Status of Ground-Water Modeling in the U. S. Geological Survey

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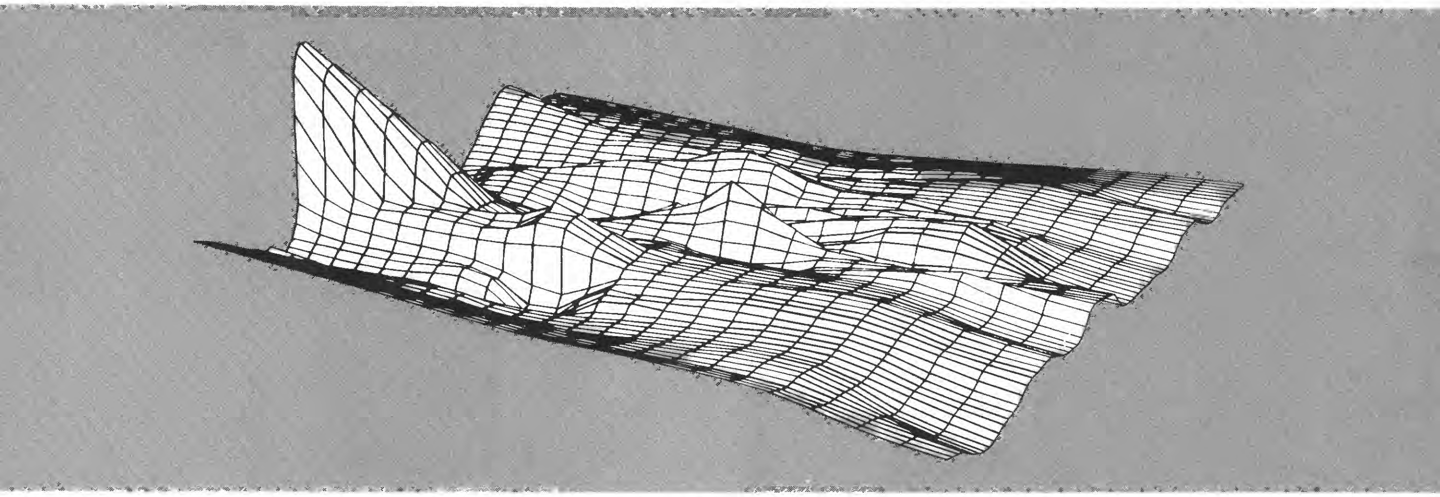

