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W. B. LANGBEIN

NTRIBUTIONS TO THE HYDROLOGY OF THE UNITED STATES

OLOGICAL SURVEY WATER-SUPPLY PAPER 1539-W

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U.S. DEPARTMENT OF THE INTERIOR
March 3, 1849

ITED STATES GOVERNMENT PRINTING OFFICE WASHINGTON : 1962

UNITED STATES DEPARTMENT OF THE INTERIOR

STEWART L. UDALL, *Secretary*

GEOLOGICAL SURVEY

Thomas B. Nolan, *Director*

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington 25, D.C.

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HYDRAULICS OF RIVER CHANNELS AS RELATED TO NAVIGABILITY

By W. B. Langbein

ABSTRACT

For any given combination of river current and depth there is a minimum specific tractive force required for upstream navigation. The ratio of the force exerted by a vessel in motion to its weight is called the specific tractive force. The force required for upstream navigation on a river depends on vessel draft, river depth, and velocity of the river current. For any given combination of current and depth there is a minimum specific tractive force—the ratio of the force exerted by a vessel in motion to its weight—required for upstream navigation. The interaction of channel depth and vessel draft produces a minimum specific tractive force when the vessel draft is about 0.7 the channel depth. Comparisons of computed values of the tractive force required for navigation on certain rivers with the minimum tractive forces developed by commercial vessels indicate that rivers that require specific tractive forces greater than 0.002 are usually considered unnavigable.

Defining the transport capacity of a vessel as the product of its speed and tonnage leads to an index of the transport capacity of a river channel in terms of its depth and current. Channel depth is the dominant hydraulic factor in determining the transport capacity of a river. Reported traffic densities of commercial inland waterways of the United States correspond with computed transport capacities.

INTRODUCTION

About 20,000 miles of river channel is used for commercial navigation in the United States. The use of rivers for the transportation of goods and people not only played a most historic role in the development of the continent, but even today the rivers carry a significant amount of commercial freight. Yet there have been few studies to test the navigability of rivers in relation to the hydraulics of vessels—if by navigability one means transport by commercial vessels: barge or river boat, as distinct from pleasure boating, or exploration. This paper is based therefore on the primary premise that navigability means two-way navigability in fact,[1] recognizing that even navigable rivers differ in their hydraulics and, to a corresponding degree, in their navigability.

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[1] For a legal definition, see decisions of U.S. Supreme Court in *The Daniel Ball*, 10 Wall. 557, 563; and in *United States* v. *Utah*, 283 U.S. 64.

The two parameters that characterize a watercourse with respect to its navigability are its depth and its velocity; too little of the first or too much of the other prohibits navigation. On this basis, some general conclusions on the navigability of rivers can be reached by connecting the newly developed ideas in the hydraulic geometry of rivers (Leopold and Maddock, 1953) with the hydraulic geometry of commercial vessels.

This study is intended to illustrate a single set of principles, defining the navigability of rivers, without detailed consideration of the engineering or economic feasibility of navigation. Moreover, in this illustration, rivers are considered in their approximate native state.

HYDRAULIC GEOMETRY OF RIVER CHANNELS

In respect to their navigability, rivers may be characterized by their depths, velocities, widths, and their meandering properties and, as now well documented in the literature, rather systematic relationships exist between these attributes.

DOWNSTREAM VARIATION IN HYDRAULIC GEOMETRY

Figure 1 shows the variation in width, mean depth, and mean velocity along the Kansas River system and along the Missouri and Mississippi Rivers based on data published by Leopold and Maddock (1953). The plotted points apply to the flow-measuring sections at gaging stations of the Geological Survey.

The variations of width, depth, and velocity in a downstream direction are linked by the condition that their product equals the discharge. Since the discharge of a river normally increases downstream, the component factors may also be expected to increase. The studies by Leopold and Maddock show that, for rivers in general, width increases as the 0.5 power of the discharge, depth as the 0.4 power, and velocity as the 0.1 power. Note that the sum of these exponents is unity, as required by the condition that the product of width, depth, and velocity must equal the rate of discharge. It is also significant that the width increases faster than the depth, which, in turn, increases faster than the velocity. Although velocity is the most conservative of the three components of the discharge carried by a river channel, there is, nevertheless, a marked tendency for it to increase downstream despite casual appearance to the contrary. The mountain torrent derives its appearance of great speed because its velocity is high relative to its shallow depth and narrow width, whereas the majestic river that drains half a continent has a greater speed but appears to move slowly because its velocity is low relative to its depth and width. The eye seems to appraise the Froude number of a stream rather than its velocity.

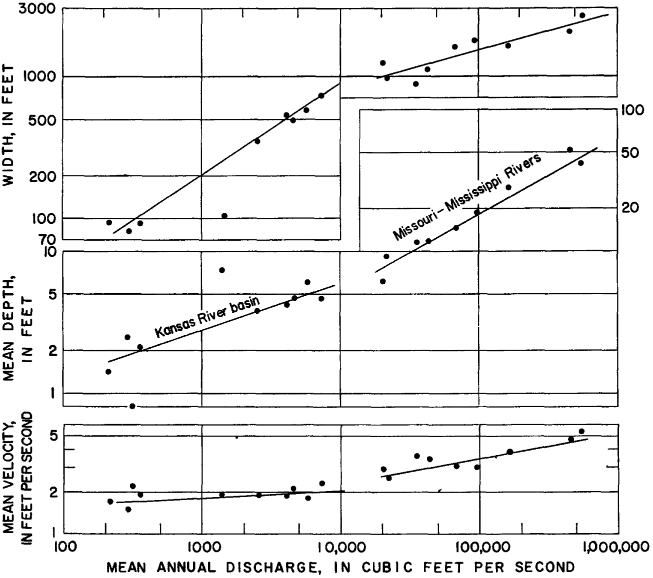

FIGURE 1.—Increase in width, depth, and velocity of rivers in a downstream direction.

In some river systems the downstream increase in discharge is accommodated in greater or lesser degree by width, depth, or velocity. For example, the graphs of figure 1 show that the depth along the Missouri and Mississippi Rivers increases faster than the width; whereas, the opposite appears to be true of the Kansas River, a fact of considerable importance to their navigability. The object of this study is to interpret data on river hydraulics in terms of river navigability.

On figure 2 are graphed depths and velocities at stations corresponding to those on figure 1. Both the Kansas and Missouri-Mississippi systems appear to define a common trend in this graph. Also shown are trend lines for the Tombigbee River, in the coastal plain of Alabama, and for the Bighorn River which drains a mountainous region in north-central Wyoming. The differing positions of these trend lines correspond in major part to differing river slopes. On figure 2 a family of lines has been drawn for equal river slopes, as defined by the well-known Manning equation

$$v=\frac{1.5}{n}r^{2/3}s^{1/2}, \tag{1}$$

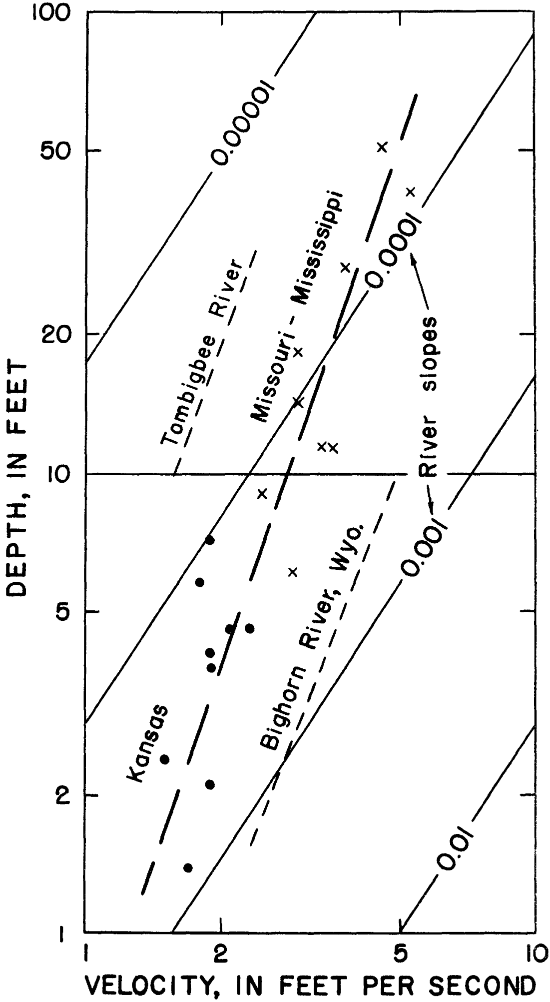

FIGURE 2.—Relations between mean depth, velocity, and river slope.

