Chemical Composition as a Guide to the Size of Sandstone-Type Uranium Deposits in the Morrison Formation on the Colorado Plateau

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GEOLOGICAL SURVEY BULLETIN 1112-B

Prepared on behalf of the U.S. Atomic
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U.S. DEPARTMENT OF THE INTERIOR
March 3, 1849

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By A. T. MIESCH, E. M. SHOEMAKER, W. L. NEWMAN, *and* W. I. FINCH

CONTRIBUTIONS TO ECONOMIC GEOLOGY

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CHEMICAL COMPOSITION AS A GUIDE TO THE SIZE OF SANDSTONE-TYPE URANIUM DEPOSITS IN THE MORRISON FORMATION ON THE COLORADO PLATEAU

By A. T. Miesch, E. M. Shoemaker, W. L. Newman, and W. I. Finch

ABSTRACT

The concentrations of uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel in 75 mill-pulp samples of uranium deposits in the Salt Wash member of the Morrison formation on the Colorado Plateau have been found, by statistical tests, to be significantly related to the size of the deposits represented by the samples.

The elements mentioned above are related to the formation of the deposits in a variety of ways. Zirconium is an intrinsic element, contained principally in the detrital syngenetic fraction of the host sandstone. Calcium, manganese, and sodium are intrinsic elements contained principally in epigenetic (diagenetic) carbonate in the host sandstone. Uranium, yttrium, and nickel are principally extrinsic elements, introduced into the host sandstone by uranium mineralization or related processes. Somewhat more than half of the iron is probably intrinsic and the remainder is extrinsic.

Three methods can be used to estimate the size of uranium deposits in the Salt Wash member within broad limits. Method 1 is based on simple linear-regression theory; method 2 is based on multiple-regression theory (long method); and method 3, on multiple-regression theory (short method).

For methods 1 and 2 the estimated log size of each deposit can be computed from tables showing the known concentration of uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel in the deposits, as determined by semiquantitative spectrographic analysis. For method 3 the estimated size or log size can be read directly from a table showing known concentration of uranium and yttrium only.

About 80 percent of the tonnage-size estimates from method 1 will be within a factor of 13 (12–14) of the true sizes. The precision of the size estimates from method 2, the long multiple-regression method, is highly variable. Some estimates from method 2 will be within a factor of 12 of the true size at the 80-percent confidence level; others, within a factor of 40 at the 80-percent confidence level. About 80 percent of the tonnage-size estimates from method 3, the short multiple-regression method, will be within a factor of 15 (13–16) of the true size.

A group of 40 deposits of known size was used to test the theoretical derivation of the confidence intervals given above. It was concluded from the test that the confidence intervals describe the precision of the methods correctly.

The methods for estimating the size of uranium deposits are useful where the ore is poorly exposed or where an independent estimate is desired. The error of the estimates may be quite large, as indicated by the confidence limits given above; but the estimates can be used to, at least, distinguish very large from very small deposits. They also may serve to encourage or discourage further expenditures in the development and exploration of ore bodies. The methods of size estimation can be particularly useful in attempts to appraise or compare groups of deposits or mining districts, inasmuch as the average estimate of size of deposits in a group is more precise than any single estimate.

The methods for estimating size are established only for deposits in the Salt Wash member of the Morrison formation. Tests indicate that the equations calculated for deposits in the Salt Wash fail completely if applied to deposits in other stratigraphic units, such as the Moss Back and Shinarump members of the Chinle formation. A further restriction, not completely evaluated at present, is that semiquantitative spectrographic analyses of mill-pulp samples are required. No tests have been made to determine the precision and accuracy of the methods when other types of samples, such as drill core, are used.

INTRODUCTION

This study is part of a broader investigation of the distribution of elements in rocks and ores of the Colorado Plateau, conducted by the U.S. Geological Survey on behalf of the Division of Raw Materials of the U.S. Atomic Energy Commission. During the investigation significant correlations were discovered between the concentration of certain elements in the uranium deposits and the calculated size[1] of the deposits. The uranium deposits discussed in this report are all in sandstone of the Salt Wash member of the Morrison formation of Jurassic age and are located in widely separated areas on the central part of the Colorado Plateau (fig. 3).

Elements whose concentrations appear to be related to the size of the deposits are uranium, nickel, yttrium, zirconium, iron, manganese, sodium, and calcium. Hence, known concentrations of these elements in a deposit in the Salt Wash member on the Colorado Plateau can be used to estimate the size of the deposit. Though such estimates have rather broad ranges of error, they are significantly more accurate than estimates obtained by considering the deposit to be of mean size, that is equal to the mean size of all deposits in the Salt Wash member. For many deposits, a knowledge of the geologic habit of the ores can facilitate estimating the size of a deposit to within a much narrower range of error than can be done with the methods given in this report. However, in areas where the deposits have a considerable range in size and where the ore is poorly exposed, the methods described here can be used to advantage.

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[1] Calculated size of a uranium deposit is determined by consideration of past production and total reserves and is assumed to be the true size. Estimated or predicted size is determined by the methods described in this report.

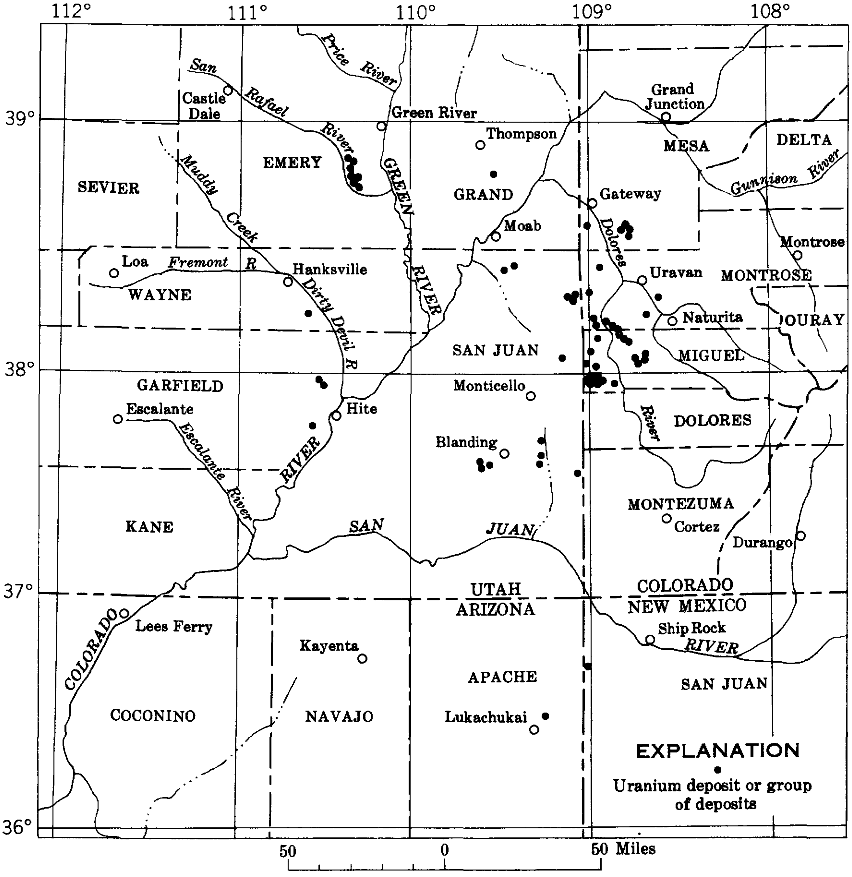

FIGURE 3.—Map of part of the Colorado Plateau showing the location of 75 uranium deposits in the Salt Wash member of the Morrison formation sampled for this investigation.

Lasky (1950) has shown an inverse relation between tonnage and grade of ore within certain porphyry-copper deposits that can be used to predict ore reserves; that is, more low-grade ore than high-grade ore is present, and that the amount of ore is inversely related to a specified grade. Lasky (1950) has also found similar relationships in bedded manganese deposits in the Artillery Mountains, Ariz.; in the Idaho-Wyoming phosphate-vanadium deposits; in the Falconbridge nickel deposit in the Sudbury district, Ontario; and in the Juneau gold deposit, Alaska. The tonnage-grade and tonnage-composition relationships described in this report differ from the type described by Lasky mainly in that they are found among, rather than within, deposits.

Three methods for estimating the size of uranium deposits in the Salt Wash member of the Morrison formation on the Colorado Plateau are given in this report. One method is based on simple linear regression and two are based on multiple regression. The multiple-regression equations were computed for the writers by statisticians of the U.S. National Bureau of Standards. Churchill Eisenhart, Chief of the Statistical Engineering Laboratory of the Bureau, suggested the methods. The computations were made by J. M. Cameron and W. S. Connor, with the assistance of Carroll Dannemiller and Marion Carson. The writers wish to express their sincere thanks to these members of the National Bureau of Standards for their very helpful consultation on the general aspect of the problem and for the great amount of time they devoted to it. The sections on size estimates by multiple regression and most of the analysis of variance (table 9) are based on their work.

Mary Epling of the National Bureau of Standards read the manuscript and brought to our attention several statistical and other problems relating to the manner of presentation. We are also indebted to W. C. Krumbein of Northwestern University who read the manuscript and helped to interpret some aspects of statistical theory in terms of this problem.

GEOLOGY OF THE URANIUM DEPOSITS

Sandstone-type uranium deposits on the Colorado Plateau consist of sandstone or conglomerate that has been impregnated, and commonly partly replaced, by uranium minerals. The deposits range in size from less than 1 ton to more than 100,000 tons. Many of them are irregularly tabular; the margins of some conform roughly to the bedding of the sandstone host rocks, whereas the margins of others cut across bedding along wavy or curved structures called rolls (Fischer, 1942, p. 367; Shawe, 1956, p. 239–241). Some uranium deposits are highly irregular in shape. The distribution of known uranium deposits on the Colorado Plateau area is shown on a map prepared by Finch (1955).

The mineralogy of uranium deposits on the Colorado Plateau has been summarized by Weeks and Thompson (1954). The uranium deposits contain two contrasting suites of ore minerals: (a) a suite of low-valent oxides and silicates, and (b) a suite of high-valent oxygen salts which may be interpreted as having been derived by oxidation of the low-valent suite. Relatively unoxidized uranium deposits in the Salt Wash member of the Morrison formation contain uraninite [UO_2] and coffinite [$U(SiO_4)_{1-x}(OH)_{4x}$]. Vanadium is generally more abundant in these deposits than uranium and occurs chiefly in montroseite [$VO(OH)$], doloresite [$3V_2O_4{\cdot}4H_2O$], and micaceous vanadium silicates. In most uranium deposits in the Salt Wash mined to date, the uranium is contained chiefly in the uranyl vanadates, carnotite [$K_2(UO_2)_2(VO_4)_2 \cdot 1\text{–}3H_2O$] and tyuyamunite [$Ca(UO_2)_2(VO_4)_2 \cdot 7\text{–}10\frac{1}{2}H_2O$].

SELECTION OF DEPOSITS FOR STUDY

In the broader investigation of the distribution of elements, mill-pulp samples from 200 deposits were analyzed by semiquantitative spectrographic methods. Of these deposits, 75 of known size were used in this study. These 75 deposits provide a range in size from very small (less than 10 tons) to very large (more than 10,000 tons), and represent a geographic sampling of uranium deposits in the Salt Wash member of the Morrison formation on the Colorado Plateau.

METHODS OF SAMPLING AND SAMPLE PREPARATION

The samples of uranium ore were obtained from mills and buying stations and are pulverized quarters of splits that were assayed to determine payment to the shippers. Most of the samples studied were obtained from the U.S. Atomic Energy Commission plant at Monticello, Utah, operated at the time by the American Smelting and Refining Co. The pulp samples were ultimately ground to 120 mesh in cast-iron disk grinders, and nearly all are probably contaminated slightly by tramp iron. The amount of ore represented by the samples from each mine ranges from 1 to more than 2,000 tons and averages about 150 tons (table 1).

Hand samples of unmineralized sandstone from the Salt Wash member were collected by the U.S. Geological Survey and furnished by Robert A. Cadigan. The samples represent the typical lithologic character of the Salt Wash member in widely separated areas on the Colorado Plateau. Analytical data from these samples are given in figures 4 to 11.

ANALYTICAL METHOD

Each of the 75 deposits was represented by a spectrographic analysis of a mill-pulp sample of one or more ore shipments from the mine. The analyses were made by a rapid semiquantitative spectrographic method under the supervision of A. T. Myers of the U.S. Geological Survey. R. G. Havens is the principal analyst, but some analyses were made by P. R. Barnett, G. W. Boyes, Jr., and P. J. Dunton.

The analytical method is described by A. T. Myers (written communication, 1957) as follows:

> In this procedure a weighed amount of the powdered sample is burned in a controlled d.c. arc and the spectrum recorded on a photographic plate. Selected lines on the resulting plate are visually compared with those of standard spectra prepared in a manner similar to that for the unknowns. The standard spectra

were prepared from mixtures of materials containing 68 elements in the following concentrations—10 percent, 4.6, 2.2, 1.0, 0.46, etc. These values were chosen so that the concentrations of the elements decrease from 10 percent to about 0.0001 percent by a factor of the reciprocal of the cube root of 10. This factor provides a geometric concentration series having three members for each order of magnitude and which is consistent with the relation between the blackness of the spectral line and the amount of an element present. By means of a comparator showing enlarged adjacent images of the sample spectra and the standard spectra, visual estimates are made of concentrations of the elements in the sample which are then reported as being between two standards in the following manner: X indicating the middle portion (5−2) of an order of magnitude; X+ the higher portion (10−5); and X− the lower (2−1).

The above method of reporting is used because the inherent limitations of this particular method of spectrographic analysis make the precision of the determinations less than the precision attained in preparing the standards. Major sources of error are (1) chemical and physical differences between the samples and the standards, (2) the omission of complete quantitative procedures for sample preparation, and plate calibration, and (3) lack of duplicate determinations. Experimental work has shown that approximately 60 percent of the reported results fall within the proper portion of an order of magnitude.

Sixty elements are detectable with one exposure by the spectrographic method used in this study. Of these elements, 20 are present in more than 70 percent of the samples in concentrations above the spectrographic limits of detectability, and 9 elements are present in less than 50 percent in concentrations above the limits of detectability. The remaining 31 elements were looked for but not detected in either the ores or the unmineralized host sandstone. Study of the composition of these ores by the rapid spectrographic method is thus limited to about 29 elements; the only elements on which enough data are provided for detailed statistical correlation studies are the 20 that are detected in more than 70 percent of the ores.

