Variable Azimuth Schlumberger Resistivity Sounding and Profiling Near a Vertical Contact

GEOLOGICAL SURVEY BULLETIN 1313-A

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By ADEL A. R. ZOHDY

NEW TECHNIQUES IN DIRECT-CURRENT RESISTIVITY EXPLORATION

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Idealized Schlumberger sounding curves, circular diagrams, and horizontal profiles are computed to assist in interpretation of electrical resistivity data obtained near a vertical plane boundary

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1970

UNITED STATES DEPARTMENT OF THE INTERIOR

WALTER L. HICKEL, *Secretary*

GEOLOGICAL SURVEY

William T. Pecora, *Director*

Library of Congress catalog-card No. 79-607759

For sale by the Superintendent of Documents, U.S. Government Printing Office
Washington, D.C. 20402—Price 65 cents (paper cover)

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NEW TECHNIQUES IN DIRECT-CURRENT RESISTIVITY EXPLORATION

VARIABLE AZIMUTH SCHLUMBERGER RESISTIVITY SOUNDING AND PROFILING NEAR A VERTICAL CONTACT

By Adel A. R. Zohdy

ABSTRACT

Generalized formulas are derived for computing apparent resistivity curves of electrical soundings of the Schlumberger type near a vertical contact separating two homogeneous and isotropic media. The azimuth angle, which is formed by the sounding line and the surface trace of a vertical contact, is considered a variable, and seven sets of theoretical sounding curves for angles 0°, 5°, 15°, 30°, 45°, 60°, and 90° are plotted for 24 values of the resistivity ratio $0 \leq \frac{\rho_2}{\rho_1} \leq \infty$.

The theoretical sounding curves indicate that when the center of the array is placed over a resistive medium in vertical contact with a conductive medium $\left(\frac{\rho_2}{\rho_1}<1\right)$, the forms of the sounding curves vary substantially as a function of the azimuth angle γ, especially for values of $\gamma<30°$, and that the smaller the value of ρ_2/ρ_1 the greater is the variation in the form of the sounding curve. Furthermore, for any given value of γ and for all values of $\frac{\rho_2}{\rho_1} \geq 20$, the theoretical sounding curves differ by less than 4 percent.

Two examples of circular sounding diagrams for $\frac{\rho_2}{\rho_1}=20$ and for $\frac{\rho_2}{\rho_1}=0.05$ illustrate the phenomenon of lateral pseudoanisotropy. Horizontal resistivity profiles made along traverses that are arbitrarily oriented with respect to the strike of a vertical contact are given for illustration of the dependency of apparent resistivity profiles on the azimuth angle and for comparison with apparent resistivity profiles that are based on sounding data.

INTRODUCTION

Electrical sounding with differently oriented lines, at a given station, may result in curves that are different from one another because of the presence of lateral heterogeneities and (or) anisotropy. The effect of a vertical contact that separates two homogeneous and isotropic media on idealized quadrupole (*AMNB*) Schlumberger sounding or horizontal profiling curves has not been studied adequately, as evidenced by the publication of only two sets of theoretical sounding curves. These two sets correspond to the parallel and perpendicular orientation of the sounding line with respect to the surface trace of the contact (Al'pin and others, 1966; Dakhnov, 1953; Golovtsin, 1963; Kalenov, 1957).

Practical experience indicates that the effect of lateral heterogeneities on sounding curves cannot always be avoided and that to a first approximation the effect is similar to that of a vertical contact. Therefore, an album of vertical electrical sounding (*VES*) curves near a vertical contact has a practical significance, especially if it is used in conjunction with methods analogous to those developed by Fomina (1958) and by Rabinovich (1962) for reducing certain *VES* curves obtained over media with horizontal and vertical boundaries to *VES* curves obtained over media with horizontal boundaries only. The theoretical sounding data also make it possible to construct circular sounding diagrams, horizontal resistivity profiles, and apparent resistivity profiles which are based on a

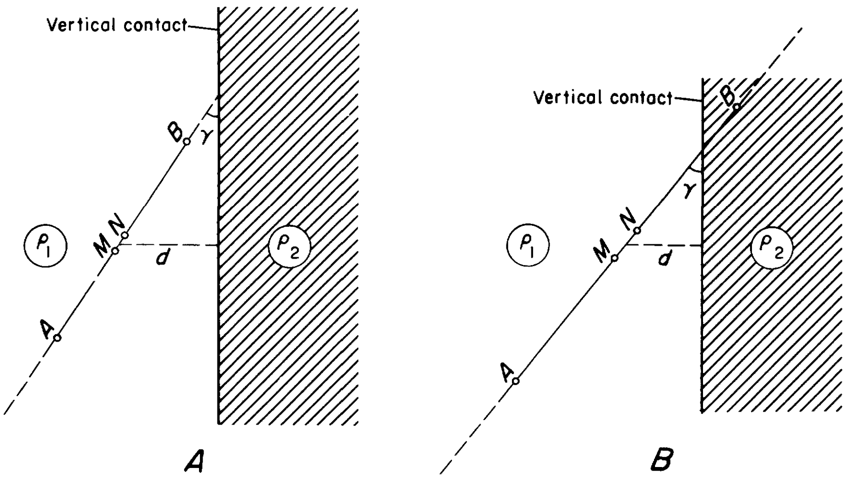

FIGURE 1.—Placement of electrodes (*A*, *M*, *N*, and *B*) in relation to a vertical contact separating two media (ρ_1 and ρ_2) of different resistivities; *d*, perpendicular distance from the center of the array to the vertical contact. *A*, All electrodes on same medium of resistivity (ρ_1); *B*, three electrodes on one medium of resistivity (ρ_1), and one electrode on second medium of resistivity (ρ_2).

set of soundings obtained near a vertical contact. Circular sounding diagrams (or polar plots of apparent resistivity) are particularly useful in studying karst structures (Ogil'vi, 1956; Arandjelovič, 1966), as well as in delineating major joint, fracture, or fault systems (Vedrintsev, 1961).

MATHEMATICAL FORMULATION

Consider a semi-infinite space that contains a vertical boundary which separates two homogeneous and isotropic media of electrical resistivities