where n is a roughness factor, v is the mean velocity in feet per second, r is hydraulic radius in feet, and s is the river slope, or more exactly, the rate of loss of head per foot of channel. Since for a river channel the hydraulic radius is nearly the same as the mean depth, and since the roughness factor of rivers usually ranges between 0.02 and 0.05, with an average of about 0.03, the Manning equation has been generalized to

$$v=50D^{2/3}s^{1/2}, \tag{1a}$$

where D is the mean depth of channel, to define the family of lines of river slope on figure 2. According to these lines, the river slope on the Kansas-Missouri-Mississippi system decreases from over 0.005 (26 feet per mile) to less than 0.00005 (3 inches per mile).

According to figure 2, river velocity in a downstream direction varies as the cube root of the depth, whereas according to the Manning equation, velocity varies as the two-thirds power of the depth. The difference corresponds to the downstream decrease in river slope.

As evident from figure 1, rivers are wide relative to their depth. In the case of the rivers shown, the width is 75 to 80 times the depth; for other rivers, the width-depth ratio may be as low as 30 or more than 100. The ratio, as shown by Schumm (1960), depends on the texture of the alluvium comprising the bed and banks. The width-depth ratio increases rather slowly downstream; thus, although rivers deepen downstream, they become more shallow in relation to their width. This fact suggests that width is not usually a limiting factor in navigation; the main concern is therefore with depth and velocity.

VARIATIONS IN VELOCITY AND DEPTH IN A RIVER REACH

The data on figures 1 and 2 describe the variation in mean depth and velocity as one moves downstream in a river system. Navigability is, however, often concerned with the depth and velocity at shallow sections. For this purpose more detailed examination of the variations in depth in river reaches must be made.

Common features of streams are deeps and shallows, or pools and riffles as they are often called. Meandering channels, for example, characteristically are deep at the outside of bends and shallow at the crossover between bends, as depicted in figure 3, which shows the variation in depth about a typical meander on the Mississippi River. Leopold and Wolman (1957) observed that straight, uniform channels rarely exist in nature, adding that the beds of straight reaches are irregular, because the thalweg wanders back and forth across the channel, simulating the pattern of a meandering channel.

The depth and velocity data plotted on figures 1 and 3 were collected at gaging stations where such measurements are made as part of the

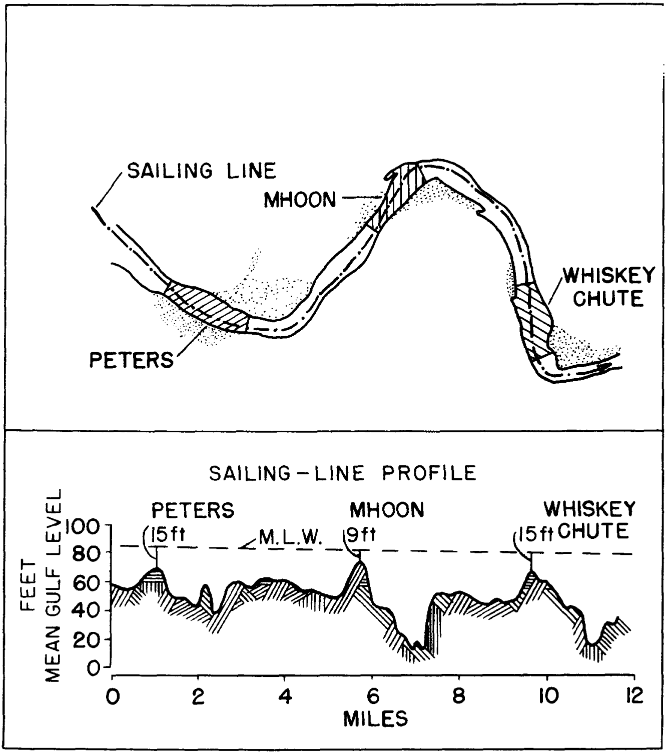

Figure 3.—A typical meander of the Mississippi River in plan and profile.

program for the collection of records of river discharge. The soundings of depth and the measurements of current speed are not made with the view of testing the navigability of the streams but are made at those cross sections where hydraulic conditions favor accurate metering. Thus, deep pools and shallow riffles are usually avoided as places for current-meter measurement of river discharge. Therefore, the data on figure 2, especially the trend line, describe a mean variation between depth and velocity and do not apply to depth and velocity in the shallows, which may control navigability. It is well to recall, too, that the data on figure 2 describe the variations in depth and velocity in an upstream-downstream direction when the river carries its mean discharge, which is a discharge that is exceeded only about

30 percent of the time. That is to say, lesser depths prevail about 70 percent of the time.

It is also necessary to consider the variations between depth and velocity with changes in discharge at a given river cross section. The relationship depends on the location of the cross section in a river reach. Figure 4 presents a schematic variation of the relations between depth and velocity in a river reach—a length of river channel between tributaries. Point A represents the mean depth and velocity at a mean cross section in a reach at mean discharge. Point A therefore represents the conditions of the data in figures 1 and 2.

Line DE represents the variation in depth and velocity at a normal section from low water to bankfull stage. Line DE corresponds to

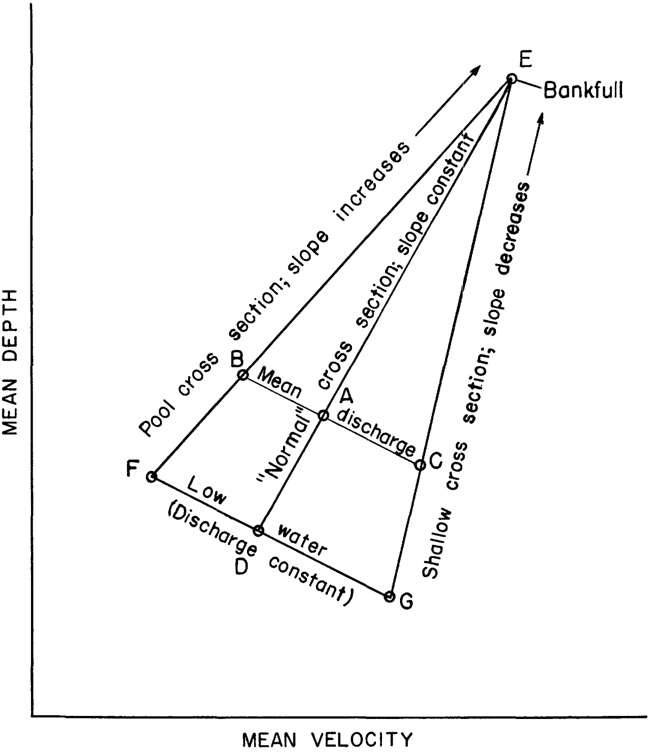

FIGURE 4.—Schematic variation of velocity and depth in a river reach.

a constant river slope, and at a normal section the velocity increases with about the two-thirds power of the depth.

Line FE represents the variation in depth and velocity at a pool section from low water to bankfull stage. Generally, in a pool above a riffle, as the discharge increases, the slope of the water surface increases as the riffle is drowned out and the slope approaches that of the longitudinal profile of the bed of the stream.

