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iron ores

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RELATION BETWEEN SPECIFIC GRAVITY AND IRON CONTENT OF ROCKS FROM THE RED MOUNTAIN FORMATION, ALABAMA

By RICHARD P. SHELDON

ABSTRACT

A balance for measuring the specific gravity of large pieces of rock rapidly, accurately, and precisely was used to determine the relation between the iron content and the specific gravity of iron-rich rocks in the Red Mountain Formation of Silurian age in Alabama. Theoretical considerations show that the reciprocal of the powder specific gravity is equal to the sum of the proportions of each mineral divided by its specific gravity. If the specific gravity of the gangue of an iron-rich rock is assumed to be constant, the reciprocal of the powder specific gravity of the rock is equal to 0.3734 minus 0.2609 times the proportion of iron. Thus, the reciprocal of the powder specific gravity is a linear function of the iron content. The experimental data fit the theoretical equation very well, although effect of differences in the gangue specific gravity does cause minor scatter about the regression line. When bulk specific gravity of the rock is used in the equation instead of powder specific gravity, porosity, which itself is variable from rock to rock, causes the relation between specific gravity and iron content to be less constant. Despite this variation, the iron content can be estimated from the bulk specific gravity with an accuracy of plus or minus about 6 percent, 95 percent of the time. With more data, the accuracy of the estimate would be improved. Thus, specific gravity can be useful in estimating the iron content of the rock and in selecting samples for further laboratory work. Statistical quality-control techniques in selecting samples for chemical analyses on the basis of specific gravity could prove very useful in exploration programs.

INTRODUCTION

During the investigation of the iron-bearing rocks of the Red Mountain Formation of Silurian age in Alabama, it became obvious that the development of a rapid and inexpensive way to estimate the iron content of the rocks was necessary. Visual estimates of the amount of hematite are usually too high because hematite, the predominant iron mineral, generally coats all other mineral grains in these rocks. Specific gravity, however, is an easily obtainable measurement that is an index to the iron content of the rock.

The use of specific gravity in determining the grade of iron ore is not a new procedure. Eckel (1910, p. 38) pointed out the approximate correlation between specific gravity of the Alabama iron ores and their iron content. Feld, Coe, and Coghill (1946) devised a rapid specific-gravity method for estimating the iron content of powdered Alabama iron ores and found that the method gave precise results.

These older studies, however, have left several important problems unsolved. First, the basic physics behind the relation between specific gravity and mineralogy of polymineral porous rocks has not been examined. Second, the relative effects of porosity, mineralogy of gangue minerals, and mineralogy of iron minerals have also not been examined.

An understanding of specific gravity is useful for purposes other than the estimation of the iron content of rocks. The bulk specific gravity of ores is needed in order to calculate ore reserves. Measurements based on differences of specific gravity of rocks are recorded by both gravity and seismic geophysical surveys; and, although successful geophysical techniques for prospecting for hematite ores have not yet been developed, a study of specific gravities of the ores and surrounding rocks will be basic to the development of future techniques. Porosity of the rocks can be quickly estimated from petrographic studies made in conjunction with specific-gravity measurements. Finally, sampling programs can be conducted more cheaply by using rapid specific-gravity determinations as the first step in the selection of samples for further work. For example, iron-rich beds from diamond-drill cores can be selected for chemical analysis on the basis of high specific gravity. The dilemma of either having to analyze large quantities of low-grade rock chemically or overlooking ore zones can be cheaply resolved by using mathematical quality-control techniques, which have revolutionized production-line control in industry.

For these reasons, it seems worthwhile to investigate further the technique of obtaining rapid specific-gravity measurement of the iron ores and associated rocks. This paper (1) reports on the technique developed, (2) attempts to explain in theoretical terms the empirical relation between specific gravity and mineralogy, and (3) outlines a quality control method of selecting for chemical analysis rocks whose iron content is greater than any given percent iron. The theoretical explanations have been tested for the most part statistically by the use of the experimental data. In order not to divert the reader's attention from the geologic results of the study, the statistics have been removed from the general discussions and have been placed in tables at the end of the appropriate sections where they may be examined if desired.

SPECIFIC-GRAVITY BALANCE

Several factors must be considered in selecting a suitable balance. First, the balance should be rugged enough to be used in the field. Second, large pieces of rock, whose size depends partly on the diameter of the core taken and partly on the size of the grains in the rock, must be measured; inasmuch as hematitic iron ores range from medium-grained sandstone to granule conglomerate, a small rock chip is commonly not representative. Finally, the balance needs to be both accurate and precise.

An inexpensive rugged apparatus suitable for determining the bulk specific gravity of hand specimens and pieces of core was constructed and is shown in figure 1. The apparatus consists of a beam balance which has a capacity of 2,000 grams and which is attached to a stand by means of the hole and thumb screw already on the balance. A tin cup is slung directly beneath the pan so that its horizontal distance from the fulcrum is the same as the distance from the center of the pan support to the fulcrum. The cup is hung inside a large beaker of water, and the beam is counterbalanced.