The limit of detectability for each element is, in general, that listed by Myers (1954, p. 195). The limit of detectability actually attained or reported for each element has varied slightly because the analytical work has extended for over a 3-year period, during which some changes were made in the details of the technique, and because of slightly different practices followed among analysts in reporting elements near the limit of detection. Histograms shown in figures 8 to 11 are cut off at the highest limit of detectability reported for each element.

Some spectrographic determinations were below the limit of detectability and were reported as questionable, "trace," or "looked for but not detected." In constructing the scatter diagrams (fig. 13) and calculating the correlation coefficients (table 3 and fig. 13) between the concentration of uranium, yttrium, nickel, and sodium and the size of the deposits, questionable determinations were used as reported; "trace" determinations were assigned to the second concentration class below the limit of detectability; and "looked for but not detected" determinations were assigned to the range of concentration immediately below that assigned to "trace" determinations. As relatively few of the spectrographic determinations were in these three catagories, this practice did not seriously bias the statistical calculations.

CLASSIFICATION OF ELEMENTS IN THE URANIUM DEPOSITS

The classification of elements in sandstone-type uranium deposits given in table 2 is from Shoemaker and others (1959) and is based largely on comparison of spectrographic analyses of 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation. The system of classification is based on the assumption that the ore minerals are epigenetic, that they were formed in the sandstone host rock some time after its original deposition, and that the epigenetic process that formed the ore minerals can be distinguished from other epigenetic process that produced changes in the sandstone, such as cementation of the sandstone with calcite.

An element whose presence in the ore is unrelated to uranium mineralization is termed "intrinsic" and is either syngenetic or epigenetic (diagenetic). Intrinsic syngenetic elements are contained in clastic fractions of the sandstone host rock and in precipitates formed simultaneously with the deposition of the clastic fraction. Intrinsic epigenetic (diagenetic) elements were precipitated later by processes unrelated to uranium mineralization. They are distributed irrespectively of uranium and are much more widely dispersed in the host rock. The deposition of intrinsic epigenetic (diagenetic) elements may have either followed or preceded the deposition of uranium, and no judgment of this is involved in the classification.

Extrinsic elements were introduced by uranium mineralization or related processes. No implication of the source of the elements is made in the classification, nor is it implied that all the extrinsic elements were deposited simultaneously. Uranium and vanadium are the only ore elements in the uranium deposits in the Salt Wash member of the Morrison formation, though the deposits contain extrinsic accessory elements, which are present in lower concentration and do not add to the economic value of the ore.

Probably no single chemical element in the sandstone-type uranium ores belongs exclusively to either the intrinsic or extrinsic components of any given ore; each element is present in some concentration, however small, in the detrital minerals of the sandstone and a certain amount, however small, was probably carried in the solutions that introduced the uranium. The problem of classification, therefore, is to assess the relative importance of the roles played by each element in the ore deposits. An element is considered dominantly in one classification if more than half its weight is believed to fall within this classification in a majority of the deposits studied (see table 2). For example, an element is considered extrinsic if it is believed to be dominantly extrinsic in more than one-half the deposits.

The eight elements whose concentrations appear to be significantly related to the size of the deposits fall into all categories of the classification (table 2); they have affected the formation of the deposits in a variety of ways. The geochemical habits of the eight elements are only briefly summarized; a more detailed account is given by Shoemaker and others (1959).

IRON

The average iron content of uranium ore in the Salt Wash member of the Morrison formation is nearly four times greater than that of unmineralized sandstone of the Salt Wash (fig. 4). Part of this difference, however, is due to the fact that the uranium ore contains, on the average, twice as much iron-bearing clay and accessory minerals as does the unmineralized sandstone. Another part is due to iron contamination of the samples during grinding at the mills (metallic iron has been removed from samples with a hand magnet). The average amount of iron thus introduced is estimated at about 0.1 percent. The remainder of the iron in the deposits, probably slightly

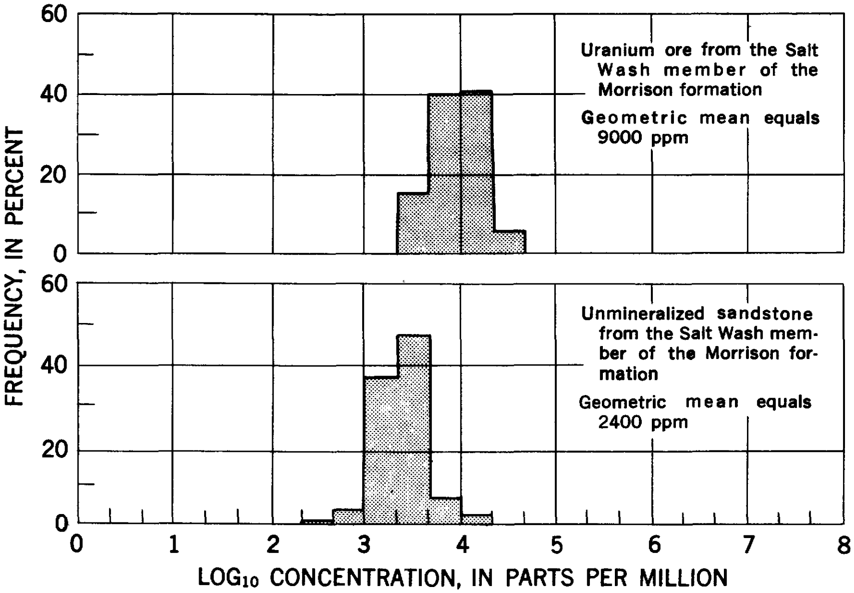

FIGURE 4.—Histograms showing the frequency distribution of iron concentration in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

less than half the total iron content, may be extrinsic, introduced by uranium mineralization or related processes. The iron content of an average uranium deposit is, therefore, a function of both iron originally present in the sandstone and to a lesser extent that introduced as an extrinsic component.

ZIRCONIUM

Zirconium is considered to be almost entirely intrinsic in the sandstone-type uranium deposit. Zirconium is not known to be an essential or major constituent of any of the ore minerals in the deposits, and only rarely is it reported even as a trace constituent in qualitative spectrographic analyses of ore minerals (Weeks and Thompson, 1954). Zirconium is more than twice as abundant in uranium deposits in the Salt Wash member of the Morrison formation than in unmineralized sandstone of the Salt Wash (fig. 5); but like iron, the abundance of zirconium varies directly with that of aluminum in the sandstone and is, therefore, more abundant in ore because of their higher clay and associated heavy-mineral content.

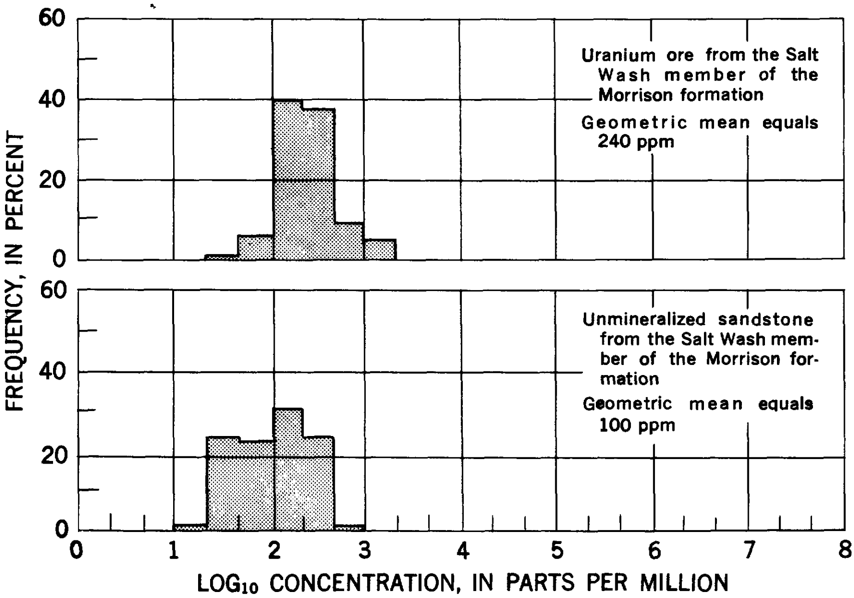

FIGURE 5.—Histograms showing the frequency distribution of zirconium concentration in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

CALCIUM, SODIUM, AND MANGANESE

Three elements contained largely in the cementing materials of the sandstone of the Salt Wash member of the Morrison formation are calcium, manganese, and—to a lesser degree—sodium (figs. 6, 7, and

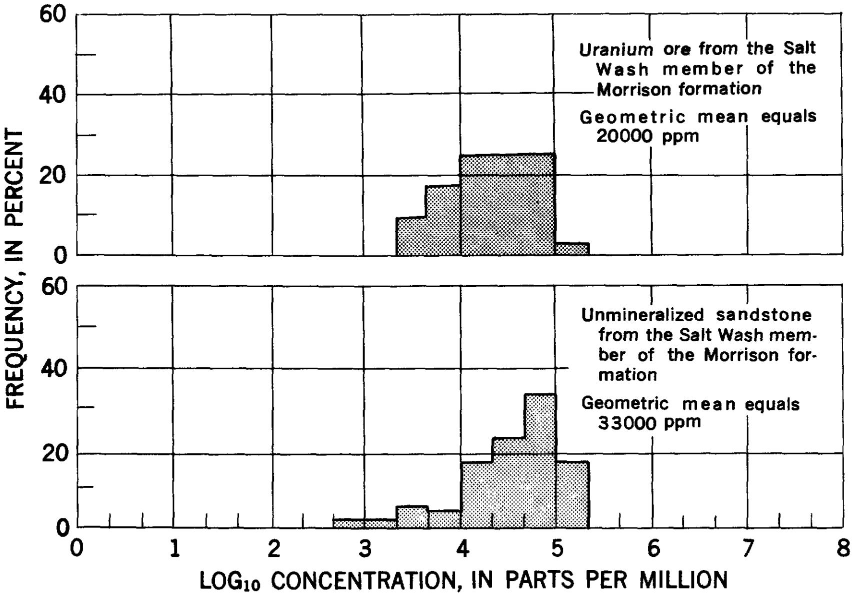

FIGURE 6.—Histograms showing the frequency distribution of calcium concentration in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

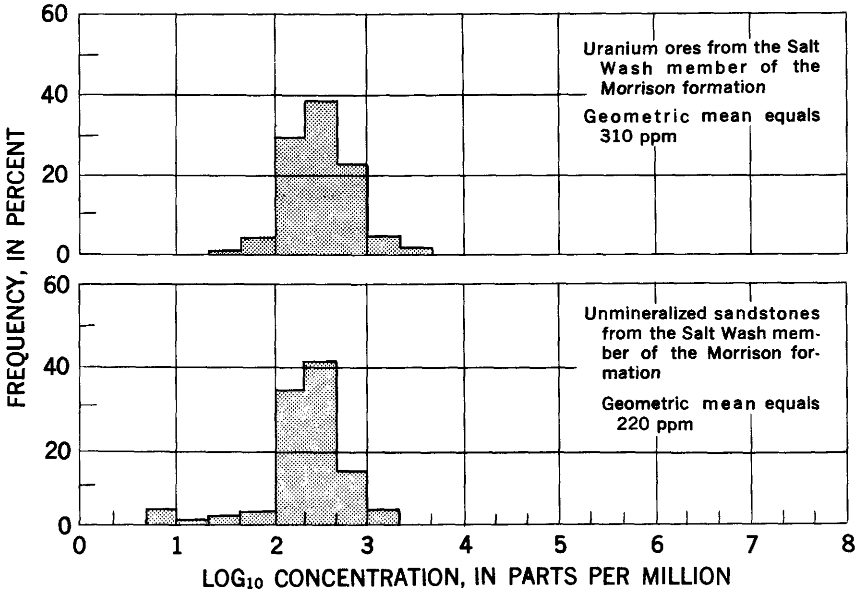

FIGURE 7.—Histograms showing frequency distribution of manganese concentration in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

8). These elements are classified as dominantly intrinsic and epigenetic.

Calcium, the second most abundant metal in the sandstone, occurs mainly in calcite, which is the dominant cement of the sandstone of the Salt Wash member (Cadigan, 1959); but a significant part of the calcium also occurs in dolomite. Locally a large part of the calcium is in gypsum.

Manganese has a very high correlation with calcium in both the unmineralized sandstone and in the uranium deposits (Shoemaker and others, 1959) and is probably contained largely in calcite. A minor part of the manganese may be present as an oxide.

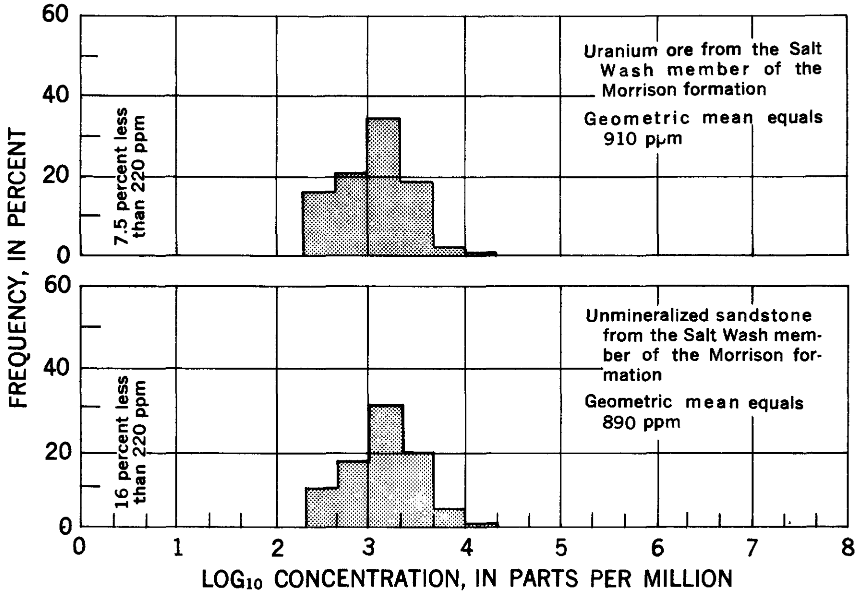

FIGURE 8.—Histograms showing frequency distribution of sodium concentration in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

Sodium is contained principally in plagioclase feldspar and in nonsilicate sodium salts in the sandstone of the Salt Wash member of the Morrison formation. Flame photometric analyses of the acid-leached sand-size fraction of 14 sandstone samples from the Salt Wash (R. A. Cadigan, written communication, 1956) suggest that the sodium in the feldspar represents about half of the total sodium in the sandstone. The remainder of the sodium is believed to be contained mainly in soluble salts, principally sodium bicarbonate. A moderate correlation of sodium with calcium and manganese in the sandstone and high correlation in the uranium deposits (Shoemaker and others, 1959) suggest that the bulk of the sodium (as bicarbonate) is physically associated with the carbonates.

