Velocity Ratio and its Application to Predicting Velocities

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Abstract

The velocity ratio of water-saturated sediment derived from the Biot-Gassmann theory depends mainly on the Biot coefficient—a property of dry rock—for consolidated sediments with porosity less than the critical porosity. With this theory, the shear moduli of dry sediments are the same as the shear moduli of water-saturated sediments. Because the velocity ratio depends on the Biot coefficient explicitly, Biot-Gassmann theory accurately predicts velocity ratios with respect to differential pressure for a given porosity. However, because the velocity ratio is weakly related to porosity, it is not appropriate to investigate the velocity ratio with respect to porosity (ϕ).

A new formulation based on the assumption that the velocity ratio is a function of $(1-\phi)^n$ yields a velocity ratio that depends on porosity, but not on the Biot coefficient explicitly. Unlike the Biot-Gassmann theory, the shear moduli of water-saturated sediments depend not only on the Biot coefficient but also on the pore fluid. This nonclassical behavior of the shear modulus of water-saturated sediment is speculated to be an effect of interaction between fluid and the solid matrix, resulting in softening or hardening of the rock frame and an effect of velocity dispersion owing to local fluid flow. The exponent *n* controls the degree of softening/hardening of the formation. Based on laboratory data measured near 1 MHz, this theory is extended to include the effect of differential pressure on the velocity ratio by making *n* a function of differential pressure and consolidation. However, the velocity dispersion and anisotropy are not included in the formulation.

Introduction

The velocity ratio has been used for many purposes, such as a lithology indicator, determining degree of consolidation, identifying pore fluid, and predicting velocities. The velocity ratio usually depends on porosity, degree of consolidation, clay content, differential pressure, pore geometry, and other factors. The velocity ratio for dry rock or gas-saturated rock is almost a constant irrespective of porosity and differential pressure, whereas the velocity ratio of wet rock depends significantly on porosity and differential pressure. The purpose of this paper is to accurately predict elastic velocities of water-saturated clastic sediments by utilizing the dependence of the velocity ratio on porosity.

Pickett's (1963) cross plot shows that P-wave to S-wave velocity ratio (V_p/V_s) for sandstone is about 1.6 in low-porosity rocks, drifting to 1.8 in relatively higher porosity rocks. His observation implies the dependence of V_p/V_s on porosity for sandstone. Gardner and Harris (1968) showed that V_p/V_s values > 2.0 are characteristic of water-saturated unconsolidated rocks, and values < 2.0 indicate either well-consolidated rock or the presence of gas in unconsolidated sands. Gregory (1976) confirmed this relationship between velocity ratio and consolidation and suggested the dependence of velocity ratio on porosity.

Hornby and Murphy (1987) and Murphy and others (1992) showed that the velocity ratio increases as clay content increases and that Biot-Gassmann theory accurately predicts the velocity ratio of unconsolidated water-saturated sand with respect to effective pressure. Castagna and others (1985) and Han and others (1986) empirically derived relationships between velocity ratio, porosity, and clay content. Han and others (1986) showed that the velocity ratio increases linearly with clay content and porosity. The equation by Castagna and others (1985) also implies an increase in velocity ratio with increasing porosity.

The prediction of S-wave velocities for water-saturated rocks based on the velocity ratio with a first-order application of Biot-Gassmann theory is given by Greenberg and Castagna (1992). Empirical relationships by Castagna and others (1985) and Han and others (1986) can be used to predict S-wave velocity either from porosity and clay content or velocity ratio and porosity. Xu and White (1996) investigated the S-wave velocity prediction based on bulk and shear moduli of the dry rock frame by a combination of Kuster and Toksöz theory (1974) and differential effective medium theory, using pore aspect ratio.

In this paper, a new method of modeling velocities for consolidated and unconsolidated sediments is presented using Biot-Gassmann theory with the assumption that the velocity ratio is related to porosity. This is an extension of Lee's (2002) theory for unconsolidated sediments. Pickett (1963), Gregory (1976), Castagna and others (1985), and Han and others (1986) each suggested a relationship between porosity and velocity ratio, but their functional relationships between porosity and velocity ratio are different from the one proposed in this paper. Using measured laboratory data, this new theory is extended to include the effect of differential (or effective) pressure on velocity ratio or velocities. This new approach is applied to the laboratory data from Gregory (1976), Domenico (1977), Han and others (1986), and that compiled by Castagna and others (1985) with good results.

Biot-Gassmann Theory (BGT)

Elastic velocities (i.e., compressional velocity (V_p) and shear velocity (V_s)) of water-saturated sediments can be computed from the elastic moduli by the following formulas:

$$V_p = \sqrt{\frac{k + 4\mu/3}{\rho}} \quad \text{and} \quad V_s = \sqrt{\frac{\mu}{\rho}} \tag{1}$$

where

k, μ, and, ρ are bulk modulus, shear modulus, and density of the formation, respectively.

The formation density is given by

$$\rho = (1-\phi)\rho_{ma} + \phi\rho_{fl} \tag{2}$$

where

ϕ, ρ_{ma} and ρ_{fl} are the porosity, matrix density, and pore fluid density, respectively.

The bulk and shear moduli based on the Biot-Gassmann theory (Biot, 1941, 1956; Gassmann, 1951; Domenico, 1977; Murphy, 1984; Krief and others, 1990) are given by the following formulas using the Biot coefficient, β:

$$k = k_{ma}(1-\beta) + \beta^2 M \tag{3}$$

$$\mu = \mu_{ma}(1-\beta) \tag{4}$$

where

$$\frac{1}{M} = \frac{(\beta-\phi)}{k_{ma}} + \frac{\phi}{k_{fl}} \quad \text{and}$$

k_{ma}, μ_{ma}, and k_{fl} are the bulk modulus of the matrix, the shear modulus of the matrix, and the bulk modulus of the fluid, respectively.

The moduli of a composite matrix including clay volume are calculated using Hill's (1952) averaging method. The Biot coefficient describes the ratio of pore volume change to total bulk volume change under dry or drained conditions (Mavko and others, 1998) and relates the moduli of a dry frame to the moduli of the matrix material.

Lee (2002) derived an alternative equation for the shear modulus

$$\mu = \frac{\mu_{ma} G^2 (1-\phi)^{2n} k}{k_{ma} + 4\mu_{ma}\left[1 - G^2(1-\phi)^{2n}\right]/3} \tag{5}$$

Equation 5 was derived with the assumption that the S-wave velocity is related to the P-wave velocity by the following equation:

$$V_s = V_p G \alpha (1-\phi)^n \tag{6}$$

where

α is the V_s / V_p ratio for the matrix material, and

G is a scale determined to fit the observation.

As the solidity (complement of porosity) approaches a value of one (porosity of zero), the shear velocity of the rock becomes equal to the shear velocity of the matrix material. As the solidity decreases, the shear velocity in a fluid-saturated rock decreases. The exponent n will be used to incorporate changes in differential pressure, which is equal to the confining pressure minus pore pressure;. exponent n is also dependent on the state of consolidation. For clean sandstone, G is close to 1, but G becomes smaller as clay content increases in the matrix. Note that Lee (2002) used n = 1 and G = 1 in equation 5 for unconsolidated sediments.

The difference between the classical Biot-Gassmann theory (BGT) and the theory proposed by Lee (2002) is in the computation of shear modulus for water-saturated sediment. The Biot theory assumes that pore fluid does not affect the shear modulus of sediment; therefore, the shear moduli of dry rock and wet rock are the same. In other words, irrespective of the fluid type, the shear modulus of the formation is given by equation 4.

In this paper, when equation 5 is used to calculate the shear modulus, it is called BGT by Lee or BGTL; when equation 4 is used to calculate the shear modulus, it is called BGT. It is emphasized that there is no difference between BGT and BGTL for gas-saturated or dry rocks, and equation 4 should be used for both cases.

Velocity Ratio

The validity of equation 5 depends on the accuracy of the assumption of velocity ratio shown in equation 6. Figure 1 shows velocity ratios with respect to the differential pressure calculated from the Gregory (1976) and Domenico (1977) data. From this figure, the following observations can be made.

