Measuring Geothermal Gradients in Drill Holes Less Than 60 Feet Deep East Tintic District, Utah

GEOLOGICAL SURVEY BULLETIN 1172

Lovering and Goode—GEOTHERMAL GRADIENTS, EAST TINTIC DISTRICT, UTAH—Geological Survey Bulletin 1172

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By T. S. LOVERING *and* H. D. GOODE

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Utilization of shallow-hole temperature measurements in finding thermal diffusivity and conductivity of rocks, temperature gradient, average surface temperature, and certain kinds of climatic data

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CONTENTS

ILLUSTRATIONS

[Plate is in pocket]

TABLES

MEASURING GEOTHERMAL GRADIENTS IN DRILL HOLES LESS THAN 60 FEET DEEP, EAST TINTIC DISTRICT, UTAH

By T. S. LOVERING and H. D. GOODE

ABSTRACT

In the East Tintic district steep temperature gradients are in part related to oxidizing sulfides at depth, and knowledge of anomalous gradients should be helpful in searching for blind ore bodies. Temperatures were measured by thermocouples in drill holes from 3 to almost 60 feet deep to establish the variation of geothermal gradients with depth and season, so that the possibility of using such measurements to deduce the deeper geothermal gradient could be appraised. Geothermal gradients, against which the shallow-drill-hole measurements could be evaluated, had already been measured to depths of more than a thousand feet in many places. The study shows that geothermal gradients can be calculated from temperature measurements in relatively shallow drill holes if the thermal diffusivity of the rocks is known. Extensive background climatic data were gathered to evaluate the reliability of the theoretical methods used to eliminate the effect of the sun's heat from near-surface measurements; such work would make procedures too expensive and time consuming for practical use but proved the soundness of the rapid mathematical methods devised. Direct measurements of gradients should be made in uncased holes more than 100 feet deep, as seasonal surface variations are negligible in their effect on rock temperature but steel casing could cause temperature disturbances well below this depth. Temperature measurements in shallow holes (40–60 ft deep) can be used to determine the diffusivity of the subsurface materials in place, where the variations in the surface temperature with time are known, or at least the spring and fall dates when the annual surface temperature wave reaches the mean annual surface temperature. These dates are essential for all the calculations relating to gradient but can be calculated from two sets of temperature measurements in the zone affected by the annual wave if the diffusivity of the rocks is already known.

In reaching these conclusions, many disturbing factors had to be evaluated. The diurnal effects of the sun's heat were observed at the surface and at shallow depths in many holes and were readily measurable in soil down to about 60 cm but are not appreciable at 1 meter. The annual temperature wave was clearly evident in several holes at depths of 10 to 15 m, but calculations indicate that in most rocks it is negligible below about 20 m.

Local heat sources such as oxidizing sulfides caused large increases in geothermal gradients, from a normal gradient of 1.5°F per 100 ft to as much as 9°F per 100 ft; such gradients were readily ascertained by direct measurement in holes 130 feet deep and were calculated with confidence from measurements in holes 40 to 60 feet deep after the diffusivity of the rocks was known.

INTRODUCTION AND ACKNOWLEDGMENTS

The East Tintic district is especially well known for two related features: its many blind ore bodies and its variation in underground temperatures. Oxidation of sulfides near the water table—commonly a thousand feet or more below the surface—is a major source of heat and gives rise to abnormally high temperature gradients in the surrounding and overlying rocks. The possibility that such gradients could be detected in shallow surface holes and used in exploration for ore at depth led to the study reported here.

The writers wish to express their appreciation of the help received from other personnel on the East Tintic project in the routine collection of temperature data and with the installation of the thermocouples. John Lemish, Hal T. Morris, and C. Albert Carlson read surface and subsurface temperatures, and Lemish assisted in wiring and placing the thermocouples.

TEMPERATURE-DETERMINING FACTORS AT AND NEAR THE EARTH'S SURFACE

The temperature of rock or soil at and near the surface of the earth results almost entirely from heating by the sun and cooling through radiation, evaporation, and various heat-absorbing processes. At any particular surface location the heat supplied from below the surface is relatively constant; it represents heat from the interior locally supplemented by heat from subsurface oxidation or other local heat sources and is responsible for rock temperatures below the zone where the effect of surface temperatures is perceptible. The temperatures at a given depth in any locality, however, depend not only on the heat flow through the rocks but on the thermal properties of the rock, and on the surface temperature with which the subsurface temperatures are in equilibrium or to which they are adjusting. On the other hand, the heat from the sun is turned on and off at varying intervals: its effect reaches a maximum in midsummer and a minimum in midwinter; its effect also differs at different times of the day. This fluctuating effect can be separated into two principal heat waves, a diurnal and an annual wave, but both may be complicated by minor waves that correspond to hot or cold spells of appreciable duration. All such periodic waves decrease in amplitude as they penetrate downward into the earth: the wave produced by the daily temperature changes dissipates rapidly and is generally negligible below 1.5 meters; nonperiodic hot or cold waves of one or two weeks' duration seldom penetrate below 5 meters; but with sensitive instruments the effect of the annual wave is measurable in rocks of average thermal properties to depths of at least 20 meters.[1] The success of any attempt to deter-

[1] In this study, it is difficult to avoid some confusion because of the use of the logically simple metric system, which is so efficient for heat calculations, and the awkward but familiar English units customarily used in the United States for depth-temperature measurements; it is hoped that the conversion factors given below will minimize the difficulties inherent in the dual use of our traditional units and those appropriate to scientific work.

Centimeter (cm) = 0.3937 inches (in.); 2.54 cm = 1 in.

Meter (m) = 3.28 feet; 1 ft = 0.3048 m

Degree Centigrade (°C) = 1.8 or 9/5 degrees Fahrenheit (°F)

°C per meter × 54.86 = °F per hundred feet

Calorie (cal) = thermal capacity of water at 15 °C per gram per °C = 0.0022 British Thermal Unit (BTU) per °F = 4.185 joules per °C.

Gram (g) = 0.0022 pound (avoirdupois) = 0.0353 ounces (avoirdupois).

mine the local geothermal gradient from temperatures measured between the surface and a depth of 20 meters depends on compensating for the variations in surface temperature.

The amount of the sun's heat that reaches any given part of the earth's surface is determined chiefly by the time of day, the surface and soil moisture, the weather on that day, the time of year, the slope of the ground, and the kind and amount of cover present. The subsurface effect of the sun's heat is determined chiefly by the thermal properties of the soil or rock and by the time of the year, but the actual surface temperature and hence the mean annual temperature depends on topographic position, the presence or lack of foliage or of snow, the moisture content of the surface and subsurface, and similar local factors.

These local characteristics also determine to a large degree the ability of the rock to dissipate heat from subsurface sources, but because such physical factors rarely cause abrupt variations in surface temperatures within small areas—say an acre or so—it might seem reasonable to believe that local subsurface heat sources, 500 meters or less below the surface, produce measurable thermal anomalies at the surface. In fact, however, the expected thermal anomalies are effectively masked by stronger heating and cooling effects at the surface. In some areas the near-surface heating effect of the sun is unexpectedly large; in deserts the daily range in surface temperature often exceeds 100° F; such temperatures and the compensating ability of the surface to dissipate heat by radiation and air cooling, and in other ways, have a far greater effect on near-surface temperatures than do local underground heat sources; the thermal anomalies due to local subsurface heat sources are thus virtually impossible to detect at the surface or at depths of only a few feet.

Our investigation in the East Tintic area did not include the study of near-surface horizontal thermal anomalies but near-surface ground temperatures are so obviously related to the complex factors noted above that areal variations in those temperatures would be most difficult to interpret at the surface or at depths where diurnal variations were still detectable. The range of the microclimate at the surface is controlled mainly by topographic position. For instance, north-facing slopes are more protected from the sun than are south-facing slopes. This protection reduces the daily and annual temperature ranges, conserves the moisture that falls as rain and snow, and consequently results in denser vegetation. Temperatures measured in holes on such slopes are likely to show minimum disturbance by surface variations in temperature.

It is evident, then, that although local subsurface heat sources must produce thermal anomalies all the way to the surface, such anomalies at depths of a few feet are completely masked by the short-term temperature fluctuations at the surface.

GEOTHERMAL GRADIENT DETERMINATION

The geothermal gradient is commonly expressed as degrees Fahrenheit per hundred feet or degrees centigrade per centimeter or kilometer. This gradient is due to dissipation of subsurface heat, which is not everywhere the same, and gradients vary from place to place because of the differences both in rock and in regional and local heat sources. In two localities where gradients in a steady-state condition are identical, the temperatures at a given moderate depth will differ by the amount that the mean surface temperatures differ. The regional gradient generally is assumed to be the result of minute but ubiquitous radioactive heat sources that are distributed through the rocks for many miles below the surface; perhaps the smallest discrete source of internal heat that could properly be called regional would be a magmatic body comparable in size to a batholith—one underlying a surface area of at least 40 square miles. A local heat source, on the other hand, might be a subterranean hot spring or a near-surface body of oxidizing sulfide a few tens or hundreds of feet in diameter.

The quantity of heat (Q) that flows from the heat source to the surface depends on the thermal conductivity (k), the geothermal gradient ($\partial T/\partial x$) and the area:
for unit area,

$$Q=k\ \partial T/\partial x$$

and if the gradient is constant through the interval x,

$$Q=kA\left(\frac{T_2-T_1}{x_1-x_2}\right)$$

where x is the depth parameter, T is temperature, $T_2>T_1$, and A is area.

If there is no other heat source between a given heat source and the surface, and no heat sink (such as moving perched ground water), the amount of heat transmitted when steady-state temperature conditions exist must be the same regardless of the differences in conductivity of the rocks:

$$k\left(\frac{\partial T}{\partial x}\right)=Q=k'\left(\frac{\partial T}{\partial x}\right)'$$

or

$$\frac{k}{k'}=\frac{\left(\frac{\partial T}{\partial x}\right)'}{\left(\frac{\partial T}{\partial x}\right)} \qquad (1)$$

and a geothermal gradient above a heat source will change in passing from one rock to another of different conductivity. This change in gradient is inversely proportional to the conductivities of the rocks; for example, a gradient of 3° per 100 feet in quartzite ($k=0.012$) would give way to a gradient of 12° per 100 feet in an overlying dry shale ($k=0.003$), though the quantity of heat passing through each would be the same. The rise in temperature per unit volume produced by a given quantity of heat, however, is inversely proportional to the specific heat (c) and the density (ρ) of the material and directly proportional to the conductivity; these relations are combined in the diffusivity constant α, where $\alpha=k/c\rho$.

If the average annual surface temperature has been nearly constant for a long time, the temperature gradient between any two depths in reasonably homogeneous rock is generally about the same as the gradient between any two other depths in the same vertical line. Measurements made in the East Tintic district indicate nearly constant gradients at depth within given rock types except near local heat sources or sinks. Ground water, especially where moving in localized channels, is the chief heat sink, and oxidizing disseminated pyrite is the chief underground heat source. The ability of most rocks to transmit heat is so small that the effect of the annual wave generated by the sun's heat is hardly perceptible below 20 meters, where the annual variation is usually no more than a few tenths of a degree Fahrenheit; accordingly measurements made below that depth record the temperatures that are due primarily to the geothermal gradient.

In that part of the subsurface measurably affected by the heat from the sun, the measurement of geothermal gradients is an entirely different matter. The temperatures due to geothermal gradient in this region must be separated from the temperatures caused by the heat of the sun, and to find the gradient one must know for two or more depths the difference between the actual temperature and the part of these temperatures that reflect variations in surface temperature. In effect, the waves generated by the sun's heat are superimposed on the nearly straight-line curve of the geothermal gradient; the problem is how to separate the two so that the gradient can be recognized.

Daily surface-temperature measurements over a period of several years would be needed to calculate accurately the effects of the sun's heat during the annual cycles that affect the temperature curves in the first 100 feet. Such measurements are hardly warranted except perhaps in an experimental investigation such as the one reported here; however, if the diffusivity of the rocks is given, a few properly spaced subsurface temperature measurements in shallow holes are all

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that are required for the calculation of gradients that approximate the deep gradient. (See p. 15, 32–35.)

TEMPERATURES IN TINTIC AREA

In the Tintic and East Tintic districts of central Utah are many mine workings and drill holes where members of the U.S. Geological Survey have measured subsurface temperatures. A local subterranean hot-spring area and several bodies of oxidizing sulfide near the level of ground water, which is commonly more than a thousand feet below the surface, provide many local heat sources of different intensity and at a sufficient depth to establish easily measurable geothermal gradients. For the study of the relevance of temperatures near the surface to deeper geothermal gradients, many shallow holes were drilled and wired for temperature measurements. (See map, pl. 1.) The deepest of these, hole 51, which was nearly 60 feet deep (17.70 meters), was 25 feet from a drill hole 1,100 feet deep where a gradient of 7.4°F per 100 feet had been measured by thermocouple, bathythermograph, thermister, and maximum-temperature thermometers. Many of the other holes were also placed so as to take advantage of available deep-gradient data. Bottom-hole temperatures were obtained subsequently in many deeper holes in the general area of the shallow thermocouple holes, especially near the site of the Burgin shaft (pl. 1). In areas lacking local heat sources, the geothermal gradient is 1.5° to 2°F per 100 feet; where sulfides are oxidizing at depth, the gradient commonly is 4° to 5°F per 100 feet and only in areas of sulfide disturbed by mine workings and close to the underground hot springs area were gradients more than 9°F per 100 feet observed.

SUN'S EFFECT ON NEAR-SURFACE TEMPERATURES: THEORETICAL CONSIDERATIONS

The sun's effect on near-surface temperatures is controlled by the local and regional geographic positions, by the thermal properties of the rocks, and by the fluctuations in the amount of solar heat that arrives at the surface. These factors are discussed in turn below.

GEOGRAPHIC POSITION

In any search for anomalous local heat sources the regional geographic factor can be disregarded, but such local factors as the sunny or shady side of a hill, the amount of vegetation, the amount of soil, depth of weathering, and the position of perched water tables, should be considered as they relate to radiation from the sun—or earth—and to the transmission of heat into or out of the earth. These geographic factors are not readily subjected to theoretical treatment, but surface-temperature measurements made on this project showed appreciable differences in temperatures measured on a ridge top and at the north side of the ravine below it (p. 23). It is evident too that subsurface temperatures would be affected by perched ground water or by the underground channels intermittently used by water moving down to the deep permanent water table.

THERMAL DIFFUSIVITY OF ROCKS

For any analysis of the effect of the fluctuating heat from the sun on subsurface temperatures the only pertinent thermal property of earth material is the diffusivity, which as noted above, is a measure of the properties that control the temperature change made when heat is applied to a body.

The range in diffusivity (table 1) of some of the rocks, such as limestone, granite, and schist, suggests that the values given should not be applied to a rock simply because its name appears in a table. Rather than use such values, it is always advisable, wherever possible, to make more precise determinations for the rock or rocks being investigated. (See p. 24 for simple field methods of doing this.)