Line GE represents the variation between depth and velocity over a riffle or shallow cross section from low water to bankfull stage. The rising river drowns out the riffle, where at low water the slope is locally higher than that of the reach as a whole. Hence, the line from G to E represents a decreasing slope of the water surface at the riffle.

Line BC represents the variation in the depth and velocity through the reach at mean discharge. Point B is in a pool, and point C is on a riffle or a shallows. Line BC slopes downward toward the right, and if the width is conservative in the reach, the product of velocity and depth along BC would be constant.

The significance of the diagram of figure 4 is twofold. First, the differences between pool and riffle decrease with increasing river stage so that at bankfull stage the whole reach is represented on the diagram by a single point, E, although this is admittedly an oversimplification. Second, at a riffle section, line GE, the variation in depth is greater than the variation in velocity.

However, there are few data available on the variation between depth and velocity at shallow cross sections. In the absence of this kind of information, the depth-velocity relations at shallows, corresponding to line GE of figure 4, are approximated on the assumption that over a period of years the conditions at gaging stations on alluvial streams vary from approximate pool to approximate riffle conditions as the bed shifts. Thus individual discharge measurements will plot within the triangle EFG of figure 4, and an envelope line corresponding to GE will approximate the relationship sought. Figure 5 shows an actual example.

The envelope GE represents the mean depth and mean velocity at shallow cross sections in a river reach. Of interest, however, are the depths and velocities in the channel, which is defined as the mean depth along that half of the cross section that contains the greatest depths. A study of the distribution of depth and velocity along river cross sections indicates that the channel depth so defined is about 1.25 times the mean depth in the section, and that the velocity in the channel is about 1.15 times the mean in the section. Inasmuch as the difference in these two ratios accords with the statement that velocity varies as the 0.62 power of the depth, which approximates the slope of line GE, the enveloping curve just described may also

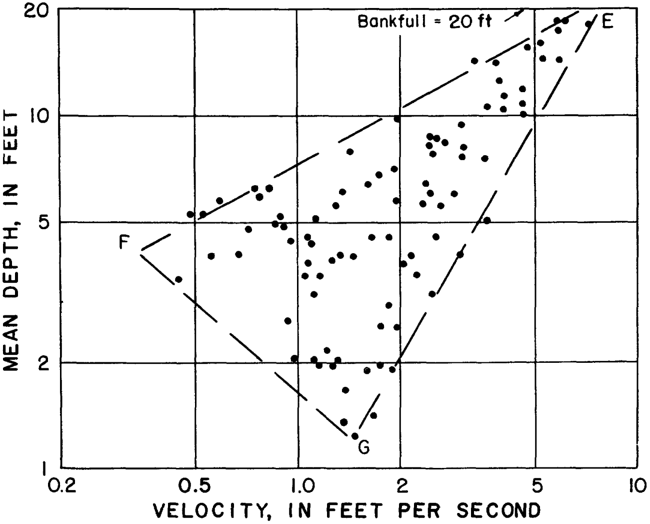

FIGURE 5.—Depth and velocity as measured at a gaging station on the Kansas River at Bonner Springs, Kans.

describe the variation between depth and velocity in the channel as defined.

HYDRAULICS OF VESSELS

As the gross character of a river may be summarized by its velocity, depth, and slope, so may the character of a vessel be summarized by a selection of comparable attributes. These are its design speed, its draft, and its specific tractive force, a dimensionless factor defined as the ratio of the thrust to the weight and in these terms is equivalent to the slope of a river, which measures the downstream component of the force of gravity per unit weight of water. The specific tractive force of a vessel may be calculated from data on its horsepower, design speed, and displacement as follows:

$$T_s = \frac{\text{Horsepower} \times 550}{\text{Speed in feet per second} \times \text{displacement in tons} \times 2240} \qquad (2)$$

Available data on these characteristics for various kinds of vessels are given in table 1. The data represent normal load conditions, and the design speed represents the speed in deep, still water at normal service draft and in normal weather. Figure 6 shows a plot of

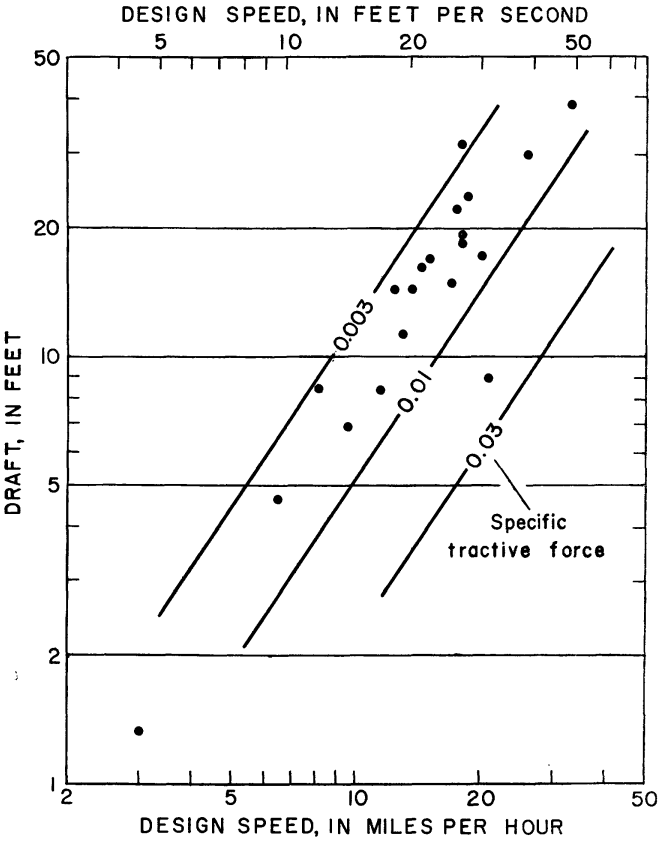

FIGURE 6.—Relation between vessel speed, draft, and specific tractive force.

the draft of vessels in relation to their design speed and is comparable to figure 2 which shows graphs of river depths and velocity. Just as rivers differ in their velocity and depth according to river slope, the distinction between vessels is mainly in their specific tractive force. Following the river analogy, the interrelationship between the several quantities may be empirically evaluated according to the Manning equation:

$$V=\frac{1.5}{n'}d^{2/3}T_s^{1/2}, \tag{3}$$

where V is in feet per second, d is draft in feet, T_s is the specific tractive force (dimensionless), and n' is a resistance factor.

RESISTANCE FORMULAS

The Manning equation assumes that the frictional resistance to motion, R, varies with the wetted surface of the hull, S, and the square of the speed, V. Thus:

$$R \propto S V^2.$$

Since the wetted surface is proportional to the ratio of the displaced volume to the draft,

$$R = c \frac{W}{d} V^2,$$

where W is displaced weight, and c is a friction factor. Solving for speed with the equation

$$V = \frac{1}{c} \sqrt{d R / W}$$

yields an adequate expression for speed except that hydraulic experience with this type of equation indicates that the friction factor should decrease with about the sixth root of the draft. With this adjustment, the above formula leads to the Manning equation

$$V = \frac{1.5}{n'} d^{2/3} \sqrt{R/W}, \tag{3a}$$

where all units are as previously given, the ratio R/W equals the specific tractive force, and where the constant 1.5 combines all the necessary conversion of units.

Values of the friction factor n' corresponding to given data on vessel speed, draft, and tractive force are also listed in table 1. These results indicate a variation of from 0.024 to 0.046 within the range observed on rivers. Considering the diversity of vessel types, the friction factor appears to be a relatively conservative property. With an average friction factor of about 0.03 inserted, the Manning equation gives the following formula for vessel design speed, V_d:

$$V_d = 50\, d^{2/3} \sqrt{R/W} = 50\, d^{2/3}\, T^{1/2}{}_s. \tag{4}$$

For a vessel moving at uniform speed, the resistance equals the thrust, so that $T_s = R/W$. Inserting T_s as defined by equation 2 in terms of horsepower, tonnage, and speed in feet per second.

$$V = 25\, d^{2/3} \sqrt{P/WV}. \tag{5}$$

Another kind of equation for resistance can be derived on the basis of the projected area in the direction of vessel movement. According to Rouse (1946, p. 243, et seq.) the force on a vessel moving with velocity V may be evaluated as follows:

$$R=C_D\,A\,\frac{wV^2}{2g},$$

where C_D is a drag coefficient, A is the projected area normal to the direction of movement in square feet, w is the specific weight (64 pounds per cubic foot), and g is the acceleration of gravity (32 feet per second).