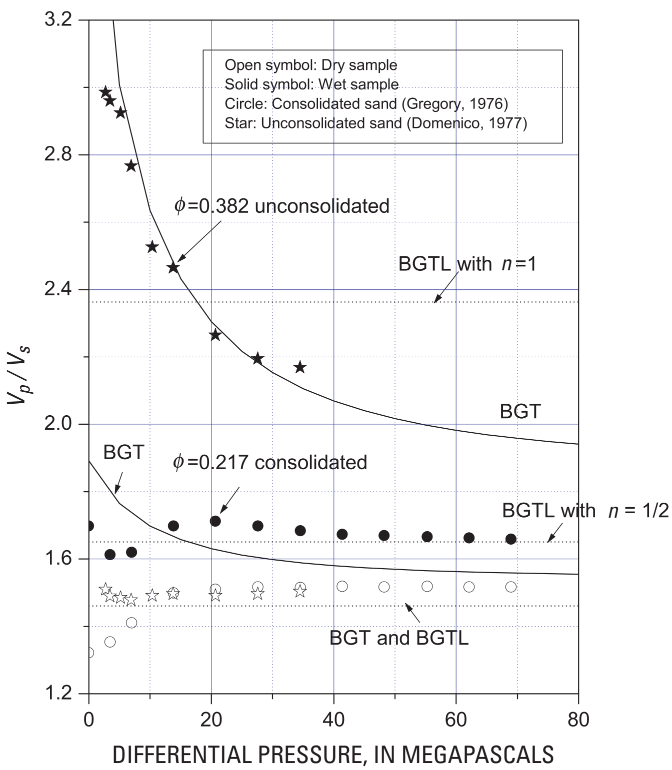

Figure 1. Measured and predicted velocity ratios for dry (open symbol) and wet rock (solid symbol) with respect to differential pressure. For consolidated sediments, data by Gregory (1976) measured at 1 MHz are used with the assumption that the uniform pressure is the same as the differential pressure—this is denoted by circles. For unconsolidated sediments, data by Domenico (1977) measured using a pulse width of 2 µs are used and denoted as stars. The predicted ratios from BGT are represented as solid lines; ratios from BGTL are shown as dotted lines. Equations 7 and 8 and the values shown in table 1 are used for predicted velocity ratios.

For dry sediments:

(a) Velocity ratio is insensitive to porosity if the differential pressure is greater than about 10 MPa. The velocity ratio of consolidated sediment that has a porosity of 0.217 is almost identical to that of unconsolidated sediment having a porosity of 0.382.

(b) The effect of differential pressure on velocity ratio is insignificant if the differential pressure is greater than about 10 MPa. There is a slight decrease of velocity ratio as the differential pressure decreases when the differential pressure is greater than 10 MPa; data by Winkler (1985) show the same trend. Scattering of the velocity ratio at less than 10 MPa may be caused by inaccuracy of measurement.

(c) Velocity ratio is insensitive to the degree of consolidation.

For wet sediments:

(a) Velocity ratio depends on porosity.

(b) For consolidated sediment, the velocity ratio increases slightly as the differential pressure decreases if the differential pressure is greater than 20 MPa; data by Winkler (1985) also indicate this trend. However, the effect of differential pressure on velocity ratio for consolidated sediment is much smaller than for unconsolidated sediment.

(c) If sediments are unconsolidated, the velocity ratio is highly dependent on differential pressure.

In general, the velocity ratio depends on matrix material, state of saturation, degree of consolidation, differential pressure, porosity, and other factors such as pore geometry.

The expected velocity ratios based on BGT or BGTL can be written as follows. Based on equations 3 and 4, BGT predicts

$$\left(\frac{V_p}{V_s}\right)^2 = \frac{4}{3} + \frac{k_{ma}}{\mu_{ma}} + \frac{\beta^2 M}{\mu_{ma}(1-\beta)} \qquad (7)$$

This indicates that the velocity ratio is a function of the Biot coefficient at a given porosity and depends on the porosity through the modulus M and β. Because the quantity $\beta^2 M/(\mu_{ma}(1–\beta))$, shown in equation 7, is small for porosity less than the critical porosity, which is 36–40 percent for well-rounded granular packs (Nur and others, 1998), the effect of porosity on velocity ratio for consolidated sediment is insignificant. Murphy and others (1992) also showed that the pore fluid contribution to velocity ratio is insignificant at low porosity. Because the Biot coefficient is a function of differential pressure at a given porosity, equation 7 could accurately model the velocity ratio with respect to differential pressure. However, it is not suitable for the prediction of velocity ratio with respect to porosity changes.

On the other hand, BGTL predicts the following velocity ratio:

$$\left(\frac{V_p}{V_s}\right)^2 = \frac{4}{3} + \frac{k_{ma}}{\mu_{ma} G^2 (1-\phi)^{2n}} + \frac{4\left[1 - G^2(1-\phi)^{2n}\right]}{3G^2(1-\phi)^{2n}} \qquad (8)$$

This equation indicates that the velocity ratio does not depend on the Biot coefficient directly. This implies that, in this formulation, whether the sediment is consolidated or not, velocity ratio is only a function of porosity. Therefore, it is not suitable to model velocity ratio with respect to differential pressure, but this equation accurately predicts velocity ratio with respect to porosity.

Modeling the Velocity Ratio

In order to model the velocity ratio with respect to the differential pressure and porosity, Biot coefficients shown in Appendix A are used with the parameters shown in table 1. Appendix A shows two examples in which the Biot coefficient depends on differential pressure and presents general Biot coefficients applicable to consolidated and unconsolidated sediments. The modeled velocity ratios are shown in figure 1 as solid and dotted lines. As mentioned previously, there is no difference between BGT and BGTL for a dry rock. Therefore, both theories accurately predict velocity ratios for dry rocks irrespective of the degree of consolidation, and velocity ratios depend only on the matrix material because *M* is negligible in equation 7.

Table 1. Elastic constants used for velocity model.

[From Helgerud and others (1999). Shear modulus and bulk modulus reported in Gpa; density reported in g/cm^3]

	Value used
Shear modulus of quartz	45
Bulk modulus of quartz	36
Shear modulus of clay	6.85
Bulk modulus of clay	20.9
Bulk modulus of water	2.29
Density of quartz	2.65
Density of clay	2.58

For wet sediments, BGTL with *n* = 0.5 accurately predicts the magnitude of velocity ratio for consolidated sediments (Gregory data), but it fails to predict an increase in velocity ratio with decreasing differential pressure. Note that *G* = 1 is used unless it is explicitly mentioned in this paper. BGT predicts the trend of velocity ratio with respect to differential pressure, but its magnitude is much higher than observed. On the other hand, BGT accurately predicts velocity ratios for unconsolidated sediments with respect to differential pressure, whereas BGTL with *n* = 1 fails to predict the velocity ratio. Constant values of velocity ratios calculated from BGTL irrespective of differential pressure are due to the fact that the variation of porosity changes with respect to differential pressure is ignored. Figure 1 also indicates that the velocity ratio for wet sediment depends on porosity. However, it is difficult to establish a relationship, partly because of the difference in consolidation and partly because only two porosities are shown.

Figure 2 shows velocity ratios calculated from published data, mostly complied by Castagna and others (1985), with predicted ratios from BGT and BGTL. Figure 2 suggests that, for all porosity ranges, the velocity ratio increases as porosity increases except for all nine gas-sand samples, which follow the trend of dry rock as BGT predicts. For unconsolidated sediments with porosities greater than 40 percent, BGTL with *n* = 1 agrees well with the measured values. For porosities less than 40 percent, BGTL with *n* = ½ and *n* = 1 performs equally well. The scattering of velocity ratios shown in figure 2 is partly due to the clay content of the samples. Figure 3 shows the measured velocity ratios using Han's data (Han and others, 1986) at the differential pressure of 30 MPa with respect to porosity and clay volume content. Predicted velocity ratios from BGT and BGTL are also shown in figure 3. It appears that, for clay content of less than 10 percent, BGTL with *n* = ½ is applicable; for high clay content, BGTL with *n* = 1 performs better.