The bulk specific gravity is obtained by two weighings, one in air and the other in water. The specific gravity is calculated by means of the equation:

$$SG=\frac{Wa}{Wa-Ww},$$

where Wa is the weight in air and Ww is the weight in water.

Water saturation causes a significant increase in the bulk specific gravity of the more porous samples. The samples used were air dried before weighing, but no particular care was taken to insure complete pore-space dryness, either in the air weight or in immersed weight. Undoubtedly, some inaccuracy and imprecision were thereby introduced, but replicate measurements, during which the sample became progressively wetter, gave about the same specific gravity each time with no progressive increases. One reason for this constant value is that because of hematite cementation, the iron-rich rocks are probably not particularly permeable. Thus, the chance of error introduced by replicate sampling does not seem to be great.

The accuracy of the balance was studied by comparison with a jolly balance, a standard instrument for determining specific gravity. The balance used was found to be at least as accurate as the jolly balance. The precision of the balance was then studied by weighing each of a number of samples twice. It was found that if the sample weighed more than 100 grams, suitable precision was obtained. The details of the statistical study of accuracy and precision follow.

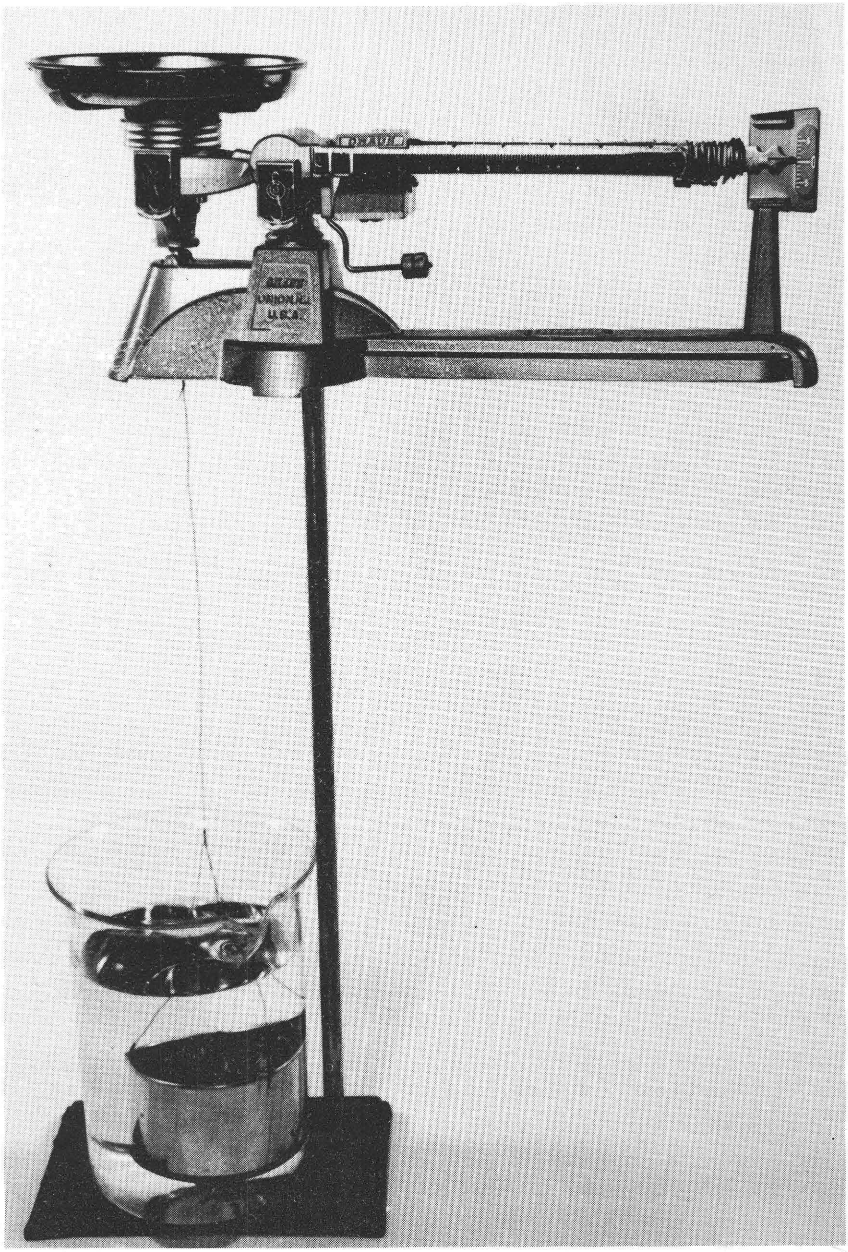

FIGURE 1.—Specific-gravity balance. Apparatus is 26 inches high.