Since area A may be approximated by the ratio of the displaced volume to the length, the above expression may be written as

$$R=C_D\frac{W}{L}\frac{V^2}{2g}$$

or

$$C_D=\frac{64L}{V^2}\frac{R}{W}, \tag{6}$$

where L is length in feet, V is in feet per second, and W is displacement in pounds.

Values of the drag coefficient as listed in table 1 show a range of from 0.15 to 1.5, a far greater range than noted for the resistance factor n'. However, it may be noted that high values tend to be associated with short vessels and low values with long vessels. Hence, the range in values would be reduced if the value of C_D were accordingly adjusted. Moreover, considering that n' enters directly in the equation for ship speed and C_D enters as the square root, the effective range is not as great as it appears.

The drag coefficient and the friction factor, n', both purport to explain the resistance against a moving vessel: the drag coefficient in terms of the projected area, and the friction factor in terms of the wetted surface. If all vessels were of identical geometric proportions, it would be immaterial which dimension were used. An equation combining both of these influences on the resistance would more satisfactorily allow for the variations in geometric proportions on the resistance. However, the equation for vessel speed in terms of draft, as in equation 5, appears more suitable for this study because of the relation of this dimension to channel depth.

SPECIFIC TRACTIVE FORCE

The specific tractive force, that is, the ratio of the thrust of a vessel in motion to the weight of the vessel, is also a measure of the horsepower-hours expended per ton-mile of transport. The latter interpretation is an evaluation of the energy consumption per unit transported, and, in this sense, the specific tractive force is a meaningful basis of comparison between different modes of transport. Specific tractive forces for common modes of transport range as follows: Commercial water transport, 0.0026 for river barges to 0.024 for express passenger vessels; naval vessels, on the order of 0.015 to 0.1; fast motorboats (planing craft), 0.2 or more; rail freight transport, 0.01; trucks, 0.04; and aircraft, 0.1 to 0.2. The specific tractive force for rockets exceeds unity. The fact that barges have a low specific tractive force contributes to the economic basis for their competitive position for movement of bulk goods.

At any state of technology there is a limit to the speed attainable for a given specific tractive force. Figure 7 shows the results of a study of this subject made by Gabrielli and von Karman (1950). The several curved lines for the different modes of transport are envelopes of the relations between speed and specific tractive force of different individual vehicles of the indicated types. In this same sense, the limiting line is an envelope (that is, the greatest speed for a given tractive force) of all types of transport.

It is interesting to note that merchant ships define this limit for low-speed transport. It is equally significant that the various kinds of water transport (that is, merchant ships, battleships, destroyers, and motorboats) form an alinement on this interesting chart but in such a way that increasing speed over water is bought dearly in terms of tractive force, so that great speed on water is usually justified only in terms of such special purposes as naval operations or sport. A similar relation can be observed among the different kinds of aircraft which define the upper range of the limiting line.

A technologic limit involves economic as well as physical factors. It would be possible, for example, to operate a fine ocean liner at a speed of 5 miles per hour with a specific tractive force of only 0.00015, which would lie well to the right of the technologic limit. But this speed would not be considered economic. Since dimensions are now limited by structural considerations, Gabrielli and von Karman suggest that the prospect for an upward shift of the technologic limit depends on use of materials with a high ratio of breaking stress to density.

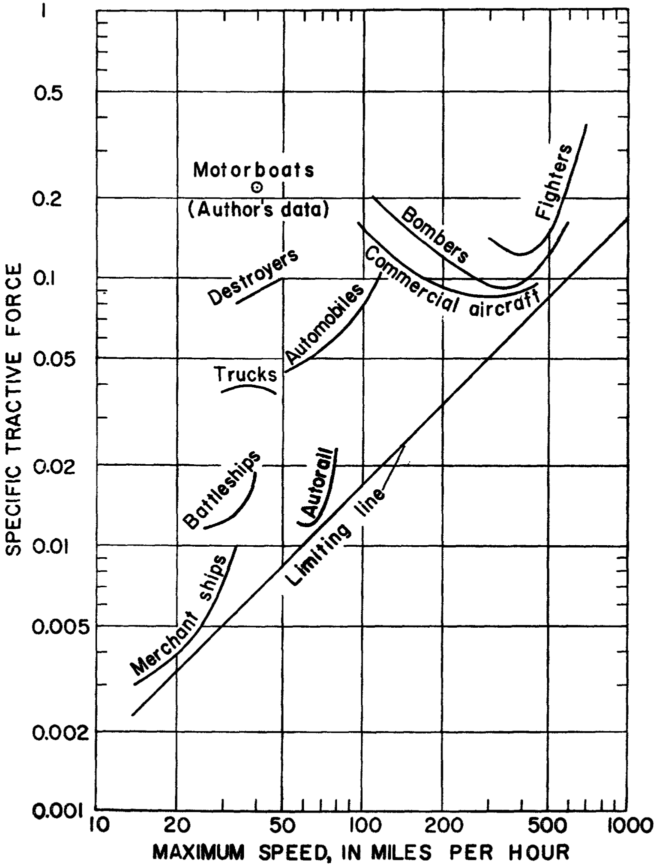

FIGURE 7.—Speed in relation to specific tractive force for various modes of transport. (After Gabrielli and von Karman, 1950.)

SHALLOW-WATER DRAG

Equation 3a leads to the following approximation for the resistance against a vessel in moving through deep still water:

$$R=W\left(\frac{n'}{1.5}\frac{V}{d^{2/3}}\right)^2 \qquad (7)$$

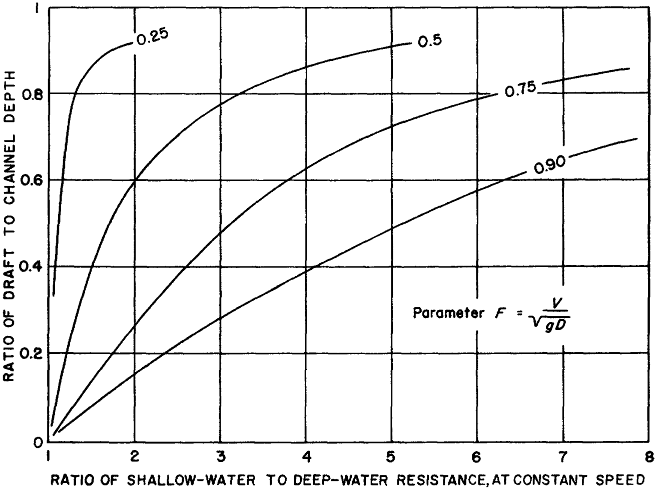

FIGURE 8.—Increase of resistance due to shallow water.

To this normal resistance must be added that produced by shallow water through which the vessel moves in proceeding along a river. The motion of the vessel induces a reverse flow of water in the confined space between the bottom of the vessel and the bed of the river, which increases the vessel's speed in relation to the water and thus adds to its resistance. Moreover, the reverse flow produces a shearing force on the bed which must be sustained by the vessel.

