DEPARTMENT OF THE INTERIOR

BULLETIN

OF THE

UNITED STATES

GEOLOGICAL SURVEY

No. 14

ON THE PHYSICAL CHARACTERISTICS OF THE IRON-CARBURETS MORE PARTICULARLY ON THE GALVANIC THERMO-ELECTRIC AND MAGNETIC PROPERTIES OF WROUGHT IRON STEEL AND CAST IRON IN DIFFERENT STATES OF HARDNESS. TOGETHER WITH A PHYSICAL DIAGRAM FOR THE CLASSIFICATION OF IRON-CARBURETS

WASHINGTON
GOVERNMENT PRINTING OFFICE
1885

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The publications of the United States Geological Survey are issued in accordance with the statute, approved March 3, 1879, which declares that—

"The publications of the Geological Survey shall consist of the annual report of operations, geological and economic maps illustrating the resources and classification of the lands, and reports upon general and economic geology and paleontology. The annual report of operations of the Geological Survey shall accompany the annual report of the Secretary of the Interior. All special memoirs and reports of said Survey shall be issued in uniform quarto series if deemed necessary by the Director, but otherwise in ordinary octavos. Three thousand copies of each shall be published for scientific exchanges and for sale at the price of publication; and all literary and cartographic materials received in exchange shall be the property of the United States and form a part of the library of the organization: And the money resulting from the sale of such publications shall be covered into the Treasury of the United States."

On July 7, 1882, the following joint resolution, referring to all Government publications, was passed by Congress:

"That whenever any document or report shall be ordered printed by Congress, there shall be printed in addition to the number in each case stated, the 'usual number' (1,900) of copies for binding and distribution among those entitled to receive them."

Under these general laws it will be seen that none of the Survey publications are furnished to it for gratuitous distribution. The 3,000 copies of the Annual Report are distributed through the document rooms of Congress. The 1,900 copies of each of the publications are distributed to the officers of the legislative and executive departments and to stated depositories throughout the United States.

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ANNUAL REPORTS.

Of the Annual Reports there have been already published:

I. First Annual Report to the Hon. Carl Schurz, by Clarence King. 1880. 8°. 79 pp. 1 map.—A preliminary report describing plan of organization and publications.

II. Report of the Director of the United States Geological Survey for 1880-'81, by J. W. Powell. 1882. 8°. lv, 588 pp. 61 pl. 1 map.

III. Third Annual Report of the United States Geological Survey, 1881-'82, by J. W. Powell. 1883. 8°. xviii, 564 pp. 67 pl. and maps.

IV. Fourth Annual Report of the United States Geological Survey, 1882-'83, by J. W. Powell. 1884. 8°. xii, 473 pp. 85 pl. and maps.

The Fifth Annual Report is in press.

MONOGRAPHS.

Of the Monographs, Nos. II, III, IV, V, VI, VII, and VIII are now published, viz:

II. Tertiary History of the Grand Cañon District, with atlas, by Clarence E. Dutton, Capt., U. S. A. 1882. 4°. xiv, 264 pp. 42 pl. and atlas of 24 sheets folio. Price $10.12.

III. Geology of the Comstock Lode and the Washoe District, with atlas, by George F. Becker. 1882. 4°. xv, 422 pp. 7 pl. and atlas of 21 sheets folio. Price $11.

IV. Comstock Mining and Miners, by Eliot Lord. 1883. 4°. xiv, 451 pp. 3 pl. Price $1.50.

V. Copper-bearing Rocks of Lake Superior, by Roland D. Irving. 1883. 4°. xvi, 464 pp. 15 l. 29 pl. Price $1.85.

VI. Contributions to the Knowledge of the Older Mesozoic Flora of Virginia, by Wm. M. Fontaine. 1883. 4°. xi, 144 pp. 54 l. 54 pl. Price $1.05.

VII. Silver-lead Deposits of Eureka, Nevada, by Joseph S. Curtis. 1884. 4°. xiii, 200 pp. 16 pl. Price $1.20.

VIII. Paleontology of the Eureka District, by Charles D. Walcott. 1884. 4°. xiii, 298 pp. 24 l. 24 pl. Price $1.10.

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The following are in press, viz:

IX. Brachiopoda and Lamellibranchiata of the Raritan Clays and Greensand Marls of New Jersey, by Robert P. Whitfield. 1885. 4°. ix, 338 pp. 35 pl.

X. Dinocerata. A Monograph of an Extinct Order of Gigantic Mammals, by Othniel Charles Marsh. 1885. 4°. —, — pp. 56 pl.

XI. Geological History of Lake Lahontan, a Quaternary Lake of Northwestern Nevada, by Israel Cook Russell. 1885. 4°. —, — pp. 46 pl.

The following are in preparation, viz:

I. The Precious Metals, by Clarence King.

Geology and Mining Industry of Leadville, with atlas, by S. F. Emmons.

Geology of the Eureka Mining District, Nevada, with atlas, by Arnold Hague.

Lake Bonneville, by G. K. Gilbert.

Sauropoda, by Prof. O. C. Marsh.

Stegosauria, by Prof. O. C. Marsh.

BULLETINS.

The Bulletins of the Survey will contain such papers relating to the general purpose of its work as do not properly come under the heads of ANNUAL REPORTS or MONOGRAPHS.

Each of these Bulletins will contain but one paper and will be complete in itself. They will, however, be numbered in a continuous series, and will in time be united into volumes of convenient size. To facilitate this each Bulletin will have two paginations, one proper to itself and another which belongs to it as part of the volume.

Of this series of Bulletins Nos. 1 to 14 are already published, viz:

1. On Hypersthene-Andesite and on Triclinic Pyroxene in Augitic Rocks, by Whitman Cross, with a Geological Sketch of Buffalo Peaks, Colorado, by S. F. Emmons. 1883. 8°. 42 pp. 2 pl. Price 10 cents.

2. Gold and Silver Conversion Tables, giving the coining value of Troy ounces of fine metal, etc., by Albert Williams, jr. 1883. 8°. ii, 8 pp. Price 5 cents.

3. On the Fossil Faunas of the Upper Devonian, along the meridian of 76° 30′, from Tompkins County, New York, to Bradford County, Pennsylvania, by Henry S. Williams. 1884. 8°. 36 pp. Price 5 cents.

4. On Mesozoic Fossils, by Charles A. White. 1884. 8°. 36 pp. 9 pl. Price 5 cents.

5. A Dictionary of Altitudes in the United States, compiled by Henry Gannett. 1884. 8°. 325 pp. Price 20 cents.

6. Elevations in the Dominion of Canada, by J. W. Spencer. 1884. 8°. 43 pp. Price 5 cents.

7. Mapoteca Geologica Americana. A catalogue of geological maps of America (North and South), 1752–1881, by Jules Marcou and John Belknap Marcou. 1884. 8°. 184 pp. Price 10 cents.

8. On Secondary Enlargements of Mineral Fragments in Certain Rocks, by R. D. Irving and C. R. Vanhise. 1884. 8°. 56 pp. 6 pl. Price 10 cents.

9. A Report of work done in the Washington Laboratory during the fiscal year 1883–'84. F. W. Clarke, chief chemist; T. M. Chatard, assistant. 1884. 8°. 40 pp. Price 5 cents.

10. On the Cambrian Faunas of North America. Preliminary studies by Charles Doolittle Walcott. 1884. 8°. 74 pp. 10 pl. Price 5 cents.

11. On the Quaternary and Recent Mollusca of the Great Basin; with Descriptions of New Forms, by R. Ellsworth Call; introduced by a sketch of the Quaternary Lakes of the Great Basin, by G. K Gilbert. 1884. 8°. 66 pp. 6 pl. Price 5 cents.

12. A Crystallographic Study of the Thinolite of Lake Lahontan, by Edward S. Dana. 1884. 8°. 34 pp. 3 pl. Price 5 cents.

13. Boundaries of the United States and of the several States and Territories, by Henry Gannett. 1885. 8°. 135 pp. Price 10 cents.

14. The Electrical and Magnetic Properties of the Iron-Carburets, by Carl Barus and Vincent Strouhal. 1885. 8°. 238 pp. Price 15 cents.

Numbers 1 to 6 of the Bulletins form Volume I, and numbers 7 to 14 Volume II. Volume III is not yet complete.

The following are in press, viz:

15. On the Mesozoic and Cenozoic Paleontology of California, by Dr. C. A. White. 1885. 8°. 33 pp. Price 5 cents.

16. On the higher Devonian Faunas of Ontario County, New York, by J. M. Clarke. 1885. 8°. — pp. 3 pl. Price — cents.

17. On the Development of Crystallization, etc., by Arnold Hague and J. P. Iddings. 1885. 8°. — pp. Price — cents.

18. On Marine Eocene, Fresh-water Miocene, and other Fossil Mollusca of Western North America, by Dr. C. A. White. 1885. 8°. — pp. 3 pl. Price — cents.

19. Notes on the Stratigraphy of California, by George F. Becker. 1885. 8°. — pp. Price — cents.

20. Contributions to the Mineralogy of the Rocky Mountains, by Whitman Cross and W. F. Hillebrand. 1885. 8°. — pp. 1 pl. Price — cents.

21. The Lignites of the Great Sioux Reservation, by Bailey Willis. 1885. 8°. — pp. 5 pl. Price — cents.

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STATISTICAL PAPERS.

A fourth series of publications having special reference to the mineral resources of the United States is contemplated.

Of that series the first has been published, viz:

Mineral Resources of the United States, by Albert Williams, jr. 1883. 8°. xvii, 813 pp. Price 50 cents.

The second volume of this series, Mineral Resources 1883 and 1884, is in preparation and will soon be put to press.

Correspondence relating to the publications of the Survey, and all remittances, which must be by POSTAL NOTE or MONEY ORDER, should be addressed

TO THE DIRECTOR OF THE
UNITED STATES GEOLOGICAL SURVEY,
Washington, D. C.

WASHINGTON, D. C., *April* 30, 1885,

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No. 14

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UNITED STATES GEOLOGICAL SURVEY

J. W. POWELL DIRECTOR

THE

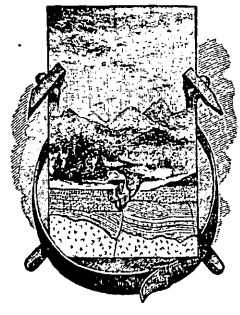
ELECTRICAL AND MAGNETIC PROPERTIES

OF THE

IRON-CARBURETS

BY

CARL BARUS and VINCENT STROUHAL



WASHINGTON

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PREFACE.

Early in 1881 one of us submitted to Prof. G. F. Becker, geologist in charge of the Division of the Pacific, a brief digest of the facts showing the singular adaptability of the electrical properties of the iron-carburets for the classification of these products, with the request that permission be given us for the extension of the work in the laboratory of the Geological Survey. Professor Becker at once indorsed the project enthusiastically, and owing to his advocacy our proposal shortly after received the assent of the Hon. Clarence King, then Director of the Survey,—given with the proviso that the electrical researches be not prosecuted to such an extent as to occupy us exclusively.

Divers special investigations and routine duties, together with the labor involved in providing for the organization of a physical laboratory, prevented us from giving the furtherance of the proposed researches the attention necessary. Nevertheless, data of a varied character continually accumulated, while the scope and the conception of our general problem enlarged at an unexpectedly rapid rate. The publication of our results in some connected form, therefore, urged itself more and more seriously upon us.

About a year ago our plan was effectually encouraged by Prof. F. W. Clarke, chief chemist of the United States Geological Survey.

The experiments to be discussed in this memoir have occupied our available time during the last five years. Notices, more or less complete, have appeared abroad from time to time in places not readily accessible to the public. Some of the papers it was deemed necessary to publish in German with considerable fullness. But all English publication has been purposely delayed, not only because we desired to reduce the results originally expressed in terms of the German standards to the more current and now legal denominations of ohm, volt, etc.,[1] but principally because it seemed expedient for facility of comparison to refer all our data to the uniform temperature, zero centigrade. This

[1] In making this reduction the legal equation, 1 ohm = 1.06 S. U., was made use of in all chapters with the exception of III and IV, the results of which were reduced at an earlier date and when 1 ohm = 1.05 S. U. appeared to be nearer the truth. This, however, is of no serious significance, because in these chapters the relative values of resistance are alone of interest. The absolute accuracy of the reduced values is of course immediately dependent on the absolute accuracy of the German standards (Siemens) at our disposal.

premised an accurate knowledge of the relation between electrical conductivity and temperature for iron, for steel in different states of temper, and for cast iron, and required excessively tedious labor.

The results in Chapter I on the electrical temperature-coefficient of iron-carburets present an unexpected range of variation, and thus possess intrinsic interest.

In Chapter II we investigate and discuss the conditions of the operation of tempering. This chapter is fundamental. Such facts as essentially sustain the argument underlying the whole of the present work are, therefore, emphasized with a larger number of experimental data than would otherwise be necessary.

In Chapter III we attempt to throw new light on the laws set forth in Chapter II by following them into their ulterior consequences. With the aid of certain allied electrical properties of alloys and of malleable cast iron, the nature of the phenomenon of hardness as presented by steel is discussed from every available physical and chemical point of view, within the scope of the present purposes.

In Chapters IV, V, and VI, the method for the accurate definition of hardness, and the scheme of operations for tempering developed in Chapter II, are consistently applied to analogous magnetic phenomena. The nature of the dependence of magnetization on the three independent variables of cylindrical rods, viz: carburation, ratio of dimensions, hardness, for given conditions of structure, is carefully discussed and in part graphically represented. Rules are finally given for the treatment of magnets, such that exceptionally great retentiveness, both as regards the hurtful effects of temperature and time and of shocks, may be conveniently attained with the least available sacrifice of magnetization.

We may add that a supplementary Bulletin is now in preparation, in which the very remarkable annealing effect of high temperatures (400°..1000°) will be magnetically discussed, and furthermore the degree of approximate coincidence between the physical state (of the necessarily linear rod) characterized by the unique maximum of magnetizability and the physical state of maximum density of steel, will be determined. That these states must be found very nearly coincident, our present results permit us to predict.

In Chapter VII, finally, we endeavor to generalize upon the foregoing results, as a whole; to restate the fundamental laws with greater accuracy and breadth of scope than was possible in the earlier chapters; and, finally, to deduce from all a method for the physical definition of iron-carburets.

Minor discussions, the relevancy of which does not justify their immediate introduction into the chapters proper, are frequently introduced as addenda.

We desire in this place particularly to emphasize that throughout the present work the terms "*thermo-electrically positive*" and "*thermo-electrically negative*" are to be understood in the way defined by the original

investigators, Seebeck,[2] Becquerel,[3] Hankel;[4] *i. e.*, with reference to the series arranged thus:

$$-\text{Bi}.... \text{Cu}... \text{Fe}.... \text{Sb}+,$$

an acceptation which we believe to be general on the Continent.[5] In England the above terms are received in a sense which is precisely the opposite of this; that is, with reference to the thermo-electric series,[6] arranged thus:

$$+\text{Bi}.... \text{Cu}... \text{Fe}.... \text{Sb}-.$$

Of the two methods of designation, the latter is obviously the more logical and consistent, as will readily be seen, for instance, if the galvanic and the thermo-element be analogously described. And if we refrained from embodying the latter acceptation in this memoir, we have done so merely because of the great liability to error encountered in changing the sense of every thermo-electric expression and diagram, as well as the signs of all of the many thermo-electric data. Isolated constructions are too apt to be overlooked, and this in a way completely to mar the drift of the context. But the English reader will find no difficulty in making this change of sign for himself in any set of thermo-electric data which may interest him. To avoid all misconception, moreover, we give the direction of current in each essential case.

Much of the work was done abroad in Professor F. Kohlrausch's laboratory. It is a pleasant duty which permits us to extend to Professor Kohlrausch, in this place, our grateful acknowledgments, not only for the kindly interest with which he regarded the progress of the experiments throughout their extent, but for much valuable advice by which the papers have materially profited.

We desire to mention, in conclusion, that work done by us conjointly, if published in German, is to be put under S. and B.; if in English, under B. and S., conformably with an original agreement.

C. BARUS.
V. STROUHAL.

PHYSICAL LABORATORY,
UNITED STATES GEOLOGICAL SURVEY,
Washington, December 1, 1884.

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[2] Seebeck: Gilb. Ann., LXXIII, pp. 115 and 430, 1823.

[3] Becquerel: Ann. de Chim. et de Phys., XLI, p. 353, 1829.

[4] Hankel: Pogg. Ann., LXII, p. 197, 1844.

[5] Cf. Wiedemann: Lehre v. d. Elektricität, II, p. 248 et seq., 1883. Mousson: Physik, III, p. 381 et seq., 1875. Jamin: Cours de Physique, 2d ed., T. III, p. 42, 1869.

[6] Cf. Jenkin: Electricity and Magnetism, p. 175 et seq., 1880. Maxwell: Electricity and magnetism, 2 ed., Vol. I, p. 338-9, 1881.

SUPPLEMENTAL.

The principal contents of the supplementary Bulletin referred to in the above preface may expediently be placed on record here:

1. The inter-dependence of density, electrical conductivity, maximum of permanent magnetization, maximum of permanent hardness of linear cylindrical steel rods, under all permissible conditions of temperature. The location of the unique magnetic maximum.

2. The bearing of temperature and time of exposure on the temper-value of the color of the oxide-films.

3. The internal structure of tempered steel.

4. Certain synthetic methods of production of iron-carburets, available for the construction of the classification-diagram.

B. & S.

WASHINGTON, *March* 27, 1885.

CONTENTS.

ILLUSTRATIONS.

INTRODUCTION.

To avoid ambiguity and vagueness we will state here, at the outset, that the considerations presented in this memoir apply to that species of hardness which can be imparted to steel by a process of tempering; *i. e.*, by a process of sudden cooling from a given temperature in red heat, accompanied by a stated amount of subsequent annealing. Such an operation, regarded as a method in virtue of which the metal acted upon experiences a particular and characteristic kind of strain, must be accompanied by a series of physical effects peculiar to itself. It is true that the conditions which determine the efficacy of tempering will themselves have to be accurately defined. Even under favorable circumstances these are not thoroughly within the observer's control, or, at best, are attainable with extreme difficulty. All results are, therefore, distorted by a variety of anomalies. But in the great number of data investigated a certain normal effect clearly appears, and it is to this that our remarks apply. In other words, we have been led in an unavoidably laborious way to the results of theoretically perfect tempering.

With regard to the hardness of steel, we cannot as yet rigidly discriminate between the effect to be ascribed to a change in the quality of carburation and that which is due to the strain simultaneously experienced; *i. e.*, the numerical value of the importance of the said effects has as yet remained undeterminable, though much has been done toward assigning to each its respective limit. But, *a priori*, in so far as the character of this strain-effect will enable us satisfactorily to interpret a majority if not all of the attendant physical phenomena, we are temporarily justified in giving the mechanical postulate preference. At all events it is frequently permissible to abstract from the chemical change altogether, and to speak of tempering a rod to glass-hardness, just as we do of magnetizing it to saturation, for instance. Both operations may be said to be such that more of a given kind of strain (hardness in the one case and magnetism in the other) is originally imparted to the rod than it is able, *of itself*, to maintain. A part of this disappears, while a residue, characteristic of the nature of the rod and of the physical circumstances under which it exists, is permanently retained. This consideration suggests some analogies between hardness and magnetism. Irrespective of its practical importance and from a purely physical point of view, tempering possesses an intrinsic interest inferior only to magnetization, or indeed comparable with it. Of the enormous stress which the former operation enables us to apply, the steel rod, in virtue of a peculiar internal structure, retains a phenomenal amount. This great intensity of available strain we may cause to disappear as gradually

or to reappear as often as we please. Conformably with the change of mechanical state, the extreme values of the electrical and magnetic properties of steel comprehend a similarly extended interval. We are thus enabled to follow these, in the case of the same material, through a range of variation that is enormous, and must throw new light on their intrinsic nature.

Another point deserves brief mention here. If we conceive a theoretically perfect process of tempering to glass-hardness, and suppose it applied to rods identical in every respect, the results must necessarily be identical. In other words, the type of internal structure presented by the first would be reproduced in all subsequent rods. Take the comparatively simple but important case of cylindrical rods: The density of the elementary cylindrical shells, coaxial with the respective cylinders, would be the same for the same radius; the distribution of density along similar radii would follow the same law. If now the rods be identically annealed the results must still be identical, and thus we arrive eventually at identical soft states. It follows, therefore, that for rods of the same dimensions (*diameter*) and composition the magnetic properties are immediately comparable as functions of hardness.

This we are no longer at liberty to assume when the (cylindrical) rods otherwise identical, have different diameters. For this reason alone the rods, though tempered alike, *i. e.*, subjected to the same operation, are to be regarded as structurally dissimilar. It is in place here to present some concise meaning for the term "structure" as applied to hard steel rods. We, therefore, define it as the law of the variation of density encountered on a passage along any radius of a given (cylindrical) steel rod from its axis to its circumference. This premised, it is not absolutely impossible (however improbable) that rods of different diameters, identically tempered, may, *cæteris paribus*, show identical structures. That such a unique condition of things cannot be postulated is obvious at once. Indeed, the manner of variation of the law of distribution of density, as we pass from rods of a given thickness to rods of any other thickness, is not even conjecturable, and we cannot, therefore, consistently compare the magnetic intensities of rods of different diameters as functions of hardness. In the most favorable case we would encounter in our passage from rod to rod, besides the difference of hardness, something of the nature of a change of parameter, expressing the necessary difference of internal structure referred to. In other words, it will be shown in the sequel that with a given thickness, a characteristic family of magnetic curves may be obtained, referred to hardness and length of rod as independent variables. We infer that such a family exists for each thickness, and that our passage from one given diameter to another is expressible by a difference in the value of a parametric constant.

We have mentioned magnetic phenomena in particular, because it is here that the evidence derived from data illustrating the influence of structure is singularly cogent.

THE ELECTRICAL AND MAGNETIC PROPERTIES OF THE IRON-CARBURETS.

By CARL BARUS and VINCENT STROUHAL.

CHAPTER I.

ON THE RELATION BETWEEN ELECTRICAL CONDUCTIVITY AND TEMPERATURE IN THE CASE OF STEEL IN DIFFERENT STATES OF HARDNESS, OF WROUGHT IRON, AND OF CAST IRON.

STEEL.

Earlier results.—The experiments made thus far on the resistance-effect of temperature[7] scarcely permit us to distinguish in this particular between iron and steel. In fact, the data in hand for the said coefficients lie within about the same interval, 0.004 to 0.005, both for the one metal and the other. We will give as examples some of the best results of earlier observers.

According to Mousson,[8] the electrical resistance s_t at the temperature t is expressible in terms of s_o, the corresponding quantity at zero, by an equation with a single constant:

$$s_t = s_o\,(1+0.00421\times t)$$

for iron;

$$s_t = s_o\,(1+0.00406\times t) \text{ to } s_t = s_o\,(1+0.00424\times t)$$

for steel.

Benoit,[9] who carried his researches to much higher degrees and through much greater intervals of temperature, finds:

$$s_t = s_o\,(1+0.00452\cdot t+0.000\,005\,83\cdot t^2)$$

in case of iron;

$$s_t = (1+0.00498\cdot t+0.000\,007\,35\cdot t^2)$$

in case of soft steel.

Nevertheless we felt justified in believing that these researches in case of a substance which, like steel, is capable of existing in so many enormously different states of hardness, which presents such an incomparably wide range of values of electrical conductivity, are far from complete; we regarded the assertion warrantable that steel cannot, under all circumstances, possess a temperature-coefficient of electrical resistance so little different from that of iron. Indeed, we had reasons to

[7]For a very complete digest of the experimental results in question, see G. Wiedemann, Lehre von der Elektricität, I, p. 502–510, 1882.

[8]Mousson: G. Wiedemann, l. c., p. 507.

[9]Benoit; Comptes. Rend., LXXVI, p. 342, 1873. Carl's Rep., IX, p. 55, 1873.

presume that between the resistance of a metal in any given physical state and the corresponding temperature-coefficient, a relation would in all probability be discoverable, and that an example of such a relation could be most satisfactorily studied with steel itself.

Analogous behavior of alloys.—Some facts lending favor to this view may be cited. It is known that alloys of two metals vary in marked degree as regards their electrical conductivity with the relative quantity of a second or foreign metallic ingredient added to the original metal. Great numbers of valuable results on these relations have been gathered by Matthiessen and Vogt.[10] Thus, for instance, the alloy silver-platinum, according to these observers, shows the following electrical behavior: If λ_t be the (relative) electrical conductivity of a given silver-platinum alloy at $t°$; if volume-percents of platinum alloyed to silver be understood; and if the wires be supposed hard drawn; then,

Platinum, 0 per cent. $\lambda_t = 100 - 0.38287 \cdot t + 0.000\,984\,8 \cdot t^2$;
Platinum, 2.51 per cent. $\lambda_t = 31.640 - 0.03936 \cdot t + 0.000\,036\,42 \cdot t^2$;
Platinum, 5.05 per cent. $\lambda_t = 18.031 - 0.01395 \cdot t + 0.000\,011\,82 \cdot t^2$;
Platinum, 19.65 per cent. $\lambda_t = 6.696 - 0.00221 \cdot t + 0.000\,001\,393 \cdot t^2$.

If we have reference to a small interval of temperature only, these results may be more perspicuously given by the aid of a single (mean) coefficient calculated from the observed conductivities. More simply, therefore,

Platinum, 0 per cent. $\lambda_t = 100\ (1 - 0.00383 \cdot t)$;
Platinum, 2.5 per cent. $\lambda_t = 31.6\ (1 - 0.00124 \cdot t)$;
Platinum, 5.1 per cent. $\lambda_t = 18.0\ (1 - 0.00077 \cdot t)$;
Platinum, 19.7 per cent. $\lambda_t = 6.7\ (1 - 0.00033 \cdot t)$.

Herefrom it is obvious that the temperature-coefficient of an alloy varies continuously and in a pronounced way with its electrical conductivity. This becomes all the more strikingly apparent when the results are represented graphically.

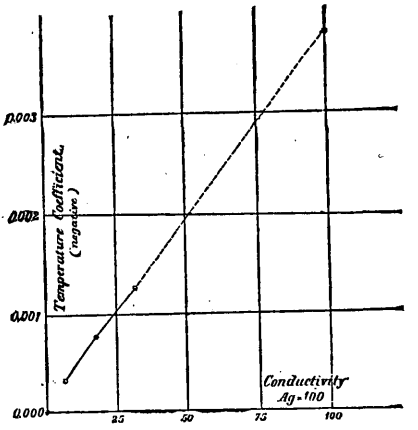


FIG. 1.—Electrical conductivity and temperature-coefficient of silver-platinum alloys.

[10] Matthiessen and Vogt: Pogg. Ann., CXXII, p. 19, 1864.

Similar results, but less complete, and therefore superfluous here, we ourselves obtained with German silver. But, to give an instance: the specific resistance, s_t, of the German silver wire of C. Vogel, Berlin, was found to be (s_t given in cm/cm^2, microhm),

$$s_t=16.4\ (1+0.00064 \cdot t),$$

whereas for that of W. Siemens, Berlin,

$$s_t=39.0\ (1+0.000\,9 \cdot t).$$

Now we shall show elsewhere that the analogy between the electrical behavior of steel in different states of hardness, and that of alloys of two metals in different proportions is, from certain points of view, very complete. It follows, therefore, that a relation similar to the one set forth is to be anticipated in case of steel, such that the variation of electrical resistance produced by tempering must be accompanied by a similarly continuous but inverse change of the values of the temperature-coefficient of steel. Our experiments fully corroborate this.

Resistance-temperature equation.—Before proceeding further, however, we may remark that it would be impossible in case of steel to adhere to the very desirable formula

$$s_t=s\ (1+at+bt^2),$$

throughout. Consistency, therefore, induces us to assume a linear relation in all cases. The reasons are these: the relation between resistance and temperature-coefficient to be investigated necessarily and primarily excludes all possibility of permanent change in the material itself. When glass-hard steel is examined, the interval of temperature within which the rod may be heated or cooled without experiencing perceptible annealing is very limited. The same is true of moderately annealed steel. We are as yet ignorant of the effects of this kind which may possibly also be produced by cooling below zero. Hence it is indispensably necessary to vary the temperature of glass-hard rods only so much as is just called for, if the measurements are to furnish satisfactorily reliable data. Should an annealing effect occur, the results would not of course be comparable. Now the available interval of temperature is fully large enough for the linear formula. It is insufficiently so in the other case; for in the quadratic formula it is possible to change the coefficient of t to quite an appreciable extent without producing marked variation in the values of s_t, if only an appropriate and compensatory change be made in the coefficient of t^2, simultaneously. Hence the simple formula

$$s_t=s_o\ (1+\alpha t)$$

has been made use of throughout, where t was permitted to vary within the interval 10° to 35° only.

Method of measurement.—The changes of temperature, and therefore also those of resistance, being very slight, amounting only to a few hundredths ohm, it was necessary to make the electrical measurements with extreme accuracy. For this purpose the method of Matthiessen and Hockin, which we have repeatedly used, proved to be admirably

serviceable. A few modifications had to be introduced. In the place of the two needles originally employed for obtaining contacts at fixed distances apart, and which if submerged in water would have introduced the disturbing effects of loose contacts, two fine copper wires were tightly wound around the extreme parts of the steel rod under experiment, and fastened in a way that made sliding impossible. The rod was then alternately placed in two vessels containing cold and warm water respectively, care being taken not in any way to strain the copper circuit wires. The short-circuiting through distilled water from one fine copper wire to the other is obviously negligible.

The temperature of the cold bath was approximately that of the room, and very constant; that of the warm bath varied during a single measurement not more than a few tenths of a degree. Temperature being read before and after the resistance measurement, the mean value could be regarded as a very satisfactory datum. The steel rod examined was placed first in the cold, then in the warm, and finally again in the cold bath. The mean of the measurements 1 and 3 was therefore to be combined with 2, 4 and 6 with 5, etc. The degree of approximate equality of the results of these distinct sets of observations, and the agreement between the measurements 1, 3, 4, 6, etc., give a good estimate of the accuracy of the work.

Material. Resistance-value of oxide-tints.—The material used was that employed in all our researches, English "silver" steel in rods 0.15 cm. in diameter. After having been suddenly chilled in great numbers, certain of them were annealed by the electrical current, in this way: Having carefully polished the hard rod, it was introduced into the circuit of a dynamo-electric machine. As the temperature of the wire increased the oxide-tints appeared in a strikingly perfect manner, and it was only necessary to regulate the current cautiously and stop the operation at a given moment to obtain any oxide-tint desired almost uniformly over the whole length of the wire. The observations of this paragraph therefore give in an approximate way the temper-value of the oxide-tint appearing in air on a bright hard steel rod, in terms of electrical resistance.

Results.—The following table, 1, contains a perspicuous comparison of the data of observation as obtained with six steel rods in different states of temper. For the diameter 2ρ (cm) of the rod and the length l (cm) direct measurement showed the resistance w (ohm) at the temperature $t°$ (*C.*). Also the resistance W (ohm) at $T°$. This is sufficient for the calculation of w_0 for 0° and a, the required coefficient. The known dimensions then enable us to deduce the specific resistance s (cm/cm² 0°). Finally, we give the specific gravity Δ of the wires, calculated for the known dimensions and the known weight. Our object in computing this constant was primarily that of checking the values for the sections of our rods as measured by the aid of the microscope. But they furnish a satisfactory corroboration of the general increase of dens-

ity of steel on passing from the hard to the soft state, conformably with the results of C. Fromme:

TABLE 1.—*Temperature-coefficient of steel.*

Rod.	2ρ, l, Δ	w	t	W	T	w_0	α	s
	cm.	*ohm.*		*ohm.*		*ohm.*		*microhm.*
Glasshard	2ρ= 0.151	0.04523	10.0	0.04685	32.9	0.04450	0.00161	45.7
	l=17.52	20	10.2	59	29.5	.....	160	.....
	Δ= 7.56	24	10.4	.....	.....	.....	.....	.....
Annealed light-yellow	2ρ= 0.148	0.03107	10.2	0.03297	35.5	0.03030	0.00250	28.9
	l=17.98	06	10.3	65	32.2	.....	238	.....
	Δ= 7.57	10	10.6	.....	.....	.....	.....	.....
Annealed yellow	2ρ= 0.150	0.02782	10.9	0.02950	33.2	0.02698	0.00278	26.3
	l=18.17	84	11.0	22	29.6	.....	279	.....
	Δ= 7.54	81	11.0	.....	.....	.....	.....	.....
Annealed blue	2ρ= 0.149	0.02043	10.0	0.02191	32.5	0.01978	0.00327	20.5
	l=16.77	47	10.1	75	29.8	.....	332	.....
	Δ= 7.56	45	10.2	.....	.....	.....	.....	.....
Annealed light-blue	2ρ= 0.148	0.01948	9.3	0.02097	31.6	0.01881	0.00357	18.4
	l=17.58	46	9.5	70	27.7	.....	363	.....
	Δ= 7.66	47	9.7	.....	.....	.....	.....	.....
Soft	2ρ= 0.146	0.01690	9.7	0.01850	32.5	0.01625	0.00428	15.9
	l=17.13	94	9.9	27	29.4	.....	419	.....
	Δ= 7.69	96	10.0	.....	.....	.....	.....	.....

If for α we take mean values, and compare these with the degrees of hardness of steel, characterized by s, we obtain more clearly:

TABLE 2.—*Oxide-tint, specific electrical resistance and electrical temperature-coefficient of steel.*

	$s\,\frac{\text{cm}}{\text{cm}^2}\,0°$	α
	microhm.	
Glasshard	45.7	0.00161
Annealed light yellow	28.9	244
Annealed yellow	26.3	280
Annealed blue	20.5	330
Annealed light blue	18.4	360
Soft	15.9	423

The electrical temperature-coefficient of steel, therefore, decreases in proportion as its specific resistance or its degree of hardness increases, at a rate diminishing as we pass from soft to hard steel.

The following little table interpolated from the above values, for practical purposes, may be put on record here:

TABLE 3.—*Specific electrical resistance and electrical temperature-coefficient of steel. Practical table.*

s	α	s	α	s	α	s	α
$\frac{\text{cm}}{\text{cm}^2}\,0°$		$\frac{\text{cm}}{\text{cm}^2}\,0°$		$\frac{\text{cm}}{\text{cm}^2}\,0°$		$\frac{\text{cm}}{\text{cm}^2}\,0°$	
microhm.		*microhm.*		*microhm.*		*microhm.*	
10	0.0050	21	0.0033	32	0.0022	43	0.0017
11	48	22	32	33	21	44	17
12	46	23	31	34	21	45	16
13	44	24	29	35	21	46	16
14	42	25	28	36	20	47	15
15	41	26	27	37	19	48	15
16	39	27	27	38	19	49	15
17	38	28	26	39	19	50	15
18	36	29	25	40	18	60	13
19	35	30	24	41	18	70	13
20	34	31	23	42	17	80	12

WROUGHT IRON.

Digest of earlier results.—The relation between electrical resistance and temperature in case of iron has been studied by a large number of observers, among whom Lenz, Becquerel, Arndtsen, Mousson, and others, are to be mentioned. But the most comprehensive and accurate data are unquestionably those given by Matthiessen and Vogt.[11] These will therefore be discussed here.

Matthiessen and Vogt assume the quadratic formula

$$\lambda=\lambda_o-at+bt^2$$

for λ, the conductivity of any given metal, relatively to hard-drawn silver ($\lambda_o=100$). Their results for a and b, in case of fifteen samples of iron, conveniently abbreviated, are contained in Table 4.

TABLE 4.—*Electrical temperature-coefficient and electrical conductivity of divers samples of iron.*

Description of sample.	No.	a	b	λ_o
Electrotype iron; Nos. 2 and 4 ignited in hydrogen and in air, respectively	1	0.512	0.00129	(16.810)[12]
	2	0.519	134	(16.810)[12]
	3	0.514	132	(16.810)[12]
	4	0.509	127	(16.810)[12]
Drawn iron wire, analyzed chemically	5	0.473	112	15.712
	6	0.472	112	15.640
	7	0.449	102	14.204
	8	0.453	112	12.132
Iron wires of different degrees of carburation	9	0.463	109	14.723
	10	0.418	092	10.666
	11	0.404	092	9.921
	12	0.397	091	9.449
Piano-forte wire	13	0.425	092	13.293
Watchspring	14	0.340	063	8.568
Commercial iron wire	15	0.428	090	18.774

For the sake of facilitating a comparison of these results with our own, we reduced them, as nearly as possible, to absolute values of $s\,\frac{\text{cm}}{\text{cm}^2}$ 0° microhm, by accepting for "silver hard," for which Matthiessen and Vogt put $\lambda=100$,

$$s=1.574.$$

Moreover, the values have been arranged, commencing with pure iron, in the order of the values for resistance, the coefficient of quadratic t being discarded and only linear d introduced. The interpretation to be given to "α interpolated" will be explained presently.

[11] Matthiessen and Vogt: Pogg. Ann., CXVIII, p. 431, 1863.

[12] Probable value for pure iron, hard, deduced from the observations with impure metal, from an inspection of the respective temperature-coefficients.

TABLE 5.—*Specific electrical resistance and electrical temperature-coefficient of different kinds of iron.*

Description of sample.	$s\frac{cm}{cm^2}$ 0°	α observed.	α interpolated.
	microhm.		
Nos. 1 to 4	9.4	0.0052	0.0052
No. 5	10.0	47	50
No. 6	10.1	47	50
No. 9	10.7	46	49
No. 7	11.1	45	48
No. 15	11.4	43	47
No. 13	11.8	42	46
No. 8	13.0	45	44
No. 10	14.8	42	41
No. 11	15.9	40	40
No. 12	16.7	40	37
No. 14	18.4	34	36
Steel, soft	15.9	42	40

Resistance-temperature equations of iron and of steel.—The first line of these data, showing the mean value of a number of observations with pure iron, is the most reliable. If this be added to our results for steel (Table 2), the whole series of results may be compared graphically, by representing specific resistance, s, as abscissa temperature-coefficient, α, as ordinate. The points lie satisfactorily on a locus of definite character, which in its turn may be utilized for purposes of interpolation. In this way the last column of the foregoing table ("α interpolated") has been deduced, the temperature-coefficient for each value of specific resistance for the sample of iron cited being selected. The discrepancies or differences between observed and calculated results are not larger than a combination of observations made on the great variety of material by different observers, together with the wide range of possible errors incident to all, would lead us to anticipate. Even the position of soft steel with reference to the curve is, in every respect, satisfactory. It is in this way, finally, that the practical results in Table 3 were derived.

Benoit[13] finds the following relations between resistance and temperature for soft iron and soft steel, respectively:

$$s_t=0.1272\ (1+0.00452\cdot t+0.0000058\cdot t^2),$$

and

$$s_t=0.1149\ (1+0.00498\cdot t+0.0000074\cdot t^2),$$

$$(Hg=1 \quad m\mid \overline{mm^2}).$$

These results referred to microhms and cm/cm² are

$$s=12.1 \qquad \alpha=0.00452$$

for iron, and

$$s=10.9 \qquad \alpha=0.00498$$

for steel. Both sets of values are in good accordance with our graphic representation. We obtain by means of this:

For $s=12.1$, the value $\alpha=0.00457$,

and

For $s=10.9$, the value $\alpha=0.00485$.

[13] Benoit: Comptes rend., LXXVI, p. 342, 1873. Wiedemann, l. c., p. 525. The small value $s=10.9$ obtained by Benoit for soft steel is remarkable and exceptional.

CAST-IRON.

Anticipative results.—Of particular interest in connection with this discussion is the behavior of the most highly carburized of commercial iron-products, cast-iron. Observations for pairs of the electrical magnitudes under consideration, for this material, are not in hand. Elsewhere we will describe certain experiments, made in some number, with reference to the thermo-electric and galvanic properties of cast-iron. Here we need only mention, that the specific resistance of this metal is very decidedly larger than the largest attainable results for glass-hard steel. If, therefore, a relation[14] between electrical conductivity and electrical temperature-coefficient of the kind premised, actually exists, then this latter quantity must, in like manner, be smaller than the smallest results arrived at in case of steel.

To test this inference, three samples were selected from our supply of cast-iron rods, Nos. 13, 14, 15, each about 25 cm. in length, and their resistance in the soft or thoroughly annealed state (annealed at red heat and cooled very slowly) determined at different convenient temperatures. But this resistance, in view of the comparatively large section of the said rods (about 0.4 cm^2) being as small as 0.004 ohms, the measurement had to be made even with greater precaution than was necessary in the case of steel.

Method of measurement.—The method of measurement was, however, essentially identical in the two sets of experiments, being Matthiessen and Hockin's. The terminal wires of copper, wherever necessary insulated by glass tubes, were wrapped around and soldered to the cast-iron rods; these, together with the insulated terminals and a good thermometer, introduced into a wide glass tube. Through the latter, closed at both ends by suitably perforated corks, securing tubes of influx and efflux, vapor at the boiling point of the respective liquids continually circulated. Methyl alcohol vapor, and steam were especially convenient. The tube itself, thickly jacketed with felt and cloth, showed a desirably constant temperature throughout the course of the work. All these precautions were necessary, for the ulterior reason of excluding possible thermo-electric action at the junctions of cast-iron and copper. Such currents are otherwise readily evoked in intensity sufficient utterly to vitiate the accuracy of the measurements.

Results.—The following table will show that the experiments conducted with this care were satisfactorily succeessful. Here a and b denote the sides of the approximately rectangular section of the cast-iron rods; l, the effective length in the resistance measurements. From an inspection of the resulting errors, the linear relation assumed to exist

[14] A detailed discussion regarding the electrical effects of the strain accompanying hardness and of carburation, respectively, will be given in Chapter III.

between resistance and temperature within the interval 0°—100°, will be found to be acceptable.

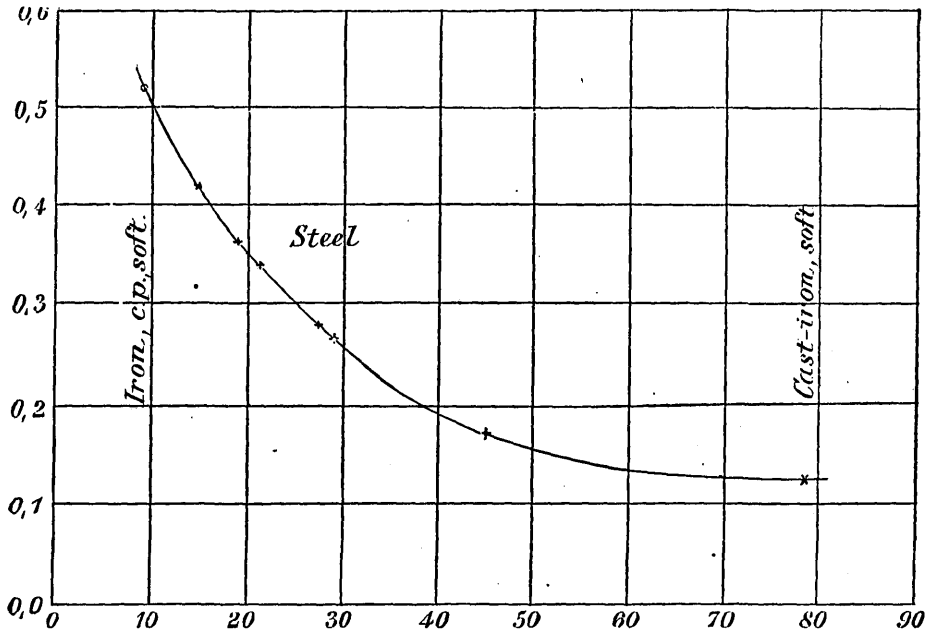


FIG. 2.—Diagram of the relation between specific electrical resistance and temperature-coefficient for wrought iron, for steel, and for cast iron.

The results show that the mean temperature-coefficient of cast iron falls very fairly on what may be considered a prolongation of the locus obtained for iron and steel (see figure), or that the electrical temperature-coefficients of iron-carburets, in general, may be regarded as a definite function of the respective specific resistances:

TABLE 6.—*Temperature-coefficient of cast-iron.*

Rod No.	Constants.	t	w	t mean.	w observed.	w calculated.	Diff.	ρ	α	β
	cm.	*C.*	*ohm.*					*ohm.*		*microhm.*
13	a= 0.640	23.8	0.003749	23.5	0.003743	0.003743	0	0.003637	0.00124	76.0
	b= 0.656	23.2	3738							
	l=20.10	67.2	3938	67.3	3941	3941	0			
		67.5	3943							
		99.4	4093	99.3	4085	4085	0			
		99.3	4077							
14	a= 0.643	22.8	3839	22.8	3837	3829	8	3712	138	76.2
	b= 0.642	22.8	3835							
	l=20.10	67.8	4041	69.5	4051	4068	—17			
		71.2	4061							
		99.5	4235	99.5	4235	4224	11			
		99.5	4235							
15	a= 0.646	20.0	4095	18.7	4099	4102	—3	4008	126	83.3
	b= 0.647	17.8	4095							
	l=20.10	18.3	4108							
		66.0	4347	66.0	4347	4341	6			
		66.0	4347							
		100.0	4500	100.0	4509	4513	—4			
		100.0	4490							
		100.0	4536							

DEDUCTIONS.

Analogous behavior of alloys and of iron-carburets.—The data contained in the above tables throw some light on the probability of an analogy between alloys generally and iron carburets, inasmuch as they show a certain similarity of behavior in both kinds of products. It is not impossible that we have here in hand examples of a general law; in other words, it may be plausibly argued that whenever the properties of a primary metal are altered by addition of various quantities of a second substance (metallic or non-metallic) alloyed thereto, that then the known variation of electrical resistance is invariably accompanied by a corresponding variation of the electrical temperature-coefficient—in such a way that an increment of the former corresponds to a decrement of the latter in accordance with some fundamental relation. In the case of ordinary alloys, small quantities of a second metal are alloyed to the original material, producing the known electrical effect. In the case of steel the process of tempering is the cause of a change in the quality of carburation, so that in a highly tempered bar more *combined* or electrically active carbon is, as it were, alloyed to iron than in one of inferior temper or in a soft rod. Hence the corresponding electrical effect. But it is well to waive this subject here to discuss it more satisfactorily in another chapter.

Temperature-coefficient and volume.—We shall show elsewhere[15] that for steel at least, and possibly for all non-electrolyzed conductors, the specific resistance may, with some fitness, be regarded as a volume-function only. Probably a similar remark may also be made with reference to the temperature-coefficient of steel, and it would appear that with this metal, the most promising of the available means for the study of the bearing of specific volume on specific resistance and temperature-coefficient, is furnished us. Clausius[16] was the first to call attention to the approximate proportionality of the resistance of most pure metals with their absolute temperatures. If we accept Mathiessen's general relation between resistance and temperature for pure metals:

$$s_t = s_o\,(1+\alpha t+\beta t^2), \qquad \alpha = 0.003824, \qquad \beta = 0.000\,001\,26$$

and put

$$\frac{ds_t}{dt} = \frac{1}{273},$$

we find that at about 60° C. the said proportionality is accurate. There would be some propriety, therefore, in considering this state of the metals in question as a normal state, and the corresponding specific

[15] Chapter III.

[16] Clausius: Pogg. Ann., CIV, p. 650, 1858. Auerbach (Wied. Ann., VIII, p. 479, 1879) has endeavored to explain this circumstance, and also to interpret the exceptional value encountered in case of iron.

resistance as their normal resistance. It may be remarked, in passing, that the temperature to which *soft* steel must be cooled in order (theoretically) to annul its resistance coincides very nearly with the absolute zero of temperature.

ADDENDUM.

STATEMENT OF A RESISTANCE METHOD FOR THE MEASUREMENT OF HEAT-CONDUCTIVITY.

The success of the above application of Matthiessen and Hockin's method for the accurate measurement of very small increments of resistance, has suggested to us the availability of the same method in determining the necessary data for the calculation of heat-conductivity.

The subject of heat-conductivity has of late been largely discussed, especially in German literature. In most of the cases new methods have been proposed and employed. Our object herewith is to offer an experimental modification of the well-known method due to Biot, but first applied by Depretz,[17] which we believe has certain practical advantages.

Depretz heats the ends of a straight rod of uniform section to different constant temperatures T and t ($T>t$). After the stationary thermal condition has set in, t_1, t_2, t_3, the temperature of any three consecutive right sections, at the same distance l apart, respectively, are related as follows:

$$e^{\mu l}+e^{-\mu l}=(t_1+t_3) : t_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where, moreover, μ for rods of the same section and of the same external conductivity is, under the assumption of constant l, inversely proportional to the square root of heat-conductivity. Suppose, however, that instead of measuring the temperatures at three consecutive equidistant sections, we propose to determine the resistances of three consecutive equal lengths l of the rod, after the thermal condition has become stationary. We have

$$dr=\frac{s}{q}(1+\alpha t)\,dx,$$

where dr is the elementary resistance corresponding to the length dx, at the temperature t, s the specific resistance of the material, q its (uniform) section, α a given constant. In view of the steady thermal flow, we may write

$$dr=\frac{s}{q}[1+\alpha(Ce^{\mu x}+C'e^{-\mu x})]\,dr.$$

If this equation is integrated successively between the limits l_2 and l_1, l_3 and l_2, l_4 and l_3, where $l_2-l_1=l_3-l_2=l_4-l_3=l$, we obtain three equa-

[17] Depretz: Ann. d. chim. XXXVI, p. 422, 1827; Cf. Langberg, Pogg. Ann., LXVI, p. 1, 1845; Wiedemann and Franz, Pogg. Ann., LXXXIX, p. 497, 1853.

tions from which the constants C and C^1 may be eliminated. The following relation results:

$$2n=e^{\mu l}+e^{-\mu l}=\frac{\frac{r_{21}}{r_o}-1+\frac{r_{43}}{r_o}-1}{\frac{r_{32}}{r_o}-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where r_o is the resistance of the length l, at zero. Equation (2) might have been more elegantly derived indirectly from equation (1) above.

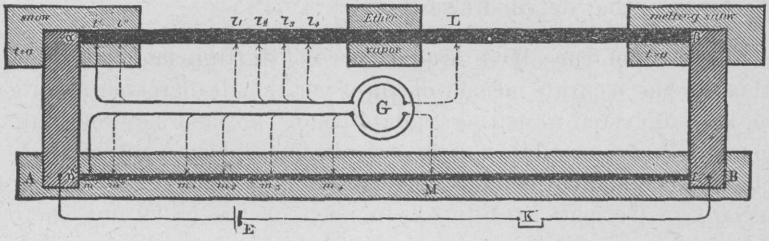


FIG. 3.—Diagram of a resistance-apparatus for measuring heat conductivity.

Suppose, now, that in connection with this result we use Hockin and Matthiessen's device of two corresponding sliding contacts in the manner indicated in the annexed diagram. Let

$$l''-l'=l_2-l_1=l_3-l_2=l_4-l_3=l$$

Let the points m', m'', $m_1 \ldots . m_4$ on the wire δe correspond to l', l'', $l_1 \ldots . l_4$ in such a way that, if connection be made between any two of them with the prolonged terminals of a galvanoscope G, the needle of the latter will receive no additional impulse on momentarily closing the key K. Then we may write

$$2n=\frac{\frac{m_2-m_1}{m''-m'}-1+\frac{m_4-m_3}{m''-m'}-1}{\frac{m_3-m_2}{m''-m'}-1};$$

whence it follows that Depretz' method by application of Hockin and Matthiessen's device may be theoretically reduced to a simple *measurement of lengths.*

It is not our object here to go into any practical details.[18] But we may remark that hurtful thermo-currents may be reduced to a minimum by using a sliding contact at the points l of a material thermo-electrically similar to the rod to be examined. Their effect would be that of changing the position of equilibrium of the needle of the galvanoscope, and they would not seriously influence the impulse given to it by momentarily closing K. Moreover, the ends, L and M, of an independent, permanently closed bridge-wire may be so adjusted as to compensate the thermo-electric disturbance entirely (end L), and

[18] The compendious form of Wheatstone's Bridge, invented by Kohlrausch, suggests itself for these measurements.

yet not interfere with the measurement (end M). In this case it is not necessary to close $l' \ldots . l_4$ for a short time prior to closing K, but both may be closed momentarily, the latter contact a little before and a little after the particular one of the former, with reference to which the observation is made.

It is our intention to endeavor to apply a procedure of this kind for the purpose of investigating whether the abnormal diminution of electrical conductivity due to tempering (about 70 per cent.) is accompanied by a corresponding variation of heat-conductivity. The plan would be the same as that detailed elsewhere.[19]

[19] Strouhal and Barus, Wied. Ann., XI, p. 976, 1880; ibid., pp. 953–4, 977.

CHAPTER II.

ON THE CONDITIONS WHICH IN THE CASE OF STEEL ESSENTIALLY DETERMINE THE EFFICACY OF THE OPERATION OF TEMPERING; THE MEASUREMENT OF THE STATE OF HARDNESS OF STEEL.

INTRODUCTORY REMARKS.

Origin of the work.—At the outset of the present series of experiments it was our object to subject the relation existing between the amount of magnetization which saturated steel rods can permanently retain, and their mechanical condition, particularly their state of hardness, to a new and rigid investigation. We were led to this undertaking by the results shown in a paper by Barus,[20] in which it appears that the electrical properties of steel—its thermo-electric power and specific resistance primarily—furnish a datum of singular sensitiveness for the hardness of this material. It therefore lay within our scope and purpose to invent a method for obtaining as many well-defined degrees of hardness between the glass hard or suddenly chilled state on the one hand, and the soft or thoroughly annealed state on the other, as would be practicable.

During the progress of the work, however, the phenomena attending the operation of tempering, as exhibited by the thermo-electric power and the specific resistances of the different stages of hardness of steel, began more and more to engross us, and eventually became of sufficient importance to occupy our attention wholly. Thus it was that a special research, though partaking of the nature of a digression and calling for a larger expenditure of time than had been allotted to this part of the projected series of experiments, became almost necessary. The data in hand throw new light on the conditions determining the temper of steel, and indeed enable us to discuss the whole subject perspicuously and from a general standpoint. These remarks will suffice to account for the origin, purpose, and disposition of the parts of the present chapter.

Material used.—The steel used in this work was of the kind known as "English silver-steel."[21] It came to our hands in the shape of rods, about 30 cm. long, varying in diameter between 0.03 cm. and 0.01 cm., drawn accurately cylindrical for the use of watchmakers. The rods were nominally identical in composition, and obtained from Cooks' Brothers, of Sheffield and Manchester. In view of the great complexity

[20] Barus, Phil. Mag. (5), VIII, pp. 341-68, 1879; Wied. Ann., VII, p. 338, 1879. Cf. Appendix to this Bulletin, p. 203.

[21] See Chapter VII.

and vagueness associated with the term steel, it appeared necessary to confine the operations to the given type of this material—an exceptionally favorable type, moreover, when considered with reference to the enormous interval of hardness comprehended between its glass-hard and soft states. Chemical analysis of this individual member of the infinite family of possible steels would have been more than useless, as will clearly appear from a perusal of the present series of papers as a whole. As a rule we draw our inferences from rods broken from a single longitudinally homogeneous sample only. The purely physical relation sought is thus investigated unaffected by secondary phenomena or distortions, while the material, as it were, is represented by a series of temporarily constant parameters. In the present chapter this method of research is suggested naturally by the experiments themselves, and satisfactory material is easily obtained. But in the later chapters on magnetism, where the necessity of using chemically and structurally identical rods is much more urgent, these essential conditions are often secured only with difficulty.

APPARATUS FOR IMPARTING GLASS-HARDNESS TO STEEL.

In view of the great number of steel wires to be tempered, the construction of an apparatus by the aid of which the operation of sudden cooling could be expeditiously and conveniently carried out, and which would impart to the wires a uniformity of hardness throughout their length, was an essential requisite. The following machine answered the purpose satisfactorily:

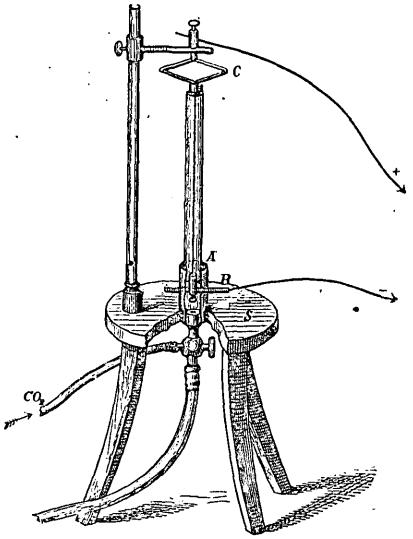


FIG. 4.—Apparatus for hardening steel rods.

In figure 4, *A* is a hollow cylinder 9 cm. long, of dense (box) wood, provided with a circular groove and slot so as to admit of its being securely fastened by a sort of bayonet-joint to a substantial tripod, or again removed, conveniently. Into the lower and wider part (3.0 cm. in diameter) of the aperture a closely-fitting faucet communicating with the water-mains by means of a hose, is inserted; into the upper opening of the same (1.5 cm. in diameter) there is fitted a glass tube of thin material and about 30 cm. long. This serves primarily as a protecting envelope for the steel wire stretched in its axis. Besides this gradually tapering canal, the box-wood cylinder is provided with a second perforation at right angles to the axis of the latter, in which a thick steel rod *B* (0.5 cm. in diameter) fits snugly.

The wire to be hardened is drawn tensely between two clamp-screws, in connection with the terminals of a powerful battery, in the following way: The lower clamp-screw possesses a longitudinal and a cross perforation. One end of the wire is fastened in the first of these holes by a lateral screw and then introduced into the wooden cylinder and attached glass tube, *A*, from below, whereupon the steel rod *B*, passed through the cross perforations in both cylinder and clamp-screw, and fastened by the vertical screw of the latter, puts the lower end of the wire in connection with one pole of the battery. As *B* may be rotated around its own axis and at the same time moved laterally, the wire may be satisfactorily centered. *A*, thus adjusted, is now attached to the tripod as above described, the faucet forced tightly in from below, and the upper end of the steel wire, which projects slightly out of the glass tube, grasped by a second clamp-screw. The latter forms a part of a peculiar spring *C*, in shape of a rhombus of very large horizontal but very short vertical diagonal. In this way a gentle tension, not so strong as to rupture the wire when red hot is constantly maintained, while the parts of the spring are thick enough to allow the passage of an intense galvanic current without becoming perceptibly heated. We have found that a wire symmetrically chilled and held tense in this way remains straight after the glass-hard temper has been imparted to it—an important desideratum. In order to centrally adjust the upper end of the wire, or to regulate the tension of the spring, the latter may be moved around and along a horizontal arm, which in its turn is similarly adjustable around a vertical post screwed to the tripod. To *C* and *B* the terminals of a powerful battery are suitably attached.

A current of dry carbonic acid gas continually circulating through the tube practically obviates the annoyance of oxidation during the heating to redness. To introduce this gas, the faucet is doubly perforated in the way devised by Senguerd. The larger of the canals is intended for the influx of water, the smaller for carbonic acid, and they are so disposed relatively to each other that if the one is open the other is necessarily closed. To manipulate the apparatus satisfactorily a considerable head of water is desirable. But even when these facilities are available,

the water, on opening the faucet, is apt to enter the tube with a squirt, chilling some parts of the wire before the main column advances, and in this way vitiating the otherwise attainable uniformity of glass-hardness. For this reason a second faucet (not shown in the figure) belonging to the hydrant was put into use. We operated in this way: The former faucet was first opened, producing no other effect than the interruption of the current of carbonic acid. Then one of us rapidly opened the second faucet, while the other, in due time, broke the galvanic circuit. As the glass tube was of comparatively small diameter, the water rushed into its interior, ascending as an unbroken column with great velocity and imparting to the wire the glass-hardness desired. According to Jarolimek[22] this rapidity of current is of paramount importance when the attainment of extreme degrees of hardness is the desideratum. In the case of quiet water a non-conducting envelope of steam is apt to inclose the wire, protecting it against instantaneous chilling. Such a layer would effectually be torn away by a very swift current, and cold water and wire remain in more intimate contact. We regarded the hypothesis, which Jarolimek believes to have verified by experiment, as plausible.

Glass tubes of thin walls were chosen, and breakage was therefore a rare occurrence. Of course we did not keep the wire in the red-hot state longer than appeared absolutely necessary.

The operation of disadjusting and drying the parts of the apparatus after each chilling proved to be a tedious annoyance. Nevertheless the efficiency of the apparatus may be said to have been demonstrated by the fact that after a little experience we were able to temper 50 to 60 wires during an interval of five hours. Of the total number of hard wires obtained (some 180), those were selected for the measurements which had been operated upon during our later and more expert manipulation. The ends were broken off and discarded, and only as much of the central part of each wire used as would warrant the assumption of uniformity of hardness throughout the lengths employed. The degree of homogeneity, moreover, admits of being specially tested, as will be explained below.

The battery used consisted of 20–30 large Bunsen cells, connected in series or in multiple arc in a way to correspond with the external resistance. In the latter case it is necessary to keep the partial circuits as nearly as possible alike. Otherwise a reverse current is apt to traverse one of them, gradually disintegrating the carbons.

MEASUREMENT OF THERMO-ELECTRIC POWER.

Thermo-element.—The thermo-electric power of our wires was deduced from measurements of electromotive force and temperature, obtained by

[22]Jarolimek: Dingler's Jour., CCXXI, pp. 436, 518, 1876.

combining them thermo-electrically with the same given normal wire. Chemically pure silver, deposited galvanoplastically, fused, drawn, and softened, appeared to be the most desirable metal-of-reference for this purpose. Two samples of this were selected and compared. From reasons of a practical character, however, we found it expedient not to use these normals in the actual work. In the measurements, a copper wire of a given kind, which had frequently and very carefully been compared with the silver, was substituted for it, and the thermo-electric powers steel-copper subsequently reduced to steel-silver, by calculation.

After testing many modifications, we adhered to the very efficient form of thermo-electric apparatus diagrammatically represented in figure 5. S_1 and S_2 are two doubly tubulated spherical receivers, of the

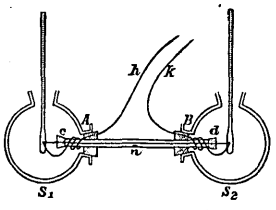


FIG. 5.—Form of thermo-element.

capacity 1 liter approximately. These were mounted on good non-conductors in such a way as to place the axes of the tubulures *A* and *B* in horizontal, the other two in vertical position. The former were provided with well-fitting corks, centrally perforated, so as to admit the glass tube *cd* (diam. 1 cm.) snugly. In this way the two receivers were held firmly together, and at distances apart adjustable at pleasure, thus adapting the arrangement for examination of a long or a short wire. Two small suitably-perforated corks, fitting the ends of the tube *cd*, gave to the wire an axial position within it. In this way the steel rods, exceedingly brittle and frail when in the state of glass-hardness, were adequately protected. Mention is still to be made of the terminals *h* and *k* of the apparatus. These were specially-selected samples of covered copper wire, as stated above. They passed through the large corks at *A* and *B*—in which they were cemented once for all—to the junctions at the center of the respective receivers. When a new rod was to be introduced, this was thrust through the tube *cd*, and the latter duly closed with the small corks, in the perforations of which the rod fitted tightly. Then the free ends of the terminals were connected with the respective ends of the steel rod, either by flat brass clamp-screws, or, where the temper permitted it, by soldering. The tube carrying the large corks was now in complete adjustment, and it was only necessary to insert these in the tubulures *A* and *B* of the receivers. One of the latter was then filled with water of the temperature of the room, the other with hot water. A woolen jacketing appropriately surrounding the hot receiver reduced the loss of heat by radiation to a minimum.

By the aid of two carefully calibrated thermometers, with their bulbs at the centers of the respective receivers, the temperature of these was read off. A small hole was drilled at n in the tube cd to allow for the expansion of the air heated by proximity to the hot water.

Method of measurement.—For the measurement of the thermo-electromotive force (expressed throughout in volts) a zero method was adopted. If E (figure 6) be the compensating (one Daniell), e the com-

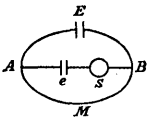


FIG. 6.—Disposition of thermo-electric apparatus.

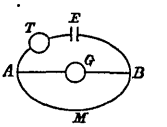


FIG. 7.—Apparatus for galvanometer-factor.

pensated element (the thermo-couple), W the resistance of AEB, w that of AMB, we shall have, when the current in AeB containing the galvanometer is zero,

$$\frac{e}{E}=\frac{w}{W+w}.$$

In our experiments, in the most unfavorable cases,

$$\frac{w}{W}=\frac{5}{10000};$$

and hence, with sufficient accuracy for the present purpose we may put

$$\frac{e}{E}=\frac{w}{W}.$$

Both w and W were furnished by Siemens' rheostats. The resistances of the connecting wires and of the Daniell are negligible. W could be increased to 30,000 ohms, w diminished as far as 0.1 ohm.

In order to eliminate such discrepancies as would arise from variations of the Daniell, the electromotive force of this element was measured before and after each observation. It is known that this source of error is by no means negligible, and that it depends on the way in which the Daniell has been put together, and on the time of use. For the purpose in question, the terminals of a Wiedemann's galvanometer of known factor A could be introduced into the circuit $EAMB$ with the aid of an appropriate key. If therefore the line ASB is broken, the resistance w excluded, we shall have a simple circuit $EAMB$ such that if n be the deflection at the galvanometer

$$E=A\,Wn.$$

Usually W=20,000 ohms was chosen, as it was through this resistance, approximately, that the Daniell acted during the measurements with the zero method.

To determine the factor A of the galvanometer we made use of a tangent-compass of known factor, C. This we calculated both from the dimensions of the coils and from voltametric observations. Figure 7

gives the disposition of apparatus diagrammatically, wherein E is the Daniell, T the tangent-compass, G the mirror-galvanometer. Let w be the resistance, i the intensity of current in the shunt $A\ M\ B$, W the resistance, and I the current in $A\ G\ B$; then

$$IW=iw;$$

whence

$$\frac{I}{I+i}=\frac{w}{W+w}.$$

But by measurement with the tangent compass we get

$$I+i=C\ \mathrm{tg}\ \varphi\,[1+f(\varphi)],$$

and with the galvanometer, simultaneously, $I=An$, whence

$$An=\frac{w}{W+w}\,C\ \mathrm{tg}\ \varphi\,[1+f(\varphi)].$$

By a proper choice of w and W, practically convenient deflections, both at the tangent compass and at the galvanometer, are obtainable. The determination of A was frequently repeated.

Essential details.—The actual arrangement of apparatus as given in figure 8 differs from the diagram (figure 6) described above, only in that serviceably-disposed commutators and keys have been introduced into the partial circuits.

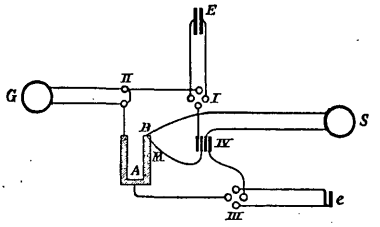


FIG. 8.—Details of thermo-electric apparatus.

The commutator I, inserted immediately after the Daniell, is used, in the first place, in determining the value of the electromotive force of this element from the double deflection of the mirror of the galvanometer G put in circuit by the key II. In the second place, when manipulated in connection with another commutator, III, which changes the direction of current of the thermo-element, it adds very materially to the attainable accuracy by enabling the observer practically to eliminate the discrepancies due to extraneous thermo-currents. These result from an irregular distribution of temperature in the connections. We actually measure not

$$\frac{e}{E}=\frac{w}{W};\ \text{but}\ \frac{e+\varepsilon}{E+\varepsilon'}=\frac{w}{W},$$

where ε and ε' are the disturbing electromotive forces in question. Now ε', which has its seat in the branch $EAMB$, is always negligible in comparison with E. A similar assumption, however, by no means applies to ε—*i. e.*, to the electromotive force which originates in the

branch ASB. This quantity, as our experiments have shown, very frequently reaches values amounting to a considerable part of e—indeed, when e is small, ε is directly comparable with it.

We therefore have

$$\frac{e+\varepsilon}{E}=\frac{w}{W}.$$

If both e and ε retained the same value before and after an observation there would result: Before commutation,

$$\frac{e+\varepsilon}{E}=\frac{w}{W};$$

after commutation,

$$\frac{e-\varepsilon}{E}=\frac{w^1}{W^1};$$

whence

$$\frac{e}{E}=\frac{1}{2}\left(\frac{w}{W}+\frac{w^1}{W^1}\right).$$

Now, although both e and ε do vary during the interval of an observation, the amount is small and the change of the former so nearly linear with the temperature T of the warm receiver, that the mean of the electromotive forces e may be regarded as coincident with the mean of the temperatures T. If the observations after commutation are rapidly made, which can easily be done because the approximate values of W and w are known from the first measurements, we may suppose ε to have remained constant.[23] A mean of the two determinations therefore will very nearly eliminate ε.

Finally, reference is still to be made to the form of Weber's commutator IV, which serves the purpose of a key of special kind. The small mercury cups are so filled that the thermo-current is not closed until a moment after the partial current from the Daniell. When w and W are properly chosen, therefore, the needle of the galvanometer S remains at rest. An exceedingly sensitive form of apparatus with an astatic needle, devised by Magnus and constructed by Sauerwald, was used here.

While one observer made the adjustments w and W, the other read off the temperatures of the thermometers in the receivers at given signals.

Calculation of constants.—If t and T be the temperatures of the thermo-electric junctions, e the corresponding electromotive force, we shall have generally[24]

$$e=a(T-t)+b(T^2-t^2) \quad \ldots\ldots\ldots \quad (1)$$

Putting $T-t=x$, $T+t=u$, $e=y$,

$$y=ax+bxu \quad \ldots\ldots\ldots \quad (2)$$

Inasmuch as the number of observations was always greater than two,

[23] By commutating again we frequently convinced ourselves that this supposition was quite permissible. The values otained for the first and third positions of the commutators were so nearly identical that we could satisfactorily accept them as such.

[24] Avenarius: Pogg. Ann., CXIX, p. 406, 1863; ibid., CXLIX, p. 374, 1873; Tait, Trans. R. Soc. Edinb., XXVII, 1872-'73, p. 125. In this memoir the above equation is accepted merely as an empirical relation. Tait has discussed it theoretically.

the constants a and b were calculated by the method of least squares, on the basis of an equation of the form

$$\frac{y}{x}=a+bu.$$

We were led to choose this form not merely because the calculations are thus essentially simplified, but principally because the form (2) gives to the values of e, corresponding to great values of $T-t$, an amount of preference which appears to us wholly unwarranted: for although e is much more accurately measurable in the case of large values of $T-t$, the observed value of T will differ the more appreciably from the true temperature of the hot junction of the thermo-element, in proportion as T is greater than the temperature of the room in which the observations are made.

By aid of the well-known formulæ developed by the method of least squares, a_o and b_o, of the thermo-couple copper-silver, were calculated from a large number of special observations.

By the same method, moreover, we derived the constants, a^1, b^1, for the elements steel-copper. To reduce from these the values a, b, which hold for the couples steel-silver, we made use of a set of tables calculated thus: If e, e^1, e_o correspond to a, a^1, a_o, we have

$$e=e^1-e_o,$$

where e^1 is dependent on the arguments T and t. Now e_o can be thus expressed:

$$e_o=(a_oT+b_oT^2)-(a_ot+b_ot^2);$$

that is, as a difference between identical functions of the same arguments T and t. A table for

$$a_oz+b_oz^2$$

is therefore calculated once for all, from which for every combination of T and t the quantity e_o, and therefore e, is easily obtained. After all the observations, e, had been referred to silver, the constants a and b were deduced by the thermo-electric formulæ above given.

MEASUREMENT OF ELECTRICAL CONDUCTIVITY.

