Dissipation of the Temperature Effect of Drilling a Well in Arctic Alaska

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By ARTHUR H. LACHENBRUCH *and* MAX C. BREWER

EXPERIMENTAL AND THEORETICAL GEOPHYSICS

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A theoretical analysis of the effects of drilling on subsequent temperature measurements in a well, with application to data obtained in permafrost near Barrow, Alaska

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SYMBOLS

Θ temperature (° C)

Θ_0 undisturbed ground temperature (° C)

z depth beneath the ground surface (cm)

r radial distance from center of drill hole (cm)

t time measured from day drill bit first reached depth in question (seconds or days)

Q total sensible heat released per unit depth by drilling (cal cm^{-1})

Q_L total latent heat released per unit depth by refreezing (cal cm^{-1})

$\bar{q}$ mean rate of sensible heat release per unit depth by drilling (cal cm^{-1} sec^{-1})

q_L mean rate of latent heat release per unit depth by refreezing (cal cm^{-1} sec^{-1})

s duration of drilling process at a given depth (seconds or days)

s_L duration of refreezing process at a given depth (seconds or days)

K thermal conductivity (cal cm^{-1} $°C^{-1}$ sec^{-1})

α thermal diffusivity (cal cm^{-2})

a radius of drill hole (about 15 cm)

$$erf\ x=\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du$$

$$erfc\ x=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-u^2}du=1-erf\ x$$

$I_0(x)$ modified Bessel function of the first kind of order zero $=J_0(ix)$

DISSIPATION OF THE TEMPERATURE EFFECT OF DRILLING A WELL IN ARCTIC ALASKA

By Arthur H. Lachenbruch and Max C. Brewer

ABSTRACT

Temperature measurements to a depth of 595 feet were made during a period of 6 years after the drilling of a 2,900-foot well near the Arctic coast at Barrow, Alaska. The temperature-time curves for depths from 295 to 595 feet fit a simple two-parameter logarithmic formula within instrumental error (±0.01° C) for data taken during the last 4½ years. This formula, which is of the type proposed by Bullard, is based upon a highly simplified theoretical model of the thermal history of the well. The general features of the temperature-time curves for earlier times can be explained in terms of the various physical factors neglected in the simplified model. The analysis indicates that the thermal disturbance decreased from about 20° C to about 0.1° C in the 6-year period following drilling. It is estimated that about 50 years will elapse between the time of drilling and the time the temperature disturbance caused by drilling diminishes to 0.01° C in the depth range studied.

This work probably forms an adequate basis for the study of secular changes in ground temperature that are measurable during the period of observation. Such changes have been observed at all depths from 75 to 275 feet.

INTRODUCTION

In the fall of 1949 two cables containing regularly spaced electrical resistance thermometers between the surface and a depth of 595 feet were installed in a recently completed 2,900-foot well 8 miles from the Arctic coast near Barrow, Alaska. Periodic measurements of the temperature in this well made during the next 6 years make it possible to study the return of the ground to thermal equilibrium as the heat introduced by drilling was dissipated by conduction. The present work is an attempt to account theoretically for the temperature disturbance due to drilling so that the effect of this disturbance can be removed and any remaining secular changes in ground temperatures may ultimately be isolated and studied.

The first part of this paper is a general analysis of the problem of dissipation of drilling heat at this and similar installations. In later sections the results of the analysis are applied to an interpretation of the data. The instrumentation, measurement, and data reduction are the responsibility of M. C. Brewer. A. H. Lachenbruch is responsible for the theoretical development and interpretation.

ACKNOWLEDGMENTS

The thermal installation at South Barrow well 3 is one of several currently under study by the U.S. Geological Survey in cooperation with the Office of Naval Research and its Arctic Research Laboratory; the Bureau of Yards and Docks of the U.S. Navy; and the Air Force Cambridge Research Center. This work represents part of a broad program of thermal measurements with thermistors that was started by Joel H. Swartz, of the Geological Survey, in 1948.

Gerald R. MacCarthy was responsible for installing the thermistor cables at South Barrow well 3 and for making some of the early observations. The work was supported by laboratory studies of thermistors by Mr. Swartz and by technical assistance and advice on instrumentation from Rudolph Raspet. The authors are grateful to Professor Francis Birch for reading the manuscript and offering many valuable suggestions.

ESTABLISHMENT OF EQUILIBRIUM TEMPERATURES

PHYSICAL CONSIDERATIONS

During the rotary drilling of a well, drilling fluid (mostly water) is pumped downward through the drill pipe to the bottom of the hole and returned to a surface reservoir through the annular region between the drill pipe and the walls of the hole. As a consequence heat is exchanged by conduction between the fluid and the walls of the hole at a rate that is dependent on the relative temperatures and physical properties of the two media and the configuration of the surface of contact between them. In addition, convective transfer sometimes occurs in varying amounts when drilling fluid permeates the wallrocks. When such fluid losses occur, cement is sometimes injected into the well and additional thermal effects are produced by the heat of hydration of the cement. A further but generally insignificant thermal effect of drilling is the generation of heat by mechanical processes near the drill bit and by deformation of the drilling pipe. This heat is rapidly transmitted to the circulating drilling fluid.

When the drilling is completed the fluid stagnates in the well and, barring secular changes, the temperature at each depth asymptotically approaches its predrilling value. The manner in which these temperatures return to equilibrium depends upon a complex interaction of many factors. Probably the most important of these factors are as follows:

1. The variation of effective source strength with time: This depends upon the variation of temperature of the drilling fluid with time and changes in the effective diameter of the hole caused by casing and washing away of weakly consolidated wallrock.
2. The variation of effective source strength with depth: The source strength varies with depth-variation of the temperature of the drilling fluid and the diameter of the hole, and with the variation of the temperature and thermal properties of the rock strata with depth. The variation with depth of the total heat supplied or removed depends upon the rate of drilling.
3. The effect of proximity of the measurement point to the ground surface: For larger values of time, the position of the measurement point, with respect to the ground surface, can influence the form of the cooling curves at shallow depths.
4. Variation in thermal properties of rock strata: The return to equilibrium at any depth is affected not only by the thermal conductivity and specific heat of the rocks at that depth but also by the properties of the strata above and below.
5. Effect of latent heat of refreezing water: At the depths considered in this study, the cooling curves cross the freezing temperature of water and the effects of latent heat enter first from the refreezing of interstitial water in the formation and later from the freezing of the drill fluid in the well.

The well under discussion, designated as South Barrow well 3, was drilled between June 23, 1949, and August 23, 1949. On August 28, 1949, the thermistor cables were installed to a maximum depth of 595 feet and allowed to freeze in place. Temperature measurements were begun on September 1, 1949, and were continued on a periodic basis for about 6 years. Since the cessation of drilling, the temperatures throughout the depth under study decreased about 20° C. A typical set of cooling data is presented in figure 29.

In the upper 595 feet the rate of advance of the drilling bit was on the order of 100 feet per day, sufficiently rapid to justify neglecting effects of preheating beneath the bit, because the times are measured only to the nearest day. The validity of this simplification may be verified by noting the order of magnitude of the radial temperature disturbance caused by a linear heat source and then observing that the longitudinal effect measured on the axis beneath a semi-infinite linear source is even less. (See Bullard, 1947.) Thus it may be assumed that the thermal disturbance due to drilling did not begin until the day that the drill bit advanced to the depth in question.

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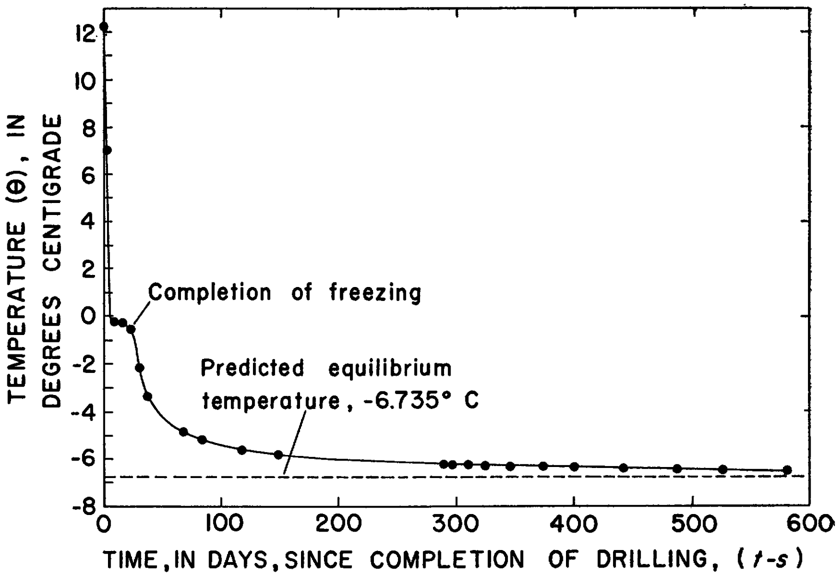

FIGURE 29.—Cooling of South Barrow well 3, 595-foot depth.

From the drilling records it is known that there was no appreciable loss of drilling fluid to the sediments. Therefore the heat transfer between the drilling fluid and the wallrocks was probably predominantly by conduction. The drilling records also show that the fluid was circulating about 60 percent of the time required to drill the well, and that these periods of circulation were more or less uniformly distributed in time. Thus the distribution of circulation time probably introduced no systematic effect on the distribution of effective source strength with time. Drilling records for South Barrow well 3 show that the temperature of the fluid leaving the hole remained almost the same as the mean daily temperature of the air in contact with the surface reservoir throughout the 2 months required to drill the well. Hence the temperature of the fluid probably did not change greatly during its traverse of the hole. Thus in the upper portion of the well at least, the temperature probably remained fairly close to that of the atmosphere throughout the periods when the fluid was circulating. The mean daily atmospheric temperature was about +10 to +15° C, and the undisturbed temperatures of the ground ranged from about −10° C near the surface to about +18 to +20° C at the bottom (2,900 feet). The predrilling temperature was about −7° C near the bottom of the thermistor cables (595 feet). From these data it seems reasonable to conclude that the drilling operation acted as a positive heat source throughout its duration in the depth range under discussion.

The surface area over which heat is transferred between the drilling fluid and the sediments is determined by the effective radius of the hole. For the first quarter of the drilling period the hole was uncased and had a radius of about 20 cm except locally where minor amounts of weakly consolidated wallrock might have been washed away. For the remaining three-quarters of the drilling time, casing fixed the effective radius of the hole at 15 cm throughout the depth range under study.

On the basis of the above information it might be expected that the rate of heat transfer from the drilling fluid to the sediments between 75 and 595 feet would generally be greater in the earlier stages of drilling because the radial thermal gradients in the surrounding sediments would tend to decrease with time. The slightly larger hole existing in the early phase of drilling would tend to accentuate this effect.

ANALYTICAL CONSIDERATIONS

This section is devoted to an analytical discussion of the thermal effects of the various physical factors outlined above. With this analysis it is not intended to treat all the physical complications in detail. It is designed to provide an estimate, for each of the major factors, of the contribution to the thermal disturbance for larger values of time.

DISTRIBUTION OF SOURCE STRENGTH WITH TIME

In an attempt to isolate the effect of variation with time of the heat supply to the drill-hole walls throughout the drilling period, it will be assumed that the medium penetrated is homogenous and infinite, and that the effect of the drilling operation can be approximated by that of a linear heat source whose strength is independent of depth.