The diffusivity of rock is greatly affected by such factors as degree of weathering, porosity, and the amount of moisture present; measurements made in the East Tintic district indicate that the diffusivity of a nearly homogeneous formation generally increases with depth but becomes nearly constant a few meters below the surface.

TABLE 1.—*Thermal diffusivities of some common rocks, in cgs units*

	1	2	3
Quartz sand, dry	0.0020		
Quartz sand, 8.3 percent moisture	.0033		
Sandy clay, 15 percent moisture	.0037		
Gravel			0.0057–0.0062
Shale		0.004	
Andesite		0.005–.006	
Tuff		.004–.009	
Basalt		.007	
Traprock	.0075		
Rhyolite		.008	
Quartz latite porphyry			.007 – .011
Dolomite		.008	
Limestone	.0081	.005–.011	.0095
Granite	.0127	.006–.013	
Marble	.0097	.0106–.011	
Sandstone	.0113	.012 – .014	
Schist		.008 – .027	
Quartzite		.023 – .031	

1. From Ingersoll, Zobel, and Ingersoll (1948, p. 244).
2. From International Critical Tables of the National Research Council (1927, p. 55, 56).
3. In-hole determinations, East Tintic district; see p. 35.

EFFECTS OF FLUCTUATION IN AMOUNT OF INCOMING SOLAR HEAT

Because the amount of heat that comes from the sun fluctuates in daily, nonperiodic, and seasonal patterns, the sun's effect on near-surface temperatures may be divided into three parts: the diurnal wave, the nonperiodic wave of several days' duration, and the annual (or seasonal) wave. The effects of the first two waves attenuate downward so rapidly that they may be disregarded for all temperature measurements below 5 meters, but the measurable effects of the annual wave commonly reach 25 meters, and in rocks of high diffusivity as deep as 40 meters (table 2).

The effect of each surface-temperature fluctuation on subsurface temperature depends on the duration of the hot or cold wave and its range in temperature as well as the thermal characteristics of the earth material. Calculations involving the form of the annual wave pose problems in data smoothing that will be discussed after the lesser problems of the diurnal and nonperiodic waves have been examined.

TABLE 2.—*Depths at which subsurface temperature range is 0.1 percent and 0.01 percent of annual effective surface range for different diffusivities* [1]

Diffusivity	Depth, in centimeters to nearest 10 cm, at which annual temperature range is—	
	0.1 percent of surface range	0.01 percent of surface range
0.0016	870	1, 170
0.0025	1, 090	1, 460
0.0036	1, 310	1, 750
0.0049	1, 530	2, 040
0.0064	1, 750	2, 330
0.0081	1, 960	2, 620
0.0100	2, 180	2, 910
0.0121	2, 400	3, 200
0.0144	2, 620	3, 490
0.0169	2, 840	3, 780
0.0196	3, 060	4, 070
0.0225	3, 270	4, 360
0.0256	3, 490	4, 650
0.0289	3, 710	4, 940
0.0324	3, 930	5, 230

[1] It is assumed that the surface wave has a symmetrical sinusoidal form, that heat transfer is by conduction only, and that the diffusivity is constant.

DAILY AND NONPERIODIC WAVES

In the mathematical treatment of dirunal temperature variations, it is customary to assume that the temperature varies with time in a regular symmetrical periodic manner, generating a heat wave at the surface that closely approximates a sine curve. The range of temperature at any point below the surface caused by such a periodic heat wave can be calculated readily from equation 2:[2]

$$T_x = T_s\, e^{-x\sqrt{\frac{\Pi}{\alpha P}}} \tag{2}$$

where T_x is temperature range at depth x; T_s, total temperature range about the mean at surface—that is, twice the amplitude; x, depth in centimeters; α, diffusivity; P, period in seconds (1 day=86,400 secs).

If T_s in equation 2 is given an arbitrary value of 100° and T_x is given a value of 0.1° (as an arbitrary value of the minimum change that is effective) the equation can be solved for x for any value of α (see table 3) to give the depth where the temperature range is 0.1 percent of the surface range—regardless of the actual value of T_s in degrees.

TABLE 3.—*Penetration of symmetrical diurnal wave in materials of different diffusivity, expressed as percent of daily range at surface*

[Calculated from equation 2, assuming diffusivity is constant and that heat moves into material (in semi-infinite body) only by conduction]

Diffusivity	Depth in centimeters at which range is—	
	0.1 percent of surface range	0.01 percent of surface range
0.0025	57	76
0.0036	59	91
0.0049	80	106
0.0064	92	121
0.0081	103	136
0.0100	114	151
0.0121	126	166
0.0225	172	226

If hourly temperatures are read at some point on the surface for a period of 24 hours, it would seem easy enough to plot the general form of the diurnal wave and to arrive at a reasonable figure for the daily range of temperature at that locality for that season of the year. The rapid perturbations in surface temperatures caused by wind and clouds, however, make it advisable to measure the diurnal wave a slight distance below the surface. In this study maximum and minimum temperatures were recorded to give the diurnal range, and measurements made at depth of 5 cm revealed a pattern that is similar to the one obtained from smoothed surface measurements. An example of the diurnal temperature wave is shown in figure 1. On

[2] Formula adapted from Ingersoll, Zobel, and Ingersoll (1948, p. 47); see also Carslaw and Jaeger (1959, pp. 64–69).

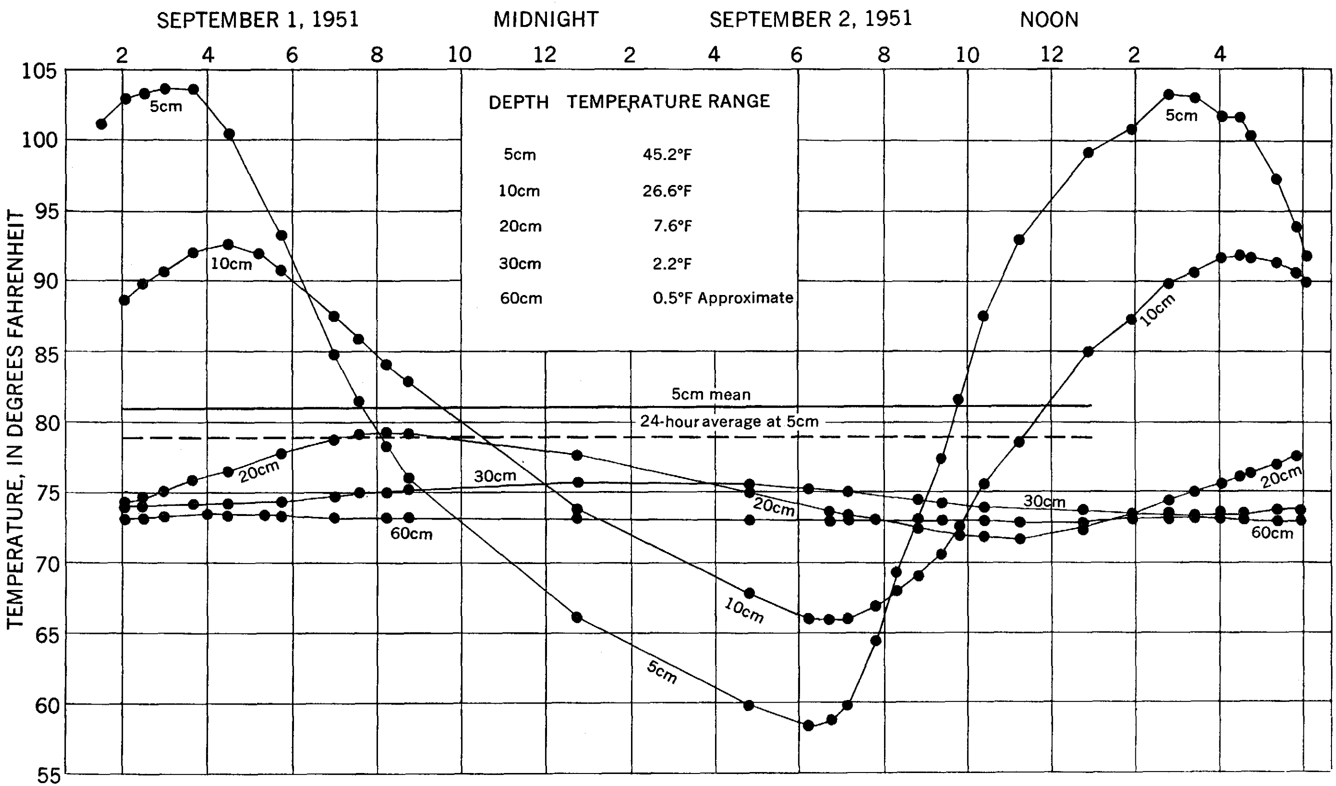


FIGURE 1.—Curves of temperatures measured in a 1-meter hole in dry soil during a 28-hour period.

the date these measurements were taken, the temperature curve was asymmetric: it had greater amplitude above the average than below and its half waves were not equal, one being about 15 hours long and the other about 9 hours long. Such asymmetry is normal, though it is different at other times of the year. As pointed out by Carslaw and Jaeger (1959, p. 69), however, this asymmetry diminishes with depth and even for a "square wave" at the surface it becomes sinusoidal as the wave moves into the solid.

In all rocks except those having exceptionally high diffusivity, such as quartzite (table 1), the bottom of a hole 5 feet (152 cm) deep will be below the depth of readily measurable penetration of the daily wave.

Equation 2 can be used also to approximate the effective depth of changes caused by longer nonperiodic hot or cold waves. For example, the values to be used in the formula for a heat wave of 1 week's duration in which the temperature remains 10° above average are:

$$T_s = 10°$$

$P = 2 \times 7 \times 86{,}400$ (since a nonperiodic heat wave must be considered as one-half of a hot and cold wave)

Table 4 shows the approximate effect of the 1-week heat wave at different depths in materials having diffusivities representative of most rocks.

TABLE 4.—*Approximate temperature range, in degrees centigrade, at various depths caused by a 1-week heat wave 10°C (18°F) above average*

Diffusivity	At depth, in centimeters—				
	100	200	300	400	500
0.0049	2.16	0.23	0.02		
0.0100		.89	.19	0.04	
0.0144		1.65	.45	.13	0.04

ANNUAL WAVE

Although the daily and short nonperiodic waves may be disregarded in the lower part of holes of very moderate depth, the effect of the annual wave is measurable to a much greater depth; the possibility of calculating the real geothermal gradient at depth by considering the temperature changes caused by the annual wave was appraised in hole 51, where excellent control was present, for both surface temperature and deep geothermal gradient. In rocks of about average diffusivity (0.0100), the annual range at a depth of 2,180 centimeters is 0.1 percent of the annual range at the surface (table 2). As hole 51 was drilled only to 1,770 centimeters it is well to consider all aspects of the annual wave so that its effects may be recognized and means devised for eliminating them mathematically from the temperatures measured, so as to leave the residual geothermal gradients.

In a mathematical treatment of the subsurface variation in temperature caused by the fluctuation in heat received at the surface during the year, it is customary to assume that the annual wave at the surface can be plotted as a simple sine curve; the half-amplitude above the average is then equal to the half-amplitude below the average, and the times when the sine curve crosses the average temperature are known and are a half-year apart.

The attenuation of such a wave with depth is readily calculated from equation 2 if the mean surface temperature of the cycle is used as the zero for the surface temperature, and a point on the subsurface curve can be calculated for any day of the year (Ingersoll, Zobel, and Ingersoll, 1948, p. 47):

$$T=T_r\left[e^{-x\sqrt{\Pi/\alpha P}}\sin\left(\frac{2\Pi t}{P}-x\sqrt{\Pi\alpha P}\right)\right] \tag{3}$$

where T is temperature at depth x; T_r, amplitude or half temperature range above and below the mean annual temperature at the surface; x, depth in centimeters; α, diffusivity; P, period of cycle (in seconds); t, time in seconds after "zero time"—that is, the time the surface sine wave crosses the average temperature line. There are other "zero times" at Π, 2 Π, 3 Π, 4 Π, and so forth. We must regard the mean annual temperature as $T=0$ (at $x=0$, when $t=0$) in equation 3, and in an arithmetical solution would add T_m (mean annual surface temperature) to the right-hand side of the equation.

More than a century ago Thompson (1859) presented a mathematical analysis of a long series of subsurface observations made by Forbes near Edinburgh, Scotland. He used a five-term Fourier series to describe the complex harmonic function that portrayed a 5-year average of the mean temperatures of 11-day periods, but found that only the first term had any weight at depths greater than 6 feet. It thus seems that the simple form of equation 3 is justified for measurements made at depths of more than 2 meters.

Inspection of equation 3 shows that in order to tie the annual surface temperature wave mathematically to subsurface temperatures, the average annual temperature range and the diffusivity of the rocks must be known. In actual practice, the average annual surface temperature is very difficult to determine, even from extensive measurements of surface and air temperatures. As will be shown later, this difficulty is due to the marked asymmetry of the annual wave in localities where frost and snow modify the surface temperatures during the winter months, as in the East Tintic district. The freezing of moisture at the surface holds the surface temperature near 32°F during winter cold waves and thus diminishes the drop in surface temperature appropriate to the air temperatures; similarly, the rise in air temperature in the spring is not immediately reflected in ground temperatures because of the lag engendered by the latent heat of fusion of the frozen soil moisture. The net effect is a winter surface temperature that does not fall as much below the mean annual temperature as does the summer temperature rise above it. The mean annual temperature must be considered as a measurement of the midpoint between the heat added to the ground during the summer season and the heat given up by the ground during the winter season. This results in a temperature wave that is asymmetric with respect to both the mean annual temperature and the length of the winter and summer portions of the annual cycle.

Formulas 2 and 3 were used to calculate the curves shown on figure 2. A diffusivity of 0.0036 corresponds to that of a rock of low diffusivity such as dry soil or shale, and a diffusivity of 0.010 is given by Ingersoll, Zobel, and Ingersoll (1948, p. 244) as the average for rock material. The outside curves, called "envelope" curves, show the diminishing annual range in temperature from the surface downward. The other two curves show the temperatures that would be read at depth 1½ months after the summer maximum and 1 month after the surface temperature had reached the mean annual temperature, on its way toward the winter minimum. For this figure it is assumed that the geothermal gradient is zero, that the surface wave is symmetrical, and that it crosses the average annual temperature every 6 months; but these assumptions must be modified appreciably in the East Tintic district.

