UNITED STATES
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A Final Report on

COMPUTED MAGNETO-TELLURIC CURVES FOR HYPOTHETICAL MODELS OF CRUSTAL STRUCTURE*

by

J. I. Pritchard**

DENVER, COLORADO

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ABSTRACT

Several mathematical models were investigated to determine the capabilities of the magneto-telluric method for determining the resistivity structure of the earth's crust. The model parameters were based on the crust model proposed by Keller (1963). The mathematical technique used was developed by Cagniard (1953).

The investigations indicate that a three-layer model approximation of the crust and mantle is the most detailed model warranted in interpreting the information provided by the magneto-telluric method about the lower crust. Only the thickness of the lower crust can be determined, and not the resistivity.

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INTRODUCTION

For the past few years, the U. S. Geological Survey has conducted investigations of the electrical resistivity in the lower part of the earth's crust and upper mantle. Two methods of investigation have been employed: the galvanic-resistivity method and the magneto-telluric method (Keller, 1963).

Briefly, the magneto-telluric method consists of recording, at the earth's surface, natural electric- and magnetic-field variations as a function of time. If the earth was completely uniform in electrical properties, the resistivity could be computed from these factors in the following manner:

$$\rho_a = 0.2\, T \left| \frac{E}{H} \right|^2 \qquad (1)$$

where

T is the period of the electromagnetic wave,
E is the magnitude of the electric field, and
H is the magnitude of the magnetic field.

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For a completely uniform medium, the value of resistivity obtained from equation 1 is the same at all frequencies. For a medium consisting of a number of horizontal layers, each with uniform electrical properties, the apparent resistivity calculated using equation 1 is a function of the period of the electromagnetic wave. The earth resistivity distribution with depth can be evaluated with the magneto-telluric method by plotting the observed values of apparent resistivity on logarithmic coordinates and comparing such plots with theoretical curves.

A series of magneto-telluric curves have been computed for models with assumed resistivity layering to a depth of 50 kilometers. A mathematical technique developed by Cagniard (1953), but using the algorithm developed by Berdichevskiy (1960) for ease of computation on an electronic computer, was used. The computations were performed by the Branch of Computations, U. S. Geological Survey. The applicability of the Cagniard equation, which was developed for plane-polarized waves penetrating a horizontally layered half-space, has been questioned by many investigators, so the following defense of this method is presented:

1. The depth of investigation is limited to 50 kilometers so that earth curvature need not be considered.

2. The depth, resistivity, and wave period parameters involved are such that the model is rigorous for the limitations stated by Price (1962).

3. Lateral changes should not be ignored for investigations to this depth, but computations for a simple horizontally layered model may serve as a first approximation for more complicated resistivity distributions.

The variations in resistivity with depth assumed in these computations are taken mostly from Keller's calculations (1963, see Fig. 1) which are based on laboratory studies of electrical properties of various rock types as a function of both temperature and frequency. The particular data utilized are those reported for measurements at a frequency of one kilocycle per second. A granite-labradorite interface is assumed to exist at 15 kilometers depth. A labradorite-eclogite phase change is assumed to exist at a depth of 35 kilometers. The resistivity below a depth of 50 kilometers is assumed constant. For the computations, the resistivities of the model above the first change in composition were varied to represent a section consisting of sediments, wet and dry granite, and/or sea water.