The shallow-water drag depends on the relative proportion of the channel occupied by the vessel and on the speed. Restricted width as well as depth can add to the drag, but in this study, since rivers are considered wide in relation to their depth, only the influence of restricted depth need be considered. Figure 8 shows the shallow-water drag as defined by the author from observations of models of towboats and barges reported by the U.S. Army Corps of Engineers (1914). The abscissa is the ratio, f, of the resistance against a vessel moving at a certain speed in shallow water to that at the same speed in deep water. The ordinate is the ratio of draft to channel depth, and the parameter, F, is the ratio of the speed of the vessel to that of a gravity wave. The curves show a marked increase in the shallow-water effect with increase in speed, and with increasing draft. The curves increase toward infinity as the draft approaches the depth, as there can be no motion where a vessel is grounded.

Thus, the total resistance as affected by shallow-water drag is

$$R = fW\left(\frac{n'}{1.5}\frac{V}{d^{2/3}}\right)^2.$$

The family of curves on figure 8 is based on only one set of data, believed to be representative of conditions treated in this report. Although the phenomenon may be more complex than shown, the curves provide working estimates of the retardance effect of shallow water in terms of the draft, depth, and speed.

SLOPE DRAG

In moving upstream, a vessel must not only overcome the frictional and other retardational forces due to its motion, but additional energy must be expended to raise the vessel. The force involved is known as the slope drag.

The existence of this force may be illustrated by the diagram in figure 9. In a floating vessel, the center of buoyancy, *CB* in the diagram, is below the center of gravity, *CG*. The buoyant force acts along the line through *CG* and *CB*. For a vessel in level water, this line is vertical, but as depicted, the buoyant force is deflected by the slope angle, *s*. The weight acting at the center of gravity remains vertical. Thus, there is a component of force equal to the product of

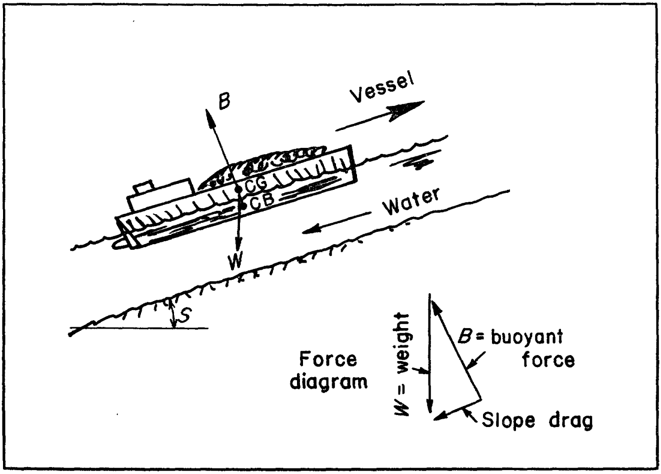

FIGURE 9.—Schematic illustration of slope drag on a vessel moving upstream.

the sine of the angle, s, and the weight of the vessel. Since for small angles, the sine equals the slope, one may write with sufficient accuracy that the slope force acting on the vessel equals s times its weight, as follows:

$$\text{Slope drag} = sW. \tag{8}$$

This force is called the slope drag for vessels moving upstream and the slope thrust for vessels moving downstream. The slope drag is usually small in relation to the hydrodynamic drag, but it may become significant for large vessels moving up steep rivers.

Consider a vessel floating downstream without power. In this case, the slope drag is a slope thrust, and therefore

$$V = \frac{1.5}{nf} d^{2/3} s^{1/2} \tag{9}$$

or, in terms of length and drag coefficient,

$$V = \frac{8}{f} \sqrt{\frac{Ls}{C_D}}. \tag{10}$$

Remembering that V is the speed relative to the water, it appears entirely possible that a vessel may have a downstream speed that is greater than that of the water in which it is floating. For example, Saunders (1957, v. 2, p. 321) reports that the slope thrust on a 1,000-ton barge may impart a speed 3 or 4 knots greater than that of the river in midstream, "sufficient to render it controllable by its own rudder."

LOSS OF BED CLEARANCE WITH SPEED

A vessel underway tends to lower the pressure field surrounding its hull. The motion induced in the water creates kinetic head at the expense of potential head, and the vessel is lower than when at rest. Moreover, variations in the distribution of the potential head over the hull, or eccentricities between center of thrust of the motive power and the center of the resistances, are reflected in a so-called change in the trim—usually a settling of the stern and a rise in the bow. Both effects—sinkage and a change in trim, sometimes collectively called squat—result in an increase in draft and a loss in bed clearance in shallow water. The squat is usually greater in shallow than in deep water because the decrease in bed clearance tends to increase the reduction in potential head, resulting in greater squat.

Squat increases with speed. Figure 10 shows a few data on squat of displacement craft in shallow water as obtained from Saunders (1957, v. 1, p. 530; v. 2, p. 330, 390) plotted in relation to the velocity

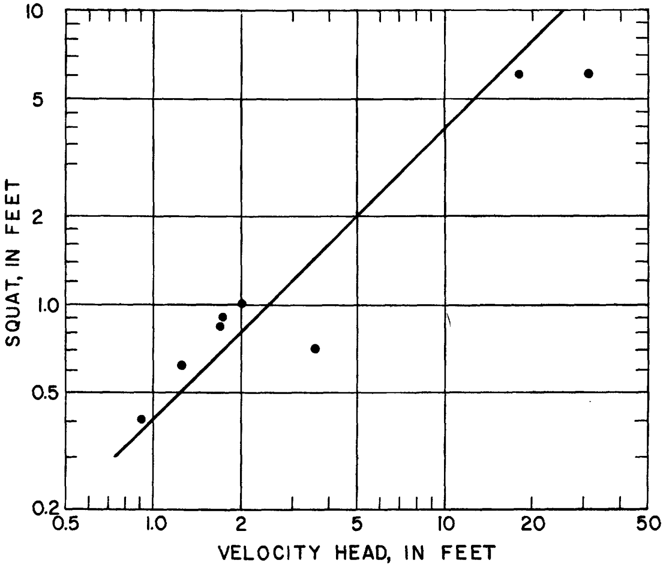

FIGURE 10.—Relation between velocity head and squat in shallow water.

head, $V^2/2g$. According to these data, squat is about 0.4 the velocity head, or 0.006 V^2.

ROUNDING RIVER BENDS

An additional source of resistance is represented by the force required in rounding river bends. This force varies with the square of the speed and inversely as the radius of curvature. Measurements reported by Leopold and Wolman (1960, p. 774) indicate that the radius of curvature of meanders averages about 2.3 times the channel width, so that as a vessel proceeds upstream and the channel narrows, the power so expended increases. Although this loss of power acts further to limit attainable speed in channels of decreasing dimensions, the loss is not sufficiently great in comparison with other factors to be controlling. Rather the dominant effect of meanders on vessel speed is through the limitations upon sight distances and maneuverability.

NAVIGABILITY OF RIVERS

MINIMUM SPECIFIC TRACTIVE FORCE

The total resistance against a vessel of weight W and draft d, moving up a river with a slope angle s, and with a speed V relative to the water, is

$$R=fW\left(\frac{n'}{1.5}\frac{V}{d^{2/3}}\right)^2+sW,$$

and the force per unit weight is

$$\frac{R}{W}=f\left(\frac{n'}{1.5}\frac{V}{d^{2/3}}\right)^2+s. \tag{11}$$

The value of river slope in equation 11 may be readily estimated for a given channel depth D and river current V_w according to the Manning equation:

$$s=\frac{n^2V_w^2}{2.25D^{4/3}}. \tag{12}$$

Inserting this value of s into equation 11 and taking $n=n'=0.03$, yields the following expression for the minimum specific tractive force;

$$\frac{R}{W}=\frac{1}{2500}\left(\frac{fV^2}{d^{4/3}}+\frac{V_w^2}{D^{4/3}}\right). \tag{13}$$

Substituting the design speed, V_d, as given by equation 4 into equation 13 yields the following expression for vessel speed:

$$V^2=\frac{V_d^2}{f}\left(1-\frac{V_w^2}{V_d^2}\frac{d^{4/3}}{D^{4/3}}\right). \tag{13a}$$