In this discussion two basic relations will be used. The first gives the axial temperature, due to an instantaneous linear source of strength, Q units of heat per unit depth released at time $t=t_0$:

$$\Theta(t)-\Theta_0=\frac{Q}{4\pi K}\frac{1}{t-t_0},\ t>t_0 \tag{1}$$

where K is the thermal conductivity of the medium, and Θ_0 is the undisturbed predrilling temperature. Throughout this paper t will represent the time elapsed since the drill bit first reached the depth in question. Hence, the origin of t will, in general, be different for each depth.

The second basic relation is obtained by integrating equation (1). It gives the axial temperature in the same medium resulting from a continuous source, $q(t)$ units of heat per unit time per unit depth, persisting throughout the period of drilling ($0<t<s$)

$$\Theta(t)-\Theta_0=\frac{1}{4\pi K}\int_0^s \frac{q(\tau)}{t-\tau}\,d\tau,\ t>s. \tag{2}$$

Here s represents the time elapsed from the day the drill bit first reached the depth in question until the drilling operation ceased. In general, s is different for each depth. If $q(t)=\bar{q}$, a constant, then equation (2) becomes

$$\Theta(t)-\Theta_0=\frac{q}{4\pi K}\log_e\frac{t}{t-s},\ t>s. \tag{3}$$

Equation (3) represents the cooling of a well in which the drilling operation acted as a constant linear heat source persisting throughout the time of drilling. This case corresponds to the assumptions made by Bullard (1947). It also forms the basis for much of the present discussion.

In table 1 this result, which is shown as case IV, is compared with 3 cases of equation (1) (cases I, II and III), and 2 other cases of equation (2) which are discussed below. In this table $\bar{q}$ represents the mean value of $q(t)$ in $0<t<s$, and Q is identified with $\bar{q}s$. The term in $\left(\frac{s}{t}\right)$ in the last column in table 1 shows the relative error that would result for large values of time ($t>>s$) by treating each case by the result for case IV, equation (3).

From the discussion of the physical problem, it was concluded that the source strength might more realistically be considered as a decreasing function of time. Case V, in table 1, represents a linear decrease in source strength to zero at the completion of drilling, an extreme case. For large values of time, this result differs from that given by equation 3 by a relative error of $-\frac{1}{6}\frac{s}{t}$.

Since the drilling data indicate that the temperature of the walls of the hole might have been fairly constant during drilling, a limiting case of the source distribution might be given by the heat flux through the surface of a region bounded internally by a circular cylinder, where this surface is maintained at constant temperature (for $0<t<s$). For small values of time, this heat flow will behave as $t^{-1/2}$, and for larger values it will approach zero more slowly (Carslaw and Jaeger, 1947, p. 282, equations (9) and (10)). Case VI, in table 1, represents the case of the source varying as $t^{-1/2}$ throughout the drilling interval and hence is more extreme than the solution for constant wall temperature. The difference between this solution and that for case IV is again represented by a relative error of $-\frac{1}{6}\frac{s}{t}$.

TABLE 1.—*Comparison of cooling formulas based on six different assumed distributions of source strength with time*

Case	Description	Axial temperature, $t>t_0$ or $t>s$	Error from treating by case IV, $t>>s$
I	Instantaneous, $t_0=0$	$\frac{Q}{4\pi K}\frac{1}{t}$	$1-\frac{1}{2}\frac{s}{t}+O\left(\frac{s}{t}\right)^2$
II	Instantaneous, $t_0=s$	$\frac{Q}{4\pi K}\frac{1}{t-s}$	$1+\frac{1}{2}\frac{s}{t}+O\left(\frac{s}{t}\right)^2$
III	Instantaneous, $t_0=s/2$	$\frac{Q}{4\pi K}\frac{1}{t-s/2}$	$1 \quad +O\left(\frac{s}{t}\right)^2$
IV	Continuous, $q=\bar{q}$, $0>t>s$	$\frac{\bar{q}}{4\pi K}\log_e\frac{t}{t-s}$	
V	Continuous, $q=2\bar{q}\left(1-\frac{t}{s}\right)$, $0<t<s$	$\frac{\bar{q}}{2\pi K}\left\{\left(1-\frac{t}{s}\right)\log_e\left(\frac{t}{t-s}\right)+1\right\}$	$1-\frac{1}{6}\frac{s}{t}+O\left(\frac{s}{t}\right)^2$
VI	Continuous, $q=\frac{\bar{q}\sqrt{s}}{2\sqrt{t}}$, $0>t>s$	$\frac{\bar{q}}{8\pi K}\left(\frac{s}{t}\right)^{1/2}\left\{\log_e\left(\frac{t}{t-s}\right)+2\log_e\left[1+\left(\frac{s}{t}\right)^{1/2}\right]\right\}$	$1-\frac{1}{6}\frac{s}{t}+O\left(\frac{s}{t}\right)^2$

From physical considerations the actual effect of variation of source strength with time in $0<t<s$ is expected to be less pronounced than in cases V and VI. Hence treating the effect by case IV should introduce a relative error not greater than $-\frac{1}{6}\frac{s}{t}$ for $t>>s$.

FINITE DIAMETER OF THE WELL

In the foregoing section the heat source has been assumed to be an infinitesimally thin linear filament. More precisely the problem deals with heat flux through the walls of a cylindrical surface to finite radius (about 15 cm in the present case). One way of generalizing the problem to estimate the error introduced by this simplification is to consider the temperature at $r=0$, $t>s$ in a homogenous infinite medium heated for $0<t<s$ by a cylindrical surface source of radius, a, and strength $\bar{q}$ heat units per unit length per unit time. The temperature at a radial distance r, due to an instantaneous cylindrical source of radius a and strength, Q heat units per unit length, released at $t=0$ is (Carslaw and Jaegar, 1947, p. 219, equation 5).

$$\frac{Q}{4\pi K} e^{-\frac{a^2+r^2}{4\alpha t}} I_0\left(\frac{ar}{2\alpha t}\right) \tag{4}$$

where $I_0(x)$ is a modified Bessel function of the first kind, of order zero. Therefore the solution sought is given by

$$\begin{aligned}\Theta(t)-\Theta_0&=\lim_{r\to 0}\frac{\bar{q}}{4\pi K}\int_0^s e^{-\frac{a^2+r^2}{4\alpha(t-\tau)}} I_0\left[\frac{ar}{2\alpha(t-\tau)}\right]\frac{d\tau}{t-\tau},\ t>s\\ &=\frac{\bar{q}}{4\pi K}\left\{\int_{\frac{a^2}{4\alpha t}}^{\infty} e^{-u}\frac{du}{u}-\int_{\frac{a^2}{4\alpha(t-s)}}^{\infty} e^{-u}\frac{du}{u}\right\}.\end{aligned}$$

In the present case $\frac{a^2}{4\alpha s}\approx 10^{-3}$ and thus for $t>>s$ we may use

$$\int_x^\infty e^{-u}\frac{du}{u}=-\gamma+\log_e\frac{1}{x}+x+\frac{x^2}{4}+\cdots,\ \gamma=\text{Euler's constant,}$$

and the result is

$$\Theta(t)-\Theta_0=\frac{q}{4\pi K}\left\{\log_e\frac{t}{t-s}-\frac{a^2}{4\alpha s}\left(\frac{s}{t}\right)^2+O\left(\frac{s}{t}\right)^3\right\}. \tag{5}$$

In some respects a more natural way to generalize the problem is to consider the temperature on the surface $r=a$, of a region bounded internally by a circular cylinder where the flux through the cylindrical boundary is constant for $0<t<s$ and zero thereafter. The same relative error to terms of order s/t is given by the simpler case of the temperature at the wall, $r=a$, of the cylindrical surface source dis-

cussed in the previous paragraph. The solution may be obtained by setting $r=a$ in equation 4 and making a change of variables. Then using $e^{-x}I_0(x)=1-x+\frac{3}{4}x^2+O(x^3)$ and integrating we obtain

$$\Theta(a,t)-\Theta_0=\frac{\bar{q}}{4\pi K}\left\{\log_e\frac{t}{t-s}-\frac{a^2}{2\alpha s}\left(\frac{s}{t}\right)^2+O\left(\frac{s}{t}\right)^3\right\}, t>>s. \tag{6}$$

Thus for observations taken sufficiently late that terms of order $(s/t)^2$ can be neglected, the effect of finite radius probably does not introduce departures from the simple logarithmic formula exceeding a factor of $1-10^{-2}\left(\frac{s}{t}\right)$; that is, a relative error of $-10^{-2}\left(\frac{s}{t}\right)$.

VARIATION OF SOURCE STRENGTH WITH DEPTH

Variation with depth in the amount of heat supplied to the ground by drilling produces a longitudinal distortion of the temperature field. The effect of this on the restoration of thermal equilibrium at any depth will now be discussed.

The temperature distribution resulting from an instantaneous point source of strength Q units of heat per unit depth at $(0, 0, z')$ in a homogenous infinite medium of conductivity K, and diffusivity α is, when expressed in cylindrical coordinates (Carslaw and Jaeger, 1947, p. 216, equation 2,

$$\frac{Q}{8\rho c(\pi\alpha t)^{\frac{3}{2}}}e^{\frac{-[r^2+(z-z')^2]}{4\alpha t}}.$$

Thus the temperature resulting in an infinite medium from an instantaneous linear source of strength $Q(z)$ is given by

$$\Theta(r,z,t)=\frac{1}{8\rho c(\pi\alpha t)^{\frac{3}{2}}}\int_{-\infty}^{+\infty}Q(z')e^{\frac{-[r^2+(z-z')^2]}{4\alpha t}}dz'. \tag{7}$$

If for any depth, z_0, the source $Q(z)$ can be expressed as

$$Q(z)=A+f(z-z_0) \tag{8}$$

where $f(x)=-f(-x)$, and A is a constant, then according to equation (7) the temperature at z_0 will be the same as that produced by a source distribution $Q=A$, independent of depth. Therefore, when considering the behavior of the temperature at any depth z_0 in a homogenous medium, it may be supposed without error that the source did not vary in strength throughout a depth interval, $(z_0-L)<z<(z_0+L)$, if the actual source strength is of the form (8) over this interval.

We now consider the axial temperature, $\Theta(t)$, due to an instantaneous linear source of strength, $Q(z)$, in a homogeneous infinite medium initially at temperature Θ_0. $Q(z)$ will be considered as the step function

$$Q(z)=Q_i,\ z_i<z<z_{i+1},\ i=0,1,2,\ldots N-1$$
$$z_0=-\infty,\ z_N=+\infty$$

For this case integration of equation 7 yields

$$\Theta(t,z)-\Theta_0=\sum_{i=0}^{N-1}\Delta\Theta_i \tag{9}$$

where

$$\Delta\Theta_i=\frac{Q_i}{4\pi Kt}\left\{\frac{1}{2}\left[\operatorname{erf}\frac{z-z_i}{2\sqrt{\alpha t}}-\operatorname{erf}\frac{z-z_{i+1}}{2\sqrt{\alpha t}}\right]\right\} \tag{10}$$

where

$$\operatorname{erf}x=\frac{2}{\sqrt{\pi}}\int_0^x e^{-u^2}du=-\frac{2}{\sqrt{\pi}}\int_0^{-x}e^{-u^2}du=-\operatorname{erf}(-x)$$

From equation (10) it is seen that the relative effect on the temperature at any level, z, of source increments distant from z, tends to become more pronounced with increasing time.

