HOW INCIPIENT MOTION DETERMINATION JUDGMENT AFFECTS DIFFERENT PARAMETERS IN SEDIMENT TRANSPORT INVESTIGATION

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Abstract: Incipient condition plays a significant role in the field of sediment transport and channel stability and different parameters based upon it are used in the sediment initiation and transport formulas both for the development and application purposes. Its determination depends upon the subjective judgment of the investigator. Effects of this judgment on sediment transport and flow parameters have been investigated in this paper by considering three flow conditions of M (when small number of particles start to move), M_1 (when large number of particles starts to move), and M_2 (when very large number of particles starts to move). Data used in this paper were collected from a sediment transport study conducted in the hydraulics laboratory of the University of Manitoba (Canada). Effects of each flow condition on sediment transport, critical discharge, critical velocity, and flow depth parameters were investigated and a significant variation in results was found when flow conditions varied. This variation was more pronounced with the smaller sediment sizes as compared to the larger ones. For the critical discharge, a difference between M and M_1, M_1 and M_2 and M and M_2 conditions ranged from 36-118%, 16-129% and 61-211%, respectively. Likewise, other parameters of flow depth, critical velocity, sediment loads were significantly affected when the conditions were altered. Among the three conditions the M_1 appeared to be more reliable and realistic to be used for the incipient motion determination.

INTRODUCTION

Owing to the variation in particle sizes and their positioning in different directions, incipient motion of sediment for all particles does not occur at one time. Vibration of bed-material particles is an indication that movement is about to begin. This indicates the response of particles to the passing flow, which causes pressure differences and shear stresses that lead to lift and drag forces. If these forces increase over time, the in-place vibration may change to motion. Meanwhile, other particles may respond to increasing shear stresses and pressure differences over their surfaces by a more-abrupt initiation of motion, without vibration. As individual particles begin to move, they leave behind vacant spaces that change the local flow field at the bed surface. These alter lift and drag forces acting on other particles and may help to mobilize several particles simultaneously (Matin 1993).

Incipient motion of sediment in channels, natural or man made, has great significance in the field of sedimentation, especially when prediction is desired. It is the beginning of movement of sediment particles that were stationary some time before. As soon as they have initiated their movement they continue to move for an unspecified time and distance. The precise discharge and time at which initial movement occurs is a subjective determination, therefore investigators have different point of views in this regard. Some argue that it takes place when a small number of particles starts to move while others disagree and say this is just a settlement stage and incipient motion takes place when a significant number of particles starts to move. Some observers consider initiation of sediment transport to occur when the first few particles start moving, whereas others may say that it occurs when a large number of particles starts to move over a large part of the bed surface.

This paper investigates this subjective determination (judgment) issue so that a possible accurate condition could be found when the incipient motion takes place. For this purpose three flow conditions M, M_1 and M_2 were assumed and their effects on flow depth, critical velocity, critical discharge, and sediment load parameters were investigated both qualitatively and quantitatively.

SIGNIFICANCE OF THE STUDY

Threshold parameters like V_c, d_c, and q_c are required in the development of incipient motion and sediment transport formulae, in their application for sediment load prediction, and other sediment related and channel stabilization studies. It is, therefore, a fundamental step to make a right judgment about the incipient motion so that these threshold parameters may be recorded correctly. Importance of the incipient motion can be imagined from the fact that most of the sediment transport formulae developed are based on the premises of sediment transport theories of excess shear $\{q_b \propto f[\tau - \tau_c]\}$, excess stream power $\{q_b \propto f[\omega - \omega_c]\}$, excess discharge $\{q_b \propto f[q - q_c]\}$, and excess velocity $\{q_b \propto f[V - V_c]\}$ which involve a parameter that rely upon the threshold condition. Therefore, a small error regarding the threshold condition judgment could seriously affect the structure of sediment transport formulae based upon them and their predicted results. Nice examples of sediment transport formulas in which threshold parameters were used are Schoklitsch (1962), Bagnold (1966), Wilcock and Southard (1989), Ashiq (1997). Likewise, the threshold condition determination is equally important for the sediment incipient motion prediction formulae (Ashiq and Bathurst (1999), irrespective of the theories used and approach followed for their development i.e. characteristic diameter size (reference particle size) approach [e.g. $q_c = 0.0345 \sqrt{g} D_{50}^{3/2} S^{-1.12}$, Milhous 1982] or fractional sizes approach [e.g. $\tau^*_{ci} = \tau^*_{c50} (D_i/D_{50}^{)-1}$, Andrews and Erman 1986].

