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GEOLOGICAL SURVEY CIRCULAR 724

In Search of a Statistical Probability Model for Petroleum-Resource Assessment

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By Bernardo F. Grossling

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A critique of the probabilistic significance of certain concepts and methods used in petroleum-resource assessment. To that end, a probabilistic model is sketched

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SYMBOLS

Basic Resource and Reserve Quantities

B	Resource base.
EVRD	Cumulative production + R_2
OIP	Initial oil in place.
p	Probability of discovery of a resource element.
p_d	Probability of locating a resource element.
p_e	Probability of existence of a resource element.
R	Recoverable resource amount
R_1	Proven reserves.
R_2	Expanded proven reserves.
R_3	Undiscovered recoverable resources.

Statistical Quantities

$f(x)$ ....Probability function.
L ....Lower bound.
$p(x \geqq 0)$ ....Probability that $x \geqq 0$.
q ....Marginal risk.
W ....Upper bound.
x ....Variate.

Quantities in Production Forecasting Model

D_n ....Exploratory drilling footage during year n.
ϵ ....Revision to prior computations of $\triangle R'$, $\triangle R''$, and $\triangle R'''$.
f ....Fraction of the current reserves to be produced each year.
λ ....Lead time in years.
m ....Finding rate in barrels of oil or thousand cubic feet of gas per exploratory foot drilled.
μ_1 ....Primary recovery factor.
μ_2 ....Secondary recovery factor.
μ_3 ....Tertiary recovery factor.
P_n ....Production in year n.
R_n ....Recoverable proven reserve at end of year n.
$\triangle R_n$ ....Increment of recoverable proven reserve established during year n.
$\triangle R'_n$ ....Revisions and extensions of earlier primary proven reserves in year n.
$\triangle R''_n$ ....Actual increment to secondary proven reserves in year n.
$\triangle R'''_n$ ....Actual increment to tertiary proven reserves in year n.

Quantities in Financial Model

α ....Ad valorem tax.
β ....Royalty rate.
$\triangle P_n(\tau)$ ....Annual production for year τ of an n-project.
E_a ....Expensed items.
E_b ....Tax credits.
E_τ ....Cash expenditures in year τ.
E_0 ....Initial cash expenditures.
F ....Present value of discounted cash flows.
γ ....Corporation tax rate.
m_0 ....Initial finding rate.
η ....Effective depletion as a fraction of the net revenue.
$\mathbf{p}$ ....Unit price of oil or gas.
Q ....Cumulative exploratory drilling.
Q_1 ....Cutoff of cumulative exploratory drilling.
r ....Annual discount rate.
S ....Cumulative supply of oil or gas.
S_n ....Oil or gas supply in year n.
T_1 ....Production decay constant.
τ ....Time counted from initial year of n-project.
θ ....Depreciation for year τ.
U ....Unit price expectation.

Quantities Used in Exploration Model

α_i ....Probability of correct recognition of trap type i.
α_j ....Probability of correct recognition of trap type j.
$\{B_i\}$ ....Set of sedimentary basins.
$\triangle S_j$ ....Average area of trap type j.
dn_j ....Incremental number of exploratory holes drilled after traps of type j.
$\{\Lambda_i\}$ ....Set of well-explored geologic domains Λ_i.
N_j ....Total number of traps type j, other than i, which exist in the region.
$r_{i,j}$ ....Ultimate recoverable resources from $v_{i,j}$.
S ....Total search area.
T ....Number of traps discovered.
$T_{i,j}$ ....Trap of type j in basin i.
T_j ....Cumulative number of traps of type j discovered.
$\{V_{i,j}\}$ ....Set of geologic compartments.
$v_{i,j}$ ....Element of geologic compartment $V_{i,j}$.

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ABSTRACT

Exploratory drilling is still in incipient or youthful stages in those areas of the world where the bulk of the potential petroleum resources is yet to be discovered. Methods of assessing resources from projections based on historical production and reserve data are limited to mature areas. For most of the world's petroleum-prospective areas, a more speculative situation calls for a critical review of resource-assessment methodology. The language of mathematical statistics is required to define more rigorously the appraisal of petroleum resources.

Basically, two approaches have been used to appraise the amounts of undiscovered mineral resources in a geologic province: (1) projection models, which use statistical data on the past outcome of exploration and development in the province; and (2) estimation models of the overall resources of the province, which use certain known parameters of the province together with the outcome of exploration and development in analogous provinces. These two approaches often lead to widely different estimates. Some of the controversy that arises results from a confusion of the probabilistic significance of the quantities yielded by each of the two approaches. Also, inherent limitations of analytic projection models—such as those using the logistic and Gompertz functions—have often been ignored.

The resource-assessment problem should be recast in terms that provide for consideration of the probability of existence of the resource and of the probability of discovery of a deposit. Then the two above-mentioned models occupy the two ends of the probability range. The new approach accounts for (1) what can be expected with reasonably high certainty by mere projections of what has been accomplished in the past; (2) the inherent biases of decisionmakers and resource estimators; (3) upper bounds that can be set up as goals for exploration; and (4) the uncertainties in geologic conditions in a search for minerals. Actual outcomes can then be viewed as phenomena subject to statistical uncertainty and responsive to changes in economic and technologic factors.

INTRODUCTION

Petroleum at present provides the major energy for driving the economies of nations. The foreseen worldwide depletion of petroleum resources in a few decades at a time of transition to new energy technologies will impose hardships on many nations but will enhance the economic opportunities of those having a petroleum potential.

The petroleum-prospective areas of the world consist of sedimentary basins and geosynclines that are not too intensely deformed tectonically, including the continental shelves down to the 200-m depth—a total area under various national jurisdictions of about 26.1 million square miles. Certain oceanic areas beyond the continental shelves—the continental slope and continental rise—also have petroleum prospects, but their eventual development may be some 10–20 years away.

Traditionally, the favorable geologic environment for petroleum has been considered to be the geosynclinal belt. Cratonic areas have been reluctantly accepted by many geologists. The French first discovered the important accumulations in Africa, actually in Algeria, in a cratonic area previously deemed inadequate. The occurrence of giant gas accumulations in the western Siberian platform also did some violence to some traditional views about areas in which petroleum should occur. Large tracts of cratonic areas now believed to be covered by not-too-thick sedimentary columns ought to be considered another frontier in petroleum geology. The continental margins—shelf, slope, and rise—are yet another petroleum frontier.

The major areas that should be considered for petroleum exploration appear to have already been outlined in the world, yet appraisals of the extent of these areas are cautious and often underestimate what subsequent exploratory work reveals. For example, estimates of the extent of the Great Artesian Basin of

Australia have been greatly increased in the course of just a decade. Similarly, I expect that the current picture of the extent of prospective areas in Africa and South America is also conservative.

The extent of prospective areas is very significant even though experience in many basins shows that only a small percentage of the total prospective area is probably actually underlain by commercial petroleum accumulations.

The larger the tract of undrilled prospective area, the greater are the chances that thick sedimentary pods may occur here and there. Even when a few scattered pieces of evidence may indicate a thin sedimentary cover, prospects for generation and primary migration of petroleum may be enhanced by the large size of a prospective area—as they are in prospective areas in the interior of Africa and in the Amazon basin. Of the total world petroleum-prospective areas, the non-OPEC, noncommunist developing countries control 48 percent; the developed countries, excluding the USSR, 30.5 percent; the USSR, 13 percent; the Middle East, 4.6 percent; and China 3.5 percent.

Next to be considered, after the extent of sedimentary areas, should be the total sedimentary volume in each region. Information on this factor either is not readily available or has not been released as yet by companies or governments that have explored certain areas. Moreover, many other geologic factors should be considered in a realistic comprehensive assessment of the world's petroleum potential.

