Use of Dar Zarrouk Curves in the Interpretation of Vertical Electrical Sounding Data

GEOLOGICAL SURVEY BULLETIN 1313-D

Use of Dar Zarrouk Curves in the Interpretation of Vertical Electrical Sounding Data

By ADEL A. R. ZOHDY

NEW TECHNIQUES IN DIRECT-CURRENT RESISTIVITY EXPLORATION

GEOLOGICAL SURVEY BULLETIN 1313-D

Dar Zarrouk curves are used to define the limits of equivalence for multilayer sections, to improve the fit between theoretical and observed VES curves, and to incorporate geological and geophysical constraints in interpreted geoelectrical models

UNITED STATES GOVERNMENT PRINTING OFFICE, WASHINGTON : 1974

UNITED STATES DEPARTMENT OF THE INTERIOR

ROGERS C. B. MORTON, *Secretary*

GEOLOGICAL SURVEY

V. E. McKelvey, *Director*

Library of Congress Cataloging in Publication Data

Zohdy, Adel A. R., 1935–
Use of Dar Zarrouk curves in the interpretation of vertical electrical sounding data.
(Geological Survey Bulletin 1313–D. New techniques in direct-current resistivity exploration)
Bibliography: p.
Supt. of Docs. No.: I 19.3:1313–D.
1. Electric prospecting. I. Title. II. Series: United States Geological Survey Bulletin 1313–D. III. Series: New techniques in direct-current resistivity exploration.
QE75.B9 no. 1313–D [TN269] 557.3'08s [622'.154] 74–14706

For sale by the Superintendent of Documents, U. S. Government Printing Office
Washington, D. C. 20402 - Price $1.45 (paper cover)

CONTENTS

ILLUSTRATIONS

TABLES

NEW TECHNIQUES IN DIRECT-CURRENT RESISTIVITY EXPLORATION

USE OF DAR ZARROUK CURVES IN THE INTERPRETATION OF VERTICAL ELECTRICAL SOUNDING DATA

By ADEL A. R. ZOHDY

ABSTRACT

Graphical and numerical methods (including programs for an electronic desk calculator) are given for the construction and inversion of DZ (Dar Zarrouk) curves. The scope of the principle of equivalence is expanded beyond its commonly stated rules, and the criteria for equivalence between multilayer sections containing different numbers of layers are clarified through the calculation of DZ curves. The use of DZ curves to obtain better fits between observed and calculated VES curves and to improve correlation between the VES interpretation and geologic or other geophysical constraints is illustrated by several examples.

INTRODUCTION

The term "Dar Zarrouk" was introduced into the literature on electrical prospecting by Maillet (1947) for describing a relationship between the longitudinal unit conductance,

$$S_i = h_i / \rho_i, \tag{1}$$

and the transverse unit resistance,

$$T_i = \rho_i h_i, \tag{2}$$

where ρ_i and h_i are the electrical resistivity and thickness of the ith layer, respectively. A DZ (Dar Zarrouk) curve for an n-layer section is a plot of the DZ resistivity

$$\rho_{m_j} = \sqrt{\sum_{i=1}^{j} T_i / \sum_{i=1}^{j} S_i}, \tag{3}$$

against the DZ depth

$$L_{mj}=\sqrt{\sum_{i=1}^{j} T_i \sum_{i=1}^{j} S_i}. \tag{4}$$

An n-layer DZ curve is composed of n branches, each of which terminates at a point whose coordinates, L_m and ρ_m, represent the thickness and resistivity of a fictitious layer that replaces all the overlying layers. According to equations 3 and 4, the coordinates of any given point on a DZ curve are a function of the thicknesses and resistivities of layers that exist above a given depth, D, but they are not related to the thicknesses and resistivities of layers beneath that depth. In contrast, on a VES (vertical electrical sounding) curve, the coordinates of a given point are calculated from an integral expression (Stefanesco and others, 1930) that involves all the thicknesses and resistivities in the section, and, therefore, they are not related to a particular depth.

Orellana (1963) described a graphical method for the simple calculation of DZ curves, and Zohdy (1965) showed the relationship between the DZ parameters L_m and ρ_m and the auxiliary-point diagrams used in the interpretation of VES curves. In spite of Maillet's and Orellana's work, the use of DZ curves as an effective tool in interpreting VES data is not widespread, primarily because neither author stressed the fact that the thicknesses and resistivities can be calculated easily from a given DZ curve (inverse problem). Orellana, for example, described several constructive applications of DZ curves, all of which were based on the calculation of DZ curves for prescribed geoelectric sections (forward problem). Similarly, Zohdy (1964) calculated a DZ curve for a complex geoelectric section composed of 17 layers to show, as Maillet and Orellana had indicated, that, for many geoelectric sections, it is possible to obtain an idea about the shape of a Schlumberger VES curve without calculating the VES curve itself. The construction of DZ curves can help in evaluating the application of the VES method for solving a given problem (Flathe, 1958).

Such applications are valuable when a digital computer is not available for the calculation of the actual VES curve or when the available computer program is unsuitable for making the required calculations. An extension of the use of DZ curves for surmounting this latter difficulty was made by Zohdy and Jackson (1973), who used the inversion of DZ curves to obtain an equivalent model that could be calculated by the available computer program. The purpose of the present paper is to outline several fundamental applications of DZ curves which lead to a greater understanding of the equivalence problems in direct-current electrical prospecting.

GRAPHICAL CONSTRUCTION OF DZ CURVES

Given a geoelectric section for which all the layer thicknesses and resistivities are specified, one can construct the corresponding DZ curve by using Orellana's template (Orellana, 1963) and the calculation of values of *T* and of *S* and their sums. However, because of the identity of the DZ point to the auxiliary-point *A*, the *A* auxiliary-point diagrams for which there are two types of representation (for example, Zohdy, 1965; Orellana and Mooney, 1966), can also be used to construct DZ curves without the calculation of the sums of *T* and *S*. These diagrams are calculated only for values of $\rho_2/\rho_1 \geq 1$ (where ρ_1 and ρ_2 are the resistivities of the first and second layers, respectively), and, therefore, they must be complemented with their mirror images, reflected across the abscissa axis, to allow the drawing of DZ curves for values of $\rho_2/\rho_1 \leq 1$. The combination of the *A* auxiliary-point diagrams and their mirror images is referred to here as DZ charts.

Plate 1 shows the DZ charts which may be used for the graphical construction of DZ curves in lieu of Orellana's template. Plate 1*A* is calculated from the equations

$$\xi_A' = \frac{\rho_m}{\rho_1} = \sqrt{\frac{1+\mu\nu}{1+(\nu/\mu)}}, \tag{5}$$

and

$$\eta_A = \frac{L_m}{h_1} = \sqrt{(1+\mu\nu)(1+(\nu/\mu))}, \tag{6}$$

and plate 1*B* is calculated from the equations

$$\xi_A' = \frac{\rho_m}{\rho_2} = \sqrt{\frac{1+\mu\nu}{1+(\nu/\mu)}}/\mu, \tag{7}$$

and

$$\eta_A = \frac{L_m}{h_1} = \sqrt{(1+\mu\nu)(1+(\nu/\mu))}, \tag{8}$$

where $\mu=\rho_2/\rho_1$=ratio of second-layer resistivity to first-layer resistivity, and $\nu=h_2/h_1$=ratio of second-layer thickness to first-layer thickness. Each chart consists of two families of curves; one is for constant values of μ, which are two-layer DZ curves, and the other is for constant values of ν, which determine the lengths of DZ branches according to the thickness of the second layer.

The graphical construction of a DZ curve for a multilayer geoelectric section using either chart of plate 1 is as follows:

1. On a transparent sheet of log-log paper (of the same scale as the DZ charts) plot the distribution of resistivity as a function of depth for the given section, with the true resistivity, ρ, on the ordinate and the depth, D, on the abscissa, as shown in figure 1.
2. The first horizontal segment on this plot represents the first branch of the required DZ curve with $\rho_1=\rho_{m_1}$ and $D_1=h_1=L_{m_1}$.
3. Calculate the ratio $\nu_1=h_2/h_1=h_2/L_{m_1}$ and place the transparent sheet on one of the DZ charts so that the second horizontal segment, representing ρ_2, parallels one of the asymptotic values of $\mu=\rho_2/\rho_1$ on plate 1*A*, or coincides with the abscissa axis on plate 1*B*.
4. Move the transparent sheet to the right or left until the abscissa value $D_1=L_{m_1}$ (the depth to the bottom of the first layer) coincides with the origin of coordinates on plate 1*A*, or falls on the ordinate axis on plate 1*B*. Trace the DZ curve (for $\mu=\rho_2/\rho_2$=constant) that starts at $D_1=L_{m_1}$, and asymptotically approaches ρ_2. Interpolate between DZ curves if necessary.
5. Terminate the DZ curve at the point where the ν=constant curve, which corresponds to the calculated value of $\nu_1=h_2/L_{m_1}$, intersects the DZ curve. The abscissa value, L_{m_2}, of this point represents the thickness of a fictitious layer that replaces the upper two layers, and its ordinate, ρ_{m_2}, represents the resistivity of that fictitious layer.
6. Read the value of the abscissa L_{m_2}, and calculate the ratio $\nu_2'=h_3/L_{m_2}$. This ratio represents the effective relative thickness of the third layer in the DZ domain.
7. For plate 1*A*, place the point (L_{m_2}, ρ_{m_2}) at the origin of coordinates of the plate and trace the DZ curve whose asymptote coincides with ρ_3. For plate 1*B*, shift the transparent sheet so that the horizontal segment for ρ_3 coincides with the abscissa axis of the plate and so that the point (L_{m_2}, ρ_{m_2}) falls on its ordinate. For either of the charts on

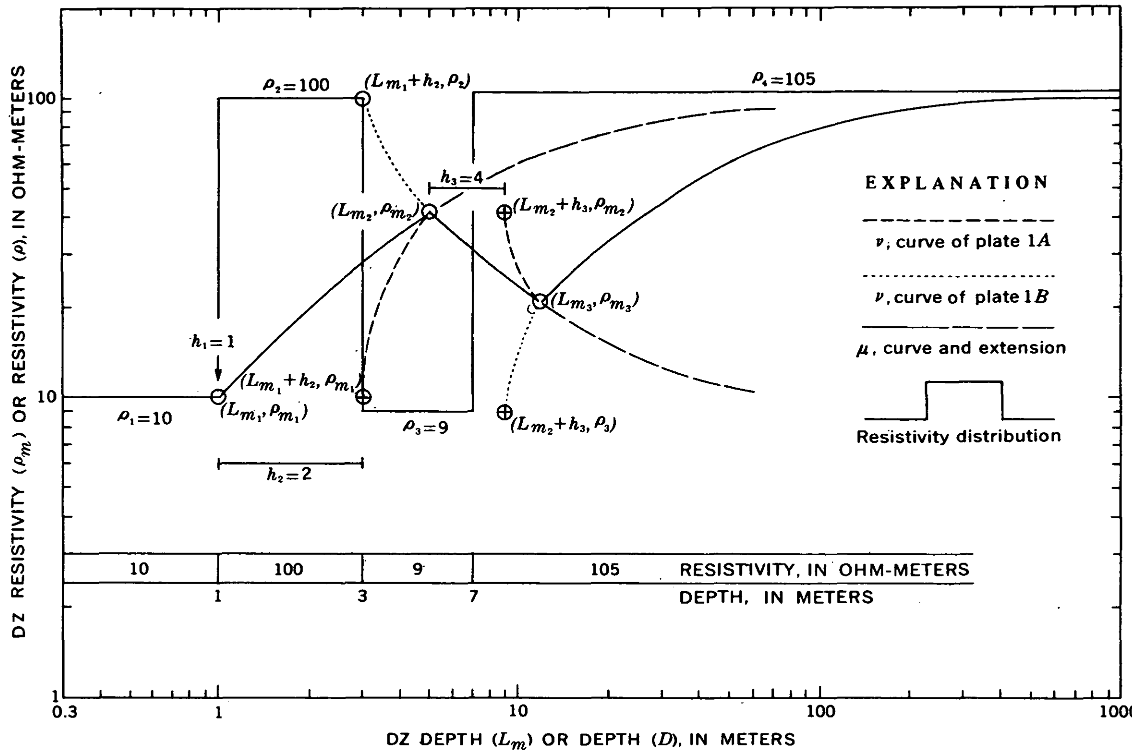

FIGURE 1.—Example showing the graphical construction or inversion of a four-layer DZ curve, using the DZ charts on plate 1. L_m and ρ_m, DZ coordinates; h and ρ, layer thickness and resistivity. $\mu=\rho_2/\rho_1$, ratio of second-layer resistivity to first-layer resistivity; $\nu=h_2/h_1$, ratio of second-layer thickness to first-layer thickness.

plate 1, trace the DZ curve that starts at (L_{m_2}, ρ_{m_2}), and terminate it where it intersects the appropriate curve for $\nu=\nu_2'=h_3/L_{m_2}$. The abscissa, L_{m_3}, of this point represents the thickness of a new fictitious layer that replaces the top three layers, and its ordinate, ρ_{m_3}, represents the resistivity of that fictitious layer.

