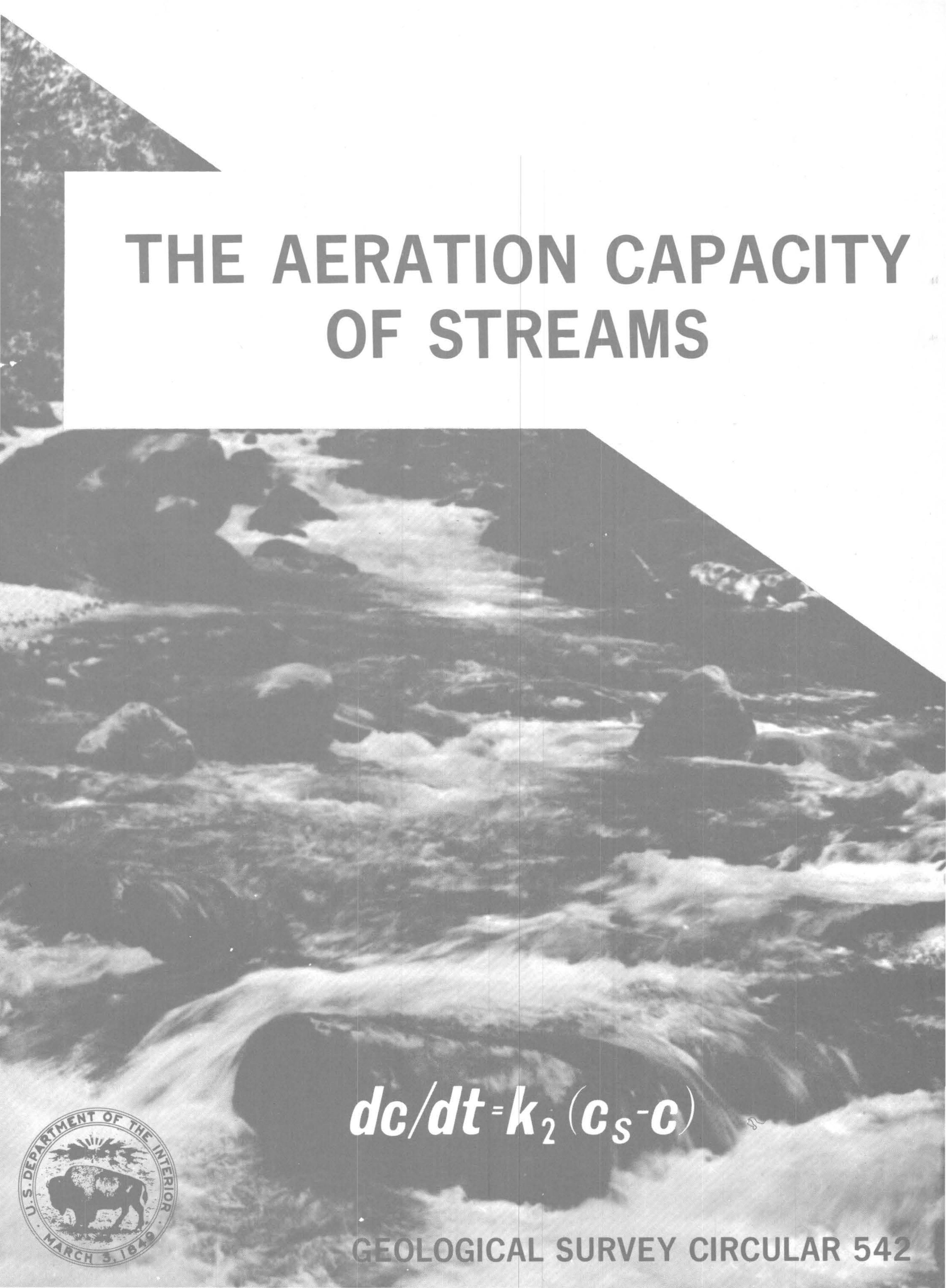
THE AERATION CAPACITY OF STREAMS



$$dc/dt = k_2(c_S - c)$$

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The Aeration Capacity of Streams