The examination of the distribution of petroleum occurrences throughout the world, in basins that have had a significant amount of exploratory drilling, indicates that roughly half the prospective basins and geosynclines do not yield any or much petroleum; in those areas having petroleum deposits, the manner of distribution, size of deposits, and geologic conditions that control the petroleum accumulations vary greatly. The situation could be described as a two-stake sequential decision game played by Nature. First, it is decided, with probability roughly ½, whether a particular basin or province will contain commercial accumulations or not. In the second stage is decided, with an underlying probability distribution, the magnitude of the petroleum resources of the basin or geosycline.

Before exploratory drilling, it would be difficult to ascertain which basins contain petroleum. Of the many prospective basins and geosynclines in developing countries, for example, roughly about half will prove disappointing and have minor petroleum accumulations or none at all. The uncertainty of the outcome at this stage is something that has to be accepted; only actual exploratory work, including drilling, can resolve the question.

Another source of surprise can be the size of the petroleum accumulations. This is especially important because of the large contribution to petroleum resources of a relatively few but very large fields. One can only speculate, before drilling, that certain geologic conditions may prevail in a given basin or geosyncline that could lead to large accumulations.

Giant oil fields discovered so far are mainly concentrated in the Middle East, and giant gas fields, in the USSR. The number of giants, per million square miles of prospective areas so far discovered, are:

	Oil	*Gas*
Middle East	45	3.3
USSR	6	11.8
Conterminous United States	3.2	7.9
Africa and Madagascar	3.2	.6
Latin America	2	0
South and Southeast Asia, mainland	0	0

About two-thirds of all the past drilling in the world for petroleum took place in the United States. Most of the petroleum-prospective areas of the developing nations are grossly underexplored. Estimates of the total number of wells drilled per square mile of prospective area in major regions are:

	Wells per square mile
Conterminous United States	1.17
USSR	.15
Argentina, Mexico, and Venezuela	.05
Middle East	.01
Latin America, except as mentioned above	.01
South and Southeast Asia and Indonesia	.01
Africa and Madagascar	.003

About the same relative values are obtained if either the number of exploration wells or the total footage drilled are considered instead of the total number of wells.

The projected drilling density in the conterminous United States provides an upper bound for the desirable drilling density. In the early development of the U.S. petroleum industry, much unnecessary drilling took place. Even allowing for this factor, however, the drilling density would be roughly about 0.5 wells per square mile of prospective area. In the Middle East, drilling density is exceptionally low because of the giant dimensions of the fields, and it cannot be considered representative of desirable drilling density elsewhere. In the USSR, petroleum exploration is intense, but the exploration is far from passing the midpoint in overall development.

For areas on the order of 1 million square miles or more, a desirable ultimate density of 0.3 wells per square mile of prospective area seems to be a reasonable target. Then the drilling density should eventually increase by a factor of about 100 in Africa and Madagascar and about 30 in South and Southeast Asia and also in Latin America, except for Argentina, Mexico, and Venezuela. Even in the last three countries, drilling density should eventually increase by a factor of about six.

The final exhaustion of worldwide petroleum resources can be foreseen to occur within a few decades. At the moment, the petroleum scene is dominated by one cartel. Looking beyond areas containing proven reserves, however, one finds a substantial amount of completely unexplored or insufficiently explored lands successful development of which could wrest the control of the petroleum scene from the cartel.

Most of the prospective acreage available belongs to some developing nations. The more important opportunities for petroleum development in developing countries that are not already playing a major role appear to be found in: Mexico, Colombia, Brazil, Peru, and Argentina in Latin America and Mali, Niger, Egypt, Mauritania, Chad, Tanzania, Somalia, and Mozambique in Africa.

Vigorous exploration in many of the promising areas should result in the discovery of substantial amounts of petroleum, which could ease the pressures for price escalation. However, whether the present petroleum seller's market will go away will depend on who controls the new petroleum provinces.

The recoverable petroleum that can be said with certainty to have been found so far in a given country is the sum of the cumulative production plus the proven reserves. Moreover, the expected value in a statistical sense from discovered fields is somewhat larger. For this purpose, I have introduced the concept of EVRD (*E*xpected *V*alue of *R*ecoverable *D*iscoveries, as defined on p. 6.

For the major regions that may be considered, the EVRD per square mile of prospective area is:

Region	Oil (bbl)	Gas (10^6 cu ft)
Middle East	496,000	510
Conterminous United States	101,600	556
USSR	49,500	453
Western Europe	27,000	207
Africa and Madagascar	30,700	92
Latin America	24,200	52
South and Southeast Asia, mainland	2,500	60

The amount of petroleum initially in the ground in a given country or region and considered to be recoverable within foreseen technological and economic limits is the *E*stimated *U*ltimate *R*ecovery (EUR) of that country or region.

By considering certain best explored areas, excluding the Middle East, as benchmarks, I have selected the following average figures for the total EUR from continental-size regions: 100,000 to 250,000 barrels of oil and 500 million to 1,300 million cubic feet of gas per square mile of prospective area. The above figures do not include the eventual occurrence of Middle East-size accumulations.

The above figures provide another upper-bound estimate for the world's EUR, namely

2,600 billion to 6,500 billion barrels of oil, and

13,000 trillion to 34,000 trillion cubic feet of gas.

In addition, some further allowance should be made for the eventual occurrence of clusters of giant accumulations, as in the Middle East. The extended reserves of the Middle East are 595 billion barrels of oil and 612 trillion cubic feet of gas. The Middle East EUR's are probably, at most, equal to a few times these amounts; that is, less than an order of magnitude larger.

Several petroleum companies are known to have pursued the search for "another Middle East." A consideration of the broad tectonic framework of various Earth regions suggests certain likely places. Some of the places that I would choose are: the north slope of the USSR, the north slope of Canada, the Gulf of Mexico, the Argentine Continental Shelf, the shelf between Mozambique and Malagasy, and the shelf between Australia and New Guinea. Of these six possibilities, four are in or near developing regions, namely: the Gulf of Mexico, the Argentine Continental Shelf, the shelf between Mozambique and Madagascar, and the shelf between Australia and New Guinea.

I propose the following upper-bound EUR's for Latin America:

490 billion to 1,225 billion barrels of oil, and

2,450 trillion to 6,370 trillion cubic feet of gas;

for Africa and Madagascar:

470 billion to 1,200 billion barrels of oil, and

2,400 trillion to 6,100 trillion cubic feet of gas; and

for South and Southeast Asia:

130 billion to 325 billion barrels of oil, and

650 trillion to 1,700 trillion cubic feet of gas.

Because of the very limited petroleum development in these prospective lands, the bulk of the above EUR's still remains in the ground to be discovered.

The above remarks show that there probably are large potential petroleum resources to be sought in the world. For most areas, drilling is only beginning. Methods of forecasting resources from projections based on historical production and reserve data (for instance, by using Gompertz and logistic functions) are limited to mature areas such as the conterminous United States and the Maracaibo basin in Venezuela. For the overwhelming majority of the world's petroleum-prospective areas, we have a more speculative situation which calls for a critical review of resource-assessment methodology.

The estimates of dimensions of prospective areas and of upper-bound EUR's in this "Introduction" have been taken from unpublished data that I submitted to the International Bank for Reconstruction and Development, Washington, D.C.; they provide an indication of the size of the exploratory tasks ahead and provide an estimate of the maximum amounts of oil and gas which would be obtained if the most favorable conditions prevail.

CONCEPTS AND PREMISES

FUNDAMENTAL CONCEPTS OF RESOURCES AND RESERVES

The span of the various resource and reserve terms for oil are shown in figure 1. The situation for gas is similar.