If some depth, z', is considered for which the source strength, Q_m, may be considered independent of depth throughout the depth interval $z_m=(z'-L)<z<(z'+L)=z_{m+1}$ then the cooling at this depth is approximated by equation (9) where

$$\Delta\Theta_i=\frac{Q_i}{4\pi Kt}\left\{\frac{1}{2}\left[\operatorname{erf}\left(\frac{z_m-z_i+L}{2\sqrt{\alpha t}}\right)-\operatorname{erf}\left(\frac{z_m-z_{i+1}+L}{2\sqrt{\alpha t}}\right)\right]\right\},m\neq i \tag{11a}$$

$$=\frac{Q_m}{4\pi Kt}\left\{\operatorname{erf}\frac{L}{2\sqrt{\alpha t}}\right\},m=i. \tag{11b}$$

If L is sufficiently large with respect to $2\sqrt{\alpha t}$ the expression in braces in (11a) becomes negligible while the corresponding expression in (11b) is approximately unity. Equation (9) then assumes the same form as equation (1) with $t_0=0$. If $L/2\sqrt{\alpha t}>1.8$ the expression in braces in equation (11b) is within one percent of unity while the corresponding expression in (11a) cannot exceed 0.01. In fact, the cumulative effect of terms represented by equation (11a) cannot contribute more than about 1 percent of the total temperature effect at z' unless $Q(z)$ deviates from

$$Q(z)=Q_m+f(z-z') \tag{12}$$

by a quantity on the order of Q_m; where $f(z-z')$ is as defined in (8). If $L/2\sqrt{\alpha t}>3.0$ the discrepancy reduces to 0.002 percent.

In the first three columns of table 2, values of L corresponding to erf $(L/2\sqrt{\alpha t})=0.99$ are given for selected values of t and α. The two values, 0.006 and 0.014 cm^2 per sec, assumed for α probably bracket most materials in the well.

TABLE 2.—*Values of L, in feet, for which* erf $\frac{L}{2\sqrt{\alpha t}}$ *equals 0.99 and 0.10 for selected values of* α *and* t

Time (t)	erf $\frac{L}{2\sqrt{\alpha t}}$=0.99		erf $\frac{L}{2\sqrt{\alpha t}}$=0.10	
	α=0.006 cm^2/sec	α=0.014 cm^2/sec	α=0.006 cm^2/sec	α=0.014 cm^2/sec
1 day	2.5	4	0.13	0.20
1 week	7	10.5	.34	.54
1 month	14	22	.71	1.1
6 months	35	54.5	1.7	2.7
1 year	50	77	2.5	3.9
5 years	111	173	5.6	8.6

Thus, if an instantaneous linear source varied in strength according to equation (12) for $-\infty<z<+\infty$, the subsequent cooling at z' would be given by equation (11b), which in this case becomes identical to equation (1); and 99 percent of the temperature disturbance existing after 5 years would be contributed by heat initially released in the 222-foot depth interval centered about z' (assuming α=0.006). Viewed another way, if the source strength were of the form (12) in the 222-foot depth interval centered about z' and zero outside of this depth interval, then the error in using equation (4) would grow to 1 percent only after 5 years. Applying cooling formulas, such as equation (3), to the data for the upper 600 feet at South Barrow well 3, it is found that after 5 years of cooling, the existing temperature disturbance is on the order of 0.1° C and a 1 percent error would amount to only 0.001° C. From equations (1) and (11) it is seen that the temperature error, in applying the uniform case to the case outlined above, varies for large times, approximately as $\frac{1}{t}\operatorname{erfc}\frac{L}{2\sqrt{\alpha t}}$. Thus after 50 years of cooling and when the predicted ground temperatures should be on the order of 0.01° C from equilibrium, the error will grow to about 0.005° C. As the observational error is about 0.01° C, the cooling can probably be treated for all time by the case of source strength independent of depth if the actual source strength may be considered to be of form (12) for a distance on the order of 175 feet above and below the observation point. This result is based on the assumption that α=0.014, which is probably somewhat large, and it makes 175 feet correspondingly large.

The effect of source strength varying simultaneously with depth and time throughout a drilling period of finite duration (s) may be studied by integrating equation (11). Integration of (11a) gives

$$\Delta\Theta_m = \frac{1}{4\pi K}\int_0^s \operatorname{erf}\frac{L}{2\sqrt{\alpha(t-\tau)}}\,\frac{q_m(\tau)}{t-\tau}\,d\tau$$

$$= \frac{1}{4\pi K}\operatorname{erf}\frac{L}{2\sqrt{\alpha(t-\bar{t})}}\int_0^s \frac{q_m(\tau)}{t-\tau}\,d\tau,\ 0<\bar{t}<s \qquad (13)$$

with a similar expression corresponding to (11b). Here $\bar{t}$ is some value of τ which gives the correct mean value of $erf\,\frac{L}{2\sqrt{\alpha(t-\tau)}}$. Thus, equation (13) bears the same relation to equation (2) as (11) bears to (1), and the same considerations apply. The principal difference is that the time as shown in table 2 is now reckoned from some time $0<\bar{t}<s$, the exact value of which depends upon the form $q(t)$.

The corresponding solution for finite radius can be deduced from the work of Blackwell (1953), and the form of the correction for longitudinal flow is identical to that in equation (13).

From the foregoing discussion it seems reasonable to conclude that, if the material surrounding the hole may be considered to be homogeneous with respect to thermal properties and if the variation in source strength with depth can be well approximated by a function of form (12) for 150 to 175 feet above and below the depth in question, then no appreciable error will be introduced by neglecting the variation of source strength with depth from its value at the point of measurement.

It was concluded above that the temperature of the drilling fluid might reasonably be assumed to be independent of depth in the upper 600 feet while the fluid was circulating. When the fluid is stagnant, the heat transfer through the walls of the well can probably be neglected. Thus, the heat transfer during the drilling process can probably be approximated by a linear combination of terms giving the flux through the walls of an isothermal cylinder embedded in an infinite medium initially at constant temperature. Such solutions are proportional to the temperature difference between the cylinder and the initial constant value for the medium (Carslaw and Jaeger, 1947, p. 282, equation (9)). Thus, to a first approximation, the source strength might be considered as a linear function of depth from about 200 feet to the bottom of the depth interval under discussion because the undisturbed ground temperatures vary linearly over this depth range. Inasmuch as a linear source distribution satisfies the condition (8), this argument suggests that neglecting source-strength variation with depth is justified in analyzing measurements in the deeper part of the depth interval studied.

PROXIMITY OF THE POINT OF MEASUREMENT TO THE GROUND SURFACE

On the basis of the discussion above, it will be assumed that the source strength varied with depth approximately as the difference between the temperature of the drilling fluid and the undisturbed ground temperature. Inasmuch as the drilling fluid had a temperature close to the mean daily atmospheric temperature during the time of drilling, the thermal disturbance at the ground surface may be considered negligible. Judging from estimated undisturbed ground temperatures at the time of drilling, the thermal disturbance at the walls of the hole rapidly increased with depth, so that its value at 25 feet was of the same order as that in the next 200 feet (about 20° C). The condition that the thermal disturbance be zero at the ground surface may be satisfied in (11) and similar equations by requiring that $Q(z)=-Q(-z)$, where the ground surface is taken as the origin of z. To get an estimate of the effect of the ground surface on the return to equilibrium at some depth z^*, we first suppose that the source strength may be considered constant to a depth $2z^*$ beneath the surface. Then if $Q(z+2z^*)\approx Q(z)$, the effects of the image source above the surface and the actual source beneath $2z^*$ will tend to nullify each other. The error introduced by neglecting the presence of the surface and assuming the source strength independent of depth would then be approximately $\left(1-\text{erf}\ \frac{z^*}{2\sqrt{\alpha t}}\right)$, according to equation (11).

The assumption that the source strength is constant to a depth $2z^*$ can lead to an underestimation of the error for small z^* inasmuch as the temperature passed continuously from about $-9°$ C at 50 feet to about $+10°$ C at the surface during the time the drilling was in progress. The discrepancy can be bracketed, however, by assuming a step distribution of source strength near the surface. For example, if it is assumed that the source strength is constant, Q, for $z>\Delta$, and Q/n for $0<z<\Delta<z^*$, then from the condition that the source be odd in z, the temperature at $z=z^*$, as given by equation (11) is

$$\frac{Q}{4\pi Kt}\left\{\frac{1}{n}\ \text{erf}\ \frac{z^*}{2\sqrt{\alpha t}}+\frac{n-1}{2n}\left[\text{erf}\ \frac{z^*-\Delta}{2\sqrt{\alpha t}}+\text{erf}\ \frac{z^*+\Delta}{2\sqrt{\alpha t}}\right]\right\}. \tag{14}$$

Setting $n=2$ and $\Delta=50$ feet$=1,525$ cm, a considerable overestimation of the error, the discrepancy, δ, resulting from neglecting the effect of the surface is bracketed as follows

$$1-\text{erf}\ \frac{z^*}{2\sqrt{\alpha t}}<\delta<1-\frac{1}{2}\ \text{erf}\ \frac{z^*}{2\sqrt{\alpha t}}-\frac{1}{4}\left[\text{erf}\ \frac{z^*-1{,}525}{2\sqrt{\alpha t}}+\text{erf}\ \frac{z^*+1{,}525}{2\sqrt{\alpha t}}\right]. \tag{15}$$

The limiting values of the discrepancy computed for two assumed diffusivities are given in table 3.

TABLE 3.—*Limiting values of relative error due to the presence of the ground surface*

Depth, z^* (feet)	$1-\text{erf}\frac{z^*}{2\sqrt{\alpha t}}$		$1-\frac{1}{2}\text{erf}\frac{z^*}{2\sqrt{\alpha t}}-\frac{1}{4}\left\{\text{erf}\frac{z^*-1525}{2\sqrt{\alpha t}}+\text{erf}\frac{z^*+1525}{2\sqrt{\alpha t}}\right\}$	
	$\alpha=0.006$	$\alpha=0.014$	$\alpha=0.006$	$\alpha=0.014$
		$t=1$ year		
75	0.00	0.05	0.05	0.11
95	.00	.00	.01	.05
115	.00	.00	.00	.01
135	.00	.00	.00	.00
		$t=5$ years		
75	0.10	0.27	0.19	0.34
95	.00	.17	.08	.22
115	.00	11	.03	.14
135	.00	.05	.01	.08
155	.00	.02	.00	.04
175	.00	.00	00	.02
195	.00	.00	.00	.01

Thus, neglecting the proximity of the ground surface can probably introduce appreciable error for large values of time at depths less than 200 feet. For rocks of low diffusivity the error after 5 years probably ranges from about 10 to 20 percent of the temperature disturbance at 75 feet to about 1 percent at 135 feet. If the diffusivity was high, however, the error after 5 years may grow to as much as 35 percent of the temperature disturbance at 75 feet and 4 or 5 percent at 155 feet. Under such circumstances an error of about 1 percent could persist to 195 feet. After only 1 year of cooling, the error is probably negligible below 95 feet, but it may grow to 5 or 10 percent at the 75-foot depth. A more precise bracketing is possible but is not necessary for the present purposes.

