38	30.269
2.64	71.684	1.66	49.045	1.00	37.793	.37	29.606
2.63	71.529	1.64	48.311	.99	37.806	.36	29.415
2.62	71.352	1.62	47.586	.98	37.632	.35350	29.320
2.61	71.173	1.60	46.872	.97	37.645	.35	29.212
2.60	71.012	1.58	46.118	.96	37.466	.34	28.996
2.59	70.851	1.56	45.371	.95	37.475	.33	28.770
2.58	70.687	1.54	44.638	.94	37.486	.32	27.956
2.57	70.521	1.52	43.854	.93	37.300	.31	27.680
2.56	70.353	1.50	43.140	.92	37.309	.30	27.386
2.55025	70.198	1.48	42.379	.91405	37.152	.29	27.072
2.50	69.303	1.46	41.627	.90	37.123	.28	26.734
2.40	67.512	1.44	40.887	.88	36.929	.27	25.695
2.30	65.623	1.42	40.091	.84	36.732	.26	25.279
2.20	63.612	1.40	39.378	.80	36.287	.25250	24.982
2.16	62.800	1.38	38.603	.78	36.047	-------	------
2.12	61.905	1.36	37.921	.76	35.794	-------	-------

Table 32.--Values of k, in feet per second, cross section 6

y	k	y	k	y	k	y	k
4.00	105.398	2.55	75.498	1.60	46.872	1.04	37.840
3.80	102.168	2.54	75.226	1.56	45.371	1.02	37.946
3.60	98.623	2.53	74.972	1.52	43.854	1.00	37.793
3.40	94.745	2.52	74.717	1.50	43.140	.99	37.806
3.30	92.695	2.51	74.456	1.48	42.379	.98	37.632
3.20	90.577	2.50	74.193	1.46	41.627	.97	37.645
3.10	88.399	2.49	73.950	1.44	40.887	.96	37.466
3.00	86.171	2.48	73.703	1.42	40.091	.95	37.475
2.90	83.885	2.47	73.453	1.40	39.378	.94	37.486
2.86	82.976	2.46	73.201	1.38	38.603	.93	37.300
2.82	82.030	2.45531	73.066	1.36	37.921	.92	37.309
2.80	81.552	2.40	71.615	1.34	37.176	.91405	37.152
2.78	81.101	2.30	68.956	1.33	36.862	.90	37.123
2.74	80.152	2.20	66.198	1.32	36.531	.80	36.287
2.70	79.160	2.10	63.348	1.31	36.186	.70	35.211
2.68	78.700	2.00	60.377	1.30694	36.097	.60	34.081
2.66	78.207	1.90	57.253	1.30	35.914	.56	33.581
2.64	77.725	1.80	53.963	1.26	35.065	.52	33.004
2.62	77.211	1.76	52.589	1.22	34.892	.51	32.577
2.60	76.705	1.72	51.212	1.18	35.522	.50	32.498
2.58	76.230	1.70	50.494	1.14	36.292	.49	32.416
2.57	75.989	1.68	49.742	1.10	36.979	.48	32.330
2.56	75.744	1.64	48.311	1.06	37.559	.47571	32.239

COMPUTED FALLS IN 400-FOOT REACH FOR SELECTED VALUES OF STAGE AND DISCHARGE

Table 33.--Computed falls, cross section 2, 400-foot reach, for selected values of stage and discharge