ACCURACY

The accuracy of the beam balance compares favorably with that of the jolly balance. Five samples ranging in specific gravity from about 2.9 to 3.5 and weighing more than 150 grams each were measured on the beam balance. The samples selected were from all indications homogeneous, and three determinations were made on each sample. The specific gravities of representative chips from these samples were determined on a jolly balance, and, as with the beam balance, three determinations were made on each chip (table 1). The results from the two balances were in excellent agreement, as shown by a t-test for the difference between mean determinations (Hald, 1952, p. 403). The hypothesis that two balances give the same results was accepted at a 0.50 two-sided probability level in order to insure against wrongly accepting the hypothesis. Thus, it can be concluded from the existing data that the beam balance is as accurate as the jolly balance in determining specific gravities.

TABLE 1.—*Comparison of specific-gravity measurements made with jolly balance and with beam balance*

Sample	Jolly balance			Beam balance		
	1	2	3	1	2	3
J12-32a	2.89	2.89	2.90	2.91	2.90	2.91
1	2.97	2.98	2.96	2.98	2.98	2.98
4	3.09	3.15	3.15	3.12	3.10	3.10
J3-32f	3.40	3.41	3.41	3.40	3.40	3.39
31f	3.47	3.55	3.55	3.54	3.52	3.52

PRECISION

The precision of the balance was found by making duplicate specific-gravity determinations on 29 samples whose weights ranged from about 75 grams to 350 grams (table 2). The variances of groups of samples in 50-gram weight intervals were computed (Hald, 1952, p. 72) and then compared by the v^2 test (or F-test of other terminology) (Hald, 1952, p. 379). The variance of samples weighing between 50 and 100 grams was 0.0014, which was significantly greater than variances of all larger weight groups. The variances of the groups larger than 100 grams were not significantly different, and their pooled variance and standard deviation were 0.0001 and 0.010, respectively. Thus, if the sample weight is more than 100 grams, measurement by the balance will give results within 0.01 for about 67 percent of the determinations and within 0.02 for about 95 percent of the determinations.

TABLE 2.—*Duplicate specific gravity determinations made with beam balance*

Weight group (grams)	Duplicate determinations		Group variance	Weight group (grams)	Duplicate determinations		Group variance
	1	2			1	2	
50–100	3.29	3.37	0.0014	>200	3.46	3.46	0.00007
	2.97	2.93			3.47	3.46	
	3.21	3.17			3.54	3.52	
	3.22	3.18			3.43	3.44	
100–150	3.32	3.30	.00015		3.45	3.45	
	3.03	3.05			2.98	2.98	
	3.16	3.16			2.85	2.85	
	2.79	2.80			3.17	3.15	
	3.06	3.04					
	2.93	2.91					
150–200	3.39	3.37	.00009				
	3.40	3.40					
	3.35	3.37					
	3.72	3.72					
	3.54	3.55					
	3.10	3.10					
	2.89	2.87					
	3.12	3.10					
	2.75	2.75					
	2.91	2.90					
	3.52	3.52					

IRON DETERMINATIONS

The iron-bearing rocks were analyzed by X-ray fluorescence spectrographic methods (table 3). The precision of the method is comparable to that of classical wet chemical methods, yet the fluorescence method is much faster (W. W. Niles, unpublished data, 1963).

TABLE 3.—*Iron content, powder specific gravity, and bulk specific gravity of iron-bearing rocks*

Sample	Fe (percent)	Powder specific gravity [1]	Bulk specific gravity	Sample	Fe (percent)	Powder specific gravity [1]	Bulk specific gravity
J3-29-1	32.2	3.42	3.31	J12-1	23.7	3.17	2.98
J3-29-2	32.4	3.44	[2] 3.39	J12-3	9.94	2.92	2.88
J3-30-1	15.7	3.03	2.95	J12-4	24.8	3.20	3.11
J3-30-2	29.7	3.39	[2] 3.20	J12-8	5.25	2.77	2.75
J3-30-3	18.2	3.12	3.04	J12-13	2.70	2.71	2.65
J3-31b	36.2	3.58	3.46	J12-17b	14.0	2.94	2.85
J3-31c	33.7	3.48	3.38	J12-20b	10.9	2.89	2.79
J3-31d	37.5	3.61	3.46	J12-28b	21.5	3.14	3.05
J3-31f	38.9	3.67	3.53	J12-30	17.0	3.04	2.92
J3-32a	36.9	3.61	3.44	J12-32a	17.0	3.03	2.91
J3-32c	37.1	3.61	3.45	J12-40h	31.6	3.39	[1] 3.23
J3-32f	34.9	3.54	3.40	M-5	34.1	3.56	3.52
J3-32g	33.1	3.50	3.36	T-6	26.7	3.33	3.16
J3-32i	44.1	3.88	3.72	C1-RPS [3]	15.77	3.03	2.95
J3-32l	37.5	3.65	3.54	C2-RPS [3]	36.46	3.63	3.33
J3-33b	26.9	3.31	3.16	C5-RPS [3]	32.05	3.45	3.40
J3-33d	24.1	3.20	3.10				

[1] Determinations by V. C. Smith. The rocks were ground to minus 80 mesh and the specific gravity of the powder was measured with a pycnometer.

[2] Jolly balance determination.

[3] Chemical determinations.