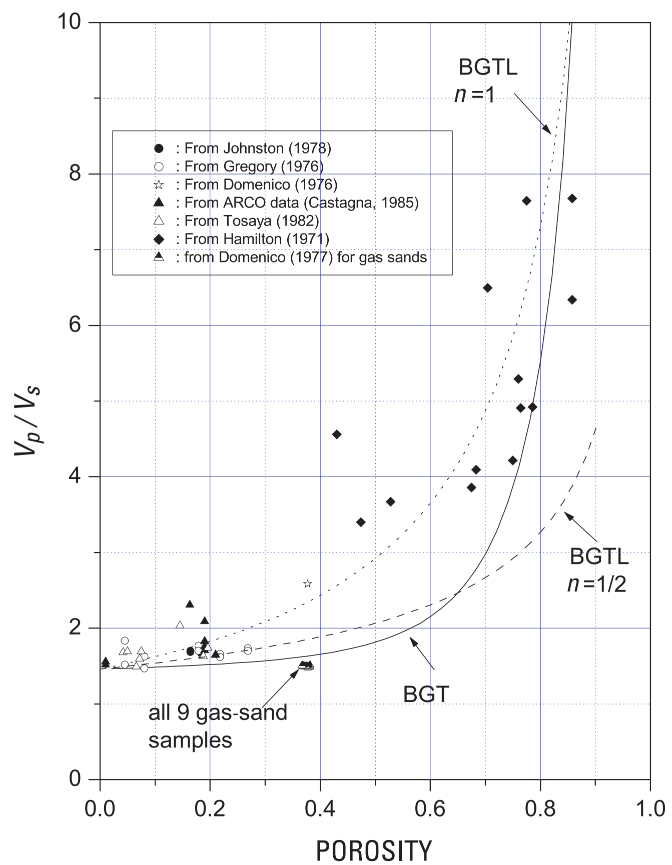

Figure 2. Measured velocity ratios with respect to porosity for data complied by Castagna and others (1985) and gas samples from Domenico (1977). The predicted velocity ratio from BGT is represented by a solid line, BGTL with *n* = 0.5 by a dashed line, and BGTL with *n* = 1 by a dotted line. Equations 7 and 8 and the values shown in table 1 are used for predicted velocity ratios. Data complied by Castagna and others (1985) are measured using frequencies ranging from 200 kHz to 2 MHz.

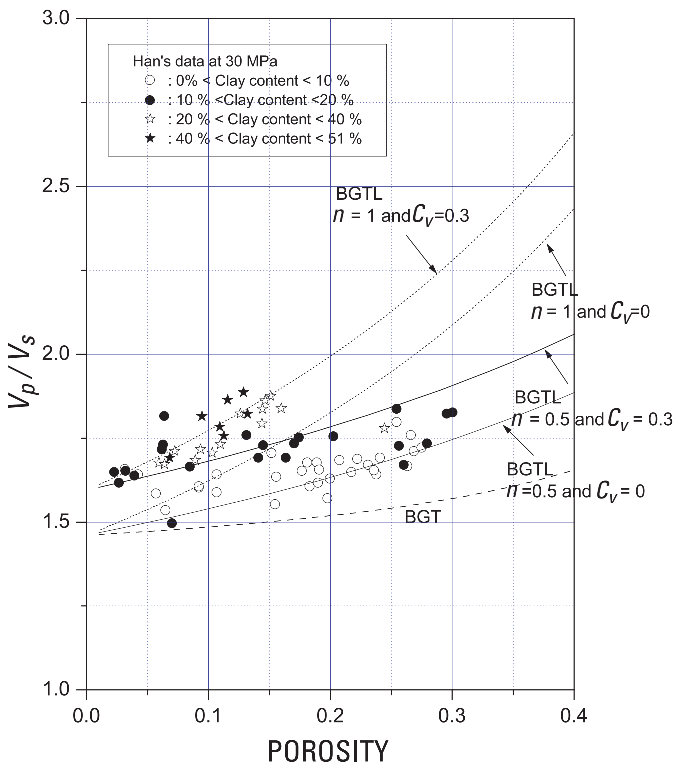

Figure 3. Measured velocity ratios with respect to porosity and clay volume content (C_V) for the data of Han and others (1986) at the differential pressure of 30 MPa. The central frequencies of transducers used for P- and S-waves are 1.0 and 0.6 MHz, respectively. Equations 7 and 8 and the values shown in table 1 are used for the predicted velocity ratios.

Based on figures 1, 2, and 3, the following general observations can be made.

1. For water-saturated and consolidated sediments, BGTL with $n = ½$ is appropriate for modeling the velocity ratio and is preferred to BGT.
2. For water-saturated unconsolidated sediments, BGTL with $n = 1$ is appropriate and is preferred to BGT.
3. BGT predicts accurate velocity ratios for dry rock and models well the effect of the differential pressure for unconsolidated sediments.
4. The exponent n appears to increase as clay content increases. However, it is shown that the effect of clay can be treated better by G.

Modeling Velocities

Figure 4 shows theoretical predictions of velocities with respect to differential pressure using the data given by Gregory (1976) and Domenico (1977). For dry rock, there is no difference between BGT and BGTL, as mentioned earlier. To compute velocities, the Biot coefficient shown in Appendix A (equation A-3 for Domenico data and equation A-4 for Gregory data) with parameters shown in table 1 is used. Figure 4*A* shows the result for the unconsolidated sediment investigated by Domenico (1977). For the dry rock, P-wave velocities by BGT are almost identical to measured velocities and computed S-wave velocities are slightly higher than measured velocities. For the wet rock, BGT (dotted line) slightly underestimates both P- and S-wave velocities, but it predicts the velocity trend with respect to differential pressure very accurately. On the other hand, BGTL with $n = 1$ (dash-dot-dash line) predicts accurate P-wave velocities for a differential pressure of less than 10 MPa, but it underestimates the P-wave velocity at higher differential pressure—the underestimation of velocity increases as the differential pressure increases. For S-wave velocity, BGTL accurately predicts the velocity only at a differential pressure of about 15 MPa; it overestimates S-wave velocity for differential pressure less than 15 MPa; and it underestimates S-wave velocity for differential pressure greater than 15 MPa.

Figure 4*B* shows similar results for the consolidated sediment investigated by Gregory (1976). For dry rock, BGT predicts accurate P-wave velocity at high differential pressure (greater than 30 MPa), but it overestimates S-wave velocity for differential pressure greater than 10 MPa. BGTL with $n = 0.5$ underestimates both P- and S-wave velocities.

Results shown in figure 4 confirm that BGT is effective in predicting velocity changes with respect to differential pressure (equation 7). This result agrees with Hornby and Murphy (1987), who demonstrated that BGT accurately predicts the effect of effective pressure on the velocity of fully water saturated unconsolidated sand. However, BGTL is not suitable to predict the effect of differential pressure, as shown in equation 8. Results denoted by heavy dashed lines in figure 4 will be discussed later.

Discussion

Fluid Effect on Shear Velocity

In poroelasticity, it is usually assumed that the fluid does not affect the shear modulus of rock unless the viscosity is high. Therefore the shear modulus of dry rock and water-saturated rock are the same, and BTG uses the same shear modulus for dry and wet rocks. However, BGTL incorporates a dependence of fluid property in the shear modulus through modulus M, which measures the variation in hydraulic pressure needed to force an amount of water into the formation without any change in formation volume (Krief and others, 1990). This dependence of fluid on shear velocity comes from the assumption shown in equation 6. Equation 5 indicates that the shear modulus depends on the bulk modulus, which, in turn, depends on the properties of the pore fluid in the sedi-