8. The above procedure is repeated to the bottom of the given geoelectric section where the last segment on the n-layer DZ curve is traced, using the coordinates of the point ($L_{m_{n-1}}$, $\rho_{m_{n-1}}$) as the thickness and resistivity of a fictitious layer that replaces all the $n-1$ layers lying above the nth layer of infinite thickness. The value of $\nu_{n-1}=h_n/L_{m_{n-1}}=\infty$, and, therefore, the last DZ curve is plotted so that it approaches the value of ρ_n asymptotically. If the value of ρ_n is assumed theoretically to be ∞ or 0, then the last DZ segment is represented by a straight line inclined to the abscissa axis at an angle of +45° or −45°, respectively. This straight line starts at the point ($L_{m_{n-1}}$, $\rho_{m_{n-1}}$) and extends to infinity.

The above graphical procedure of constructing a DZ curve differs from the one proposed by Orellana in that the DZ depths L_{m_i} are determined graphically, and only the ratios h_{i+1}/L_{m_i} are calculated for all the layers, one at a time; but no summations of S or T are required.

CALCULATION OF DZ CURVES

The graphical construction of DZ curves for geoelectric sections composed of more than four or five layers is time consuming and subject to graphical errors. Instead, it is recommended to calculate the coordinates of the successive DZ points by means of a programable electronic desk calculator. A program based on the DZ equations

$$\rho_{m_j}=\sqrt{\sum_{i=1}^{j}\rho_i h_i \Big/ \sum_{i=1}^{j}(h_i/\rho_i)}, \tag{9}$$

and

$$L_{m_j}=\sqrt{\sum_{i=1}^{j}\rho_i h_i \cdot \sum_{i=1}^{j}(h_i/\rho_i)}, \tag{10}$$

was written for the Wang 370 calculator and is given in table 1. (For code listing, see table 4.) With this program, if the user

TABLE 1.—*Wang 370 program for terminal DZ points*

(THIS SPACE FOR INSTRUCTIONS AND NOTES)

ρ_i = Resistivity of ith layer

D_i = Depth of ith layer

L_{m_i} = DZ depth of ith layer

ρ_{m_i} = DZ resistivity of ith layer

OPERATING INSTRUCTIONS

(1) CLEAR ALL REGISTERS

(2) PRESS CONTINUE

(3) INDEX D_1, PRESS CONTINUE

(4) INDEX ρ_1, PRESS CONTINUE

(5) COPY L_{m_1}, PRESS CONTINUE

(6) COPY ρ_{m_1}, PRESS CONTINUE

(7) REPEAT STEP (3) TO STEP (6) WITH D_2, ρ_2, ... etc.

Total of coding numbers = 1,540

(SEE OTHER SIDE FOR PROGRAM CODES)

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	MARK	07		40	÷	47	
01	1	61		41	√	44	
02	STOP	01	Index D_i	42	STOP	01	COPY ρ_{m_i}
03	CL.R+	50		43	SEARCH	02	
04	R+	52		44	1	61	
05	CL.L+	54		45			
06	L+	56		46			
07	R.R0	14		47			
08	R-	53		48			
09	St.R1	11		49			
10	ENT	41		50			
11	R.L+	55		51			
12	St.R0	10		52			
13	STOP	01	Index ρ_i	53			
14	CL.L+	54		54			
15	L+	56		55			
16	÷	47		56			
17	CL.R+	50		57			
18	R+	52		58			
19	R.R2	16		59			
20	R+	52		60			
21	St.R2	12		61			
22	R.R1	15		62			
23	ENT	41		63			
24	R.L+	55		64			
25	X	46		65			
26	CL.R+	50		66			
27	R+	52		67			
28	R.R3	17		68			
29	R+	52		69			
30	St.R3	13		70			
31	R.R2	16		71			
32	ENT	41		72			
33	R.R3	17		73			
34	X	46		74			
35	√	44		75			
36	STOP	01	COPY L_{m_i}	76			
37	R.R3	17		77			
38	ENT	41		78			
39	R.R2	16		79			

indexes the depths (instead of the thicknesses) and the resistivities of the successive layers, the coordinates of the successive DZ points (which terminate each DZ branch) are calculated. If some of these terminal DZ points are widely separated, then one can use the DZ charts or the Orellana template, or can calculate other intermediate points, to connect these points with accurately drawn DZ branches.

The coordinates of intermediate DZ points may be calculated using the same program after subdividing the layers, which are represented by long DZ branches, into several sublayers of the same resistivity. However, this procedure has two drawbacks. First, the abscissas (DZ depths) of the intermediate DZ points generally are not round numbers and will not be equally spaced on the logarithmic scale when the layer in question is subdivided into sublayers of linearly equal thicknesses. Second, the intermediate points can only be calculated in an increasing order of layer depths.

The following procedure makes it possible to calculate ρ_m values at any prescribed L_m values and regardless of the order of L_m. A formula that relates ρ_m to L_m directly is derived as follows.

For a two-layer section with $n=2$ in equation 9,

$$\rho_m=\sqrt{\frac{\rho_1 h_1+\rho_2 h_2}{\frac{h_1}{\rho_1}+\frac{h_2}{\rho_2}}}. \tag{11}$$

Solving for h_2 we get

$$h_2=h_1\cdot\frac{\rho_2}{\rho_1}\cdot\left(\frac{\rho_m^2-\rho_1^2}{\rho_2^2-\rho_m^2}\right), \tag{12}$$

but it can be shown also from equations 9 and 10 that for $n=2$,

$$\rho_m=\frac{\rho_1 h_1+\rho_2 h_2}{L_m}. \tag{13}$$

Substituting for h_2 from equation 12 in equation 13, rearranging and simplifying, we obtain the quadratic equation

$$(L_m\rho_1)\rho_m^2+h_1(\rho_2^2-\rho_1^2)\rho_m-L_m\rho_1\rho_2^2=0, \tag{14}$$

which we solve for ρ_m to obtain

$$\rho_m=\frac{-h_1(\rho_2^2-\rho_1^2)\pm\sqrt{(h_1(\rho_2^2-\rho_1^2))^2+4(L_m\rho_1\rho_2)^2}}{2L_m\rho_1}. \tag{15}$$

Equation 15 defines the ordinate ρ_m at any prescribed value of L_m for a two-layer DZ curve which starts at the point ($L_{m_1}=h_1$, $\rho_{m_1}=\rho_1$) and asymptotically approaches ρ_2. For the calculation of intermediate points on the jth DZ branch, the values of $\rho_{m_{j-1}}$ and $L_{m_{j-1}}$ (which terminate the $j-1$ branch) are used in equation 15 in lieu of ρ_1 and h_1, respectively, and ρ_j is used in

TABLE 2. – *Wang 370 program for prescribed DZ points.*

(THIS SPACE FOR INSTRUCTIONS AND NOTES)

ρ_i = Resistivity of ith layer

h_i = Thickness of ith layer

L_{m_i} = DZ depth of ith layer

ρ_{m_i} = DZ resistivity of ith layer

OPERATING INSTRUCTIONS

(1) CLEAR ALL REGISTERS

(2) STORE h_i in Register 1

STORE ρ_1 in Register 2

STORE ρ_2 in Register 3

STORE L_{m_i} in Register 0

(3) PRESS CONTINUE

(4) COPY ρ_m

(5) STORE NEXT VALUE OF L_m IN REGISTER 0 AND REPEAT STEP (3) TO STEP (5)

Total of coding numbers = 2,384

(SEE OTHER SIDE FOR PROGRAM CODES)

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	MARK	07		40	R-	53	
01	1	61		41	ENT	41	
02	R.R3	17		42	R.R1	15	
03	x^2	45		43	X	46	
04	CL.R+	50		44	L-	57	
05	R+	52		45	R.R2	16	
06	R.R2	16		46	ENT	41	
07	x^2	45		47	R.R0	14	
08	R-	53		48	X	46	
09	x^2	45		49	ENT	41	
10	CL.R+	50		50	2	62	
11	R+	52		51	X	46	
12	R.R1	15		52	÷	47	
13	x^2	45		53	ENT	41	
14	ENT	41		54	R.L+	55	
15	R.R+	51		55	X	46	
16	X	46		56	STOP	01	COPY ρ_{m_i}
17	CL.L+	54		57	SEARCH	02	
18	L+	56		58	1	61	
19	R.R2	16		59			
20	ENT	41		60			
21	R.R3	17		61			
22	X	46		62			
23	ENT	41		63			
24	R.R0	14		64			
25	X	46		65			
26	x^2	45		66			
27	ENT	41		67			
28	4	64		68			
29	X	46		69			
30	L+	56		70			
31	$\sqrt{}$	44		71			
32	CL.L+	54		72			
33	L+	56		73			
34	R.R3	17		74			
35	x^2	45		75			
36	CL.R+	50		76			
37	R+	52		77			
38	R.R2	16		78			
39	x^2	45		79			

lieu of ρ_2. The positive sign of the square root is used so that positive, physically meaningful values of ρ_m are obtained. A program for the Wang 370 calculator was written for equation 15 and is given in table 2.

INVERSION OF DZ CURVES

The inversion of DZ curves into layer thicknesses and resistivities is a simple, important, and powerful tool in the interpretation of VES data. The significance of this technique has not been recognized, probably because of the notion that any given DZ curve must have been constructed from known thicknesses and resistivities, and, therefore, it is absurd to recalculate a layering that is already known. It is important, however, to be able to determine several combinations of layer thicknesses and resistivities that are electrically equivalent and are all solutions, therefore, to the same VES curve. Also, the inversion of DZ curves is fundamental in making adjustments in layer thicknesses and resistivities to produce closer fits between observed and calculated VES curves.

The DZ charts shown on plate 1*A* and 1*B* can be used effectively to calculate the layer thicknesses and resistivities from a given DZ curve in the same manner in which the auxiliary-point diagrams are used in the method of partial curve matching (Kalenov, 1957; Keller and Frischknecht, 1966). For the use of plate 1*A*, the graphical inversion of a DZ curve is as follows: The thickness, h_1, and resistivity, ρ_1, of the first layer are equal to L_{m_1} and ρ_{m_1}, respectively. The thickness and resistivity of the second layer are obtained by placing the point (L_{m_1}, ρ_{m_1}) at the origin of coordinates of plate 1*A*, keeping axes parallel, and tracing the ν curve that passes through the point (L_{m_2}, ρ_{m_2}). The intercept of this ν curve with the abscissa axis, whose ordinate is ρ_{m_1}; less the value of L_{m_1} equals h_2, and the asymptote of the μ=constant curve, which coincides with the second DZ branch, is equal to ρ_2. This procedure is repeated to the *n*th DZ branch for which $h_n=\infty$ and ρ_n is obtained by tracing the asymptote of the appropriate μ=constant curve on the plate. Figure 1 shows an example of the graphical inversion of a four-layer DZ curve.

The above graphical procedure may be substituted by the following formulas for the determination of the layer thicknesses and resistivities with greater accuracy. From equations 3 and 4, it can be shown that

$$L_{m_j}\rho_{m_j}=\sum_{i=1}^{j} T_i, \tag{16}$$

and

$$L_{m_j}/\rho_{m_j}=\sum_{i=1}^{j} S_i. \tag{17}$$

Therefore, for $j=1$ (first layer),

$$\rho_1=\rho_{m_1}, \tag{18}$$

and

$$h_1 = L_{m_1}, \tag{19}$$

and for $j=2$ to n,

$$\rho_j = \sqrt{\frac{L_{m_j}\rho_{m_j} - L_{m_{j-1}}\rho_{m_{j-1}}}{\dfrac{L_{m_j}}{\rho_{m_j}} - \dfrac{L_{m_{j-1}}}{\rho_{m_{j-1}}}}}, \tag{20}$$

and

$$h_j = \rho_j\left(\frac{L_{m_j}}{\rho_{m_j}} - \frac{L_{m_{j-1}}}{\rho_{m_{j-1}}}\right). \tag{21}$$

By substituting the coordinate values of any two DZ points in equations 20 and 21, we can calculate the resistivity and thickness of the jth layer, which corresponds to the DZ segment between the $(j-1)$ and jth DZ points. Equations 20 and 21 were programed for the Wang 370 calculator to simplify the computation of the geoelectric layering. (See tables 3, 4.)

PRINCIPLE OF EQUIVALENCE

Two geoelectric sections are said to be "practically equivalent" when the measurement or calculation of the combined effect of the layering in each section results in two curves that are "practically coincident." Therefore, in practice, the phenomenon of equivalence depends on the resolving power of the measuring technique, and, in theory, it depends on the form of the equation by which the combined effect of the layering is calculated. For example, two geoelectric sections that produce practically coincident kernel function curves (Ghosh, 1971; Koefoed, 1968; Shkabarnia and Gritsenko, 1971; Strakhov and Karelina, 1969) may not produce practically coincident Schlumberger VES curves; therefore, although these two geoelectric sections are practically equivalent in the kernel domain, they are not equivalent in the Schlumberger VES domain. Similarly, because of the greater resolving power of differential soundings (Rabinovich, 1965; Zohdy, 1969) certain geoelectric sections that are considered equivalent in the Schlumberger VES domain may be nonequivalent in the differential-sounding domain.