EXPERIMENTAL METHODOLOGY

For the purpose of data collection an experimental setup (Figure 1) was re-designed and constructed in the hydraulics laboratory of the University of Manitoba, Department of Civil Engineering (Winnipeg, Canada) during 2003. At completion, an adjustable slope flume channel of 10 m long, 0.7 m wide, and 0.945 m deep was available to be used in this study. Further details regarding the construction, equipment used, and problems faced during the study may be found in Ashiq et. al (2004). During this study, five different sediment sizes were used and for each size slopes were varied 6 times. The sediment material sizes used and the corresponding slopes adopted are given in Table 1, whereas the collected sediment data and other related parameters may be seen in Table 2.

For determining the incipient condition at which movement of sediment starts, the data corresponding to three different flow conditions were recorded: a) when small number of particles started to move, M; b) when large number of particles started to move, M_1; and c) when very large number of sediment particles started to move, M_2. Values of the critical velocity, flow rates, flow depth and sediment load parameters were recorded for the M, M_1 and M_2 conditions and are given in Table 2.

Figure 1 Measurements in progress with a flume channel in the hydraulics lab of the University of Manitoba (2003).

Table 1 Sediment sizes used and the corresponding slopes adopted during the experimentation [University of Manitoba, Canada 2003].

Sr. No.	Channel Slope (%)	Bed Material Size (mm)
1	1.16 - 2.13	6.350 - 7.938
2	1.16 - 2.75	4.763 - 6.350
3	1.30 - 3.03	3.175 - 4.763
4	1.14 - 2.86	1.588 - 3.175
5	1.17 - 2.90	0.794 - 1.588

EFFECTS OF THE FLOW CONDITIONS' SELECTION ON FLOW DEPTHS

The general trend of variation between the depths recorded under all the three flow conditions (M, M_1, and M_2), for the five bed sediment sizes used, is evident from the bar chart heights and trend lines as depicted in Figs. 2-6. According to that the depths recorded under the M condition are less than the M_1 and M_2 for all the sediment sizes, except for 6.350-7.938 mm size (largest size, Fig. 2) for which they are greater than the M_2 condition but smaller than M_1 condition. The reason for this anomaly is that during the M and M_1 flow (conditions) period, the transported sediment from the upstream reach deposited at the location of measurements, which reduced the flow depth. Likewise, for the 4.763-6.350 mm size the trend line for the M_1 crossed M_2 when depth exceeded 40 mm, due to the reason that bed surface which was a bit uneven some time ago changed to a plane one, resulting in a decrease in the flow depth under the M_2 condition. Other minor variations in the flow depths for the M, M_1 and M_2 flow conditions for the 3.175-4.763 mm and 1.588 - 3.175 mm sizes were due to: i) a series of pits in the channel bed in the longitudinal direction (Fig.7); and ii) a wavy bed and water surfaces during measurements (Fig.8). Likewise, fluctuations in depths under the M and M_2 conditions for the smallest size (0.794-1.588 mm) were due to the wavy water surface and sediment deposition at the point of measurement (Fig.6).

Table 2 Sediment transport and other related data collected during experimentation at the University of Manitoba (2003).