CORRELATION OF IRON CONTENT WITH POWDER SPECIFIC GRAVITY

THEORETICAL RELATIONS

The relation of specific gravity to the composition of a rock which consists of hematite and a gangue mineral and which has no porosity is:

$$SG_r=\frac{W_r}{V_r}=\frac{W_r}{V_h+V_g}=\frac{W_r}{\frac{W_h}{SG_h}+\frac{W_g}{SG_g}},$$

where SG_r, SG_h, and SG_g are the specific gravities of the rock, hematite, and gangue, respectively; W_r, W_h, and W_g are the weights; and V_r, V_h, and V_g are the volumes of the rock. Then, where H and G are the proportions by weight of hematite and gangue, respectively, and Fe is the proportion of iron in the rock,

$$\frac{1}{SG_r}=\frac{\frac{W_h}{SG_h}+\frac{W_g}{SG_g}}{W_r}=\frac{W_h}{W_rSG_h}+\frac{W_g}{W_rSG_g}=\frac{H}{SG_h}+\frac{G}{SG_g}.$$

Substituting $G=1-H$ and $H=1.43$ Fe and collecting terms,

$$\frac{1}{SG_r}=\frac{1}{SG_g}+1.43\left(\frac{SG_g}{SG_h}-\frac{SG_h}{SG_g}\right)\text{Fe}. \tag{1}$$

Thus, $1/SG_r$ is a linear function of the iron content of a rock as long as the specific gravities of the gangue and hematite are constant.

EXPERIMENTAL RESULTS

Equation 1 can be used to analyze the experimental data. The powder specific gravity and iron content were determined on 33 samples ranging in iron content from 2.70 to 44.11 percent (table 3). The reciprocal of the powder specific gravity was plotted against the iron content (fig. 2). The points fell on a straight line with little scatter about the line.

A statistical analysis of the data gives equation 2 (reported in the following section), which is the equation of the line shown on figure 2. Substitution of the coefficients in equation 1 gives a specific gravity of 5.241 for hematite and of 2.678 for the gangue. The specific gravity of crystalline hematite ranges from 5.20 to 5.25 (Ford, 1945, p. 483) and the specific gravities of quartz and calcite, the main gangue minerals in these rocks, are 2.653–2.660 and 2.710, respectively (Ford, 1945, p. 470, 513). Thus the data fit the theoretical equation fairly well.

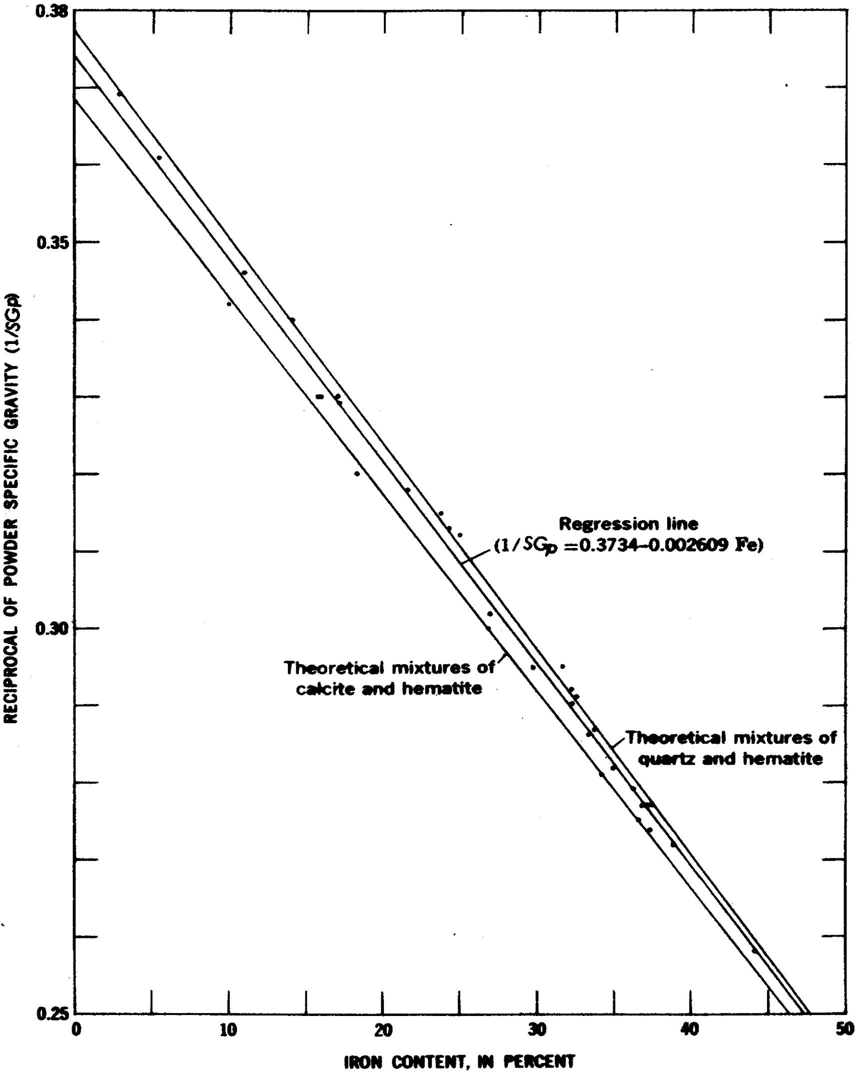

FIGURE 2.—Correlation between the reciprocal of the powder specific gravity and the iron content.

