

ESTIMATION OF PARTICLE SIZES FOR A RANGE OF NARROW SIZE DISTRIBUTIONS OF NATURAL SANDS SUSPENDED IN WATER USING MULTI-FREQUENCY ACOUSTIC BACKSCATTER

Christopher K. Smith, Associate R&D Engineer, National Center for Physical Acoustics, University, MS, cksmith@olemis.edu; Daniel Wren, Hydraulic Engineer, USDA-ARS-NSL, Oxford, MS, wren@msa-oxford.ars.usda.gov; Jim Chambers, Senior Research Scientist, National Center for Physical Acoustics, University, MS, chambers@olemiss.edu

Abstract: The measurement of particle size using multiple, megahertz range acoustic frequencies has been focused on particles with radii of 50 μ m -150 μ m. The present study seeks to extend the applicability of the technique to particles with radii ranging from 50 μ m -425 μ m. A single acoustic transducer, transmitting a waveform with three peak frequencies, was used to measure backscatter from natural sand particles entrained in a turbulent jet. An approach similar to that taken by Sheng (1991) was used in the data processing, where particle size estimates were created by comparing theoretical ratios of the backscatter form factor and size density to measured ratios. It was found that the method produces 0.1%-36% error for particles with radii of 150 μ m - 425 μ m and 24%-160% error for particles with radii of 75 μ m - 150 μ m, depending on the concentration of the suspended particles. This work has been supported by the USDA.

INTRODUCTION

The use of single frequency ultrasonic acoustics to measure the concentration of suspended sediment is an established practice (Lynch, 1985, 1991; Hanes *et al.*, 1988; Libiki *et al.*, 1989; Thorne, 1993, 1994, 2000; Hardenberg *et al.*, 1991; Lee and Hanes, 1995); however, this method requires previous knowledge of the sediment size characteristics to be accurate (Ma *et al.*, 1983). To measure suspended sediment loads with no prior knowledge of the sediment sizes, a system that uses multiple frequencies must be used (Yu-shih Liu, 1965). Previous research using multi-frequency acoustics to estimate particle sizes has occurred primarily in shallow marine environments (Sheng, 1991; Crawford and Hay, 1993; Schat, 1997; Thosteen and Hanes, 1998) with sand of radii ranging from 50-150 μ m. Most of these studies used three single frequency transducers. The focus of the current study was to extend the measured range of radii of the sand particles to 50-425 μ m, and to establish the feasibility of using a single transducer with a composite waveform of three frequencies to estimate the size characteristics of the suspended sediment.

EXPERIMENTAL SETUP

The multi-frequency backscatter system was tested in a jet tank, Figure 1, similar to the one used by Hay (1991). The tank, hereafter referred to as the calibration tank, was constructed at the University of Mississippi National Center for Physical Acoustics. It was designed to produce a turbulent sediment carrying jet, by way of a pumping system that re-circulated sediment introduced into the tank. The pump speed could be varied to obtain optimum sediment jet conditions. A vacuum/j-tube system was also placed in the tank to collect physical samples for concentration measurements to be compared with acoustical backscatter data from a single transducer ultrasonic system.

The acoustic signals used by the backscatter system were produced by an arbitrary waveform generator. The composite waveform was created by adding together three different sine waves, each of a different frequency (1.4 MHz, 2.5 MHz, and 3.2 MHz). A broadband transducer with a center frequency of approximately 2.25 MHz was used to transmit and receive the signal. The transducer was placed 20cm from the centerline of the sediment jet. The input amplitude of each frequency was adjusted until the return echo from the back wall of the sediment free tank for each frequency was approximately equal. The return signal passed through a 34 dB preamp and was then digitized by a 50 MHz oscilloscope card.

Backscatter data was taken before any physical samples, and consisted of 200 bursts from the transducer. Each burst was composed of twenty pings separated by a 5ms delay. Each ping was composed of ten cycles of the composite waveform. The jet's centerline concentration was then determined by collecting and analyzing an isokinetic sample of the fluid/particle mixture.