The ratios of abundance of the elements in the uranium deposits to their abundance in unmineralized sandstone in the Salt Wash member indicate that virtually no calcium or sodium, and only very little, if any, manganese, is extrinsic. The low abundance ratio of calcium (0.6, see table 2) may be attributed to the fact that the upper sandstone strata of the Salt Wash member, which include most of the uranium deposits, contain about half as much calcium as does the Salt Wash as a whole. The higher frequency of occurrence of uranium deposits in the upper sandstone strata of the Salt Wash member may be partly a function of a relatively low carbonate-cement content and consequent greater permeability. It is not improbable, however, that some leaching of calcium from the mineralized host rock occurred with the precipitation of the ore metals, thereby causing a relatively low abundance ratio.

URANIUM

Uranium is not only the principal ore element in the deposits but also has the highest ratio of abundance in the deposits to abundance in unmineralized sandstone (table 2 and fig. 9). The uranium content of the samples varies considerably (fig. 9); but no systematic or orderly regional pattern of variation is apparent, either because it is highly complex or because it was obscured by the selectivity of mining or ore mixing or both. Fortunately, the uranium content of the mill-pulp

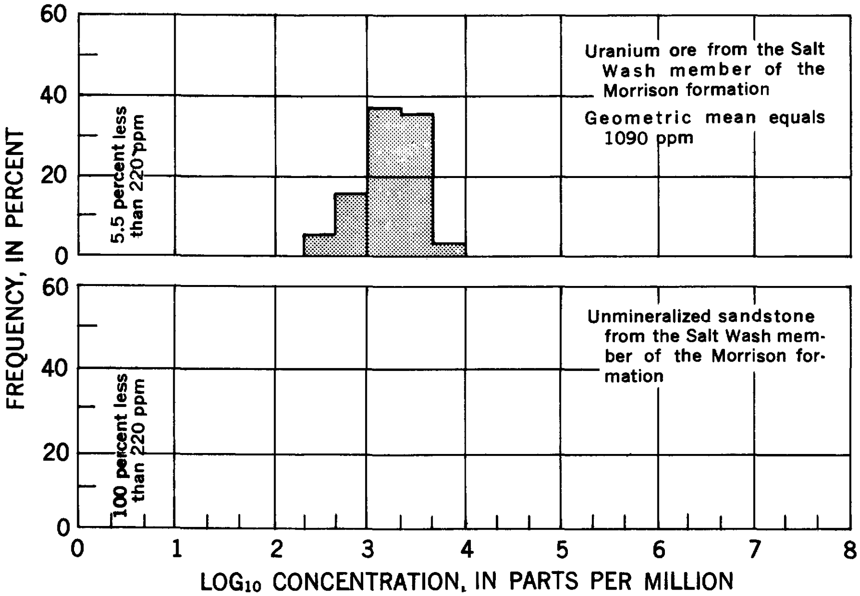

FIGURE 9.—Histograms showing frequency distribution of uranium concentration in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

samples has no pronounced relationship to the content of any of the other detected elements, except vanadium (Shoemaker and others 1959). Therefore, selective mining has probably not appreciably biased the samples with regard to the other elements.

NICKEL

The mean nickel content of uranium deposits in the Salt Wash member of the Morrison formation is about 20 times higher than that of unmineralized sandstone of the Salt Wash (table 2 and fig. 10).

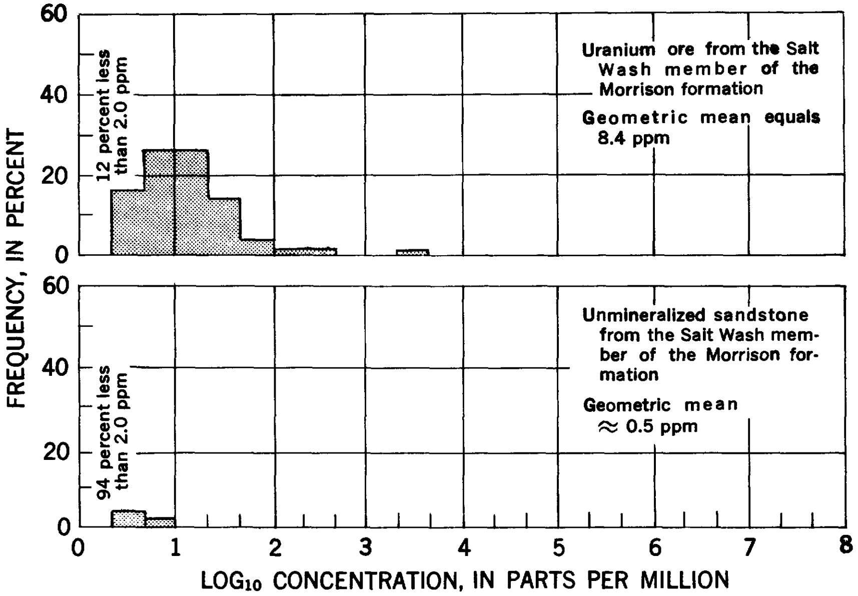

FIGURE 10.—Histograms showing the frequency distribution of nickel concentration in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation.

Nickel and iron have a moderately high correlation in uranium deposits in the Salt Wash (Shoemaker and others, 1959), probably reflecting an association of nickel with iron sulfides in low-valent ore minerals, perhaps bravoite or nickeliferous pyrite. The concentration of nickel in the uranium deposits of the Salt Wash (and in deposits of the Chinle formation of Triassic age) tends to be broadly zoned across the central part of the Colorado Plateau. Nickel is more abundant in ores toward the western or northwestern part of the region and less abundant in ores toward the eastern or southeastern part.

YTTRIUM

Yttrium is a dominantly extrinsic element in the uranium deposits of the Salt Wash member of the Morrison formation. It is estimated to be about seven times more abundant in the deposits than in unmineralized sandstone of the Salt Wash (table 2 and fig. 11). Correlation studies suggest some affinity of yttrium for the nickel-cobalt-molybdenum group of extrinsic elements (Shoemaker and others, 1959); and, therefore, the regional distribution of yttrium in ores in the Salt Wash (and in ores in the Chinle formation) is similar, though more erratic, to the regional distribution of nickel, cobalt, and molybdenum.

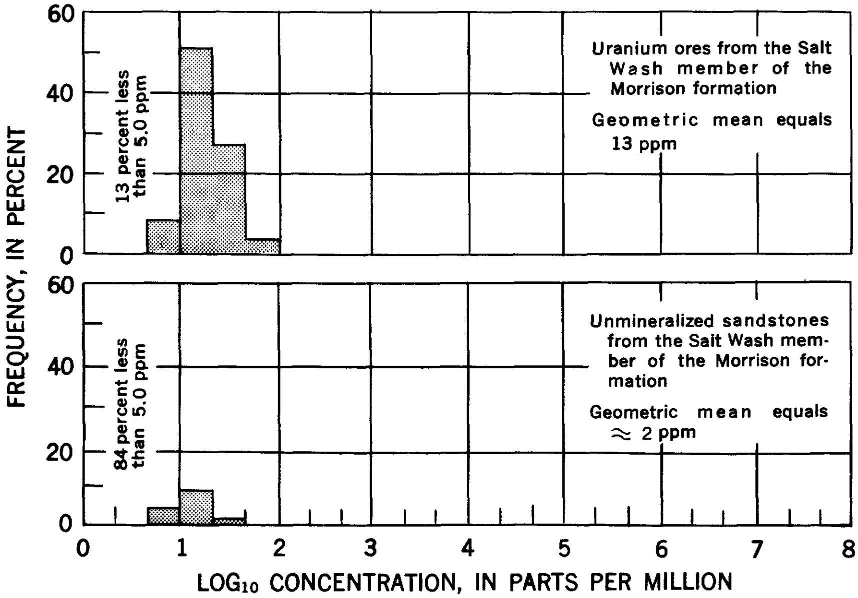

FIGURE 11.—Histograms showing the frequency distribution of yttrium concentration in 211 samples of uranium ore and 96 samples of unmineralized sandstone from the Salt Wash member of the Morrison formation

SIZE OF URANIUM DEPOSITS

The size of each of 75 uranium deposits in the Salt Wash member of the Morrison formation was calculated by adding its total production to its indicated and inferred reserves. The production data were compiled from records of the U.S. Geological Survey for the time prior to 1948 and from records at the operations office of the Finance Division of the U.S. Atomic Energy Commission in Grand Junction, Colo., for the period April 1948 through December 1953. The data for the reserves were compiled from records of the Ore Reserve Section, also at the Grand Junction operations office, and from many reports of the Geological Survey.

The frequency distribution of the calculated size of the 75 deposits is given in figure 12. The calculated size of each deposit was assigned

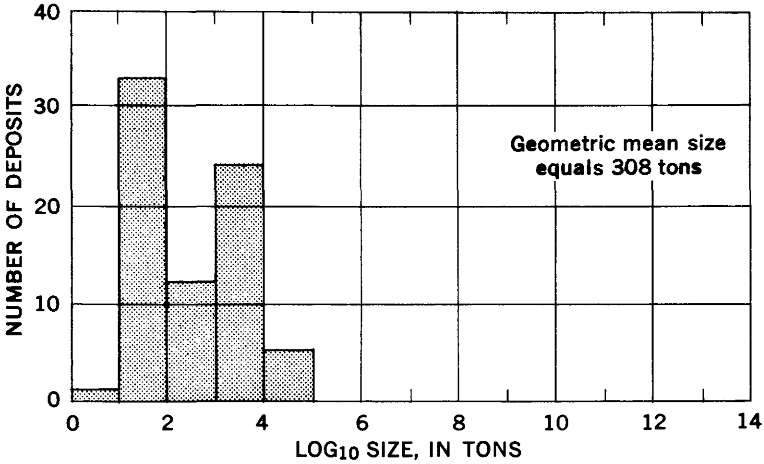

FIGURE 12.—Histogram showing frequency distribution of log size of 75 uranium deposits in the Salt Wash member of the Morrison formation

to logarithmic classes with limits of integral powers of 10, as in the following table:

Tons	*Log size*
1–10	0–1
10–100	1–2
100–1,000	2–3
1,000–10,000	3–4
10,000–100,000	4–5

A sandstone-type uranium deposit (as the term "deposit" is used here) is not in every case a continuous body of mineralized rock but may be composed of more or less discontinuous, but closely spaced, ore bodies of varying dimensions. The specification term, "closely spaced," is not rigid and is determined to some degree by mining practice. Groups of ore bodies that are or can be mined as one deposit may generally be considered as one deposit in estimating reserves. Rarely, closely spaced ore bodies that may or may not be connected are mined by two or more companies through different adits and are considered as separate deposits. Had they been mined by one company they might have been considered as a single deposit.

Estimation of the extent of incompletely mined deposits or of deposits not completely blocked out by drilling involves considerable personal judgment. Normally the third dimension of an exposure of ore in a mine or on an outcrop is considered nearly equal to the length of the exposure, unless some idea of the third dimension can be obtained in crosscuts in the mine, irregularities on the outcrop, or from drill holes.

The error involved in calculating the size of a uranium deposit is undoubtedly large, but probably only a few sizes are misclassified in the broad class intervals given on figure 12.

In estimating tons of ore reserves at particular mines no consideration was given to the grade of the ore, except that the ore matched or exceeded the minimum requirements of 0.10 percent U_3O_8 or 1.0 percent V_2O_5 in continuous layers 1 or more feet thick.

CORRELATION OF ELEMENT CONCENTRATION WITH SIZE OF THE DEPOSITS

The relation between size and composition of the uranium deposits was judged to be statistically significant on the basis of computed correlation coefficients, wherein certain assumptions are necessary regarding the form of the bivariant frequency distributions (fig. 13). We recognize that our assumptions may be invalid, but this possibility has no bearing on the completely separate regression analyses on which the size-prediction methods are actually based. Correlation studies served only to point out elements which could be useful in regression analyses. The usefulness of these elements is verified in an analysis of variance given on page 46 and in table 9, and also in tests of the methods made on 40 other deposits of known size.

Using semiquantitative spectrographic analyses of samples from uranium deposits for which the size could be calculated, simple linear-correlation coefficients were computed between the log of the element concentration and the log size of the deposits. These are given in table 3. The bivariant frequency distributions or scatter diagrams of element concentration in relation to size are shown in figure 13 for the eight elements that correlate significantly with the size of the deposits.

The computed linear-correlation coefficients of the log concentration of the eight elements (nickel, calcium, manganese, zirconium, iron, sodium, yttrium, and uranium) in relation to the log size of the deposits are higher than the lowest significant coefficient for 75 pairs at the 95-percent level of confidence, that is less than 5 out of 100 coefficients of this magnitude would be expected to occur by chance alone.

Significant correlations of the size of the uranium deposits with element concentration do not indicate, necessarily, that elemental composition controls the size or that size controls the composition. The probability values given in table 3, however, indicate that most of the statistically significant correlation coefficients are not due to chance, and that the size of the deposits and the concentration of the eight elements are in some way related. The low values of the coefficients indicate that the relationships are not strong.

