Estimation of Petroleum Exploration Success and the Effects of Resource Base Exhaustion Via a Simulation Model

GEOLOGICAL SURVEY BULLETIN 1328

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CONTENTS

ILLUSTRATIONS

TABLES

ESTIMATION OF PETROLEUM EXPLORATION SUCCESS AND THE EFFECTS OF RESOURCE BASE EXHAUSTION VIA A SIMULATION MODEL

By Lawrence J. Drew

ABSTRACT

A random-walk model was used to study several aspects of the process of exploring for petroleum deposits. The Powder River Basin, Wyo., was chosen as a control area for this study.

The outcome of exploration is expressed by performance criteria; these include the average quantity of petroleum discovered, the average number of deposits discovered, the probability of "gambler's ruin," and the probability of attaining various levels of success. The incremental return per exploratory well was determined.

Several of the more significant results predicted by the model are:

1. Increasing the number of exploratory wells per program increases, but at a decreasing rate, the average number of deposits and the average quantity of petroleum expected to be discovered. (Ten wells yield an average of 0.22 deposits containing an average of 13.1 million barrels of petroleum; 100 wells yield an average of 2.11 deposits containing an average of 122.8 million barrels of petroleum.)
2. The probability of gambler's ruin (drilling consecutive unsuccessful wells) decreases as the number of wells per program increases. (Ten wells have a probability of gambler's ruin of 0.801; with 100 wells, the probability is 0.103)
3. The expected incremental return per exploratory well decreases as the number of exploratory wells drilled increases. (At the 5-well intensity, the incremental (next) well returns 1.31 million barrels; at the 450-well intensity, the incremental well returns 0.77 million barrels)

A resource base exhaustion study determined the level at which different sizes of exploration programs will not achieve a specified rate of return.

Random-walk allocation (which excludes geologic information from influencing discovery potential) of the number of exploratory wells to be drilled in an area was used to estimate the probability of discovering specific deposits during various time periods. If the exploratory drilling effort expended in the Powder River Basin between 1952 and 1968 (2,812 wells) was allocated according to the random-walk strategy, the probability of discovering the Hilight deposit would be 0.999+, a virtual certainty.

INTRODUCTION

A computer simulation model was used to study several aspects of exploring for petroleum deposits within the Powder River Basin, Wyo. The purpose of this study is to estimate the performance of the random-walk exploration model in the hypothetical exploration of a large region in which a substantial number of deposits has been discovered, such as the Powder River Basin. When a technique becomes available which will yield a reliable estimate of the total frequency distributions for the size, shape, reserves, and depths of the deposits contained in an area, a valid comparison can be made between the performance of the random-walk model and the success rates actually achieved in the basin. Such a comparison cannot be made here because an estimate of the total resource base of deposits contained in the basin was not available for this study; only the suite of deposits discovered with the network of exploratory wells drilled through the year 1970 was available. If these exploratory wells had been located differently, a somewhat different suite of deposits would almost surely have been discovered.

The term "gambler's ruin" is used in this report to denote the probability of not discovering a single deposit during the execution of a given exploratory drilling program. Several additional terms used in this paper are defined below:

1. Simulation model.—A device (a computer program) which is used to mimic the exploration process.
2. Random-walk allocation rule.—A rule making it equally likely for any point in the basin to receive a well in an exploration program, given that the point is not closer than a specified distance (lag) from the location of the last well drilled or that it is not within the boundary of a previously discovered deposit.
3. Resource base exhaustion.—The sequential depletion of deposits from a resource base through the exploration process.
4. Hindsight simulation model.—A model which assumes that an analysis of historical data can predict the outcome of future exploration. A static time reference is used—that is, time is not a factor in itself (in that it passes) but only in that it refers to a point at which the full resource base is present. In this study, 1899, the year when the first deposit was discovered in the basin, was chosen as the time reference point.
5. Sequential simulation model.—A variant of the basic hindsight model which employs a dynamic time element—that is, at

any given time only those deposits remaining to be discovered between that year and the year 1971 are retained in the resource base.

The hindsight simulation model was used to demonstrate the relationships between the intensity of exploratory drilling (the number of wells per program) and the outcome of exploration, as summarized by a suite of performance criteria. The criteria include (1) the average quantity of petroleum discovered, (2) the average number of deposits discovered, (3) the probability of gambler's ruin, and (4) the probabilities of obtaining specific levels of success.

The hindsight model simulated the discovery of that part of the entire petroleum resource base discovered in the Powder River Basin (approximately 27,000 mi^2) during the 1889–1970 time interval. During this period 154 deposits were discovered in the basin. This suite of deposits is conservatively estimated to contain 1.6 billion equivalent barrels of producible petroleum; approximately 35 percent of this quantity is contained in a single deposit, the Salt Creek Deposit.

In the hindsight simulation model the exploratory wells were allocated according to the random-walk strategy. The intensity of exploratory drilling was varied from a minimum level of 10 wells to a maximum level of 500 wells. From the raw simulation results, tables and graphs were constructed which display the response characteristics of the exploration model for each of the performance criteria within this range of exploratory drilling. The effect of incremental exploratory drilling upon the average quantity of petroleum discovered was also examined.

The sequential variation of the exploration model was used to isolate the effect of resource base exhaustion upon the exploration performance criteria. The same method was used to allocate the exploratory wells in the sequential model as was used in the hindsight model. The resource base on which exploratory drilling was applied in this model was the suite of 111 deposits discovered during the actual exploration of the basin between the years 1952 and 1971. The effects of exhaustion of this resource base upon the exploration performance criteria were determined by removing each deposit discovered during each yearly time period and redrilling the basin with the same intensity as actually used.

The study also estimated the probability of certain compound discovery events, the discovery of a deposit at least once during a sequence of time intervals.

ACKNOWLEDGMENTS

The input data were collected and the majority of the computer simulation runs were completed while the writer was a research geologist with Cities Service Oil Company, Tulsa, Okla.; the writer wishes to gratefully acknowledge the support he received from Cities Service. Particular acknowledgment goes to Joseph Huffstetler who prepared several of the basic documents for this study and to Phillip Fincannon who wrote the majority of the logic for the basic simulation model.

EXPLORATORY WELL ALLOCATION SCHEME AND INPUT DATA ARRAYS

In both model variations the random-walk method of allocation was used to locate exploratory wells. Using this allocation rule, any point within the basin is equally likely to receive a well, given that it is not closer than a specified distance (lag) from the location of the last well drilled or that it is not within the boundary of a previously discovered deposit. The magnitude of the lag was chosen arbitrarily to range from a maximum of 10 miles (16 km) for a 10-well exploration program to a minimum of 2 miles (3.2 km) for exploration programs with 100 or more wells.

Two types of input data arrays are required by the models. First, the configuration of the boundary of the basin and the geometry of the target deposits must be input in the form of a grid. This formulation allows the boundaries of the target area and the geometry of the target deposits to be approximated to any desired degree of precision by simply varying the spacing of the input array. The second type of data input required is estimates of the quantity of petroleum that can ultimately be produced from each target deposit. When such estimates were unavailable, cumulative production figures through February 28, 1971, were used.