By W. B. Langbein and W. H. Durum

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THE AERATION CAPACITY OF STREAMS

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Introduction

Flowing bodies of water such as streams have the inherent ability to assimilate organic pollution from domestic and industrial waste-water discharges, from agricultural runoff, and from various natural sources. The sanitary engineer and the hydrologist are searching for ways to define the ability of a water body to assimilate waste, the proportion of capacity being used at present, and the indices by which the capacity can be measured. One of these indices is dissolved oxygen, the fuel required for destroying organic waste. Although there are other factors—such as the amount of dissolved solids present, temperature, suspended sediment, biological organisms, and the amount of flow—the amount of dissolved oxygen is a useful measure of the capacity of streams to assimilate waste.

When polluted water is exposed to the air, oxygen is absorbed to replace that consumed in the slow combustion of the organic matter. This process of reoxygenation, or reaeration, goes on at a rate that is proportional to the deficit of oxygen—that is, the difference between the amount of oxygen the stream can hold at a given temperature and the actual content.

The Coefficient of Reaeration

The rate of absorption of oxygen per unit of time is often expressed by the simple equation,

$$dc/dt = k_2(c_s - c),$$

where c is the concentration of oxygen (milligrams per liter), c_s is the concentration for saturation at the given temperature, t is time (days), and k_2 is the coefficient of reaeration. The subscript 2 denotes this coefficient as the second coefficient in the Streeter-Phelps (1925) formulation for the deoxygenation (k_1) and reoxygenation (k_2) of streams.

The oxygenation of a stream is a function of the biologic, physical, and hydraulic properties of the stream. Oxygen may be added by such processes as photosynthesis of aquatic vegetation and by mechanical aeration of the flowing water. Oxygen may be removed by such processes as vegetal decay and plant respiration, as well as by the oxidation of pollutants. In flowing streams mechanical aeration may be the dominant factor.

The effect of hydraulic properties of rivers on the coefficient of reaeration is usually expressed as the coefficient of reaeration, k_2. There are available a few measurements of the coefficient of reaeration that indicate a rough sort of relation with the mean velocity, v, and depth, H. Two sets of river data and two sets of laboratory results plotted in terms of $v/H^{1.33}$ are shown in figure 1.

This ratio seems to accommodate both the river and the laboratory data referenced and graphed in figure 1, which suggest that $k_2 = 3.3v/H^{1.33}$.

The correspondence shown in figure 1 is evidence that velocity and stream depth are highly significant factors, although measurements of these factors alone are incomplete estimators of reaeration. Other hydraulic properties, such as the occurrence of pools and riffles and the degree of meandering, also affect the rate of reaeration.

Other relations have been prepared but none for the set of available data in this simple form. The empirical nature of the formula (fig. 1) limits its application to the range of data on which it is based; fortunately, the range happens to be fairly large.