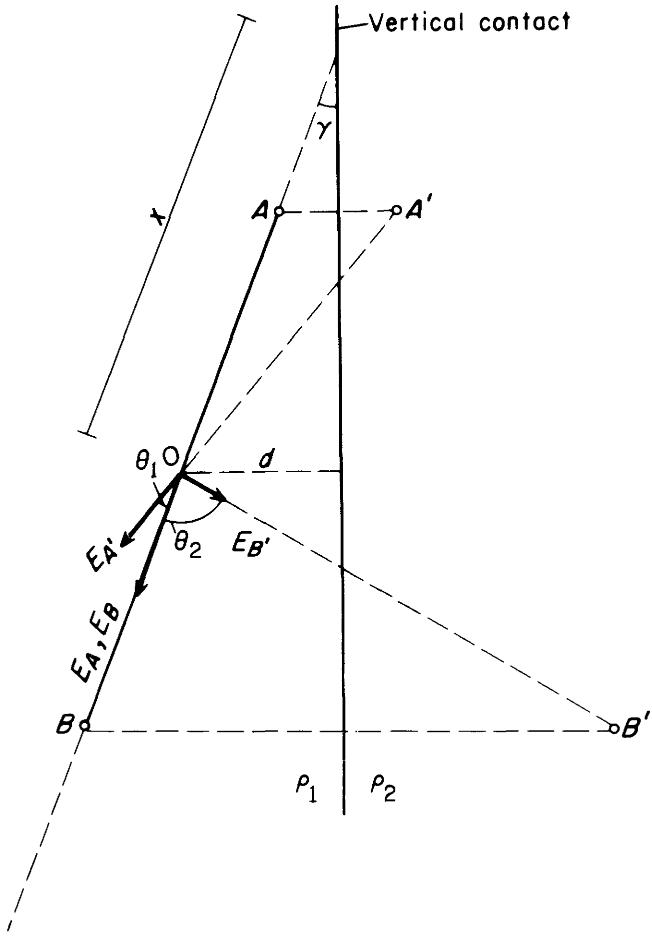

FIGURE 2.—Electric field (components E_A, E_B, $E_{A'}$, and $E_{B'}$) caused by the source at electrode A, its image at A', the sink at electrode B, and its image at B'. O, center of array; x, distance from center of array along sounding line to the vertical contact; ρ_1 and ρ_2, resistivities of medium 1 and 2; d, perpendicular distance from the center of the array to the vertical contact.

ρ_1 and ρ_2. If a set of four electrodes, A, M, N, and B are placed along a straight line with their center, O, over the medium with resistivity ρ_1 so that the sounding line forms an arbitrary angle, γ, with the surface trace of the vertical contact, then two general equations must be derived for computing the Schlumberger apparent resistivity. The first equation is applicable when all four electrodes are over the medium with resistivity ρ_1 (fig. 1A), whereas the second equation is applicable when one of the current electrodes (A or B) is placed across the boundary over the medium with resistivity ρ_2 (fig. 1B).

For an ideal Schlumberger array the spacing between the potential electrodes, M and N, is infinitesimal ($\overline{MN}=0$); therefore, the possibility that one of the potential electrodes (M or N) may be placed across the boundary during the sounding process will not be considered here.

1. *First formula*: When all four electrodes are on the medium of resistivity ρ_1, the magnitude of electric field E at the center of the array may be expressed by

$$E=\vec{E}_A\cdot\hat{\imath}+\vec{E}_B\cdot\hat{\imath}+\vec{E}_{A'}\cdot\hat{\imath}+\vec{E}_{B'}\cdot\hat{\imath} \tag{1}$$

where $\vec{E}_A$, $\vec{E}_B$, $\vec{E}_{A'}$ and $\vec{E}_{B'}$ are the electric fields due to the point source at A, the point sink at B, the source image at A', the sink image at B', respectively, and where $\hat{\imath}$ is a unit vector at the center, O, in the direction from A to B.

Considering figure 2, and making use of the image theory (Keller and Frischknecht, 1966), the following relations may be derived:

$$\vec{E}_A\cdot\hat{\imath}=\frac{\rho_1 I}{2\pi}\times\frac{1}{(AB/2)^2}, \tag{2}$$

$$\vec{E}_B\cdot\hat{\imath}=\frac{\rho_1 I}{2\pi}\times\frac{1}{(AB/2)^2}, \tag{3}$$

$$\vec{E}_{A'}\cdot\hat{\imath}=E_{A'}\cos\theta_1=\frac{\rho_1 I}{2\pi}\times\frac{k}{d^2}\times\frac{\dfrac{AB}{2d}+2(\sin\gamma)\left(1-\dfrac{AB}{2d}\sin\gamma\right)}{\left[\left(\dfrac{AB}{2d}\cos\gamma\right)^2+\left(2-\dfrac{AB}{2d}\sin\gamma\right)^2\right]^{3/2}} \tag{4}$$

$$\vec{E}_{B'}\cdot\hat{\imath}=E_{B'}\cos\theta_2=\frac{\rho_1 I}{2\pi}\times\frac{k}{d^2}\times\frac{\dfrac{AB}{2d}-2(\sin\gamma)\left(1+\dfrac{AB}{2d}\sin\gamma\right)}{\left[\left(\dfrac{AB}{2d}\cos\gamma\right)^2+\left(2+\dfrac{AB}{2d}\sin\gamma\right)^2\right]^{3/2}} \tag{5}$$

and therefore

$$E=\frac{\rho_1 I}{2\pi}\left\{\frac{2}{(AB/2)^2}+\frac{k}{d^2}\left[\frac{\frac{AB}{2d}+2(\sin\gamma)\left(1-\frac{AB}{2d}\sin\gamma\right)}{\left[\left(\frac{AB}{2d}\cos\gamma\right)^2+\left(2-\frac{AB}{2d}\sin\gamma\right)^2\right]^{3/2}}\right.\right.$$

$$\left.\left.+\frac{\frac{AB}{2d}-2(\sin\gamma)\left(1+\frac{AB}{2d}\sin\gamma\right)}{\left[\left(\frac{AB}{2d}\cos\gamma\right)^2+\left(2+\frac{AB}{2d}\sin\gamma\right)^2\right]^{3/2}}\right]\right\}, \tag{6}$$

where

I = electric current intensity;

$\frac{AB}{2}$ = half the distance between current electrodes;

d = perpendicular distance from the center of the array to the surface trace of the vertical contact;

$k=\frac{\rho_2-\rho_1}{\rho_2+\rho_1}=\frac{\mu-1}{\mu+1}$ = reflection factor, and μ is the resistivity ratio $\frac{\rho_2}{\rho_1}$;

γ = angle formed by the sounding line and the surface trace of the vertical contact.

θ_1 and θ_2 = angles formed by sounding line and electric fields $E_{A'}$ and $E_{B'}$, respectively.

The apparent resistivity, $\bar{\rho}_s$, for the ideal Schlumberger array is given by

$$\bar{\rho}_s=\pi\left(\frac{AB}{2}\right)^2\times\frac{E}{I}. \tag{7}$$

Substituting equation 6 in 7 and rearranging we get

$$\frac{\bar{\rho}_s}{\rho_1}=1+\frac{k}{2}\left(\frac{AB}{2d}\right)^2\left\{\frac{\frac{AB}{2d}+2(\sin\gamma)\left(1-\frac{AB}{2d}\sin\gamma\right)}{\left[\left(\frac{AB}{2d}\cos\gamma\right)^2+\left(2-\frac{AB}{2d}\sin\gamma\right)^2\right]^{3/2}}\right.$$

$$\left.+\frac{\frac{AB}{2d}-2(\sin\gamma)\left(1+\frac{AB}{2d}\sin\gamma\right)}{\left[\left(\frac{AB}{2d}\cos\gamma\right)^2+\left(2+\frac{AB}{2d}\sin\gamma\right)^2\right]^{3/2}}\right\}. \quad (8)$$