First we have the initial Oil In Place (OIP), that is, the amount prior to any exploitation that is to be found in undiscovered and discovered fields. Obviously, the OIP is difficult to estimate. As for the undiscovered (unknown) fields, an estimate has to be made on the basis

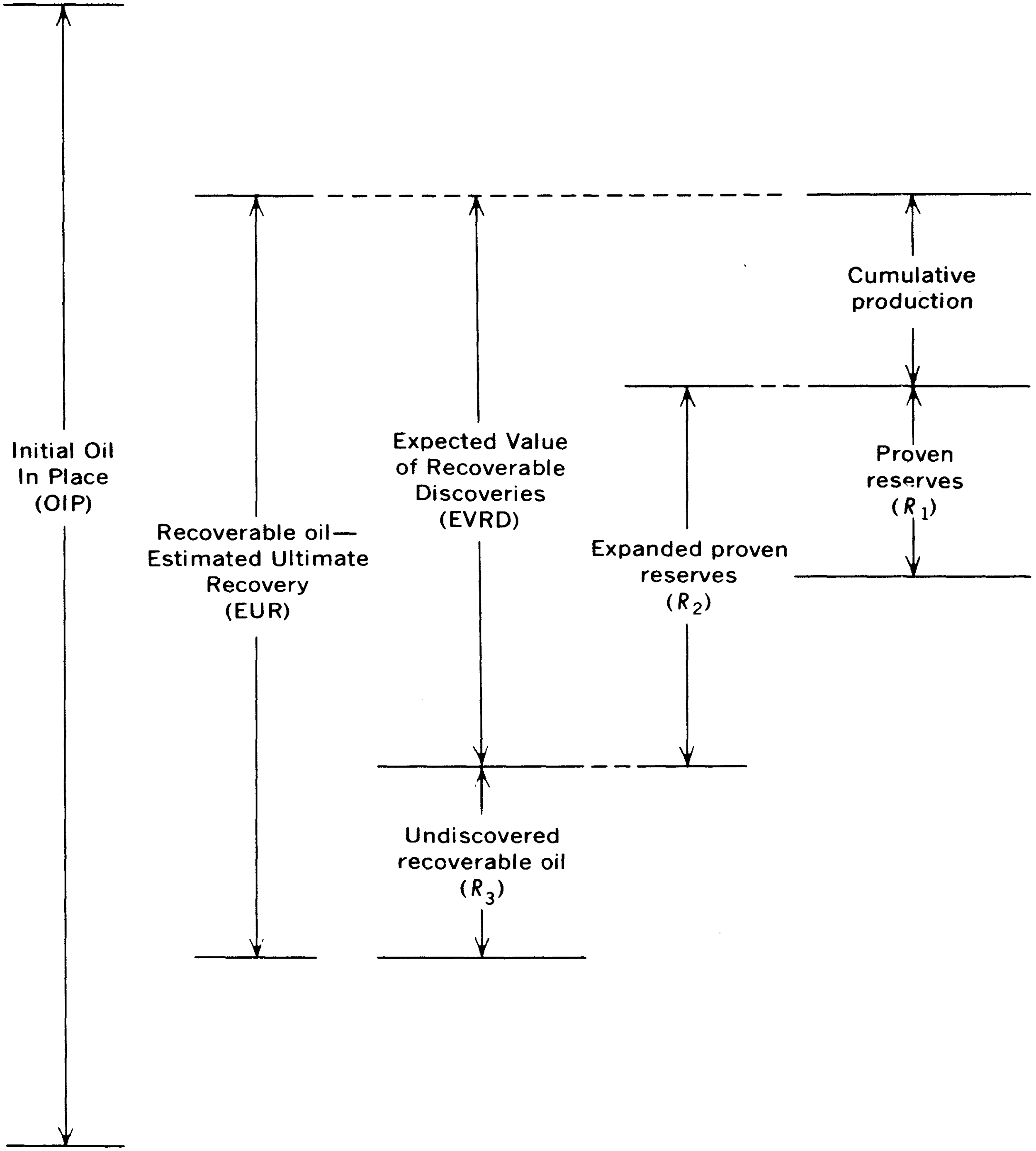

FIGURE 1.—Span of the various resource and reserve terms for oil.

of the discovered (known) fields in relation to the geologic setting. This estimate would not be too difficult to make if the unknown fields belong to the same geologic groups as the known fields or if both known and unknown fields belong to a well-defined statistical population. Actual exploration, however, shows that often a certain discovery alters the concept of the petroleum accumulations to be found in a given area. Projections from an old model can thus become obsolete. This logical difficulty is hard to overcome. Perhaps the best we can hope for with respect to the OIP value is to establish lower bounds for it and to raise the lower bound whenever wider knowledge about the petroleum geology of a region being considered justifies it.

Second comes the concept of recoverable resources; that is, the amount of oil that can be recovered, within technological and economic limits, from both undiscovered and discovered fields—often denoted as the *E*stimated *U*ltimate *R*ecovery (EUR). The relative amount of oil that is recoverable varies greatly and has not been well established on a worldwide basis. As a result of modern production practices, primary and secondary methods have become well integrated, and a sharp distinction between them is not justifiable. At the moment, 40 percent probably represents a realistic target figure for these two integrated phases of recovery. How much oil still remains in the ground, even after the best production practices, is not really known. Perhaps the actual recovery can be as high as 80 percent. Economic limits constitute another factor that conditions the estimation of the amount of recoverable resources. These limits will change with time, so a forecast of the recoverable resources should vary with the time span of the forecast.

Third comes the concept of cumulative past production, that is, the total oil that has been produced from the discovered fields. This is the figure that can be ascertained most readily.

Fourth comes the concept of proven reserves R_1. Proven reserves, designated here as R_1, are defined as the amounts of petroleum that can be considered certain to be producible from explored acreage within present economic and technological limits. To convey a qualitative sense to the above definition, I would say that in known fields, the amounts of petroleum that can be obtained with certainty, say with a higher than 90 percent probability, can be estimated within ± 25 percent or so. The main uncertainty in this estimation process would be covered by the span of the error (± 25 percent). The increments of oil to be expected from known fields would fall off rapidly below a 90 percent probability. Essentially then, we would have a rather narrow estimation distribution function for the proved reserves—the estimation of the magnitude of a quantity that is known to exist.

Fifth comes the concept of expanded proven reserves R_2, which represents the expected amounts, in a statistical sense, of oil from revisions and extensions of discovered fields. We deal here with a different kind of uncertainty—the speculation that some petroleum may exist beyond the known parts of fields and as extensions of them. For most countries, the following generalization can be made: An additional quantity equal to the proven reserves can be obtained with probability of 80 percent, and another equal additional quantity, with probability of 50 percent. Hence, the expected value of the expanded proven reserves, designated here as R_2, would be $(1+0.8+0.5)R_1=2.3\ R_1$.

The recoverable petroleum that with certainty can be said to have been found so far in a given country is the sum of the cumulative production plus the proven reserves. The expected value from discovered fields is somewhat larger. For this purpose, I have introduced the concept of EVRD (*E*xpected *V*alue of *R*ecoverable *D*iscoveries), defined as

$$\text{EVRD}=\text{cumulative production}+\text{expanded proven reserves } R_2.$$

Finally, the difference between the recoverable resources and the sum (cumulative production+expanded proven reserves) is the undiscovered recoverable resources R_3. Hence, an estimation of R_3 involves an estimation of recoverable resources.

ASSESSMENT OF THE PETROLEUM-RESOURCE BASE

The main issue underlying the sudden increase in oil prices is a wide, sudden realization that world petroleum resources will be depleted within a few decades. How to appraise the extent of the remaining petroleum resources is a source of confusion. For long-range economic planning, it would be useful to have rather accurate estimates of the total amount of recoverable oil in a given country. However, because of the high degree of unpredictability of the actual location of petroleum deposits and because of technological limitations in the search techniques, it is not economically viable or technically possible to discover, ahead of development, all the fields containing the remaining world petroleum resources.