For direct-current measurements and for horizontally stratified media, exact equivalence (in all domains) exists only between isotropic and microanisotropic media, or between

TABLE 3.—*Wang 370 program for inversion of DZ curves*

(THIS SPACE FOR INSTRUCTIONS AND NOTES)

ρ_i = Resistivity of ith layer

h_i = Thickness of ith layer

L_{m_i} = DZ depth of ith layer

ρ_{m_i} = DZ resistivity of ith layer

OPERATING INSTRUCTIONS

(1) CLEAR ALL REGISTERS

(2) PRESS CONTINUE

(3) INDEX L_{m_1}, PRESS CONTINUE

(4) INDEX ρ_{m_1}, PRESS CONTINUE

(5) COPY ρ_1, PRESS CONTINUE

(6) COPY h_1, PRESS CONTINUE

(7) REPEAT STEP (3) TO STEP (6) WITH L_{m_2}, ρ_{m_2} TO CALCULATE ρ_2, h_2, ... etc.

Total of coding numbers = 1,646

(SEE OTHER SIDE FOR PROGRAM CODES)

No.	Cmd	Code	Comment	No.	Cmd	Code	Comment
00	MARK	07		40	STOP	01	COPY h_i
01	1	61		41	R.R0	14	
02	STOP	01	Index L_m	42	St.R2	12	
03	CL.L+	54		43	R.R1	15	
04	L+	56		44	St.R3	13	
05	ENT	41		45	SEARCH	02	
06	STOP	01	Index ρ_m	46	1	61	
07	CL.R+	50		47			
08	R+	52		48			
09	X	46		49			
10	St.R0	10		50			
11	R.L+	55		51			
12	ENT	41		52			
13	R.R+	51		53			
14	÷	47		54			
15	St.R1	11		55			
16	R.R0	14		56			
17	CL.R+	50		57			
18	R+	52		58			
19	R.R2	16		59			
20	R-	53		60			
21	ENT	41		61			
22	R.R1	15		62			
23	CL.R+	50		63			
24	R+	52		64			
25	R.R3	17		65			
26	R-	53		66			
27	÷	47		67			
28	$\sqrt{\ }$	44		68			
29	STOP	01	Copy ρ_i	69			
30	CL.L+	54		70			
31	L+	56		71			
32	R.R1	15		72			
33	CL.R+	50		73			
34	R+	52		74			
35	R.R3	17		75			
36	R-	53		76			
37	ENT	41		77			
38	R.L+	55		78			
39	X	46		79			

microanisotropic media having different coefficients of microanisotropy (Cagniard, 1948). But even this type of equivalence can be resolved by comparing direct-current soundings with electromagnetic soundings, at least theoretically. In this paper the equivalence between geoelectric sections is referenced to the coincidence of corresponding VES curves of the Schlumberger type.

The principle of equivalence (Bhattacharya and Patra, 1968; Dakhnov, 1953; Kalenov, 1957; Keller and Frischknecht, 1966;

TABLE 4.—*Code listing for program control*

PROGRAM CODE	CODE LISTING FOR PROGRAM CONTROL		
	CP - 1 and 371		CP - 2
	200 thru 360/370/380	362/370-2/380-2	200 thru 362
00			
01	Stop		
02	•Search		
03	•Search & Return		
04	•Return	Same as Column 1	Same as Column 1
05	•Skip if +		
06	•Continue		
07	•Mark		
10	Store Reg. 0	Store Half B	
11	Store Reg. 1	Store Half A	
12	Store Reg. 2	Add Full	Either column as determined by keyboard
13	Store Reg. 3	Store Full	
14	Recall Reg. 0	Recall Half B	
15	Recall Reg. 1	Recall Half A	
16	Recall Reg. 2	Subtract Full	
17	Recall Reg. 3	Recall Full	
20	•Skip if Overflow		Start master programmer at step 00
21	•Control		Start slave programmer No. 1 at step 00
22	•Read 1		Start slave programmer No. 2 at step 00
23	•Read 2	Same as Column 1	Start slave programmer No. 3 at step 00
24	•Write 1		Continue master programmer at next step
25	•Write 2		Continue slave programmer No. 1 at next step
26	•Store Direct		Continue slave programmer No. 2 at next step
27	•Recall Direct		Continue slave programmer No. 3 at next step
30	•Skip if 0		Go to step 00 if W is negative
31			Go to step 10 if W is negative
32	•Store Indirect		Go to step 20 if W is negative
33	•Recall Indirect	Same as Column 1	Go to step 30 if W is negative
34			Go to step 40 if W is negative
35			Go to step 50 if W is negative
36	•Group 1		Go to step 60 if W is negative
37	•Group 2		Go to step 70 if W is negative
40	Print		
41	Enter		
42	$\mathrm{Log}_e X$		
43	e^x		
44	$\sqrt{x}$	Same as Column 1	Same as Column 1
45	x^2		
46	×		
47	÷		
50	Clear Right Adder		
51	Recall Right Adder		
52	+ Right Adder		
53	− Right Adder	Same as Column 1	Same as Column 1
54	Clear Left Adder		
55	Recall Left Adder		
56	+ Left Adder		
57	− Left Adder		
60	Numeral 0	Numeral 0 + Reg. 0	
61	Numeral 1	Numeral 1 + Reg. 1	
62	Numeral 2	Numeral 2 + Reg. 2	
63	Numeral 3	Numeral 3 + Reg. 3	Either column as determined by keyboard
64	Numeral 4	Numeral 4 + Reg. 4	
65	Numeral 5	Numeral 5 + Reg. 5	
66	Numeral 6	Numeral 6 + Reg. 6	
67	Numeral 7	Numeral 7 + Reg. 7	
70	Numeral 8	Numeral 8 + Reg. 8	
71	Numeral 9	Numeral 9 + Reg. 9	
72			Either column as determined by keyboard
73			
74			
75	Decimal Point	Decimal Point	
76	Clear Display	Clear Display + Reg. 10	
77	Change Sign	Change Sign + Reg. 11	

•370 & 380 Program functions only.

Lasfargues, 1957) states that: Two three-layer geoelectric sections of the H-type ($\rho_1 > \rho_2 < \rho_3$), or of the A-type ($\rho_1 < \rho_2 < \rho_3$), may be equivalent if the thickness of the second layer is relatively small and the value of $S_2=h_2/\rho_2$ is equal in both sections (equivalence by S). It also states that two three-layer sections of the K-type ($\rho_1<\rho_2>\rho_3$), or of the Q-type ($\rho_1>\rho_2>\rho_3$), may be equivalent if the thickness of the second layer is relatively small and the value of $T_2=h_2\rho_2$ is equal in both sections (equivalence by T). Nomograms showing the range of validity of this principle were prepared by Pylaev and copies of these nomograms were reproduced in all the above-cited textbooks except that by Lasfargues.

The above-stated conditions on which the principle of equivalence for three-layer sections is based, and which are often quoted in the literature, have several limitations and are neither sufficient (without the Pylaev nomograms) nor necessary for all geoelectric sections. For example, when the thickness of the second layer is reduced, there are no stated guidelines to indicate (1) whether this reduction should result in making the depth to the top of the third layer shallower or the depth to the bottom of the first layer deeper, or (2) whether it should be a combination of both, and, if it is a combination, how the proportion of increase or decrease to the top of the second and third layers should be distributed. In practice, and for almost all geoelectric sections which are defined as equivalent by the Pylaev nomograms, it is sufficient to change the thickness of the second layer by changing only the depth to the top of the third layer (Zohdy, 1968a).

The study of sets of theoretical sounding curves (Compagnie Générale de Géophysique, 1963; Orellana and Mooney, 1966; Rijkswaterstaat, 1969) indicates that the Pylaev nomograms provide only a sufficient condition for equivalence, but not a necessary one. Figure 2*A* shows two geoelectric sections of the K-type for which the value of T_2 is identical ($T_2=250$ ohm-m^2). The corresponding VES curves, which were taken from the Orellana-Mooney album, do not coincide; therefore, the two geoelectric sections are not equivalent. The use of the Pylaev nomograms confirms this observation and indicates that the change in resistivity and thickness of the second layer is beyond the range of applicability of the principle of equivalence. However, this does not mean that the second layer in model I, which has a thickness of 5 meters and a resistivity of 50 ohm-meters, cannot be replaced by a layer having a resistivity of 100 ohm-meters and a thickness of less than 5 meters. By using the same set of theoretical sounding curves in the Orellana and Mooney album, we can find the VES curve for model III (with $\rho_2=100$ ohm-m, $h_2=2$ m, and $T_2=200$ ohm-m^2)

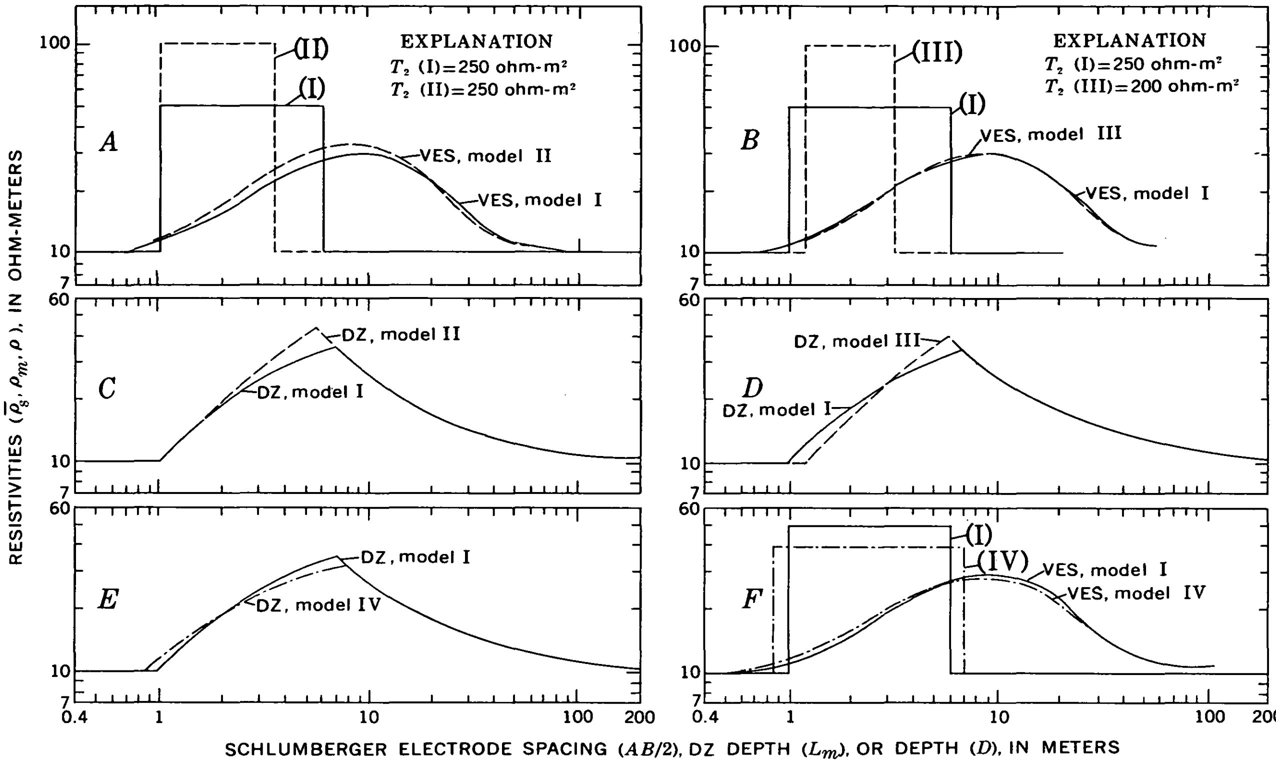

FIGURE 2.—Use of DZ curves in extending the known limits of the principle of equivalence for three-layer sections of the K-type ($\rho_1 < \rho_2 > \rho_3$). T_2, second-layer transverse resistance, in ohm-m².

which practically coincides with the VES curve for model I, as shown in figure 2*B*. Therefore, the geoelectric section of model III must be considered to be equivalent to that of model I. This equivalence is valid despite the fact that the value of T_2 is not identical for both sections.

The comparisons of the DZ curves for models I and II and for models I and III are shown in figure 2*C* and 2*D*, respectively, and they help explain the phenomenon of equivalence for sections of the K type with different values of T_2. In figure 2*C*, the origin of the second DZ branch of model II coincides with that of the second DZ branch of model I, then the two DZ branches diverge continuously until the magnitude of their divergence becomes significant near their terminal points. In figure 2*D*, however, the second DZ branch for model III originates at a different point from that of model I; consequently, it lies partly below and partly above the second DZ branch of model I. This "cancellation effect" resulted in minimizing the differences between the corresponding VES curves, and, therefore, it can be used as an index that helps predict the equivalence between models I and III.

Figure 2*E* shows how the above observation can be used to replace the second layer of model I by an equivalent layer of larger thickness and smaller resistivity through the construction of the DZ curve for model IV and its subsequent solution for the layering. (See equations 18-21.) For the determination of this equivalence, it was unnecessary to search through the albums of theoretical VES curves or to consult the Pylaev nomograms. Furthermore, the proof that the two geoelectric sections for models I and IV are equivalent is shown in figure 2*F* by the practical coincidence of their corresponding VES curves.