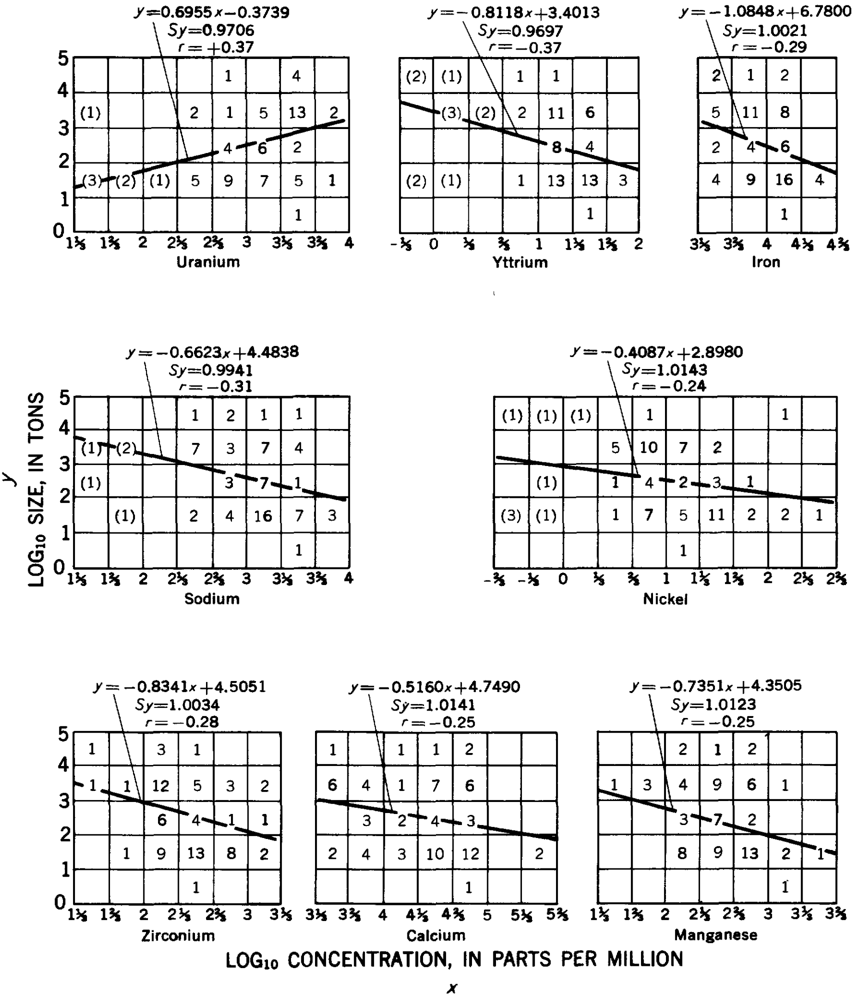

FIGURE 13.—Scatter diagrams of minor-element concentration in 75 uranium deposits and the size of the deposits, showing regression lines and regression equations; standard errors, S_y, of the equations; and correlation coefficients, r. The positions of numbers in parentheses on the scatter diagrams are based on determinations reported as "trace," "looked for but not detected," or "questionable" (see p. 22).

Uranium is the only element of those studied (table 3) whose concentration has a significant positive correlation with the size of the deposits. The relationship indicates that the higher grade ore shipments tend to come from the larger ore deposits. As the analyzed samples are from ore shipments, there may be some doubt as to whether this observed relationship is a function of larger deposits containing a greater concentration of uranium or of mining practices in which higher grade ore in larger deposits is selectively mined and shipped. The latter reason seems unlikely to be important because mining practices are highly variable and are possibly selected without consideration for the size of the deposit. The mining practice most pertinent here is the mixing of ore, not to reach a high grade, but to maintain a grade that will give the highest return under the various payment procedures administered by the U.S. Atomic Energy Commission. It seems likely that mine operators shipped higher grade ore from the larger deposits, not because they could afford to leave low-grade material in the mines, but because these deposits contained less low-grade material. A possible exception to this practice may occur where a large deposit consisting of zoned high- and low-grade ore is mined in the high-grade zones and, therefore, the low-grade ore is not conveniently available for dilution of grade by mixing.

The reason why larger uranium deposits contain higher average grades of uranium ore is not clear, but it may be partly due to the distribution of a uranium-precipitating agent, such as organic carbon, in the host rock (McKelvey, Everhart, and Garrels, 1955, p. 506–507).

Nickel and yttrium tend to be more highly concentrated in uranium deposits in the Salt Wash member of the Morrison formation in the western and northwestern parts of the Colorado Plateau where the deposits tend to be smaller. This fact alone might account for the significant negative correlation between nickel and yttrium concentration and the size of the deposits. However, molybdenum shows a regional variation within the uranium deposits similar to that of nickel and yttrium but does not exhibit a significant correlation with the size of the deposits.

Calcium, manganese, and sodium are present mainly in carbonate cement of the host-rock sandstone; in uranium deposits their significant negative correlation with size of the deposits might be explained by the hypothesis that larger amounts of cement material in the host rock are unfavorable to the formation of large ore deposits.

The reasons for the significant negative correlation between size of the deposits and their concentration of iron[2] and zirconium are unknown.

ESTIMATION OF SIZE OF DEPOSITS

Regardless of the reasons for the correlations between element concentrations and the sizes of the deposits, the relationships provide a basis for estimation or prediction of the sizes of uranium deposits. In the absence of any other geologic information, the best estimate[3] of the size of any single deposit would be the geometric mean size of all the deposits in the group—in this case, the Salt Wash member of the Morrison formation. The indices of prediction[4]

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[2] Subsequent study indicates that the regional variation of iron concentration in uranium deposits is similar to those of nickel and yttrium. Thus, the negative correlation of iron concentration with size may also be related to regional variation.

[3] Estimate with the smallest standard error.

[4] Index of prediction $=1-\sqrt{1-r^2}$, where r is the correlation coefficient.

(Davies and Yoder, 1941, p. 348) reveal that estimation or prediction of the size of deposits based on the concentration of any 1 of the 8 elements will improve the estimate by about 3 to 7 percent. This gain is tangible, but small. Estimates based on the concentration of one of these elements would have about 93 to 97 percent of the standard error that would have obtained if every deposit were estimated to be of mean size. Size estimates can be improved considerably, however, if based simultaneously on the concentration of all the eight elements—uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel.

Three methods of estimating size of the deposits from elemental composition are given. The first method is based on simple linear regression; the statistics were computed by the writers. The other two methods are based on multiple regression; these statistics were computed at the Statistical Engineering Laboratory of the U.S. National Bureau of Standards.

SIMPLE LINEAR REGRESSION (METHOD 1)

Regression lines and equations expressing the variations of size as a function of element concentration are given in figure 13. The standard error of estimate (Waugh, 1952, p. 445) for each equation is also given. When the concentrations of uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel in a uranium deposit are known, it is possible to estimate eight different sizes for any given deposit. If the eight estimates are each weighted by the z value[5] corresponding to the correlation coefficient of the element with size of the deposits, a weighted average-size estimate can be computed. In other words, an average estimate is computed from a group of estimates that are weighted according to the degree of correlation of the element on which each is based. Thus a size estimate based on the concentration of uranium would receive more weight than one based on manganese. Table 4 has been prepared to facilitate calculation of the weighted average-size estimate. The constants in the table have been weighted by the proper z values, and it is necessary only to sum the eight constants that correspond to the proper elements and their concentration to obtain the log of the weighted estimate of the size. It is important that all eight elements be considered. The sum of the upper constants in table 4 is the $\log_{10}$ of the weighted average-size estimate.

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[5] z is a function of the correlation coefficient, r; but unlike r, z is approximately normally distributed (Fisher, 1950, p. 197–201). The equation for z may be written

$$z = 1.1513 \log_{10} \frac{1+r}{1-r}.$$

The log of the weighted average-size estimate is compared graphically on figure 14 with the log of the calculated or true size for each of the 75 deposits on which the study is based. The correlation coefficient of these two variables is +0.61. The index of prediction calculated from this coefficient indicates that by using all eight elements, a size estimate can be improved by about 21 percent as opposed to about 3 to 7 percent by using only a single element. The regression line of y on x is,

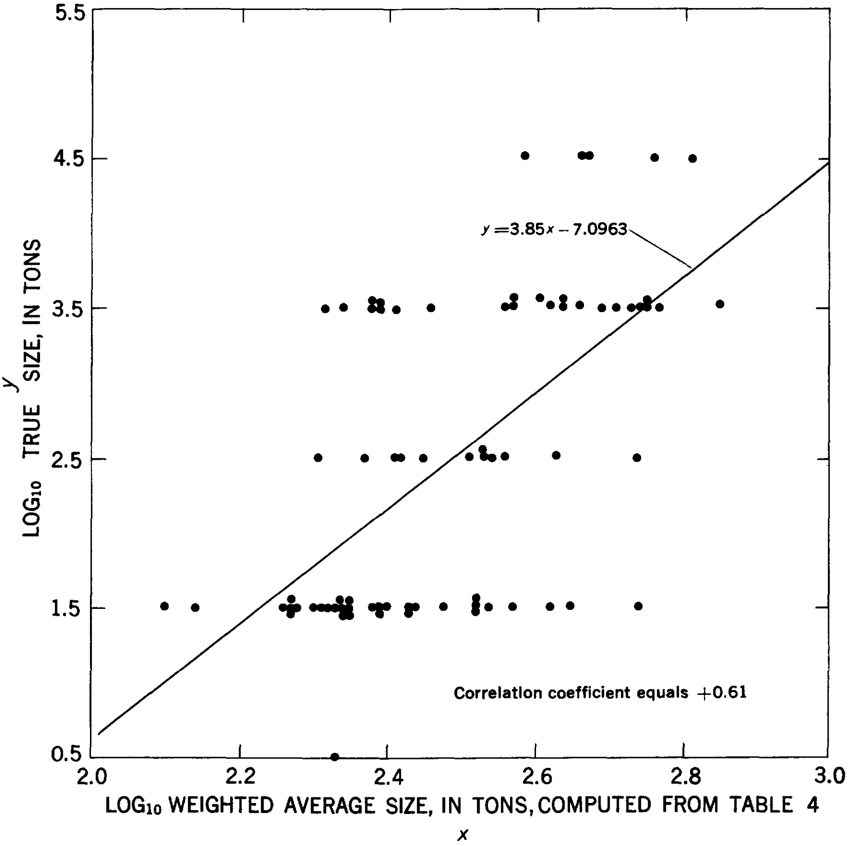

FIGURE 14.—Scatter diagram of log weighted average-size estimate of 75 uranium deposits and log true size of the deposits.

$$y = 3.85x - 7.0963, \qquad (1)$$

where y is a second estimate of $\log_{10}$ of the size and x is $\log_{10}$ of the weighted average-size estimate. Thus, if the log of the weighted average-size estimate is computed from table 4 and used in equation 1, another weighted average-size estimate is obtained. This second estimate is herein called the expanded weighted average-size estimate, as the use of equation 1 increases the range of sizes that may be estimated. For example, log weighted average sizes of 2 and 3 (computed from table 4), which correspond to 100 and 1,000 tons, will be expanded with the equation to 0.6037 and 4.4537 respectively, corresponding to 4 and 28,000 tons.

The log of the expanded weighted average-size estimate is computed by summing the eight appropriate lower values from table 4, which is constructed to include all computational steps. For example, if a spectrographic analysis of a mill-pulp sample from a mine in the Salt Wash member of the Morrison formation gives U=0.X−, Y=0.00X−, Na=0.X−, Fe=0.X+, Zr=0.0X, Mn=0.0X, Ca=X.0, and Ni=0.00X−, the respective constants are 0.6919, 0.6172, 0.3127, 0.3485, 0.2160, 0.1394, 0.0659, and 0.0640. The sum of these constants (2.4556) is the estimated size of the deposit, in log tons.

The standard error *Sy* of the regression line (equation 1) is 0.836 (table 9). The standard error measures the deviation of points from the fitted regression line in terms of y (fig. 14).

The error involved in the prediction of y from any specific value of x may be obtained by the following expression:

$$\pm\left[t^2(Sy)^2\left\{1+\frac{1}{n}+\frac{(x-\bar{x})^2}{\sum(x-\bar{x})^2}\right\}\right]^{\frac{1}{2}} \qquad (1a)$$

(Anderson and Bancroft, 1952, p. 156–164), where t is taken from a table of t for a given confidence level, Sy is the standard error of estimate (0.836), n is the number of pairs (75) of values of x and y, and $\bar{x}$ is the mean x. To estimate the error of a size prediction from equation 1 at the 95-percent confidence level, constants are inserted and the expression becomes

$$\pm 1.99(0.836)\left\{1.0133+\frac{(x-2.489)^2}{2.09}\right\}^{\frac{1}{2}}, \qquad (1b)$$

where x is the log of the weighted average-size estimate obtained by summing the appropriate upper values from table 4. Thus the error of a prediction varies with the difference between the mean x on figure 14 and the value of x that is actually used to predict. This is understandable when it is considered that an error in the slope of the fitted regression line (fig. 14) is most serious in terms of the dependent variable at its extremities and least serious near the mean.

Size predictions and errors for two examples are given below. In example A, all the element concentrations are in the same spectrographic group or frequency class as the mean of the element[6] in the

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[6] For the mean case (example A): U=0.X−, Y=0.00X−, Na=0.X−, Fe=0.X+, Zr=0.0X, Mn=0.0X, Ca=X., and Ni=0.00X−.

75 deposits. In example B, the concentrations are in extreme classes.[7] The error of size estimates at the 95-percent confidence level will almost always be intermediate between these two examples.

Example	Log of weighted average size [1] (x)	Log of expanded weighted average size [1] (y)	Error of y (95-percent confidence level)
A	2.4809	2.4556	[2] $\pm 1.6746\left(\substack{\times \\ \div}47.3\right)$
B	2.9776	4.3677	$\pm 1.7646\left(\substack{\times \\ \div}58.2\right)$

[1] Computed from table 4.

[2] The range $y-1.6746$ to $y+1.6746$ is equivalent to the range antilog $y \div 47.3$ to antilog $y \times 47.3$, where antilog y=predicted size of the deposit in tons rather than in log tons.

To obtain the 80-percent, rather than the 95-percent, confidence interval the expression becomes

$$\pm 1.29(0.836)\left\{\frac{1.0133+(x-2.489)^2}{2.09}\right\}^{\frac{1}{2}}, \qquad (1c)$$

and the errors given for examples A and B would be $\pm 1.0856\left(\substack{\times \\ \div}12.2\right)$ and $\pm 1.1439\left(\substack{\times \\ \div}13.9\right)$, respectively.

LONG MULTIPLE REGRESSION (METHOD 2)

The multiple correlation coefficient between the log size of the 75 deposits and the logs of their concentrations of uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel—as determined by semiquantitative spectrographic analysis—is 0.67. The index of prediction calculated from this value indicates that a size estimate can be improved by about 26 percent in comparison to about 21 percent by using the simple linear-regression method and from 3 to 7 percent improvement by using a single element.