The grid characterizing deposit geometry and the configuration of the boundary of the basin was obtained by digitizing a base map with a ¼ by ¼-mile (0.4 by 0.4-km) grid. This density is equivalent to one datum point per 40 acres, which is the smallest of the commonly used development drilling spacings. Each grid point of the basin was classified as being either barren or within a productive petroleum deposit. Those points located within the boundaries of productive deposits were assigned a code number which was keyed to the estimated productivity of the deposit. In simulating the exploration of the basin, these code numbers served as the basis for an accounting system utilized to summarize the performance of each exploration program. An idealization of this digitization process is given in figure 1. The principle advantage of formulating the input data array in this manner is that an explicit relationship between the intensity of exploration and both the probabilities of discovering each target deposit and the values of each performance criterion were determined.

THE HINDSIGHT MODEL

Two central elements are common to any variation of a hindsight model. First, the spatial distribution of the targets and the distribution of their values are derived from records of past exploration activity in mature or exhausted areas and not from predictions based upon geologic or geophysical data of the resource potential remaining to be discovered within the target area. Second, to predict future performance from historical data, a similarity assumption is made. The similarity assumption nearly always is an assertion that the resource base discovered in the past in one geographic region (the control area) is similar to the resource base remaining to be discovered in another geographic area (the target area) and henceforth the control area resource base can be

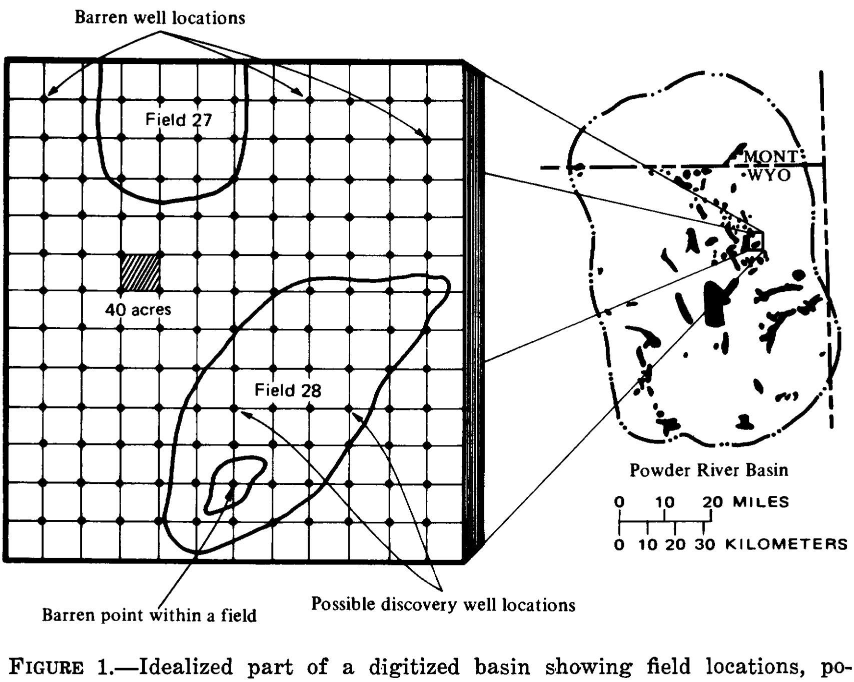

FIGURE 1.—Idealized part of a digitized basin showing field locations, potential well sites, and barren points within a field.

used as a basis for predicting the results of future exploration in the target area. For example, Allais (1957) used an assumption of this type to justify the use of several sets of historical resource data from regions within the United States and Europe as a basis for predicting the outcome of exploring the Algerian Sahara for base and precious metals.

HISTORICAL ASPECTS

At least three reasons can be suggested to explain the extensive use of the hindsight model. First, the exploration data commonly available to exploration analysts are historical records summarizing past exploration performance. These data, which are useful only for retrospectively describing exploration, are usually available for a region in the form of lists of past discoveries and yearly drilling intensity statistics. Second, a hindsight model alone will usually yield adequate estimates of future exploration performance if the principal exploration objective is to predict the expected gross return or range of return. Third, several analysts who have studied the exploration process have done so by adapting specific hindsight models developed in the field of military operations–research to search for hostile submarines, antiship mines, or pilots downed at sea (Engel, 1957; de Guenin, 1962; Griffiths and Drew, 1964; Griffiths and Singer, 1972).

RESPONSE CHARACTERISTICS OF THE HINDSIGHT MODEL

The response characteristics of the random-walk exploration model are the values, particular to this study, which are associated with the performance criteria. Table 1 shows the relationship

TABLE 1.—*Summary statistics and probabilities of discovering various quantities of petroleum, in millions of equivalent barrels*

[Gas converted to barrels at 26 cents per 1,000 ft ^{3}]

Number of wells	Summary statistics, quantity of petroleum		Probability of discovering zero petroleum	Probability of discovering indicated minimum quantity of petroleum				
	Mean	Standard deviation		1	2	5	10	20
10	13.1	59.1	0.801	0.194	0.191	0.179	0.159	0.121
20	26.0	86.7	.639	.350	.343	.324	.291	.226
30	38.9	102.4	.509	.484	.476	.440	.418	.345
40	51.2	116.0	.415	.567	.558	.536	.490	.405
50	63.3	128.5	.326	.665	.657	.631	.589	.504
75	91.8	153.7	.188	.804	.795	.769	.727	.642
100	122.8	176.8	.103	.890	.882	.858	.813	.739
150	178.1	203.1	.036	.958	.952	.939	.914	.864
200	228.9	226.4	.013	.982	.984	.969	.956	.904
300	327.3	254.9	.003	.993	.994	.991	.983	.973
400	405.6	265.9	.000	.999	.998	.996	.996	.993
500	483.0	274.1	.000	.999	.999	.999	.999	.998

TABLE 1.—*Summary statistics and probabilities of discovering various quantities of petroleum, in millions of equivalent barrels*—Continued

Number of wells	Probability of discovering indicated minimum quantity of petroleum							
	30	40	50	100	200	400	600	800
10	0.086	0.067	0.051	0.037	0.013	0.008	0.000	0.000
20	.156	.132	.113	.054	.026	.012	.002	.000
30	.271	.203	.177	.113	.036	.027	.003	.000
40	.320	.237	.223	.140	.051	.030	.005	.000
50	.419	.335	.295	.175	.067	.056	.007	.000
75	.557	.468	.420	.259	.104	.077	.017	.000
100	.660	.575	.538	.337	.160	.120	.048	.000
150	.814	.764	.701	.490	.254	.171	.065	.003
200	.870	.843	.815	.624	.358	.214	.124	.010
300	.961	.952	.939	.800	.552	.318	.237	.033
400	.988	.982	.977	.903	.714	.405	.330	.058
500	.997	.996	.995	.953	.818	.507	.428	.157

between the values of the performance criteria describing the distribution of the quantity of petroleum discovered and the intensity of exploratory drilling. The values of the performance criteria which describe the distribution of the number of deposits discovered at each level of drilling intensity are shown in table 2, as is the probability of encountering a given number of unsuccesful exploratory wells in succession before the first discovery. A

TABLE 2.—*Summary statistics and probabilities of discovering various numbers of commercial deposits*

Number of wells	Summary statistics, number of deposits		Probability of discovering exactly zero deposits	Probability of discovering indicated minimum number of deposits					
	Mean	Range		1	2	3	4	5	6
10	0.22	0–3	0.801	0.199	0.018	0.001	0.000	0.000	0.000
20	.43	0–4	.639	.361	.071	.011	.001	.000	.000
30	.67	0–5	.509	.411	.146	.028	.005	.001	.000
40	.843	0–6	.415	.575	.212	.051	.009	.002	.001
50	1.08	0–6	.326	.674	.294	.096	.024	.005	.001
75	1.58	0–7	.188	.812	.477	.207	.066	.019	.004
100	2.11	0–8	.103	.897	.635	.352	.152	.055	.016
150	3.06	0–11	.036	.964	.836	.616	.371	.183	.068
200	3.98	0–13	.013	.987	.930	.783	.586	.366	.197
300	5.72	0–15	.003	.997	.988	.950	.860	.711	.521
400	7.34	1–17	.000	1.000	.998	.989	.960	.893	.785
500	8.93	1–18	.000	1.000	.999	.998	.990	.969	.926