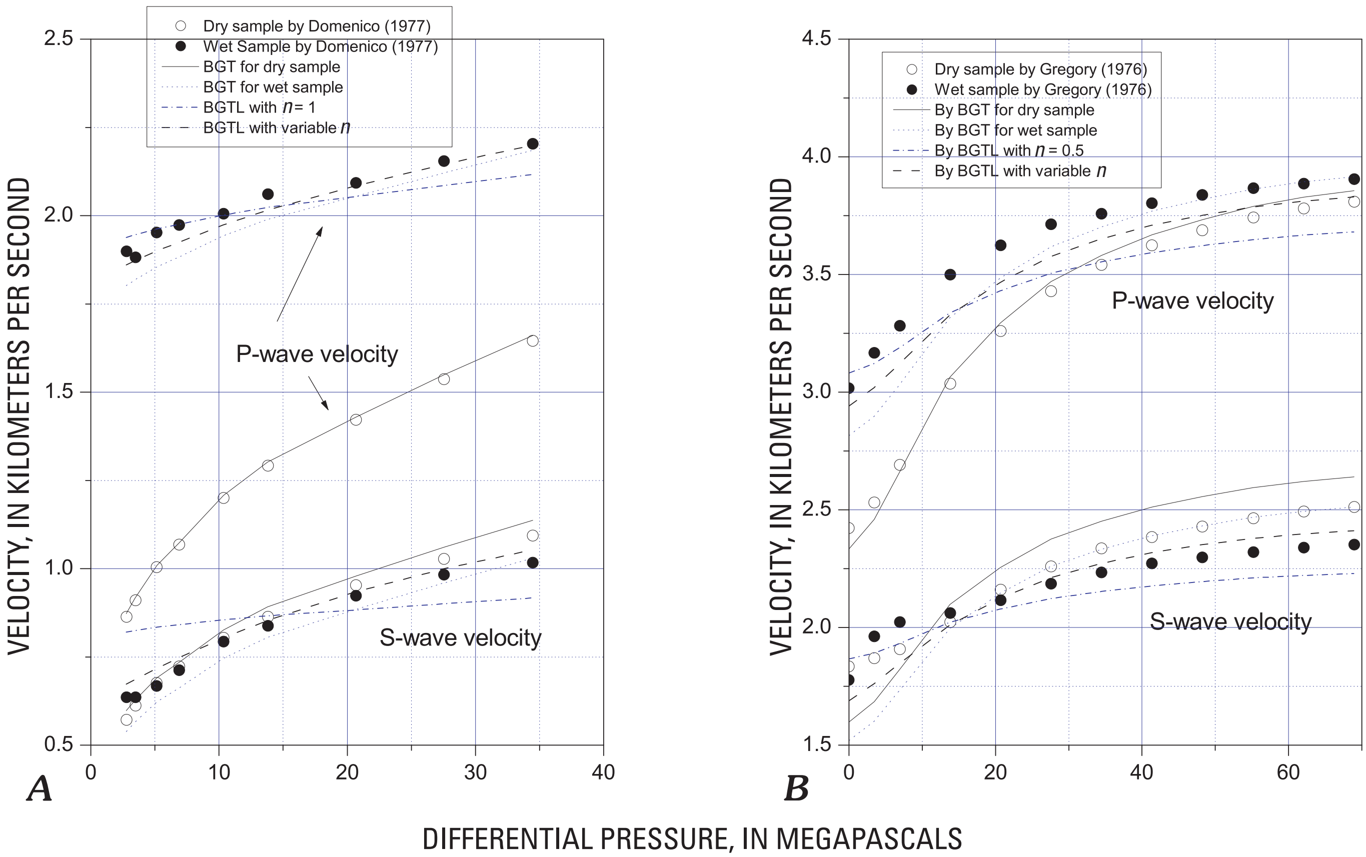

Figure 4. Measured and modeled velocities based on BGT and BGTL with respect to differential pressure. For elastic moduli, the values shown in table 1 are used and the Biot coefficient is computed using Appendix A, equations A-3 and A-4. *A*, Unconsolidated sediments measured by Domenico (1977). *B*, Consolidated sediments measured by Gregory (1976).

ment. On the other hand, BGT indicates that bulk modulus is related to the shear modulus through the Biot coefficient, which is independent of pore fluid.

Equation 5 indicates that, when *n* is greater than about 0.25 and the porosity is less than the critical porosity, the shear modulus computed from BGTL is less than that from BGT. In other words, the formation becomes softer. The degree of softness increases with increasing *n*. If *n* is near 0.25, the difference between BGT and BGTL is negligible.

According to Biot (1956), the fluid actually affects S-wave velocity through coupling between the pore fluid and rock frame. The coupling constant, χ, appears in the following equation:

$$V_s = \left[\frac{\mu}{\rho[1-\rho_{fl}\phi/(\rho\chi)]} \right]^{1/2} \quad (9)$$

where the coupling constant, χ, ranges from 1 for no coupling at infinite frequency to ∞ for perfect coupling at zero frequency.

The usual S-wave velocity or velocity of BGT is given by equation 9 at zero frequency. This kind of dependence of S-wave velocity on the presence of fluid is different from the result of BGTL. Note that the S-wave velocity is always greater than the S-wave velocity at zero frequency; the pore fluid increases the S-wave velocity as the coupling constant decreases.

One of the assumptions in Gassmann's equation and implicit in the BGT is that the pore fluid does not interact with the solid in a way that would soften or harden the frame. In reality, the pore fluid will inevitably interact with the rock's solid matrix to change the surface energy. When a rock is saturated by a fluid, the fluid may either soften or harden the matrix—a shaly sandstone is often softened because of clay swelling (Wang, 2000). Therefore, the exponent *n* could be interpreted as a parameter controlling the degree of the softening/hardening effect of fluid on the formation. One problem with this interpretation is that the softening/hardening occurs only for the shear modulus, not for the bulk modulus. Therefore, this may not be a problem for the velocity ratio (i.e., equation 8), but it may produce asymmetrical error in predicting P- and S-wave velocity from the porosity.

Another interpretation of fluid effect on S-wave velocity is velocity dispersion. BGT is a theory for low frequency, so there is no interaction between fluid and solid in rocks because fluid and solid in rocks moves in phase. However, at high frequencies, fluid interacts with the solid in such a way that velocity increases as the frequency increases (i.e., velocity dispersion) (Dvorkin and Nur, 1992; Dvorkin and others, 1993, 1994; Mavko and Jizba, 1993). In Biot's model, pore fluid participates in a solid's motion by viscous friction and inertia coupling (Biot's flow), and a small amount of velocity dispersion occurs for frequencies of less than 1 MHz (Dvorkin and others, 1993). A different mechanism of fluid flow (the squirt flow), perpendicular to wave propagation, is associated with the squirting of pore fluid out of cracks and results in high velocity dispersion for frequencies greater than sonic frequencies (kHz range). As apposed to BGT, BGTL is based on the velocity ratio (measured at any frequency), so its formulation implicitly includes the effect of velocity dispersion.

Scale *G*

The accuracy of BGTL depends on how accurately the observed V_p/V_s of sediments agrees with the velocity ratio of matrix material at zero porosity. Many factors, such as the aspect ratio of pore space, clay volume content, degree of compaction, and differential pressure, affect the velocity ratio and cause discrepancy between predicted and observed velocities. In BGTL formulation, the exponent n accounts for the effect of differential pressure and the degree of consolidation/compaction and scale G attempts to account for the effect of clay (or gas hydrate) on velocity. The effect of clay on velocity is partially corrected by using Hill's (1952) averaging method of elastic moduli. The other effect of clay on velocity by BGTL comes from the fact that the shear modulus of the matrix used for the derivation of the Biot coefficient (which used a clean sandstone with the shear modulus shown in table 1) is different from the shear modulus of matrix of the sediment, such as a shaly sandstone. Because the Biot coefficient depends on the shear modulus of the matrix, a different Biot coefficient should be used for different matrix material. Also, the presence of clay in the matrix reduces the overall aspect ratio of the pore space (Xu and White, 1996) and affects the velocity ratio. Introducing a scale, G, in the formulation of BGTL is an attempt to correct the discrepancy between observed and calculated velocity ratios. There are two ways of implementing G in BGTL formulation. One approach is to use the same Biot coefficient that was derived from a clean sandstone (such as in equation A-6) for shaly or gas-hydrate-bearing sediments. Numerically this is the same as treating αG as a modified velocity ratio of matrix material. The other method is to use a different Biot coefficient depending on G.