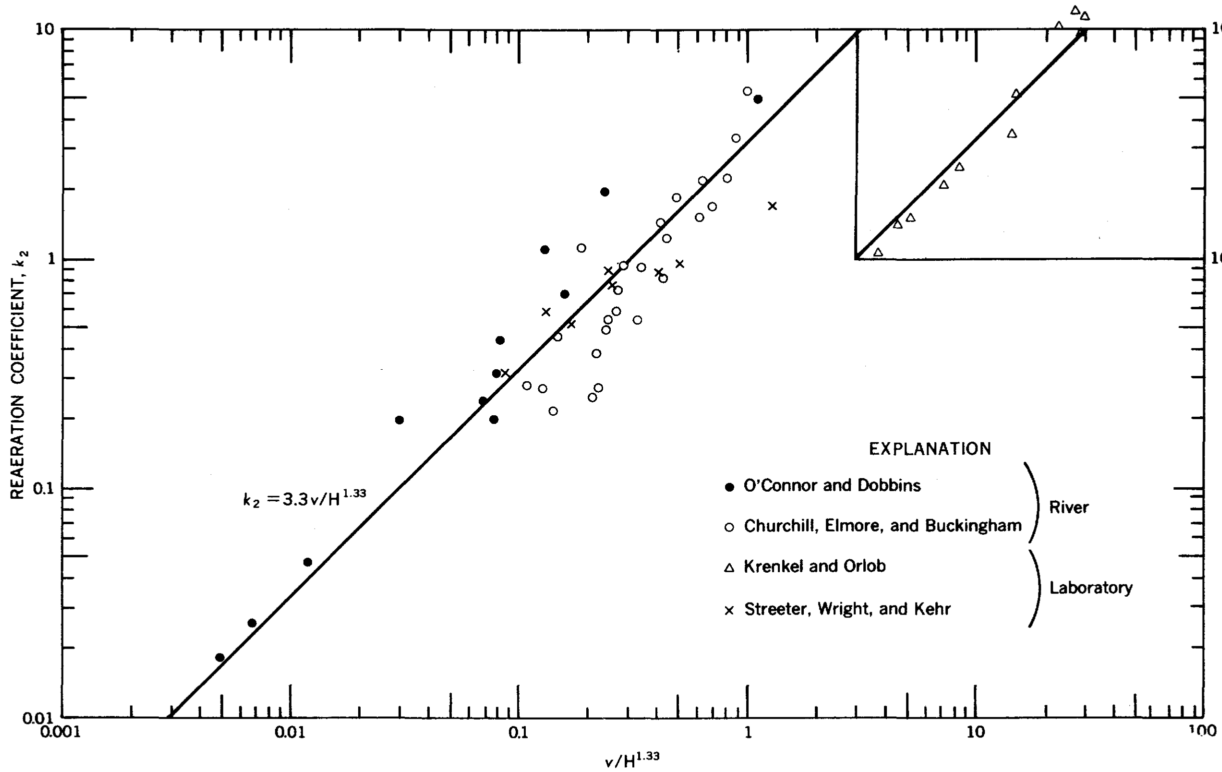

FIGURE 1.—Relation of velocity and depth to reaeration coefficient.

Other Influences on the Reaeration of Streams

In general, mechanical reaeration is probably a dominant factor in shallow, swift streams. In deep, sluggish estuaries and lakes, dissolved oxygen is a product of photosynthesis of phytoplankton and benthic flora (floating and bottom plant life).

Water temperature affects the reaeration coefficient such that it is somewhat greater in warm waters than in cold waters. In normal practice a temperature correction is applied to the basic equation. (A 1°F [0.55°C] change modifies the reaeration coefficient by about 1 percent.) However, since this paper is concerned with the influence of the hydraulic properties, all coefficients are corrected to a common base of 68°F (20°C).

The reaeration coefficient is also affected, usually in an adverse direction, by a pollution load, including sediment, that alters the physical and chemical properties of the stream.

To sum up, the reaeration coefficient of a stream is a property of its velocity and depth, and many pertinent data are available in the records of the U.S. Geological Survey to examine the extent and nature of the variations in this coefficient.

Downstream Variations

As it flows downstream, a river increases in its discharge and, with the increase in discharge, it increases also in velocity and depth. However, as shown by Leopold and Maddock (1953, p. 16), the velocity increases only as the 0.10 power of the discharge, whereas the depth increases as the 0.40 power. Inserting these relations in the formula in figure 1 indicates that the reaeration coefficient decreases in the downstream direction at the 0.43 power of the discharge. Thus the reaeration coefficient of large rivers, despite their greater velocity, may be less than that of small streams.

The downstream variation in the reaeration coefficient may be computed from the set of data for the Kansas-Missouri-Mississippi Rivers (Leopold and Maddock, 1953). As shown in table 1 and figure 2, the reaeration coefficient of the lower Mississippi River is only a tenth or less of that of the Kansas River. The data indicate that the reaeration coefficient decreases at about the square root of the discharge, in approximate conformance with that previously indicated (0.43 power of discharge).