The multiple-regression equation calculated at the U.S. Bureau of Standards is:

$$y=-0.0170+0.1579x'_{\mathrm{U}}-0.0908x'_{\mathrm{Y}}-0.0709x'_{\mathrm{Na}}-0.0494x'_{\mathrm{Fe}} \\ -0.0854x'_{\mathrm{Zr}}+0.0139x'_{\mathrm{Mn}}+0.0098x'_{\mathrm{Ca}}-0.0400x'_{\mathrm{Ni}}, \qquad (2)$$

where y is the estimated $\log_{10}$ size of the deposit, in tons, and x'_{U}, x'_{Y} . . . x'_{Ni} are respectively 6 times the $\log_{10}$ of the spectrographic

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[7] For the extreme case (example B): U=0.X+, Y=0.0000X+, Na=0.00X, Fe=0.X, Zr=0.00X−, Mn=0.X+, Ca=XX.−, and Ni=0.0000X. The values for Y, Na, and Ni are below the spectrographic limits of detectability. However, spectrographic reports of "looked for but not detected" for these elements were assigned to these respective classes for use in all calculations. See section on statistical treatment of data (p. 22).

determinations,[8] in percent, for uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel.

The same value of estimated log size may be obtained with less computation by summing the eight appropriate values from table 5.

The standard error of equation 2 is 0.820 (table 9). The range of error involved in the prediction of y from any specific combination of $x'_U, x'_Y \ldots x'_{Ni}$ is given by

$$\pm\left[t^2(0.672049)\left\{1+[1, x'_U, x'_Y \ldots x'_{Ni}]\, c_8 \begin{bmatrix} 1 \\ x'_U \\ x'_Y \\ \cdot \\ \cdot \\ \cdot \\ x'_{Ni} \end{bmatrix}\right\}\right]^{\frac{1}{2}}. \qquad (2a)$$

The values of the matrices C_8 are given in table 6. Expansion of this expression is a tedious process,[9] which is impractical. The error in prediction will generally be within the ranges given below for the 95- and 80-percent confidence levels.

Confidence level (percent)	Range From— Example A	 To— Example B
95	$\pm 1.6532 \left(\overset{\times}{\div}45.0\right)$	$\pm 2.4654 \left(\overset{\times}{\div}292.0\right)$
80	$\pm 1.0717 \left(\overset{\times}{\div}11.8\right)$	$\pm 1.5982 \left(\overset{\times}{\div}39.7\right)$

Examples A and B were calculated using values of $x'_U, x'_Y \ldots x'_{Ni}$ corresponding to values in the same spectrographic classes as those used in the sample calculation with equation 1b.

SHORT MULTIPLE REGRESSION (METHOD 3)

Inasmuch as the logs of the uranium and yttrium contents of the deposits show the highest linear correlation with the log size of the deposits (table 3), a multiple regression equation which includes only uranium and yttrium contents as the independent variables, was calculated at the U.S. Bureau of Standards. The equation is

$$y=0.5074+0.1326x'_U-0.1594x'_Y, \qquad (3)$$

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[8] As the $\log_{10}$ of the midpoints of the semiquantitative spectrographic ranges, in which the analyses are reported in percent, form a series, such as $8\frac{1}{6}-10$, $8\frac{3}{6}-10$, $8\frac{5}{6}-10$, $9\frac{1}{6}-10$ . . ., multiplication of each value by a factor of 6 reduces the series to one of simple whole numbers, such as −11, −9, −7, −5. . . , which are easier to handle in calculation.

[9] Procedure for expansion of equation 2a and 3a is given on page 40.

where x'_U is 6 times the log of the percentage of uranium, x'_Y is 6 times the log of the percentage of yttrium, and y is the $\log_{10}$ of the estimated size of the deposit, in tons. To eliminate calculation, table 7 is given from which the estimated log size or size may be read directly, using semiquantitative spectrographic determinations of uranium and yttrium.

The standard error of equation 3 is 0.866 (table 9). The range of error involved in the prediction of y from any specific combination of x'_U and x'_Y is given by

$$\pm\left[t^2(0.749634)\left\{1+[1,x'_U,x'_Y]\,C_2\begin{bmatrix}1\\x'_U\\x'_Y\end{bmatrix}\right\}\right]^{\frac{1}{2}}. \qquad (3a)$$

The values of the matrices C_2 are given in table 6. Expanded equation 3a becomes

$$\pm t[0.749634\ (1+0.495495+0.002528x'_U +0.027117x'_Y+0.002528x'_U+0.001140x'^2_U-0.000237x'_Ux'_Y +0.027117x'_Y-0.000237x'_Ux'_Y+0.001653x'^2_Y)]^{1/2};$$

when like terms are combined the expression is

$$\pm t[1.121074+0.003790x'_U+0.040656x'_Y -0.000355x'_Ux'_Y+0.000855x'^2_U+0.001239x'^2_Y]^{1/2}.$$

The error in prediction will generally be within the ranges given below for the 95- and 80-percent confidence levels.

Confidence level (percent)	Range	
	From—	To—
	Example A	Example B
95	$\pm 1.7351\ \left(\overset{\times}{\div}54.3\right)$	$\pm 1.8533\ \left(\overset{\times}{\div}71.3\right)$
80	$\pm 1.1248\ \left(\overset{\times}{\div}13.3\right)$	$\pm 1.2014\ \left(\overset{\times}{\div}15.9\right)$

Example A was computed with equation 3a using values of x'_U and x'_Y which correspond to the respective log midpoints of the classes in which the mean log concentration of the elements fall (U=0.X− and Y=0.00X−). Calculated errors for example A are about the minimum that will be obtained from equation 3a at the 95- and 80-percent confidence levels with any combination of uranium and yttrium concentrations. Example B was computed with equation

3a using values of x'_U and x'_Y which correspond to the respective log midpoints of extreme classes (U=0.X+, Y=0.000X+). Unless a higher concentration of uranium or a lower concentration of yttrium is detected in the sample, the computed range of error for size estimates at the given confidence levels cannot be larger than those for example B.

SUMMARY OF PROCEDURES FOR ESTIMATING SIZE OF DEPOSITS

The type of sample and analysis required for use in the equations and tables given in this report are as follows:

1. The uranium deposit must be in the Salt Wash member of the Morrison formation on the Colorado Plateau.
2. A mill-pulp sample representing several tons or more of the deposit is required to estimate the error of the prediction. Other types of samples, such as drill core, could be used but the precision and accuracy of estimation for such samples is indeterminate.
3. The analyses for uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel must be made by the semiquantitative spectrographic method used at the Denver, Colo., laboratory of the U.S. Geological Survey.

Simple linear-regression method.—A constant corresponding to the semiquantitative spectrographic determination of the concentration for each element is obtained from table 4. The eight constants are totaled, using the appropriate constants for expanded weighted-average size. The antilog of this sum is the estimated size, in tons. About 95 percent of such estimates will be within a factor of 47–58 of the true size of the deposits, and about 80 percent of the estimates will be within a factor of 12–14 of the true size.

Long multiple-regression method.—A constant corresponding to the semiquantitative spectrographic determination of the concentration for each element is obtained from table 5. The eight appropriate constants are summed. The antilog of this sum is the estimated size, in tons. About 95 percent of these estimates will be within a factor of 45–292 of the true size of the deposits. About 80 percent of the estimates will be within a factor of 12–40 of the true size.

Short multiple-regression method.—The log size or size of the deposit, in tons, is read directly from table 7 for the concentration of uranium and yttrium determined by semiquantitative spectrographic analysis. About 95 percent of the estimated sizes, in tons, will be within a factor of 54–71 of the true size of the deposit. About 80 percent of the estimated sizes will be within a factor of 13–16 of the true size.

SUMMARY AND CONCLUSIONS

Three methods have been given for estimating the size of a uranium deposit in the Salt Wash member of the Morrison formation on the Colorado Plateau. Two of the methods are based on the concentration of uranium, yttrium, sodium, iron, zirconium, manganese, calcium, and nickel in the deposit and the third only on the concentration of uranium and yttrium. The element concentrations are determined by semiquantitative spectrographic analyses of mill-pulp samples obtained from uranium-ore mills and buying stations. The samples are splits of those assayed to determine payment to the ore shipper; each represents several tons or more of ore. The spectrographic analyses were made by the U.S. Geological Survey.

The methods of size estimation given in this report are not very precise; each can be used to estimate size, in tons, only to within a factor of 12 or more at the 80-percent confidence level. For example, if the size of a deposit were estimated with equation 1 to be 285 tons (as in example A by method 1), one could be only 80 percent confident that the true size is somewhere between 23 and 3,477 tons. Thus appraisal of a single ore deposit by method 1 appears to be of only little use. However, 285 tons is the best estimate if no other information regarding the size of the deposit is available, as may often be true where mine development or exploratory drilling is meager and the deposits in the area have a large range in size. If the size of a deposit were estimated with method 1 to be 23,300 tons (as in example B) one could be only 80 percent confident that the true size is somewhere between 1,700 and 324,000 tons. However, the best estimate is still 23,300 tons, which may encourage further expenditure in exploration development. The method has indicated that a relatively large deposit is present. Where the estimated size of a deposit is small, the range of error is also small, in terms of tons, because the error is a function of the estimated size. For example, if the size of a deposit were estimated with equation 1 to be 43 tons, one could be 80 percent confident that the true size is less than 600 tons. Known production or reserves may serve to place a lower limit on the size of the deposit.

As applied to individual deposits the methods may be useful in appraisal of old mining property. A deposit thought to have been mined out may be tested in order to judge the likelihood of undiscovered ore. For example, a deposit may have produced 2,000 tons of ore and is then believed to have been exhausted. If its composition indicates that the deposit is 20,000 tons, further exploration may be encouraged. However, if the deposit is estimated to be near 2,000 tons or less, the methods would tend to confirm the belief that the deposit is mined out.

For individual deposits the methods of size estimation given here are useful in distinguishing only the general range of size, such as small compared to intermediate or large. The precision of estimation is greater, however, for the average of a number of deposits than for single deposit. If replicate estimates of a single deposit have a frequency distribution with a standard deviation of σ, then the mean of N independent estimates for this deposit has a frequency distribution with a standard deviation of $\sigma_{\bar{x}}=\sigma/\sqrt{N}$. Thus, if the precision of estimation does not vary too widely for deposits of different estimated size, the confidence interval (expressed in log values) for the estimated mean of 10 deposits, for example, is about one-third of the confidence interval for a single estimate. The precision of the mean in terms of tons of ore, in other words, is greatly increased. The methods of size estimation given in this report, therefore, may be most useful in appraising groups of deposits or ore districts.

The precision and accuracy of the methods of size estimation have not been determined for ore samples other than mill pulp. However, the principles underlying the methods may serve to establish similar methods for other types of samples, such as drill core. Such methods may be extremely useful in gathering maximum data from costly drilling exploration. Size estimation of ore deposits from the composition of drill cores is expected to be considerably less precise than estimation from mill pulp.

The controls of the relations between size and composition of uranium deposits are largely unknown, though some speculation is given on pages 33–34. More thorough understanding of these controls may lead to more precise methods of estimating the size of uranium deposits in the Salt Wash member of the Morrison formation.

Similar relations may also be found among other types of ore deposits and may be useful in estimating their size. A search for such relationships may not only be economically rewarding but may also add to our knowledge of the nature of ore deposits and the controls of their localization.

TESTS OF THE METHODS OF ESTIMATION

The log size for each of 40 uranium deposits in the Salt Wash member of the Morrison formation was estimated from semiquantitative spectrographic determinations of the 8 elements in mill-pulp samples using equations 1, 2, and 3. These estimated log sizes are given in table 8 together with the range of the calculated or true log size for each deposit. The range of error for each estimate from equations 1 and 3 at the 95- and 80-percent confidence levels is also given. None of the 40 deposits listed in table 8 is part of the original 75 deposits on which the methods of size estimate are based.

Like the original 75 deposits, the true log sizes of the 40 deposits listed in table 8 are known only within broad ranges. For comparison, the true log size of each deposit may be taken as the midpoint of the range of the true log size. Errors arising from this assumption are probably compensating.

The methods of size estimation are not tested to determine their precision, which may be calculated from the statistical theory (equations 1a, 2a, and 3a), but to determine whether their precision for a particular group of deposits is similar to, or greatly different from, that calculated for the method. Theoretically, about 38 out of 40 estimates for each equation should be within the 95-percent confidence interval and about 32 out of 40 should be within the 80-percent confidence intervals. Actually, 39 out of 40 estimates (97.5 percent) made with equation 1 proved to be within the calculated 95-percent confidence interval and 28 out of 40 (70 percent) proved to be within the 80-percent confidence interval. Similarly, 37 out of 40 estimates (92.5 percent) made with equation 3 proved to be within the 95-percent confidence interval and 31 out of 40 (77.5 percent) proved to be within the 80-percent confidence interval. Confidence intervals for estimates made with equation 2 were not calculated except for two extreme cases. Had equation 2a been used to calculate the confidence interval for each estimate, 38–40 out of 40 estimates (95–100 percent) would be within the 95-percent confidence interval and 29–38 (72.5–95 percent) would be within the 80-percent confidence interval. Thus the calculated confidence intervals describe fairly well the precision of size estimates made for this group of 40 deposits.

The accuracy of the 40 size estimates may be judged by observing the deviation of the estimates from the true log sizes or by comparing the mean of the estimates with the mean true log size of the 40 deposits. Deviation of the estimates from the true log sizes may be expressed in terms of $[\Sigma(y-y')^2/(n-1)]^{\frac{1}{2}}$, which is equivalent to the standard errors of the regression equations (table 9). The term y is the true log size of a deposit, y' is the estimated log size, and n is the number of estimates. For the 40 estimates made with each of the three equations this value is as follows:

Equation	Deviation of estimate from true log size	Standard error
1	0. 905	0. 836
2	. 944	. 820
3	1. 017	. 866

For each equation the value $[\Sigma(y-y')^2/(n-1)]^{1/2}$ is somewhat higher than the calculated standard error. This may be due primarily to the fact that the mean log size (3.05 tons) of the 40 deposits is greater than the mean log size (2.49 tons) of the 75 deposits on which the calculations of the regression equations are based. Log-size estimates that average nearly 2.49 tons will have a value $[\Sigma(y-y')^2/(n-1)]^{1/2}$ that is near the standard error. Log-size estimates greater or less than 2.49 tons will have a value $[\Sigma(y-y')^2/(n-1)]^{1/2}$ that is greater than the standard error because the errors in slopes of the regression lines are greater at some distance from the mean when the error is measured in terms of y.