Therefore, to appraise the long-range supply of oil and gas, we need to go beyond mere projections made from present trends. *First,* one should estimate the magnitude of the resource base, regardless of economics and uncertainty of discovery. And then one should subdivide the resource base according to various intervals of unit costs, in increasing order starting from present levels. Future technological developments or changed market conditions probably will permit the exploitation of certain resources that currently are uneconomical. *Second,* the resource base should be subdivided according to certainty of occurrence. Improvement of exploration techniques and increasing knowledge of actual geologic conditions would allow some resources whose existence now is considered uncertain to be incorporated into the available supply. Such a conceptual framework for the long-range appraisal of resources has already been proposed by McKelvey (1972). It provides a much more meaningful basis for long-range forecasts than before.

The recoverable reserves for one fully developed oil field can be estimated with relatively great accuracy (say within ±25 percent), but the estimation for a new field, a petroleum district, a sedimentary basin, a nation, or the Earth constitutes a series of exercises that are increasingly difficult and that have correspondingly wider ranges of uncertainty. A few basically original estimates have been made of the world petroleum resources, but a vast amount of published data in fact amounts to "regurgitations" of someone else's data, or someone else's "regurgitations" of someone else's, etc. The language of mathematical statistics is required to define more rigorously the problem of appraisal of petroleum resources.

To carry the analysis a step farther, one can pose the question of whether the amount of recoverable oil and gas of a given part of a region could be of a given magnitude. The answer can be given only in probabilistic terms. That is, we are faced here with the a priori probability density function of the recoverable amount of petroleum for an undrilled area. Such a probability density function is really not known, for example, for many undrilled areas in developing nations, and for most of the continental shelves, and it can only be conjectured.

Conceptually, at a given time, one could first classify the remaining resources F according to the probability of being found as exploration continues. This probability p can be considered to be the product of the probability of existence of a field p_e and of the probability of actually locating the undiscovered field p_d. The resources that at a given time we know with certainty to exist are a certain amount. One could conceptually proceed to classify the undiscovered resources as incremental quantities $\Delta\mathbf{R}$ corresponding to ranges of the probability p down to some low value of the probability. The resource base **B** could be defined as the expected value of the resources, that is, the summation of the incremental resource amounts multiplied by the probability of finding them; that is

$$\mathbf{B} = \sum_{\substack{\text{for all} \\ \Delta\mathbf{R}}} p\Delta\mathbf{R}. \qquad (1)$$

Upon this first classification scheme, we have now to impose the constraints that result from economics. Only a fraction of the segment of resources within a certain probability range can be considered to be economically recoverable and discoverable, corresponding to the conditions at the time when the assessment is made. In the future, the economic limits may widen (although not necessarily so), thus permitting a larger proportion of each segment to be exploitable.

Moreover, it would seem that the a priori probability density function is not narrow and would definitely become small only beyond the largest conceivable size of the recoverable amount of petroleum. Furthermore, the probability density corresponding to a very small size of the recoverable oil, or gas, is significant and different from zero. Moreover, the probability density for, let us say, an amount of recoverable oil comparable on a unit area basis with a given known oil basin is significant and different from zero. As we do not know the shape of this probability density function for undrilled basins and as it appears to be quite broad, it is not proper to give only one value for the amount of recoverable oil or gas or to expect that its standard deviation is a small fraction of the magnitude of the recoverable amount. As a first approximation, a uniform, or flat, probability density function could be taken for the petroleum estimates of an unexplored area. To give one figure for the petroleum resources of an unexplored area would be a futile undertaking.

Although this scheme might appear to be conceptually clear, operationally it is very difficult. Below a given probability, say 60 percent, the situation becomes highly speculative. There is very little basis on which to construct the actual scheme, and yet the largest expected contributions to the resource base **B** should come from resources having low probabilities of eventually being found. However, one could strive to perfect such a picture gradually, considering the past record of discovery as a basis for estimating parameters for theoretical statistical models.

Perhaps one of the most perverse effects of the conceptual difficulties of petroleum-resource assessment is what I call the accuracy delusion. By that I mean the misconception that published figures for undiscovered resources have a somewhat narrow distribution function; that is, that the possible values form a Gaussian distribution about the published figure and have not too great a standard deviation, say 20 percent. For new undrilled tracts of territory, as exemplified by the continental shelves, such a Gaussian distribution cannot yet be provided. A team of company specialists might agree among themselves on a "most probable" value, but from team to team the "most probable value" will be found to vary substantially. The wide scatter observed in bids for offshore petroleum leases could well be attributed to these variations.

A better approximation to the underlying uncertainty function of resource estimates than the Gaussian curve that leads to the accuracy delusion is a modified uniform distribution function. It would extend, with uniform probability, from zero up to a figure somewhat greater than that for the richest known similar tract elsewhere and then would drop rapidly back to zero probability for greater amounts. By analogy with other tracts and from knowledge of adjoining areas, one might justify modifying the uniform distribution on the low side also; that is, to drop quickly to zero probability for resource amounts smaller than a certain amount.

The estimated magnitude of the resource base, and its subdivision according to economics and degree of uncertainty in finding, sets the targets for long-range petroleum exploration. In this selection of targets we are not restricted by the high degree of certainty required by short-range considerations because, in the exploration of large unknown areas, the uncertainties diminish as the exploration proceeds.

To appraise the petroleum-resource potential of new tracts of territory, one should consider the basic scheme (table 1) that underlies petroleum-resource exploration and development. After the sedimentary basins have been identified, a predrilling estimate of potential is made on the basis of factors such as the area, the maximum thickness of sediments, the type of sediments, the existence of structural traps, the existence of stratigraphic traps, the reconstruction of geologic history, the occurrence of oil and gas seeps, the adjoining petroleum provinces, and so on.

An exploratory well is aimed at a very specific target which has been identified from similar previously undrilled, unrecognized targets on the basis of (1) the prior data obtained in a region plus (2) the specific geological and geophysical surveys in the particular prospect. The various targets that exist in the basin may be categorized in various groups such as foreland anticlines, hinge-belt anticlines, platform anticlines, fault traps, reefs, pinnacle reefs, domes over salt domes, regional pinchouts, shoestring sands, etc. Moreover, the geologic definition of the targets in each group could be quite specific.

When the petroleum industry in a given basin is pursuing a given "play" it is in fact running after targets in one of these groups. The actual existence of petroleum in one of these targets

TABLE 1.—*Stages in petroleum-resource development*

Geological and geophysical exploration
Existence of sedimentary basins — YES ↓ — NO: OUT
Predrilling information:
- Area of the basins
- Maximum thicknesses of the sediments
- Type of sediments
- Existence of structural traps
- Existence of stratigraphic traps
- Reconstruction of the geologic history
- Oil or gas seeps
- Adjoining petroleum provinces

Predrilling potential → (end of first phase of appraisal)
Exploratory drilling campaign
Postdrilling estimate of petroleum potential → (end of second phase of appraisal)
Further geological and geophysical work
Further drilling
Development of oil potential (subsequent phases of appraisal)

can only be ascertained with a given statistical probability even after consideration of all the information that can be gathered before drilling. One could say, for example, that one out of four structures of a certain type in a certain part of a basin would contain oil. Moreover, the magnitude of the accumulation would be essentially determined by the group type, the actual size being almost unpredictable.