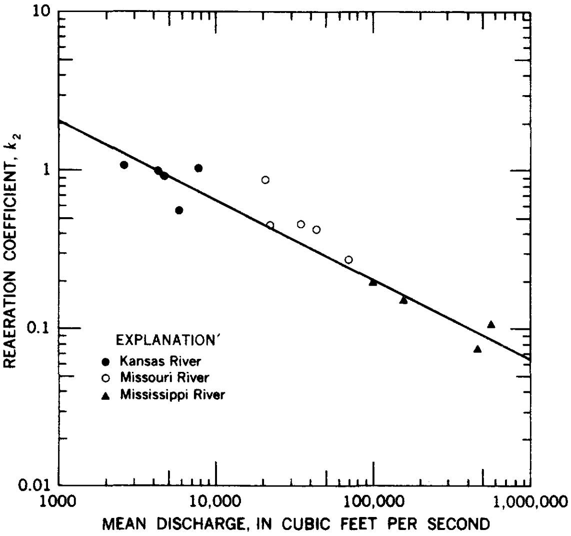

FIGURE 2.—Decrease in the reaeration coefficient with increase in size of river, Kansas-Missouri-Mississippi Rivers.

TABLE 1.—*Hydraulic data for Kansas-Missouri-Mississippi Rivers*

[From Leopold and Maddock, 1953]

Locality	Mean discharge (cfs)	Mean velocity (ft per sec)	Mean depth (ft)	k_2 (k_2=3.3 $v/H^{1.33}$)
Kansas River at—				
Ogden, Kans	2,514	1.9	3.8	1.07
Wamego, Kans	4,114	1.9	4.1	.96
Topeka, Kans	4,655	2.1	4.6	.92
Bonner Springs, Kans	5,874	1.8	5.9	.56
Lecompton, Kans	7,838	2.3	4.6	1.0
Missouri River at—				
Bismarck, N. Dak	20,320	2.9	6.1	.87
Pierre, S. Dak	22,080	2.5	9.1	.44
St. Joseph, Mo	35,440	3.6	11.5	.45
Kansas City, Mo	43,710	3.4	11.7	.42
Hermann, Mo	69,170	3.0	14.5	.28
Mississippi River at—				
Alton, Ill	96,670	3.0	18.6	.20
St. Louis, Mo	166,700	3.8	28.0	.15
Memphis, Tenn	454,900	4.6	51.0	.073
Vicksburg, Miss	554,600	5.3	40.1	.11

Regional Variations at Mean Flow

The different geomorphic character of rivers is reflected in their reaeration coefficients when the data are compared in graphs similar to figure 2. Figure 3 shows a substantial contrast between the streams of the Coastal Plain and those of the basins of the Bighorn and Powder Rivers in the North-