Method calculation.—For the measurement of resistances we employed Kirchhoff's form of Wheatstone's bridge. The resistances w and δ to be compared could be exchanged by the aid of a mercury commutator. Heavy copper plates, the resistance α and β of which was such as to introduce a very small correction only, connected the latter with an interposed commutator and finally with the end points of the bridge. The current was advantageously furnished by Weber's magnetic inductor. Sauerwald's sensitive apparatus, already referred to, was used as a galvanoscope.

The following is the method of calculation adopted:

$$\text{First position of commutator, } \frac{w+a}{\delta+b}=\frac{a_1}{b_1}=n_1;$$

$$\text{Second position of commutator, } \frac{w+b}{\delta+a}=\frac{b_2}{a_2}=n_2:$$

whence

$$w=n_1\delta+n_1b-a, \qquad w=n_2\delta+n_2a-b,$$

and therefore, if $\frac{1}{2}(n_1+n_2)=n$,

$$w=n\delta+(n-1)\frac{a+b}{2}-\frac{n_1-n_2}{2}\frac{a-b}{2}.$$

In the case of our bridge, at $t=10°$

$$\frac{a+b}{2}=0{\cdot}00194, \qquad \frac{a-b}{2}=-0{\cdot}00016.$$

Now, in so far as n_1 and n_2 are always nearly identical, a sufficient approximation for w is furnished by the formula

$$w=n\delta+(n-1)\frac{a+b}{2} \quad \ldots\ldots\ldots \quad (1)$$

In this way the calculations were greatly simplified, since n could be immediately obtained from Obach's[25] tables. A small table of our own contained the respective values of the term $(n-1)\frac{a+b}{2}$.

In a part of the measurements we obtained excellent results with the beautiful method due to Hockin and Matthiessen.

From this value of w, the length and the diameter 2ρ—the latter dimension being determined microscopically—the specific resistance of the given wire at the temperature t followed at once.

The normal resistance δ was given by a specially constructed étalon of heavy German silver wire, soldered to stout terminals of copper properly amalgamated. We were in possession of six of these, such that by suitable combinations, either in series or in multiple arc, resistances as high as 0·6 ohms and as low as $\frac{1}{60}$ ohm were available. We were thus able to keep the sliding contact near the middle of the bridge wire, and the value of the factor $(n-1)$ in equation (1) was therefore invariably small.

Resistances at points of contact.—In view of the very small resistances (0.5 to 0.05 ohms) to be measured, great care had to be taken to reduce the resistances at the places where the steel wires were inserted to a minimum. To obtain an estimate as to the value of the discrepancies produced in this way we made a determination of the resistance of a soft steel wire for three different methods of insertion. In the first of these the wires were firmly held between flat clamp-screws; in the second, the ends of the wires were covered galvanoplastically with a thin even coat of copper, which was then amalgamated; finally, the contact was obtained by actually soldering the steel wires to pieces of heavy

[25] E. Obach: Hülfstafeln für Messungen elektrischer Leitungswiderstände mittelst der Kirchoff-Wheatstone'schen Drahtcombination, 1879.

copper wire. Only the last of these methods of insertion was found to be thoroughly satisfactory under all circumstances. In the first the error was quite large, while the second proved to be unsafe; for the amalgamated copper coat was apt to become insufficiently adhesive. In the case of glass-hard wire expeditious soldering is essential, otherwise the ends are annealed to an extent that will seriously affect the results. We avoided this at least partially by cooling the wire, immediately after the soldering had been effected, by a jet of water from a small wash-bottle. With a little practice the whole operation is finished in a few seconds. It is to be noted that if an error had been introduced in this way its effect would only have been that of giving further emphasis to the abnormally large results discussed in the sequel, since the values for specific resistance as thus actually found can only be short of the true values. In the later experiments all these inconveniences were avoided by the use of Hockin and Matthiessen's method.

Criterion for homogeneity.—The method just mentioned is peculiarly adapted for the determination of the degrees of uniformity of hardness along a given length of wire, because it enables the observer to compare the resistances of different parts of the said length with facility. The process as actually employed will be discussed in another chapter.

The method used for calibrating the essential wire of the bridge has been described elsewhere. (See appendix to this chapter, p. 72.)

THE OPERATION OF SUDDEN COOLING. GLASS-HARDNESS.

Effect of chemical composition.—The steel rods tempered to glass-hardness in the manner described were found, on examination, to be thermo-electrically negative[26] relatively to silver.

In so far as the same process had been applied to rods nominally of like composition, it appeared plausibly presumable that the degrees of hardness attained would be nearly identical; in other words, that the tempering would have furnished us with a set of glass-hard rods of nearly the same thermo-electric power. This was not the case. A graphic representation of the relation between thermo-electromotive force and temperature showed this remarkable result, that the wires tempered were readily referable to a few distinct groups; for the position of the different thermo-electric curves was such as to permit them all to be comprehended by a few narrow and well-defined sectors. This implies that though the maximum of attainable hardness is different in the different wires, particular groups possessing nearly identical properties are readily discernible. We were unable to discover any relation between the individual members of the said groups and the

[26] Cf. preface, p. 6.

respective diameters of the wires. Nor did the wires hardened on any particular day show like thermo-electric qualities. It is conceivable, for instance, where currents of different intensity are employed, that the resulting difference in the degrees of red heat imparted to the wires would find its expression in a correspondingly marked difference in the degrees of hardness. The true cause of the difference of character observed must therefore be ascribed to chemical composition. In other words, the groups are distinguishable one from another by the respective amounts of carbon contained in the wires. The rods were not all received at the same time. Without doubt the maximum of hardness attainable by sudden chilling is essentially conditioned by the degree of carburation of the steel rod, and to a very small degree only by the impurities present. In a general way we may state that the maximum in question is a characteristic datum for the type of steel under experiment. In Chapter VII of the present memoir we shall have occasion to discuss this subject in detail.

Maximum hardness reached.—The greatest hardness met with in this work was that possessed by two rods of the diameters 0.056 cm. and 0.073 cm., respectively. The thermo-electric constant a here reached the exceptionally small value $a = -2{\cdot}60$ (microvolts), and the specific resistance, at ordinary temperatures, was as large as 45 microhms, cm/cm^2. Unfortunately most of these very hard wires had to be discarded, because in the earlier experiments boiling water had been used in investigating their thermo-electric powers. We subsequently found that 100° produces a very pronounced annealing effect. In the later experiments with glass-hard wires water of only 40° was employed, and this but for a very short period of time.

Temperature of ignition.—Jarolimek and Ackermann[27] in their experiments on steel arrived at the important result that the rapidity of the first part of the chilling of red-hot steel, say from 600° or 700° to 300° or 400°, is far more essential as regards the degree of hardness obtained than the further cooling from 300°–400° to zero. It is easily possible to harden a rod very perceptibly by cooling it from bright redness in a metallic bath (Zn, Pb) at 400°. Such a process combines in one the operations of chilling and of annealing. Cooling from 300° or 400°, however, produces no effect. Again, Chernoff had previously found that if the temperature from which steel is chilled be supposed to increase continuously, no observable effect will be apparent until a temperature in dark cherry-red heat is reached, when glass-hardness is suddenly attained.[28]

Our experience is in perfect accord with these results. The phenomenon was strikingly manifested both in rods of the same diameter and in

[27] Jarolimek u. Ackermann, Zeitschr. für das chem. Grossgewerbe, 1880. Similar results, we believe, were published by the distinguished American engineer, Mr. Joshua Rose, but we have been unable to find them.

[28] D. Chernoff: Vortrag gehalten in der russischen Technischen Gesellschaft, im April u. Mai 1868.

those of different diameters. In the first instance we may cite our observations with comparatively thick steel rods, which the available current was able to heat to dark redness only. After chilling these remained soft and pliable for more than one-third of their length; but at a particular point of the wire its mechanical condition changed suddenly to brittle hardness, and this despite the fact that during the heating a change of intensity of redness along the parts of the wire in question was scarcely discernible. In the second instance we frequently noted that where the intensity of current used was sufficient to impart hardness readily to a given comparatively thick class of wires, this was no longer possible for the next larger dimension, notwithstanding the fact that the respective thicknesses varied as little as 0.125 cm. and 0.145 cm. It is furthermore to be added that the sudden change in question is equally apparent both in the mechanical as well as in the electrical properties of steel.

The experiments made would certainly not warrant the forced assumption that the phenomenon in question partakes of the nature of a true discontinuity. Some molecular change occurring at an extremely rapid rate is alone to be understood. In a general way, however, it may be stated that a certain critical temperature in red heat must be surpassed if sudden cooling is to produce glass-hardness; otherwise the steel remains soft.

Thermo-electric maxima and neutral points.—In this place a final remark may be added. During the experiments we had frequent occasion to observe "neutral points" (occurring at the mean temperature $\frac{1}{2}(T+t)=\frac{1}{2}\frac{a}{b}$ of the junctions of the thermo-element, the temperature at which the electromotive force passes with a change of sign through zero) of comparatively low value. Many of the hard rods differed but slightly from silver as regards their thermo-electric properties. Maxima of electromotive force at low temperatures were even more frequently obtained. A number of examples of this kind will be found in the tables below.

BEHAVIOR OF HARD STEEL RODS ANNEALED IN HOT OIL BATHS.

Manipulation.—Having thus in hand an assortment of glass-hard rods of excellent quality, it was our next endeavor to reduce the degrees of hardness of these consecutively by equal amounts. In other words, the problem was so to anneal the rods that between the glass-hard and the soft states as extremes a great number of intermediate approximately equidistant states might be obtained. We commenced our operations with this end in view by heating the hard steel in a large bath of linseed oil very gradually to different temperatures, re-

moving samples of the series of immersed rods at different stages of the process. Inequalities of temperature in the bath were reduced to a minimum by constant stirring. We also adopted an inverse method of procedure, in which, when a desired temperature was reached, the hard wires selected were submerged in the oil on a false bottom of wire gauze, and the whole allowed to cool. After several days they were examined both as to their thermo-electric power and their specific resistance.

Results.—The results of these measurements are given in the following table. It will be remembered that e (observed or calculated as specified) is the electromotive force in microvolts for the temperature T and t (centigrade) of the junctions. a and b are the thermo-electric constants of Avenarius, s_t $\left(\frac{\text{cm}}{\text{cm}^2} t^\circ \text{ microhm}\right)$ the specific resistance at the temperature t, ρ (cm.) finally the radius of the wires:

TABLE 7.—*Thermo-electrics and conductivity of steel wires, annealed in oil baths.*

Annealed at—	No.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s_t	t
		°C.	°C.	*microvolts.*	*microvolts.*	*microvolts.*	*microvolts.*	$\frac{\text{cm}}{\text{cm}^2} t$ *microhm.*	°C.
I. 300°....	No. 1 (2ρ=0.0968)	19.3	88.1	4.418	4.429	7.80	—0.0127	18.96	19
		19.3	74.0	3.622	3.615				
		19.3	63.3	2.982	2.971				
		19.3	50.1	2.125	2.131				
	No. 2 (2ρ=0.0900)	19.5	89.5	4.619	4.622	8.03	—0.0130	18.49	19
		19.5	76.2	3.849	3.843				
		19.5	62.0	2.961	2.959				
		19.5	54.3	2.457	2.458				
	No. 3 (2ρ=0.0721)	19.6	89.8	4.052	4.050	6.99	—0.0111	20.54	19
		19.7	73.3	3.194	3.191				
		19.6	59.1	2.406	2.414				
		19.7	50.1	1.893	1.888				
	No. 4 (2ρ=0.0568)	18.9	87.9	4.497	4.503	7.78	—0.0118	20.00	19
		18.9	70.3	3.468	3.462				
		18.9	57.4	2.657	2.651				
		18.9	48.3	2.052	2.056				
	No. 5 (2ρ=0.0345)	19.7	86.7	4.787	4.789	8.52	—0.0130	17.64	19
		19.7	72.1	3.847	3.843				
		19.7	59.4	2.975	2.977				
		19.7	50.6	2.353	2.352				
II. 275°...	No. 6 (2ρ=0.0903)	20.0	89.2	3.987	3.985	7.24	—0.0135	20.75	20
		20.0	78.2	3.437	3.436				
		20.0	64.9	2.731	2.733				
		20.0	50.9	1.941	1.940				
	No. 7 (2ρ=0.0558)	20.0	90.1	3.553	3.562	6.15	—0.0097	22.74	19
		20.0	71.9	2.737	2.729				
		20.0	59.7	2.141	2.134				
		20.0	48.3	1.529	1.533				
III. 250°..	No. 8 (2ρ=0.0880)	19.9	88.0	3.429	3.427	6.46	—0.0132	23.20	19
		19.9	74.8	2.863	2.860				
		19.9	61.6	2.237	2.244				
		19.9	51.9	1.767	1.763				
	No. 9 (2ρ=0.0720)	20.0	80.5	3.462	3.457	6.96	—0.0124	20.47	19
		20.0	65.8	2.697	2.701				
		20.0	55.3	2.124	2.128				
		20.0	49.9	1.827	1.822				

TABLE 7.—*Thermo-electrics and conductivity of steel wires, annealed in oil baths*—Cont'd.

Annealed at—	No.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s_t	t
		°C.	°C.	*microvolts.*	*microvolts.*	*microvolts.*	*microvolts.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
	No. 10 (2ρ=0. 0720)	20. 0	79. 9	2. 161	2. 159	4. 67	—0. 0007	25. 66	19
		20. 0	68. 4	1. 803	1. 804				
		20. 0	57. 3	1. 335	1. 340				
		20. 0	49. 0	1. 144	1. 141				
	No. 11 (2ρ=0. 0565)	20. 1	87. 1	2. 663	2. 672	5. 15	—0. 0109	24. 81	19
		20. 1	65. 9	1. 948	1. 931				
		20. 1	53. 9	1. 464	1. 469				
		20. 1	41. 8	0. 971	0. 972				
	No. 12 (2ρ=0. 0548)	20. 1	89. 4	3. 442	3. 434	6. 40	—0. 0131	22. 93	19
		20. 1	67. 9	2. 494	2. 503				
		20. 1	55. 3	1. 900	1. 902				
		20. 1	44. 8	1. 372	1. 369				
	No. 13 (2ρ=0. 0337)	20. 1	87. 4	3. 770	3. 769	6. 89	—0. 0121	20. 66	19
		20. 1	74. 0	3. 105	3. 107				
		20. 1	62. 4	2. 498	2. 496				
		20. 1	52. 0	1. 922	1. 923				
IV. 225°...	No. 14 (2ρ=0. 0900)	19. 9	89. 2	2. 059	2. 068	4. 06	—0. 0098	27. 36	20
		19. 9	75. 6	1. 751	1. 737				
		19. 9	64. 4	1. 434	1. 437				
		19. 9	54. 9	1. 161	1. 163				
	No. 15 (2ρ=0. 0558)	20. 1	89. 5	1. 989	1. 989	3. 85	—0. 0090	28. 11	19
		20. 1	68. 8	1. 484	1. 489				
		20. 1	69. 1	1. 231	1. 225				
		20. 1	50. 1	0. 963	0. 965				
V. 200°....	No. 16 (2ρ=0. 0974)	18. 9	89. 8	2. 524	2. 528	4. 67	—0. 0102	26. 70	19
		18. 9	74. 3	2. 063	2. 063				
		18. 9	61. 1	1. 631	1. 626				
		18. 9	51. 7	1. 294	1. 297				
	No. 17 (2ρ=0. 0882)	18. 9	82. 5	2. 056	2. 056	4. 26	—0. 0102	27. 93	19
		19. 0	68. 8	1. 681	1. 678				
		18. 9	59. 8	1. 414	1. 417				
		19. 0	51. 9	1. 166	1. 165				
	No. 18 (2ρ=0. 0723)	19. 1	78. 8	1. 314	1. 312	2. 79	—0. 0060	31. 32	19
		19. 1	68. 2	1. 109	1. 112				
		19. 1	57. 8	0. 899	0. 900				
		19. 1	48. 7	0. 706	0. 705				
	No. 19 (2ρ=0. 0560)	18. 1	82. 0	1. 859	1. 862	3. 83	—0. 0091	28. 68	19
		18. 1	62. 8	1. 387	1. 381				
		18. 1	47. 8	0. 954	0. 958				
		18. 2	39. 8	0. 714	0. 713				
	No. 20 (2ρ=0. 0336)	18. 3	76. 8	2. 723	2. 715	5. 45	—0. 0086	24. 34	19
		18. 3	62. 3	2. 088	2. 098				
		18. 3	52. 9	1. 680	1. 677				
		18. 3	42. 1	1. 177	1. 176				
VI. 175°...	No. 21 (2ρ=0. 0908)	18. 8	86. 4	1. 822	1. 809	3. 76	—0. 0103	29. 34	19
		18. 8	67. 1	1. 366	1. 388				
		18. 8	53. 8	1. 061	1. 054				
		18. 8	43. 3	0. 765	0. 763				
	No. 22 (2ρ=0. 0571)	18. 8	87. 2	1. 734	1. 729	3. 41	—0. 0083	30. 94	19
		18. 9	72. 4	1. 414	1. 417				
		18. 8	59. 7	1. 122	1. 126				
		18. 9	48. 8	0. 854	0. 850				
VII. 150°.	No. 23 (2ρ=0. 0335)	18. 5	73. 7	1. 745	1. 744	4. 24	—0. 0117	27. 93	19
		18. 5	60. 4	1. 387	1. 390				
		18. 5	52. 1	1. 146	1. 147				
		18. 5	45. 4	0. 941	0. 939				

Digest.—If we arrange these different degrees of hardness, expressed both thermo-electrically and in terms of their specific resistance, with reference to continuous variation of the former quantity, we obtain

the following perspicuous tabular comparison. The table shows that our endeavor to reach a great number of symmetrically distributed degrees of hardness systematically, was only partially successful.

TABLE 8.—*Thermo-electric position and conductivity of annealed steel.*

No.	a	s	Annealed at	No.	a	s	Annealed at
	microvolts.	*microhms.*	°C.		*microvolts.*	*microhms.*	°C.
5	8.52	17.64	300	11	5.15	24.81	250
2	8.03	18.49	300	16	4.67	26.70	200
1	7.80	18.96	300	10	4.67	25.66	250
4	7.78	20.00	300	17	4.26	27.93	200
6	7.24	20.75	275	23	4.24	27.93	150
3	6.99	20.54	300	14	4.06	27.36	225
9	6.96	20.47	250	15	3.85	28.11	225
13	6.89	20.66	250	19	3.83	28.68	200
8	6.46	23.20	250	21	3.76	29.34	175
12	6.40	22.93	250	22	3.41	30.94	175
7	6.15	22.74	275	18	2.79	31.32	200
20	5.45	24.34	200				

ON THE BEARING OF THE TIME OF EXPOSURE ON THE EFFICACY OF ANNEALING.

Low annealing temperatures.—In the foregoing experiments the lowest temperature employed was 150°. It will be seen that the annealing effect thus produced is strikingly large. This observation naturally suggested the question as to what results are to be expected when the annealing is conducted even at lower temperatures. The inquiry would have an immediate practical bearing: It will be remembered that in the thermo-electric measurements it is desirable to raise one of the junctions to a high temperature relatively to the other. We are led to ask, therefore, how high this temperature may be chosen without destroying uniformity of temper and producing partial annealing at one end of the rod.

Results for 100°.—We began the preliminary experiments with the two rods, Nos. 24 and 25, of nearly the same thicknesses, 0.0574 cm. and 0.0554 cm., respectively, but of different degrees of glass-hardness. Thermo-electric measurements only were made, with results for the glass-hard state as follows:

TABLE 9.—*Thermo-electric power of glass-hard wires.*

	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*
No. 24	12.5	88.1	−2.809	−2.793		
	12.5	78.8	−2.376	−2.396		
	12.5	58.1	−1.572	−1.567	−2.83	−0.0086
	12.6	44.1	−1.139	−1.139		
No. 25	12.3	89.0	−0.707	−0.697		
	12.4	80.1	−0.552	−0.554		
	12.4	71.2	−0.415	−0.428	+0.13	−0.0103
	12.4	59.8	−0.294	−0.289		

These wires were now exposed to the action of steam at 100° for a period of one hour, in an ordinary boiling-point apparatus. The measurement of thermo-electric position made on the following day showed these values:

TABLE 10.—*Hard wires, 1h at 100°.*

	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*
No. 24[1]	16.9	59.4	−0.069	−0.069		
	16.9	50.7	−0.025	−0.025	+0.61	−0.0102
	16.9	45.2	−0.005	−0.005		
	16.9	39.9	+0.008	+0.008		
No. 25	16.9	76.1	1.030	1.029		
	16.9	65.2	0.893	0.890	+2.75	−0.0109
	16.9	54.7	0.750	0.746		
	16.9	46.0	0.599	0.600		

[1] Neutral point: $(T+t)=-\frac{a}{b}=60°.2$; $\therefore T=43°.3$.

From a comparison of the values of a before and after annealing at 100°, a variation of thermo-electric power, therefore also of mechanical state, is very strikingly apparent; whence it follows at once that in the measurements of e, the use of boiling water in the hot receiver in the case of very hard wires is under no circumstances permissible. Indeed, from the magnitude of the variation in question, we infer that temperatures even much lower than 100° will show a tendency to produce an annealing effect seriously detrimental to a uniformity of the temper of the wire.

The above experiment was repeated. The rods were again annealed at 100° for a period of one hour, and thermo-electrically examined on the following day.

TABLE 11.—*Hard wires, 2h at 100°.*

	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*
No. 24[2]	17.4	73.8	0.142	0.152		
	17.4	64.7	0.184	0.174	1.26	−0.0110
	17.4	56.1	0.183	0.178		
	17.3	49.3	0.162	0.167		
No. 25	17.3	71.1	1.256	1.249		
	17.3	61.6	1.093	1.092	3.64	−0.0150
	17.4	53.4	0.916	0.932		
	17.4	46.7	0.794	0.787		

[2] Maximum at $T=-\frac{1}{2}\frac{a}{b}=57°.8$; observed, $y=18.5$; calc., $y=17.8$.

The results again show a smaller but nevertheless clearly pronounced variation of thermo-electric power, and we arrive at the important conclusion that, in addition to the temperature of the annealing bath, a second factor is essential in conditioning the resultant hardness, namely, the interval of *time* during which the annealing is prolonged, or the rod exposed to the given temperature.

With the object of discovering the nature of this time-effect, the experiments were continued in the same way. The results are contained in the following tables:

TABLE 12.—*Hard wires* 3^h *at* 100°.

	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*
No. 24	17.5	74.1	0.329	0.327	0.161	−0.0111
	17.5	62.0	0.316	0.318		
	17.5	54.2	0.294	0.294		
	17.5	48.6	0.270	0.269		
No. 25	17.5	74.3	1.499	1.497	3.55	−0.0100
	17.5	62.7	1.242	1.245		
	17.5	53.8	1.033	1.033		
	17.5	49.2	0.926	0.926		

TABLE 13.—*Hard wires* 4^h *at* 100°.

	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*
No. 24	17.7	86.4	0.484	0.487	1.76	−0.0101
	17.7	77.5	0.480	0.477		
	17.7	66.4	0.444	0.445		
	17.7	54.7	0.379	0.379		
No. 25	17.7	87.4	1.886	1.887	3.77	−0.0102
	17.7	76.1	1.660	1.648		
	17.7	67.7	1.447	1.453		
	17.7	56.5	1.177	1.182		

TABLE 14.—*Hard wires* 5^h *at* 100°.

	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*
No. 24	17.0	71.6	0.560	0.562	1.70	−0.0075
	17.0	58.4	0.477	0.468		
	17.0	44.5	0.326	0.340		
	17.0	34.5	0.233	0.228		
No. 25	17.1	75.0	1.661	1.665	3.90	−0.0111
	17.1	62.1	1.363	1.358		
	17.1	53.6	1.136	1.138		
	17.1	47.0	0.952	0.954		

TABLE 15.—*Hard wires* 6^h *at* 100°.

	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*
No. 24	17.5	82.6	0.637	0.641	1.92	−0.0093
	17.5	57.1	0.483	0.483		
	17.5	40.7	0.319	0.329		
	17.5	33.9	0.230	0.236		
No. 25	17.4	72.0	1.628	1.632	4.02	−0.0115
	17.4	48.7	1.018	1.019		
	17.4	36.3	0.648	0.642		
	17.4	27.5	0.351	0.355		

Digest.—The relation between a and the time of exposure is more perspicuously apparent in the following comparison:

	0^h	1^h	2^h	3^h	4^h	5^h	6^h
(24) ..... $a=$	−2.83	+0.61	1.26	1.61	1.76	1.70	1.92
(25) ..... $a=$	+0.13	2.75	3.64	3.55	3.77	3.90	4.02

From this grouping of parallel results, or, more evidently still, from a graphic representation (time as abscissa, thermo-electric constant as ordinate, mean values of Nos. 24 and 25, figure 9), it will be seen that

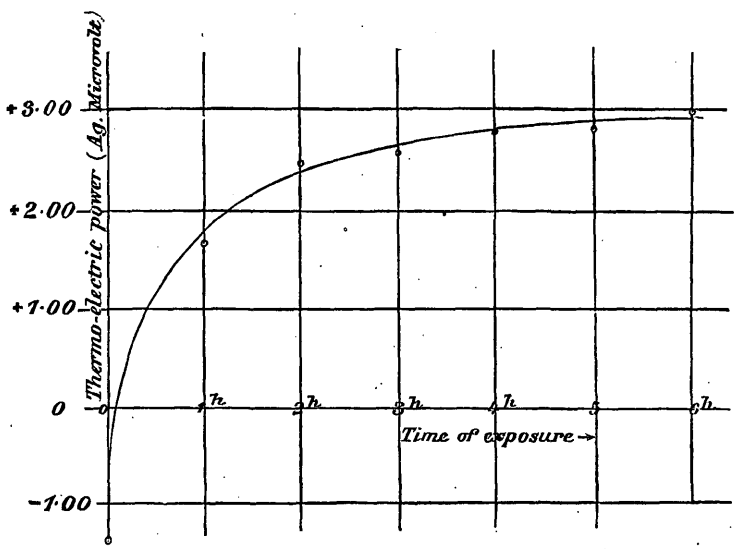


FIG. 9.—Hard wires annealed for six consecutive hours in steam at 100°.

hardness varies continuously with the time of exposure of the glass-hard rod to the given annealing temperature; that the amount of thermo-electric change rapidly decreases as the time increases, until finally a definite and superior limiting value is asymptotically reached.[29]

After these introductory experiments we determined to investigate

[29]Among the above results the individual values of a, 1.70 for the first and 3.77 for the second rod, require comment. It is obvious that from the four observations for each, probably affected with larger errors than usual, the method of least squares has not derived the constants in closest accordance with fact. This betrays itself in the values obtained for b, which differ largely from the average values. The constant b varies but slightly (as a general mean $-b=0.0100$ may be accepted). Associated with the values of a in question, however, we have $-b=0.0075$ (corresponding to the small result $a=1.70$) and $-b=0.0102$ (corresponding to the large result $a=3.77$). But in view of the fact that the function of which a is the coefficient is positive, and that in which b is involved, negative, the errors of both a and b are partially eliminated in the sum, so that observed e and calculated e are again in satisfactory accordance.

the phenomena of annealing more rigidly, giving the effect of time of exposure due prominence. Besides the boiling point of water, that of methyl alcohol at 66°.0 and that of aniline at 185° were chosen for annealing, the wires being suspended in vapors of these substances circulating in a glass boiling-point apparatus. A still higher temperature (330°) was furnished by the melting point of lead. Batches of three wires of different diameters and different degrees of glass-hardness were selected to be annealed respectively at each of these temperatures. To trace the effects resulting, simultaneous determinations, both of thermo-electric power and conductivity, were made at as many stages of the process as appeared desirable. The data are contained in the tables 16–28.

BEHAVIOR OF HARD STEEL ANNEALED IN VAPOR OF BOILING METHYL ALCOHOL (66°).

The rods selected were:

No. 28; diameter $(2\rho)=0.0827$ cm.,
No. 29; diameter $(2\rho)=0.0631$ cm.,
No. 30; diameter $(2\rho)=0.0479$ cm.

After the thermo-electric power (microvolts) and specific resistance (microhms) for the glass-hard state had been determined, the wires were subjected to the annealing influence of vapor of methyl alcohol at 66° for three consecutive hours, and at the end of each hour examined by the aid of the electrical qualities in question. The results are these:

TABLE 16.—*Hard steel annealed at* 66°.

[Rod No. 28. $2\rho=0.0827$.]

Remarks.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s	t
	°C.	°C.	microvolt.	microvolt.	microvolt.	microvolt.	$\frac{cm}{cm^2 t°}$ microhm.	°C.
Glass-hard	18.7	54.8	−1.003	−0.999				
	18.7	48.9	−0.907	−0.904				
	18.7	43.4	−0.721	−0.731	−2.56	−0.0064	42.64	18
	18.7	35.6	−0.495	−0.492				
1h in vapor of methyl alcohol, t=66°.0.	18.1	58.2	−1.033	−1.027				
	18.1	52.4	−0.932	−0.937				
	18.1	47.9	−0.792	−0.798	−1.94	−0.0111	42.55	18
	18.1	42.0	−0.629	−0.626				
1h more in vapor of methyl alcohol, t=66°.0.	18.3	58.5	−0.968	−0.964				
	18.4	49.4	−0.785	−0.787				
	18.3	42.9	−0.601	−0.608	−1.81	−0.0107	42.45	19
	18.4	38.7	−0.497	−0.492				
1h more in vapor of methyl alcohol, t=66°.0.	20.0	60.6	−0.956	−0.947				
	20.0	51.5	−0.767	−0.774				
	20.0	46.4	−0.625	−0.632	−1.58	−0.0122	42.55	19
	20.0	39.4	−0.453	−0.448				

TABLE 17.—*Hard steel annealed at* 66°.

[Rod No. 29. 2ρ=0.0631.]

Remarks.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s_t	t
	°C.	°C.	microvolt.	microvolt.	microvolt.	microvolt.	$\frac{cm}{cm^2 t°}$ microhm.	°C.
Glass-hard	18.7	50.4	−0.696	−0.697	−1.90	−0.0044	42.08	17
	18.8	44.5	−0.564	−0.559				
	18.7	38.8	−0.429	−0.432				
	18.8	34.8	−0.343	−0.342				
1h in vapor of methyl alcohol, t=66°.0.	18.2	57.7	−0.704	−0.702	−1.25	−0.0076	41.70	18
	18.2	51.7	−0.579	−0.581				
	18.2	45.8	−0.467	−0.467				
	18.2	40.4	−0.378	−0.378				
1h more in vapor of methyl alcohol, t=66°.0.	18.5	57.2	−0.636	−0.639	−0.91	−0.0098	41.52	19
	18.5	49.4	−0.487	−0.486				
	18.5	43.9	−0.385	−0.387				
	18.5	40.1	−0.324	−0.321				
1h more in vapor of methyl alcohol, t=66°.0.	19.9	61.4	−0.624	−0.620	−0.70	−0.0098	41.42	19
	19.9	53.2	−0.469	−0.471				
	19.9	45.3	−0.334	−0.340				
	19.9	41.1	−0.280	−0.275				

TABLE 18.—*Hard steel annealed at* 66°.

[Rod No. 30. 2ρ=0.0479.]

Remarks.	t	T	$e:10^2$ observed.	$e:19^2$ calculated.	a	b	s_t	t
	°C.	°C.	microvolt.	microvolt.	microvolt.	microvolt.	$\frac{cm}{cm^2 t°}$ microhm.	°C.
Glass-hard	18.7	54.2	−0.335	−0.329	−0.42	−0.0071	37.08	18
	18.7	45.3	−0.223	−0.229				
	18.7	40.2	−0.175	−0.178				
	18.7	34.4	−0.126	−0.124				
1h in vapor of methyl alcohol, t=66°.0.	17.8	56.9	−0.209	−0.209	−0.01	−0.0070	36.80	18
	20.1	45.1	−0.117	−0.117				
1h more in vapor of methyl alcohol, t=66°.0.	19.2	59.1	−0.105	−0.103	+0.30	−0.0074	36.51	18
	19.2	54.5	−0.079	−0.079				
	19.2	49.0	−0.054	−0.055				
	19.2	42.6	−0.031	−0.032				
	19.2	38.4	−0.023	−0.021				
1h more in vapor of methyl alcohol, t=66°.0.[1]	19.8	54.6	−0.003	−0.002	+0.47	−0.0065	36.42	19
	19.9	49.7	+0.009	+0.008				
	19.8	45.0	0.015	0.014				
	19.9	38.2	0.018	0.018				

[1] Neutral point: $(T+t)=-\frac{a}{b}=72.5$; $\therefore T=52°.7$.

BEHAVIOR OF HARD STEEL ANNEALED IN STEAM (100°).

The rods selected were:

No. 31; diameter (2ρ)=0.0839 cm.,
No. 32; diameter (2ρ)=0.0616 cm.,
No. 33; diameter (2ρ)=0.0491 cm.

Our earlier experiments with rods Nos. 24 and 25 had shown that the effect of annealing in steam is particularly marked during the first hour of exposure. We therefore decided to obtain intermediate stages by making interruptions and examinations after each of the first 10, 30, and 60 minutes of this interval. After this the rods were annealed during two hours more, as above. The results for the hardness in the original (glass-hard) and subsequent states are fully given in the following tables:

TABLE 19.—*Hard steel annealed at* 100°.

[Rod No. 31. $2\rho=0.0839$.]

Remarks.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s_t	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
Glass-hard.....	22.5	55.2	−0.621	−0.624				
	22.5	51.3	−0.542	−0.539	−1.20	−0.0093	40.47	21
	22.4	44.9	−0.410	−0.409				
	22.4	39.6	−0.303	−0.304				
10m in steam at 100°.....	19.7	61.0	−0.335	−0.332				
	19.7	55.4	−0.266	−0.269	−0.08	−0.0090	39.05	20
	19.7	48.7	−0.202	−0.202				
	19.7	42.9	−0.150	−0.150				
20m more in steam at 100°.	18.5	76.0	−0.177	−0.170				
	18.5	65.7	−0.094	−0.095	+0.55	−0.0090	37.64	19
	18.5	49.4	−0.009	−0.018				
	18.5	37.5	+0.006	+0.009				
30m more in steam at 100°.	18.1	63.5	+0.122	+0.124				
	18.1	56.9	0.126	0.125	0.86	−0.0072	36.33	19
	18.1	50.0	0.118	0.118				
	18.1	44.5	0.107	0.107				
1h more in steam at 100°..	18.2	72.4	0.330	0.329				
	18.2	58.8	0.304	0.304	1.55	−0.0104	35.38	18
	18.3	51.0	0.270	0.271				
	18.3	45.4	0.242	0.241				
1h more in steam at 100°..	19.2	71.4	0.457	0.455				
	19.3	61.4	0.410	0.410	1.81	−0.0103	34.72	18
	19.2	55.2	0.371	0.374				
	19.3	47.2	0.314	0.312				

TABLE 20.—*Hard steel annealed at* 100°.

[Rod No. 32. $2\rho = 0.0616$.]

Remarks.	t	T	e: 10^2 observed.	e: 10^2 calculated.	a	b	s_t	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
Glass-hard	20.7	50.7	−0.659	−0.659	−1.44	−0.0106	41.42	21
	20.7	46.0	−0.545	−0.544				
	20.7	41.9	−0.437	−0.448				
10m in steam at 100°	19.2	56.7	−0.269	−0.266	−0.39	−0.0042	39.53	20
	19.3	53.4	−0.235	−0.238				
	19.2	44.7	−0.168	−0.168				
	19.3	40.4	−0.137	−0.136				
20m more in steam at 100°[1]	18.5	70.1	0.075	0.075	0.80	−0.0075	38.02	19
	18.5	63.6	0.090	0.088				
	18.6	56.0	0.094	0.094				
	18.6	44.8	0.088	0.088				
30m more in steam at 100°	18.0	59.1	0.291	0.294	1.39	−0.0086	36.33	19
	18.0	49.6	0.260	0.253				
	18.0	44.7	0.225	0.225				
	18.0	41.4	0.201	0.203				
1h more in steam at 100°	17.7	55.7	0.519	0.519	2.01	−0.0089	35.19	18
	17.7	49.5	0.451	0.451				
	17.7	41.8	0.357	0.359				
	17.7	36.4	0.288	0.287				
1h more in steam at 100°	19.2	67.3	0.754	0.754	2.26	−0.0080	34.34	18
	19.3	58.9	0.650	0.648				
	19.2	52.0	0.556	0.556				
	19.2	45.8	0.463	0.463				

[1] Maximum at $T = -\frac{1}{2}\frac{a}{b} = 53°.1$.

TABLE 21.—*Hard steel annealed at* 100°.

[Rod No. 33. $2\rho = 0.0491$.]

Remarks.	t	T	e: 10^2 observed.	e: 10^2 calculated.	a	b	s_t	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
Glass-hard	20.5	57.0	−0.379	−0.381	−0.07	−0.0122	36.92	21
	20.6	52.3	−0.307	−0.306				
	20.6	45.9	−0.226	−0.224				
	20.6	41.2	−0.170	−0.171				
10m in steam at 100°	19.2	58.6	0.151	0.150	0.97	−0.0076	35.00	20
	19.2	51.7	0.142	0.142				
	19.2	44.4	0.124	0.124				
	19.2	37.8	0.100	0.100				
20m more in steam at 100°	18.4	53.2	0.361	0.364	1.50	−0.0062	33.58	19
	18.4	44.7	0.294	0.290				
	18.4	37.5	0.221	0.218				
	18.4	32.5	0.163	0.166				
30m more in steam at 100°	18.0	56.3	0.632	0.634	2.25	−0.0079	32.26	19
	18.1	47.8	0.517	0.510				
	18.0	41.9	0.418	0.423				
	18.1	32.0	0.256	0.257				
1h more in steam at 100°	18.0	66.1	0.881	0.893	2.67	−0.0097	31.22	18
	18.0	58.6	0.795	0.783				
	18.0	51.4	0.670	0.668				
	18.0	42.1	0.500	0.504				
1h more in steam at 100°	19.3	67.3	1.071	1.070	3.00	−0.0090	30.67	18
	19.3	58.1	0.895	0.897				
	19.3	50.2	0.737	0.737				
	19.3	40.8	0.521	0.521				

BEHAVIOR OF HARD STEEL ANNEALED IN VAPOR OF BOILING ANILINE (185°).

The details of the operations in this case were the same as those described in the foregoing paragraph. The rods used were:

No. 34; diameter $(2\rho)=0.0835$ cm.,
No. 35: diameter $(2\rho)=0.0627$ cm.,
No. 36; diameter $(2\rho)=0.0481$ cm.

The results obtained are the following:

TABLE 22.—*Hard steel annealed at* 185°.

[Rod No. 34. $2\rho=0.0835$.]

Remarks.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s_t	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{\text{cm}}{\text{cm}^2 t}$ *microhm.*	°C.
Glass-hard	22.1	59.2	−0.573	−0.569	−0.83	−0.0087	39.34	21
	22.1	51.1	−0.422	−0.425				
	22.1	44.2	−0.313	−0.313				
	22.0	39.5	−0.239	−0.238				
10^m in vapor of aniline at 185°.	19.5	71.0	1.414	1.423	3.59	−0.0090	29.25	20
	19.5	62.8	1.235	1.228				
	19.5	52.8	0.981	0.975				
	19.5	43.6	0.722	0.726				
30^m more in vapor of aniline at 185°.	18.7	77.5	2.694	2.700	4.10	−0.0116	28.02	19
	18.7	59.8	1.321	1.312				
	18.7	49.7	1.025	1.027				
	18.7	42.9	0.817	0.820				
30^m more in vapor of aniline at 185°.	18.2	71.0	1.717	1.721	4.33	−0.0121	27.17	19
	18.2	65.1	1.568	1.562				
	18.2	56.9	1.325	1.327				
	18.2	45.9	0.986	0.986				
1^h more in vapor of aniline at 185°.	18.2	76.9	2.054	2.052	4.61	−0.0118	26.32	18
	18.2	68.2	1.895	1.899				
	18.2	61.1	1.579	1.581				
	18.2	52.5	1.298	1.297				
1^h more in vapor of aniline at 185°.	19.3	88.5	2.453	2.454	4.84	−0.0121	25.85	18
	19.3	74.3	2.043	2.045				
	19.3	65.9	1.782	1.779				
	19.3	58.4	1.526	1.528				

TABLE 23.—*Hard steel annealed at* 185°.

[Rod No. 35. $2\rho=0.0627$.]

Remarks.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s_t	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
Glass-hard	22.1	56.5	−0.847	−0.846	−1.97	−0.0059	42.45	21
	22.2	50.4	−0.680	−0.684				
	22.1	44.4	−0.538	−0.534				
	22.1	39.1	−0.402	−0.402				
10m in vapor of aniline at 185°.	19.8	65.0	1.531	1.532	4.22	−0.0097	28.58	20
	19.9	57.0	1.292	1.297				
	19.9	49.2	1.032	1.038				
	19.9	42.6	0.821	0.819				
20m more in vapor of aniline at 185°.	18.5	75.6	2.078	2.081	4.75	−0.0118	27.26	19
	18.5	66.6	1.812	1.808				
	18.6	56.2	1.453	1.455				
	18.6	45.7	1.082	1.078				
30m more in vapor of aniline at 185°.	18.1	72.7	2.189	2.189	5.18	−0.0128	26.04	19
	18.1	63.2	1.861	1.863				
	18.1	56.0	1.604	1.602				
	18.1	48.0	1.294	1.294				
1h more in vapor of aniline at 185°.	18.3	69.4	2.226	2.223	5.42	−0.0122	25.29	18
	18.3	62.6	1.962	1.964				
	18.3	55.4	1.682	1.686				
	18.3	45.3	1.256	1.254				
1h more in vapor of aniline at 185°.	19.2	73.5	2.452	2.450	5.64	−0.0122	24.72	18
	19.3	66.2	2.155	2.158				
	19.2	57.6	1.806	1.807				
	19.3	48.7	1.415	1.414				

TABLE 24.—*Hard steel annealed at* 185°.

[Rod No. 36. $2\rho=0.0481$.]

Remarks.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s_t	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
Glass-hard	20.3	51.2	−0.279	−0.279	−0.05	−0.0121	37.17	21
	20.4	46.6	−0.222	−0.222				
	20.3	41.8	−0.171	−0.170				
	20.4	38.6	−0.137	−0.138				
10m in vapor of aniline at 185°.	19.2	71.8	2.069	2.071	4.76	−0.0091	25.85	20
	19.2	63.7	1.793	1.787				
	19.2	57.4	1.552	1.555				
	19.2	49.4	1.251	1.250				
20m more in vapor of aniline at 185°.	18.5	78.5	2.485	2.483	5.26	−0.0115	24.91	19
	18.5	64.3	1.968	1.971				
	18.5	54.9	1.609	1.605				
	18.5	45.2	1.207	1.208				
30m more in vapor of aniline at 185°.	17.8	78.8	2.605	2.603	5.50	−0.0125	24.15	19
	17.9	65.2	2.115	2.108				
	17.9	55.5	1.726	1.720				
	17.9	44.7	1.259	1.262				
1h more in vapor of aniline at 185°.	17.9	72.0	2.447	2.461	5.50	−0.0106	23.58	18
	17.9	64.7	2.178	2.163				
	17.9	53.8	1.703	1.696				
	17.9	46.5	1.374	1.371				
1h more in vapor of aniline at 185°.	19.2	68.3	2.327	2.325	5.74	−0.0114	23.12	18
	19.3	58.0	1.880	1.876				
	19.2	52.6	1.632	1.641				
	19.3	44.7	1.273	1.269				

BEHAVIOR OF HARD STEEL ANNEALED IN MOLTEN LEAD (330°).

From an inspection of the foregoing families of curves it appeared probable, in view of this relatively high temperature, that annealing effects of great magnitude would occur during the first minutes of exposure. Conformably herewith, the rods were subjected to 330° during consecutive intervals of 1^m, 30^m, and 1^h, and examined at the end of each of these times. The rods selected were:

No. 37; diameter $(2\rho)=0.0820$ cm.
No. 38; diameter $(2\rho)=0.0616$ cm.
No. 39; diameter $(2\rho)=0.0483$ cm.

The following results were obtained:

TABLE 25.—*Hard steel annealed at* 330°.

[Rod No. 37. $2\rho=0.0820$.]

Remarks.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s_t	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
Glass-hard	18.7	56.1	−0.248	−0.248	0.00	−0.0089	36.51	18
	18.8	52.7	−0.215	−0.215				
	18.8	51.8	−0.206	−0.206				
1^m in molten lead at 330°	17.8	77.5	3.898	3.898	7.74	−0.0126	18.96	18
	17.8	67.5	3.308	3.308				
	17.8	59.2	2.800	2.799				
	17.8	46.9	2.012	2.012				
30^m more in molten lead at 330°.	18.5	86.7	4.491	4.491	7.77	−0.0113	18.78	19
	18.5	75.2	3.806	3.808				
	18.5	62.3	3.003	3.005				
	18.5	50.7	2.252	2.251				
1^h more in molten lead at 330°.	18.9	69.0	3.471	3.458	7.71	−0.098	18.78	19
	18.9	57.2	2.667	2.685				
	18.9	47.6	2.045	2.038				
	18.9	39.4	1.472	1.470				

TABLE 26.—*Hard steel annealed at* 330°.

[Rod No. 38. $2\rho=0.0616$.]

Remarks.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	s_t	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
Glass-hard	18.8	50.2	−0.577	−0.573	−1.19	−0.0093	40.38	18
	18.8	44.8	−0.459	−0.461				
	18.8	39.6	−0.357	−0.360				
	18.8	34.8	−0.271	−0.269				
1^m in molten lead at 330°	17.6	67.6	4.012	4.015	8.96	−0.0111	17.55	18
	17.7	61.3	3.532	3.531				
	17.7	53.1	2.998	2.998				
	17.6	42.7	2.083	2.084				
30^m more in molten lead at 330°.	18.8	87.8	5.470	5.470	9.24	−0.0124	17.36	19
	18.9	73.4	4.413	4.417				
	18.9	63.7	3.794	3.784				
	18.9	50.3	2.631	2.633				
1^h more in molten lead at 330°.	19.1	84.4	5.204	5.201	9.39	−0.0138	17.27	19
	19.1	63.3	3.649	3.649				
	19.1	50.1	2.610	2.615				
	19.1	41.7	1.936	1.932				

TABLE 27.—*Hard steel annealed at* 330°.

[Wire No. 39. 2ρ=0.0483.]

Remarks.	t	T	$e:10^2$ observed.	$e:10^2$ calculated.	a	b	st	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
Glass-hard	18.0	53.8	−0.254	−0.254				
	18.1	50.7	−0.221	−0.221	0.08	−0.0110	36.04	18
	18.1	48.1	−0.195	−0.194				
1m in molten lead at 330°.	17.5	56.6	2.647	2.646				
	17.5	51.7	2.335	2.334	7.61	−0.0114	18.30	18
	17.5	45.8	1.943	1.948				
	17.5	39.6	1.540	1.537				
30m more in molten lead at 330°.	19.1	85.6	4.372	4.372				
	19.1	72.9	3.610	3.615	7.76	−0.0113	18.11	19
	19.1	61.9	2.937	2.929				
	19.1	50.4	2.180	2.182				
1h more in molten lead at 330°.	19.0	82.5	4.218	4.217				
	19.0	66.7	3.255	3.254	7.80	−0.0115	18.02	19
	19.0	57.8	2.687	2.687				
	19.0	49.5	2.142	2.142				

GENERAL DISCUSSION OF THE RESULTS OF THIS ANNEALING.

Digest.—The results thus far given adequately exhibit the general physical character of the process of tempering. For the sake of clearness, and with a view of partially eliminating such discrepancies as are due to incidental errors, the three individual values of thermo-electric power and specific resistance for each of the temperatures of annealing will be combined and their mean chosen for discussion. We thus arrive at the following relations:

TABLE 28.—*Mean results:*

I.—*For annealing in vapor of boiling methyl alcohol* (66°).

Time of annealing=	0h	1h	2h	3h
a=	−1.62	−1.07	−0.80	−0.60

II.—*For annealing in steam* (100°).

Time of annealing=	0h	$\frac{1}{6}$h	$\frac{1}{2}$h	1h	2h	3h
a=	−0.91	0.17	0.95	1.50	2.08	2.36

III.—*For annealing in vapor of boiling aniline* (185°).

Time of annealing=	0h	$\frac{1}{6}$h	$\frac{1}{2}$h	1h	2h	3h
a=	−0.95	4.19	4.71	5.00	5.18	5.40

IV.—*For annealing in molten lead* (330°).

Time of annealing=	0h	$\frac{1}{10}$h	$\frac{1}{2}$h	$\frac{3}{4}$h
a=	−0.37	8.11	8.26	8.30

Deduction.—If these functionalities are constructed graphically, as has been done in Fig. 10 (time as abscissa, thermo-electric constant as ordi-

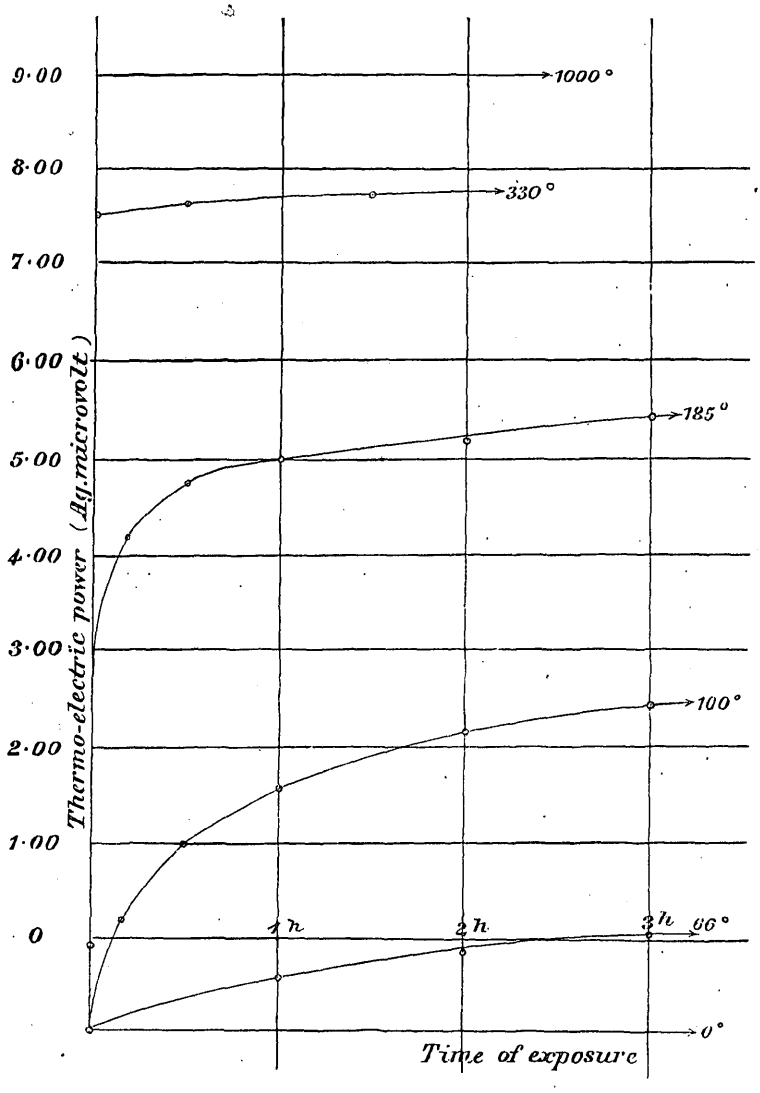


FIG. 10.—Hard wires annealed continuously at 0°, 66°, 100°, 185°, 330°, and 1,000°, respectively.

nate), we obtain a family of typical curves, the general character of which is distinctly pronounced and may be thus expressed:

The degree of hardness retained by a glass-hard rod, after having been subjected to the operation of annealing, is dependent both on the temperature to which it has been exposed and on the interval of time during which this exposure has taken place, in such a way that the effect of time, though of predominating importance in the case of small values of temperature, is more and more negligible in proportion as

these values increase. The operation is always most effective in its earlier stages, and this efficiency decreases very slowly where the temperatures are low—very rapidly, indeed almost suddenly, where they are high. If the action of any temperature be indefinitely prolonged, the rod under its influence ultimately reaches an inferior and limiting degree of hardness characteristic both of the temperature chosen and the type of steel under experiment. In other words: the annealing effect of any temperature increases gradually at a rate diminishing continuously through infinite time, very slowly in case of low temperature ($<100°$), with extreme rapidity in case of high temperature ($>200°$), so that the highest of the inferior states of hardness possible at any given temperature is approached asymptotically. The effect of the intensity of glass-hardness (strain) of the originally hard rod is not readily discernible.

The considerations set forth readily suggest the terminology: "*maximum of permanent hardness for the temperature t*"—an expression which will be used throughout the sequel to designate the highest of the inferior states of hardness, persistent at the said temperature, *t*.

The relatively large effect produced by low temperatures ($<100°$), when a glass-hard rod is subjected to their influence for a long period of time, is deserving of special mention. It follows that a perceptible annealing effect must be attainable with temperatures much lower than the lowest above employed; indeed that the only inferior limit in this respect is probably the temperature of the water in which the rod was originally chilled,—while even the chilled rod, though kept at constant temperature, cannot be regarded as existing in a state of thorough molecular equilibrium until the lapse of a considerable interval of time after the hardening has taken place. Herefrom it appears that in the thermo-electric experiments the danger of destroying the uniformity of temper of a glass-hard rod by overheating the hot junction is greatly to be apprehended. Quick work and low temperatures are, therefore, to be preferred at an unavoidable sacrifice of accuracy. Nor is soldering of the ends permissible, except with the most extreme and intelligent caution. We desire to advert, in conclusion, to the important bearing of this unlooked-for sensitiveness of hard steel, even to low temperatures, on all other physical properties depending on the temper of this material—permanent magnetism for instance. It is obvious that the nature of a purely magnetic phenomenon can only be satisfactorily investigated when the material carrying the magnetic quality, throughout the course of the experiments undergoes no permanent change, otherwise we virtually commence our investigation with one rod and finish it with another, obtaining data which are not immediately, if at all, comparable.

ON THE EFFECT OF HIGHER AND OF LOWER TEMPERATURES ON THE TEMPER OF STEEL ORIGINALLY ANNEALED AT A GIVEN INTERMEDIATE TEMPERATURE.

Effect of lower temperatures.—The marked tendency of glass-hard rods to suffer diminution of hardness, even when exposed to temperatures but slightly above mean atmospheric temperature, naturally led to another important inquiry almost the converse of the preceding. We refer to the behavior of a rod annealed at a given temperature to the prolonged effect of lower temperatures subsequently applied.

For the purpose of obtaining experimental data a steel wire (No. 26, $2\rho=0.085$), which had on a former occasion been annealed in an oil bath at 250°, was exposed to steam at 100° for one hour. The results of the thermo-electric measurements made before and after this final exposure were these:

TABLE 29.—*Steel annealed at* t_2° *exposed to* t_1°, $t_2 > t_1$

Before.			After.			
t	T	$e:10^2$	t	T	$e:10^2$	$e:10^2$
16.1	89.1	4.05	16.9	73.2	3.17	(3.22)
16.1	80.7	3.62	16.9	63.2	2.69	(2.72)
16.2	68.1	2.99	16.9	56.5	2.32	(2.35)
16.2	54.6	2.29	17.0	50.2	1.98	(1.99)
16.2	44.5	1.72				
16.2	37.9	1.34				

The temperatures t in both series are approximately identical. The electromotive force e may therefore be regarded as dependent on the difference of temperature $T-t$ only,[30] and with this presumption graphically represented. If, with the object of interpolating graphically, the two curves be carefully constructed, and the values of e for the temperatures of the one set of observations be derived from the curve for the other, then a comparison of corresponding values of e shows clearly that no annealing due to 100° is appreciable. In the table (29) the interpolated electromotive forces are distinguished by parentheses.

[30] For $e=a(T-t)+b\,(T^2-t^2)=(T-t)\,(a+b\,[T-t]+2\,bt)=f\,(T-t)$ if t is constant.

This result was corroborated by a second experiment made in the same way with rod No. 27 ($2\rho=0.085$), which had been annealed in oil at 200°. The thermo-electric data obtained before and after the final annealing at 100° were:

TABLE 30.—*Steel annealed at t_2° exposed to t_1°, $t_2>t_1$.*

Before.			After.			
t	T	$e:10^2$	t	T	$e:10^2$	$e:10^2$
16.3	88.5	2.62	17.0	84.2	2.45	(2.45)
16.4	80.8	2.38	17.0	71.7	2.08	(2.08)
16.4	70.0	2.06	17.1	62.3	1.76	(1.77)
16.4	61.6	1.77	17.2	43.4	1.09	(1.09)
16.4	53.1	1.48				
16.4	46.4	1.22				

Plainly these results admit of the following generalized interpretation: A steel rod, annealed at a given temperature, will remain the more nearly passive as regards the effect of an inferior temperature, the lower its value and the shorter its time of application on the one hand, and the nearer the mechanical state of the rod to the limit of hardness for the said temperature on the other. Where the limit in question has been fully reached the rod is unaffected by the action of lower temperature, however prolonged.

Effect of higher temperatures.—There is another question of a similarly important bearing on the nature of the phenomena of annealing. In the above we drew the general inference that in the case of a given rod possessing a given degree of glass-hardness, the mechanical state resulting after annealing is dependent on the temperature and the time of exposure only; that when the latter is indefinitely prolonged, a particular and characteristic limit of hardness is reached for each temperature of the annealing bath. If this be the case, then it must be immaterial, in so far as the identity of the final states is concerned, whether, for instance, a rod is first annealed in steam at 100° and then in aniline vapor at 185° to a limit, or whether it is annealed to a limit in aniline vapor at once. In other words, the maximum reduction of hardness attainable by the action of any given temperature on glass-hard steel is independent of the manner of variation of hardness (the path as it were) by which this final state is reached. On this point the following experiments were made:

Three steel wires of different thicknesses (Nos. 40, 41, 42), carefully tested for longitudinal homogeneity, were first examined in the glass-hard state. They were then broken in two, and one of the halves of each was annealed directly in aniline vapor at 185° for 10 minutes; the others, however, were first exposed in steam at 100° for 40 minutes, and after this, as in the former case, for 10 minutes in aniline. Hereupon the six rods were tested for hardness, with the following results. It

was expedient to make measurements of resistance only, Hockin and Matthiessen's method being employed:

In steam and in aniline vapor.		In aniline vapor only.	
Rod No. 40 (2ρ=0.085)	Glass-hard ..s=41.3	s=40.6	(t=14°)
	Annealed ...s=31.0	s=30.6	.....
Rod No. 41 (2ρ=0.064)	Glass-hard ..s=43.0	s=42.9	(t=14°)
	Annealed ...s=30.0	s=29.8	.....
Rod No. 42 (2ρ=0.049)	Glass-hard ..s=36.4	s=36.5	(t=14°)
	Annealed ...s=26.0	s=26.0	.....
Mean	Glass-hard ..s=40.2	s=40.0	(t=14°)
	Annealed ...s=29.0	s=28.8	.....

A second experiment was made with three other wires (Nos. 43, 44, 45). After the hardness for the glass-hard state had been obtained, the rods were again broken in two; one of the halves of each was annealed for 40 minutes in ethylic alcohol vapor at 78°, and then for 6 hours in steam; the others in steam only during the same interval. The results obtained are these:

In vapor of alcohol and in steam.		In steam only.	
Rod No. 43 (2ρ=0.085)	Glass-hard ..s=40.2	s=40.6	(t=10°)
	Annealed ...s=31.9	s=31.8	.....
Rod No. 44 (2ρ=0.066)	Glass-hard ..s=40.5	s=41.2	(t=10°)
	Annealed ...s=29.4	s=29.8	.....
Rod No. 45 (2ρ=0.049)	Glass-hard ..s=35.5	s=35.8	(t=10°)
	Annealed ...s=27.5	s=27.9	.....
Mean	Glass-hard ..s=38.7	s=39.2	(t=10°)
	Annealed ...s=29.8	s=29.7	.....

Both of these series of experiments are in perfect accord, and we therefore infer: The specific effect of a given temperature of annealing on the state of hardness of steel is independent of the possibly pre-existing effects of a lower temperature, to the extent that the result of the influence of the latter is the more fully annulled the greater the period of action of the former.

Anomalous feature.—This result is remarkable, inasmuch as by a sudden exposure of a glass-hard rod to (high) temperature, a very large amount of potential energy is *instantaneously* released. If, therefore, two identical glass-hard wires be annealed, the one by raising the temperature very gradually as far as t^0, the other by being suddenly exposed to t^0, we would infer, *ceteris paribus*, that a difference of hardness must result. For, in view of the sudden conversion of the available mechanical energy into heat in the latter case, we anticipate an equivalent increase in the temperature of the rod. Virtually, therefore, this steel is annealed at a temperature somewhat above that of the annealing bath, or above that of the other rod. So far as our experiments went this was not detected; but the test made is not exhaustive. Our attention was centered on other matters at the time. Possibly, however, the anomalous distribution of hardness obtained by annealing in oil may be partially accounted for in this way.

BEHAVIOR OF SOFT STEEL RODS.

Object of the measurement.—The above values for thermo-electric power all refer to elements in which steel is thermo-electrically combined with pure soft silver; but the latter metal, though chosen expediently, is otherwise arbitrary and without inherent bearing on the subject in hand. We will undoubtedly obtain data more in harmony with the purpose of the present investigation, and far more readily intelligible, by eliminating this foreign metal from our considerations entirely. It is to be our endeavor throughout the following paragraphs not only to add to the perspicuity of our data, but also to give them possibly immediate theoretical significance, by referring all measurements to a particular and normal state of steel. The soft state would appear to suggest itself in so far as it occupies an extreme and inferior position in the scale. If, however, this is to be chosen as a point of departure, the rods in this state should possess normal and fixed thermo-electric qualities, even for different kinds of steel. This is by no means the case, as the special experiments presently to be detailed will show. Indeed, even in the case of soft rods nominally of the same material, at all events nearly of identical composition, the difference of thermo-electric power is often very marked.

Method of softening.—With the object of reaching extreme values of the soft state, the rods were embedded in artificial powdered ferro-ferric oxide, as it falls from the anvil, and surrounded by a closed gas-pipe; the latter in turn enveloped in a thick coating of fire-clay, protected externally with thin sheet-iron. The whole was heated to intense redness, and then allowed to cool very slowly in the ashes of the smoldering fire[31]. The non-conducting envelopes insured a very gradual subsidence of temperature.

Data for soft steel and for soft iron.—As examples the results obtained with rods Nos. 46, 47, and 48 will be given:

TABLE 31.—*Behavior of soft steel rods.*

Remarks.	t	T	e observed.	e calculated.	a	b	st	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2}t$ *microhm.*	°C.
No. 46. $2\rho=0.0843$	18.7	87.7	486.3	486.2				
	18.7	67.3	354.8	354.5	8.33	−0.0121	17.08	19
	18.7	54.2	264.0	264.6				
	18.7	45.5	202.8	202.6				
No. 47. $2\rho=0.0625$	23.0	85.7	543.5	544.8				
	22.7	63.1	406.9	405.5	10.17	−0.0137	15.09	19
	22.6	51.7	266.9	266.4				
	22.4	40.6	169.0	169.3				
No. 48. $2\rho=0.0485$	19.0	84.0	474.7	472.5				
	19.0	64.3	338.4	340.1	8.52	−0.0121	16.41	19
	19.0	50.8	242.6	243.9				
	19.0	37.9	148.9	148.0				

[31] With regard to the decarbonization possible or incident to this operation see Chapter III.

For the sake of comparison iron wires of different kinds were treated in the same way and subsequently examined. The results obtained with Nos. I, II, and III will be cited as examples:

TABLE 32.—*Behavior of soft iron wire.*

Remarks.	t	T	e observed.	e calculated.	a	b	st	t
	°C.	°C.	*microvolt.*	*microvolt.*	*microvolt.*	*microvolt.*	$\frac{cm}{cm^2 t}$ *microhm.*	°C.
No. I. $2\rho=0.0966$	18.7	79.6	581.3	582.0	11.25	−0.0174	13.02	19
	18.8	66.3	465.4	464.9				
	18.7	54.3	356.3	355.8				
	18.8	44.9	264.7	265.0				
No. II. $2\rho=0.0630$	18.6	78.0	550.2	549.5	10.56	−0.0135	12.74	19
	18.6	64.2	430.1	430.3				
	18.6	53.4	332.6	333.6				
	18.6	43.4	241.6	241.1				
No. III. $2\rho=0.0312$	18.5	86.9	560.2	560.0	9.66	−0.0140	13.87	19
	18.5	68.1	418.0	419.1				
	18.5	54.3	310.3	309.3				
	18.5	40.1	190.7	190.9				

Impurities in steel and iron.—Now, there is reason to suppose that in the case of iron carburets generally the electrical effect of foreign impurities is partially masked by the phenomenally great effect of combined carbon.[32] Conformably with the usually accepted views, therefore, the shifting of thermo-electric position due to the presence in iron of materials (S., P., Si., etc.) other than carbon will be particularly marked for the soft states. For it is here that the quantity of combined carbon contained is at a minimum, while the steel itself is in a natural homogeneous state. We have analogies in the case of alloys. For instance, on alloys of gold and silver small additional quantities of foreign admixtures are relatively without marked electrical effect, whereas if pure gold or pure silver be adulterated with the same quantities of the same material the result is pronounced. If we compare the values of a and s for soft iron with those obtained for soft steel the specific effect of carbon is strikingly obscure. We are induced to refer the comparatively small differences apparent to the effect of foreign impurities in the iron. At least, in contrast with the phenomenal electrical interval between the glass-hard and the soft states of steel, the electrical effect of carbon is here wholly masked by that of the incidental ingredients. But it is herewith fully demonstrated that the steel rod possessing the qualities of a normal must be sought for elsewhere than in the soft state.

[32] Directly or indirectly. It will be shown in the sequel that the electrical variation discussed in this chapter is probably due to the strain accompanying hardness, but this in its turn is conditioned by the presence of carbon in iron.

THE RELATION EXISTING BETWEEN THE THERMO-ELECTRIC POWER AND THE SPECIFIC RESISTANCE OF STEEL.

Reduction of data.—In the above experiments we have thus far met with 86 pairs of correlative values of specific resistance and thermo-electric constant. We will supplement these results by values obtained with four rods subjected to six hours of annealing in steam. They are as follows:

Rod.	2ρ	a	s_t	t
No. 49.....	0.0574	1.90	35.75	19
No. 50.....	0.0554	4.08	29.34	19
No. 51.....	0.0531	4.06	27.08	19
No. 52.....	0.0344	3.90	28.68	19

In all, therefore, we are in possession of 90 pairs of values. These amply suffice to decide in how far an intrinsic relation between thermo-electric power and specific resistance, in case of one and the same metal (steel), in different states of temper, demonstrably exists. Now we have

$$\left(\frac{de}{d(T-t)}\right)_{T=t}=a;$$

whence it appears that the thermo-electric constant a primarily refers to the temperature zero, centigrade. It is, therefore, consistent to reduce the observed values of specific resistance (18°—21°) to the same temperature (zero). This is at best a laborious piece of work, and moreover presupposes a knowledge of the relation of resistance and temperature (the coefficients vary within the large range of 0.1 per cent. nearly to above 0.4 per cent.) for each of the 90 degrees of hardness encountered. But with the aid of the results contained in Chapter I of the present memoir this reduction may be satisfactorily made. The results are given in the following table, where $s_t=s(1+\alpha t)$:

TABLE 33.—*Specific resistance (cm/cm² 0° microhm) of steel.*

No.	s_t	t	$\alpha\times10^3$	s	No.	s_t	t	$\alpha\times10^3$	s	No.	s_t	t	$\alpha\times10^3$	s
1	18.96	19	3.4	17.81	29	41.32	19	1.7	40.03	35	28.58	20	2.4	27.27
2	18.49	19	3.5	17.34	30	37.08	18	1.8	35.92	35	27.26	19	2.5	26.03
3	20.54	19	3.1	19.40	30	36.80	18	1.9	35.58	35	26.04	19	2.6	24.81
4	20.00	19	3.3	18.22	30	36.51	18	1.9	35.30	35	25.29	18	2.7	24.11
5	17.64	19	3.6	16.51	30	36.42	19	1.9	35.15	35	24.72	18	2.7	23.57
6	20.75	20	3.2	19.51	31	40.47	21	1.7	39.08	36	37.17	21	1.8	35.82
7	22.74	19	2.9	21.55	31	39.05	20	1.8	37.70	36	25.85	20	2.6	24.58
8	23.20	19	2.9	21.99	31	37.64	19	1.8	36.40	36	24.91	19	2.7	23.69
9	20.47	19	3.2	19.30	31	36.33	19	1.9	35.06	36	24.15	19	2.8	22.93
10	25.67	19	2.6	24.46	31	35.38	18	1.9	34.21	36	23.58	18	2.9	22.41
11	24.81	19	2.7	23.60	31	34.72	18	2.0	33.50	36	23.12	18	2.9	21 97
12	22.93	19	2.9	21.73	32	41.42	21	1.7	39.99	37	36.51	18	1.9	35.30
13	20.66	19	3.2	19.48	32	39.53	20	1.7	38.23	37	18.96	18	3.4	17.87
14	27.36	20	2.5	26.06	32	38.02	19	1.8	36.76	37	18.78	19	3.4	17.64
15	28.11	19	2.4	26.88	32	36.33	19	1.9	35.06	37	18.78	19	3.4	17.64
16	26.70	19	2.5	25.49	32	35.19	18	1.9	34.03	38	40.38	18	1.7	39.17
17	27.93	19	2.4	26.71	32	34.34	18	2.0	33.14	38	17.55	18	3.6	16 48
18	31.32	19	2.2	30.06	33	36.98	21	1.8	35.64	38	17.36	19	3.6	16.25
19	28.68	19	2.4	27.43	33	35.00	20	1.9	33.72	38	17.25	19	3.6	16.16
20	24.34	19	2.8	23.11	33	33.58	19	2.0	32.35	39	36.04	18	1.9	34.85
21	29.34	19	2.3	28.11	33	32.26	19	2.1	31.02	39	18.30	18	3.5	17.22
22	30.94	19	2.2	29.70	33	31.22	18	2.2	30.03	39	18.11	19	3.5	16.98
23	27.93	19	2.4	26.71	33	30.67	18	2.2	29.49	39	18.02	19	3.5	16.89
28	42.64	18	1.6	41.45	34	39.34	21	1.7	37.98	46	17.08	19	3.6	15.98
28	42.55	18	1.6	41.36	34	29.25	20	2.3	27.96	47	15.09	19	3.9	14.05
28	42.45	19	1.6	41.20	34	28.02	19	2.4	26.80	48	16.41	19	3.7	15.34
28	42.55	19	1.6	41.30	34	27.17	19	2.5	25.94	49	35.75	19	1.9	34.51
29	42.08	17	1.6	40.95	34	26 32	18	2.6	25.14	50	29.34	19	2.3	28.11
29	41.70	18	1.6	40.53	34	25.85	18	2.6	24.69	51	27.08	19	2.5	25.85
29	41.52	19	1.7	40.12	35	42.45	21	1.6	41.07	52	28.68	19	2.4	27.43

Fundamental constants.—For the sake of greater clearness in designation, let a be replaced by y', s_0 by x. Parallel variations of y' and x we have had frequent occasion to notice. But we have now the data in hand for a complete discussion of the nature of their mutual dependence, and this will manifest itself perspicuously and at once in a graphic representation with x as abscissa, y' as ordinate. Figure 11 contains the 90 points in question. They lie within a band or pathway which, as a rule, is narrow, especially so near its middle, but widens somewhat as we approach the lower extremity, where $y'=0$. Nevertheless, the character of the functionality involved within the given range of possible variation appears with due prominence. *It is unquestionably linear.*

We may, therefore, with great probability premise that between y' and x a relation of the form

$$y'=m-nx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

exists. If we make this equation the basis of a calculation in which the fundamental constants m and n are to be determined by the method of least squares we arrive at the values

$$m=15.176 \qquad n=0.4123$$

by solving the normal equations

$$P=mA-nB \qquad Q=mB-nC$$

where

$$A=\Sigma X^0 = 90$$
$$B=\Sigma X = 2536.55$$
$$C=\Sigma X^2 = 773.21$$
$$P=\Sigma y^1 = 319.91$$
$$Q=\Sigma X y^1 = 6612.1.$$

New thermo-electric data.—Of these constants, m commands only secondary interest, in so far as it depends on the metal (silver in our case)

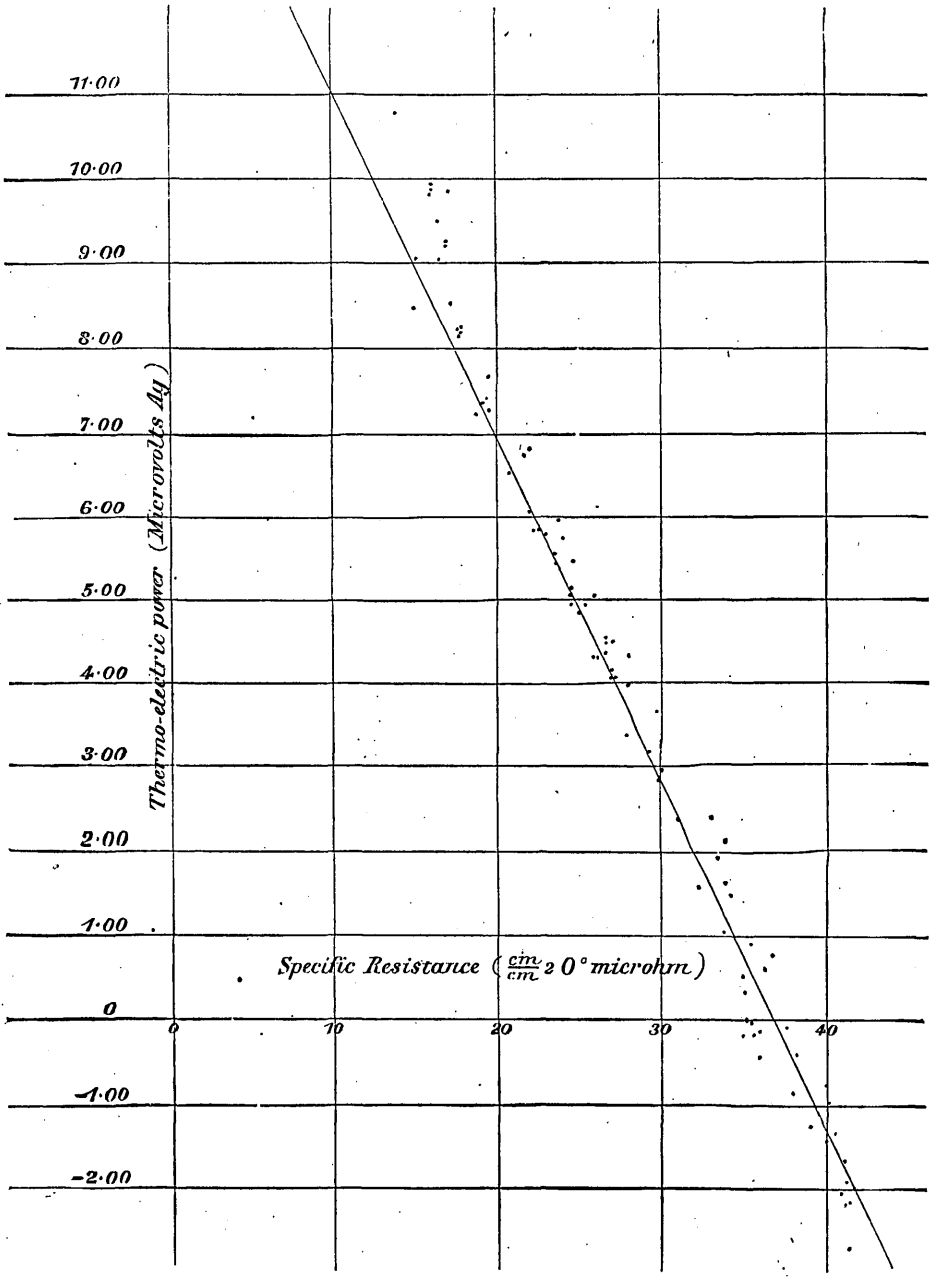


FIG. 11.—Diagram of the relation between the thermo-electric power of steel in different states of temper, and its specific resistance.

arbitrarily chosen to fix a datum or point of departure for thermo-electric measurements. This, however, is not true of $m-y'$, a quantity wholly

independent of the arbitrary element in question. It therefore appears expedient to select $m-y'=h$ as a new thermo-electric variable, in which case (1) reduces to the simpler form $h=nx$. The new thermo-electric data (h, nx) thus obtained are exclusively dependent on qualities inherent in steel. They have no reference to metals (silver) foreign to the present purpose. From a mathematical standpoint m is the experimental constant by which a specially selected set of co-ordinates is determined. From a physical standpoint m would represent the thermo-electric constant of a couple, one member of which is silver, the other the imaginary steel rod, whose specific resistance, supposed diminishable indefinitely in accordance with the above law, is zero. It is this initial state of hardness ($h=o$) at which the new variable h originates.