A different Biot coefficient for shaly sandstone can be derived as follows.
The equation for β can be written, at zero porosity, as

$$\mu \approx \mu_{ma}(1-\beta)G^2 \tag{10}$$

If β_1 is used as β computed with G = 1 and β_G is the β computed with G other than 1, the Biot coefficient with G other than 1 is given by

$$\beta_G \approx 1-\left(\frac{1-\beta_1}{G^2}\right) \tag{11}$$

Introducing different Biot coefficients depending on G in BGTL results in computed velocities that are closer to measured velocities, but it also limits the application of the theory. Because the frame shear modulus cannot be greater than the shear modulus of the matrix, the lower limit of β is zero. Usually G is less than 1, so equation 11 yields β values less than 0, which is not physically possible. The porosity that yields negative β using equation 11 is defined as a cut-off porosity, ϕ_{co}. Therefore, the use of equations 3 and 5 to compute the moduli of sediments is limited to porosities greater than ϕ_{co}. For example, when G = 0.8, a negative β_G results when β is less than $(1–G^2)$ or 0.36. Therefore, from equation A-6, the cut-off porosity is about 5 percent. In other words, BGTL given in equations 3 and 5 is correct only for porosities greater than about 5 percent. At the cut-off porosity, the Biot coefficient is zero and the following relation exists.

$$k = k_{ma} = k_{sk}$$

$$\mu = \frac{k_{ma}\mu_{ma}G^2(1-\phi_{co})^{2n}}{k_{ma}+4\mu_{ma}\left[1-G^2(1-\phi_{co})^{2n}\right]/3} \approx \mu_{ma}G^2(1-\phi_{co})^{2n} \quad \text{when } G \approx 1 \tag{12}$$

It is shown that, at the cut-off porosity, the bulk modulus of the formation is the same as the bulk modulus of the matrix or the skeleton, but the shear modulus of the formation is less than the shear modulus of the matrix.

At ϕ = 0, the modulus, M, is given by $M = k_{ma} / \beta$. This modulus is negative when β is negative, so there is no physical meaning to M. However, when M is inserted into equation 5, the shear modulus is given by $\mu \approx G^2\mu_{ma}$ at ϕ = 0 and $G \approx 1.0$. This relationship with equation 12 indicates that the shear modulus reasonably increases when the porosity decreases from the cut-off porosity to zero porosity. Therefore, for this porosity range (zero to cut-off porosity), the shear modulus of the formation has a physically realizable value and S-wave velocity can be predicted from the BGTL. It is proposed that the shear-wave velocity can be obtained beyond the cut-off porosity by interpolating the shear modulus at ϕ = 0 and $\phi = \phi_{co}$. It is noted that the bulk modulus cannot be interpolated between ϕ = 0 and $\phi = \phi_{co}$ because, at both porosities, the bulk modulus is given by $k = k_{ma}$.

The difference between the two approaches can be examined from figure 5. Figure 5 indicates that both velocities, V_p and V_s, predicted from the first approach are lower than velocities using G = 1. However, velocities predicted from second approach show that P-wave velocity with G = 0.9 is almost the same as that computed with G = 1 and V_s is lower than that with G = 1. Both approaches yield the same velocity ratios, as shown in figure 5. The first approach compensates for the discrepancy between measured and calculated velocity ratio by lowering both V_p and V_s, whereas the second approach compensates primarily by lowering V_s. Both approaches can be applied to analyze observed data. In the case that BGTL overestimates both V_p and V_s, the first approach is preferable. On the other hand, in the case that BGTL predicts accurate P-wave velocity but overestimates S-wave velocity, the second approach is suitable. All examples in this paper are calculated using the first approach of implementing G in BGTL.

The magnitude of G depends on the matrix. The more the shear modulus of the matrix deviates from the shear modulus of clean sandstone, the smaller G becomes. For a clean sandstone, G is close to 1 because the shear moduli of various kinds of quartz are similar to each other and to the value given in table 1. Based on first-approach numerical experiments on the data of Han and others (1986), it is suggested that

$$G = 0.9552 + 0.0448 e^{-C_v / 0.06714} \tag{13}$$

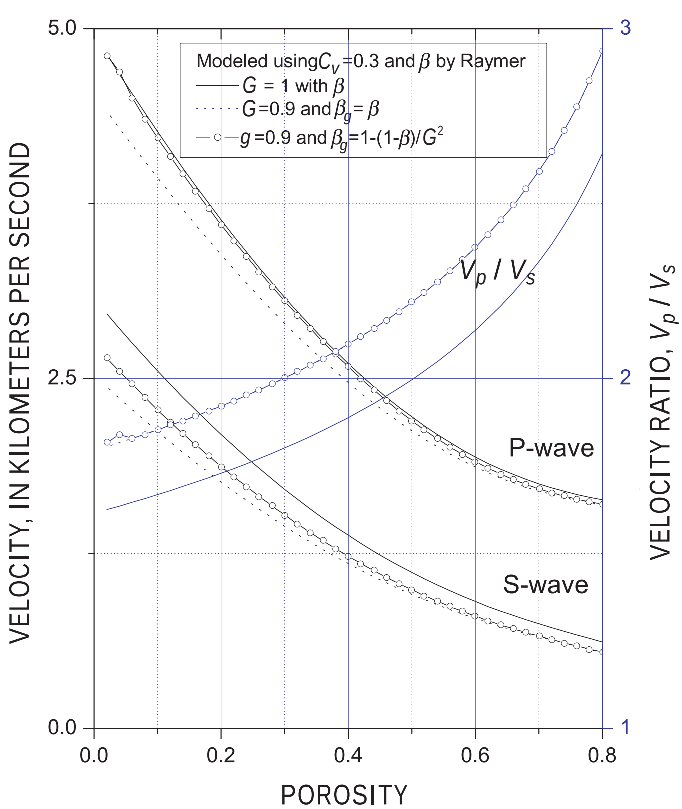

Figure 5. Graph showing the effect of scale G on velocities. Note that the second approach of implementing G in BGTL changes P-wave velocity slightly, but S-wave velocity changes significantly.

for consolidated sediments having an average clay volume content of 15 percent. For practical applications of BGTL, G may be treated as a free parameter to fit the observation. For unconsolidated sediments, data at the Mallik 2L-38 well indicate that G becomes small as the gas-hydrate concentration increases; G is close to 0.85 for sediments with high gas-hydrate concentration (more than 80 percent).

Analysis of Exponent *n*

The exponent n is a free parameter in BGTL that can be chosen to fit the measured data. In the case that BGTL is used to predict S-wave velocity from porosity and P-wave velocity, an optimum n can be estimated from the P-wave data by fitting the observation to the BGTL. However, if the purpose is modeling of P- and S-wave velocities from porosity, n should be known beforehand.

General trends of the exponent n can be inferred from figures 1, 2, and 3 and from data presented by Han and others (1986). Analysis of Han's data and figures 1, 2, and 3 suggest that (1) as the consolidation increases, the exponent n decreases. (2) as the clay content increases, n increases. and (3) as the differential pressure decreases, n increases.

As can be inferred from equation 8, as n increases, V_p/V_s increases. An increase of V_p/V_s is related to the consolidation of the sediment and also to clay content. Therefore, a decrease of n with increasing consolidation agrees with the observation of Gregory (1976) that V_p/V_s increases as consolidation decreases. The effect of clay can be modeled by using a clay-dependent exponent, but it is more effective to use a scale G, as discussed in the previous section.

In order to derive a functional relation between the exponent and the differential pressure for clean sandstone, results shown in figure 4 were carefully examined. By fitting the predictions of BGTL with different exponents to the velocities of unconsolidated wet sediment at different differential pressures (figure 4*A*), the following equation is derived for the exponent:

$$n = 0.67 + 0.77 e^{-p/17.78} \tag{14}$$

where

p is the differential pressure in MPa.

Based on equation 14 and the above observations, a general formula for exponent n as a function of differential pressure and degree of consolidation is proposed as follows:

$$n = \left[\ 0.67 + 0.77 e^{-p/17.78}\ \right] / m \tag{15}$$

where

m is a constant incorporating the effect of consolidation.

Because equation 14 is derived from data on unconsolidated sediment (Domenico, 1977), *m* = 1 is appropriate for unconsolidated sediment and *m* = 3 is appropriate for consolidated sediment at high differential pressure, as demonstrated in the following section.

The result of using *n* as a function of differential pressure is shown in figure 4*A* as heavy dashed lines. As indicated in figure 4*A*, BGTL with a variable *n*, which is computed from equation 15 with *m* = 1, accurately predicts both P- and S-wave velocities. In order to model the effect of differential pressure on the velocity of consolidated sediment (Gregory, 1976), an exponent calculated with *m* = 2 is tried and the result is shown in figure 4*B*. (Note that *m* = 2 instead of *m* = 3 is used for the Gregory data. The reason is that the Biot coefficient calculated for the Gregory data is higher than the one calculated from the Raymer and others (1980) data (Appendix A). This implies that the sample used by Gregory (1976) is less consolidated than the sample used by Raymer and others (1980)). Figure 4*B* indicates that BGTL with the variable exponent, shown as heavy dashed lines, accurately predicts the effect of differential pressure on velocities, albeit predicted P-wave velocities are slightly lower than measured velocities.