Number of wells	Probability of discovering indicated minimum number of deposits								
	7	8	9	10	12	14	16	18	20
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	.000	.000	.000	.000	.000	.000	.000	.000	.000
30	.000	.000	.000	.000	.000	.000	.000	.000	.000
40	.000	.000	.000	.000	.000	.000	.000	.000	.000
50	.000	.000	.000	.000	.000	.000	.000	.000	.000
75	.001	.000	.000	.000	.000	.000	.000	.000	.000
100	.003	.001	.000	.000	.000	.000	.000	.000	.000
150	.020	.007	.002	.001	.000	.000	.000	.000	.000
200	.091	.035	.014	.004	.001	.000	.000	.000	.000
300	.341	.191	.100	.040	.005	.001	.000	.000	.000
400	.627	.466	.306	.179	.044	.007	.001	.000	.000
500	.838	.704	.552	.392	.148	.034	.006	.001	.000

minimum of 3 million simulation cycles was used to obtain the results shown in these tables.

Inspection of table 1 suggests some general conclusions about the response characteristics of the model. First, the relationship between the average quantity of petroleum discovered per program and the intensity of drilling is nonlinear. Thus, doubling the size of the exploration program does not cause the average quantity of petroleum discovered to double. Second, as the intensity of exploration is doubled at higher and higher levels, the deviation from nonlinearity increases. This phenomenon of increased deviation from linearity of response is usually referred to as the exponential saturation effect or the effect of diminishing returns. This effect is a common consequence arising from the normal use of many search procedures actually used in exploration.

The response characteristics of the exploration model are shown in figures 2 through 4. In figure 2, the probability of gambler's ruin and the probabilities of attaining six specific levels of discovery are plotted as functions of the intensity of exploration. In figure 3, the average quantity of petroleum discovered and the standard deviation of the distribution of the quantity of petroleum discovered are plotted as functions of the intensity of exploration. In figure 4, the average incremental quantity of petroleum discovered is plotted as a function of the intensity of drilling.

Within the 10- to 100-well range, the probability of gambler's ruin, the probability of not making a single discovery, declines very rapidly as the intensity of exploration is increased (fig. 2); from 0.801 for 10 wells to 0.103 for 100 wells. Further increases in the intensity of drilling (beyond 100 wells) cause little additional decrease in the probability of gambler's ruin. Thus, the probability of discovering one or more deposits is strongly affected by changes in the intensity of exploration drilling at the lower intensities.

Six curves in figure 2 show how the probabilities of achieving or exceeding six levels of discovery (1, 10, 50, 100, 200, and 400 million barrels) varies in response to changes in drilling intensity. The curve for the 1-million-barrel level is essentially the complement of the gambler's-ruin curve because less than 15 percent of the deposits in the resource base have estimated potentials of less than 1 million barrels. Therefore, when a deposit is discovered there is at least an 85 percent chance that the 1-million-barrel level has been attained or exceeded.

The form of the response curves changes significantly as the discovery goal is set at successively higher levels. At the 200-

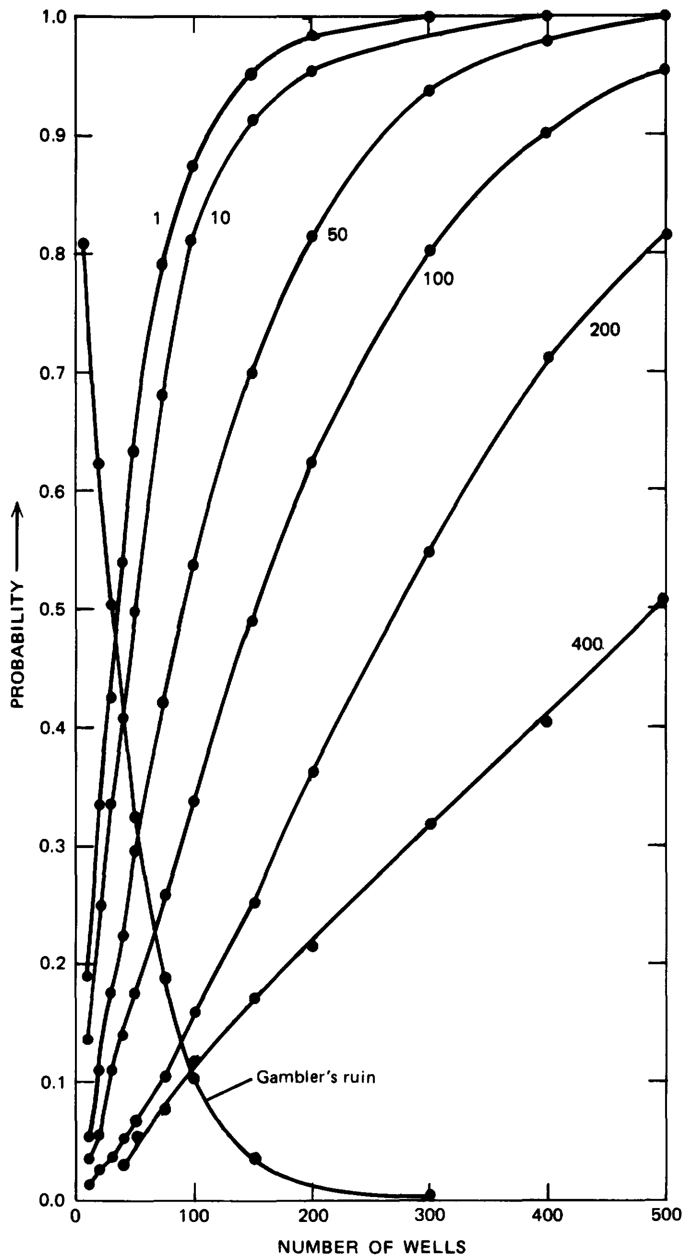

FIGURE 2.—Probability curves for the discovery of specific quantities of petroleum, in millions of barrels.

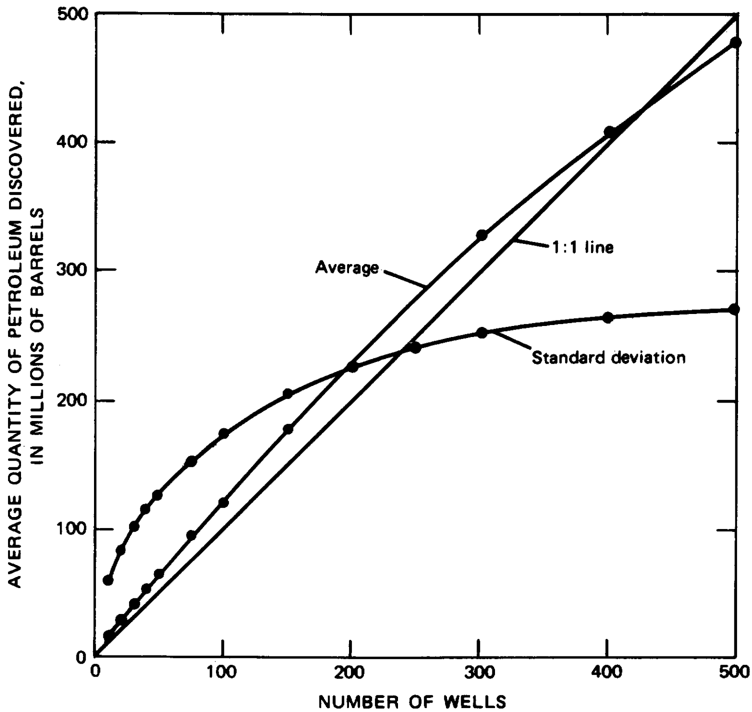

Figure 3.—Average quantity of petroleum discovered versus exploratory drilling intensity.

million-barrel level and beyond, the response of the probability curve is essentially linear over the entire 10- to 500-well range; additional increments of exploration drilling return nearly equal increments in the probability of achieving the goal.