Thermo-electric hardness defined.—The new thermo-electric variable, h, will, throughout the present memoir, be designated by the term *thermo-electric hardness*. The introduction of this quantity was suggested to us by considerations similar to those which led to the conception of absolute temperature. To the absolute zero of temperature, however, Thomson has been able to give a concrete physical interpretation; whereas the zero of thermo-electric hardness, so far as the results of the present chapter go, furnishes no more than a point of departure, at once convenient and free from arbitrary assumptions. It is hardly necessary to cite as a second example the hypothesis defining the modulus of elasticity, for instance.

The following is a brief summary of the advantages immediately derived from the introduction of thermo-electric hardness:

1. Hardness in the case of steel is expressed for the soft as well as for the hard rod in significant and at the same time convenient figures, which increase as this quality increases. The scale is, therefore, adapted to circumstances.

2. A change of signs is avoided, the current invariably flowing from the given specimen through hot to the normal. In other words, the normal is thermo-electrically positive to the whole family of iron-carburets, iron, steel and cast iron, no matter what be their mechanical state.

3. The results are independent of the shiftings of thermo-electric position common to all metals, due to small amounts of impurity or difference of mechanical state.

4. By the aid of the coefficients discussed in chapter I and for the sake of concreteness of conception only, we may state that, theoretically, the normal condition of steel, to which thermo-electric hardness refers, would nearly be given by soft steel cooled down as far as the absolute zero of temperature. (See Chapter III.)

Tabular comparisons.—A complete table follows, showing the differences between observed ($h=m-y'$) and calculated (nx) values of thermo-

electric hardness. As has been stated, the results are deduced by the method of least squares:

TABLE[31] 34.—*Comparison of the values of the thermo-electric hardness and specific resistance.*

No.	h	nx	Diff.	No.	h	nx.	Diff.	No.	h	nx	Diff.
1	7.38	7.34	0.04	29	15.88	16.50	—0.62	35	10.96	11.24	—0.28
2	7.15	7.15	0.00	30	15.60	14.81	0.79	35	10.43	10.73	—0.30
3	8.19	8.00	0.19	30	15.19	14.67	0.52	35	10.00	10.23	—0.23
4	7.40	7.76	—0.36	30	14.88	14.56	0.32	35	9.76	9.94	—0.18
5	6.66	6.81	—0.15	30	14.71	14.49	0.22	35	9.54	9.72	—0.18
6	7.94	8.04	—0.10	31	16.38	16.11	0.27	36	15.23	14.77	0.46
7	9.03	8.88	0.15	31	15.26	15.54	—0.28	36	10.42	10.13	0.29
8	8.72	9.07	—0.35	31	14.63	15.01	—0.38	36	9.92	9.77	0.15
9	8.22	7.96	0.26	31	14.32	14.45	—0.13	36	9.68	9.45	0.23
10	10 51	10.08	0.43	31	13.63	14.10	—0.47	36	9.68	9.24	0.44
11	10.03	9.73	0.30	31	13.37	13.82	—0.45	36	9.44	9.06	0.38
12	8.78	8.96	—0.18	32	16.62	16.49	0.13	37	15.18	14.56	0.62
13	8.29	8.03	0.26	32	15.57	15.76	—0.19	37	7.44	7.37	0.07
14	11.12	10.74	0.38	32	14.38	15.16	0.78	37	7.41	7.27	0.14
15	11.33	11.08	0.25	32	13.79	14.45	—0.66	37	7.47	7.27	0.20
16	10.51	10.51	0.00	32	13.17	14.03	—0.86	38	16.37	16.15	0.22
17	10.92	11.01	—0.09	32	12.92	13.66	—0.74	38	6.22	6.79	—0.57
18	12.39	12.39	0.00	33	15.25	14.69	0.56	38	5.94	6.70	—0.76
19	11.35	11.31	0.04	33	14.21	13.90	0.31	38	5.79	6.66	—0.87
20	9.73	9.53	0.20	33	13.68	13.34	0.34	39	15.10	14.37	0.73
21	11.42	11.59	—0.17	33	12.93	12.79	0.14	39	7.57	7.10	0.47
22	11.77	12.25	—0.48	33	12.51	12.38	0.13	39	7.42	7.00	0.42
23	10.94	11.01	—0.07	33	12.18	12.16	0.02	39	7.38	6.97	0.41
28	17.74	17.09	0.65	34	16.01	15.66	0.35	46	6.85	6.59	0.26
28	17.12	17.05	0.07	34	11.59	11.53	0.06	47	5.01	5.79	—0.78
28	16.99	16.99	0.00	34	11.08	11.05	0.03	48	6.66	6.32	0.34
28	16.76	17.03	—0.27	34	10.85	10.69	0.16	49	13.28	14.23	—0.95
29	17.08	16.89	0.19	34	10.57	10.37	0.20	50	11.10	11.59	—0.49
29	16.43	16.71	—0.28	34	10.34	10.18	0.16	51	11.12	10.66	0.46
29	16.09	16.54	—0.45	35	17.15	16.93	0.22	52	11.28	11.31	—0.03

[31] The second decimal place (h, nx, diff.) serves only, of course, to give greater accuracy to the first and might perhaps have been advantageously suppressed.

Distribution of errors.—If this series of results is examined critically, that is with especial reference to magnitude and location of the errors, relatively large discrepancies encountered at the beginning of the table, in the case of very soft rods, are conspicuous. It is not, therefore, to be considered as finally decided whether the relation $h=f(x)$ may not for small values of x be other than linear; such, however, as rapidly to approximate to the latter form of function as x increases. The other errors in general are as liable to assume positive as negative values; they lie within sufficiently narrow limits, and do not, with one exception, exhibit any characteristic progress for consecutive states. In the case of wires annealed in methyl alcohol a uniform succession of errors is, however, apparent. For continually decreasing values of h we have:

No. 28.....	7	1	0	—3
No. 29.....	2	—3	—5	—6
No. 30.....	8	5	3	2

In other words, with continually augmenting time of exposure, the temperature, 66°, produces an annealing effect such that the linear relation between thermo-electric hardness and specific resistance is not rigidly maintained. Here we have unquestionably the evidences of a

structural phenomenon. The inference is plausible that the annealing influence of this comparatively low temperature is largely confined to those parts of the rod where the strain is most intense. This is probably the case with the superficial layers; for, according to Fromme's datum the density of these has been increased enormously. Now, it would seem that it is upon the surface layers of a rod that thermo-electric power is principally dependent, whereas specific resistance varies with the mean hardness of rod taken as a whole. The discrepancy in question is repeated with such uniformity in each of the three wires, that it is obvious we have in hand more than a mere error of observation. Furthermore, the similar variation which occurs in the case of wires annealed at 100° shows that the anomaly exists in steel itself, and is deserving of special inquiry.

Consequences of an elementary law, $h=ns$.—A more detailed analysis of the thermo-electric effects of structure does not substantiate the stated inference that the thermo-electric data are influenced to a greater extent by the condition of the surface layers of steel than the resistance data.

From the very general evidence obtained in Table 34 in favor of the law $h=ns$, we are justified in giving it fundamental or elementary importance. In other words, the supposition is warranted that, for each of the coaxial necessarily homogeneous elementary shells of which we may suppose the rod to be made up, the relation

$$h=ns \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

applies. Suppose, now, we take into consideration any two such shells.

Let $s_1, s_2, h_1, h_2, q_1, q_2$, be their specific resistances, their thermo-electric hardness, and their right sections, respectively.

Let $s_{12}, h_{12}, q_{12}=q_1+q_2$, be the specific resistance, thermo-electric hardness, and right sections, respectively, of a single shell, which is capable galvanically and thermo-electrically to replace the two original shells (they are actually combined in multiple arc) in every respect. Then

$$h_{12}=\frac{h_2\frac{s_1}{q_1}+h_1\frac{s_2}{q_2}}{\frac{s_1}{q_1}+\frac{s_2}{q_2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$s_{12}=\frac{\frac{s_1}{q_1}\frac{s_2}{q_2}}{\frac{s_1}{q_1}+\frac{s_2}{q_1}}q_{12} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It therefore follows, in view of equation (1), that

$$h_{12}=\frac{n}{q_{12}}(q_1+q_2)\,s_{12}=n\,s_{12}$$

and in the same way generally

$$h_{1,2,3\ .\ .\ .}=\frac{n}{q_{1,2,3\ .\ .\ .}}\Sigma(q_k)\,s_{1,2,3\ .\ .\ .}=ns_{1,2,3\ .\ .}$$

That is, if the law $h=ns$ be true for the elementary (necessarily homo-

geneous) layers of the rod, then must it be true also for the rod taken as a whole, however complicated its structure may be, and whatever be the difference in the variation of the density of successive layers while annealing is in progress. The converse of this does not follow so readily.

The superficial layers of the steel, being the ones directly and immediately operated upon in sudden cooling, will probably be the ones experiencing greatest intensity of strain. It may be plausibly argued, therefore, that the effect of incipient annealing is confined to these, or at least that the said layers expand at a far more rapid rate than the rod as a whole contracts. The law $h=ns$ may therefore be considered to apply with good approximation, both in the case of volume expansion and of volume contraction.

We may account for the progressive errors encountered in the case of hard wires annealed at low temperatures ($<100°$) from two points of view: Either the elementary law $h=ns$ is not *rigidly* true throughout the phenomenal variation of density which the individual layers experience on tempering (probably from 7 to 10); in other words, $h=f(s)$ is no longer rigidly linear when we approach the enormous condensation near the surface, for instance. Or, the discrepancy may be due to the facts observed by Caron[32] that a hard-steel rod during the annealing contracts æolotropically; *i. e.*, not in like ratio both in the direction of its longitudinal and its radial axes. In such a case the law $h=ns$, conceded rigidly to hold good for isotropic strains, need no longer do so in the case in hand. This discussion will be advantageously resumed, in the light gained from further relevant data, in chapter III.[33]

SOURCES OF ERROR.

The difference between the observed and the calculated results of thermo-electric hardness, as exhibited in the last paragraph, pertinently suggest an especial inquiry in regard to how far they are referable to errors of observation. This involves an examination into the possible descrepancies incident to the various methods of measurement used.

Thermo-electric measurement.—If we commence with the thermo-electric measurement we at once encounter a series of obvious sources of error which may be thus classified: Variation of the factor of reduction of the galvanometer, to be referred in part to fluctuations in the intensity of terrestrial magnetism, in part to the immediate influences due to changes of atmospheric temperature; the inconstancy of the electromotive force of the Daniell between the observations made for the determinations of the same; the effect of extraneous thermo-currents in

[32] Caron: Comptes rendus, LVI, p. 211, 1863.

[33] Chapter III, p. 88.

the galvanic connections; and, finally, the difficulties surrounding an accurate measurement of the temperatures of the thermo-electric junctions, especially when they are high. The first two of these may be satisfactorily avoided by frequent repetitions of a series of systematic and corroborative measurements. The devices[34] by which we endeavored to eliminate the third have already been fully described; nevertheless the distortion due to this cause is not thoroughly eliminable, and its effect will be most apparent in the case of small values of thermo-electric force. Steel wires lying very near pure silver in the thermo-electric scale are therefore apt to be seriously influenced. Conformably herewith, the column of "differences" in the foregoing table (34) shows large values when y' is approximately zero. But there is another reason for these relatively large discrepancies, which deserves mention as being even of more importance than the one just referred to. The thermo-electromotive force is expressed by the aid of two constants, thus—

$$e=a\,(T-t)+b\,(T-t^2)$$

whence it follows that the accuracy of the constant a, resulting from the calculation, is conditioned to some extent by the accuracy of b; for, although the latter is usually small relatively to the former, it enters the equation

$$e=[a+b\,(T+t)]\,(T-t)$$

as a co-efficient of the sum of the temperatures T and t. Now, an accurate measurement of b presupposes a large range of variation of the difference of temperatures T and t; and it is just here that in the case of glass-hard rods we encounter an unavoidable difficulty. Large values of T are inadmissible, because they change the temper of the exposed junction of the rod. It is equally inexpedient to cool t below zero. At least our attempts in this direction served only to convince us that the errors encountered in consequence of insufficient constancy of t more than counterbalance the advantages gained, to say nothing of annoyances of a practical kind. An inspection of some of the earlier tables will show that wherever the value of a is larger or smaller than was with good reason to be anticipated, this discrepancy is compensated by a similar error of b. Indeed the calculations from which the constants of the 90 series of experiments given above were derived, clearly show that a number of observations greater than four for each value of a is indispensable for an advantageous application of the method of least squares. This did not urgently appeal to us until the measurements were well in progress, and we were equally undesirous both of changing the routine adopted and of encountering additional labor in the calculations. Four observations, however, will not suffice where an accuracy of a within one per cent. is demanded. We also observed that

[34] Cf. pp. 34-35.

the correlative values of a and b were materially different, according as the equation

$$y=ax+bxu$$

or

$$\frac{y}{x}=a+bu$$

was assumed for the application of the method of least squares.

Resistance measurement.—The values for specific resistance are liable to two sources of error. The first of these, which is due to variations of temperature, the results contained in Chapter 1 have enabled us largely to eliminate. The second error is of a more serious kind, and embraces the difficulties encountered in endeavoring to obtain values for the sections of (thin) rods. As the average thickness of our wires was only about 0.05 centimeter, it would have been necessary to measure the mean diameter to $\frac{1}{4000}$ centimeter in order to arrive at a mean section correct to within one per cent. This is not feasible, except with extreme precautions, either by the use of microscopic or gravimetric methods. In the case of thinner wires the difficulty is proportionately increased. Such errors as are due to imperfections of contact at the places of insertion of steel wires, etc., have been already touched upon.[35] They were wholly avoided eventually by a change of method.

Other errors.—In addition to the sources of errors just mentioned, such as are more intrinsically connected with the subject discussed, deserve consideration. It has been stated that the inference is warranted that the rods used were not quite identical as regards chemical composition, and that the electrical behavior of these, though alike in kind, may differ in degree.

CONCLUDING REMARKS.

A brief review of the general tenor of the results discussed in the above, in so far as they have a bearing on certain important consequences which we desire now to touch upon, will be in place here. We purposely avoid all remote theoretical speculations and confine ourselves to inferences immediately deducible from the facts.

Tempering.—The physics of the process of tempering have thus far been but slightly understood; and it is therefore to the new light which the present chapter throws upon the essential features and results of this operation that we desire first to advert. Only cursory attention[36] has hitherto been given to the phenomenal difference between

[35] Cf. pp. 37–38.

[36] In certain thermo-electric measurements incidentally made by Joule (Phil. Trans., 1859, I, pp. 95–97). The results for variation of specific resistance of all earlier observers (Mousson, Chwolson) are erroneously small.

the extremes of mechanical state: the glass hard and the soft. The largest thermo-electric interval observed in the present work is 10.17—(−2.60)=12.8; and the maximum ratio of the respective values of specific resistance, $\frac{45}{15}=3.0$. It is this enormous range of variation of the electrical properties, together with the facility with which *absolute* data dependent merely on qualities essentially inherent in steel are obtainable, that would seem emphatically to recommend the scale of hardness here introduced to the attention of physicists. Furthermore, the method of annealing adopted enables the observer to carry the temper from any given initial state to any desirable final state within the range of possible variation in a safe and systematic way. Of the two factors which condition the degree of hardness reached—the temperature of the annealing bath and the time of exposure of the hard steel rod—the latter if sufficiently prolonged is particularly effective for very small values of the former. If annealing be continued through infinite time, the ultimate states (measured as thermo-electric hardness), would appear to decrease continuously as the temperature increases, though there is reason to predict the existence of a minimum between 400° and 600°. We had commenced a special investigation of this functionality, but the work was forcibly interrupted. It is still to be noted that the maximum annealing effect of a temperature t' is independent of the possibly pre-existing effects of a temperature t, and is not in any way influenced by subsequent applications of the latter, provided $t'>t$.

Scheme for magnetic and other allied researches.—The results above detailed open up a large field for further research in directions differing from those of the present chapter. Indeed, wherever we have to deal with those properties which depend essentially on the hardness of steel, the experiments discussed in the above formally prescribe, as it were, a method of procedure at once expedient and safe.

From a physical point of view, steel possesses a special and peculiar interest in virtue of its pronounced magnetic properties. The importance of the influence of hardness on the intensity of permanent magnetization has long been known. But the confusion and discordance in the results of all observers bear witness to the difficulties encountered, not only from the vagueness which attaches to the term steel, but principally out of the want of a method by which identical mechanical states can be rigidly defined. As a consequence the data of many laborious researches do not readily admit of direct comparison, and are therefore nearly valueless. Indeed, we may add that even in the case of one and the same type of steel each rod must be regarded as an individual.

It follows, therefore, that a plan of research in which a given steel rod is carried, by annealing, from an initial hard to a final soft state, through all intermediate states, is alone adapted to the class of experiments under discussion. And here again the peculiar mechanical condition

which we have above characterized as the limit of hardness for a given temperature of the annealing bath will appear saliently important. The reasons for this preference are obvious. We have in hand the highest of the inferior states of hardness permanently unaffected by the given circumstances of exposure; the maximum of *permanent* hardness, in other words. We are justified in ascribing to it greater uniformity of temper and greater definiteness of internal structure than is possessed by steel tempered in any other way whatsoever. Thus it will be possible, independently of the degree of carburation of steel, or of its chemical composition in general, independently, moreover, of the geometrical figure of a given sample, to investigate the magnetic variation due to the the change of a single variable hardness alone. With this as a point of departure, the inquiry leads to the effect of form or dimensions, and ultimately to that of carburation.

ADDENDUM: ON A SIMPLE METHOD FOR THE GALVANIC CALIBRATION OF A WIRE.

The methods in vogue for the calibration of an electrical measuring wire, one, for instance, to be used in the Wheatstone-Kirchhoff bridge, are without doubt inconvenient in so far as they presuppose a set of resistance coils of known simple ratios. In other words, they call for the preliminary work of comparing with great care the said auxiliary coils. The accuracy of the result immediately sought is thus dependent on the accuracy of the earlier measurement, and the anticipative caliber errors of the wire are obscured by the errors possible in the accepted ratios of the coils themselves—particularly so where, as is generally the case, the discrepancies primarily in question are small. In view of the importance of the very accurate and convenient method of determining resistances by means of the sliding-contact bridge, the endeavor to overcome the obstacles mentioned as completely as possible is therefore at once justifiable. A method moreover which can accomplish this will be all the more acceptable in proportion as the auxiliary devices to be adopted are simple and readily available. We believe the following plan partakes of these advantages.

The method now to be described, and which we have profitably employed in determining the sectional errors of an electrical measuring wire, is a complete analogon of the procedure usually employed for the calibration of thermometers. The immediate principles and the point of departure are the same as those utilized in the well-known Matthiessen-Hockin method of resistance measurement. In the annexed figure, in which the ordinary bridge adjustment is diagrammatically given, let $A\,N\,B$ and $A\,M\,B$ be the two branches terminating at A and B, where

$A\,M\,B$ is the wire to be calibrated, $A\,N\,B$ a series of resistances, whose sum in any given case remains constant.

Now, if M_1 and N_1, M_2 and N_2 are, respectively, pairs of points of like

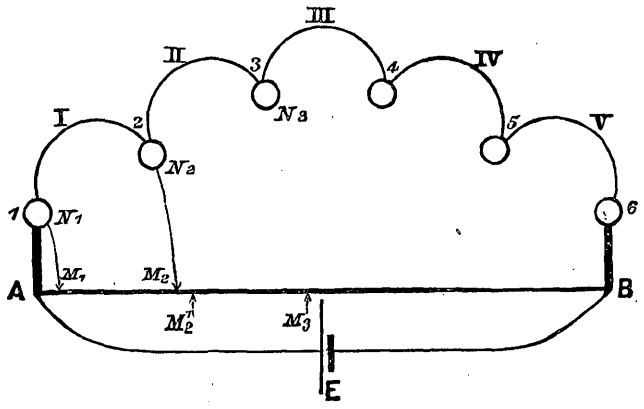


FIG. 12.—Disposition of apparatus for calibration.

potential, and if the resistance of $N_1\,N_2$ be Y, and the length of the part $M_1\,M_2$ be x, then

$$Y = C\,x,$$

where C is a constant (sensitiveness) dependent only on the sum of the resistances $A\,N\,B$. Indeed, if W be this sum, and L the total length of wire $A\,M\,B$, then

$$W = C\,L.$$

Furthermore, let the interval of calibration a be so chosen that $\frac{L}{a} = n$, where n is an integer.

Suppose, now, that n approximately equal resistances are either specially constructed or at the observer's disposal. In our case a number of tenths of Siemens mercury units, which had been made for the purpose of serving as normals for resistance comparisons, were available. But in general it is wholly sufficient to select a german silver wire of appropriate length and thickness, to cut it into n approximately equal parts, and then to solder the ends of each of these to terminals of thick copper, the free ends of which, in their turn, are to be amalgamated so that mercury contacts may be used at N_1, $N_2 \ldots$, $N_6 \ldots$ throughout. Further knowledge of the resistance ratio of these auxiliary wires is quite unnecessary.

These n approximately equal resistances I, II, III . . . are joined in series by mercury cups, as shown in the diagram. To have a concrete case, let $n=5$, for which assumption the figure applies. The extreme points A and B of the measuring wire are connected by pieces of thick copper with the extreme mercury cups. The calibration is then effected as follows:

One of the prolonged terminals of a sensitive galvanoscope is con-

nected consecutively with the points N_1, N_2; and by means of the other terminal the corresponding positions of the sliding contact on the wire $A\,M\,B$, viz, M_1, M_2, respectively, determined.

Then *I* and *II* are exchanged, so that the resistance wire *I* now occupies the former position of *II*, and *vice versa;* and while one of the terminals of the galvanoscope is connected consecutively with the mercury cups N_2 and N_3, the other enables us to determine the corresponding positions of the sliding contact on the wire $A\,M\,B$, viz, M_2' and M_3, respectively.

After this, *I* and *III* are exchanged. The contacts of one galvanoscope terminal at N_3 and N_4 fix the positions of the sliding contact at M_3' and M_4, respectively. And so on until *I* has passed consecutively from its original first position to the final portion among the whole series of resistances $A\,N\,B$. In this way we obtain on the wire $A\,M\,B$ the nearly consecutive parts $M_1\,M_2$, $M_2'\,M_3$, $M_3'\,M_4$ ... of like resistance.

Herewith it is obvious that this method of galvanic calibration is strikingly analogous to the method adopted in thermometer calibration. In the latter case a mercury thread of a given constant convenient volume and the approximate length $a=\frac{100}{n}$ is moved from place to place in the tube, and its length for each of these positions determined in terms of the thermometer scale—lengths which under assumption of a constancy of caliber would be proportional to the volume of the thread, and hence identical. In the former electrical measurement, a wire of constant resistance is moved from place to place, and its value, as it were, determined in terms of lengths of the measuring wire, which again under conditions of constant sections would be proportional to the said moving resistance, and therefore identical. In both cases, finally, the sectional error of the tube or wire is shown by the discrepancies or differences in the observed lengths of mercury thread or wire-part, respectively. In thermometry, the two fixed points, with reference to which the calibration is completed, are the freezing and the boiling points of water. On the bridge the corresponding points coincide with the extremities *A* and *B* of the wire.

Let a_1, a_2, a_3, ... be the equivalent wire parts of $A\,M\,B$, determined by direct observation. Then will the mean length of these, which under assumption of a mean section of the calibrated wire corresponds to the calibration resistance, be

$$\frac{a_1+a_2+a_3+\ .\ .\ .\ .\ a_n}{n}$$

But as a_1, a_2, a_3 differ but slightly from the calibration interval $a=\frac{L}{n}$, it is practically convenient not to introduce into the calculation the

whole lengths $a_1 \ . \ . \ . \ a_n$, but merely their (positive or negative) differences from a. Therefore let

$$a_1 = a + \delta_1; \ a_2 = a + \delta_2 \ . \ . \ . \ a_n = a + \delta_n$$

and similarly

$$\frac{a_1 + a_2 + a_3 + \ . \ . \ . \ a_n}{n} = a + \alpha.$$

Then, more simply, there follows

$$\alpha = \frac{\delta_1 + \delta_2 + \delta_3 + \ . \ . \ . \ \delta_n}{n}$$

and therefore for the table of calibration of the wire,

from 0 to a	$\alpha - \delta_1$
from a to $2a$	$\alpha - \delta_2$
from $2a$ to $3a$	$\alpha - \delta_3$, etc.

Therefore by summation:

At a, correction $= \alpha - \delta_1$
At $2a$, correction $= 2\alpha - \delta_1 - \delta_2$
At $3a$, correction $= 3\alpha - \delta_1 - \delta_2 - \delta_3$, etc.

As the main feature of the present method, we desire again to advert to the simple means by which the calibration is effected. The only error to be guarded against is the possibility of variation of temperature during the course of the consecutive adjustments of the calibration resistance. But the work can be accomplished rapidly—all the more as the approximate positions of M are readily predeterminable with considerable accuracy. If a fairly sensitive galvanoscope be employed the errors of adjustment are of the same order as the errors of observation. In case of our special bridge $2\frac{1}{2}$ meters long, very ordinary means enabled us to secure an accuracy of adjustment within 0.1 millimeter.

By means of the principle here enunciated any of the other methods of thermometer-calibration would be equally applicable. The convenient one given, however, is sufficient for all practical purposes.

CHAPTER III.

THE NATURE OF THE PHENOMENON OF TEMPER AS OBSERVED IN STEEL DISCUSSED FROM AN ELECTRICAL STANDPOINT—PARTICULARLY IN REFERENCE TO THE ANALOGOUS BEHAVIOR OF MALLEABLE CAST-IRON AND OF ALLOYS OF SILVER.

INTRODUCTION.

General inferences deducible from the cast-irons.—If we abstract from grossly impure material, details and ulterior complications, the different kinds of cast-iron may be conveniently classed with reference to two well-known types—the gray cast-iron and the white cast-iron—which are thoroughly distinct and well marked, as regards both their chemical and their mechanical properties. Reasons for this classification, it is believed, are not far to seek, and they are such as depend on the manner in which the carbon contained is combined with the iron. White cast-iron is known to contain carbon in the chemically combined state only. In gray cast-iron, however, the uncombined or mechanically admixed forms of carbon are present in much the greater proportion. Furthermore, it is known that a given specimen of molten iron may be changed into either the gray or the white type, respectively, according as the cooling, after casting, is permitted to take place very gradually or very rapidly.[37] In this respect an experiment of Karsten's[38] is peculiarly instructive. Karsten poured molten iron into a thick iron mold. The outer layers of the cold mass, which in this way had cooled at a comparatively rapid rate, were found to contain combined carbon only. In the inner and axial layers, however, where cooling took place gradually, only about one-sixth of the total carbon occurred in the combined form.[39] Suppose we compare the process of fast and slow cooling in the case of cast iron, with sudden chilling and very gradual cooling in case of steel, respectively; suppose, furthermore, we contrast the very high degree of hardness possessed both by white cast iron and chilled steel with the readiness with which gray cast-iron and slowly

[37] The remarks here made present the facts under consideration only in a brief and introductory way. For full and exhaustive discussions the reader is referred to Percy-Wedding, Eisenhüttenkunde, Bd. II of the German Percy, pp. 130–186, Braunschweig, 1864. Cf. particularly critical remarks, pp. 165–167.

[38] Karsten: Eisenhüttenkunde, Bd. I, 3 Aufl., 1841, p. 581 ff.

[39] The results of Karsten's analysis are these:

Before melting (iron homogenous): C combined, 0.8 per cent.; C uncombined, 3.2 per cent.

After melting (outer layers): C combined, 5.1 per cent.; C uncombined, 0.0 per cent. Inner layers: C combined, 0.6 per cent.; C uncombined, 3.2 per cent.

cooled or thoroughly annealed steel yield even to an ordinary file; then the inference is apparently unavoidable that the hardness which may be imparted to steel by a process of tempering is to be referred to the transfer of carbon from the uncombined to the combined state.[40]

Data corroborating this analogy.—If, however, what may be called the chemical method of accounting for the hardness in steel were alone dependent on analogies of the kind just sketched, we would not be inclined to accept it immediately and with full confidence. In glass-hard steel we encounter strains of a peculiar character and of enormous intensity. It is upon such observations that a purely physical explanation of the hardness in question might be based—an explanation much better adapted for the prediction of certain physical phenomena to be mentioned below than the chemical view under consideration. At the very outset we come upon difficulties. Suppose, for instance, we attempt to draw further inferences from Karsten's experiment. We find that the glass-hard external layers of his cylinder, consisting wholly of white cast iron, possess the specific gravity 7.55, whereas the specific gravity of the gray nucleus was only 7.18. The effect of sudden chilling in case of steel,[41] however, is a very marked diminution of density of the rod, as a whole. Even if we pursue our inference further, and endeavor to compare the external and internal layers of a glass-hard steel rod with the corresponding parts of Karsten's chilled iron cylinder, respectively, we fail to arrive at satisfactory results; for the density of the external layer of a steel bar (about 10) is incommensurately large when compared with the above datum for white cast-iron. The chemical explanation, however, at once becomes of incontestible importance when it is found that in addition to the given analogies we have in hand a number of data which go to prove that carbon exists differently, as regards its mode of occurrence, in hard and in soft steel. The data are derived from the chemical behavior of soft and hard steel toward acids. In this place the extensive experiments of Caron[42] need alone be mentioned. The results of this observer with steel in the commercial, the hammered, and the glass-hard states, respectively, furnish striking evidence of the fact that the mode of occurrence of carbon in

[40] Cf. Karsten: Karsten u. v. Dechen's Archiv., XXV, p. 223, 1853. Cf., also, p. 103 of the present paper.

[41] Karsten: op. cit., Bd. I, p. 193–4, experiments of Hausmann and Karsten. Cf. also ibid., p. 184, ff. For a table for Swedish Bessemer steel, see "Steel, its history, etc.," J. S. Jeans, Sec. Br. Iron and Steel Inst., London, Spon, 1880; Cf. ibid., p. 612–615. On the mean specific gravity of iron-carburets, cf. Karsten, op. cit., p. 183.

[42] Caron: Comptes rend., LVI, p. 43, 1863. The residues in the cases referred to, viz., 1.62 per cent., 1.24 per cent., 0.24 per cent., were found to contain 0.83 per cent., 0.56 per cent., 0.00 per cent. of carbon, respectively. The following remark of Caron's moreover, is of especial importance here: "Ainsi de l'acier trempé ayant été recuit pendant un temps variant entre quelques heures et plusiers jours a donné après dissolution des quantités de carbon libre qui ont augmenté en même temps que la durée et l'intensité des chauffes." pp. 45–46.

soft steel is thoroughly different from the chemical condition of this element in hard steel. In so far, then, as it is necessary to associate with this difference of chemical state of the carbon in steel some corresponding difference in the mechanical properties of this material, the validity of the chemical explanation of tempering may be said to have received its first important vindication.

Results at variance with chemical hypotheses.—Of late many facts have been adduced by various observers and in different ways against the hypothesis that the union of carbon and iron is ever of a thoroughly chemical character. Singularly clear in this respect are the views of Matthiessen,[43] who supposes that iron-carburets are to be regarded either as solidified solutions of carbon in iron, or as solutions containing carbon mechanically admixed. To these new opinions Matthiessen is led by a study of the electrical conductivity of these materials. With this general hypothesis the results of the calorimetric researches of Troost and Hautefeuille[44] are in strict accordance. These observers find that not only carbon but silicon show such thermo-chemical relations toward iron as call for a classification of iron-carburets with the category of solutions. Forquignon,[45] finally, who has lately been occupied with similar researches, gives to this view his unqualified assent.

Ulterior consequences of the chemical theory in case of steel.—In connection with our researches on the hardness of steel a careful consideration of the subject in hand appeared necessary. Our attention was not, however, attracted to the questions just developed, viz., whether a chemical or mechanical explanation for the mode of occurrence of carbon in steel is more in coincidence with facts. The subject-matter of importance to us is sketched in all its essential points in the earlier sections. It is our endeavor to investigate whether a given change in the state of hardness of a steel rod is to be ascribed to a corresponding change in the mode of occurrence of the carbon in this material, or whether the presence of carbon imparts to iron certain distinct and unique properties ("steel") in such a way that the whole series of the phenomena observed in tempered steel are to be regarded as purely mechanical in their character. It may be observed that the first manner of explanation does not necessarily conflict with the second. Both chemical and mechanical phenomena may coexist. But it is best in this place to hold the two hypotheses strictly apart. This is readily possible, because the different states of hardness obtainable from a glass-hard steel rod by annealing are all reached, practically, when the temperature of the annealing bath is still below 350°, a temperature insufficiently high for the conversion of uncombined into combined carbon.

[43] Matthiessen: Rep. Br. Assoc. Adv. Sc., 1866, p. 15.

[44] Troost and Hautefeuille: Ann. de ch. et de phys. (5), IX, 1876, p. 70.

[45] Forquignon: Ann. de ch. et de phys. (5), XXIII, 1881, pp. 531–536.

Suppose now that we abstract from physical considerations altogether for the time being, and endeavor to obtain from the older and more thoroughly investigated chemical hypothesis an explanation for the details of the phenomena of annealing discussed in an earlier chapter.[46] If sudden chilling is accompanied by chemical combination of the carbon in steel, then must the operation of annealing, which reverses the effects of the former, enable us to reconvert combined carbon into its original uncombined state.[47] But we have shown that for each temperature of exposure during the annealing of a glass-hard rod there exists a distinct degree of hardness, *cæteris paribus*, characteristic of the temperature in question alone. It follows, therefore, that to each temperature of the annealing bath there must correspond a certain fixed ratio of combined to uncombined carbon, the time of exposure being indefinite. From the known behavior of steel in different states of temper, moreover, combined carbon, must be looked upon as electrically active, uncombined carbon as electrically neutral. Thus we have it in our power, with the aid of the simple process of sudden chilling combined with subsequent annealing, in so far as in this way, within certain limits, any given amount of an electrically active ingredient may be converted into an electrically passive form, to reach the same results which, in the case of alloys, for instance, are obtainable only by melting the two component metals together in a ratio which may be called for. We said "within certain limits"; the superior limit here meant, is the total amount of carbon present in the given type of (normal) steel; the inferior limit, the small amount of combined carbon which even after most prolonged and gradual cooling cannot be made to appear in the uncombined state. With this chemical interpretation, finally, the remarkable linear relation between the thermo-electric constant and the specific resistance of steel passing continuously from the glass-hard to the soft state would at once appear to possess broader significance than that of being a special peculiarity of steel.

These are the reflections which induced us to undertake the present investigation. We were in hopes, moreover, that the results would prove of such a character as to enable us to derive inferences from them on the chemical nature of steel. At all events, the subject, considered from the standpoint of its own merits, is not undeserving of detailed attention. The number of data in hand on the relation in question is almost insignificant.

[46] Strouhal u. Barus: Wied. Ann., XI, p. 962, 963, 1880. Chapter II, pp. 43 et seq.

[47] Caron: l. c., pp. 45, 46. Unfortunately we have not been able to find any data relative to the effect of ordinary annealing (exposures below 400°) on the mode of occurrence of carbon in steel.

EXPERIMENTS WITH ALLOYS.

Earlier results inadequate.—The literature on the electrical conductivity of alloys is very voluminous, but the exhaustive researches of Matthiessen[48] contain all the essential data. At least, little has been developed that has a bearing on the present paper since the date of Matthiessen's main research.[49] From the tables and graphical constructions there given, the conductivity of a large number of alloys can be at once deduced when the proportion in which the ingredients of the alloy are mixed is known. Similar remarks by no means, however, apply to our knowledge of the thermo-electric properties of alloys. Qualitative results are not lacking. Available quantitative data are, however, very rare.[50] Pairs of values of both the electrical conductivity and the thermo-electric power of alloys are only to be found in isolated examples, if at all. At least we were not able to obtain other data than a few measurements incidentally made by Sundell.[51]

Matthiessen's numerous results made a special determination of data on the relation between percentage composition and either of the electrical constants superfluous. Such measurements presuppose chemical analysis, which could not have been made without unduly sacrificing the greater part of our wires and material. By weighing out the ingredients, and careful fusion, we were able to obtain from Matthiessen's tables a satisfactory corroboration of our results.

Material, fusion, preparation, etc.—The general plan adopted in our researches with alloys was such as would correspond to a progressive increase in the state of hardness of steel from the soft or thoroughly annealed to the glass-hard condition, premising the views discussed in the last section. We commenced with pure silver, to which more and more of a second metal was successively added, until finally mechanical difficulties were encountered such as prevented a drawing of the alloy to wire. In case of silver-gold alloys, a complete series, commencing with silver and ending with gold, were obtainable. The alloys examined are those of silver with gold, platinum, copper, and zinc. Silver and gold were obtained pure from the mint at Frankfort; platinum, from the shops of Heraeus in Hanau. The copper was deposited electrolytically from a copper-sulphate solution. In making the alloys we proceeded thus: A weighed amount of silver having been well fused on a bone-ash cupel by the aid of a blast-lamp, the proper quantity of the second metal was added to it. Solution is usually immediate. Having mixed

[48] Matthiessen: Pogg. Ann., CX, pp. 190–221, 1860.

[49] Matthiessen and Holzmann: Pogg. Ann., CX, pp. 222–234, 1860; Rep. Br. Assoc. Adv. Sc., 1863, p. 37.

[50] We may, perhaps, mention Joule: Phil. Trans., 1859, I, p. 96 (Bi, Sb); Sundell: Pogg. Ann., CXLIX, 1873 (Bi, Sb, Sn).

[51] Sundell: *Ibid.*, pp. 154–170.

the two component metals as thoroughly as possible by chasing the globule over the surface of the cupel, the flame was withdrawn and a glass bell-jar filled with hydrogen (the gas being continually resupplied through an aperture in the neck of the jar), quickly placed over it. Oxydation, as well as absorption of oxygen by the melted globule, was thus avoided, the button cooling in an atmosphere of hydrogen. Only in the case of silver-copper alloys is this simple process to be regarded insufficient for the attainment of very accurate results.[52] But the discrepancies thus introduced into our work, and which are due to absorption of oxygen by copper, are quite insignificant. Of this fact a comparison of Matthiessen and Holzmann's data with our own has fully convinced us. On the other hand, the variation of thermo-electric power and conductivity of alloys of silver and copper takes place within limits so nearly coincident that the general character of the diagram which we will endeavor to construct below is in nowise distorted.

The buttons[53] were hammered and drawn down to wires of appropriate diameter. These were then annealed at high red heat in a current of hydrogen by the aid of a dynamo-electric machine. We were able to regulate the intensity of current, and with it the glow, by introducing into the circuit a solution of copper-sulphate, in which the copper electrodes were kept at suitable distances apart. Accidental fusion of the hot wires was thus not to be apprehended. After this annealing they appeared soft and flexible. Only the middle parts of the wires, over the whole length of which the glow had been uniform, and which, for other reasons, were apt to be homogeneous, were reserved for the measurements.

Method of thermo-electric measurement.—Results.—The methods of measurement of thermo-electric power and specific resistance are identical with those employed in our earlier researches.[54] The former is throughout referred to pure silver as a datum, *i. e.*, denotes the power of a thermo-element, one part of which is always silver; the other, however, the given alloy. Junctions were carefully soldered. The composition of all alloys is given in volume per cents, the data expressing the volume of the metal alloyed to silver in 100 volumes of alloy. It has been stated that these numbers are only approximate, since during fusion the two metals will hardly have been volatilized in like ratios.

With these remarks the following tables (35–39) will be readily intelligible. The thermo-electric constants a and b are calculated on the basis of Avenarius'[55] formula, $e=a\ (T-t)+b\ (T^2-t^2)$, from six corre-

[52] Cf. Matthiessen u. Holzmann: *l. c.*, pp. 222–224.

[53] It would have been desirable to operate with larger quantities of metal, but these were not at our disposal.

[54] See Chapter II, p. 31 *et seq.*

[55] Avenarius: Pogg. Ann., CXLIX, p. 374, 1873. e is the electro-motive force for the temperatures T and t of the junctions of the given thermo-element whose constants are a and b.

sponding values of e, T, t, for each alloy, by the method of least squares. The measurements were originally made in Weber-Siemens' units, and then reduced to current values by the relation ohm=1.06 Siemens.

TABLE 35.—*Thermo-electric power of alloys.*

Alloy.	Vol. P. ct.	t	T	e observed.	e calculated.	Diff.	a	b
		C.	C.	*microvolt.*	*microvolt.*			
Silver-platinum	2	16.5	85.6	−308.6	−308.8	0.2		
		16.6	75.7	−261.3	−261.1	−0.2		
		16.7	64.1	−206.7	−206.7	0.0	−3.95	−0.0051
		16.7	53.6	−159.1	−159.1	0.0		
		16.8	43.1	−111.7	−111.9	0.2		
		16.8	36.3	− 82.4	− 82.3	−0.1		
Do	5	15.8	78.6	−413.0	−412.5	−0.5		
		15.9	69.3	−346.3	−346.4	0.1		
		15.9	60.9	−288.2	−288.5	0.3	−5.72	−0.0090
		16.0	52.3	−230.1	−230.1	0.0		
		16.1	45.8	−186.4	−186.5	0.1		
		16.1	37.5	−132.9	−132.7	−0.2		
Do	10	17.1	90.6	−604.4	−603.4	−1.0		
		17.1	79.1	−498.3	−499.6	1.3		
		17.1	68.3	−405.2	−405.0	−0.2	−6.77	−0.0164
		17.1	55.4	−296.8	−296.4	−0.4		
		17.1	45.2	−213.8	−213.7	−0.1		
		17.2	35.2	−134.4	−134.6	0.2		
Do	16	16.1	90.2	−710.6	−709.4	−1.2		
		16.1	74.0	−539.7	−539.5	−0.2		
		16.2	62.9	−425.6	−427.1	1.5	−7.90	−0.0158
		16.2	52.4	−324.9	−324.9	0.0		
		16.3	43.4	−239.4	−239.5	0.1		
		16.3	36.2	−174.0	−173.6	−0.4		

TABLE 36.—*Thermo-electric power of alloys.*

Alloy.	Vol. P. ct.	t	T	e observed.	e calculated.	Diff.	a	b
		C.	C.	*microvolt.*	*microvolt.*			
Silver-gold	5	15.5	85.1	−132.9	−132.9	0.0		
		15.5	75.5	−114.1	−113.7	−0.4		
		15.4	68.3	− 99.1	− 99.8	0.7	−1.77	−0.0014
		15.4	62.0	− 87.8	− 87.5	−0.3		
		15.4	50.6	− 65.5	− 65.5	−0.0		
		15.4	40.0	− 45.4	− 45.4	−0.0		
Do	15	15.4	89.5	−209.2	−209.5	0.3		
		15.5	80.6	−181.5	−181.6	0.1		
		15.5	71.5	−154.1	−154.0	−0.1	−2.38	−0.0043
		15.5	63.2	−129.7	−129.3	−0.4		
		15.6	52.8	− 99.3	− 99.3	0.0		
		15.6	44.1	− 74.8	− 75.0	0.2		
Do	25	15.7	90.2	−227.0	−227.0	0.0		
		15.7	82.6	−201.1	−201.3	0.2		
		15.8	66.3	−148.3	−148.0	−0.3	−2.54	−0.0048
		15.8	54.3	−110.8	−110.7	−0.1		
		15.8	44.9	− 82.5	− 82.3	−0.2		
		15.8	35.9	− 55.9	− 56.1	0.2		
Do	35	13.9	88.3	−223.6	−223.7	0.1		
		13.9	79.7	−195.8	−195.6	−0.2		
		13.8	72.3	−172.3	−172.1	−0.2	−2.61	−0.0039
		13.8	63.4	−144.2	−144.2	0.0		
		13.8	51.9	−109.2	−109.2	0.0		
		13.8	42.4	− 80.9	− 80.9	0.0		

TABLE 37.—*Thermo-electric power of alloys.*

Alloy.	Vol. P. ct.	t	T	e observed.	e calculated.	Diff.	a	b
		C.	C.	*microvolt.*	*microvolt.*			
Silver-gold	50	15.0	87.1	−202.3	−202.6	0.3		
		15.0	78.2	−175.4	−175.2	−0.2		
		14.7	65.6	−138.7	−138.5	−0.2	−2.39	−0.0041
		14.7	55.3	−108.9	−108.8	−0.1		
		14.7	44.4	− 78.2	− 78.3	0.1		
		14.6	36.2	− 56.1	− 56.1	0.0		
Do	75	14.1	86.1	−151.5	−154.4	+2.9		
		14.1	77.5	−135.1	−133.5	−1.6		
		14.1	68.0	−111.9	−111.4	−0.5	−1.71	−0.0043
		14.1	53.3	− 79.3	− 78.5	−0.8		
		14.1	42.1	− 55.8	− 54.7	−1.1		
		14.0	30.5	− 30.7	− 31.5	0.8		
Do	90	17.3	91.0	−116.0	−117.7	1.7		
		17.3	80.2	−100.8	− 99.2	−1.6		
		17.3	68.1	− 79.3	− 79.3	0.0	−1.41	−0.0017
		17.3	57.0	− 61.2	− 61.0	−0.2		
		17.4	45.7	− 43.3	− 43.1	−0.2		
		17.4	34.6	− 25.6	− 25.8	0.2		
Gold	100	15.3	89.1	19.3	19.3	0.0		
		15.4	80.7	17.0	17.2	−0.2		
		15.5	70.9	14.8	14.7	0.1	+0.275	−0.00012
		15.6	60.5	12.2	12.0	0.2		
		15.6	51.4	9.6	9.5	0.1		
		15.7	40.3	6.5	6.6	−0.1		

TABLE 38.—*Thermo-electric power of alloys.*

Alloy.	Vol. P. ct.	t	T	e observed.	e calculated.	Diff.	a	b
		C.	C.	*microvolt.*	*microvolt.*			
Silver-copper	2	14.6	83.7	0.3	−0.6	0.9		
		14.6	77.7	−1.2	−0.6	−0.6		
		14.6	68.2	−0.7	−0.6	−0.1	−0.023	+0.0001
		14.7	57.7	−1.2	−0.6	−0.6		
		14.7	48.0	−0.4	−0.5	0.1		
		14.7	39.8	−0.2	−0.4	0.2		
Do	5,6	13.7	89.5	1.6	1.0	0.6		
		13.7	80.4	1.2	0.9	0.3		
		13.8	72.8	0.6	0.9	−0.3	+0.021	−0.0001
		13.9	62.8	−0.2	0.8	−1.0		
		13.9	52.2	0.5	0.6	−0.1		
		13.9	39.2	0.8	0.4	0.4		
Do	15	14.3	84.8	4.0	3.6	0.4		
		14.3	75.5	2.6	3.0	−0.4		
		14.3	65.3	2.5	2.4	0.1	+0.041	+0.0001
		14.4	56.5	2.0	2.0	0.0		
		14.4	46.3	1.4	1.5	−0.1		
		14.4	35.5	1.0	0.9	0.1		
Do	50	14.3	83.3	11.6	11.8	−0.2		
		14.3	72.9	9.0	9.1	−0.1		
		14.4	63.4	7.1	7.1	0.0	+0.045	+0.0013
		14.4	53.8	5.5	5.2	0.3		
		14.5	43.9	3.7	3.5	0.2		
		14.5	33.5	1.9	2.0	−0.1		
Do	75	16.2	86.9	8.7	9.2	−0.5		
		16.3	74.0	6.8	6.7	0.1		
		16.3	64.8	5.2	5.2	0.0	+0.020	+0.0011
		16.3	52.4	4.1	3.4	0.7		
		16.3	41.7	2.0	2.1	−0.1		
		16.3	33.0	1.0	1.2	−0.2		

TABLE 39.—*Thermo-electric power of alloys.*

Alloy.	Vol. P. ct.	t	T	e observed.	e calculated.	Diff.	a	b
		C.	C.	*microvolt.*	*microvolt.*			
Silver-zinc, with larger amount of zinc.	.....	15.5	87.0	− 8.8	− 8.6	−0.2		
		15.5	80.8	− 8.0	− 7.4	−0.6		
		15.5	71.5	− 4.9	− 6.0	+1.1	−0.024	−0.0009
		15.5	60.5	− 4.3	− 4.3	0.0		
		15.8	49.1	− 3.2	− 2.8	−0.4		
		15.8	35.3	− 1.3	− 1.4	+0.1		
Silver-zinc, with smaller amount of zinc.	.....	15.7	90.6	−12.8	−12.2	−0.6		
		15.7	81.0	−11.1	−10.7	−0.4		
		15.8	72.2	− 8.2	− 9.4	−1.2	−0.176	+0.0001
		15.8	58.3	− 7.1	− 7.1	0.0		
		15.8	48.0	− 5.7	− 5.4	−0.3		
		15.9	40.7	− 4.1	− 4.2	0.1		

Specific resistance.—The following table contains our data for the specific resistance of the same alloys to which the foregoing tables (35–39) refer. The first two columns need no explanation. Column third contains the resistance in ohms per meter of wire at the temperature in column fourth. Under 2ρ the diameter and under s_t the specific resistance in micro-ohms at the temperature t is given. The constant α, finally, whose signification is apparent from $s_t = s\,(1+\alpha t)$, enables us to calculate s, the specific resistance at zero centigrade, in micro-ohms.

It is to be remarked that all the measurements were originally made in Siemens' units (ohm = 1.06 Siemens), and subsequently reduced. The temperature-coefficients used, α, are obtained from the results of Matthiessen, C. Vogt, and v. Bose,[56] by calculating with the aid of their formulæ, s_t for 100° and 0°, and then finding the mean coefficient (α) between these limits. This is obviously permissible in our case.

TABLE 40.—*Electrical resistance of alloys.*

Alloy (soft).	Vol. P. ct.	W 1 m	t	2ρ	$s_t \frac{\text{cm}}{\text{cm}^2} t°$	a	$s \frac{\text{cm}}{\text{cm}^2} 0°$
		ohm.	C.	*cm.*	*microhm*		*microhm*
Commercial silver	100	0.2023	15.2	0.0319	1.619	0.00398	1.521
Electrolytic silver	100	0.1777	15.4	0.0337	1.589	0.00398	1.492
Do	100	0.1802	15.0	0.0335	1.586	0.00398	1.491
Silver-platinum	2	0.2431	11.3	0.0495	4.67	0.00130	4.60
Do	5	1.0802	15.2	0.0330	9.21	0.00072	9.11
Do	10	0.8054	11.0	0.0496	15.53	0.00043	15.46
Do	15	2.4027	15.2	0.0347	22.74	0.00033	22.63
Silver-gold	5	0.3887	15.5	0.0337	3.463	0.0025	3.329
Do	15	0.7483	15.6	0.0337	6.686	0.0012	6.561
Do	25	0.9886	15.7	0.0339	8.906	0.0008	8.790
Do	35	1.1350	15.1	0.0338	10.188	0.0007	10.079
Do	50	1.2098	15.4	0.0337	10.700	0.0007	10.585
Do	50	1.2190	15.2	0.0337	10.782	0.0007	10.667
Do	75	0.9177	16.2	0.0339	8.295	0.0009	8.174
Do	90	0.2772	11.2	0.0493	5.283	0.0016	5.188
Gold	100	0.1137	18.0	0.0495	2.193	0.00395	2.037
Silver-copper	2	0.2049	15.0	0.0338	1.844	0.0036	1.744
Do	5,6	0.1036	11.0	0.0496	1.999	0.0034	1.924
Do	15	0.2292	16.8	0.0344	2.131	0.0032	2.016
Do	50	0.2370	16.2	0.0358	2.383	0.0029	2.271
Do	75	0.2314	16.7	0.0352	2.255	0.0031	2.138
Silver-zinc	.....	0.1733	17.4	0.0500	3.403	(0.0020)	(3.28)
Do	.....	0.2114	15.6	0.0500	4.151	(0.0015)	(4.06)

[56] Matthiessen u. v. Bose: Pogg. Ann., CIII, p. 412, 1858.

The following table (41) contains results of Matthiessen and others; a and b here are Matthiessen's constants for his relation between electrical conductivity and temperature. By means of these λ and λ', the electrical conductivity at 0° and 100°, respectively, were calculated. But, according to Matthiessen, $\lambda=100$ for hard silver; $\lambda=1.656$ for mercury, (0°). Hence

$$s_t=\frac{100}{\lambda_t}\,\frac{1.656}{1.06} \qquad \log s_t=2.19375-\log \lambda_t.$$

From these values of s_t the constant α was then obtained, in the way already given, $s_t=s\,(1+\alpha t)$.

TABLE 41.—*Specific resistance of alloys of silver.*

[Results of Matthiessen (and C. Vogt).]

Metal.	Hard or soft.	Vol. Per ct.	λ at 0° Ag (h) =100	a	b	λ' at 100° Ag (h)=100	$s\frac{cm}{cm^2}0°$	$s'\frac{cm}{cm^2}100°$	$\frac{a}{0°....100°}$
							microhm.	*microhm.*	
Silver	h	100	100	0.38287	0.0009848	71.561	1.5623	2.1831	0.003974
Do	s	100	108.74	0.41570	0.0010624	77.794	1.4367	2.0082	0.003978
Silver-platinum	h	2.51	31.640	0.039363	0.00003642	28.068	4.938	5.566	0.001272
Do	h	5.05	18.031	0.013949	0.00001182	16.754	8.664	9.325	0.000763
Do	h	19.65	6.696	0.002214	0.00000139	6.489	23.331	24.075	0.000319
Silver-gold	h	19.86	21.684	0.019185	0.00001152	19.881	7.205	7.858	0.000906
Do	h	52.08	15.030	0.010120	0.00000370	14.055	10.394	11.115	0.000694
Do	h	79.86	21.335	0.023212	0.00001694	19.183	7.322	8.144	0.001123
Do	s	19.86	21.746	0.019753	0.00001395	19.910	7.184	7.847	0.000923
Do	s	52.08	15.080	0.010864	0.00000746	14.068	10.360	11.105	0.000719
Do	s	79.86	21.584	0.024539	0.00002506	19.381	7.238	8.061	0.001137
Gold	h	100	77.964	0.28648	0.0006582	55.898	2.0038	2.7948	0.003947
Do	s	100	79.327	0.29149	0.0006697	56.875	1.9694	2.7468	0.003947
Silver-copper	h	1.53	79.708	0.32868	0.0006965	53.805	1.960	2.904	0.004816
Do	h	8.25	80.284	0.22101	0.0003503	61.686	1.946	2.533	0.003017
Do	h	46.67	74.940	0.21011	0.0003961	57.890	2.085	2.699	0.002946
Do	h	77.64	69.811	0.21194	0.0004240	52.857	2.238	2.956	0.003208
Do	h	95.17	82.300	0.26758	0.0005717	61.269	1.898	2.550	0.003442
Do	h	98.35	89.544	0.30886	0.0007155	65.813	1.745	2.374	0.003606
Copper	h	100	99.947	0.38681	0.0009004	70.270	1.5631	2.2232	0.004223
Do	s	100	102.213	0.39557	0.0009208	71.864	1.5297	2.1739	0.004211

Digest. Diagram.—For the sake of perspicuity, the final results are tabulated again as follows.

TABLE 42.—*Corresponding values of the galvanic and thermo-electric constants.*

Alloy.	Volume Per cent.	$s\frac{cm}{cm^2}0°$	a $\left(\frac{de}{d(T-t)}\right)_{(T+t)=0}$
		microhm.	
Silver-platinum	0	1.49	0.00
	2	4.60	—3.95
	5	9.11	—5.72
	10	15.46	—6.77
	15	22.63	—7.90
Commercial-platinum	100	*(8.67)	(0.33)
		13.44	
Silver-copper	0	1.49	0.000
	2	1.74	—0.023
	5, 6	1.92	0.021
	15	2.02	0.041
	50	2.27	0.045
	75	2.14	0.020
	100	1.53	(—0.114)

*Applies for chemically pure metals. These values in parentheses are approximate, and are not immediately derived from observations.

TABLE 42.—*Corresponding values of the thermo-electric and galvanic constants*—Cont'd.

Alloy.	Volume. Per cent.	$s\frac{cm}{cm^2}0°$	$a\left(\frac{de}{d(T-t)}\right)_{(T+t)=0}$
		microhm.	
Silver-gold	0	1,49	0.00
	5	3.33	—1.77
	15	6.56	—2.38
	25	8.79	—2.54
	35	10.08	—2.61
	50	10.63	—2.39
	75	8.17	—1.71
	90	5.19	—1.41
	100	2.04	0.27
Silver-zinc	0	1.49	0.000
	<	(3.28)	—0.024
	>	(4.06)	—0.176
	100	5.38	(—0.098)

The results contained in the last table (42) are given graphically in figure 13, specific resistance being represented as abscissa, thermo-elec-

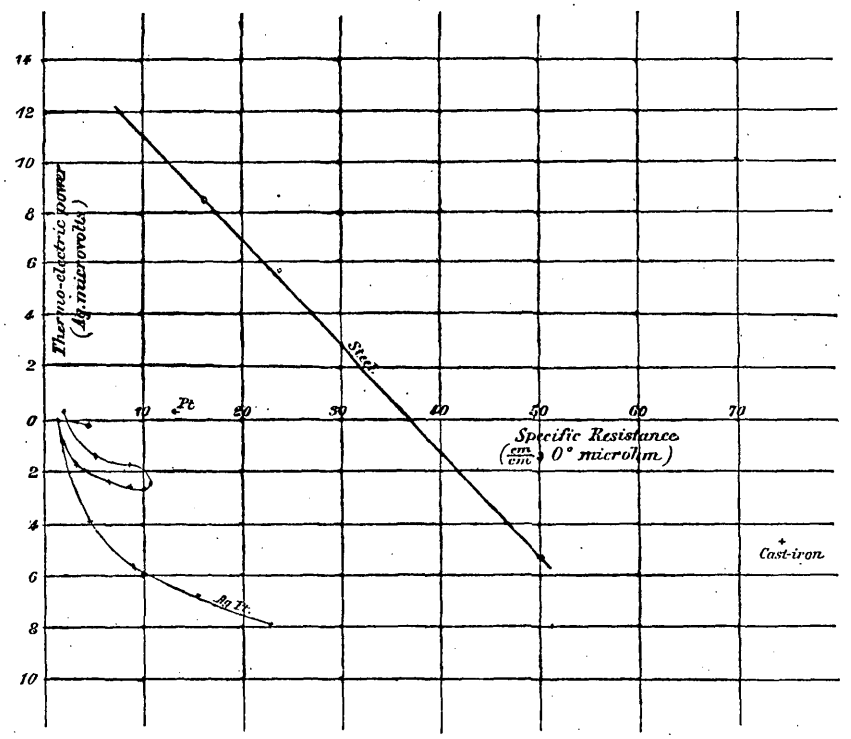


FIG. 13.—Specific resistance and thermo-electric constant: steel, silver-gold, silver-platinum, silver-zinc.

tric constant as ordinate. In order to facilitate comparison the linear relation which holds for steel (see chapter II, page 63) is also drawn,

and the interval of variation[57] of the constants in question indicated by two small circles near the ends of the line. All the constants are given in microhms and micro-volts, respectively, and the thermo-electric position is referred to silver.

Discussion.—A mere glance at the curves tells us that the relations under consideration are by no means of a simple character. Both properties, as exemplified by the thermo-electric constant and specific resistance, pass through maxima or minima, in such a way, however, that the said singular points do not coincide—*i. e.*, are not properties of one and the same wire as regards composition. Equally difficult is it to observe any perspicuous law for the direction of the line joining the extreme points (those corresponding to pure metals) of the curves. The very simple relations which obtain in case of steel would apparently lead to the anticipation of some result of this kind.

The given curves, however, apply only for fixed conditions of temperature; the galvanic constant s only for the temperature $t=0$; the thermo-electric constant a for the mean temperature $\frac{1}{2}(T+t)=0$. But the temperature-coefficient, both for thermo-electric power[58] as well as for specific resistance, varies in a marked degree as we pass from alloy to alloy. Hence it follows that for each temperature there exists a set of curves of the kind in question, characteristic of the said temperature. Change of temperature, therefore, involves a change in the curves both in figure and in position. It follows herefrom that a certain temperature must exist for which the figure of the curves is the simplest possible; our knowledge of the effects of temperature is, however, as yet insufficiently accurate to enable us to adduce further inferences. It is to be noticed that, at least as far as the above experimental matter is concerned, the magnitude of the given curves—*i. e.*, the range of variation of the involved electrical constants—is qualitatively in the order of the differences of the specific volumes of the metallic components of the respective alloys. The very large loop, obtained in the experiments with silver-platinum alloys, corresponds to the marked increase of hardness which these materials experience with increasing additions of platinum to silver. On the other hand, in the case of silver-gold and silver-copper alloys, a very great difference in the extent of the loop is apparent, unaccompanied by corresponding differences in the degrees of hardness[59] of the respective alloys. That the bulge in the different curves must be wholly to the right of the lines joining the extreme

[57] In later researches the superior limit was found to be even above 50.

[58] In reference to thermo-electric power we consider the quantity $2b$ in the equation $e=a\tau+b\tau\sigma$ as a kind of temperature coefficient, since the thermo-electromotive force e for a given difference τ, of temperature, increases linearly with $\frac{\sigma}{2}$, the mean temperature of the junctions.

[59] We shall find below that hardness may also exist in iron-carburets without a marked electrical equivalent.

points is known. Finally it may be called to mind that the direction of the initial tangent at the point "silver" $\left(\frac{da}{ds}\right)_{s=0.017}$ throughout the above results is negative.[60] The thermo-electric effect of the addition of small amounts of other metals to silver would therefore appear to be a progression in a thermo-electrically negative direction.[61] The number of alloys here examined is too small to permit us to attach much significance to this fact. Nevertheless it follows that the thermo-electric relation of any given metal to another does not, so far as can now be seen, enable us to predict the effect produced when the first metal is alloyed to the second. Gold and platinum, for instance, are both positive as regards silver; the initial alloys silver-gold, silver-platinum, however, are negative as regards silver.

THE GENERAL PHENOMENON OF TEMPER REGARDED FROM THE CHEMICAL AND FROM THE MECHANICAL STANDPOINT.

Inferences from the behavior of alloys.—We now desire to consider the bearing of the above results on the electrical properties of steel. From the standpoint of the chemical theory of tempering, it is permissible to regard the linear locus, thermo-electric power varying with resistance, as the initial tangent of a loop of comparatively enormous magnitude. (See figure 13.) The increment of resistance with increasing hardness, or, in other words, with an increase of the ratio of combined to uncombined carbon, is to be considered analogous to the increase of resistance encountered on alloying small amounts of any metal to any given other metal. The linear relation can hardly be regarded striking, since in the case of steel the maximum amount of available carbon does not exceed one to two per cent. Finally, the magnitude of the variation or the extent of the loop might again have been anticipated, from the fact that here a metalloid is alloyed to a metal. The question, therefore, at once suggests itself, in how far these results are corroborated by the electrical behavior of cast-iron. In another paper we will show that the specific resistance of cast-iron, when in a condition corresponding to that of hard steel, *i. e.*, when by means of sudden cooling as much carbon has been converted into the combined form as possible, may be increased to $100\frac{cm}{cm^2}0^{\circ}$ microhm. That is, the specific resistance of chilled cast-iron is nearly as

[60] The isolated negative value for the two per cent. copper alloy might be regarded suspiciously in consequence of the difficulties encountered in endeavoring to obtain these alloys free of oxygen. But its specific resistance shows no anomalous behavior.

[61] Thermo-electrically negative in Seebeck's sense. Cf. Wiedemann, Galvanismus, 3 ed., p. 248, 1883.

large as that of mercury. Suppose, now, we compare the variation of specific resistance $\left(\text{superior limit } s = 50\frac{cm}{cm^2}0^\circ \text{ microhm}\right)$ in the case of steel, where the amount of combined carbon cannot exceed 1 to 2 per cent., with the corresponding variation of specific resistance $\left(\text{superior limit } 100\frac{cm}{cm^2}0^\circ \text{ microhm}\right)$ in the case of cast-iron, where the combined carbon can be increased as high as 5 per cent.; then it apparently follows at once that the electrical behavior of cast-iron is such as we would be justified in predicting from the known relations in case of steel, if we accept the chemical hypothesis of tempering.[62] Hence neither the simple relation of thermo-electric power and specific resistance discovered for steel, nor the phenomenal variation of these constants with the state of hardness of this material, are further to be looked upon as remarkable. We will return to this subject again, below.

It follows from what has been said, that our hope of being able to throw additional light on the nature of the phenomenon of temper in case of steel from a detailed study of the analogous electrical behavior of alloys was not realized. For although on the one hand the new data furnish no results antagonistic to the chemical theory, on the other they leave us without satisfactory or additional reasons for accepting the same. Under these circumstances it is necessary to contrast the theories which may be advanced for the explanation of temper with full regard to all known facts. This we will endeavor to do in the following, particularly so as the discussion will show where good work has been done done and where crucial data are still lacking.[63]

Electrical resistance a volume function.—The discussion may be appropriately commenced with the examination of certain mechanical phenomena which accompany hardness in steel. It is known that the effect of tempering to glass-hardness is an increase both of the specific resistance and the specific volume of steel. This, however, is analogous to the effect of temperature in the case of metals generally. The question is therefore at once suggested, in how far a given increment of volume in the case of steel, whether produced by temperature or by tempering will be accompanied by the same increment of specific resistance. We

[62] Cf., however, p. 186.

[63] In this place it will be well to refer to certain memoirs which have a definite bearing on the subject-matter in hand. In addition to Karsten, Hausmann (Molecularbewegungen, 1856, p. 48), Tunner (Leob. Jahrbuch, x, p. 480, 1861), Jullien (Bullet. de la Soc. de l'industrie minérale, I, p. 566; II, p. 202, 644), Hezner (Berg- u. Hüttenm. Ztg., 1862, p. 140), Deville (Dingler's Journal, clxviii, p. 174), Caron (ibid., p. 36) have developed theories on the causes of hardness in steel. For a discussion the reader is referred to Kerl's Metallurgy, 1870, Vol. III, pp. 25–7, translated by Crookes and Röhrig. It is not our object to enter into a critical examination and comparison of these, as we propose simply to consider the inferences which may be drawn from the electrical behavior of iron-carburets.

were fortunate in finding a sufficient number of data in the literature of steel to enable us to make a preliminary calculation or estimate.