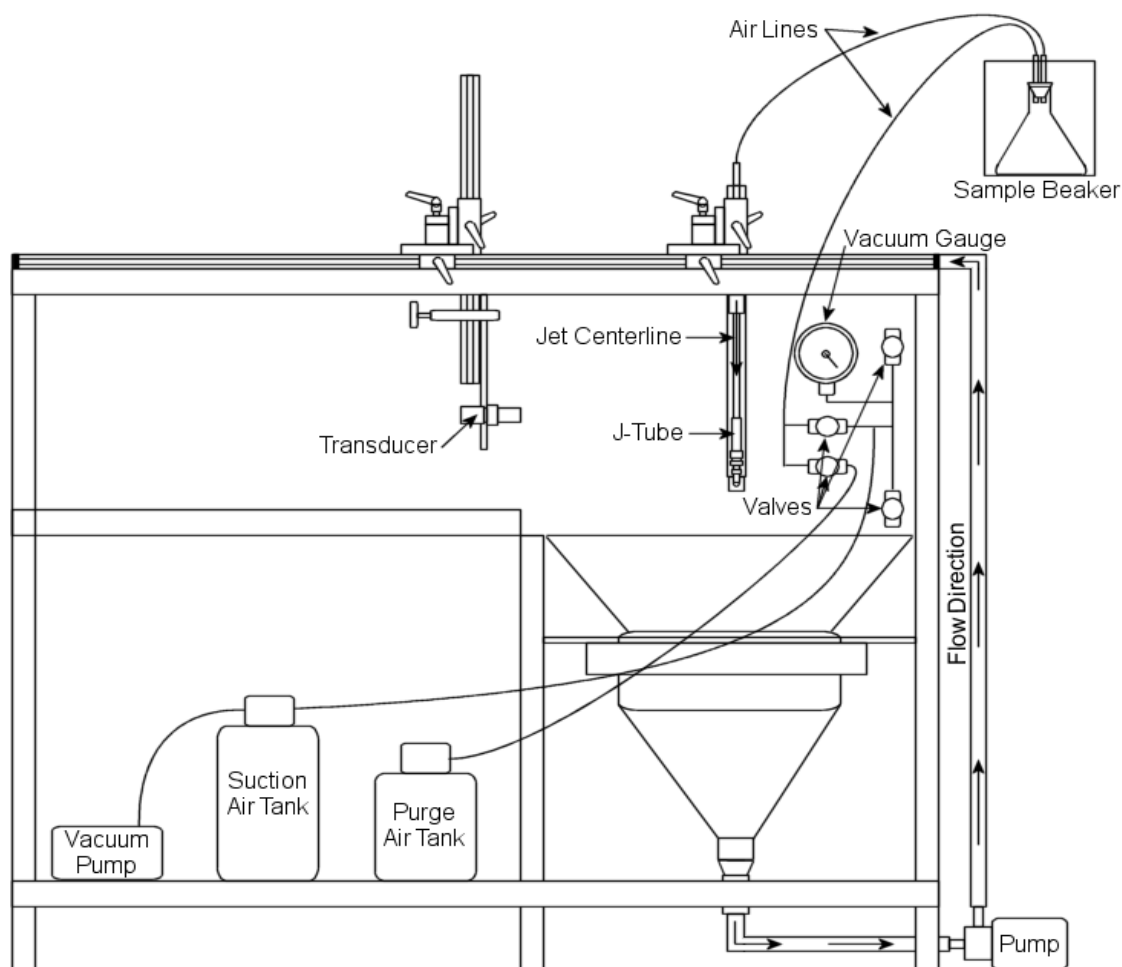


Figure 1. The calibration tank.

The sand used in the experiment was sieved into ten ranges of particle radius. Table 1 shows the ranges and their corresponding mean radii. The geometric standard deviation was equal to 1.05. Four different masses of each size range were used in the calibration tank: 5g, 10g, 20g, and 40g. The large size ranges yielded the highest concentrations for each mass of sand and the small size ranges yielded the smallest.