Feld and others (1946) calculated from their data specific gravities of 5.39 for hematite and 2.63 for gangue. Substituting these data in equation 1 gives

$$\frac{1}{SG_r}=0.3802-0.2784\ \mathrm{Fe}.$$

There is some scatter about the regression line. This scatter may be due to random errors in either the specific gravity or iron determinations, or it may be due to small differences in mineralogy. Some iron is known to be present in minerals other than hematite, and the ratio of calcite to quartz is known to be variable. Thus, in applying the theoretical equation, the assumption that the mineralogy of the iron mineral and the gangue is constant is not entirely correct.

STATISTICAL ANALYSIS

How well the data fit the theoretical equation is best determined by statistical analysis. A regression analysis (Hald, 1952, chap. 18) of the data gives the regression equation:

$$\frac{1}{SG_p}=0.3734-0.2609\ \mathrm{Fe}, \qquad (2)$$

where SG_p is the powder density and Fe is the proportion of iron in the rock. The variance of $1/SG_p$ is 6.48×10^{-5}, the variance of the coefficient 0.3734 is 1.96×10^{-7}, and the variance of the coefficient -0.2609 is 1.75×10^{-9}. These data are plotted on figure 2, where iron is given in percent and not in proportion so that its coefficient is -0.002609. The coefficients 0.3734 and -0.2609 when compared with the coefficients of Feld are significantly different at a 0.01 level, as shown by a *t*-test. Thus these data for powder specific gravity and iron content differ from the results obtained by Feld and others owing possibly to either bias in method of determination of specific gravity and iron, or possibly to differences in the gangue. However, because the data of Feld and others are not published, a rigorous comparison could not be made to assess the variability of their coefficients.

EFFECT OF IRON MINERALS ON SPECIFIC GRAVITY

Many thin sections and polished sections of the rocks show small amounts of chamosite associated with the hematite. The iron ores also invariably contain a few percent aluminum, most of which is probably present in chamosite or altered chamosite. These small amounts of chamosite, however, were not detected by X-ray methods.

The presence of small amounts of chamosite in the rocks would cause a negligible amount of scatter about the regression line. A sample of Silurian oolitic chamosite from the Westmoreland Formation in Oneida County, N.Y., contains about 50 percent chamosite and no hematite, as determined by modal analysis.[1] Yet the reciprocal of the powder specific gravity of this rock is 0.330, and the rock contains 15.8 percent iron; thus the rock falls almost exactly on the regression line of $1/SG_p$ on iron (fig. 2). Furthermore, the specific gravity and iron content for chamosite reported by Hallimond (1925) range from 3 to 3.5 ($1/SG$=0.333 to 0.286) and from 26.26 to 30.75 percent, respectively, which fall close enough to the regression line of $1/SG_p$ or $1/SG_b$ on iron.

The main effect of small amounts of chamosite in the rocks is to decrease slightly the accuracy of the specific-gravity estimates for hematite made from these data.

EFFECT OF GANGUE MINERALS ON SPECIFIC GRAVITY

The theoretical linear relation between iron content and specific gravity is based on the assumption that the specific gravity of the gangue is constant. Because the gangue is composed mainly of varying mixtures of quartz and calcite, this assumption is not strictly correct. The variation of the mixture of quartz and calcite causes some scatter in the regression of $1/SG_p$ on iron. When this variation is considered, the equation becomes:

$$\frac{1}{SG_p}=\frac{H}{SG_h}+\frac{C}{SG_c}+\frac{Q}{SG_q}, \tag{3}$$

where C and Q are the proportions of calcite and quartz in the rock and SG_c and SG_q are the specific gravities of calcite and quartz, respectively. Because the specific gravities of quartz (2.65) and calcite (2.71) are about the same, the effect would be small but measurable.

Lines showing the theoretical relation between $1/SG_p$ and hematite in gangues of either quartz or calcite are plotted on figure 2. The regression line and most of the points fall between these two lines.

The gangue of the 33 samples used in constructing the $1/SG_p$-iron regression line was determined by means of X-ray analysis. The ratio of amplitudes of the 3.339–A diffraction peak for quartz to the 3.028–A diffraction peak for calcite is an approximate function of the proportion of quartz to calcite, and this ratio was determined for each sample. The mean of the peak ratios of points above the regression line is 2:416, and below the line it is 1:204; this fact indicates that the rocks having a quartz-rich gangue are somewhat lower in specific gravity than rocks having a calcite-rich gangue, if the hematite content is equal. The variation of specific gravity due to differences in gangue mineralogy

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[1] Hunter, R. H., 1960, Iron sedimentation in the Clinton group of the Central Appalachian Basin: Baltimore, Md., Johns Hopkins Univ., Ph. D. dissertation.

is small in comparison to the variation due to differences in hematite content.