Let v_2 and v_1 denote the specific volume of steel in the glass-hard and soft states respectively. Then the results of Fromme[64] with steel rods whose diameters (0.27 centimeter and less) are nearest those of our own (0.10 centimeter and less) may be thus expressed:[65]

$$\frac{1013}{1000} > \frac{v_2}{v_1} > \frac{1012}{1000} \quad \ldots\ldots\ldots \quad (1)$$

Suppose we put $v_2 = v\,(1+3\beta t)$, where $\beta = 12 : 10^6$, meaning thereby that the volume-increment $v_2 - v$ is due to an increase of temperature t. Then we find

$$360^\circ > t > 330^\circ \quad \ldots\ldots\ldots \quad (2)$$

Since β will probably increase with temperature, this inequality might approximately be expressed $t = 300^\circ$.

Experiments on the resistance effect of high temperatures were originally made by Müller.[66] By the aid of his data for the resistance of iron at 21°, 285°, "oxide-tints appear" (300° to 350°), we deduce, if s' and s be the specific resistances of steel at 300° and 0°, respectively,

$$3.6 > \frac{s'}{s} > 2.5 \quad \ldots\ldots\ldots \quad (3)$$

where the superior limit is possibly too small. The more recent results of Benoit,[67] however, furnish us more accurate values. Let $\frac{s'}{s}$ refer to an interval of temperature between 0°–330° and 0°–360°. Then Benoit's special formulæ and constants for steel give us the relation:

$$3.7 > \frac{s'}{s} > 3.4 \quad \ldots\ldots\ldots \quad (4)$$

If, finally, s' and s denote the specific resistance of good steel in the glass-hard and soft states, respectively, then our results have shown[68]

$$3.0 > \frac{s'}{s} > 2.5 \quad \ldots\ldots\ldots \quad (5)$$

If we compare the estimates expressed in the inequalities (3), (4), (5), it becomes at once apparent that the results of Fromme, together with those of Müller and Benoit, are more than sufficient for the explanation of the enormous increase of resistance which a steel rod tempered to glass-hardness, experiences. It follows that in the case of steel, a given increment of specific volume, whether produced by temperature

[64] Fromme: Wied. Ann., VIII, p. 354, 1879.

[65] These inequalities are not general in their signification. Their purpose is merely that of expressing the extent of the interval within which the cited data were observed to lie.

[66] Müller: Pogg. Ann., CIII, p. 176, 1858. It is hardly necessary to distinguish between iron and steel here.

[67] Benoit: Comptes rend., LXXVI, p. 342, 1873; cf. Carl's Rep., IX, pp. 55–9, 1873.

[68] The predominant influence of carbon on the ratio $\frac{s'}{s}$ will be referred to elsewhere.

or by tempering, is accompanied by increments of specific resistance of the same order. Of course, a result of this kind, since the rod remains homogeneous throughout in one case and is subjected to extremely complicated structural modification in the other, cannot be rigidly true. Nevertheless, the approximate data are of the greatest importance, because they demonstrate conclusively that the effects of the operation of tempering, even if regarded as of a purely mechanical character, must be attended by electrical phenomena of the same order as those which are actually observed.

(2) There is another question intimately connected with the one just discussed, viz, whether a given decrement of volume,[69] produced either by pressure alone or by temperature alone, is always attended by the same diminution of resistance for the same substance (metal); or whether it is necessary to postulate the existence of a purely thermal influence in the second case. Chwolson,[70] who has subjected wires of copper, brass, and lead to pressure, infers that the resistance and the specific volume of these substances vary in like ratio.

In Chwolson's experiments the effects due to pressure are so nearly commensurate with the errors possibly introduced by temperature that we are justified in approaching his results with some diffidence. Marked compression is not readily attainable by piezometric methods. With steel, however, the result, which more than verifies Chwolson's inference on the resistance-effect of volume-reduction, may be thus illustrated.

Let $s_t = s_0 (1+\alpha t)$. Specific resistance (s_t) is here regarded as a temperature (t) function. Then, if t be supposed to decrease indefinitely, the equation $s_t = 0$ would be satisfied at about

$$t = -250^\circ \quad \ldots\ldots\ldots \quad (6)$$

provided, of course, the given relation holds.[71]

On the other hand, let

$$s_2 = s_1 \left(1 + k\frac{v_2 - v_1}{v_1}\right) \quad \ldots\ldots\ldots \quad (7)$$

where the specific resistances s_2 and s_1 correspond, respectively, to the volumes v_2 and v_1, and k is a constant. Specific resistance is here regarded as a volume-function.

[69] In the case of mercury, for instance. The results for solids would be distorted by the loss of structural homogeneity of the compressed solid resulting from external pressure. Large increment of volume is probably only obtainable by tempering steel, and to this enlargement of mean specific volume the remarks already made apply.

[70] Chwolson, Carl's Rep., XIV, 26, 27, 1878.

[71] The temperature coefficient here accepted is the one found for soft steel in chapter I, p. 19, viz, $\alpha = 0.004$.

In order to estimate the value of k, we made a few preliminary experiments, the results of which are contained in this table:[72]

No.	Temper.	Diameter.	Length.	Specific gravity.	Specific volume.	Specific resistance.	Specific resistance.	k
		cm.	*cm.*	at 20°.	$v_2 : v_1$ at 20°.	$\frac{cm}{cm^2}0°$ *microhm.*	$\frac{cm}{cm^2}20°$ *microhm.*	
2	Soft.....	0.575	10.11	7.6977	1.0000	14.3	15.5	} 146
2	Glass-hard....	.....	.....	7.6145	1.0109	38.6	40.2	}
3	Soft.....	0.385	10.02	7.6117	1.0000	14.9	16.2	} 149
3	Glass-hard....	.....	.....	7.5289	1.0110	41.0	42.7	}

It will be observed that, in the calculation of k, both $\frac{v_2-v_1}{v_1}$ and $\frac{s_2-s_1}{s_1}$ were taken at 20°, because we as yet possess no data for the reduction of the former ratio to zero. Equation (7) implies, of course, that the same temperature (desirably 0° C.) occurs throughout.

Suppose, now, that by any mechanical means whatever the volume of steel has been isothermally so far decreased that in equation (7), $s_2=0$. Then

$$-\frac{1}{k}=\frac{v_2-v_1}{v_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Such a volume decrement, if the result of diminution of temperature, would correspond to a cooling of steel as far as $[v_t=v_0\,(1+3\,\beta\,t)]$,

$$t=\frac{1}{3\beta}\;\frac{v_t-v_0}{v_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

i. e., if the volume decrements in (8) and (9) be identical,

$$t=-\frac{1}{3\beta}\;\frac{1}{k}=-190°\text{ (nearly)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

From a comparison of equations (6) and (10), we therefore again infer that variation of resistance is a necessary concomitant of variation of volume. No matter how the latter may have been produced, whether by temperature or by tempering, the increments of resistance due to a given increment of volume are of the same order. It is probable that this relation will apply to the resistance of non-electrolyzed solids generally. The results for steel are as nearly coincident as the excessively large sources of errors unavoidably encountered permit us to anticipate. In addition to those already enumerated, we need only mention that Fromme worked with a different kind of steel from that used in our researches; that neither the coefficient of heat-expansion nor the galvanic temperature coefficient applies accurately for the large intervals of temperature met with, &c. A special error is introduced in consequence of the difficulty encountered in endeavoring to define values for the soft state.

[72] Rods not thoroughly homogeneous, tempered after heating in a blast-flame.

(3) The results of Fromme[73] and Chwolson finally contain a remarkable analogy, to which attention has not yet been given. If the hardness of a glass-hard steel rod be supposed to decrease continuously, then the specific volume will be found to pass through a minimum, as Fromme has emphasized, at a state of hardness corresponding to the annealing tint, "gray" (annealed at 300°–500°). Chwolson experimented on a very large number of metals and alloys. He found that the electrical effect of "drawn" hardness as well as the hardness due to sudden chilling vanishes on exposure of the hard wire to high temperature, in such a way that after the first gentle ignition at low red heat the specific resistance of all metals passes through a minimum.[74] Unfortunately Fromme has not interpolated a temperature which would produce an effect between "gray annealed" and "soft." Nevertheless it is difficult to avoid the conclusion that in all these examples of minima we encounter phenomena essentially of the same kind. Chwolson remarks that in case of steel his results were singularly striking. It would appear, therefore, that the mechanical effect (electrically measured) of sudden cooling is the same for almost all metals; that the magnitude of this effect differs from metal to metal, and is enormously pronounced in steel. This is the first evidence adduced that the given operation, tempering, produces qualitatively like effects on metals generally, the difference being in degree only.[75]

(4) In view of the increasing importance of density or of the values of specific volume in all these instances, the fact that in the above experiments with alloys the ranges of variation were found to be in the order of the differences of specific volume of the metals composing the alloy may again be called to mind. In case of steel the intimate relationship between specific resistance and thermo-electric power, described elsewhere, suggests that the cause of the increase of volume consequent upon sudden cooling is of a purely mechanical character, *i. e.*, that it is not the direct result of the change of chemical condition of steel which tempering probably produces.

"*Drawn*" *hardness.*—Matthiessen[76] found that a hard-drawn wire of silver could be very perceptibly annealed (conductively decreased) by continued boiling in water. We obtained a good corroboration of this

[73] Fromme, l. c.

[74] Lead and German silver are exceptions.

[75] It must be borne in mind that this remark refers to the electrical indication of the (mechanical) effect of tempering. It is well known that while sudden chilling hardens steel, it is said actually to soften some of the other metals. It is, however, exceedingly difficult to distinguish between the degrees of hardness of brass or copper suddenly chilled or slowly cooled from red heat, by ordinary mechanical means; and, moreover, sudden cooling may be accompanied by purely chemical effect in alloys and (not chemically pure) metals as it is in case of steel.

[76] Matthiessen, und v., Bose: Pogg. Ann., CXV, p. 363, 366, 370, etc., 1862.

result with drawn German-silver wire,[77] in which, after an exposure to steam at 100° of only an hour, a change of resistance of about 0.25 per cent. was perceptible. It follows, therefore, that not only tempered but drawn wires may be annealed to a very marked degree at comparatively low temperature.

Inferences from the behavior of iron-carburets.—(1) In chapter I we saw that the temperature-coefficient of cast-iron is in good accordance with the coefficients of wrought iron and of steel in different states of temper, so that in this respect all the iron-carburets form a single connected series.

(2) This is by no means the case if we compare the relative galvanic and thermo-electric behavior of these products. From an inspection of figure 13 it is at once obvious that the position of cast-iron is isolated relatively to the linear locus which obtains for steel. This observation shows conclusively that in the case of steel the cause of the electrical variation in question must be essentially different from that which determines the position of cast-iron. Below it is shown to be necessary to explain the distinctive electrical qualities of cast-iron as effects of chemical composition. Hence we infer with some assurance that the electrical behavior of steel on passing from one state of temper to another, since the succession of values in no way suggests the electrical qualities of cast-iron, is conditioned by the cotemporaneous and purely mechanical changes which steel undergoes.

Phenomena of annealing chemically interpreted.—On endeavoring to use the chemical hypothesis of tempering to account for the phenomena of annealing,[78] we at once encounter serious difficulties. On the basis of this theory there must exist a fixed ratio of combined to uncombined carbon for each temperature of the annealing bath. Moreover, for a given temperature this ratio must be approached gradually (asymptotically) as time of exposure is prolonged indefinitely; for different temperatures it must decrease as temperature increases indefinitely, until finally a minimum value wholly independent of temperature is asymptotically reached.[79] The minimum of the ratio of combined to uncombined carbon need not, of course, be zero. The last phase of this species of decomposition is in many respects similar to the phenomena of dissociation. The resemblance can, however, only be apparent, since continuous dissociation has not been observed except in the case of gases. At least, in our knowledge, there are no chemical examples in

[77] The results are these: Before boiling: 12°.4, r=6.8522; 12°.5, r=6.8523; 12°.6, r=6.8524. During boiling: 99°.8, r=7.0167, r=7.0182, r=7.0204; resistance (r) measured successively. After boiling: 13°.5, r=6.8691; 13°.6, r=6.8694; 13°.7 r=6.8696.

[78] Strouhal und Barus: Wied. Ann., XI, p. 962, 1880.

[79] In these cases of annealing slow cooling of the annealed rod after exposure to the annealing temperature is always presupposed. It is thus that annealing at red heat is synonymous with softening ("Ausglühen"). This operation does not, however, convert all the carbon in steel to the uncombined form, in which case the said ratio would be zero.

which solids are found to dissociate in accordance with the laws to which gases, in virtue of their physical state, must conform; and for this reason the explanation of the phenomena of annealing given by the chemical theory is remote and forced, and to be discarded. We gain no more by adopting Matthiessen's hypothesis, which considers all iron-carburets as solidified, more or less thorough solutions of carbon in iron.

Annealing physically interpreted.—It is not difficult, however, to find for the phenomena of annealing a satisfactory explanation of a purely mechanical character. We proceed in a manner analogous to the way in which, in the case of soft rods of the same dimensions, what is known as coercive force[80] in magnetism is sometimes defined. To this analogy we will recur again.

(1) It will be permissible in this place to introduce a definition of viscosity which is conveniently adapted to our present purposes. We will suppose the viscosity of steel to be measured by the maximum of strain of a given kind (in the present research the strain accompanying hardness) which a steel rod of given dimensions, after previous condition of supersaturation, of itself, permanently retains. The process of tempering to glass-hardness, therefore, is essentially one by which energy is stored up, whereas during the operation of annealing the stored energy is again more or less gradually expended; and the maximum energy permanently potentialized, under given circumstances of temperature (the same rod always presupposed), increases directly with the maximum intensity of permanent strain under the same circumstances. Practically, however, we possess no means for the absolute evaluation of either of these quantities. But in so far as both thermo-electric hardness and specific resistance (the rod being in the state of the maximum of permanent hardness as regards the given temperature) vary directly with strain and stored energy, we may accept either of the former quantities as furnishing a good general estimate of the magnitude of the latter. In the case of one and the same steel wire viscosity is an inverse function of temperature and of temperature only. In the case of wires which differ quantitatively though not qualitatively as regards carburation, and are otherwise identical, viscosity is a function both of temperature and carburation (carburation < 2 per cent.), decreasing with the former and increasing with the latter magnitude.

(2) The difference between the phenomena of annealing and the phenomena of viscosity, as ordinarily studied and observed, is succinctly this, that in the latter case stress or a single force is applied from without to produce the gradual and permanent deformation through infinite time; in the former, however, stress exists in the rod itself (strain of hardness), but can be made to disappear in the asymptotic way characteristic of these phenomena by decreasing the viscosity of the rod as a whole; *i. e.*, increasing its temperature. In other words, in the ordinary

[80] Cf., for instance, Mousson: Physik, 2 ed., Vol. III, p. 86.

case of viscosity-measurement the phenomenon is evoked by sudden application of stress (torsion, tension, flexure, volume-compression) at constant viscosity; in the present case by sudden decrement of viscosity at initially constant stress. For instance, given a glass-hard rod at ordinary temperatures. Let its temperature be raised to 100°. The stress which was just permanently maintained at the lower temperature is in excess at the higher, because of the diminished viscosity of the rod, as a whole; and this excess must therefore disappear gradually through infinite time, precisely as we have observed it. If, furthermore, the diminution of viscosity be sudden and very large (annealing at high temperatures 200°, 300°, &c.), stress will disappear at a correspondingly rapid rate; etc.

In this way we are able, as a first approximation, to refer the phenomena of annealing to the general category of phenomena of viscosity. But for the reason of the deformation accompanying the existence of internal stress, for the reason that the degree of heterogeneity of a hard rod varies to a greater or less extent with each variation of stress as well as temperature, it is difficult to form any clear conception of the viscosity of the rod, as a whole, except by the aid of such a definition as is given at the beginning of the present paragraph. We will waive a further development of these views here for the reason that the details of operation of tempering itself are in need of further experimental elucidation, in various directions. Here, however, an account of the occurrences which rob the theory just sketched of much of its clearness and perspicuity is in place.

(3) The most satisfactory conception which we can form of the nature of the strain of a glass-hard steel rod is that of abnormally condensed external layers surrounding, like an arch, an abnormally rare core. These two abnormal states of density mutually condition each other. It is the stress under which the internal layers exist which is the cause of the condensation of the relatively thin external shell, and the extreme limit of condensation of the latter, again, which prevents the internal layers from falling back to normal density. This view has much experimental evidence to support it,[81] though it must be regarded as a mere *diagram* of the essential features of the vastly more complicated structure of the glass-hard rod. An increase of the temperature of a hard rod (annealing) is antagonistic to the existence of a strain of this character in two respects. In the first place it is obvious that the rare core will be in a condition of less intensity of strain at a high than at a low temperature, and this because of the volume-expansion due to temperature. For instance, the temperature may readily be chosen so high that an identical hot rod would normally have the same low density that originally existed in the strained core of the cold glass-hard rod. Increase of temperature, on the other hand, diminishes the den-

[81] Cf. Mousson: Physik, 2. ed., pp. 224, 225, Vol. I, 187. Fromme: l. c.

sity of the external layers. In proportion, therefore, as the temperature of the annealing bath is higher, the inner layers of the originally hard rod approach the normal density for the respective temperatures more and more nearly, and the surface condensation, because the conditions of its existence are being annulled, must therefore also disappear at a corresponding rate. When the temperature, t^0, at which the stress in the inner layers is wholly gone has been reached, only as much of the surface condensation can have remained as is in conformity with the ordinary viscosity of steel at t^0. The rod if cooled from t^0 will therefore in the cold state be of greater density than an otherwise identical and homogeneous rod—it being postulated, of course, that with the disappearance of strain the tempered rod itself approaches nearer and nearer the homogeneous state. If the annealing be carried to even higher temperatures than t^0, the surface condensation will also more and more fully vanish, and the density of the cold rod as a consequence again decrease. Now, it is exceedingly remarkable and significant that the electrical effects due to the annealing of a glass-hard steel rod, enormous as they are, vanish almost entirely after the temperature of the annealing bath exceeds 300° to 400°, that, is at a temperature at which the density of a homogeneous hot rod is the same as the density of an identical cold glass-hard rod,[82] or, in other words, very nearly at the temperature t^0 just discussed.[83] Magnetic effects, however, as will be shown elsewhere, are still very marked for rods annealed at temperatures higher than t^0. This is in conformity with the hypothesis of residual surface condensation, because magnetic phenomena are so largely dependent on the external layers of a rod. Finally, the actual existence of a maximum of density for rods annealed at temperatures near t^0 has been experimentally shown by Fromme.[84]

(4). With these remarks, however, the subject is by no means fully exhausted, even so far as our present purposes go. It is easily conceivable that a rod at 400° may be brought into a condition of strain (strain of hardness) which bears the same relation to an identical homogeneous rod at 400° that an ordinary glass-hard rod does to an identical cold soft rod. We believe, however, that the viscosity of the hot rod is too insignificant to admit of the permanent retention of more than reduced intensities of stress, no matter how much of the strain peculiar to temper may by some ideal means[85] have been imparted to it. In other words, suppose that a sufficient and very great intensity of

[82] Cf. pp. 90–93.

[83] The condensed layers of the rod being thin in comparison with the rarefied parts.

[84] Fromme: l. c., p. 355.

[85] To form a definite conception of an ideal process of tempering like the one in question, suppose the observer to have in control a set of forces, by aid of which he is able to move each of the molecules of a (soft) steel rod into such positions that the rod, as a whole, experiences a strain of glass-hardness of very great intensity. When this characteristic distribution of molecules within the volume of the (now) hard rod has been effected, let the action of the said forces be discontinued and the molecules

the strain in question is imparted both to an iron and to a steel rod of the same dimensions. When the tempering influence has ceased, the steel rod will have retained a phenominal amount and exhibit glass-hardness. The iron rod, on the contrary, will have lost nearly the whole of it. Analogous to the latter, the steel rod when at 400°, no matter how much stress may be imparted to it, will permanently retain no more at 400° than the small amount which is consistent with the definition of viscosity above given.[86] The present consideration, therefore, culminates in an experimental inquiry which may be formulated thus: Will a steel rod, if chilled from a given temperature in red heat in a liquid at any temperature below t^0, and subsequently annealed by an exposure to t^0, indefinite as regards time, always exhibit the same final intensity of stress or thermo-electric hardness—barring of course all secondary effects, such as are due to decarburation, etc.

THE PHENOMENON OF GLASS-HARDNESS DISCUSSED FROM THE CHEMICAL AND FROM THE MECHANICAL STANDPOINT.

Chemical interpretation.—We now proceed to the discussion of the effects of sudden chilling on steel. If we endeavor to explain the known phenomena mechanically, we encounter two grave difficulties. In the first place, we fail to find any obvious reason why the hardness of a chilled rod is not, as Chernoff[87] discovered, a uniformly continuous function of the temperature of the rod before sudden cooling. In the second place, the important function of carbon in the process of tempering, and the necessary presence of this element in steel if the effects of sudden cooling are to be permanent, is not easily discernible. Both these apparently weak points of the mechanical theory may be very easily explained chemically. It is only necessary to suppose that a certain high temperature must be reached before the uncombined carbon of soft or annealed rod is again either converted into the combined form or is dissolved.[88] That such a temperature would lie, at lowest, in the

left to themselves. Then will the excess of stress gradually disappear, until, through infinite time, the intensity of strain which the given rod under the given circumstances of temperature of itself can just permanently retain, is reached. The (hard) rod is now in a condition of stable molecular equilibrium. The close analogy between this ideal method of imparting temper and magnetization is at once apparent.

[86] Cf. p. 95, (1).

[87] Chernoff: Vortrag geh. in der Russ. Tech. Gesell., April u. Mai, 1878; cf. Jeans, op. cit., p. 644–7. A reprint of Chernoff's lecture kindly sent by the author showed us that Chernoff was the first to discover the remarkable phenomenon in question. Our own experiments, made independently of Chernoff and shortly after him, led to the same result, both as regards mechanical and thermo-electric hardness. (Cf. Barus; Wied. Ann., VII, p. 405, 1879; Strouhal and Barus: Wied. Ann., XI, p. 946, 1880.)

[88] Cf. Karsten's theory on the existence of a "polycarburet," for instance, in Perey-Wedding, op. cit., p. 167; Jeans, op. cit., p. 646; etc.

region of red heat is immediately probable. Sudden cooling from temperatures below this critical value, in so far as carbon would remain undissolved or uncombined, could not impart hardness to steel, but would merely anneal it the more thoroughly—which is actually observed. It is particularly to be noted here, moreover, that, according to Jeans, in the cementation process iron will not absorb carbon until a certain definite high temperature has been reached.

Physical interpretation.—(1) The first of the above-mentioned difficulties encountered by a mechanical theory must here be discussed with greater detail. The phenomenon is not without remarkable physical analogies. Cumming,[89] for instance, and after him others, called attention to the very sudden expansion exhibited by iron at a certain temperature in red heat. Tait[90] signalizes the abnormal thermo-electric behavior of iron under the same circumstances. Finally, the discovery due to Gore,[91] which Baur[92] has recently studied with considerable detail, is to be cited here, viz., that the temporary magnetism of saturated iron at the said temperature suddenly vanishes from a foregoing very large value. There appears to be little reason to doubt that all of these phenomena are different manifestations of the same cause.[93]

(2) If, now, we bear in mind that a glass-hard rod has retained a very large part of the increment of volume due to the heat expansion, at the temperature from which it was chilled; that furthermore very rapid cooling is the necessary condition of the appearance of glass-hardness; then the remarks just made at once suggest the inference that Cumming's phenomenon plays a very important part in the process of tempering. It would follow, furthermore, that the said phenomenon must first fully have appeared if sudden cooling is to be accompanied by hardness. Hence the necessity of a critical temperature. Here, therefore, we encounter the first difference between iron and steel on the one hand, and all other metals and alloys on the other (these being without the said phenomenon), as regards conditions favorable to the production of hardness by chilling.

(3) The difference between iron and steel, however, appears even at ordinary temperatures whenever both materials are subject to the same kind of stress. If, for instance, we expose two like rods, one of iron and

[89] Cumming: Cf. Tait, l. c. Gore: Proc. Roy. Soc., XVII, 260, 1869.

[90] Tait: Trans. Roy. Soc., Edinbgh., XXVII, 1872-73, p. 125; Proc. R. S. E., VIII, p. 32, 1872-73.

[91] Gore: Phil. Mag., (4), XXXVIII, p. 59, 1869; *ibid.*, XL, p. 170, 1870.

[92] Baur: Wied. Ann., XI, p. 408, 1880.

[93] Tait refers the sinuous and broken character of the iron line in his thermo-electric diagram to Cumming's phenomenon. Gore, and more thoroughly Baur, ascribe the anomalous behavior of temporary magnetism at red heat to the same cause, in both cases attributed to Gore. The same irregular results would probably recur in the variation of electrical conductivity on passing the temperature in question. The method of experimentation adopted by Benoit (l. c.) led him to overlook this. With the discovery of Chernoff these interesting analogies are enriched by a new fact.

one of steel, to the influence of the same sufficiently intense magnetic field, the steel rod, after withdrawal, is found to have retained a very considerable part of its original magnetism permanently, whereas the iron is comparatively unmagnetic. Analogously we find that, although both iron and steel possess the essential property of sudden expansion at red heat discovered by Cumming, steel after sudden cooling has retained a phenomenal amount of the peculiar and characteristic strain accompanying hardness, while iron remains nearly soft and comparatively homogeneous. It appears, therefore, that the effect of sudden cooling on steel also admits of a physical explanation.

Physical and chemical changes simultaneous.—There is one other consideration to be added here. If we compare the difference between steel and iron as regards their power for the retention of a given mechanical strain, with the respective retentive properties of steel at different temperatures, we may readily infer that under all circumstances the absolute amount of combined carbon in iron or steel is the essential factor. From this standpoint every gradually progressive change of strain from any original to any final value is to be looked upon as a physical phenomenon; the fundamental cause of such gradual change of strain, in other words the cause for the necessary change of viscosity, is to be regarded as only dependent on temperature in such measure as temperature itself determines the ratio of combined to uncombined carbon; hence, also, the amount of combined carbon in the steel rod. Here, therefore, the thermo-electric hardness of a rod annealed for an indefinite length of time at t degrees is an indication of the amount of combined carbon in the rod at this temperature.

EXPERIMENTS WITH MALLEABLE CAST-IRON.

Above inferences not decisive.—Endeavoring to review the above matter with the object of obtaining data for the discrimination between the physical and the chemical features of the process of tempering, we fail to find criteria sufficiently decisive to enable us to draw our inferences satisfactorily. We are induced even to acknowledge that there exists a remarkably intimate relation between the two methods of interpreting the various phenomena; for not only are we able to explain even the more important details both from the chemical and the physical standpoint, but the development of either hypothesis continually suggests new means for the development of the other.

Electric behavior of malleable cast-iron.—At this stage of our research it appeared probable that further and perhaps critical information might be obtained from a study of the electrical behavior of different species of cast-iron. We commenced our inquiry with an examination of what

is known as "malleable cast-iron,"[94] a material which from its exceptional position in the series of iron-carburets seemed particularly well adapted for the elucidation sought. This product is tough and tenaceous, though imperfectly malleable in the soft state, but after sudden cooling from red heat becomes almost as hard and certainly quite as brittle as hard steel. Forquingnon[95] has shown that it is distinguished from steel, chemically, by the relatively very large amount of graphite which it contains. We were fortunate in obtaining rods of malleable cast-iron from M. E. Hartmann, in Würzburg.

The results of our measurements of thermo-electric power are given in the following table (42) on a plan identical to that adopted above for alloys. The rods were examined in three states of hardness, viz., in the commercial condition in which they reached our hands, then immediately after sudden chilling, finally after thorough annealing (softening) by slow cooling from red heat:[96]

TABLE 42.—*Thermo-electric power of malleable cast-iron.*

Rods.	t	T	e observed.	e calculated.	Diff.	a	b
ROD No. 1.	C.	C.	*microvolt.*	*microvolt.*			
Original condition	18.3	79.7	−57.3	−57.2	−0.1		
	18.4	68.8	−39.4	−39.2	−0.2		
	18.5	58.3	−25.0	−25.1	0.1	+0.45	−0.014
	18.6	47.4	−13.4	−13.7	0.3		
	18.7	38.2	− 7.0	− 6.8	−0.2		
Chilled hard	15.7	59.0	−81.6	−80.8	−0.8		
	15.7	51.2	−62.5	−62.7	0.2		
	15.7	43.4	−45.6	−46.2	0.6	−0.90	−0.013
	15.7	36.7	−33.0	−33.2	0.2		
	15.7	30.1	−21.8	−21.6	−0.2		
Soft	17.9	86.1	−69.8	−68.5	−1.3		
	17.9	77.2	−52.3	−53.0	0.7		
	17.9	66.5	−36.3	−36.9	0.6	+0.30	−0.013
	17.9	53.9	−21.5	−21.6	0.1		
	17.9	43.1	−11.9	−11.7	−0.2		
ROD No. 2.							
Original condition	16.8	75.9	−14.3	−15.2	0.9		
	16.9	66.3	− 7.1	− 6.4	−0.7		
	17.0	56.1	0.0	+ 0.1	−0.1	+0.98	−0.013
	17.0	45.2	+ 3.7	+ 4.2	−0.5		
	17.1	37.2	+ 5.5	+ 5.1	0.4		
Chilled hard	12.0	53.9	−25.5	−24.4	−1.1		
	12.2	46.3	−15.4	−16.3	0.9		
	12.3	40.9	−11.1	−11.5	0.4	+0.37	−0.014
	12.4	35.5	− 7.6	− 7.4	−0.2		
	12.5	30.3	− 4.6	− 4.4	−0.2		
Soft	18.2	86.1	−43.7	−43.8	0.1		
	18.2	72.3	−25.5	−25.2	−0.3		
	18.3	55.5	− 9.5	− 9.1	−0.4	+0.72	−0.013
	18.3	45.1	− 2.4	− 3.0	0.6		
	18.3	35.1	+ 0.1	+ 0.3	−0.2		

[94] Cf. Perry-Wedding, *op. cit.*, p. 143.

[95] Forquingnon, *l. c.*, p. 536.

[96] The usual process of surrounding the rods with ferro-ferric oxide in gas-pipe (closed), then heating the latter to redness in a charcoal furnace, in which the tube is left until the fire is completely extinguished, being adopted.

TABLE 42.—*Thermo-electric power of malleable cast iron*—Continued.

Rods.	t	T	e observed.	e calculated.	Diff.	a	b
ROD No. 3.	C.	C.	*microvolt.*	*microvolt.*			
Original condition	16.1	67.3	7.6	6.8	0.8		
	16.2	62.0	8.3	8.6	−0.3		
	16.2	50.3	9.6	10.6	−1.0	1.00	−0.010
	16.3	43.7	10.5	10.4	0.1		
	16.3	37.3	9.5	9.2	0.3		
Chilled hard	17.8	58.2	−22.9	−22.1	−0.8		
	17.9	51.1	−14.0	−14.4	0.4		
	17.9	44.9	− 8.8	− 9.0	0.2	0.73	−0.017
	17.9	37.9	− 4.2	− 4.2	0.0		
	17.9	31.2	− 1.4	− 1.3	−0.1		
Soft	18.9	80.2	−31.0	−29.6	−1.4		
	18.9	66.1	−10.8	−12.0	1.2		
	18.9	53.1	− 0.6	− 1.3	0.7	1.15	−0.016
	18.9	42.1	+ 2.8	+ 3.3	−0.5		
	18.8	33.2	+ 4.1	+ 4.1	0.0		

The following and final tables (43) contain our results for the specific resistance of malleable cast iron in each of the three states—commercial, chilled hard, annealed soft (at red heat). In column third is given the actual resistance per meter of rod at the temperature t. The sections in cm^2 are found under q. The specific resistances at t and 0 degrees are arranged under s_t and s, respectively, while the intermediate column α contains the temperature-coefficients for this reduction:

TABLE 43.—*Electrical resistance of malleable cast iron.*

Rod.	Remarks.	W 1_m	t	q	$s_t \frac{cm}{cm^2} t°$	α	$s \frac{cm}{cm^2} 0°$
		ohm.	C.	cm^2	*microhm.*		*microhm.*
1	Original condition	0.0274	15.5	$(0.308)^2$	26.0	0.004	24.4
	Chilled hard	377	11.5	$(0.302)^2$	34.3	0.004	32.7
	Soft	290	14.5	$(0.290)^2$	24.4	0.004	23.0
2	Original condition	0.0262	15.5	$(0.307)^2$	24.8	0.004	23.3
	Chilled hard	323	11.5	$(0.300)^2$	29.2	0.004	27.9
	Soft	285	14.5	$(0.296)^2$	25.0	0.004	23.6
3	Original condition	0.0262	15.5	$(0.303)^2$	24.1	0.004	22.6
	Chilled hard	316	11.8	$(0.297)^2$	27.9	0.004	26.6
	Soft	290	14.5	$(0.290)^2$	24.4	0.004	23.0

In the following digest the main data obtained for malleable cast-iron are given in more perspicuous form. Δa contains the range of variation of the thermo-electric constant a from soft to hard. Δs has the same signification as regards specific resistance:

TABLE 44.—*Thermo-electric power and specific resistance of malleable cast-iron.*

Rod number.	Thermo-electric power (a).		Specific resistance.		Δa	Δs
	Soft.	Hard.	Soft.	Hard.		
1	+0.30	−0.90	23.0	32.7	−1.20	+9.7
2	+0.72	+0.37	23.6	27.9	−0.35	+4.3
3	+1.15	+0.73	23.0	26.6	−0.42	+3.6

Decisive character of the evidence.—The first striking feature of the last set of results is the slight thermo-electric difference between the malleable cast-iron rods and silver. Hence the large values of thermo-electric hardness corresponding to the large values of specific resistance.

Far more remarkable, however, are the almost insignificant variations (Δa and Δs) produced by sudden cooling. Here, therefore, we have a valuable example of an iron-carburet in which the interval of mechanical hardness due to tempering; *i. e.*, the difference of mechanical hardness between hard and soft is very great, whereas the corresponding change of the electrical constants scarcely reaches the electrical difference between "blue annealed" and "soft" for steel. In this respect, therefore, these measurements have critical value. It follows that the mechanical hardness possessed by chilled steel cannot be the only or even the principal cause of the enormous variation of the electrical constants; that the said cause of these variations must be sought in the secondary phenomena which accompany the presence of hardness; that they are therefore very probably due to the strains manifesting themselves, as a whole, in the volume-expansion of hard steel.

This important deduction is further substantiated by certain experiments made with cast-iron by Joule.[97] His results showed that cast-iron in the white, hard condition is thermo-electrically nearest antimony; in the black,[98] graphitic condition, however, nearest bismuth. All other samples of cast iron examined occupied thermo-electric positions between these limits. If, therefore, the shifting of thermo-electric position produced by sudden chilling were to be ascribed to the conversion of uncombined into combined carbon, hard steel, like white cast-iron, would lie nearest antimony, soft steel nearest bismuth, whereas the actual positions of steel in these two extreme states of hardness is just the reverse of this. Hence it would follow, again, that the change of thermo-electric power of steel due to tempering must be referred to the increase of volume simultaneously experienced. In striking accordance with this inference are the results of Forquignon:[99] "Le fonte malléable se distingue de l'acier par ses *faible allongements* et sa forte teneur en graphite." It appears, therefore, that here again the absence of marked volume increase is the concomitant of the absence of marked electrical variation.

If we add to these deductions the remarks made in pages 89, 95, and 99, we arrive at the following final result: The existence of the peculiar strain accompanying hardness in steel is the cause of electrical effects so enormous that such additional effects which any possible change of carburation may involve can be wholly disregarded, and all electrical results interpreted as due solely to variations in the intensity of the said strain.

[97] Joule: Phil. Trans., 1859, I, p. 97.

[98] That is, black at the surface of fracture, known to be rich in graphite.

[99] Forquignon, l. c., p. 536.

CHAPTER IV.

ON THE THERMO-ELECTRIC EFFECT OF MAGNETIZATION.

Earlier results.—A number of facts go to prove that the molecular structure of an iron-carburet is materially affected both by the intensity and by the kind of magnetization which it receives. We know, moreover, from experiments of Joule,[100] Wertheim,[101] Mayer,[102] that the existence of magnetism in a bar exerts a stress bearing some analogy to a pull in the direction of the magnetic axis. This strain seems to be an inseparable concomitant of the magnetic quality. We therefore infer at once that if two identical steel rods be magnetized to different intensities, their original thermo-electric powers will have changed; that, moreover, the increments of this quantity in the two cases, in accordance with the difference of magnetic state premised, will be unequal. Similar remarks possibly apply to other physical constants of steel, in particular to its electrical conductivity. With regard to the latter, the results of earlier observers are either unsatisfactory or discordant.[103] The experiments of Beetz[104] furnish the first evidence of a decisive character. This physicist found that the resistance effect of longitudinal magnetization is an increment of from 0.03 per cent. to 0.06 per cent.; that in the case of transverse magnetization, however, no variation (greater than 0.0005 per cent.) is appreciable. Beetz's result has been corroborated and supplemented by experiments of Auerbach,[105] who maintains that while the specific resistance of hard steel increases continuously from the longitudinally to the circularly magnetized state, soft iron and soft steel possess a minimum of resistance when unmagnetic.

The discovery of a thermo-electric effect of magnetization is due to Sir W. Thomson,[106] and his experiments furnish the only results on the subject which have thus far been obtained. He found that an iron wire in the longitudinal or in the transversely magnetized state is thermo-electrically positive or negative,[107] respectively, when compared with the

[100] Joule: Phil. Mag., XXX, pp. 76, 225, 1847.

[101] Wertheim: Ann. d. Chim. et de. Phys. (3), XXIII, p. 302, 1848.

[102] Mayer: Phil. Mag. (4), XLVI, p. 177, 1873.

[103] Cf. digest and partial discussion in Wiedemann's Galvanismus, II *a*, p. 586 (ed. 1874).

[104] Beetz: Pogg. Ann., CXXVIII, p. 202, 1866.

[105] Auerbach: Wied. Ann., V, p. 289, 1878.

[106] Thomson: Phil. Trans., III, p. 722, 1856.

[107] In Seebeek's sense. See preface.

same wire in the unmagnetic state; that is, there would be a thermo-current from transversely magnetic to unmagnetic through hot, or from unmagnetic to longitudinally magnetic through hot.

Relative electrical effects of hardness and magnetization.—These facts have an immediate bearing on the question in how far the method for the measurement of hardness which we employed in the case of unmagnetic rods is to be modified when we deal with magnets. For it is obvious, if the hardness of a steel rod is to be expressible in terms of its thermo-electric power, that magnetization, in so far as it produces no appreciable change of the mechanical quality, must to an equal extent be without influence on the value of its electrical equivalent.

The effect of transverse and of circular magnetization on the specific resistance is not distinctly marked. Beetz, *l. c.*, obtained no indication whatever. We are justified in accepting a similarly small interval of variation for the thermo-electric power, although Thomson, *l. c.*, was able to adduce conclusive experimental proof of its existence. In its bearing on the especial purposes of the present series of papers, this effect is, however, of secondary importance; for which reason, and in view of the difficulties with which an exact measurement would be surrounded, we determined to neglect it in favor of the much more pronounced phenomenon now to be considered.

It is the evaluation of the electrical effect of longitudinal magnetization, therefore, which is carefully to be essayed. As regards the specific resistance, the results of Beetz furnish the datum of only about a half-tenth per cent. for iron in the most favorable case. Now the resistance effect of glass-hardening is as high as 300 per cent. It is obvious, therefore, that where the specific resistance is used for the definition of hardness of steel, the effect of magnetization is thoroughly negligible, since for steel it will probably fall even below the small value for iron; particularly so where permanent magnetism is alone of interest. Consequently this mode of measurement of hardness may be immediately applied to steel irrespective of its magnetic state.

In considering the thermo-electric behavior of steel in its dependence on magnetization, the following striking peculiarity is conspicuous at the outset. When the simultaneous variation of thermo-electric power and specific resistance is produced by tempering, the result is such that steel for continually increasing resistance moves constantly toward greater thermo-electrically negative values. When the simultaneous variation in question is produced by magnetization, however, a positive increment of resistance (Beetz) corresponds to a shifting of steel towards thermo-electrically positive values (Thomson). This at once points out a radical difference between these phenomena, a difference, however, which might have been anticipated both from the utter dissimilarity of the strains, as well as from the fact that we necessarily deal with steel only in the first case, but with iron and steel (soft) respectively, in the

second—where, moreover, the behavior of soft and of hard steel is not even qualitatively the same (Auerbach).

The magnetic result admits of the following interpretation. It has been stated that one of the results of magnetization is elongation in the direction of the magnetic axis and contraction at right angles to it in such proportions as leave the volume of the iron unchanged (Joule). In this respect the stress is analogous to a pull. Now, Thomson discovered in the one case a current from transversely magnetized to longitudinally magnetized iron through hot; in the other a current from transversely strained to longitudinally strained iron through hot. Possibly, therefore, we have in hand more than an incidental coincidence. It is to be noted that both results apply for elongation within the elastic limits.[108]

Method of experimentation.—Thomson's results are qualitative. For our purposes, however, quantitative results are a necessity. With the object of deriving these the following experiments were made. The disposition of apparatus is diagrammatically given in Fig. 14. A soft iron

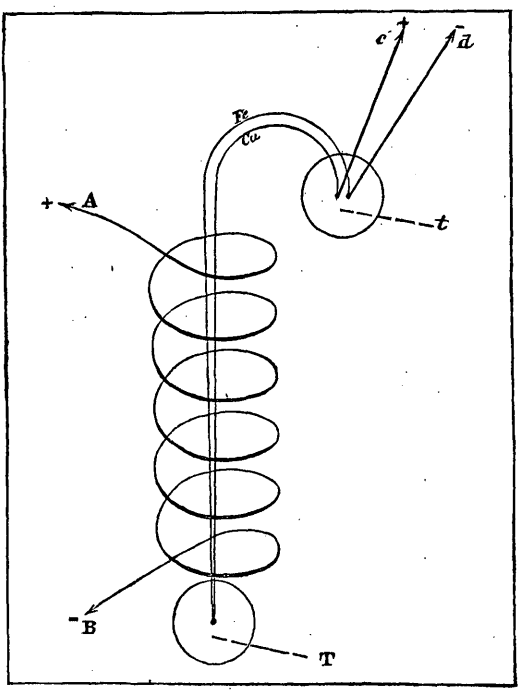


FIG. 14.—Disposition of apparatus.

wire about 0.08 centimeter thick and 40 centimeters long, the ends of which had been fastened to long terminals (*c*, *d*) of copper wire, was

[108] Thomson, *l. c.*, pp. 712, 713. Cohn, Wied. Ann., VI, p. 385, 1879. Beetz (Pogg. Ann., CXXVIII, p. 193, 1866) remarks that the resistance effect of magnetization is probably to be referred to the occurrence of the magnetic strain, and in so far anticipates the above statement. But he possesses no direct evidence.

thrust through a helix (AB) together with one of the copper wires, and fixed in its axis. At each of the ends of the helix, and as near its center as possible, a doubly tubulated spherical receiver, containing petroleum at the temperatures t and T, respectively, was placed. These receivers contained the junctions of the thermo-element (copper-iron). In other respects the disposition and method of measurement was the same as that employed in the previous chapter II, and the reader desiring further information is referred thither.[109]

The helix in question had a length of 22.3 centimeters, contained ten layers, each with about 55 windings of thick copper wire 0.3 centimeter in diameter.

A Siemens dynamo-electric machine was at our disposal as a source of strong current, the intensity of which, obtained by the aid of a large tangent compass, under the given circumstances proved to be

$$i=3.12\frac{g^{\frac{1}{2}}\,cm^{\frac{1}{2}}}{sec}$$

Let $2a$ be the length, r the radius of a given layer of the helix, n the number of turns of wire in the layer, $2b$ the length of the rod to be magnetized. Then will the magnetizing force on the axis of the helix and at a distance b from its center be

$$X_b=\frac{\pi ni}{a}\left[\frac{a+b}{\sqrt{(a+b)^2+r^2}}+\frac{a-b}{\sqrt{(a-b)^2+r^2}}\right]$$

and the mean magnetizing force of this layer referred to the given length ($2b$) of wire

$$X=\frac{\pi ni}{ab}\left[\sqrt{(a+b)^2+r^2}-\sqrt{(a-b)^2+r^2}\right]$$

The following are the data describing the field actually used:

$$\Sigma X_b=35 \qquad \Sigma X=528 \qquad \frac{g^{\frac{1}{2}}}{cm^{\frac{1}{2}}\,sec}$$

It is unfortunate that the conditions of a more uniform field could not be secured. But in view of the very large central intensities ($X_0=940$) the belief is warranted that the maximum of magnetization must at least have been very nearly attained. A uniform field of the intensity, 130 to 140, is usually regarded as sufficient to magnetically saturate iron. Our mean value (528) is about four times as large.

The actual measurement of the influence of magnetization on the thermo-electric power of iron was made in such a way that two separate series, each of five observations of electro-motive force and temperature for the rod free from magnetism (*i. e.*, without magnetizing current), included a single series of the same number of observations for the magnetized rod (*i. e.*, with current). The thermo-electric constants of the first and third series were therefore to be regarded as identical,

[109] Chap. II –36.

while those of the second would differ from them by an amount equivalent to the magnetic effect to be measured. In order that all direct action of the helix on the galvanometers or parts of the circuit might be avoided, the former with its attached thermo-element was placed at a great distance (100 feet) from the latter. These remote parts of the set of apparatus being connected by copper wire of a given kind, no discrepancies due to extraneous thermo-currents in the connections were to be apprehended. On all these points we satisfied ourselves by special tests.

Results.—(1) The results of the three series of observations in question are contained in the following table. If e be the electro-motive force for the temperatures t and T of the junctions, we have

$$e=a(T-t)+b(T^2-t^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

a formula to be applied to the three cases. We proceeded as follows: Having calculated the constants a and b for the series of observations *I* and *III* (unmagnetic iron-copper) by the method of least squares, the mean of the two values of each was taken, and together with the temperatures T and t belonging to the series *II*, was used in deducing values of e for this series, by the aid of formula (1). In the table the latter are given in parentheses. They are therefore such as would hold for the element unmagnetic copper-iron under circumstances identical with those in which the element magnetic copper-iron was actually placed.

TABLE 45.—*Thermo-electric power of magnetic and unmagnetic iron.*

	t	T	e observed.	e calculated.	Diff.	
	°C.	°C.	*microvolts.*	*microvolts.*		
I. Iron wire not magnetic[1]	15.5	71.9	564.1	564.0	+0.1	$a=$ 11.860
	15.4	60.1	456.9	458.3	−1.4	$b=-$ 0.0213
	15.3	50.0	364.7	363.3	+1.4	
	15.3	39.7	261.7	260.8	+0.9	
	15.2	31.0	171.3	171.9	−0.6	
II. Iron wire magnetic	15.1	83.6	674.2	(669.3)	+4.9	
	15.0	72.5	578.8	(575.5)	+3.3	
	15.0	58.2	448.9	(445.5)	+3.4	
	15.0	46.2	332.5	(329.6)	+2.9	
	14.9	35.9	229.5	(226.5)	+3.0	
III. Iron wire not magnetic	14.8	90.8	733.8	732.3	+1.5	$a=$ 11.863
	14.8	79.3	635.9	637.2	−1.3	$b=-$ 0.0211
	14.8	67.8	536.2	536.3	−0.1	
	14.7	54.0	408.8	409.2	−0.4	
	14.7	45.0	325.7	325.2	+0.5	

[1] Electro-motive force relatively to copper (given sample), as zero, in each of the three cases.

An inspection of the column of differences proves conclusively that longitudinally magnetic iron wire is thermo-electrically more positive than iron free from magnetism. We have, therefore, a corroboration of Thomson's result.

Nevertheless, the differences in question are small, and if we compare them with the set of values for the series I and III in the same column we infer that they must be largely distorted by incidental errors. It appears, therefore, that the observations are not adequately exact for

the calculation of the constants of the element magnetic-unmagnetic iron with the desirable accuracy, though they do furnish a good estimate of their value.

(2) For this reason we made a second set of experiments differing from the above in so far as much greater intervals ($T-t$) of temperature were chosen. T and t being kept as nearly constant as possible, measurements of e for the magnetized and unmagnetized state of iron were alternately made. Supposing the resistances of Bosscha's zero method to have been so adjusted that for given values of T and t and unmagnetic iron no current was appreciable in the galvanometer, it was found that a permanent deflection at this instrument immediately occurred on closing the magnetizing circuit through the helix, of such a kind as to indicate an *increase* of thermo-electromotive force. That this effect is of purely thermo-electric origin, and is not to be directly ascribed to extraneous influences, follows conclusively from the regularity of increase of e with increasing values of T. To produce different but temporarily constant values of the latter quantity the boiling point of water and of aniline and the melting point of lead were found serviceable.

The table below contains mean values of electro-motive force and temperature, the results being derived from a great number of measurements of e, with alternately open and closed magnetizing current, for each pair of values of T and t. The differences between the values of e corresponding to magnetic and unmagnetic iron respectively, enable us to calculate the constants a and b for the thermo-couple composed of two identical iron rods, one of which, however, is magnetically saturated, the other free from magnetism.

TABLE 46.—*Thermo-electric power of magnetic and unmagnetic iron.*

	t	T	e observed.	e calculated.	Diff.	
	°C	°C	*microvolts.*	*microvolts.*		
I. Iron wire, not magnetic[1]	16.5	99.3	785	787	−2	$a=$ 12.49
	15.4	184.2	1256	1251	+5	$b=-$ 0.0240
	17.	328.	1173	1178	−5	
II. Iron wire, magnetic[1]	16.5	99.2	788	790	−2	$a=$ 12.43
	15.4	184.2	1267	1261	+6	$b=-$ 0.0248
	17.	328.	1199	1204	−5	
Thermo-electric couple: magnetic-unmagnetic iron[2]	16.5	99.2	4.0	4.2	−0.2	$a=+$ 0.035
	15.4	184.0	11.3	10.5	+0.8	$b=+$ 0.00014
	17.	328.	25.4	25.8	−0.4	

[1] Thermo-electromotive force relatively to copper (given sample) as zero.
[2] Thermo-electromotive force relatively to unmagnetic iron as zero.

Value of the effect.—The thermo-electric effect of magnetization, as given by the constant a, is therefore +0.035. The corresponding effect due to tempering—*i. e.*, the difference between the constants a for the soft and for the glass-hard state—attains values as high as 13.0. Hence,

even if the behavior of iron and steel were qualitatively the same—an unfavorable supposition not in accord with Thomson's observations—the increment of thermo-electric power resulting from complete saturation would only amount to a few tenths per cent. of the thermo-electric interval between soft and glass-hard steel. But in our experiments we shall have to do with permanent magnetism only, whence it follows that the discrepancy will be reduced to a much smaller value. We conclude that the thermo-electric hardness of a steel rod is practically *independent* of its magnetic state.

A remark on the general character of the strains met with in this chapter is in order here:

(1.) In the case of steel (cylindrical rods) hardened by tempering, we observe a dense external shell surrounding an abnormally rare core, in such a way that greatest intensity of stress is exerted in the radial direction; *i. e.*, at right angles to the axis of figure. Tempered steel has been shown to be thermo-electrically negative towards soft steel.

(2.) In the case of steel hardened by drawing through a wire-iron, we observe a compressed external shell (though of far lesser density than in the first case) surrounding a slightly rarified core, in such a way that greatest intensity of stress is exerted in an axial direction. Hard-drawn steel is thermo-electrically positive toward soft steel.[110]

(3.) If an iron wire is longitudinally stretched and the deformation temporary, then the strain, so far as its electrical indications are concerned, is similar to the magnetic strain in iron.

[110] Magnus: Pogg. Ann., LXXXIII, p. 469, 1851.

CHAPTER V.

ON THE INFLUENCE OF HARDNESS ON THE MAXIMUM OF MAGNETIZATION WHICH THIN CYLINDRICAL STEEL RODS OF DIFFERENT DIMENSIONS PERMANENTLY RETAIN.

PLAN AND PURPOSE OF THE PRESENT EXPERIMENTS.

Earlier results.—The relation existing between the magnetic properties and the state of hardness of steel has been made the subject of a very large number of investigations.[111]

Dimension-ratio and hardness.—After the apparently discordant results of J. Müller, Plücker, and Wiedemann, on the one hand, and Hansteen and Lamont, on the other, had been interpreted by Ch. Ruths in a very thorough investigation—subsequently corroborated by Fromme—the present question may be said to have been answered in so far as the practically important factor, the ratio of dimensions of the material experimented upon, is concerned. It was herewith conclusively proven that both temporary[112] and permanent magnetism show a totally different kind of dependence on the state of hardness of steel when the saturated rods are long than when they are short. On the basis of these researches our knowledge of the practically more important permanent magnetism, more particularly the maximum possible for a given cylindrical steel rod, may be summarily stated as follows:

So long as the ratio of length to diameter is smaller than a certain

[111] Digests of the earlier papers may be found in J. Lamont, Handbuch des Magnetismus, p. 249, 1867; G. Wiedemann, Galvanismus, II *a*, p. 340, 1874. Among the more recent publications (since 1876) the following are especially to be mentioned: Ch. Ruths, Inaug. Dissertion, p. 34, Darmstadt, 1874; Ch. Ruths, Ueber den Magnetismus weicher Eisencylinder, etc., Dortmund, 1876; J. M. Gangain, Comptes Rend., LXXXII, p. 144, 1876; C. Fromme, Göttinger Nachrichten, p. 157, 1876; Trève et Durassier, Comptes Rend., LXXXII, p. 145, 1876; A. v. Waltenhofen, Dingler's Journal, CCXVII, p. 357, 1876; ibid., CCXXXII, p. 141, 1879; Gray, Phil. Mag. (5), VI, p. 321-332, 1878; A. Righi, Beiblätter, V, p. 62, 1881; W. Metcalf, Beiblätter, V, p. 895, 1881; A. Pictet, Arch. de Gen. (3), VI, p. 113-125, 1881.

The remarks made in this and the following chapters were originally published in the Verhandl. der phys.-med. ges. zu Würzburg," N. F. XVII, p. 19, 1882. The present paper differs from these in form, the observations having been reduced throughout to the current absolute electromagnetic units, and by containing a large number of experiments made since the date of original publication. It will be found that the latter have added essential corroboration to the inferences drawn. The reduction in question, presupposing a knowledge of the contents of Chapter I, has not until lately been feasible.

[112] The terms "permanent" and "temporary" as used here have the signification proposed by G. Wiedemann in Beiblätter I, p. 67, 1877.

limiting value, apparently characteristic of the material chosen, cylindrical rods are capable of retaining a greater intensity of magnetism permanently when in the glass-hard than when in the annealed state; but after the ratio in question increases to values above this limit, hard rods are overtaken in this respect by the yellow annealed; these in turn by the blue annealed. Soft rods, *cæteris paribus*, usually retain more magnetism than hard or annealed rods of the same dimensions, except when the ratio of axes is small.

Carburation.—The results in hand for the specific magnetic effect of carburation are even more unsatisfactory. We possess at best cursory or isolated data, due to Barlow,[113] Müller,[114] Jamin,[115] v. Waltenhofen,[116] Pictet.[117] To our knowledge the only attempt at a systematic research with reference to the effect of carburation was made by Trève and Durassier.[118] But the results of these observers are far from being even approximately complete. They are, moreover, seriously obscure, because insufficient attention is paid both to hardness and dimensions.

Taken as a whole, therefore, the work done, however valuable, can serve for orientation only. It is of the nature of a mere estimate. The full and minute analysis must introduce all those complications of a metallurgical kind which render the term "steel" itself indefinite. In this place the effect of carburation cannot even be discussed with advantage. It will, therefore, be waived, to be resumed in Chapter VII of the present memoir, where we shall have occasion to review the physical characteristics of the iron carburets in general.

Critical discussion.—Returning from this digression to a discussion of the magnetic effect of the hardness and of the dimensions of steel rods, we find it undeniable, although the main features of the subject may be said to be roughly outlined, that the information gained is almost intangibly vague and unsatisfactory. In fact, the solution is at most a qualitative one. The data of the above researches, in other words, do not enable us to trace the continuous change of magnetic intensity of thin saturated steel rods, considered for the time being as a function of a single independent variable hardness only—the steel passing from an initial (soft) to a final (hard) state through every intermediate state, dimensions and carburation being regarded as parameters. And yet it is only from results of this character that an intelligent insight into these complicated phenomena can be obtained.

There are two difficulties which have thus far stood in the way of more exact research in this direction. The first of these was the want of a sufficiently sensitive method for the discrimination between degrees of

[113] Barlow: Phil. Trans. p. 117, 1822.
[114] Müller: Pogg. Ann., LXXXV, p. 157, 1852.
[115] Jamin: Comptes Rend., LXXVII, p. 89, 1873.
[116] V. Waltenhofen: Pogg. Ann., CXXI, p. 431, 1864.
[117] Pictet: Arch. de Genève (3), VI, p. 113, 1881.
[118] Trève et Durassier, Mondes, XXVIII, pp. 587, 667, 1875.

hardness. The estimate furnished by the oxide tints,[119] if indeed rigidly reliable, is much too crude. The same would be true of any other of the more current methods. After we had found, therefore, how well the electrical properties of steel can be thus utilized—after we had seen, for instance, that the annealing effect of temperature as low as 50° C. can be pursued and studied with the utmost nicety—the application of our method to magnetic phenomena at once forcibly presented itself. Subsequently certain analogies, which appear to exist between hardness itself when measured electrically and permanent magnetic moment, rendered the application of our method even more desirable. With regard to the second difficulty, we want in this place again to emphasize that the behavior of rods of different dimensions cannot by any means be immediately compared unless, *cæteris paribus*, the same thickness occurs throughout. Most of the observers cited, however, have obtained their results with rods for which different values of $L : D$ were secured[120] by retaining L constant and varying D. Now, it is, *a priori*, to be anticipated that glass-hardening, accompanied or not by subsequent annealing, will be productive of different effects, of states of hardness, which, even if temperature and time are identical, are structurally[121] the more dissimilar in proportion as the rods subjected to these operations differ in diameter. Nor is this all. Difference of thickness usually implies a difference in chemical composition, unless, indeed, the specimens chosen are cut from the same piece of steel. To our knowledge, in none of the above researches (rods from 0.2 centimeter to 0.7 centimeter in diameter) was this done. How very different the behavior of rods frequently is, even when coming from the same source and nominally of the same kind of steel, will appear from several examples to be cited in the next paragraph.

Thomson's law, in accordance with which geometrically similar rods show equal moments per unit of mass when placed in identical magnetic fields, is probably true for iron, but cannot be immediately applied to the case of steel: possibly it may be applied to very soft rods; to a lesser extent to those in the annealed state; certainly not to glass-hard rods, inasmuch as here the peculiarities of internal structure resulting from the process of tempering manifest themselves, even in the case of

[119] It is highly probable that a given oxide tint can be obtained in an infinite number of ways by varying temperature and time of exposure simultaneously: lower temperatures acting for very long intervals of time producing the same color effect as higher temperatures for short periods of exposure. Researches on this subject are in progress. Possibly the variation of color may correspond very closely with the actual variation of hardness, as discussed in Chapter II.

[120] All the above remarks apply to cylindrical rods of length L and diameter D.

[121] Structure, as used here and elsewhere, refers to the variation of density encountered on passing along a radius from the axis of the rod to the circumference. In how far the structure of rods of different diameters will vary cannot even be conjectured.

geometrically similar figures, in a way that compels us to treat each thickness individually.[122]

The present experiments.—The above remarks, we believe, clearly show that the whole of our knowledge on the relation of magnetism to the variables, carburation, hardness, and dimensions, is urgently in need not only of reconstruction, but of further elucidation; and we took great delight in welcoming this opportunity for the practical application of the experience which the attention of many years to the physical properties of steel has given us. The first part of our projected magnetic researches, referring principally to long, thin rods, to annealing temperatures below 500° C., and to the soft state, are herewith made public. The second part, on the behavior of short, thick rods, and on the influence of temperatures between 400° and 1,000°, will have to be reserved for another communication. We may add, more specifically, that for the 30 rods examined in the present experiments the ratio of dimensions, $\alpha = \frac{L}{D}$, varied between $\alpha = 120$ and $\alpha = 10$. But the data in this chapter enable us to predict the magnetic behavior, even for magnets in which the said ratio (α) lies below 10 or above 120, with a degree of probability amounting almost to certainty. But, in view of the importance of thick magnets, now so largely used in magnetometric and galvanometric work, a new research on this subject seems pertinent. Similar remarks apply to high annealing temperatures.

We conclude this paragraph with the appropriate remark that our knowledge of the excessively complicated phenomena under consideration can be advanced by tentative methods alone. The mathematical theory of the subject, even in case of homogeneous material, encounters formidable and almost insuperable difficulties. But we will be able to show in the sequel that the condition of internal structure is one of the most essential factors which determine the magnetic character of the (tempered) rod. Again, the investigation of the true nature of the structural properties of hard steel is surrounded with difficulties which will long baffle the experimentalist's skill. For in all such methods, like the one which Fromme pursued in order to obtain preliminary data on the condition of the interior of a steel rod, and in which the external layers are gradually and consecutively removed by acids or otherwise, we apprehend that the rod, for the very reason of this removal, will change its physical condition (strain) while in the hands of the operator. Long before we reach the core of the rod its density and electrical properties must be conceded to have changed materially. Nevertheless, we infer that the structure of tempered steel is of a thoroughly definite and distinct character; that this is emphatically

[122] This inference, which we published in the same form in 1882, has since been corroborated by H. Meyer, in his paper "Ueber die Magnetisirungsfunction von Stahl u. Nickel," Wied. Ann., XVIII, p. 248, 1883. Meyer contends that Thomson's law is true neither for soft nor for hard steel rods.

demonstrated by beautiful consistency and uniformity of our different series of results, obtained with different steel rods, tempered at different times.

ON THE MATERIAL USED.

The steel.—The steel chosen was that described in the previous chapter. It was received in the shape of rods 30 centimeters long and from 0.084 centimeter to 0.150 centimeter in diameter. These were hardened as shown above (Chap. II, pp. 29–31). Analyses are superfluous, as it is our intention at present to study the results for a single value of the variable parameter, carburation, only; or, in other words, to investigate the magnetic behavior of a given type of steel. From the large supply of tempered rods such were chosen as not only showed a maximum of thermo-electric hardness for the glass-hard state, but also, when tested for longitudinal homogeneity in the manner described in pages 38, 121, gave the most satisfactory results.

Rods of like composition, thickness, temper.—At the outstart we were inclined to infer that from rods of the same kind of steel (composition), of the same thickness, and tempered in the same manner—in so far as the details of this process are at the observer's control—comparable values of the maximum of magnetic intensity could be immediately derived. For the purpose of testing this supposition, rods of nearly the same thermo-electric hardness (II, p. 65), but of different lengths, were broken from different samples of the class of rods experimented upon. These, after having been subjected alike to the action of the same magnetic field—in our case of unnecessarily great intensity and fairly uniform—according to Thomson's law ought to have retained a moment per unit of mass such as would be expressible by the same function of the ratio of length to diameter. Moreover, as the thickness of the rods chosen was practically the same, this function would have to increase to a certain limiting value, as the corresponding lengths increased from zero indefinitely.

These anticipations were, however, by no means realized. In this place it will be well to add, by way of example, such results as are particularly applicable for the illustration of the consideration just made:

A magnet 4.1 centimeters long, 0.084 centimeter thick, weighing 0.173 gram, specific resistance $s=39.8$ at 17.6 degrees, after saturation was found to have the specific magnetism $m=43.0$; whereas another specimen taken from a different rod, the magnet being only 3.05 centimeters long, 0.084 centimeter thick, weighing 0.131 gram, specific resistance $s=39.4$ at 18.5 degrees, after saturation had retained $m=52.0$; whence it appears that in the second case, notwithstanding the fact that the length (or ratio of dimensions) was only about three-fourths of that

in the first, a very decidedly greater intensity of magnetization was attainable. This is hardly referable to a difference of composition. In all probability the result is to be associated with peculiarities of the internal structure of the rods.

The following example is still more to the point:

A magnet (*a*) 5.0 centimeters long, 0.084 centimeter thick, weighing 0.214 gram, of the specific resistance $s=40.4$ at 18.5 degrees, had retained the specific magnetism $m=45.8$; another taken from a second rod of the same thickness, the magnet (*b*) being 5.9 centimeters long, as before 0.084 centimeter thick, weighing 0.249 gram, of the specific resistance $s=39.7$ at 18.3 degrees, gave, after saturation, $m=45.0$.

In order to arrive at some inference as to the cause of this difference of magnetic state, both the magnets last mentioned were slowly annealed, at first for a period of ten hours in steam, thereupon for an additional period of ten hours in aniline vapor at 185 degrees, finally for one hour in melted lead at 330 degrees. After this both were softened by heating to redness and cooling slowly. For each of the particular states of hardness thus obtained the specific resistance was measured. The rods were then magnetized to saturation and their magnetic moment determined. The results of these precursory experiments are given in the following table:

	(*a*)			(*b*)		
	s	*t*	*m*	*s*	*t*	*m*
Glass-hard	40.6	18	45.8	39.9	18	45.0
10^h in 100°	35.2	20	43.1	35.1	20	42.7
10^h in 185°	26.6	20	62.2	26.4	20	61.7
1^h in 330°	20.3	20	84.3	20.1	20	86.4
Soft	16.9	20	46.8	16.7	20	56.0

From an inspection of the table, or of Fig. 15, it is obvious that at 330° (lead bath) the magnetic intensity of the longer magnet (*b*) has overtaken that of (*a*), but it is not until both magnets are quite soft (homogeneous) that the normal behavior fully appears. It is to be noted, however, that this experiment is not in itself perfectly conclusive. The influence of carburation is principally efficient in the glass-hard state; $L:D$, if its value be as large as in the present case, less so. In the soft state, however, the effect of $L:D$ predominates. It would be possible, therefore, with due consideration of the values of *s*, to ascribe the results obtained to a difference of carburation between (*a*) and (*b*). The nearly equal values of specific resistance for the soft state, especially when viewed in the additional light given by the first example, make the latter inference improbable.

Each series of magnets originally integrant parts of the same hard rod.—The individuality of behavior of different rods (*i. e.*, such as are not

pieces of the same homogeneous rod, though otherwise identical) appears, therefore, to be conclusively demonstrated.[123]

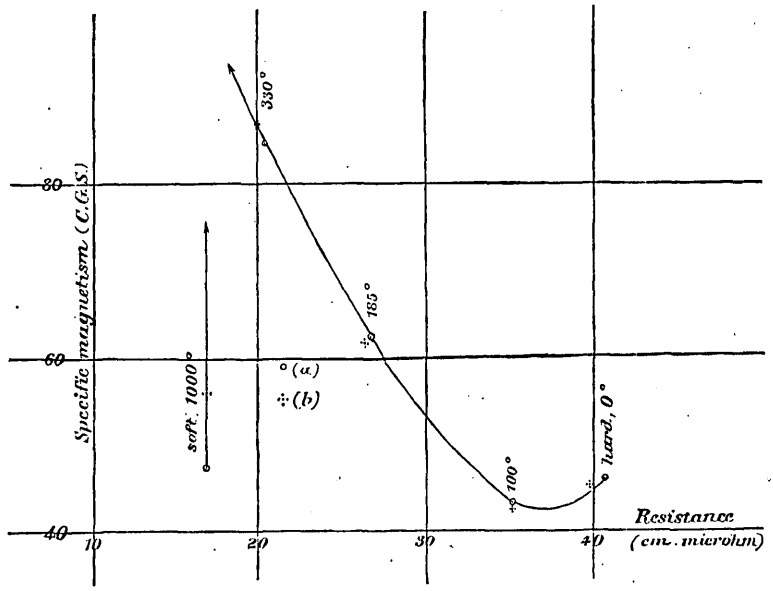


FIG. 15.—Specific magnetism of rods annealed from 0° to 1000°.

Results of this kind—a number of other examples might be cited—urged the inference upon us that only in the case where magnets are broken from one and the same longitudinally homogeneous hard rod, can comparable and harmonious results be expected. Add to this the fact that the frequent reheatings to redness, necessary in drawing a wire to small diameter, imply a difference of carburation in different samples, for this reason alone. However insignificant this may appear to be, we were convinced that its effect, where accurate results are called for, is by no means negligible. These instances are adequate to give emphasis to the statement above made, that each rod must be examined and treated separately. Indeed, a better opinion of the longitudinal homogeneity of a wire can be formed subsequently from an inspection of the behavior of the different magnets taken out of it, than could be formed at the outset by the aid of the electrical properties of different parts of the rod in question.

Unfortunately, thin (diameter $= 2\rho = 0.084$ centimeter) glass-hard rods, of the same hardness throughout their length, out of which magnets of the kind desired, having different values for the ratio of dimension ($L : D$), can be broken, are only obtainable with great difficulty.

[123] It is quite clear that even the details of the process of tempering have an important bearing on the magnetic intensity of hard rods. A single rod repeatedly rehardened shows very marked differences in the maximum of permanent magnetization for nominally the same state.

Even those which were here finally accepted as the best for the present purposes, after the test for homogeneity was applied, proved to be satisfactory for only two-thirds of their length. The remaining third had to be discarded. There are some drawbacks in the way of heating a thin steel rod uniformly by means of the electric current. Greater difficulty is encountered in chilling it suddenly and alike throughout its length, in consequence of the large rate at which radiation takes place. Our thick rods (diameter $= 2\rho = 0.150$ centimeter) were much more uniform. The samples selected from our supply showed differences of electrical conductivity for different parts of their lengths, amounting to only a few tenths of one per cent.

METHOD OF MAGNETIZATION.

Helix. Magnetic field.—Magnetization was effected by the inductive action of a galvanic current in a helix. As a source of current the Hefner-Alteneck dynamo-electric machine of the Physical Institute of Würzburg was at our disposal. A magnetic field of very great intensity could thus easily be produced, and discrepancies due to non-saturation were therefore not to be apprehended.

The helix referred to (length $2a=22.3$ centimeters, inner radius $r=2.1$ centimeters, outer radius $r=5.3$ centimeters) contained ten layers, in each of which there were $n=55$ turns of thick copper wire.

If a current, i, circulates through the helix, the magnetizing force X_b at a point on its axis at a distance b from its center will be

$$X_b = \frac{\pi n i}{a}\left[\frac{a+b}{\sqrt{(a+b)^2+r^2}} + \frac{a-b}{\sqrt{(a-b)^2+r^2}}\right]$$

In our case the mean value of current was

$$i = 3.0 \frac{g^{\frac{1}{2}} cm^{\frac{1}{2}}}{sec} = 30 \text{ Ampères}$$

If this value is introduced into the formula specified and the aggregate of the value of X_b calculated, the total intensity ΣX_b of the magnetic field at different points b (cm) on the axis varied as follows:

$b=$	0	1	2	3	4	5	(cm)
$\Sigma X_b=$	884	882	879	874	865	851	$\left(\frac{g^{\frac{1}{2}}}{cm^{\frac{1}{2}}\, sec}\right)$

If it be remembered that the diameter of our magnets was less than 1 millimeter, these figures show in how far the field by which magnetization was effected may be said to have been uniform. The longest magnet was 10 centimeters. In this extreme case the variation of force throughout its length amounted to less than 4 per cent. In the more

usual case of magnets of lesser length much smaller variations present themselves. On the other hand, where permanent magnetism is alone of interest and the field of great intensity, the desideratum of uniformity is perhaps of minor importance.

The mean intensity X of the magnetizing force due to a single layer of wire and corresponding to a length $2b$ (cm) symmetrically located with respect to the center of the helix is

$$X=\frac{\pi ni}{ab}\left[\sqrt{(a+b)^2+r^2}-\sqrt{(a-b)^2+r^2}\right]$$

In the present case the aggregate ΣX of these means, or the mean intensity of the field due to the whole helix, corresponding to the part $2b$ of the axis situated as specified was

$b=$	0	1	2	3	4	5	(cm)
$\Sigma X=$	884	883	882	880	877	874	$\frac{g^{½}}{cm^{½}\,sec}$

The largest values of ΣX employed by Ruths, for instance, in his researches on temporary magnetism, were $\Sigma X=40$ for iron and $\Sigma X=147$ for steel. There can be no doubt, therefore, that in our case the most complete saturation possible was actually attained. Indeed, we had frequent occasion to convince ourselves that successive magnetization produced no additional effect equivalent to one per cent. of the whole moment.