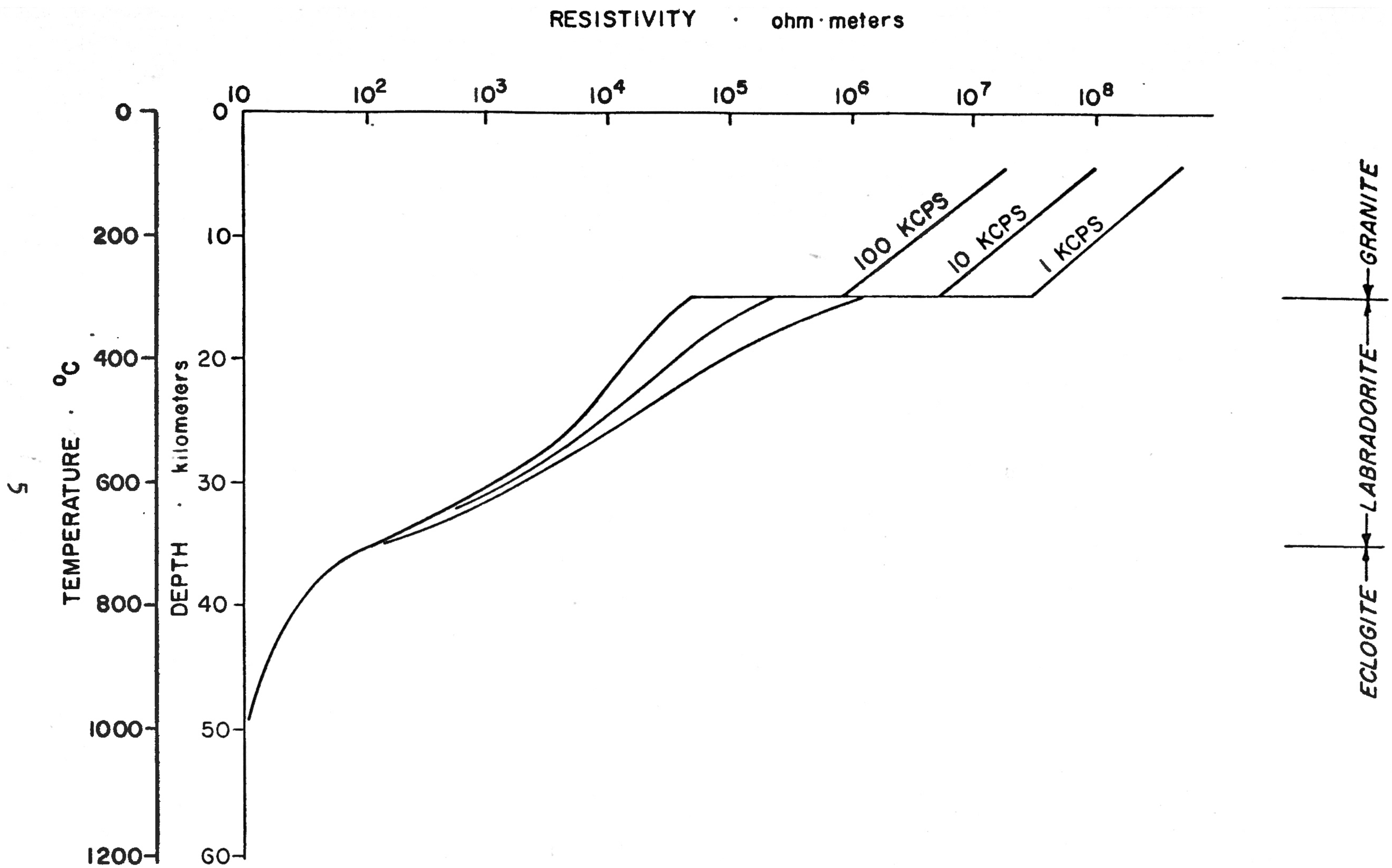

FIGURE 1: Variation of resistivity with depth in an assumed model of the crust. (after Keller, 1963)

The computational technique required that the continuous changes of resistivity be approximated by a series of discrete steps. The number of steps used ranged from three to seventy. The results of the computations are presented as apparent resistivity values plotted as a function of the square root of the wave period, plotted on bio-logarithmic coordinates.

SEDIMENTARY MODELS

Most of the computations were performed for the sedimentary-layering model shown in Fig. 2 which includes a sedimentary layer 5-kilometers thick and a wet granite layer 6-kilometers thick.