Table 1 The range of sand particle diameters and their corresponding mean radii.

Sand Particle Radius Range (μm)	Mean Sand Particle Radius (μm)
75.00-90.00	85.20
90.00-106.00	98.00
106.00-125.00	115.50
125.00-150.00	137.50
150.00-177.50	163.75
177.50-212.50	195.00
212.50-250.00	231.25
250.00-300.00	275.00
300.00-355.00	327.50
255.00-425.00	390.00

THEORY

The derivation of the acoustic equations used in this work has been established elsewhere (Sheng, 1991), and only the relevant equations are presented here. The backscatter voltage from the centerline of a sediment carrying jet (V_o) can be expressed by

$$V_o = B \sqrt{\frac{M_o}{\rho}} \frac{f^\infty}{\sqrt{x}} \sqrt{\frac{\sinh \zeta}{\zeta}} \exp\left[-2 \int_0^{r_0} \alpha_s dr\right], \quad (1)$$

where B is a system constant based on the frequency, directivity pattern, pulse duration, and beam-width of the acoustic system, M_o is the mass concentration at the centerline of the sediment jet, ρ is the density of the suspended sediment (2.65 g/cm^3), f^∞ is the backscatter form factor equation for uniformly sized suspended sediment, r_o is the range from the transducer to the centerline of the jet, and α_s is the attenuation due to the particles in the sediment jet. The term x is equal to the product of the wave number (k) and the particle radius (a).

The $\sqrt{\frac{\sinh \zeta}{\zeta}}$ term corrects for attenuation across the detected volume of the sediment jet (Hay, 1991). If the centerline concentrations of the sediment jet are small then the correction for attenuation across the detected volume and the attenuation due to the particles in the jet can be neglected (Sheng, 1991; Hay 1991). This gets rid of the $\sqrt{\frac{\sinh \zeta}{\zeta}}$ and $\exp\left[-2 \int_0^{r_0} \alpha_s dr\right]$ terms, Equation (1) then becomes

$$V_o = B \sqrt{\frac{M_o}{\rho}} \frac{f^\infty}{\sqrt{x}}. \quad (2)$$

The backscatter form factor equation is a rational fraction fit developed by Crawford and Hay (1993). The system constant, B can be solved for using Equation (2), if the backscattered voltages, sediment concentrations, and particle sizes at the centerline of the sediment jet are known.

A ratio of the backscattered voltages and system constants for two of the three frequencies (Equation 3) can be formed by rearranging and combining Equation (2) for each of the three frequencies.

$$\frac{V_{0i} B_j}{V_{0j} B_i} = \frac{f^\infty_i \sqrt{x_j}}{f^\infty_j \sqrt{x_i}}, \quad i \neq j; \quad i = 2.5, 3.2, 3.2; \quad j = 1.4, 1.4, 2.5, \quad (3)$$

where i and j are subscripts for each of the three frequencies. This equation no longer has a dependence on the mass concentration as does Equation (2).

The left-hand side of the equation is known from the backscattered voltage data, and the previously determined system constants. The right-hand side of the equation can be rewritten as

$$\frac{f^\infty_i \sqrt{x_j}}{f^\infty_j \sqrt{x_i}} = \frac{F(x_i, \sigma_g)}{F(x_j, \sigma_g)}, \quad i \neq j; \quad i = 2.5, 3.2, 3.2; \quad j = 1.4, 1.4, 2.5, \quad (4)$$

where $F(x, \sigma_g) = F[n(a), |f_\infty|]$, which is a function of the size distribution and the backscatter form factor (Sheng, 1991), σ_g is the geometric standard deviation of the particle sizes and $n(a)$ is the size spectral density. $F[n(a), |f_\infty|]$ can be written as

$$F[n(a), |f_\infty|] = \frac{1}{\sqrt{k}} \left[\frac{\int_0^\infty a^2 |f_\infty|^2 n(a) da}{\int_0^\infty a^3 n(a) da} \right]^{\frac{1}{2}}. \quad (5)$$