Sr. No.	Remarks	Channel Width (m)	Temp. (C)	Slope (%)	Flow Depth (mm)	Flow Depth (m)	Mini Flow Reading	Critical Vel. (Vc) (cm/s)	Critical Vel. (Vc) (m/s)	Flow Rate (Q) (m3/sec)	Duration of Sediment Collection (sec)	Wt. of Sed. (kg)	Sediment Load Qs (kg/min)
1- Bed Material Size 6.350 -7.938 mm							28 Aug. 2003						
1 iv	M	0.945	19.5	1.17	80	0.080	38.0	105.0	1.050	0.0794	948	17.273	1.0932
2 iii	M	0.945	19.5	1.3	55	0.055	30.0	84.0	0.840	0.0437	617	3.436	0.3342
3 iii	M	0.945	20	1.55	50	0.050	31.0	86.5	0.865	0.0409	1003	9.150	0.5474
4 iii	M	0.945	20	1.76	44	0.044	30.0	84.0	0.840	0.0349	1243	10.341	0.4992
5 iii	M	0.945	19.5	2.24	34	0.034	27.0	75.5	0.755	0.0243	1119	10.314	0.5530
6 iii	M	0.945	20	2.9	25	0.025	27.0	75.5	0.755	0.0178	542	9.118	1.0094
1 v	M1	0.945	19.5	1.17	85	0.085	40.0	111.0	1.110	0.0892	948	17.273	1.0932
2 iv	M1	0.945	19.5	1.3	62	0.062	33.0	92.0	0.920	0.0539	617	3.436	0.3342
3 iv	M1	0.945	20	1.55	55	0.055	34.0	94.5	0.945	0.0491	1003	9.150	0.5474
4 iv	M1	0.945	20	1.76	47	0.047	32.0	89.0	0.890	0.0395	1243	10.341	0.4992
5 iv	M1	0.945	19.5	2.24	37	0.037	31.0	86.5	0.865	0.0302	1119	10.314	0.5530
6 iv	M1	0.945	20	2.9	28	0.028	29.0	81.0	0.810	0.0214	542	9.118	1.0094
3 v	M2	0.945	20	1.55	59	0.059	38.0	105.0	1.050	0.0585	1003	9.150	0.5474
4 v	M2	0.945	20	1.76	50	0.050	34.0	94.5	0.945	0.0447	1243	10.341	0.4992
5 v	M2	0.945	19.5	2.24	40	0.040	33.0	92.0	0.920	0.0348	1119	10.314	0.5530
6 v	M2	0.945	20	2.9	33	0.033	32.0	89.0	0.890	0.0278	542	9.118	1.0094
2- Bed Material Size 4.763 - 6.350 mm							21 Sept. 2003						
1 iv	M	0.945	19.5	1.14	55	0.055	28.0	78.0	0.780	0.0405	787	8.695	0.6629
4 iii	M	0.945	20	2.13	30	0.030	26.0	73.0	0.730	0.0207	688	7.359	0.6418
5 iv	M	0.945	19.5	2.35	31	0.031	23.5	66.5	0.665	0.0195	890	8.636	0.5822
6 iii	M	0.945	20	2.86	29	0.029	23.5	66.0	0.660	0.0181	766	11.277	0.8833
1 v	M1	0.945	19.5	1.14	58	0.058	30.0	84.0	0.840	0.0460	788	8.695	0.6621
2 v	M1	0.945	20	1.44	45	0.045	29.0	81.0	0.810	0.0344	567	5.568	0.5892
3 iv	M1	0.945	20	1.75	36	0.036	28.0	78.5	0.785	0.0267	444	7.586	1.0252
4 iv	M1	0.945	20	2.13	32	0.032	28.0	78.0	0.780	0.0236	689	7.359	0.6408
5 v	M1	0.945	19.5	2.35	32	0.032	26.5	74.5	0.745	0.0225	891	8.636	0.5816
6 iv	M1	0.945	20	2.86	30	0.030	24.5	69.0	0.690	0.0196	767	11.277	0.8822
1 vi	M2	0.945	19.5	1.14	63	0.063	32.0	89.5	0.895	0.0533	789	8.695	0.6613
2 vi	M2	0.945	20	1.44	48	0.048	31.5	88.0	0.880	0.0399	568	5.568	0.5882
4 v	M2	0.945	20	2.13	34	0.034	29.0	81.0	0.810	0.0260	690	7.359	0.6399
5 vi	M2	0.945	19.5	2.35	34	0.034	28.5	80.0	0.800	0.0257	892	8.636	0.5809
6 v	M2	0.945	20	2.86	31	0.031	27.0	76.0	0.760	0.0223	768	11.277	0.8810
3- Bed Material Size 3.175 - 4.763 mm							2 Oct. 2003						
1 v	M	0.945	20	1.3	45	0.045	24.5	69.0	0.690	0.0293	835	7.132	0.5125
2 iii	M	0.945	19.5	1.61	30	0.030	24.5	66.5	0.665	0.0189	909	5.909	0.3900
3 iv	M	0.945	20	1.95	25	0.025	22.5	64.0	0.640	0.0151	954	17.445	1.0972
4 iii	M	0.945	19.5	2.27	25	0.025	20.5	58.5	0.585	0.0138	304	6.223	1.2282
5 iii	M	0.945	19.5	2.62	17	0.017	19.0	54.0	0.540	0.0087	580	6.336	0.6555
6 ii	M	0.945	20	3.03	16	0.016	16.0	46.0	0.460	0.0070	651	10.427	0.9610
1 vi	M1	0.945	20	1.3	47	0.047	26.5	74.5	0.745	0.0331	836	7.132	0.5119
2 iv	M1	0.945	19.5	1.61	32	0.032	25.5	72.0	0.720	0.0218	910	5.909	0.3896