In this manner, the statistical success of drilling would be about the same almost to the very end of the play, except for the effect of the enhancement of knowledge because of interaction with previously obtained data. One could thus describe the outcome, in barrels of oil or thousand cubic feet of gas found per foot drilled, as a random sample from a normal distribution, having a certain mean and a certain variance that characterize the play. When several plays are being pursued, the outcome would consist of random samplings from the various normal distributions corresponding to each group.

In none of these models would there be a decreasing finding rate as a normal situation. The exploration would reach the limits of the resource with few warning signals from the finding rate, and the bottom would be hit rather unexpectedly.

An analogy here may help to visualize the problem. Let us suppose that an experienced hunter with a shotgun is hunting rabbits in a large enclosed field. Let us further assume that 20 rabbits are in the field and that the hunter requires three shots per rabbit, on the basis of earlier experiences in similar fields. One would expect that on the average he will require three shots per rabbit from the first one he downs until the very last rabbit. Moreover, when a series of exploratory wells is drilled in a given region, the aim could actually improve because, as the data from an increasing number of wells and exploration surveys become available, the geologic picture is clarified.

OBJECTIVE AND BIAS IN RESOURCE ASSESSMENT

When faced with the problem of estimating the magnitude of undiscovered petroleum resources, one needs to analyze the objectives of making such estimates. In situations that involve uncertainty, one can rely heavily on the tools of mathematical statistics as long as the statistical properties of the quantities involved are well defined. But such is not the case for most of the petroleum-prospective areas of the world. We are not dealing with a single statistically well defined population of petroleum fields. As exploration encroaches upon the undrilled areas, one needs to introduce new statistical categories of fields, and, because of this, statistical methods have to be used with caution, and their limitations must be understood.

Of course, situations vary from the statistically well-defined—as to size of accumulation and location—to the very fuzzy. Unfortunately, for most of the undrilled areas, the situation is very fuzzy. For each petroleum province there is a certain histogram of petroleum accumulation ranked according to a sequence of size intervals which range from zero to the largest conceivable value. This histogram—or its conceptual limit, the probability density distribution—can be estimated for certain limited regions, but we have at present very little to go on in constructing one for a sedimentary basin in the interior of Africa, for example.

We are forced then to a blend of objective and subjective judgments. This situation has to be kept in mind because the mere fact that a probability density function is formulated mathematically does not necessarily mean that subjective elements have been removed. Ignoring these underlying logical difficulties, and also ignoring the various points of view that can legitimately be used in resource appraisal, has given rise to much confusion and argument. Petroleum-resource data have been misused by being taken out of context.

The amount of recoverable resources in a certain geologic domain can be viewed as a variate x, which could conceivably attain any value within a certain range $L \leq x \leq W$. By analysis of the outcomes of the same estimation process as applied to a suite of well-explored geologic domains $\{\Lambda_i\}$, one could construct a probability function $f(x)$ so that

$$p(x \geq 0) = \int_0^x f(x)\,dx.$$

Alternatively, one could estimate $f(x)$ by using a model that involves subjective probabilities.

Another way of looking at the problem is to aim at the estimation of limits for x and for the range of x, within assigned probabilities. In practice, it would be more meaningful, as we have explained, to target on certain limits of x, rather than on $f(x)$.

What use is to be made of a petroleum-resource assessment is one of the crucial questions. Some of the possibilities are that:

1. A bank is to decide on the financing of an exploration and development project.
2. An oil company is to decide on beginning an exploration project.
3. An oil company is to decide on continuing an exploration project.
4. A government is to define a short-term national energy policy.
5. A government is to define a mid-term national energy policy.
6. A government is to define a long-term national energy policy.

Each application involves: (1) a decision-maker who will use the resource estimate and (2) a specific decision that must be made and that hinges on the resource estimate. Both are important in defining the nature of the petroleum-resource assessment required.

For certain decision, one needs a floor, or lower bound, for the resource estimate that has a high probability of being either fulfilled or overpassed. A bank, for example, will require almost a 100 percent certainty. By using the language of mathematics, this can be explained as follows:
For deciding on the financing of a specific investment for resource exploration and development, the bank needs to know the amount x_L of the resource so that the relationship

$$L \leq x \leq x_L$$

will be fulfilled with a low probability q. This probability q determines the marginal risk of losing the investment. Conventional financing, moreover, would require (1) that the bulk of the investment be recoverable on the basis of an amount of resources that is almost certain to exist and (2) that only a marginal fraction of the investment be risked. Such an estimation problem could be described by a probability density distribution such that

$$p(x \geq L) = 1,$$

and

$$p(L \leq x \leq u) = \int_L^u f_L(x)\,dx.$$

A government, as another example, when analyzing the various outcomes of alternative energy postures will need an indication of the minimum amount that with reasonable certainty can be counted on to be available from the domestic sources.

An economic forecaster may want to focus his discussion on a most likely value of the recoverable resources. Here, a word of caution is in order. The "most likely" value should be sought in most resource assessments, but seldom is it really available. What we often find is an intermediate value that has been *termed* "the most likely value."

For other decisions, one needs a roof, or upper bound, for the recoverable resources characterized by a low probability of being surpassed. When appraising, for example, the possible impact of a new energy resource, one may want to know how large its contribution to the total energy supply could be. It may be enough to know that the upper bound is at the most a few percent of the total energy supply; however, if the calculated percentage turns out to be substantial, then a subsequent estimation aimed at the midpoint of the likely outcomes may be in order.

An important bias arises out of competitive considerations. For instance, when appraising the petroleum prospects of specific undrilled potential areas throughout the world, an oil company may not want to encourage its competitors by aiming its appraisal at the upper range of the possible outcomes; rather, it may want to aim at the lower range while still staying within the range of what the company considers viable.

Now that there is widespread recognition of the energy crisis, national economic planners have been asking how much oil is left in their country. They want to know this with precision, in order to make firm policy decisions. We meet here several problems, one of which is to make the planners understand that counting barrels of undiscovered recoverable oil in the ground is not as simple as counting sheep grazing on the land. Let us examine what kinds of answers are both viable and useful to economic planners.

One needs to distinguish the various types of countries as to their initial petroleum posture, namely: (a) no exploration and no development; (b) unsuccessful exploration; (c) incipient exploration and incipient development; (d) limited exploration and limited development; and (e) intensive exploration and intensive development.

For countries groups (a), (b), and (c), the first legitimate question for the planner is whether to act so as to permit exploration to proceed. In this decision problem, one has to compare the two alternatives:

Go: that is, proceed with exploration,

and

No Go: that is, no exploration allowed.

This is not an unlikely situation. In fact it is the type of decision confronting national planners in some developing nations having a petroleum potential. What they must weigh here is the (1) economic prize that might be obtained if exploration turns out to be successful against (2) the costs of exploring plus possible indirect economic losses incurred because of having proceeded with the exploration. For those countries, estimates at the center of the range and at the upper bound would be most useful to them in making, their decision.

For countries in group (d), however, further limited exploration and development may be discouraged by an estimate that is aimed at the lower end of the range.

A country in group (e) that has had intensive exploration and development may want, in the first place, an estimate of the minimum of recoverable resources that are left in order to proceed cautiously. For comprehensive contingency planning beyond this minimum, they would need also the most likely outcome and the upper bound.

The requirements on which resource-appraisal methods should be judged are several: (1) the consistency with observations; (2) the degrees of objectivity and, its counterpart, of subjective judgment; (3) the stability of forecast; (4) reproducibility; and (5) the adequacy, as to the true answer.

Often, forecasting methods have been advanced because they satisfy some of these criteria, but they fail to demonstrate adequacy. Objectivity and reproducibility, for instance, are not enough. The final criterion is adequacy, which is difficult to achieve and even more difficult to demonstrate.