Assuming a lognormal distribution of the sand particles yields

$$F[n(a), |f_\infty|] = \frac{1}{\sqrt{k}} \left[\frac{\int_0^\infty a^2 |f_\infty|^2 \exp\left[\frac{-(\ln a - \ln a_g)^2}{2 \ln^2 \sigma_g}\right] da}{\int_0^\infty a^3 \exp\left[\frac{-(\ln a - \ln a_g)^2}{2 \ln^2 \sigma_g}\right] da} \right]^{\frac{1}{2}}, \quad (6)$$

where a_g is the mean particle radius and σ_g is equal to 1.05 in this experiment. Equation (6) can be solved to generate a set of theoretical values for each of the three frequencies used in the experiment using the rational fraction fit of the backscatter form factor and the known values of a_g , σ_g , and k . These theoretical values can then be used with Equation (4) to produce three theoretical ratios. A commercially available software package was used to produce an equation to fit the data points produced by each of the three theoretical ratios.

The method of estimating the size of the particle radius was to use one ratio (Equation 4) as a first estimate and one of the other two ratios (Equation 4) as a second estimate, depending on the estimated particle radius from the first estimate. This second estimate would then be averaged with the first to get the actual estimate of the particle's size. An experimental value using a ratio of backscattered voltages from the centerline of the jet and the system constants for two of the frequencies was substituted for the left hand side of Equation (4) and combined with the theoretical ratio to be used as the first estimate by substituting the ratio's curve fit equation for the right hand side of Equation (4). The roots of the resulting equation were then found. The polynomial equations used for the curve fits often resulted in more than one root being found. In order to account for this the median of the resulting roots was taken as the first estimate. This first estimate was then compared to a breakpoint. The breakpoint determined which of the other two ratios to use as the second estimate. If the first estimate was larger than the breakpoint a specific ratio was used, and if it was smaller than the breakpoint the other ratio was used. Estimates that were less than or equal to the breakpoint were considered "small". The second estimates were determined in the same manner as the first by finding the median of the roots for the second ratio's curve fit equation and the same experimental value used for the first estimate. Outlined below is the procedure using the 2.5/1.4 ratio as the first estimate, 164 μ m as a breakpoint, and the 3.2/1.4 ratio for estimating small particles, which is similar to the method described by Sheng (1991).

1. Determine a first estimate using the 2.5/1.4 ratio, a_1 .
2. If $a_1 \leq 164\mu\text{m}$, determine a second estimate using the 3.2/1.4 ratio, a_2 .
3. If $a_1 > 164\mu\text{m}$, determine a second estimate using the 3.2/2.5 ratio, a_2 .
4. Average a_1 and a_2 to get the final estimate, a .

Unlike the theoretical values published in Sheng's paper the theoretical ratios produced with the combination of frequencies used in this experiment have multiple inflection points and overlap in some places. A likely breakpoint is not obvious from inspection of the equation plots. Consequently multiple combinations of the three ratios as first estimates and different breakpoints were tried until the best possible particle size estimates were realized. The breakpoint that produced the best estimates across all four masses of sand was 164 μ m.

RESULTS

The theoretical ratios produced using Equation (5) for each of the three ratio combinations used in the experiment and the corresponding curve fit equation are shown in Figure 2. All the curve fit equations had R^2 values greater than 0.9.