The average quantity of petroleum discovered per program increases as the intensity of drilling increases but at an ever-decreasing rate (fig. 3). The incremental return is seen (fig. 4) to decline from 1.31 million barrels per (incremental) exploration well to 0.77 million barrels per (incremental) exploration well over the 10- to 500-well range. At approximately the 250-well level, the rate of change of this curve is 1 million barrels per exploraton well.

THE SIMILARITY ASSUMPTION AND AN EXAMPLE

The similarity assumption is required to convert the results obtained from a hindsight model into a basis for predicting the outcome of future exploration in a target area similar to the control area. Hindsight model results as they stand, however, are

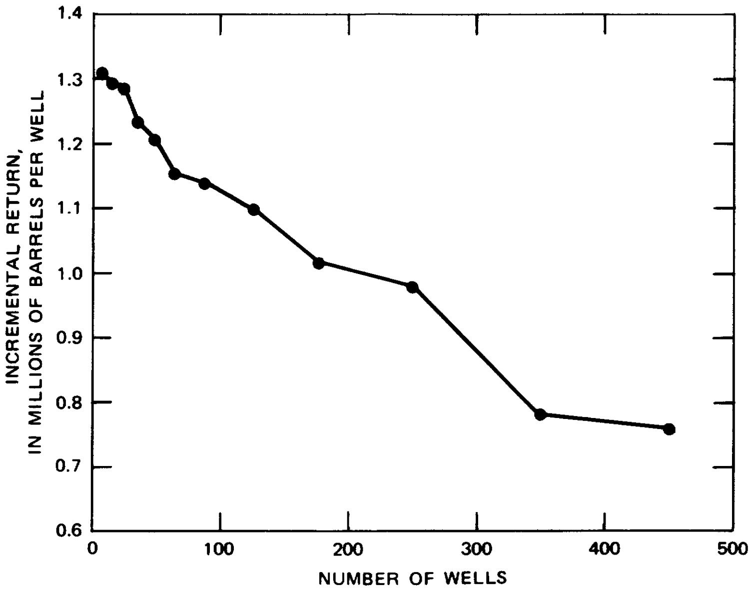

FIGURE 4.—Average incremental quantity of petroleum discovered.

completely adequate to determine what would have happened if actual past exploration had been carried out according to the tenets of the hindsight model. Extrapolation from hindsight summary to the prediction of future discovery without adequate consideration being given to the similarity assumption is a short but peril-laden leap.

Often it is possible to support the use of the similarity assumption with specific geologic data. Criteria which should be examined in judging the validity of the similarity assumption for any specific target area are (1) the thickness of the stratigraphic section, (2) the presence of transgressive and regressive sediments, (3) the source bed characteristics, and (4) the types of structures within the basin.

In other cases, the exploration analyst may have to rely on the fact that no serious objection can be raised to negate the use of the assumption. In those cases where little explicit data exists to support the use of the assumption, the analyst is forced into the position of making the assumption knowing that subsequent evidence may be obtained which would severely compromise the validity of the predictions.

Once the control and target areas are selected and the assumption of similarity is made, however, the outcome of future exploration can be predicted from the model results. The model results displayed in tables 1–3 and figures 2–4 can be used for predictive purposes if it can be assumed that the undiscovered deposits in the target area are similar to the deposits discovered in the past in the Powder River Basin and if the method of allocating exploration wells in the target area is similar to that used to develop the Powder River Basin model results.

The question of how deep exploration wells should be drilled must be resolved by establishment of a rule. Such a rule might specify drilling to a target formation, say the Minnelusa (Pennsylvanian) or its equivalent. An additional cost restraint might produce a joint rule which might specify drilling each exploration well to the Minnelusa or to 13,000 feet (3,950 m) whichever is shallower.

The information contained in table 1 suggests probabilistic statements about the relationship between the quantity of petroleum discovered and the size of the exploration program. For example, if only 10 wells can be allocated for exploration the risk of gambler's ruin is very large (80.1 percent). The chance of discovering a million barrels or more is only 19.4 percent. Ten million barrels or more will be discovered only 15.9 percent of the time, 50 million barrels or more only 5.1 percent of the time, and 100 million barrels or more only 3.7 percent of the time. On the average a total of 13.1 million barrels will be discovered when

TABLE 3.—*Estimates of the probabilities of discovering any of six deposits within each year according to random-walk allocation of actual drilling intensities*

Year	Wells drilled	Deposit					
		Hilight	Springen Ranch	Gas Draw	Recluse	Bell Creek	Kitty
1952	38	0.122	0.010	0.012	0.015	0.037	0.042
1953	59	.183	.018	.018	.023	.059	.068
1954	62	.191	.018	.019	.023	.062	.070
1955	56	.175	.015	.017	.022	.051	.059
1956	76	.231	.021	.023	.029	.078	.074
1957	98	.285	.026	.030	.039	.098	.110
1958	93	.272	.025	.027	.035	.091	.103
1959	138	.378	.036	.040	.052	.133	.151
1960	227	.542	.056	.066	.083	.210	.230
1961	217	.524	.054	.062	.082	.203	.226
1962	188	.480	.052	.056	.070	.182	.197
1963	182	.466	.049	.053	.069	.171	.188
1964	168	.444	.045	.049	.063	.163	.177
1965	205	.511	.054	.061	.075	.192	.213
1966	183	.471	.049	.053	.069	.174	----
1967	236	.557	.060	.070	.090	.221	----
1968	586	.868	.154	.162	----	----	----
1969	445	.785	----	----	----	----	----
1970	292	----	----	----	----	----	----
1971	142	----	----	----	----	----	----
Total	3,691	----	----	----	----	----	----

10 exploratory wells are allocated. At first glance this average may appear to be disconcertingly large when compared with the probabilities of reaching or exceeding specific levels of discovery. The apparently large size of this average relative to the probabilities of achieving various discovery goals is caused by the large skewness in the right tails of the distribution of petroleum deposits lying within the basin. For example, the largest deposit, the Salt Creek field, is nearly three times larger than the next larger deposit. As a result, the distribution of the quantity of petroleum discovered is highly skewed to the right when a small exploration program is employed because the probability of discovering the largest field, or one or more of the other large fields, is small ($P=0.14$ for discovering the Salt Creek Field with a 10-well exploration program). Thus, the discovery of the Salt Creek Field "pushes" the frequency distribution of petroleum discovered far to the right. When this field is not discovered the aggregate quantity of petroleum discovered with a 10-well program is far smaller, usually less than a third as large.

The skewness of the distribution of the quantity of petroleum discovered decreases as the intensity of exploratory drilling is increased because the chance of discovering the larger deposits approaches certainty.