Although the standard error for equation 2 is less than those for equations 1 and 3, the deviation of log sizes estimated with equation 2 from the true log sizes is greater than that for equation 1. However, this was only found to be true for this group of 40 deposits and cannot be considered representative of what might be found with other tests.

The mean true log size of the 40 deposits listed on table 8 is 3.05. The mean of the estimates made with equation 1 is 2.73, with equation 2 is 2.80, and with equation 3 is 2.70. Thus the estimates from all three equations appear to be too low on the average, the bias becoming respectively more serious with estimates made with equations 2, 1, and 3.

The accuracy of the equations used in estimating the log size of particularly large or small deposits, or deposits of near average size, is shown by the following data.

The 40 deposits are grouped according to log-size classes (p. 46); for example, the 8 deposits with a log size ranging from 4 to 5 tons are considered to have a true mean log size of 4.50 tons. The mean of the log-size estimates made with equation 1 is 3.37 tons, with equation 2 is 3.40 tons, and with equation 3 is 3.07 tons. The mean log sizes estimated with equations 1 and 2 are somewhat nearer to the true mean-log size for each class than are those estimated with equation 3. For the test group of 40 deposits, log-size estimates of large deposits made with equation 3 are considerably less accurate than those made with equations 1 or 2; log-size estimates of small deposits made with equation 1 appear to be more accurate than those made with either equations 2 or 3.

For the 40 deposits listed on table 8, slightly more accurate size estimates can be made with equation 1, based on the simple linear-regression method, and equation 2, based on the long multiple-regression method, than with equation 3.

Number of deposits in group	Mean true log size (tons)	Mean estimated-log size (tons)		
		Eq 1	Eq 2	Eq 3
8	4. 50	3. 37	3. 40	3. 07
12	3. 50	2. 78	2. 76	2. 64
15	2. 50	2. 58	2. 71	2. 67
5	1. 50	2. 05	2. 24	2. 34

COMPARISON OF THE METHODS OF ESTIMATION

The precision with which estimates of size can be made with each of the methods is dependent on two factors: (a) The degree of relationship found between size and the independent variable or variables used in the method, and (b) the error or uncertainty of the equation used to express the relationship between size and the independent variable or variables. The first factor, the actual degree of relationship between size and x or x'_{U} . . . x'_{Ni}, may be examined for the three methods by a comparison of the correlation coefficients and an analysis of variance (table 9). The highest correlation coefficient ($r^2=0.67$) given in table 9 is that between the log size of a deposit and the logs of the eight element concentrations. Equation 2 reduces the sum of squares of the true log size more than either of the other two methods and has a smaller standard error. The reduction in the sum of squares with equation 1 is somewhat less than that obtained with equation 2, and the standard error of equation 1 is slightly greater. Equation 3 causes the least reduction in the sum of squares and has the greatest standard error. The F ratios for all three methods indicate that each of the equations reduce the variance by statistically significant amounts.

Most of the reduction of sum of squares with the long multiple-regression method (eq 2) is due to the variables x'_1 and x'_2 (functions of uranium and yttrium concentrations). However, the reduction in variance owing to inclusion of the variables x'_2 . . . x'_8 in equation 2 is significant at the 95-percent probability level.

It may be concluded from the analysis of variance in table 9 that the variance of the log size of uranium deposits is significantly reduced by each of the methods and, so, each method will be useful in estimating size.

Equations 1a, 2a, and 3a give the ranges of error for individual size estimates at specified confidence levels. Both the degree of relationship found between log size and the independent variable or variables and the uncertainty of the equation used to express this relationship are considered in these equations. The ranges of error at the 95- and 80-percent confidence levels have been computed for each of the three methods for the case where all elements are at their mean concentration (example A) and for the case where all elements are at extreme concentration (example B). These are given in the appropriate sections of the report but are brought together below for comparison.

Method and equation	Estimated log size (tons)		Confidence level			
			95 percent		80 percent	
	Example A	Example B	Example A	Example B	Example A	Example B
1	2.4556	4.3677	±1.6746	±1.7646	±1.0856	±1.1439
2	2.4939	5.8933	±1.6532	±2.4654	±1.0717	±1.5982
3	2.5542	4.3598	±1.7351	±1.8533	±1.1248	±1.2014

Example	Log of weighted average size (x)	Element concentration (percent)	$\log_{10}$ of element concentration×6 ($x'_U \ldots x'_{Ni}$)
A	2.4809	U=0.X−, Y=0.00X−, Na=0.X−, Fe=0.X+, Zr=0.0X, Mn=0.0X, Ca=X., Ni=0.00X−	−5, −17, −5, −1, −9, −9, 3, −17
B	2.9780	U=0.X+, Y=0.000X+, Na=0.00X, Fe=0.X, Zr=0.00X−, Mn=0.X+, Ca=XX.−, Ni=0.0000X	−1, −25, −15, −3, −17, −1, 7, −27

It appears from these sample calculations, wherein the uncertainty of the regression equation is considered, that the range of error for individual estimated sizes is generally smaller for equation 1 than for equations 2 and 3. The range of error for log size estimates made with equation 2 may be especially large if the eight elements are at their extreme concentration, that is, the deposit is extremely large or small.

It is suggested on the basis of these comparisons and the tests of the methods described previously on pages 43–46 that the simple linear-regression method (eq 1) and the long multiple-regression method (eq 2) are more accurate for estimating the size of deposits than is the short multiple-regression method (eq 3). Usually, estimates made with equation 1 are more precise than those made with equation 2, except where the deposit is estimated to be of near-average size. Equation 1 has the additional advantage over equation 2 that the confidence interval of an estimate can be calculated more easily.

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TABLE 1.—*Semiquantitative spectrographic analyses of samples of uranium deposits in the Morrison formation on the Colorado Plateau*

[Semiquantitative spectrographic determinations made by the rapid visual-comparison method. Comparisons of similar data with those obtained by quantitative methods show that the assigned semiquantitative class interval includes the quantitative value in about 60 percent of the determinations. Analysts: R. G. Havens, P. R. Barnett, G. W. Boyes, Jr., and P. J. Dunton, U.S. Geological Survey]

Deposit	Sample	Tons of ore from which sample was split	Size of deposit [2] (tons)	Semiquantitative spectrographic analyses [1]							
				U	Y	Na	Fe	Zr	Mn	Ca	Ni
1	D-86736	1.3	10-100	0.X+	0.00X+	0.X−	X.−	0.0X	0.0X−	X.	0.00X
2	D-68706	194.3	1,000-10,000	0	.00X−	0	.X	.0X−	.0X−	.X	.000X
3	D-87672	19.2	100-1,000	.X−	.00X	.0X+	.X+	.0X	.0X−	.X+	.000X+
4	D-68711	172.6	1,000-10,000	.X	.00X	.0X+	.X	.0X−	.0X−	.X+	.000X+
	D-68712	210.0		.X−	.000X	.0X	.X	.0X−	.0X−	.X+	.000X
Average [3]			1,000-10,000	.X−	.000X+	.0X	.X	.0X−	.0X−	.X+	.000X
5	D-86233	30.4	10-100	.X−	.00X−	.0X	X.	.00X+	.0X−	X.−	.0X−
6	D-86234	10.8	10-100	.0X+	.00X	.0X+	X.	.0X−	.0X−	.X+	.0X
7	D-87777	149.0	1,000-10,000	.X	Tr	.0X+	.X	.0X−	.0X−	.X	.000X+
	D-87778	28.0		.X+	.00X−	.0X+	.X+	.0X−	.0X−	.X+	.000X+
Average			1,000-10,000	.X	Tr	.0X+	.X	.0X−	.0X−	.X	.000X+
8	D-87760	14.6	10-100	.0X+	.00X−	.X−	.X+	.0X+	.0X−	.X+	0
9	D-88075	36.0	10,000-100,000	.X	.000X	.0X	.X+	.0X−	.00X+	X.−	.000X−
	203418	61.8		.X	.00X−	.0X	X.−	.0X−	.0X−	X.	.000X−
Average			10,000-100,000	.X	.000X+	.0X	X.−	.0X−	.0X−	X.	.000X−
10	D-88076	270.0	1,000-10,000	X.−	.00X	.0X	.X+	.0X−	.00X+	.X	.000X+
	D-88077	5.0		X.+	.00X+	0	X.−	.0X	.00X+	X.−	.000X−
	D-86700	309.8		X.−	.00X	0	.X+	.0X	.0X	X.−	.00X−
	D-86701	205.8		.X−	.00X−	Tr	.X	.0X−	.0X−	.X+	.000X+
Average			1,000-10,000	.X+	.00X	Tr	.X+	.0X−	.0X	.X+	.000X+
11	D-68703	151.6	100-1,000	.X−	.00X−	.X−	X.−	.0X−	.0X−	.X+	.00X−
12	D-72760	12.2	10-100	.X−	.00X−	.0X	.X	.0X−	.0X−	X.	.000X+
13			10,000-100,000	.X	0	.0X+	X.−	.00X	.0X−	.X	.0X−
14	D-68715	373.1	100-1,000	.X−	.00X−	0	.X	.0X−	.0X−	X.−	.000X
	D-68716	366.5		.0X	.00X−	0	.X	.0X−	.0X−	X.−	.000X
	D-86769	45.5		.0X+	Tr	Tr	.X	.0X−	.0X	X.	.000X+
Average			100-1,000	.0X+	.00X−	0	.X	.0X−	.0X−	X.−	.000X

See footnotes at end of table.

TABLE 1.—*Semiquantitative spectrographic analyses of samples of uranium deposits in the Morrison formation on the Colorado Plateau*—Continued

Deposit	Sample	Tons of ore from which sample was split	Size of deposit [2] (tons)	Semiquantitative spectrographic analyses [1]							
				U	Y	Na	Fe	Zr	Mn	Ca	Ni
15	D-68700	42.7	10-100	0.0X+	0.00X−	0.X−	X.−	0.0X−	0.0X−	X.−	0.00X−
16	D-86743	13.1	10-100	Tr	.00X−	Tr	.X	.0X−	.0X−	.X	.000X+
17	D-88072	280.0	1,000-10,000	.X	.00X−	.0X	.X+	.0X	.00X+	.X	.000X+
18	55259	72.8	1,000-10,000	.X	.00X	.0X−	.X	.0X	.0X	.X	.000X
	D-87795	33.2		.X−	.00X−	.X−	X.−	.0X−	.0X−	.X+	.00X
	205345	44.1		.X+	.00X−	.0X+	X.−	.0X−	.0X	.X+	.00X−
Average			1,000-10,000	.X	.00X−	.0X	.X+	.0X−	.0X	.X+	.00X−
19	D-87754	318.1	1,000-10,000	.X	0	.X−	X.−	.0X	.0X	X.	.00X−
	D-87755	228.0		.X−	0	.X−	X.−	.0X	.0X+	X.+	.00X
	D-88079	290.0		.X−	.00X−	.0X	X.−	.0X−	.0X−	X.−	.000X
Average			1,000-10,000	.X−	Tr	.X−	X.−	.0X	.0X	X.	.00X−
20	D-86744	24.2	100-1,000	.X	.00X	.X−	X.−	.0X	.0X	.X+	.00X
21	D-68707	390.9	100-1,000	.0X	.00X	.0X	.X	.0X−	.0X−	X.	.000X
	D-86950	248.0		.X−	.00X−	.0X+	.X+	.0X	.0X	X.+	.00X−
	D-86951	330.4		.X	.00X	.0X+	.X+	.0X	.0X+	X.+	.00X
	D-86952	9.5		0	.00X+	.0X+	.X+	.0X	.0X	X.	.00X
Average			100-1,000	.0X+	.00X	.0X+	.X+	.0X	.0X	X.+	.00X−
22	D-88078	75.0	1,000-10,000	.X	.00X−	.0X	.X+	.0X	.0X−	.X+	.000X
	D-86972	72.7		.X	.00X	.0X+	.X+	.0X	.0X	X.+	.00X−
	D-86973	120.1		.X	.00X−	.0X+	.X+	.0X	.0X	X.	Tr
Average			1,000-10,000	.X	.00X−	.0X+	.X+	.0X	.0X	X.	.000X
23	55281	28.9	10-100	.X	.00X	.X	.X+	.0X	.0X+	X.	.000X+
	D-86219	65.0		.X−	.00X−	.X	.X+	.0X−	.0X	X.−	.000X+
	D-86750	28.9		.X−	Tr	.X	.X	.0X−	.0X+	X.	.000X+
Average			10-100	.X−	.00X−	.X	.X+	.0X−	.0X	X.−	.000X+
24	D-86228	47.6	100-1,000	.0X+	.00X−	.X−	.X+	.0X−	.0X	X.−	.000X+
25	D-87715	6.5	10-100	.X	.00X	.X+	X.	.0X	.0X	X.	.00X+
	55273	9.8		.X	.00X+	.X+	X.	.0X	.0X	X.	.00X+
Average			10-100	.X	.00X+	.X+	X.	.0X	.0X	X.	.00X+