Curve 1 (Fig. 3) shows the apparent resistivity which would be measured when the model in Fig. 2 is approximated by 25 or more steps, each step being two kilometers or less in thickness. It may be assumed that curve 1 (Fig. 3) very closely approximates the values obtained with a computational technique that would handle rigorously the exponential changes in resistivity of the model in Fig. 2. Computations for a larger number of thinner layers did not change the results appreciably.

Several three-layer approximations to the model shown in Fig. 2 were used as a basis for computation to determine the errors involved in using a three-layer approximation. In these computations, the first layer was assumed to extend to a depth of 5 kilometers and the third layer was assumed to start at a depth of 50 kilometers. The resistivity of these two layers was assumed to be 10 ohm-meters. The resistivity of the second layer was varied from 10^3 to 10^8 ohm-meters. None of the curves computed for these three-layered models varied enough from curve 1 (Fig. 3) to be discernible even in matching high-quality field data. The same conclusion was reached for all approximations involving more than three layers.

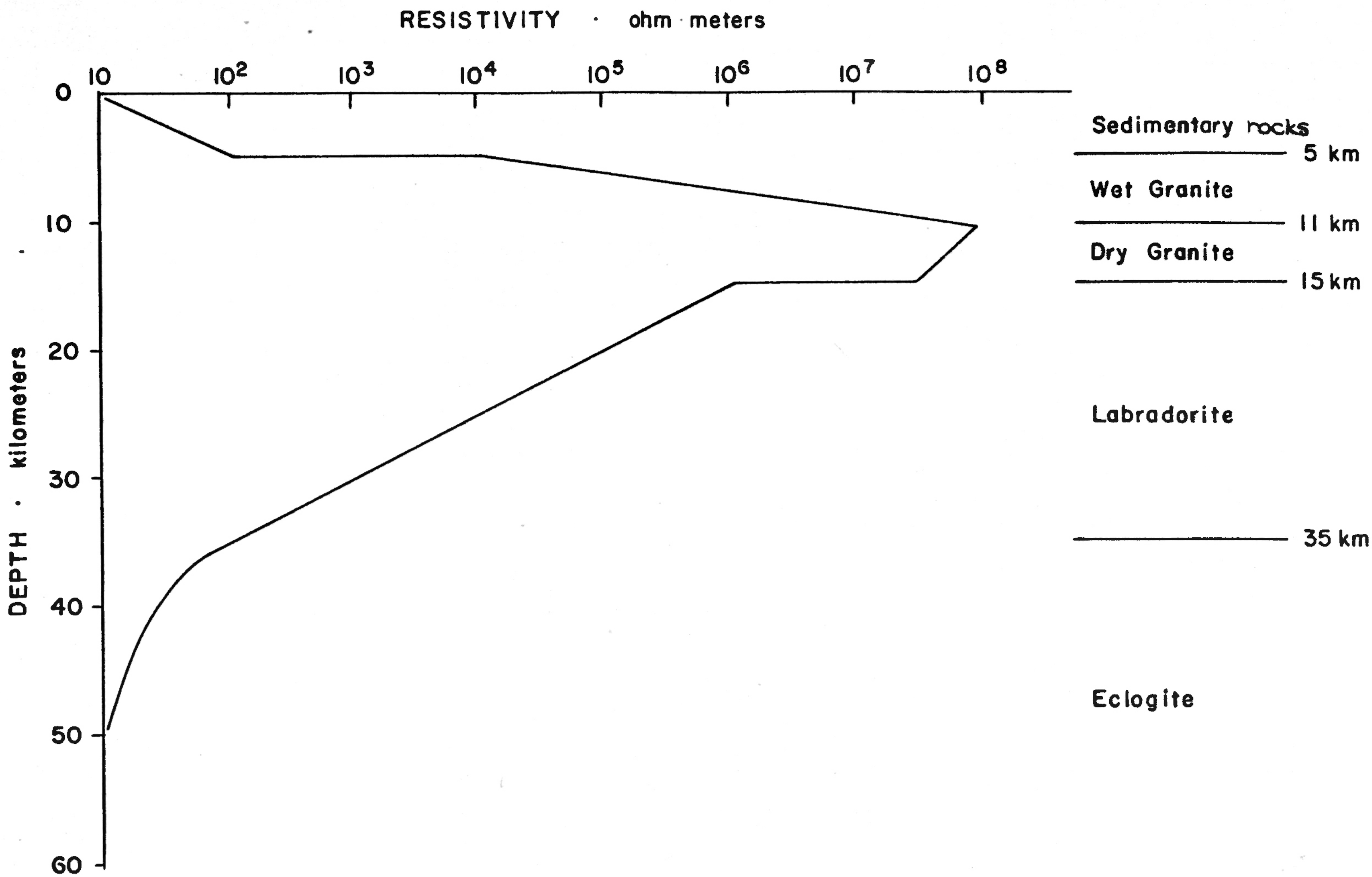

FIGURE 2: Variation of resistivity with depth in the sedimentary-rock model.