The most likely use that an exploration analyst may choose to make of the information displayed in table 1 is to determine the intensity of exploratory drilling required to achieve a specific goal and at the same time not to violate a specified risk level. For example, if the management of an exploration company sets a goal of discovering at least 100 million barrels of petroleum in the target area and at the same time is not willing to assume more than a 20 percent risk of not reaching that goal, then a 300-well exploration program must be scheduled (fig. 2). It does not follow, however, that one out of five times the firm must face gambler's ruin in the exploration project. On the contrary, the probability of gambler's ruin is only 0.003, three chances in a thousand. The probabilities of achieving lesser levels of discovery ranges from 0.996 for the 1-million-barrel level to 0.939 for the 50-million-barrel level.

The expectation is that a 300-well exploration program would result in the discovery of 327.3 million barrels in the target area (table 1), that this average quantity would be contained in 5.72 deposits (table 2), and that the number of deposits discovered would range from 0 to 15. The probability of discovering at least one deposit is then 0.997, a virtual certainty. The probability of discovering at least five deposits with a program of this size is 0.711, or about seven chances in ten. The probability of discovering 10 or more deposits is only 0.040, or four chances in one hundred.

The probability of encountering a specific sequence of unsuccessful exploration wells in the normal course of exploration in the target area prior to the first discovery can be computed by interpolating the results listed in the fourth column of table 2. The probability of drilling a series of 10 unsuccessful wells in a row is 0.801, a rather large risk. Approximately half of the time ($P=0.509$) 30 consecutive unsuccessful exploration wells would be encountered prior to the first discovery.

EXHAUSTION

The effect of exhaustion of the petroleum resource base of the basin upon several exploration performance criteria is investigated. (The sequential model must be used to develop the results discussed in this and the following sections.) The 20-year time interval 1952–71 was chosen for study. During this period, 111 deposits were discovered which were conservatively estimated to have contained 545,727,000 equivalent barrels of petroleum. It is pointed out that this resource base is only a part (approximately 40 percent) of the total petroleum resource base of the basin discovered since 1889; the total resource base could not be utilized because of restraints on the availability of computer time. The relationship between the exhaustion of the resource base and the values of each performance criterion were computed by diminishing the resource base by yearly time increments (exhaustion sequence) and simulating the exploration of the basin with programs of 10 to 500 exploratory wells at the start of each new time increment. The discovery sequence actually encountered during the exploration of the basin during the 1952–1971 period was chosen as the exhaustion sequence. The quantity of petroleum and the number of undiscovered deposits remaining in the resource base at the beginning of each yearly time increment are shown in figures 5 and 6, respectively. The results of this exhaustion study are again hindsight, and extrapolation of these results into a different petroleum province must be made cautiously.

The effect of the exhaustion of the basin's resource base upon three exploration performance criteria were evaluated and are displayed in figures 7 through 12. Figures 7 through 9 show the surfaces for the expected value of the quantity of petroleum discovered, expected numbers of deposits discovered, and the prob-

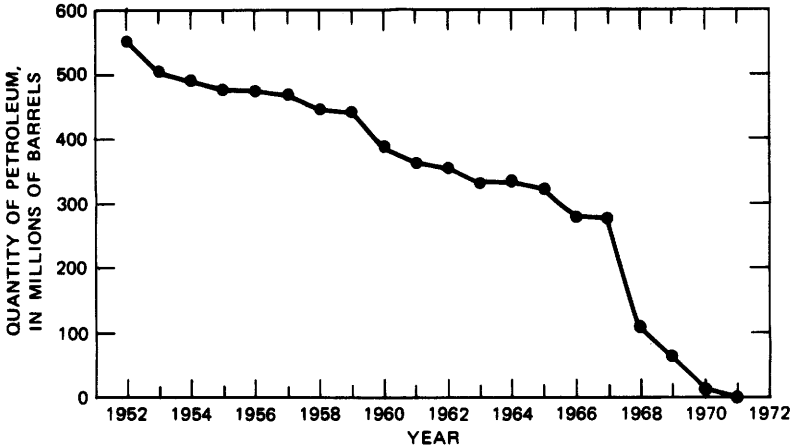

FIGURE 5.—Quantity of petroleum in resource base.

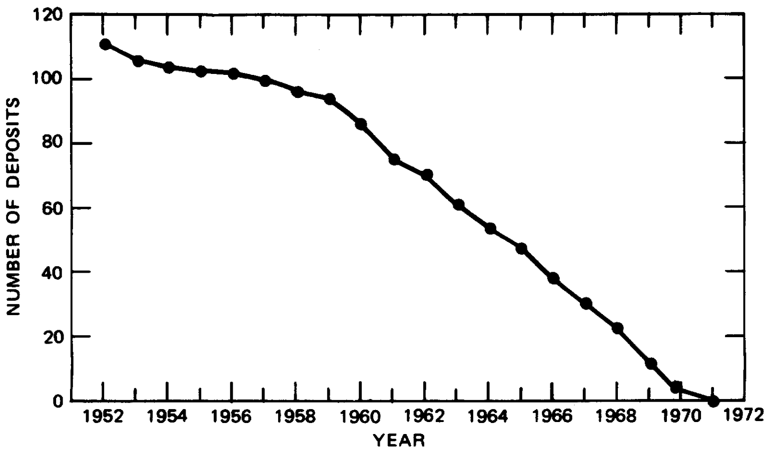

FIGURE 6.—Number of deposits in resource base.

ability of gambler's ruin. Figures 10 and 11, respectively, show the expected value of petroleum to be discovered for three intermediate exhaustion levels and the partial derivative curves for the expected quantity of petroleum discovered with respect to the size of the exploration program.

From figures 7 through 9, the performance of various exploration programs can be determined with respect to the level of exhaustion. For example, an operator planning, say, a 20-well

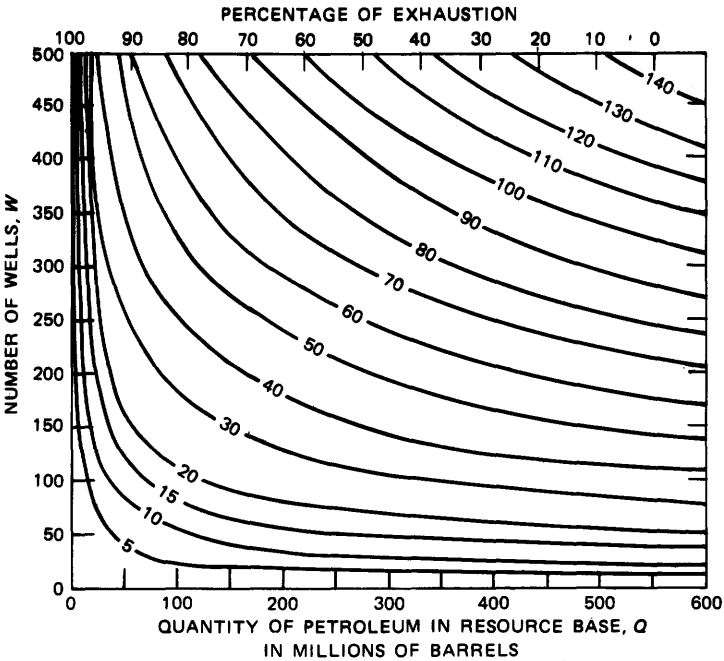

FIGURE 7.—Average quantity of petroleum discovered, $E(q) = 10^{-1.341} Q^{0.419} W^{0.876}$, in millions of barrels.

exploration program can estimate what effect exhaustion of the resource base will have upon the expected results of such a program. At the start of the 20-year interval a 20-well program will on the average discover 0.303 deposits containing 8.7 million barrels. The probability of gambler's ruin (not discovering a single deposit) is 0.601. When the resource base is at the 50 percent exhaustion level, 0.21 deposits with 6.5 million barrels are expected to be discovered when 20 wells are drilled. The chance of ruin is 0.926 for this situation.