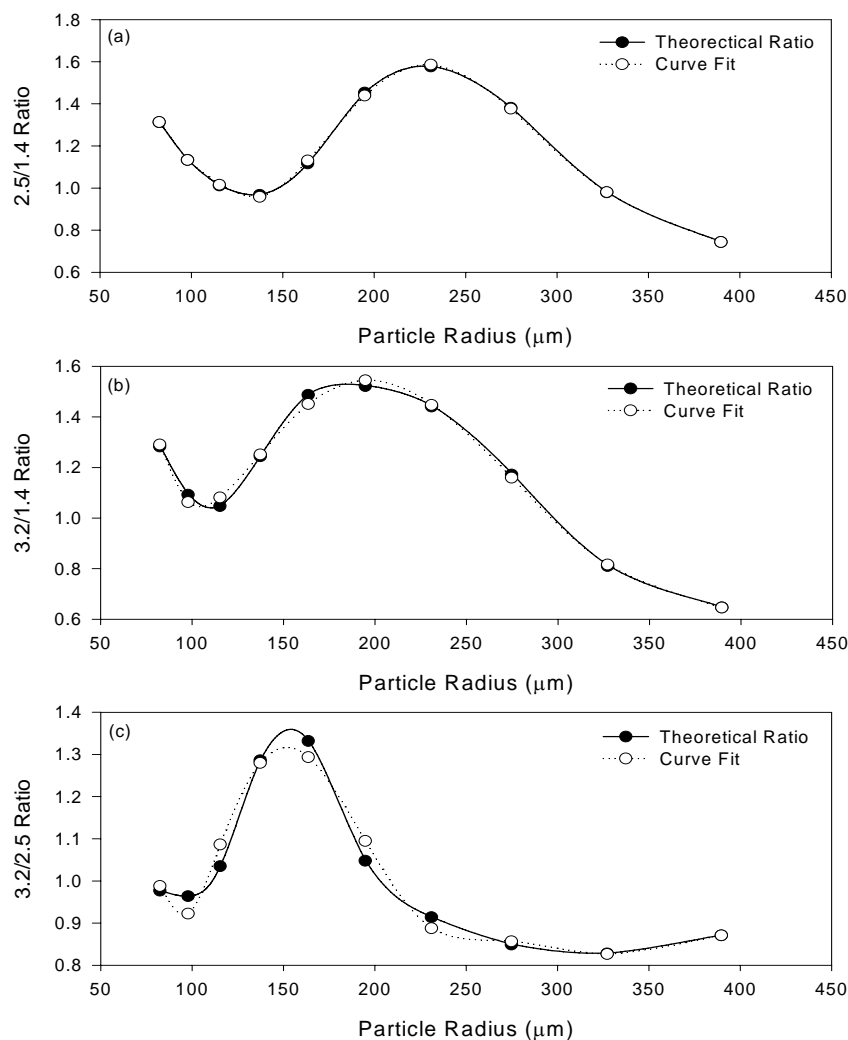


Figure 2 The theoretical values produced by Equation (5) for each of the three frequency ratios: (a) 2.5/1.4 Ratio, (b) 3.2/1.4 Ratio, (c) 3.2/2.5 Ratio.

The plots of true particle radius vs. estimated particle radius for each of the ratio combinations show the estimated particle radius for each of the four masses of sand, relative to a perfect agreement line, Figure 3. Each data point represents the mean estimated particle radius from a 200 burst experimental run for a particular mass of sand and size distribution, excluding radius estimates of zero. The exclusion of zero estimates caused the actual number of estimates for some of the size ranges be less than 200. The fewest number of estimates used to find a mean particle radius was 25, and the average number of estimates used to find a mean particle radius was 148. The previously established method estimates well the mean radius of particles in the range from 150 μm - 425 μm . However, the estimated mean radius for particles in the range from 75 μm - 150 μm are larger than their actual mean particle radius. The combination of the 3.2/2.5 and 3.2/1.4 ratios, Figure 3(f), yields reasonable results for particles in the 125 μm -

150 μ m range. The standard deviation of the estimates for all the ratio combinations ranged from 25 to 100, with a median of 54 and a mean of 56.