The above estimation of *n* is based on sediment having a porosity of 0.382 (Domenico data), and it also appears to be reasonable for sediment having a porosity of 0.217 (Gregory data). Does this hold for sediments with other porosities? The next section investigates this question.

Application to Velocity Prediction

In order to see the effectiveness of a variable *n* defined in equation 15, data by Han and others (1986) are used. Han's data includes porosities that range from 2 to 30 percent and C_v that ranges from 0 to 51 percent. Both BGT and BGTL can be used to predict velocities from porosity, and either can be used to predict S-wave velocity from porosity and P-wave velocity. With BGTL, the S-wave velocity can be predicted from porosity and P-wave velocity using the following formula, which is derived from equations 1 and 4:

$$\mu = \frac{\mu_{ma}(1-\phi)^{2n}\rho V_p^2}{k_{ma} + 4\mu_{ma}/3} \qquad (16)$$

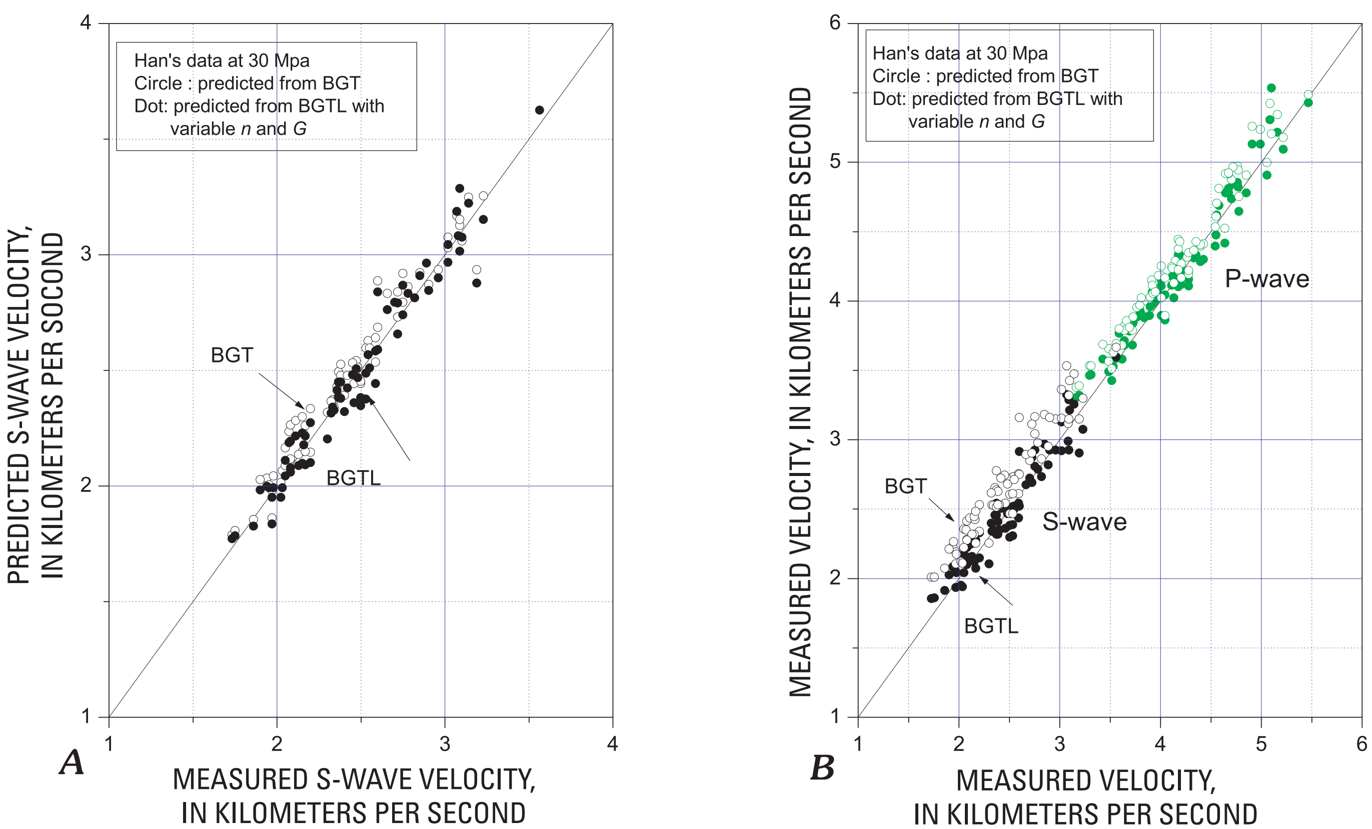

Figure 6. Measured velocities by Han and others (1986) at the differential pressure of 30 MPa and predicted velocities from BGT and BGTL using elastic moduli shown in table 1. *A*, S-wave velocity predicted from measured P-wave velocity and porosity. For BGT, the Biot coefficient is calculated from Appendix B. *B*, P- and S-wave velocities predicted from porosity. Biot coefficient by Raymer and others (1980)—equation A-6 is used.

In BGT, the S-wave velocity can be derived from the P-wave velocity through the Biot coefficient. Therefore, in order to predict S-wave velocity from P-wave velocity and porosity using BGT, equation B-1, shown in Appendix B, is solved for the Biot coefficient using the measured P-wave velocity. In the following examples, when BGT is used to predict S-wave velocity from P-wave velocity and porosity, the Biot coefficient calculated from Appendix B is used. When predicting velocities from porosity only, the Biot coefficient shown in equation A-6 (Appendix A) is used because the Biot coefficient cannot be calculated without velocity data.

Figure 6*A* shows S-wave velocities predicted from P-wave velocities and porosities using Han's data at 30 MPa based on BGTL and BGT. A variable exponent *n* from equation 15 with $m = 3$, $p = 30$ MPa, scale *G* given in equation 13, and the values shown in table 1 are used for BGTL. The fractional S-wave velocity error ($\Delta V_S / V_S$) from BGTL is 0.01±0.03, whereas it is 0.06±0.03 from BGT.

Figure 6*B* shows P- and S-wave velocities predicted using only porosities. Figure 6*B* indicates that both BGTL and BGT predict reasonable velocities, but velocities predicted from BGTL are more accurate. Overall, BGT slightly overestimates the velocities and BGTL slightly underestimates velocities. The fractional errors for P-wave velocities predicted from BGTL and BGT are –0.01±0.02 and 0.03±0.03, respectively, indicating the same amount of error for both methods. However, the fractional errors for S-wave velocities predicted from BGT and BGTL are 0.09±0.05 and 0.00±0.05, respectively, indicating more accurate S-wave velocities from BGTL.

Figure 7 shows predictions using a different set of data compiled by Castagna and others (1985). As pointed out by Koesoemadinata and McMechan (2001), the data were missing some explicit auxiliary information. It is assumed that the data were acquired at a differential pressure of 50 MPa and the average clay volume content is 15 percent. Equation 15 with $m = 3$ and $p = 50$ MPa yields $n = 0.25$. Using table 1, $n = 0.25$, and *G* given by equation 13, S-wave velocities predicted from BGT and BGTL, with predictions by Castagna and others (1985), are shown in figure 7*A*. All three predictions are similar, but the predictions by Castagna and others (1985) are the most accurate. However, their accuracy was achieved by using the wet bulk modulus calculated from both P- and S-wave velocities. Predicted P- and S-wave velocities from BGTL, shown in figure 7*B*, are more accurate than those predicted from BGT. The fractional errors for P- and S-wave velocities predicted from BGTL are 0.03±0.11