For a petroleum company or group of companies undertaking a 200-well exploration program, it is expected that an average of 2.49 deposits containing 66.3 million barrels would be discovered when the full resource base is present; the chance of gambler's ruin is estimated to be only 3.6 percent. If the resource base is at 50 percent exhaustion level, a 200-well program is expected to lead to the discovery on the average of 1.75 deposits containing an average of 49.6 million barrels; the chance of gambler's ruin for this case is 5.8 percent.

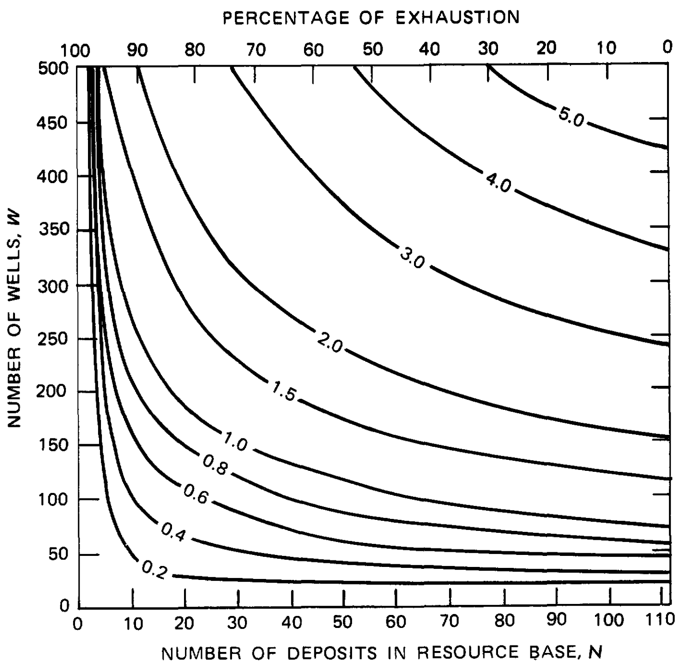

FIGURE 8.—Average number of deposits discovered, $E(d) = 10^{-2.715}N^{0.493}W^{0.916}$.

It is noteworthy that a reduction of the resource base by half (reduced to the 50-percent exhaustion level) does not reduce the levels of the performance criteria proportionally. This phenomenon is perhaps better illustrated by constructing plots like those given in figures 10 and 11. In figure 10, the expected quantity of petroleum to be discovered is plotted against exploration intensity for three intermediate levels of exhaustion, 500 (8.4 percent exhaustion), 300 (45.0 percent exhaustion), and 100 million barrels (81.7 percent exhaustion). Note that, irrespective of the size of the exploration program, increasing the quantity of petroleum in the resource base by a given factor does not cause a similar increase of the expected quantity of petroleum discovered. This is again a result of the exponential saturation effect (the effect of diminishing returns). Figure 11 gives the partial derivative functions $(\partial E(q)/\partial W)$ for the same three intermediate exhaustion levels given in figure 10. The symbol $E(q)$ denotes the expected quantity of petroleum to be discovered by a given program. The symbols W and Q denote, respectively, the number of exploratory

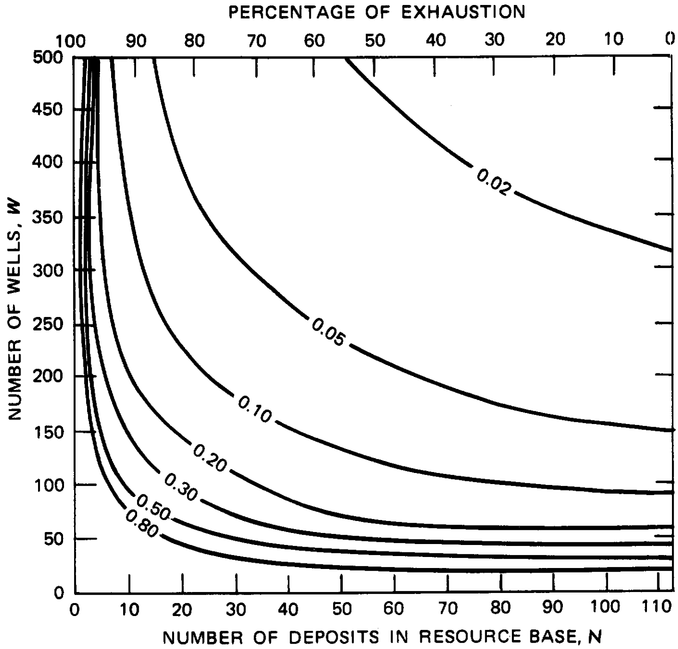

FIGURE 9.—Probability of gambler's ruin, $P(ruin) = 10^{2.802} N^{-0.701} W^{-1.223}$

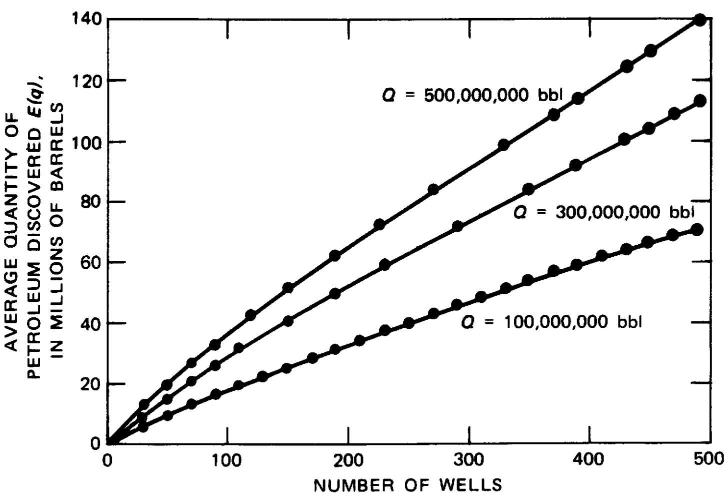

FIGURE 10.—Average quantity of petroleum discovered for three levels of exhaustion.

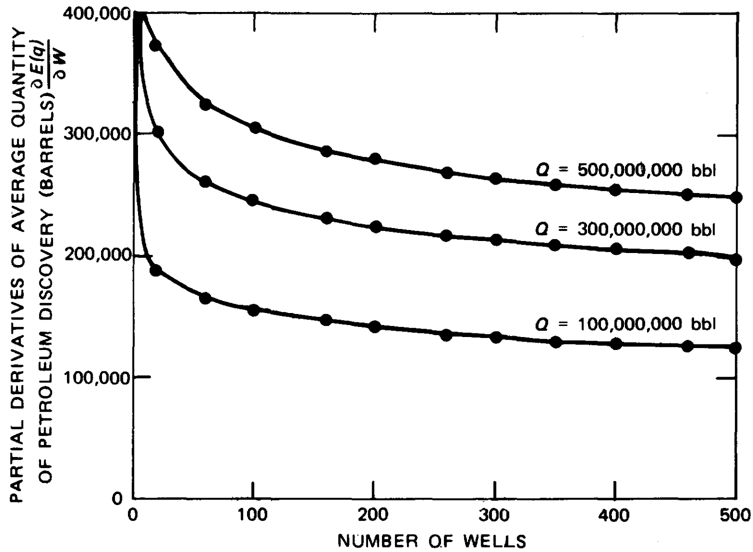

FIGURE 11.—Partial derivatives of the average quantity of petroleum discovered for three levels of exhaustion.

wells drilled and the quantity of petroleum remaining in the resource base at a particular time. The variation in the rate of diminishing return per well is most extreme for those exploration programs with less than 50 wells. For example, when the resource base contains 100 million barrels, the rate declines from 220,000 barrels per well for a 10-well program to 170,000 barrels per well for a 50-well program. Beyond this initial range the rate still diminishes, but much more slowly. At the 500-well level, the rate has only declined to the 125,000 level.