In the examples in figure 2, it is important to compare the directions and the magnitudes of deviation of the second DZ branches for models I and II, models I and III, and models I and IV, and to note that the directions are the same as those between the corresponding VES curves, but that the magnitudes are a function of the cancellation effect. Figure 3 shows a comparison between five pairs of equivalent and nearly equivalent VES curves and their corresponding DZ curves. These comparisons and the study of several others indicated that the following rules generally can be applied.

1. For monotonically deviating DZ branches which are ascending, horizontal, or gently descending (negative slopes of less than about −30°, the corresponding VES

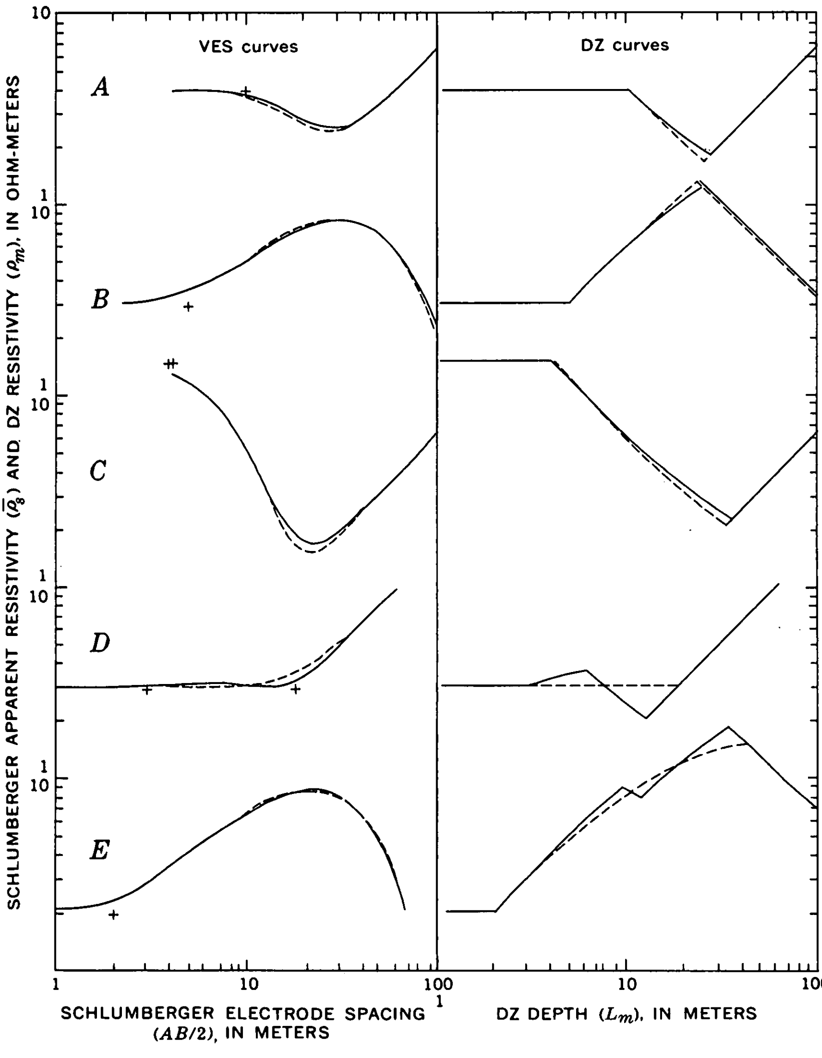

FIGURE 3.—Equivalent and nearly equivalent VES curves and their corresponding DZ curves. Pairs of nearly coinciding solid and dashed curves were calculated for different but practically equivalent models.

curves will deviate in the same direction and by about the same magnitude or less (curves *A* and *C* in fig. 2, and curves *A* and *B* in fig. 3).

2. For multilayer sections that contain low-resistivity layers which have a sufficiently large thickness and a sufficiently low resistivity to be represented on the corresponding DZ curve by a long steeply descending branch (length

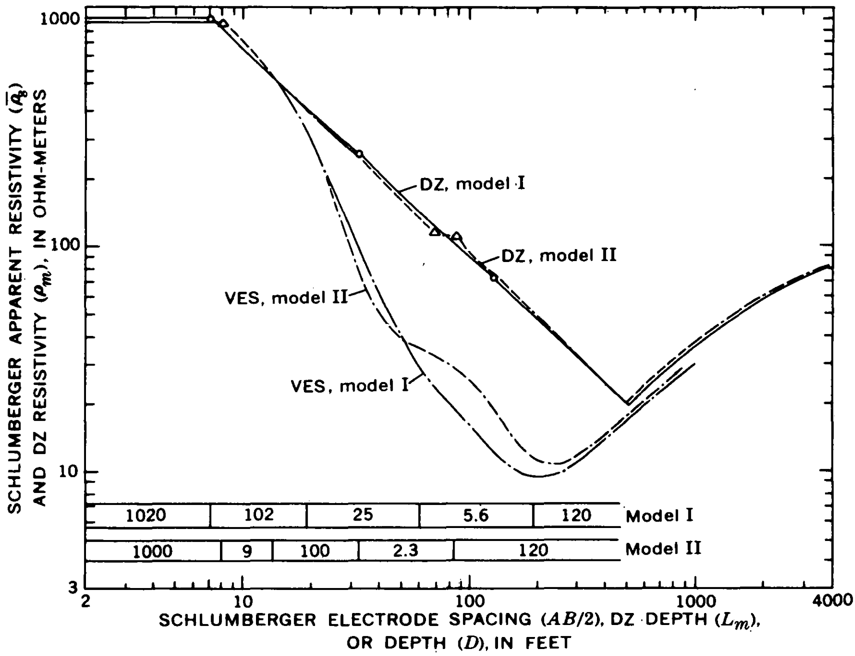

FIGURE 4.—Two geoelectric sections which are seemingly equivalent on DZ curves but are not equivalent on VES curves. Numbers in bars designate true resistivities, in ohm-meters.

approaching or exceeding a full logarithmic cycle and a negative slope approaching −45°), the maximum difference between two seemingly equivalent DZ curves is reflected by an almost equal or even much larger difference on the corresponding VES curves (curves *C*, fig. 3). Figure 4 shows an example encountered during the interpretation of VES data from Yellowstone National Park (Zohdy and others, 1973), where two DZ curves differ by about 10 percent, but the corresponding VES curves differ by about 35 percent. This example shows that DZ curves of geoelectric sections containing an HK segment with long steeply descending branches cannot always be smoothed into an equivalent Q or QQ curve.

3. For multilayer sections that contain several layers of alternating high and low resistivities, if these layers are represented on the corresponding DZ curves by a set of ascending, almost horizontal, or gently descending, saw-edge segments, then that set of segments can be replaced by a single DZ branch. The saw-edge DZ segments may

oscillate about the single DZ branch with amplitudes of as much as 20 percent without the corresponding VES curves varying by more than about 3 percent. (For example, see curves *D* and *E*, fig. 3.)

The stated properties of DZ curves make them very useful in the search for subtle solutions of VES curves that cannot be obtained easily by using partial curve-matching procedures.

PRINCIPLE OF SUPPRESSION

The equivalence of three-layer VES curves of the A-type and of the Q-type is explainable more often by the principle of suppression than by the rules of the principle of equivalence. For example, two three-layer VES curves of the Q-type may be equivalent, not necessarily because the values of T_2 are approximately equal in both models, but because the effect of the middle layer on the VES curve is so suppressed that the curve could be mistaken for a two-layer curve. This phenomenon is illustrated well by DZ curves. Figure 5 shows two equivalent

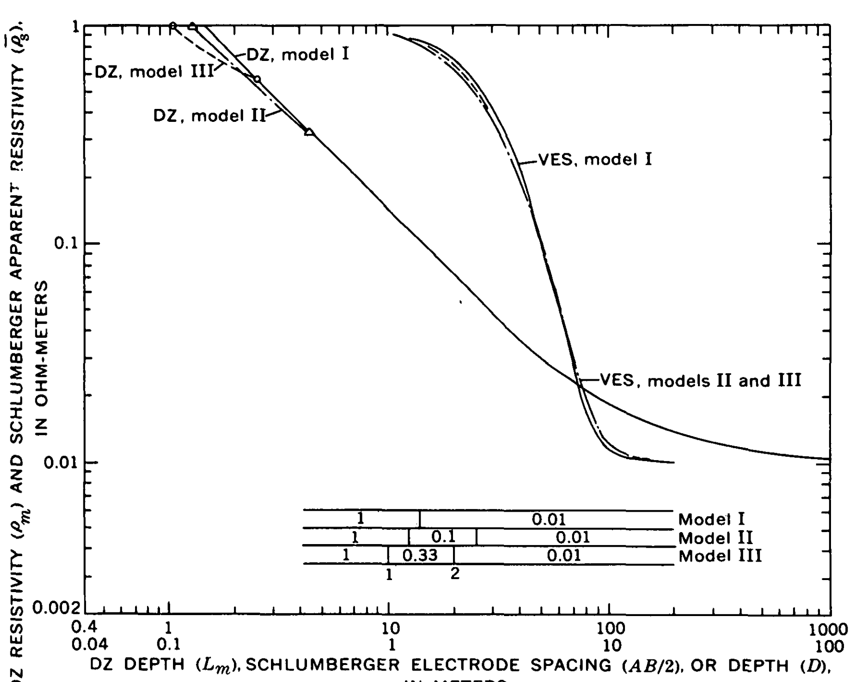

FIGURE 5.—Equivalence between two three-layer sections of the Q-type (models II and III) and one two-layer section (model I). Numbers in bars designate true resistivities. T_2 for model II=0.125 and T_2 for model III=0.33, but models are practically equivalent for VES measurements.

three-layer curves of the Q-type (models II and III) for which the value of T_2 in model III is about 2.6 times larger than it is in model II. The VES curve for the almost equivalent two-layer model (model I) is shown also to illustrate the extent of the suppression of the low-resistivity second layer by the much lower resistivity third layer. The corresponding DZ curves (fig. 5) are almost coincident except near the beginnings of the second branches—the same area where the three VES curves deviate most from one another.

Inasmuch as the value of T_2 is not nearly the same for the two three-layer Q-type sections, one could not use the principle of equivalence by T to calculate this equivalence between these two Q-type sections. By drawing DZ curves that are within about 3 percent of either of the DZ curves and solving for the layering using equations 18, 19, 20, and 21, one can create several nearly equivalent geoelectric sections.

ADJUSTMENT OF INTERPRETED RESISTIVITIES AT CONSTANT THICKNESSES

A four-layer VES curve of the KH-type is shown in figure 6. This curve is taken from the Orellana and Mooney album (1966) for an earth model (model I, fig. 6) with the arbitrary units of:

$$\begin{aligned}
&\rho_1=1, && h_1=1, && D_1=1;\\
&\rho_2=5, && h_2=2, && D_2=3;\\
&\rho_3=0.4, && h_2=10, && D_3=13;\\
&\rho_4=\infty, && h_4=\infty, && D_4=\infty.
\end{aligned}$$

Assuming that the curve represents field data for an unknown geoelectric section, it was interpreted using the three-layer album of the Rijkswaterstaat (1969) and the auxiliary-point diagrams. The Rijkswaterstaat album does not contain a set of curves for $\mu=\rho_2/\rho_1=5$. The results of the interpretation yielded the following section (model II, fig. 6):

$$\begin{aligned}
&\rho_1=1, && h_1=1, && D_1=1,\\
&\rho_2=6, && h_2=1.4, && D_2=2.4,\\
&\rho_3=0.56, && h_3=13.7, && D_3=16.1,\\
&\rho_4=\infty, && h_4=\infty, && D_4=\infty.
\end{aligned}$$

Assuming that the auxiliary-point model (model II, fig. 6) is an equivalent model to the reference model and assuming that the thicknesses given for the reference model I ($h_1=1$, $h_2=2$, $h_3=10$) were encountered in a drill hole near the VES station, the interpreted thicknesses of the second and third layers must

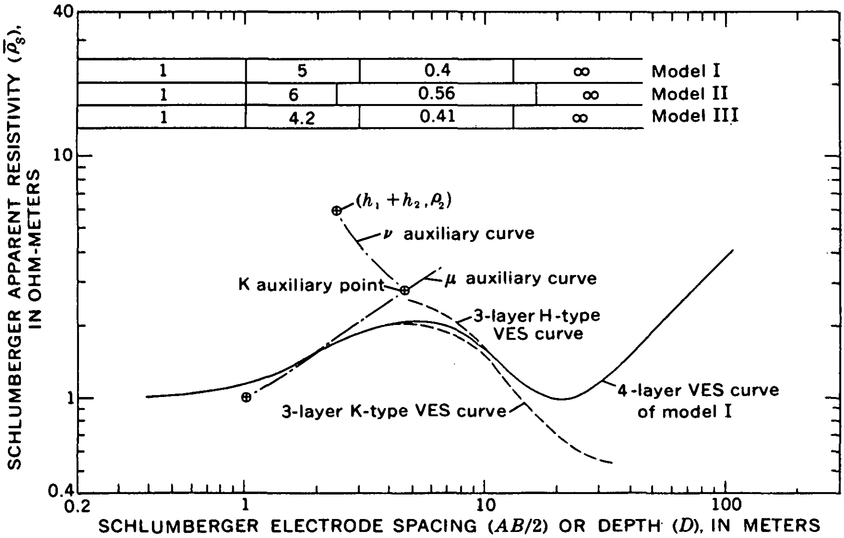

FIGURE 6.—Auxiliary-point interpretation of a four-layer VES curve of the KH-type. Numbers in bars designate true resistivities. K is the K-auxiliary point. Model I, reference model from Orellana and Mooney (1966); model II, auxiliary-point interpretation; model III, equivalent model for model II with depths to interfaces equal to those of model I, as determined from the use of the principle of equivalence.