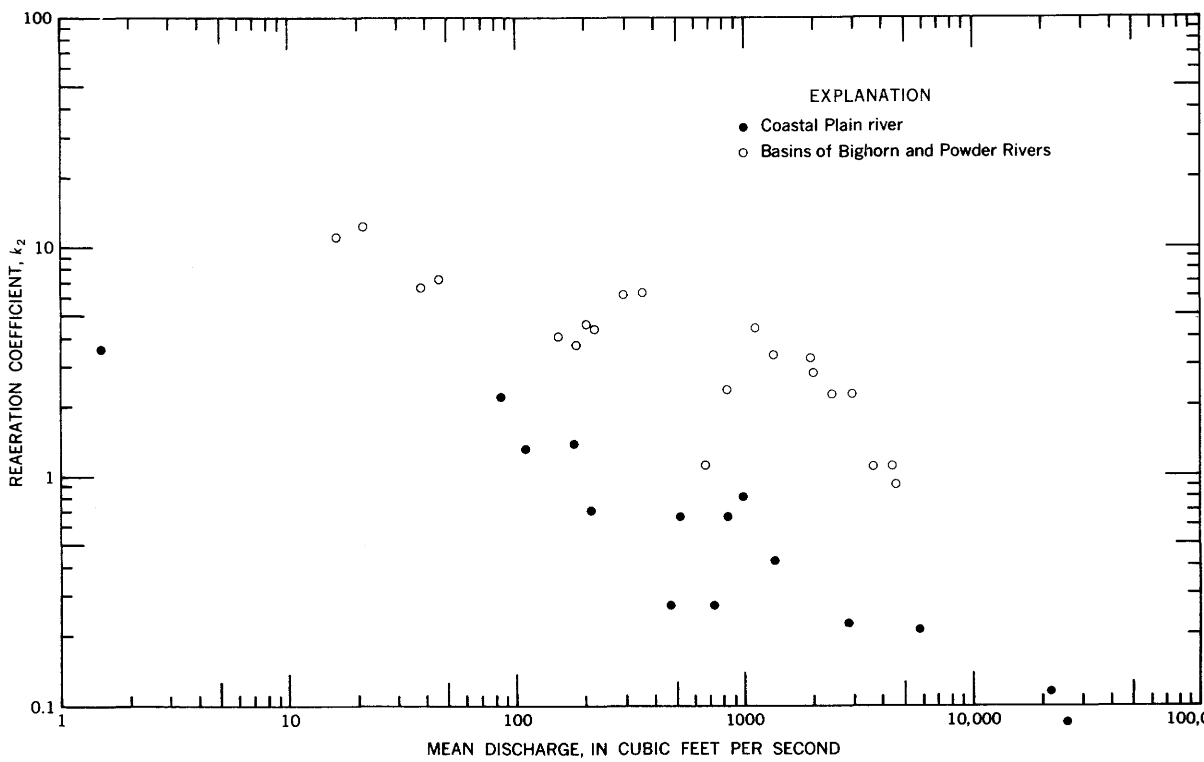

FIGURE 3.—Regional contrast in the reaeration coefficient.

ern Rocky Mountains region. According to inferences from river depths and velocities, the reaeration coefficient, k_2, varies as $15/\sqrt{Q}$ (where Q is mean discharge, in cubic feet per second) in the Coastal Plain and as $80/\sqrt{Q}$ in the Northern Rocky Mountains region; other regions have values in between. For example, the imputed values of the reaeration coefficient in the Appalachian Plateaus region are given by $50/\sqrt{Q}$. A look at the regional variations suggests that river slopes are the dominant factor and point to the obvious but perverse fact that the streams of the populated areas—those of lesser slopes—have reaeration capacities that are low relative to their rates of discharge.

The values of the coefficient of reaeration for smaller rivers derived from this formula seem to exceed those customarily used in engineering practice. There may be valid reasons for the aeration values of polluted streams to be less than the values projected from experimental data. The answer is that more field data are needed.

The textbook classification of rivers for estimation of the reaeration properties does not seem to be consistent with their geomorphic properties or the factors suggested by the formula. For example, in the guide that is commonly reported in the literature (Linsley and Franzini, 1955, p. 502), values are given for "sluggish streams" and "swift streams," and for "large streams." However, depth is a more significant parameter than velocity, and "swift" streams shown as having large values of the coefficient of reaeration are usually large streams which, in turn, are deep and should therefore have low values of the coefficient.

Local Variations

As a river rises in response to an increase in discharge, it increases its depth and velocity, a condition causing the reaeration coefficient to decrease. In general, rivers increase in depth and velocity at about the 0.4 power of the discharge. Hence, at any given location, the coefficient of reaeration decreases at about the 0.13 power of the discharge.