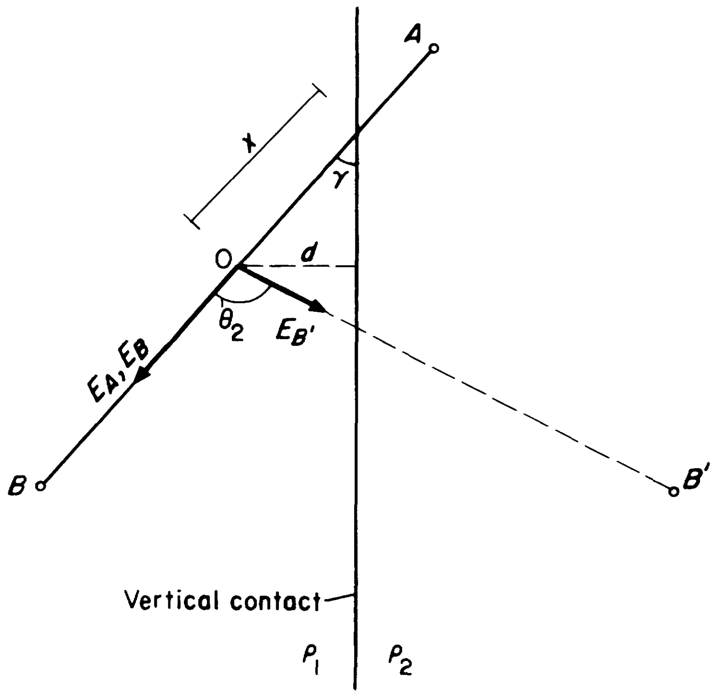

FIGURE 3.—Electric field (components E_A, E_B, and $E_{B'}$) caused by the point electrodes A and B, and the image at B'. O, center of array; x, distance from center of array along sounding line to the vertical contact; ρ_1 and ρ_2, resistivities of medium 1 and 2; d, perpendicular distance from the center of the array to the vertical contact.

The computation of the reduced resistivity $\frac{\bar{\rho}_s}{\rho_1}$ by equation 8 is only valid when all electrodes are on the medium of resistivity ρ_1, that is $\frac{AB}{2d} \leq \operatorname{cosec} \gamma$ or $\frac{AB}{2x} \leq 1$, where x is the distance measured along the sounding line from the center, O, to the surface trace of the contact (fig. 2). It is only at $\gamma=0$ that equation 8 is applicable for computing the complete form of the sounding curve; because with the array oriented parallel to the contact and with the center of the array placed on the medium with resistivity ρ_1, no electrode can be placed on the medium with resistivity ρ_2. Therefore at $\gamma=0$ the condition $\frac{AB}{2d} \leq \operatorname{cosec} \gamma$ or $\frac{AB}{2x} \leq 1$ is always fulfilled.

2. *Second formula*: When the current electrode A is placed over the medium with resistivity ρ_2 (fig. 3), the strength of the image at A' must be identically zero, because the potential in the medium where a measurement is made must be finite everywhere except at a real source or sink. Therefore, the magnitude of the electric field at the center of the array is given by

$$E=\vec{E}_A \cdot \hat{\imath}+\vec{E}_B \cdot \hat{\imath}+\vec{E}_{B'} \cdot \hat{\imath}, \tag{9}$$

where

$$\vec{E}_A \cdot \hat{\imath}=\frac{\rho_1 I}{2\pi} \times \frac{2\rho_2}{\rho_2+\rho_1} \times \frac{1}{(AB/2)^2}, \tag{10}$$

$$\vec{E}_B \cdot \hat{\imath}=\frac{\rho_1 I}{2\pi} \times \frac{1}{(AB/2)^2}, \tag{11}$$

$$\vec{E}_{B'} \cdot \hat{\imath}=E_{B'} \cos \theta_2=\frac{\rho_1 I}{2\pi} \times \frac{k}{d^2} \left\{ \frac{\frac{AB}{2d}-2(\sin \gamma)\left(1+\frac{AB}{2d} \sin \gamma\right)}{\left[\left(\frac{AB}{2d} \cos \gamma\right)^2+\left(2+\frac{AB}{2d} \sin \gamma\right)^2\right]^{3/2}} \right\}; \tag{12}$$

and therefore

$$E=\frac{\rho_1 I}{2\pi}\left\{\frac{1}{(AB/2)^2}\times\left(1+\frac{2\mu}{\mu+1}\right)\right.$$

$$\left.+\frac{k}{d^2}\left[\frac{\frac{AB}{2d}-2(\sin\gamma)\left(1+\frac{AB}{2d}\sin\gamma\right)}{\left[\left(\frac{AB}{2d}\cos\gamma\right)^2+\left(2+\frac{AB}{2d}\sin\gamma\right)^2\right]^{3/2}}\right]\right\}. \quad (13)$$

Substituting equation 13 in equation 7 and rearranging we get

$$\frac{\bar{\rho}_s}{\rho_1}=\frac{\mu}{\mu+1}+\frac{1}{2}\left\{1+\left(\frac{AB}{2d}\right)^2\left(\frac{\mu-1}{\mu+1}\right)\frac{\frac{AB}{2d}-2(\sin\gamma)\left(1+\frac{AB}{2d}\sin\gamma\right)}{\left[\left(\frac{AB}{2d}\cos\gamma\right)^2+\left(2+\frac{AB}{2d}\sin\gamma\right)^2\right]^{3/2}}\right\}. \quad (14)$$

Equation 14 applies for all values of $\frac{AB}{2d}\geq\operatorname{cosec}\gamma$ or $\frac{AB}{2x}\geq 1$.

ASYMPTOTIC FORMULAS

Unlike theoretical sounding curves over horizontally stratified media, in the presence of a vertical contact the value of the reduced resistivity $\frac{\bar{\rho}_s}{\rho_1}$ at large values of $\frac{AB}{2d}$ does not asymptotically approach the value of the resistivity ratio $\mu=\frac{\rho_2}{\rho_1}$. This phenomenon makes interpolation between theoretical sounding curves difficult. Interpolation can, however, be considerably facilitated if the value of the asymptote of $\frac{\bar{\rho}_s}{\rho_1}$ is known.