3 v	M1	0.945	20	1.95	27	0.027	24.0	68.0	0.680	0.0174	955	17.445	1.0960
4 iv	M1	0.945	19.5	2.27	27	0.027	22.5	63.5	0.635	0.0162	305	6.223	1.2241
5 iv	M1	0.945	19.5	2.62	19	0.019	21.0	59.5	0.595	0.0107	581	6.336	0.6544
6 iii	M1	0.945	20	3.03	18	0.018	19.0	54.0	0.540	0.0092	652	10.427	0.9596
1 vii	M2	0.945	20	1.3	55	0.055	28.5	80.0	0.800	0.0416	837	7.132	0.5112
2 v	M2	0.945	19.5	1.61	34	0.034	27.0	75.5	0.755	0.0243	911	5.909	0.3892
3 vi	M2	0.945	20	1.95	29	0.029	27.0	75.5	0.755	0.0207	956	17.445	1.0949
4 v	M2	0.945	19.5	2.27	28	0.028	26.5	74.5	0.745	0.0197	306	6.223	1.2201
5 v	M2	0.945	19.5	2.62	20	0.020	24.0	68.0	0.680	0.0129	582	6.336	0.6532
6 iv	M2	0.945	20	3.03	20	0.020	20.0	57.0	0.570	0.0108	653	10.427	0.9581
4- Bed Material Size 1.588 - 3.175 mm							10 Oct. 2003						
1 iv	M	0.945	20	1.16	19	0.019	11.0	32.0	0.320	0.0057	813	7.614	0.5619
2 ii	M	0.945	20	1.47	13	0.013	11.5	34.0	0.340	0.0042	873	4.659	0.3202
3 iii	M	0.945	19.5	1.81	15	0.015	11.5	34.0	0.340	0.0048	768	7.586	0.5927
1 v	M1	0.945	20	1.16	21	0.021	14.0	40.5	0.405	0.0080	814	7.614	0.5612
2 iii	M1	0.945	20	1.47	15	0.015	14.0	40.5	0.405	0.0057	874	4.659	0.3198
3 iv	M1	0.945	19.5	1.81	17	0.017	14.0	40.5	0.405	0.0065	769	7.586	0.5919
4 iii	M1	0.945	19.5	2.16	14	0.014	13.5	39.5	0.395	0.0052	808	14.973	1.1118
5 ii	M1	0.945	20	2.42	9	0.009	10.0	30.0	0.300	0.0026	570	11.991	1.2622
6 iii	M1	0.945	19.5	2.75	11	0.011	9.5	28.5	0.285	0.0030	586	10.936	1.1198
1 vi	M2	0.945	20	1.16	25	0.025	17.0	49.0	0.490	0.0116	815	7.614	0.5605
2 iv	M2	0.945	20	1.47	16	0.016	15.5	45.0	0.450	0.0068	875	4.659	0.3195
3 v	M2	0.945	19.5	1.81	18	0.018	18.0	51.5	0.515	0.0088	770	7.586	0.5911
4 iv	M2	0.945	19.5	2.16	16	0.016	15.0	43.5	0.435	0.0066	809	14.973	1.1105
5 iii	M2	0.945	20	2.42	10	0.010	15.0	43.5	0.435	0.0041	571	11.991	1.2600
6 iv	M2	0.945	19.5	2.75	13	0.013	16.0	46.0	0.460	0.0057	587	10.936	1.1179
5- Bed Material Size 0.794 - 1.588 mm							25 Oct. 2003						
1 iii	M	0.945	20	1.16	11	0.011	8.5	26.0	0.260	0.0027	1544	2.868	0.1115
3 iii	M	0.945	20	1.72	11.5	0.012	6.5	20.5	0.205	0.0022	1060	5.173	0.2928
4 iii	M	0.945	19.5	1.84	8	0.008	7.5	23.5	0.235	0.0018	1044	5.682	0.3265
5 iv	M	0.945	20	1.96	10	0.010	7.0	21.5	0.215	0.0020	800	6.309	0.4732
6 ii	M	0.945	19.5	2.13	10	0.010	6.0	19.0	0.190	0.0018	690	9.914	0.8621
1 iv	M1	0.945	20	1.16	13	0.013	11.0	32.5	0.325	0.0040	1545	2.868	0.1114
2 iii	M1	0.945	20	1.46	14	0.014	10.0	30.0	0.300	0.0040	821	5.768	0.4215
3 iv	M1	0.945	20	1.72	13	0.013	10.5	31.5	0.315	0.0039	1061	5.173	0.2925
5 v	M1	0.945	20	1.96	12	0.012	11.5	34.0	0.340	0.0039	801	6.309	0.4726
6 iii	M1	0.945	19.5	2.13	11	0.011	7.5	23.5	0.235	0.0024	691	9.914	0.8608
1 v	M2	0.945	20	1.16	15	0.015	12.5	37.0	0.370	0.0052	1546	2.868	0.1113
2 iv	M2	0.945	20	1.46	16	0.016	14.0	40.0	0.400	0.0060	822	5.768	0.4210
3 v	M2	0.945	20	1.72	14	0.014	11.5	34.0	0.340	0.0045	1062	5.173	0.2922
4 iv	M2	0.945	19.5	1.84	11	0.011	10.5	31.5	0.315	0.0033	1045	5.682	0.3262
5 vi	M2	0.945	20	1.96	15	0.015	13.5	39.5	0.395	0.0056	802	6.309	0.4720
6 iv	M2	0.945	19.5	2.13	13	0.013	10.0	30.0	0.300	0.0037	692	9.914	0.8596
Where M = condition when small number of particles started to move.													
M1 = condition when large number of particles started to move.													
M2 = condition when very large number of particles started to move.													