PROBLEMS WITH HISTORICAL PROJECTION MODELS

LINEAR "DRILLING INPUT-PETROLEUM OUTPUT" MODEL

A linear input-output model of petroleum exploration and development may be used to gage the economic impact of the finding rate.

The input into the model is a schedule of exploratory drilling per year, and the main output is the amount of oil or gas discovered per year. Such a model has been used in the United States, for example, by the National Petroleum Council (1973) and the U.S. Federal Energy Administration (1974).

One assumption in this model is that the amount discovered per year is simply proportional to the drilling per year. The amount of oil or gas discovered per foot drilled can be taken to be a parameter that is a function of the cumulative drilling and that varies from region to region. Another basic assumption made is that the production per year is a constant fraction of the proven reserves.

The calculation starts with the reserves at the beginning of the period of projection and updates them year by year as the result of the drilling discoveries and the yearly production withdrawals. A separate tally is kept of primary, secondary, and tertiary reserves.

The basic equations for this model would be as follows:

$$\Delta R_n = \mu_1 m D_{n-\lambda} + \Delta R'_n + \Delta R''_n + \Delta R'''_n + \epsilon, \quad (2)$$

$$P_{n+1} = f R_n, \quad (3)$$

and

$$R_n = R_{n-1} + \Sigma \Delta R_n - P_n, \quad (4)$$

where

ΔR_n = (recoverable) proven reserve established during the n-th year in a region,

$D_{n-\lambda}$ = exploratory drilling footage undertaken during the n—λ year,

m = finding rate in barrels of oil or thousand cubic feet of gas per foot drilled, where m is a function of ΣD_n,

μ_1 = primary recovery factor,

λ = lead time in years,

$\Delta R'_n$ = revisions and extensions of earlier primary proven reserves on n-th year,

$\Delta R''_n$ = actual increment to secondary proven reserves on the n-th year, μ_2 = secondary recovery factor,

$\Delta R'''_n$ = actual increment to tertiary proven reserves on the n-th year, μ_3 = tertiary recovery factor,

ϵ = revisions to prior computations of $\Delta R'$, $\Delta R''$, and $\Delta R'''$,

P_n = production in n-th year,

f = fraction of the current reserves to be produced each year, and

R_n = recoverable proven reserve as of the end of the n-th year.

The starting conditions are the set of the R_0 values as of the beginning of the period of projection. The parameters μ_1, μ_2, μ_3, m, f, and λ are assumed to be known in the model and would be estimated by studying the historical data for the region. The schedule of primary, secondary, and tertiary recovery factors is also assumed for the model.

Then an exploratory drilling schedule is assumed for the region; that is, the footage D_n to be drilled in each region in each year.

Equations (2), (3), and (4) express the aproaches used by the National Petroleum Council (1973) for oil and gas calculations. For the oil model it is assumed that a certain proportion of associated and dissolved gas will be found per barrel of oil reserve discovered, and this gas will be passed to the gas model. A constant fraction of the amount of all exploratory drilling is assumed to be allocated to gas exploration.

Up to this point the big uncertainty is the role of the market price in the assumed drilling schedule. Actual economic costs of various oils vary widely, and because some producers would lose money at the average price, they would not have undertaken the assumed scheduled exploration and development. The market price required to yield the assumed drilling schedule should be higher than the average cost, or, said another way, the actual drilling schedule and petroleum outcome would be smaller than indicated by the above model if the market price is the same as the average cost.

What is needed, then, is a way to preestimate the supply curves for oil and for gas—that is, the amounts of oil and gas that would become available as a function of their market price. This preestimate thus requires consideration of the variety of the individual development projects ranked as to their profitability.

The exploratory effort D_n in a given province on a given year n can be viewed as the initial point of an investment decision. Let us define this as an "n-project." The exploratory effort D_n would result in the discovery of a proven reserve $\Delta R_{n+\lambda}$ where λ is an exploration lead time, but only if the industry actually decides to make the necessary investments. An entrepreneur would decide whether to undertake the project, depending on his assessment of the project's financial outcome. For this decision, the discounted cash-flow method, having an assumed discount rate, r, can be used. This is the procedure adopted, for example, in Project Independence (U.S. Federal Energy Admin., 1974), which can be used to develop a mathematical formulation.

Let $\Delta P_n(\tau)$ be the annual production to be obtained from an n-project, τ being the time counted from its initial year n. For a given discount rate r, there would be a minimum unit price $\mathbf{p}_n(r)$ of the oil, or gas, that the entrepreneur would require for deciding to make the investment on the n-project. This unit price is a function of the discount rate r and also of the specific n-project considered. As a basic characteristic of an n-project, we can take the finding rate m. Hence, we can write

$$\mathbf{p}_n(r)=\mathbf{p}(m,r). \tag{5}$$

That is, given the finding rate m and the discount rate r, the minimum required price can be calculated.

Consider now the supply situation in a given year n as being the result of all prior n-projects undertaken. For a unit market price U we can ask what would be the supply P_n of oil, or gas, that would be available. The supply P_n would be the sum of the productions for the year n of all prior n-projects that would be undertaken. The supply for a given year would thus consist of the addition of the contributions drawn various years after discovery from a set of n-projects, that, is of several "vintages." The question, therefore, is which n-projects would have been undertaken.

If the future-price expectation of all entrepreneurs prior to year n had been U, then all those n-projects for which

$$U(m,r)\leqq U \tag{6}$$

would have been carried out, and the supply would be

$$S_n=\sum_{\tau=n}^{\tau=L} \delta_{m,p}\Delta P_n(n-\tau), \tag{7}$$

where the operator, $\delta_{m,p}$, is equal to 1 if (6) is satisfied, and equal to 0 if it is not.

The finding rate is defined as the ratio

$$m=\frac{\Delta R_{n+\lambda}}{D_n}. \tag{8}$$

For a given region, this finding rate may vary in some manner as a function of the cumulative drilling, that is $m=f(\Sigma D_n)$.

Now I would like to discuss whether the value S_n above, as a function of U, would provide an estimate of the supply curve. The most critical input data to the calculation of this estimate of the supply curve appears to be the finding rate. We have

$$\Delta R_{n+\lambda}=mD_n. \tag{9}$$

First, the future-price expectation in the past may be different from the actual price at a later time. If the future-price expectation in the past would have been higher than the actual current price, then the actual current supply would be higher than indicated by the equation (7) because more entrepreneurs would have decided, some mistakenly, to carry out their n-projects. On the other hand, if the future-price expectation in the past would have been lower than the actual current price at a later time, then the actual supply would be lower than indicated by (7) because less n-projects would have been carried out. If however, it is assumed that the future-price expectation of entrepreneurs is correct, then the above difficulty disappears.

Second, an n-project would generally encompass several petroleum fields of various degrees of profitability, yet all of them are linked in this simple model as one project. This naturally tends to blur the significance of the supply curve as calculated above.

Now we assume that the production rate obtained from $\Delta R_{n+\lambda}$ decreases exponentially with time and that the decay constant is T_1, so that

$$\Delta P_n(\tau)=(\Delta R_{n+\lambda}/\mathrm{T}_1)e^{-\tau/T_1}. \tag{10}$$

Hence

$$\Delta P_n(\tau)=m(D_n/\mathrm{T}_1)e^{-\tau/T_1}. \tag{11}$$

The development and exploitation of an n-project gives origin to a sequence of annual investments, expenditures, and incomes throughout the life of the project. In the discounted cash-flow method, the cash proceeds and cash outlays are discounted to the initial time. If the discount rate is assumed, then the unit required price to make the present value of the proceeds equal to the present value of the outlays can be calculated.