If the azimuth angle γ is zero (array parallel to contact), then the asymptotic value of $\frac{\bar{\rho}_s}{\rho_1}$ may be derived from equation 8 and is given by

$$\frac{\bar{\rho}_s}{\rho_1}=1+\left(\frac{\mu-1}{\mu+1}\right)\frac{\left(\frac{AB}{2d}\right)^3}{\left[4+\left(\frac{AB}{2d}\right)^2\right]^{3/2}}; \tag{15}$$

therefore the limit as $\frac{AB}{2d}\rightarrow\infty$ of equation 15 is

$$\begin{aligned}\frac{\bar{\rho}_{s\,\mathrm{asymptote}}}{\rho_1}&=1+\frac{\mu-1}{\mu+1}\\&=1+k\\&=\frac{2\rho_2}{\rho_2+\rho_1},\quad (\text{for } \gamma=0).\end{aligned} \tag{16}$$

If $\gamma\neq0$, then at some large value of $AB/2d$ a current electrode will be placed across the contact and the asymptotic value of $\frac{\bar{\rho}_s}{\rho_1}$ must be derived from equation 14, which is valid for all values of $\frac{AB}{2d}\geq\operatorname{cosec}\gamma$. Therefore, rearranging equation 14 and taking the limit as $\frac{AB}{2d}\rightarrow\infty$, we get

$$\frac{\bar{\rho}_{s\,\mathrm{asymptote}}}{\rho_1}=\frac{\mu}{\mu+1}+\frac{1}{2}\left[1+\left(\frac{\mu-1}{\mu+1}\right)(1-2\sin^2\gamma)\right],\quad (\text{for } \gamma\neq0). \tag{17}$$

The variation of the asymptotic value of $\frac{\bar{\rho}_s}{\rho_1}$ as a function of μ for various values of γ is shown in figure 4, and its variation as a function of γ for various values of μ is shown in figure 5.

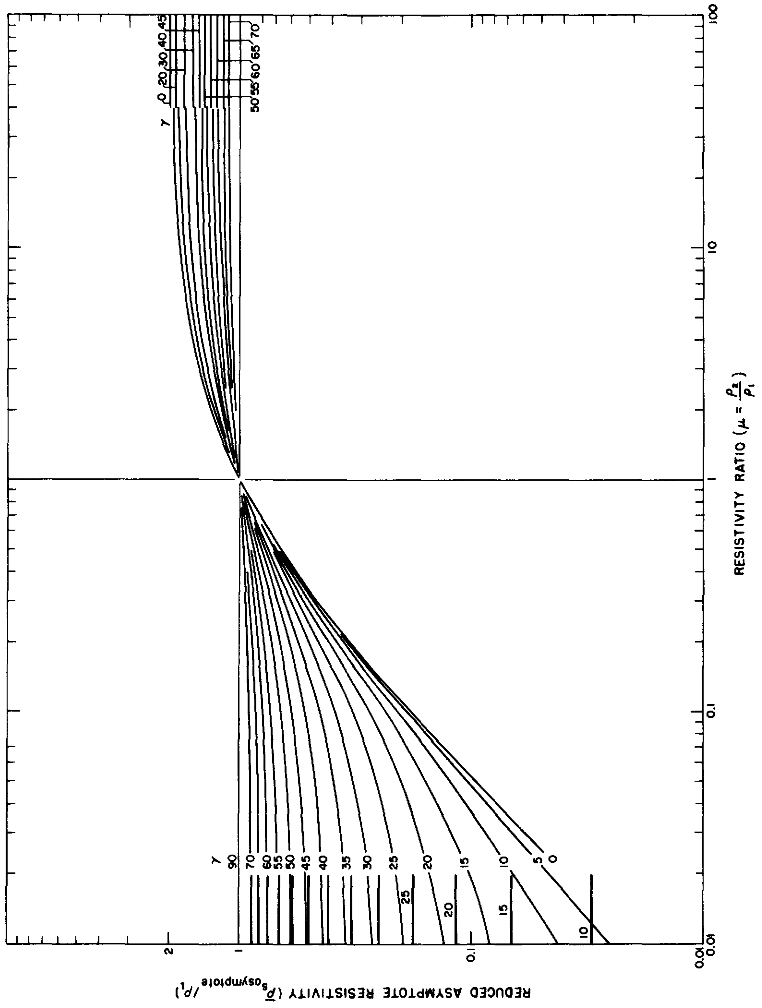

FIGURE 4.—Variation of the reduced asymptote resistivity as a function of the resistivity ratio for various values of the angle formed by the sounding line and the surface trace of the vertical contact (γ).

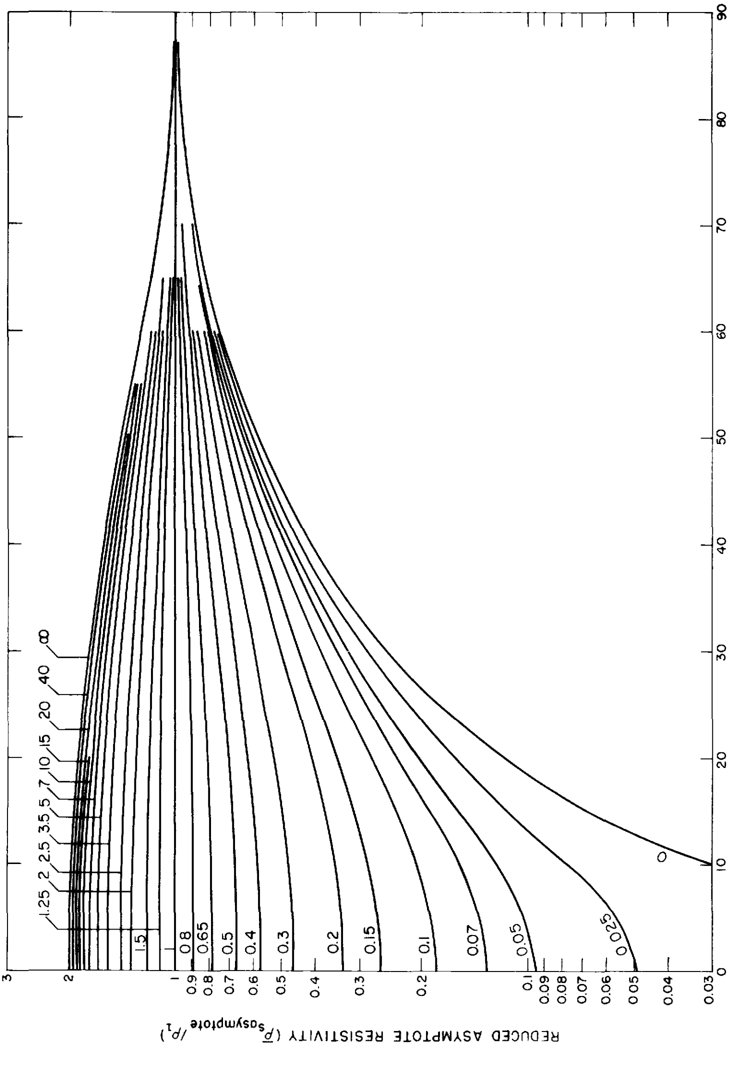

FIGURE 5.—Variation of the reduced asymptote resistivity as a function of the angle formed by the sounding line and the surface trace of the vertical contact (γ) for various values of the resistivity ratio $\left(\mu \text{ or } \frac{\rho_2}{\rho_1}\right)$

The value of the left asymptote of $\frac{\bar{\rho}_s}{\rho_1}$ obtained as $\frac{AB}{2d} \to 0$ is equal to unity and is independent of the values of γ or μ, as it can be proved from equation 8.