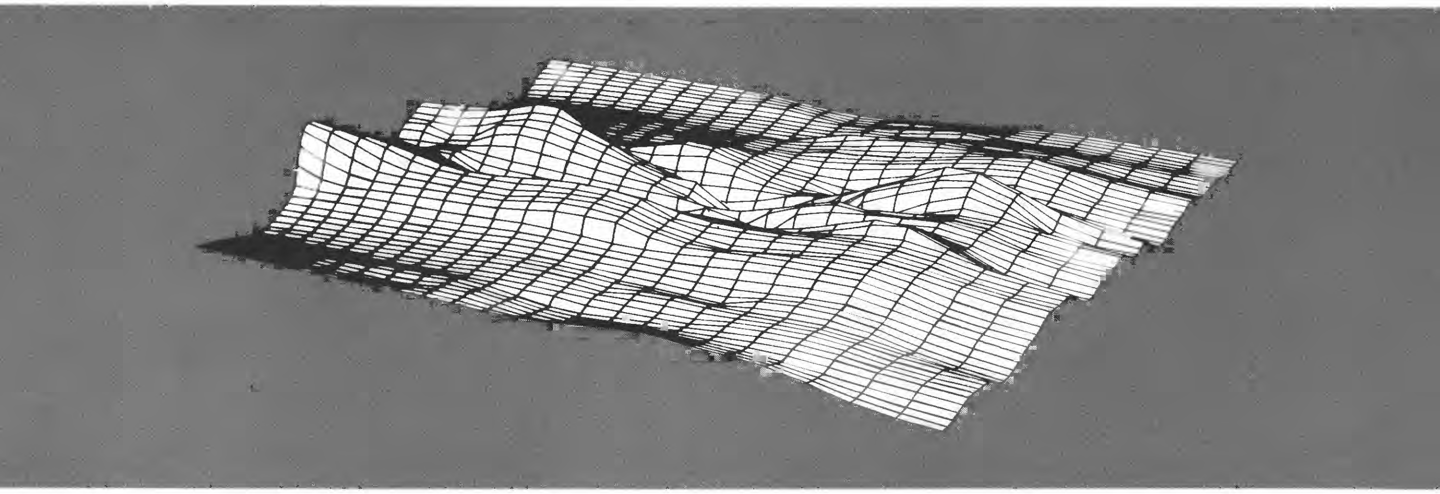

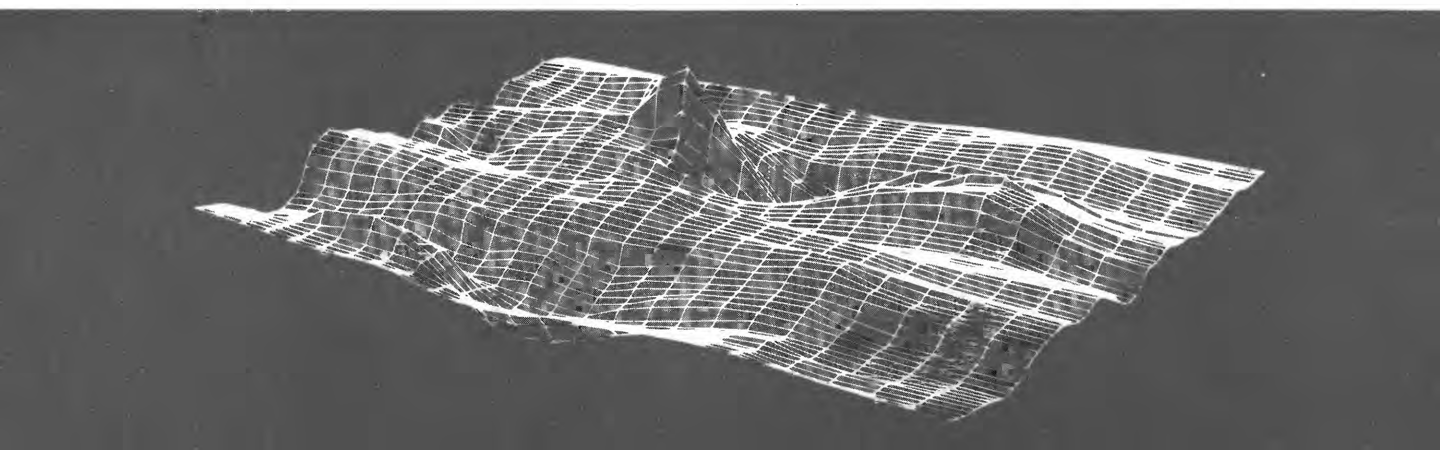

Status of Ground-Water Modeling in the U.S. Geological Survey

By Charles A. Appel and John D. Bredehoeft

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Status of Ground-Water Modeling in the U.S. Geological Survey

BY CHARLES A. APPEL AND JOHN D. BREDEHOEFT

ABSTRACT

The U.S. Geological Survey is active in the development and use of models for the analysis of various types of ground-water problems. Types of problems for which models have been, or are being, developed include: (1) ground-water flow in saturated or partially unsaturated materials, (2) land subsidence resulting from ground-water extraction, (3) flow in coupled ground water-stream systems, (4) coupling of rainfall-runoff basin models with soil moisture accounting and aquifer flow models, (5) interaction of economic and hydrologic considerations, (6) predicting the transport of contaminants in an aquifer, and (7) estimating the effects of proposed development schemes for geothermal systems. The status of modeling activity for various models is reported as being in a developmental, verification, operational, or continued improvement phase. Recently published references that provide useful details on the characteristics of the models are identified.

INTRODUCTION

The use of models to aid in the analysis of ground-water problems has increased significantly in recent years. Both the applied researcher interested in establishing theoretical relationships of the dependence of a solution on problem variables and the field hydrologist interested in developing the capability to predict effects of stresses on an aquifer system find models useful.

