Determination of Quantitative Geologic Data with Stereometer Type Instruments

By WILLIAM R. HEMPHILL

PROCEDURES AND STUDIES IN PHOTOGEOLOGY

GEOLOGICAL SURVEY BULLETIN 1043-C

A discussion of the applications and limitations of stereometer-type instruments as applied to geologic problems

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1958

UNITED STATES DEPARTMENT OF THE INTERIOR

FRED A. SEATON, *Secretary*

GEOLOGICAL SURVEY

Thomas B. Nolan, *Director*

For sale by the Superintendent of Documents, U. S. Government Printing Office
Washington 25, D. C. - Price 35 cents (paper cover)

CONTENTS

ILLUSTRATIONS

PROCEDURES AND STUDIES IN PHOTOGEOLOGY

DETERMINATION OF QUANTITATIVE GEOLOGIC DATA WITH STEREOMETER-TYPE INSTRUMENTS

By William R. Hemphill

ABSTRACT

A stereometer-type instrument is a photogrammetric measuring device used to determine differences in elevation between features as seen on vertical aerial photographs. The operation of stereometer-type instruments and some of their uses in obtaining quantitative geologic data are described. Measurement of stratigraphic thickness and strike and dip are emphasized because such measurements not only demonstrate general stereometer procedures but also are basic in many geologic field-mapping programs. Necessary photogrammetric terms, normally not familiar to many geologists, are defined and their use explained.

INTRODUCTION

Quantitative geologic data often may be obtained quickly and reliably with stereometer-type instruments from overlapping vertical aerial photographs. Where evidence of rock units is discernible on aerial photographs, these instruments may aid in determining strike and dip, in obtaining amount of fault throw, in measuring stratigraphic thickness, and in obtaining the difference in elevation between selected features of topographic significance.

The purpose of this paper is to discuss the procedures for applying stereometer-type instruments to geologic problems. The advantages as well as the limitations of these instruments are discussed in some detail. Only vertical aerial photographs are considered.

Photogrammetric terms as used in this paper are defined as follows:

Parallax:—Apparent displacement of an object with respect to a point of reference owing to a shift in the point of observation (Am. Soc. Photogrammetry, 1952, p. 829).

Stereoscopic pair:—"Two photographs of the same area taken from different camera stations so as to afford stereoscopic vision" (idem, p. 838).

Stereoscope:—"An optical instrument for assisting the observer to view two * * * photographs" in order "to obtain the mental impression of a three dimensional model" (idem).

Stereoscopic model:—The impression of three dimensions received when viewing a stereoscopic pair beneath a stereoscope.

Image point:—Point at which the image of an object on the ground appears on one photograph of a stereoscopic pair. Images of the same object on both photographs making up the stereoscopic pair are referred to as conjugate images or conjugate image points.

Stereoscopic image point:—Point at which the image of an object on the ground appears in the stereoscopic model.

Fiducial mark:—Reference mark imprinted midway along each margin of an aerial photograph at the time of exposure.

Photograph center:—Point located at the intersection of lines drawn between fiducial marks on opposite margins of the photograph. For the purposes of this paper, the photograph center may be considered to be coincident with the plumb point.

Transferred center:—Point on a photograph at which the photograph center of the adjacent photograph of the stereoscopic pair appears.

Photobase:—Distance on one photograph between the photograph center and transferred center.

Camera focal length:—Distance measured along the lens axis between the nodal point of the lens and the plane of best average definition (positive plane or negative plane) (idem, p. 820).

General photograph scale:—Photograph scale as stated on the photograph or photograph index.

Applicable photograph scale:—Scale at a particular datum in the stereoscopic model as determined from a reliable base map or from field control.

Tilt:—Angle of departure, expressed in degrees, of the camera axis from true vertical.

Relief exaggeration:—Apparent exaggeration of the vertical scale with respect to the horizontal scale in the stereoscopic model.

Relief displacement:— The horizontal linear distance an image point is displaced on the photograph radially from the plumb point, owing both to the difference in elevation between that image point and the plumb point and to the horizontal distance between that image point and the plumb point. For the purposes of this paper, the plumb point may be considered coincident with the photograph center.

WORKING PRINCIPLE OF STEREOMETER-TYPE INSTRUMENTS

A stereometer-type instrument is used to measure the difference between the absolute parallax of two or more stereoscopic image points of different elevations. Absolute parallax of a stereoscopic image point may be defined as the algebraic difference of the distances measured parallel to the photobase between the two image points and their respective photograph centers. The distance measured to the right of the photograph center is positive; the distance measured to the left of the photograph center is negative. This definition assumes that the photographs making up the stereoscopic pair are truly vertical and identical in scale. Tewinkel (1952, p. 325) expresses absolute parallax algebraically:

$$p=x-x'$$

where p is the absolute parallax, x is the distance from the photograph center to the image point on the left photograph, and x' is the corresponding distance on the right photograph; both distances are measured parallel to the photobase. For example, in figure 13

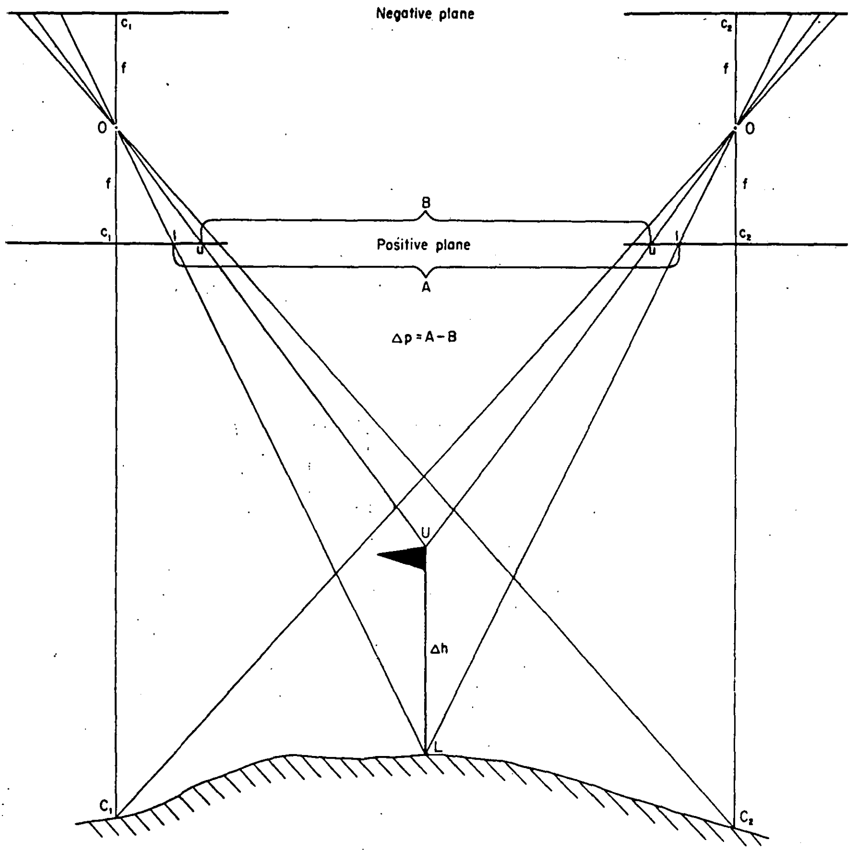


FIGURE 13.—Diagram showing difference in elevation (Δh) on ground expressed as difference in parallax (Δp) on photographs.