y_m	Q_m	F_m	y_m	Q_m	F_m	y_m	Q_m	F_m
2.90	40.39	0.188	1.60	3.90	0.00542	1.00	12.38	0.188
2.80	30.22	.108	1.60	7.99	.0229	.94	12.38	.235
2.50	3.90	.00215	1.60	12.38	.0569	.90	3.90	.0181
2.50	7.99	.00905	1.60	20.13	.169	.90	7.99	.0845
2.50	12.38	.0219	1.60	30.22	.568	.90	12.38	.283
2.50	20.13	.0595	1.56	30.22	.663	.87	12.38	.340
2.50	30.22	.142	1.54	30.22	.731	.85	12.38	.399
2.50	40.39	.280	1.53	30.22	.774	.84	12.38	.442
2.28	40.39	.369	1.52	30.22	.826	.83	12.38	.504
2.20	40.39	.416	1.51	30.22	.894	.820	12.38	.611
2.08	40.39	.513	1.504	30.22	.933	.812	12.38	1.200
2.00	3.90	.00341	1.489	30.22	1.200	.80	3.90	.0238
2.00	7.99	.0144	1.40	3.90	.00712	.80	7.99	.119
2.00	12.38	.0351	1.40	7.99	.0304	.74	7.99	.153
2.00	20.13	.0979	1.40	12.38	.0768	.70	3.90	.0326
2.00	30.22	.252	1.40	20.13	.247	.70	7.99	.190
2.00	40.39	.611	1.30	20.13	.318	.68	7.99	.216
1.98	40.39	.643	1.24	20.13	.388	.66	7.99	.254
1.91	40.39	.799	1.20	3.90	.00977	.65	7.99	.280
1.90	40.39	.831	1.20	7.99	.0424	.64	7.99	.314
1.89	40.39	.867	1.20	12.38	.111	.63	7.99	.364
1.88	40.39	.908	1.20	20.13	.460	.62	7.99	.454
1.87	40.39	.957	1.17	20.13	.546	.618	7.99	.486
1.86	40.39	1.016	1.15	20.13	.638	.612	7.99	1.200
1.858	40.39	1.027	1.14	20.13	.711	.60	3.90	.0479
1.840	40.39	1.200	1.132	20.13	.797	.50	3.90	.0811
1.80	3.90	.00422	1.121	20.13	1.200	.46	3.90	.109
1.80	7.99	.0179	1.10	3.90	.0117	.42	3.90	.168
1.80	12.38	.0440	1.10	7.99	.0517	.41	3.90	.199
1.80	20.13	.126	1.10	12.38	.141	.40	3.90	.254
1.80	30.22	.350	1.00	3.90	.0144	.395	3.90	.315
1.68	30.22	.452	1.00	7.99	.0648	.391	3.90	1.200

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Table 34.--Computed falls, cross section 3, 400-foot reach, for selected values of stage and discharge

y_m	Q_m	F_m	y_m	Q_m	F_m	y_m	Q_m	F_m
2.50	3.77	0.00345	1.55	29.80	0.942	0.90	7.37	0.0817
2.50	7.37	.0132	1.512	29.80	1.200	.90	10.90	.225
2.50	10.90	.0303	1.40	3.77	.00854	.80	3.77	.0255
2.50	18.10	.0825	1.40	7.37	.0333	.80	7.37	.113
2.50	29.80	.242	1.40	10.90	.0800	.80	10.90	.427
2.00	3.77	.00497	1.40	18.10	.239	.79	10.90	.490
2.00	7.37	.0191	1.20	3.77	.0110	.781	10.90	.593
2.00	10.90	.0451	1.20	7.37	.0436	.773	10.90	1.200
2.00	18.10	.123	1.20	10.90	.104	.70	3.77	.0353
2.00	29.80	.396	1.20	18.10	.373	.70	7.37	.179
1.80	3.77	.00582	1.10	3.77	.0130	.66	7.37	.239
1.80	7.37	.0226	1.10	7.37	.0517	.63	7.37	.330
1.80	10.90	.0532	1.10	10.90	.126	.612	7.37	.484
1.80	18.10	.148	1.10	18.10	.620	.606	7.37	1.200
1.80	29.80	.521	1.082	18.10	.781	.60	3.77	.0526
1.60	3.77	.00699	1.071	18.10	1.200	.50	3.77	.0928
1.60	7.37	.0270	1.00	3.77	.0157	.45	3.77	.143
1.60	10.90	.0635	1.00	7.37	.0634	.42	3.77	.224
1.60	18.10	.184	1.00	10.90	.161	.408	3.77	.328
1.60	29.80	.788	.90	3.77	.0196	.404	3.77	1.200