HISTORICAL BACKGROUND

The U.S. Geological Survey (USGS) has contributed significantly to the development and use of models for the analysis of ground-water problems. The history of its contributions goes back at least as far as C. S. Slichter's classic analysis of the mathematics of steady-state flow through porous media, which was published in the U.S. Geological Survey's 19th Annual Report, 1897–98. C. V. Theis' development of the nonequilibrium formula for transient ground-water flow toward a discharging well in a confined aquifer, published in 1935 [1], is credited as being "one of the most important recent milestones in the development of ground-water hydrology." [2] This paper preceded a similar analysis of transient conditions in an infinite aquifer by petroleum reservoir engineers.[3]

Following Theis' 1935 paper, a number of analytical solutions were developed for differing geologic and pumping situations. These were closed-form analytical solutions. Although they often predicted the response of individual wells accurately, they were much less effective in the analysis of aquifer systems. The closed-form analytical solutions generally require: (1) homogeneous and isotropic aquifer properties, and (2) simple aquifer geometry; but real aquifers commonly differ markedly from these idealizations.

In the early 1950's the pulsed resistor-capacitor (R–C) electronic network was developed as a direct analog for the equation of ground-water flow. A pulsed R–C network permitted solution of the general flow problem. Problems involving non-homogeneity, irregular boundary geometry, and pumping complexity could be analyzed. Petroleum reservoir engineers also adapted R–C networks for analysis of

[1] Theis, C. V., 1935, Relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage: Am. Geophys. Union Trans., pt. 2, p. 519–524.

[2] Ferris, John G., and Sayre, A. Nelson, 1955, The quantitative approach to ground-water investigations: Econ. Geol., Fiftieth Anniv. Volume, 1905–1955, pt. II, p. 714–747.

[3] Muskat, M., 1937, The flow of homogeneous fluids through porous media: McGraw-Hill Book Co., Inc., 763 p.

flow problems. Research in the Geological Survey improved the methodology by (1) high speed pulsing of the networks, thus reducing the data acquisition time needed to complete a model analysis, and (2) demonstrating that low-cost, low-precision components gave results compatible with the accuracy of hydrologic data. With the development of the R–C network, most linear ground-water problems involving the flow of a single fluid could be readily solved in two or three space dimensions. The Geological Survey established an operational facility devoted exclusively to application of this methodology to aquifer analysis. About 100 hydrologic analog models have been developed and analyzed by Survey personnel in the past 15 years.

RECENT DEVELOPMENT

The R–C network is ideally suited for solving flow problems involving a single fluid of uniform density—the usual ground-water supply problem. Most petroleum reservoir problems involve two or more fluids—problems which are not particularly well suited to R–C analog solution. In the early 1950's reservoir engineers looked to numerical methods adapted for digital computers to solve large scale flow problems. Early numerical reservoir simulators were severely limited by the size and speed of the digital computers of the period. By the late 1960's digital computers could compete with the analog for the solution of transient problems involving a few thousand nodes in two space dimensions. With today's digital computers and powerful numerical techniques, three-dimensional problems involving up to 10,000 nodes can be solved. Nearly all of the offices of the Geological Survey are now tied to a central computer through field terminals, and numerical ground-water models are used rather routinely for flow problems. Numerical models have been used for about 90 two-dimensional flow problems in the past 6 years. Due largely to the widespread availability of the digital computer, numerical methods have replaced the analog in most applications. However, the analog model remains a very useful tool for problems involving large multi-aquifer hydrologic systems which must be simulated using more than 10,000 nodes.

With the development of high-speed computers, it has become feasible to develop numerical models that consider more realistic representations of complex hydrologic systems than was possible earlier. Types of problems that have been analyzed include: (1) flow in water-table aquifers in which relatively large changes in saturated thickness take place, (2) flow in unsaturated or partially unsaturated materials, (3) nonrecoverable compaction of fine-grained materials in response to pumping-related stresses, (4) flow in coupled ground water-stream systems, (5) coupling of rainfall-runoff basin models with soil moisture accounting and aquifer-flow models, (6) interaction of economic and hydrologic considerations, (7) predicting the transport of contaminants in an aquifer, and (8) estimating the effects of proposed development schemes for geothermal systems.

Many of these types of problems require that more than one equation be solved simultaneously. For example, general transport problems require the coupling and simultaneous solution of the partial differential equations that describe three (or more) components of a non-steady flow system. Typically, these would include, (1) an equation for pressure, (2) an equation for heat, in terms of temperature or enthalpy, and (3) an equation for the concentration of each chemical constituent of interest.