Magnetization.—In the case where magnetism is induced by means of a helix, the question in reference to the manner in which the magnets are to be introduced and withdrawn from its action is always of importance. Some observers, among them J. Wiedemann and C. Fromme, placed their magnets in the helix after the current had been closed and withdrew them before breaking it. In this way the complications due to certain undesirable induced currents are avoided. At the same time, however, the advantages of a uniform field are lost and a normal distribution of magnetism possibly interfered with. For the last mentioned reasons, Holz and Fromme preferred to open and close the current while the rod to be magnetized lay in place. Both methods have been made the subject of an experimental comparison by Fromme.[124]

We gave preference to neither of these methods, but chose a plan which, under the circumstances, was also the most convenient. It consisted in allowing the current to increase from zero to its maximum value gradually and then again to wane in the same manner. This may be satisfactorily accomplished by appropriately applying the power to the dynamo-electric machine. We think we are justified in the belief that in this way the permanent magnetic moment is finally weakened only to a negligible extent, at least to a smaller amount than in either of the other methods.

[124] C. Fromme, Wied. Ann., V, p. 345, 1878.

In order to give the magnets the desired symmetrical position, relatively to the helix, a glass tube was secured coaxially with it. Into one end of this the little rods were introduced to be drawn into its center by the force of magnetic attraction. Each rod was magnetized twice, an interval of about 30 seconds[125] intervening.

MEASUREMENT OF MAGNETIC MOMENT.

Apparatus.—The magnetic moments of the wires were derived from the deflection of a small magnetized steel mirror, observed after the manner of Poggendorf with telescope and scale. We made use of an appliance due to Professor Kohlrausch, by means of which the attached magnets could always again be brought into the same position relatively to the mirror, and rotated 180° around the vertical. Both the first and the second method of Gauss were applied.

The apparatus consisted of a solid brass ring provided with leveling screws, from which two vertical columns of the same metal arose supporting a circle graduated in degrees. An alhidade, with two verniers, resting on conical bearings, occupied a central position within the latter. A horizontal brass bar, in connection with the middle of the alhidade, and adjustable with reference to the vertical, carried on its lower edge the special device above referred to, by the aid of which the magnets were appropriately fastened in position. This apparatus was fixed in place and adjusted at the outset, and the distance between the axis and that of the needle determined once for all. In this way the relative values of the moments of our magnets (and it is from these that the results of the present paper are principally deduced) are independent of discrepancies introduced by errors in the determination of the distance between magnet and mirror.

Method.—In calculating the results we made use of the ordinary formulæ

$$M=\tfrac{1}{2}\frac{r^3\,T\,\text{tang}\,\varphi}{1+\tfrac{1}{2}\frac{l^2}{r^2}}$$

for the first, and

$$M=\frac{r^3\,T\,\text{tang}\,\varphi}{1-\tfrac{3}{8}\frac{l^2}{r^2}}$$

for the second of the positions of Gauss, where T is the horizontal intensity of terrestrial magnetism at the place where the observations are made $\left(\text{in our case } T=0.196\,\frac{g^{\frac{1}{2}}}{cm^{\frac{1}{2}}\,\text{sec.}}\right)$, r the distance between magnet and needle (in our adjustment $r=26.34$ centimeters), φ the angle of de-

[125] Frankenheim was the first to show that the number of times a magnet is exposed in a given field, and not the duration of such exposure, is of marked influence on the resultant moment (Pogg. Ann., LXXIII, p. 49, 1864).

flection, l the distance between the poles. We assumed $l=0.85\,L$ where L is the length of the magnet. This approximation is sufficient because the term of the above formulæ involving l is only of the nature of a correction.

The measurements were made at ordinary temperatures, varying between 18° and 21°. The effects of the changes in the magnetic moment, as well as those due to variation of terrestrial magnetism, we regarded negligible. This is also true with reference to the torsional discrepancy of the silk fiber (the torsion-coefficient was found to be only 0.00032).

In addition to the magnetic moment M of the whole rod we shall give throughout the moment m per unit of mass (1 g.). To this quantity the name *specific magnetism* has been given. In the case where a number of different magnets are compared it is a more perspicuous result than the former.

DETERMINATION OF THE DEGREE OF HARDNESS.

Resistance measurement.—Of the two methods of estimating the hardness of the rods proposed in Chapter II, one of which depends upon thermo-electric properties, the other upon the specific resistance of steel, we chose the latter because of its greater simplicity. Here again it was our object, primarily, to arrive at accurate relative values, and this we were able to accomplish satisfactorily by the aid of Matthiessen and Hockin's modification of the bridge. In one of the branches of Wheatstone's combination, the bridge cylinder of F. Kohlrausch (diagramatically represented in Fig. 16 by the straight line AB) was inserted; in the other the steel wire D_1D_2, whose resistance is to be measured, and a tenth Siemens unit Z_3Z_4, the whole being appropriately connected with thick copper wire. Flat clamp-screws of brass pressed the steel rod down firmly upon a plane surface. In this way the danger of breaking the very brittle magnets was less seriously to be apprehended. The normal Z_3Z_4 was made of heavy German silver wire soldered to thick terminals. These were amalgamated and in galvanic connection with the bridge by means of mercury. From the points A and B the copper-connecting wires lead to a galvanometer, the deflections of which are observed with telescope and scale. As a source of current two Smee elements were found sufficient.

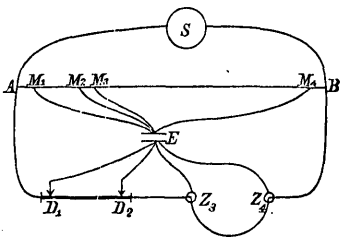


FIG. 16.—Disposition of apparatus for resistance measurement.

In figure 16, if the points D_1, D_2, Z_3, Z_4 correspond to the points M_1, M_2, M_3, M_4, in such a way that when connected respectively with the

terminal of the battery, the current in the galvanometer (S) is zero, we have

$$\frac{D_1, D_2}{Z_3, Z_4} = \frac{M_1, M_2}{M_3, M_4}$$

To obtain the points of contact D_1 and D_2, we made use of two steel needles, the points of which had been flattened wedge-shaped. These were fastened parallel to each other, and at a fixed distance apart, in a small piece of wood. To their tops the ends of two long thin copper wires were soldered. This arrangement, after having been screwed to a longer ⌐-shaped piece of wood, along which the copper wires were led, represented as a whole a kind of tripod, two of the feet being the steel needles mentioned. During the measurements the latter stood on the steel rod to be tested. Small clamp-screws served for connecting the battery-wires with the terminals of this apparatus, these being sufficiently long and thin that the successive clamping and unclamping did not interfere with the adjustment in such a way as to slide the points D_1, D_2 along the steel rod, for instance.

This simple contrivance, which is easily improvised, gave us results of the most satisfactory kind. Our resistances were rarely larger than a few hundredths ohm. Nevertheless they were determinable with an accuracy of a few tenths per cent. with certainty. Our mirror-galvanometer enabled us to adjust the sliding contact on the bridge-cylinder of Kohlrausch to $\frac{1}{10}$ a scale-part, or $\frac{1}{10000}$ of the length of the wire A, B. In virtue of a fixed distance D_1, D_2—we possessed a number of these arrangements to be used according as a larger or a smaller wire was to be operated upon—in view of the stability of the apparatus as a whole, and of the fact that all operations were conducted in a room of practically constant temperature, an unusual accuracy in the relative data was attained.

It is obvious that this method is applicable at once as a test for the degree of uniformity of hardness along the length of the wire. By placing the contrivance just described on different consecutive lengths without making any further alteration in the adjustments, the respective resistances of these parts could be subjected to a minute comparison. This furnishes us with the criterion desired.

Great difficulties were encountered in the measurement of the sections of the steel wires. We determined the diameter microscopically, with the aid of an ocular micrometer of known factor. Two readings, corresponding to diameters at right angles to each other, each pair taken at a number of different, approximately equidistant parts of the wires were made. The mean of these individual values was accepted. This plan was fairly good in so far as the error made in the measurement of the sections does not influence the relative values of resistance.

From the observed resistance of any wire at t degrees, the value of this quantity per meter, in ohms (w), was calculated. A knowledge of the tem

perature-coefficients of steel[126] then enabled us to deduce herefrom the specific resistance (resistance in microhms between opposite faces of a cubic centimeter) for the temperature 0°. This value, $s\left(\frac{\text{cm}}{\text{cm}^2}\,0°\text{ microhm}\right)$, has been adopted as a measure of hardness.

From s, the thermo-electric hardness of our wires may be calculated by the aid of the formula

$$h=0.412\,s$$

In this way the present scale of hardness is made identical with that previously employed in our work on the phenomena of annealing. The data h are therefore proportional to the specific resistances of the steel rods and have a very simple connection with the respective thermo-electric constants a, as has been shown elsewhere.[127]

METHOD OF ANNEALING.

Systematic plan.—In the different series of experiments the glass-hard state was chosen as the point of departure. After the rods had been examined with reference to their hardness in the way described, they were magnetized to saturation and their magnetic moment determined. Thereupon we exposed them to the annealing effect of steam at 100°. A sufficiently small decrement of hardness resulted by continuing the operation for one hour at first, and then for additional two, three, and four hours, amounting in the aggregate to ten hours of exposure. This is sufficient to bring the rod as near the limiting state, or the maximum of permanent hardness corresponding to 100° as is easily practicable.

The magnets were now subjected to the action of aniline vapor at 185°, during successive intervals of twenty minutes, forty minutes, two, four, and finally six hours.

A lead bath at 330° was next chosen. At first the magnets were annealed therein for a time of one minute only. Then the action was prolonged to one hour. In the later experiments the melting point of tin (240°) as well as that of zinc (420°) were found serviceable as temperatures of annealing.

Finally the rods were thoroughly softened in the usual way, by heating to redness and cooling very slowly. To avoid oxidation of the magnets we imbedded them in pulverized artificial ferro-ferric oxide, surrounded this with fire-clay, and finally enveloped the whole with a piece of gas-pipe.

In this way we obtained about twelve distinct states of hardness, distributed with fair uniformity between the glass hard and soft states, as

[126] Cf. Chapter I.

[127] Chapter II, p. 63.

extremes. It is only between the soft condition and that corresponding to the temperature 300° of the annealing bath that a greater number of degrees of hardness, as our results eventually showed, would have been desirable. Particularly in the case of long magnets there appear within this interval changes of magnetic capacity of a very significant character, changes, moreover, which cannot be satisfactorily supplied by a process of interpolation based on the data for neighboring states.

After the magnets had been taken out of the annealing bath it was our custom to put them aside for some time in order to allow a complete cessation of possible "after" effects. Thereupon the test for hardness was applied.

Results of the annealing.—The progress of annealing in the different experiments is fully given in the following tables:

TABLE 47.—*Thin wire.*

[2ρ=0.084 centimeter.]

Description of temper.	Specific resistance $\frac{cm}{cm^2}$ 0° microhm.	
	Rod I.	Rod II.
Glass-hard	38.5	37.3
Annealed 1 hour in steam at 100°	36.3	34.7
Annealed 3 hours in steam at 100°	34.8	33.0
Annealed 6 hours in steam at 100°	33.9	32.2
Annealed 10 hours in steam at 100°	33.4	31.6
Annealed 20 minutes in aniline vapor at 185°	29.5	27.4
Annealed 1 hour in aniline vapor at 185°	28.4	26.3
Annealed 3 hours in aniline vapor at 185°	27.1	24.9
Annealed 7 hours in aniline vapor at 185°	25.9	23.7
Annealed 13 hours in aniline vapor at 185°	25.0	22.8
Annealed 1 minute in melting lead at 330°	20.4	18.9
Annealed 1 hour in melting lead at 330°	18.8	17.4
Annealed at red heat (soft)	15.7	14.5

TABLE 48.—*Thick wire.*

[2ρ=0.15 centimeter.]

Description of temper.	Specific resistance $\frac{cm}{cm^2}$ 0° microhm.			
	Rod IX.	Rod X.	Rod XI.	Rod XII.
Glass-hard	47.2	46.8	43.8	43.5
Annealed 1 hour in steam at 100°	42.0	41.7	38.6	38.4
Annealed 3 hours in steam at 100°	39.5	39.3	36.5	36.2
Annealed 6 hours in steam at 100°	38.2	38.0	35.1	34.9
Annealed 10 hours in steam at 100°	37.4	37.1	34.3	34.0
Annealed 20 minutes in aniline vapor at 185°	31.5	31.4	29.0	28.7
Annealed 1 hour in aniline vapor at 185°	29.7	29.6	27.5	27.3
Annealed 3 hours in aniline vapor at 185°	27.9	27.7	25.6	25.5
Annealed 7 hours in aniline vapor at 185°	26.2	26.0	24.2	23.9
Annealed 13 hours in aniline vapor at 185°	24.8	24.6	22.9	22.7
Annealed 10 minutes in melting tin at 240°	24.0	23.8	22.2	22.0
Annealed 1 minute in melting lead at 330°	20.3	20.1	19.0	18.7
Annealed 1 hour in melting zinc at 420°	17.5	17.2	16.2	16.0
Annealed at red heat (soft)	15.7	15.6	14.9	14.9

In how far we are justified in regarding the scale of hardness for rods in the magnetized state as *cæteris paribus* identical with the scale corresponding to the unmagnetized state has been fully discussed in the previous chapter on the thermo-electric effect of magnetization.[128]

MAGNETIC RESULTS FOR RODS OF LARGE DIMENSION-RATIO.

Description of magnets.—The results of the experiments, the plan of which has been fully detailed in the preceding paragraphs, are given in the following tables.

The first experiments were made with wire of smaller diameter. Breaking rod No. I into three parts, we obtained the magnets Nos. 1, 2, 3; and, similarly, the magnets Nos. 6, 9, 10, from rod No. II. Accidentally, the longest of these, No. 10, during the course of the experiments, was further broken into parts, which, however, in their turn, were available for magnets. Nos. 5, 7, 8 were thus obtained. All magnets are numbered in the order of their respective lengths. The following table (49) contains certain important physical constants of the ten rods. We measured their dimensions first while they were in the glass-hard state; then after the various stages of annealing; finally, before and after thorough annealing at red heat. In this way we were able to arrive at numerical data for the simultaneous volume-contraction with some accuracy. Mean values are given in the tables. In calculating the specific gravity, Δ, from the mass and dimensions of the individual rods, our main object was that of checking the measurement. In view of the small diameter, Δ will not be correct within 1 per cent.

TABLE 49.—*Constants of magnets.*

Rod 2ρ cm, Δ	Magnet No.	Mass μ	Length L	Dim. ratio. α
		g	*cm.*	
I.	1	0.086	2.00	23.9
2ρ=0.0838 cm	2	172	4.05	48.4
Δ=7.70	3	249	5.85	69.8
	4	336	7.90	94.3
	5	068	1.63	19.6
II.	6	126	3.03	36.5
2ρ=0.0831 cm	7	197	4.72	56.8
Δ=7.69	8	236	5.66	68.1
	9	413	9.92	119.5
	10	502	12.06	145.1

The degree of homogeneity of these rods may be satisfactorily inferred from the data for s. In the case of Nos. 1, 5, 6, the method for the measurement of resistance could not readily be applied, the rods being too short. We, therefore, preferred to accept in place of such direct

[128] Wied. Ann., XIV, p. 54, 1881. Cf. W. Beetz, Pogg. Ann., CXXVIII, p. 193, 1866.

results, the values deducible by interpolation from those obtained from rods of like origin. Data of this kind are, however, distinguished in the next table by an asterisk (*) preceding the numbers.

Magnetic data.—The magnetic measurements were made in the second position of Gauss. The numerical values of the constants occurring in the formulæ—

$$m=\frac{M}{\mu} \qquad M=\frac{r^3 T \operatorname{tg}\varphi}{l^2\left(1-\frac{3}{8}\frac{r^2}{l^2}\right)} \qquad 2\operatorname{tg}2\varphi=\frac{n}{R}$$

are as follows:

$$T=0.196\ \frac{\text{cm}^{\frac{3}{2}}}{\text{sec}} \qquad r=26.34\text{ cm} \qquad R=207.4\text{ cm}$$

The results of the magnetic measurements are these:

TABLE 50.—*Relation between the magnetic constants of steel and hardness.*

1. *Magnets in the glass-hard state.*

Rod.	Magnet No.	a	W 1 m	t	$s\frac{\text{cm}}{\text{cm}^2}$ 0°	n	M	m
			ohm.	°C.	*microhm.*	*cm.*	*abs. E.*	*abs. E.*
I...	1	24	*0.726	*18.2	*38.5	1.34	2.89	33.7
	2	48	0.734	17.6	38.8	3.46	7.41	43.0
	3	70	0.727	18.3	38.6	5.28	11.20	45.0
	4	94	0.716	18.7	38.1	7.68	16.06	47.9
II...	6	36	*0.719	*18.5	*37.3	2.58	5.54	43.9
	9	120	0.723	18.5	37.5	10.77	22.11	53.5
	10	145	0.714	18.5	37.1	13.78	27.65	55.1

2. *Magnets annealed 1 hour in steam.*

I...	1	24	*0.687	*18.4	*36.3	1.33	2.85	33.3
	2	48	0.695	18.2	36.7	3.38	7.23	42.0
	3	70	0.688	18.4	36.4	5.19	11.01	44.3
	4	94	0.679	18.6	35.9	7.51	15.70	46.8
II...	6	36	*0.670	*18.6	*34.7	2.53	5.44	43.1
	9	120	0.673	18.6	34.8	10.61	21.78	52.8
	10	145	0.667	18.6	34.6	13.53	27.15	54.1

3. *Magnets annealed 3 hours in steam.*

I...	1	24	*0.662	*20.1	*34.8	1.30	2.80	32.7
	2	48	0.669	20.0	35.1	3.30	7.06	41.0
	3	70	0.665	20.2	34.9	5.08	10.78	43.3
	4	94	0.653	20.2	34.4	7.41	15.49	46.2
II...	6	36	*0.641	*20.1	*33.0	2.50	5.36	42.5
	9	120	0.644	20.2	33.2	10.41	21.38	51.8
	10	145	0.638	20.1	32.9	13.29	26.67	53.2

4. *Magnets annealed 6 hours in steam.*

I...	1	24	*0.648	*21.1	*33.9	1.28	2.76	32.2
	2	48	0.654	20.9	34.2	3.23	6.91	40.1
	3	70	0.651	21.3	34.1	5.00	10.60	42.7
	4	94	0.638	21.2	33.5	7.29	15.24	45.4
II...	6	36	*0.626	*21.0	*32.2	2.47	5.31	42.0
	9	120	0.630	21.0	32.3	10.22	20.98	50.8
	10	145	0.623	21.0	32.1	13.05	26.19	52.2

TABLE 50.—*Relations between the magnetic constants of steel and hardness*—Continued.

5. *Magnets annealed 10 hours in steam.*

Rod.	Magnet No.	a	W 1 m	t	$s\frac{cm}{cm^2}0°$	n	M	m
			ohm.	°C.	*microhm.*	*cm.*	*abs. E.*	*abs. E.*
I....	1	24	*0.637	*20.1	*33.4	1.28	2.75	32.1
	2	48	0.645	20.2	33.7	3.24	6.93	40.2
	3	70	0.640	20.1	33.6	5.00	10.60	42.7
	4	94	0.627	20.0	33.0	7.30	15.25	45.4
II....	5	20	*0.614	*20.1	*31.6	0.93	2.01	29.6
	6	36	*0.614	*20.1	*31.6	2.42	5.19	41.1
	7	57	0.614	20.2	31.6	4.28	9.13	46.4
	8	68	0.612	20.3	31.4	5.30	11.20	47.8
	9	120	0.617	19.8	31.7	10.26	21.06	51.0

6. *Magnets annealed 20 minutes in aniline vapor.*

I....	1	24	*0.566	*20.1	*29.5	1.28	2.76	32.2
	2	48	0.572	20.1	29.7	3.45	7.38	42.8
	3	70	0.568	20.1	29.6	5.38	11.41	45.9
	4	94	0.559	20.1	29.2	7.85	16.41	48.9
II....	5	20	*0.536	*20.0	*27.4	0.92	1.98	29.1
	6	36	*0.536	*20.0	*27.4	2.56	5.51	43.6
	7	57	0.534	20.0	27.3	4.51	9.62	48.9
	8	68	0.536	20.0	27.4	5.69	12.12	51.3
	9	120	0.539	20.0	27.5	11.05	22.69	55.0

7. *Magnets annealed 1 hour in aniline vapor.*

I....	1	24	*0.545	*18.9	*28.4	1.36	2.93	34.1
	2	48	0.550	19.0	28.7	3.63	7.77	45.1
	3	70	0.546	19.0	28.5	5.64	11.96	48.1
	4	94	0.538	18.8	28.1	8.27	17.29	51.5
II....	5	20	*0.514	*18.8	*26.3	0.94	2.04	30.0
	6	36	*0.514	*18.8	*26.3	2.68	5.75	45.6
	7	57	0.515	19.0	26.4	4.75	10.14	51.5
	8	68	0.512	19.0	26.2	5.96	12.69	53.7
	9	120	0.514	18.5	26.3	11.66	23.94	58.0

8. *Magnets annealed 3 hours in aniline vapor.*

I....	1	24	*0.520	*18.9	*27.1	1.44	3.09	36.1
	2	48	0.523	18.8	27.2	3.93	8.41	48.8
	3	70	0.523	18.9	27.3	6.17	13.08	52.6
	4	94	0.514	19.0	26.8	8.99	18.80	53.0
II....	5	20	*0.488	*18.8	*24.9	1.00	2.16	31.8
	6	36	*0.488	*18.8	*24.9	2.90	6.23	49.3
	7	57	0.487	18.7	24.9	5.16	11.01	55.9
	8	68	0.486	18.8	24.8	6.46	13.76	58.2
	9	120	0.490	19.0	24.9	12.68	26.03	63.1

9. *Magnets annealed 7 hours in aniline vapor.*

I....	1	24	*0.500	*20.0	*25.9	1.51	3.25	37.9
	2	48	0.503	20.0	26.1	4.29	9.19	53.3
	3	70	0.502	20.0	26.0	6.72	14.26	57.3
	4	94	0.495	20.0	25.7	9.89	20.67	61.6
II....	5	20	*0.466	*20.0	*23.7	1.09	2.36	34.7
	6	36	*0.466	*20.0	*23.7	3.11	6.69	53.0
	7	57	0.465	20.0	23.6	5.63	12.02	61.0
	8	68	0.467	20.0	23.7	7.11	15.14	64.0
	9	120	0.467	20.0	23.7	13.88	28.50	69.0

TABLE 50.—*Relations between the magnetic constants of steel and hardness*—Continued.

10. *Magnets annealed 13 hours in aniline vapor.*

Rod.	Magnet No.	a	W 1 m	t	s cm / cm² 0°	n	M	m
			ohm.	°C.	*microhm.*	*cm.*	*abs. E.*	*abs. E.*
I....	1	24	*0.483	*19.9	*25.0	1.59	3.42	39.9
	2	48	0.483	19.9	24.9	4.60	9.85	57.1
	3	70	0.485	19.9	25.1	7.19	15.25	61.3
	4	94	0.480	19.9	24.9	10.50	21.95	65.4
II....	5	20	*0.449	*19.9	*22.8	1.11	2.39	35.2
	6	36	*0.449	*19.9	*22.8	3.30	7.09	56.2
	7	57	0.445	19.8	22.6	6.01	12.83	65.1
	8	68	0.449	19.0	22.7	7.66	16.31	69.1
	9	120	0.454	20.0	23.0	14.69	30.16	73.1

11. *Magnets annealed 1 minute in melting lead.*

I....	1	24	*0.397	*18.6	*20.4	1.57	3.38	39.4
	2	48	0.399	18.4	20.5	5.68	12.16	70.5
	3	70	0.397	18.7	20.4	9.44	20.02	80.5
	4	94	0.394	18.7	20.3	14.06	29.40	87.6
II....	5	20	*0.374	*18.5	*18.9	0.99	2.15	31.5
	6	36	*0.374	*18.5	*18.9	3.58	7.68	60.9
	7	57	0.374	18.5	18.9	7.25	15.47	78.5
	8	68	0.373	18.7	18.8	9.32	19.85	84.0
	9	120	0.375	18.4	18.9	18.91	38.82	94.0

12. *Magnets annealed 1 hour in melting lead.*

I....	1	24	*0.367	*18.8	*18.8	1.45	3.11	36.3
	2	48	0.370	18.8	18.9	5.96	12.76	74.0
	3	70	0.367	18.8	18.8	10.13	21.48	86.4
	4	94	0.365	18.7	18.7	15.27	31.92	95.2
II....	5	20	*0.346	*18.8	*17.4	0.94	2.03	29.8
	6	36	*0.346	*18.8	*17.4	3.59	7.71	61.1
	7	57	0.345	18.8	17.4	7.62	16.25	82.5
	8	68	0.345	18.8	17.3	9.96	21.22	89.8
	9	120	0.348	18.7	17.5	20.62	42.34	102.6

13. *Magnets annealed at red heat (soft).*

I....	1	24	*0.302	*15.1	*15.7	0.32	0.69	8.0
	2	48	0.310	18.2	15.7	2.57	5.70	31.9
	3	70	0.308	18.2	15.7	6.21	13.18	53.0
	4	94	0.299	9.0	15.6	10.85	22.68	67.6
II....	5	20	*0.288	*15.4	*14.6	0.31	0.25	3.7
	6	36	*0.288	*15.4	*14.6	1.39	2.99	23.7
	7	57	0.292	18.3	14.6	4.22	9.00	45.7
	8	68	0.292	18.3	14.6	6.07	12.93	54.7
	9	120	0.279	9.6	14.5	16.23	33.55	80.7

RESULTS WITH RODS OF SMALLER DIMENSIONAL RATIO.

Description of magnets.—The rods used in the present experiments differ from the others principally in two respects: in the first place, the maximum of attainable hardness is here of much larger value than in

the earlier work; in the second, the degree of longitudinal homogeneity in the glass-hard state (and we may accept this as a fair evidence of structural uniformity throughout the chosen lengths) far surpasses that hitherto obtained. Hardness was carried even as far as $s = 47.5 \left(\frac{\text{cm}}{\text{cm}^2}\, 0^\circ \text{ microhm}\right)^{129}$. In consequence of the exceptional homogeneity mentioned, we find that the magnetic data are in excellent accordance.

With the object of arriving at corroborative results, two pairs of wires of approximately identical degrees of galvanic hardness were selected. These are numbered IX, X, and XI, XII.[130] Each of these was broken into magnets such that the dimension-ratios $\alpha = 10, 20, 30, 40, 50$, obtained.[131] Certain important physical constants of these 20 magnets, Nos. 21 to 40, have for convenience of reference here been tabulated. The remarks which precede Table 49 apply to Table 51.

TABLE 51.—*Constants of magnets.*

Rod 2ρ cm, Δ	Magnet No.	Mass μ	Length L.	Dim. ratio α
		g	*cm.*	
IX $2\rho=0.147$, $\Delta=7.46$	21	0.184	1.45	9.9
	22	0.372	2.94	20.0
	23	0.570	4.50	30.6
	24	0.764	6.04	41.1
	25	0.953	7.52	51.2
X $2\rho=0.149$, $\Delta=7.58$	26	0.194	1.46	9.8
	27	0.386	2.92	19.6
	28	0.593	4.49	30.1
	29	0.789	5.99	40.2
	30	0.998	7.56	50.7
XI $2\rho=0.148$, $\Delta=7.70$	31	0.194	1.47	9.9
	32	0.401	3.01	20.3
	33	0.578	4.33	29.2
	34	0.799	6.00	40.5
	35	0.979	7.38	49.8
XII $2\rho=0.151$, $\Delta=7.65$	36	0.208	1.51	10.0
	37	0.413	3.03	20.1
	38	0.634	4.62	30.6
	39	0.827	6.04	40.0
	40	1.040	7.58	50.2

Homogeneity.—The criterion for homogeneity was applied in the manner described in p. 121 by carefully examining the rod under experiment with reference to the electrical resistance of the parts between six equidistant points, about 5 centimeters apart. We believe it expedient to

[129] The maximum hardness observed with the older material was $s = 0.48$ Siemens units at 18°, therefore about $s = 44.5 \left(\frac{\text{cm}}{\text{cm}^2}\, 0^\circ \text{ microhm}\right)$. Cf. Wied. Ann., xi, p. 945, 1880.

[130] The rods III to IX and the magnets Nos. 11 to 20 are discussed in the following chapter.

[131] Of course this can only be done approximately. It is not easy to break a glass-hard rod accurately at a given point.

take the present opportunity of communicating some of these results by way of example. Table 52 contains (columns 2 to 5) the readings immediately taken from a Kohlrausch bridge. The measuring wire of this had been previously calibrated, with due allowance for the small corrections thus resulting. Column 1 shows which of the consecutive lengths (5.07 centimeters) of the steel rod corresponds to the given bridge reading, and the portion of the bridge-wire equivalent to 0.1 Siemens unit. Herefrom the absolute resistances (ohms) of the steel rods per 5.07 centimeters at $t°$ are calculated, and the values given in columns 6 to 9. In view of these very satisfactory results, we were justified in simplifying the subsequent work by testing the homogeneity of the four longest magnets only.

TABLE 52.—*Results for homogeneity.*

	Bridge reading; scale parts.				Resistance per length 5.07 cm. (ohm).			
	IX.	X.	XI.	XII.	IX.	X.	XI.	XII.
.....	.....	.....	.....	.....	0.017	0.013	0.013	0.012
1	70.1	69.0	67.2	66.1	0.026	0.085	0.011	0.054
2	70.4	68.9	67.1	66.1	0.033	0.083	0.009	0.054
3	70.6	69.0	67.2	66.2	0.037	0.085	0.011	0.056
4	70.8	68.9	67.5	66.3	0.040	0.083	0.016	0.058
5	70.7	69.0	67.5	66.3	0.039	0.085	0.016	0.058
0.1 S. U.	468.1	474.3	488.3	502.0	.....	.....	.....	.....
$t=$	10.8	10.8	11.0	11.0	10.8	10.8	11.0	11.0

Magnetic data.—In making the magnetic measurements we used the first position of Gauss. The numerical values of the constants occurring in the formulæ

$$m=\frac{M}{\mu} \qquad M=\tfrac{1}{2}\frac{r^3\,T\,\mathrm{tg}\,\varphi}{1+\tfrac{1}{2}\frac{l^2}{r^2}} \qquad 2\,\mathrm{tg}\,2\,\varphi=\frac{n}{R}$$

are as follows:

$$T=0.194\,\frac{g^{\frac{1}{2}}}{\mathrm{cm}^{\frac{1}{2}}\,\mathrm{sec.}} \qquad r=24.90\ \mathrm{cm.} \qquad R=232.8\ \mathrm{cm.}$$

The magnetic results of our experiments, finally, are given in the series forming table (53):

TABLE 53.—*Relation between the magnetic constants of steel and hardness.*

1. *Magnets in the glass-hard state.*

Rod.	Magnet No.	a	W 1 m	t	$s\frac{\mathrm{cm}}{\mathrm{cm}^2}0°$	n	M	m
			ohm.	°C.	*microhm.*	*cm.*	*abs. E.*	*abs. E.*
IX..	21	9.9	0.283	10.8	47.2	2.60	4.17	22.7
	22	20.0	0.283	10.8	47.2	8.32	13.31	35.8
	23	30.6	0.283	10.8	47.2	14.86	23.62	41.4
	24	41.1	0.283	10.8	47.2	21.37	33.64	44.0
	25	51.2	0.283	10.8	47.2	28.18	43.86	46.0

TABLE 53.—*Relation between the magnetic constants of steel and hardness*—Continued.

1. *Magnets in the gluss-hard state*—Continued.

Rod.	Magnet No.	a	W 1 m	t	$s\frac{cm}{cm^2}0°$	n	M	m
			ohm.	°C.	*microhm.*	*cm.*	*abs. E.*	*abs. E.*
X..	26	9.8	0.273	11.2	46.8	2.82	4.53	23.3
	27	19.6	0.273	11.2	46.8	8.88	14.21	36.8
	28	30.1	0.273	11.2	46.8	15.82	25.14	42.4
	29	40.2	0.273	11.2	46.8	22.45	35.36	44.8
	30	50.7	0.273	11.2	46.8	29.75	46.30	46.4
XI..	31	9.9	0.259	11.0	43.8	2.99	4.80	24.8
	32	20.3	0.259	11.0	43.8	10.27	16.43	41.0
	33	29.2	0.259	11.0	43.8	17.16	27.30	47.2
	34	40.5	0.259	11.0	43.8	25.77	40.59	50.8
	35	49.8	0.259	11.0	43.8	32.98	51.40	52.5
XII..	36	10.0	0.248	11.0	43.5	2.98	4.79	23.0
	37	20.1	0.248	11.0	43.5	9.53	15.24	36.9
	38	30.6	0.248	11.0	43.5	17.32	27.50	43.4
	39	40.0	0.248	11.0	43.5	24.33	38.30	46.3
	40	50.2	0.248	11.0	43.5	32.28	50.22	48.3

2. *Magnets annealed* 1 *hour in steam.*

IX..	21	9.9	0.252	11.8	42.0	2.43	3.90	21.2
	22	20.0	0.252	11.8	42.0	7.80	12.48	33.6
	23	30.6	0.252	11.8	42.0	13.92	22.12	38.8
	24	41.1	0.252	11.8	42.0	20.05	31.56	41.3
	25	51.2	0.252	11.8	42.0	26.47	41.21	43.2
X..	26	9.8	0.244	12.0	41.7	2.66	4.27	22.0
	27	19.6	0.244	12.0	41.7	8.30	13.28	34.4
	28	30.1	0.244	12.0	41.7	14.88	23.65	39.9
	29	40.2	0.244	12.0	41.7	21.08	33.20	42.1
	30	50.7	0.244	12.0	41.7	27.95	43.50	43.6
XI..	31	9.9	0.229	12.0	38.6	2.82	4.53	23.3
	32	20.3	0.229	12.0	38.6	9.83	15.73	39.2
	33	29.2	0.229	12.0	38.6	16.42	26.12	45.2
	34	40.5	0.229	12.0	38.6	24.69	38.89	48.7
	35	49.8	0.229	12.0	38.6	31.63	49.29	50.4
XII..	36	10.0	0.219	12.0	38.4	2.86	4.59	22.1
	37	20.1	0.219	12.0	38.4	9.20	14.72	35.6
	38	30.6	0.219	12.0	38.4	16.70	26.52	41.8
	39	40.0	0.219	12.0	38.4	23.43	36.89	44.6
	40	50.2	0.219	12.0	38.4	31.15	48.47	46.6

3. *Magnets annealed* 3 *hours in steam.*

IX..	21	9.9	0.238	11.9	39.5	2.46	3.94	21.4
	22	20.0	0.238	11.9	39.5	7.72	12.34	33.2
	23	30.6	0.238	11.9	39.5	13.69	21.76	38.2
	24	41.1	0.238	11.9	39.5	19.72	31.05	40.6
	25	51.2	0.238	11.9	39.5	25.89	40.30	42.3
X..	26	9.8	0.230	11.8	39.3	2.70	4.34	22.4
	27	19.6	0.230	11.8	39.3	8.21	13.13	34.0
	28	30.1	0.230	11.8	39.3	14.62	23.23	39.2
	29	40.2	0.230	11.8	39.3	20.65	32.52	41.2
	30	50.7	0.230	11.8	39.3	27.34	42.55	42.6
XI..	31	9.9	0.216	12.0	36.5	2.83	4.54	23.4
	32	20.3	0.216	12.0	36.5	9.68	15.49	38.6
	33	29.2	0.216	12.0	36.5	16.16	25.70	44.5
	34	40.5	0.216	12.0	36.5	24.21	38.13	47.7
	35	49.8	0.216	12.0	36.5	30.94	48.22	49.8
XII..	36	10.0	0.207	12.0	36.2	2.88	4.63	22.2
	37	20.1	0.207	12.0	36.2	9.12	14.59	35.3
	38	30.6	0.207	12.0	36.2	16.50	26.21	41.3
	39	40.0	0.207	12.0	36.2	23.09	36.35	43.9
	40	50.2	0.207	12.0	36.2	30.57	47.57	45.7

TABLE 53.—*Relation between the magnetic constants of steel and hardness*—Continued.

4. *Magnets annealed 6 hours in steam.*

Rod.	Magnet No.	a	$W1m$	t	$s\frac{cm}{cm^2}$	n	M	m
			chm.	°C	*mic rohm.*	*cm.*	*abs. E.*	*abs. E.*
IX..	21	9.9	0.230	11.9	38.2	2.48	3.98	21.6
	22	20.0	0.230	11.9	38.2	7.72	12.35	33.2
	23	30.6	0.230	11.9	38.2	13.70	21.77	38.2
	24	41.1	0.230	11.9	38.2	19.63	30.90	40.4
	25	51.2	0.230	11.9	38.2	25.80	40.16	42.1
X..	26	9.8	0.223	11.9	38.0	2.70	4.34	22.4
	27	19.6	0.223	11.9	38.0	8.21	13.13	34.0
	28	30.1	0.223	11.9	38.0	14.53	23.09	38.9
	29	40.2	0.223	11.9	38.0	20.55	32.37	41.0
	30	50.7	0.223	11.9	38.0	27.24	42.39	42.5
XI..	31	9.9	0.209	11.8	35.1	2.82	4.53	23.3
	32	20.3	0.209	11.8	35.1	9.68	15.49	38.6
	33	29.2	0.209	11.8	35.1	16.09	25.60	44.3
	34	40.5	0.209	11.8	35.1	24.05	37.88	47.4
	35	49.8	0.209	11.8	35.1	30.82	48.03	49.1
XII..	36	10.0	0.199	11.8	34.9	2.84	4.56	21.9
	37	20.1	0.199	11.8	34.9	9.12	14.59	35.3
	38	30.6	0.199	11.8	34.9	16.39	26.03	41.1
	39	40.0	0.199	11.8	34.9	22.92	36.68	43.6
	40	50.2	0.199	11.8	34.9	30.41	47.32	45.5

5. *Magnets annealed 10 hours in steam.*

IX..	21	9.9	0.226	11.8	37.4	2.46	3.95	21.5
	22	20.0	0.226	11.8	37.4	7.73	12.37	33.3
	23	30.6	0.226	11.8	37.4	13.69	21.76	38.2
	24	41.1	0.226	11.8	37.4	19.66	30.95	40.5
	25	51.2	0.226	11.8	37.4	25.83	40.21	42.2
X..	26	9.8	0.218	11.9	37.1	2.70	4.34	22.4
	27	19.6	0.218	11.9	37.1	8.19	13.10	33.9
	28	30.1	0.218	11.9	37.1	14.57	23.16	39.0
	29	40.2	0.218	11.9	37.1	20.60	32.45	41.1
	30	50.7	0.218	11.9	37.1	27.25	42.41	42.5
XI..	31	9.9	0.204	11.9	34.3	2.81	4.51	23.3
	32	20.3	0.204	11.9	34.3	9.61	15.37	38.3
	33	29.2	0.204	11.9	34.3	16.06	25.54	44.2
	34	40.5	0.204	11.9	34.3	24.03	37.85	47.4
	35	49.8	0.204	11.9	34.3	30.74	47.91	48.9
XII..	36	10.0	0.195	11.9	34.0	2.87	4.61	22.2
	37	20.1	0.195	11.9	34.0	9.09	14.54	35.2
	38	30.6	0.195	11.9	34.0	16.34	25.95	40.9
	39	40.0	0.195	11.9	34.0	22.87	36.01	43.5
	40	50.2	0.195	11.9	34.0	30.37	47.25	45.4

6. *Magnets annealed 20 minutes in aniline vapor.*

IX..	21	9.9	0.191	12.0	31.5	2.31	3.71	20.2
	22	20.0	0.191	12.0	31.5	7.97	12.75	34.3
	23	30.6	0.191	12.0	31.5	14.60	23.21	40.7
	24	41.1	0.191	12.0	31.5	21.15	33.30	43.6
	25	51.2	0.191	12.0	31.5	28.02	43.62	45.8
X..	26	9.8	0.185	12.0	31.4	2.60	4.18	21.5
	27	19.6	0.185	12.0	31.4	8.45	13.52	35.0
	28	30.1	0.185	12.0	31.4	15.48	24.60	41.5
	29	40.2	0.185	12.0	31.4	22.22	34.99	44.4
	30	50.7	0.185	12.0	31.4	29.58	46.04	46.1
XI..	31	9.9	0.173	12.0	29.0	2.54	4.08	21.0
	32	20.3	0.173	12.0	29.0	9.75	15.60	38.9
	33	29.2	0.173	12.0	29.0	16.70	26.56	46.0
	34	40.5	0.173	12.0	29.0	25.52	40.20	50.3
	35	49.8	0.173	12.0	29.0	32.87	51.23	52.3
XII..	36	10.0	0.165	12.0	28.7	2.67	4.29	20.6
	37	20.1	0.165	12.0	28.7	9.28	14.84	35.9
	38	30.6	0.165	12.0	28.7	17.31	27.49	43.7
	39	40.0	0.165	12.0	28.7	24.52	38.60	46.0
	40	50.2	0.165	12.0	28.7	32.75	50.96	49.4

TABLE 53.—*Relation between the magnetic constants of steel and hardness*—Continued.

7. *Magnets annealed 1 hour in aniline vapor.*

Rod.	Magnet No.	a	$W1m$	t	$s\frac{cm}{cm^2}0°$	n	M	m
			ohm.	°C.	*microhm.*	*cm.*	*abs. E.*	*abs. E.*
IX..	21	9.9	0.179	12.0	29.7	2.33	3.75	20.4
	22	20.0	0.179	12.0	29.7	8.30	13.28	35.7
	23	30.6	0.179	12.0	29.7	15.38	24.45	42.9
	24	41.1	0.179	12.0	29.7	22.37	35.22	46.1
	25	51.2	0.179	12.0	29.7	29.75	46.31	48.6
X..	26	9.8	0.175	12.0	29.6	2.52	4.05	20.9
	27	19.6	0.175	12.0	29.6	8.79	14.06	36.4
	28	30.1	0.175	12.0	29.6	16.31	25.92	43.7
	29	40.2	0.175	12.0	29.6	23.60	37.17	47.1
	30	50.7	0.175	12.0	29.6	31.46	48.97	49.1
XI..	31	9.9	0.164	12.0	27.5	2.57	4.13	21.3
	32	20.3	0.164	12.0	27.5	10.08	16.13	40.2
	33	29.2	0.164	12.0	27.5	17.49	27.82	48.1
	34	40.5	0.164	12.0	27.5	26.83	42.26	52.9
	35	49.8	0.164	12.0	27.5	34.64	53.99	55.1
XII..	36	10.0	0.157	12.0	27.3	2.66	4.27	20.5
	37	20.1	0.157	12.0	27.3	9.60	15.36	37.2
	38	30.6	0.157	12.0	27.3	18.19	28.89	45.6
	39	40.0	0.157	12.0	27.3	25.88	40.75	49.3
	40	50.2	0.157	12.0	27.3	34.63	53.89	51.8

8. *Magnets annealed 3 hours in aniline vapor.*

IX..	21	9.9	0.169	11.7	27.9	2.41	3.87	21.0
	22	20.0	0.169	11.7	27.9	8.99	14.38	38.7
	23	30.6	0.169	11.7	27.9	16.93	26.91	47.2
	24	41.1	0.169	11.7	27.9	24.75	38.97	51.0
	25	51.2	0.169	11.7	27.9	32.94	51.27	53.8
X..	26	9.8	0.164	11.8	27.7	2.62	4.21	21.7
	27	19.6	0.164	11.8	27.7	9.56	15.30	39.6
	28	30.1	0.164	11.8	27.7	18.08	28.73	48.5
	29	40.2	0.164	11.8	27.7	26.23	41.31	52.4
	30	50.7	0.164	11.8	27.7	35.06	54.56	54.7
XI..	31	9.9	0.153	11.8	25.6	2.61	4.19	21.6
	32	20.3	0.153	11.8	25.6	10.71	17.13	42.7
	33	29.2	0.153	11.8	25.6	18.98	30.19	52.2
	34	40.5	0.153	11.8	25.6	29.25	46.07	57.7
	35	49.8	0.153	11.8	25.6	37.83	58.95	60.2
XII..	36	10.0	0.147	11.9	25.5	2.75	4.42	21.2
	37	20.1	0.147	11.9	25.5	10.29	16.46	39.8
	38	30.6	0.147	11.9	25.5	19.81	31.46	49.6
	39	40.0	0.147	11.9	25.5	28.37	44.67	54.0
	40	50.2	0.147	11.9	25.5	38.16	59.37	57.1

9. *Magnets annealed 7 hours in aniline vapor.*

IX..	21	9.9	0.159	12.0	26.2	2.53	4.06	22.1
	22	20.0	0.159	12.0	26.2	9.80	15.68	42.2
	23	30.6	0.159	12.0	26.2	18.73	29.76	52.2
	24	41.1	0.159	12.0	26.2	27.65	43.53	57.0
	25	51.2	0.159	12.0	26.2	37.03	57.65	60.5
X..	26	9.8	0.154	12.0	26.0	2.68	4.30	22.2
	27	19.6	0.154	12.0	26.0	10.36	16.58	42.9
	28	30.1	0.154	12.0	26.0	20.06	31.88	53.8
	29	40.2	0.154	12.0	26.0	29.31	46.16	58.5
	30	50.7	0.154	12.0	26.0	39.39	61.31	61.4
XI..	31	9.9	0.145	12.0	24.2	2.63	4.22	21.8
	32	20.3	0.145	12.0	24.2	11.40	18.24	45.5
	33	29.2	0.145	12.0	24.2	20.69	32.92	56.9
	34	40.5	0.145	12.0	24.2	32.19	50.70	63.5
	35	49.8	0.145	12.0	24.2	41.96	65.39	66.8
XII..	36	10.0	0.138	12.0	23.9	2.80	4.50	21.6
	37	20.1	0.138	12.0	23.9	11.01	17.61	42.6
	38	30.6	0.138	12.0	23.9	21.73	34.51	54.4
	39	40.0	0.138	12.0	23.9	31.32	49.31	59.6
	40	50.2	0.138	12.0	23.9	42.37	65.93	63.4

TABLE 53.—*Relation between the magnetic constants of steel and hardness*—Continued.

10. *Magnets annealed 13 hours in aniline vapor.*

Rod.	Magnet No.	a	W 1 m	t	$s\frac{cm}{cm^2}0°$	n	M	m
			ohm.	°C.	*microhm.*	*cm.*	*abs. E.*	*abs. E.*
IX..	21	9.9	0.151	12.0	24.8	2.50	4.02	21.8
	22	20.0	0.151	12.0	24.8	10.25	16.40	44.1
	23	30.6	0.151	12.0	24.8	20.09	31.93	56.0
	24	41.1	0.151	12.0	24.8	30.02	47.26	61.9
	25	51.2	0.151	12.0	24.8	40.25	62.66	65.8
X..	26	9.8	0.146	12.0	24.6	2.64	4.24	21.9
	27	19.6	0.146	12.0	24.6	10.79	17.26	44.7
	28	30.1	0.146	12.0	24.6	21.59	34.32	57.9
	29	40.2	0.146	12.0	24.6	31.87	50.20	63.6
	30	50.7	0.146	12.0	24.6	42.96	66.87	67.0
XI..	31	9.9	0.138	12.0	22.9	2.53	4.06	20.9
	32	20.3	0.138	12.0	22.9	11.59	18.54	46.2
	33	29.2	0.138	12.0	22.9	21.68	34.49	59.7
	34	40.5	0.138	12.0	22.9	34.53	54.39	68.1
	35	49.8	0.138	12.0	22.9	45.08	70.26	71.8
XII..	36	10.0	0.132	12.0	22.7	2.73	4.38	21.1
	37	20.1	0.132	12.0	22.7	11.30	18.08	43.8
	38	30.6	0.132	12.0	22.7	22.90	36.37	57.4
	39	40.0	0.132	12.0	22.7	33.50	52.74	63.8
	40	50.2	0.132	12.0	22.7	45.60	70.96	68.2

11. *Magnets annealed 10 minutes in melting tin.*

IX..	21	9.9	0.147	11.7	24.0	2.46	3.95	21.5
	22	20.0	0.147	11.7	24.0	10.37	16.59	44.6
	23	30.6	0.147	11.7	24.0	20.64	32.80	57.5
	24	41.1	0.147	11.7	24.0	30.97	48.75	63.8
	25	51.2	0.147	11.7	24.0	41.92	65.25	68.5
X..	26	9.8	0.141	11.5	23.8	2.60	4.18	21.5
	27	19.6	0.141	11.5	23.8	10.98	17.57	45.5
	28	30.1	0.141	11.5	23.8	22.28	35.41	59.7
	29	40.2	0.141	11.5	23.8	33.11	52.16	66.1
	30	50.7	0.141	11.5	23.8	44.78	69.70	69.8
XI..	31	9.9	0.134	11.6	22.2	2.46	3.95	20.4
	32	20.3	0.134	11.6	22.2	11.70	18.72	46.7
	33	29.2	0.134	11.6	22.2	22.17	35.27	61.0
	34	40.5	0.134	11.6	22.2	35.60	56.07	70.2
	35	49.8	0.134	11.6	22.2	46.70	72.78	74.3
XII..	36	10.0	0.127	11.7	22.0	2.68	4.30	20 7
	37	20.1	0.127	11.7	22.0	11.37	18.19	44.0
	38	30.6	0.127	11.7	22.0	23.47	37.27	58.8
	39	40.0	0.127	11.7	22.0	34.58	54.44	65.8
	40	50.2	0.127	11.7	22.0	47.29	73.59	70.8

12. *Magnets annealed 1 minute in melting lead.*

IX..	21	9.9	0.124	11.9	20.3	2.26	3.60	19.7
	22	20.0	0.124	11.9	20.3	10.88	17.40	46.8
	23	30.6	0.124	11.9	20.3	24.04	38.19	67.0
	24	41.1	0.124	11.9	20.3	38.43	60.51	79.2
	25	51.2	0.124	11.9	20.3	52.53	81.77	85.8
X..	26	9.8	0.120	11.3	20.1	2.33	3.74	19.3
	27	19.6	0.120	11.3	20.1	11.14	17.83	46.2
	28	30.1	0.120	11.3	20.1	25.34	40.27	67.9
	29	40.2	0.120	11.3	20.1	40.08	63.12	80.0
	30	50.7	0.120	11.3	20.1	55.46	86.32	86.5
XI..	31	9.9	0.115	11.9	19.0	2.30	3.69	19.0
	32	20.3	0.115	11.9	19.0	11.71	18.72	46 7
	33	29.2	0.115	11.9	19.0	24.71	39.30	68.0
	34	40.5	0.115	11.9	19.0	42.45	66.87	83.7
	35	49.8	0.115	11.9	19.0	56.53	88.10	90.0
XII..	36	10.0	0.109	11.9	18.7	2.46	3.95	19.0
	37	20.1	0.109	11.9	18.7	11.18	17.89	43 3
	38	30.6	0.109	11.9	18.7	25.95	41.21	65.0
	39	40.0	0.109	11.9	18.7	41.39	65.16	78.8
	40	50.2	0.109	11.9	18.7	58.01	90.26	86.8

TABLE 53.—*Relation between the magnetic constants of steel and hardness*—Continued.

13. *Magnets annealed 1 hour in melting zinc.*

Rod.	Magnet No.	a	W 1 m	t	$s\frac{cm}{cm^2}0°$	n	M	m
			ohm.	°C.	*microhm.*	*cm.*	*abs. E.*	*abs. E.*
IX..	21	9.9	0.107	10.3	17.5	1.85	2.98	16.2
	22	20.0	0.107	10.3	17.5	10.15	16.24	43.7
	23	30.6	0.107	10.3	17.5	26.02	41.35	72.5
	24	41.1	0.107	10.3	17.5	43.90	69.12	90.5
	25	51.2	0.107	10.3	17.5	61.65	95.96	100.7
X..	26	9.8	0.102	10.3	17.2	1.83	2.94	15.2
	27	19.6	0.102	10.3	17.2	9.82	15.71	40.7
	28	30.1	0.102	10.3	17.2	26.89	42.64	72.1
	29	40.2	0.102	10.3	17.2	46.03	72.49	91.9
	30	50.7	0.102	10.3	17.2	65.10	101.32	101.5
XI..	31	9.9	0.098	10.3	16.2	1.74	2.79	14.4
	32	20.3	0.098	10.3	16.2	10.06	16.09	40.1
	33	29.2	0.098	10.3	16.2	26.87	42.74	72.6
	34	40.5	0.098	10.3	16.2	47.63	75.03	93.9
	35	49.8	0.098	10.3	16.2	65.29	101.74	103.9
XII..	36	10.0	0.093	10.3	16.0	1.93	3.10	14.9
	37	20.1	0.093	10.3	16.0	9.95	15.91	38.5
	38	30.6	0.093	10.3	16.0	27.14	43.10	68.0
	39	40.0	0.093	10.3	16.0	46.73	73.57	89.0
	40	50.2	0.093	10.3	16.0	67.60	105.17	101.1

14. *Magnets annealed at red heat (soft).*

IX..	21	9.9	0.096	10.0	15.7	0.52	0.83	4.5
	22	20.0	0.096	10.0	15.7	2.72	4.35	11.7
	23	30.6	0.096	10.0	15.7	7.94	12.62	22.1
	24	41.1	0.096	10.0	15.7	10.87	26.56	34.8
	25	51.2	0.096	10.0	15.7	28.49	44.35	46.5
X..	26	9.8	0.093	10.1	15.6	0.46	0.74	3.8
	27	19.6	0.093	10.1	15.6	2.52	4.03	10.5
	28	30.1	0.093	10.1	15.6	7.59	12.06	20.3
	29	40.2	0.093	10.1	15.6	15.72	24.73	31.4
	30	50.7	0.093	10.1	15.6	29.15	45.56	45.4
XI..	31	9.9	0.096	10.2	14.9	0.52	0.84	4.3
	32	20.3	0.096	10.2	14.9	2.55	4.08	10.2
	33	29.2	0.096	10.2	14.9	6.80	10.82	18.7
	34	40.5	0.096	10.2	14.9	15.13	23.83	29.8
	35	49.8	0.096	10.2	14.9	26.63	41.50	42.4
XII..	36	10.0	0.087	10.2	14.9	0.57	0.92	4.4
	37	20.1	0.087	10.2	14.9	3.15	5.04	12.2
	38	30.6	0.087	10.2	14.9	8.25	13.10	20.7
	39	40.0	0.087	10.2	14.9	16.32	25.69	31.1
	40	50.2	0.087	10.2	14.9	29.42	36.36	44.0

DISCUSSION.

Digest.—Our present purpose will be greatly facilitated by a tabular comparison of the essential features of the results in hand. The following digest (Tables 54 to 59) contains, besides brief descriptions of temper, the hardness as represented by the specific resistance s, together with the corresponding magnetic moments per unit of mass, m, of the several rods.

TABLE 54.—*Rod I.*

Description of temper.	Hardness, s.	Specific magnetism, m.			
		No. 1.	No. 2.	No. 3.	No. 4.
Glass-hard	38.5	33.7	43.0	45.0	47.9
Annealed 1 hour in steam 100°	36.3	33.3	42.0	44.3	46.8
Annealed 3 hours in steam 100°	34.8	32.7	41.0	43.3	46.2
Annealed 6 hours in steam 100°	33.9	32.2	40.1	42.7	45.4
Annealed 10 hours in steam 100°	33.4	32.1	40.2	42.7	45.4
Annealed 20 minutes in aniline vapor 185°	29.5	32.2	42.8	45.9	48.9
Annealed 1 hour in aniline vapor 185°	28.4	34.1	45.1	48.1	51.5
Annealed 3 hours in aniline vapor 185°	27.1	36.1	48.8	52.6	56.0
Annealed 7 hours in aniline vapor 185°	25.9	37.9	53 3	57.3	61.6
Annealed 13 hours in aniline vapor 185°	25.0	39.9	57.1	61.3	65 4
Annealed 1 minute in lead bath 330°	20.4	39.4	70.5	80.5	87.6
Annealed 1 hour in lead bath 330°	18.8	36.3	74.0	86.4	95.2
Soft	15.7	8.0	31.9	53.0	67.6

TABLE 55.—*Rod II.*

Description of temper.	Hardness, s.	Specific magnetism, m.					
		No. 5.	No. 6.	No. 7.	No. 8.	No. 9.	No. 10.
Glass-hard	37.3	.....	43.9	.....	.....	53.5	55.1
Annealed 1 hour in steam 100°	34.7	.....	43.1	.....	.....	52.8	54.1
Annealed 3 hours in steam 100°	33.0	.....	42.5	.....	.....	51.8	53.2
Annealed 6 hours in steam 100°	32.2	.....	42.0	.....	.....	50.8	52.2
Annealed 10 hours in steam 100°	31.6	29.6	41.1	46.4	47.8	51.0	.....
Annealed 20 minutes in aniline vapor 185°	27.4	29.1	43.6	48.9	51.3	55.0	.....
Annealed 1 hour in aniline vapor 185°	26.3	30.0	45.6	51.5	53.7	58.0	.....
Annealed 3 hours in aniline vapor 185°	24.9	31.8	49.3	55.9	58.2	63.1	.....
Annealed 7 hours in aniline vapor 185°	23.7	34.7	53.0	61.0	64.0	69.0	.....
Annealed 13 hours in aniline vapor 185°	22.8	35.2	56.2	65.1	69.1	73.1	.....
Annealed 1 minute in lead bath 330°	18.9	31.5	60.9	78.5	84.0	94.0	.....
Annealed 1 hour in lead bath 330°	17.4	29.8	61.1	82.5	89.8	102.6	.....
Soft	14.5	3.7	23.7	45.7	54.7	80.7	.....

TABLE 56.—*Rod IX.*

Description of temper.	Hardness, s.	Specific magnetism m, for the dimension-ratio=				
		9.9	20.0	30.6	41.1	51.2
Glass-hard	47.2	22.7	35.8	41.4	44.0	46.0
Annealed 1 hour in steam 100°	42.0	21.2	33.6	38.8	41.3	43.2
Annealed 3 hours in steam 100°	39.5	21.4	33.2	38.2	40.6	42.3
Annealed 6 hours in steam 100°	38.2	21.6	33.2	38.2	40.4	42.1
Annealed 10 hours in steam 100°	37.4	21.5	33.3	38.2	40.5	42.2
Annealed 20 minutes in aniline vapor 185°	31.5	20.2	34.3	40.7	43.6	45.8
Annealed 1 hour in aniline vapor 185°	29.7	20.4	35.7	42.9	46.1	48.6
Annealed 3 hours in aniline vapor 185°	27.9	21.0	38.7	47.2	51.0	53.8
Annealed 7 hours in aniline vapor 185°	26.2	22.1	42.2	52.2	57.0	60.5
Annealed 13 hours in aniline vapor 185°	24.8	21.8	44.1	56.0	61.9	65.8
Annealed 10 minutes in tin bath 240°	24.0	21.5	44.6	57.5	63.8	68.5
Annealed 1 minute in lead bath 330°	20.3	19.7	46.8	67.0	79.2	85.8
Annealed 1 hour in zinc bath 420°	17.5	16.2	43.7	72.5	90.5	100.7
Soft	15.7	4.5	11.7	22.1	34.8	46.5

TABLE 57.—*Rod X.*

Description of temper.	Hardness, *s*.	Specific magnetism *m*, for the dimension-ratio=				
		9.8	19.6	30.1	40.2	50.7
Glass-hard	46.8	23.3	36.8	42.4	44.8	46.4
Annealed 1 hour in steam 100°	41.7	22.0	34.4	39.9	42.1	43.6
Annealed 3 hours in steam 100°	39.3	22.4	34.0	39.2	41.2	42.6
Annealed 6 hours in steam 100°	38.0	22.4	34.0	38.9	41.0	42.5
Annealed 10 hours in steam 100°	37.1	22.4	33.9	39.0	41.1	42.5
Annealed 20 minutes in aniline vapor 185°	31.4	21.5	35.0	41.5	44.4	46.1
Annealed 1 hour in aniline vapor 185°	29.6	20.9	36.4	43.7	47.1	49.1
Annealed 3 hours in aniline vapor 185°	26.7	21.7	39.6	48.5	52.4	54.7
Annealed 7 hours in aniline vapor 185°	26.0	22.2	42.9	53.8	58.5	61.4
Annealed 13 hours in aniline vapor 185°	24.6	21.9	44.7	57.9	63.6	67.0
Annealed 10 minutes in tin bath 240°	23.8	21.5	45.5	59.7	66.1	69.8
Annealed 1 minute in lead bath 330°	20.1	19.3	46.2	67.9	80.0	86.5
Annealed 1 hour in zinc bath 420°	17.2	15.2	40.7	72.1	91.9	101.5
Soft	15.6	3.8	10.5	20.3	31.4	45.4

TABE 58.—*Rod XI.*

Description of temper.	Hardness, *s*.	Specific magnetism *m*, for the dimension-ratio=				
		9.9	20.3	29.2	40.5	49.8
Glass-hard	43.8	24.8	41.0	47.2	50.8	52.5
Annealed 1 hour in steam 100°	38.6	23.3	39.2	45.2	48.7	50.4
Annealed 3 hours in steam 100°	36.5	23.4	38.6	44.5	47.7	49.3
Annealed 6 hours in steam 100°	35.1	23.3	38.6	44.3	47.4	49.1
Annealed 10 hours in steam 100°	34.3	23.3	38.3	44.2	47.4	48.9
Annealed 20 minutes in aniline vapor 185°	29.0	21.0	38.9	46.0	50.3	52.3
Annealed 1 hour in aniline vapor 185°	27.5	21.3	40.2	48.1	52.9	55.1
Annealed 3 hours in aniline vapor 185°	25.6	21.6	42.7	52.2	57.7	60.2
Annealed 7 hours in aniline vapor 185°	24.2	21.8	45.5	56.9	63.5	66.8
Annealed 13 hours in aniline vapor 185°	22.9	20.9	46.2	59.7	68.1	71.8
Annealed 10 minutes in tin bath 240°	22.2	20.4	46.7	61.0	70.2	74.3
Annealed 1 minute in lead bath 330°	19.0	19.0	46.7	68.0	83.7	90.0
Annealed 1 hour in zinc bath 420°	16.2	14.4	40.1	72.6	93.9	103.9
Soft	14.9	4.3	10.2	18.7	29.8	42.4

TABLE 59.—*Rod XII.*

Description of temper.	Hardness, *s*.	Specific magnetism *m*, for the dimension-ratio=				
		10.0	20.1	30.6	40.0	50.2
Glass-hard	43.5	23.0	36.9	43.4	46.3	48.3
Annealed 1 hour in steam 100°	38.4	22.1	35.6	41.8	44.6	46.6
Annealed 3 hours in steam 100°	36.2	22.2	35.3	41.3	43.9	45.7
Annealed 6 hours in steam 100°	34.9	21.9	35.3	41.1	43.6	45.5
Annealed 10 hours in steam 100°	34.0	22.2	35.2	40.9	43.5	45.4
Annealed 20 minutes in aniline vapor 185°	28.7	20.6	35.9	43.4	46.7	49.0
Annealed 1 hour in aniline vapor 185°	27.3	20.5	37.2	45.6	49.3	51.8
Annealed 3 hours in aniline vapor 185°	25.5	21.2	39.8	49.6	54.0	57.1
Annealed 7 hours in aniline vapor 185°	23.9	21.6	42.6	54.4	59.6	63.4
Annealed 13 hours in aniline vapor 185°	22.7	21.1	43.8	57.4	63.8	68.2
Annealed 10 minutes in tin-bath 240°	22.0	20.7	44.0	58.8	65.8	70.8
Annealed 1 minute in lead-bath 330°	18.7	19.0	43.3	65.0	78.8	86.8
Annealed 1 hour in zinc-bath 420°	16.0	14.9	38.5	68.0	89.0	101.1
Soft	14.9	4.4	12.2	20.7	31.1	44.0

Plan of the discussion.—These results may be discussed from two distinct stand-points. In case of one and the same wire, the specific magnetism of the magnets taken out of it varies both with their degree

of hardness and with the ratio of length and diameter. In place of a diagram in three dimensions, however, it is expedient to regard the one or the other variable, as an arbitrary constant, while the other passes through all possible values. We thus represent all relations by plane diagrams. Our results are compared in this way. It is, however, convenient to replace Tables 54 and 55 by others satisfactorily deducible from them by graphic interpolation, since the ratio of dimensions in the earlier experiments (Tables 54 and 55) do not compare well with those in the later (Tables 56 to 59). In other words, without such reconstruction of the former results, the difference of magnetic behavior of the two kinds of steel employed cannot be readily discerned. For this reason the Tables 60 and 61, following, have been prepared. The data contained are such as would have been found if the ratio of dimensions had been α=20, 40, 60, 80, and 100, respectively, in place of the values which actually obtained.

TABLE 60.—*Rod I.*

Description of temper.	Hardness, s.	Specific magnetism m, for the dimension-ratio=				
		20	40	60	80	100
Glass-hard	38.5	31.4	40.2	44.3	46.5	48.0
Annealed 1 hour in steam 100°	36.3	30.5	39.2	42.9	44.7	46.0
Annealed 3 hours in steam 100°	34.8	30.1	38.5	42.2	43.9	45.2
Annealed 6 hours in steam 100°	33.9	29.9	38.3	42.0	43.8	45.0
Annealed 10 hours in steam 100°	33.4	29.8	38.0	41.8	43.8	45.2
Annealed 20 minutes in aniline vapor 185°	29.5	29.6	40.0	44.5	47.2	49.4
Annealed 1 hour in aniline vapor 185°	28.4	30.3	42.3	47.4	50.3	52.3
Annealed 3 hours in aniline vapor 185°	27.1	31.8	46.2	51.6	54.4	56.5
Annealed 7 hours in aniline vapor 185°	25.9	33.6	50.4	56.0	59.1	61.4
Annealed 13 hours in aniline vapor 185°	25.0	34.7	52.6	60.0	63.4	65.8
Annealed 1 minute in lead-bath 330°	20.4	32.2	63.4	77.3	84.2	89.2
Annealed 1 hour in lead-bath 330°	18.8	26.0	65.3	81.4	90.3	97.2
Soft	15.7	6.0	21.3	43.0	59.4	70.3

TABLE 61.—*Rod II.*

Description of temper.	Hardness, s.	Specific magnetism m, for the dimension-ratio=				
		20	40	60	80	100
Glass-hard	37.3	31.5	45.0	49.5	51.3	52.4
Annealed 1 hour in steam 100°	34.7	30.4	43.4	47.8	49.7	50.5
Annealed 3 hours in steam 100°	33.0	30.0	42.8	47.3	49.0	49.9
Annealed 6 hours in steam 100°	32.2	29.9	42.7	47.2	48.8	49.8
Annealed 10 hours in steam 100°	31.6	29.9	42.7	47.2	48.8	49.7
Annealed 20 minutes in aniline vapor 185°	27.4	29.4	45.3	50.1	52.5	54.0
Annealed 1 hour in aniline vapor 185°	26.3	30.0	47.3	52.2	55.0	56.6
Annealed 3 hours in aniline vapor 185°	24.9	32.8	50.9	57.3	60.0	62.2
Annealed 7 hours in aniline vapor 185°	23.7	34.3	54.7	62.3	66.0	68.0
Annealed 13 hours in aniline vapor 185°	22.8	35.0	58.0	66.4	70.8	72.0
Annealed 1 minute in lead-bath 330°	18.9	33.0	64.3	80.7	88.6	92.2
Annealed 1 hour in lead-bath 330°	17.4	30.0	65.3	84.6	95.0	100.8
Soft	14.5	6.0	27.8	47.5	63.3	74.0

General inferences.—A detailed comparison of the results expressed in Tables 60 and 61 with those in Tables 56, 57, 58, 59, and all of these among themselves, shows conclusively that the magnetic behavior of

each particular wire is of a distinct and individual character. More perspicuously, even, the same fact is observable in a graphic representation of the data. The curves obtained for the several wires are by no means coincident. The thicker wire shows greater magnetizability than the wire of smaller diameter. In the case of both kinds of steel, galvanically hard rods are magnetizable to a lesser degree than soft rods. Moreover, there appears to exist a relation between the density and maximum permanent moment, in so far as the latter in general increases directly with the former. This effect is particularly apparent for the thicker wire. But the fundamental inference is this, that the relation between specific magnetism and hardness expressed galvanically, as exhibited by all the wires, is essentially of the same kind.

Magnetism and hardness.—The general contours of the families of curves expressing the functionality between magnetic moment, per unit of mass, hardness, ratio of dimensions, are of very great interest. It appears clearly that the latter variable by no means deserves the exclusive importance which has hitherto been attributed to it, at least not in the accepted sense that a certain critical value exists (Ruths, for instance, puts it, $\alpha=35$) below which magnets differ in a pronounced way as regards magnetizability, from magnets with a larger dimension-ratio. The variations of the series of curves belonging to magnets taken from one and the same wire are obviously of like nature; in other words, obtained from a definite law by the mere change of a parametric constant, a. In general, all magnets, whether long or short, after incipient annealing diminish in magnetizability, until a well-marked minimum of this quality is reached. If thereafter annealing be continued magnetizability again increases, attains a maximum, and finally decreases again to the value for the soft state.

The causes which led earlier observers into partially erroneous inferences are not far to seek. With the ordinary very rough method of estimating the degree of hardness of steel by the aid of the oxide tints, the character of the curves, as a whole, did not appear. Hitherto data for only four points for each rod, corresponding to the hard, the yellow, the blue, and the soft states, were available. Irrespective of the vagueness of degrees of hardness thus defined it will readily be seen that where the full contours of the curves are unknown the grouping of these points is such as to suggest the said imperfect inference of a critical dimension-ratio.

The effect of a variation of dimensions is particularly apparent in its bearing on the position and value of the minima and maxima of the permanently saturated magnetic state. In proportion as the length of the magnets, or more accurately the ratio of dimensions increases, these singular points, which are nearly coincident in very short rods, move apart, the minimum occurring in the glass-hard state, the maximum passing at a very rapid rate toward the soft state. At the same time the maximum increases enormously in value, and for long, thin rods is

greater than twice the equivalent of the permanently saturated magnetic moment of the same rod when hard. As we approach short rods, as will be seen, the maximum flattens out, moving toward the minimum at a more rapid rate than the minimum progresses in the opposite direction. But the variations are such that for very short magnets, *i. e.*, such for which the ratio of dimensions is below 5, the minimum will probably lose its marked character altogether, and eventually come into full coincidence with the maximum. Here, therefore, a uniformly continuous decrease of magnetizability from the glass-hard to the soft states is to be anticipated.

Graphic representation.—All these results may be made to appear with striking clearness by representing them graphically. It will be sufficient for this purpose to accept the mean of the two series of results for the thin sample of steel wire, and then to compare this with the mean or combined results (four series) obtaining for the thicker sample of wire. These mean values are given in the following tables, 62 and 63:

TABLE 62.—*Progressive states of magnetizability, varying with hardness.*

[Thick wire, 2ρ=0.15 centimeter.]