be corrected. If the thickness of the second layer must be changed from 1.4 to 2, then according to the principle of equivalence by T, its resistivity, ρ_2', must be calculated from

$$\rho_2 h_2 = \rho_2' h_2',$$

or

$$\rho_2' = \frac{6 \times 1.4}{2} = 4.2.$$

With the second layer having a resistivity of 4.2 and a thickness of 2, the third-layer thickness is reduced from 13.7 to 13.1 (owing to the increase in thickness of the second layer by 0.6); therefore, its resistivity, ρ_3, must be adjusted by using the principle of equivalence by S

$$\frac{h_3}{\rho_3} = \frac{h_3'}{\rho_3'},$$

or

$$\rho_3' = \frac{13.1 \times 0.56}{13.7} = 0.5354.$$

Making the correction for the thickness of the third layer because its thickness should be reduced from 13.1 to 10, we use the principle of equivalence by S, once more, and calculate ρ_3'' from

$$\frac{h_3'}{\rho_3'}=\frac{h_3''}{\rho_3''},$$

or

$$\rho_3''=\frac{10\times 0.5354}{13.1}=0.4087\approx 0.41.$$

Therefore, by using the principle of equivalence we obtain a value for ρ_2 of 4.2 instead of 5 (error of about 16 percent) and a value for ρ_3 of 0.41 instead of 0.4 (error of about 2.5 percent). There is nothing in the principle of equivalence, as it is stated in the literature, to indicate that it is possible to calculate a resistivity of 5 for the second layer (without changing its thickness) so that it would have the correct thickness of 2.

Assuming that the auxiliary point solution (model II) is an equivalent solution to the reference model (model I) and using DZ curves we can obtain several practically equivalent four-layer solutions, with $h_1=1$, $h_2=2$, and $h_3=10$, as follows.

1. Plot the DZ curve for the section (model II) obtained from the auxiliary point interpretation as shown by the solid line in figure 7.
2. Using the DZ chart shown on plate 1*A*, place the point α at the origin of coordinates and trace the curve for $\nu=2$. If you use the DZ chart on plate 1*B*, place the point α (with the coordinates $L_m=1$, $\rho_m=1$) on the ordinate axis of the lower half of the DZ curve so that it coincides with the origin of one of the DZ curves and mark the point where this DZ curve intersects the dashed curve for $\nu=2$. Shift the point α upward, or downward, to the origin of another DZ curve and again mark the intersection of the DZ curve with the curve for $\nu=2$. Repeat this procedure with several DZ curves on the lower half of the DZ chart of plate 1*B*, then join the marked points with a smooth curve. Designate this curve as the $\nu_\alpha=2$ curve. This curve represents the termination of various DZ curves that originate at the point α, rise to different resistivity values, and terminate for a thickness of $h_2=2h_1$. Instead of the preceding graphical procedures, the Wang 370 program in table 1 can be used effectively to calculate the curve $\nu_\alpha=2$.

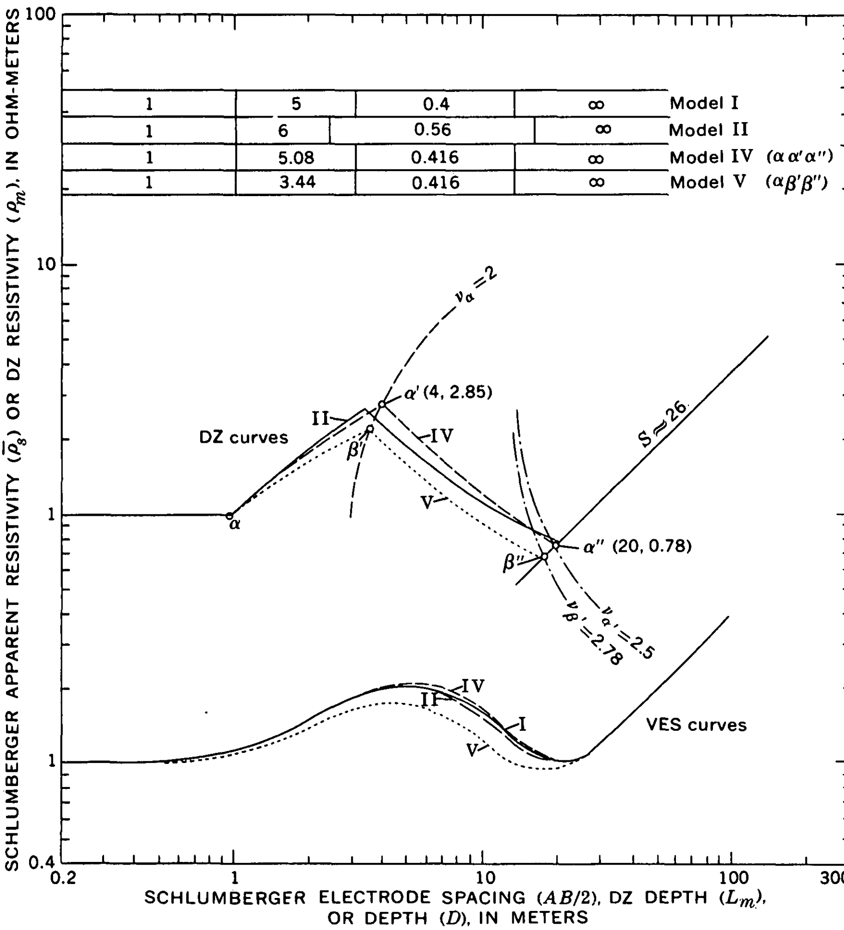

FIGURE 7.—Graphical evaluation of equivalence for models with constant layer thicknesses ($h_1=1$, $h_2=2$, $h_3=10$) from DZ curves. Numbers in bars designate true resistivities. Solid-line DZ curve is based on the layering of model II. VES curves are calculated for models I, II, IV, and V. α, α', α'', β', β'' are DZ points; ν_α, $\nu_{\alpha'}$, $\nu_{\beta'}$ are curves terminating DZ curves for constant thickness of subsequent layer. Model I, reference model; model II, auxiliary point model; models IV and V, equivalent and nonequivalent models of model II, with depths to interface equal to those of model I, as determined from the inversions of the DZ curves $\alpha\alpha'\alpha''$ and $\alpha\beta'\beta''$, respectively.

3. Mark the two points α' and β' on the $\nu_\alpha=2$ curve so that they fall within about 10 percent above and below the third branch of the DZ curve, respectively. For the point α', the pseudothickness of the layer that replaces the top two layers is equal to the abscissa, $L_{m_{\alpha'}}\approx 4$. Similarly for the

point β' the pseudothickness is about 3.6. Therefore, the effective relative thickness of the third layer with respect to the DZ points α' and β' is $10/4=2.5$ and $10/3.6=2.78$, respectively.

4. Place the point α' at the origin of coordinates of plate 1*A*, or on the ordinate axis of plate 1*B*, and, as explained in step 2, graphically construct or calculate (table 1) the line for $\nu'_\alpha=2.5=\text{constant}$. Then repeat the process with the point β' and draw the line for $\nu'_\beta=2.78=\text{constant}$.
5. Inasmuch as the value of the total conductance S must remain constant, ($S\approx 26$), extend the S line backward and mark the points α'' and β'' where the S line intersects the $\nu'_\alpha=2.5$ line (at α'') and the $\nu'_\beta=2.78$ line (at β''), respectively.

   The DZ curves marked by the points α, α', α'' and α, β', β'', represent two geoelectric sections in which the thicknesses h_1, h_2, and h_3 have the correct values of 1, 2, and 10 (which are the respective thicknesses of the reference model I), and for which the total conductance S is the same as observed on the VES curve. However, the second and third branches of the $\alpha\beta'\beta''$ curve fall completely below the DZ curve of model II (solid-line curve) which is based on the auxiliary point interpretation. Therefore, it is to be expected that a VES curve calculated for the section obtained from the inversion of the $\alpha\beta'\beta''$-DZ curve will not adequately fit the original VES data. The DZ curve marked by the points α, α', α'', however, better approximates the auxiliary point DZ curve (solid-line curve).

By using the following graphically determined coordinates from the $\alpha\alpha'\alpha''$-curve:

$$L_{m_1}=1,\ \rho_{m_1}=1,$$
$$L_{m_2}=4,\ \rho_{m_2}=2.85,$$
$$L_{m_3}=20,\ \rho_{m_3}=0.78,$$

the following section (model IV, fig. 7) is calculated from equations 19 to 21 (table 3):

$$\rho_1=1,\quad h_1=1,$$
$$\rho_2=5.076,\quad h_2=2.05,$$
$$\rho_3=0.416,\quad h_3=10.09,$$
$$\rho_4=\infty,\quad h_4=\infty.$$

The above thicknesses and resistivities are in excellent agreement with the reference model (model I) taken from the Orellana and Mooney album.

From the foregoing example, the following observations and predictions can be made:

1. The inversion of the $\alpha\alpha'\alpha''$-DZ curve resulted in model IV, which is almost identical to the reference model (model I), and, therefore, the corresponding VES curves must practically coincide.
2. By comparing the $\alpha\alpha'\alpha''$-DZ curve (model IV) with the solid-line DZ curve (model II), we can predict that the corresponding VES curves should coincide except to the right of the maximum, where the VES curve of model II should fall slightly below the VES curve of model IV.
3. By comparing the $\alpha\beta'\beta''$-DZ curve with the $\alpha\alpha'\alpha''$-DZ curve and with the solid-line DZ curve, we can predict that the maximum and the minimum on the VES curve for model V would fall below the corresponding maximums and minimums on the VES curves of models I, II, and IV. These predictions were confirmed by the calculation of VES curves, as shown in the lower part of figure 7.

With the above technique, the investigator can determine several other resistivity values (including the one obtained earlier by using the principle of equivalence) for the same thicknesses of $h_1=1$, $h_2=2$, and $h_3=10$. These different equivalent solutions cannot be obtained by using the simple concept of the principle of equivalence.

ADJUSTMENT OF INTERPRETED THICKNESSES AT CONSTANT RESISTIVITIES

The same reference VES curve (model I) and the same auxiliary point interpretation (model II) of the previous section are considered here. However, let us assume that from the interpretation of other VES data and (or) from electrical logging data, the second and third layer resistivities are known to be 5 and 0.4 instead of 6 and 0.56, respectively. Therefore, the thicknesses of the second and third layers must be adjusted.

By using the principle of equivalence by T, as in the first example, we calculate the thickness of the second layer to be

$$h_2=\frac{6\times 1.4}{5}=1.68.$$

This reduces the thickness of the third layer from 13.7 to 13.42, and, therefore, its resistivity must be adjusted (by the principle of equivalency by S) to

$$\rho_3=\frac{0.56\times 13.42}{13.7}=0.5485.$$

However, as ρ_3 should be made equal to 0.4, then the thickness of the third layer must be readjusted to the value

$$h_3' = \frac{13.42 \times 0.4}{0.5485} = 9.78.$$

As in the previous section, the calculated second-layer parameter, which here is the thickness, is underestimated by about 16 percent of what it could be. By using DZ curves, a range of thicknesses can be determined for the second and third layers with their resistivities remaining constant (5 and 0.4, respectively).

In figure 8 the DZ curve for model II (figs. 6, 7), which is based on the auxiliary-point interpretation, is shown as the solid curve. The other DZ curves in the figure are constructed as follows:

1. Place the point $\alpha(L_m=1, \rho_m=1)$ at the origin of coordinates of plate 1*A*, or on the ordinate axis of plate 1*B* so that the abscissa axis of plate 1*B* coincides with the ordinate of the resistivity value of 5.
2. Trace the DZ curve for $\mu=5$. This curve also can be calculated by using the Wang 370 program given in table 2.
3. Select a point on the DZ curve for $\mu=5$ (such as the point α_1, α_2, or α_3 in fig. 8), and place it on the origin of coordinates of plate 1*A*, or on the ordinate axis of plate 1*B* and shift this point upward or downward so that the abscissa axis of plate 1*B* coincides with the resistivity value of 0.4. Trace the descending DZ curve for $\mu=0.4$ to where it intersects the *S* line or its backward extension.
4. Repeat step 3 for other α points that fall on the $\mu=5$ curve so that the traced DZ curve does not depart appreciably from the solid-line DZ curve.