- c_1 Left photograph center.
- c_2 Right photograph center.
- f Camera focal length.
- O Camera lens or theoretical position of eyes when viewing stereoscopic model.
- L Bottom of flagpole in nature.
- l Image of the bottom of the flagpole on the photographs.
- U Top of flagpole in nature.
- u Image of the top of the flagpole on the photographs.
- A Distance between conjugate image points l.
- B Distance between conjugate image points u.

The difference between distances A and B is the change in parallax (Δp) between the top and bottom of the flagpole.

absolute parallax ($p=x-x'$) of the top of the flagpole (U) can be shown:

$$p=c_1u-(-c_2u)$$
$$p=c_1u+c_2u$$

Similarly, the absolute parallax of the bottom of the flagpole can be shown:

$$p=c_1l-(-c_2l)$$
$$p=c_1l+c_2l.$$

Determining differences in absolute parallax with stereometer-type instruments is based on the so-called "floating-dot" principle. A target consisting of a dot, cross, or small circle scribed on transparent plastic or glass is mounted on each end of a supporting bar as shown on plate 1. The horizontal distance between the dots can be varied by manipulating a reading drum graduated to either hundredths millimeter or thousandths inch. When the horizontal distance between the dots equals the distance between conjugate image points on a properly alined stereoscopic pair (fig. 14), the dots will fuse and

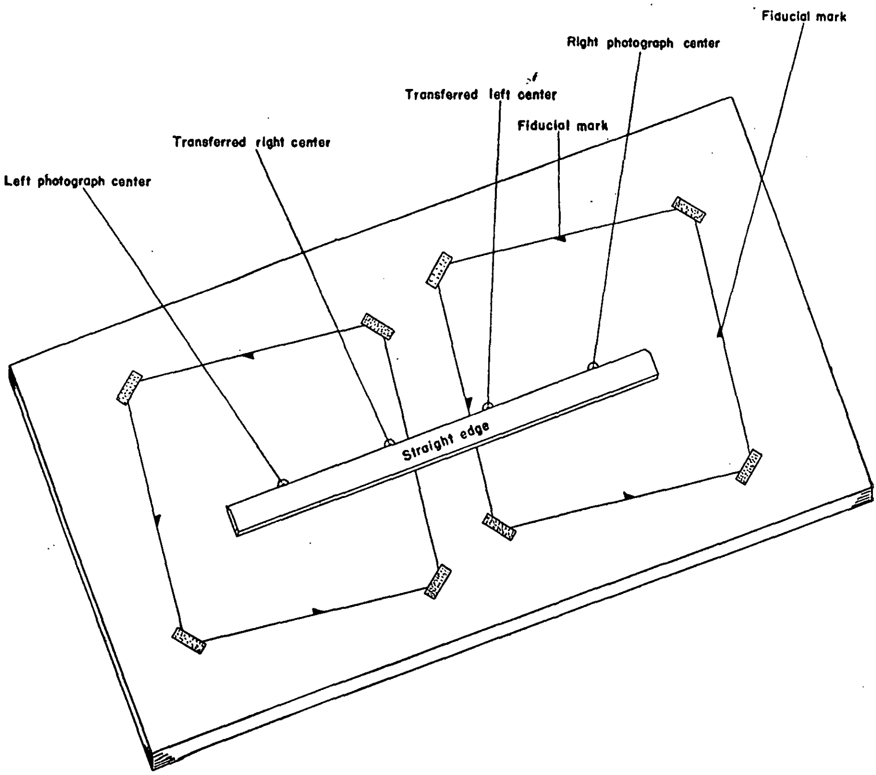
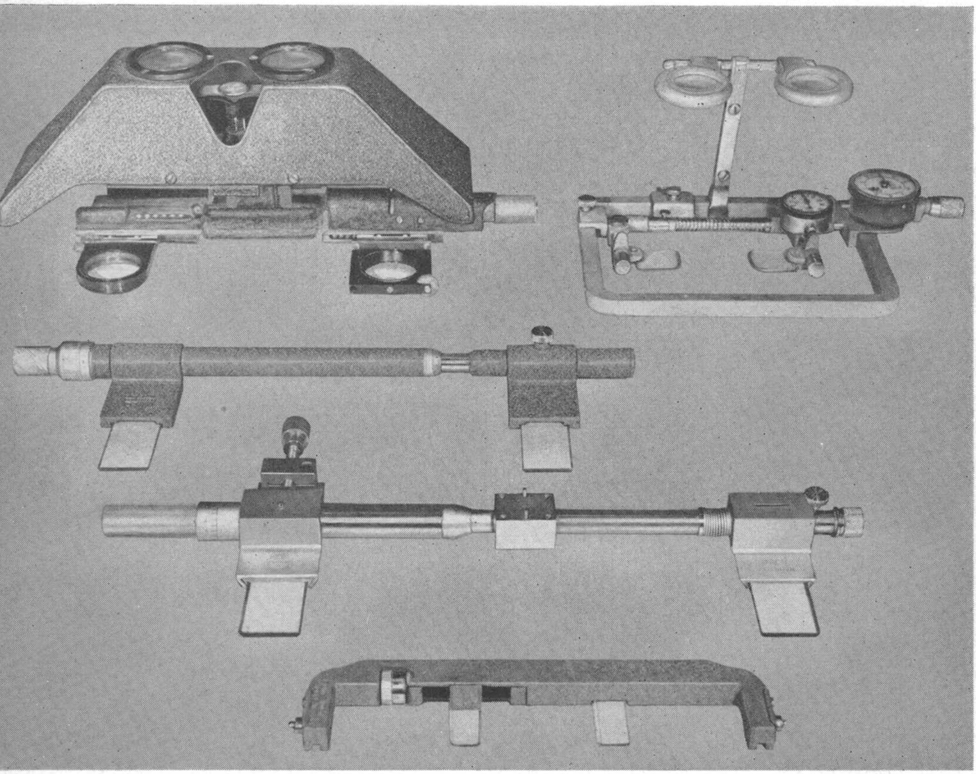


FIGURE 14.—Properly alined stereoscopic pair.