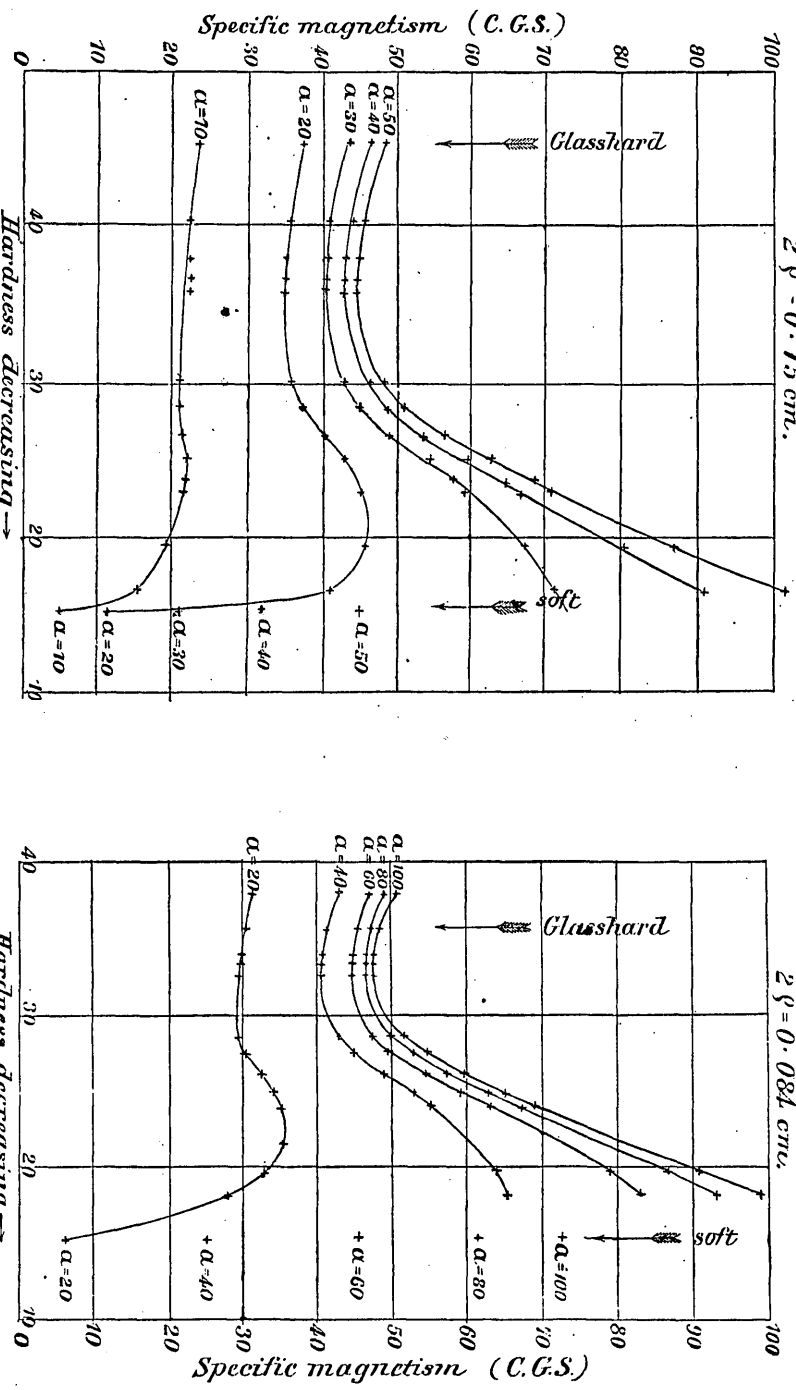
Mean hardness (galvan).	Mean specific magnetism m, for the dimension-ratio =				
	10	20	30	40	50
45.3	23.5	37.6	43.6	46.5	48.3
40.2	22.2	35.7	41.3	44.2	46.0
37.9	22.3	35.3	40.8	43.3	45.0
36.6	22.3	35.3	40.6	43.1	44.8
35.7	22.3	35.2	40.6	43.1	44.8
30.2	20.8	36.0	42.9	46.2	48.3
28.5	20.8	37.4	45.1	48.8	51.2
26.7	21.4	40.2	49.4	53.8	56.5
25.1	21.9	43.3	54.3	59.6	63.0
23.8	21.4	44.7	57.8	64.3	68.2
23.0	21.0	45.2	59.3	66.5	70.9
19.5	19.3	45.8	67.0	80.4	87.3
16.7	15.2	40.8	71.3	91.3	101.8
15.3	4.3	11.2	20.5	31.8	44.6

TABLE 63.—*Progressive states of magnetizability, varying with hardness.*

[Thin wire, 2ρ=0.084 centimeter.]

Mean hardness (galvan).	Mean specific magnetism m, for the dimension-ratio =				
	20	40	60	80	100
37.9	31.5	42.6	46.9	48.9	50.2
35.5	30.5	41.3	45.4	47.2	48.3
33.9	30.1	40.7	44.8	46.5	47.6
33.1	29.9	40.5	44.6	46.3	47.4
32.5	29.8	40.3	44.5	46.3	47.4
28.5	29.5	42.6	47.3	49.8	51.7
27.4	30.1	44.8	49.8	52.7	54.5
26.0	32.3	48.6	54.4	57.2	59.4
24.8	33.9	52.6	59.2	62.6	64.7
23.9	34.8	55.3	63.2	67.1	68.9
19.7	32.6	63.8	79.0	86.4	90.7
18.1	28.0	65.3	83.0	92.7	99.0
15.1	6.0	24.5	45.3	61.4	72.2

On the basis of these tables the two families of curves, Figs. 17 and 18, have been constructed.



FIGS. 17 and 18.—Relation between specific magnetism and hardness of steel for different dimension-ratios.

Magnetism and dimension-ratio.—Of equally great interest, moreover, is the other phase of these phenomena in which magnetizability is regarded as a function of the independent variable, the dimension-ratio, whereas hardness appears merely as an arbitrary constant. We will endeavor in this place to give an account of our data, sufficiently detailed to admit satisfactorily of a comparison with results of earlier observers. In the absence of a better means of estimating hardness the oxide tints were largely made use of, and the wires examined, therefore, classified as hard, yellow annealed, blue annealed, and soft. In conformity with this method of description, we will attempt to give the parameter (hardness) in our own results, such values as will correspond with the said four States. The following special experiments furnish approximate data for the expression of the degree of hardness corresponding to any given oxide tint in our own galvanic scale:

Cm / Cm², 0° microhm.

Glass-hard ..... s=45.7
Yellow annealed ..... s=26.3
Blue annealed ..... s=20.5
Soft ..... s=15.9

It is well to note that the logarithmic interval between yellow annealed and blue annealed is about equivalent to the same interval between blue annealed and soft, and about twice as large as that between glass-hard and yellow annealed. Referred to our method of tempering, the values for yellow annealed and blue annealed would agree very nearly with three hours of annealing in aniline vapor (mean s=26.7 and 26.0) and one minute of annealing in melting lead (mean s=19.5 and 19.7), respectively.

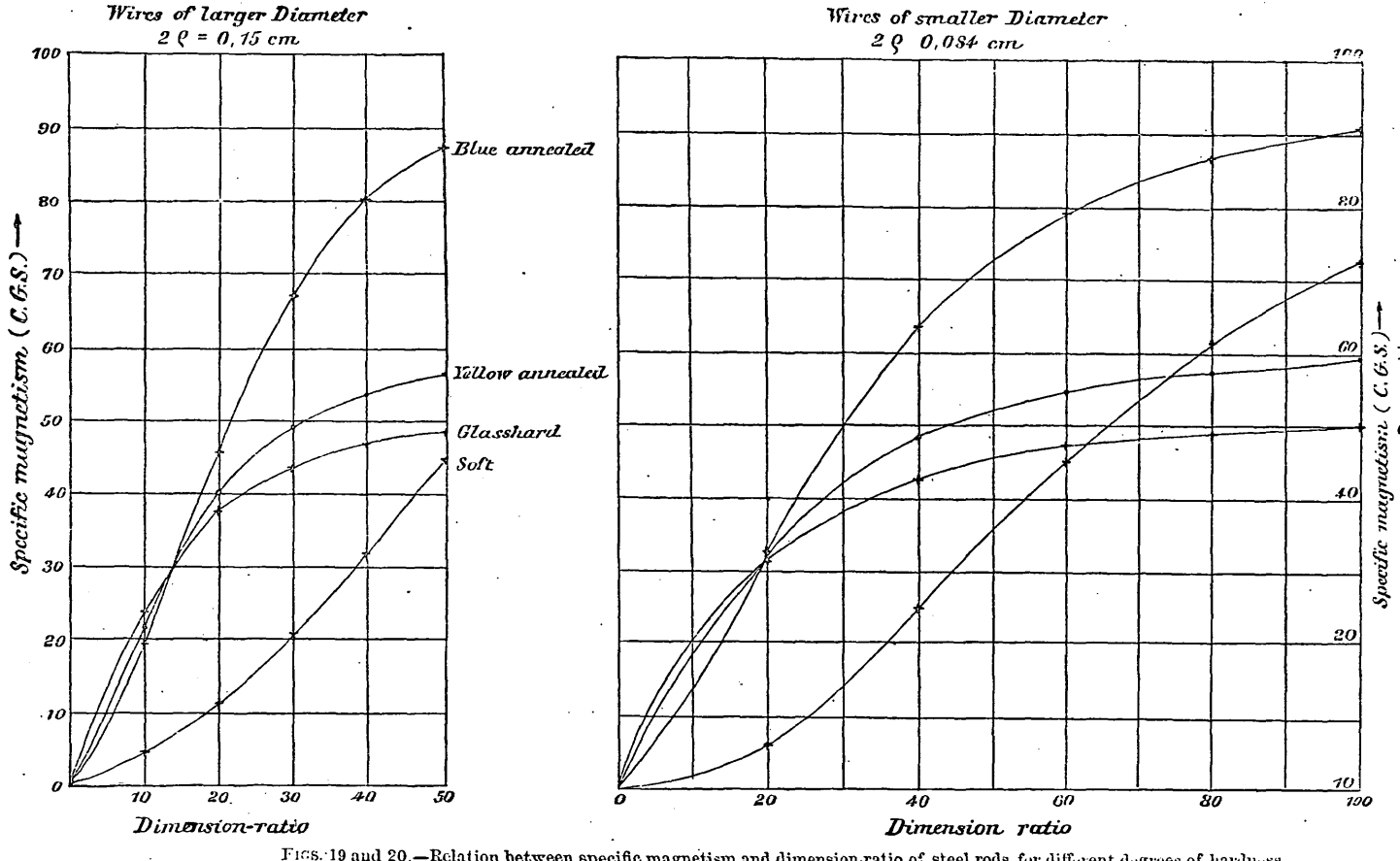
The smaller galvanic values for these states correspond to the smaller values for the soft state (mean s=15.3 and 15.1), respectively, and the difference is, therefore, obviously due to carburation. Hence for our present purposes it is fully sufficient to select from tables 62 and 63 the following mean values of magnetism for the degrees of hardness in question:

TABLE 64.—*Thick wire.*

	Mean specific magnetism m, for the dimension-ratio =				
	10	20	30	40	50
Glass-hard	23.5	37.6	43.6	46.5	48.3
Yellow annealed	21.4	40.2	49.4	53.8	56.5
Blue annealed	19.3	45.8	67.0	80.4	87.3
Soft	4.3	11.2	20.5	31.8	44.6

TABLE 65.—*Thin wire.*

	Mean specific magnetism m, for the dimension-ratio =				
	20	40	60	80	100
Glass-hard	31.5	42.6	46.9	48.9	50.2
Yellow annealed	32.3	48.6	54.4	57.2	59.4
Blue annealed	32.6	63.8	79.0	86.4	90.7
Soft	6.0	24.5	45.3	61.4	72.2



FIGS. 19 and 20.—Relation between specific magnetism and dimension-ratio of steel rods, for different degrees of hardness.

Graphic representation.—These data enable us to construct the curves in Figs. 19 and 20.

The curve glass-hard is concave as regards the axis of abscissæ throughout. Rising very rapidly at first it finally ascends to a distinct limiting value or horizontal asymptote. The curve soft, on the other hand, rises very slowly in its earlier stages, and is convex as regards the axis of abscissæ. From here it passes rapidly upward through a point of circumflexion into concavity towards X, and then above the former curve. Finally the rate of ascent again decreases so that a horizontal asymptote must eventually also be reached, but apparently at a later stage of the progress than is the case with hard wire. From either of these two curves, which describe the variations of the extreme states, we may, by the process of annealing, pass continuously to the other; but the manner of such passage from the one to the other, resulting from a continuous change of parameter (hardness), is excessively complicated. Incipient annealing of a glass-hard rod produces a distinct though small descent of the original curve as a whole. As annealing progresses, the farther end of the curve is always the first to rise and pass above the original curve in such a way that the point of intersection of the new curve and the original curve, glass-hard, moves rapidly from greater to smaller values of the dimension-ratio. When the curve "yellow annealed" is reached the part between $\alpha=14$ or 18, respectively, and $\alpha=\infty$ has already been elevated above the curve glass-hard. In the case of the "blue annealed" curve we observe the part between $\alpha=15$ or 20, respectively, and $\alpha=\infty$ to have risen enormously; but on the other hand the part of this curve between $\alpha=0$ and $\alpha=15$ or 20, respectively, having descended gradually, is now distinctly convex as regards the axis of abscissæ. Passing from blue-annealed to soft, we find the left-hand part of the curve falling at a more rapid rate, while the right-hand branch still rises slowly, reaching a superior limit and then falling rapidly into coincidence with the curve for the soft state, already discussed.

We have remarked that the phenomenon of variation of magnetizability, regarded as a whole, does not by any means seem to present a certain critical value for the dimension-ratio, below which magnets behave differently than above it. If only the four curves hard, yellow annealed, blue annealed and soft were known, as they appear in Figs. 19 and 20, there would appear to be reasons for considering $\alpha=15$ or 20, respectively, as a critical value of this kind; for it is here that the three curves approximately intersect each other. How unjustifiable, however, such conclusions are, will be obvious from a mere glance at Figs. 17 and 18, in which the curves for this value of α manifest no change of character whatever.

Magnetism and density.—We may remark here that when the rod is very long in comparison with its diameter, its specific magnetism (saturation presupposed) becomes constant or independent of the dimensions.

In Green's equation for the distribution of magnetism in a cylinder of finite length, $2l$, and radius r.

$$\lambda = \pi KX \operatorname{pr} \frac{e^{\frac{px}{r}} - e^{-\frac{px}{r}}}{e^{\frac{pl}{r}} + e^{-\frac{pl}{r}}}$$

where λ is the linear density of free magnetism at a distance x from the middle of the cylinder, K the coefficient of magnetization, X the magnetizing force, p a numerical quantity related to K; let KX be accepted as the expression for coercive force, F, and put for greater simplicity

$$r \, lg \frac{1}{\nu} = p.$$

If now we evaluate

$$M = 2 \int_0^l \lambda x dx = -\frac{2\pi p F r}{lg\, \nu} \left[l + \frac{1}{lg\, \nu} \frac{\nu^{-l} - \nu^{+l}}{\nu^{-l} + \nu^{+l}} \right]$$

and furthermore put

$$m = \frac{M}{\mu} = \frac{M}{\pi r^2 l d}$$

we find that if l (or better $\frac{l}{r} = \alpha$) be supposed to increase indefinitely, m will pass through

$$m = \frac{2F}{d} \left\{ 1 - \frac{1}{p\alpha} \right\}$$

into a constant value, depending only on the material. Biot[132] put $p = 0.124$, approximately.

Now it is remarkable that in this case, in which moreover the expression for the linear distribution of magnetism attains its simplest form, we are led to infer that the (linear) rods are capable of retaining more magnetism permanently in proportion as they are softer. But this general deduction will not hold good as far as the soft state. Figure 20 gives as yet no decisive answer to this question. However, in passing from the blue annealed to the soft condition, steel passes through a state of maximum density; and our results thus give warrant to the surmise that the greatest attainable magnetization can be imparted to steel, when the hardness of the necessarily linear rod has that particular value which is characterized by maximum density. This digression suggests itself naturally here, but will be made the subject of a special inquiry. From figure 17, however, it is already quite apparent that the said unique maximum will be considerably larger than 785 c. g. s. units of intensity (magnetic moment per unit of volume), the value thus far assumed and derived from an incidental result of Kohlrausch.[133]

[132] Biot: Traité de Phys., III, p. 76, 1816.

[133] Cf. Gordon: Electricity and Magnetism, I, p. 156, London, 1880.

Earlier results interpreted.—It will now be in place to compare our results with those of Ruths. We will select from his data the series contained on p. 47 of his memoir, as these may be regarded typical. Here again, those alone can be considered which refer to magnets as nearly as possible in the permanently saturated state. In the place of the absolute magnetic moment of his steel rods we have deduced the corresponding values for specific magnetism from his data. This greatly facilitates comparison.

TABLE 66.—*Ruths' magnetic results.*

Length = 12.

I.	II.	III.	IV.	V.	VI.		
0.10	0.24	0.29	0.38	0.49	0.59	Diameter.	
70	50	40	30	25	20	Dim. ratio, approx.	
1.99	4.31	6.22	10.55	17.50	25.65	Mass.	
68.7	66.1	74.7	61	66.6	51	Glass-hard.	Spec. Magnetism.
88.6	71	79.1	52	38.6	29.7	Yellow annealed.	
92.0	71	90.2	61.3	48.2	28.4	Blue annealed.	

The first comment to be made in this connection has reference to our remarks given in p. 117, on the structural influence of tempering. Ruths' magnets were all of the same lengths, whereas the thickness varied from 0.2 centimeters to 0.6 centimeters. To impart like degrees or similar states of hardness to rods varying in thickness to this very large extent, is manifestly impossible, even if the material were throughout perfectly identical. It follows that the divers values of specific magnetism obtained cannot even be compared one with another. Hence the attempt to arrive at the nature of the functionality between permanent magnetism and ratio of dimensions from these results, must necessarily be futile. Ruths' data are unavailable, therefore, for the construction of curves, corresponding to those in figure 19. We infer from Ruths' table that in general the specific magnetism increases with the dimension-ratio. In the case of the thicker magnets (IV, V, VI) Ruths found values for specific magnetism much larger than our own. In accordance with these, the intersection of the curves glass-hard and blue annealed would occur for $\alpha=30$; the corresponding point for blue annealed and yellow annealed lying at $\alpha=28$. In accordance with Ruths' results, moreover, figure 19 would have to be changed in such a way that the curve glass-hard (to agree with Ruths' large values) rise at a more rapid rate, thus falling short of intersection with the other two curves until these have intersected each other at about $\alpha=20$. Then its passage is through blue annealed at about $\alpha=25$, and through yellow annealed at about $\alpha=40$, where the latter curve is also supposed to rise more rapidly than is the case in our results. But with this apparently satisfactory interpretation, the equivalence of blue annealed and yellow annealed for $\alpha=50$, is wholly in discordance.

Fromme uses eight rods, all of the same length, four of which are of

like small diameter, the other four of like larger diameter, to corroborate Ruths' results. The dimensions are these:

$$\text{First four: } \alpha=\frac{100}{7}=15;\ \text{second four: } \alpha=\frac{100}{2}=50$$

His results for the ratio of mass (μ) and square of the time (t) of vibration are the following:

		$\alpha=15$	$\alpha=50$
$\mu:t^2$	Glass-hard	1982	413
	Yellow annealed	1508	448
	Blue annealed	1118	440

These values would put the point of intersection of blue annealed and yellow annealed at $\alpha=50$, agreeing with Ruths' result for his magnet II. But this by no means removes the discordance between the data for $\alpha=50$ and those of Ruths for $\alpha<50$, because the latter refer this point to $\alpha=20$.

An extended series of observations on the relation between moment of permanent magnetism and hardness, of exceptional accuracy, has been published by Gray[134]. All the rods have the same dimension-ratio, $a=50$. Unfortunately he does not carry one and the same hard rod through all possible states of inferior temper. Although the material is identical throughout, the results are therefore necessarily distorted by structural discrepancies, in addition to the effects due to differences of hardness in the glass-hard state. In operating with low temperatures Mr. Gray neglects the (then unknown) time-effect of annealing.

In Chapter II, pp. 40–43, we discussed the results obtained with rods tempered in a way nearly identical with that employed by Mr. Gray, hard wires being immersed in linseed oil, the temperature of which was raised very gradually by the aid of a Bunsen flame, and batches of wires removed from the bath at different stages of the heating. If we take into consideration the great irregularity of distribution of temper which the rods so treated manifest, the causes of relatively large discrepancies occurring in Mr. Gray's results are apparent at once. For the same reasons we are not able to compare them in detail with our own.

One striking difference between the two sets of results is, however, to be noticed. Mr. Gray has observed no minimum of magnetizability in the region of glass-hardness. The specific magnetism, in general, is found to increase rapidly between glass-hard and annealed at 100°; is fairly constant between the latter state and annealed at about 280°; and after this increases rapidly again until the highest annealing temperature employed (310°) is reached. As a whole, however, the observed range of variation (72 to 80 C. G. S. units of magnetic moment per gramme) is remarkably small when compared with our own (45 to 85

[134] Gray: l. c.

and 42 to 75, respectively, C. G. S. units of magnetic moment per gramme). The following tabular comparison between Mr. Gray's mean results and our own will exhibit this perspicuously:

Table comparing the mean results of Gray and of B. and S.

[Gray. l=5.000, 2ρ=0.097 centimeters.]			[B. and S. l=7.50 centimeters, 2ρ=0.15 centimeters.]				
Rod. No. —	Annealed at—	Mag. mom. (C. G. S.), per gramme.	Rod No.	Annealed at—	During the time.	Mag. mom. (C. G. S.), per gramme.	Hardness, s.
1 to 5	15°	71.82 74.33					
			IX to XII	20°		48.3	45.3
6 to 10	100°	75.64 76.98	IX to XII	100°	1h	46.0	40.2
			IX to XII	100°	3h	45.0	37.9
			IX to XII	100°	6h	44.8	36.6
			IX to XII	100°	10h	44.8	35.7
11 to 15	150°	75.60 76.50					
			IX to XII	185°	20m	48.3	30.2
			IX to XII	185°	1h	51.2	28.5
			IX to XII	185°	3h	56.5	26.7
			IX to XII	185°	7h	63.0	25.1
			IX to XII	185°	13h	68.2	23.8
16 to 20	200°	75.76 76.04					
			IX to XII	210°	10m	70.9	23.0
21 to 25	240°	77.42					
26 to 30	250°	76.89					
31 to 35	260°	75.92					
36 to 40	270°	77.59					
41 to 45	280°	76.51					
46 to 50	290°	76.14					
51 to 55	300°	78.60 79.09					
56 to 60	310°	79.24					
			IX to XII	330°	1m	87.3	19.5
			IX to XII	440°	1h	101.8	16.7
			IX to XII	1000°		44.6	15.3

The large though unavoidable discrepancies encountered by Mr. Gray, notwithstanding his careful manipulation and precision of measurement clearly show the difficulties with which the problem in question was then surrounded. We possess no data for the magnetic effect produced by tempering in oil. The only satisfactory method of accounting for the above discordance would be that of supposing that Mr. Gray operated upon a less highly carburized steel than was used in our experiments. In such a case, greater values of specific magnetism for the hard state and small values for the soft state in Mr. Gray's results, as distinguished from our own, are anticipative.

CONCLUSION.

The general problem.—The results in this paper offer only a partial solution of the general problem. Irrespective of the effect of carburation, concerning which remarks have already been made, the temporary and permanent magnetization induced by magnetic fields of an intensity insufficient to saturate steel, are new subjects of importance. A fifth

variable, the intensity of the field, therefore, suggests itself. If we remember how intimately magnetic phenomena are associated with the structural condition of a rod, and how absolutely devoid of reliable facts our knowledge of this actually is, we see how little advancement can be hoped from theory. The mathematical analysis, again, even when starting from the simplified premises of homogeneity, encounters formidable difficulties. On the other hand, the possibility of a solution of the general problem, by the application of the method which we have endeavored to develop in the above, we dare say is undeniable. Such an attack we have in view.

Practical bearing.—The present investigation has a practical bearing of some importance. For cylindrical rods it gives a satisfactory solution of the problem as to what degree of hardness is to be chosen in order that a given steel rod may possess the maximum magnetizability. It is a remarkable result, that for very small values of the dimension-ratio considered as one extreme case, the rods can be most intensely magnetized when in the very hard states, whereas for very large values of the dimension-ratio regarded as the other extreme case, similarly favorable conditions are offered by the softer states, with the probability that a state of singular excellence in this respect lies between blue annealed and soft.

But it does not by any means follow herefrom that in the construction of magnets for practical purposes they are to be made glass-hard if short and thick, and soft if long. This question involves elements of quite another character. The problem is so to construct a magnet that with a maximum of intensity it may be best qualified to withstand the hurtful influence of atmospheric changes of temperature, of shocks, and such like effects; in short, to produce a magnet which, under like circumstances, shall always show practically identical values of the intensest available magnetization. This will be made the subject of the next chapter.

ADDENDUM: ON THE DENSITY-EFFECT OF INCIPIENT ANNEALING OF HARD STEEL.

In Chapter V we adverted to the relation probably existing between the maximum of permanent magnetism of linear steel rods and their density. We there, moreover, state the grounds why the conditions most favorable for the appearance of such a relation are encountered in the case of linear rods. The question is therefore immediately suggested whether the characteristic minimum of magnetizability of steel rods corresponds to an analogously singular point in the variation of density. We commenced experiments with the object of discussing the

matter, with a steel rod 1.91 centimeters in diameter and 5.40 centimeters long, tempered glass-hard in the usual way.

State of hardness.	Specific gravity at 20°.	Specific volume.
Commercial	7.8320	1.00000
	7.8313	
Glass-hard	7.8035	1.00363
	7.8033	
Annealed 1h at 100°	7.8032	1.00365
Annealed 4h at 100°	7.8040	1.00355
	7.8039	
Annealed 8h at 100°	7.8052	1.00339
	7.8052	
Annealed 12h at 100°	7.8058	1.00332

The first result obtained from these data is a striking corroboration of a result of Fromme's, viz, that the maximum increase of specific volume experienced by a hard-tempered steel rod diminishes as the diameter of the rod increases. Fromme's results for thickness on specific volume of tempered steel rods are these:

2ρ = Thickness (cm.) =	0.7	0.42	0.265	0.255
v = Sp. volume =	1.00772	1.01000	1.01285	1.01210

The result for the above rod $2\rho = 1.91$ and $v = 1.00363$ is in excellent accordance with these data. In drawing this inference it is necessary to bear in mind that serious discrepancies may be introduced by the possible differences in *carburation* of the respective rods. Nevertheless, all these results taken together are so satisfactorily consistent that it is difficult to avoid the deduction made.

The second result shows that the increment of specific volume due to tempering in general decreases as time increases. But this variation takes place at a more rapid rate at the middle stages of the operation than either at its early or closing stages. The curve, therefore, contains a point of circumflection, and the possibility of a maximum near the inception is by no means excluded. To obtain definite and decisive results, however, it will be necessary to operate with thin rods, for which the density effect of tempering is so much more clearly pronounced.

The plausible inference that the anomalous electrical behavior of steel on incipient annealing, discussed in Chapter II (page 67), and the minimum of magnetizability investigated in this chapter, may find an analogous variation in the density of similarly treated hard steel, cannot as yet, therefore, be said to have been disproved.

CHAPTER VI.

THE TEMPERING OF STEEL CONSIDERED IN ITS BEARING ON THE POWER OF MAGNETIC RETENTION, OR ON THE CONDITIONS OF MAGNETIC STABILITY OF THIS MATERIAL.

INTRODUCTION.

Temporary and permanent magnetic effects of annealing.—The influence of temperature on the moment of permanent magnetism of steel rods is characterized by a temporary and a permanent effect, usually superimposed. If a steel rod magnetized to saturation at the temperature t° is heated to T° ($T>t$) and then again cooled to t°, it will be found to have experienced a loss of magnetic moment. If the process is repeated—the temperatures t and T being the same as before—an additional diminution, decidedly smaller, however, than the first, will manifest itself. Continuing in this way we shall find that the permanent loss converges to zero. The rod is now in a condition for which the magnetization lost during the passage from t° to T° is again restored when the original temperature t° is regained.

Researches on this important subject have been made in great numbers.[135] The result has usually been that magnets which are to be used between the atmospheric temperaturess t and T should be heated and cooled between these or greater limits for an indefinite number of times, in order that a condition of magnetic permanence or of perfect magnetic elasticity, as it has been called, may be assumed.

Simultaneous mechanical effects hitherto disregarded.—To the venture of resuming a topic which has been so elaborately and apparently so exhaustively discussed, we were primarily induced by certain new and important facts which our researches on the hardness of steel had developed. Curiously enough among those who operated with glass-hard rods—and it was to these that we, at first, desired to confine our attention—only a few have given even cursory consideration to the important factor, the change of the mechanical state of the material. We have shown that temperatures 20° or 30° above that of the water in which during tem-

[135] The earlier literature is systematized in J. Lamont, Magnetismus, p. 386, 1867; G. Wiedemann, Galvanismus, II a, p. 603, 1874; A. Mousson, Physik, 3 Aufl., III., p. 110. Among the more recent papers (since 1876) we desire to mention: G. Wiedeman, Pogg. Ann., CLVII, p. 257, 1876; J. M. Gaugain, Comptes rend., LXXXII, p. 1422; LXXXIII, p. 661, 1876; LXXXVI, p. 536, 1878; G. Poloni, Beiblätter V, p. 67, p. 802, and p. 614, 1881; Elettricista, II, p. 193, 1878; J. Trowbridge, Am. Journal (3), XXI, p. 316, 1881

pering the rod was chilled, when acting on glass-hard rods can be made to produce annealing effects of definite and accurately definable magnitude.

If, therefore, it is our object to investigate the functionality between magnetization and temperature, it is manifestly necessary at the very outset to exclude all permanent and simultaneous changes in the material carrying the magnetic quality. This is what none of the former observers have done. Temperatures as high as 100° are frequently applied. Under these circumstances the hard rod assumes different mechanical properties while in the hands of the operator, and the results will necessarily be stripped of the claims to accuracy which the care frequently bestowed would otherwise justify.

Retentiveness.—There was a second consideration which suggested the present series of experiment. This was the desire to utilize certain earlier data in the endeavor to construct magnets possessing great power of retention. It is not necessary to advert to the important bearing of this problem, not only on all absolute magnetic measurements, but more particularly even on those of a relative character. That the methods now employed are inadequate is conceded by the observers of highest authority and experience. Old magnets subserve best the purposes of measurement; but even these, where a constancy of moment under like circumstances is to be presumed are carefully to be protected from shocks and larger changes of temperature.

The present work.—In the following pages our experiments will be cited chronologically. Some of them are merely corroborative as regards results which have already been pronounced by others. But the intimate connection between these and our subject proper, together with the new interpretation which has frequently been given them, vindicates their appearance here.

The method of magnetization and the calculation of the magnetic moment as well as the measurement of hardness was identical with that detailed in Chapter V.

RETENTIVENESS AS REGARDS VARIATION OF TEMPERATURE.

Preliminary experiments.—Our first experiments were incidentally made with six little steel parallelpipedons, of the same material which had been used in a series of experiments described elsewhere. The dimensions (cm.) and mass (*g*) of these, after being tempered to glass-hardness, were as follows:

	I.	II.	III.	IV.	V.	VI.
Length	3.0	2.5	3.0	2.5	3.0	2.5
Breadth	0.5	0.5	0.4	0.4	0.3	0.3
Height	0.3	0.3	0.3	0.3	0.2	0.2
Mass	0.328	0.278	0.278	0.232	0.133	0.113

When magnetized with a large Funkler's magnetic battery (50 kilos portative force) of the horseshoe form, these retained the following quantities of specific magnetism:

	I.	II.	III.	IV.	V.	VI.	
$m=$	16.4	12.9	18.6	14.8	25.0	20.7	$\frac{cm^{\frac{1}{2}}}{g^{\frac{1}{2}}\,sec}$

Hereupon the magnets were transferred from a water-bath at 15° C. to another at 50° C. and then returned. This was repeated ten times, with an allowance of about ten seconds of immersion for each. After this process the respective values of specific magnetism were found to be—

	I.	II.	III.	IV.	V.	VI.
$m=$	16.1	12.7	18.4	14.6	24.6	20.3

The losses are therefore small—as a rule, only about $\frac{3}{2}$ per cent. The temperature 50° appears to be relatively low in so far as its effect in producing the variation in question is concerned. If it should be our purpose to reach a limiting value in this way the operation would have to be repeated a great number of times. Possibly several hundred repetitions would even be inadequate.

For this reason we decided to continue the work with the aid of higher temperatures. The magnets were exposed in steam. Previously, however, we remagnetized them to saturation. This was done with a large and powerful helix, through which the current of a dynamo-electric machine circulated. The mean intensity of the magnetizing force when referred to the lengths 2.5 cm. to 3.0 cm. of the magnets, and for the current 3.0 $cm^{\frac{1}{2}}$ $g^{\frac{1}{2}}$/sec, was found to be—

$$\Sigma X = 885\ g^{\frac{1}{2}}/cm^{\frac{1}{2}}\,sec.$$

It is obvious that these powerful forces[136] were far more than sufficient to effect the saturation.

The magnets were now exposed in steam at 100° for consecutive intervals of 20, 40 minutes, then for 1, 2, 3, and finally 4 hours. After each withdrawal from the steam-bath they were laid aside for some time in a room in which the temperature varied between 10° and 15°. After this we made the measurement of the magnetic moment. The

[136] For divers other data relative to this helix conf. Chapter V.

results of this series of experiments are shown in the following table for *m*:

TABLE 67.—*Limits of magnetic state.*

Description of temper.	I.	II.	III.	IV.	V.	VI.
Original condition (hard)	18.97	14.90	21.34	17.08	28.70	23.55
Annealed 20m in steam 100°.....	14.08	10.74	15.80	12.42	20.94	16.66
Annealed 40m more in steam 100° .	11.68	9.05	13.45	10.42	17.60	13.92
Annealed 1h more in steam 100°...	10.34	8.11	12.21	9.60	15.68	12.22
Annealed 2h more in steam 100°...	9.42	7.32	11.21	8.85	14.16	11.04
Annealed 3h more in steam 100°...	8.86	7.07	10.68	8.21	13.63	10.24
Annealed 4h more in steam 100°...	8.65	6.69	10.29	8.02	12.98	9.91

If these results be constructed graphically, time of exposure as abscissa and specific magnetism (at ordinary temperature) as ordinate, we obtain a series of curves of regular and similar contour. Their general characteristic is that of an initially rapid descent, which as annealing continues is soon retarded until finally an asymptote parallel to the axis X, is approximately reached. These results corroborate the older results for the effect of the time of exposure on the variations of the magnetic state of a bar. The phenomena were first studied by Moser and Riess,[137] somewhat later by Holmgren,[138] and particularly emphasized by the latter in a way at variance to the views of the former observers. Physicists were inclined, however, to refer Holmgren's apparently anomalous results to other causes,[139] whereas Lamont[140] pointed out that Holmgren had operated with glass-hard rods, while the magnets of Moser and Riess were untempered. Quite recently G. Poloni[141] has given the effect of the time of exposure due prominence. He fails, however, clearly to discriminate between the results obtained in cases of differently tempered bars, or, in other words, confounds the effect due to change of temper with the magnetic effect. We shall show in the sequel that they are entirely distinct and can be separately studied.

Simultaneous magnetic and mechanical effect of annealing.—But there is another inference of greater relevancy to our immediate inquiry. The curves under consideration are strikingly similar to those formerly obtained while investigating the electrical effect produced by a change of temper of steel (annealing).[142] In fact, the analogy is sufficiently evident to induce us to associate the observed decrement of magnetic intensity and the cotemporaneous change of mechanical state of glass-hard rods as phenomena intrinsically related. In other words, we were inclined to regard annealing as the primary, and the diminution of magnetic intensity (in by far the greater part) as the secondary occurrence; or more

[137] L. Moser and P. Riess: Pogg. Ann., XVII, p. 403, 1829.
[138] K. A. Holmgren: Acta Soc. Scient. Upsala (3) I; Fortschritte d. Physik, p. 536, 1856.
[139] Cf., G. Wiedemann: Galvanismus, II a, p. 614, 1874.
[140] J Lamont: Magnetismus, p. 385, 1867.
[141] G. Poloni: Elletricista, II, p. 139.—Beibl. II, p. 523, 1878.
[142] Strouhal and Barus: Wied. Ann., XI, p. 930, 1880. Chap. II.

accurately, to distinguish between a primary or purely magnetic (permanent) effect of temperature, a *direct* effect; and an *indirect* effect of temperature, due to the influence of the rearrangement of molecules in consequence of mechanical annealing, upon the existing intensity of magnetism of the rod.

With the object of verifying this hypothesis we selected a sample out of our supply of glass-hard rods which, when tested, showed great uniformity of hardness throughout its length. This was broken into two nearly equal parts Nos. 11 and 12.[143] The constants of each were found to be—

	No. 11.	No. 12.	
Length ...	10.0	10.0	cm.
Mass.....	0.417	0.418	*g.*

These were now magnetized to saturation with the helix, and then annealed in steam at 100° during consecutive intervals of 10, 20, and 30 minutes, 1, 2, 3, 4, 5, and 6 hours. After each of these we made a determination of hardness using specific resistance *s* at the atmospheric temperature *t* as a datum for this purpose. Furthermore, in order to arrive at the desired comparison, No. 12 was remagnetized to saturation after each interval of annealing, whereas No. 11 was tested for specific magnetism in the condition in which it left the annealing bath, that is without repeated magnetization.

The results of these experiments are contained in the following table. The specific magnetism is represented by m^* and m, according as the magnetic datum in question was obtained with or without remagnetization respectively. W here denotes the observed electrical resistance of the respective rods per meter of length at the temperature *t*, *s* the corresponding specific resistance (cm./cm.^{2} 0° microhm).

TABLE 68.—*Diminution of hardness, magnetization, and magnetizability.*

Description of temper.	Magnet No. 11.				Magnet No. 12.				
	W	*t*	*s*	*m*	*W*	*t*	*s*	*m*	m^*
Glass-hard	0.755	18.3	39.5	62.6	0.761	18.5	39.9	.....	62.5
Annealed 10^m in 100°	741	18.7	38.7	59.5	746	18.7	39.1	59.7	62.4
Annealed+20^m in 100°	724	20.1	37.6	56.0	728	20.1	38.0	58.2	61.9
Annealed+30^m in 100°	708	21.0	36.7	52.6	712	21.0	37.1	57.5	60.6
Annealed+ 1^h in 100°	689	19.9	35.7	50.0	693	20.0	36.1	56.5	60.2
Annealed+ 2^h in 100°	671	20.2	34.8	47.3	676	20.1	35.2	56.1	59.5
Annealed+ 3^h in 100°	656	18.7	34.1	46.1	660	18.7	34.5	56.4	59.4
Annealed+ 4^h in 100°	646	19.2	33.5	45.1	651	19.0	33.9	56.5	59.3
Annealed+ 5^h in 100°	639	20.0	33.1	44.3	644	20.0	33.5	56.3	59.1
Annealed+ 6^h in 100°	634	19.9	32.8	43.8	638	19.9	33.2	56.5	59.0

[143] The magnets Nos. 1–10, were used in the work detailed in Chapter V.

Mere inspection of this table discovers at once the striking parallelism in the successive values of specific resistance s and specific magnetism m. If the two series of results be represented graphically, time of exposure as abscissa, s and m respectively as ordinate, we obtain (Fig. 21) two curves of an obviously allied character. Indeed there can be

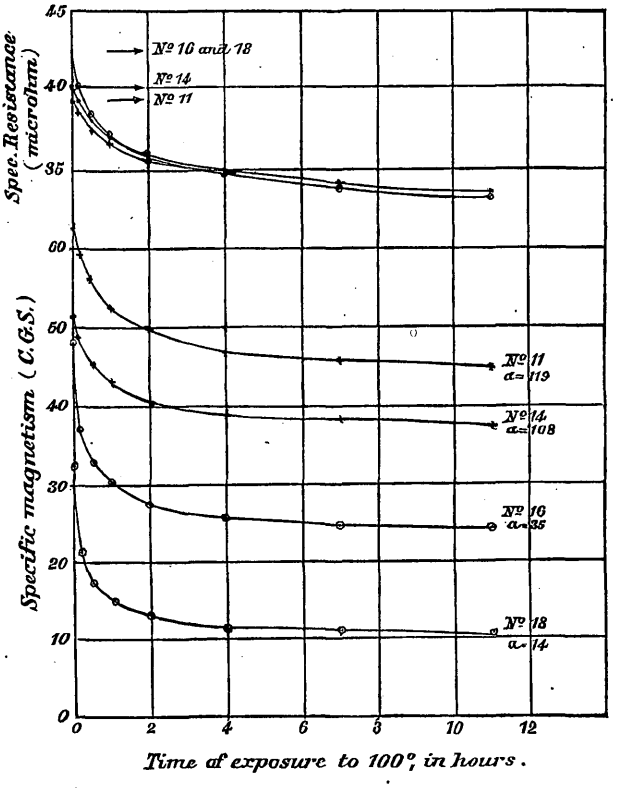


FIG. 21.—Diminution of specific magnetism and specific resistance produced by continued annealing (low temperatures).

no doubt that the occurrence of the continuous diminution of magnetic moment here observed is largely conditioned by the variation of hardness which occurs contemporaneously. It is particularly to be noted that in both instances the asymptotic limit is reached after the lapse of the same time, approximately.

If now we regard the values for m^* as obtained with the magnet No. 12, there appears in the first place a diminution of the quantity of magnetism which the saturated rod permanently retains, which soon however, as annealing continues, reaches a certain minimal value in a way consistent with the results of the former experiments. Furthermore, from a comparison of the values of m and m^*, it follows that during the progress of the annealing the influence of the higher temperature (100°), regarded in connection with the time during which it acts, becomes less and less pronounced, in proportion as the magnet itself has reached

the limiting state of hardness for the temperature (100°) under consideration. But it also appears that the same limiting value of specific magnetism gradually reasserts itself, no matter how often the combined process of magnetization and subsequent indefinite annealing may be repeated. Moreover, a magnet which has approximately reached the limiting hardness for the given temperature, if remagnetized to saturation and annealed again at the same temperature, reaches a limiting magnetic condition, the value of which is nearly independent of the time of exposure.

The purely magnetic effect (permanent).—The inference enunciated at the end of the last paragraph still needs additional proof. Accordingly, our magnets Nos. 11 and 12 were now magnetized afresh and then exposed to steam in the uniform manner described. We thus arrived at the following results:

TABLE 69.—*Specific magnetism, m, of saturated rods successively annealed at* 100°.

Time of exposure to 100°.	No. 11.	No. 12.
Rods remagnetized	59.0	59.0
10 minutes in steam at 100°	57.3	57.1
20 minutes more in steam at 100°	56.6	56.6
30 minutes more in steam at 100°	55.7	55.9
1 hour more in steam at 100°	55 7	55.6
2 hours more in steam at 100°	55.7	55.6
Rods remagnetized	59.0	58.7
10 minutes in steam at 100°	57.2	57.1
20 minutes more in steam at 100°	56.6	56.4
30 minutes more in steam at 100°	56.0	55.9
1 hour more in steam at 100°	56.0	55.9
2 hours more in steam at 100°	56.0	55.8

We also remagnetized the steel parallelopipedons Nos. I . . . VI, with which the original experiments were made, and then annealed them in steam in the usual way, repeating the whole process a number of times. The following table contains the values of specific magnetism obtained:

TABLE 70.—*Specific magnetism, m, of saturated rods successively annealed at* 100°.

Time of exposure to 100°.	I.	II.	III.	IV.	V.	VI.
Rods remagnetized	17.82	13.96	20.14	16.22	27.20	22.58
1 hour in steam at 100°	16.34	12.78	18.54	14.75	25.44	21.07
1 hour more in steam at 100°	16.24	12.69	18.33	14.60	25.24	20.93
2 hours more in steam at 100°	16.12	12.66	18.29	14.54	25.12	20.41
Rods remagnetized	17.84	13.94	20.12	16.03	27.40	22.80
1 hour in steam at 100°	16.28	12.69	18.47	14.73	25.42	20.93
1 hour more in steam at 100°	16.24	12.66	18.37	14.69	25.34	20.97
Rods remagnetized	17.77	13.91	20.09	16.00	27.25	22.71
10 minutes in steam at 100°	16.25	12.73	18.31	14.64	25.60	21.01
50 minutes more in steam at 100°	16.10	12.59	18.24	14.54	25.34	20.93
1 hour more in steam at 100°	16.07	12.54	18.26	14.54	25.31	20.86

If the present behavior of the magnets, where the steel has practically reached its limiting mechanical state for 100°, is contrasted with the above, where temper varies simultaneously with magnetism, a much

smaller and thoroughly uniform loss of specific magnetism is everywhere apparent. In the case of No. 11 the original loss amounted to $\frac{62.6-43.8}{62.6}=30$ per cent. nearly; whereas at present the average loss is only $\frac{59.0-55.9}{59.0}=5.3$ per cent. Similarly, the average loss of specific magnetism of the rods I . . . VI was originally $\frac{20.76-9.42}{20.76}=55$ per cent. nearly. Now we have only an average loss of $\frac{19.62-17.93}{19.62}=8.6$ per cent. But owing to the fact that these magnets had been specially treated at the outstart—repeatedly heated from 15° to 60°—an immediate comparison between the last results and those for No. 11 is not to be made.

We have thus arrived at a partial corroboration of certain results obtained by Moser and Riess,[144] and subsequently also by Dufour.[145] Following Moser and Riess, we have for the successive losses of a hard needle:

	Per cent.
After the first magnetization	44.0
After the second magnetization	6.1
After the third magnetization	4.4

But with this last result our observations are at variance. As has been stated, our data have invariably shown that when the maximum of permanent hardness corresponding to any temperature has once been attained, then the magnetic effects of repeated application of the same annealing process are identical, the losses of specific magnetism experienced by saturated rods constant.

The direct and indirect effect of temperature.—We conclude, therefore, that if it be our object to perspicuously represent the law of the phenomena in question, it is essential to discriminate between two species of magnetic loss. If the magnet is in such a condition—for instance glass-hard—that the higher temperature (T) produces a mechanical effect, then this is invariably accompanied by a magnetic effect peculiar to itself, and as experiment has shown, of relatively very large intensity. The reasons for this behavior are obvious. The existence of magnetism is conditioned by a strain of a particular and characteristic kind. The same is true of hardness. It is very probable, therefore, that the partial disappearance of one of these strains from any cause whatsoever will materially interfere with the intensity of the other.[146] Why the influence of the time of exposure to 100° is marked when the state of hardness is such that annealing produces both a mechanical and a mag-

[144]Moser u. Riess: Pogg. Ann., XVII, p. 403, 1829.

[145]Dufour: Fortschr. der Physik, 1857, p. 438.

[146]Whether mere magnetization produces a change in the temper of glass-hard steel is still to be investigated. In consequence of the very small variation of dimensions the anticipative effect must, of course, necessarily be small.

netic effect, is readily seen. For the latter effect must continue to vary until the limit of variation of the former has been fully reached; and the annealing effect of 100° in case of glass-hard steel is a diminution of hardness occurring at a very gradually decreasing small rate through infinite time.

When this has occurred—*i. e.*, when the final state of hardness due to an exposure to T has been reached—we have to do with a purely magnetic phenomenon only. A rod magnetized to saturation and annealed at $T°$ experiences a direct effect—a loss of specific magnetism which is relatively small, nearly independent of the time during which T acts, and the cause of which may be loosely ascribed to a smaller coercive force at T and to the effect produced by the thermal expansion on the magnetic strain. We may add that while in the first case, where the rod itself undergoes a change of state, a limiting value of specific magnetism was not fully reached even after 22 hours of annealing; in the second, the action is certainly complete after the lapse of an hour, and occurs in such a way that the principal part of the magnetism is lost within the first ten minutes.

The reasons fully appear why Moser and Riess found that when soft and annealed rods were used the losses were not only small, but occurred with the characteristic rapidity of those here enunciated. In this case an annealing effect due to 100° is manifestly impossible, and the peculiarities of the purely magnetic phenomenon are alone observed. It would moreover appear that the latter for a given temperature, T, is independent of the material used, of an intrinsically magnetic nature. At least Moser and Riess found for this loss

	Per cent.
When the needle was soft	13.6
When tempered blue	13.4
When tempered cherry red	13.7

We will waive this matter here, as it is our object to investigate it specially, paying particular attention, moreover, to the effect incident upon a variation of the dimensions L / D. Such an effect is already, though somewhat obscurely, apparent.

Pre-existing magnetization.—If the inference derived in the foregoing paragraph be correct, then must it be immaterial whether a glass-hard unmagnetic steel rod is first annealed, say in steam, at 100°, until the final mechanical state for this temperature has been practically reached and then magnetized to saturation, or whether the rod, *originally* saturated, is annealed and then remagnetized, as in the previous case. The ultimate result must, in other words, be independent of pre-existing conditions so long as these are effects of a lower order than correspond to the given temperature.

In order to give this question, which partakes of the nature of a crucial test, due experimental consideration, two rods, Nos. 13 and 14, of equal length, were broken from a glass-hard sample of the same

thickness (0.084 cm.) and material as Nos. 11 and 12. The constants of the new magnets are:

	No. 13.	No. 14.
Length	9.1	9.1 *cm.*
Mass	0.379	0.381 *g.*

Of these, No. 14 was magnetized to saturation; No. 13, however, left unmagnetized. Both were then exposed to the action of steam, and the progress of the annealing investigated by repeated measurements of the specific resistances of the rods. Unfortunately, a piece of No. 13 was accidentally broken off in clamping. The new rod (No. 13) was 8.7 cm. long and weighed 0.363 *g*. The two wires in their present condition would not, however, permit us to discuss the question from a sufficiently broad standpoint, and we, therefore, selected three other wires of the diameter 0.2 cm., so chosen as to present nearly the same specific resistances, viz:

No. V.....	$s = 42.4$ $\left(\frac{\text{cm}}{\text{cm}^2}\ 0^0\ \text{microhm.}\right)$
No. VI.....	$s = 42.6$ Do.
No. VII.....	$s = 42.2$ Do.

The rods were tempered by sudden cooling after heating to redness in the flame of a blast-lamp. Out of the first but a single magnet was taken, No. 15, and but one, No. 16, from the second; while to the third and most homogeneous of the three the two shorter magnets, Nos. 17, and 18, owe their origin. It was intended to have the lengths of Nos. 15 and 16 and of Nos. 17 and 18 identical, but it is difficult in the case of wires of this thickness to break them off at a prescribed mark. Small variations of length are, therefore, unavoidable. The constants of the four magnets (0.21 cm. thick) are:

	No. 15.	No. 16.	No. 17.	No. 18.
Length *cm.*	7.2	7.3	2.90	2.95
Mass ..... *g.*	1.90	1.92	0.773	0.776

These were now treated in a manner analogous to that applied to Nos. 13 and 14; 11 hours of annealing in steam at 100° transferred them into the final state of temper for this temperature, not completely, it is true, but sufficiently so for the purposes. All were now remagnetized. Nos. 15, 16, 17, 18, acted on by steam for some time, and their magnetic behavior examined, were then remagnetized again, and once more annealed. Nos. 13 and 14, however, were first exposed to a lower temperature, that of boiling methyl alcohol at 66°, during a certain interval; and not until the magnetic limit for 66° had been fully reached

were they exposed in steam, in order that the limiting value of permanent specific magnetism corresponding to the new temperature (100°) might in its turn appear. The data expressing the magnetic effect of these operations are detailed in the following tables, 71 and 72. As before, W denotes the resistance per meter of length of rod at $t°$, s the corresponding specific resistances at zero.

TABLE 71.—*Limits of magnetic state at* 100°.

Time of exposure.	Magnet No. 13.				Magnet No. 14.			
	W	t	s	m	W	t	s	m
Glass-hard.	*ohm.*	C.	*microhm.*	*abs. E.*	*ohm.*	C.	*microhm.*	*abs. E.*
0^m in 100°	0.734	9.0	39.7	.....	0.741	9.1	40.1	51.4
10^m in 100°	721	9.0	39.0	.....	726	9.1	39.3	48.7
$+20^m$ in 100°	701	9.0	37.9	.....	707	9.0	38.2	45.8
$+30^m$ in 100°	683	8.0	37.0	.....	688	8.1	37.2	43.4
$+ 1^h$ in 100°	666	9.0	35.9	.....	669	9.0	36.1	40.9
$+ 2^h$ in 100°	646	9.0	34.9	.....	650	9.0	35.0	39.2
$+ 3^h$ in 100°	632	9.0	34.1	.....	634	9.0	34.2	38.
$+ 4^h$ in 100°	622	9.3	33.5	.....	626	9.8	33.6	37.(
Rod remagnetized	.....	.....	.....	49.9	.....	.....	.....	49.'
1^h in 66°	.....	.....	.....	48.3	.....	.....	.....	48.(
$+ 2^h$ in 66°	.....	.....	.....	48.0	.....	.....	.....	47.1
$+ 3^h$ in 66°	.....	.....	.....	48.0	.....	.....	.....	47.4
1^h in 100°	.....	.....	.....	47.5	.....	.....	.....	47.1
$+ 3^h$ in 100°	.....	.....	.....	47.5	.....	.....	.....	47.1

TABLE 72.—*Limits of magnetic state at* 100°.

Time of exposure.	Magnet No. 15.				Magnet No. 16.				No. 17.	No. 18.
	W	t	s	m	W	t	s	m	m	m
Glass-hard.										
0^m in 100°	.....	.....	.....	.....	.....	.....	.....	48.1	.....	32.3
One day after tempering	0.1277	8.7	42.4	.....	0.1282	8.8	42.5	47.2	.....	31.5
10^m in 100°	1219	9.3	40.4	.....	1218	9.3	40.3	37.4	.....	21.4
$+20^m$ in 100°	1165	9.6	38.6	.....	1167	9.8	38.6	33.2	.....	17.5
$+30^m$ in 100°	1128	9.7	37.3	.....	1129	9.7	37.3	30.3	.....	15.2
$+ 1^h$ in 100°	1087	9.0	35.9	.....	1084	9.0	35.9	27.8	.....	13.4
$+ 2^h$ in 100°	1048	9.1	34.6	.....	1049	9.1	34.7	26.0	.....	11.6
$+ 3^h$ in 100°	1021	9.3	33.7	.....	1024	9.5	33.0	24.8	.....	11.2
$+ 4^h$ in 100°	1011	10.0	33.3	.....	1068	10.00	33.2	24.5	.....	10.7
Rod remagnetized	.....	.....	.....	43.6	.....	.....	.....	45.3	29.2	30.1
10^m in 100°	.....	.....	.....	41.8	.....	.....	.....	43.7	27.8	28.6
$+20^m$ in 100°	.....	.....	.....	41.7	.....	.....	.....	43.6	27.6	28.2
$+30^m$ in 100°	.....	.....	.....	41.6	.....	.....	.....	43.6	27.4	28.2
Rod remagnetized	.....	.....	.....	43.4	.....	.....	.....	45.2	29.3	30.0
10^m in 100°	.....	.....	.....	42.1	.....	.....	.....	44.0	27.8	28.8
$+20^m$ in 100°	.....	.....	.....	41.9	.....	.....	.....	43.7	27.6	28.4
$+30^m$ in 100°	.....	.....	.....	41.7	.....	.....	.....	43.5	27.4	28.4
$+ 3^h$ in 100°	.....	.....	.....	41.6	.....	.....	.....	43.4	27.3	28.3

Magnetic effect of annealing—Final result.—In this series of results our views are fully corroborated. When the limiting state of hardness conditioned by the temperature of annealing T has once been reached, then it is wholly immaterial, in so far as the subsequent magnetic behavior is concerned, whether the rod was originally a magnet or not. The consecutive values of m for Nos. 15 and 16, as well as Nos. 17 and 18, after 11 hours of annealing and remagnetization have been applied, manifest a perfectly similar progress throughout. The same is true of Nos. 13 and 14, both while in vapor of methyl alcohol and while in

steam. It will be observed that the limiting magnetic state for 66° differs from that of 100°, although neither is able to effect any further change in the qualities of the material. Whence it follows, in complete analogy with the results formerly found in the case of simple annealing, that in the present case each given temperature, after having been applied to a glass-hard rod to produce and consummate the mechanical effect, and thereupon to the remagnetized rod for magnetic effect, has its particular and characteristic magnetic limit.

Simultaneous variation of specific resistance and magnetism (low temperature).—In Fig. 21 the diminution of specific resistance, together with the simultaneous variation of specific magnetism due to annealing at 100°, has been graphically represented. The results are those above given for the magnets Nos. 11, 14, 16, 18. The similarity in the two sets of curves suggests an inquiry into the mutual dependence of these quantities. In figure 22 we have plotted specific resistance as abscissa

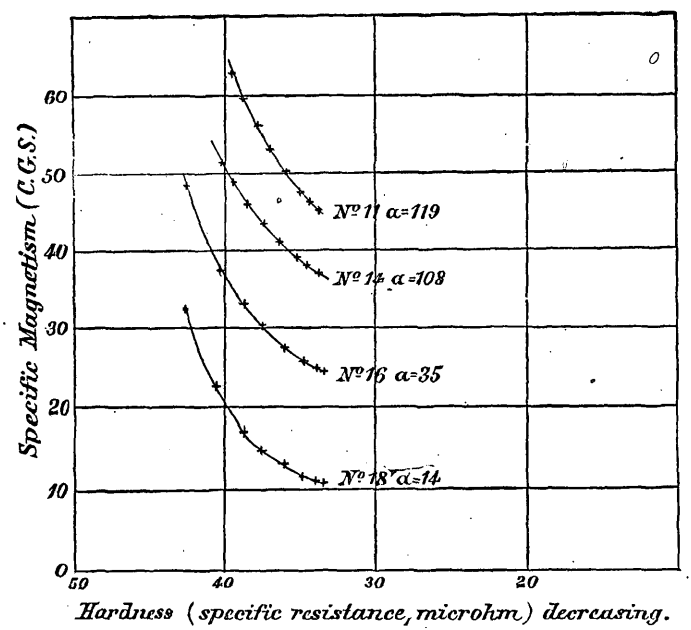


FIG. 22.—Simultaneous variation of specific magnetism and specific resistance in case of continued annealing (low temperatures).

and specific magnetism as ordinate. The points are found to lie on loci of small curvature, approximately parallel to one another. The members of the family appear the more nearly linear the greater $\alpha=\frac{L}{D}$ and therefore the simpler the linear distribution of the magnetism. All are, of course, only the initial parts of far more extensive curves to be obtained by continuing the annealing at temperatures higher than 100°.

The data for Nos. 11, 14, 16, and 17, moreover, show in how far the diminution of specific magnetism from the original to the final value, as

resulting from annealing, is dependent on the ratio of dimensions α. We have the following values for the amounts lost:

No. 11 ..... $\frac{62.6-43.8}{62.6}=30$ per cent.; $\frac{L}{D}=\alpha=\frac{10}{0.084}=119$;

No. 14 ..... $\frac{51.4-37.0}{51.4}=28$ per cent.; $\frac{L}{D}=\alpha=\frac{9.1}{0.084}=108$;

No. 16 ..... $\frac{48.1-24.5}{48.1}=49$ per cent.; $\frac{L}{D}=\alpha=\frac{7.3}{0.207}=35$;

No. 18 ..... $\frac{32.3-10.7}{32.3}=67$ per cent.; $\frac{L}{D}=\alpha=\frac{2.95}{0.207}=14$.

It appears, therefore, that long, thin magnets lose decidedly less than those of small length. But the initial intensity is not without influence. For instance, although both No. 11 and No. 14 were originally saturated, the former retained a larger quantity, whether in virtue of its state of hardness or from small differences of chemical composition does not appear; for the loss of No. 11 is greater, or else that of No. 14 smaller, than the other values would indicate.

In a similar manner the influence of hardness appears in a series of experiments made with ten small steel parallelopipedons of nearly the same dimensions. These magnets (designated by No. VII to No. XVI), 2.5 cm. long, 0.4 cm. broad, and 0.3 cm. thick, of the same material, were glass-hardened in the same way—in so far as this is possible in the ordinary method of tempering—and finally magnetized to saturation by the action of the same magnetic field (helix). The following tabular comparison of the results, however, shows variations of a kind such that few consistent inferences, with the exception of that emphasizing the effect of hardness, can be deduced from them. But from this very fact the importance of structural effects is again clearly indicated.

TABLE 73.—*Limits of magnetic state at* 100°.

	VII.	VIII.	IX.	X.	XI.
Weight (g)	2.53	2.45	2.48	2.47	2.52
Specific magnetism:					
Glass-hard, saturated	11.18	14.11	14.03	15.11	14.15
4^h in steam	2.70	2.89	3.93	4.34	4.06
Rods remagnetized	10.05	12.26	12.31	13.25	12.79
2^h in steam	7.91	11.06	10.93	12.09	11.44
Loss in per cent. of original value (m):					
First loss	76	80	72	71	71
Second loss	21	10	12	9	11

TABLE 74.—*Limits of magnetic state at* 100°.

	XII.	XIII.	XIV.	XV.	XVI.
Weight (g)	2.51	2.41	2.48	2.39	2.43
Specific magnetism:					
Glass-hard, saturated	12.47	14.05	14.64	16.56	15.76
4^h in steam	4.09	3.08	4.53	5.55	4.55
Rods remagnetized	11.50	13.00	13.61	15.72	14.85
2^h in steam	9.80	11.82	12.45	13.70	13.34
Loss in per cent. of original value (m):					
First loss	67	78	69	67	71
Second loss	15	9	9	13	10

Specific resistance and specific magnetism—Higher temperatures of annealing.—The following final experiments on the relation between magnetic moment per unit of mass and the respective specific resistance, in case where a diminution of both qualities is effected by annealing, give further insight into the nature of this functionality. Two magnets, Nos. 19 and 20, were broken from a glass-hard rod, No. VIII (diam. 0·147 cm.), and exposed for different lengths of time in hot baths. The constants of the magnets are these:

Length	$L = 1.50$	9.09 cm.
Mass	$\mu = 0.193$	1.172 *g*
Dimension ratio	$\alpha = 10.2$	61.9

The results obtained by examination at different stages of the operation of annealing are given by the following table:

TABLE 75.— *Simultaneous variations of magnetization and galvanic hardness, for continued annealing.*

Magnets Nos. 19 *and* 20.

Time of exposure.	$W1\,m$	t	s	m No. 19	m No. 20
	Ohm	°C.	*microhm.*		
Glass-hard	0.283	11.5	47.0	21.25	39.1
20^m in 100°	0.267	12.0	44.2	12.56	29.5
+ 2^h in 100°	0.245	12.0	40.5	7.74	22.3
+10^m in 185°	0.202	11.6	33.4	3.93	18.8
+ 6^h in 185°	0.162	11.3	26.7	2.45	17.5
+ 1^m in 330°	0.120	10.0	19.6	0.25	14.4

These curves (figure 23) pass from convexity as regards the axis x through a point of circumflexion into concavity. Both loci are very

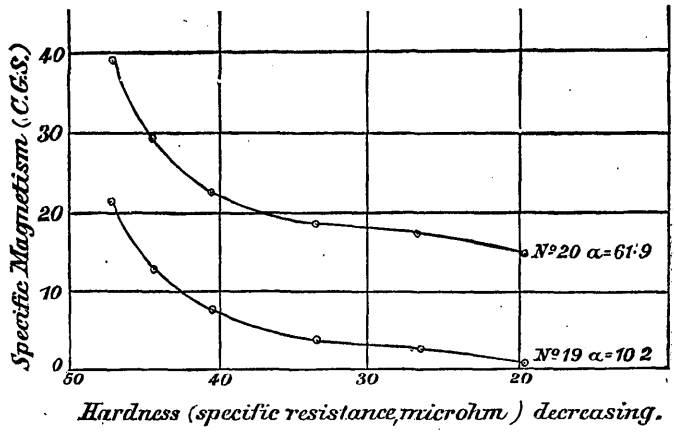


FIG. 23.—Simultaneous variation of specific magnetism and specific resistance in case of continued annealing (high temperatures).

much alike, decreasing rapidly near the origin as well as near their extreme points (annealed at 330°). Short magnets lose their magnetiza-

tion when exposed to temperatures at more rapid rates than long ones. The latter, on the whole, show greater magnetic permanence than the former.

The interpretation of these phenomena is difficult. But it is in place to present the salient points to be considered here. For the reasons given on page 145, a perspicuous relation between magnetism and resistance can only be anticipated in case of linear rods. The results in hand conform with this view to the extent that the observed curvature diminishes in marked degree in proportion as α increases (cf. Fig. 22). In general, moreover, there are two phenomena superimposed: the direct magnetic effect of temperature and a much larger indirect effect, the latter being the magnetic effect of a change of hardness (annealing) produced by temperature, simultaneously. In the case of a given rod (α constant and large) the direct effect is a function of time and temperature. The indirect effect, on the other hand, must be expressible as a function of s since this quantity shows the amount of change of mechanical state. Take rod No. 11, in which α is large, for instance, and deduct the direct effect given in Table 69 from the superimposed effects given in Table 68; the variation of magnetic moment (m') and hardness for the indirect effect alone is given in this table:

Time of annealing.	Direct effect.	Indirect effect.	s
	$m=$	$m'=$	
0	62.6	62.6	39.5
10 minutes	59.5	61.3	38.7
20 minutes	56.0	58.4	37.6
30 minutes	52.6	55.6	36.7
1 hour	50.0	53.6	35.7
2 hours	47.3	50.3	34.8
3 hours	46.1	49.1	34.1
4 hours	45.1	48.1	33.5
5 hours	44.3	47.3	33.1
6 hours	43.8	46.8	32.8

Here, therefore, the relation between m' and s is approximately linear. Thus far, however, magnetizability has decreased; its variation is small.

A passage to higher annealing temperatures is accompanied by a marked *increase* of magnetizability. The actual magnetism possessed by the rod after thorough annealing in the lower temperature (100°), may, therefore, fall so far short of the attainable saturation at a higher temperature as not to be affected sensibly, either by the direct or the indirect influence of the said higher temperature. This inference is substantiated by the data in Table 75 for the rods Nos. 19 and 20. The slight variation of specific magnetism in the case of higher annealing temperatures (185°, 330°) is exhibited in Fig. 23 by the approximately horizontal parts of the corresponding curves. It follows, therefore, that before the relation of magnetism and resistance can be thoroughly discussed, the character of the relation between maximum of permanent magnetism and hardness, which in Figs. 17 and 18 is

graphically given for ordinary temperatures, must be known for all other temperatures. In other words, a family of curves such as given by Figs. 17 to 20 exists with equal definiteness for higher temperatures (100°, 200°, 300°, etc.) and from the co-ordination of such series of families the value of the direct or purely magnetic effect of temperature is deducible for any given circumstances.

Furthermore, the increments of the superimposed magnetic effects and the simultaneous increments of specific resistance should be compared when obtained under circumstances such that the (linear) rod remains, as nearly as possible, in a state of magnetic saturation. It is, therefore, essentially necessary to compare increments. The character of the direct effect being known, that of the indirect effect follows.

It will be observed that the difficulty encountered here is the result of variation of magnetizability. Moreover, the direct and indirect effects need not necessarily vary independently of each other when superimposed.

MAGNETIC RETENTIVENESS AS REGARDS THE EFFECTS OF PERCUSSION AND TIME.

Effects of temperature, percussion, time.—The use of annealed in the place of glass-hard magnets for magnetic instruments, procures for us the decided advantages of a diminished sensitiveness as regards the influence of temperature. If a magnet, which is thus to be used, is annealed at a given higher temperature $T°$— we will continue to suppose it produced with steam, a method both convenient and satisfactory—until the limit of hardness characteristic of this temperature has been attained, then will this material be perfectly passive in so far as temperatures below $T°$ are concerned. If the rod is then magnetized to saturation—how often soever this may have been previously done is entirely without consequence—and again thoroughly annealed at $T°$, then the magnet will in a comparatively short interval of time arrive at a stable and limiting magnetic state, which when exposed to temperatures below $T°$ will in its turn be equally passive, *i. e.*, suffer no *permanent* magnetic variations. But where magnets are to be used for purposes of measurement, a full guarantee for their stability as regards permanent effects from changes of atmospheric temperature is not the only desideratum. The efficiency of the magnet is conditioned almost to an equal extent by its power of magnetic retention against such influences as percussion or rapid vibration, or indeed the prolonged effect of time. That the methods previously employed for the construction of magnets for practical purposes are invariably deficient in this respect is well known, and it will therefore be adequate to add in this place a single authoritative statement only.

Observations of Wild.—In the "Annalen des physikalischen Centralobservatoriums," St. Petersburg, p. 63, 1878, H. Wild discusses the efficiency of Edelmann's bifilar magnetometer and refers to the magnet of this apparatus as follows:

Obgleich der Magnet nach seiner Magnetisirung abwechselnd einer Temperatur von 0° und 30° ca. 30 Mal ausgesetzt worden war, um ihn permanenter zu machen, fand doch eine schnelle Abnahme des magnetischen Momentes statt, so dass sie mehrfache Verstellungen und Veränderungen erforderte. Damit nämlich die Scala noch nicht ganz aus dem Gesichtsfelde herausrücke, musste schon am 17. April (seit Anfang December) der Torsionswinkel um 1° 51′.5 vermindert werden, um wieder die Mitte der Scala in das Gesichtsfeld des Fernrohrs zu bringen . . . Da die Verminderung des Magnetischen Momentes des Stabes auch in folgenden Monaten ungeschwächt fortdauerte, so befürchtete ich, es sei der Magnet schlecht und liess nach dem Muster desselben einen neuen herstellen.

A numerical estimate of these variations is furnished by the following data: The magnet in question was 8.0 centimeters long, 2.1 centimeters broad, and 0.22 centimeter thick; its magnetic moment M and specific magnetism m were found initially, after the change of temperature mentioned, to be—

$$M=954.2 \qquad m=28.31;$$

after 9 months:

$$M=914.5 \qquad m=27.14$$

The mean loss per month was therefore as high as 0.46 per cent. of the original value, but the new magnet was hardly found to be preferable to the old. On page 8 of the "Annalen des Centralobservatoriums, etc.," 1879, H. Wild reports as follows:

Auch der neue Magnet verlor nach der Aufhängung am Bifilar noch fortwährend bis zum Schlusse des Jahres so viel Magnetismus, dass eine Bearbeitung der Beobachtung an diesem Bifilar nicht erfolgen konnte.

This magnet had also been subjected in the usual manner to 16 successive changes of temperature between 0° and 56°.

The following data show the extent of these variations accurately:

On the 29th of December, 1878, immediately after magnetization the magnetic moment M and the specific magnetism m of the bar were

$$M=1852 \qquad m=55.0$$

Some time after, on the 4th of February, 1879,

$$M=1756 \qquad m=52.1$$

After 16 changes of temperature between 0° and 55°,

$$M=1694 \qquad m=50.3$$

The explanation of these results on the basis of our investigation is no longer difficult. It was customary—following Riess and Moser, who considered the time of exposure to the higher temperature as of no consequence—to put the principal stress on the condition of a change of temperature. As a result, the magnet was subjected to the influence of the higher temperature only long enough to heat it uniformly throughout. Even Holmgren contended that permanent and hurtful effects of

temperatures between given limits (0° and 100°) could be made to vanish by repeated alternations of temperature from the inferior (0°) to the superior (100°) limit, and with this end in view heated and recooled his magnet fully 213 times. Possibly this might actually suffice. If, however, as is usual, only 20–40 alternations are made (and even this is tedious and troublesome, consuming much time), a sufficiently advanced state of annealing can hardly have been attained. To a much smaller extent even will this be the case where temperatures only as high as 50° are employed. The magnet therefore practically remains in an extreme of glass-hard state, a condition in which strains of an intensity so enormous exist in the rod, that irrespective of other causes, changes of mechanical state purely the results of time may be anticipated. By the process of annealing, these abnormal strains, so to speak, are diminished to values which insure far greater stability of mechanical state.