26	D-72763	50.5	1,000-10,000	.0X	.000X	.X−	.X	.00X	.0X	X.	.000X
27	D-72759	87.5	1,000-10,000	.0X	.00X−	.0X	.X	.0X−	.0X	X.	.000X+
28	D-86752	12.4	100-1,000	.0X+	.00X−	.0X+	.X	.0X−	.0X	X.	.000X+
29	D-87710	1,415.1	10,000-100,000	.X	.00X−	.0X+	.X+	.0X	.0X	X.−	.000X+
30	55246	100.3	1,000-10,000	.X−	.00X	.X−	X.−	.0X	.0X	X.	.000X
31	D-86919	169.4	1,000-10,000	.X	.00X−	.X	X.−	.X−	.0X	X.+	.000X+
32	D-86749	5.3	10-100	0	Tr	.X−	.X	.0X−	.0X	X.	0
33	55276	14.9	10-100	.X	.00X	.X−	.X+	.X−	.0X	X.	.00X
34	55270	9.4	10-100	.0X	.00X	.X−	.X	.0X	.0X	.X+	.00X
35	55297	24.2	10-100	.X	.00X	.X−	X.	.0X	.0X	.X	.00X
36	D-87788	18.8	100-1,000	.X−	.00X−	.X−	.X+	.0X−	.0X	X.	.000X+
37	55265	32.5	10-100	.0X	.00X	.X	X.−	.0X	.0X	X.	.00X
38	D-87785	6.6	10-100	0	.00X−	.X	.X+	.0X−	.0X+	X.+	.00X−
	55268	9.4		.0X	.00X	.X−	X.−	.0X	.0X	X.	.000X
Average			10-100	.0X−	.00X	.X−	X.−	.0X	.0X	X.	.000X+
39	D-86748	38.1	1,000-10,000	.X−	Tr	Tr	.X+	.0X−	.0X	X.	.000X+
40	D-88051	375.0	1,000-10,000	.X	.00X−	.0X	.X+	.0X−	.00X+	.X	.00X−
41	D-86920	9.5	100-1,000	.X	.00X	.X−	X.−	.0X+	.0X	X.	.00X+
42	D-88053	375.0	1,000-10,000	.X	.00X−	.0X	.X+	.0X−	.00X+	.X	.000X+
43	205349	37.2	1,000-10,000	.X	.00X−	.0X+	.X+	.0X−	.0X	.X+	.00X−
44	D-68710	235.2	100-1,000	.X−	.00X−	.X−	X.−	.0X−	.0X	X.	.00X−
	D-87790	41.4		.X	.00X−	.X−	X.−	.00X+	.0X	X.	.00X+
	D-87791	79.5		.X	.00X−	.X	X.−	.0X−	.0X	X.	.00X
	D-87796	37.8		.X−	.00X−	.0X	X.−	.00X+	.0X	X.	.00X+
	D-87797	8.1		.0X+	.00X	.X−	X.−	.0X−	.0X	X.	.00X+
Average			100-1,000	.X−	.00X−	.X−	X.−	.0X	.0X−	X.	.00X
45	D-86712	11.5	10-100	.0X+	.00X	.X−	.X+	.0X	.0X	X.	.00X−
46	D-86957	242.0	1,000-10,000	.X	.00X	.X−	X.−	.X−	.0X+	X.+	.00X−
47	D-87677	21.1	10-100	.X−	.00X−	.0X+	.X+	.X−	.X−	X.+	.000X+
48	D-86943	4.8	10-100	0	.00X	.X	X.−	.0X−	.0X+	X.+	.00X+
49	D-87746	13.0	10-100	.X	.00X−	.X+	X.−	.0X	.0X+	X.+	.000X+
50	D-87697	8.5	10-100	.0X	.00X	.X	X.−	.0X+	.0X+	X.+	.000X+
51	D-87724	55.4	1,000-10,000	.0X+	.000X+	.X	X.−	.0X	.0X+	X.+	.000X+
52	55248	111.9	1,000-10,000	.X	.00X+	.X	X.−	.00X	.0X+	X.	.00X
	D-86915	173.3		.X−	.00X	.X	.X+	.0X−	.X−	XX.	.00X−
Average			1,000-10,000	.X−	.00X	.X	.X+	.00X+	.X−	X.+	.00X−
53	D-86956	9.4	10-100	.0X+	.00X	.X−	X.−	.0X+	.0X+	X.+	.00X
54	D-86940	12.0	10-100	.0X	.00X−	.X−	X.−	.0X+	.0X+	X.+	.00X
55	D-87727	7.1	1-10	.X	.00X	.X	X.−	.0X	.X−	X.+	.00X−
56	D-87748	8.2	1,000-10,000	.X+	.00X	.X	X.−	.0X	.0X	X.−	.00X
	D-87749	78.8		.X	.00X−	.X	X.−	.0X+	.0X	X.−	.00X
	D-87750	104.9		.X	.00X−	.X	X.−	.0X+	.0X+	X.	.00X−
Average			1,000-10,000	.X	.00X−	.X	X.−	.0X+	.0X+	X.	.00X−

See footnotes at end of table.

TABLE 1.—*Semiquantitative spectrographic analyses of samples of uranium deposits in the Morrison formation on the Colorado Plateau*—Continued

Deposit	Sample	Tons of ore from which sample was split	Size of deposit [2] (tons)	Semiquantitative spectrographic analyses [1]							
				U	Y	Na	Fe	Zr	Mn	Ca	Ni
57	D-86980	150.9	100-1,000	0.X−	0.00X−	0.X	X.−	0.0X	0.0X+	X.+	Tr
	D-86981	32.9		.0X+	.00X−	.X	.X+	.0X	.0X+	X.+	Tr
Average			100-1,000	.X−	.00X−	.X	X.−	.0X	.0X+	X.+	Tr
58	D-86756	12.3	10,000-100,000	.X	0	.X	.X	.0X−	.0X+	X.+	0
59	D-87726	11.5	10-100	0	0	.X	X.−	.0X	.X	XX.	0
60	D-86949	9.4	10-100	.X−	.000X+	.X−	.X+	.0X	.0X+	XX.	Tr
61	D-86945	10.1	10-100	.0X	.00X+	.X+	X.−	.0X+	.X−	X.+	.00X
62	D-86942	6.7	10-100	.X	.00X	.X	X.−	.0X	.0X+	X.−	.00X
	D-86946	28.3		.X−	.00X−	.X	X.−	.0X+	.0X+	X.	.00X
Average			10-100	.X−	.00X−	.X	X.−	.0X+	.0X+	X.	.00X
63	D-86921	14.7	100-1,000	.X−	.00X−	.0X+	X.−	.X−	.0X	X.+	.00X
	D-86922	17.5		.X	.00X	.X−	X.−	.X−	.0X+	XX.	.00X
	D-87733	11.7		.0X	.00X−	.X−	X.−	.0X+	.0X+	X.+	.00X−
Average			100-1,000	.X−	.00X−	.X−	X.−	.X−	.0X+	X.+	.00X
64	D-87743	14.2	1,000-10,000	.X−	.00X	.X	.X+	.0X	.0X	X.+	Tr
	D-87744	128.8		.X	.000X	.X−	.X+	.0X−	.0X+	X.−	.000X+
Average			1,000-10,000	.X	.000X	.X−	.X+	.0X−	.0X+	X.−	.000X+
65	D-86947	26.0	1,000-10,000	.X+	.00X	.X−	X.−	.0X+	.0X+	X.+	.00X
66	D-88052	375.0	1,000-10,000	.X	.00X−	.0X	.X+	.0X−	.00X	.X	.000X+
67	55250	316.0	10,000-100,000	.X−	.000X+	.0X+	.X+	.00X+	.0X	X.	.000X
	D-87698	428.1		.0X+	0	.X−	.X	.0X−	.0X+	X.+	Tr
Average			10,000-100,000	.0X+	Tr	.X−	.X	.0X−	.0X+	X.+	Tr
68	D-86733	19.4	10-100	.0X+	.00X−	.X−	.X+	.0X	.0X+	X.+	.00X
69	D-86713	41.2	10-100	Tr	.00X−	.X−	.X+	.0X−	.0X+	X.+	.000X
70	D-87756	190.4	1,000-10,000	.X	.00X	.X−	X.−	.0X+	.0X+	X.+	.00X
71	D-86711	55.9	10-100	.0X+	0	.0X+	.X+	.0X−	.0X−	.X+	.00X−
72	D-87751	6.2	10-100	.X	.00X	.X	X.−	.0X	.0X	X.+	.00X−
	D-87752	35.0		.X−	.00X−	.X	X.−	.0X	.0X+	X.+	.00X−
Average			10-100	.X−	.00X−	.X	X.−	.0X+	.0X+	X.+	.00X−

73	D-86948	10.0	10–100	.X	.00X	.X−	X.−	.0X+	.0X+	X.+	.00X−
74	D-86232	10.2	10–100	.0X+	.00X	.0X+	X.−	.0X	.0X+	X.	.0X−
75	D-87784	11.3	10–100	.0X+	.00X	.X−	X.−	.0X	.0X+	X.+	.00X

---

[1] Tr, near threshold(below limit of sensitivity); 0, looked for but not detected.

[2] Size determined from unpublished production and reserves information.

[3] Average analysis throughout table is computed by weighting the logs of the midpoints of the spectrographic classes by tons of ore represented by the sample. See p. 22 for explanation of values.

TABLE 2.—*Classification of elements in sandstone-type uranium deposits in the Salt Wash member of the Morrison formation on the Colorado Plateau*

Elements are—		Element	Abundance ratio[1]
Dominantly intrinsic	Dominantly syngenetic	Al	2. 1
		Fe[2]	3. 8
		K	≈1
		Ti	2. 0
		Zr[2]	2. 4
		B	≈2
		Cr	2. 6
		Zn	≈2(?)
		Ag	≈2(?)
		Sb	≈. 5(?)
	Dominantly epigenetic (diagenetic).	Mg	3. 0
		Ca[2]	. 6
		Na[2]	≈1
		Mn[2]	1. 4
		Ba	2. 4
		Sr	2. 5
Dominantly extrinsic	Ore elements	U[2]	>1, 000
		V	500
	Accessory elements	Co	≈20
		Ni[2]	≈20
		Cu	7
		As	>17
		Se	>6
		Y[2]	≈7
		Mo	>3
		Pb	>9

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[1] Ratio of estimated geometric mean concentration in uranium deposits to estimated geometric mean concentration in unmineralized sandstone.

[2] Concentration in uranium deposits significantly related to the size of deposits (see table 3).

TABLE 3.—*Correlation coefficients between elements in 75 uranium deposits in the Salt Wash member of the Morrison formation and sizes of the deposits*

Element	Correlation coefficient[1] (r)	Probability level[2]
Lead	+0.02	-----------------
Magnesium	+.03	-----------------
Chromium	−.03	-----------------
Titanium	−.04	-----------------
Molybdenum	−.04	-----------------
Barium	−.04	-----------------
Potassium	−.05	-----------------
Boron	+.08	-----------------
Aluminum	−.08	-----------------
Strontium	+.08	-----------------
Copper	−.13	-----------------
Vanadium	+.17	-----------------
Nickel	−.24	$0.05>P>0.02$
Calcium	−.25	$.05>P>.02$
Manganese	−.25	$.05>P>.02$
Zirconium	−.28	$.02>P>.01$
Iron	−.29	$.02>P>.01$
Sodium	−.31	$.01>P>.002$
Yttrium	−.37	$P<.001$
Uranium	+.37	$P<.001$

[1] Correlation coefficient, r, between log element concentration and log size of the deposits is given by

$$r=\frac{\Sigma xy-n(\bar{x})(y)}{\sqrt{(\Sigma x^2-n\bar{x}^2)(\Sigma y^2-n\bar{y}^2)}},$$

where x is the log of the concentration of the element in a sample from the deposit and y is the log of the size of the deposit, in tons; $\bar{x}$ is the mean x and $\bar{y}$ is the mean y for the 75 samples; n is the number of pairs (75). Values of r may range from +1.00, indicating a perfect direct correlation, to −1.00, indicating a perfect inverse correlation. Lowest significant coefficient at 95-percent level of confidence is 0.228. Lowest significant coefficient at 90-percent level of confidence is 0.192.

[2] Probability of values greater than absolute r occurring by chance. Probability values derived from table of t (Fisher, 1950, p. 193). The equation for t is

$$t=\frac{r\sqrt{n-2}}{\sqrt{1-r^2}}.$$

TABLE 4.—*Size-estimation factors for use in method 1*

[Upper figures are constants for weighted average size; lower figures, for expanded weighted average size. Values in parentheses are for extrapolated element concentrations beyond the spectrographic limit of detectability]

Concentration [1] (percent)	Log concentration [2] (ppm)	Uranium	Yttrium	Sodium	Iron	Zirconium	Manganese	Calcium	Nickel
XX.–	5–5⅓	-------- --------	-------- --------	-------- --------	-------- --------	-------- --------	-------- --------	(0.2125) (–.0689)	-------- --------
X.+	4⅔–5	0.5946 1.4022	–0.0832 –1.0983	0.1675 –.2421	0.1881 –.1628	0.0561 –.6710	0.0847 –.5609	.2300 –.0015	0.0942 –.5243
X.	4⅓–4⅔	.5577 1.2601	–.0401 –1.0414	.1963 –.1312	.2324 .0077	.0890 –.5443	.1107 –.4608	.2475 .0659	.1080 –.4712
X.–	4–4⅓	.5208 1.1181	.0030 –.8754	.2251 –.0204	.2766 .1779	.1220 –.4173	.1367 –.3607	.2651 .1336	.1219 –.4177
.X+	3⅔–4	.4839 .9760	.0461 –.7095	.2540 .0909	.3209 .3485	.1549 –.2906	.1627 –.2606	.2826 .2010	.1358 –.3642
.X	3⅓–3⅔	.4470 .8340	.0892 –.5436	.2828 .2018	.3651 .5186	.1878 –.1640	.1886 –.1609	.3002 .2688	.1497 –.3107
.X–	3–3⅓	.4101 .6919	.1322 –.3780	.3116 .3127	.4094 .6892	.2207 –.0373	.2146 –.0608	.3177 .3361	.1636 –.2571
.0X+	2⅔–3	.3733 .5502	.1753 –.2121	.3405 .4239	.4537 .8597	.2536 .0894	.2406 .0393	.3353 .4039	.1775 –.2036
.0X	2⅓–2⅔	.3363 .4078	.2184 –.0462	.3693 .5348	.4979 1.0299	.2865 .2160	.2666 .1394	.3528 .4713	.1914 –.1501
.0X–	2–2⅓	(.2994) (.2657)	.2615 .1198	(.3981) (.6457)	.5422 1.2005	.3195 .3431	.2926 .2395	.3704 .5390	.2053 –.0966
.00X+	1⅔–2	(.2625) (.1236)	.3045 .2853	(.4270) (.7570)	.5864 1.3706	.3524 .4697	.3186 .3396	.3879 .6064	.2192 –.0431
.00X	1⅓–1⅔	(.2256) (–.0184)	.3476 .4513	(.4558) (.8678)	.6307 1.5412	.3853 .5964	.3446 .4397	.4055 .6742	.2331 .0104
.00X–	1–1⅓	-------- --------	.3907 .6172	-------- --------	.6750 1.7118	.4182 .7231	.3706 .5398	.4230 .7416	.2470 .0640
.000X+	⅔–1	-------- --------	.4338 .7831	-------- --------	(.7192) (1.8819)	(.4511) (.8497)	.3966 .6399	(.4405) (.8089)	.2609 .1175
.000X	⅓–⅔	-------- --------	(.4769) (.9491)	-------- --------	(.7635) (2.0525)	(.4840) (.9764)	(.4226) (.7400)	(.4581) (.8767)	.2748 .1710
.000X–	0–⅓	-------- --------	(.5199) (1.1146)	-------- --------	(.8077) (2.2226)	(.5169) (1.1031)	(.4486) (.8401)	(.4756) (.9441)	(.2886) (.2241)
.0000X+	–⅓–0	-------- --------	(.5630) (1.2806)	-------- --------	-------- --------	-------- --------	(.4746) (.9402)	-------- --------	(.3025) (.2776)
.0000X	–⅔––⅓	-------- --------	-------- --------	-------- --------	-------- --------	-------- --------	-------- --------	-------- --------	(.3164) (.3311)

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[1] See page 22 for explanation of values.