STEREOMETERS USED BY THE GEOLOGICAL SURVEY appear to "float" in the stereoscopic model at the elevation of the conjugate image points or stereoscopic image point in question (distance A, fig. 13). The reading on the drum is noted and the procedure may be repeated at a stereoscopic image point of different elevation (distance B, fig. 13). The difference between these readings is the change in absolute parallax or simply the parallax change (Δp) between the two stereoscopic image points. Thus,

$$\Delta p = (c_1 l + c_2 l) - (c_1 \mu + c_2 \mu) = \mathrm{A} - \mathrm{B}.$$

A mathematical relationship exists between this parallax change (Δp) and change in elevation (Δh) between the two points on the ground. This relationship is not affected by relief exaggeration; that is, the amount of measured parallax change (Δp) between stereoscopic image points of different elevation is not exaggerated with respect to the change in elevation (Δh) on the ground.

FORMULAS USED TO CONVERT CHANGE IN PARALLAX TO CHANGE IN ELEVATION

A simple formula for converting change in parallax to change in elevation is as follows:

$$\Delta h = \frac{H}{b} \Delta p \tag{1}$$

where Δh is the change in value of h or the difference in elevation between two stereoscopic image points, b is the average of the photobases of the left and right photographs, and H is the height of the airplane above the ground based upon camera focal length and general photograph scale. This formula is commonly incorrectly applied. It will provide satisfactory measure of Δh in areas of low relief; if applied without discrimination to all types of topography, considerable error may result because the numerator, H, and denominator, b, as used in the formula are only approximations.

The numerator, H, is commonly found by dividing camera focal length by general photograph scale. This scale may be locally inaccurate; for example, photographs of an area in the plateau country of the Western United States with a general photograph scale of 1:20, 000 have been found to have an applicable photograph scale of 1:29,000 at a particular datum in the stereoscopic model, owing either to variations in the flying height of the airplane, to extreme topographic relief of the photographed area, or to a combination of both. If a 6-inch focal-length lens were used with a general photograph scale of 1:20,000, H would have been computed to be 10,000 feet. The applicable photograph scale of 1:29,000 should be used, in which case H would be computed correctly at 14,500 feet. The 30-percent error in H in this example is extreme, but 10-percent error in using H as the numerator is common and will cause similar error in the resulting value of Δh. Therefore, H should be based upon the applicable photograph scale as determined from a reliable base map or from field control.

Reliable base maps from which to determine applicable photograph scale at a particular datum may not be available. This poses no problem for some types of quantitative geologic determinations. A strike and dip, for example, does not require absolute differences in elevation; any convenient scale can be used as long as this same scale is used to calculate both the vertical and horizontal components of the tangent of dip. Also, where relative differences in elevation are required, such as for comparison of relative stratigraphic thickness, and base maps or field control are not available, appropriate steps may be taken to obtain the most reliable results. These steps are discussed on page 54.

The other factor that can be responsible for appreciable error in Δh is b, the average of the photobases of the left and right photographs.

If the elevation of the lower of the two stereoscopic image points, between which the difference in elevation is being measured, is approximately midway between the elevations of the left and right photograph centers, an average of the photobases will be adequate for most geologic measurements. Likewise, in areas of low relief the error incurred will generally be insignificant if the lower point is at an elevation other than approximately midway between the elevations of the two photograph centers. However, in areas of extreme relief the error in using the average photobase (b) as the denominator commonly exceeds 5 percent and will cause an approximately equivalent percentage of error in Δh.

Thus, formula (1) should be used to provide rapid computation of Δh for areas of low topographic relief only; significant error may result when this formula is applied to rugged terrain. To compensate for the effect of high relief on the measurement of Δh, the following formula (modified from Tewinkel, 1952, p. 325–326) should be used:

$$\Delta h = \frac{H'}{ab + \Delta p} \Delta p \qquad (2)$$

where H' is the height of the camera above the lower of the two stereoscopic image points between which an elevation difference is desired, and ab is the photobase adjusted to the photograph scale at the lower stereoscopic image point. Factor H' is found by applying the following formula:

$$H' = \frac{f}{S} \qquad (3)$$

where f is the camera focal length in feet and S is the applicable photograph scale determined from a reliable base map or field control at or near the elevation of the lower stereoscopic image point.

Adjusted photobase (ab) may be found by the following method:[1]

1. Orient the photographs (fig. 14).
2. With a millimeter scale, measure the distance between the two photograph centers.
3. Similarly, measure the distance between the conjugate images of the lower of the two stereoscopic image points between which an elevation difference is desired.
4. Subtract the distance obtained in step 3 from the figure obtained in step 2 (round off to nearest millimeter). The difference is the photobase adjusted to the photograph scale at, or the elevation of, the lower stereoscopic image point.

Where several differences in elevation are to be determined in the same stereoscopic model, the necessity of measuring with a scale, for each calculation, the distance between the lower conjugate image points in step 3 above may be omitted by using the following procedure:

1. Orient the photographs (fig. 14).
2. With a millimeter scale, measure either of the two photobases of the stereoscopic pair to the nearest half millimeter.
3. With a stereometer, measure Δp to the nearest half millimeter between the lower stereoscopic image point and the photograph center of the photograph whose photobase was not measured in step 2.
4. If the lower stereoscopic image point is below the photograph center used in step 3, subtract (and round off to nearest half millimeter) the figure obtained in step 3 from the figure obtained in step 2. If the lower stereoscopic image point is above the photograph center used in step 3, add (and round off to nearest half millimeter) the figure obtained in step 3 to the figure obtained in step 2. The result is the photobase adjusted to the elevation of the lower stereoscopic image point.[2]

Both the photobase obtained in step 2 and the stereometer reading on the photograph center in step 3 should be recorded and used in determining adjusted photobase for all additional computations of difference in elevation in the same stereoscopic model. Because the stereometer reading at the lower stereoscopic image point in step 3 is made in the course of determining Δp between the lower and upper stereoscopic image points, it does not represent an additional measurement in obtaining the adjusted photobase.

[1] Modified from Instruction manual for the stereo elevation meter, 1954, Photogrammetry, Inc., Silver Spring, Md., p. 2.

[2] This method for obtaining adjusted photobase is similar to a method described by Elliot (1952, p. 10).

An adjusted photobase need not be computed where the lower of the two stereoscopic image points appears at or near the elevation of one of the photograph centers; the photobase of the adjacent photograph of the stereoscopic pair will suffice for the adjusted photobase. Similarly, where the lower of the two stereoscopic image points appears at a position approximately midway between the elevations of the two photograph centers, the average of the two photobases may be used for the adjusted photobase.