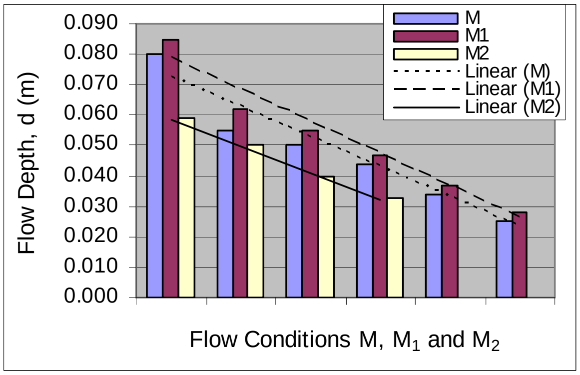


Figure 2 Variation in flow depth for the sediment size of 6.350 -7.938 mm.

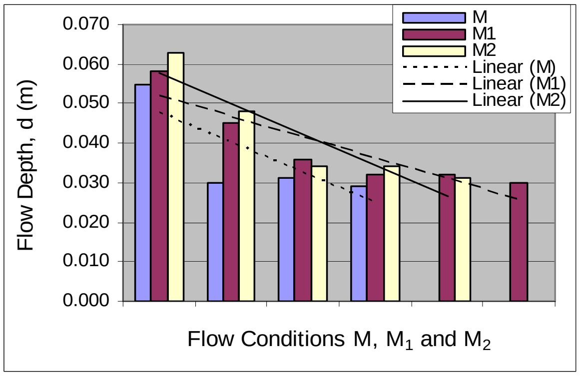


Figure 3 Variation in flow depth for the sediment size of 4.763 - 6.350 mm.