Although the calculation of actual temperatures at depth would require surface temperatures recorded many times a day through a period of a year or more, the geothermal gradient can be derived with far less information. If there were no temperature gradient the depth temperature curve caused by the annual wave would cross the mean annual temperature at successive points a half wavelength apart at depths dependent on the time at which the depth temperatures were measured. If the temperature at the surface varies from $+T_r$ to $-T_r$, then the proposition stated above is:

$$T=0 \text{ at } x_0=f(t, \lambda)\pm\frac{n\lambda}{2} \qquad (4)$$

where λ is the wavelength of the subsurface temperature wave measured in cm between two depths where the temperatures are at exactly the same point in the annual cyclic change for these different depths, and t is the time after the annual wave of surface temperature passes

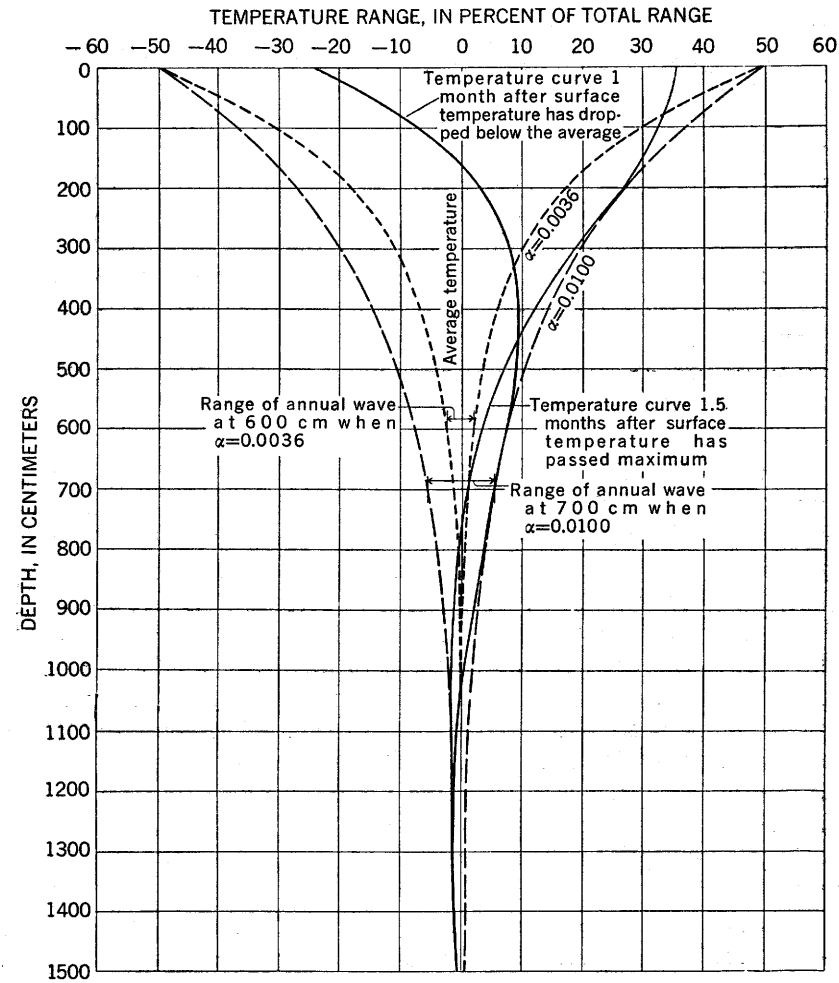


FIGURE 2.—Subsurface envelope curves showing attenuation with depth of effect of annual surface temperature in media having diffusivity (α) of 0.0036 and 0.0100. Theoretical depth temperature curves when α=0.0100 are shown for dates 1.5 months after maximum, and for 1 month after annual average is passed in the fall. The envelope curves for diffusivity 0.0036 and 0.0100 show the temperature range at depth in percent of total temperature range at surface

through the mean annual surface temperature, taken at April 10 in the East Tintic district. From equation 3 it can be shown that x_0 of equation 4 can be calculated from equation 5 (see equation 26, explanatory notes, p. 43, for derivation of equation 5).

$$x_0=\sqrt{\alpha\Pi P}\left(\frac{2t}{P}\pm n\right) \tag{5}$$

where $T=0$ at x_0. As $\lambda=2\sqrt{\alpha\Pi P}$ the conditions specified in equation 4 are fulfilled.

A temperature gradient will displace the subsurface gradient; assuming that the gradient is constant between the points used, then the values above zero gradient at x_0 points give the gradient:

$$\frac{T_{x_0''}-T_{x_0'}}{x_0''-x_0'}=\frac{\partial T}{\partial/x} \qquad (6)$$

where $x_0''=x_0'+\frac{\lambda}{2}$, $T_{x_0''}$ and $T_{x_0'}$ are the temperatures measured at the depth x_0'' and x_0' at the time t.

Equations 5 and 6 were used to calculate gradient determinations that seem satisfactory, but these determinations were checked further both by gradients in deep holes and by equation 3, using climatic and surface-temperature records to establish the character of the asymmetric annual surface wave.

PROCEDURE

FIELDWORK

As the main purpose of the study was to test the possibility that anomalous geothermal gradients could be detected by measuring temperatures in shallow drill holes, 58 shallow holes were drilled to depths ranging from 3 to 58.5 feet in various rocks, in many different topographic positions, above known sulfide bodies and away from them. Temperature measurements were made by means of thermocouples and a potentiometer. Most of the holes (pl. 1) were in the Packard Quartz Latite, an Eocene lava series, but some holes were in Quaternary alluvium and gravel, and Cambrian limestone, dolomite, and shale; the surface distribution of these rocks is shown on the geologic and alteration maps of the district (Lovering and others, 1960). The program was so timed that the thermocouples were placed in the holes soon after they were drilled. Standard Weather Bureau weather shelters were set up at holes 51 and 12 so that air- and ground-temperature measurements could be compared for these stations, and these temperatures were read daily by various members of the project between September 1949 and November 1950, except for short interruptions during the severe blizzards in the winter of 1949–50.

FIELD PROCEDURE

Plate 1 shows the location of 58 holes, and table 5 shows the kind of rock, the depth of hole, and the positions of thermocouples in typical holes. Of the 58 holes drilled, the deepest one was 58.5 feet deep, 23 were 33–52 feet deep, and nearly all the rest were about 15 feet deep. Most of the holes were completely dry, but a few penetrated perched water tables and were left shallower than was

TABLE 5.—*Kind of rock, depth of drilling, and placement of thermocouples in 12 representative shallow drill holes*

[(____) indicates drill hole number as shown on pl. 1; *indicates 2 thermocouples placed at this depth]

(48) Altered rhyolite	(43) Alluvium	(49) Altered rhyolite	(6) Fresh rhyolite	(12) Fresh rhyolite	(51) Rhyolite	(36) Limestone	(24) Altered rhyolite	(29) Alluvium for 16 ft, rhyolite	(34) Altered rhyolite	(53)[1] Altered rhyolite	(58) Alluvium
Depth of hole (ft)											
50	33	33	10	33	58.5	33	33	50	17	24	3
Depth (cm) at which thermocouples were placed											
2	2	2	2	2	2	2	*2	2	2	2	5
10	10	10	10	10	10	10	*10	10	10	10	10
30	30	30	30	30	30	30	*30	30	30	30	15
60			60	60			*60	60			20
100	100	100			100	100			100	100	25
200											30
300	300	300	300	*300	300	300	*300	*300	300	300	35
400										450	40
500	500	500		*500	500	500	*500	*500	500		
625											45
750	750	750			750	750				740	50
875											60
1,000	1,000	*1,000		*1,000	*1,000	1,000	*1,000	*1,000			70
1,125											
1,250					1,250						90
1,375											
1,500					1,500			*1,500			102
1,500					*1,770						

[1] Water in bottom 8 ft.

originally planned because of the difficulty of drilling into wet rock with the percussion air drill that was used.

A 2-man team operated a shop for wiring the sets of thermocouples needed for each hole, timing the wiring so that a set of thermocouples having the required spacing was available for inserting into each hole soon after it was drilled.

The thermocouples were of copper and constantan, and the wires used had lacquer and cotton insulation. Two methods of wiring sets of thermocouples were tried (fig. 3). In one method, each thermocouple was wired separately, whereas in the other a common constantan lead was used; the latter method proved to be unsatisfactory, however, as it gave poor results in some holes. After the wires were soldered together the thermocouples were painted with shellac to provide insulation and to make the thermocouples reasonably waterproof.

The lead wires and the thermocouples were securely fastened to ½- by 1-inch wooden rods 12 feet long. The thermocouples were spaced along the rods so that each one was at its proper depth when the rods and attached thermocouples were inserted into the (uncased) drill holes. The dry cuttings from the drilling operation were then poured into the hole firmly imbedding the rods and protecting the thermocouples from everything except the curiosity of sheepherders and hunters.

The above-ground ends of each pair of wires were terminated on wooden stakes 1 by 2 by 18 inches that had copper lugs on one side and one or more constantan lugs on the other side (fig. 3). Sufficient lead wire was allowed during construction of the thermocouples to permit the terminal stake to be set about 3 feet from the top of the rod inserted into the hole. The leads from the thermocouples were attached to the lugs on the stakes so that the shallowest thermocouple was attached to the topmost lug, and the deeper thermocouples were attached to consecutively lower lugs.

For holes more than 12 feet deep two or more wooden rods were fastened together end-to-end by flexible hinges that were tightened as the rods were lowered into the holes. This method insured that the thermocouples were actually set at the desired depths. The error in setting the deep thermocouples was no more than ± 0.1 percent, or less than 2 cm for the deepest thermocouple at 1,770 cm; the error in the final position of the junctions between the surface and 100-cm depth was less than 0.3 percent (3 mm).

A Leeds and Northrup portable precision potentiometer (model 8662) was used to measure the temperatures of the thermocouples. All temperatures were measured against the same reference thermocouple (in ice water) and the same two lead wires from the measuring equipment were used to make all connections (fig. 4). The readings

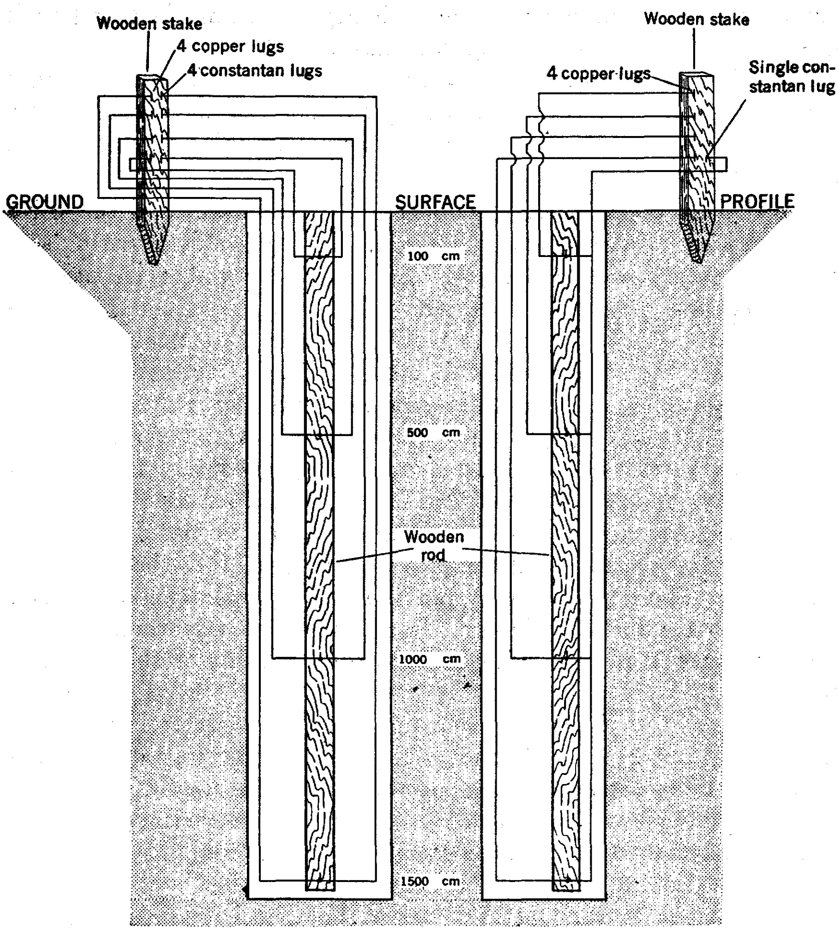


FIGURE 3.—Simplified schematic drawing of wiring of thermocouples on wooden rods in drill holes and on wooden stakes above ground surface.

were recorded in the field in millivolts and were converted in the office into degrees Fahrenheit or centigrade as required. The form of the subsurface temperature wave can be adequately determined if thermocouples are placed at or near the following depths in cm—100, 250, 500, 750, 1,000, 1,250, 1,750, 2,000; or in feet—3, 7.5, 15, 22.5, 30, 37.5, 45, 52.5, 60.

AIR AND GROUND TEMPERATURES FOR BACKGROUND DATA

Surface temperatures were recorded for more than a year, to relate the daily air and surface variations in temperature to the annual temperature wave as it controlled subsurface effects of the sun's heat.

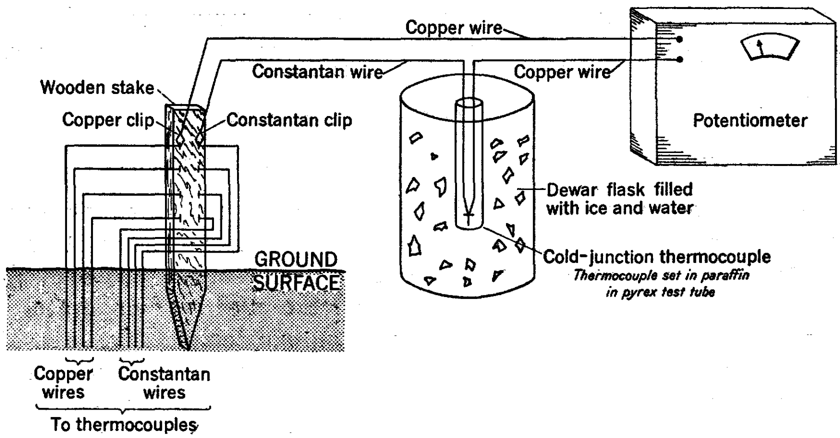


FIGURE 4.—Simplified schematic drawing of connections between measuring equipment and connector stakes that mount above-ground terminations of subsurface thermocouples.

Standard Weather Bureau weather shelters lent by the Salt Lake City office of the U.S. Weather Bureau were placed near two shallow holes; one, next to hole 51 near the bottom of an easterly trending ravine; the other, near hole 12 a few hundred feet east-southeast of hole 51 atop the low ridge south of the ravine. Standard Weather Bureau maximum and minimum thermometers were placed inside the shelters mounted in the approved Weather Bureau manner. In addition, minimum and maximum thermometers were set outside the shelters; one pair at each shelter mounted so as to be in the direct sunlight as much as possible each day, and another pair placed on the ground below the shelters. The bulbs of the ground thermometers were covered by very thin layers of soil (less than 1 mm) but were otherwise exposed to direct sunlight. The readings of these thermometers were recorded daily between September 1949 and November 1950, except for a few interruptions caused by blizzards in the winter of 1949–50.