It will be noted that factor Δp has been added to the denominator in formula (2). Δp is generally very small compared to factor ab; therefore, the error in Δh is not generally appreciable if Δp is omitted. Nevertheless, the ease of computing factor $(ab+\Delta p)$ to the nearest half millimeter makes including it desirable. If Δp is comparatively large, say more than 2 millimeters, Δp should be included in the denominator if the most reliable results are to be obtained.[3]

To facilitate computation of Δh, part of this formula has been reduced to a variable called V so that

$$V=\frac{H'}{ab+\Delta p} \qquad (4)$$

As

$$\Delta h=\frac{H'}{ab+\Delta p}\,\Delta p,$$

$$\Delta h=V\Delta p. \qquad (5)$$

V may be determined rapidly by referring to a graph (fig. 15) so constructed that V varies as a function of H' and $(ab+\Delta p)$. V as determined from this graph represents the number of feet on the ground for each millimeter of parallax change (Δp) in the stereoscopic model. Some of the flight altitudes (H') for photograph scales of 1:12,000 to 1:50,000 are included. These are scales of photographs commonly used by geologists. Additional flight altitudes may be added when other photograph scales are met. A similar graph may be constructed when $(ab+\Delta p)$ and V are required in terms of inches.

An inexpensive photogrammetric computer, currently manufactured by a commercial company, may also be used to determine V. This computer is particularly advantageous because it may be operated like a circular slide rule to multiply change in parallax (Δp) by the feet per unit of parallax (V). In addition, the computer may be used as an aid in determining photograph scale (S), aerial camera altitude (H), and aerial camera focal length (f).

[3] Inasmuch as factor $(ab+\Delta p)$ is equivalent to photobase adjusted to the upper stereoscopic image point, a small amount of calculation could be avoided by adjusting photobase to the upper stereoscope image point instead of the lower. However, in keeping with normal practice, adjusted photobase is computed in the present paper with respect to the lower stereoscopic image point.

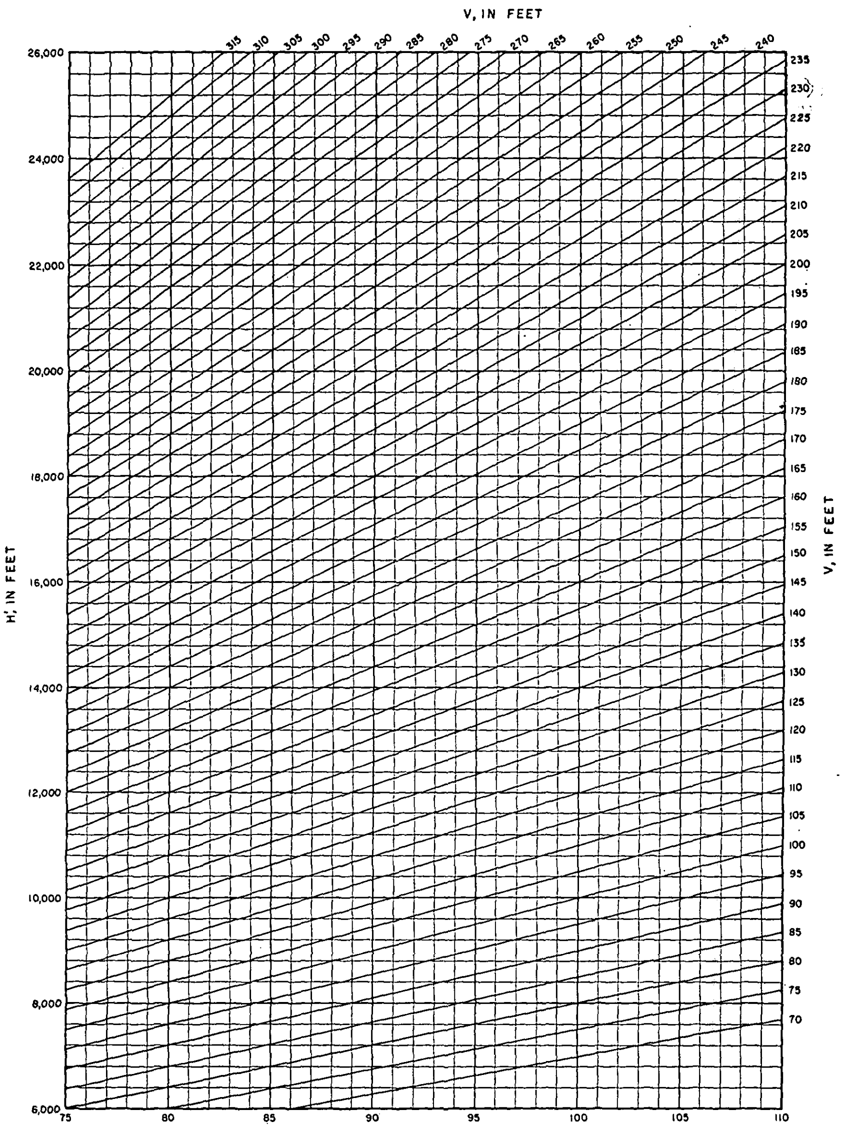


FIGURE 15.—Graph showing V varying as a function of H' and $(ab+\Delta p)$.

To illustrate the use of the formula (2) in an area of high relief, a sample problem is presented below. An extreme example was purposely chosen in order to emphasize the fact that large errors may result from indiscriminate use of formula (1).

Problem:

A stereoscopic image point has been located on each of two flat-lying beds cropping out in an area of rugged relief. The photobase of the right photograph is 92.5 millimeters; the photobase of the left photograph is 97.5 millimeters. When the dot is "floated" on the lower and upper stereoscopic image points and the left photograph center, the stereometer reads 30.50, 28.30, and 21.30 millimeters, respectively. (Both stereoscopic image points are at lower elevation than the left photograph center.) Applicable photograph scale at the lower stereoscopic image point is 1:23,400, and the photographs were taken with a 6-inch focal-length lens. What is the thickness of section between the two beds?

Solution:

Applying formula (3) to find H':

$$H'=\frac{f}{S}$$

$$=\frac{0.5\text{ ft}}{\frac{1}{23{,}400}}$$

$$=11{,}700.$$

Adjusted photobase is found by subtracting the difference in parallax (Δp) between the left photograph center and the lower stereoscopic image point from the photobase of the right photograph. (Adjusted photobase could also be found by subtracting the difference in parallax (Δp) between the right photograph center and the lower stereoscopic image point from the photobase of the left photograph.)

92.5 mm	Photobase of right photograph
9.2 mm	Δp between left photograph center and lower stereoscopic image point
83.3 mm	Adjusted photobase (adjusted to lower stereoscopic image point).

Applying formula (2) to find thickness of the stratigraphic section (Δh):

$$\Delta h = \frac{H'}{ab+\Delta p}\,\Delta p$$
$$= \left(\frac{11{,}700}{83.3+2.2}\right) 2.2$$
$$= \left(\frac{11{,}700}{85.5}\right) 2.2.$$

From the reference graph (fig. 15):

$$V = 137$$
$$\Delta h = V\Delta p \qquad (5)$$
$$= 137 \times 2.2 \text{ mm}$$
$$= 301 \text{ ft.}$$

For comparison, formula (1) is applied to the same problem. General photograph scale is 1:20,000.