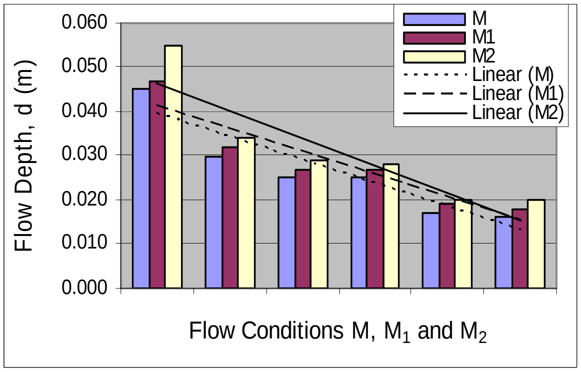


Figure 4 Variation in flow depth for the sediment size of 3.175 - 4.763 mm.

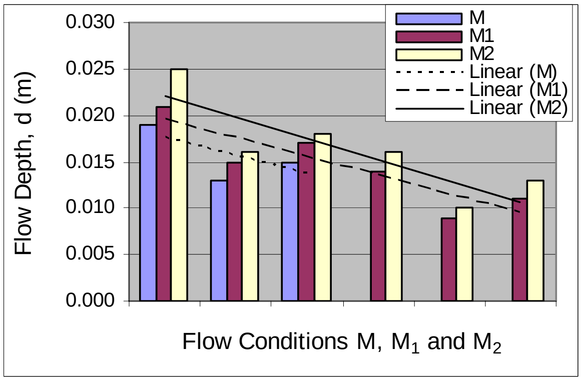


Figure 5 Variation in flow depth for the sediment size of 1.588 - 3.175 mm.

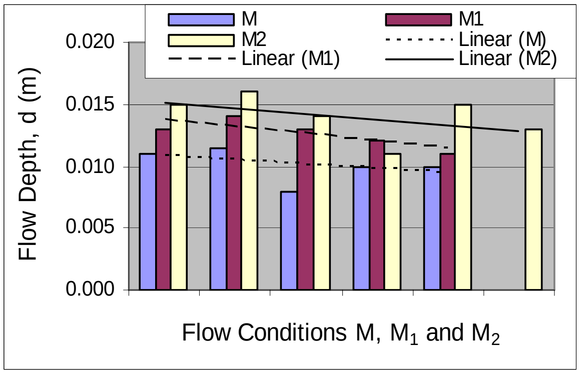


Figure 6 Variation in flow depth for the sediment size of 0.794 - 1.588 mm.

Figure 7 A series of pits in the longitudinal direction observed during the experimentation.

EFFECTS OF FLOW CONDITIONS' SELECTION ON THE SEDIMENT LOAD AND CORRESPONDING FLOW PARAMETERS

To investigate effects of incipient condition determination judgment on the depth, critical vertical, discharge, and sediment load, these parameters were analyzed for all the five sizes used in the study. In this analysis critical discharge vs. sediment load, slope vs. critical velocity, and sediment load vs. critical velocity were plotted and then trend lines were fitted to find the general

trend of relationships among the respective variables. Out of all these plots only 0.794-1.588 mm size plots are presented here for the purpose of demonstration (Figs. 9-11).
In critical discharge vs. sediment load plots, for the largest four sizes which almost belonged to the coarser size of sediment, the relationships were found to be of polynomial nature. While for the smallest size, a size which belonged to the fine sediment material, the linear relationship was found more appropriate. Both for the coarse and fine sizes the sediment loads under the three flow conditions (M, M_1 and M_2) increased from M towards M_2. The only size for which (partly) deviation recorded was the largest size (i.e. 6.350-7.938 mm), for which somewhere in the middle of the plot the M and M_1 lines crossed each other. This deviation from the general trend could be due to aggradations at the point of measurements.

On the other hand for the slope vs. critical velocity plots, for all the five sizes, a linear relationship was found more appropriate and clearly the critical velocity values increased with the flow conditions i.e. from M toward M_2. Nonetheless, in the case of the sediment load vs. critical velocity plots, the best relationships were of polynomial nature for the four coarser sizes and for the fine size the linear relationship was found to be the best one.