Better results are obtained by application of higher temperatures than 50°, if the action of these is sufficiently prolonged. For instance, a magnet, destined to become a part of the bifilar at the Physical Institute at Würzburg, was, after magnetization, held for ten minutes in boiling water and then adjusted in place. The original specific magnetism was found to be

$$m = 28.95 \qquad M = 2397$$

After the annealing

$$m = 24.43 \qquad M = 2023$$

Since that time the bifilar has been in continual use, and the behavior is quite satisfactory. If we observe how great is the initial change of hardness, relatively speaking, during the first ten minutes of exposure to 100°, we infer that even this amount of annealing is sufficient to reduce the hurtful excess of strain to a degree that insures fair stability. A constancy of magnetic moment under like conditions is the result.

Obviously, however, results of better permanence will be reached where the annealing at the practically convenient temperature of 100° is sufficiently prolonged to leave the magnet in the limiting mechanical state characteristic of this temperature. But unfortunately such a process would diminish the magnetic intensity very materially; indeed, in the usual case of a ratio of dimensions $\alpha = 10 \ldots 20$, a loss of even more than 70 per cent. is frequently met with in the above. If, however, the bar is again magnetized to saturation, the original intensity will be very nearly regained; whereupon, if the process of annealing is once more applied, a limiting magnetic state possessing the desired stability, is reached, with a loss of magnetization amounting to *only* 5 to 10 per cent.

Maximum hardness and magnetization for 100°, *stable at* 0°.—Now, there is ample reason for the belief that rods in this singular magnetic state possess the best available qualities of magnetic retention. If a glass-hard saturated magnet be dropped, the result is invariably a loss

of magnetic moment. This is by no means the case with rods subjected to the treatment specified. Even if a sharp blow be intentionally administered, or the magnet be thrown with violence upon the floor, the action is without apparent magnetic effect. As an example we will cite the following observations made with a short and thick magnet. It is known that the retentiveness of such is much inferior to that of long, thin rods.

The magnet was 2.5 centimeters long, 0.4 centimeter broad, and 0.3 centimeter thick. At the outset it was purposely boiled for but 4 hours in water; then magnetized to saturation, and subjected during 2 hours to the action of steam. Our magnetometer showed the following average deflection of five readings (scale-parts millimeters):

$$n = 27.00$$

Then the magnet was placed on a block of wood, and, with the aid of a second block, sharply struck 30 times at right angles to the direction of its magnetic axis, and 20 times in a direction with it. After placing the magnet aside for a time, in order that its original temperature might be reassumed, the reading at the magnetometer was

$$n = 26.97$$

The same process was repeated, with the result

$$n = 26.93$$

Even if the slight diminution from 27.00 to 26.93—about 0.3 per cent.—had existence in fact—that is, was not due to errors of observation, but to magnetic moment lost—still, as it represents the magnetic effect of 100 powerful blows, it is certainly negligible. But this magnet has not yet reached the maximum of permanent hardness for 100°. Whence it follows that, from a rigid application of our method, results more satisfactory even than this are to be looked for.

Experiments of a determinate and final character were made upon a hollow cylindrical magnet, weighing 109.32 *g*. The length of the tube was 16 centimeters, its outer radius 1.6 centimeters, its inner radius 1.2 centimeters. After tempering to glass-hardness it was magnetized to saturation, then annealed in steam at 100° for a period of 30 hours. After this the cylinder was once more magnetically saturated and thereupon reannealed in steam for 10 hours. The specific magnetism, determined from time to time, showed the following values:

Magnet glass-hard, saturated	$m = 41.0$
10 hours in steam at 100°	$m = 26.1$
20 hours in steam at 100°	$m = 25.2$
30 hours in steam at 100°	$m = 24.8$
Magnet annealed, resaturated	$m = 39.9$
5 hours in steam at 100°	$m = 33.8$
10 hours in steam at 100°	$m = 33.1$

In the last instance the needle of our magnetometer for a distance of 72.9 centimeters of the magnet and a rotation of the same of 180°, was deflected over $n=475.6$ scale-parts (millimeters), where 250 centimeters intervened between mirror and scale and the whole measurement was subject to the conditions of Gauss's second position.

The magnet was now introduced into a long and wide glass tube and allowed to fall vertically for a distance of 1.5 meters, impinging on a block of wood. This was done once with the north pole and again with the south pole downward, whereupon the deflection was found to be $n=475.2$ scale-parts, and 10 minutes later $n=475.6$ scale-parts.

We then allowed the same magnet to fall in horizontal position from a height of ½ meter upon the floor. After ten of these descents the magnetometer showed $n = 474.7$ scale-parts, and five minutes later $n=475.0$ scale-parts.

Finally the magnet was again introduced into the glass tube and dropped in vertical position from a height of 1.5 meters, with the north and south poles alternately foremost. After ten repetitions of this treatment we observed the deflection $n = 473.3$; after three minutes, $n=474.0$; after thirty minutes, $n=475.5$.

The observed difference may therefore safely be referred to temporary thermal variations, partly incident to the percussion experienced by the rod, partly due to contact of the latter with the hand of the operator. A destructive effect due to percussion cannot be said to be apparent at all, despite the intense shocks to which the magnet was exposed. The temperature of the room (=6°.0) did not vary during experiment.

After these results, the inference is warranted that the magnetic retentiveness of rods tempered and saturated in the manner set forth will be proof against effects of cold of the same order, such for instance as are met with by observers in the polar regions. Direct researches on this point are contemplated. Whether or not cold is capable of producing a *mechanical* annealing effect is unknown. Certain it is that it would be discernible only by such sensitive methods as are employed in Chapter II. But the marked magnetic effect produced by reduction of temperature has long been understood. Indeed, J. Trowbridge, by the use of carbonic acid and ether, was able to diminish the magnetic moment in this way fully 66 per cent. In the case, however, where a rod is in the magnetically stable condition as regards an elevation of temperature of say 100°, it is altogether probable that if cooled to a similar extent it will continue to possess the desirable quality in question.

We believe, therefore, in the method described, actually to have found a process for the construction of magnets of exceptional constancy and of powerful magnetic retentiveness. In how far this quality may be preserved in the lapse of time, will have to be deferred to the verdict of observers by whom such magnets may possibly be used. How much more reliable and satisfactory measurements made in different parts of the earth will be, when the magnets are no longer liable to injury from

the shocks and blows unavoidably encountered during transportation, needs no further comment.

In conclusion, we desire to add the following rules for the practical construction of magnets:

1. Rods tempered glass-hard are never to be used as essential parts of magnetic instruments.

2. After having tempered the rod in a way that insures a uniformity of glass-hardness throughout its length, expose it for a long time (say 20 to 30 hours; in the case of massive magnets even longer intervals are preferable) to the action of steam at 100°. The operation may be interrupted as often as desirable. The magnet has now reached the highest or hardest of the mechanical states which are no longer influenced by temperatures below 100°.

3. Magnetize the rod, no matter whether originally a magnet or not, to saturation, and then subject it during a period of about 5 hours (in the case of large massive magnets even longer intervals are preferable) to the action of steam at 100°. Then the magnet will have reached the highest and most powerful of the magnetic states which are no longer influenced by temperatures equal to or less than 100°. The magnet is now ready for use.

It may be added that the advantages of using steam are two-fold: In the first place, the process is exceedingly convenient and economical.[144] In the second, it will be remembered that the temperature 100° ultimately leaves the rod in a condition in which the change of capacity for magnetization, as hardness decreases, takes place along the contours of a very flat minimum.[145] Even if slight changes of hardness should subsequently occur, their magnetic effect would be reduced to insignificance, in virtue of the singular variation just mentioned.

ADDENDUM.

RESULTS OF PROF. H. WILD, OF ST. PETERSBURG, WITH REFERENCE TO MAGNETS TEMPERED AND MAGNETIZED BY THE METHOD PROPOSED IN THIS CHAPTER.[146]

We are fortunate in being able to cite in this place some results confirmatory of the efficiency of the method for the treatment of magnets proposed in this chapter bearing the authority of Professor Wild. In his work on the absolute value of Siemens' mercury unit, Professor Wild

[144] A flask with a long neck will be found serviceable. The steam condensing in the latter runs back into the bulk of boiling water. Or the suspended magnet may simply be boiled. The prolonged action of steam is in no way deleterious or corrosive. The rods are etched uniformly black and may thus be polished.

[145] Cf. Chapter V, p. 141.

[146] H. Wild: Mémoires de l'Académie impériale des sciences de St.-Pétersb., VII^e sér., T. XXXII, No. 2, p. 36.

made use of a large magnet constructed with exceptional care by H. Freiberg, out of "Eibiswalder naturhartem Wolfram-Stahl." Its dimensions (rectangular parallelopipedon) and weight are these: length, 29 centimeters; breadth, 3.6 centimeters; thickness, 1.2 centimeters; weight, 1,030 grams. This magnet was carefully tempered by heating to low redness and sudden cooling in lime-water at 20°. After being magnetized to saturation between the poles of a powerful electro-magnet, it was kept at 100° for 35 hours; thereupon again magnetized and exposed to steam for 10 hours more. The following are the observed magnetic moments (C. G. S.) at 20°:

1883.		
April 27	After the first magnetization	32.60×10^3
April 30	After 35 hours' exposure to 100°.....	24.56×10^3
April 30	After the second magnetization.....	31.48×10^3
May 1	After 10 hours' further exposure to 100°.	29.15×10^3

On June 21, 1883, at 20°, Wild obtained for the same quantity, 28.8×10^3. The magnet, despite the treatment which it had experienced, therefore, still lost about 1 per cent. of its total moment, during the intervening three months. But this variation is little more than 0.0001 of the total intensity per day, if the loss were proportional to time. Referred to July 1, however, when the actual measurements were commenced, the said decrement cannot be estimated as above 0.00005 per day—particularly so if it be borne in mind that the diminution in question must gradually vanish at a continually decreasing rate through infinite time, or that a final and definite moment is being asymptotically approached. Indeed, the said limit was practically reached in the second third of July, as the following results show. Wild further remarks that final magnetization of 28 C.G.S. units of magnetism per gramme is to be considered as a satisfactory value. The results in question are as follows:

Date.	$M: 10^3=$	Differs from the mean by
July 21	28.8018	
July 22	28.7846	+0.00043
July 26	28.7749	— 54
Aug. 3	28.7760	— 43
Aug. 4	28.7771	— 32
Aug. 5	28.7834	+ 31
Aug. 10	28.7844	+ 41
Aug. 11	28.7754	— 49
Aug. 13	28.7870	+ 67
	Mean $M: 10^3=28.7803$	±0.00045

The value for July 21 is not included in this mean.

We venture to remark that with so unusually large a magnet even better results could have been obtained by repeated boiling and remagnetization. In consideration of the dimensions, the decrement of 1 per cent. is by no means surprising. The tubular magnet discussed in the above was operated upon three successive times by our method, before definite adjustment for absolute work. We are not aware, however, whether it will not take some time, in order that a magnet kept at 100° may thoroughly regain the state of molecular equilibrium for 20°.

CHAPTER VII.

A PHYSICAL DEFINITION OF STEEL BASED ON THE ELECTRIC BEHAVIOR OF IRON WITH GRADUALLY INCREASING DEGREES OF CARBURATION.

INTRODUCTION.

Nature of the problem.—A detailed and thoroughly exhaustive study of the problem in hand, viz, in how far the effect of carburation on the galvanic and thermo-electric properties of iron is available for the general classification of iron-carburets, calls for working facilities at a puddling furnace, for instance, or other technically satisfactory agency for the decarburation or carburation of iron. Possibly, however, similar work might be done on a small scale in the laboratory, if the necessarily complicated apparatus or opportunities for constructing the same were at the observer's disposal. These advantages were not within our reach.

The problem is of a kind, moreover, which is apt to mislead the investigator into insuperable and almost infinite complications. To avoid these it is absolutely necessary to conduct the experiments with reference to some thoroughly preorganized plan. In the absence of the above-mentioned metallurgical apparatus we were obliged to content ourselves with commercial products, and the main purpose of the present memoir has therefore been restricted to the development of a plan or scheme of operations for the general and tentative study of the electrical behavior of iron-carburets. These efforts have not been unsuccessful; indeed they appear to be of considerable promise. They have already afforded us a method for the physically exact definition of steel which we regard as important. As a whole, the present chapter furnishes an essential and interpretative sequel to our researches on the hardness of steel.

Electrical manifestation of mechanical properties.—The very remarkable effect of rapid and of prolonged cooling from red heat, respectively, on the physical and chemical properties of iron-carburets has always been a subject of great metallurgical interest.[147] In the case of steel the contrast between the two states or conditions thus produced is particularly well marked and of the greatest practical value. Experience has shown, however, that the said processes may be applied, with much advantage, to most of the other iron-carbon products. It is thus that

[147] On Karsten's theory, relative to the nature of these effects, see Percy's Metallurgy, edited by Wedding and others, Vol. II, p. 167, *et seq.*, Braunschweig, 1864.

the question naturally suggested itself to us, whether the remarkable parallelism discovered in the variation of the degree of hardness of steel and its galvanic and thermo-electric properties was not to be considered as only a special case of the behavior of iron-carburets generally, under analogous circumstances. In this respect we believed ourselves justified in predicting that those characteristic mechanical qualities which distinguish steel from other iron-carburets must necessarily be sharply outlined in a general electrical diagram adapted to the classification of iron-carburets as a whole. A similar idea, as we subsequently found, seems incidentally to have occurred to Joule,[148] since he remarks, after having given the necessary experimental data: "I believe the excellence of the latter metal (steel) might be tested by ascertaining the amount of change in thermo-electric condition which can be produced by the process of hardening." But neither Joule nor others have given the subject more than this inadequate consideration, and it was not until our investigations on steel had been fully developed that the problem attracted our attention.

Critical operations.—At first sight a comparison of the electrical intervals comprehended between the hard tempered, and soft annealed states, appeared to be rich in promise; but the processes of slow and of most rapid cooling possible, from red heat, are as yet not sufficiently defined, even if we abstract from decarburation, etc., for obtaining iron carburets in two characteristic physical states. In the case of slow cooling the temperature in red heat to which the specimen has been exposed, as well as the time during which exposure takes place are important items, particularly when the cast-irons are under examination. In the case of rapid cooling the temperature to which the red-hot rod is suddenly and permanently lowered is additionally to be considered. It would not, however, be difficult to define the two processes in question succinctly. A rod, for instance, suddenly chilled from red heat in water at ordinary temperature and then annealed by long exposure in ether vapor at 35° might appropriately be termed glass-hard; if annealed in vapor of sulphur (500°) or of cadmium (700°), soft. For the very large and physically important class of iron-carburets, wrought-iron, low-carbureted steel, and steel, these details, fortunately, do not produce any serious distortions; the thermo-electric hardness and the specific resistance of steel, no matter what the process may have been by which a given rod was softened, remain very nearly constant in value—at least when compared with the enormous range of variation of these qualities due to tempering. The same is true for the hard condition of the carburets between iron and steel, where it is only necessary to choose the temperature before sudden cooling sufficiently high to insure the appearance of hardness, and to chill in water at ordinary

[148] Joule: Phil. Trans. 1859, I, p. 96.

room-temperature. Decarburation is, however, under all circumstances to be avoided,[149] and the exposures to high temperature must not be prolonged. It follows therefore that it will be expedient to commence the present investigation by a consideration of the electrical properties of the carburets in question, that is such in which the total carbon is less than about 2 per cent. To this may be added that within the interval (0—2 per cent.) those modifications in the mode of occurrence of the carbon in iron, which are the cause of such great diversity in the character of the different species of cast iron, are as yet comparatively without marked influence. Thus it appears that our results for this set of products may be considered as satisfactory and definite. In order to complete the discussion conveniently, however, it is desirable to include certain essential properties of the cast-irons, or in other words to prolong the loci of our diagrams into the region of cast-iron, without going into any details. That this is readily possible will appear in the sequel.

A further introductory remark may be added here. Commercial iron-carburets are never pure, but contain in greater or smaller amounts vitiating impurities like phosphorus, sulphur, silicon, and the like. Each of these produces its own electrical effect, as has been seen in the earlier chapter (III) on alloys. The discrepancy thus introduced need not by any means be negligible, and full consideration is given to it in a later paragraph. For the present it will be expedient to suppose this secondary electrical effect to be absent, or that the material in hand is a pure iron-carbon product.

Nomenclature.—As a convenient nomenclature to be used throughout, we will designate the process of *softening* steel, that is cooling from red heat as slowly as necessary by the Roman numeral "*I*"; the process of sudden cooling from the same temperature (*hardening*, tempering, glass-hard) by the Roman numeral "*II*." In like manner all constants which refer to *I* or *II*, are to be marked with the subscript 1 or 2, respectively. For instance, $h_1, s_1 \ldots h_2, s_2 \ldots$ In like manner we may, without confusion, consider "soft state" and "glass-hard state," "state *I*" and "state *II*," respectively identical, etc.

[149] In this place it is well to call to mind an important result of Forquignon's (Ann. de Chim et de Phys. (5) XXIII, p. 538, 1881). He found that steel kept at red heat for seventy-two hours, in an envelope of hematite, lost nearly one-half of its total carbon. This corresponds to the loss of nearly four-fifths of total carbon, due to continued exposure of cast iron to red heat in the preparation of malleable cast iron (Percy op. cit., p. 143.) In many physical experiments with steel, particularly in magnetic work, it is often necessary to soften steel by heating it to redness for some time. It is therefore obvious that the product thus obtained cannot be considered identical, as regards total carburation, with the original carburet. Possibly this method of eliminating carbon may be available for obtaining points in an electric diagram corresponding to iron-carburets lying between iron and steel.

WROUGHT-IRON.

Electrical data.—Pure iron subjected to the process *II* is mechanically indistinguishable from the same metal when subjected to *I*. So also the electrical difference between these two states is practically insignificant. This is true approximately for commercial iron with less than 0.2 per cent. of carbon, and the more nearly true, moreover, the smaller the amount of this element.

For the electrical conductivity of iron Chwolson[150] gives the following results: If a hard-drawn wire is ignited at low redness its electrical resistance is found to vary about −0.4 per cent. If the ignition be intense about +5.3 per cent. The process *II* produces a variation of only 0.7 per cent. in comparison with the hard-drawn state. If, therefore, we compare the rod in the state *II* with the same rod in the state *I*, we find a total electrical change about +1.1 per cent. or −4.6 per cent., respectively, according as *I* was produced by gentle or by intense ignition.

For results of this kind, with reference to the thermo-electric behavior of iron, we searched in vain. But the relation between the variations of thermo-electric and galvanic constants is initially (*i. e.* for very small amounts of a foreign element alloyed to any given metal) linear, as we proved both in the case of steel and of alloys of silver. Hence results of the same order as Chwolson's may be at once predicted for the thermo-electric behavior of commercial iron. Sir William Thomson[151] found that the thermo-electric hardness[152] of iron, like that of steel, is increased by the process *II*. Joule[153] finally remarks, "I find that in steel the (thermo-electric) change is in the same direction as in iron, but of enormously greater magnitude."

These small variations amounting to less than 2 per cent. of the total resistance or thermo-electric hardness, are for the present purposes at least, quite negligible; particularly so when contrasted with the corresponding change of the electrical constants of steel (200–300 per cent.). Where great accuracy is sought for, special measurements may be made. For this reason we accept for wrought iron the values for the electrical constants given in the following table (76). Here thermo-electric hardness is represented by h, specific resistance by s; h_1 refers to iron subjected to process *I*; h_2 to the same wire subjected to process *II*, etc., as has been stated. Furthermore $\Delta h = h_2 - h_1$; $\Delta s = s_2 - s_1$; Δlog

[150] Chwolson: Bulet. de St. Petersb., X, p. 379, 1877; also Carl's Rep., XIV, p. 26, 1878.

[151] Thomson: Phil. Trans., 1856, III, p. 722.

[152] On the definition of thermo-electric hardness, see our paper in Wied. Ann., XI, p. 970, 1880, or this memoir, Chapter II, p. 65.

[153] Joule: Phil. Trans., 1859, I, p. 95–97.

$h = log\ h_2 - log\ h_1$; $\Delta log\ s = log\ s_2 - log\ s_1$. The object of these differences will appear below. For h and s the values obtained elsewhere[154] are given. Microvolts, microhms, and square centimeters are the fundamental units: *log* refers to Brigg's logarithms.

TABLE 76.—*Electrical constants of wrought-iron.*

Material.	Thermo-electric hardness.		Specific resistance.		Δh	Δs	$\Delta \log h$	$\Delta \log s$
	h_1	h_2	$s_1 \frac{cm}{cm^2} 0°$	$s_2 \frac{cm}{cm^2} 0°$				
Wrought iron.....	*Microvolt.* 4.7	*Microvolt.* 4.7	*Microhm.* 12.2	*Microhm.* 12.2	Zero.	Zero.	Zero.	Zero.

STEEL.

Electrical data.—If we suppose the degree of carburation of iron to increase continuously from zero to about 1.5 per cent., the difference in mechanical hardness between states *I* and *II* will likewise increase continuously until steel is reached. For the purpose of defining the relatively enormous interval peculiar to the latter substance technically, metallurgists are in the habit of using some empirical criterion—for instance, the power to give sparks with flint.[155] It is, moreover, of the greatest practical importance that this phenomenal change of mechanical condition is confined to state *II*, and that state *I*, as regards hardness at least, is not readily distinguishable from wrought iron.

A large number of results on the electrical behavior of steel were discussed above. It will therefore only be necessary, in this place, to recall to mind the electrical interval *II–I* for this substance in a table constructed on the plan of the preceding, and therefore needing little further elucidation. For state *II* the largest values[156] occur in case of rod No. 28.

$$h = 17.7 \text{ and } s = 41.5$$

But later experiments[157] furnished considerably harder wires, the results

[154] Chapter II, p. 61. The mean values for the three iron wires there examined are here given.

[155] Cf. Karsten: Karsten's und v. Dechen's Archiv, XXV, p. 223 et seq., 1853; also Jeans, "Steel, its history," etc., London, Spon, 1880, pp. 533–542.

[156] Cf. Chapter II.

[157] See our paper, Wied. Ann., XX, p. 640, 1883.

for which are not given in the digest in question. The hardest of these showed $s=47.5$. If we put $h=ns$, there follows,

$$h=19.6 \text{ and } s=47.5$$

The minimal values for state *II* are those of rod No. 39:

$$h=15.1 \text{ and } s=34.9$$

The largest values for state *I* are given by No. 46:

$$h=6.9 \text{ and } s=16.0$$

and the smallest by No. 47:

$$h=5.0 \text{ and } s=14.0$$

The following table (77) contains both extreme and mean values for the different constants, the largest being put on the horizontal row *l*, the smallest on the row *s*, the mean on the row *m*. All of these are obtained from a combination of the data just cited.

TABLE 77.—*Electrical constants of steel.*

	Thermo-electric hardness.		Specific resistance.		Δh	Δs	$10^3 \times \Delta \log h$	$10^3 \times \Delta \log s$
	h_1	h_2	$s_1 \frac{\text{cm}}{\text{cm}_2} 0°$	$s_2 \frac{\text{cm}}{\text{cm}^2} 0°$				
	Microvolt.	*Microvolt.*	*Microhm.*	*Microhm.*				
l	5.0	19.6	14.0	47.5	14.6	33.5	.....	.....
s	6.9	15.1	16.0	34.9	8.2	18.9	.....	.....
m	6.0	17.3	15.0	41.2	11.3	26.2	460	440

From the remarks made in the above it follows obviously that the magnitude of the interval *II–I* will depend very essentially on the degree of carburation of the steel tested. We experimented with silver steel[158] of excellent quality. But commercial varieties of steel may be readily found in which the said interval *II–I* is even less than one-half that given in the table.

CAST-IRON.

Thermo-electric data.—In his experiments on cast-iron Joule[159] found that the difference of thermo-electric position between state *I* and state *II* is about $\frac{1}{200}$ of the thermo-electric interval bismuth-antimony. In case of steel he found the change of thermo-electric position *II–I* to be in the same sense as in cast-iron and as large as $\frac{1}{15}$ of the said interval. Furthermore, "that the metal (cast-iron) is brought nearer bismuth (*i. e.*, thermo-electric hardness is increased) as the quantity of carbon in combination is increased," and in much larger ratio than would be commensurate with the additional amounts of carbon.

[158] For silver steel see Percy, op. cit., pp. 237–240; also Faraday and Stoddard, Quar. Jour. Science, 1820, p. 325.

[159] Joule: l. c., p. 96.

As accurate a knowledge as possible of the electrical qualities of cast-iron is a matter of such importance as to call for a special examination of a variety of products. These were made with as much material as we found available. The results for thermo-electric power are given in the following tables (78–79), where e denotes the electromotive force referred to silver in microvolts, observed or calculated as specified, corresponding to the temperature T and t of the junctions. On the basis of the formula of Avenarius, $e=a(T-t)+b(T^2-t^2)$, the constants a and b were calculated, usually from five sets of observations, by an application of the method of least squares. In how far the measurements are satisfactory may be seen by consulting the column of differences (Diff.) between e observed and e calculated. Rods No. 1, 2, 3 were of German cast iron; No. 1 soft and of excellent quality; Nos. 2 and 3, though also of good iron, so hard[160] as not to yield readily to a file. White cast-iron cannot be put in the form necessary for measurements like the present without encountering very great mechanical difficulties. But its properties are well represented by rods Nos. 2, 3, which were specially cast thin. Rods No. 4 . . . . 12 are of good American cast-iron, so soft as to be easily touched with a file. They were planed down to approximately square and uniform sections for us by Mr. William Grunow, of New York. All the rods were examined in three states: the original or commercial condition in which they reached our hands; after sudden cooling from red heat (*II*); after annealing soft at red heat (*I*).[161] We considered chemical analyses superfluous for the reasons given near the beginning of this paper. A few isolated results of this kind are valueless.

TABLE 78.—*Thermo-electric power of cast-iron. German material.*

Rod.	Remarks.	t	T	e observed.	e calculated.	Diff.	a	b
		° C.	° C.	*microvolt.*	*microvolt.*			
1	Original condition, soft	19.2	83.4	—385.5	—386.4	0.9	—4.94	—0.0105
		19.8	70.1	—296.9	—296.0	—0.9		
		19.9	59.9	—230.8	—231.1	0.3		
		20.1	50.2	—171.7	—171.0	—0.7		
		20.3	40.2	—110.7	—111.0	0.3		
	Suddenly cooled (II), hard	15.7	59.9	—343.2	—342.8	—0.4	—7.04	—0.0095
		15.9	51.6	—274.1	—274.1	0.0		
		16.0	44.4	—215.8	—216.1	0.3		
		16.1	37.9	—164.3	—164.7	0.4		
		16.2	32.2	—120.3	—120.0	—0.3		
	Annealed at red heat (I)	Rod broken. Too short for measurement.						
2	Original condition, hard	16.7	76.9	—512.6	—512.3	—0.3	—7.89	—0.0067
		16.7	69.5	—446.2	—446.6	0.4		
		16.8	59.7	—360.2	—360.1	—0.1		
		16.8	49.9	—275.5	—275.7	0.2		
		16.8	39.1	—184.2	—184.1	—0.1		

[160] Rods Nos. 2 and 3 being only 0.2 . . 0.3 cm. in diameter, it was found impossible to cast them without the appearance of a steel-like and brittle hardness.

[161] Cf. Chapter II, p. 60.

TABLE 78.—*Thermo-electric power of cast-iron. German material*—Continued.

Rod.	Remarks.	*t*	*T*	*e* observed.	*e* calculated.	Diff.	*a*	*b*
		° C.	° C.	*microvolt.*	*microvolt.*			
		18.1	55.2	—339.7	—341.0	1.3		
		18.1	49.6	—289.2	—287.5	—1.7		
2	Suddenly cooled (II), hard..	18.0	44.7	—236.4	—237.1	0.7	—8.30	—0.0121
		18.0	39.3	—192.4	—191.7	—0.7		
		18.0	34.8	—149.8	—150.3	0.5		
		17.5	84.5	—551.9	—551.7	—0.2		
		17.5	74.5	—461.6	—461.6	0.0		
	Annealed at red heat (I), hard	17.6	63.9	—368.1	—368.3	0.2	—6.84	—0.0136
		17.6	54.8	—291.4	—291.3	—0.1		
		17.7	42.9	—193.3	—193.2	—0.1		
		17.8	79.8	—526.0	—527.2	1.2		
		17.9	70.8	—446.4	—448.2	1.8		
3	Original condition, hard	18.0	59.7	—354.5	—350.9	—3.6	—8.08	—0.0043
		18.1	49.3	—261.1	—261.3	0.2		
		18.2	39.6	—187.6	—178.3	0.7		
		17.9	58.8	—377.2	—377.8	0.6		
		17.9	52.6	—318.4	—317.4	—1.0		
	Suddenly cooled (II), hard ..	17.8	46.1	—255.7	—256.1	0.4	—8.12	—0.0145
		17.8	38.9	—189.0	—188.8	—0.2		
		17.8	33.0	—140.1	—140.2	0.1		
		18.0	88.2	—569.2	—568.4	—0.8		
		18.0	78.6	—481.6	—482.6	1.0		
	Annealed at red heat (I), hard	18.1	66.4	—376.7	—377.1	0.4	—6.61	—0.0140
		18.1	52.2	—259.6	—259.1	—0.5		
		18.1	44.9	—200.6	—200.8	0.2		

TABLE 79.—*Thermo-electric power of cast-iron. American material.*

Rod.	Remarks.	*t*	*T*	*e* observed.	*e* calculated.	Diff.	*a*	*b*
		° C.	° C.	*microvolt.*	*microvolt.*			
		24.0	89.0	—500	—499	—1		
		24.4	74.7	—374	—375	1		
4	Original condition, soft	25.0	49.9	—175	—176	1	—5.88	—0.0158
		25.1	39.9	—103	—102	—1		
		23.3	31.6	— 56	— 56	0		
		20.5	55.6	—245.0	—244.7	—0.3		
		20.4	49.5	—201.5	—201.6	0.1		
	Suddenly cooled (II), hard ..	20.2	43.5	—159.0	—160.4	1.4	—6.42	—0.0073
		20.2	38.9	—129.0	—128.1	—0.9		
		20.1	32.4	— 83.5	— 83.6	0.1		
		19.1	88.7	—428.7	—429.7	1.0		
		19.1	77.5	—351.2	—349.5	—1.7		
	Annealed at red heat (I), soft	19.1	62.1	—246.0	—246.6	0.6	—4.42	—0.0162
		19.2	53.8	—193.0	—193.8	0.8		
		19.3	42.1	—123.8	—123.4	—0.4		
		12.8	80.8	—486	—484	—2		
		13.1	68.0	—380	—382	2		
5	Original condition, soft	13.4	56.0	—290	—290	0	—5.96	—0.0123
		13.7	47.0	—224	—223	—1		
		13.9	35.2	—140	—140	0		
		13.5	55.8	—315.6	—317.1	1.5		
		14.0	49.8	—268.2	—266.4	—1.8		
	Suddenly cooled (II), hard..	14.3	39.6	—184.6	—185.6	1.0	—6.78	—0.0104
		14.6	33.4	—137.5	—136.8	—0.7		
		15.0	27.6	— 90.7	— 91.0	0.3		
		16.0	79.0	—360.5	—361.3	0.8		
		16.4	65.7	—274.5	—272.4	—2.1		
	Annealed at red heat (I), soft	16.7	56.6	—213.1	—214.8	1.7	—4.21	—0.0161
		17.0	43.9	—140.1	—139.4	—0.7		
		17.4	35.3	— 90.3	— 90.5	0.2		
		14.9	83.2	—462	—462	0		
		15.3	75.4	—399	—400	1		
6	Original condition, soft	15.5	63.2	—310	—309	—1	—5.32	—0.0146
		15.6	50.0	—215	—216	1		
		15.7	39.6	—147	—147	0		

TABLE 79.—*Thermo-electric power of cast-iron. American material*—Continued.

Rod.	Remarks.	t	T	e observed.	e calculated.	Diff.	a	b
		°C.	°C.	*microvolt.*	*microvolt.*			
		14.4	57.7	—270.6	—269.7	—0.9		
		15.0	48.9	—206.6	—207.1	0.5		
6	Suddenly cooled (II), hard..	15.3	40.7	—151.4	—152.2	0.8	—5.17	—0.0147
		15.9	34.8	—111.7	—111.8	0.1		
		16.2	29.4	— 77.4	— 77.1	—0.3		
		22.8	83.3	—361.2	—361.6	0.4		
		23.0	75.2	—308.0	—305.4	—2.6		
	Annealed at red heat (I), soft	23.0	63.2	—225.5	—227.4	1.9	—4.27	—0.0161
		23.0	53.8	—168.7	—169.5	0.8		
		23.0	41.5	— 98.8	— 98.1	—0.7		
		18.2	71.9	—372	—371	—1		
		18.3	67.3	—335	—336	1		
7	Original condition, soft	18.4	58.8	—273	—272	—1	—5.72	—0.0132
		18.6	51.3	—217	—217	0		
		18.6	44.5	—170	—170	0		
		14.7	57.9	—321.2	—320.4	—0.8		
		15.2	50.6	—260.0	—258.8	—1.2		
	Suddenly cooled (II), hard, warped.	15.6	43.0	—195.2	—197.3	2.1	—6.29	—0.0155
		16.0	37.1	—149.5	—150.1	0.6		
		16.4	31.7	—108.6	—107.7	—0.9		
		21.4	82.6	—359.0	—358.9	—0.1		
		21.4	69.4	—273.3	—272.1	—1.2		
	Annealed at red heat (I), soft	21.6	56.5	—189.5	—191.2	1.7	—4.32	—0.0148
		21.6	47.0	—135.6	—135.6	0.0		
		21.6	39.5	— 94.0	— 93.6	—0.4		
		17.4	86.9	—502	—500	—2		
		17.4	80.5	—445	—447	2		
8	Original condition, soft	17.4	72.3	—382	—381	—1	—5.34	—0.0178
		17.0	52.9	—236	—236	0		
		17.0	38.9	—139	—139	0		
		17.5	57.1	—261.6	—259.8	—1.8		
		17.9	50.8	—212.0	—213.3	1.3		
	Suddenly cooled (II), not very hard.	18.2	45.3	—172.1	—173.9	1.8	—5.60	—0.0129
		18.7	40.1	—137.2	—136.0	—1.2		
		19.0	35.4	—103.4	—103.3	—0.1		
		10.5	87.7	—437.3	—436.9	—0.4		
		10.8	75.3	—350.7	—352.0	1.3		
	Annealed at red heat (I), soft	11.1	62.9	—274.8	—272.3	—2.5	—4.03	—0.0166
		11.2	52.0	—205.7	—207.2	1.5		
		11.5	42.7	—154.1	—153.7	—0.4		
		12.7	72.3	—411	—411	0		
		12.8	64.0	—347	—347	0		
9	Original condition, soft	12.8	52.2	—262	—261	—1	—5.72	—0.0138
		12.9	43.0	—195	—196	1		
		12.9	37.0	—154	—154	0		
		18.9	58.3	—300.4	—299.5	—0.9		
		19.2	52.6	—251.1	—251.4	0.3		
	Suddenly cooled (II), hard, fissured, warped.	19.5	44.9	—187.2	—188.7	1.5	—6.54	—0.0137
		19.9	37.1	—126.6	—126.0	—0.6		
		20.2	33.6	— 97.5	— 97.0	—0.5		
		16.2	89.8	—457.8	—454.5	—3.3		
		16.4	81.8	—388.1	—393.1	5.0		
	Annealed at red heat (I), soft	16.6	72.5	—327.7	—325.6	—2.1	—3.98	—0.0208
		16.8	60.7	—245.2	—245.1	—0.1		
		16.9	51.6	—187.3	—187.3	0.0		
		19.4	86.4	—473	—472	—1		
		19.3	73.6	—372	—372	0		
10	Original condition, soft	19.1	56.1	—243	—244	1	—5.51	—0.0145
		18.8	43.8	—160	—160	0		
		18.7	35.0	—103	—102	—1		
		23.4	57.8	—235.3	—234.8	—0.5		
		23.4	50.9	—185.0	—185.6	0.6		
	Suddenly cooled (II), hard..	23.5	47.1	—157.8	—158.3	0.5	—5.95	—0.0108
		23.7	42.6	—126.5	—126.0	—0.5		
		23.8	37.4	— 89.8	— 89.9	0.1		

TABLE 79.—*Thermo-electric power of cast iron. American material*—Continued.

Rod.	Remarks.	t	T	e observed.	e calculated.	Diff.	a	b
		°C.	°C.	*microvolt.*	*microvolt.*			
		20.1	86.2	—383.8	—386.2	2.4		
		20.0	78.3	—335.1	—333.0	—2.1		
10	Annealed at red heat (I), soft.	19.1	69.2	—275.7	—274.2	—1.5	—4.12	—0.0163
		19.7	59.6	—214.5	—215.6	1.1		
		19.6	50.1	—160.0	—160.0	0.0		
		21.6	74.9	—364	—364	0		
		21.6	67.4	—309	—308	—1		
11	Original condition, soft.....	21.5	60.3	—258	—258	0	—5.59	—0.0129
		21.4	51.4	—195	—196	1		
		21.4	43.3	—141	—141	0		
		25.8	57.6	—234.8	—234.6	—0.2		
		25.7	50.5	—181.1	—182.0	0.9		
	Suddenly cooled (II), hard..	25.6	46.3	—151.7	—151.4	—0.3	—6.93	—0.0054
		25.6	42.6	—124.7	—124.1	—0.6		
		25.5	36.8	— 81.8	— 82.1	0.3		
		19.0	89.6	—388.1	—389.9	1.8		
		19.0	80.3	—331.1	—329.6	—1.5		
	Annealed at red heat (I), soft.	18.8	67.8	—254.3	—253.8	—0.5	—3.83	—0.0156
		18.7	60.6	—211.8	—212.2	0.4		
		18.5	51.6	—162.6	—162.9	0.3		
		22.5	86.2	—449	—449	0		
		22.5	75.9	—369	—368	—1		
12	Original condition, soft.....	22.5	63.9	—279	—279	0	—5.53	—0.0139
		22.4	51.6	—190	—192	2		
		22.4	40.7	—118	—117	—1		
		12.1	57.9	—312.7	—311.0	—1.7		
		12.5	50.0	—249.5	—250.8	1.3		
	Suddenly cooled (II), hard..	12.7	44.5	—208.9	—210.3	+1.4	—5.83	—0.0137
		13.1	38.2	—164.3	—164.0	—0.3		
		13.2	34.3	—137.3	—136.7	—0.6		
		19.5	87.9	—399.4	—399.9	0.5		
		19.5	80.2	—348.3	—348.3	0.0		
	Annealed at red heat (I), soft.	19.4	67.3	—267.8	—266.1	—1.7	—4.34	—0.0141
		19.4	54.1	—184.5	—186.3	1.8		
		19.3	45.9	—140.4	—139.8	—0.6		

Resistance.—We give in the following table (80) the results for the electrical resistance of cast-iron. The rods are respectively identical to those for which the Tables 78 and 79 apply. Under W are contained the resistances per meter of length at the temperature t, in ohms; under q the two sides of the rectangular section of the rods, in centimeters; s_t and s are the specific resistances at t^0 and 0^0, the reduction having been effected by aid of the constant α, determined elsewhere.[162] The rods are tested in the three states: original or commercial; after sudden cooling from red heat (*II*); after annealing at red heat (*I*).

[162] Cf. Chapter I, p. 22.

TABLE 80.—*Electrical resistance of cast-iron.*

GERMAN IRON.

Rod.	Condition.	$W1m$	t	q	$s_t \frac{cm}{cm^2} t°$	a	$s \frac{cm}{cm^3} 0°$
		ohm.	°C.	*cm²*	*microhm.*		*microhm.*
1	Original state	0.01235	16.0	0.771 · 0.771	73.4	0.0013	71.9
	Suddenly cooled (II)	1537	11.2	0.771 · 0.771	91.4	12	90.2
	Thoroughly annealed (I)	1070	14.5	0.758 · 0.758	61.5	13	60.4
2	Original state	0.0729	15.5	0.309 · 0.309	69.8	0.0013	68.4
	Suddenly cooled (II)	791	10.6	0.309 · 0.309	75.8	13	74.8
	Thoroughly annealed (I)	674	14.5	0.297 · 0.297	59.5	13	58.4
3	Original state	0.0750	15.5	0.307 · 0.307	70.6	0.0013	69.2
	Suddenly cooled (II)	790	11.4	0.307 · 0.307	74.4	13	73.3
	Thoroughly annealed (I)	683	14.5	0.293 · 0.293	58.8	13	57.7

AMERICAN IRON.

4	Original state	0.00981	23	0.977 · 0.936	89.7	0.0012	87.3
	Suddenly cooled (II)	1200	29	0.977 · 0.936	109.7	12	106.0
	Thoroughly annealed (I)	0848	16	0.977 · 0.936	77.5	12	76.1
5	Original state	0.00991	25	0.959 · 0.957	91.0	0.0012	88.3
	Suddenly cooled (II)	1244	25	0.959 · 0.957	114.2	12	110.9
	Thoroughly annealed (I)	0872	16	0.959 · 0.957	80.0	12	78.5
6	Original state	0.00931	25	0.951 · 0.950	84.1	0.0012	81.6
	Suddenly cooled (II)	1025	29	0.951 · 0.950	92.6	12	89.5
	Thoroughly annealed (I)	0877	16	0.051 · 0.950	79.3	12	77.8
7	Original state	0.01377	23	0.813 · 0 803	89.9	12	87.5
	Suddenly cooled (II)	1720	25	0.813 · 0.803	112.3	12	109.0
	Thoroughly annealed (I)	1214	16	0.813 · 0.803	79.2	12	77.7
8	Original state	0.01352	25	0.807 · 0.805	87.8	0.0012	85.3
	Suddenly cooled (II)	1561	29	0.807 · 0.805	101.4	12	98.0
	Thoroughly annealed (I)	1267	20	0.807 . 0.805	82.3	12	80.4
9	Original state	0.01403	25	0.790 · 0.789	87.4	0.0012	84.9
	Suddenly cooled (II)	1720	18	0.790 · 0.789	107.2	12	104.9
	Thoroughly annealed (I)	1265	16	0.790 · 0.789	78.9	12	77.4
11	Original state	0.02072	23	0.656 · 0.640	87.0	0.0012	84.7
	Suddenly cooled (II)	2510	25	0.656 · 0.640	105.4	12	102.3
	Thoroughly annealed (I)	1950	20	0.656 · 0.640	81.9	12	80.0
11	Original state	0.02173	25	0.643 · 0.642	89.7	0.0012	87.1
	Suddenly cooled (II)	2662	18	0.643 · 0.642	109.9	12	107.5
	Thoroughly annealed (I)	2075	20	0.643 · 0.642	85.7	12	83.7
12	Original state	0.02173	25	0.647 · 0.646	90.8	0.0012	88.2
	Suddenly cooled (II)	2624	25	0.647 · 0.646	109.7	12	106.5
	Thoroughly annealed (I)	2060	20	0.647 · 0.646	86.1	12	84.1

Digest.—In the following table the important results in Tables 78, 79 and 80, above, are systematically arranged for convenience in reference. It will be at once intelligible. We need only remark that in case of No. 1 the measurement of a was no longer possible for the soft annealed (I) condition, the rod having been accidentally broken. For this reason the value of a for the original state is taken for a_1 in the formation of Δa, and Δh. This, however, is quite permissible since the rod was originally soft. It may be added that in the tables the logarithms are primarily deduced, the antilogarithms from these. This table furnishes us with mean values for cast-iron. They are given in the final horizontal row.

TABLE 81.—*Electrical constants of cast-iron.*

Rod.	Thermoel. constant a, referred to soft silver, for $\frac{1}{2}(T+t)=0$ (in microvolt).			Thermoel. hardness, deduced from a and $h=15.18-a$, for $\frac{1}{2}(T+t)=0$ (in microvolts).						Specific electrical resistance $s\frac{cm}{cm^3}0°$ (in microhms).					
	a_1	a_2	Δa	h_1	h_2	Δh	$\log h_1$	$\log h_2$	$10^3\Delta \log h$	s_1	s_2	Δs	$\log s_1$	$\log s_2$	$10^3\Delta \log s$
No. 1	(—4.94)	—7.04	2.10	(20.12)	22.22	2.10	1.3036	1.3467	43.1	60.4	90.2	29.8	1.7809	1.9551	174
No. 2	—6.84	—8.30	1.46	22.02	23.48	1.46	.3428	.3707	27.9	58.4	74.8	16.4	.7663	.8736	107
No. 3	—6.61	—8.12	1.51	21.79	23.30	1.51	.3383	.3674	29.1	57.7	73.3	15.6	.7610	.8651	104
No. 4	—4.42	—6.42	2.00	19.60	21.60	2.00	.2923	.3345	42.2	76.1	106.0	29.9	.8812	.0255	144
No. 5	—4.21	—6.78	2.57	19.39	21.96	2.57	.2876	.3416	54.0	78.5	110.9	32.4	.8949	.0448	150
No. 6	—4.27	—5.17	0.90	19.45	20.35	0.90	.2889	.3086	19.7	77.8	89.5	11.7	.8907	.9517	61
No. 7	—4.32	—6.29	1.97	19.50	21.47	1.97	.2900	.3318	41.8	77.7	109.0	31.3	.8906	.0375	147
No. 8	—4.08	—5.60	1.57	19.21	20.78	1.57	.2835	.3176	34.1	80.4	98.0	17.6	.9052	.9911	86
No. 9	—3.98	—6.54	2.56	19.16	21.72	2.56	.2824	.3369	54.5	77.4	104.9	27.5	.8885	.0209	132
No. 10	—4.12	—5.95	1.83	19.30	21.13	1.83	.2856	.3249	39.3	80.0	102.3	22.3	.9029	.0100	107
No. 11	—3.83	—6.93	3.10	19.01	22.11	3.10	.2790	.3446	65.6	83.7	107.5	23.8	.9225	.0316	109
No. 12	—4.34	—5.83	1.49	19.52	21.01	1.49	.2905	.3224	31.9	84.1	106.5	22.4	.9246	.0273	103
Means	—4.66	—6.58	1.92	19.84	21.76	1.92	.....	.....	40.3	74.4	97.8	23.4	.....	.....	119

DISCUSSION.

Plane diagram.—The experimental material in hand is sufficient for the discernment of the main and characteristic features of the relation which exists between the mechanical and chemical properties of iron-carburets, and their electrical behavior. We will arrive at our end soonest by proceeding graphically, using as the basis for our construction the general mean values given in Tables 76–77 and at the end of Table 81. As regards the interpretation of these data we derive very valuable clews from our earlier researches on the electrical properties of alloys. If to a given metal small amounts of a second metal be alloyed, the electrical constants are found to vary in a uniformly continuous way. Now, the total carbon in steel amounts to only 1–2 per cent; in cast iron to less than 5 per cent. We may therefore predict a mode of variation of specific resistances and thermo-electric power which shall be analogous to that observed for alloys. In other words, a uniformly continuous change in the electrical qualities of iron-carburets with the amount of carbon contained may justifiably be assumed. With the ultimate object in view of developing our diagram, let the percentage of total carbon, therefore, be represented as abscissa, the electrical constants h and s as ordinate. Carbon, however, occurs differently in iron-carburets when in the condition *I* than when in the other extreme condition, *II*. It is therefore necessary sharply to distinguish between the constants h_1 and s_1 belonging to the first or thoroughly annealed state, and the constants h_2 and s_2 for the hard state. Our data therefore furnish points belonging to two essentially different loci. Unfortunately for each of these, only three such points are in hand: the first

corresponding to wrought-iron and the abscissa, $x=0$; the second to steel where, approximately, $x=1.5$; the third to cast-iron, where we may roughly put $x=5.0$. The mean values of ordinate belonging to these abscissæ may be taken from the following little table:

TABLE 82.—*Mean electrical constants of iron-carburets.*

Material.	x	h_1	h_2	Δh	$10^3\times\Delta\log h$	s_1	s_2	Δs	$10^3\times\Delta\log s$
Wrought iron	0	4.7	4.7	zero	zero	12.2	12.2	zero	zero
Steel	1.5	6.0	17.3	11.3	460	15.0	41.2	26.2	440
Cast iron	5.0	19.8	21.8	1.9	40	74.4	97.8	23.4	120

With these data and the qualitative results just referred to, our attention will be first directed to the hard state, *II*, of iron-carburets. Between $x=0$ (iron), and $x=1.5$ (steel), iron-carburets vary in marked degree as regards hardness, a circumstance observable both in the mechanical and electrical properties (h_2, s_2) of these products. Beyond this, approaching castiron, the variations are slight. The curve rises rapidly at first, finally at a gradually decreasing rate, and is therefore as a whole concave towards the axis of abscissæ.

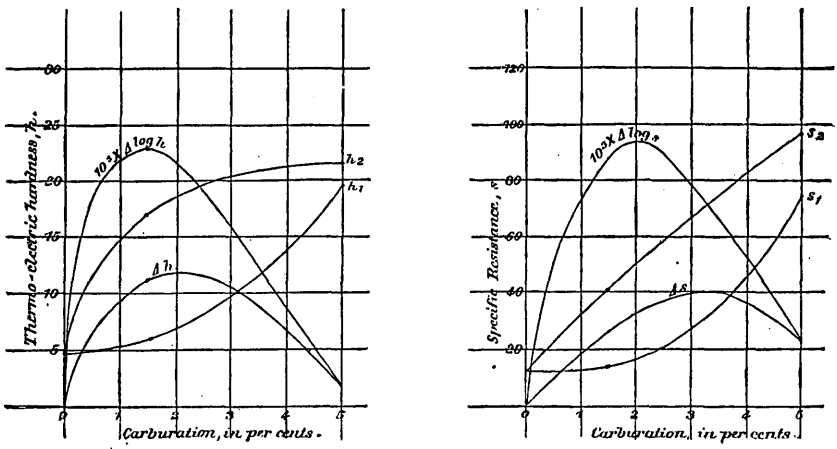


FIG. 24.—Diagram of the mean variation of the thermo-electric hardness and the specific electric resistance of iron-carburets, with their degrees of carburation; 1, thoroughly annealed state; 2, suddenly cooled state.

A result almost the inverse of this is encountered in the case of the "soft" state *I*. Here the initial variations are insignificant, soft iron and soft steel differing slightly, not only in their mechanical states of hardness, but also in the electrical manifestation (h_1, s_1) of this quality. It is not until we pass beyond $x=1.5$ and enter the region of the cast-irons that the present curve *I* rises at a relatively rapid or accelerated rate. The curves, *I*, are therefore on the whole convex as regards the axis of abscissæ.

From these points of view the curves in the above diagram h_2, s_2

and h_1, s_1 have been drawn. It may be plainly argued, moreover, that their general character does not change during the progress from $x=0$ to $x=5$; in other words, we accept concavity and convexity respectively throughout the given extent of the curves, to the exclusion of points of circumflexion. With this probable inference postulated, we observe that the two sets of curves h_2, s_2 and h_1, s_1 must intersect in the region of cast-iron, that is, near $x=5$.

Curiously enough we found in certain data of Joule[163] the evidence for such intersection. If we reduce Joule's figures to our scale of thermo-electric hardness, his results for white cast-iron and gray cast-iron are approximately these:

First extreme: cast-iron, black fracture (with much graphite), $h=25$.

Second extreme: cast-iron, white fracture (very hard), $h=12$. All other samples of cast-iron which he examined lay between these limits. The large values for h and s obtained[164] for malleable cast-iron (known to be graphitic[165]) are also in accordance with Joule's results. The first mentioned of the cast-irons above ($h=25$) is therefore to be referred to condition *I* (soft, carbon uncombined), the second ($h=12$, carbon combined, hard) to condition *II*, which gives evidence in favor of the intersection of the respective curves.

Superimposed electrical effects.—To this inversion of the electrical properties of iron-carburets as the quantity of total carbon is increased we have already adverted,[166] and it is therefore essential to any discussion of the phenomena before us to take cognizance of the mechanical as well as the chemical causes of the same. These are three in number: (1) Effect of uncombined carbon; (2) effect of combined carbon; (3) effect of the peculiar strain accompanying temper.

In the curves h_1 and s_1 the first of these factors is primarily active, the second of small importance, the third inactive. The material is approximately homogeneous. In the curves h_2 and s_2, however, all factors produce their own specific results, though the influence of the first is relatively small. Here, therefore, we are confronted by a complex superposition of effects; such however, that between wrought-iron and steel the mechanical cause (3) is almost solely efficient; between steel and cast-iron the chemical causes (1, 2) predominate.

Character of Δh.—As the result of the opposed character of the curvatures of the loci *I* (soft state) and *II* (hard state), the first being convex, the second concave, as regards the axis of abscissæ, we are able to draw an inference of fundamental importance. For if we put $\Delta h=h_2-h_1$ and $\Delta s=s_2-s_1$, then it follows obviously that the new variables Δh and Δs, regarded as functions of x, must each pass through a maximum. This remarkable result may be formulated thus:

The diagram which expresses the electrical behavior of commercial

[163] Joule: l. c.

[164] Chapter III, p. 102.

[165] Forquignon: l. c.

[166] Chapter III, p. 103.

iron-carburets distinctly indicates the existence of a singular product, possessing the unique capability of occurrence in the greatest number possible of mechanical states—a member, in other words, for which the difference of thermo-electric hardness between condition *II* and condition *I* is a *maximum*. This is a necessary but not a sufficient condition for the definition of that valuable product to which the term "steel" is applicable. For, irrespective of the fact that Δh=max. and Δs=max. need not necessarily select the same product, a definition based solely on the absolute value of the interval of variation does not necessarily exclude the occurrence of any marked change of hardness as regards the soft state. Yet this is essential. The mere fact, in other words, that for the unique iron-carburet in question the state *II* is furthest removed with reference to hardness from the state *I*, might even be true of a product brittle in the latter (soft) state.

Character of $\Delta \log h$.—To meet this objectionable feature of the above definition, therefore, it is expedient to give preference not to the absolute variations of h and s, but to the relative variations $h_2 : h_1$, $s_2 : s_1$, of pairs of values of h and s. For this reason the *logarithmic* interval is considered. If $\Delta h = \log h_2 - \log h_1$ and $\Delta s = \log s_2 - \log s_1$, then we have for the definition of steel $\Delta \log h$=max., or $\Delta \log s$=max. Between these we have yet to decide. In the first place it is to be noted that the equations Δh=max. and Δs=max., determine $x=2$ and $x=3$, respectively, and so far as can as yet be foreseen; whereas $\Delta \log h$=max. and $\Delta \log s$=max., apply for $x=1.5$ and $x=2.0$, respectively. In the latter case, therefore, not only are the two carburets defined chemically much more nearly coincident, but their mean position in the diagram is such as to select from all the iron-carburets the one which may stand as a *type* for the products commercially termed steel. And we may add here that, in so far as the variation of a function like the one in hand in the neighborhood of a maximum is usually small, a group of iron carburets disposed on either side of the said maximum will possess properties sufficiently alike to enable us to consider them as practically and commercially identical with the accurately defined type. Now it is very probable, if material of sufficient purity could be obtained, that both the equations $\Delta \log h$=max. and $\Delta \log s$=max. will be found satisfiable by the same x, and this from the fact that as far as steel, at least, the relation between thermo-electric hardness and specific resistance is linear. But after subjecting any iron-carburet to the process of sudden cooling, the external layers will usually have been transferred into the state *II* more thoroughly than the core, particularly in case of thick rods and highly carburized iron. But it is from the external layers that $\Delta \log h$ is practically determinable, irrespective of the figure and dimensions of the carburet operated upon. $\Delta \log s$ is not readily determinable except for the mean condition of a carburet of definite figure.

Definition of steel.—Preference is, therefore, to be given to the former of the two critical equations and $\Delta \log h = \text{maximum}$ to be accepted as the physical definition of steel. This equation is to be interpreted thus:

Let each member of the whole series of non-carburets be subjected successively to the following two operations:

1. A process of very slow cooling from a given temperature in red heat.

2. A process of most rapid cooling possible from the same temperature.

If now the carburets be examined with reference to the hardness produced in the two instances, there will be found among them a certain unique member whose properties are such that while process *I* has more nearly identified it with pure soft iron; process *II* will have moved it farther away from this initial carburet[167] than is simultaneously the case with any other iron-carbon product; or which, in other words, is capable of occurring in the greatest number of states of hardness *relative to the soft state* possible. To this unique product the term "steel" is to be applied.

In so far, however, as the softest steel, for the very reason of its being essentially an iron-carburet, can never reach pure iron, it is obvious that there must remain a *difference* between the mechanical properties (Young's modulus, simple rigidity, tenacity, etc.) in general of soft iron and soft steel, on account of which preference will be given to one material or the other, as the needs of the engineer suggest. To this carburet ($\Delta \log h = \text{max.}$), finally, the maximum capacity for the retention of a strain of any given kind which may be imparted to it, seems also to belong. The strain accompanying magnetism, and the strain peculiar to hardness, of each of which steel retains a phenomenal amount, may be cited as examples.

COMMERCIAL OR IMPURE IRON-CARBURETS.

Series of iron-carburets.—Thus far no attention has been paid to the unavoidable impurities like sulphur, silicon, phosphorus, manganese, etc., which in addition to carbon are always present in commercial iron-carbon products. But it is easy to extend the considerations just made in such a way as will make them applicable to iron-carburets, pure and impure, generally. Let any iron-carburet be given. To this belong a whole series of iron-carburets so constituted that, while in other respects the composition is identical throughout, carbon alone passes through the interval from zero to about six per cent.[168] To each such series (see

[167] That is, soft iron for which $x=0$.

[168] Of course, the increment in C in this case presupposes a decrement in total *Fe*, so that there may be no variation in the percentage presence of the impurities.

below, p. 191) a definite and characteristic pair of curves, h_2 and h_1, will correspond, and that particular impure iron-carburet which is selected by the equation

$$\Delta \log h = \text{maximum}$$

is *the* steel for the given class of impure iron-carbon products. These relations may be tersely exhibited as follows. Let

$$z_1 = f_1(a, b, c \ldots . x)$$

where z_1 denotes the thermo-electric hardness of a given sample of impure iron-carburet, a, b, c . . . . , are parameters expressing the respective amounts of the impurities present in per cents. by weight of the whole, x (independent variable) representing the total carbon present, also in per cents. of the whole.

Let

$$z_2 = f_2(a, b, c, \ldots x)$$

be interpreted in like manner. Now suppose the functions z_1 and z_2 to belong simultaneously to the same product, or in other words, to apply when the said given product is in the state *I* and the state *II*, respectively.[169] Then will these equations, if x be allowed to increase continuously from zero to about six, express the critical electrical properties of the whole given series of impure iron-carburets. Again by varying any one or all of the parameters, a, b, c . . ., we may pass from a given series of this kind to any other.[170] Finally the function, in the case of a given series,

$$\Delta \log z = \log z_2 - \log z_1$$

possesses the important property that for wrought iron its value approaches zero; for cast-iron its value is some positive number, and for steel we shall have $\Delta \log z = \text{maximum}$. In order to determine the position of any iron-carburet in its own series a knowledge of the functions z_1 and z_2 is fully sufficient.

Thermo-electric hardness—general interpretation.—With these new inferences we are able to give the results of an earlier chapter (II), a much more comprehensive interpretation. The extreme values of thermo-electric hardness, z_1 and z_2, corresponding to thorough annealing and sudden cooling, respectively, describe any given member of any given series of iron-carburets with reference to its chemical properties; *i. e.*, z_1 and z_2, together, determine its position in a diagram of classification peculiar to iron-carburets. The intermediate values of z, in other words, those lying between z_1 and z_2, and obtainable by the annealing of the previously chilled carburet, describe the same product with reference to its mechanical properties; *i. e.*, the value of z deter-

[169] In practice f_1 and f_2 will be interpolatory functions of like character.

[170] It is to be borne in mind here that these remarks apply principally for iron-carburets, in which x does not exceed 2. The final generalization is given below.

mines its position in a scale of hardness peculiar to the particular iron-carburet (z_1, z_2) in hand.

Herewith we have succeeded in expressing the general conception of a problem, of which our research on the hardness of steel (Chapter II) is to be regarded as only a special solution.[171]

FINAL GENERALIZATION.

Qualitative and quantitative carburation.—In the above discussion the general features of the electrical behavior of iron carburets, whose degree of carburation is above 2 per cent., has already been given. But a more detailed consideration must take cognizance of the fact that even in case of the same chemical composition we encounter iron-carburets differing enormously in their physical properties, and we are led into a host of complications not readily to be surveyed. The mode of occurrence of carbon in iron, or what may be called the quality of carburation, becomes more and more dominant in effecting changes of physical character of these products, as the amount of total carbon present is increased. The temperature in red heat from which the conditions *I* (thoroughly annealed) and *II* (suddenly cooled) are reached, as well as the time during which exposure to the same takes place, here possesses essential importance.

Classification-function.—The prolongation of the critical curves h_1 and h_2 from steel into the region of cast-iron can therefore be made in a great variety of ways. For each particular path definite premises must be laid down, viz., that the increase of carbon shall take place, qualitatively as well as quantitatively, in a way such as is in accordance with the manner of carburation of the sample of cast-iron, in which the particular path in question is to terminate. Perhaps these relations are expressible with greater clearness in this wise: Suppose that with reference to the height and duration of the temperature from which thorough annealing (*I*) and sudden cooling (*II*) is to be effected, etc., certain fixed assumptions have been made. But beyond this let it be unrestricted, so that it may even lie beyond the melting point of the carburets operated upon. Furthermore, suppose the percentage amount (parameter) of foreign admixtures and impurities other than carbon present in iron be constant throughout. Then let the relations here involved be represented graphically in three dimensions: Thermo-electric hardness parallel to the axis Z; carburation quantitatively considered (percentage

[171] A good example of the difference between mechanical and thermo-electric hardness is given by malleable cast-iron, results for which are given elsewhere (Chapter III, p. 102).

of total carbon) parallel to the axis X; finally qualitative carburation,[172] *i. e.*, a quantity which expresses the mode of occurrence of carbon in iron, parallel to the axis Y. With this understanding a characteristic surface will correspond both to condition I and condition II; and if analogously to the above (p. 189), we deduce the difference $\Delta \log z$, its form will be

$$\Delta \log z = F(a, b, c, \ldots . y, x)$$

where the parameters $a, b, c, \ldots$ refer to the impurities present.

Every plane parallel to XZ cuts this surface in a curve, expressing the electrical behavior of a series of iron carburets as above defined (p. 188). All such curves of intersection are characterized by the presence of a maximum defining the steel corresponding to the impure iron, $F(a, b, c, \ldots y, 0)$, the initial carburet of the particular series under consideration.

Every plane parallel to YZ cuts the surface $\Delta \log z$ in a curve, expressing the electrical behavior of all carburets possible in case of a selected fixed amount of total carbon in the iron carburet. Two such curves will differ more as their distance apart as well as from the initial plane YZ is greater. It will be seen that the further step in the present research must be that of suitably varying the elements of the operations I and II[173] in such a way as to throw additional light on all these complications, among which those last mentioned are as yet the most obscure and imperfectly understood.

Classification diagram.—Notwithstanding the dangers encountered in representing views in part theoretical with the aid of a diagram, we are able to bring to the mind of the reader all that has been said so perspicuously in this way, as readily vindicates the venture of a rough construction of the probable contour of the surface $\Delta \log z$. In the following figure (25) the XZ plane cuts the surface $\Delta \log z$ (*rstno*) in a line coinciding with the axis X. This is supposed to be an hypothetical carburet, iron-graphite, which remains iron-graphite both in state I and in state II. The extreme plane parallel to XZ represents the series of

[172] As an example of the nature of the variable y, suppose it, for instance, to be the mean ratio of combined to uncombined carbon for the two states, I and II, or, $y = \frac{1}{2}\left\{\left(\frac{\text{combined } C}{\text{uncombined } C}\right)_I + \left(\frac{\text{combined } C}{\text{uncombined } C}\right)_{II}\right\}$. Should a single quantity, y, be insufficient to express the qualitative occurrence of carbon, so that a number of such variables $y, y', y'', \ldots\ldots$ are necessary, then while any given one, y for instance, passes all values, the others are to be regarded as parameters—*i. e.*, constants during this variation. The presence of graphite, amorphous carbon, and combined carbon in iron may suggest two variables, y, y'.

[173] Much, for instance, could be learned from continuous electrical ignition of iron-carburets, in vacuo and gases, respectively, and examination of the test sample from time to time; more from a detailed exploration of these phenomena, made conjointly by a physicist and a chemist. Carburation produced by ignition in gaseous hydrocarbons, and subsequently in vacuo, suggests itself as a first convenient method of experimental attack.

iron-carburets (*rst*) in which all carbon is combined, at least in state *II*. White cast-iron may be supposed to belong to this series. The plane

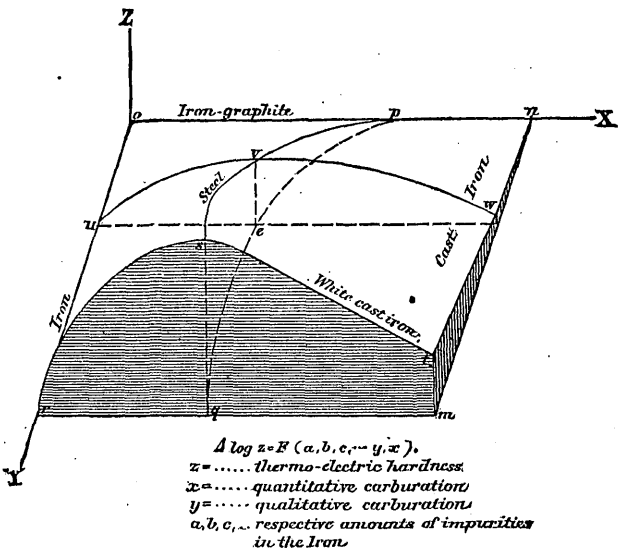


FIG. 25.—Classification diagram.

YZ intersects the surface $\Delta \log z$ in the iron line approximately coinciding with the axis *Y*. The extreme plane (*mtn*) parallel to *YZ* corresponds to about $x=6$. The most interesting feature of the diagram is the totality of steel maxima (*svp*). If ordinates be let fall from each of these, the resulting cylinder will intersect the plane *XY* in a curve (*peq*), expressing both qualitatively and quantitatively the carburation of the infinite steels possible in case of a given impure iron-carburet $F(a, b, c, \ldots y, x)$. It is hardly necessary to add that the number of such possible surfaces (*rstno*) is again infinite; since any variation of the parameters $a, b, c, \ldots$ or the accessary variables $y,' y,'' \ldots$ involves the construction of a new one. Practically, however, a finite number of typical surfaces suffice.

The figure finally contains an illustrative intermediate section, in which *v* is the steel of the series of iron-carburets *uvw*.

Concluding remarks.—The perusal of the above pages will have shown that the problem in hand, considered in its full generality, is exceedingly complicated. Nevertheless, we presume to believe that the method of attack which has been briefly developed in this chapter contains promise of success. Both the intimate relationship between the mechanical properties of iron-carburets and their electrical behavior, as well as the incomparable sensitiveness of the functions involved, emphatically commend our electrical diagram to the metallurgical engineer. Indeed, it is remarkable that metallurgists have thus far given no attention to a class of physical properties which, from their simplicity

and pronounced character, seem above all others to be adapted for purposes of discrimination and classification. Magnetic functions, as we have shown elsewhere, admit much less readily of satisfactory interpretation, varying as they do enormously with the figure and dimensions of the sample under examination. The interest of the present chapter, however, centers in the important product, steel. Curiously enough, the large range of variation of mechanical properties which renders this substance so indispensably useful have not as yet been found available for its accurate definition. The electrical qualities of steel, however, furnish a means to this end, which can conveniently be pushed to greater detail and nicety than will be necessary for any metallurgical or even physical purposes. It is in this respect that we believe with the present publication to have opened a new field of research, useful alike in its bearing on the practical and the theoretical problems concerning iron-carburets.

CHAPTER VIII.

BRIEF SUMMARY OF THE PRINCIPAL DATA.

Introduction.—The remarks in this memoir refer almost exclusively to the species of hardness known as temper, and which may be imparted to iron-carburets by sudden cooling from red heat combined with more or less subsequent annealing.

It falls within the scope of the work to develop so far as possible the very close analogy which exists between tempering and magnetization.

If we define the structure of a hard cylindrical steel rod as being the law of variation of density encountered on a passage from axis to circumference along any radius of the rod, then structural identity in case of two given geometrically similar rods of the same composition *a priori* implies identity of diameter. In what way structure may vary with diameter is not even conjecturable. It follows that immediately comparable magnetic data are to be anticipated only where the rods of variable hardness and length retain the same thickness and composition throughout the course of the experiments.

Chapter I.—Like the specific resistance, the galvanic temperature-coefficient of the iron-carburets exhibits a phenomenal range of variation, passing from the values for wrought iron and soft steel, 0.0052 and 0.0043, respectively, to the values for hard steel and cast iron, 0.0016 and 0.0013, respectively. The said coefficient decreases continuously and uniformly as resistance increases, more rapidly than the latter during the earlier stages, much more slowly during the later stages of a progress from iron to cast iron. An inferior limit of the temperature-coefficient would therefore seem to appear much before the iron-carburet reaches the superior limit of resistance. If classified with reference to the relation between resistance and temperature, the iron-carburets as a whole form one continuous series.

Chapter II.—If the temperature from which steel is suddenly cooled be supposed to increase continuously from a very low value to the highest admissible, the hardness of the chilled rod will remain comparatively inappreciable until a certain critical temperature in red heat is reached. At this stage and a little beyond hardness increases at exceedingly rapid rates with temperature, after which the rate again decreases.

The largest observed variation of thermo-electric power produced by

tempering is 12.8 microvolts per degree centigrade at 0°. The largest ratio of the respective resistances of hard and soft steel

$$45\ (\text{cm} \,/\, \text{cm}^2, 0°, \text{microhm}) : 15\ (\text{cm} \,/\, \text{cm}^2, 0°, \text{microhm}) = 3.$$

Since hard and soft steel lie on opposite sides of pure silver in the thermo-electric scale, maxima and neutral points of electromotive force (thermo-electric inversions) are a common occurrence.

The annealing effect of any temperature acting on glass-hard steel increases gradually at a rate diminishing continuously through infinite time—diminishing very slowly in case of low temperatures ($<100°$), very rapidly at first, then again slowly in the case of high temperatures ($>200°$); so that the highest and hardest of the inferior states of hardness possible at any given temperature is approached asymptotically.

The ultimate annealing effect of any temperature $t°$ is independent of the possibly pre-existing effects of a temperature $t'°$, and is not in any way influenced by subsequent application of the latter, provided $t>t'$. In case of partial annealing at $t°$ (time finite) this law applies more fully the more nearly the said ultimate effect of $t°$ is reached.

If hardness of steel is to be expressed thermo-electrically, it is inexpedient to use the soft state as a point of departure. The thermo-electric difference between soft steel and soft iron (say 1) when compared with the corresponding difference between the extreme states of steel (say 10) is small. In soft steel, however, the effect of foreign ingredients (impurities: S, P, Si, Mn, etc.) is still too pronounced to admit of a desirably accurate determination of the thermo-electric position[174] of this metal.