Applying formula (3) to find H:

$$H = \frac{f}{S}$$ (Here, S is the general photograph scale.)

$$= \frac{0.5}{\frac{1}{20{,}000}}$$
$$= 10{,}000.$$

Average photobase: 95 mm.

Applying formula (1) to find Δh:

$$\Delta h = \frac{H}{b}\,\Delta p$$
$$= \left(\frac{10{,}000}{95}\right) 2.2$$
$$= 231 \text{ ft.}$$

Errors in both the numerator (H) and the denominator (b) are not necessarily mutually compensating. Neither is the sum of their errors necessarily the percentage of error to be found in the result; for example, a 10-percent error in both numerator and denominator may not mean a 20-percent error in Δh. In application of formula (1) to the problem above, a 15-percent error in the numerator (H) and an 11-percent error in the denominator (b) caused a 23-percent error in the result (Δh).

TILT

Tewinkel (1952, p. 319) estimates that with modern-day methods, 90 percent of vertical aerial photographs are tilted less than 2° and 50 percent are tilted less than 1°.

The method of detection of such small amounts of tilt must be commensurate with the accuracy of the instrument used to measure parallax change (Δp). Double-projection stereoplotting instruments are constructed to correct mechanically for tilt; stereometers, however, are not so constructed and depend solely upon the alertness and understanding of the operator to notice manifestations of tilt as they occur in the stereoscopic model.

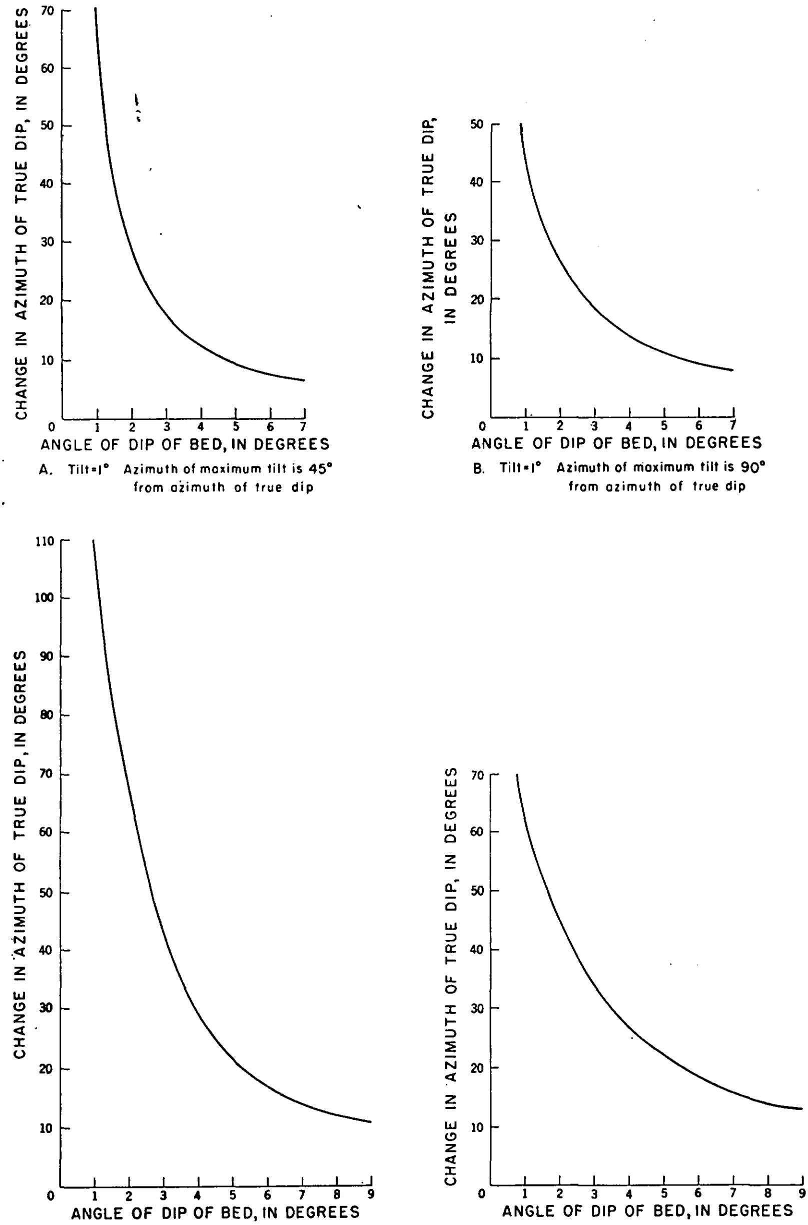
Tilt manifests itself in the stereoscopic model in several ways. If applicable photograph scales of several image points, all at the same elevation, are computed from a topographic base map, the scale throughout the tilted stereoscopic model will be different for points of equal altitude. Also, streams may appear to flow uphill or their gradients may appear too steep or not steep enough. Meandering rivers or swamps, features that should be level or nearly level, may appear inclined in the stereoscopic model. Strike of low-dipping strata may appear to be different when the same area is viewed on adjacent stereopairs. These and additional geomorphic manifestations of tilt are more easily detected and interpreted with the aid of a mirror-type stereoscope than with a lens-type stereoscope. The mirror type has a large field of view and local evidence of tilt can be viewed with respect to the stereoscopic model as a whole. The lens type has a comparatively small field of view. Also, stereoscope distortion of mirror-type stereoscopes is negligible, whereas, with some lens-type stereoscopes, distortion produces a noticeable "bowllike effect" in the stereoscopic model. For example, flat terrain would appear inclined inward toward the center of the field of view, thus complicating and perhaps obscuring evidence of any tilt that may be present.

Tilt of two photographs comprising a stereoscopic pair is seldom the same in either amount or direction; therefore, tilt in a stereoscopic pair must be considered in terms of two independently tilted photographs whose net effect in the stereoscopic model produces (1) an inclined surface or datum and (2) an unevenness or warpage of this inclined surface or datum. For example, different points on the shoreline of a large lake may have several hundredths millimeter of parallax change in the stereoscopic model owing to the warping effect of tilt, in addition to possibly several millimeters of parallax change owing to tilt's physical inclination of the shoreline.