The effect of exhaustion of the resources upon the quantity of petroleum expected to be discovered is shown in yet another manner in figure 12, where this performance criterion is expressed as the average quantity of petroleum discovered per well and plotted against the quantity of petroleum remaining in the resource base for each of four exploration programs. For example, assume that a discovery rate of 250,000 barrels per well must be attainable to attract funds for exploration. Given such a level, it can be determined from this graph that a 20-well exploration program can be carried out successfully down to the point where only 145 million barrels (73.4 percent) is left in the basin. If a 100-well exploration program is planned, then at least 225 million barrels (58.8 percent exhaustion) must remain undiscovered in

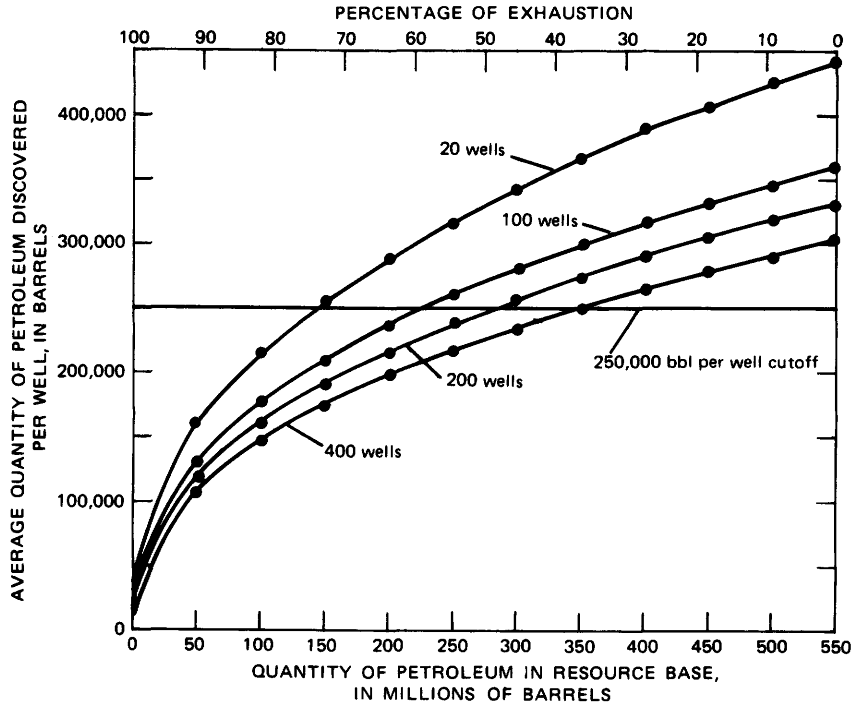

FIGURE 12.—Average quantity of petroleum discovered per well versus level of exhaustion for four levels of exploratory drilling.

the basin to attain or surpass the desired goal. For a 200-well program, 280 million barrels or more (48.7 percent exhaustion or less) must be present; and 345 million barrels (36.8 percent exhaustion or less) must be left for the goal to be achieved if a 400-well program is to be undertaken.

A tendency therefore exists for the larger operators to decrease and finally cease exploration as the level of exhaustion increases in a region. This occurs because the larger operators cannot reasonably expect to discover, at an acceptable rate of return, the volume of petroleum necessary to induce them to continue to explore in the region. Despite the fact that the larger operators leave a region because of the effects of exhaustion, a smaller operator can still obtain an acceptable rate of return, assuming that the risk of not finding the remaining deposits is not too great.

COMPOUND-DISCOVERY-EVENT PROBABILITIES

The efficiency of the aggregate exploration process in discovering specific deposits is often a topic of debate among petroleum explorationists. It is often the case that the larger deposits in a basin are discovered early in the exploration history of the basin. In other cases, a major deposit may remain undiscovered for decades even through significant exploration activity has been undertaken within a short geographic or stratigraphic distance of its location. As a first approximation to the solution of estimating the chance of compound discovery events, the probabilities of discovering six of the larger deposits actually discovered during the 1952–71 time span were estimated. The actual intensities of exploratory drilling expended during this period were used in these computations. The results (table 3) are again based upon the random-walk method of allocating exploration wells. These results therefore represent the worst that could be expected to occur because no information was used to remove those regions with low discovery potential.

The number of exploratory wells actually drilled from 1952–71 are shown in figure 13 (see also table 3). During this period the yearly drilling rates ranged from a minimum of 38 for the year 1952 to a maximum of 586 for 1968. The probabilities of discovering each of the six deposits within a particular year are shown in table 3. Note that these probabilities vary as a direct function of the drilling intensity. For example, the year in which the probability of discovering the Hilight deposit is the largest ($P = 0.868$) is 1968 when the greatest number, 586, of exploratory wells were drilled and smallest ($P = 0.122$) in 1952 when only 38 exploratory wells were drilled.

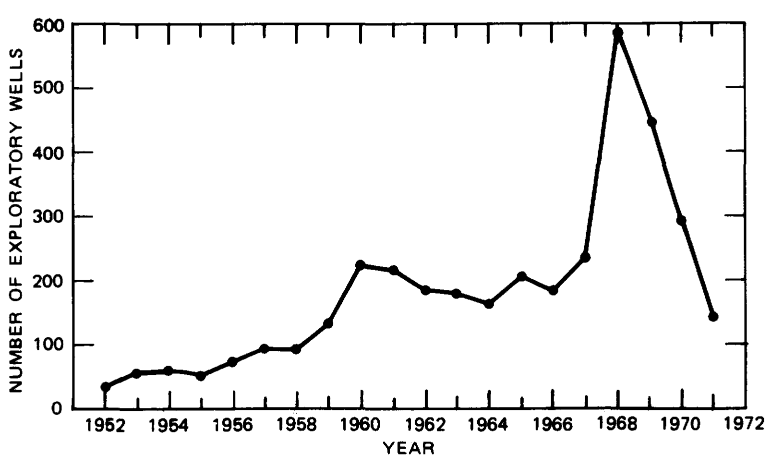

FIGURE 13.—Number of exploratory wells drilled in the Powder River Basin, 1952–71.

The probabilities of discovering each deposit in at least one year during the period beginning in 1952 and up to and including the year of discovery are shown in table 4. These computations were made with a computer program designed to solve the binominal equation for the case where the probabilities of success vary from trial to trial. The probability of discovering the Hilight deposit at least once when the random-walk strategy is used to allocate exploration wells is 0.99998, a virtual certainty. In fact, the chance of discovering this deposit at least four times with the 3,257 exploratory wells drilled between 1952 and 1969 is also nearly certain ($P = 0.985$). The chance of discovering the Bell Creek deposit, the largest deposit discovered in the basin in the last several decades, at least once with the random-walk strategy is estimated to be 0.903. The discovery of relatively small deposits, such as the Gas Draw or Springen Ranch fields, at least once using the random-walk strategy is estimated to occur at least 50 percent of the time in each case.