Table 35.—Computed falls, cross section 4, 400-foot reach, for selected values of stage and discharge

y_m	Q_m	F_m	y_m	Q_m	F_m	y_m	Q_m	F_m
3.178	39.98	0.378	2.00	29.91	0.768	1.40	19.10	0.852
3.02	29.91	.221	1.96	29.91	.856	1.39	19.10	.920
3.00	39.98	.441	1.92	29.91	.977	1.383	19.10	.985
2.90	29.91	.241	1.90	29.91	1.060	1.369	19.10	1.200
2.80	29.91	.262	1.896	29.91	1.081	1.20	3.79	.0226
2.80	39.98	.541	1.877	29.91	1.200	1.20	7.62	.0987
2.64	39.98	.660	1.80	3.79	.00935	1.20	12.19	.318
2.50	3.79	.00479	1.80	7.62	.0385	1.10	3.79	.0277
2.50	7.62	.0189	1.80	12.19	.103	1.10	7.62	.125
2.50	12.19	.0503	1.80	19.10	.289	1.10	12.19	.479
2.50	19.10	.128	1.80	20.15	.327	1.08	12.19	.542
2.50	20.15	.144	1.60	3.79	.0120	1.06	12.19	.634
2.50	29.91	.355	1.60	7.62	.0500	1.04	12.19	.809
2.50	39.98	.817	1.60	12.19	.136	1.028	12.19	1.200
2.46	39.98	.881	1.60	19.10	.422	1.00	3.79	.0350
2.42	39.98	.956	1.60	20.15	.494	1.00	7.62	.167
2.40	29.91	.399	1.56	20.15	.552	.90	3.79	.0460
2.40	39.98	1.001	1.54	19.10	.487	.90	7.62	.245
2.38	39.98	1.051	1.52	20.15	.629	.84	7.62	.347
2.37	39.98	1.080	1.48	19.10	.585	.80	3.79	.0635
2.36	39.98	1.110	1.48	20.15	.744	.80	7.62	.508
2.358	39.98	1.115	1.44	19.10	.683	.788	7.62	.654
2.335	39.98	1.200	1.44	20.15	.944	.78	7.62	1.200
2.28	29.91	.465	1.433	20.15	1.002	.70	3.79	.0967
2.16	29.91	.558	1.42	19.10	.754	.60	3.79	.175
2.00	3.79	.00750	1.419	20.15	1.200	.57	3.79	.231
2.00	7.62	.0307	1.41	19.10	.798	.55	3.79	.301
2.00	12.19	.0811	1.40	3.79	.0160	.54	3.79	.367
2.00	19.10	.218	1.40	7.62	.0681	.533	3.79	.494
2.00	20.15	.243	1.40	12.19	.194	.528	3.79	1.200

Table 36.--Computed falls, cross section 5, 400-foot reach, for selected values of stage and discharge