VARIATION OF THERMAL PROPERTIES WITH DEPTH

It is difficult to present a direct quantitative discussion of the effect of stratification with respect to thermal properties, on the restoration of thermal equilibrium. This is because the solution of the heat-conduction equation for radial flow in a stratified medium is difficult and because the stratification itself generally produces a change in the effective source strength with depth. It should be possible, however, to draw certain semiquantitative conclusions from consideration of the homogeneous case. After heating by an infinite linear source, the cooling at a point interior to a homogeneous stratum of finite thickness will, for sufficiently small values of time, behave as if the medium were infinite and homogeneous, possessing the thermal properties of that stratum. As time increases, the percentage contribution of heat initially liberated in adjacent strata to the temperature disturbance at the observation point will increase. Thus, the form of the cooling curve at any depth will depend upon the thermal properties of all strata in which the initially liberated heat contributes appreciably to the temperature disturbance at that depth; and this will, in general, change with time. If the observed temperatures were plotted against $\log_e \left(\frac{t}{t-s}\right)$ in accordance with equation (3), for small values of time, one might obtain a straight line of slope $\frac{\bar{q}}{4\pi K}$ where K is the conductivity of the stratum containing the thermistor. If this line were extrapolated to $t=\infty$, the equilibrium temperature so determined might be seriously in error if the stratum were not sufficiently thick and the adjacent strata had different thermal properties, because the slope of the curve would, in general, change for larger times.

To estimate how thick a homogeneous bed must be before the cooling at its center can be treated by the homogeneous case without appreciable error, reference is made to table 2. If a homogeneous medium were heated initially by an instantaneous linear source extending only 173 feet above and below the measurement point, the error in computing the temperature disturbance on the assumption that the source extended to plus and minus infinity would grow to 1 percent after 5 years of cooling (for $\alpha=0.014$). Thus, if heat were released initially in strata of different properties beyond 173 feet from the measurement point, the error in treating the medium as homogeneous and infinite probably would not exceed about 1 percent after 5 years and would be within the margin of instrumental error for all time.

If the point of observation lies at the interface of two semi-infinite media of differing thermal properties, by symmetry the form of the cooling curve will not change with time, and the homogeneous case may be applied directly. Thus, the conclusion of the previous paragraph can be refined to state that to apply the homogeneous problem to the cooling of a stratified medium in the present case, the material for 175 feet above the observation point and that for 175 feet below the observation point must each be virtually homogeneous but they need not have the same properties.

It is most unlikely that the stratigraphic section would be homogeneous with respect to thermal properties for distances on the order of a few hundred feet. It might be expected, however, that the effects of thinly interbedded strata would average themselves out after the passage of sufficient time; and that thereafter the medium will cool as if homogeneous, with properties intermediate between those of its extreme constituents. To get an estimate of the time necessary for this "averaging out," reference is made again to the homogeneous case. Inasmuch as erf y is approximately linear for $y<0.4$, it follows from equation (10) that heat initially liberated by a linear source of constant strength in equal depth increments near the observation point contributes equally to the temperature disturbance there. Thus, from table 2 it can be seen that, after 6 months of cooling, the stratum 3.4 feet thick centered at the observation point contributes 10 percent of the existing temperature disturbance. Similarly the stratum 13.6 feet thick centered at the point contributes about 40 percent of the disturbance when it is assumed that $\alpha=0.006$. Assuming $\alpha=0.014$, the corresponding thickness over which the percent contribution is proportional to distance is 21.6 feet. For thinly interbedded strata it might be expected that the effects are averaged out where the cycle of repetition is small with respect to the depth interval in which the contributions are linear in the homogeneous case. Thus, after 6 months of cooling, it is probably permissible to treat as homogeneous, a section composed of uniformly interbedded layers not exceeding about 2 feet in thickness. Similarly, after 1 year, interbeds on the order of 3 feet thick and after 5 years those on the order of 6 to 8 feet thick can probably be neglected without appreciable error.

The above analysis is crude and serves only to give an order of magnitude in limiting cases. An evaluation of the effect of stratification boundaries at intermediate distances from the observation point is not possible by reference to the homogeneous case. It is quite possible, however, that for special distributions of strata of differing thermal properties, appreciable errors can be introduced with the assumption of homogeneity.

LATENT HEAT

As the well cools after cessation of drilling, the freezing isotherm moves radially inward, and latent heat is released as first the interstitial water in the wallrocks, and later the drilling fluid, freezes.

A precise analytical treatment of the thermal effects of refreezing is difficult because of the variation of moisture content from one stratum to the next and from the wallrocks to the drilling fluid. It should be possible, however, to determine for the effect an order of magnitude valid for large values of time by assuming the latent heat to be released on the axis at a constant rate, $\bar{q}_L$ heat units per unit time per unit depth, throughout the freezing period, which is defined to be $s<t<s+s_L$. In this case the contribution of the latent heat to the temperature at subsequent times is given by

$$\frac{\bar{q}_L}{4\pi K}\log_e\frac{t-s}{t-(s+s_L)}=\frac{\bar{q}_L}{4\pi K}\frac{s_L}{t-(s+\frac{1}{2}s_L)}+O\left(\frac{s_L}{t}\right)^3, t\gg s+s_L.$$

Using equation (3) to approximate the temperature effect of the sensible heat introduced during drilling gives

$$\begin{aligned}\Theta(t)-\Theta_0&=\frac{\bar{q}}{4\pi K}\left\{\log_e\frac{t}{t-s}+\frac{\bar{q}_L}{\bar{q}}\left[\frac{s_L}{t-(s+\frac{1}{2}s_L)}+O\left(\frac{s_L}{t}\right)^3\right]\right\}\\&=\frac{\bar{q}}{4\pi K}\left\{\log_e\frac{t}{t-s}+\frac{Q_L}{Q}\left[\frac{s}{t-(s+\frac{1}{2}s_L)}+O\left(\frac{s}{t}\right)^3\right]\right\}\\&=\frac{\bar{q}}{4\pi K}\left\{\log_e\frac{t}{t-s}+\frac{Q_L}{Q}\left[\frac{s}{t}+\left(\frac{s}{t}\right)^2+\frac{s_Ls}{2t^2}+O\left(\frac{s}{t}\right)^3\right]\right\}\\&=\frac{\bar{q}}{4\pi K}\frac{Q+Q_L}{Q}\left\{\log_e\frac{t}{t-s}+\frac{Q_L}{Q+Q_L}\left[\frac{1}{2}\frac{s}{t^2}(s+s_L)+O\left(\frac{s}{t}\right)^3\right]\right\}\\&=\frac{\bar{q}}{4\pi K}\frac{Q+Q_L}{Q}\log_e\frac{t}{t-s}\left\{1+\frac{1+\frac{s_L}{s}}{2\left(1+\frac{Q}{Q_L}\right)}\frac{s}{t}+O\left(\frac{s}{t}\right)^2\right\}.\end{aligned}\qquad(16)$$

To estimate the order of magnitude of the amount of heat supplied by the drilling process, the observed temperatures are plotted against $\log_e\left(\frac{t}{t-s}\right)$ for each depth, and the slope of the resulting curve is identified with the quantity $\frac{Q+Q_L}{Q}\frac{\bar{q}}{4\pi K}$ in accordance with equation (16). It is found that the value of these slopes for all depths is about 3 or 4° C. Assuming that K is about 0.004 cal cm^{-1} ° C^{-1} sec^{-1}, gives $Q+Q_L$ on the order of 10^6 cal per cm.

To estimate the order of magnitude of the total latent heat supplied, consider the temperature in an infinite medium heated by an infinite linear source of constant strength $\bar{q}$, which is given by (Carslaw and Jaeger, 1947, p. 221, equation (4))

$$\Theta(r,t)-\Theta_0=\frac{\bar{q}}{4\pi K}\int_{\frac{r^2}{4\alpha t}}^{\infty}e^{-u}\frac{du}{u}.\qquad(17)$$

The corresponding equation taking account of latent heat is more complicated; but since the radius of the thawed cylinder increases as $\sqrt{t}$ (Ingersoll, Zobel, and Ingersoll, 1954, p. 265, equations 9 and 12), the net rate of absorption of latent heat is constant during heating by a constant source. Thus, as an approximation, the source strength may be reduced by the rate of latent-heat absorption in equation (17) for this calculation. Using the empirically determined value of $\frac{Q+Q_L}{Q}\frac{\bar{q}}{4\pi K}\approx 3\frac{1}{2}$° C and assuming $\alpha=0.01$ cm² per sec, it is found at the 595-foot depth that if the moisture content of the rocks is 30 percent by volume, then the radius of the thawed cylinder at the completion of drilling is about $4\frac{1}{2}$ times the radius of the hole, and the value of Q_L/Q is about one-third. Water contents exceeding 30 percent by volume are not expected at depth at this locality. Lower assumed water contents result in smaller values of Q_L/Q. At the 295-foot depth, a similar calculation gives $Q_L/Q<\frac{1}{4}$ for rocks containing 30 percent water by volume. These results are sensitive to the choice of thermal constants, but those assumed above tend to favor a higher value for Q_L/Q and hence probably give an approximate upper limit of this ratio.

The heat released by refreezing the drilling fluid is on the order of 5×10^4 cal per cm ($\pi a^2\times 80$ cal per cm³) or only about 5 percent of that introduced by drilling. It is, therefore, probably of minor importance with regard to the present considerations.

The ratio $\frac{s_L}{s}$ varies from about one-fourth near the surface to about one-half at 595 feet. Thus, from equation (16) the error introduced by latent heat in supposing the ground temperature to return to equilibrium as $\log_e\left(\frac{t}{t-s}\right)$ probably does not exceed a factor of $1+\frac{1}{5}\frac{s}{t}+O\left(\frac{s}{t}\right)^2$.

EFFECT OF IDEALIZING THE PROBLEM

If it were supposed that the temperature at each depth returned to equilibrium as $\log_e(t/t-s)$, the foregoing discussion gives an idea of the error that would arise for $t>>s$ from various neglected factors.

1. From the variation of source strength with time, the relative error is negative and probably does not exceed $-\frac{1}{6}\left(\frac{s}{t}\right)$.
2. From the finite radius of the hole, the relative error is negative and probably does not exceed $-10^{-2}\frac{s}{t}$.
3. The relative error introduced by neglecting the effect of latent heat of refreezing is positive and probably does not exceed $+\frac{1}{5}\left(\frac{s}{t}\right)$, but it may well be considerably less.
4. Neglecting appreciable modification of effective source strength due to stratification, the error introduced by variation of source strength with depth at depths greater than about 200 feet is probably less than 1 percent of the temperature disturbance for

the first 5 years of observations and less than 0.01 °C for all time.

5. At depths less than about 200 feet, neglecting the effect of the ground surface can introduce significant error for large values of time. However, at depths greater than 200 feet, this effect probably never exceeds the probable error inherent in the temperature measurement.
6. The effect of stratification is probably negligible for all time if the strata may be considered homogeneous for 175 feet above and below the observation point. Uniformly distributed interbeds on the order of 6 to 8 feet thick are probably unimportant for times exceeding 5 years. The effect of stratification boundaries at intermediate distances from the observation point can probably introduce appreciable errors under unfavorable circumstances.