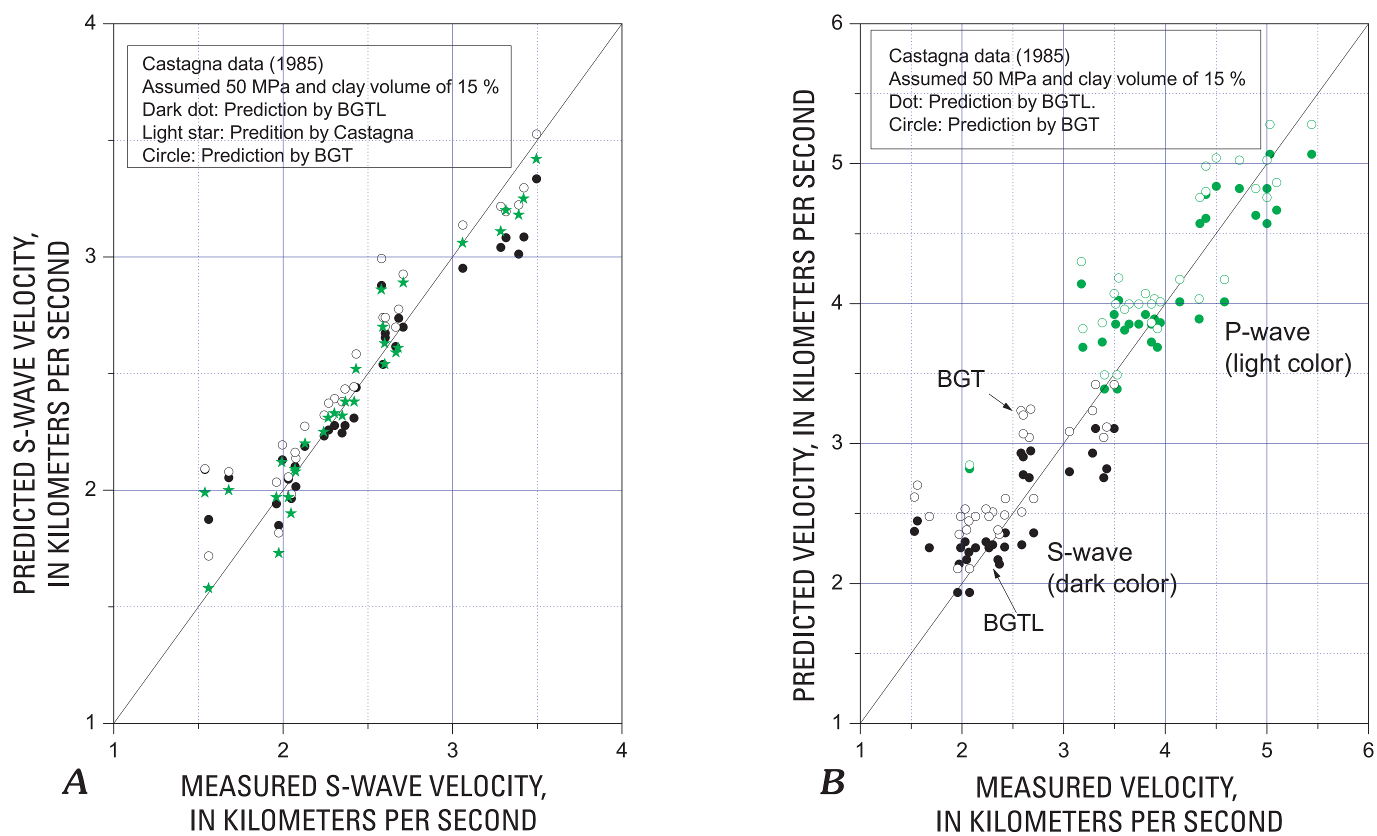

Figure 7. Predicted and measured velocities for data compiled by Castagna and others (1985). For the prediction, a differential pressure of 50 MPa and a clay volume content of 15 percent are assumed using elastic moduli shown in table 1. *A*, S-wave velocity predicted from measured P-wave velocity and porosity. For BGT, the Biot coefficient is calculated using Appendix B. *B*, P- and S-wave velocities predicted from porosity. Biot coefficient by Raymer and others (1980)—equation A-6 is used.

and 0.06±0.24, respectively, whereas they are 0.07±0.11 and 0.17±0.25, respectively, for velocities predicted from BGT. More scattering in figure 7*B* compared to figure 6*B* may be caused by using average values for differential pressure and clay volume content.

Velocity Dispersion and Velocity Ratio

The velocity ratio mainly depends on porosity, differential pressure, clay content, and frequencies of measurements. It has been known that elastic wave velocities in water-saturated sediments are dispersive (cf. references in Mavko and others, 1998), so the velocity ratio would be dependent on the frequency of measurement. In the BGTL formulation, the exponent n incorporates the effect of differential pressure and the scale G compensates for the effect of clay on the velocity ratio. However, the proposed BGTL does not include the effect of velocity dispersion. The exponent n and scale G were based on the data by Gregory (1976) measured at 1 MHz, by Domenico (1977) measured at about 0.5 MHz (pulse width of 2 µs), and by Han and others (1986) measured at 1 MHz for the P-wave velocity and at 0.6 MHz for the S-wave velocity. The question is how much error is introduced by ignoring velocity dispersion in BGTL.

In order to examine the magnitude of error in BGTL by ignoring the velocity dispersion, an empirical formula by Koesoemadinata and McMechan (2001), who used a variety of data measured with frequency ranges from 380 Hz to 1 MHz, is used. Their analysis indicates that the dispersive part of P-wave velocity is 0.338f, where f is the frequency in MHz; it is 0.368f for the S-wave velocity. The P-wave velocity of a sandstone having a porosity of 15 percent is 4.237 km/s, and it is 2.532 km/s for the S-wave at 0 Hz and p = 40 MPa. These velocities are 4.575 km/s and 2.9 km/s for P- and S-wave velocity, respectively, at 1 MHz. Therefore, V_p/V_s at 0 Hz is 1.673 and is 1.578 at 1 MHz, which is about a 6 percent decrease in V_p/V_s going from 0 Hz to 1 MHz. However, velocity itself has a larger error, namely 8 percent for the P-wave velocity and 15 percent for the S-wave velocity.

Velocity dispersion requires that the exponent n should be dependent on frequency as well as differential pressure to accurately predict velocities. If the exact amount of velocity dispersion is known, the exponent n can be adjusted to fit the measurements. However, quantifying velocity dispersion is difficult. The velocity dispersion in sediments is very important and yet poorly understood (Wang and Nur, 1990). Note that Wang and Nur (1990) stated “We still do not know exactly how much velocity dispersion occurs in fluid-saturated rock from seismic to laboratory ultrasonic frequencies.” McDonal and others (1958) observed that there appears to be no detectable dispersion of velocity with frequency in consolidated sedimentary rocks at frequencies of less than 1 MHz. Blangy and others (1993) analyzed Troll sandstone and concluded that ultrasonic measurements can be used directly for detailed seismic work without correction for frequency dispersion, probably because of high permeability of Troll sands and the lack of grain-scale local flow effect (Mavko and Jizba, 1991).

Therefore, the proposed BGTL is optimum for velocities measured at ultrasonic frequencies and for sandstones with high permeability without grain-scale local flow; it is also optimum for consolidated sediments with no appreciable velocity dispersion. Because the exponent n is based on ultrasonic frequency, the appropriate n for velocities measured at frequencies less than 1 MHz would be a little larger than n calculated from equation 15.

Error Analysis

For a given Biot coefficient, predicted errors in BGT come from the error in the calculation of matrix material. In BGTL, in addition to error in the calculation of matrix material, errors depend on exponent n and scale G . The error in the predicted S-wave velocity from BGTL can be written in the following form using equation 6:

$$\frac{\Delta V_s}{V_s} = \frac{\Delta V_p}{V_p} + \frac{\Delta\alpha}{\alpha} + n\ln(1-\phi)\frac{\Delta n}{n} + \frac{\Delta G}{G} \qquad (17)$$

When the measured P-wave velocity is used for S-wave prediction, the first term can be ignored. Therefore, the fractional error in S-wave velocity is proportional to the fractional error in the matrix material, to the fractional error in scale G, and is linearly proportional to the fractional error in the exponent. The error caused by n depends on the magnitude of n and porosity and increases as the porosity and exponent increase. For consolidated sediments, n is usually 0.5. Assume that a fractional error in n is ±20 percent. Then the fractional error in S-wave velocity using BGTL is about ±4 percent for a porosity of 30 percent and ±1.6 percent for a porosity of 10 percent. The average porosity reported in the data of Han and others (1986) is 15 percent, and the fractional S-wave velocity error from BGTL is less than 1 percent (figure 6*A*). Therefore, the exponent used for the data of Han and others (1986) at 30 MPa has an error of about 12 percent, assuming no other errors.