STATUS OF MODELING ACTIVITY

The Geological Survey continues to be active in modeling various types of ground-water problems. The status of our modeling activity is briefly summarized in the following table. With few exceptions the models handle transient conditions. Note that for many types of ground-water problems, models have been developed using more than a single solution technique. This is done so that the comparative strengths of different methods can be evaluated and used to advantage. Such evaluations provide a rational basis for determining the method likely to be most efficient for particular types of problems.

Some comments are in order on the classification of phases of modeling activity. To state that a model is in the "verification" phase is intended, in most cases, to mean that development of a basic computer program has been completed and tests are being made to determine how well a model-derived solution represents the solution to the related differential equations used to describe the basic problem. For models of complex phenomena, the analog or numerical models often are tested by comparing the model results with analytical solutions. A difficulty here is that the most complex problem for which an analytical solution can be obtained usually is of trivial complexity compared to the degree of complexity of problems for which the model is developed to handle. A method used to check the usefulness of a model for complexity typical of realistic problems is to compare model-derived and observed responses to known stresses on the ground-water system for which the dependent variables and the system parameters are well known. In some cases, the required degree of certainty of these data in unavailable. In those cases one often relies on satisfaction of balance computations of mass, energy, and so forth to reflect the "correctness" of the computed values.

Models in the "operational" phase are being used to evaluate field problems. In some cases, experimental work is being done on operational models to develop useful improvements in model accuracy, flexibility, or convenience of application.

In many cases, investigators other than those given in table 1 are, or have been, involved in the listed modeling activity. The investigators listed can be contacted [4] for information on the subject models beyond that given in available publications.

The "recently published" references noted in table 1 provide useful details on the characteristics of the subject models and indicate the model capabilities. A more extensive, but not exhaustive, list of recently published references follows table 1.

[4] The investigators can be located through the U.S. Geological Survey, Water Resources Division, National Center, Reston, Va. 22092.

TABLE 1.—*Status of ground-water modeling, U.S. Geological Survey*

	Phase of activity: Developmental	Verification	Operational	Continued improvement	Principal U.S. Geological Survey investigators	Recently published selected references
FLOW						
Saturated						
Two-dimensional						
Analytical	--	--	×	×	S. S. Papadopulos, R. L. Cooley.	Cooper (1966), Papadopulos (1967), Papadopulos (1966), Cooper and others (1965).
R-C Analog Networks	--	--	×	×	S. M. Longwill	Skibitzke (1960), Patten (1965), Stallman (1963b).
Numerical—Finite difference	--	--	×	×	P. O. Trescott	Trescott (1973), Pinder (1969), Maddock (1970).
Finite element—Galerkin	--	--	×	×	G. F. Pinder [1], R. L. Cooley	Pinder and Frind (1972), Frind and Pinder (1973).
Finite element—Variational	--	--	×	--	R. T. Hurr	Hurr (1972).
Three-dimensional						
R-C Analog Networks	--	--	×	×	S. M. Longwill	Skibitzke (1960), Stallman (1963a), Patten (1965).
Numerical (Finite difference)	--	--	×	×	P. C. Trescott	Trescott (1975), Bredehoeft and Pinder (1970).
Partly (or entirely) unsaturated						
One-dimensional						
Analytical [2]	--	--	×	--	C. D. Ripple, J. Rubin, T. E .A. Van Hylckama.	Ripple, Rubin, and Van Hylckama (1972), Stallman and Reed (1966).
Numerical—Finite difference	--	--	×	×	J. Rubin and C. D. Ripple	Rubin (1967, 1968a).
Finite element—Galerkin	--	×	--	--	----do	
Two-dimensional						
Numerical—Finite difference	--	--	×	×	----do	Rubin (1968b).
Cylindrical Region						
Numerical—Finite element—Galerkin	--	×	--	--	----do	Doherty (1972).
LAND SUBSIDENCE—Induced by ground water extraction						
Analytical	--	--	×	--	F. S. Riley	Riley (1969).
R-C Analog and Analytical	--	--	×	--	D. G. Jorgensen	Jorgensen (1975).
Numerical and Analytical	--	--	×	×	D. C. Helm	Helm (1974, 1975).
COUPLED GROUND WATER—*stream systems*						
Numerical—Finite difference	--	×	--	--	G. F. Pinder [1] and S. P. Sauer	Pinder and Sauer (1971).
Numerical and Analytical	--	--	×	--	A. F. Moench, V. B. Sauer, M. E. Jennings.	Moench, Sauer, and Jennings (1974); Luckey and Livingston (1975).
COUPLED GROUND WATER—RAINFALL-RUNOFF MODELS—Numerical	--	×	--	--	J. E. Reed and M. S. Bedinger and John Terry.	
COUPLED GROUND WATER—ECONOMIC SYSTEMS—Numerical	--	--	×	×	T. Maddock, III and J. D. Bredehoeft.	Bredehoeft and Young (1970), Young and Bredehoeft (1972), Maddock (1972, 1973, 1975).
COUPLED FLOW AND TRANSPORT OF CHEMICAL CONSTITUENTS						
Saturated system						
Conservative (or nonconservative trace constituents)						
Uniform density, inorganic						
Two-dimensional						