Referring again to equation 13, if we consider that the limit of navigability is achieved when the vessel speed is just sufficient to make headway upstream against the river current, then V must equal the water velocity. In other words, the value of R/W evaluated for the water velocity gives the value of the minimum specific tractive force that must be developed by a vessel to maintain upstream headway:

$$T_s=\frac{V^2}{2500}\left(\frac{f}{d^{4/3}}+\frac{1}{D^{4/3}}\right). \tag{14}$$

Examination of this equation indicates that there is an optimum draft for a given channel. A vessel with a draft nearly equal to the channel depth would be retarded by bottom drag; that is, the value

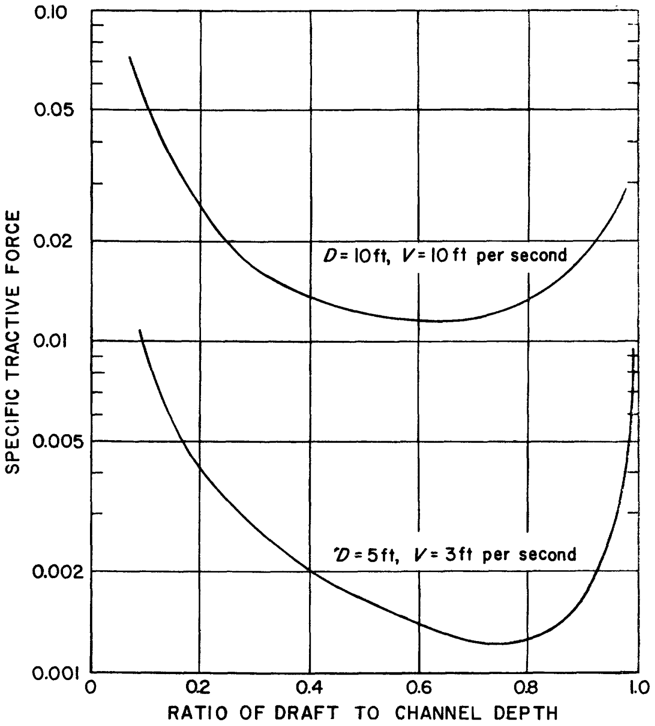

FIGURE 11.—Variation of minimum specific tractive force with ratio of draft to channel depth.

of f would be great, tending to increase T_s. On the other hand, T_s also tends to increase as the draft becomes smaller. The two factors f and d react so that T_s is infinite when $d=0$ and when $d=D$, and they react to define an intermediate minimum value of the specific tractive force. For example, figure 11 shows the variation in transport capacity as defined by equation 14, with various ratios of draft to channel depth. The curves indicate that the specific tractive force is at a minimum when the draft is about 0.7 the channel depth. Accordingly, equation 14 can be simplified as follows:

$$T_s=\frac{V^2(f+0.6)}{1600D^{4/3}}. \qquad (15)$$

As previously explained, the specific tractive force as defined by equations 13 or 14 equals that minimum which must be expended to maintain two-way navigation in a channel of depth *D* and river velocity *V*. Figure 12 shows the computed specific tractive force of various channels in terms of the channel depth and river velocity as computed by equation 15. This diagram defines the physical conditions that must be met to sustain upstream river navigation.

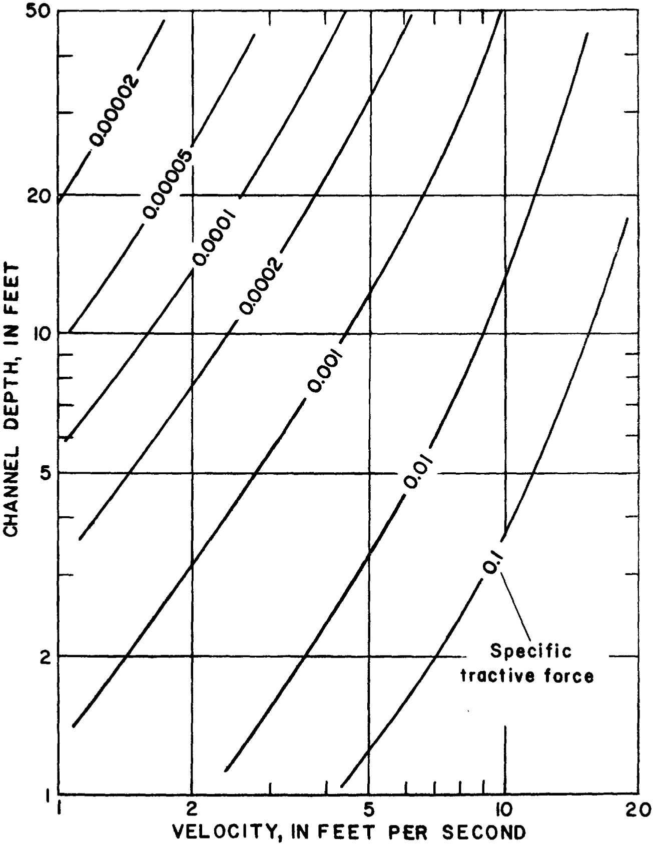

FIGURE 12.—Minimum specific tractive force required to sustain upstream navigation, in terms of velocity and depth.

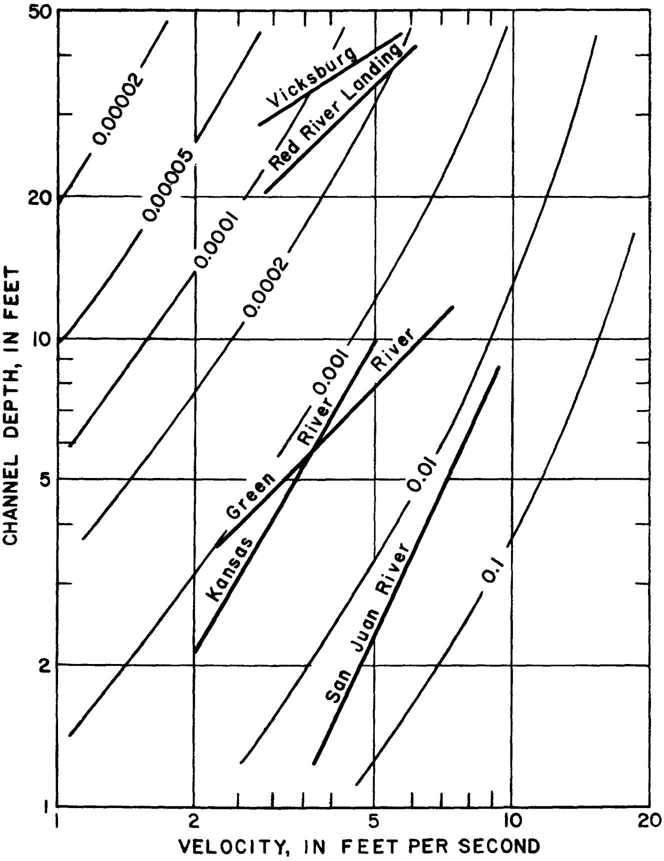

FIGURE 13.—Depth-velocity curves for several rivers in relation to minimum specific tractive force required for upstream navigation.

Figure 13 shows depth-velocity curves for several river sections drawn upon a field representing the minimum tractive force as shown on figure 12. In each case the depth-velocity curve is defined as the enveloping line GE of figure 4 as demonstrated on figure 5. As previously explained, this enveloping line represents the variation between depth and velocity in shallow river sections up to bankfull stages.

Inspection of the river data on figure 13 shows that the minimum tractive force tends to be conservative over the range of depths shown. The channel depths and velocities for the San Juan River near Bluff, Utah, indicate a required tractive force of about 0.02, an amount greater than is customary for commercial vessels.

As an opposite extreme, figure 13 also shows the depths and velocities at two gaging stations, Red River Landing and Vicksburg, on the Mississippi River. The specific tractive force for upstream movement at these sections is only on the order of 0.00015, compared with 0.004 customarily available to commercial craft on this river. At crossovers where the minimum depth is only 9 feet, the required tractive force is on the order of 0.002 times the displacement.