Usually, when computing stratigraphic thickness, these effects of tilt can be sufficiently minimized by choosing positions of stereoscopic image points not more than 1 inch apart on photographs of say, 1:20,000 scale (a correspondingly shorter distance should be used on photographs of smaller scale). In this manner the effect of tilt will be much the same on all points involved. Stratigraphic thicknesses should not be computed between stereoscopic image points located on opposite sides of the stereoscopic model unless control is available or the vertical distance to be measured is large and only an approximate value is required. Only 1° of tilt in photographs of 1:20,000 scale could result in an error of as much as 280 feet if the upper and lower stereoscopic image points were located on opposite sides of the stereoscopic model.

When determining strike and dip, the most serious effect of tilt is on the azimuth of true dip of strata dipping less than 6° or 7°. Tilt in these circumstances can swing the azimuth of true dip in the stereoscopic model, the amount of swing depending upon the direction and amount of tilt relative to the direction and amount of true dip (fig. 16). By being aware of this effect of tilt when working with low-dipping strata, one can take the precaution of carefully inspecting the stereoscopic model for whatever geomorphic manifestations of tilt may be present. Where low-dipping strata crop out as dip slopes, their true dip may be nearly the same as the direction and gradient of intermittent streams flowing on their surfaces. Where this is so, one should suspect presence of tilt if the direction of true dip appears to be transverse or opposite to the direction of streamflow. The direction of true dip, when recorded on the photograph or base map, should be adjusted with respect to the stream or other geomorphic indications of tilt that may be present. Where stream gradients are steep or where meandering rivers, lakes, or swamps are absent, it is difficult to detect evidence of tilt in the stereoscopic model; consequently, azimuth of true dip of low-dipping beds in these areas should be regarded with suspicion. The effect of tilt on strata dipping more than 10° probably is not significant for most geologic problems.

Tilt may be inadvertently introduced into the stereoscopic model by careless alinement of the stereoscopic pair. To demonstrate this point, it is suggested that two photographs of a stereoscopic pair be rotated or skewed in opposite directions beneath a stereoscope, preferably of mirror type, with the photograph centers functioning as the axes of rotation. The stereoscopic model will be observed to tilt alternately away from, then toward the viewer, as the direction of rotation is changed. Because of this tilting effect, it is important that the photograph centers and the transferred centers be positioned accurately. The transferred centers should be located stereoscopically and not with respect to surrounding conjugate images observed with the naked eye. The photographs should be alined in such a



A. Tilt=1° Azimuth of maximum tilt is 45° from azimuth of true dip

B. Tilt=1° Azimuth of maximum tilt is 90° from azimuth of true dip

C. Tilt=2° Azimuth of maximum tilt is 45° from azimuth of true dip

D. Tilt=2° Azimuth of maximum tilt is 90° from azimuth of true dip

FIGURE 16.—Graphs showing effect of tilt on low-dipping beds.

manner that the two centers and two transferred centers are in a straight line (fig. 14). The largest amount of image displacement due to tilt is in the extreme corners of the photographs; consequently, determination of quantitative data should be avoided, if possible, in the corners of the stereoscopic model.

Camera-lens distortion and stereoscope-lens distortion probably have no significant effect on stereometer measurements. The effect of differential shrinkage of film and photograph print paper may be minimized somewhat by positioning stereoscopic image points, between which differences in elevation are being measured, as close together as possible. Vertical exaggeration and magnification have no distorting effect on differences in elevation (Δh) as determined from stereometer measurements.

MEASUREMENT OF STRIKE AND DIP

To determine the attitude of a bed, field geologists often use the "three-point method." The procedure for determining the attitude of a bed on a vertical aerial photograph is basically the same as this field method except that a stereometer, rather than a barometer or alidade and stadia rod, is used to obtain the differences in elevation (Δh). This procedure takes into account three factors influencing strike and dip determinations on aerial photographs: (1) effect of surface expression of the strata, (2) effect of photograph scale, and (3) effect of relief displacement.

EFFECT OF SURFACE EXPRESSION OF THE STRATA

Where individual beds crop out as dip slopes, strike and dip determinations can be based upon elevations at only two stereoscopic image points located on the azimuth of true dip. The "three-point method" is unnecessary because the azimuth of true dip can be determined by stereoscopic inspection. For example, where strata are well exposed, attitudes of low-dipping beds are easily detected in the stereoscopic model because of an apparent exaggeration of the model's vertical scale relative to its horizontal scale. A dip of 6° may appear to be as much as 20° or more. Moreover, evidence of dip is indicated in the stereoscopic model by the three-dimensional image of each discernible bedding plane both in the local area in which a strike and dip determination is desired and in nearby areas. Stream patterns also may reflect either strike or dip of the strata. In short, azimuth of true dip can be determined by stereoscopic inspection because vertical aerial photographs provide an overall picture of an area and indicate the direction of dip at many places in the stereoscopic model. However, where evidence of bedding is only along canyon walls or hillsides, the three-point solution must be used.

To simplify the discussion of the effect of scale and relief displacement on strike and dip determinations, reference will be made to only two points on a bedding plane chosen along the azimuth of true dip. These points in the stereoscopic model will be referred to as the upper (u) and lower (l) stereoscopic image points.

EFFECT OF PHOTOGRAPH SCALE

Formula (2) should be used for strike and dip determinations in areas of high relief and the photobase should be adjusted to the lower of the two stereoscopic image points on the bedding plane. However, it is not necessary to know the applicable photograph scale at the lower stereoscopic image point; instead, any convenient scale may be chosen. In spite of the fact that the scale may be erroneous, its effect upon the length of the vertical and horizontal distances will be proportional; that is, the percentage of error in the vertical distance (Δh) will be compensated by an equal percentage of error in the horizontal distance. The tangent of the dip in question will remain the same.

EFFECT OF RELIEF DISPLACEMENT

A photograph differs from a map in that horizontal distances cannot be reliably measured directly from the photograph with a rule because the position of any image point does not necessarily correspond to its correct orthographic position. Instead, a point on the photograph may be displaced from its orthographic position, the amount of displacement depending upon its vertical and horizontal distance from the plumb point or point directly beneath the camera lens. (For the purposes of the present paper, the plumb point may be considered as the photograph center.) In figure 17, A–A′ is a horizontal reference datum at the elevation of the photograph center (c). Because point E on the flagpole lies on reference datum A–A′, its image (e) will appear on the photograph in its correct orthographic position with respect to that datum. Points U and L do not lie on datum A–A′; therefore, their images u and l do not appear on the photograph in their correct orthographic positions. Thus, although points U and E occupy the same geographic position on the ground, their images are separated by the distance eu; similarly, although E and L are in the same geographic position, their images are separated by the distance el. These distances are image displacements due to relief of points U and L relative to datum A–A′.