The error in the matrix can be reduced by a proper use of elastic moduli of the matrix material based on well-log analysis, laboratory measurements, or published data. However, the error associated with n and G is more complicated and difficult to estimate because optimum n and G are difficult to compute without velocity data. Therefore, a judicious choice of G and n, guided from equations 13 and 15, is important to accurately predict velocities.

Conclusion

The classical Biot-Gassmann theory (BGT) accurately predicts the effect of differential pressure on velocities or the velocity ratio. However, explicit Biot coefficients as a function of differential pressure are required to predict velocities. Without an explicit Biot coefficient for a particular data set, the Biot coefficient given by Raymer and others (1980) provides an accurate P-wave velocity at differential pressures higher than about 30 MPa. Because the velocity ratio derived from BGT is not sensitive to porosity variation at low porosities, it is not suitable to model the velocity ratio with respect to porosity. BGT is easy to use, but it usually overestimates velocities, particularly the S-wave velocity.

Biot-Gassmann theory by Lee (BGTL), which is formulated under the assumption that the velocity ratio is a function of porosity (ϕ) such that $(1-\phi)^n$ accurately predicts velocity ratio or velocities with respect to porosity, but not with respect to differential pressure. In order to overcome this problem, a variable exponent n as a function of the differential pressure, which is estimated from measured velocities of unconsolidated sediment, is proposed and applied to the consolidated sediment. However, velocity dispersion is not included in the BGTL formulation. This investigation suggests that BGTL is preferable to BGT in most cases.

One of the consequences of the BGTL formulation is the dependence of S-wave velocity on pore fluid. This coupling of pore fluid to the S-wave velocity is interpreted as a manifestation of the effect of fluid interaction on the solid matrix.

The prediction of S-wave velocity based on BGTL requires properties of matrix material, including clay content, porosity (or porosity and P-wave velocity), the Biot coefficient such as proposed by Raymer and others (1980) or Lee (2002), and differential pressure. The accuracy of prediction of velocity increases as differential pressure increases because the error associated with the exponent decreases as n decreases or as differential pressure increases.

Exponent n is based on data measured at ultrasonic frequencies—the proposed BGTL is optimum for velocity data measured in the range of 1 MHz. However, the effect of velocity dispersion could be incorporated by using a frequency-dependent n if the exact amount of velocity dispersion is known.

The application of BGTL is more complex than the application of BGT because the exponent n depends on many factors. A judicious choice of n and G is essential to accurately predict velocities, and feasible n and G values can be estimated from the general guidelines and examples presented in this paper.

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Appendix A

Biot Coefficient

Wave velocity in a dry rock depends on differential pressure as well as porosity. Therefore, the Biot coefficient depends on porosity and differential pressure. Two examples of Biot coefficients depending on differential pressure are calculated from measured velocities.

The Biot coefficient can be calculated from equations 3 and 4 using the following equation:

$$\beta = 1 - \frac{\rho_d (V_{pd}^2 - 4V_{sd}^2 / 3)}{k_{ma}} \qquad \text{(A-1)}$$

or

$$\beta = 1 - \frac{\rho_d V_{sd}^2}{\mu_{ma}} \qquad \text{(A-2)}$$

In equations A-1 and A-2, ρ_d, V_{pd}, V_{sd}, are the density of dry rock, P-wave velocity of dry rock, and S-wave velocity of dry rock, respectively.

Figure A-1 shows calculated Biot coefficients using unconsolidated sands measured by Domenico (1977) with respect to differential pressure. Equations A-1 and A-2 yield slightly different Biot coefficients, so the average is taken as the Biot coefficient in this study. The porosity of the dry rock sample is 0.3817 at 2.7 MPa and decreases to 0.3672 at 34 MPa. A least-squares fitting curve to the calculated Biot coefficient is also indicated and is given by

$$\beta = 0.92767 + 0.0635 e^{-p/36.61} \qquad \text{(A-3)}$$

where

p is the differential pressure in MPa.

Figure A-1 also shows calculated Biot coefficients using consolidated sediment having a porosity of 0.217, which was measured by Gregory (1976). A least-squares fitting curve is given by

$$\beta = 0.65948 + 0.23256 e^{-p/27.41} \qquad \text{(A-4)}$$

In the case that there is no measured Biot coefficient, the following Biot coefficient can be used. For unconsolidated sediments, the following Biot coefficient is suggested (Lee, 2002)

$$\beta = \frac{-184.05}{1 + e^{(\phi + 0.56468)/0.10817}} + 0.99494 \qquad \text{(A-5)}$$

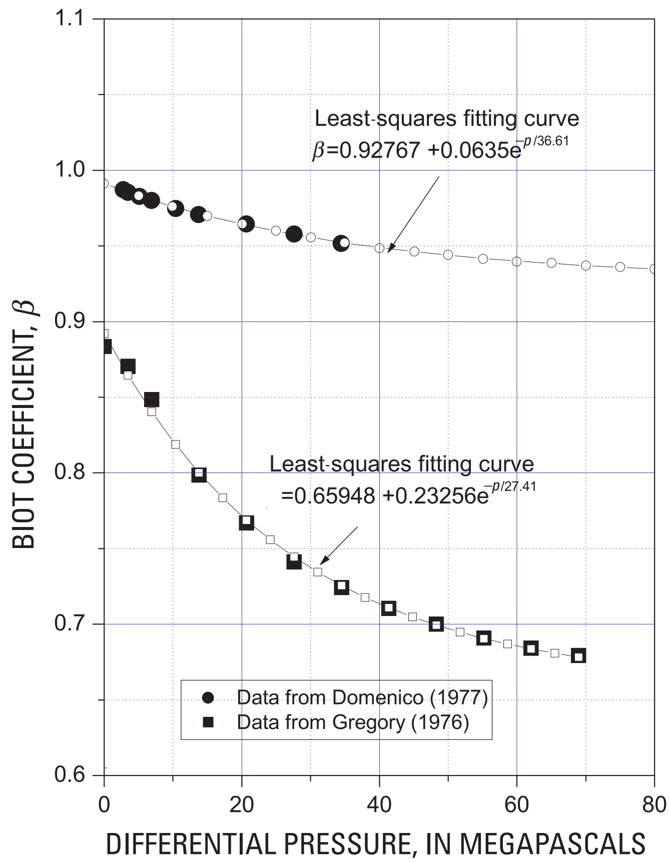

Figure A-1. Computed Biot coefficients from velocities of dry samples and least-squares fits to calculated coefficients. The computed Biot coefficient is the average of Biot coefficients computed from P- and S-wave velocities.

For hard formations, the equation by Raymer and others (1980), which is written in the following form by Krief and others (1990), can be used.

$$\beta = 1 - (1 - \phi)^{3.8} \qquad \text{(A-6)}$$

The Biot coefficient predicted from equation A-5 is 0.966 at a porosity of 0.382. The computed Biot coefficient using equation A-5 agrees with the measured Biot coefficient from the Domenico (1977) data around the differential pressure of 18 MPa. Therefore, the Biot coefficient proposed by Lee (2002) is adequate for a differential pressure of about 20 MPa. For consolidated sediments with a porosity of 0.217, equation A-6 yields β = 0.605. The measured Biot coefficient from Gregory (1976) data is always greater than that predicted by equation A-6 for all ranges of differential pressure. This may suggest that the sample used by Gregory is less consolidated than the sample used by Raymer and others (1980).

Appendix B

Prediction of S-Wave Velocity Based on BGT

P-wave and S-wave velocities are related through the Biot coefficient as shown in equations 1 through 3. If these equations are solved using measured P-wave velocities, the following quadratic equation results:

$$a\beta^2 + b\beta + c = 0 \qquad \text{(B-1)}$$

where

$a = 4\mu_{ma} k_{fl} / 3$,

$b = \rho k_{fl} V_p^2 - k_{ma}\left[k_{fl}(1+\phi \ - k_{ma}\phi\right] - 4\mu_{ma}\left[k_{fl}(1+\phi \ - k_{ma}\phi\right] / 3$, and

$c = \phi(k_{fl} - k_{ma})(-\rho V_p^2 + k_{ma} + 4\mu_{ma} / 3)$.

The computed Biot coefficient from equation B-1 with equation 4 can be used to calculate the S-wave velocity.

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