THEORETICAL SOUNDING CURVES

A program based on formulas 8, 14, 16, and 17 was written for the IBM 360/65 computer, and curves were computed for 19 values of γ, 24 values of μ, and 30 values of $AB/2d$. These values of γ, μ, and $AB/2d$ are

$\gamma = 0°$, 5°, 10°, 15°, 20°, 25°, 30°, 35°, 40°, 50°, 55°, 60°, 65°, 70°, 75°, 80°, 85°, and 90°

$\mu = 0$, 0.025, 0.05, 0.07, 0.1, 0.15, 0.2, 0.3, 0.4, 0.5, 0.65, 0.8, 1.25, 1.5, 2, 2.5, 3.5, 5, 7, 10, 15, 20, 40, and 1,000 ($\approx \infty$).

$\frac{AB}{2d} = 0.3$, 0.4, 0.5, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 14, 16, 20, 25, 30, 40, 50, 60, 80, 100, ∞, and cosec γ.

The value of the resistivity at $\frac{AB}{2d} = \text{cosec } \gamma$ is obtained when a current electrode is placed exactly at the surface trace of the vertical contact. Only seven sets of curves for $\gamma = 0°$, 5°, 15°, 30°, 45°, 60°, and 90° (pl. 1), and one set of curves for $\mu = 0$ and ∞ and $0 \leq \gamma \leq 90$ (fig. 6) are given here.

If the need arises for plotting a specific curve for a value of μ (or γ), which is not included in the given sets, then one can use logarithmic interpolation in conjunction with the diagrams in figures 4 or 5. For example, if one wishes to construct the curve for $\gamma = 15°$ and $\mu = 0.035$, then one determines the value of $\frac{\bar{\rho}_{s\,\text{asymptote}}}{\rho_1} = 0.13$ from the diagram in figure 4 and uses the master curves for $\mu = 0.025$ and 0.05 (at $\gamma = 15°$) to construct the required curve as shown in figure 7.

PROPERTIES OF THEORETICAL VES CURVES NEAR A VERTICAL CONTACT

1. The formation of a cusp on a *VES* curve is a function of the values of μ and γ. The greater the departure of the value of μ from unity ($0.5 > \mu > 5$) and the larger the value of γ ($\gamma > 20°$) the better is the development of the cusp.
2. The abscissa of the apex of a cusp determines the distance of the surface trace of the vertical contact from the center of the electrode array.

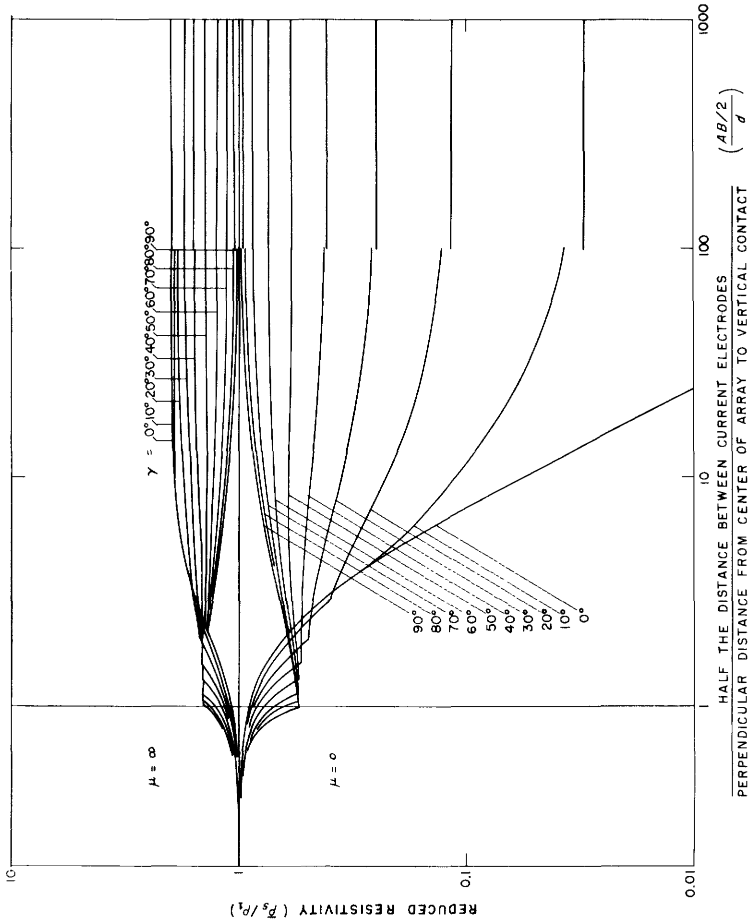

FIGURE 6.—Theoretical *VES* curves near a vertical contact (resistivity ratio of 0 or ∞) for various values of γ.