Analytical	--	--	×	--	A. Ogata	Ogata (1976, 1970), Grove (1970).
Numerical—Characteristics	--	--	×	×	L. F. Konikow and J. D. Bredehoeft.	Konikow and Bredehoeft (1973), Robertson (1974), Bredehoeft and Pinder (1973).
Finite difference	--	×	--	--	D. B. Grove	
Finite element—Galerkin	--	--	×	×	D. B. Grove, J. Rubin, G. F. Pinder.	Pinder (1973).
Finite difference and finite element—Galerkin	--	×	--	--	D. B. Grove	
Variable Density						
Two-dimensional						
Analytical [2]	--	×	--	--	A. Ogata	Henry (1964).
Analog and Analytical [2]	--	--	×	--	G .D. Bennett	Bennett and others (1968).
Numerical—Characteristics	--	--	×	--	G. F. Pinder [1]	Pinder and Cooper (1970).
Finite difference [2]	--	×	--	--		Henry (1964).
Finite element—Galerkin	--	×	--	--	do	Segol, Pinder, and Gray (1975).
Three dimensional						
Numerical—Finite element—Galerkin	--	×	--	--	do	
Nonconservative, major constituents						
Uniform density						
Inorganic constituents (one-dimensional)—Numerical	--	×	--	--	D. B. Grove, W. W. Wood, J. Rubin, and R. V. James.	Rubin and James (1973).
Organic constituents (two-dimensional)—Numerical	×	--	--	--	J. B. Robertson and D. F. Goerlitz.	
Unsaturated						
Nonconservative, Major constituents						
Uniform density, inorganic constituents—Numerical	×	--	--	--	J. Rubin and R. V. James	
COUPLED FLOW AND TRANSPORT OF HEAT						
Single phase (hot water)						
Two dimensional						
Numerical—Finite difference	--	×	--	--	C. R. Faust and J. W. Mercer	Faust and Mercer (1976).
Integrated Finite difference	--	×	--	--	M. L. Sorey	Sorey (1975).
Finite element—Galerkin	--	×	--	--	J. W. Mercer and G. F. Pinder [1].	Mercer, Pinder ,and Donaldson (1975); Mercer and Pinder (1975).
Three dimensional						
Numerical—Finite element—Galerkin	×	--	--	--	G. F. Pinder [1]	
Two Phase (steam-water)						
Two-dimensional						
Numerical—Finite difference	--	×	--	--	C. R. Faust and J. W. Mercer	Faust and Mercer (1976).
Finite element—Galerkin	--	×	--	--	J. W. Mercer and C. R. Faust	Mercer and Faust (1975), Faust and Mercer (1975).
Three-dimensional						
Numerical—Finite element—Galerkin	×	--	--	--	G. F. Pinder [1]	
COUPLED FLOW AND TRANSPORT OF CONSERVATIVE (OR NONCONSERVATIVE TRACE) CONSTITUENTS AND HEAT (SINGLE PHASE)						
Two-dimensional						
Numerical—Finite difference [2,3]	--	×	--	--		Henry and Hilleke (1972).
Three-dimensional						
Numerical—Finite difference [3]	--	×	--	--	D. B. Grove, S. P. Larson	Intercomp (1976).

[1] Part-time investigator with USGS.
[2] Model limited to steady-state conditions.
[3] Model prepared for USGS under contract.

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