After the measurements taken inside and outside the weather shelters had been tabulated, time-temperature curves were plotted. The resulting erratic curves were smoothed by using calculated moving averages, at the suggestion of the U.S. Weather Bureau officials at Tucson, Ariz. The analysis and correlation of the shallow-hole temperature with the surface temperature led to the development of both mathematical and graphic methods of determining diffusivities from subsurface temperatures and the determination by graphic methods of the subsurface effects of the irregular asymmetric surface temperature changes in rocks of known diffusivity. It was then possible to check the geothermal gradients calculated from the shallow-hole measurements against the irregular weather pattern of the year

1949–50. Both techniques also were checked by comparing gradients measured by shallow-hole methods with those measured in deep holes (p. 35).

SURFACE-TEMPERATURE MEASUREMENTS

ANNUAL WAVE AT THE SURFACE AND ITS ASYMMETRY

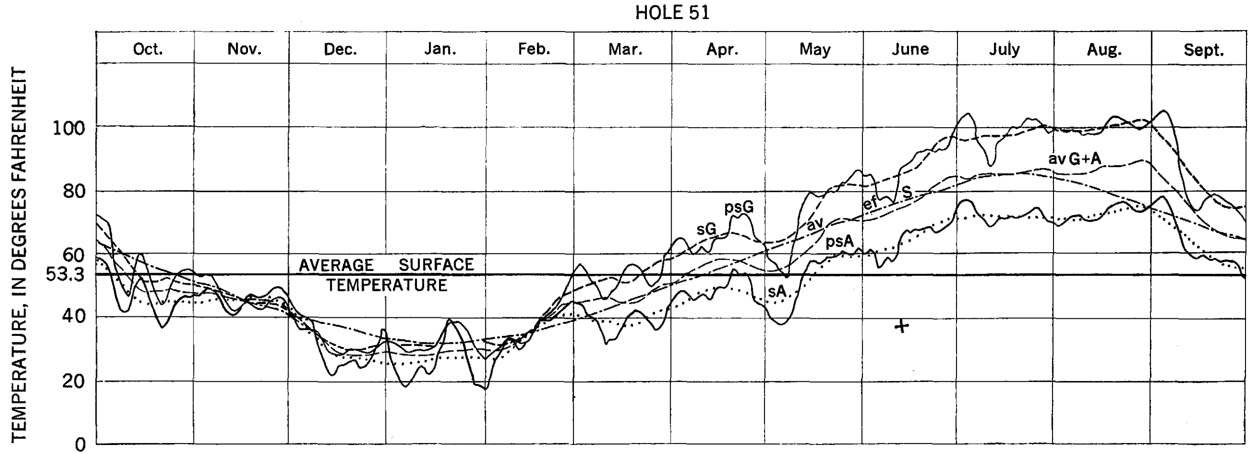
The day-to-day fluctuations in temperature as measured by any pair of thermometers at the Weather Bureau shelters at holes 51 and 12 were so erratic and so great that the daily mean temperatures proved useless for determining the annual plot of the effective temperature wave at the surface, or even its average annual temperature.

The first "smoothing" of the curve plotted from the daily mean temperature was done by calculating a 7-day moving average of the raw data. The resulting curve for the shelter and the ground temperatures at hole 51 is shown by the solid-line curves on figure 5. It was evident, however, that this smoothing had not accomplished the desired result, and so a 21-day moving average was calculated from the 7-day data (referred to hereafter as a 21-of-7-day moving average). The curves resulting from these calculations are shown as the dotted-line and dashed-line curves in figure 5. These curves are believed to be reasonably good representations of the effects of the sun's heat at the ground surface and in sheltered air about 5 feet above the surface.

As neither the 7-day nor the 21-of-7-day moving average curve had the sine-wave shape that had been anticipated, the actual average surface temperature was ascertained by projecting to the surface the deep-hole geothermal gradient measured in Tintic Standard Mining Company's churn-drill hole T.S.C.H. 15 close to hole 51; this procedure indicated that the average surface temperature was 53.3°F, a figure that compares well with the 53.42°F and 53.34°F that were found later by projecting the shallow-hole gradient as calculated mathematically (p. 35). These figures show that the deviation from a sine wave is caused by the lack of symmetry about the average temperature. The actual *average* ground temperature was 3.5°F below the mean of the ground and air shelter means.

Penrod, Elliott, and Brown (1960) measured soil temperature at various depths at 2-hour intervals over a period of 5 years near Lexington, Ky. Weather Bureau air shelter measurements showed that the mean annual air temperature differed from the mean annual ground temperatures by 1.20°F over the 5-year period, with a range of 0.48°F to 1.96°F in different years; however, the ground temperature was measured at about one-half inch beneath the sod.

The average of the mean daily temperatures measured at the surface at hole 51 from October 1, 1949, through September 30, 1950, was 63.75°F, and that of the average mean daily air temperatures was



EXPLANATION

psG

Partly smoothed mean ground temperature curve, 7-day moving average

sG

Smoothed mean ground temperature curve, 21-of-7-day moving average

psA

Partly smoothed mean air (shelter) temperature curve, 7-day moving average

sA

Smoothed mean air (shelter) temperature curve, 21-of-7-day moving average

av ef S

Inferred average effective surface temperature

av G+A

Average of sG and sA

FIGURE 5.—Chart showing relation of inferred effective surface temperature curve to known factors: average surface temperature, mean ground temperature, and mean air temperature.

49.85°F, a difference of 13.90°F in contrast to a difference of 1.20°F for the Kentucky measurements.

The low conductivity of the centimeter or so of soil lying on the bedrock at hole 51 tends to insulate the underlying rock somewhat in summer, whereas in winter the moisture present as snow insulates against the cold, and the latent heat of freezing water and melting ice further minimize surface-temperature variation. The final smoothed surface-temperature curve was made with these factors in mind and is somewhat below the mean of air and ground for the summer season, and well above their mean for the winter. The resulting average-ground-temperature curve is the sinelike but asymmetric dot-dash curve in figure 5, which is called the curve of inferred effective annual temperature. This curve is similar to a sine curve but is asymmetric in both amplitudes and half-wavelength. As noted earlier the half-wavelength and amplitude are greater for the summer than for the winter, because the annual average temperature is not the mean between the summer maximum (that is, the maximum average temperature) and the winter minimum. This curve is idealized in that it does not show nonperiodic fluctuations and probably approximates the average of many years' temperatures. In any particular year the curve probably varies slightly from this average.

Weather Bureau shelter temperatures have been taken for many years at Elberta, Utah, about 4 miles east of the area studied, and the average of the annual temperatures there from 1938 to 1948 was 50.4°F. In 8 of these 10 years the mean annual temperature ranged from 0.0° to ±1.0°F about the average, and in two of the years the mean annual temperature differed from the average by −2.1° and +2.6°F. The average change from one year to the next was ±0.9°F. Such differences in the average shelter temperature would be greatly minimized a short distance below the surface by the insulating properties of the dry surface in the summer and the snows of winter. No systematic or periodic perturbation of mean annual temperatures is evident in the Elberta weather record.

LOCAL DIFFERENCES IN SURFACE TEMPERATURES

The choice of holes 51 and 12 for the shelter-and-ground-temperature study was prompted by the desire to ascertain if an appreciable difference in surface temperature would be caused by a small local geographic difference: hole 51 is near the bottom of a 75-foot-deep ravine and hole 12 is atop the adjacent ridge to the south but is only about 25 feet higher than hole 51 (pl. 1).

The table below shows temperatures taken from the 21-of-7-day moving average curves of air (inside shelter) and ground temperatures at holes 51 and 12, and the difference from the average annual surface temperature at hole 51 (53.3°F), and from probable average surface temperature at hole 12 (53.7°F).

	Maximum temperature, in degrees Fahrenheit			
	Aug. 27, 1950		Jan. 5, 1950	
	51	12	51	12
Shelter	76	77.5	25	25
Ground	103	99	31	29
Difference from average annual surface temperatures				
Shelter	+22.7	[1] +23.8	−28.3	[1] −28.7
Ground	+49.7	[1] +45.3	−22.3	[1] −24.7

[1] Probable.

The ground temperature at hole 51 is 4°F higher in the summer and 2°F higher in the winter than similar temperatures at hole 12. This can not be ascribed even in part to the abnormally large geothermal gradient below hole 51, as the gradient below hole 12 (6.3°F per 100 ft) is nearly as high as at 51 (7.4°F per 100 ft). The higher temperatures recorded at hole 51 are due to the geographic location of the stations; the thermometers at hole 51 were afforded protection from north winds by shrubbery and from west winds by the adjacent slope, whereas the ground thermometers at hole 12 were exposed to the cooling effects of all winds, and were less protected from low winter temperatures because of a thinner snow cover. No doubt snowfall was the same at both locations but the snow was more readily removed from near hole 12 by the wind.

The difference in the temperatures measured inside the shelters at the two localities is much less than the difference in ground temperatures; the winter minimums, in fact, are only a few tenths of a degree apart. For an explanation of the difference in summer maximums it is necessary to examine the raw data from which the highly smoothed 21-of-7-day moving average was constructed. The shelter and ground temperatures were recorded as maximums and minimums, and the means of the maximums and minimums provided the basic data for the moving-average curve. The actual summer temperatures measured show that the mean for the shelter temperature at hole 12 is higher than the mean at hole 51 because the nighttime minimum at 12 is 1°F to 3°F higher than the minimum at hole 51. Wind currents probably cause the difference, as hole 12, atop a hill, was exposed to rising warm air currents whereas hole 51, near the bottom of a ravine, was exposed to cooler air currents.

CALCULATION OF DIFFUSIVITY FROM SUBSURFACE-TEMPERATURE MEASUREMENTS

As the subsurface wave generated by the annual temperature changes at the surface is dependent not only on the effective annual surface-temperature wave, but also on the diffusivity of the subsurface material, it is essential to know the value of this thermal constant. There are many laboratory methods of determining diffusivity, but all have the disadvantage that the test specimen must be removed from its original surroundings; the diffusivity as thus determined may be valid for the small sample, but may be somewhat in error when assigned to the rock mass that it represents. A very few in-hole techniques have been described, but with the exception of those devised for hole 51 and described below, most of the methods require lengthy temperature records or artificial perturbation of temperature by introduction of known amounts of heat, and accurate results require much more elaborate instrumentation than the method used in the East Tintic district. Perhaps the most notable of these methods is the one devised by Thompson (1859) (later Lord Kelvin) in which the variation in amplitude and range of the temperature at the surface and at one or more depths during the annual cycle is used to fix the value of a term analogous to diffusivity—that is, $\sqrt{\Pi\sigma\rho/k}$. Comparison of the annual wave at different depths allows the determination of the diffusivity appropriate to the rock between any two depths. This method of course requires records of temperature throughout the year.

Because the diffusivity of the subsurface material must be known before the annual surface wave can be used to predict subsurface temperatures, the determination of diffusivity from subsurface-temperature measurements is described before the method of calculating gradients from shallow subsurface temperatures.

The diffusivity of a rock controls the wavelength of any heat wave that penetrates the rock, and the times (t_1, t_2, ***) of measurement of the subsurface-temperature curve determine where the curves will cross at depth for any specific value of α. The measurement of the half or full wavelength or of the depth of the crossing point of two subsurface temperatures thus provides data for calculating diffusivity.

Figure 6 has been constructed to show the subsurface curves at various times of the year and their envelope curves in rock having a diffusivity of 0.0049. The temperatures may be regarded as percentiles of the surface-temperature range, which may therefore be in either centigrade or Fahrenheit units, but the zero point must be taken at the average temperature. The subsurface effects of the annual wave are calculated on the assumption that the plotted form of the annual temperature wave is that of a sine curve, and that no geothermal gradient is present. All the curves in figure 6 except the two defining the envelope cross all other curves, and the properties of the subsurface temperature wave (p.12) cause successive crossings of any two curves to be vertically equidistant so long as the diffusivity is constant; the separation is one-half the wavelength and where $\alpha=0.0049$, $\frac{1}{2}\lambda=697$, and curves 0 and 7 cross at 58 cm and 755 cm. A striking feature of the curves is the specific vertical regularity with which each curve crosses the center line—and in direct proportion to the time intervals represented.

It is interesting to observe that the temperature curves do not touch the envelope curves at the point where the slope of the temperature curve changes sign; instead the maximum or minimum for a given depth is the point where the curve is tangent to the envelope (fig. 6). These minima and maxima are a half wavelength apart,

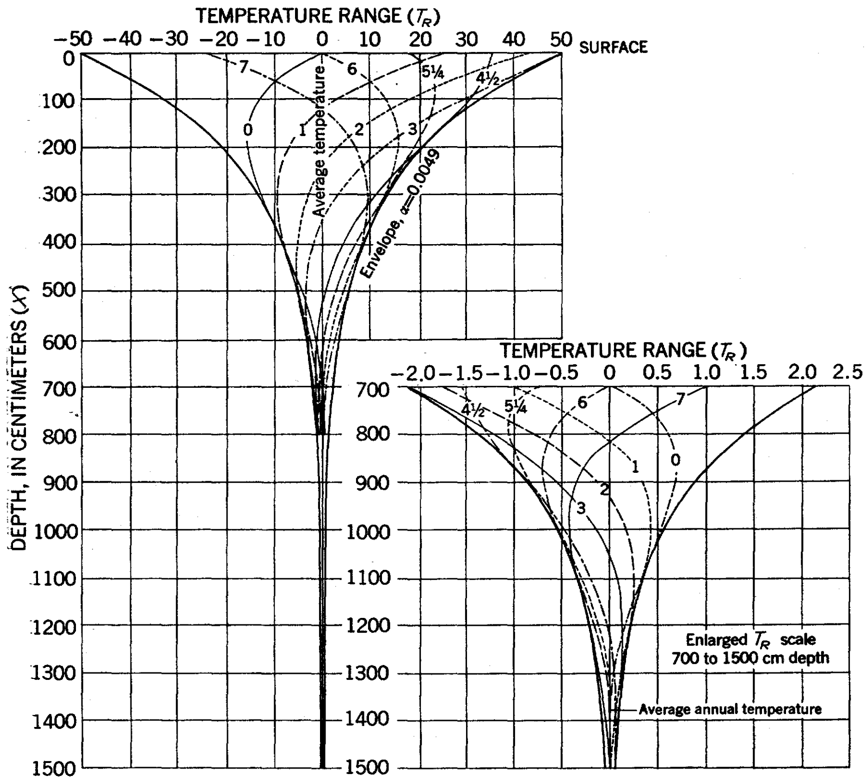


FIGURE 6.—Curves showing theoretical subsurface temperatures at different times of the year. Diffusivity is 0.0049 and temperatures are in percent of annual range at surface. Numerals on curves show time, in months, after annual surface-temperature wave has passed the average temperature on way to maximum. Envelope curves show total subsurface-temperature range between surface and 1,500 cm.

as are the points where the slope of the curve changes from positive to negative. Either of the two relations—(1) that the slope reversal points on the temperature curve (where the slope changes sign) are separated by a half wavelength, or (2) that the vertical separation of successive crossings of any two curves is always equal to one-half the wavelength—can be used to calculate the diffusivity. Although half wavelengths can be read directly from successive crossings, the small range of actual temperatures at depths of the second crossing makes the precise crossing point difficult to ascertain. If half wavelengths can be measured satisfactorily the diffusivity is calculated as follows:

$$\lambda=2\sqrt{\Pi\alpha P} \tag{7}$$

or

$$\alpha=\frac{\lambda^2}{4\Pi P} \tag{8}$$

and

$$\alpha=\lambda^2(25.22\times10^{-10})$$

where λ is wavelength in cm; α, diffusivity; P, 1 year (365 days) in seconds (31,536,000; a sidereal year is 31,558,150 sec).