For the discounted cash flow of a given n-project, and taking the U.S. type of taxation as an example, we can use the expression

$$F=(1-\alpha)(1-\beta)U\sum_{\tau}\Delta P_\tau e^{-r\tau} +\gamma(E_a+E_b)-E_0-\sum_{\tau}E_\tau e^{-r\tau} -\sum_{\tau}\gamma\{(1-\alpha)(1-\beta)p\Delta P_\tau -E_\tau-\eta-\theta\}e^{-r\tau}, \tag{12}$$

where

F = present value of the discounted cash flows,

U = unit price of oil (or gas),

P_τ = production per year on τ year of project,

α = ad valorem tax,

β = royalty rate,

E_a = expensed items: dry holes + δ% of successful wells + lease rentals + overhead,

E_b = tax credits: ϵ% of successful wells + environment and safety + gas plant + lease equipment,

γ = corporation tax rate,

E_0 = initial cash expenditures,

E_τ = cash expenses on τ year of project,

η = effective depletion rate for τ year of project, as a fraction of the net revenue,

θ = depreciation for τ year of project, and

r = annual discount rate.

The first term in the summation of expression (12) corresponds to the present value of the net revenues; the second term, to tax credits; the third, to the initial expenditure; the fourth, to the present value of the annual expenses; and the last, to the present value of the income taxes.

Equation (12) is to be solved for U, namely

$$U = \frac{E_0 + \gamma \sum_\tau (E_\tau - \eta - \theta) e^{-r\tau} - \gamma (E_a + E_b)}{\gamma (1-\alpha)(1-\beta) \sum_\tau \Delta P_\tau e^{-r\tau}}. \quad (13)$$

Some of the quantities in the numerator of (13) correlate strongly with D, and others, with ΔR, but the predominant effect appears to be a linear dependence of E_0 on D. Moreover

$$\Delta P_\tau = m(D/T_1) e^{-\tau/T_1} \quad (14)$$

because of equation (11), and thus ΔP_τ is also proportional to D. Therefore, the factor D tends to cancel out in (13), and the price U would turn out to be inversely proportional to the finding rate m, namely

$$U = \frac{c}{m}, \quad (15)$$

where c is a constant which depends on the investment and operating expense coefficients, tax rates, and production decay constant T_1.

DRILLING FINDING RATE

The density of drilling, that is, the number of exploratory wells drilled per square mile of prospective area, is a useful indicator of what needs to be done in young petroleum provinces, as, for instance, in most developing countries. Because of this, it is necessary to review the relationship between wells drilled and petroleum found. Total footage is a measure of the amount of drilling.

A common assumption is that the average amount of proven reserve found each year is proportional to the amount of exploratory drilling in that year. Moreover, it has been assumed that this finding rate, in barrels of oil or thousand cubic feet of gas per foot, decreases as the cumulative drilling increases. Offhand, this appears to be a reasonable assumption.

Why should the finding rate decrease steadily as the cumulative drilling increases? It cannot be simply because the oil resources in a region are being depleted as the exploratory drilling increases because the finding rate could remain constant or even increase during most of the exploratory phase of a region and then drop rapidly to zero as the limits of the resource base are finally approached.

It has been claimed that the finding rate should decrease because the larger fields will be discovered first. Of course, this is what one would like to have happen, but the record of exploration in basin after basin reveals that this is not so. The discovery of giant fields typically occurs some 30 years after exploration begins in a region. There does not appear to be a dominating "pickle-barrel effect" to thus justify a decrease of the finding rate.

Maximum depth of drilling has been increasing worldwide. For given basins and time lapses of 10–20 years, the average drilling depth may show a trend of increase that would introduce a decreasing trend of the finding rate with time. The year-to-year fluctuations of the finding rate, however, can be considerably larger than the effect per year of the basic trend due to drilling depth.

As an example, let us examine how the finding rate behaves for the National Petroleum Council (U.S.) regions (table 2). For the period 1956–70, the finding rates for regions 1, 2, 2A, 6, 6A, 7, 8, 9, 10, and 11 vary rather randomly from year to year and do not demonstrate any trend of decreasing values as drilling footage increases. On the contrary, for regions 1, 2A, the aggregate of 8, 9, and 10, and 11, the basic trends indicate that the finding rate increases as cumulative drilling footage increases. Only for regions 3, 4, and 5 is a declining trend indicated.

TABLE 2.—*New oil in place (bbl/ft) added per foot of exploratory drilling*
[From Natl. Petroleum Council, 1973, p. 212–222]

	U.S. region										
Year	1	2	2A	3	4	5	6	6A	7	8,9,10	11
1956	0	191	33	93	94	201	86	1,239	97	106	0
1957	0	163	13	188	115	254	92	1,027	79	90	0
1958	0	186	38	227	107	274	80	4,037	61	90	0
1959	214	143	34	113	86	329	103	17,451	65	132	0
1960	3,968	222	522	159	100	314	87	-----	75	112	0
1961	2,178	151	774	138	99	273	146	-----	76	68	0
1962	0	545	365	69	54	249	135	751	92	75	22
1963	43	263	1,206	71	42	213	81	460	72	84	0
1964	366	1,360	163	56	52	149	102	323	76	82	138
1965	2,975	226	1,063	40	71	184	93	257	83	69	569
1966	2,256	40	1,262	80	40	134	62	487	55	81	0
1967	4,492	93	1,867	112	69	136	71	282	46	106	0
1968	11,233	411	640	68	75	95	41	391	41	144	16
1969	128	43	1,695	45	25	87	59	415	51	149	129
1970	463	77	0	44	41	121	77	1,423	78	175	72

ECONOMIC ROLE OF THE FINDING RATE

To gage the quantitative importance of the finding-rate function on the supply curve, let us assume a linearly decreasing function, namely

$$m = m_0 - hQ, \tag{16}$$

where m_0 is the finding rate at the initial point considered, Q is the cumulative exploratory drilling measured from the initial point, and h is a constant.

The supply S is obtained by integrating the outcome of the drilling effort, namely

$$S = \int_0^{Q_1} m\,dQ = \int_0^{Q_1} (m_0 - hQ)\,dQ, \tag{17}$$

or

$$S = m_0 Q_1 - \frac{hQ_1^2}{2}. \tag{18}$$

The cutoff point of the drilling, namely the Q_1 value, would depend on the economics. The cumulative amount of drilling Q determines m by (16), and m determines the minimum required price from equation (15). Thus

$$\frac{c}{U_1} = m_0 - hQ_1, \tag{19}$$

and

$$Q_1 = \frac{m_0 - c/U_1}{h}. \tag{20}$$

So that the supply S is

$$S = \frac{m_0^2}{2h} - \frac{c^2}{2hU_1^2} \tag{21}$$

Equation (21) demonstrates that the supply function S has an asymptote. No matter how large the price U_1 the quantity S would be no larger than $m_0^2/2h$. As U_1 increases, S increases at a slower and slower pace to its ultimate asymptotic value. The asymptotic value of S varies with the square of the initial finding rate and also is inversely proportional to the downward slope of the finding-rate line.

A quantitative assessment of the role of the finding-rate function, which I have made briefly here, reveals that the finding rate plays a major role in defining the supply curve for petroleum. A decreasing finding rate, even a mild linear decrease as cumulative drilling increases, imposes a definite roof (asymptotic value) to the supply curve. No matter how high the price goes, the supply does not go above it. As the price increases, the supply increases at a more and more sluggish pace.

NOTE ON THE LOGISTIC METHOD OF PROJECTION

Application of the logistic function to the estimation of petroleum resources has been made by M. K. Hubbert in an extensive series of papers since 1956 or so (see, for example, Hubbert, 1969). A similar approach is the use of the Gompertz curve (Moore, 1965). These methods have the advantage of their simplicity and of being based on published statistical data. They are intended to be used in areas where exploratory drilling has reached a mature stage.