Table 5 gives another sequence of probabilities which show the progressive increase in the probability of discovering the six deposits over the time period starting in 1952 and ending in the year prior to actual discovery of each deposit. The probabilities shown in this table were computed sequentially by re-exploring the basin according to random-walk strategy and by increasing the intensity of drilling by yearly increments. The probability of discovering the Bell Creek deposit when 38 exploration wells are drilled (1952) is 0.037. If the two years 1952 and 1953 are considered as a single time unit, the probability of discovery increases to 0.093; for the time unit ending in 1960 the probability increases to 0.590 and it increases further to 0.763 for the time unit ending in 1963. When the total period from 1952 through 1966 (the year prior to the actual discovery of the deposit) is considered as a single time unit, the probability increases to

TABLE 4.—*Estimates of the probabilities of compound discovery events using random-walk allocation of exploratory wells*

[Reserves in millions of equivalent barrels]

Deposit	Year of discovery	Ultimate primary reserves	Total wells drilled between 1952 and year of discovery	Number of years through year of discovery	Probabilities of discovering deposit at least these number of times during time period: 1	2	3	4
Hilight	1969	55.4	3,257	18	0.99998	0.9997	0.997	0.985
Gas Draw	1968	12.8	2,812	17	.573	.195	.043	.007
Springen Ranch	1968	9.7	2,812	17	.541	.170	.035	.005
Recluse	1967	27.0	2,226	16	.580	.203	.047	.008
Bell Creek	1967	121.4	2,226	16	.903	.654	.362	.152
Kitty	1965	28.9	1,807	14	.879	.597	.300	.111

TABLE 5.—*Estimates of the probabilities of discovering any of six deposits with the actual drilling effort expended between 1952 and the actual year of discovery of each deposit*

Years	Wells drilled	Deposit					
		Hilight	Springen Ranch	Gas Draw	Recluse	Bell Creek	Kitty
1952	38	0.122	0.010	0.012	0.015	0.037	0.042
1952–53	97	.279	.027	.030	.039	.093	.106
1952–54	159	.420	.043	.050	.063	.146	.178
1952–55	215	.543	.061	.063	.086	.204	.226
1952–56	291	.632	.077	.083	.102	.259	.294
1952–57	389	.736	.108	.115	.134	.326	.362
1952–58	482	.804	.134	.148	.165	.389	.440
1952–59	670	.887	.154	.180	.217	.472	.517
1952–60	847	.942	.216	.238	.287	.590	.631
1952–61	1,064	.975	.257	.286	.329	.664	.715
1952–62	1,252	.990	.297	.317	.373	.713	.765
1952–63	1,434	.990	.335	.355	.413	.763	.806
1952–64	1,602	.996	.361	.388	.467	.809	.849
1952–65	1,807	.998	.401	.428	.502	.855	----
1952–66	1,990	.999+	.436	.458	.554	.886	----
1952–67	2,226	.999+	.467	.487	----	----	----
1952–68	2,812	.999+	----	----	----	----	----

0.886. Thus, when the wells are allocated according to the random-walk strategy, there is an 88.6 percent chance of discovering the Bell Creek deposit with the 1,990 exploratory wells drilled in the time period 1952–66.

Caution must be observed in using the probability estimates developed from this model study to evaluate the effectiveness of historical performance in discovering specific deposits; these estimates are subject to both positive and negative biases, the magnitudes of which can be only roughly estimated. For example, these probability estimates are conservative on the one hand because the target area in the simulation study is the entire basin, in which an exploration well can be located anywhere, even near the boundary of the basin where potential is low. Such areas are roughly estimated to comprise 150 townships, or more than 5,000 mi^2. Exploration wells can also be located within the relatively unexplored central and north-central parts of the basin where, in 1970, 139 townships, or about 5,000 mi^2, were untested. Thus, of the basin's 27,000 mi^2, approximately 40 percent which did not physically contain any of the deposits used to make the probability estimates and which would not be given much consideration by the industry as having discovery potential was considered to be in the target area. The magnitude of this negative bias upon the estimates would, however, be less than 40 percent because of the effect of diminishing returns. On the other hand, the discovery probabilities are larger than those to be expected in reality because the depth to which each exploration well needs to be drilled to insure discovery is not explicitedly considered in the model. A rule such as the one mentioned earlier (drill to the Minnelusa or 13,000 feet (3,950 m), whichever is shallower) would be completely effective but such a rule might take a great deal of exploration effort to establish. In the case of the six deposits in this sample, establishing a rule would be a somewhat easier task because the depths of these deposits range from 4,500 feet (1,360 m, Bell Creek) to 9,600 feet (2,910 m, Hilight).

Other problems of biases arise from land-leasing practices and variations in exploration economics.

CONCLUSIONS

The process of exploring for petroleum deposits with a random-walk strategy in the Powder River Basin, Wyo., was investigated over a range of drilling intensity. The effectiveness of this strategy on the outcome of exploration was analyzed with respect to a suite of performance criteria. Examples of the criteria employed are (1) the average quantity of petroleum discovered, (2) the average number of deposits discovered, (3) the probability of gambler's ruin, and (4) the probabilities of obtaining various levels of success. Before the model results can predict future exploration results in a target area, it is necessary to make an assumption of similarity between the resource base in the control area (in which the probabilities of success were generated) and the resource base remaining to be discovered in the target area. If this assumption can be justified for the target area in question, the following are several results predicted by the model:

1. Increasing the number of exploratory wells per program increases, but at an ever-decreasing rate, the average number of deposits and the average quantity of petroleum expected to be discovered. (Ten wells yield an average of 0.22 deposits containing an average of 13.1 million barrels of petroleum; 100 wells yield an average of 2.11 deposits containing an average of 122.8 million barrels of petroleum.)
2. The probability of drilling consecutively unsuccessful wells (gambler's ruin) decreases as the number of wells per program increases. (Ten wells have a probability of gambler's ruin of 0.801; with 100 wells it is 0.103.)
3. The expected incremental return per exploratory well decreases as the number of exploratory wells drilled increases. (At the 5-well intensity, the incremental (next) well returns 1.31 million barrels; at the 450-well intensity, the incremental well returns 0.77 million barrels.)

The simulation model also isolated the effect of resource base exhaustion upon the exploration performance criteria; this effect was found to progressively force the larger exploration interests out of the target area even though a large percentage of the resource base remains. For example, if the acceptable rate of return is set at 250,000 barrels per well and a 200-well program is required by the firm to obtain the volume of discovery necessary to command investment in an area, the firm would be forced out when only 48.7 percent of the resource base was exhausted. In contrast, an operator planning to drill only 20 wells can continue to obtain an average rate of return of 250,000 barrels per exploratory well until 73.4 percent of the resource base is exhausted.

Another result obtained from the model is the estimation of the probability of various intensities of exploratory drilling discovering specific deposits over various time periods. For example, it is estimated that if the exploratory drilling effort expended in the Powder River Basin between 1952 and 1968 (2,812 wells) was allocated according to the random-walk strategy, the probability of discovering the Hilight deposit would be 0.999, a virtual certainty. The probability of discovering this deposit with 1,252 exploratory wells drilled between 1952 and 1962 would also be nearly certain ($P=0.990$).

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