Table 6 shows the wavelengths of the annual wave as calculated for diffusivities that span all values recorded for the rocks listed in table 1.

If the wavelength could be measured accurately from a plot of the subsurface temperatures, the diffusivity could be quickly determined, but unfortunately the attenuation of the wave with depth makes it difficult to fix even the half wavelength with satisfactory accuracy, especially where a marked geothermal gradient is present. The method, however, is useful for getting a rude approximation of λ and from it an approximate value for α.

A more accurate in-hole technique is available where two or more depth-temperature curves are available and the times t_1, t_2 *** (after $t=0$) at which they were measured are known. It is basic to these and other calculations that the date for $t=0$ be known, the time in the spring when the annual temperature wave passes through the average surface temperature, and also the half periods of the asymmetric annual temperature cycle. In the East Tintic district $t=0$ on April 10; the summer half period $P_s/2$ from April 10 to October 24 is 395+ /2 and $P_s=34.2+10^6$ sec, the winter half period $P_w/2$ from October 25 to April 9 is 334/2 days and $P_w=28.9+10^6$ sec.

Given the values of t_1 and t_2 and the depth x_c where the two curves cross, it is possible to calculate the average diffusivity of the rock between the surface and the crossing points, using equation 9. (See

TABLE 6.—*Wavelength of the annual ground-temperature wave, in centimeters, in materials of different diffusivities, in cgs units*

Diffusivity α	Wavelength λ	Diffusivity α	Wavelength λ
0.0016	797	0.0121	2, 190
0.0025	996	0.0144	2, 389
0.0036	1, 195	0.0169	2, 588
0.0049	1, 394	0.0196	2, 788
0.0064	1, 593	0.0225	2, 987
0.0081	1, 792	0.0256	3, 186
0.0087	1, 860	0.0289	3, 386
0.0100	1, 991	0.0324	3, 585

Explanatory notes page 42, for derivation of equation 9 from equation 3, and a chart, fig. 14, for graphical solutions.)

$$\alpha=\frac{\frac{4x_c^2\Pi}{P}}{\left[2\left(\frac{t_1\Pi}{P}+\frac{t_2\Pi}{P}\right)\pm(2n+1)\Pi\right]^2} \qquad (9)$$

where n is zero or any integer and P is the appropriate period equal to twice the summer or winter half period.

The regularity of the events shown graphically on figure 6 is even more striking in table 7, which has been calculated to show the depths at which temperature curves (α=0.0049) calculated a month apart cross each other the first time and the depths for other significant events such as maximum, minimum, and average of the yearly range. The depth at which a curve crosses the average is useful in selecting times for temperature measurements if the gradient is to be calculated from equations 5 and 6.

It is possible to use the relations of crossing points in materials of different diffusivities to quickly approximate an unknown diffusivity, given x_c, t_1, and t_2 for the materials of both known and unknown diffusivity, if a table of crossing points and their depths is available such as that given for α=0.0049 in table 7. By interpolating between the appropriate figures in the vertical and horizontal columns of table 7, one can quickly find x_c for t_1 and t_2 to the nearest tenth of a month for α=0.0049. The value for the unknown diffusivity α' given by 7.1:

$$\frac{x_c\text{ (known)}}{x_c'}=\frac{\sqrt{\alpha}\text{ (known)}}{\sqrt{\alpha'}} \qquad (10)$$

where x_c' is the crossing point in the material of unknown diffusivity for t_1 and t_2. From equation 10 we have

$$\alpha'=\alpha\left(\frac{x_c'}{x_c}\right)^2 \qquad (11)$$

and for table 7

$$\alpha' = \left(\frac{.07x'_c}{x_c} \right)^2 \tag{12}$$

where x_c is the crossing point for t_1 and t_2 as interpolated in table 7.

The use of a chart similar to the one shown in figure 12, constructed for the appropriate period (P), is even more satisfactory for speed, accuracy, and versatility.

TABLE 7.—*Depth, in centimeters, of first crossings of subsurface-temperature curves calculated for monthly intervals $\alpha = 0.0049$, $\lambda = 1{,}394$ cm*

[Dates equivalent to the months show sixths of the unequal East Tintic summer and winter half periods equivalent to months in 12-month year.

Numerals at left and at top of table identify subsurface-temperature curves in equilibrium with surface temperature at times (in months) after zero time, April 10, when surface temperature reaches average annual temperature. Avg, min, and max indicate depths at which subsurface-temperature curves cross the average or touch the minimum or maximum envelope curves. Minus signs indicate crossing points below average surface temperature. Plus signs indicate crossing points above average surface temperature. The dates have been taken from the asymmetric inferred effective surface temperature curve of fig. 5]

	Apr. 10	May 13	June 15	July 18	Aug. 20	Sept. 22	Oct. 25	Nov. 22	Dec. 20	Jan. 17	Feb. 13	Mar. 13
Months	0	1	2	3	4	5	6	7	8	9	10	11
0	min 348	− 407	− 465	− 523	− 581	− 639	avg 0	− 58	− 116	− 174	− 232	− 290
1	− 407	min 465	− 523	− 581	− 639	− 697	+ 58	avg 116	− 174	− 232	− 290	− 348
2	− 465	− 523	min 581	− 639	− 697	− 58	+ 116	+ 174	avg 232	− 290	− 348	− 407
3	− 523	− 581	− 639	min 697	+ 58	+ 116	+ 174	+ 232	+ 290	avg 348	− 407	− 465
4	− 581	− 639	− 697	+ 58	max 116	+ 174	+ 232	+ 290	+ 348	+ 407	avg 465	− 523
5	− 639	− 697	+ 58	+ 116	+ 174	max 232	+ 290	+ 348	+ 407	+ 465	+ 523	avg 581
6	avg 0	+ 58	+ 116	+ 174	+ 232	+ 290	max 348	+ 407	+ 465	+ 523	+ 581	+ 639
7	− 58	avg 116	+ 174	+ 232	+ 290	+ 348	+ 407	max 465	+ 523	+ 581	+ 639	+ 697
8	− 116	− 174	avg 232	+ 290	+ 348	+ 407	+ 465	+ 523	max 581	+ 639	+ 697	− 58
9	− 174	− 232	− 290	avg 348	+ 407	+ 465	+ 523	+ 581	+ 639	max 697	− 58	− 116
10	− 232	− 290	− 348	− 407	avg 465	+ 523	+ 581	+ 639	+ 697	− 58	min 116	− 174
11	− 290	− 348	− 407	− 465	− 523	avg 581	+ 639	+ 697	− 58	− 116	− 174	min 232

Figure 7 shows curves plotted from actual temperature measurements made at hole 51 on widely separated dates in 1949 and 1950 The average temperature at the surface (as extrapolated from depth) and the shape of the inferred effective annual surface wave (fig. 5) show that the effective annual surface wave crosses the average temperature line on April 10 (and again on October 25); by translating the dates of the temperature measurements at hole 51 into numbers corresponding to months after zero time, April 10, and then interpolating between the crossing points given in table 7, we find that the crossing points for those dates in material of diffusivity 0.0049 and at hole 51 are as shown in table 8.

Using table 8, the values for α for successively greater depths as given by equation 12 are shown in column 3 of table 9, and those calculated directly from equation 9 are given in column 4 of table 9. The agreement is satisfactory.

TABLE 8.—*Depths of crossing points in hole 51 compared to corresponding crossing point in material of diffusivity 0.0049*

Temperature measurements in drill hole 51			Depth, in centimeters, of crossing points obtained from table 7 (T7) and observed in hole 51 (H51) for indicated months after zero time								
No.	Date	Approximate months after zero time	T7	H51	T7	H51	T7	H51	T7	H51	T7
			3.7		5.1		6		7.7		10.5
t_1	Aug. 10	3.7	------	------	162	170	215	255	313	395	477
t_2	Sept. 26	5.1	162	120	------	------	296	365	394	545	558
t_3	Oct. 24	6	215	255	296	365	------	------	448	------	610
t_4	Dec. 11	7.7	313	395	394	545	448	610	--------	------	709
t_5	Feb. 27	10.5	477	635	558	775	610	890	12/709	------	------

NOTE.—T7 columns give interpolated figures from table 7, and H51 columns give depths of crossing points in hole 51.

If we regard the obviously anomalous crossing point at 120 cm—a depth still within the influence of nonperiodic surface-temperature fluctuations—as reflecting a relatively low but not precisely determined diffusivity characteristic of the surficial layer of rock, the other values of α correspond well to the gradual increase in average value for the unweathered quartz latite of a diffusivity about 0.010 having a surficial layer about 1 meter thick of diffusivity about 0.003. It is interesting to note that the diffusivity of gravel as determined by crossing points in hole 55 (45 ft deep) is 0.0057 ± 0.0004, as given by equation 9 for 8 crossing points at depths ranging from 5.2 to 13.4 meters.

It should be pointed out that raw subsurface-temperature data can be used for diffusivity determinations by crossing-point methods without correction for geothermal gradient, but the "zero times" must be known. The gradient is a constant factor (if the conductivity

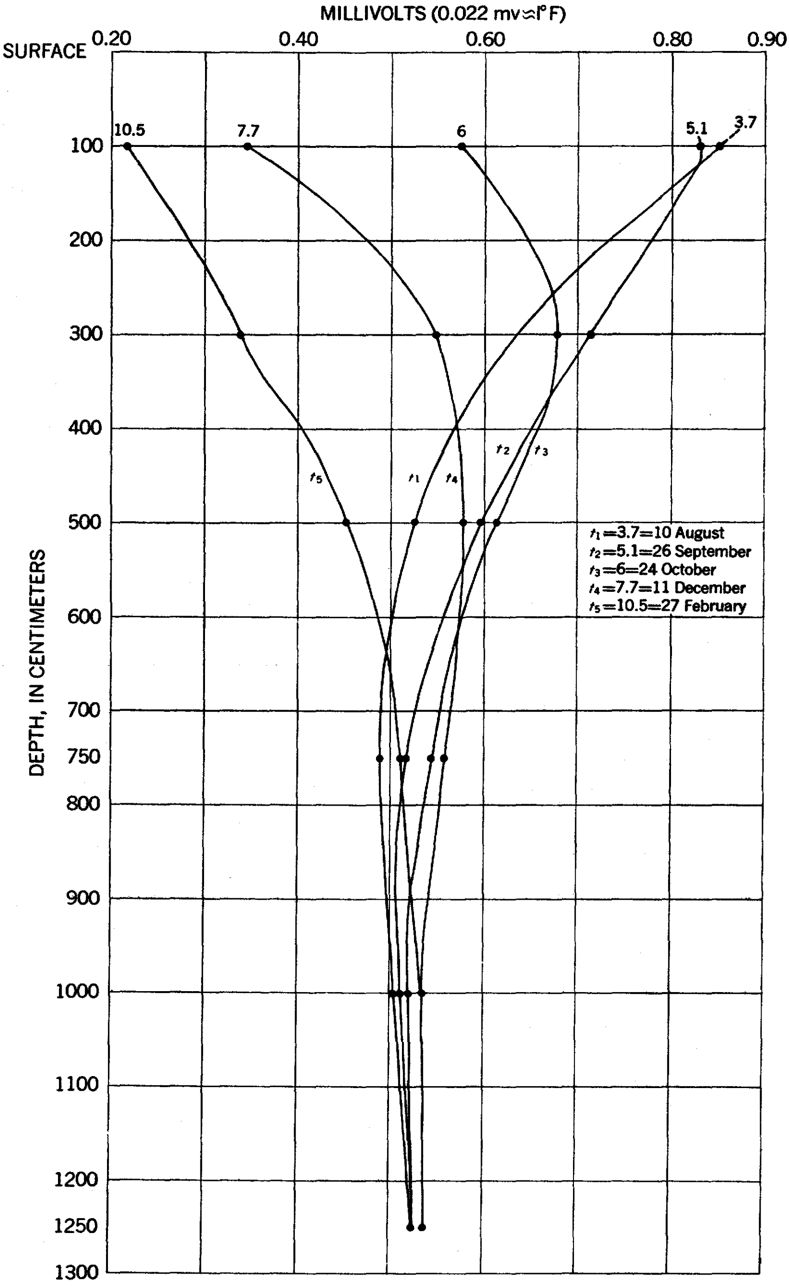


FIGURE 7.—Subsurface temperature curves plotted from potentiometer readings to show "crossing points" at depth.

is constant) and it affects all temperature measurements at a particular depth by the same amount. The difficulty of using the apparent slope-reversal points on a depth-temperature curve where a marked

TABLE 9.—*Diffusivities α of quartz latite porphyry as calculated by different in-hole methods at hole 51*

Zero times and crossing depths from table 8		α calculated by method of—		
Time	x_c, crossing depth	Table 7 and equation 12	Equation 9	Crossing points and equation 8
t_1, t_2	120	0.0027	0.0026	0.012, $\lambda=2250$
t_1, t_3	255	.0069	.0068	.010, $\lambda=1990$
t_2, t_3	365	.0074	.0072	.008, $\lambda=1770$
t_1, t_4	395	.0077	.0077	
t_2, t_4	545	.0094	.0092	
t_3, t_4	610	.0090	.0093	
t_5, t_1	635	.0086	.0086	
t_5, t_2	775	.0094	.0094	
t_5, t_3	890	.0104	.0103	
t_5, t_4	1000	.0098	.0097	

NOTES.—
t_1 $\lambda/2=750$ measured between slope reversal points, $\alpha=0.005$.
t_2 $\lambda/2=600$ measured between slope reversal points, $\alpha=0.004$.
t_3 $\lambda/2=770$ measured between slope reversal points, $\alpha=0.007$.
t_1=Aug. 10 $\approx$3.7 months or $1.054+10^7$ sec ($P_s=3.4236+10^7$ sec).
t_2=Sept. 26 $\approx$5.1 months or $1.460+10^7$ sec ($P_s=3.4236+10^7$ sec).
t_3=Oct. 24 $\approx$6.0 months or $1.702+10^7$ sec ($P_s=3.4236+10^7$ sec).
t_4=Dec. 11 $\approx$7.7 months or $2.108+10^7$ sec ($P_w=2.8879+10^7$ sec).
t_5=Feb. 27 $\approx$10.5 months or $2.790+10^7$ sec ($P_w=2.8879+10^7$ sec).
x_c is depth (cm) of crossing points.

gradient is present is indicated by the poor value obtained by this method as shown in column 5 of table 9.

Although equation 24 is derived on the assumption that α is a constant, it is evident that this equation will yield an average value where the diffusivity changes in value between two crossing points. The increase with depth of the values computed for diffusivity in hole 51 can be explained if we assume that α for the first meter is about 0.0030, that from 1 meter to 4 meters α is about 0.0092, and that at greater depths it is about 0.011. Table 10 compares the diffusivities computed on these assumptions with the values taken from column 4, table 9.