These methods have been called "mathematical" because they use a formula to fit the cumulative-production and proven-reserve data. In fact they are empirical because no theory justifies the use of either the logistic function or the Gompertz curve.

When these methods are used, it is difficult to recognize when the final maximum has been reached. The historical oil-production curves of several countries show well-defined maxima. Had the logistic, or Gompertz, projection method been used, there would have been a gross underestimation of the undiscovered oil resources. For example, the annual oil-production curve for Mexico in the period 1918–32 has a maximum in 1921; for Austria in the period 1946–61, a single maximum in 1955; for France in the period 1946–73, a maximum in 1965; for Rumania in the period 1918–47, a maximum in 1936; and for the USSR in the period 1918–45, a maximum in 1941.

These methods provide a firm estimate of the *minimum* amount of petroleum that ought to be found if the industry continues doing what it has been doing in the past—what is already in the bag, so to speak. They cannot, however, predict new plays in a basin.

The application of these methods is limited to mature petroleum regions and thus cannot be used in most of the partially explored and unexplored petroleum-prospective areas of the world.

NOTE ON RANDOM SEARCH

In petroleum resource appraisal, two basic questions are to be answered: (1) How much petroleum is there? and (2) where is it? If we assume the answer to the first of these questions, we have a problem of search only.

Some random-drilling models purport to show that the outcome of past exploration is to be explained by random drilling. But even a superficial acquaintance with the history of petroleum exploration reveals that such is not the case. The overwhelming majority of exploratory drilling decisions are made on the basis of all information available to the decision-maker, and the choices of exploratory sites are those that currently appear to be most promising. The aggregate nature of the data used and the naivete of some decision models used might explain why this exercise seems to prosper.

A major logical difficulty in the modelling of past exploration is the introduction of a Sunday morning quarterback or post-mortem point of view. When exploration begins in a new province, it does so with no assurance that commercial accumulations exist and with no assurances as to the size of such accumulations. If, in fact, there are, let us say, 100 fields in a province at the beginning of the exploration, a search strategy cannot assume this but must assume the variety of possible ultimate outcomes: 0 field, 1 field, 2 fields . . . *n* fields. On the other hand, if we assume that at the beginning of the exploration, we are provided with a statement saying that somewhere in the basin are 100 basins and that we ought to find them, the situation would be different. Then we could engage in a systematic campaign of grid drilling, using closer and closer grids, until the fields are found. Thus, a model of random or grid drilling loses meaning when examined against the exploration that has to unfold the plot.

In order to explain the outcome of exploratory efforts, one must take into consideration both the search strategy of predrilling surveys (geology and geophysics) and the exploratory drilling strategy. By trying different composite strategies the petroleum industry has come to settle on quite different strategies for the above two phases of exploration.

Predrilling surveys, not being so much hindered by logistic limitations as drilling, can systematically cover wide-ranging areas. The purpose of this predrilling phase is to find likely traps. A probability of trap recognition could be associated with the outcome of each predrilling exploratory method. Whether the behavior of predrilling surveys could be simulated by a random search is not clear.

Exploratory drilling, on the other hand, cannot roam freely over the prospective area. Logistic considerations of the exploratory activity itself constrain its freedom of movement. Moreover, logistic considerations of the petroleum industry itself also modify and restrict the choice of the areas for more intensive exploratory drilling. As soon as a discovery is made, a reassessment has to be made about the exploratory/development drilling strategy. Because of the above considerations, it would be uneconomical to assume a random exploratory drilling strategy. Exploratory drilling is focused on specific traps; as soon as a discovery is made, the drilling is concentrated on traps that belong to the same geologic family, or what is called a play. While a systematic search is being made for a given type of trap, unforeseen conditions may be met which may lead to a random disturbance, which in turn may trigger another sequel of exploratory drilling.

The result of this dual strategy can be gaged by a simple model. Let us assume that a number dn_i of exploratory holes are drilled after traps of type i; then the number dT of traps discovered is

$$dT=\alpha_i dn_i+\sum_j (1-\alpha_j)(N_j-T_j)\frac{\Delta S_j}{S}dn_j, \qquad (22)$$

where

α_i=probability of correct recognition of trap type i,

α_j=probability of correct recognition of trap type j,

ΔS_j=average area of trap type j,

S=total search area,

dn_j=number of exploratory holes drilled after traps of type j,

N_j=total number of traps type j, other than i, which exist in the regions, and

T_j=cumulative number of traps of type j discovered.

If n_i and n_j are independent variables, then the solution of equation (22) is

$$T=\alpha_i n_i+\sum_j [N_j-\text{k}\exp\{-\frac{\Delta S_j(1-\alpha_j)}{\text{S}}n_j\}]+\text{c}, \qquad (23)$$

where c and k are constants of integration.

The above formula reveals the two elements of the exploratory drilling outcome namely, a hit with constant probability α_i represented by the first term on the right side, plus a term with a negative exponential represented by the second term.

If we were to focus only on the rate of discovery of traps type j, accidentally while running after type i, one gets

$$\frac{dT}{dn_j}=\frac{\text{k}\Delta S(1-\alpha)}{S}\exp\{-\frac{\Delta S(1-\alpha)n}{S}\}. \qquad (24)$$

H. W. Menard and George Sharman (written commun., 1975) have proposed a model of search which in effect takes into account only the second term of equation (22). That is, they assume a purely random search.

SKETCH OF A PROBABILISTIC MODEL OF PETROLEUM-RESOURCE ASSESSMENT

The petroleum-prospective regions of the world consist of a set of sedimentary basins $\{B_i\}$. Each basin may be segmented in geologic compartments $\{V_{i,j}\}$, characterized by a homogeneous probability space as to size and location of a given type of petroleum trap $T_{i,j}$. The $V_{i,j}$ may overlap in part with each other, and the union of the $\{V_{i,j}\}$ may be smaller than the space occupied by B_i.

The exploration strategy aims at identifying these geologic compartments, which provide lanes for further discoveries. The probability of discovery of a field is enhanced when the definition of a geologic compartment is established. Exploration then can move to a higher probability path.

Some of the statistical distribution functions of the $T_{i,j}$ may be correlated or functionally dependent on those of other $T_{i,k}$, but many are totally independent. Therefore, the statistical properties of some $T_{i,j}$ may be inferred statistically from those of known $T_{i,j}$, but there are others for which the known $T_{i,j}$ will provide no information.

For a given B_i, the domains of some of the $V_{i,j}$ may have been defined in part or in total by the exploration effort. That is, an element $v_{i,j}$ of $V_{i,j}$ would be known. The ultimate recoverable resources $r_{i,j}$ from $v_{i,j}$ may be estimated from field-production data, success ratios, and even logistic or Gompertz type of projections.

For those already identified $V_{i,j}$ having a known element $v_{i,j}$, an estimate of the recoverable resources $r_{i,j}$ in $V_{i,j}$ may be made:

1. By scaling $r_{i,j}$ in terms of the significant dimensional parameters of $v_{i,j}$ and $V_{i,j}$, or

2. By an econometric analysis based on the dimensional parameters, which would allow expression of the results in terms of their statistical significance, or
3. By formulating a field of probability occurrences over $V_{i,j}$ based on an extension of what may be known in $v_{i,j}$.

For those $V_{i,j}$ that are already identified but that have no known element, an estimate of the recoverable resources would have to be made by comparison with other $V_{i,j}$ and by the introduction of a subjective judgment of the field of trap occurrences throughout $V_{i,j}$.

For the remaining parts of B_i where no $V_{i,j}$ have been identified a broader range of estimates would have to be made based on more subjective judgments as to the likelihood of occurrence of fields and the statistical properties of their size and locational parameters.

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