These are general averages. On alluvial streams with shifting beds, the coefficient of reaeration changes in rather complex ways. For example, over a period of several years, the depths and velocities of the Kansas River, as measured at a given section at Bonner Springs, Kans., changed in the following way:

	Depth (ft)	Velocity (ft per sec)	Reaeration coefficient (day^{-1})
Low water (riffle)	1.2	1.5	4.1
Low water (pool)	4.5	.3	.13
Mean flow (riffle)	2.5	2.2	2.2
Mean flow (pool)	6.5	.85	.23
Bankfull (riffle and pools "drowned" out)	20	7	.43

At low water the reaeration coefficient ranged between 0.13 and 4.1; the range is somewhat less at mean flow, whereas at bankfull the reaeration coefficient centered about 0.43. It must be noted that these changes in depths and velocities were measured at a given section. Considering the river as a whole, it is possible that the changes at one section may be compensated by changes in the opposite direction at another section.

In many rivers the water flow is alternately through shallows and deeps—or riffles and pools, as they are often called. The contrast between riffles and pools is especially marked at low water; when the river reaches bankfull, these features are said to be "drowned out." Figure 4 shows the

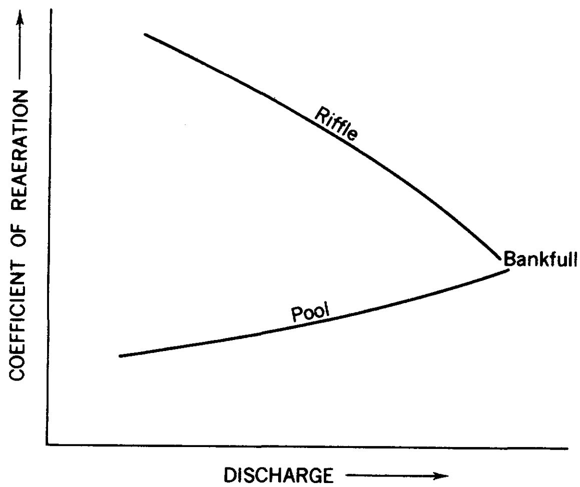

FIGURE 4.—Schematic variation in reaeration coefficient in pools and riffles.

schematic variation of the coefficient of reaeration with discharge. The relations between velocity and depths in these reaches are such that the reaeration coefficient increases slowly with increasing stage in the pool, but it decreases rapidly in the riffle.

The general trend of the coefficient of reaeration is downward with increasing stage, as remarked

TABLE 2.—*Hydraulic factors and total assimilative capacity of streams of different orders*

Stream order	Average discharge (cfs)	Average depth (ft)	Average velocity (ft per sec)	Coefficient of reaeration (day⁻¹)	Total length of streams (miles)	Total assimilative capacity [1] (Tons per day per unit deficiency in dissolved oxygen)	River representative of each order
1	0.6	....	....	....	1,570,000	....	....
2	2.8	....	....	....	810,000	....	....
3	14	0.55	1.2	9.3	420,000	16,300	....
4	65	.95	1.6	5.5	220,000	19,000	....
5	310	1.8	1.8	2.6	116,000	20,000	Pecos
6	1,500	2.7	2.0	1.8	61,000	30,000	Shenandoah, Raritan.
7	7,000	5	2.5	1.0	30,000	31,000	Allegheny, Kansas, Rio Grande.
8	33,000	12	3.0	.37	14,000	21,000	Tennessee, Wabash.
9	160,000	25	4.0	.19	6,200	18,000	Columbia, Ohio
10	700,000	45	5.0	.10	1,800	9,400	Mississippi

[1] $\frac{Q\,L\,k_2}{v}$, where k_2 is the proportion of natural logarithmic units. However, the values of k_2 given in the table are in terms of common logarithmic units, hence assimilative capacity equals $\frac{2.3\,Q\,L\,k_2}{v}$. Since Q is given in cubic feet per second, L and v in miles, and k_2 in days⁻¹, the formula used is $\frac{1}{2700}\frac{Q\,L}{v}k_2$. The quantity $Q\,L/v$ is the volume of water in the stream at mean flow.

before; but since rapid aeration at riffles is at the expense of lesser aeration in the pools, it is less evident whether the pool and riffle combination is more or less efficient than a relatively uniform channel might be.