y_m	Q_m	F_m	y_m	Q_m	F_m	y_m	Q_m	F_m
3.00	31.28	0.306	1.60	1.32	0.00474	0.93	1.84	0.705
2.88	31.28	.370	1.60	1.84	.00886	.92	1.84	.841
2.80	11.30	.0339	1.60	2.71	.0196	.914	1.84	.954
2.80	31.28	.429	1.60	3.87	.0415	.905	1.84	1.200
2.72	31.28	.510	1.52	3.87	.0562	.90	.34	.00726
2.65	31.28	.615	1.44	3.87	.0820	.90	.56	.0206
2.61	31.28	.705	1.40	.34	.00066	.90	.79	.0437
2.59	31.28	.763	1.40	.56	.00177	.90	1.32	.166
2.57	31.28	.840	1.40	.79	.00345	.80	.34	.0108
2.56	31.28	.887	1.40	1.32	.00986	.80	.56	.0309
2.55	31.28	.943	1.40	1.84	.0192	.80	.79	.0669
2.525	31.28	1.200	1.40	2.71	.0440	.80	1.32	.292
2.50	.34	.00008	1.40	3.87	.103	.75	1.32	.418
2.50	.56	.00014	1.36	3.87	.138	.73	1.32	.496
2.50	.79	.00028	1.34	3.87	.165	.71	1.32	.628
2.50	1.32	.00077	1.32	2.71	.0685	.70	.34	.0156
2.50	1.84	.00154	1.32	3.87	.207	.70	.56	.0453
2.50	2.71	.00325	1.31	3.87	.239	.70	.79	.101
2.50	3.87	.00680	1.30	1.84	.0323	.70	1.32	.744
2.50	11.30	.0590	1.30	2.71	.0783	.694	1.32	.847
2.20	11.30	.0979	1.30	3.87	.289	.687	1.32	1.200
2.00	.34	.00015	1.298	3.87	.310	.60	.34	.0224
2.00	.56	.00033	1.285	3.87	1.200	.60	.56	.0670
2.00	.79	.00063	1.26	2.71	.106	.60	.79	.160
2.00	1.32	.00174	1.22	2.71	.155	.54	.79	.227
2.00	1.84	.00330	1.20	.34	.00165	.50	.34	.0329
2.00	2.71	.00724	1.20	.56	.00459	.50	.56	.104
2.00	3.87	.0150	1.20	.79	.00920	.50	.79	.302
2.00	11.30	.151	1.20	1.32	.0275	.48	.79	.376
1.90	11.30	.201	1.20	1.84	.0592	.47	.79	.441
1.84	11.30	.250	1.20	2.71	.197	.461	.79	.578
1.80	.34	.00021	1.19	2.71	.228	.456	.79	1.200
1.80	.56	.00049	1.18	2.71	.271	.44	.56	.145
1.80	.79	.00095	1.17	2.71	.343	.40	.34	.0518
1.80	1.32	.00267	1.158	2.71	1.200	.40	.56	.196
1.80	1.84	.00509	1.10	.34	.00270	.38	.56	.240
1.80	2.71	.0112	1.10	.56	.00754	.37	.56	.277
1.80	3.87	.0232	1.10	.79	.0154	.36	.56	.346
1.80	11.30	.299	1.10	1.32	.0486	.354	.56	.443
1.76	11.30	.378	1.10	1.84	.119	.350	.56	1.200
1.75	11.30	.407	1.05	1.84	.186	.30	.34	.102
1.74	11.30	.444	1.00	.34	.00456	.28	.34	.127
1.73	11.30	.493	1.00	.56	.0129	.27	.34	.149
1.721	11.30	.565	1.00	.79	.0268	.26	.34	.191
1.704	11.30	1.200	1.00	1.32	.0921	.252	.34	.266
1.60	.34	.00033	1.00	1.84	.307	.250	.34	1.200
1.60	.56	.00087	.96	1.84	.473	....	......	....
1.60	.79	.00168	.94	1.84	.608	....	......	....

Table 37.--Computed falls, cross section 6, 400-foot reach, for selected values of stage and discharge

y_m	Q_m	F_m	y_m	Q_m	F_m	y_m	Q_m	F_m
2.989	30.63	0.189	1.80	1.84	0.00472	1.00	0.83	0.0298
2.82	30.63	.253	1.80	3.98	.0222	1.00	1.84	.307
2.66	30.63	.359	1.60	.83	.00177	.97	1.84	.422
2.57	30.63	.469	1.60	1.84	.00857	.95	1.84	.535
2.50	.83	.00023	1.60	3.98	.0419	.93	1.84	.705
2.50	1.84	.00119	1.40	.83	.00375	.914	1.84	.954
2.50	3.98	.00538	1.40	1.84	.0190	.905	1.84	1.200
2.50	30.63	.622	1.40	3.98	.110	.90	.83	.0489
2.48	30.63	.693	1.35	3.98	.164	.80	.83	.0752
2.46	30.63	.800	1.32	3.98	.236	.70	.83	.115
2.455	30.63	.834	1.307	3.98	.305	.60	.83	.185
2.431	30.63	1.200	1.294	3.98	1.200	.50	.83	.388
2.00	.83	.00057	1.20	.83	.0102	.49	.83	.449
2.00	1.84	.00291	1.20	1.84	.0590	.48	.83	.551
2.00	3.98	.0134	1.10	.83	.0170	.476	.83	.635
1.80	.83	.00093	1.10	1.84	.119	.471	.83	1.200

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