Figure 8 shows three DZ branches that were constructed according to the above procedure. The $\alpha_1\alpha_1'$ branch originates at the point α_1, which lies very close to the solid-line DZ curve, but it departs from that curve appreciably at its terminal point α_1'. Conversely, the $\alpha_3\alpha_3'$ branch starts at the point α_3, which does not lie sufficiently close to the solid-line DZ curve, but it terminates at the point α_3' which almost coincides with the third terminal point on the solid-line DZ curve. Neither the $\alpha_1\alpha_1'$ branch nor the $\alpha_3\alpha_3'$ branch approximates the second branch of the solid-line DZ curve adequately; but the $\alpha_2\alpha_2'$ branch does, and it represents one of the closest fits to the solid-line DZ curve. The geoelectric sections which are based on the inversion of the three DZ

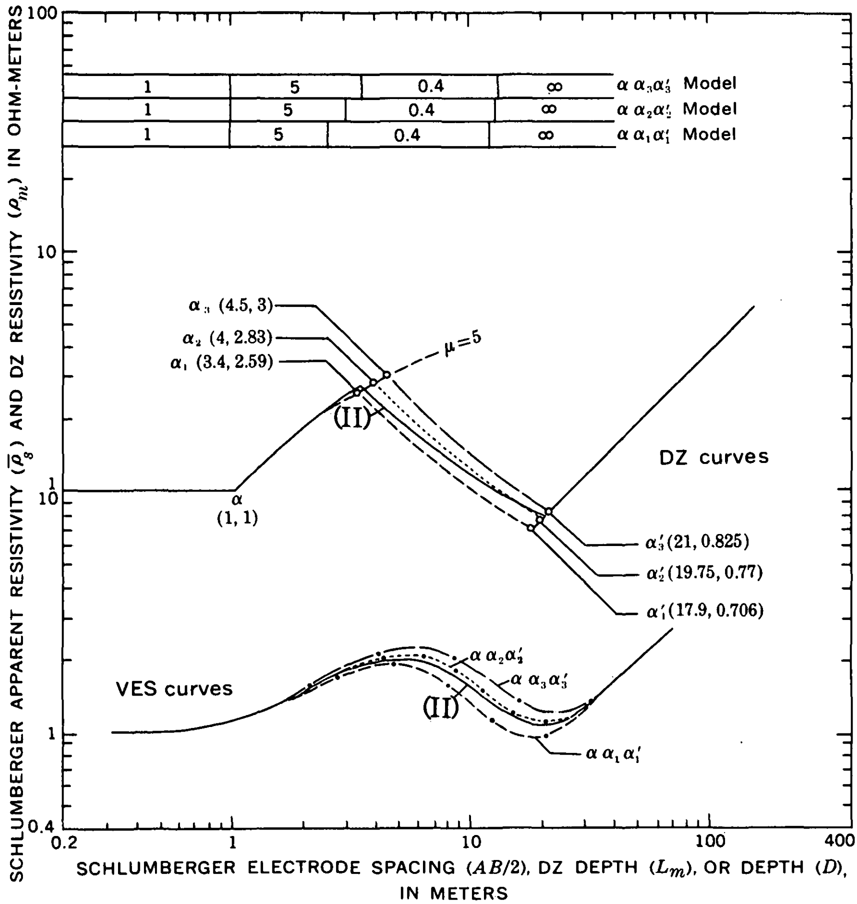

FIGURE 8.—Graphical evaluation of equivalence for models with constant layer resistivities ($\rho_1=1$, $\rho_2=5$, $\rho_3=0.4$) from DZ curves. Numbers in bars designate true resistivities, in ohm-meters; $\mu=\rho_2/\rho_1$; α, α_1, α_2, α_3, DZ points. Solid-line DZ curve and solid-line VES curve correspond to model II in figures 6 and 7.

curves $\alpha\alpha_1\alpha_1'$, $\alpha\alpha_2\alpha_2'$, and $\alpha\alpha_3\alpha_3'$ are shown at the top of figure 8, and the VES curves which were calculated for these sections are compared with the VES curve for model II at bottom of the figure.

IMPROVEMENT OF THEORETICAL FIT TO OBSERVED VES CURVES

The VES curve depicted by long dashes in the upper part of figure 9 was taken from the collection of theoretical curves published by Flathe (1963). It represents the following five-layer geoelectric section of the QHK type:

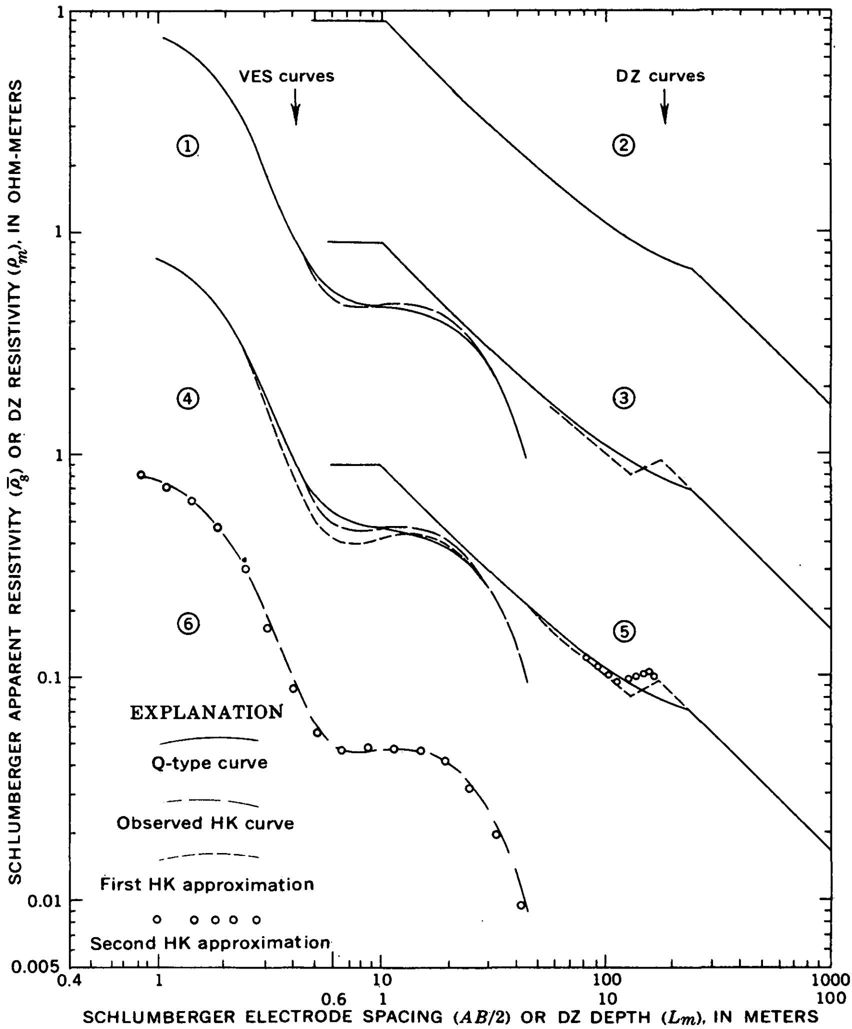

FIGURE 9.—DZ curves used to adjust a preliminary interpretation of a multilayer VES curve. Numbers in circles designate steps, as explained in the text, to achieve an almost perfect match between observed and calculated VES data.

$$
\begin{aligned}
h_1 &= 1, \rho_1 = 9, \\
h_2 &= 1, \rho_2 = 1, \\
h_3 &= 1, \rho_3 = 0.11, \\
h_4 &= 7, \rho_4 = 1, \\
h_5 &= \infty, \rho_5 = 0.
\end{aligned}
$$

Only four layers are visually distinguishable because the effect of the second layer is suppressed. Let us assume that this curve was obtained in the field and that it is to be interpreted as a four-layer curve of the HK type. Ordinarily, we would proceed to interpret the curve by partial curve matching, using the auxiliary point method. Instead, a different method is considered, as shown by the sequence of numbered steps in figure 9.

Step 1 in figure 9 shows an approximate match between the "observed" HK curve and a three-layer curve of the Q type taken from the C.G.G. album (Compagnie Générale de Géophysique, 1963), which is depicted by the solid curve. This Q-type curve corresponds to the following section:

$$h_1 = 1, \rho_1 = 9,$$
$$h_2 = 16, \rho_2 = 0.474,$$
$$h_3 = \infty, \rho_3 = 0.$$

The two curves match well except near the minimum and the maximum of the "observed" curve, which are not particularly well developed. This type of low-quality match is analogous to what is observed occasionally when an auxiliary point solution is checked by the calculation of the corresponding VES curve. The required adjustments (in thicknesses, resistivities, and, sometimes, the number of layers) can be made effectively by the use of DZ curves.

Step 2 in figure 9 shows the solid-line DZ curve constructed for the three-layer Q-type section given above. Inasmuch as the three-layer Q-type VES curve does not fit the "observed" curve near the minimum and maximum but fits the first and last branches sufficiently well, a minimum and a maximum are constructed on the three-layer DZ curve to change it into a four-layer DZ curve, as shown in step 3 in figure 9. Solving for the thicknesses and resistivities from equations 18 to 21 (table 3) we get

$$h_1 = 1, \rho_1 = 9,$$
$$h_2 = 5, \rho_2 = 0.32,$$
$$h_3 = 4, \rho_3 = 1.53,$$
$$h_5 = \infty, \rho_4 = 0.$$

The VES curve for this four-layer section was calculated, and in step 4 of figure 9 it is depicted as the short-dashed curve and is compared with the "observed" VES curve and to the three-layer Q-type VES curve. The comparison indicates that the calculated minimum is too low and that the maximum is not sufficiently high. Therefore, by reconsidering the DZ curves in step 3, two new DZ points are selected, as shown in step 5 of figure 9 so that the amplitude of the DZ minimum is reduced and that of the DZ maximum is increased. Solving for the new thicknesses and resistivities from the new set of DZ points, the following section is obtained:

$$h_1=1,\quad \rho_1=9,$$
$$h_2=4.5,\ \rho_2=0.38,$$
$$h_3=4.4,\ \rho_3=1.35,$$
$$h_4=\infty,\quad \rho_4=0.$$

The calculation of the VES curve for this four-layer section fits the "observed" curve within about 1 percent, as shown in step 6 of figure 9.

SIMPLIFICATION OF AN OVERCOMPLICATED LAYERING

A parametric VES (Kalenov, 1957; Zohdy, 1968a) is a VES made near a well to evaluate the parameters (thicknesses and resistivities) of the geoelectric section. Well-logging data in the form of lithologic, water-quality, and (or) electric logs should be available for correlation with the interpretation of the VES curve.

Digitized electric logs of many wells commonly contain so many layers with highly contrasting resistivities that in using certain computer programs, the time to calculate the corresponding VES curve can take more than 15 minutes and can even reach several hours. The large number of layers on an electric log can be reduced by combining several layers, which form an anisotropic layer, into a single equivalent isotropic layer, and then repeating the process for other groups of layers. This type of calculation (Kalenov, 1957) is tedious without the aid of a desk calculator, and if many layers are involved, it can be difficult to decide which layers should be combined. The use of DZ curves is very effective in making this decision.

Figure 10 shows the distribution of resistivity with depth, plotted on log-log coordinates, for data obtained with a Schlumberger laterolog LL–7 in a deep well north of Aspermont, Tex. The resistivity distribution in the upper 80 feet is based on the interpretation of a few VES curves that were obtained in the survey area (Zohdy and Jackson, 1973). The well was drilled to a depth of 4,700 feet and did not penetrate high-resistivity basement rocks. However, an infinitely thick basement having a resistivity greater than 500 ohm-meters was assumed to exist at a depth of 4,700 feet. The layering depicted in figure 10 shows an extremely anisotropic geoelectric section consisting of very high

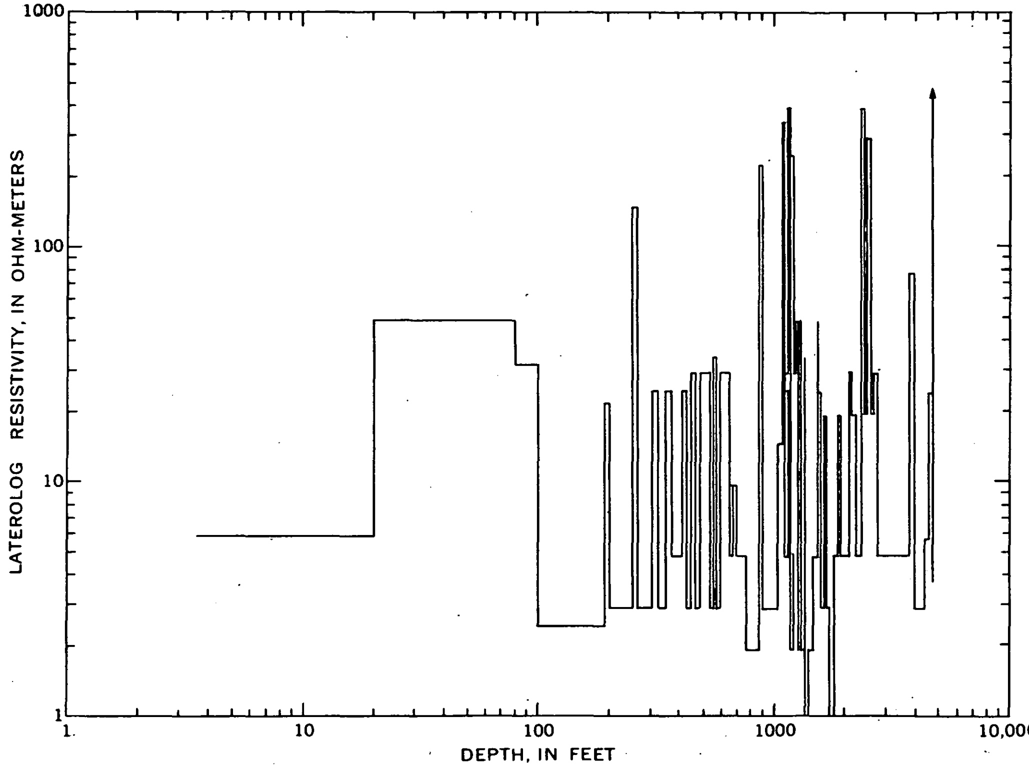

FIGURE 10. — Resistivity distribution with depth in a 4,700-foot-deep oil well near Aspermont, Texas. From VES data, for the upper 80 feet, and from a Schlumberger laterolog LL–7, for the remainder of the section.

resistivity layers of 200 and 400 ohm-meters, alternating with very low resistivity layers of 1 and 2 ohm-meters. An attempt to calculate the VES curve on the IBM 360/65 computer for this 75-layer model, using a program based on the work of Mooney, Orellana, Pickett, and Tornheim (1966), was terminated after 20 minutes of computer time had elapsed without obtaining the required apparent resistivities.