TABLE 10.—*Computed diffusivity in relation to depth, hole 51*

	Diffusivity values at indicated depth$\times 10^4$ [obtained from table 9]									
	120	255	365	395	545	610	635	775	890	1000
α is constant[1]	26	68	72	77	92	93	86	94	103	97
α increases with depth[2]	(40)	(68)	(74)	(77)	(86)	(89)	(90)	(93)	(95)	(97)

[1] Values obtained from table 9, col. 4.
[2] Computed on assumed α values: for depths of 0–1 meter, $\alpha=0.0030$; for depths of 1–4 meters, $\alpha=0.0092$; for depths >4 meters, $\alpha=0.011$.

DETERMINATION OF GEOTHERMAL GRADIENTS FROM SHALLOW-DRILL-HOLE-TEMPERATURE MEASUREMENTS

The problem of obtaining geothermal gradients at depths affected by the annual wave from the sun is primarily that of determining what part of the measured temperature at any depth is due to the heat from the sun and what part is due to heat from the interior. The determination of the subsurface effect of the sun's heat, and the determination of gradient by separating that effect from the temperatures measured, are discussed below.

If we use equations 5 and 6 to determine the average geothermal gradient between given points on the subsurface temperature curve as plotted from the subsurface temperature measurements, we can neglect the many minor perturbations of temperature in the annual temperature cycle at the surface; this is obviously an overwhelming advantage. It is mandatory, however, to know the dates when the annual surface wave passes through the average annual surface temperature. If crossing points x_c and $(x'_c = x_c + \lambda/2)$ of two subsurface temperature curves are known and the average diffusivity has been found within narrow limits, as by laboratory methods from drill core, the date of $t=0$ can be calculated:

$$\lambda = 2\sqrt{\alpha \Pi P}$$

and

$$P = \frac{\lambda^2}{4\alpha\Pi} \tag{13}$$

From equation 9 we have

$$\sqrt{\alpha} = \frac{2x_c\sqrt{\Pi/P}}{(2\Pi/P)(t_1+t_2) \pm n\Pi} \tag{14}$$

$$t_1+t_2 = \frac{x_c}{\sqrt{\alpha}}\sqrt{\frac{P}{\Pi}} \pm \frac{nP}{2}. \tag{15}$$

Adding $t_2 - t_1$ to both sides of the equation and solving for t_2,

$$t_2 = \frac{1}{2}\left(t_2 - t_1 + \frac{x_c}{\sqrt{\alpha}}\sqrt{\frac{P}{\Pi}} \pm \frac{nP}{2}\right), \tag{16}$$

whence t_1 can be found by subtracting the difference between the times of measurement,

$$t_1 = t_2 - (t_2 - t_1) \tag{17}$$

As the difference between t_2 and t_1 is known from the dates of measurement and P_s or P_w is given by 13, all parameters on the right-hand side of equation 16 are known. Because of the approximation of λ that is involved in the measurement of the second crossing point, the value of P may be only approximate.

Nevertheless, the values obtained for $t=0$ (the date from which t_1, t_2 * * *, are measured, as explained on page 26) and for P_w and P_s are very satisfactory. The use of this method is illustrated below for data obtained from hole 51 and the values are compared with those obtained from the smoothed curves for the daily surface temperature.

Using the crossing points t_2 t_3 of figure 7, $\lambda/2=1250-365=885$, and $\lambda=1770$. Assuming that the value of $\alpha=0.0072$ (table 9) is correct but was found by experimental methods, then from equation 13

$$P_w=\frac{1770^2}{4\times 0.0072\Pi}=28{,}345{,}800 \text{ sec.}=328 \text{ days}$$

$$P_s=365+(365-328)=402 \text{ days}$$

As $t_2-t_1=$Oct. 24—Sept. 26=28 days, equation 16 now can be used to calculate t=0; converting from seconds to days we have for equation 16

$$t_3=\frac{365\sqrt{402/\Pi}}{2\times\sqrt{86{,}400}\times\sqrt{.0072}}+\frac{28}{2}+\frac{402}{4}$$

$$=\frac{365\times 11.31}{2\times 294\times 0.085}+14+100.5,$$

$$t_3=83+14+100=197.$$

$t=0$ on Oct. 24—197=April 10.

For comparison, the climatic data give $t=0$ on April 10 and $P_s=395$. The dates that the annual wave crosses the average ground temperature are April 10 and October 24 from smoothed climatic data, and April 10 and October 28 by the calculations based on the two subsurface-temperature curves measured about a month apart—agreement that is satisfactory for calculations involving anomalous temperature gradients from equations 5 and 6. Using the rapid graphic method described on page 44 for solving equations 9 and 5, diffusivities and gradients were calculated for the shallow holes 33 feet and deeper, using the values of 402 days for P_s as calculated above. The results are compared in table 11, with the actual average geothermal gradients as given by temperatures measured at depth and the average surface temperature given by the shallow-hole gradient, a temperature that apparently is in error by less than 1°F.

The specific heat σ and bulk density ρ on representative samples of rock together with the in-hole diffusivity α determinations were used to calculate the approximate thermal conductivity k of the rocks where $k=\alpha\ \sigma\ \rho$. (See column 6 table 11, and column 2 table 12.)

The agreement between calculated and observed gradient is satisfactory when the possible variations in conductivity of the deeper rocks are considered (table 11).

TABLE 11.—*Diffusivity α, gradient, conductivity* k, *and surface temperatures calculated for shallow holes 10 meters or more in depth*

Hole No.	Depth (meters)	$\alpha \times 10^4$	Gradient (degrees F/100 feet)		$k \times 10^4$ in col/cm sec deg[1]	Surface temperature	Alteration	Unit	Age
			Calc.	Observed					
12	10	110	6.1	5.8	51	53.7	Fresh	Packard Quartz Latite	Tertiary.
13	10	74	6.0	5.5	35	53.8	Pyritic to calcitic	do	Do.
14	10	58	4.7	4.7	24	56.9		Colluvium	Quaternary.
15	10	95	5.0	6±	45	52.5	Fresh	Packard Quartz Latite	Tertiary.
18	10	60	5.5	5.4		52.7		Colluvium	Quaternary.
19	10	60	4.4	5.0		53.6		Packard Quartz Latite and colluvium	Tertiary and Quaternary.
24	10	56	3.7	4.5	27	52.4	Pyritic and weathered	Packard Quartz Latite	Tertiary.
25	10	72	4.8	4.5		54.9		Packard Quartz Latite and colluvium	Tertiary and Quaternary.
26	10	68	4.8	4.8	33	51.8	Pyritic and weathered	Packard Quartz Latite	Tertiary.
27	10	60	3.0	4±		53.7		Alluvium	Quaternary.
28	10	60	4.7	5.0	29	53.9	Pyritic and weathered	Packard Quartz Latite	Tertiary.
29	15	57	5.9	4.6	24	53.8		Gravel	Quaternary.
31	10	62	3.9	4.5	26	55.3		do	Do.
32	12.8	58	3.0	2.1+		51.3		Colluvium	Do.
36	10	95	4.1	5	56	54.8		Limestone, Ophir Formation	Cambrian.
37	10	75	4.7	5.7		50.5	Calcitic	Gravel and Packard Quartz Latite	Quaternary and Tertiary.
38	10	106	6.9	7.1	52	54.5	Calcitic	do	Do.
43	10	[2] 35	11.7	7.2		55		Alluvium and colluvium	Quaternary.
43	10	[2] 65	5.4	7.2		55		do	Do.
44	10	68	6.6	7.3		55.3		Packard Quartz Latite and thin colluvium.	Tertiary and Quaternary.
45	8.8	70	5.1	5.0	33	51.0	Weathered	Packard Quartz Latite	Tertiary.
47	10	65	5.8	5.3	29	52.5	Chloritic	do	Do.
48	15	96	4.6	5±	47	55.0	Calcitic	do	Do.
49	10	96	7.5	7.4	47	53.2	do	do	Do.
50	10	70	9.3	7.7	31	54.6	Chloritic	do	Do.
51	17.7	[3] 68	6.7	7.4	33	53.3	Pyritic, weathered	do	Do.
51	17.7	[3] 100	6.7	7.4	49		Pyritic	do	Do.
52	10	64	7.7	7.4	31	52.0	Calcitic	Sheared Packard Quartz Latite	Do.
54	10	78	4.4	6.8	32	54.7	Pyritic, calcitic, weathered	Packard Quartz Latite	Do.
55	15.5	57	6.3	4.6	22	54.2		Gravel	Quaternary.
57	10	78	5.5	6.3		53.6	Calcitic	Packard Quartz Latite and colluvium	Tertiary and Quaternary.

[1] "k" is calculated from: $k=\alpha\sigma\rho$ where σ is specific heat and ρ is density. σ for fresh quartz latite was determined experimentally as 0.193 (average of 3 specimens) and ρ is 2.43 (average of 3). Measured values of ρ for other types of quartz latite: weathered 2.35; fresh pyritic 2.45; weathered pyritic 2.40; calcitic 2.45; chloritic 2.24. Assumed value of σ for fresh pyritized quartz latite is 0.193 but for all other varieties value assumed is 0.20; this value is too low, however, if appreciable moisture is present. Assumed values of ρ and σ for gravel are 1.90 and 0.23.

[2] α is 0.0065 from 1 to 7 m, and 0.0035 below 7 m.

[3] α is 0.007 above 3 m and 0.01 below.

It is possible to determine the gradient by means of subsurface-temperature curves and smoothed climatic data, and a graphic method for doing this has been devised, which is described on page 45, and it was used for checking the data of hole 51. The gradient is determined by comparing the actual subsurface temperatures measured on a particular day with a curve constructed to show the subsurface temperatures caused by the annual wave as of that same day.

TABLE 12.—*Average approximate conductivity* k *(cal per cm sec deg) of different materials in place*

Age	Unit	Alteration	Thermal conductivity, $k \times 10^4$	Range	Number of items averaged	Experimental determination [1] of thermal conductivity, $k \times 10^4$
Cambrian	Ophir Formation (limestone).	Fresh	56		1	[1] (65.6)
Tertiary	Packard Quartz Latite.	Fresh	48	6	2	48.1
		Weathered	33		1	
		Calcitic, fresh, unweathered, unsheared.	49	5	3	48.2
		Pyritic, calcitic, weathered.	33	3	2	
		Pyritic, fresh	47		1	44.7
		Pyritic, weathered to fresh.			1	43.6
		Pyritic, weathered	31	6	3	
		Chloritic, weathered	30	2	2	
Quaternary	Gravel		24	±2	3	

[1] Thermal conductivity determined experimentally by E. C. Walker on drill core or other subsurface specimens of similar rock from Burgin mine, except limestone, which is Teutonic Limestone (Middle Cambrian) from outcrop in Homansville Canyon 2 miles northwest.

Figure 8 compares such a calculated curve for hole 51 and the actual temperatures measured. The difference between them should be due to the gradient. The reliability of such calculations increases greatly with depth, as is shown in figure 9, in which several determinations of gradients in hole 51 are compared with the deep-hole gradient measured about 25 feet away.

The results of the different methods of determining the geothermal gradient at hole 51 are compared in table 13.

The agreement of methods 3, 4, and 5 is satisfactory and the average of the results of both 4 and 5 is less than 10 percent below the gradient found in the deep hole 25 feet from hole 51. The average surface temperature as determined by the gradient data for columns 3, 4, and 6 is 53.42°, 53.34°, and 53.3°F.

It should be pointed out in passing that the range of the effective annual temperatures can be calculated also from one depth-temperature curve if α, λ, t, P, $\frac{dT}{dx}$, and the dates of the summer maximum and winter maximum are known. As the speed of downward movement

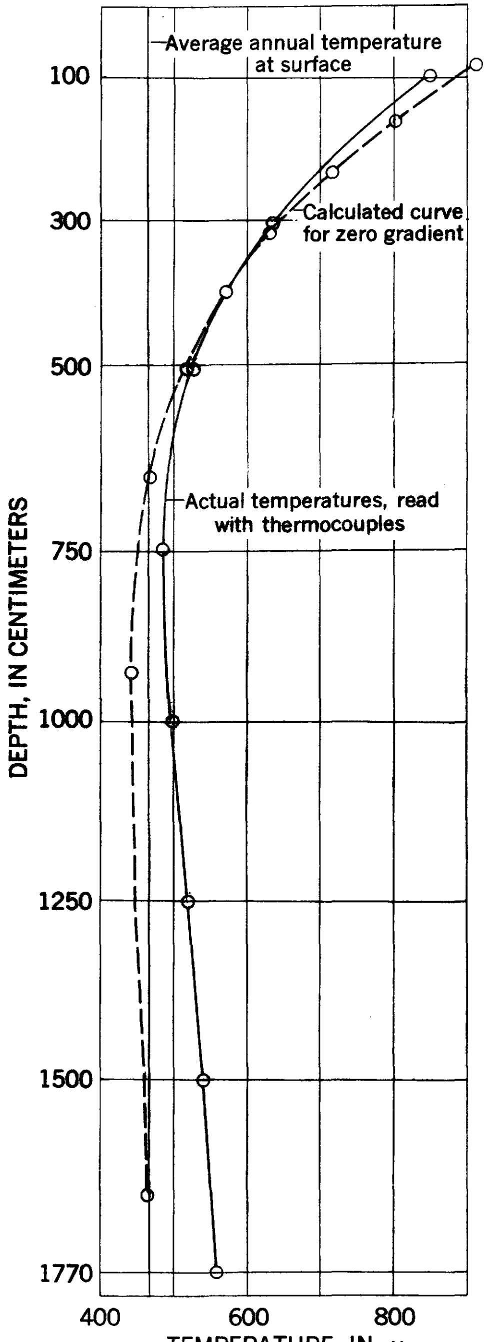


FIGURE 8.—Comparison of calculated curve and one showing actual temperature at depth in hole 51, August 10, 1950.