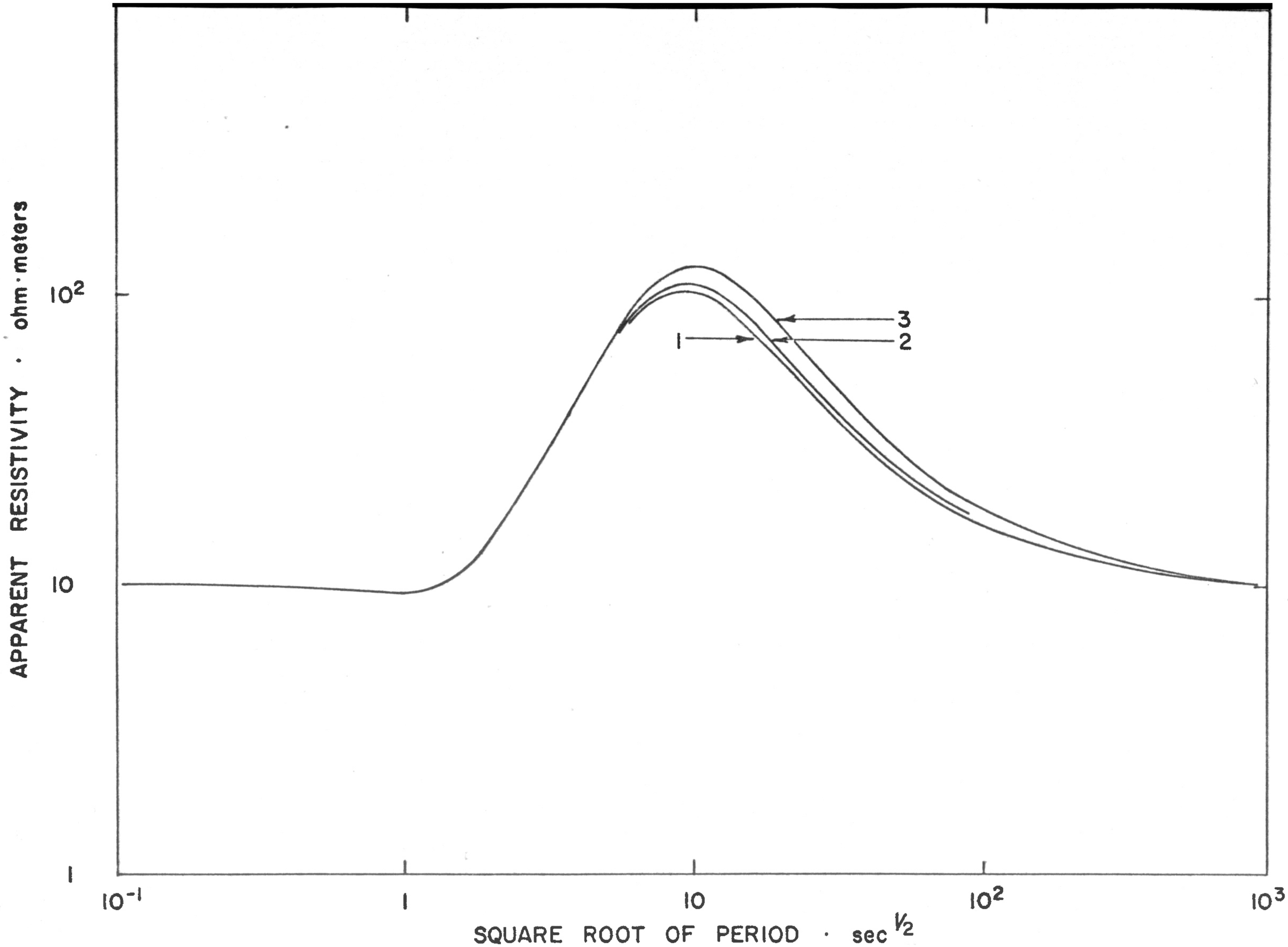

FIGURE 3: Magneto-telluric resistivity curves – models 1,2, & 3.

This same lack of resolution is clearly shown in two-layer curves computed by Cagniard (1953); magneto-telluric measurements do not accurately determine the resistivity of a layer having a resistivity of more than 100 times greater than those of adjoining layers. Apparently, the only information about the lower crust obtainable by use of the magneto-telluric method will be the thickness of the lower crust.

Curves 2 and 3 (Fig. 3) show computed resistivity values for cases in which the layers below the sedimentary-basement layer boundary are thicker than shown in Fig. 2. The thickness of the sedimentary layer was held constant at 5 kilometers, while the thicknesses of the lower layers were increased by proportionate amounts to increase the effective depth of the crust-mantle boundary. The models were approximated by 50 discrete layers and all the layer resistivities were the same as used in the model for curve 1, Fig. 3. Curve 2 was computed for a model with the crust-mantle boundary at 52 kilometers and curve 3, for a model with the boundary at 60 kilometers.

The curves in Fig. 3 illustrate that the translation of maximum point and the descending portion of the curves is small even for 10-kilometer changes in depth of the crust-mantle boundary. This translation is difficult to recognize in field data.

Curves showing the phase relation between the electric- and magnetic-field components for these same three models are presented in Fig. 4. The phase relation curves serve to show the range in frequencies (or wave periods) for which the boundaries have an appreciable effect on the magneto-telluric field. The phase difference between the two field components is 45° for frequencies where the boundaries have no effect on the fields. As is shown in Fig. 4, the phases for the frequencies that penetrate into the lower