Factors (1), (2), and (3) tend to cause a departure from the idealized temperature distribution along the radial coordinate only. These will be referred to below as "radial effects." Factors (4), (5) and (6), the "longitudinal effects," tend to produce a longitudinal distortion of the idealized temperature field. In general, the relative error in temperature calculations resulting from neglecting radial effects tends to decrease with increasing time. The corresponding discrepancies resulting from neglecting longitudinal effects in many cases increase with increasing time.

If it were assumed that the cooling proceeds as $\frac{\bar{q}}{4\pi K}\frac{s}{t-s/2}$, case III, table 1, the results would agree with those obtained by the use of $\frac{\bar{q}}{4\pi K}\log_e\left(\frac{t}{t-s}\right)$, case IV, table 1, to terms of order $(s/t)^2$, and the choice between the two forms is somewhat arbitrary. Case III corresponds to the cooling of a homogeneous infinite medium heated initially by an instantaneous linear heat source released in the center of the drilling period ($t=s/2$). The logarithmic formula describes the cooling of the same medium heated initially by a linear source of constant strength persisting throughout the drilling period ($0<t<s$). In the analysis of data that follows, the logarithmic form is used.

ACCURACY AND METHOD OF TEMPERATURE MEASUREMENT

The temperature-sensitive elements used are small disc-type thermistors that have been individually calibrated in the Geological Survey laboratories. The thermistors have a negative temperature coefficient of resistance of about 6 percent per degree centigrade and a total resistance of about 5,000 ohms at −10° C. They were spliced into multiconductor cables which were installed in the well in such a way that temperatures could be measured at 20-foot intervals between the depths of 75 and 595 feet. The measurement circuit permits an independent determination of the lead resistance with each set of observations (Swartz, 1954), making it possible to determine the net resistance of each thermistor in place. Under the field conditions prevailing when the earlier observations were made, it is estimated that temperature changes determined for any one thermistor were measured to within 0.01° C of their true value except in rare cases. As a result of refinements in observational procedure instituted by Brewer in December 1950, the precision of these measurements has been increased slightly, and the number of erratic readings reduced for measurements made since that time.

Less accuracy can be attached to the measurements of temperature than to those of temperature change with time. The individual temperature measurements are estimated to be accurate to ±0.1° C. Inasmuch as temperature changes measured on any one thermistor of only 0.01° C are treated as significant in this study, the stability of the thermistor as a temperature-measuring device must be considered. From 1948 to 1957 Max C. Brewer and J. H. Swartz have observed about 4,000 of these thermistors (Western Electric disc-type 17A) in the course of geothermal studies carried out by the Geological Survey. They have found that the calibration curves of some thermistors undergo a uniform offset which generally changes with time. A thermal shock usually initiates such "drift effects," and hence they are most common in thermistors that have been sealed into geothermal cables by vulcanizing (at 130° to 150° C). These results are in qualitative agreement with those obtained by Beck (1956) for bead-type thermistors. The thermistors used in the present study were not vulcanized, their resistance was measured by using d-c equipment with very low amperage; and as far as is known, they have not been subjected to temperatures greater than 40° C. Their thermal environment has been uniform since installation. The internal consistency of the data presented below provides perhaps the best justification for the present assumption with regard to the accuracy of measurement of temperature change.

The effect of pressure on calibration of Western Electric type 12A(s) thermistors was investigated by Misener and Thompson (1952). They found no measurable effect of temperature on the pressure coefficient of resistance. Assuming their results apply to the Western Electric type 17A thermistors, the effect of pressure on measurement of temperature change can be neglected. For the range of pressure anticipated at South Barrow well 3, Misener and Thompson found pressure coefficients of 1 to 3×10^{-6} in.^{2} per pound, or about one ten-thousandth of the temperature coefficient (in $°C^{-1}$) for the thermistors used in South Barrow well 3. Assuming again that the response to pressure is the same for the two types of thermistors, the temperature effect of pressure probably does not exceed the estimated error in temperature measurement (±0.1° C) throughout the depth penetrated by the thermal cables at South Barrow well 3.

ANALYSIS OF DATA

In analyzing the data, observed temperatures were plotted against $\log_e \frac{t}{t-s}$, where s, the drilling time, was determined to the nearest day from the driller's log, for each depth, and t is the time elapsed since the day the drill bit reached the depth in question.

Of the 27 depths from 75 feet to 595 feet, three were selected for detailed presentation. These are the 595-, 475-, and 355-foot depths represented, respectively, by tables 4, 5, 6 and by figures 30, 31, 32.

TABLE 4.—*Cooling data for 595-foot depth, South Barrow well 3*

[s=50 days, temperatures given in degrees centigrade]

t/s	Θ (observed)	Θ(computed) [=3.35 $\log_e (t/t-s)$ −6.735]	Θ(computed)− Θ(observed)	Estimated temperature dissurbance [=Θ(observed) +6.735]
1.60	−2.163	−3.448	−1.285	+4.570
1.74	−3.345	−3.870	−.525	+3.390
2.34	−4.829	−4.867	−.040	+1.905
2.66	−5.155	−5.155	.000	+1.580
3.34	−5.542	−5.543	.000	+1.195
3.94	−5.764	−5.760	+.005	+.970
6.76	−6.191	−6.199	−.010	+.545
6.92	−6.216	−6.212	+.005	+.520
7.20	−6.228	−6.234	−.005	+.505
7.48	−6.245	−6.254	−.010	+.490
7.90	−6.275	−6.281	−.005	+.460
8.46	−6.300	−6.313	−.015	+.435
9.00	−6.332	−5.340	−.010	+.405
9.84	−6.374	−6.376	.000	+.360
10.74	−6.408	−6.407	.000	+.325
11.54	−6.437	−6.431	+.005	+.300
12.62	−6.462	−6.458	+.005	+.275
13.34	−6.475	−6.474	.000	+.260
13.98	−6.487	−6.486	.000	+.250
14.62	−6.500	−6.498	.000	+.235
15.38	−6.508	−6.510	.000	+.230
16.96	−6.529	−6.531	.000	+.205
20.14	−6.563	−6.564	.000	+.170
23.92	−6.597	−6.592	+.005	+.140
28.86	−6.618	−6.617	.000	+.115
39.70	−6.651	−6.650	.000	+.085
40.46	−6.651	−6.651	.000	+.085

The 595-foot curve is one of the better ones, showing early agreement with the logarithmic formula. The curve for the 475-foot depth is more typical, and the curve for the 355-foot depth is among those showing the worst agreement. The solid line on figures 30, 31, and 32 represents a formula of the form

$$\Theta = A \log_e \frac{t}{t-s} + \Theta_0$$

TABLE 5.—*Cooling data for 475-foot depth, South Barrow well 3*

[s=51 days, temperatures given in degrees centigrade]

t/s	Θ (observed)	Θ(computed) [=3.80 log$_e$ $(t/t-s)$ −7.830]	Θ(computed) − Θ(observed)	Estimated temperature dissurbance [=Θ(observed) +7.830]
1.46	−2.741	−3.380	−0.910	+5.360
1.59	−3.780	−4.052	−.270	+4.050
1.72	−4.452	−4.534	−.080	+3.380
2.31	−5.751	−5.677	−.075	+2.080
2.63	−6.079	−6.008	+.070	+1.750
3.29	−6.517	−6.454	+.065	+1.315
3.90	−6.752	−6.704	+.050	+1.080
6.64	−7.219	−7.210	+.010	+.610
6.80	−7.235	−7.225	+.010	+.595
7.08	−7.254	−7.251	+.005	+.575
7.35	−7.281	−7.274	+.005	+.550
7.76	−7.312	−7.306	+.005	+.520
8.31	−7,347	−7,342	+.005	+.485
8.84	−7,376	−7,374	.000	+.455
9.66	−7,422	−7.415	+.005	+.410
10.54	−7.457	−7.451	+.005	+.375
11.33	−7.484	−7.479	+.005	+.345
12.39	−7.508	−7.510	.000	+.320
13.09	−7.527	−7.528	.000	+.305
13.72	−7.543	−7.542	.000	+.285
14.35	−7.554	−7.555	.000	+.275
15.09	−7.566	−7.569	−.005	+.265
16.64	−7.589	−7.594	−.005	+.240
19.75	−7.628	−7.633	−.005	+.200
23.46	−7.667	−7.664	+.005	+.165
28.30	−7.690	−7.693	−.005	+.140
38.92	−7.733	−7.731	.000	+.095
39.67	−7.733	−7.733	.000	+.095

TABLE 6.—*Cooling data for 355-foot depth, South Barrow well 3*

[s=56 days, temperatures given in degrees centigrade]

t/s	Θ (observed)	Θ(computed) [$=3.89 \log_e \frac{t}{t-s}$ −8.935]	Θ(computed) − Θ(observed)	Estimated temperature dissurbance [=Θ(observed) +8.935]
1.41	−4.550	−4.132	+0.420	+4.385
1.54	−5.565	−4.835	+.730	+3.370
1.66	−6.122	−5.347	+.775	+2.815
2.20	−7.141	−6.570	+.570	+1.795
2.48	−7.380	−6.928	+.450	+1.555
3.09	−7.706	−7.412	+.295	+1.230
3.64	−7.890	−7.686	+.205	+1.045
6.14	−8.289	−8.243	+.045	+.645
6.29	−8.308	−8.261	+.050	+.625
6.54	−8.326	−8.288	+.040	+.610
6.79	−8.345	−8.314	+.030	+.590
7.16	−8.370	−8.349	+.020	+.565
7.66	−8.404	−8.391	+.015	+.531
8.14	−8.445	−8.425	+.020	+.490
8.89	−8.485	−8.471	+.015	+.450
9.70	−8.518	−8.511	+.005	+.415
10.41	−8.548	−8.542	+.005	+.385
11.38	−8.577	−8.577	.000	+.360
12.02	−8.599	−8.597	.000	+.335
12.59	−8.618	−8.613	+.005	+.315
13.16	−8.636	−8.627	+.010	+.300
13.84	−8.643	−8.643	.000	+.290
15.24	−8.669	−8.671	.000	+.265
18.09	−8.710	−8.714	−.005	+.225
21.47	−8.750	−8.749	.000	+.185
25.88	−8.779	−8.782	−.005	+.155
35.56	−8.824	−8.824	.000	+.110
36.24	−8.827	−8.826	.000	+.110