Distribution of Assimilative Capacity

The quantity k_2QL/v has the dimension of weight per unit of time and represents the assimilative capacity of a stream. The formula for the reaeration coefficient k_2 can be used with data available for river discharges (Q), lengths (L), and velocities (v). Leopold (1962) has listed the number, lengths, and drainage areas of streams in the United States and has classified them as to their order.

The drainage areas given by Leopold have been converted to mean rates of discharge on the assumption of a rate of 0.6 cfs per square mile, the national average. Average depths and velocities corresponding to rivers having these discharges are then determined from the data given by Leopold and Maddock (1953). The total lengths of rivers are those given by Leopold (1962). Pertinent data are listed in table 2.

The coefficients of reaeration are those calculated by the formula given previously, and the assimilative capacity for each stream of given order has been computed by the formula given in the footnote of table 2. The values in the column for total assimilative capacity refer to the total load, in tons of oxygen per day, at mean flow that could be absorbed from the air by the river system for each unit (part per million) of oxygen less than the saturation value. This follows the original premise in the paper that reaeration occurs at a rate that is proportional to the deficit of oxygen.

One could compute similar values for, say, average low flow for the systems (lower 25-percent quartile) when oxygen levels are minimized if the hydraulic characteristics of the channel are known. However, the main purpose of the computations is to give some order of magnitude of the capacity of the stream for "reconditioning" itself when oxygen-consuming substances are encountered. The computations presuppose a synoptic condition, an unlikely situation in any total river system which is subject to the vagaries of nature and whose regimen is undergoing continuous change by man.

The results, as shown in the column for assimilative capacity, indicate that, although the total assimilative capacity among the several orders is roughly of corresponding magnitude, most of the assimilative capacity occurs in streams of the sixth and seventh orders, not the largest or the smallest.

The above discussion on hydraulic factors helps to explain why large streams that might be sources of copious supply are not equally effective in disposing of wastes through self-purification. Because population growth tends to develop around large rivers or bodies of water, cities in the lower reaches of a basin are inherently at some disadvantage in view of the relatively low natural assimilative capacity of their adjacent streams. Of course, in the final analysis, the extent of waste loading in relation to total oxygen available at any given time is the balancing factor.

Summary

The well-established equation for expressing the rate of absorption of oxygen per unit of time in water, $dc/dt = k_2(c_s - c)$, involves the concentration of oxygen, the concentration for saturation at the given temperature, the time in days, and the coefficient of reaeration, k_2.

This study examines in more detail the effects of known hydraulic properties of rivers on the coefficient of reaeration (k_2). A few sets of available field and laboratory experimental data indicate that $k_2 = 3.3v/H^{1.33}$, where v is mean velocity of the stream, in feet per second, and H is the mean depth, in feet.

It is shown that the reaeration coefficient (k_2) decreases in the downstream direction at the 0.43 power of the discharge; k_2 is less for large rivers than for small rivers, despite the greater velocity of the large rivers.

The reaeration coefficient is inferred from river depths and velocities and varies as $15/\sqrt{Q}$ (Q is discharge in cfs) in the Coastal Plain streams and as $80/\sqrt{Q}$ in the Northern Rocky Mountains region. Thus streams of lesser slopes characteristic of populated areas have reaeration capacities that are low relative to their rates of discharge. For example, k_2 of the lower Mississippi River is only a tenth, or less, of that of the Kansas River.

As the river rises in response to a change in discharge, depth and velocity change also, and the reaeration coefficient decreases at about the 0.13 power of the discharge. Where river flow is alternately through shallows and deeps, k_2 increases slowly with increasing stage in the pool, but it decreases abruptly in the riffle. By use of the formula k_2QL/v to give the dimension of weight of oxygen per unit time, computations are made for the total load, in tons of oxygen per day at mean flow, that could be absorbed from the air by the river system for each unit of oxygen less than saturation value. Most of the assimilative capacity occurs in streams of the sixth and seven order, not in the largest or the smallest.

The above discussion of hydraulic factors helps to explain why cities in the lower reaches of a basin are inherently at some disadvantage in view of the relatively low natural assimilative capacity of their adjacent streams.

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