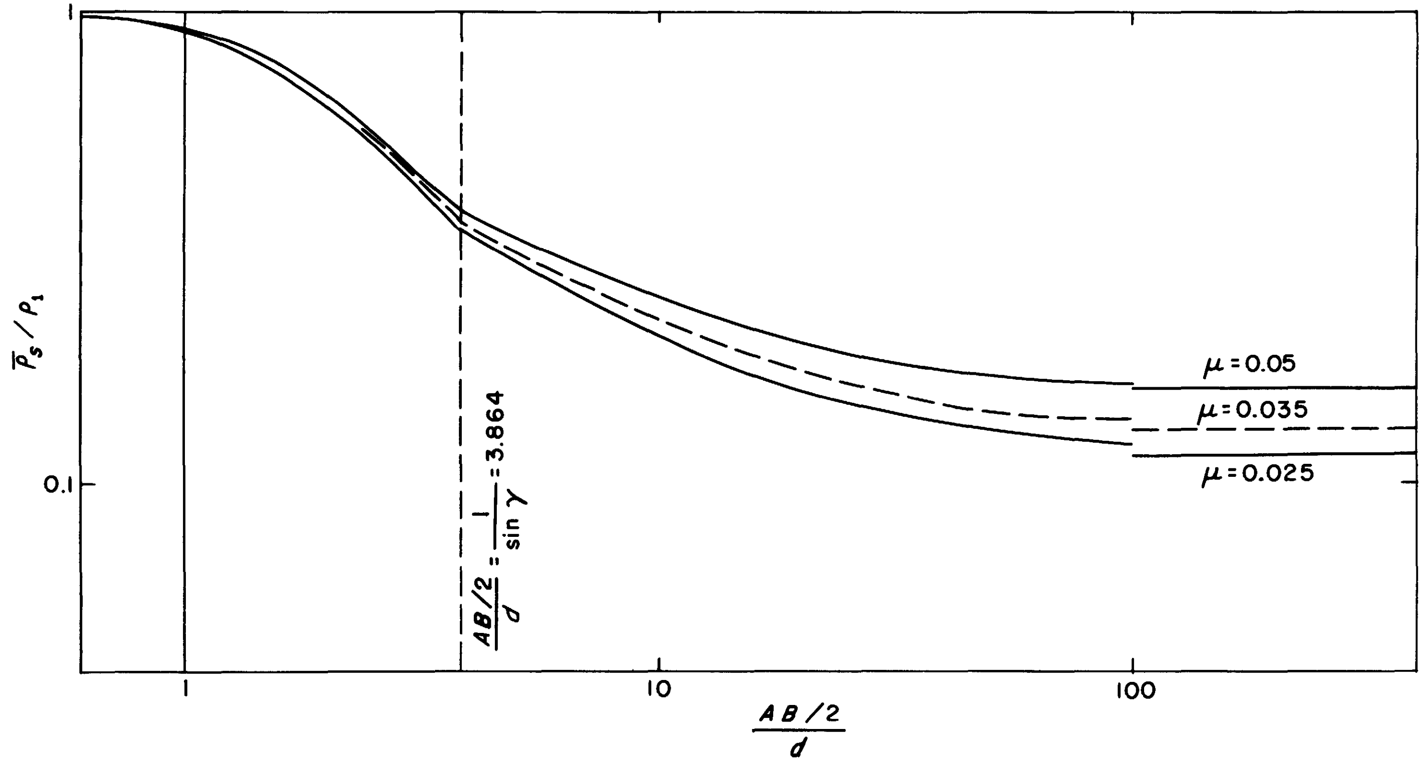

FIGURE 7.—Graphical construction (dashed line) of a sounding curve for a resistivity ratio of 0.035 at $\gamma = 15°$. Curves for $\mu = 0.05$ and $\mu = 0.025$ from plate 1.

3. The maximum and minimum values of $\bar{\rho}_s$ are 2 and 0 and are obtained when $\gamma=0$ for $\mu=\infty$ and $\mu=0$, respectively.

4. The asymptote of $\frac{\bar{\rho}_s}{\rho_1}$ for $\mu>1$ is always less than the actual value of μ, irrespective of the value of γ, whereas for $\mu<1$, it is always greater than μ except at $\mu=0$ and provided that $\gamma=0$ where the value of the asymptote of $\frac{\bar{\rho}_s}{\rho_1}=\mu=0$.

5. The rate of change of the asymptote of $\frac{\bar{\rho}_s}{\rho_1}$ with respect to γ as a function of γ, $\frac{\partial \bar{\rho}_{s\,\text{asymptote}}}{\partial \gamma}$, is much larger for $\mu<1$ than for $\mu>1$ (fig. 5). Consequently the shape of the sounding curve varies as a function of γ more significantly when $\mu<1$ than when $\mu>1$.

6. The value of $\frac{\partial \bar{\rho}_{s\,\text{asymptote}}}{\partial \gamma}$ is negative for $\mu>1$ and is positive for $\mu<1$.

7. The forms of sounding curves obtained with the array oriented parallel to the vertical contact resemble those of sounding curves obtained over horizontally stratified media. In general, the equivalent geoelectric section is composed of at least three horizontal layers. For $\mu>1$, the equivalent horizontally stratified geoelectric section is of the K-type ($\rho_1<\rho_2$ and $\rho_2>\rho_3>\rho_1$), whereas for $\mu<1$, the equivalent section is of the Q-type ($\rho_1>\rho_2>\rho_3$). An example is shown in figure 8, which illustrates the almost complete equivalence

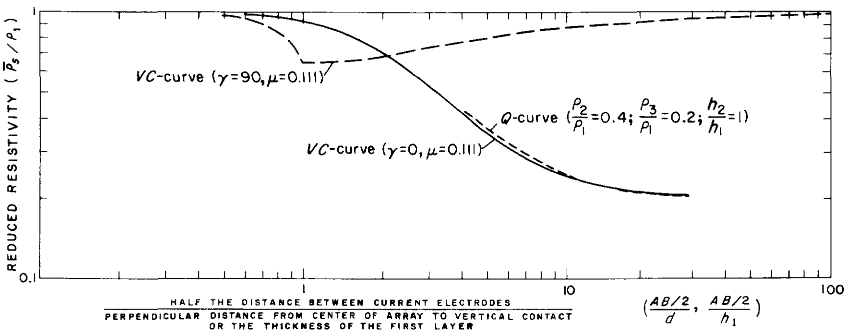

FIGURE 8.—Equivalence of sounding curves near a vertical contact (VC-curves) and over horizontal layers (Q-curve); h_1 is thickness of first layer, h_2 of second layer.

of the curve obtained near a vertical contact (VC-curve) at $\gamma=0$ and $\mu=0.111$ to the Q-curve (Orellana and Mooney, 1966) obtained over a horizontally stratified medium with $\frac{\rho_2}{\rho_1}=0.4$, $\frac{\rho_3}{\rho_1}=0.2$, and $\upsilon=\frac{h_2}{h_1}=1$. The left and the right asymptotes of both curves are 1 and 0.2, respectively. The maximum deviation of the Q-curve from the VC-curve is observed at $\frac{AB}{2d}=\frac{AB}{2h_1}\approx 6$ and amounts to less than +3 percent (h_1 in $AB/2h_1$ is the thickness of the first layer). Complete equivalence can probably be obtained by matching the given VC-curve to a four-layer curve of the QQ-type with $\frac{\rho_2}{\rho_1}=0.4$, $0.4<\frac{\rho_3}{\rho_1}<0.2$, and $\frac{\rho_4}{\rho_1}=0.2$. This phenomenon of equivalence can be easily resolved, however, by making crossed soundings.

CIRCULAR SOUNDING DIAGRAMS

A circular sounding diagram is obtained by making several soundings at the same sounding station by means of differently oriented sounding lines. The values of resistivities at predetermined spacings are then plotted on a set of polar coordinates in the form of "vectors" as a function of the azimuth angle γ. Each curve plotted by the tip of the resistivity "vector" for a given spacing may be called a resistivity hodograph.

Resistivity hodographs obtained by means of the symmetric $AMNB$ array near a vertical contact display a quadrilateral symmetry. The two planes of symmetry are parallel and at right angles to the plane of the vertical contact. This type of symmetry arises because for a given $AB/2$ spacing and a given resistivity contrast the same value of apparent resistivity is measured for γ, $(\pi-\gamma)$, $(\pi+\gamma)$, and $(2\pi-\gamma)$. Therefore, the side on which the vertical contact lies with respect to any sounding line cannot be determined from a single circular sounding diagram, but the strike of the contact can be determined from the symmetry planes and from the form of the resistivity hodographs. The position of the vertical contact can be determined, however, from two circular diagrams obtained at two different sounding stations or even from two hodographs obtained with the same spacing but at two different sounding stations.

Theoretical circular sounding diagrams near a vertical contact for $\mu=20$ and for $\mu=\frac{1}{20}=0.05$ are shown in figures 9 and 10, respectively.[1]

In practice, it is very time consuming to make a complete and continuous circular sounding, but it is recommended that at least three, preferably four, sounding lines be made that are differently oriented with respect to a given geographic direction.