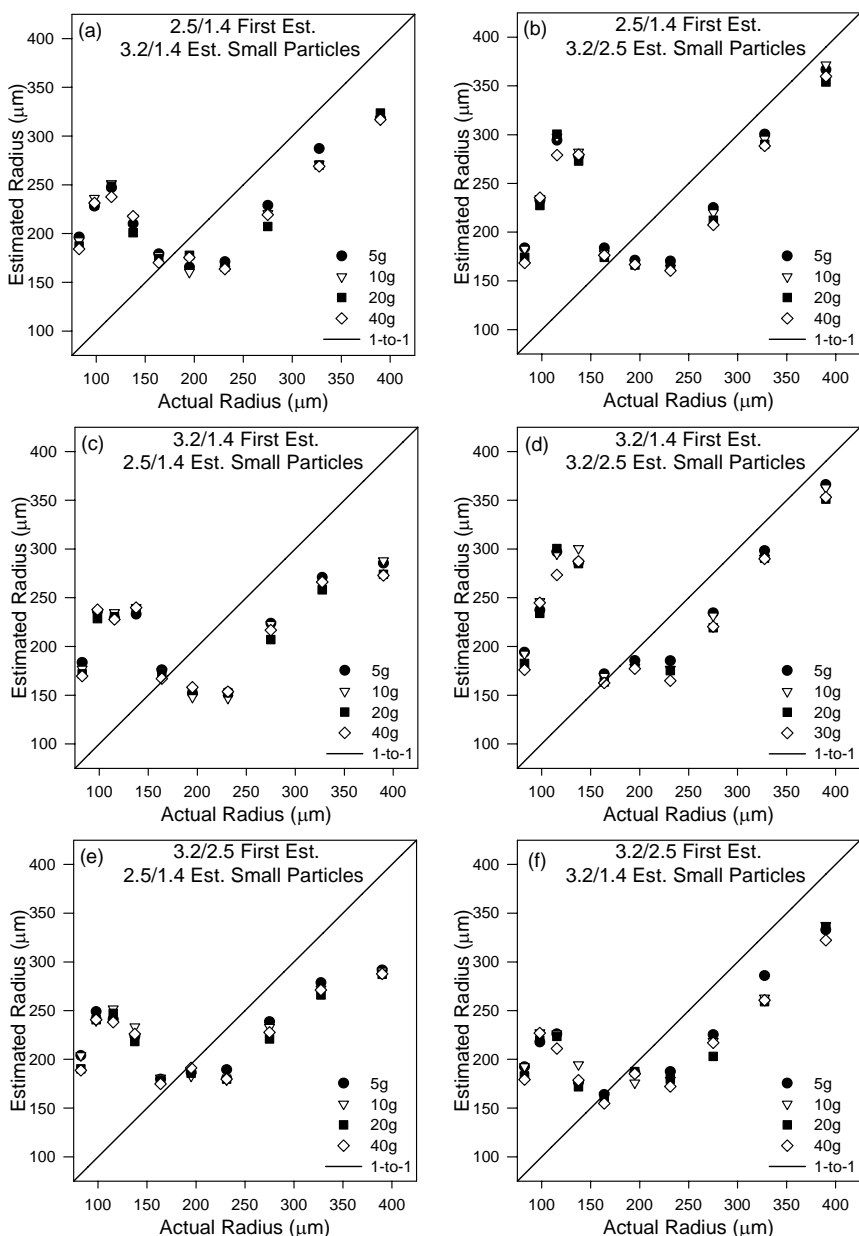


Figure 3 Actual mean particle radius compared to the estimated mean particle radius for each of the ratio combinations.

The plots of the true particle radius vs. percent error of the estimated mean particle radius for each of the ratio combinations show the percent error of the estimated particle radius for each of the four masses of sand, as well as a 0%, 20%, and -20% error lines, Figure 4. Each data point represents the percent error of the mean estimated particle radius of all the non-zero estimates from each experimental run for a particular mass of sand and size distribution. The majority of the estimates for particles in the 150 μ m - 425 μ m range fall within 20% of the actual mean particle radius. The error for estimates of the mean radius for particles in the range from 75 μ m-150 μ m are all between 25% and 160%. The combination of the 3.5/2.5 and 3.2/1.4 ratios, Figure 4(f), had errors from 25%-41% in the 125 μ m-

150 μ m range, depending on the mass of sand. This was the best of any of the ratio combinations at estimating particle sizes in this particular size range.