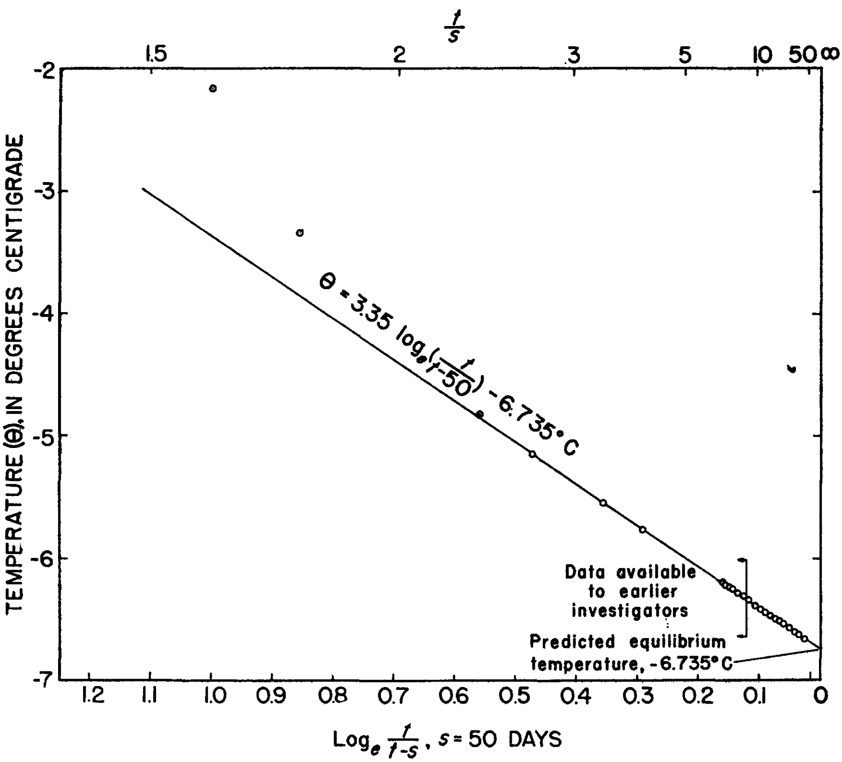

Figure 30.—Observed temperature plotted against $\log_e \left(\frac{t}{t-s}\right)$, 595-foot depth, South Barrow well 3.

where the two parameters A and Θ_0 were determined graphically from the observational data, and s was determined from the drilling record. It is to be noted that in the presentation of these data the temperatures have been entered to the thousandths decimal place. This is to make possible the determination of temperature difference to the nearest 0.005° C in order to avoid introducing rounding errors greater than the observational error in the measurement of such differences. The second and third decimal places have little or no significance with respect to absolute temperature values.

In table 7 the data are summarized for depths from 295 feet to 595 feet. The third column in this table shows the time, measured in multiples of the drilling period (t/s), of the first observation taken after the observational curve joins the logarithmic curve. In the fourth column is listed the number of subsequent points that do not lie within 0.01° C of the logarithmic curve. Most of the points represented in column 4 probably correspond to erratic readings for which the observational error was unusually large. Column 5 represents the equilibrium temperatures obtained by extrapolating to infinite

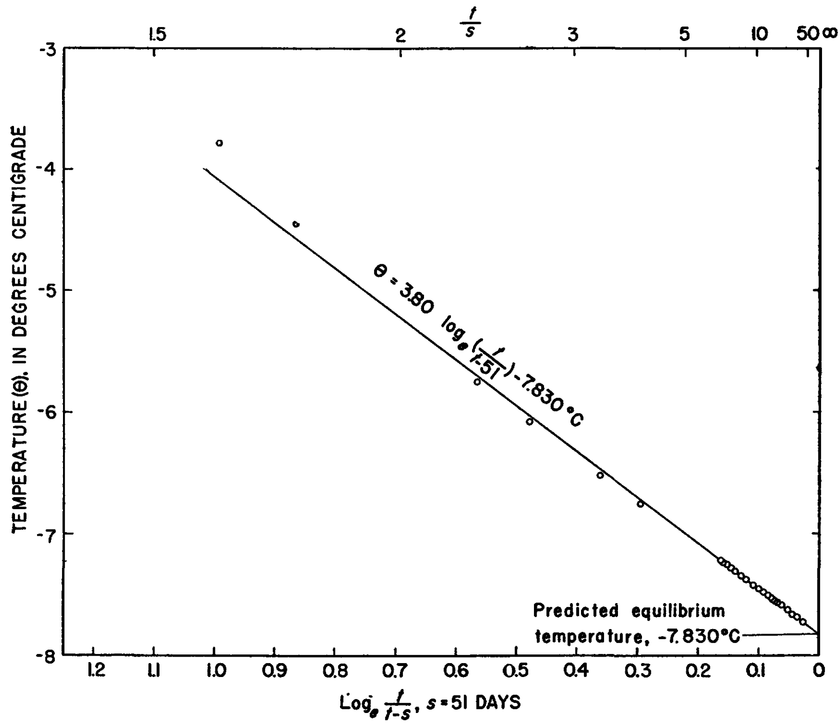

Figure 31.—Observed temperatures plotted against $\log_e\left(\frac{t}{t-s}\right)$, 475-foot depth, South Barrow well 3.

time using only the data obtained between $2\frac{1}{2}s$ and $4s$. In column 6 the equilibrium temperatures determined for data taken between $6s$ and $10s$ are tabulated. Column 7 represents the equilibrium temperatures determined from the data taken between $10s$ and $40s$. Assuming the equilibrium temperatures tabulated in column 7 to be approximately correct, column 8 gives the error that would have resulted from computing the equilibrium temperature without data taken after $4s$ (on the order of 6 months after the completion of drilling). Column 9 lists t/s at the time of the last observation, and column 10 lists the estimated temperature disturbance persisting at that time.

The data from 75 to 275 feet show indications of a secular change and require special treatment. They will be discussed separately in a later section.

In general, the observational curves fall off more steeply than their logarithmic approximations for observations taken shortly after refreezing. This is evidently the result of the influence of latent heat which introduces appreciable terms of higher order in $(s_L/(t-s+s_L)$ for small times $(t \approx s+s_L)$. For larger values of time the slope of the observational curve at some depths diminishes monotonically with time and reaches a constant value when the observational curve eventually joins the logarithmic curve from above. This is shown by the 595-foot depth, figure 30. More typically, the slope of the observational curve, reaches a minimum value and eventually joins the logarithmic curve from below (figures 31 and 32). Whether the first or second type of behavior occurs probably depends primarily upon whether or not the effect of variation of source strength with time ever dominates the effect of latent heat. The possibility that either effect can dominate is borne out by results of the theoretical analysis, which indicate that the magnitude of the two effects can be the same but that their signs are opposite. The few depths where the observational curve joins the logarithmic curve at very early times ($t \approx 2\frac{1}{2}s$), are attributed to a coincidental nullification of the effect of variation in source strength with time by the effect of latent heat. For all such depths, the observational curve joins the logarithmic curve from above as would be expected. (See figure 30.)

From the results of the preceding analysis, it seems unlikely that the combined effect of the factors tending to produce a distortion of

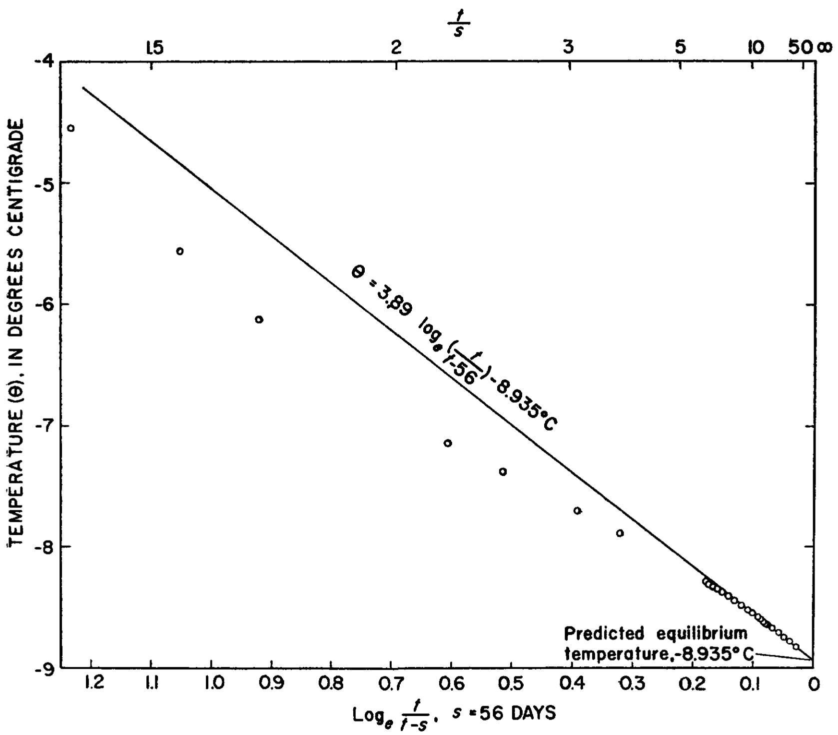

Figure 32.—Observed temperatures plotted against $\log_e \left(\frac{t}{t-s}\right)$, 355-foot depth, South Barrow well 3.

TABLE 7.—*Summary of cooling data for 295- to 595-foot depths, South Barrow well 3*

[Temperatures given in degrees centigrade]

1	2	3	4	5	6	7	8	9		10
Depth (feet)	Drilling period, s (days)	First observation falling on log curve $\left(\frac{t}{s}\right)$	Number of later points off curve	Θ_0(top), $2\frac{1}{2}<\frac{t}{s}<4$	Θ_0(middle), $6<\frac{t}{s}<10$	Θ_0(bottom), $10<\frac{t}{s}<40$	Θ_0(bottom), $-\Theta_0$(top)	Last observation		Estimated temperature disturbance at last observation
								$\frac{t}{s}$	Temperature	
295	57	8.0	1	−9.37	−9.39	−9.360	0.00	35.5	−9.260	0.100
315	57	7.1	2	−9.16	−9.26	−9.250	−.09	35.5	−9.144	.105
335	57	11.2	2	−8.97	−9.07	−9.105	−.14	35.5	−8.989	.115
355	56	9.7	0	−8.73	−8.87	−8.935	−.21	36.1	−8.824	.110
375	56	6.5	0	−8.60	−8.73	−8.730	−.13	36.1	−8.633	.095
395	56	6.1	2	−8.32	−8.53	−8.530	−.21	36.1	−8.428	.100
415	52	6.5	2	−8.34	−8.37	−8.365	−.03	38.8	−8.271	.095
435	52	2.6	2	−8.18	−8.180	−8.180	.00	38.8	−8.086	.095
455	51	6.6	1	−7.94	−7.950	−7.960	−.02	39.6	−7.861	.100
475	51	6.6	0	−7.85	−7.830	−7.830	+.02	39.6	−7.733	.095
495	51	6.6	0	−7.60	−7.61	−7.590	+.01	39.6	−7.500	.090
515	51	6.6	2	−7.48	−7.46	−7.450	+.03	39.6	−7.351	.100
535	51	8.3	0	−7.29	−7.26	−7.250	+.04	39.6	−7.158	.090
555	51	7.4	1	−7.14	−7.10	−7.110	+.03	39.6	−7.012	.100
575	50	3.3	1	−6.91	−6.870	−6.865	+.05	40.0	−6.777	.090
595	50	2.7	1	−6.765	−6.735	−6.735	+.03	40.1	−6.651	.085

the temperature field along the radial coordinate would, for large times produce a negative relative error exceeding about $-\frac{1}{6}\frac{s}{t}$. Such a discrepancy would first fall within instrumental error (±0.01° C), at times on the order of 8s (about 1 year after the completion of drilling). It is shown in column 3, table 7, that the observational curve does actually coincide with the logarithmic approximation, beyond times of about 8s for all depths except 335 and 355 feet. For many depths the first agreement is shown for the observation taken at about 6⅓s, and this corresponds to a relative error of about $-\frac{1}{10}\frac{s}{t}$. Inasmuch as as $-\frac{1}{6}\frac{s}{t}$ is a fairly liberal estimate of the upper limit of this radial error, it is probable that minor stratification effects are present at depths showing first agreement, at times on the order of 7s or 8s.