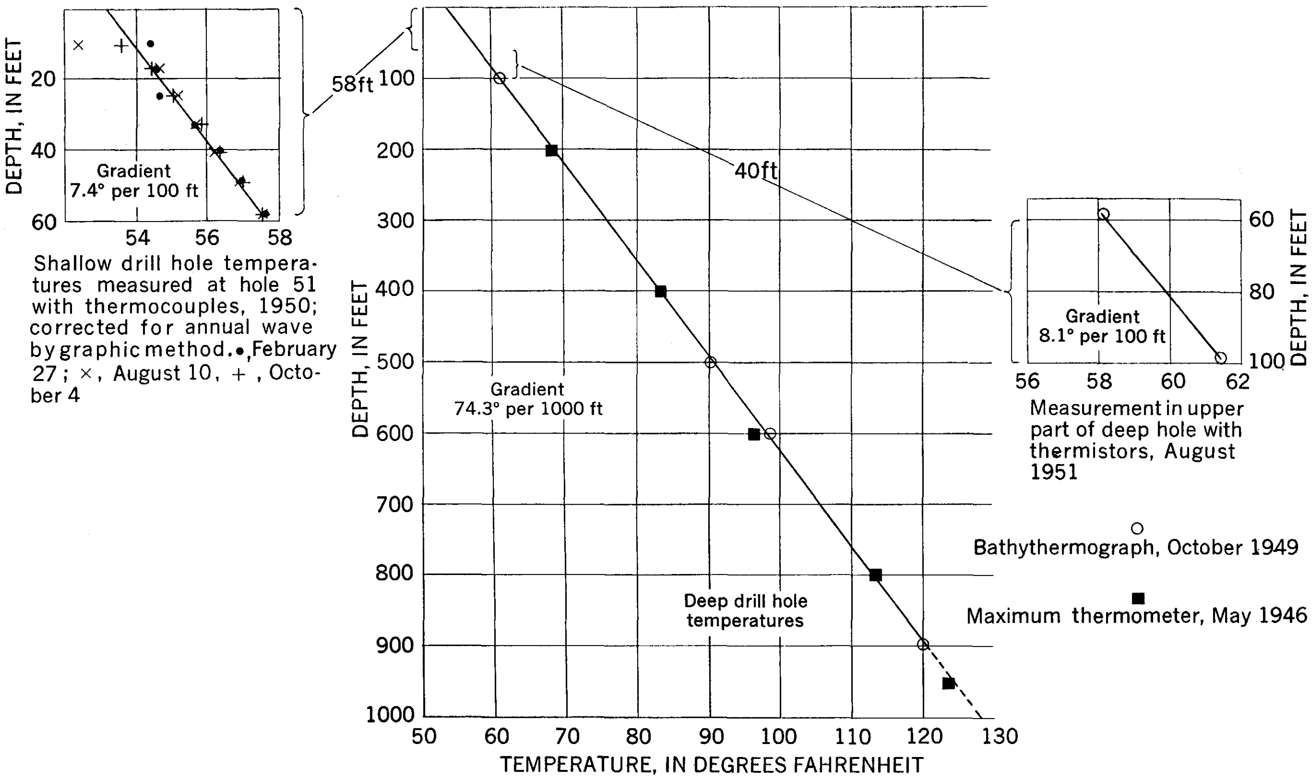


Figure 9.—Chart showing comparison of gradients measured by different methods.

TABLE 13.—*Comparison of gradients determined by different methods for hole 51*

Time (date)	Method 1		Method 2		Method 3		Method 4		Method 5		Method 6	
	x, T=0	°F per 100 ft	x, T=0	°F per 100 ft	x, T=0	°F per 100 ft	x, T=0	°F per 100 ft	X	°F per 100 ft	Depths, in feet	°F per 100 ft
t_1	757	+10. 1	650	+6. 9	619	6. 56	601	6. 72	500	6. 23		
	1, 731		1, 624		1, 542		1, 592		1, 500		200	7. 43
t_2	20		901	+5. 0	858	6. 70	854	6. 68			900	
	1, 080		1, 875		1, 781		1, 885					
t_3	96	−8. 4	41	−0. 2	0	6. 51	1, 008	7. 06				
	1, 157		1, 051		1, 000		1, 937					
t_4	381	−4. 8	275	−0. 3	216	6. 23	209	6. 23				
	1, 408		1, 301		1, 220		1, 228					
t_5	855	+5. 4	748	+6. 0	691	6. 23	670	6. 79	500	7. 06		
	1, 829		1, 722		1, 696		1, 680		1, 500			
t_6									500	6. 37		
									1, 500			
t_7									500	7. 60		
									1, 500			
Average		+0. 6		+3. 5		+6. 44		6. 70		6. 82		7. 43

t_1=Aug. 10, 1950; t_2=Sept. 26, 1949; t_3=Oct. 24, 1949; t_4=Dec. 11, 1950; t_5=Feb. 27, 1950; t_6=Apr. 29, 1950; t_7=Oct. 4, 1950.

Method 1: Depth x in cm of where T=0 calculated for T=0 and t=0 on Mar. 21, and $t=\frac{1}{2}P$ on Sept. 21. (At midpoint of range for zero gradient).

Method 2: Depth x in cm of where T=0 calculated for t=0 on Apr. 10, $t=\frac{1}{2}P$ on Oct. 10.

Method 3: Depth x in cm of where T=0 calculated for t=0 on Apr. 10, $t=\frac{1}{2}P$ on Oct. 25.

Method 4: Depth x in cm of where T=0 calculated for t=0 on Apr. 10, $t=\frac{1}{2}P$ on Oct. 28.

Method 5: Gradient determined by graphic method, described in explanatory notes.

Method 6: Temperatures measured in deep hole adjacent to hole 51.

of the maximum or minimum is $\frac{\lambda}{P}$, the position of these points on the subsurface wave is readily calculated; the difference between the temperature measured at such a point and the temperature of the mean for this depth as calculated from the geothermal gradient gives the amplitude of the wave at this depth for the corresponding season. The attenuation of the surface wave with depth is known from equation 2, and accordingly the amplitude of the surface wave can be readily calculated from the amplitude at depth:

$$T_s = \frac{T_x}{e^{-x\sqrt{\Pi/\alpha P}}} \tag{18}$$

where T_s is the surface temperature amplitude, T_x is the amplitude at depth x, α is diffusivity, and P is the period.

SUMMARY AND EVALUATION

Efforts have been made from time to time by various geologists and geophysicists to decipher geologic conditions or to find anomalous geothermal gradients by measuring earth temperatures in holes 1 to 2 meters deep. Such measurements would be so greatly influenced by local conditions unrelated to gradients that any correlation with the factors sought would be only fortuitous. Short-range hot or cold spells and small local variations in weather, geography, vegetation, soil moisture, the diffusivity of the subsurface material, or in the time of temperature measurement, could generate perturbations far greater than those caused by the geologic factors. In our opinion such work is a waste of time.

Temperature measurements made to determine gradient directly should be taken below 50 feet in uncased holes at least 100 feet deep, and preferably in the lower 50 feet of holes about 150 feet deep. Where a low geothermal gradient obtains in rocks of high diffusivity, temperature measurements made at depths of less than 100 feet can produce very erroneous results if used to determine the gradient. As shown on figures 10 and 11, however, little or no annual change in the temperature occurs in rocks of average diffusivity at a depth of 18 meters, and a reasonably accurate determination of gradient can be made from measurements at depths of 18 and, say, 30 meters. As suggested by the right side of figure 10, an appreciable error could be made in measuring a low gradient in a rock of high diffusivity even at these depths.

The various techniques devised for determining geothermal gradients from shallow-hole measurements were first tried for hole 51 because it was the deepest of the holes equipped with thermocouples and was located where the best deep-hole and surface measurements

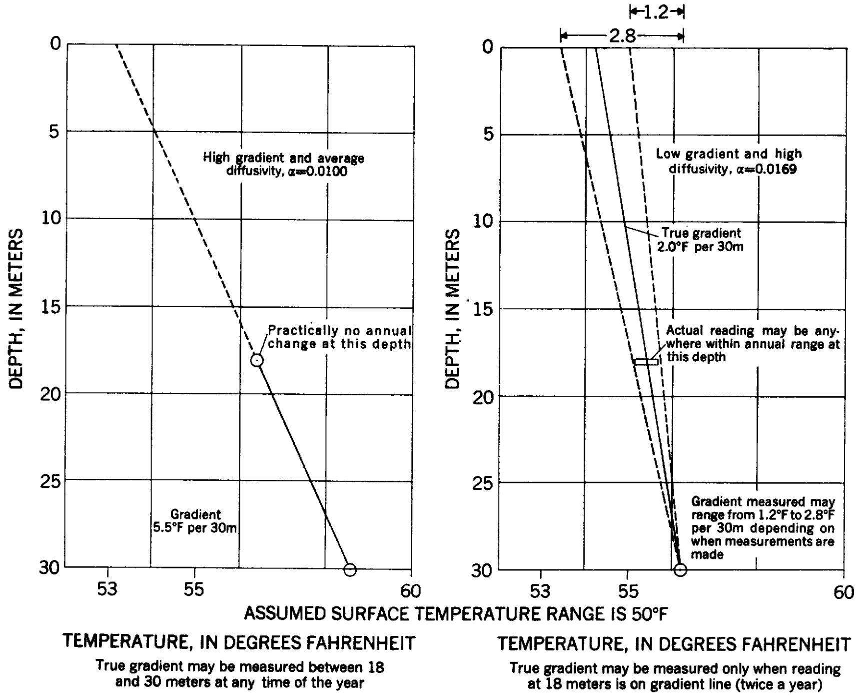


FIGURE 10.—Charts showing how accuracy of shallow-hole gradient measurements depends on diffusivity of rock and amount of gradient.

had been made. Of the in-hole methods used, the one that depends on two or more sets of temperature readings is by far the most accurate. This method requires a hole deep enough to ascertain where two depth-temperature curves cross, and also knowledge of the spring and fall "zero time" dates—that is, when the annual surface temperature wave passes through the average annual surface temperature. We believe that the spring "zero time" for hole 51 was estimated rather closely as April 10 and the fall "zero time" was on or slightly later than October 24, but if these dates were in error by as much as 10 days it would change the diffusivity calculation by only a few percent so long as the interval between the two dates was known accurately.

A method based on two sets of in-hole temperature measurements was devised for ascertaining the spring and fall "zero time" dates, and as it eliminates the necessity of obtaining extensive background weather data it should be of considerable value. The calculation requires only (1) that the diffusivity of the rock be determined experimentally within narrow limits—preferably on drill core, and (2) that two sets of temperature measurements be made to or below the depth where the resulting depth-temperature curves cross. From these data the spring and fall zero-time dates and the length of the summer and winter half periods are readily computed, and these dates and time intervals should hold for a considerable area surrounding the drill hole that provided the basic information.

With the zero times established, it is then possible to ascertain readily from two sets of temperature measurements the diffusivity of the subsurface material for other shallow holes in the nearby region and to calculate the geothermal gradient and both the mean annual temperature and the annual temperature range at the surface for each hole.

Table 13 and figure 11 show that gradients calculated from the measurements made in hole 51 are very close to the one measured in the nearby 1,000-foot hole. The best value of the gradient determined at hole 51 is closer to the apparent true gradient in the deep hole than is that measured in the upper part of the deep hole, where the temperatures were affected by steel casing, the diffusivity and conductivity of which are much greater than those of the bedrock.

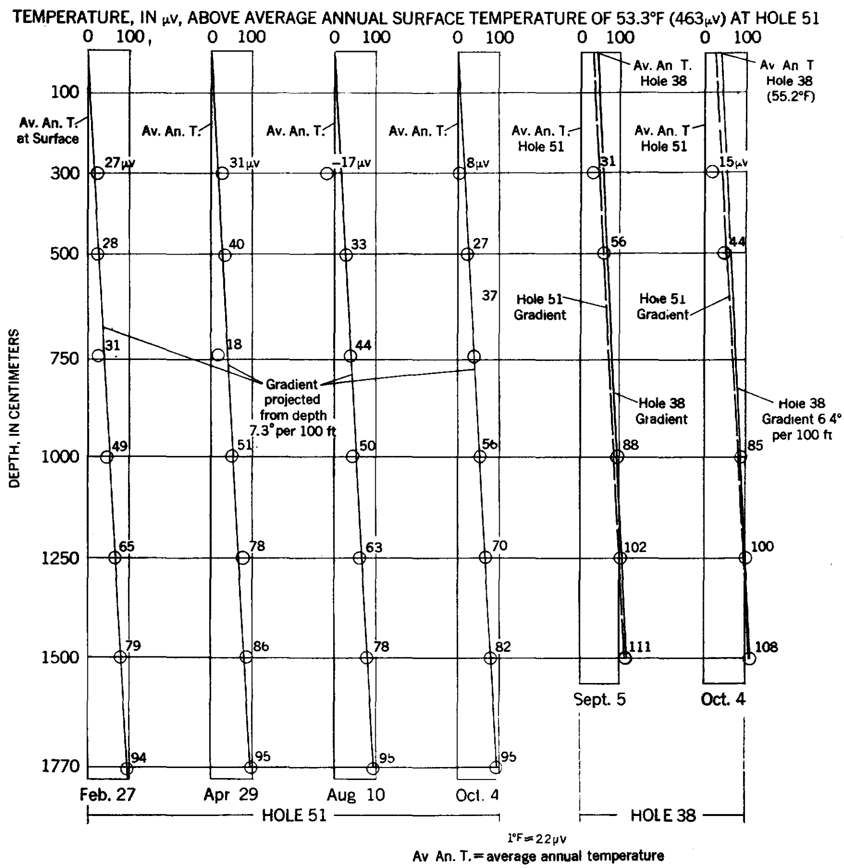


FIGURE 11.—Deep-hole gradients compared with shallow temperatures corrected for variation of effective annual surface temperature, 1950.

EXPLANATORY NOTES

CALCULATION OF DIFFUSIVITY FROM MEASURED CROSSING POINTS OF TWO SETS OF DEPTH-TEMPERATURE MEASUREMENTS

The diffusivity α can be calculated from crossing points of depth-temperature curves. We assume that the temperature T_x at depth x, at times t_1 and t_2 after t_0, for surface-temperature wave of period P is known. Starting with equation 3 (Ingersoll, Zobel and Ingersoll, 1948)

$$T_x = T_0 e^{-x\sqrt{\Pi/\alpha P}} \sin\left(\frac{2\Pi t}{P} - x\sqrt{\frac{\Pi}{\alpha P}}\right) \tag{3}$$

let

$$A=\frac{2\Pi t_1}{P},\quad B=\frac{2\Pi t_2}{P},\quad y=\sqrt{\frac{\Pi}{\alpha P}}.$$

Then where the subsurface-temperature waves generated during P as measured at times t_1 and t_2, cross at x, which is also known, all parameters in the equations are identical except t_1 and t_2; as T_x for t_1 and T'_x for t_2 are equal, we have

$$T_0 e^{-xy} \sin(A-xy) = T_0 e^{-xy} \sin(B-xy), \tag{19}$$

or

$$\sin(A-xy) = \sin(B-xy). \tag{20}$$

If A, B, and xy are in radians, the points corresponding to A and B on the sine curve must be displaced so that they are symmetrical with respect to $\frac{n\Pi}{2}$ when 20 is true. This symmetry is determined by xy, where

$$xy=\frac{A+B}{2} \pm \frac{n\Pi}{2} \tag{21}$$

where n is an odd integer.