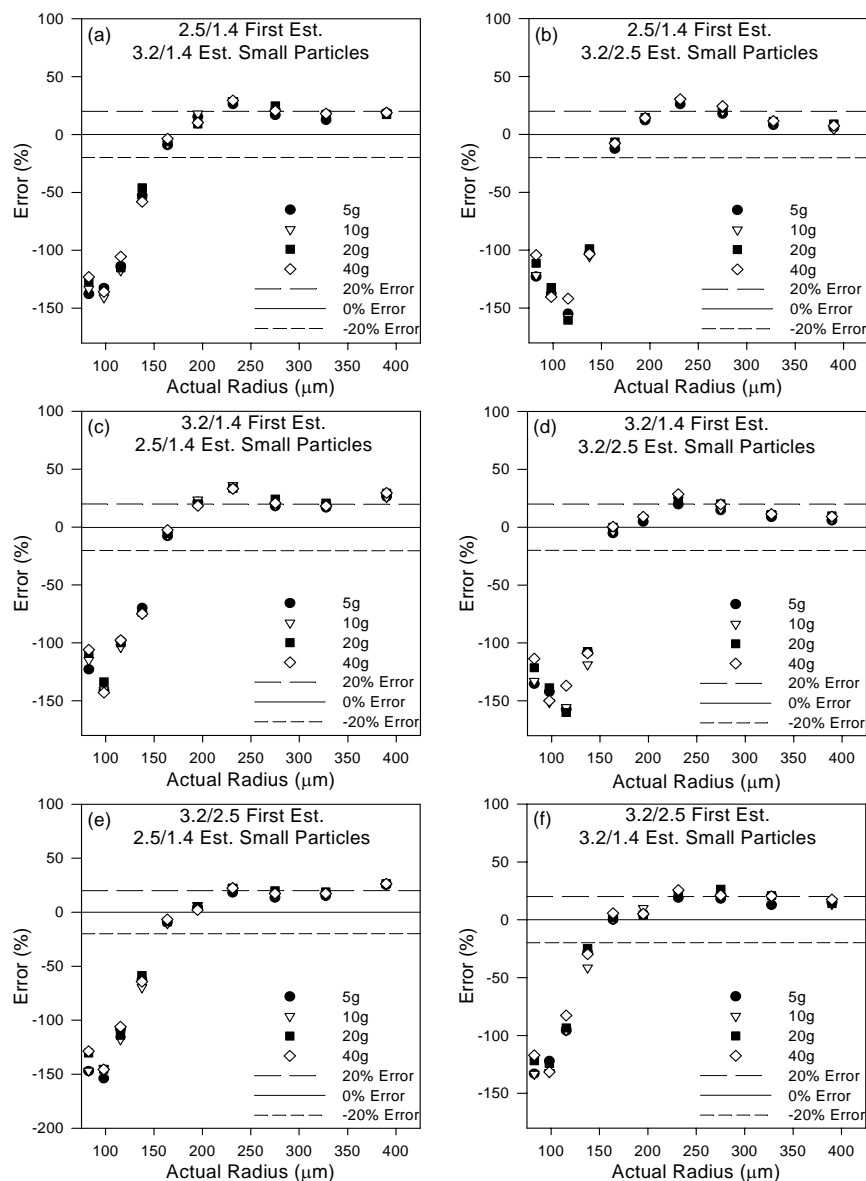


Figure 4 Actual mean particle radius compared to the percent error for the estimated mean particle radius for each of the ratio combinations.