Some of the samples that were used in studying the relation between bulk specific gravity and iron content, reported in following section, were chemically analyzed for CaO, MgO, and SiO_2. The approximate quartz and calcite contents of these samples were calculated from their chemical analyses. The scatter due to differences in gangue mineralogy was almost completely masked by the scatter due to porosity. The mean calcite content of rocks above the $1/SG$-iron regression line is 16.92 percent and below the line 24.09 percent. This evidence indicates that scatter due to differences in gangue minerals is apparent but negligible.

CORRELATION OF IRON CONTENT WITH BULK SPECIFIC GRAVITY

Bulk specific gravity is generally a more desirable measurement than powder specific gravity because of the speed of determination and the lack of damage to the specimen. However, for estimating iron content, it is not as desirable because porosity, which affects the results, must be considered. The relation between bulk specific gravity and iron content was studied to determine whether usable results could be obtained, and some degree of success was achieved.

The equation relating the bulk specific gravity and iron content for 91 samples is

$$1/SG_b = 0.3821 - 0.2681 \text{ Fe}.$$

where SG_b is the bulk specific gravity and Fe is the proportion of iron in the rock. The points are plotted on figure 3. The relation is linear, but the points show more scatter than in the powder specific gravity-iron content relation. The line shown on figure 2, which is also plotted on figure 3 for comparison, is the underlying function of the bulk specific gravity-iron content relation and falls below but nearly parallel to the bulk specific gravity-iron line. This position indicates that the higher reciprocal of bulk specific gravity (due to the lower bulk specific gravity) is caused by porosity. As with the reciprocal of the powder specific gravity and iron content, the scatter is partly due to differences in gangue mineralogy and to the presence of chamosite as a minor iron-bearing mineral.

STATISTICAL ANALYSIS

The bulk specific gravity and iron content were found for 91 samples of iron-bearing rocks from the Red Mountain Formation in Alabama. The data for 33 of these samples were determined during this study (table 3); and the data for the remaining 58 samples were collected from the literature and industrial sources.

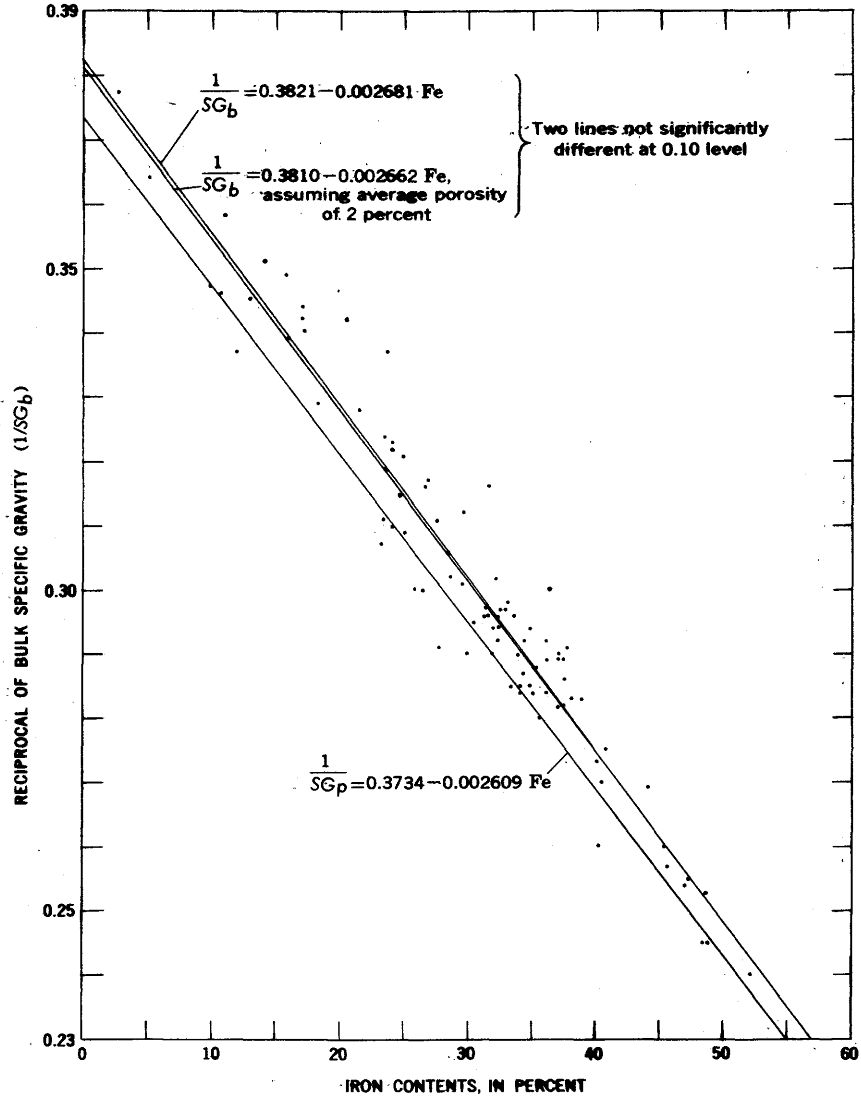

FIGURE 3.—Correlation between the reciprocal of the bulk specific gravity and the iron content. The line of equation $1/SG_p = 0.3734 - 0.002609$ Fe from figure 2.