A list of some values of the required tractive force are given below for several rivers for purposes of comparison. Three of the rivers listed are not known to have any commercial navigation, but they are typical of rivers whose navigability is tested in the courts, usually without the benefit of a standard of comparison.

River and location	*Commercial use*[1]	*Minimum specific tractive force required for two-way navigation*
Mississippi River at Vicksburg, Miss	A	0.00015
Tombigbee River at Columbus, Miss	A	.0002
Red River at Arthur City, Tex	B	.001
Missouri River at Williston, N. Dak	B	.001
Green River at Green River, Utah	B	.002
Yellowstone River near Sidney, Mont	B	.002
Missouri River at Bismarck, N. Dak	B	.002
Kansas River at Bonner Springs, Kans	B	.002
Red River at Terral, Okla	C	.005
Rio Grande at Bernalillo, N. Mex	C	.02
San Juan River near Bluff, Utah	C	.02

[1] A, commercial waterway of the United States; B, ferry and other short-run navigation; C, no known commercial navigation.

Examination of these data for the several rivers in relation to what is known of their use for navigation indicates that rivers with specific tractive forces above 0.002 are not used for navigation. As one may note in table 1, tractive forces developed by commercial craft are as low as 0.002. Thus, to navigate rivers that require tractive forces near or more than this amount would require most of the developed energy to be expended to breast the current rather than for transport. Within the range from 0.002 to 0.001, navigation is usually limited to ferry or short-run operations. Major navigation appears to be associated with rivers that require tractive forces less than 0.001.

TRANSPORT CAPACITY

The transport capacity of a vessel may be defined as the product of its tonnage, W, and its speed, V, relative to the water. The product WV can be expressed in terms of the design speed, V_d, from equation 13a as follows:

$$WV=WV_d\sqrt{\frac{1-(V_w/V_d)^2(d/D)^{4/3}}{f}}.$$

This equation can be simplified further. As shown on figure 14, which is based on data given in table 1, the product WV_d varies as the cube of the draft; since the permissible draft is proportional to the channel depth, WV_d is proportional to D^3. Moreover, as suggested by the trend of the data shown on figure 6, $V_d^2=10d^{4/3}$. With these two substitutions, a transport-capacity index, C, can be related to the channel properties as follows:

$$C=D^3\sqrt{\frac{1-V_w^2/10D^{4/3}}{f}}. \qquad (16)$$

In this form, the expression can be interpreted as an index of the transport capacity of a river channel. A negative sign under the radical describes upstream navigation, and a positive sign, downstream navigation. The second term is zero for slack-water navigation where $V_w=0$.

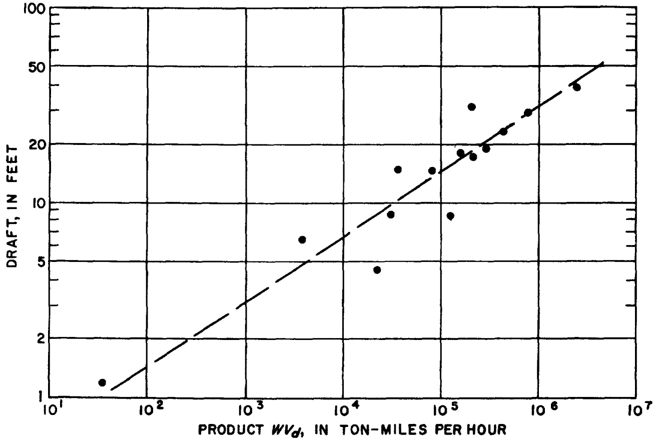

FIGURE 14.—Relation between WV_d and draft.

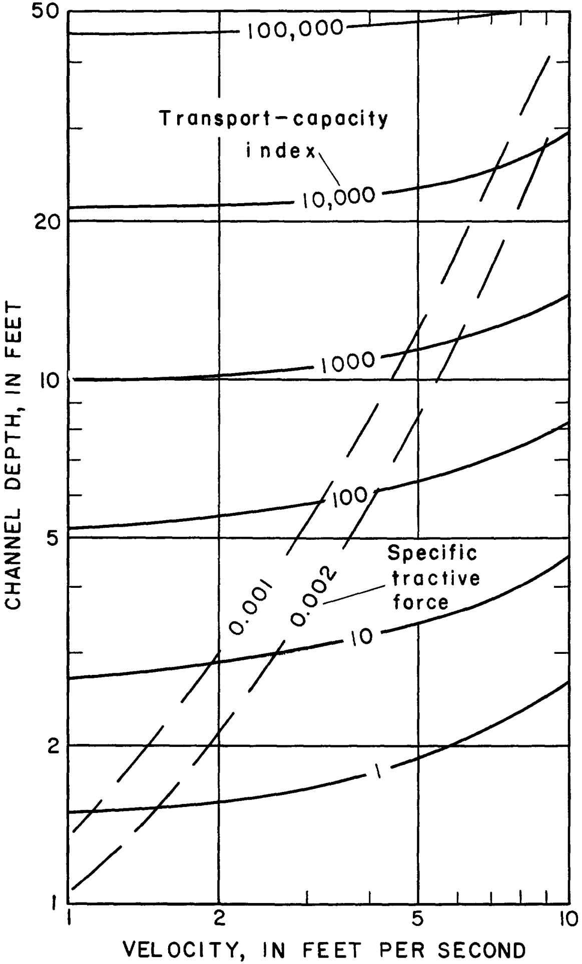

FIGURE 15.—Transport-capacity index in terms of channel depth and velocity.

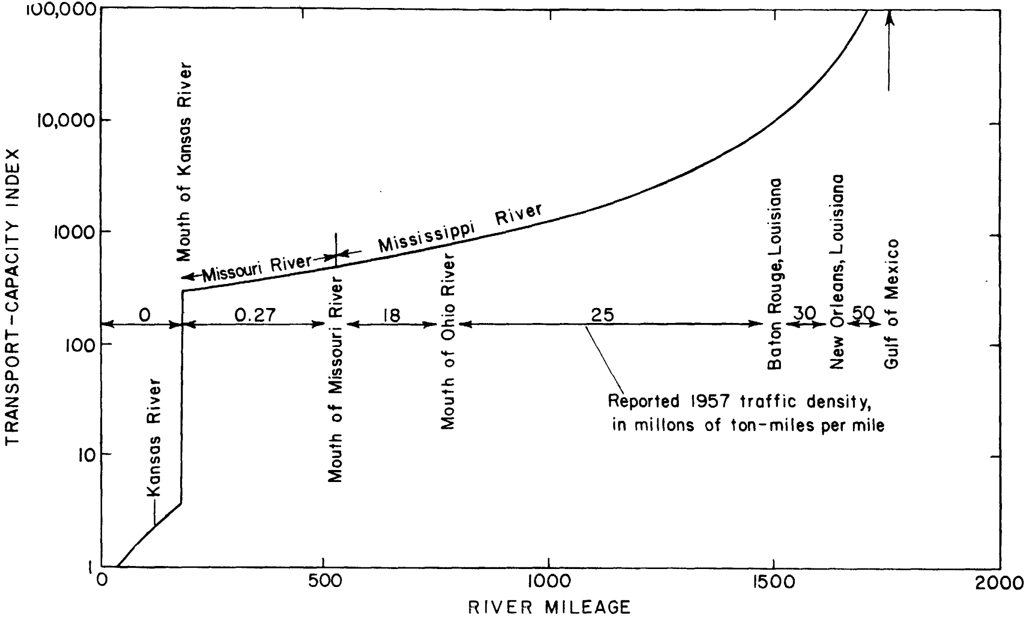

FIGURE 16.—Relation between transport-capacity index and traffic density along the Kansas, Missouri, and Mississippi Rivers.