In the case of steel, the relation between thermo-electromotive force per degree centigrade at 0°, and specific resistance at 0° (s), is linear throughout the whole of the phenomenal range of variation of these qualities with hardness. This law suggests the introduction of the new variable *thermo-electric hardness* (h), defined thus: Suppose the said law of linear variation to be true indefinitely; then will the electromotive force (microvolt) per degree centigrade at 0° of a thermo-element consisting of steel in the imaginary normal state whose specific resistance (cm / cm² 0° microhm) is zero, and steel in any given state, be the thermo-electric hardness of the latter. The thermo-electric position of steel in the stated normal condition, with reference to pure soft silver, is

$$m=15.18 \text{ microvolts}$$

per degree C. at 0°. Finally in the fundamental equation $h=ns$,

$$n=0.412$$

Chapter III.—The chemical theory of the phenomenon of temper leads to this ulterior inference. By the simple process of sudden cool-

[174] It may be added here that, even if chemically pure steel were available, its thermo-electric position would, very probably, be seriously variable in consequence of unknowable differences in the mode of occurrence of the carbon contained.

ing, combined with subsequent annealing, applied to steel, inasmuch as in this way, within certain limits, any given amount of an electrically active ingredient (carbon) may be converted into an electrically passive form, the same results are reached which in the case of alloys are obtainable only by melting the two component metals together.

In the case of alloys, the plane locus determined by thermo-electric power (microvolts) per degree centigrade, at 0°, and specific resistance (cm / cm² 0° microhm) shows maxima (or minima) for both properties, so disposed that the said singular points, at the temperature 0° at least, do not coincide. The range of variation of the electrical properties in question appears to increase with the difference of specific volume of the (two) ingredients of the alloy. Applied to steel, the results interpret the observed linear locus for the metal (steel) as being the initial tangent of a curve of comparatively enormous magnitude.

Variation of resistance is a necessary concomitant of variation of volume; no matter how the latter may have been produced—whether by temperature or by tempering—the increments of resistance (s) due to a given increment of volume (v) are of the same order. A purely thermal effect in the one case does not therefore appear. If we put

$$s_2 = s_1\left(1+k\frac{v_2-v_1}{v_1}\right)$$

$$k = \frac{s_2-s_1}{s_1} : \frac{v_2-v_1}{v_1} = 150, \text{ a first approximation.}$$

Gentle ignition after a previous state of (hard) temper appears to be generally accompanied by a passage of both resistance and volume through minima.

The annealing effect of temperature and time in the case of drawn hardness is quite analogous to the said effect in the case of temper.

The position of cast iron is isolated with respect to the locus expressing the simultaneous variation of thermo-electric power and specific resistance due to changes of temper in case of the other carburets (steel). Cf. this résumé, Chapter I.

The chemical theory does not suggest nor account for the observed phenomena of annealing. Considered physically these are at once referable to the category of viscous phenomena. In the ordinary cases of viscosity measurement, the phenomenon is evoked by sudden application of stress (torsion, flexure, tension, volume compression or extension, etc.) under conditions of constant viscosity; in the case of annealing, by sudden decrease of viscosity under conditions of initially constant stress. Thermal expansion interferes with the purity of these phenomena by destroying the conditions of existence of the characteristic strain which accompanies hardness, and this in proportion as the expansion is greater. The final evidence in favor of the given interpretation (viscosity) of the phenomena of annealing is this: that the maximum of permanent hardness which can in any way be imparted to steel

(*i. e.*, the maximum intensity of strain which a steel rod is of itself able to maintain) decreases rapidly as temperature increases. (Cf. Chap. III, p. 97.)

The temperature condition (cf. Chapter II of this résumé) to which the appearance of glass-hardness is subject, is readily suggested by the chemical theory. Physically the state of red heat of iron and of steel is remarkable for the occurrence of certain characteristic phenomena: sudden volume-expansion (Cumming); anomalous thermo-electric behavior (Tait); disappearance of the magnetic quality (Gore); sudden appearance of glass-hardness in steel chilled from this temperature (Chernoff). An anomalous variation of resistance may be additionally inferred. As regards conditions favorable to glass-hardness Cumming's phenomenon distinguishes iron and steel from all other substances. The difference between iron and steel, finally, is exhibited in like degree by the maximum of permanent magnetization, by the maximum of permanent hardness (Strain), and probably by the maxima of other strains which these metals, under like conditions, respectively retain.

In the substance malleable cast-iron there is encountered a remarkable example of the occurrence of mechanical hardness unaccompanied by an equivalent variation of the electrical properties.

The existence of the characteristic strain in glass-hard steel is the cause of electrical effects so enormous that such additional effects which any change in carburation may involve can be wholly disregarded, and all electrical and magnetic results interpreted as due solely to variations in the intensity of the said strain.

Chapter IV.—Both the galvanic and the thermo-electric effects of magnetization are negligible in comparison with the corresponding electrical effects of tempering—the former amounting to less than 0.3 per cent. of the latter in the most unfavorable case.

The absolute value of the said thermo-electric effect for magnetically saturated iron is $+0.035$ microvolts per degree centigrade, at zero.

The thermo-electric effects of a temporary tensile strain in iron and of magnetization are qualitatively alike. Hence we infer that the latter effect is to be attributed to the strain which accompanies magnetism.

Chapter V.—Rigidly comparable data of the relation between magnetism and hardness are not readily obtainable except with magnets which were originally integrant parts of the same (hard) steel rod of uniform temper throughout its length. The plan of experimentation is expediently made to conform with a passage from hard to soft.

If magnetic moment (C. G. S.) per unit of mass (g) be regarded as a function of hardness, the family of curves obtained exhibits the following general character: Magnets, whether long or short, after incipient annealing from the glass-hard state diminish in magnetizability to a pronounced minimum of this quality. If the annealing be continued magnetizability again increases to an enormously developed maximum in case of rods of large dimension-ratio, to a flat or indistinct maximum in case of small dimension-ratio. On passing from long to short steel rods

the minimum is found to move in a direction from hard to soft, at very slow rates, thus remaining in the region of glass-hardness; the maximum, on the other hand, in a direction from soft to hard, at somewhat more rapid rates. The unique maximum of permanent magnetizability will probably be exhibited by a linear steel rod, annealed from glass-hardness as far as the physical state of maximum density. The value of the unique maximum is demonstrably much above 785 C. G. S. units of intensity, or 100 C. G. S. units of moment per gramme-mass. Continued diminution of the dimension-ratio finally, will probably bring the said minima and maxima into coincidence in such a way that permanent magnetizability decreases uniformly from hard to soft.

The family of magnetic curves must be separately investigated for each given diameter (structure) and each given degree of carburation. If magnetic moment per unit of mass be regarded as a function of the dimension-ratio ($\alpha =$ length $/$ diameter), the family of curves obtained (conveniently described with the aid of the four type curves: "glass-hard," "yellow annealed," "blue annealed," "soft,") exhibit the following general character:

The curve "glass-hard" is concave as regards the axis of abscissæ (dimension-ratio) throughout. Rising very rapidly at first, it finally ascends to a distinct limiting value or horizontal asymptote. The curve "soft," on the other hand, rises very slowly in its earlier stages, and is convex as regards the axis of abscissæ. From here it passes rapidly through a point of circumflexion into concavity, and then above the former curve. Finally, the rate of ascent again decreases, so that a horizontal asymptote is also reached, but apparently at a later stage of progress than is the case with hard steel.

From either of these two loci expressing the variations of the extreme states, we may by annealing pass continuously to the other. But the manner of such passage, from the one curve to the other, in consequence of the continuous change of parameter (hardness), is exceedingly complicated. Incipient annealing of glass-hard steel produces a distinct, though relatively small, descent of the original curve as a whole. As annealing progresses, the farther end of the curve is always the first to rise and to pass above the original curve in such a way that the point of intersection of the new curve and the original curve (glass-hard) moves along the latter with great rapidity, from greater to smaller values of the dimension-ratio. When the curve "yellow annealed" is reached, the part of it between a small value of the dimension-ratio, α ($= 14$ and 18, in the above measurements, for diameters $2\rho = 0.08$ cm. and 0.15 cm.), and $\alpha = \infty$, has been already elevated above the curve "glass-hard." At the stage of progress given by the blue annealed curve, the part between another small value of α ($\alpha = 15$ and 20 in the above results) and $\alpha = \infty$ has risen far more rapidly than before, while, on the other hand, the advancing part of curve between $\alpha = 0$ and the said small value ($\alpha = 15$ or 20, respectively) having descended very grad-

ually, is now distinctly convex downward. Passing from "blue annealed" to "soft," the part of the curve above smaller dimension-ratios continues to fall, in general at greater rates, finally to merge into the curve "soft." The remaining part, above greater dimension-ratios, still rises slowly, reaching its superior elevation, from which it then falls rapidly into coincidence with the extreme curve "soft" also. During this last phase of progress the point of intersection of the advancing curve and the curve glass-hard passes along the latter from smaller to larger values of the dimension-ratio.

Chapter VI.—In considering the permanent magnetic effect of temperature on steel permanently saturated, it is necessary to discriminate sharply between two species of magnetic loss:

1. The direct effect, due simply to the action of temperature, and to be ascribed to diminution of coercive force and to interference of thermal expansion with the magnetic strain.

2. The indirect effect, due to the action of temperature in producing mechanical annealing, and to be ascribed to the interference of the rearrangement of molecules resulting, with the magnetic strain.

The two effects are frequently superimposed. Considered separately, the latter (indirect effect) is by far the greater in amount, and its character, with regard to magnitude and duration, fully typified by the concomitant phenomenon of ordinary mechanical annealing. The former (direct effect) is not only of smaller magnitude, but subsides completely within a very much smaller interval of time. A third (temporary) effect of temperature does not fall within the scope of the present work. If the contemporaneous effects of the action of temperature on permanently saturated glass-hard magnets, viz., reduction of magnetic moment per unit of mass (ordinates) and of specific resistance (abscissæ), be compared graphically, the loci of the relation pass from pronounced convexity, as regards the axis of abscissæ, almost horizontally through a point of circumflexion into pronounced concavity. It must be borne in mind that both the direct and indirect effects are here superimposed. The said curves, if constructed for different dimension-ratios, are approximately parallel, presenting greater curvature, however, for smaller dimension-ratios than for larger. The immediate bearing of temper on the indirect effect is strikingly shown by the fact that, in the case of long, hard, permanently saturated steel rods, the relation between permanent magnetism per unit of mass and resistance, where both variations are simultaneous and due to changes of temper only, and where the latter occurs between the maximum of permanent hardness for ordinary temperature and the maximum of the same quality for 100°, is ultimately ($a = \infty$) linear.

The maximum of permanent magnetization for any given temperature, $t°$, which can be imparted to a steel rod exhibiting the maximum of permanent hardness for the same temperature, $t°$, is wholly independent of the possibly pre-existing states of magnetization. If $t = 100°$,

such magnets possess exceptional retentiveness both as regards effects of (atmospheric) temperature and time and of percussion.

The following rules for the practical treatment of magnets, where great retentiveness is the principal desideratum, we believe to be justified in submitting:

1. Rods tempered glass-hard are not to be used as essential parts of magnetic instruments.

2. Having tempered a given steel rod in such a way as insures uniformity of glass-hardness throughout its length, expose it for a long time (say 20–30 hours; in case of massive magnets even longer intervals of exposure are preferable) to the annealing effect of steam (100°). The operation may be interrupted as often as desirable. The magnet will then exhibit the maximum of permanent hardness for 100°.

3. Magnetize the rod—whether originally a magnet or not is quite immaterial—to saturation, and then expose it again for about 5 hours (in case of massive magnets even larger intervals of exposure are preferable) to the annealing effect of steam (100°). The operation may be interrupted as often as desirable. The magnet will then exhibit both the maximum of permanent magnetization as well as the maximum of permanent hardness corresponding to 100°. Its degree of magnetic retentiveness against effects of temperature ($< 100°$), time, and percussion is probably the highest conveniently attainable.

Chapter VII.—Given any iron-carburet, pure or impure.

Suppose carburation to vary continuously from 0 per cent. to 6 per cent. by weight of the whole, in any definite manner, and in such a way that increment in total carbon is compensated by decrement in total iron. Let there be no further change in the ingredients of the carburet. In this way we generate a special *series* of iron-carbides of which the given carburet is a particular member.

Such a series of iron-carburets exhibits the following characteristics:

If cooled from a temperature in red heat as rapidly as possible, thermo-electric hardness varies continuously with carburation, at a gradually retarded rate, and in such a way that the locus ascends rapidly during the earlier stages of progress, very slowly during the later stages. As a whole, therefore, the curve shows pronounced concavity downward.

If cooled from the same temperature in red heat with all desirable slowness, thermo-electric hardness varies continuously with carburation, at a gradually accelerated rate, and in such a way that the locus ascends slowly during the earlier stages of progress, very rapidly during the later. As a whole, therefore, the curve shows pronounced convexity downward.

The difference between the values of thermo-electric hardness for the same carburation passes through a maximum in the region of steel.

The difference between the logarithms of the respective values of thermo-electric hardness for the same carburation passes through a pro-

nounced maximum, defining a carbide, the mechanical properties of which are those of a type steel, and may be fully given thus:

Let each member of the whole series of iron-carburets be subjected successively to the following operations:

I. A process of very slow cooling from a given temperature in red heat.

II. A process of most rapid cooling possible from the same temperature.

If now the carburets be examined with reference to the hardness produced in the two instances, there will be found among them a certain unique member, whose properties are such that while process *I* has more nearly identified it with pure soft iron, process *II* will have moved it further away from this initial carburet than is simultaneously the case with any other iron-carbon product; a unique member, in other words, which is capable of occurring in the greatest number of states of hardness relative to the soft state, possible. To the said product the term "steel" is to be applied.

Similar deductions may be made from the critical values of specific resistance of iron-carburets.

The variation of a function like the one in hand in the neighborhood of a maximum is small. Hence a group of iron-carburets disposed on either side of the said maximum will possess properties sufficiently alike to be practically or commercially identical with the accurately defined type.

The general statement of the problem of which a special solution is contained in Chapter II is this:

The extreme values of thermo-electric hardness, *i. e.*, the values corresponding to sudden cooling and thorough annealing, respectively, describe any given member of any given series of iron-carburets with reference to its chemical properties. In other words, the said extreme values fix its position in a diagram of classification peculiar to iron-carburets. The intermediate values of thermo-electric hardness, *i. e.*, the values lying between the extremes and obtainable by annealing of the previously chilled carburet, describe the same product with reference to its mechanical properties. In other words, the said intermediate values determine its position in a scale of hardness peculiar to the particular iron-carburet in hand.

Given the same iron-carburet pure or impure, as before. To this belong another series of carburets, so generated that while chemical composition remains unchanged, the mode of occurrence of carbon passes from an initial extreme phase to a final extreme phase. In the simple case, where carbon is present in the combined and uncombined modifications only, $\frac{\text{combined } C}{\text{Total } C}=0$ may be regarded as the initial phase, $\frac{\text{combined } C}{\text{Total } C}=1$, as the final phase of occurrence. The character of the variation of thermo-electric hardness with the generating variable—con.

veniently termed qualitative carburation—cannot as yet be enunciated. The totality, or whole class of iron-carburets of which the given sample is a particular member, finally, may be expressed by a diagram in three dimensions, in which thermo-electric hardness appears as ordinate, quantitative carburation (total carbon in per cents.) as one independent variable, x, qualitative carburation as the other independent variable, y. In the general equation of such a surface, impurities (S. P. Si. Mn. etc.) including any further modification of carbon, are indicated by arbitrary constants (parameters). Any variation of each or all of these is equivalent to a passage from one definite classification-surface to another.

In constructing the diagram use was made of a class of easily measurable physical properties, thermo-electric hardness and specific resistance, which in the case of iron-carburets, from their simplicity and pronounced character, seem above all others to be adapted for purposes of discrimination and classification. In addition to these, however, both the density and magnetic quality (magnetic moment per unit of mass of permanently saturated *linear* rods) of iron-carburets are similarly, though less conveniently, available. On the basis of the above magnetic researches it may be safely affirmed that the given plan of discussion, if applied in turn to these functions, must lead to a classification-diagram differing in no essential respect from the above.

APPENDIX.

ON THE RELATION BETWEEN THE THERMO-ELECTRIC PROPERTIES, THE SPECIFIC RESISTANCE, AND THE HARDNESS OF STEEL (1879).[175]

1.—INTRODUCTORY REMARKS.

The experiments which gave rise to the following paper were commenced with the view of further studying the relation between the maximum of permanent magnetism, hardness, and form of steel, a subject proposed for inaugural work by Professor Kohlrausch.

Although this question has elicited considerable experimentation ever since Coulomb's[176] time, it was not until comparatively recently that harmonious results were arrived at, chiefly through the labors of Ruths,[177] Rowland,[178] Gaugain,[179] Fromme,[180] Trève and Durassier,[181] and Gray.[182] All these observers, however, classified steel, with reference to its hardness, either simply into hard and soft, or accepted the colors of the oxide film on the tempered bar as a criterion of distinction sufficient for their purposes. It seemed, therefore, that the most probable method of further elucidating the magnetic subject referred to would consist in attempting to find some method by which the hardness of steel can be more distinctly and more rationally expressed. My endeavor was, in other words, to give the very vague notion hardness, as applied to steel, a *quantitative* signification. So long, however, as the ultimate nature of hardness does not admit of accurate definition, it is sufficient for the accomplishment of this end to examine some of the other properties of steel, which likewise vary with its hardness, and by considering the magnetic moment, *cæteris paribus*, as dependent on the former, to eliminate, as it were, the notion of hardness between them. My attempt is, in short, to find an expression of the more complicated functions of

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[175] This is here printed for its bearing upon the discussion of steel, in the form of its original publication in the Phil. Mag. (5), VIII, pp. 341-368, 1879.

[176] Coulomb: Biot. Phys. III, p. 108, &c.; Hansteen: Pogg. Ann., III, p. 236, 1825; Müller: Pogg. Ann., LXXXV, p. 157, 1852; Plücker: Pogg. Ann., XCIV, p. 28, 1855; Wiedemann: Pogg. Ann., CVI, p. 169, 1859; Lamont: Handbuch. d. Magnet., pp. 223, 249-253.

[177] Ruths: Inaugural Dissertation, p. 34. Darmstadt, 1874.

[178] Rowland: Phil. Mag. (4), L, p. 361, 1875.

[179] Gaugain: Comptes rend., LXXXII, p. 145, 1876.

[180] Fromme: Gött. Nachr., No. 7, 1876, p. 157 *et seq.*

[181] Trève and Durassier: Ann. de Chim. et de Phys. (5), V, p. 266, 1875.

[182] Gray: Phil. Mag. (5), VI, pp. 321-3, 1878.

hardness, *cæteris paribus*, in terms of the more simple. Of the latter, the thermo-electric properties and the specific resistance of steel, both admitting of accurate and easy determination, appear most suitable.

But the experiments on hardness and the electrical properties alluded to, although only introductory in their character, gave rise to a number of new results. I determined, therefore, to publish them separately. To obtain as complete a picture as possible of these phenomena I have made free use of all the information on the subject within my reach. In each case the author borrowed from is cited.

2.—APPARATUS FOR HARDENING THIN STEEL WIRE.

For reasons which become apparent below[183] the principal experiments of the following paper are confined to thin rods cut from the same coil. The rather difficult task of hardening these homogeneously throughout their length, without giving rise to a change in their chemical composition (either from oxidation or carburation), I believe to have accomplished by the aid of the following apparatus:

A glass tube 200 to 300 millimeters long, 8 millimeters wide, was provided at a distance of about 80 millimeters from one end, with two opposite apertures *aa*, Fig. 26, each about 3 millimeters in diameter. This part was then surrounded with a cork, *A*, perforated perpendicularly to the axis of the tube in a manner to correspond with the holes *aa*. This arrangement is fastened vertically in a suitable iron stand (not shown in the figure). The wire to be hardened is introduced into the tube and fastened below to a brass rod, *bh*, fitting tightly in the perforation *baab*, and above to the spring *dK*. For the purpose of fastening the lower end it was found sufficient, after having previously wound it around the rod *bh* so as to form a coil which could easily be made to slide off, to push the rod through *baab* and the coil, the latter having been introduced into the tube from the top. The spring *dK*, round the lower half of which the other end of the wire was wound, the upper half being provided with a clamp-screw *gg*, was fastened to a second arm of the stand (also omitted in the figure). By properly adjusting the rod *bh* and the arms of the stand the wire could be brought into coincidence with the axis of the tube and stretched as far as was necessary.

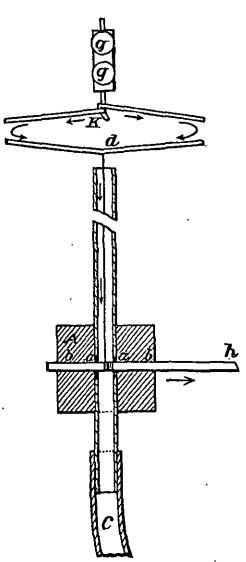

FIG. 26.—Apparatus for hardening.

[183] Difficulties due to structure, *vide* § 7, d.

A powerful galvanic current heated the wire to the degree of redness desired. The former entered *gg* and passed back to the battery through *bh* (as shown in the figure). To prevent the oxidation of the wire during the heating, a current of dry CO_2-gas,[184] was passed through the tube, entering by means of the hose *C* attached to the lower end.

After the wire had attained a steady uniform glow the hose *C* was closed by the fingers, its connection with the carbonic-acid apparatus disadjusted, the open end being connected with a neighboring hydrant instead; hereupon the faucet of the latter was quickly opened, the galvanic current being at the same time interrupted; the water dashing up the tube (as an unbroken column, however) with great velocity, imparts to the wire the hardness desired. Before each experiment the parts of the apparatus were well dried in a current of air.

The apparatus described presents the following advantages:

(1) By employing currents of different intensity, thus heating the wire to different degrees of redness, we are able to obtain corresponding degrees of hardness, which, though scarcely distinguishable mechanically (all appearing equally hard and brittle), have very different effects on the magnetic and electrical properties of steel.[185]

(2) From the fact that the wire is kept in a state of continual tension by the spring *dK*, and from the particular method of chilling, the wires remain straight after being hardened.

(3) The very slight oxidation noticeable on the hard wires is probably due to the contact of water and steel in the act of hardening.

Disadvantages, however, arise from the fact that the use of the apparatus is confined to thin bars, and that the wires obtained may be in a state of circular magnetization. This would partly prevent their employment in (certain) subsequent magnetic experiments. The difficulty may however be avoided by opening the galvanic current a little before opening the faucet.

3.—METHODS OF MEASURING THE HARDNESS OF STEEL ELECTRICALLY.

(*a*) *Thermoelectric position and hardness of steel.*—In this place it will be expedient to leave the special consideration of steel for the moment,

[184] Having accidentally employed moist carbonic-acid gas, a small flame was observed at the top of the tube. This is probably due to the combustion of H_2 and CO, the former being generated by the decomposition of aqueous vapor by the hot steel, the latter by the action of the nascent H produced on the CO_2.

[185] See IX. The coercive force of steel being a minimum at a point in incipient redness, it is possible that this apparatus might be used in obtaining intense circularly or longitudinally magnetic wires. In the first case, the wires should be cooled without breaking the current; in the second, the wires surrounded by a tube through which, during the act of hardening, a powerful galvanic current flows. (See Holtz, Wied. Ann., VII, p. 71, 1879.)

giving attention to the electromotive force of a thermo-element composed of any two different metals A′ and A″.

Kohlrausch[186] has shown that the phenomena included under the head of thermo-electricity can be explained on the hypothesis that the heat current is always accompanied by an electric current, whose intensity is proportional to the number of caloric units passing the same section. He thus arrives at an expression for the electromotive force between any two metals (A′ and A″) which, if for simplicity we suppose the cold end to be kept at zero,[187] has the following form:

$$E_\tau=[\vartheta'-\vartheta'']\ \tau\ [1+\dot{f}(\tau)]$$

where $E\tau$ is the electromotive force corresponding to the difference of temperature, τ, of the ends; $(\vartheta'-\vartheta'')$, a constant, specific for the combination.

This expression of Kohlrausch is very convenient, inasmuch as it allows us to separate the actual electromotive force into two terms, of which the first,

$$\vartheta'\tau\ [1+f(\tau)]$$

is dependent only on the metal A′ and τ, the second, $\vartheta''\ \tau\ [1+f(\tau)]$, only on A″ and τ.

Now, we know that the thermo-electric position of a metal is dependent not only on its chemical nature, but also on its mechanical condition (hardness). Let us therefore put

$$\vartheta'=\vartheta_0'+\theta' \qquad \vartheta''=\vartheta_0''+\theta''$$

where ϑ_0' and ϑ_0'' are to represent the (absolute) constants dependent on the chemical nature of A′ and A″, respectively, θ' and θ'', however, varying with the hardness of the metals. Thus the above equation becomes:

$$E_\tau=[\vartheta_0'-\vartheta_0'']+[\theta'-\theta'']\tau[1+f(\tau)]$$

But suppose that A′ and A″ are not different metals, but represent two rods to which different degrees of hardness have been imparted, which were originally, however, cut from the same wire. In this case $[\vartheta_0'-\vartheta_0'']=0$, whence

$$E_\tau=[\theta'-\theta'']\ \tau\ [1+f(\tau)]$$

dependent only on τ and the difference of hardness of the rods.

It will be shown below that the electromotive force of an element of soft and hard steel varies continuously with the difference of temperature τ and with the difference of hardness of the rods. We will therefore put θ'', the constant belonging to the soft bar (*i.e.*, one which has been heated above redness and allowed to cool slowly in a badly conducting medium), equal to zero, as it is in this that the molecules will

[186] Kohlrausch: Pogg. Ann., CLVI, p. 601, 1875.

[187] The thermo-electromotive force being, according to Tait, Avenarius, Hankel, and others, a function of the temperature of the two ends.

most probably have assumed normal positions. If, furthermore, we replace $E_\tau=\theta'\,\tau\,[1+f(\tau)]$ by $E_\tau=a\tau+b\tau^2$, a sufficient approximation for practice, we derive $\frac{dE_\tau}{d\tau}=a+2b\tau$ and $\left[\frac{dE_\tau}{d\tau}\right]_0=a$, for $\tau=0$.

This expression, *i. e.*, the limiting value of the electromotive force of a thermo-element composed of a soft rod and one of any degree of hardness to the corresponding difference of temperature[188] when the latter converges towards zero, will, in the sequel, be taken as the measure of hardness of the harder bar. I shall apply the term thermo-electric hardness abbreviated (T. E. H.) to it, throughout the following paper.

The relation between the thermo-electric properties and the hardness of steel, notwithstanding its comparative importance, has never to my knowledge been made the subject of detailed and exclusive study. All the experiments thus far published (the principal being those of Magnus,[189] Sir William Thomson,[190] and E. Becquerel[191]), are of a qualitative nature, the results being derived from the direction of the current observed on bringing together, in one way or another, wires of different hardness.

The experiments of Magnus, being limited to hard-drawn wires, do not properly fall within the scope of the present paper. The same is true of a number of the experiments in the excellent paper of Sir William Thomson. With regard to the effects of annealing Professor Thomson observes: "In cases of round steel wire, of steel wire flattened through its whole length by hammering, and of steel watch-spring, the thermo-electric effect of annealing portions after the whole had been suddenly cooled, was a current from unannealed to annealed through hot." This result comprehends all that has thus far been done.

(*b*) *Specific resistance and hardness of steel.*—With reference to the specific resistance and the hardness of steel, I shall proceed in a manner analogous to the preceding. Denoting the observed specific resistance of a bar by S, that part of S which is due only to the chemical nature of the rod by S_0, that due to hardness by ΔS_0, I have

$$S=S_0+\Delta S_0$$

Now, as it follows from results given below that the specific resistance of steel increases continuously with its hardness, it will be convenient to put ΔS_0 for the soft bar equal to zero. The value of ΔS_0 for a bar of any degree of hardness, thus numerically determined, will in the following be accepted as a second measure of that property. The work thus far published on the relation between specific resistance and

[188] The colder end being supposed at 0°.

[189] Magnus: Pogg. Ann. LXXXIII, p. 486, 1851.

[190] Thomson: Phil. Trans., III, pp. 709–727, 1856.

[191] Becquerel: Ann. de Chim. et de Phys., (4), VIII, p. 402, 1866.

hardness of steel is due principally to Mousson.[192] Of late, results have been announced by Chwolson.[193] The data of both observers agree only qualitatively with mine, and among themselves.

4.—DETERMINATION OF THERMO-ELECTRIC HARDNESS. APPARATUS.

(*a*) *Method of measurement.*—In the determination of thermo-electric force the procedure known as Ohm's method was first employed. Afterwards it was found expedient to measure these forces (as Kohlrausch and Ammann[194] had done in similar experiments before) by a zero method, the object being to avoid the species of polarization due to Peltier's phenomenon. My method can easily be deduced from that proposed by Bosscha, the latter, in the case where small electromotive forces are to be measured, admitting of simplification. In the diagram, Fig. 27, E denotes the compensating element (1 Daniell's element = 11.7 Weber × Siemens' units), e the thermo-couple whose electromotive force is to be determined, both acting as shown in the figure, C a Weber's commutator (employed for reasons given below), G the galvanoscope. Let the resistance of the branch ab be represented by x, that of the branch aEb by $W+k$, where W represents the large resistance of a rheostat interposed, k that of the remainder of the branch (about equal to 1 Siemens' unit) including E. When the current in G is zero we have,

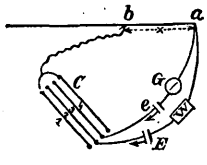

FIG. 27.—Disposition of apparatus.

$$\frac{e}{E} = \frac{x}{W+k+x}$$

But as $\frac{e}{E}$ is small, and therefore of necessity also x (maximum value =10 S. U.), in comparison with $W+k+x$ (20,000 Siemens' units), I may with sufficient approximation, put

$$e = \frac{E}{W}\, x$$

the experimental accuracy attainable allowing me to neglect $(k+x)$ in comparison with W. In the experiments the branch ab was a small Siemens' rheostat.

The precise moment at which the current in the galvanoscope is zero can be best determined by observing whether the needle on closing and opening the circuit remains at rest. This, however, is only possible when the opposed currents from e and E which pass through the galvanoscope are closed simultaneously. To accomplish this the little

[192] Mousson: N. Deuksch. d. Schw. Gesellsch. (8), XIV, pp. 1–90, 1855.

[193] Chwolson: Mél. Phys. de St. Pétersbourg, X, p. 379, 1877. See also Carl's Repert., XIV, p. 15, 1878.

[194] Kohlrausch and Ammann: Pogg. Ann., CXLI, p. 450, 1870.

cups at the end of the rods 1 and 2 of the commutator were quite filled with mercury, those of 3 and 4 only partially. By this device, on closing, C_1 and C_2 are first brought into contact, and the current $EC_2\ C_1\ ba\ WE$, not passing through the galvanometer, comes into action; in the next moment (C_3 and C_4 being joined) the current from e and the partial current from E referred to are closed simultaneously. In this way also induction-currents which may possibly be generated in the rheostat, are without disturbing effect. The commutator merely serves the purpose of a double key.

As the electromotive forces to be measured were all very small, the large resistance W could be left unaltered, so that $e = const.\ x$. Now, the resistance x was so chosen that the intensity of the current from e exceeded that of the partial current from E by the minimum possible. The thermo-electric force, however, decreasing with the temperature, $T-t=\tau$ of the ends, a moment soon arrives at which the intensities of the two currents are equal, and the deflection of the needle = 0, in consequence. At this point the thermometers are read off.

(*b*) *Description of thermo-element.*—Instead of measuring the electromotive force of soft and hard steel directly, it was found expedient to compare all the rods with one and the same piece of copper wire. By this means the apparatus could be considerably simplified and many practical difficulties avoided. The construction of the copper-steel couple is given in vertical section in Fig. 28.

To raise the ends of the rod to different temperatures two doubly-tubulated receivers, each about 10 centimeters in diameter, were used. These, held in position by movable supports of poorly-conducting material, and so placed that the tubulures *A* and *B* were horizontal, the other two vertical, were connected by a glass rod *cd* fitting water-tight in the perforated corks adapted to the horizontal tubulures. This rod served a double purpose. By uniting the receivers as one it prevented breakage of the very brittle steel rods, at the same time allowing the receivers to be easily adjusted and removed; on the other hand the receivers could by means of it be placed at any distance apart, this being necessary, as the rods to be examined were of very different lengths.

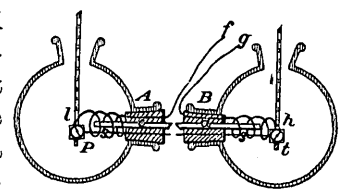

FIG. 28.—Original form of thermo-element.

On one side of the glass rod the copper wires which acted as poles of the instrument were inserted once for all; on the other two appropriate holes served for the introduction of the steel rods *ss* to be tested. The ends of the latter were connected with the corresponding ends of the copper wires by small flat clamp-screws of brass.

The apparatus being thus ready for experiment the two receivers were filled with distilled water at $T°$ and $t°$, respectively, where t was so chosen as to differ but slightly from the temperature of the room.

The thermometers (introduced through the vertical tubulures) were read off with a telescope as follows: The deflection of the needle having become very small, t was determined, after which, when the current in $G = 0$, T, whereupon another check-reading of t was made. Before each observation the water in the receivers was well stirred.

(*c*.) *Galvanoscope.*—The galvanoscope used was a very delicate instrument of Sauerwald's, provided with mirror and astatic needle. The deflection of the latter was read off with telescope and scale. As the telescope of this instrument stood side by side with the telescope for the thermometers, both readings could conveniently be made by the same observer.

5.—DETERMINATION OF SPECIFIC RESISTANCE.

For determining the specific resistance of steel rods use was made of Wheatstone-Kirchhoff's bridge. An appropriate commutator allowed the observer to exchange the unknown resistances without altering the value of those belonging to the bridge proper. Thermo-electric disturbances were avoided as far as possible by closing the current (1 Smee with large resistance) only for very short intervals of time. Finally they were eliminated completely by replacing the hydro-electromotive force by Weber's magneto-inductor.[195] The resistances of all rods were determined in terms of an arbitrary standard δ (0.0312 Siemens unit at 0°), chosen to correspond in magnitude with the unknown resistances. The galvanometer used was the one already mentioned above.

As the resistances to be measured were all very small (0.1 to 0.01 Siemens unit), great care had to be taken to exclude all disturbing resistances due to insufficient contact. Soldering could not be resorted to, as it was believed that the ends would thereby be annealed. The method adopted was as follows: The ends of the rods having been well cleansed were covered to about 1 centimeter with a thin adhesive film of galvanically deposited copper, which was thereupon amalgamated (easily accomplished by plunging the freshly-covered part in mercury). The rod thus prepared was then fixed together with a glass rod in two corks, in a manner similar to that employed in the case of the thermo-element, and the whole, except the amalgamated ends, covered with a thick coat of varnish. Suitable wooden cups, provided with horizontal and vertical apertures, completed the connection of the rods with the respective parts of the bridge by means of mercury.

The efficient length was determined by deducting from the total length that of the amalgamated ends. For the measurement of diameter use

[195] Weber: Pogg. Ann., CXLII, p. 418, 1871. The use of the magneto-inductor in connection with the bridge was suggested by Kohlrausch. This physicist also showed that the method is applicable even when the resistances to be determined are in form of coils, the extra currents produced being calculable. I found the application of great convenience, inasmuch as the observer always has the needle of a delicate galvanometer completely under his control.

was made of the microscope. A determination of this dimension from known weight, specific gravity, and length was impracticable, inasmuch as the use of the pycnometer, which alone would have given sufficiently accurate results, would have compelled me to break the rods.

6.—EXPERIMENTAL RESULTS.

A. Thermo-electric hardness of rods suddenly immersed in cold water while in different states of red heat.—The following (older) results were obtained directly by Ohm's method. E_τ was put = constant τ, a condition nearly fulfilled by couples of soft and hard steel between zero and 80°, in which case T. E. H. and the constant of proportionality are identical. The rods were hardened in the apparatus described in § 2; diameter = 0.0678 cm.; the numbers preceded by the point (.) were afterwards checked by the method of compensation. T. E. H. is expressed in Weber × Siemens units.

The third column of the following table contains the number of large Bunsen cells employed in heating the wire, the fourth the observed degree of redness at the time of sudden cooling.

TABLE I.

No.	T. E. H.	Bunsen elements.	Remarks.
1	0.0000004	.....	Soft, cooled slowly from red heat.
.2	0.000003	4	Below redness.
3	0.000012	5	
4	0.000000	5	
.5	0.000000	6	
6	0.000000	6	Ignited dark red.
7	0.000052	7	
.8	0.000049	7	
9	0.000054	7	
.10	0.000056	8	
11	0.000057	8	
12	0.000063	8	
.13	0.000064	9	Ignited brick red.
14	0.000065	9	
15	0.000064	9	

The Bunsen cups were introduced without altering the remaining part of the circuit. The length of the wire heated was not in all cases the same.

B. Further thermo-electric data. Hard steel.—The following determinations were made in the summer of 1878; temperature of the room very constant and at about 20°, this, as already observed, being nearly the same as t, the temperature of the water in the colder receiver. The method of compensation was employed throughout. The determination of T was effected by a Geissler normal thermometer (graduated in 0°.1), that of t by an ordinary instrument (graduated in 0°.2) which had, however, been carefully compared with the former.

In Tables II and III the difference of temperature of the ends of the steel rod is given under τ, the corresponding electromotive force for the elements copper-steel under E_τ; α and β are constants which satisfy

the equation $E_\tau = a\tau - \beta\tau^2$. These were calculated by first computing their approximate values out of two distant observations, and then adding to them corrections deduced from five of the most satisfactory observations, by the method of least squares.

If, now, we denote by a and b the constants of an element soft-steel and hard-steel, corresponding to those α, β, α', β' of the same rods, when compared with copper, we shall have, since $E_\tau = a\tau - \beta\tau^2$ and $E'_\tau = a'\tau - \beta'\tau^2$,

$$E_\tau - E_\tau' = (a - \alpha)\tau - (\beta - \beta')\tau^2$$

But $E_\tau - E_\tau' = \mathrm{E}_\tau$, the electromotive force of the element steel-steel; so that, since also $\mathrm{E}_\tau = a\tau - b\tau^2$,

$$a = a - \alpha^1 \qquad b = \beta - \beta^1$$

The constants a and b are given in the last two columns. a may be regarded as numerically equal to the T. E. H. above defined, as E_τ is nearly a linear function of τ.

TABLE II.—*Rods 0.0678 centimeters thick.*

No.	τ	E_τ Observed.	E_τ Calculated.	α	β	a=T. E. H.	b
[1]I	13.92	0.001464	0.001457	0.0001071	0.0000001733	Nil	Nil.
	25.68	2635	2636				
	39.78	3981	3985				
	49.34	4859	4862				
	62.65	6031	6030				
[2]II	17.90	0.001815	0.001807	0.0001038	0.0000001629	0.0000031	0.0000000104
	31.39	3104	3100				
	47.58	4567	4572				
	56.53	5328	5349				
	63.72	5972	5956				
[3]IV	20.25	0.001112	0.001095	0.00005811	0.0000002006	0.0000490	—0.0000000273
	29.20	1522	1526				
	46.50	2283	2268				
	59.90	2752	2761				
	61.35	2810	2810				
[4]V	18.07	0.000878	0.000881	0.00005156	0.0000001568	0.0000555	0.0000000165
	21.93	1054	1056				
	47.76	2108	2105				
	61.32	2576	2571				
	63.44	2635	2640				
[5]VI	15.75	0.000703	0.000700	0.00004660	0.0000001390	0.0000605	0.0000000343
	31.84	1347	1343				
	43.51	1756	1765				
	45.06	1815	1818				
	59.42	2283	2278				
[6]VII	19.04	0.000761	0.000764	0.00003807	0.0000001529	0.0000641	0.0000000204
	34.08	1288	1288				
	53.64	1874	1867				
	63.40	2108	2112				
	68.30	2225	2225				
[7]VIII	18.42	0.000644	0.000658	0.00003807	0.0000001279	0.0000690	0.0000000454
	31.15	1054	1062				
	45.00	1464	1454				
	62.10	1874	1871				
	65.40	1932	1943				
[8]IX*	.....	.....	.....	.....	.....	0.000780	.....

[1] Heated above redness and allowed slowly to cool in wood ash; "soft."
[2] Heated galvanically below redness, then suddenly cooled; "soft."
[3] Heated to dark redness, then suddenly cooled; "glass-hard."
[4] Heated to redness, then suddenly cooled; "glass-hard."
[5] Heated galvanically to brick-redness, then suddenly cooled; "glass-hard."
[6] Heated galvanically to brick-redness, then suddenly cooled; "glass-hard."
[7] Heated to yellow-ignition, then suddenly cooled; "glass-hard."
[8] Heated to yellow-ignition, then suddenly cooled; "glass-hard."
* This determination was made at a later date; with reference to its accuracy the remarks under *C* and *D* apply.

TABLE III.—*Rods 2.65 millimeters thick, tempered by Mr. Barth, mechanician.*

No.	τ	E_τ Observed.	E_τ Calculated.	a	β	$a=T.E.H.$	b
[1]*D*	32.28	0.002928	0.002918	0.00009447	0.0000001257	0.00001263	0.0000000476
	45.55	4040	4042				
	46.88	4157	4153				
	55.54	4860	4859				
	56.24	4918	4915				
[2]*C*	20.32	0.001639	0.001628	0.00008265	0.0000001229	0.00002445	0.0000000504
	31.98	2518	2517				
	32.90	2576	2586				
	47.93	3689	3678				
	49.80	3806	3811				
[3]*B*	32.40	0.001991	0.002004	0.0000646	0.0000000864	0.0000424	0.0000000869
	33.08	2049	2044				
	50.47	3045	3043				
	61.00	3630	3623				
	62.25	3689	3690				
[4]*A*	10.86	0.000117	0.000121	0.00000999	0.0000001068	0.00011710	0.0000000665
	27.20	351	351				
	43.75	644	642				
	58.12	937	942				
	60.37	995	993				

[1] Soft; gently ignited, slowly cooled.
[2] Tempered blue.
[3] Tempered dark yellow.
[4] Glass-hard.
* This rod (as also those of the following section *C*) is electronegative with reference to copper.

B'. Specific resistance of the same material.—The following Tables IV and V contain the specific resistances of the rods already cited in Tables II and III. The data are given in terms of mercury. The column S contains the total specific resistances, ΔS_0 the corresponding excess of the latter over that of the normal rod I. Of the ratio $\frac{\Delta S}{T.E.H.}$ mention will be made hereafter.

Let the resistances of the two parts of the first branch of the bridge, on each side of the sliding contact, be denoted by a and b; let the resistances of the corresponding parts (thick copper wire) of the second branch of the bridge be denoted by K' and K''. Into the latter the unknown resistances W and R are to be inserted, respectively. Finally, let δ be the standard of comparison above referred to, in terms of which W and R are to be measured. When the current in the galvanometer is zero, we shall have for a particular position of the commutator:

(1) W and R alone—

$$\frac{a}{b}=\frac{W+K'}{R+K''}$$

(2) W and R with δ on the right—

$$\frac{a'}{b'}=\frac{W+K'}{R+K''+\delta}$$

(3) W and R with δ on the left—

$$\frac{a''}{b''}=\frac{W+K+\delta}{R+K''}$$

Three similar equations may also be derived from the other position of

the commutator, as only W and R are interchanged. From these six equations we deduce:

First position—

$$\frac{W}{\delta}+\frac{K'}{\delta}=\frac{1}{\frac{b'}{a'}-\frac{b}{a}} \qquad \frac{R}{\delta}+\frac{K''}{\delta}=\frac{1}{\frac{a''}{b''}-\frac{a}{b}}$$

Second position—

$$\frac{W}{\delta}+\frac{K''}{\delta}=\frac{1}{\frac{a'}{b'}-\frac{a}{b}} \qquad \frac{R}{\delta}+\frac{K'}{\delta}=\frac{1}{\frac{b''}{a''}-\frac{b}{a}}$$

$\frac{K'}{\delta}$ and $\frac{K''}{\delta}$ were determined in the same way previous to the experiments, and their values checked from time to time. In comparing the resistances $I \ldots IX$ and $A \ldots D$, the following plan was observed:

$W:\delta$	I	IV	IV	.....	VIII	VIII	I	And in the same way with A to D.
$R:\delta$	II	II	V	.....	VII	IX	IX	

Each of the data is therefore derived as a mean of four determinations. Possible heterogeneity of the wire $a+b$ thus becomes less effective.

As an example, I will add results obtained for the rod C (resist.= 0.0051 Siemens units):

		Compared with A.	Compared with B.	Compared with D.
1st position.....	$W:\delta=$	0.1648	0.1654	0.1649
2d position.....	$W:\delta=$	0.1630	0.1632	0.1639
	Mean..	0.1639	0.1643	0.1644

For the rod I (resistance=0.0542 Siemens units) on different occasions, 1.735 and 1.741 were found for $W:\delta$.

As will be seen, the principal stress was put on relative values of the resistances. To facilitate orientation, however, the results were approximately reduced to Siemens units by determining the standard δ in that denomination.

Assuming the coefficient for temperature for steel to be the same as that for copper, the results obtained will be good for 0°C directly, δ having been previously reduced. Although this is only approximately true, the influence of this difficulty on the relative values of the resistances will be but slight, as the temperature of the room remained nearly constant.

TABLE IV.

No.	Resistance.	S	ΔS_0	$\frac{\Delta S_0}{T.E.H.}$
I..	0.05417	0.1361	Nil.	.....
II..	0.04305	0.1400	0.0039	1260
*IV[1]	0.11130	0.2337	0.0976	1990
V..	0.11060	0.2483	0.1122	2020
VI..	0.07846	0.2592	0.1231	2030
VII..	0.08740	0.2648	0.1287	2010
VIII..	0.15330	0.2779	0.1418	2050
*IX..	0.09417	0.2810	0.1449	2070

[1]The resistances marked with an asterisk were determined by using the magneto-inductor as a current generator. In the others a Smee element was employed.

TABLE V.

No.	Resistance.	S	ΔS_0	$\frac{\Delta S_0}{T.E.H.}$
D	0.00209	0.1654	0.0293	2320
C	0.00512	0.2065	0.0704	2860
B	0.00595	0.2271	0.0910	2150
A	0.00908	0.3804	0.2443	2090

The experiments now following, under the heads *C* and *D*, have more a descriptive character than one of precise measurement; the results are therefore given with one decimal less. For the determination of T.E.H. the rods coming under these heads were compared indirectly with rod VIII (Table II) for the T. E. H., of which the number 0.000069 (as above found) was assumed.

C. T. E. H., *and* ΔS_0 *for glass-hard rods, diameter*=2.30 *millimeters.*— These were hardened by the aid of the blast lamp. The flame of the latter was for this purpose directed horizontally and placed directly over a trough containing cold water. In this way the red-hot rod could be transferred with great rapidity out of the flame into the water.

TABLE VI.

No.	*T. E. H.*	Resistance.	S	ΔS_0	$\frac{\Delta S_0}{T.E.H.}$	Remarks.
*[I]	0.000138	0.0121	0.421	0.285	2100	Ignited yellow and chilled.
*[II]	131	.....	.....	.....	.....	Do.
*[III]	130	.....	.....	.....	.....	Do.
*[IV]	130	0.0129	0.442	0.306	2300	Do.
*[V]	116	107	0.336	0.230	2000	Ignited red and chilled.
*[V]′	136	123	0.430	0.294	2200	[III] u. [V] Twice re-ignited yellow and chilled.
*[III]′	133	.....	.....	.....	.....	

D. T. E. H. and ΔS_0, of rods 0.678 millimeter in diameter, hardened[196] in the apparatus §2, afterwards *annealed* by immersion in hot linseed oil.

[196]The T. E. H. of these rods in the glass-hard condition varied from $50:10^6$ to $60:10^6$.

TABLE VII.

No.	*T. E. H.*	Resistance.	*S*	Δ *S*₀	$\frac{\Delta S_0}{T.\ E.\ H.}$	Remarks.
*1	0.000010	0.0540	0.157	0.021	2100	Glass-hard and gradually heated to 300° in linseed oil.[1]
*2	13	0616	158	022	1700	
*3	11	0537	159	023	2100	
*4	14	0691	163	027	1900	
*5	12	0592	166	030	2500	
*6	18	0763	177	041	2300	Similarly annealed at 280°.
*7	24	0659	185	049	2000	
*8	33	0746	205	069	2100	Similarly annealed at 240°.
*9	31	0807	206	070	2200	
*10	36	0731	209	073	2000	Similarly annealed at 200°.
*11	57	0.1088	250	114	2000	Similarly annealed at 150°.
*12	58	0.1155	261	125	2200	
Soft iron.	−0.000004	0.0426	0.133	−0.003	750	Heated to redness in Bunsen's blast-lamp and slowly cooled.

[1] Rods 1 to 5 remained in the bath during the whole process; the others were immersed but for 2ᵐ to 3ᵐ. The temperature at which the rods 6 and 7 were annealed could not be determined with certainty, a microscopic air-bubble in the neck of the mercury reservoir having given rise to rupture of the thread.

E. Corroborative measurements.—The resistances thus far determined being very small, it was feared that in spite of the continuity apparent in their variation, errors from insufficient contact might have conspired in producing illusory results. For these reasons check-experiments with longer wires were made, the resistance of some of which (I to V) is sufficiently great to admit of direct comparison with the Siemens mercury unit (étalon). To insure a more homogeneous hardening, these wires were spirally wrapped around a round stick of wood, and the length and diameter of the coils resulting so determined that the whole during the process of heating could be brought within the mantle of the blast-flame. In other respects the method given under *C* was pursued.

TABLE VIII.

No.	Resistance.	Length.	Diameter.	*S*	Δ *S*₀	Remarks.
I	0.643	0.880ᵐ	0.67	0.260	0.124	Ignited yellow; then suddenly cooled.
II	379	917	96	296	160	Do.
III	245	928	1.19	293	157	Do.
IV	121	932	1.83	342	206	Ignited red; then suddenly cooled.
V	105	556	1.62	391	255	Ignited light yellow; then suddenly cooled.
*VI	0169	733	2.15	354	218	Ignited red-yellow; then suddenly cooled.
*VII	0155	732	2.15	325	189	Ignited red-yellow; but ends darker.
*VIII	0169	740	2.15	352	216	Ignited red-yellow; then suddenly cooled.

Spirals II and IV were afterwards softened by heating to redness in a Bunsen burner. Their specific resistance in this condition was found to be

*II. S_0=0.154 *IV. S_0=0.159

Finally, in order to compare the results obtained with induction currents with those in which a Smee element was employed, certain of the experiments were repeated. The agreement was entirely satisfactory.

7.—HARDNESS AND THERMO-ELECTRIC PROPERTIES OF STEEL: DEDUCTIONS AND SUPPLEMENTARY EXPERIMENTS.

(*a*) *Thermo-electric and mechanical hardness.*—From the data contained in Tables II, III, and VII, we derive that the thermo-electric position of steel progresses continuously with its degree of hardness, or, in other words, thermo-electric and mechanical hardness are direct functions one of another.

This statement involves the assumption that the rod cannot pass from the glass-hard (maximum) to the soft state (minimum) without passing through every intermediate stage; or that by proper methods of annealing, every state between the maximum and minimum can be produced. This, I dare say, will generally be admitted.

As of further interest I may add: (1) that rods cut from the same wire and glass-hardened in the same way possess also the same thermo-electric hardness (Table VI); (2) this is the case even when the rods are carefully rehardened (rods [III]′, [V]′, Table VI); (3) that if we start from like maxima, the thermo-current always passes from less to more annealed, through warm. (The direction of the current was independently observed.)

(*b*) *Variation of T. E. H. with differences of material and of thickness.*—From an examination of the data obtained for different material, we infer that the T. E. H. of soft and similarly annealed rods approximates to the same value;[197] that the value of this constant for glass-hard rods is remarkably different. The rods in Table VI, for instance, possess a T. E. H. amounting nearly to $140 : 10^6$; whereas in the rods in Table II the maximum[198] value found does not exceed $70 : 10^6$. This may be due to a difference in thickness, or, more probably, to a difference in the composition of the rods examined.

These phenomena were further studied through the following experiments:

1. Commercial rods of different diameters were glass-hardened and examined with reference to the current produced when one end of a couple was cooled with a wedge-shaped piece of ice. In general a maximum of T. E. H. was observed in rods whose diameters lay between 1 and 2 millimeters. These experiments, however, are unsatisfactory, inasmuch as the composition of the rods enters as an element of dis-

[197] In order to compare the degree of hardness corresponding to a particular oxide tint with that corresponding to a given temperature of the oil bath, use was made of the tables found in Frick's Physikal. Technik, 3 ed., p. 377; also Wagner, Chem. Tech., 8 ed., p. 29. On the authority of these works 230° corresponds to yellow, 290° to blue, annealed. We should therefore expect rods 6, 7, and 8, 9 (Table VII) to agree with the rods *C* and *B* (Table III) respectively. This is sufficiently the case.

[198] These rods (Table II), even when heated to the utmost white and suddenly cooled, remained strongly electro-positive towards copper. T. E. H., even in this extreme case, was much less than $107 = 10^6$.

turbance which cannot be allowed for. For this reason experiments were made on thick bars, the parts of which had been filed to different diameters.

2. The halves of each of two pieces cut from the same rod, 5 millimeters in diameter, were reduced[199] by filing to thicknesses of 3 millimeters and 1 millimeter, respectively, and glass-hardened. During the process of heating care was taken to raise all parts of the bars to the same degree of redness. On connecting the ends with the galvanometer and applying the ice wedge at the middle (where the diameter enlarged), very decided currents were observed, passing from thin to thick through warm. Hereupon two cones were filed of the same material (5 millimeters thick at base and 50 millimeters long). Point and base of the cones (previously glass-hardened) being connected with the galvanoscope, the ice wedge applied at any point produced in each case currents from apex to base through warm, thus harmonizing with the previous experiments. Near the points only the results became uncertain. On bringing together the cones with the rod [IV] (Table VI) the points were found to be thermo-electrically harder, the base softer than the rod. On the other hand, the point of a fine needle prepared from the same material gave contrary indications. The point, therefore, was apparently softer than rod IV.

3. Finally two very gradually tapering cones were prepared from another steel rod, 2.8 millimeters in diameter. Connection with the galvanometer being made, the application of the ice wedge to parts near the base generated a current from thin to thick through warm; to parts near the apex, a current in the opposite direction. Finally, between these a position was found at which the application of the ice wedge produced no current at all. This occurred at parts about .5 millimeters in diameter. The other cone gave like results.

4. When the bases of two similar cones of the same materials are connected with the galvanometer and their apices brought in contact, upon warming the latter, a current in one direction or another will be produced; this from the fact that the points are rarely equally hard. Experiment shows that by consecutively warming parts which lie symmetrically to the right and left of the apices in contact, currents in opposite directions are the effect. Herefrom it follows that the currents originate each in a single cone.[200]

5. In endeavoring to generalize from these experiments, attention must be paid to the following points: *a*, the maximum value of T. E. H. attainable is dependent on the quantity of carbon contained in the steel. The thermo-electric difference between rods of soft and sud-

[199] It is to be observed that the thinner parts of these pieces sooner arrive at red heat and remain longer in this condition than the thicker parts. This applies equally to the cones.

[200] The experiments with two separate cones were equally successful when the conical forms were only part of the same continuous piece.

denly cooled wrought iron can, for instance, be neglected in comparison with the corresponding difference between hard and soft steel. *b*, T. E. H. is influenced by the temperature of the rod when suddenly chilled (VII *c*), as well as by the time of heating, the latter affecting the composition. *c*, By the form of the piece of steel and the method of sudden cooling, the internal structure of the mass being thereby modified (VII *d*). *d*, Finally, we might add that, in the time elapsing between the removal of the rod out of the fire and the subsequent immersion, the loss of heat by radiation will be relatively greater in the case of thin than in the case of thick rods.[201] With these facts in mind we may conclude that the maximum values of T. E. H. attainable by glasshardening rods of the same composition increases as thickness diminishes; that as this dimension continues to decrease a diameter is reached at which the negative effects of decarburization are equal and finally overcome by the positive effects due to diminution of diameter.

(*c*) *Effect of the temperature from which steel is suddenly cooled in hardness. Chemical and mechanical hardness.*—From the results contained in Table I there follows: The hardness of steel does not increase continuously with its temperature at the moment of sudden cooling, but at a point in dark red heat the glass-hard state is suddenly attained. From this point on, however, the degree of glasshardness (measured electrically) continues to increase with the temperature. This observation conduces to the conclusion that the change of state due to glass-hardening is chemical in its nature. In this opinion, I believe, most chemists at present also concur, it being assumed that the uncombined carbon in soft steel is, by sudden immersion, converted into the chemically combined. In summing up the facts by which the hypothesis is furthermore supported, I will mention in the first place the detailed analogy[202] which exists between the white pig (used in the manufacture of Bessemer steel) and glass-hard steel, on the one hand, and ordinary (gray) cast-iron and soft steel on the other; the former containing carbon only in the combined, the latter also in considerable proportion in the uncombined, state. Secondly, hardening by a process of wire-pulling, hammering, etc., will, in all probability increase the specific gravity of steel; hardening by sudden cooling, as is well known, di-

[201] I will add here (1884) that another possible source of error was overlooked in the above. I refer to the electro-motive force of superficial layers in case of loose contracts, as investigated by Franz (Pogg. Ann., LXXXV, p. 388, 1852), Jenkin (Rep. Br. Assoc., 1861, (2), p. 34), Gaugain (C. R., XXXVI, p. 612-16, 1853, Ann. de Chim. et de Phys. (3), LXV, p. 75-102, 1862) and others. But I feel confident that these phenomena are not contained in the above: 1, because loose contacts were avoided; 2, the metals were always bright and polished; 3, the results were the same when difference in diameter (double cone) occurred in one and the same continuous piece of steel; 4, from the variety of experiments made and the uniformity of the results obtained.

[202] See Wagner: Chem. Tech., 8 ed., pp. 14, 15, 29. These analogies refer to mechanical properties, method of preparation, color.

minishes it. In the former case the thermo-electric current usually passes from soft to hard through warm;[203] in the latter, in the contrary direction. In drawn wire the specific resistance is smaller,[204] in hard tempered greater than that of the same wire in the soft state.

Considering these facts as a whole, we are perhaps justified in distinguishing between a process of chemical and a process of mechanical hardening. This, however, does not prevent us from paying due regard to a series of physical phenomena which accompany the former. To these the peculiar internal structure of glass-hard bars, the warping which frequently attends sudden cooling, etc., are to be referred. We conclude, therefore, that the cause chiefly influential in bringing about glasshardness in steel is chemical in its nature, and that, in consequence of the physical phenomena which invariably accompany it, the degree of glasshardness is more or less modified. On the latter ground the continual increase of the T. E. H. after the critical temperature above referred to has been reached is to be explained.

(*d*) The observed T. E. H. can, of course, only be assumed as directly expressive of its hardness when the rod under experiment is homogeneous throughout. In thick bars, which in the glass-hard state may be considered as made up of a series of concentric cylindrical shells, the hardness of which decreases rapidly as we pass from the exterior to the interior, the circumstances become more complicated.

Furthermore, suppose the ends of a thick glass-hard cylinder to be kept at temperatures T and t ($T>t$). In this case, since each of the infinitely thin cylindrical shells has a particular T. E. H. corresponding to its hardness, we are led to infer that thermo-currents, closing themselves in the interior of the cylinder, are the result—the direction of these in the outer harder parts being from t to T, in the inner from T to t. In figure 29 (vertical section) the hypothetical condition of the cylinder in this case is indicated. As will be seen, its electrical state corresponds to that of a rod circularly magnetized.

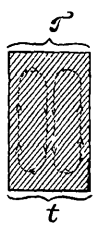

Fig. 29.—Thermo-currents evolved in a hard thick cylinder.

For the purpose of studying this question experimentally, a steel cylinder, 30 millimeters in diameter and 50 millimeters long, was turned and glass-hardened. This was placed vertically directly before the needle of a magnetometer (the deflection of which could be read off with telescope and scale) in such a manner that the position of equilibrium of the needle was left unaltered. The relative position of the needle and cylinder, in other words, was such that the axis of the former, if prolonged, would intersect the axis of the latter if prolonged at its middle point. Upon now cooling the

[203] Magnus: Pogg. Ann., LXXXIII, p. 469, 1851; Sir W. Thomson, Phil. Trans., III, p. 722, 1856.

[204] Mousson: N. Denkschr. d. Schwz. Gesell., XIV, 8, p. 1-90; Chwolson: Carl's Rep., XIV, p. 15.

upper end of the cylinder with a piece of ice, or warming by projecting a jet of steam against it, very decided deflections were observed toward the right or left, respectively, which increased with the difference of temperature $T-t$, and vanished as this difference became nil.

As the cylinder was not magnetic, it is improbable that these phenomena can be referred to a change in the state of magnetic distribution. With reference to the direction of the currents, however, no simple results could be arrived at.

(*e*). *Relative thermo-electric qualities of soft steel and soft iron.*—In § 7, *c*, we ascribed the very high value of the T. E. H. of a glass-hard steel rod to the large proportion of chemically combined carbon contained therein. If this be true, the thermo-electric difference between soft steel and soft iron, in both of which combined carbon is either wholly absent or exists only in traces, must be quite small. This inference is supported by the data actually found for soft iron. Making allowance for the difference of circumstances involved, the result to be derived from the experiments of Kohlrausch and Ammann also agrees sufficiently herewith.

On the other hand, Joule[205] has long since shown that ordinary cast-iron is thermo-electrically negative towards copper, all the more, therefore, towards soft steel—a result which I should be inclined to predict from the quantity of combined carbon contained.

The minimum values of T. E. H. (obtained by cooling the red-hot bar as slowly as possible) of different kinds of steel[206] and of soft iron are approximately the same; whereas the maximum values of this constant (obtained by cooling the highly heated bar as rapidly as possible) differ enormously; this difference being a direct function of the quantity of carbon contained.

(*f*) *Thermo-electric effect of magnetization.*—Sir William Thomson[207] has shown that in a thermo element, consisting of magnetized and unmagnetized steel of the same hardness and form, thermo-currents due only to magnetic difference are generated. The direction of these currents was found to be in one case from unmagnetized to longitudinally magnetized through warm, in another from transversely magnetized to unmagnetized through warm, therefore also from transversely to longitudinally magnetized through warm.

For the purpose of informing myself of the magnitude of the thermo-electric difference thus produced, the following experiment was made: A soft rod (I, Table II) was tested for its electro-motive force when combined with copper (as described above, p. 348), and the locus of the

[205] Joule: Phil. Mag. (4), XV, pp. 538–'39, 1857.

[206] Steel is here used as distinguished from iron only by containing a greater proportion of carbon. No attention has been paid in this paper to the effects of P, S, Si, Mn, etc., so often present in both.

[207] Thomson: Phil. Trans. III, pp. 722–'7, 1856.

equation $E\tau = a\tau - \beta\tau^2$ constructed from *fifteen* very carefully made observations.

Hereupon a large magnetic battery, weighing 40 pounds, was so placed that each of the ends of the horseshoe touched a receiver. The distance between the poles of the magnet and the corresponding ends of the steel rod was thus about 50 millimeters. A second series of fifteen observations was now made. Upon comparing the locus of the latter with that that of the former, the two curves coincided so completely that no influence could be discerned.

Herefrom I conclude that the thermo-electric effect due to difference of magnetic state may, in comparison with that which can be produced by a difference of hardness, be completely neglected.

(*g*) *Relation between T. E. H. and density of steel.*—A very curious analogy was found in comparing the results at which Fromme[208] arrived, in studying the specific gravity of differently tempered steel, with the T. E. H. of similar rods as found in my experiments. Dr. Fromme, if I infer correctly, limited his experiments to rods whose diameter was greater than 2 millimeters, and less than 7 millimeters. His results are contained in the following table, the volume of the soft bar being put = unity.

TABLE IX.

	Volume.	Volume-increase.	T. E. H.
Soft	1.000	0.000	0.000 000
Annealed blue	1.002	0.002	0.000 024
Annealed yellow	1.005	0.005	0.000 042
Glass-hard	1.010	0.010	0.000 117

In the fourth column the T. E. H. of rods cited in Table III. (these, as I believe, corresponding very closely to those for which the data of Fromme apply) is added. If we take into consideration that the results were obtained from different material, the parallelism observable is striking. To the very large difference between glass-hard and yellow annealed, when compared with the much smaller difference between yellow and blue, blue and soft, as seen in both observations, I would once more call attention. The fact that glass hard rods can be considerably annealed at comparatively low tempeartures must be regarded as an adequate indication of their unnatural (strained) condition.

A second result of Fromme, that the specific gravity of thin rods suffers greater loss by glass-hardening than that of thick rods, also harmonizes with the conclusions drawn in § 7 (*b*) with reference to the T. E. H. of such rods.

[208] Fromme: Götting. Nachr. p. 165, 1876.

8.—HARDNESS AND SPECIFIC RESISTANCE OF STEEL: DEDUCTIONS.

From the data for the specific resistance of rods of different hardness inferences analogous to the above may be deduced.

(*a*) The specific resistance of steel increases continuously with its mechanical hardness.

(*b*) Rods like annealed differ but slightly, glass-hard rods considerably, with respect to their specific resistance.

(*c*) *Relation between the thermo-electric hardness and the specific resistance of steel.*—On comparing the values found for the ratio $\Delta S_0 / \text{T. E. H.}$, we infer that the specific resistance of steel is approximately a linear function of its thermo-electric hardness. In Fig. 30 these results are graphically represented.

I will remark, however, that the assumption of proportionality as based on the above figures is to be regarded as a first approximation

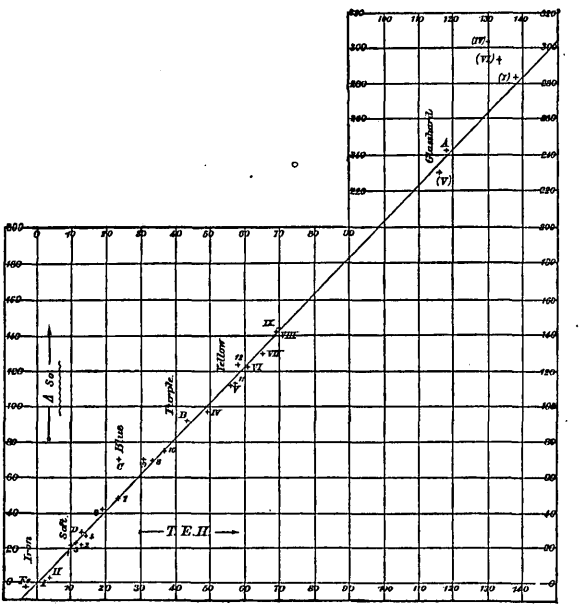

FIG. 30.—Relation between thermo-electric power and specific resistance.

only, notwithstanding the fact that the discrepancies fall within the errors of experiment. A rigid discussion of the latter comes more appropriately within the scope of another paper soon to appear. In this place I would only call attention to the following: In the thick bars

experimented upon the value of $\Delta S_0 / \text{T. E. H.}$ is usually too large; a fact which is easily accounted for, as the unavoidable resistances of contact above referred to will in this case have a relatively great effect, the resistances of the bars themselves being very small. In the rods included in Table VI it was impossible to secure uniform redness of ignition throughout; the ends invariably remained darker. As T. E. H. depends principally on the warm end, ΔS_0, however, on the mean hardness of the whole bar, we have thus a second cause for an overlarge ratio.

So much I think to have fully established, that T. E. H. and specific resistance throughout their variations are very simple functions one of another. T. E. H. and specific resistance must therefore be looked upon as effects of the same cause, as *phenomena having some very intimate connection.*

(*d*) Particular attention must here be called to the remarkable result, that the specific resistance of steel can by a process of glass-hardening be increased to *nearly three times* its value in soft steel.[209] As this datum far exceeds that determined by Mousson (about 25 per cent.), it is not without some hesitation that I make it public. The care bestowed on the experiments, however, together with the regularity observed in the variation of the results, I believe sufficiently insure their correctness. See, moreover, § 6, E.[210]

(*e*) As deserving special notice, I will further add that the thermo-current always passes from the bar with greater to the bar with less resistance.[211] The few exceptions to this fact in the tables were afterwards found to be referable to errors of experiment by direct observation.

(*f*) Like T. E. H., so also the specific resistance of steel approximates to the value of this constant for soft iron. Upon the value found for

[209] It is to be observed, however, that the difference between the specific resistance of steel in the soft and hard states is dependent on the composition, increasing with the quality of carbon contained from a very small value in soft iron to the very large value above announced for steel.

[210] Chwolson reports the increase of resistance due to glass-hardening to be only 0.6 per cent. This I can only explain by supposing the results of this observer to have been obtained with wires suddenly cooled at a temperature below that referred to in § 7 (*c*).

[211] I would here again refer to the fact that, according to Magnus, Thomson, and Mousson, drawn-steel wire and hard-tempered steel are on opposite sides of soft steel, both with respect to their thermo-electric properties and their specific resistance. Thomson furthermore finds transversely magnetized steel electro-negative towards soft steel, this again towards longitudinally magnetized steel. Auerbach (Wied. Ann., V, p. 316, 1878), analogously, that the specific resistance of hard steel continuously increases as the rod passes from a condition of saturated longitudinal to a condition of saturated circular magnetization. The result above enunciated has therefore probably even a more general signification.

the ratio $\Delta S_0 / \text{T. E. H.}$, however, not much reliance can be placed, the factors involved being too small to admit of accurate determination.

With cast-iron no experiments were made.

9.—REMARKS ON THE ABOVE, CONSIDERED AS AUXILIARY TO THE DETERMINATION OF THE RELATION BETWEEN HARDNESS AND MAGNETIC MOMENT.

In this place I will avail myself of the experiments of Ruths on the relation between hardness and magnetic moment, these being perhaps the most comprehensive. With sufficient approximation for the purpose I will put the T. E. H. of glass-hard rods $= 120 : 10^6$; that of the yellow annealed, $= 40 : 10^6$; of the blue, $= 20 : 10^6$; and with these as abscissæ and Ruths' values for the corresponding permanent magnetic moments ("in millions of absolute units," milligrammes-millimeters-seconds) as ordinates, suppose the curves belonging to each of the bars to be constructed. These curves are given in figure 31, the attached number referring to the ratio between the length and the diameter of each rod.[212]

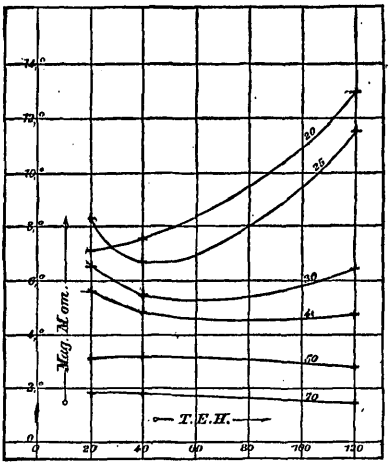

Fig. 31.—Diagram of Ruths' results.

I will omit the interesting deductions which Ruths makes from his data, these going beyond our present purpose.

From an inspection of the curves we derive the following important results, viz: that, like the electrical properties of steel and its specific gravity, so also the maximum of permanent magnetism is largely modified by the different degrees of glass-hardness of the bar (*i. e.*, those lying above yellow annealed, and scarcely distinguishable mechanically). In the second place, the results of Ruths, being obtained from comparatively thick rods, are largely influenced by structure. In view of these facts, I deem the hope by no means too sanguine that if, to avoid complications from structure, we experiment on thin rods, the maximum of permanent magnetism may be empirically expressed in terms of the dimensions and T. E. H. only; that, furthermore, from the parallelism discovered in the variation of specific gravity and the electrical prop-

[212] The data employed are those obtained by Ruths for rods 120 millimeters long, and 1.7, 2.4, 2.9, 3.8, 4.9, 5.9 millimeters, respectively, in diameter, with very powerful magnetizing forces.

erties of steel, the T. E. H., the specific resistance, and the magnetic moment are in some very intimate way connected with the volume of a unit of mass. This would imply a connection of these phenomena with the intermolecular spaces of steel.

In conclusion, it gives me great pleasure to acknowledge my indebtedness to Professor Kohlraush for much kind assistance throughout the course of the experiments.

PHYSICAL INSTITUTE, UNIVERSITY OF WÜRZBURG,
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