A regression analysis of these data yielded the following equation:

$$\frac{1}{SG_b}=0.3821-0.2681\ \mathrm{Fe},$$

where SG_b is the bulk specific gravity and Fe is the proportion of iron in the rock. The variances are as follows: For $1/SG_b$, 4.44×10^{-5}; for 0.3821, 4.88×10^{-7}; for -0.2681, 4.78×10^{-5}. These data are plotted on figure 3; as iron is given in percent and not in proportion, its coefficient is -0.002681. The regression of $1/SG_b$ upon iron is clearly poorer than that of $1/SG_p$ upon iron, as shown by the variances of the dependent variables; the variance of $1/SG_b$ is 6.48×10^{-6}, and the variance of SG_b is 4.44×10^{-5}; standard deviations are 0.00255 and 0.00666, respectively.

POROSITY

Where the rock is made up wholly of hematite, calcite, and quartz, the relation between the porosity of a rock and its bulk specific gravity is given by the equation

$$\frac{1}{SG_b}=\frac{H}{SG_h}+\frac{C}{SG_c}+\frac{Q}{SG_q}+\frac{p}{SG_b}, \tag{4}$$

in which p is the proportion porosity (percent porosity divided by 100). Combining equation 4 with equation 3,

$$\frac{1}{SG_p}=\frac{(1-p)}{SG_b}. \tag{5}$$

The porosities of the 33 samples that were analyzed for both SG_p and SG_b (table 3) were calculated by means of equation 5. The mean porosity of these rocks is 3.49 percent; it ranges from 0.7 to 8.2 percent. The porosity of the rest of the samples used in the $1/SG_b$ to iron regression could not be calculated from equation 5, but the porosity of 51 of these samples, for which chemical analyses were available, was determined by means of equation 4. The mean porosity of these 51 samples is 1.84 percent and the standard deviation is about 0.75 percent. This technique is not very satisfactory because equation 4 gave negative values of porosity for about 20 percent of the samples, an impossible result which was probably due to uncertainties in the specific gravity and proportion values assigned to the calcite and quartz and also to the assumption that gangue is made up solely of calcite and quartz. A plot of $1/SG_p$, which was calculated from equation 5 which indicates the iron content for these samples, shows that the points fall on a straight line with no more scatter than on figure 2, but the slope and intercepts of the line are different from those shown on figure 2. Thus the constants used in equation 3, rather than the technique, are probably in error.

As was demonstrated by equation 5, the reciprocal of the powder specific gravity is smaller than the reciprocal of the bulk specific gravity by the factor p/SG_b. If an average porosity of 2 percent is assumed for the rocks, the previously derived regression equation

$$\frac{1}{SG_p}=0.3734-0.002609\ \mathrm{Fe\ (percent)}$$

becomes

$$\frac{1}{SG_b}=0.3810-0.002662\ \mathrm{Fe\ (percent)}. \qquad (6)$$

The experimentally derived regression equation was previously shown to be:

$$\frac{1}{SG_b}=0.3821-0.002681\ \mathrm{Fe\ (percent)}. \qquad (7)$$

All these lines are plotted on figure 3.

The coefficients of equation 7 were compared to the theoretical coefficients of equation 6 by means of *t*-test and were found not to be significantly different at a 0.10 two-sided level of probability. The high probability level was chosen to minimize the chance of wrongly accepting the hypothesis that the coefficients are the same. This comparison satisfactorily shows that porosity is the main factor which causes the iron-content plots to scatter about the regression line of $1/SG_b$ and that the average porosity is about 2 percent.

QUALITY-CONTROLLED SELECTION OF IRON ORES BY THEIR BULK SPECIFIC GRAVITY

An approximate estimate of the amount of iron in a sample of iron ore can be made from the bulk specific gravity of the sample. The precision of such an estimate can be rigorously examined by means of a regression analysis in which the iron content is used as the dependent variable. Such an analysis, made with the data used in the $1/SG_b$-iron regression, gave the following equation,

$$\mathrm{Fe\ (percent)}=136.2479-352.1876\frac{1}{SG_b}.$$

The sample variance and standard deviation of the mean value of iron are 5.8312 and 2.415, respectively, with 89 degrees of freedom. The sample variances and standard deviations of the coefficients (fig. 4) are as follows:

Coefficient	*Variance* (s^2)	*Standard deviation* (s)
136.2479	0.06408	0.2532
352.1876	82.333	9.074

Confidence intervals for the scatter of points about the regression can be drawn. For values of iron close to the mean value of iron or near the center of the line, the standard deviation of iron is approximately $s=2.415$. Because this value is only an estimate of the true standard deviation and, as such, has a mean and a variance of its own, the variance of the standard deviation must be considered in drawing the confidence interval lines. If lines $2.25s$ above and below the regression line are drawn close to the mean iron value, 95 times out of a hundred 95 percent of the points will fall between the two lines (Hald, 1952, p. 311, 540). These lines are shown on figure 4.