Figure 8 A wavy bed surface observed during the experimentation.

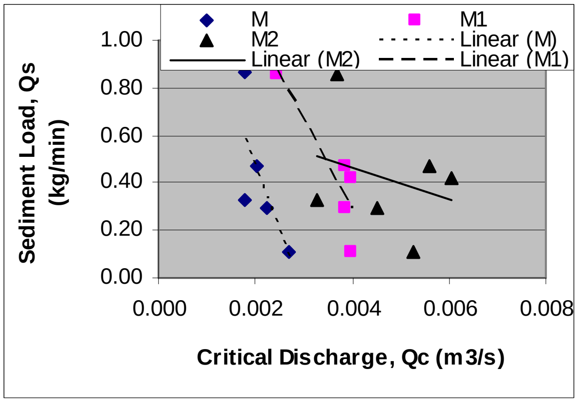


Figure 9 Variation in sediment load under M, M_1, & M_2 flow conditions for sediment size of 0.7941-0.588mm.

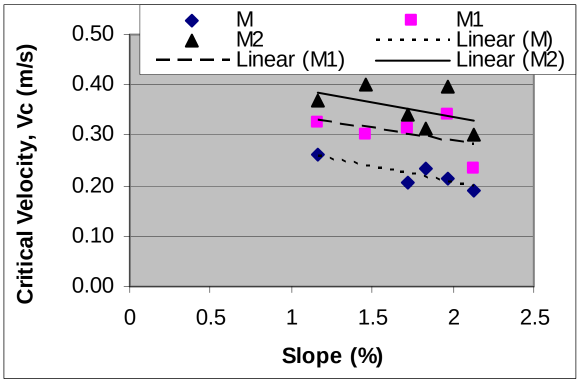


Figure 10 Variation in critical velocity under M, M_1, & M_2 flow conditions for sediment size of 0.794 - 1.588 mm.

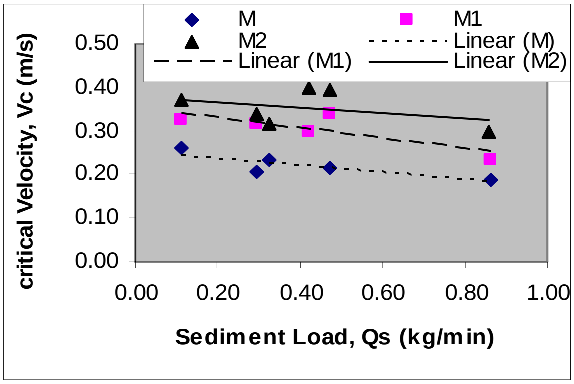
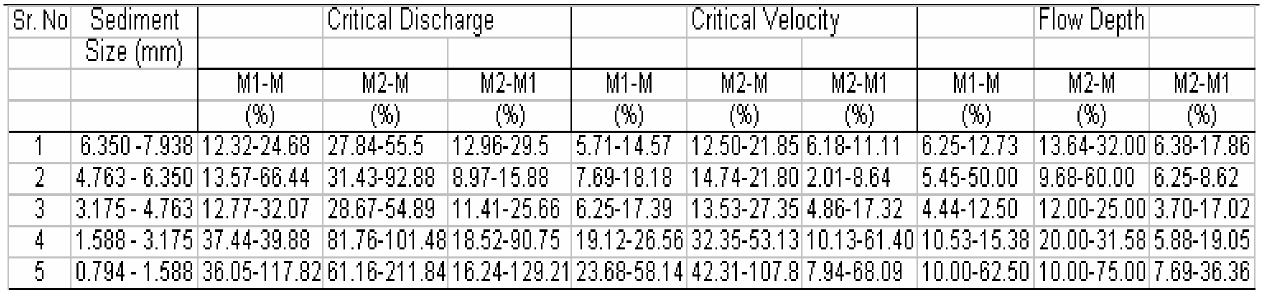


Figure 11 Variation in critical velocity under M, M_1, & M_2 flow conditions for sediment size of 0.794 - 1.588 mm.