Solving for y, and substituting the value of y in equation 21,

$$\sqrt{\Pi/\alpha P} = \frac{A+B \pm n\Pi}{2x}. \tag{22}$$

Substituting the values of A and B, and solving for $\sqrt{\alpha}$ in equation 22

$$\sqrt{\alpha} = \frac{2x\sqrt{\Pi/P}}{\frac{2\Pi}{P} t_1 + \frac{2\Pi}{P} t_2 \pm n\Pi} \tag{23}$$

$$\alpha = \frac{4x^2\Pi/P}{[(2\Pi/P)(t_1+t_2) \pm n\Pi]^2}. \tag{24}$$

TEMPERATURE GRADIENT

Where there is no temperature gradient, an annual surface-temperature wave generates a subsurface-temperature wave which reaches the mean annual surface-temperature at depths $x+n$ $(\lambda/2)$ that are a function of the time, t_1, after the surface temperature has passed through the mean annual temperature, where n is any positive integer.

A temperature gradient will displace the subsurface temperatures to higher values than those for zero gradient, and if the temperatures for two points at the calculated mean temperatures are measured, the gradient can be determined readily. The depths at which the subsurface temperatures at time t_1 reach the mean annual temperature, given a zero gradient, follow from the special condition that $T_x=0$ in equation 3, where the annual range is from $+T_0$ to $-T_0$:

$$T_x = T_0 e^{-x\sqrt{\Pi/\alpha P}} \sin\left(\frac{2\Pi}{P} t_1 - x\sqrt{\frac{\Pi}{\alpha P}}\right)=0,$$

and hence

$$\sin\left(\frac{2\Pi}{P} t_1 - x\sqrt{\frac{\Pi}{\alpha P}}\right)=0=\sin\left(\pm n\Pi\right) \tag{25}$$

where $n=0$ or any integer,

$$\frac{2\Pi}{P} t_1 - x\sqrt{\frac{\Pi}{\alpha P}} = \pm n\Pi;$$

$$x\sqrt{\frac{\Pi}{\alpha P}} = \frac{2\Pi}{P} t_1 \pm n\Pi$$

$$x=\frac{\Pi\left(\frac{2t}{P}\pm n\right)\sqrt{\alpha P}}{\sqrt{\Pi}}$$

$$x=\sqrt{\Pi\alpha P}\left(\frac{2t}{P}\pm n\right) \tag{26}$$

and as $\lambda=2\sqrt{\Pi\alpha P}$ it is evident that $T=0$ at intervals of $\frac{1}{2}\lambda$ below the shallowest zero point, which is determined by $\frac{2t}{P}\frac{\lambda}{2}$ or $T=0$ where $x=\frac{\lambda t}{P}$.

As $\sqrt{\Pi P}$ is a constant for a given locality, the values of x for $T=0$ may be shown by straight lines in a graph, computed for different values of α according to the equation

$$x=C\sqrt{\alpha}\left(\frac{2t}{P}\pm n\right). \tag{27}$$

Such a chart computed for $P_s=402$ days is shown in figure 12. This chart may also be used for finding the value of α from crossing points of depth-temperature curves, by merely revaluing the abscissae and moving the zero time as shown. On this chart information for scale A is derived from equation 24, and for scale B from equation 27.

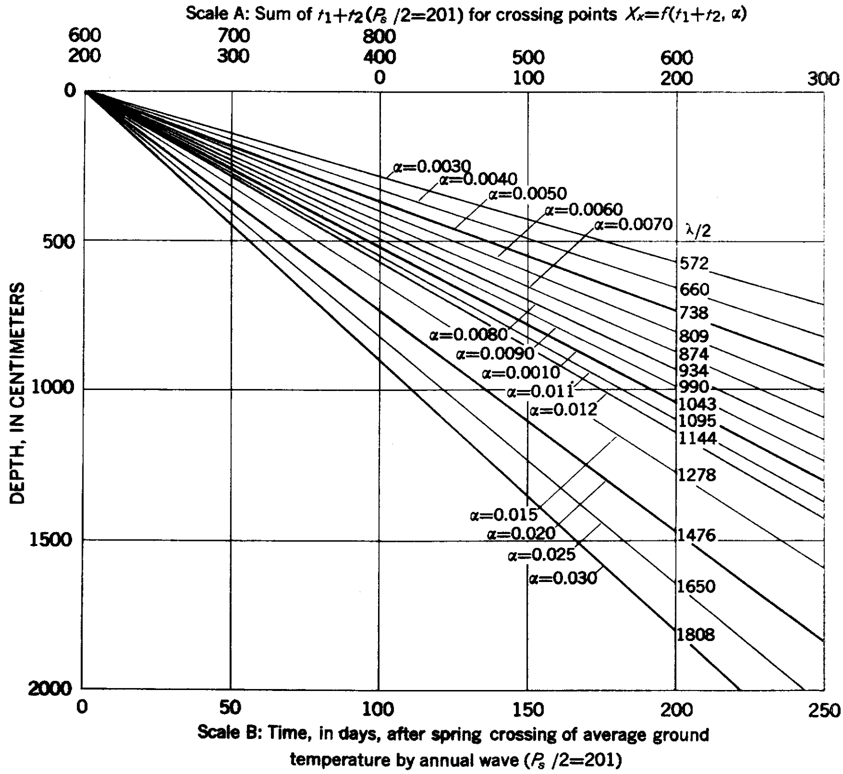


FIGURE 12.—Chart for rapid graphical solution of equations 5 and 9 where P_s is 402.

Scale A: Diffusivities (α) corresponding to crossing points of depth-temperature curves measured at times t_1 and t_2 after the surface temperature reaches the average annual surface temperature in the spring; assumed length of summer is 201 days (P_s=402).

Scale B: Depth x_0, in centimeters, where temperatures are at midpoint of annual range, for materials of different diffusivities (α). Gradient is determined by temperatures at x_0 and $x_0+\lambda/2$, or by x_0 for t_1 and x_0 for t_2.

SUBSURFACE EFFECT OF A KNOWN ASYMMETRIC SURFACE-TEMPERATURE WAVE OF KNOWN FORM IN MATERIAL OF KNOWN DIFFUSIVITY

As a mathematical solution of the Fourier heat conduction equation for an asymmetric surface heat wave is complicated and cumbersome, a graphic solution of the problem has been devised; the time coordinates of the surface-temperature cycle are plotted on the same scale as the depth coordinates and P (one period) is made equal to one subsurface wavelength, and the same temperature coordinates are used for both surface and subsurface waves.

Every point on a subsurface-temperature wave reflects the temperature at the surface at some time in the past. The depth at which the temperature is measured and the time corresponding to the generation of that part of the wave by temperatures at the surface in the past, may both be regarded as functions of wavelength; the wavelength of subsurface temperatures is expressed as depth, whereas for surface temperatures the equivalent of this wavelength may be expressed as time, 1 year. For convenience in projecting surface temperatures below the surface, the time wavelength of the surface wave is scaled onto cross-section paper so that it occupies the same space as the depth wavelength of the subsurface-temperature wave.

The method is illustrated by the data from hole 51. The average wavelength of the subsurface wave is about 1,860 cm (from table 7), and the average value of the diffusivity is taken as 0.0087. For the purposes of this illustration the use of average diffusivity and wavelength seems justified. As shown in figure 13, the scale used for

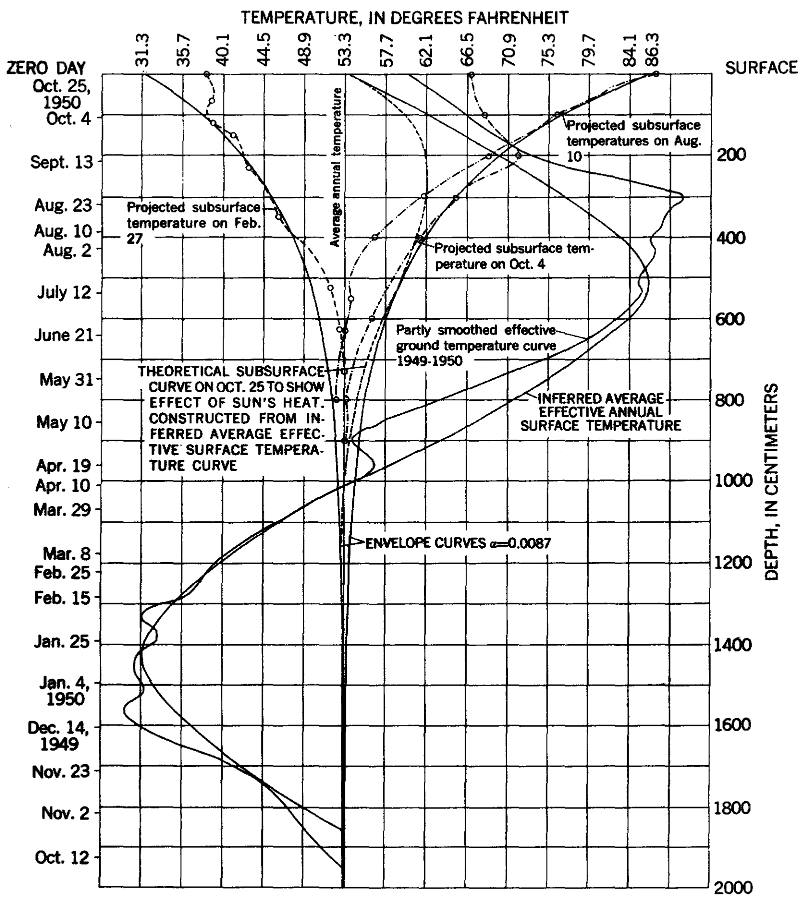


FIGURE 13.—Composite drawing showing (1) relation of the theoretical subsurface effect of the sun's heat to inferred effective surface temperature and (2) the subsurface projection of ground temperatures that produce subsurface-temperature curves for particular dates, Oct. 4, Aug. 10, and Feb. 27, 1950.

1,860 cm, the average subsurface wavelength, is made equal to the scale used for one year's time.

In constructing any subsurface curve from a surface-temperature curve, the required subsurface curve is named for the date of its starting point at the surface; on the graph this starting point on the subsurface curve is the temperature at the surface on this "zero day." "Zero day" for the theoretical curve in figure 13 was October 25, 1950 and "zero day" for the other curves in figure 13 were Oct. 4, Aug. 10, and Feb. 27, of the same year.

The steps in constructing figure 13 follow:

1. The envelope curves in a rock with a diffusivity of α=0.0087 are constructed, using for the base the range given by the curve plotted for the inferred effective annual surface temperature in figure 5, which is also used to determine P and t_0.
2. The partly smoothed effective ground temperature curve of 1949–50 is plotted so that its time "wavelength" (one year) is scaled to equal the wavelength of the subsurface wave; in figure 13 this curve is taken as slightly less than the mean of the smoothed air and ground temperatures (21-of-7-day moving averages) for the summer, and approximating the ground temperatures in the winter (fig. 5).
3. A "zero day" is chosen and the temperature for that day as shown on the asymmetric smoothed but actual surface wave is projected to the surface line for the start of the subsurface wave.
4. After the ground temperature wave for the time *preceding* this "zero day" is examined, the first subsurface point to be located, preferably about 100 cm below the surface, is chosen so as to correspond to the depth that the annual wave would penetrate by "zero day" after a selected past date. To find the temperature of this point—and the temperature of other points required to construct the subsurface curve—the following procedure is used:
   a. Project to the surface line the past surface temperature on the day (earlier than "zero day") corresponding to the time required for the wave to penetrate to the depth selected (point D in figure 14).
   b. Locate point B on the envelope curve at the same depth as the point chosen, which is the depth to which the annual wave would penetrate between the "zero day" and the selected past date.
   c. Extend line A–B to point C on average annual temperature line.
   d. Construct line C–D.

Point E, where line C–D crosses the horizontal line equivalent in depth to B, is the temperature at depth generated by the surface temperature on the selected past day.

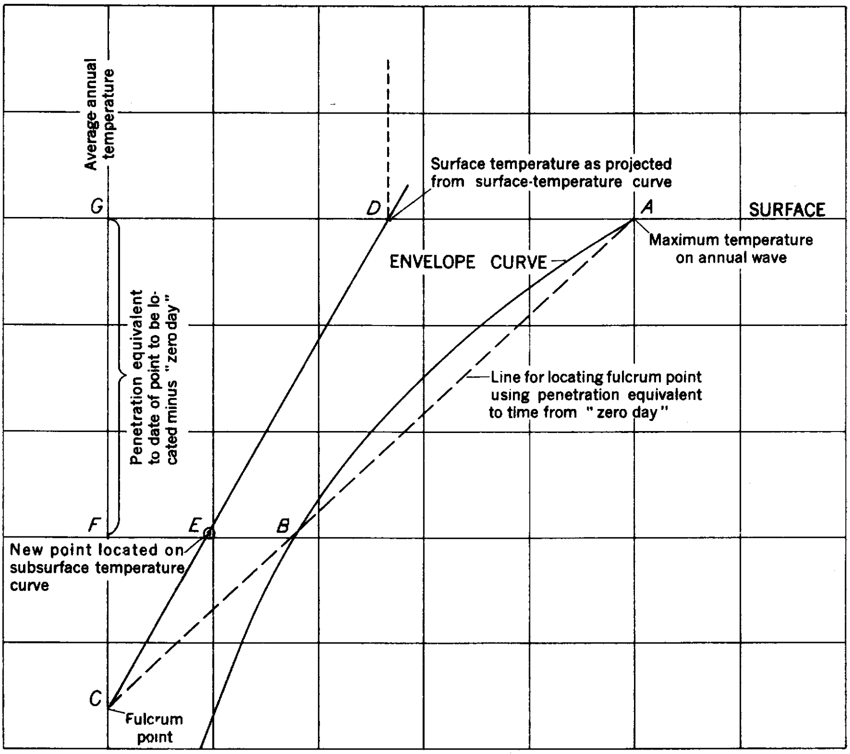


FIGURE 14.—Graphic method of locating one point on a subsurface-temperature curve of figure 13.

The geometry of the method is based on the similarity of triangle *CFE* to *CGD* and of triangle *CFB* to *CGA*, where *AG* equals the half range at the surface and *FB* equals the half range at depth; point *E* at depth is the equivalent of point *D* on the surface.

Once the curves for the surface temperature and the envelope of the subsurface temperatures are plotted, the construction of a subsurface curve by this method is relatively fast, but the method requires temperature measurements made at the surface over a long period of time. The curves constructed by this method for hole 51 (fig. 13) are based on daily surface-temperature measurements for about 14 months, an average surface temperature extrapolated from measurements made in a nearby deep drill hole, and the preparation of a smoothed annual surface-temperature curve which is taken as the mean of the smoothed average shelter and ground temperatures.

In figure 13 the envelope curves were constructed from the inferred average effective annual temperature curve of figure 5, whereas the subsurface curves of figure 13 were constructed from the much less regular partly smoothed curve of the inferred effective ground temperatures of 1949–50, as explained on page 46.

Geothermal gradient.—The gradient is determined by comparing the actual subsurface temperatures measured on a particular day with a curve constructed to show the subsurface temperatures caused by the annual wave as of that same day.

Figure 8 shows the comparison between such a calculated curve for hole 51 and the temperatures measured. The difference between them should be due to the gradient. The reliability of such calculations increases greatly with depth, as is shown in figure 9, in which several determinations of gradients in hole 51 are compared with the deep-hole gradient measured about 25 feet away.

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