Circular soundings may be made at different sounding stations placed along a profile which is approximately at right angles to a presumed

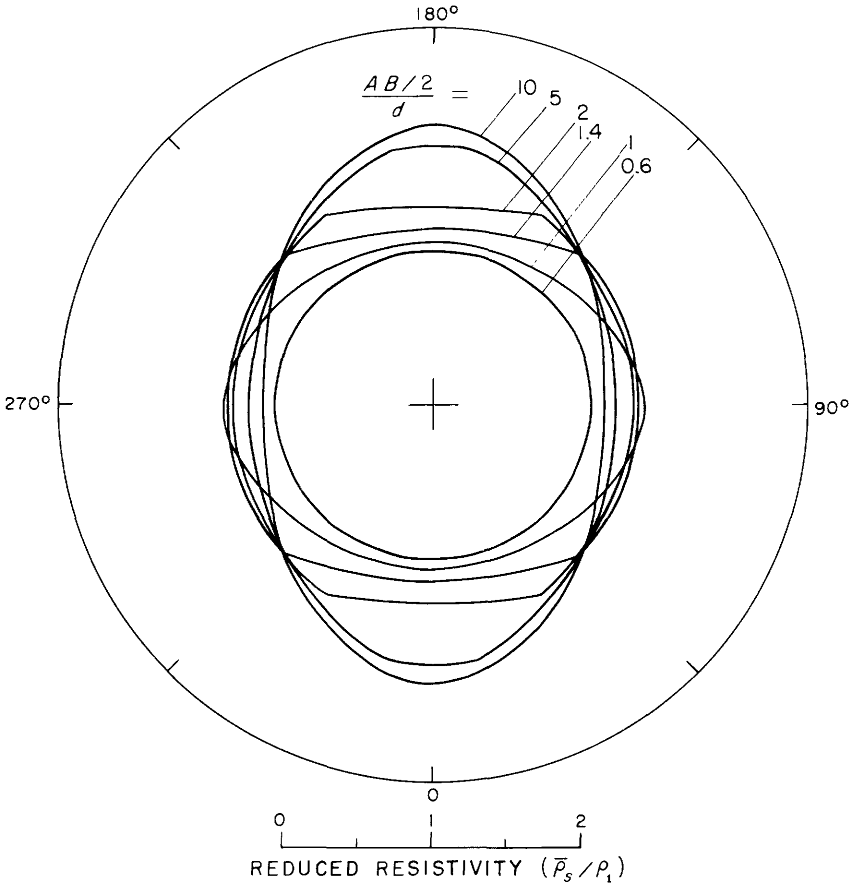

FIGURE 9.—Circular sounding diagram near a vertical contact for a resistivity ratio of 20.

[1] Similar circular diagrams for $\mu=0$ and ∞ were given by Ogil'vi (1956); however, his diagrams for $\mu=0$ is in error owing to the insufficient number of values of γ that he used in plotting the diagram.

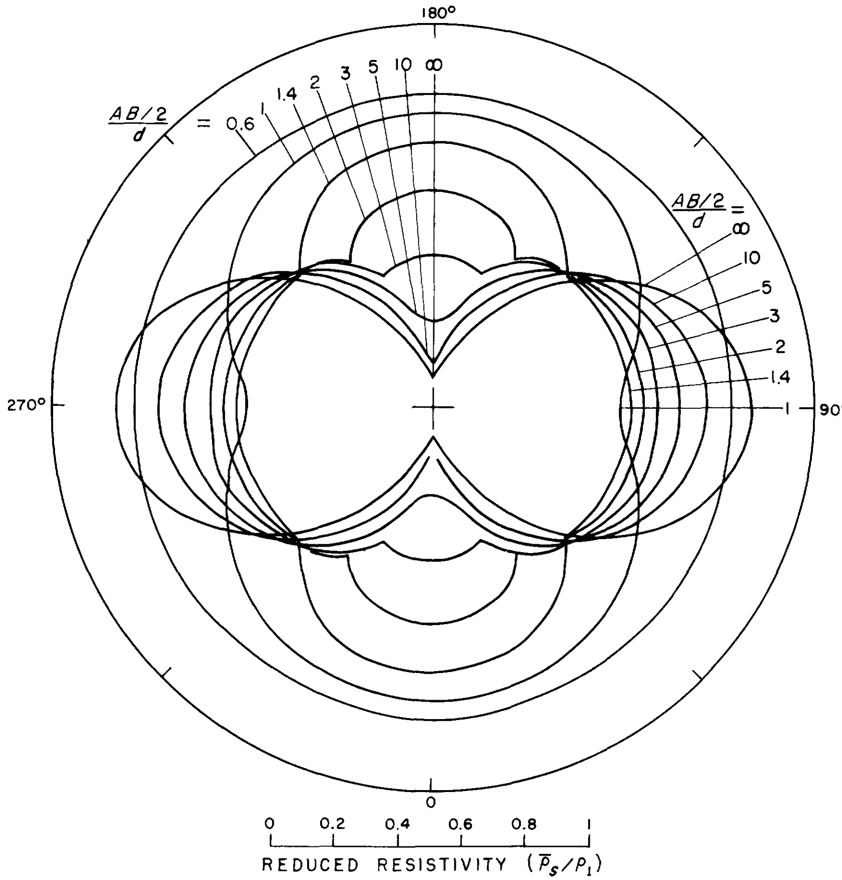

FIGURE 10.—Circular sounding diagram near a vertical contact for a resistivity ratio of 1/20.

vertical contact. For example, consider a vertical contact separating two media that have resistivities of 10 and 50 ohm-meters. The direction of the profile of circular soundings is made normal to the vertical contact. By using computed sounding data for $\mu=5$ and 0.2 for various values of γ and at specific values of $AB/2d$, one can construct the required profile of resistivity hodographs as shown in figure 11.

RESISTIVITY PROFILING

There are two types of apparent resistivity profiles. The first type, horizontal profiling, is obtained by displacing the electrode array along a given traverse. The second type of apparent resistivity profile is con-

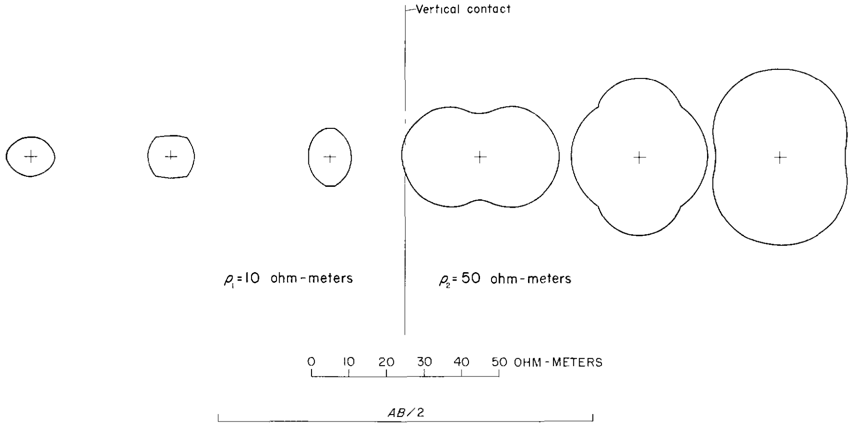

FIGURE 11.—Profile of resistivity hodographs near a vertical contact separating two media of resistivities of 10 and 50 ohm-meters.

structed by plotting the apparent resistivity at a given $AB/2$ spacing as obtained from a profile of electrical soundings where the sounding lines may form one or more arbitrary angles with the surface trace of the vertical contact.