QUANTITATIVE EFFECTS OF FLOW CONDITIONS' SELECTION ON CRITICAL DISCHARGE, VELOCITY, AND DEPTH

When the critical discharges were compared under the M and M_2 flow conditions, the variations of 27.84-55.5%, 31.43-92.88%, 28.67-54.89%, and 81.76-101.48% respectively were recorded for the coarse sizes (size 1-4) and 61.16-211.84% for the fine (size 5). The variations between the M and M_1 were found to be 12.32-24.68%, 13.57-66.44%, 12.77-32.07%, and 37.44-39.88% respectively for the coarse sizes and 36.05-117.82% for the fine size of 0.794-1.588 mm. While for the M_1 and M_2 flow conditions the variations ranged between 16.24-129.21% for the fine size and 12.96-29.5%, 8.97-15.88%, 11.41-25.66%, and 18.52-90.75% respectively for the coarse sizes. It showed that the discharge significantly changed with the selection of flow condition when altered from M to M_1, M_1 to M_2 and M to M_2. Likewise, variation in discharge increased with the decrease in sediment size which means that the finer sizes are more susceptible to the variation as compared to the coarse ones. A similar, general trend of variation was also found with the other two parameters of velocity and depth, with a few exceptions. A numeric set of values for variations in critical discharges, velocity and depth under the three flow conditions may be seen in Table 3.

Table 3 Variations (in percentage) in critical discharge, critical velocity, and flow depth under the M, M_1 and M_2 flow conditions for the five sediment sizes used.



Sr. No	Sediment Size (mm)		Critical Discharge			Critical Velocity			Flow Depth	
		M1-M (%)	M2-M (%)	M2-M1 (%)	M1-M (%)	M2-M (%)	M2-M1 (%)	M1-M (%)	M2-M (%)	M2-M1 (%)
1	6.350 -7.938	12.32-24.68	27.84-55.5	12.96-29.5	5.71-14.57	12.50-21.85	6.18-11.11	6.25-12.73	13.64-32.00	6.38-17.86
2	4.763 - 6.350	13.57-66.44	31.43-92.88	8.97-15.88	7.69-18.18	14.74-21.80	2.01-8.64	5.45-50.00	9.68-60.00	6.25-8.62
3	3.175 - 4.763	12.77-32.07	28.67-54.89	11.41-25.66	6.25-17.39	13.53-27.35	4.86-17.32	4.44-12.50	12.00-25.00	3.70-17.02
4	1.588 - 3.175	37.44-39.88	81.76-101.48	18.52-90.75	19.12-26.56	32.35-53.13	10.13-61.40	10.53-15.38	20.00-31.58	5.88-19.05
5	0.794 - 1.588	36.05-117.82	61.16-211.84	16.24-129.21	23.68-58.14	42.31-107.8	7.94-68.09	10.00-62.50	10.00-75.00	7.69-36.36

CONCLUSIONS

Based upon the study results, the following specific conclusions may be drawn from this study:

- When the determination judgment regarding the incipient motion is not made accurately the recorded data could be significantly underestimated/overestimated, therefore a very careful decision has to be made.

- The coarser sizes were found to be less affected with the subjective judgment as compared to the finer one under these flow conditions. Therefore, for studies in which finer sediments are to be dealt with, more care should be followed. However, it is important to mention that only one fine size sediment material was available for this investigation and further tests with more fine sizes would be helpful to further generalize this part of the conclusion.

- It seemed more appropriate to take the M_1 flow condition for the incipient motion determination, which represents a condition when large number of sediment particles start to move over a large part of the bed surface.

NOTATIONS

D_i = particle size for ith size fraction;
D_{50} = particle size for which 50% of the material is finer.
V = flow velocity;
V_c = critical flow velocity (i.e. velocity at threshold point);
q = water discharge per unit width;
q_c = critical discharge per unit width;
q_b = bed load discharge per unit width;
S = slope;
τ = shear stress;
τ_c = critical shear stress
τ^*_{ci} = Shield's parameter for particle size D_i;
τ^*_{c50} = reference shear stress parameter used in Parker et. al (1982) for D_{50} size;
ω = stream power; and
ω_c = critical stream power.

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