CONCLUSIONS

The method of using ratios of backscattered voltages and system constants combined with theoretical ratios works well at estimating 150 μ m - 425 μ m particles, but overestimates 75 μ m - 150 μ m particles. This method yielded similar results for all four masses of sand used in the experiment which shows its robust nature in estimating mean particle radii for many different concentrations of suspended sediment composed of particles with radii larger than 150 μ m. The inability of the single transducer system to estimate particle sizes less than 150 μ m was problematic. Therefore, more work needs to be done on refining the method to reduce the error associated with smaller particles. One possible alternative will be to test different frequencies than the ones used in the experiment to produce theoretical ratio curves that have fewer inflection points. This may require the uses of two or more transducer

instead of a single transducer. This will allow lower order polynomials to be used as curve fit equations, yielding fewer possible roots.

REFERENCES

- Crawford A. M. and Hay. A. E. (1993) "Determining suspended sand size and concentration from multifrequency acoustic backscatter," *Journal of the Acoustic Society of America*. Volume 94. Number 6. Pp. 3312-3324.
- Hanes D. M. *et al.* (1988) "Acoustic measurements of suspended sand concentration in the C²S² experiment at Stanhope Lane, Prince Edward Sound," *Marine Geology*. 81. Pp. 185-196
- Hardenberg B. *et al.* (1991) "Field measurements of the vertical structure of suspended sediment," *Coastal Sediments*. Pp.300-312.
- Hay A. E. (1991) "Sound scattering from a particle-laden turbulent jet," *Journal of the Acoustic Society of America*. Volume 90. Number 4. Pp. 2055-2074.
- Lee T. H. and Hanes D. M. (1995) "Direct inversion method to measure the concentration profile of suspended particles using backscattered sound," *Journal of Geophysical Research*. Volume 100. Number C2. Pp. 2649-2657.
- Libiki C. *et al.* (1989) "The interpretation and evaluation of a 3-Mhz acoustics backscatter device for measuring benthic boundary layer sediment dynamics," *Journal of the Acoustic Society of America*. Volume 85. Number 4.
- Lynch J. F. (1985) "Theoretical analysis of ABSS data for HEBBLE," *Marine Geology*. 66. Pp. 277-289.
- Lynch J. *et al.* (1991) "Sediment concentration profiling in HEBBLE using a 1-Mhz acoustic backscatter system," *Marine Geology*. 99. Pp. 361-385.
- Ma Y. *et al.* (1983) "Multifrequency remote acoustic sensing of suspended materials in water," *Journal of the Acoustic Society of America*. Volume 74. Number 2. Pp. 581-585.
- Schat J. (1997) "Multifrequency acoustic measurement of concentration and grain size of suspended sand in water," *Journal of the Acoustic Society of America*. Volume 101. Number 1. Pp. 209-217.
- Sheng J. (1991) "Remote determination of suspended sediment size and concentration by multi-frequency acoustic backscatter," PhD. Dissertation. Memorial University. St. Johns, Newfoundland.
- Thorne P. D. (1993) "Analysis of acoustics measurements of suspended sediment," *Journal of Geophysical Research*. Volume 99. Number C1. Pp. 899-910.
- Thorne P. D. *et al.* (1994) "On the use of acoustics for measuring shallow water suspended sediment processes," *IEEE Journal of Oceanic Engineering*. Volume 19. Number 1. Pp. 48-57.
- Thorne P. D. (2000) "Acoustic measurements of boundary layer flow and sediment flux," *Journal of the Acoustic Society of America*. Volume 108. Number 4. Pp.1568-1581.
- Thosteson E. D. and Hanes D. M. (1988) "A simplified method for determining sediment size and concentration from multiple frequency acoustic backscatter measurements," *Journal of the Acoustic Society of America*. Volume 104. Number 2. Pp. 820-830.
- Yu-shih Liu E. (1965) "Ultrasonic measurement of the size distribution and concentration of natural sediments in suspension," M. S. Thesis. Utah State University. Logan, UT.