The first agreement between the observational curve and the logarithmic approximation for the 355- and 335-foot depths does not occur until $t \approx 10s$ and 11s, respectively. If these discrepancies were to be accounted for solely by radial disturbances of the temperature field, they would correspond to relative errors of about $-\frac{1}{4}\frac{s}{t}$ and $-\frac{1}{3}\frac{s}{t}$, respectively. The preceding analysis is not so precise that negative errors of this order can be ruled out for radial effects, but it suggests that they are extremely unlikely. If for the moment it were supposed that these errors did result from radial effects alone, it would be necessary to assume an extremely low moisture content in this depth range because the sign of the discrepancy requires that it be attributed to a strong dominance of the effect of variation of source strength with time over the effect of latent heat. But an abnormally low moisture content itself suggests a change in lithologic character and a corresponding change of thermal properties. From table 7, column 8, it is seen that the only depths at which the equilibrium temperatures determined from early observations differ by more than 0.05° C from those determined from late observations are those in the depth range from 315 to 395 feet. In the light of the discussion in the section on the effect of change in thermal properties with depth, these lines of evidence indicate that significantly different thermal properties occur in the 300- to 400-foot depth range. According to this interpretation, the 335- and 355-foot depths are farthest from the effective stratification interfaces; and the stratification effects there are profound, producing late agreement with the logarithmic curve (column 3, table 7). The outer beds (315-, 375-, and 395-foot depths), being closer to the effective interface (probably in zones of interstratification) show an earlier change of slope on the logarithmic cooling curves, as shown by the large value in column 8 and the normal values in column 3, table 7. In an earlier discussion it was concluded that strata of this thickness might introduce significant departures from a cooling formula based on the homogeneous case.

From table 7 it is seen that the estimated temperature disturbance was about 0.1° C at the time of the last observation. Application of equation 3 shows that about 50 years of cooling will be required before the drilling disturbance diminishes to 0.01° C in the depth range studied.

An additional set of observations was taken 9 years after the completion of drilling, or more than 3 years after the last observation available when this paper was prepared. They agree, within observational error, with values predicted on the basis of the theory presented above.

COMPARISON OF RESULTS WITH EARLIER WORK

MacCarthy (1952) and Nakaya (1953) have both presented formulas to describe the restoration of thermal equilibrium in South Barrow well 3. At the time of their work, however, only about 1 year's data were available (to $t \approx 8\frac{1}{3}s$). Nevertheless, it is informative to review their conclusions in the light of the additional 5 years' data that have since become available.

MacCarthy observed that the temperature at any depth could be closely approximated as a function of time by an empirical relation of the form

$$\Theta(x) = \frac{x}{ax+b} + c \tag{18}$$

where x is the time in days referred to an arbitrary origin, and the three parameters a, b, and c are adjustable, taking on different values for each depth. On the basis of the success of this formula in the time range for which observations were available, he concluded that the equilibrium temperature should be given with "fair accuracy" by formally letting x become infinite in equation (18).

Using the values of the parameters given by MacCarthy for the 595-foot depth and rewriting equation (18) in the notation of this paper we obtain

$$\Theta(t) = 106.54 \frac{1}{t-54} - 6.57 \tag{19}$$

where t is measured in days and Θ in ° C. By comparing equations (1) and (19) it is seen that the empirical formula is equivalent to one based on the assumption that the drilling operation behaved as an instantaneous linear heat source released 54 days after the bit first reached the 595-foot depth. At this depth $s=50$ days, and hence

MacCarthy's formula corresponds approximately to case II in table 1. It has been shown that case IV provides a reliable representation of the drilling operation, and hence we should have expected the empirically determined value of t_0 to be about $s/2=25$ days (case III, table 1). The large values of t_0 in (19) evidently arose from an attempt to fit 18 to the data for times so small that terms of higher degree representing latent heat effects were still appreciable. In fact from equation (16) it can be shown that if MacCarthy's empirical formula, which had the approximate form $\frac{s}{t-s}$, were valid to terms in $(s/t)^2$, it would be required that $Q/Q_L \approx s_L/s$; that is, that the latent heat liberated on refreezing be on the order of 10 times larger than its probable value.

Nakaya (1953) pointed out the danger of large error in extrapolating an empirical formula to infinity, and the desirability of determining equilibrium temperatures to within the margin of instrumental error in order that the results might be applied to the study of secular changes in permafrost temperatures. To supply a "theoretical" extrapolation formula he wrote down the series

$$-\frac{d\Theta}{dt}=\sum_{n=1}^{\infty} k_n\left[\Theta(t)-\Theta(\infty)\right]^n$$

$$k_n=\text{undetermined constants} \tag{20}$$

and observed that after the passage of sufficient time, terms of higher degree than the first can be neglected. Integrating the simplified expression gives his formula:

$$\Theta(t)=Ae^{-k_1 t}+\Theta(\infty) \tag{21}$$

where the three parameters A, k_1, and $\Theta(\infty)$ are adjusted to give the best fit to the data.

It is clear that a series of the form (20) would converge, and that terms of higher degree might ultimately be neglected. Inasmuch as the form of the function $\frac{d\Theta}{dt}$ is unknown, however, the k_n are unknown, and there is no indication of how large t must be before equation (21) applies. Therefore, equation (21) seems to have little more theoretical justification in the present application than does the original empirical formula, equation (18), given by MacCarthy.

In figure 33 the temperatures predicted by MacCarthy's empirical formula and Nakaya's theoretical formula are compared with the observed temperatures. The formula derived from the present analy-

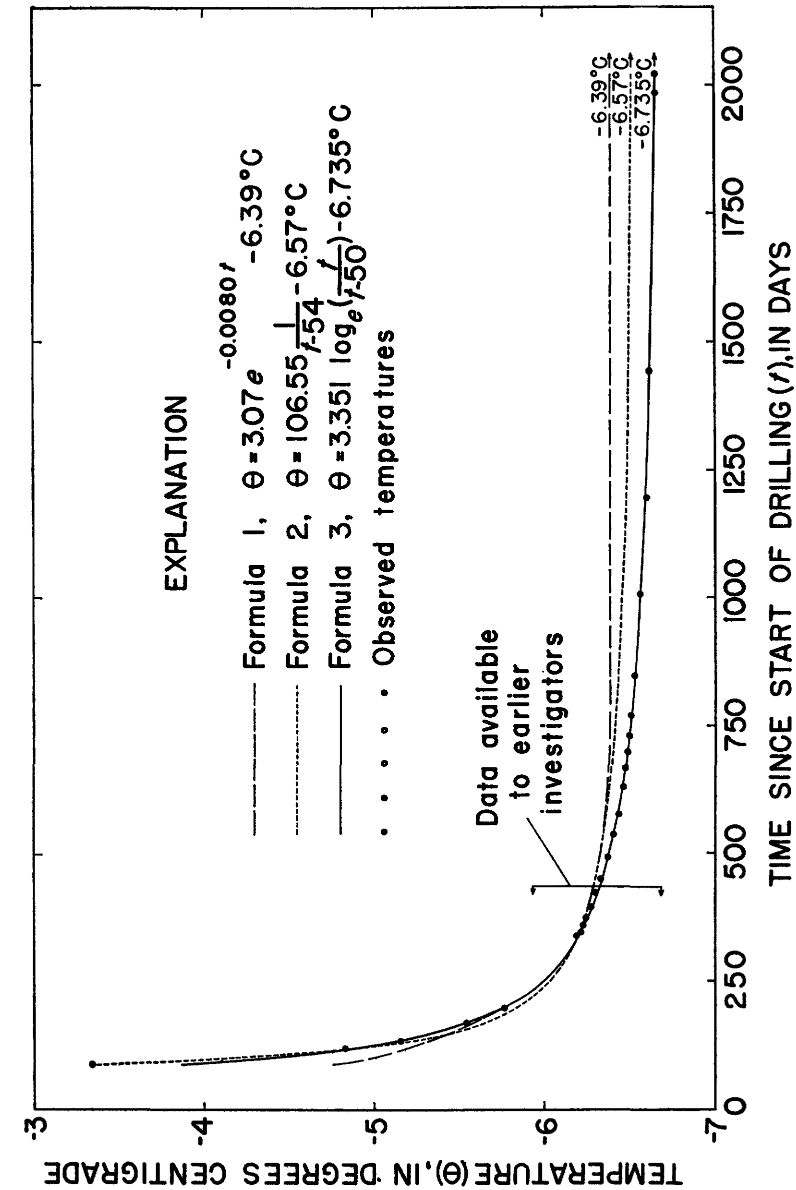

FIGURE 33.—Comparison of cooling formulas, 595-foot depth, South Barrow well.

sis is also shown. The present formula coincides with the observed temperatures within instrumental error (±0.01° C) except for times less than about $2\frac{1}{3}s$. From figure 30 it can be seen that if only the data available to earlier investigators were used, the results of the present analysis would not differ appreciably from those obtained using the additional 5 years' data. It is also to be noted that the earlier formulas each contain 3 adjustable parameters, whereas the present formula contains only 2.

It must be pointed out that between the time of the earlier work and the present work the temperature values have been refined by the application of small instrumental corrections and carried out to the third decimal place in degrees centigrade. This work was done before the present interpretative study. A few of the temperatures used by the earlier workers have been changed by as much as 0.015° C, but they do not seriously effect the fit of the earlier formulas to the observed data.

In retrospect it can be seen that MacCarthy's empirical formula gave remarkably good predictions of later temperatures. It has been shown above that the basic form of his expression is theoretically sound. Its ultimate failure evidently resulted from an attempt to fit it to the data while the effects of latent heat were still appreciable.

Nakaya's insistence that the extrapolation formula have theoretical justification was a valuable contribution. However, his formula evidently does not apply to the present problem.

COMPUTATION OF SECULAR TRENDS

The primary application of the foregoing discussion is to the study of secular trends in earth temperatures that are measurable over the period during which observations are available. Such trends reflect changes in temperature at the earth's surface and hence are related to climatic variation. It has been mentioned earlier that such a secular change is indicated by temperatures obtained at the 11 measurement points from depths of 75 to 275 feet. A detailed discussion of the climatic implication of these trends must await the completion of a systematic analysis of the data. In this brief section a typical set of data will be presented only to show the application.

On the basis of the previous section, if the temperature at a point is experiencing a linear secular variation, m° C per day, in addition to the thermal effects of drilling, it should be given to a good approximation by a relation of the form

$$\Theta = A \log_e \frac{t}{t-s} + \Theta_0 + mt, \; t/s \text{ large.} \qquad (22)$$

Here A is a constant which at South Barrow well 3 has a value of about 3° or 4° C, depending on the depth. Setting

$$y = -\log_e \frac{t}{t-s}$$

we obtain on differentiating

$$\frac{d\Theta}{dy} = -A + m\,\frac{t(t-s)}{s}. \qquad (23)$$

The second term in equation (23) represents the effect of linear secular change on the slope of the curve of the temperature plotted in the manner of figures 30, 31, and 32. Inasmuch as this error grows as t^2, it will introduce a progressively increasing slope for larger values of time, but small secular changes will go unnoticed for intermediate values of time. For example, at South Barrow well 3, a positive secular change at the rate of 0.01° C per year will effect the slope of curves like those in figures 30, 31, and 32 by 10 percent only after 2½ years have elapsed since drilling. After 7½ years, however, it will cause the slope to change sign.