Figure 15 shows a graph of the transport-capacity index, *C*, as defined by formula 16 for upstream navigation. The graphs indicate that channel depth is the dominant hydraulic factor in determining the transport capacity of a river.

Also shown on figure 15 are lines transferred from figure 13, showing the river tractive forces of 0.001 and 0.002 required for upstream navigation. As pointed out, river tractive forces of about these amounts are near the maximum feasible for commercial navigation. Therefore, only the region above these lines on figure 15 may be considered to define the class of navigable rivers.

Figure 1 shows the mean depths and velocities along the Kansas, Missouri, and Mississippi Rivers. Figure 16 shows for these same rivers the variation in the transport-capacity index based on controlling depths along the rivers. The diagram shows a sharp contrast between these three rivers, as well as the downstream increase in the transport capacity and traffic density along the Missouri and Mississippi Rivers.

Along the Kansas River the minimum specific tractive force required for upstream navigation ranges from 0.01 to 0.002 at the mouth. The low transport capacity and the high tractive force required explain why this river is not used for commercial navigation. Along the Missouri and Mississippi Rivers the minimum specific tractive force is less than 0.001.

The significance of the index of transport capacity may be tested by comparisons with the reported traffic densities of commercial waterways. Table 2 represents a summary of the 1957 traffic densities of inland waterways of the United States, classified by depth. Also listed in table 2 are estimates of the corresponding transport-capacity indexes. A plot of the transport-capacity index against the reported traffic density on figure 17 indicates a proportional relationship between these two factors. Considering the fact that the actual traffic in waterways reflects many factors of an economic or a commercial rather than a hydraulic nature, the correspondence seems to justify the use of the index of transport capacity as a base for comparing the navigability of rivers.

TABLE 1.—*Characteristics of commercial vessels*

Vessel	Horsepower	Displacement (long-tons)	Length (feet)	Draft (feet)	Design speed (miles per hour)	Specific tractive force	Friction factor	Drag coefficient	Reference
Mississippi River towboats and barges.	630	3, 500	-------	4. 7	6. 5	0. 0046	0. 024	------	U.S. Army Corps of Engineers (1922, table 2).
Do	3, 200	15, 000	750	8. 5	8. 1	. 0044	. 033	1. 5	United Nations (1954, p. 17–18).
Seine River barges	200	400	130	6. 9	9. 5	. 009	. 037	. 40	Office de l'Navigation de la Seine (oral communication).
Fast river steamers	4, 500	1, 500	400	9. 0	21. 0	. 024	. 035	. 60	Saunders (1957, v. 2, p. 664).
Trawlers	-------	-------	130	11. 4	13	-------	------	------	Saunders (1957, v. 2, p. 773).
Great Lakes cargo ships.	7, 000	24, 000	650	24	18. 5	. 0026	. 031	. 15	Saunders (1957, v. 2, p. 758–760).
Car ferry	3, 000	2, 022	310	15. 0	17	. 015	. 045	. 45	Saunders (1957, v. 2, p. 792).
Do	7, 000	8, 860	410	18. 5	18	. 0073	. 034	. 27	Saunders (1957, v. 2, p. 791).
Cargo ships	-------	-------	-------	8. 5	11. 5	-------	------	------	U.S. Army Corps of Engineers (1938, p. 387).
Do	-------	-------	-------	14. 5	12. 2	-------	------	------	Do.
Do	-------	-------	-------	16. 5	14. 4	-------	------	------	Do.
Do	-------	-------	-------	22. 5	17. 4	-------	------	------	Do.
Tankers	10, 000	14, 000	550	32	18	. 0067	. 046	. 37	Lloyd's Register.
Canal barge (mule)	2	25	45	1. 35	3	. 0046	. 03	. 7	Author's measurements.
Minelayer	5, 700	6, 700	-------	17. 5	20. 0	. 007	. 028	------	Saunders (1957, v. 2, p. 379).
Concrete ship	1, 600	6, 200	350	14. 5	13. 5	. 0032	. 025	. 175	Saunders (1957, v. 2, p. 378).
Cargo vessel	4, 500	9, 920	438	19. 2	18	. 0041	. 026	. 15	Saunders (1957, v. 2, p. 377).
S.S. Queen Elizabeth.	160, 000	14, 000	965	39. 5	33	. 0096	. 034	. 25	Lloyd's Register.
S.S. Independence	37, 000	30, 000	663	30	26	. 008	. 033	. 15	Do.
Average	-------	-------	-------	-------	-------	-------	. 033	------	

TABLE 2.—*Traffic density of commercial inland waterways classified by channel depth*

[U.S. Congress (1960, tables 3, 7.) Transport-capacity index estimated by author]

Controlling channel depth (feet)	Estimated transport-capacity index	Total length (miles)	Annual traffic density (millions of ton-miles per mile)
<6	75	4, 181	0. 20
6–9	285	2, 936	. 25
9–12	850	6, 397	5. 0
12–14	1, 600	4, 018	7. 0
14–20	4, 000	2, 621	12.
20–37	20, 000	644	28.

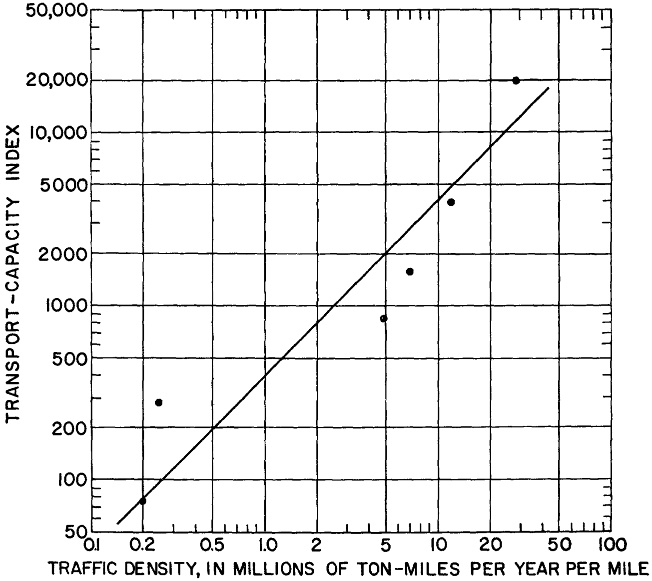

FIGURE 17. Comparison of transport-capacity indices with reported density of river traffic.

REFERENCES

Gabrielli, Guiseppe, and von Karman, Theodore, 1950, What price speed?: Mech. Engineering, v. 72, p. 775–781.

Leopold, L. B., and Maddock, Thomas, Jr., 1953, The hydraulic geometry of stream channels and some physiographic implications: U.S. Geol. Survey Prof. Paper 252.

Leopold, L. B., and Wolman, M. G., 1957, River channel patterns: braided, meandering, and straight: U.S. Geol. Survey Prof. Paper 282–B.

———, 1960, River meanders: Geol. Soc. America Bull., v. 71, p. 769–794.

Rouse, Hunter, 1946, Elementary mechanics of fluids: New York, John Wiley & Sons.

Saunders, H. E., 1957, Hydrodynamics in ship design: Soc. Naval Architects and Marine Engineers, v. 1, 648 p., v. 2, 980 p.

Schumm, S. A., 1960, The effect of sediment type on the shape and stratification of some modern fluvial deposits: Am. Jour. Sci., v. 258, p. 177–184.

United Nations, 1954, Inland water transport in Europe and the United States of America: ST/TAA/Ser. C/9.

U.S. Army Corps of Engineers, 1914, Experimental towboats: U.S. 63d Cong., 2d sess., House Doc. 857.

——— 1922, Experimental towboats: U.S. 67th Cong., 1st sess., House Doc. 108.

——— 1938, Atlantic-Gulf Ship Canal, Fla.: U.S. 75th Cong., 1st sess., House Doc. 194.

U.S. Congress, Senate Committee on National Water Resources, 1960, Water resources activities in the United States, future needs for navigation: U.S. 86th Cong., 2d sess., Committee Print 11.

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