The necessary information for plotting either type of resistivity profile across a vertical fault is obtainable from the computed tables of resistivity soundings. Theoretical horizontal resistivity profiles over a vertical con-

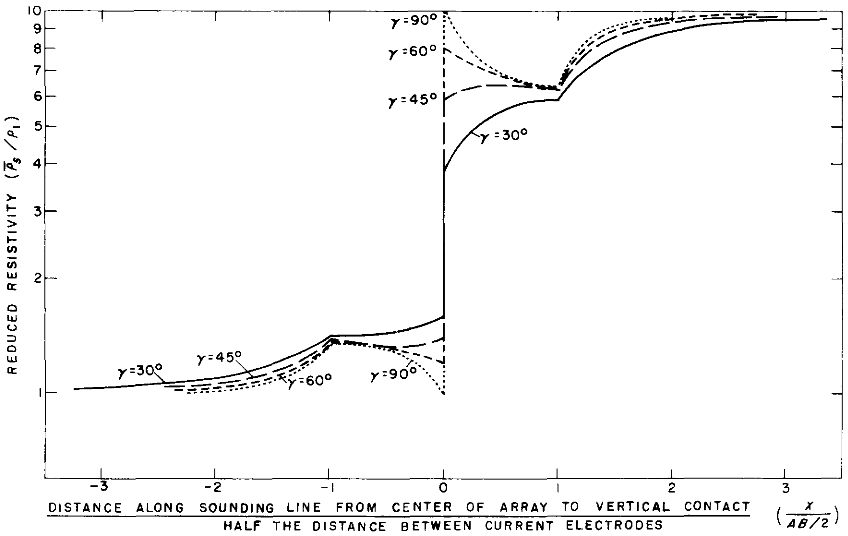

FIGURE 12.—Theoretical horizontal resistivity profiles across a vertical contact (resistivity ratio of 10).

tact that separates two media with resistivities $\rho_1=10$ ohm-meters and $\rho_2=100$ ohm-meters are shown in figure 12. The horizontal resistivity profiles shown are for traverses crossing the contact at angles of $\gamma=30°$, 45°, 60°, and 90°. The curves shown in figure 13 represent data for continuous sets of sounding points whose sounding lines make an angle $\gamma=0°$, 45°, or 90° with the surface trace of a vertical contact separating two media with resistivities of $\rho_1=10$ ohm-meters and $\rho_2=100$ ohm-meters.

Figure 13 shows that for $\gamma\neq0$, $d=0$ (center of sounding is on the surface trace of vertical contact), and $MN=0$ (ideal Schlumberger array), the apparent resistivity has a double value. These two values depend on whether the limit of $\dfrac{d}{AB/2}$ approaches zero from the left (using a resistivity ratio $=\mu$) or from the right (using the corresponding reciprocal resistivity ratio $\left.\dfrac{1}{\mu}\right)$.

For $\gamma=0$, however, the value of the apparent resistivity at $d=0$ is a unique constant and is the same for μ and $\dfrac{1}{\mu}$. This unique apparent re-

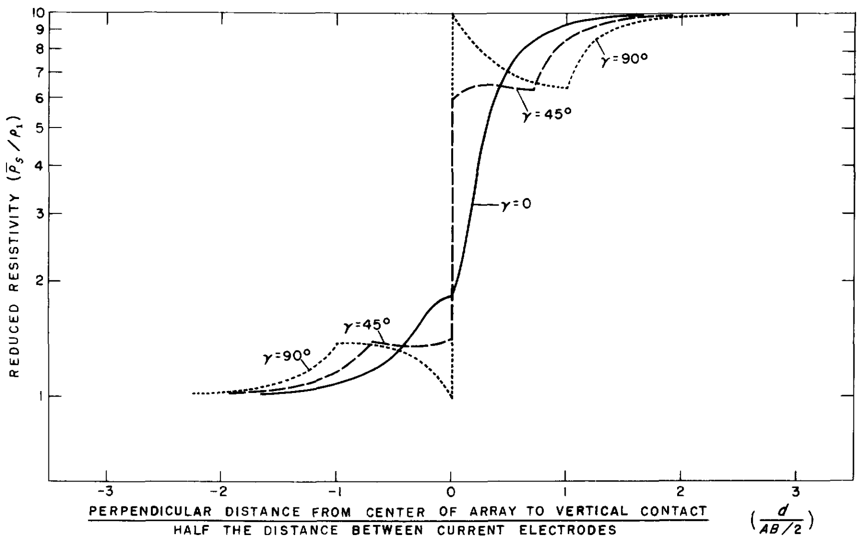

FIGURE 13.—Theoretical resistivity profiles from sounding data (resistivity ratio of 10).

sistivity value can be evaluated from

$$\bar{\rho}_{s_{\gamma=0,d=0}}=\bar{\rho}_{s_{\text{asymptote for }\mu}}\cdot\rho_1,$$

or

$$\bar{\rho}_{s_{\gamma=0,d=0}}=\bar{\rho}_{s_{\text{asymptote for }\frac{1}{\mu}}}\cdot\rho_2,$$

which can be easily computed from equation 16 or determined from the curve for $\gamma=0$ on figure 4. For example, if a hypothetical sounding is made exactly on the surface trace of a vertical contact that separates two media with resistivities $\rho_1=15$ and $\rho_2=60$, then $\mu=4$ or $\mu'=\frac{1}{\mu}=\frac{15}{60}=$ 0.25; and as seen in figure 4, $\frac{\bar{\rho}_{s_{\text{asymptote}(\mu)}}}{\rho_1}\approx 1.6$, or $\frac{\bar{\rho}_{s_{\text{asymptote}(\mu)'}}}{\rho_1}\approx 0.4$. Therefore, the sounding curve will consist of a horizontal straight line showing a constant apparent resistivity value of $\bar{\rho}_s=1.6\times15=0.4\times60=24$ ohm-meters.

SUMMARY AND CONCLUSIONS

The effect of the variation in the azimuth angle between a Schlumberger sounding line and the surface trace of a vertical contact has been studied in detail. Two formulas were derived for computing theoretical sounding curves of the Schlumberger type near a vertical contact. The theoretical sounding curves indicate that for a given angle, γ, the form of a sounding curve changes considerably more as a function of μ when μ is less than unity than when μ is larger than unity. For values of $\mu\geq20$, the sounding curves for a given value of γ are almost identical. Furthermore, for values of $\mu\ll1$, the theoretical sounding curves are significantly distinct from one another when γ is varied (especially at small values of γ). The results of the computations were used not only to draw sounding curves at certain azimuth angles but also to construct circular sounding diagrams and resistivity profiles. The form of resistivity hodographs which are obtained from circular soundings near a vertical contact differ in shape depending on whether the center of the array is placed on the conductive or on the resistive medium. Furthermore, because of the complexity of the resistivity hodographs when the center of the array is placed over a resistive medium, the exact form of most hodographs can be realized only from three or more soundings of different azimuths.

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