The DZ curve for the 75-layer model is shown in figure 11. By visual inspection, we can reduce the number of layers from 75 to 10 by marking the DZ points 1 through 10, as shown in the graph, which fall on, or very close to, the 75-layer DZ curve. These points, which define what is to be the smoothed DZ curve, are chosen so that a two-layer DZ curve that joins any two of these points would pass through, or within about 5 percent of, all intermediate DZ points. With this type of manual smoothing, which preferably should be made with the help of one of the DZ charts or with the two DZ master curves of Orellana (1963), the interpreter may choose points that are already on the complicated DZ curve, or he may select points that are not on the DZ curve but that fall very close (within about 5 percent) to it. The interpreter must not generate new DZ points which would be connected with a line inclined to the abscissa axis at an angle greater than ±45°, because these are the limiting angles of any branch on a DZ curve. If this rule is accidentally overlooked for an ascending branch, then the use of the Wang program given in table 3 to calculate thicknesses and resistivities will result in a negative thickness, which is a good indicator that an error was committed. For a descending branch, however, positive but erroneous resistivities and thicknesses are obtained with the given Wang program.

By substituting the coordinates L_m and ρ_m of the 10 DZ points in equations 18–21 (see table 3 for the Wang program), we can solve for the thicknesses and resistivities of a 10-layer geoelectric section which is electrically equivalent to the 75-layer section obtained from the electric log. Figure 12 shows this equivalent geoelectric section (10-layer model), which was calculated, using the Wang program, in about 5 minutes. The VES curve for this 10-layer section was calculated on the IBM 360/65 computer, using the Mooney program, in less than 5 minutes; it also is shown in figure 12 (10-layer VES curve).

The visual inspection and subsequent interpretation, by partial curve matching, of the calculated 10-layer VES curve indicated that the curve essentially reflects the presence of only 6 layers, not 10. Therefore, the representation of the 75 layers, from the electric log, amounts to only 6 layers on the VES curve. In figure

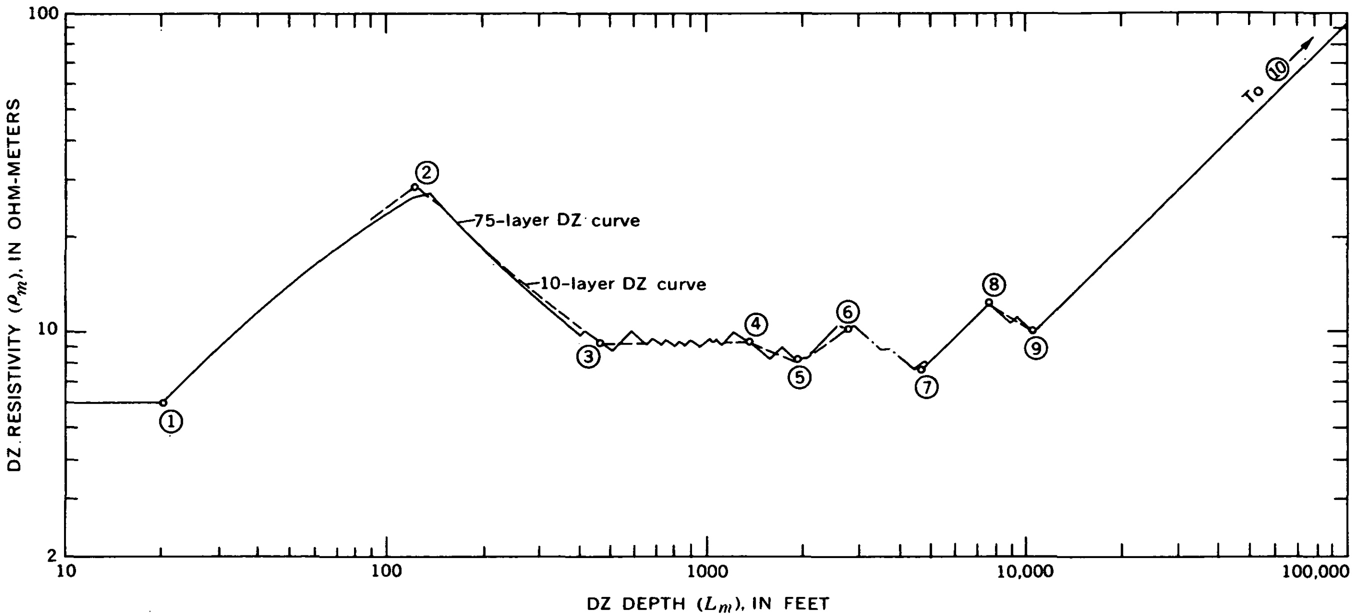

FIGURE 11.— Seventy-five-layer DZ curve for the resistivity–depth distribution shown in figure 10. Numbers in circles designate layers of the smoothed 10-layer DZ curve.

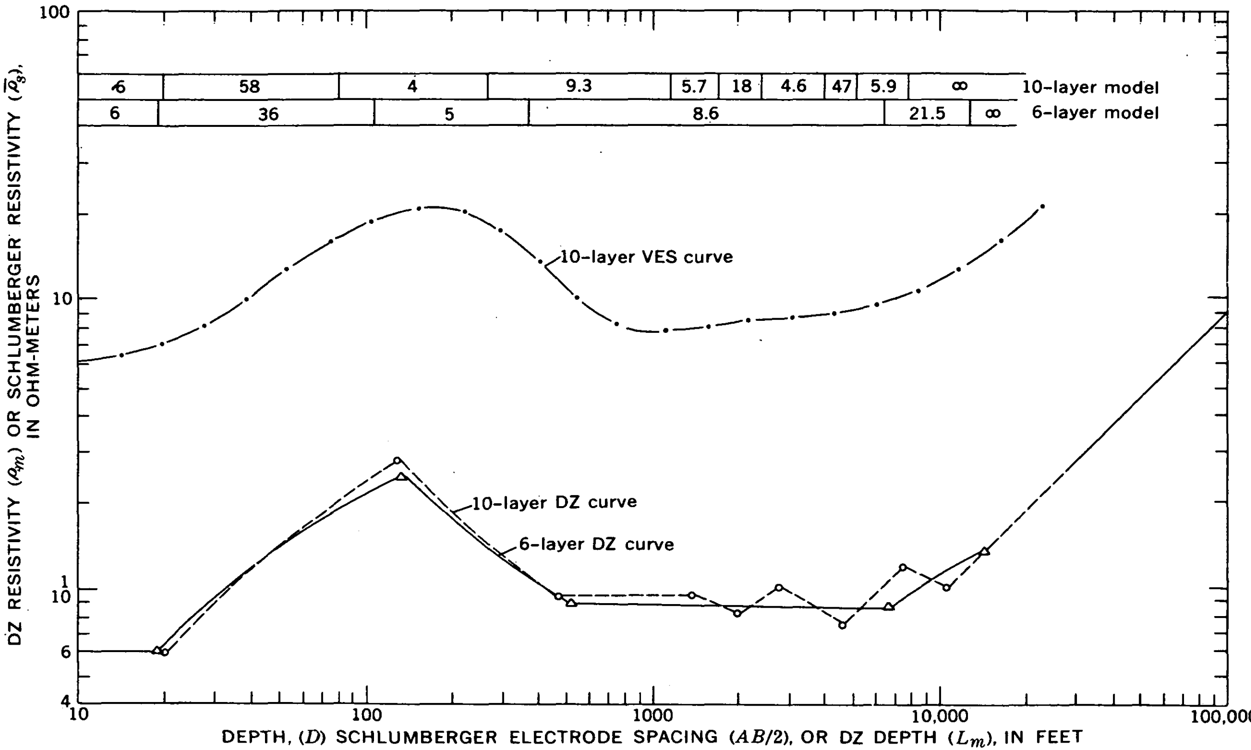

FIGURE 12. — Ten-layer model obtained from solution of smoothed DZ curve shown in figure 11. Six-layer model obtained from graphical interpretation of computed 10-layer VES curve. Comparison of 10-layer and 6-layer equivalent DZ curves. Numbers in bars designate interpreted true resistivities, in ohm-meters.

12 also, the DZ curve for the 6 layers interpreted from the synthetic VES curve is compared with the 10-layer DZ curve. The comparison indicates that (as mentioned earlier) two DZ curves may be separated by more than 5 percent and still represent equivalent geoelectric sections. In this example the 10-layer DZ curve oscillates around the 6-layer DZ curve and departs from it on the horizontal and ascending segments by as much as 15 percent.

When a DZ curve or a VES curve is calculated from a digitized electric log, it is assumed that all the layers on the electric log have very large lateral extents and that the deeper these layers are, the larger their lateral extent must be. Because of this assumption, large values of the coefficient of anisotropy are often calculated from the analysis of electrical logging data. For example, the depth to basement from the smoothed 10-layer DZ curve is about 7,700 feet, as compared with 4,700 feet from the 75-layer log. Therefore, the total coefficient of anisotropy, λ, is equal to about 1.64. The graphical interpretation of the 10-layer VES curve, in terms of a 6-layer section, results in an even larger value for λ of about 2.76.

The justification for the assumption that all the layers have very large lateral extents can be examined by comparing the values of the total longitudinal conductance, S, as determined from the DZ curve of the digitized log and from a parametric VES curve obtained near the well. If the two values are almost equal, then the validity of the assumption that all the layers on the electric log have very large lateral extent is possible but is not necessarily established.

If the value of S is significantly smaller on the field VES curve than on the DZ curve of the electric log, then this may result from one or more of the following causes: (1) The layers on the digitized log do not all have large lateral extent, and, therefore, an unnecessarily large coefficient of anisotropy has been calculated in smoothing the digitized section, (2) the S line on the VES curve has been falsely measured, possibly because of current leakage from damaged insulation on the current cables (Zohdy, 1968b), (3) the well was drilled in a depression in the basement whereby it has penetrated a larger thickness of conductive materials near the bottom than the average thickness of the section beneath the VES line, and (4) the true resistivity of conductive beds on the digitized log was underestimated, and that of resistive beds was overestimated.

If the value of S is larger on the VES curve than on the DZ curve of the digitized log, then this may be attributed to one or more of the following causes: (1) Some of the layers on the electric log are microanisotropic and, therefore, appear to be thicker on a VES curve than on the electric log, (2) resistivities of resistive layers on the electric log were underestimated, and those of conductive layers overestimated, and (3) the well was drilled on a high in the basement surface, whereby it penetrated a smaller total thickness of sedimentary rocks than the average total thickness beneath the VES line.

COMPLICATION OF AN OVERSIMPLIFIED LAYERING

The preceding section showed how a complicated geoelectric section, consisting of many layers, was simplified and reduced to an equivalent section consisting of only a few layers. This section examines the possibilities for reversing this process, whereby an oversimplified graphical interpretation of a VES curve is made more complex by the use of DZ curves.

Near the margins of sedimentary basins, coarse high-resistivity materials are commonly intercalated with fine low-resistivity materials and where volcanic flows, if present, generally terminate and are intertongued with sedimentary deposits. Consequently, VES curves obtained near basin margins are often difficult to interpret. The interpreter, though he may be able to obtain a good match for these VES curves, may find it difficult to correlate his interpreted layer thicknesses and resistivities for these soundings with his interpretations of other soundings. His failure to correlate some of his VES-curve interpretations indicates that he should interpret certain VES curves in terms of five or six layers instead of three and four layers. The use of the auxiliary point method (Kalenov, 1957; Keller and Frischknecht, 1966; Zohdy, 1965) and other similar graphical methods may not be adequate to achieve the sought-for solution. This inadequacy of the auxiliary-point method is caused primarily by the effects of the principle of suppression and by the smallness of the effective relative conductances or effective relative resistances[1] of some of the layers in the section.

Figure 13*A* shows a VES curve which was obtained by Zohdy and Eaton (1971) along a profile of soundings north of Bowie, Ariz. The curve was interpreted (using the auxiliary-point method) in terms of the five-layer section shown below the curve. The theoretical five-layer VES curve, which was calculated to verify the accuracy of the graphical interpretation, fitted the ob-

[1]The effective relative conductance, or resistance, of a layer is the longitudinal conductance or the transverse resistance, respectively, defined in terms of the effective relative thickness (Flathe, 1963) and resistivity, instead of its thickness and resistivity only. Flathe (1963, 1967) presented a few examples of the inadequacy of the use of the auxiliary-point method for approximating predetermined solutions.