[2] To obtain ranges of log concentration, in percent, add –4 to each limit given here.

TABLE 5.—*Size-estimation constants for use in method 2*

[Leaders indicate concentration beyond spectrographic limit of detectability. Values in parentheses are for extrapolated element concentrations beyond the spectrographic limit of detectability.]

Concentration[1] (percent)	Log concentration (ppm)	Uranium[2]	Yttrium	Sodium	Iron	Zirconium	Manganese	Calcium	Nickel
XX.−	5-5⅓	----------	----------	----------	----------	----------	----------	(0.0686)	----------
X.+	4⅔-5	0.7725	−0.4540	−0.3545	−0.2470	−0.4270	0.0695	.0490	−0.2000
X.	4⅓-4⅔	.4567	−.2724	−.2127	−.1482	−.2562	.0417	.0294	−.1200
X.−	4-4⅓	.1409	−.0908	−.0709	−.0494	−.0854	.0139	.0098	−.0400
.X+	3⅔-4	−.1749	.0908	.0709	.0494	.0854	−.0139	−.0098	.0400
.X	3⅓-3⅔	−.4907	.2724	.2127	.1482	.2562	−.0417	−.0294	.1200
.X−	3-3⅓	−.8065	.4540	.3545	.2470	.4270	−.0695	−.0490	.2000
.0X+	2⅔-3	−1.1223	.6356	.4963	.3458	.5978	−.0973	−.0686	.2800
.0X	2⅓-2⅔	−1.4381	.8172	.6381	.4446	.7686	−.1251	−.0882	.3600
.0X−	2-2⅓	(−1.7539)	.9988	(.7799)	.5434	.9394	−.1529	−.1078	.4400
.00X+	1⅔-2	(−2.0697)	1.1804	(.9217)	.6422	1.1102	−.1807	−.1274	.5200
.00X	1⅓-1⅔	(−2.3855)	1.3620	(1.0635)	.7410	1.2810	−.2085	−.1470	.6000
.00X−	1-1⅓	----------	1.5436	----------	.8398	1.4518	−.2363	−.1666	.6800
.000X+	⅔-1	----------	1.7252	----------	(.9386)	(1.6226)	−.2641	(−.1862)	.7600
.000X	⅓-⅔	----------	(1.9068)	----------	(1.0374)	(1.7934)	(−.2919)	(−.2058)	.8400
.000X−	0-⅓	----------	(2.0884)	----------	(1.1362)	(1.9642)	(−.3197)	(−.2254)	(.9200)
.0000X+	−⅓-0	----------	(2.2700)	----------	----------	----------	(−.3475)	----------	(1.0000)
.0000X	−⅔-−⅓	----------	----------	----------	----------	----------	----------	----------	(1.0800)

[1] See p. 22 for explanation of values.

[2] The y intercept of equation 2 is included with all factors listed for uranium.

TABLE 6.—*Values of the matrices C_8 and C_2 for equations 2a and 3a*

Matrices C_8				
Column 1	**Column 2**	**Column 3**	**Column 4**	**Column 5**
1.7400 998	0.0063 5119 72	0.0247 0828 9	0.0060 7675 46	−0.0425 6645 3
.0063 5119 71	.0014 4998 92	.0000 9537 4681	−.0001 0718 575	−.0005 3859 083
.0247 0828 8	.0000 9537 4679	.0024 1109 32	.0000 9868 9965	−.0003 3651 925
.0060 7675 43	−.0001 0718 575	.0000 9868 9960	.0030 3101 63	−.0017 0291 47
−.0425 6645 2	−.0005 3859 083	−.0003 3651 925	−.0017 0291 47	.0080 2480 91
.0197 6463 1	−.0007 6943 667	−.0012 5080 55	.0000 3643 5302	−.0009 9022 844
.0897 7376 7	.0007 7461 663	.0003 2079 755	−.0016 0953 64	−.0002 6955 054
−.0619 2568 7	.0002 1164 578	−.0000 3164 3963	−.0006 6375 175	.0003 0703 967
.0071 7884 35	−.0001 0565 453	−.0005 0361 664	−.0000 1028 1318	−.0007 8078 125

Matrices C_8—Continued				Matrices C_2
Column 6	**Column 7**	**Column 8**	**Column 9**	**Column 1**
0.0197 6463 0	0.0897 7376 7	−0.0619 2568 7	0.0071 7884 36	0.4954 9473
−.0007 6943 667	.0007 7461 662	.0002 1164 579	−.0001 0565 453	.0025 2836 06
−.0012 5080 55	.0003 2079 750	−.0000 3164 3933	−.0005 0361 664	.0271 1699 0
				Column 2
.0000 3643 5302	−.0016 0953 64	−.0006 6375 174	−.0000 1028 1318	0.0025 2836 06
−.0009 9022 843	−.0002 6955 050	.0003 0703 964	−.0007 8078 126	.0011 4016 08
.0049 3239 01	−.0009 5422 205	−.0009 3374 780	.0002 9117 682	−.0002 3692 412
				Column 3
−.0009 5422 208	.0097 5397 26	−.0044 3768 66	.0001 6653 749	0.0271 1699 0
−.0009 3374 778	−.0044 3768 66	.0049 8278 68	.0000 2077 6805	−.0002 3692 412
.0002 9117 682	.0001 6653 749	.0000 2077 6807	.0007 1798 808	.0016 5261 89

TABLE 7.—*Log size and size, in tons, of uranium deposits, estimated with equation 3, based on uranium and yttrium contents*

[Upper figures are log size based on concentration class; lower figures give size in tons]

Concentration classes, yttrium [1]:		Concentration classes, uranium [1]							
		0.00X [2]	0.00X+ [3]	0.0X−	0.0X	0.0X+	0.X−	0.X	0.X+
	x'_U x'_Y	−15	−13	−11	−9	−7	−5	−3	−1
0.00X+	−13	0. 5906 4	0. 8558 7	1. 1210 13	1. 3862 24	1. 6514 45	1. 9166 83	2. 1818 152	2. 4470 280
.00X	−15	. 9094 8	1. 1746 15	1. 4398 28	1. 7050 51	1. 9702 93	2. 2354 172	2. 5006 317	2. 7658 583
.00X−	−17	1. 2282 17	1. 4934 31	1. 7586 57	2. 0238 106	2. 2890 195	2. 5542 358	2. 8194 660	3. 0846 1215
.000X+	−19	1. 5470 35	1. 8122 65	2. 0774 120	2. 3426 220	2. 6078 405	2. 8730 747	3. 1382 1375	3. 4034 2532
.000X	−21	1. 8658 73	2. 1310 135	2. 3962 249	2. 6614 459	2. 9266 845	3. 1918 1555	3. 4570 2864	3. 7222 5275
.000X− [3]	−23	2. 1846 153	2. 4498 282	2. 7150 519	2. 9802 955	3. 2454 1760	3. 5106 3240	3. 7758 5968	4. 0410 10, 990
.0000X+ [2]	−25	2. 5034 319	2. 7686 587	3. 0338 1081	3. 2990 1991	3. 5642 3666	3. 8294 6752	4. 0946 12, 430	4. 3598 22, 900

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[1] See p. 22 for explanation of values.
[2] Looked for but not detected.
[3] Trace.

TABLE 8.—*Comparison of the log size of 40 uranium deposits, estimated with equations 1–3, with the range of the true log size*

[Estimated log size: Upper figures preceded by ± are ranges of error at 95-percent confidence level; lower figures, at 80-percent confidence level]

Deposit	True log size (tons)	Estimated log size (tons)		
		Eq 1	Eq 2	Eq 3
1	2–3	2.66±1.68 ±1.09	2.64±1.65–2.47 ±1.07–1.60	2.82±1.75 ±1.14
2	3–4	2.54±1.67 ±1.09	2.79	2.50±1.75 ±1.14
3	3–4	2.26±1.68 ±1.09	2.55	2.50±1.75 ±1.14
4	3–4	2.91±1.68 ±1.09	2.82	2.87±1.74 ±1.13
5	2–3	2.94±1.68 ±1.09	2.80	2.55±1.74 ±1.13
6	2–3	2.54±1.67 ±1.09	2.46	2.29±1.74 ±1.13
7	1–2	2.04±1.68 ±1.09	2.35	2.55±1.74 ±1.13
8	2–3	2.67±1.68 ±1.09	2.75	2.50±1.75 ±1.14
9	4–5	3.89±1.73 ±1.12	4.21	4.09±1.83 ±1.19
10	4–5	3.34±1.69 ±1.10	3.78	4.09±1.83 ±1.19
11	3–4	1.90±1.68 ±1.09	2.00	2.61±1.74 ±1.13
12	3–4	2.96±1.68 ±1.09	3.00	2.55±1.74 ±1.13
13	1–2	2.11±1.68 ±1.09	2.38	2.18±1.76 ±1.14
14	2–3	4.24±1.76 ±1.14	4.37	3.51±1.78 ±1.16
15	2–3	2.86±1.68 ±1.09	2.95	2.55±1.74 ±1.13
16	2–3	1.70±1.69 ±1.10	2.40	2.50±1.75 ±1.14
17	3–4	2.00±1.68 ±1.09	2.13	2.18±1.76 ±1.14
18	3–4	2.25±1.68 ±1.09	2.43	2.55±1.74 ±1.13
19	2–3	1.63±1.69 ±1.10	1.84	2.24±1.76 ±1.14
20	2–3	2.27±1.68 ±1.09	2.23	2.55±1.74 ±1.13
21	3-4	3.44±1.70 ±1.10	3.20	3.51±1.78 ±1.16
22	4–5	2.81±1.68 ±1.09	2.97	2.82±1.75 ±1.14
23	4–5	4.28±1.76 ±1.14	3.95	2.82±1.75 ±1.14
24	3–4	2.20±1.68 ±1.09	1.26	1.23±1.81 ±1.18
25	2–3	2.20±1.68 ±1.09	2.03	2.55±1.74 ±1.13
26	3–4	3.02±1.68 ±1.09	3.02	2.50±1.75 ±1.14
27	2–3	3.13±1.69 ±1.09	3.55	4.04±1.81 ±1.17
28	1–2	2.28±1.68 ±1.09	2.43	2.50±1.75 ±1.14
29	4–5	2.98±1.68 ±1.09	3.06	2.82±1.75 ±1.14
30	1–2	1.63±1.69 ±1.10	1.69	1.97±1.74 ±1.13
31	2–3	2.51±1.67 ±1.09	2.54	2.29±1.74 ±1.13
32	1–2	2.20±1.68 ±1.09	2.34	2.50±1.75 ±1.14
33	3–4	4.46±1.78 ±1.15	4.60	3.83±1.84 ±1.19
34	4–5	3.37±1.70 ±1.10	3.06	2.55±1.74 ±1.13
35	2–3	2.41±1.67 ±1.09	2.54	2.61±1.74 ±1.13
36	4–5	3.19±1.69 ±1.09	2.93	2.55±1.74 ±1.13
37	3–4	3.46±1.70 ±1.10	3.31	2.82±1.75 ±1.14
38	4–5	3.08±1.68 ±1.09	3.24	2.82±1.75 ±1.14
39	2–3	1.92±1.68 ±1.09	2.38	2.24±1.76 ±1.14
40	2–3	3.02±1.68 ±1.09	3.21	2.87±1.74 ±1.13

TABLE 9.—*Analysis of variance of log size of uranium deposits for comparing three methods of predicting size of deposits*

[Variance ratio (F): Significant at 0.95 level of probability; critical F ratios: 0.95 (1, 73)=3.9; 0.95 (8, 66)=2.0; 0.95 (2, 72)=3.1; 0.95 (6, 66)=2.2; 0.995 (1, 73)=8.4; 0.995 (8, 66)=3.0; and 0.995 (2, 72)=5.7]

	Sum of squares	Degrees of freedom	Mean square	$\sqrt{\text{Mean square}}$	Variance ratio (F)
Total	$\Sigma y^2=544.75$	$n=75$			
Mean	$n(\bar{y})^2=463.7633$	1			
Deviation from mean	$\Sigma(y-\bar{y})^2=80.9866$	$n-1=74$	1.0944	1.05	
Reduction due to x with eq 1($r_1=0.61$).	$r_1^2[\Sigma(y-\bar{y})^2]=29.9326$	1	29.9326		42.8
Deviation from eq 1	$(1-r_1^2)[\Sigma(y-\bar{y})^2]=51.0540$	$n-2=73$	0.6994	[1] 0.836	
Reduction due to $x_1'\ldots x_8'$ with eq 2 ($r^2=0.67$).	$r_2^2[\Sigma(y-\bar{y})^2]=36.6314$	8	4.5789		6.8
Deviation from eq 2	$(1-r_2^2)[\Sigma(y-\bar{y})^2]=44.3552$	$n-9=66$	0.6720	[1] 0.820	
Reduction due to x_1' and x_2' with eq 3 ($r_3=0.58$).	$r_3^2[\Sigma(y-\bar{y})^2]=27.0130$	2	13.5065		18.0
Deviation from eq 3	$(1-r_3^2)[\Sigma(y-\bar{y})^2]=53.9736$	$n-3=72$	0.7496	[1] 0.866	
Reduction due to $x_2'\ldots x_8'$ with eq 2.	$(r_2^2-r_3^2)[\Sigma(y-\bar{y})^2]=9.6184$	6	1.6030		2.4

[1] Standard error of the equation, or standard error of estimate, which may also be obtained from methods given by Waugh (1952, p. 445) or by Dixon and Massey (1957, p. 197). The standard error is equivalent to $[\Sigma(y-y')^2/\text{d.f.}]^{\frac{1}{2}}$, where y is the true size of a deposit, y' is the estimated or predicted size $[\Sigma(y-y')^2=(1-r^2)\Sigma(y-\bar{y})^2]$, and d.f. is the number of degrees of freedom.

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