Hald (1952, p. 540) gave the sample variance of iron for all values of $1/SG_b$ by

$$s^2_{Fe}=s^2\left[1+\frac{1}{n}+\frac{\left(\frac{1}{SG_b}-\frac{\overline{1}}{SG_b}\right)^2}{\sum_{i=1}^{n}\left(\frac{1}{SG_{bi}}-\frac{1}{SG_b}\right)^2}\right].$$

Again, because this value is a sample variance, it has a variance of its own, and a correction factor must be used when the sample variance is substituted for the true variance in computing confidence limits on the regression line. The size of the standard deviation depends on the

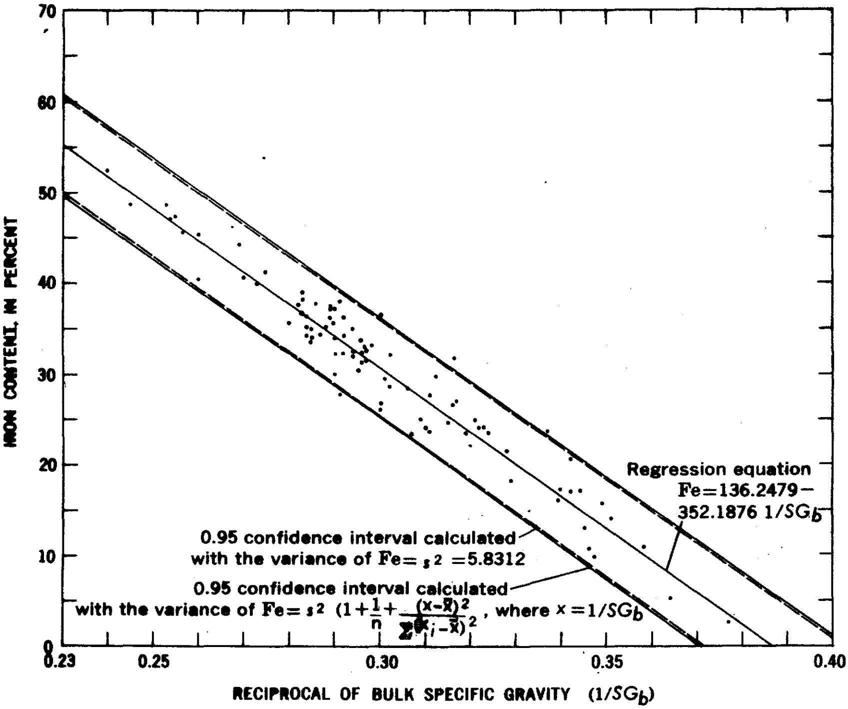

FIGURE 4.—Quality-control chart for selection of iron ore from the reciprocal of bulk specific gravity.

value of $1/SG$; that is, it varies with the value of the independent variable. The confidence intervals are indicated by lines $2.25s_{Fe}$ drawn above and below the regression line for each value of $1/SG_b$ (Hald, 1952, p. 315). These lines are curves which diverge from the regression line, away from the mean value of $1/SG_b$ (fig. 4). As is evident on figure 4, there is little difference between the two confidence intervals, and so the straight-line approximations to the more precise confidence intervals can be used.

This graph (fig. 4) can be used in the following manner: Let us assume that one wants to select for chemical analysis all samples of iron-bearing rock that have an iron content of at least 30 percent. This percentage corresponds to a high value of 0.318 for $1/SG_b$; that is, if a rock has a reciprocal bulk specific gravity of 0.318, the chances that its iron content is greater than 30 percent are 2.5 in 100. For rocks with a larger $1/SG_b$, the chance that their iron contents are larger than 30 percent becomes smaller and smaller. Thus, if all rocks having a $1/SG_b$ less than or equal to 0.318 are selected for chemical analysis, the chance that a rock with an iron content of 30 percent or greater will be overlooked is no greater than 2.5 in 100. The graph can be used in a similar way for any minimum iron value. Other confidence levels or coefficients desired between 0.90 and 0.999 can be obtained from the report by Hald (1952, table 11.8, p. 315).

As experience is acquired from analyzing many rocks, estimation of the standard deviation will become more accurate and fewer standard deviations will be used in drawing the confidence intervals. For example, the interval would have been drawn plus and minus $2.04s$ away from the regression line if 1,000 samples had been used. Furthermore, it is very likely that visual estimates or simple tests of the porosity can be made in order to narrow down the variation of the iron estimate, thereby narrowing the confidence interval.

By use of statistical quality-control techniques (Grant, 1952), rock samples or core chips may be selected for chemical analysis without making too large an error of over or under sampling. Just how large the margin of error may be is left to the discretion of the exploration geologist.

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