In figure 34 the temperatures observed at the 135-foot depth, plotted against $\log_e \frac{t}{t-s}$, are represented by solid circles. The curvature for large values of time can be explained in terms of a linear secular increase of temperature at that depth at a rate of 0.025° C per year. The temperatures corrected for this effect are plotted as crosses in figure 34. These crosses are colinear within observational error (±0.01° C). Barring an anomalous behavior of the drilling disturbance at this depth, the error in the estimate of rate of secular change is less than 10 percent. From table 3 it can be seen that the effect of the proximity of the measurement point to the ground surface is not great enough to affect appreciably the behavior of the drilling disturbance over this time range at 135 feet.

It might be argued that the curvature could result from a linear calibration drift in the temperature-measuring element. This alternative is rendered unlikely however, by the fact that the effect is clearly a function of depth. A preliminary analysis shows that the rate of secular change decreases systematically from the 75-foot depth to the 275-foot depth. From 295 to 595 feet no measurable curvature is found in the logarithmic cooling curves for larger values of time.

By repeating the above analysis the current rate of secular change can be calculated at the depths at which it is measurable. By extrapolating the logarithmic cooling curves to infinite time, the predrilling thermal profile to 595 feet can be reconstructed. Then by upward extrapolation of the linear portion of the profile (below 315 feet) the total temperature change at each depth can be estimated.

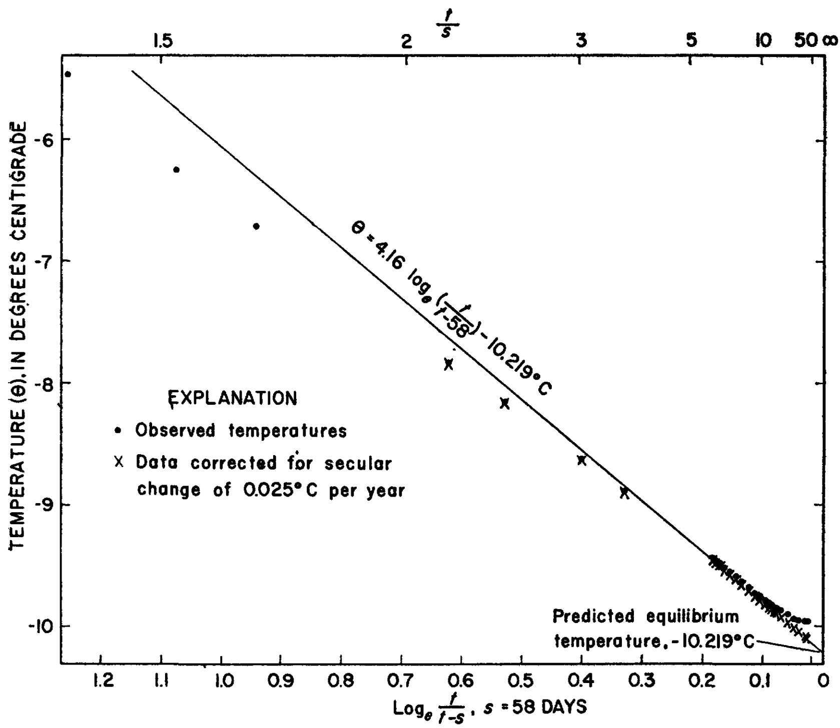

Figure 34.—Observed temperatures (circles) plotted against $\log_e \left(\frac{t}{t-s}\right)$, 135-foot depth, South Barrow well 3. Crosses represent data corrected for secular change of 0.025° C per year.

This yields 23 independent data from which to reconstruct the thermal history of this recent event at the earth's surface. It is hoped that this work, which is in progress, will give some new information on the widely discussed problem of the warming of the Arctic.

Preliminary results indicate that the mean annual ground surface temperature has increased on the order of 3° C in the last 50 to 75 years at South Barrow well 3. Furthermore the data suggest that the greater part of the change occurred in the second-half of this time interval. Since these results are preliminary, however, they must be treated with caution. Furthermore the regional significance of such local information can be ascertained only after considerably more study. It is reassuring however, that a preliminary analysis of similar data from Cape Simpson, 50 miles east of Barrow, yields comparable results. It is important to emphasize that such changes in ground surface temperature do not necessarily reflect equal changes in air temperature, for systematic changes in any of several factors involved in heat exchange at the ground surface can exercise an appreciable influence on mean annual temperatures. One of the more important of these factors, the geothermal effect of winter snow cover, is discussed in another paper (Lachenbruch, 1959).

ESTABLISHMENT OF AN EQUILIBRIUM GRADIENT

When geothermal data are to be used to determine the outward flux of earth heat, the equilibrium thermal gradient rather than the equilibrium temperature is of principal importance. Inasmuch as logistic considerations frequently require that temperature measurements be made shortly after the completion of drilling, an unknown transient anomaly is commonly present in the observed gradient. This in turn can introduce considerable uncertainty into the determination of heat flow.

A general analytical treatment of the restoration of equilibrium thermal gradients after drilling is beset with many difficulties, and it will not be attempted here. It is informative, however, to look at the special case of the transient behavior of the gradient at depths not affected by secular change in South Barrow well 3. Selected results are plotted in figure 35 and summarized in table 8. The straight lines represent the least-square approximations to the data. It is interesting that only 67 days after the completion of drilling, the gradient defined by the least-square line is only 5 percent greater than the probable equilibrium value. The individual temperatures at that time are still about 2° C greater than the values predicted for equilibrium. The general trend in gradients, decreasing from 35° C per km at 37 days to 29° C per km at 400 days, can be explained by the more rapid refreezing at the shallow positions owing to the lower

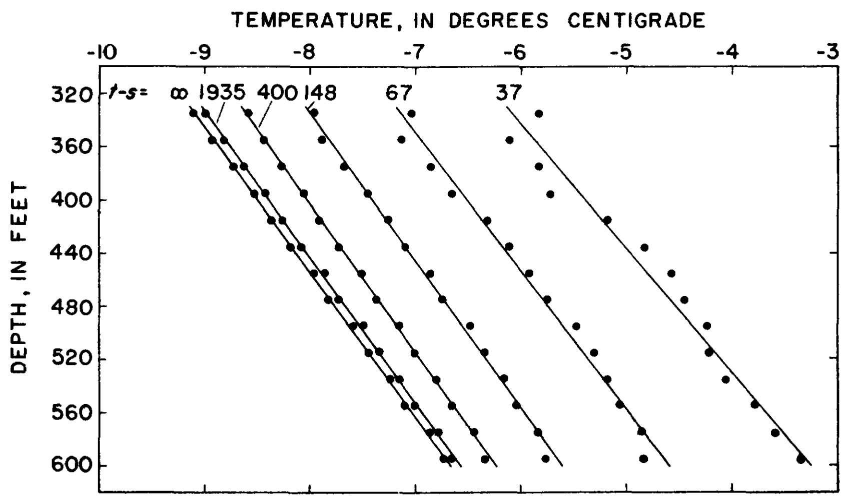

Figure 35.—Return of the geothermal gradient to equilibrium, South Barrow well 3.

predrilling temperatures there. An interesting feature of figure 35 is the progressive improvement of the linear approximation for times up to about 400 days ($t/s \approx 9$). This is shown in table 8 by the standard deviations of temperature, or more properly, the correlation coefficients. It has been shown that in this time range the erratic effects of higher order terms due to latent heat, distribution of source strength with time, and stratification are dominant. For times exceeding 400 days the linear approximation shows no significant improvement, but the gradient undergoes a measurable increase from about 29° C per km to 30° C per km.

TABLE 8.—*Return of the geothermal gradient to equilibrium, South Barrow well 3*

Postdrilling period, $t-s$ (days)	Gradient (° C/Km)	Extrapolated surface temperature (° C)	Standard deviation of Θ (° C)	Correlation coefficient
37	35.0	−9.67	0.17	0.984
67	31.6	−10.37	.10	.988
148	29.3	−10.99	.05	.998
400	29.2	−11.59	.02	.999
1935	29.9	−12.04	.02	.999
∞	30.1	−12.17	.02	.999

This is explained in terms of the earlier discussion by recalling that in this time range the temperature at any depth is given to a good approximation by

$$\Theta = \frac{q}{4\pi K} \log_e \frac{t}{t-s} + \Theta_0$$

hence

$$-\frac{d\Theta}{dt} = \frac{qs}{4\pi K} \frac{1}{t^2} \left(1 + \frac{s}{t} + \ldots\right).$$

As explained earlier, q generally decreases with increasing predrilling temperatures and hence with depth, and s decreases with depth. Thus, the rate of cooling generally decreases with depth for large values of time.

HEAT FLOW FROM THE EARTH'S INTERIOR

It has been suggested (Misener, 1955) that the large thermal gradients reported for Barrow wells (MacCarthy, 1952; Brewer, 1958) are indicative of an anomalously large flow of heat from the earth's interior in the Barrow area. This inference would seem consistent with the anomalous value computed for Resolute Bay, Canada (Misener, 1955). However, it is likely that the anomalous gradient at Resolute Bay, and at some of the Barrow wells, is the result of thermal effects of surface bodies of water (Lachenbruch, 1957). South Barrow well 3, however, is far from bodies of water, but the gradient of 30° C per km would lead to normal heat flow if the conductivity ranged between 3 and 4×10^{-3} cal ° C^{-1} sec^{-1}. Although no conductivity data are available, the section under study is predominantly shale and hence conductivities of this magnitude are to be expected. Thus there seems to be no reason at present, to suspect anomalous heat flow in the Barrow area.

CONCLUSION

Consistency of evidence obtained from empirical and theoretical sources regarding the cooling of South Barrow well 3 suggests that the major features of the cooling curves can be accounted for theoretically. For times exceeding about eight times the duration of the drilling period the temperature effect of drilling can probably be accounted for to within about 0.01° C by a simple logarithmic formula with two adjustable parameters, provided profound effects of stratification do not exist. With the exception of five depths, presumably influenced by stratification effects, the equilibrium temperatures predicted by the present method from 6 months' cooling data are within 0.05° C of those predicted by the use of 6 years' data. (See lines 10–14, p. 100.)

After 6 years of cooling, the drilling disturbance diminished from about 20° C to about 0.1° C in the upper 600 feet of the well under study. It is expected that about 50 years will elapse between the time of drilling and the time that the drilling disturbance decreases to 0.01° C in this depth range.

This work forms an adequate basis for the study of secular changes in ground temperature that are measurable during the period of observation. By correcting the observed temperatures for the effects of drilling, it is possible to detect residual secular changes in natural earth temperatures as small as 0.01° C per year long before thermal equilibrium is restored. Changes of this order have been observed in the upper portion of the depth interval studied. Preliminary results suggest that such changes correspond to an increase in temperature at the ground surface on the order of 3° C during the past 50 to 75 years.

It is highly unlikely that the thermistors used in this study experienced any significant calibration shift during the 6-year period of observation. If they did, they would have to drift in a very special way indeed to fit the internally consistent and theoretically rational pattern exhibited by the data.

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