The amount of relief displacement of an image point is directly proportional to the horizontal and vertical distance between that point and the photograph center and inversely proportional to the height of the camera above the center. With reference to a horizontal datum at the elevation of the photograph center, any other point at lower elevation will be displaced toward the photograph center; any

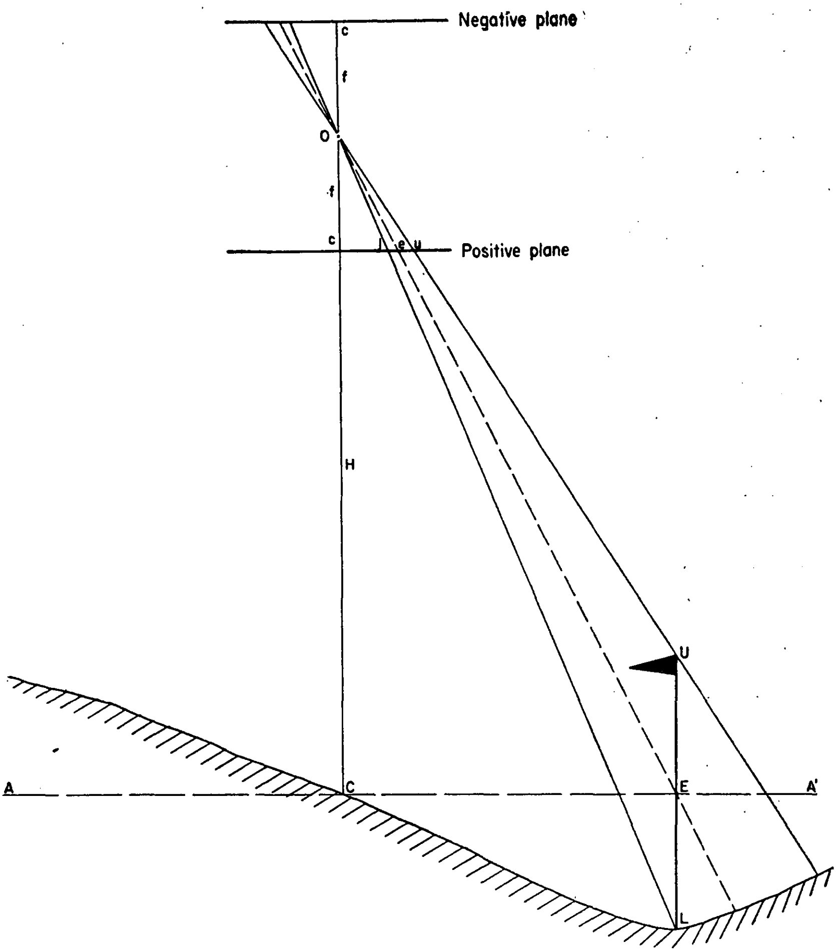


FIGURE 17.—Diagram showing image displacement of a flagpole due to the height of the flagpole.

A-A′ Horizontal datum at elevation of *C*.
L Base of flagpole.
U Top of flagpole.
E Point on ground on datum A-A′.
C Point on ground on datum A-A′ and appearing on the photograph at *c*.
c Photograph center.
l Image point of *L*.
u Image point of *U*.
e Image point of *E*.

Image point *e* is the orthographic position of *U* and *L* on the photograph relative to datum A-A′. Distance *eu* is the relief displacement of image point *u* away from the photograph center, and distance *el* is the relief displacement of image point *l* toward the photograph center.

point at higher elevation will be displaced away from the photograph center.

Thus, an image point will appear on the photograph in its correct orthographic position with respect to a datum passing through the photograph center only if its elevation or its location is coincident with the photograph center. Where an image point is located at any other position, relief displacement, if appreciable, will render any horizontal distances measured relative to that point invalid. Dip measurements based on these horizontal distances will be unreliable.

CORRECTION FOR RELIEF DISPLACEMENT

On plate 2*A* upper and lower points *U* and *L* of dip angle Θ appear on the left and right photographs at *lu* and *ll* and *ru* and *rl*, respectively (pl. 2*A* and *B*). The relief displacement of the upper point *U* relative to the lower point *L* is clearly seen if the discrepancy between the photograph distances *lu*–*ll* and *ru*–*rl* is noted and compared with the corrected or true horizontal photograph distance represented by *ul* on plate 2*C*. A method for correcting this displacement and obtaining true horizontal distance between points *U* and *L* is as follows:

1. Label and mark with a fine dot on each photograph of the stereoscopic pair the following image points: the center (c_1 and c_2), transferred center (*ltc* and *rtc*), upper points (*lu* and *ru*), and lower image points (*ll* and *rl*). Connect the photograph center and transferred center of each photograph with a line (pl. 2*B*).
2. Determine the photobase adjusted to the elevation of the lower point *L* (pl. 2*A*) by following the procedure outlined in the discussion of formula (2).
3. Mark a line, the length of which is equal to the length of the adjusted photobase, on transparent overlay material (such as dyrite, acetate, or tracing paper). Label the left terminus of this line c_1 and the right terminus c_2.
4. Position the overlay on the right photograph in such a manner that point c_2 on the overlay is coincident with point c_2 on the photograph and line c_1–c_2 on the overlay is coincident with c_2–*rtc* on the photograph.
5. On the overlay, plot point *rl* and label it *l*. Draw a fine line on the overlay from c_2 through *ru* on the photograph beneath (pl. 2*B* and 2*C*).
6. Position the overlay over the left photograph in the same manner as in step 4. If no tilt is present, the position of the point *ll* on the photograph should be coincident with the position of *l* plotted on the overlay in step 5. If tilt is present, point *ll* on the photograph will appear a short distance from point *l* on the overlay. The overlay should then be shifted in such a manner that point *l* becomes coincident with image point *ll* while line c_1–c_2 is maintained parallel to the photobase of the left photograph. The error introduced by this shift is seldom significant.
7. Draw a fine line from c_1 through *lu* (pl. 2*B* and *C*) as in step 5.
8. The intersection of lines c_2—*ru* and c_1—*lu* (pl. 2*C*) is the corrected position of the upper stereoscopic image point. Label that intersection *u*. Distance *ul* is the corrected horizontal photograph

distance on the overlay between ground points *U* and *L*. Horizontal distance as it is to be used in determination of dip may be obtained by dividing the photograph distance *ul* by the same photograph scale used in measurement of the vertical distance (see "Effect of photograph scale," page 50). Similarly, horizontal ground distance between points *U* and *L* may be determined by dividing photograph distance *ul* by the applicable photograph scale at the lower point (*rl* or *ll*).

Where three stereoscopic image points on the bedding plane are necessary, relief displacement of each of the upper two points is determined relative to the lower point. A small amount of calculation may be omitted if two lower stereoscopic image points can be positioned at the same elevation; this allows the relief displacement to be determined relative to both lower points in only one operation.

The overlay procedure may be eliminated under certain conditions. Where beds are either low dipping or located near one of the photograph centers, the amount of relief displacement of the upper image point relative to the lower is negligible. Also, where strike is parallel to a radial line connecting the photograph center and the lower image point, the effect of relief displacement upon horizontal distance is insignificant. Under any of these conditions, horizontal distance may be measured directly from the photograph.