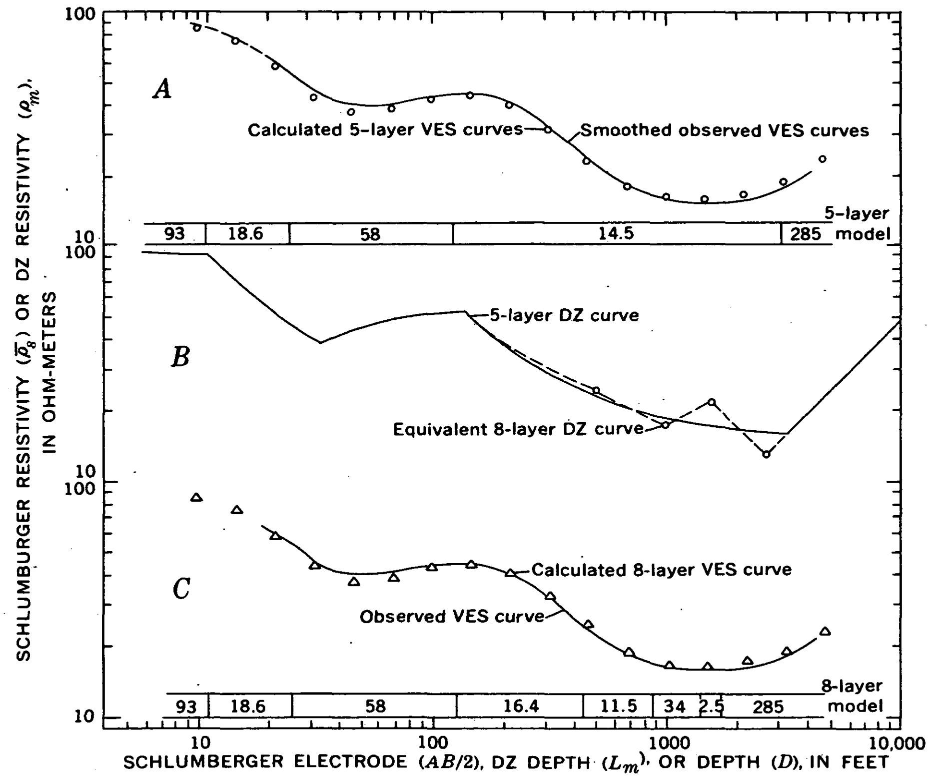

FIGURE 13. — Complicating a five-layer preliminary interpretation into an equivalent eight-layer interpretation through the use of DZ curves. Numbers in bars designate interpreted true resistivities, in ohm-meters.

served curve very well. However, it was difficult to correlate this solution with the interpretation of other VES curves and with the interpretation of gravity and seismic-refraction data. The thick fourth layer of 14.5 ohm-m resistivity correlated poorly with the interpretation of VES curves obtained at neighboring VES stations, and it caused the calculated depth to basement to be about 3,080 feet. The DZ curve for the five-layer model was constructed and (fig. 13*B*) modified into a more complex, but equivalent, eight-layer section. This modification was made so that the section obtained from inverting the eight-layer DZ curve could be correlated satisfactorily with the interpretation of other VES curves. The depth to basement was reduced from about 3,080 feet to about 1,650 feet, and a layer of about 34 ohm-meter resistivity (silicic volcanic rocks(?)) was included in the section from a depth of about 870 feet to a depth of about 1,350 feet. Figure 13*C* illustrates that the theoretical eight-layer VES curve fits the observed VES curve just as closely as did the five-layer curve.

In addition to the just-stated solution, other equivalent eight-layer sections were determined through the manipulation of DZ curves to obtain solutions that were in better agreement with all the available geologic and geophysical information. One of the most logical eight-layer solutions, which agrees well with the seismic-refraction interpretation, is shown in figure 14 together with the five-layer and eight-layer DZ curves, and the observed and calculated best fitting eight-layer VES curves. This particular solution was, in part, pointed out by the method of automatic interpretation of VES curves (Zohdy, 1973).

SUMMARY AND CONCLUSIONS

The methods of the construction and inversion of DZ curves were presented in detail. The use of DZ curves were shown to have significant value in expanding the known limits of the principle of equivalence beyond the constraints of the Pylaev nomograms, especially with respect to the equivalence between multilayer geoelectric sections containing different numbers of layers. For most geoelectric sections, the modulation of the VES curve and especially its deviation from another VES curve can be predicted by comparing the corresponding DZ curves. This property can be used advantageously by the interpreter (especially if an interactive computer-graphics system is available) either to improve the quality of a theoretical fit to an observed VES curve, or to obtain a layering which is correlatable with the interpretation of other VES curves. If geologic information is available from drilling data prior to the interpretation of the VES curves, then DZ curves

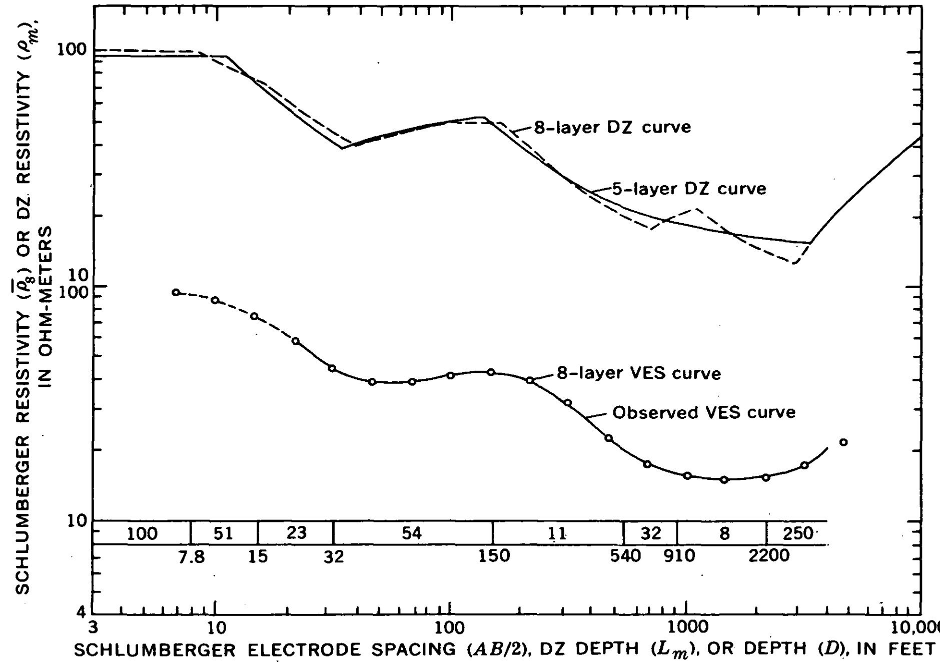

FIGURE 14. — Evaluation of a second eight-layer interpretation of the VES curve shown in figure 13. Numbers in bar designate interpreted true resistivities, in ohm-meters; those beneath designate depth, in feet.

can be used to implement those interpretations so that the geological constraints on the number of layers, their thicknesses, and resistivities can be accounted for in the interpreted geoelectric sections. Similarly, if drilling data become available after the preliminary interpretation of the VES curves is completed, then DZ curves can be instrumental in modifying the interpretations of certain VES curves, in order to remove any major discrepancies between geoelectrical predictions and geologic findings.

REFERENCES CITED

Bhattacharya, P. K., and Patra, H. P., 1968, Direct-current geoelectric sounding, principles and interpretation: New York, Elsevier, 135 p.

Cagniard, Louis, 1948, Importance of the anisotropy phenomena in the problem of interpretation of electrical sounding data—practical consequences: Inst. Physique Globe [Paris] Annales, New Ser., v. 4, pt. 3, p. 3–28 [in French].

Compagnie Générale de Géophysique, 1963, Master curves for electrical soundings [2d rev. ed.]: The Hague, European Assoc. Explor. Geophysicists.

Dakhnov, V. N., 1953, Electrical prospecting for petroleum and gas deposits: Moscow, Gostoptekhizdat, 497 p. [in Russian].

Flathe, Herbert, 1958, Geoelectrical investigations on clay deposits in Western Germany, *in* Geophysical surveys in mining, hydrological, and engineering projects: The Hague, European Assoc. Explor. Geophysicists, p. 170–185.

———1963, Five-layer master curves for the hydrogeological interpretation of geoelectric resistivity measurements above a two-story aquifer: Geophys. Prosp. [Netherlands], v. 11, p. 471–508.

———1967, Interpretation of geoelectrical resistivity measurements for solving hydrogeological problems: Mining and Groundwater Geophysics/1967, Canada Geol. Survey Econ. Geology Rept. 26, p. 580–597.

Ghosh, D. P., 1971, The application of linear filter theory to the direct interpretation of geoelectrical resistivity sounding measurements: Geophys. Prosp. [Netherlands], v. 19, p. 192–217.

Kalenov, E. N., 1957, Interpretation of vertical electrical sounding curves: Moscow, Gostoptekhizdat, 472 p. [in Russian].

Keller, G. V., and Frischknecht, F. C., 1966, Electrical methods in geophysical prospecting: New York, Pergamon Press, 517 p.

Koefoed, O., 1968, The application of the kernel function in interpreting geoelectrical measurements: Stutgart, Geoexplor. Mon., ser. I, no. 2, Gebruder Borntraeger, 111 p.

Lasfargues, Pierre, 1957, Prospection électrique par courants continus: Paris, Masson et Cie, 290 p.

Maillet, Raymond, 1947, The fundamental equations of electrical prospecting: Geophysics, v. 12, no. 4, p. 529–556.

Mooney, H. M., Orellana, Ernesto, Pickett, Harry, and Tornheim, Leonard, 1966, A resistivity computation method for layered earth models: Geophysics, v. 31, no. 1, p. 192–203.

Orellana, Ernesto, 1963, Properties and drawing of the so-called Dar Zarrouk curves: Geophysics, v. 28, no. 1, p. 99–110.

Orellana, Ernesto, and Mooney, H. M., 1966, Master tables and curves for vertical electrical sounding over layered structures: Madrid, Interciencia, 150 p., 66 tables.

Rabinovich, B. I., 1965, Fundamentals of the method of field difference [in Russian]: Prikladnaya Geofizika, v. 43, p. 47–59 [English translation by I. Mittin, U.S. Geol. Survey, Denver, Colo., 21 p.].

Rijkswaterstaat, 1969, Standard graphs for resistivity prospecting: The Netherlands, European Assoc. Explor. Geophysicists.

Shkabarnia, N. G., and Gritsenko, V. G., 1971, Interpretatsiia kriv'ikh elektricheskogo zondirovaniia s primeneniem EVM: Izdatel'ctvo "Nedra," Moscow, 113 p.

Stefanesco, S. S., Schlumberger, Conrad, and Schlumberger, Marcel, 1930, Sur la distribution électrique potentielle autor d'une prix de terre ponctuelle dans un terrain à couche horizontales, homogènes et isotrope: Jour. Physique et le Radium, v. 11, no. 1, p. 132–140.

Strakhov, V. N., and Karelina, G. N., 1969, On the interpretation of vertical electrical sounding data with an electronic calculating machine [in Russian]: Prikladnaia Geofizika, v. 56, p. 118–129.

Zohdy, A. A. R., 1964, Earth resistivity and seismic refraction investigations in Santa Clara County, California: Stanford Univ. unpub. Ph. D. thesis, 132 p.

———1965, The auxiliary point method of electrical sounding interpretation and its relationship to the Dar Zarrouk parameters: Geophysics, v. 30, no. 4, p. 644–660.

———1968a, A rapid graphical method for the interpretation of A- and H-type electrical soundings: Geophysics, v. 33, p. 822–833.

———1968b, The effect of current leakage and electrode spacing errors on resistivity measurements, *in* Geological Survey research 1968: U.S. Geol. Survey Prof. Paper 600–D, p. D258–D264.

———1969, A new method for differential resistivity sounding: Geophysics, v. 34, p. 924–943.

———1973, Automatic interpretation of resistivity sounding curves using modified Dar Zarrouk functions [abs.]: Geophysics, v. 38, no. 1, p. 196–197.

Zohdy, A. A. R., and Eaton, G. P., 1971, Electrical resistivity sounding of the aquifer system near Bowie, Cochise County, Arizona: Geol. Soc. America Abs. with Programs, v. 3, no. 2, p. 223–224.

Zohdy, A. A. R., and Jackson, D. B., 1973, Recognition of natural brine by electrical soundings near the Salt Fork of the Brazos River, Kent and Stonewall Counties, Texas: U.S. Geol. Survey Prof. Paper 809–A, 14 p.

Zohdy, A. A. R., Anderson, L. A., and Muffler, L. J. P., 1973, Resistivity, self potential, and induced polarization surveys of a vapor-dominated geothermal system: Geophysics, v. 38, no. 6, p. 1130–1144.

☆ U.S. GOVERNMENT PRINTING OFFICE: 1974—677-306/18