LIMITATION OF RELIEF DISPLACEMENT CORRECTION

To minimize error in computing the tangent of the angle of dip, corrected horizontal distance should probably measure at least 0.2 inch on the overlay. This horizontal distance requirement will vary, depending upon clarity of rock outcrop and quality of the photographs and stereometer. It will vary also with the magnitude of dip; dips less than 5° require a longer horizontal distance so that any error in differences in elevation (Δh) between stereoscopic image points will be minimized. Dips more than 50° also require a longer horizontal distance if accuracy is to be maintained, because the change in the tangent function per degree of dip is greater at this magnitude. Dips more than 70° should probably not be measured with stereometers because the change in tangent per degree of dip is too great. In fact, beds dipping over 60° or 65° rarely crop out with enough areal extent in the direction of true dip to provide sufficiently accurate measurement of horizontal distance from aerial photographs.

MEASUREMENT OF STRATIGRAPHIC INTERVALS

Measurement of stratigraphic intervals or differences in elevation between topographic features involves the same "floating-dot" technique described on page 38. The photobase is adjusted to the level of the lower point, and formula (2) is applied to convert change in parallax (Δp) to change in elevation (Δh).

When measuring stratigraphic intervals, variations of procedure are often necessary, depending on the geologic and photogrammetric characteristics of the individual problem. In areas where strata are flat lying it is not necessary to correct for relief displacement. However, in areas where dip is significant it is necessary first to correct for relief displacement and determine strike and dip, and then correct for the effect of dip on the stratigraphic thickness.

Although general photograph scale or any other convenient scale can be used in determining strike and dip, an error which may be significant, is commonly incurred by using general photograph scale in measuring stratigraphic intervals. For example, if scale is erroneous in a strike and dip determination, the percentage of error in the vertical distance will be compensated by an equal percentage of error in the horizontal distance. In measuring stratigraphic intervals there is no such compensating effect; the percentage of error in an erroneous scale will cause an equal percentage of error both in the height of the camera (H) as used in the formula and in Δh, the stratigraphic interval in question. Therefore, it is essential for accurate measurement of stratigraphic intervals that applicable photograph scale at the lower point be computed from reliable base maps or field control.

If only relative thicknesses of a rock unit are desired in an area where base maps are not available, a value can be assigned to H'. This value is obtained by arbitrarily selecting a horizontal datum within the stereoscopic model (the datum chosen is usually at or near mean relief), and then relating the general photograph scale and H' to that datum. For example, suppose it is necessary to obtain the rate of thinning along the outcrop of a flat-lying stratigraphic unit: the photographs on which this rock unit appear have a general photograph scale of 1:20,000 and were taken with a 6-inch focal-length camera. First, a datum is chosen (in this example, the datum happens to be below the outcrop in question) and from the general photograph scale it is assumed that the camera is 10,000 feet above that datum. Second, the photobase is adjusted to the datum and found to be 100 millimeters. Next, difference in parallax (Δp) between the datum and the lower bed, measured with a stereometer, is found to be 4.25 millimeters. From the graph it is noted that $V=100$; therefore, the lower rock unit appears about 425 feet above the datum, or approximately 425 feet nearer the camera. For determining relative stratigraphic thickness of the beds in question, 9,575 feet (which may be rounded off to 9,600 feet) will be assumed to be the value of H'. H' assigned for additional stratigraphic thickness measurements in the area must be determined with respect to the same datum.

It is not recommended that this method of assigning a value to H' be applied to an area larger than that covered by one stereoscopic model because considerable variance in scale, due to fluctuation in height of the airplane above the terrane or to tilt in the photographs, may be present between photographs of the same and adjacent flight strips. It is not practical to compensate for differences in scale in this method of assigning a value to H'.

SUMMARY

The generalized procedures for obtaining quantitative geologic data with stereometer-type instruments are summarized as follows:

1. Locate and mark the photograph center and transferred center on each photograph of the stereoscopic pair. Carefully aline the photographs in such a manner that a straight line may be drawn between the photograph centers intersecting the transferred centers (fig. 14).
2. Locate and mark on the photographs the stereoscopic image points between which differences in elevation are to be measured. Tilt may be present in some stereoscopic models, and in order to minimize its effect, the points should be positioned as close together as possible, particularly when measuring stratigraphic intervals.
3. Compute adjusted photobase (ab) at the elevation of the lower of the two stereoscopic image points. If formula (1) can be used, an average of the photobases of each photograph of the stereoscopic pair is sufficient.
4. Obtain applicable photograph scale at the lower stereoscopic image point. This step is very important when measuring stratigraphic intervals or differences in elevation. Any assumed scale is acceptable for determining strike and dip.
5. Determine H' from the photograph scale obtained in step 4.
6. Measure Δp with the stereometer and convert to Δh by applying formula (1) if the area is of low relief or formula (2) if the area is of high relief.
7. In determining strike and dip, correct for relief displacement and obtain the correct horizontal distance along the azimuth of true dip. The effect of the tilt of azimuth of true dip should be considered when measuring dips less than 6° or 7° (fig. 16).
8. Compute the angle of dip.

Unless compensatory steps are taken, two geometrical relationships of vertical aerial photographs may prevent obtaining quantitative data with stereometer-type instruments:

1. Relief displacement: Relief displacement affects horizontal distance in dip measurements.
2. Tilt: The most serious effect of tilt is on the azimuth of true dip of low-dipping beds. The amount of image displacement due to tilt is comparatively large in the corners of the stereoscopic model.

Stereometers may be used to:

1. Obtain reliable differences in elevation between geologic features when applicable photograph scale is known.
2. Obtain approximate or relative differences in elevation between geologic features when only general photograph scale is known.
3. Determine the attitude of beds dipping less than 70°, regardless of whether or not applicable photograph scale is known.

Stereometers normally should not be used to:

1. Obtain reliable measurement of differences in elevation between geologic and topographic features when applicable photograph scale is not known.
2. Determine strike and dip when the horizontal distance along the azimuth of true dip is of insufficient length on the photograph.
3. Measure dips greater than 70°.

Utilization of stereometers is advantageous to the geologist because:

1. Stereometers are simple in construction and usage.
2. Stereometers are portable—they may be set up and used in any field camp.
3. Stereometer techniques, if properly applied, provide reliable data.
4. Stereometric data can often be used to supplement certain phases of fieldwork in the office.

The stereometer procedures described in the present paper apply to a wide variety of geologic and photogrammetric conditions. Modifications of these procedures generally may be made to accommodate the photogrammetric conditions and the geologic aspects of the individual problem.

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U. S. GOVERNMENT PRINTING OFFICE: 1959