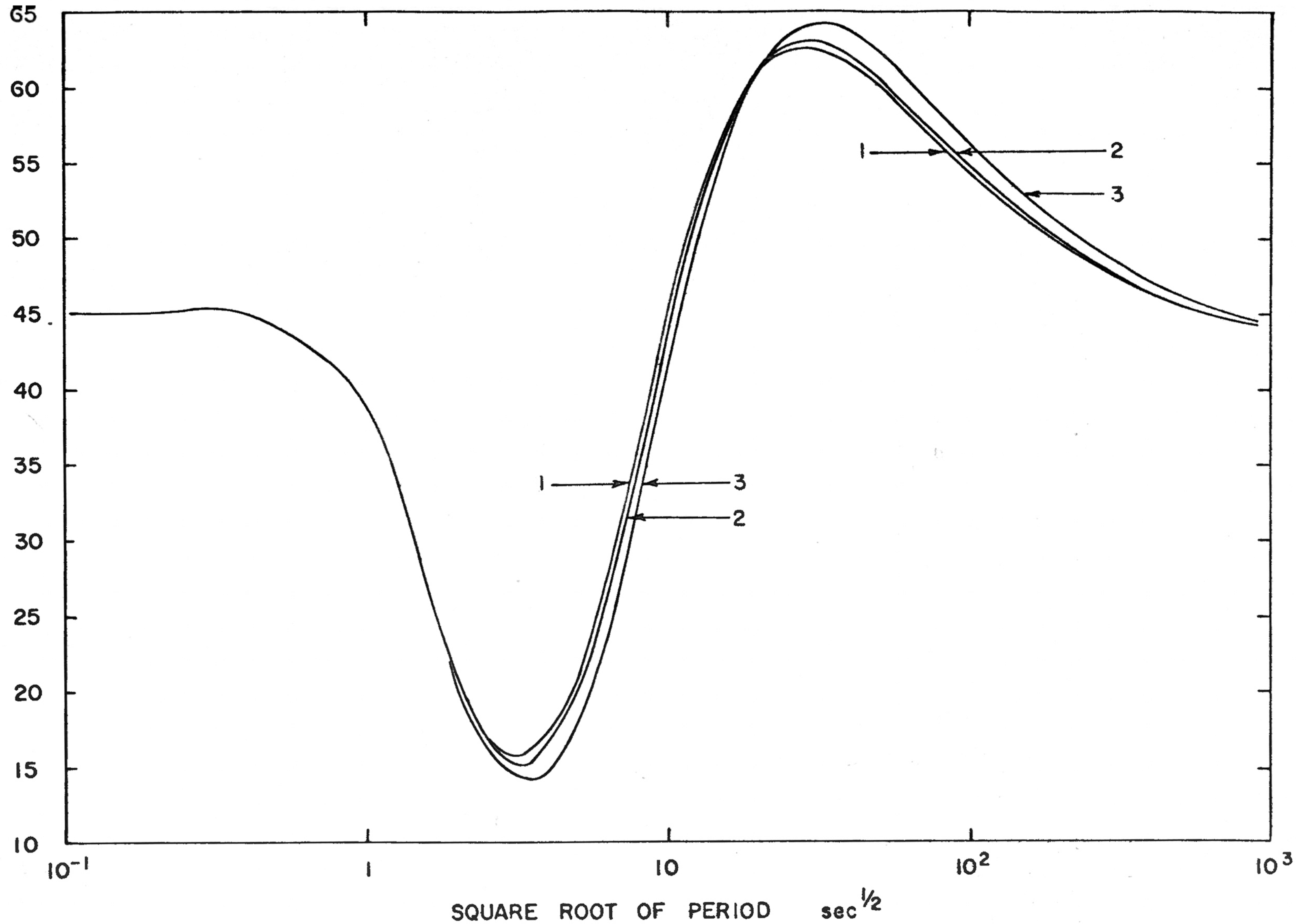

FIGURE 4: Magneto-telluric phase curves - models 1,2,& 3

crust only differ from 45° over a narrow range of periods indicating that the effect of the boundary between the sedimentary layer and the lower crust overlaps the effect of the boundary between the lower crust and mantle. This also indicates the difficulty met in determining the resistivity of the lower crust with the magneto-telluric method.

Computations were also made for a variety of models having different resistivity and thickness of the sedimentary layer. Progressive thinning of the sedimentary layer resulted in a translation of the resistivity maximum point to shorter periods, but caused little change in the computed amplitude of the resistivity maximum. Increasing the resistivity of either or both the sedimentary layer and the mantle layer to 100 ohm-meters (from 10 ohm-meters) changed the amplitude and position of the maximum computed resistivity only slightly, though the slopes of the curves were decreased. Decreasing the resistivity of these layers gave the opposite effect.

MODELS OF SPECIAL RESISTIVITY LAYERING SECTIONS

Models for resistivity sections other than the normal sedimentary sections previously described were also investigated. The parameters for these models are listed in table 1 and the computed resistivity curves are shown in Fig. 5. These models are variations of the one shown in Fig. 1.

In model 4, the temperature gradient below sea water was assumed to be 70°C per kilometer, with corresponding rapid decrease in resistivity with depth. The curve for model 4 looks much like a two-layer curve except for its steep slope. To establish this curve the periods of the magneto-telluric field must be observed for longer than 24 hours.

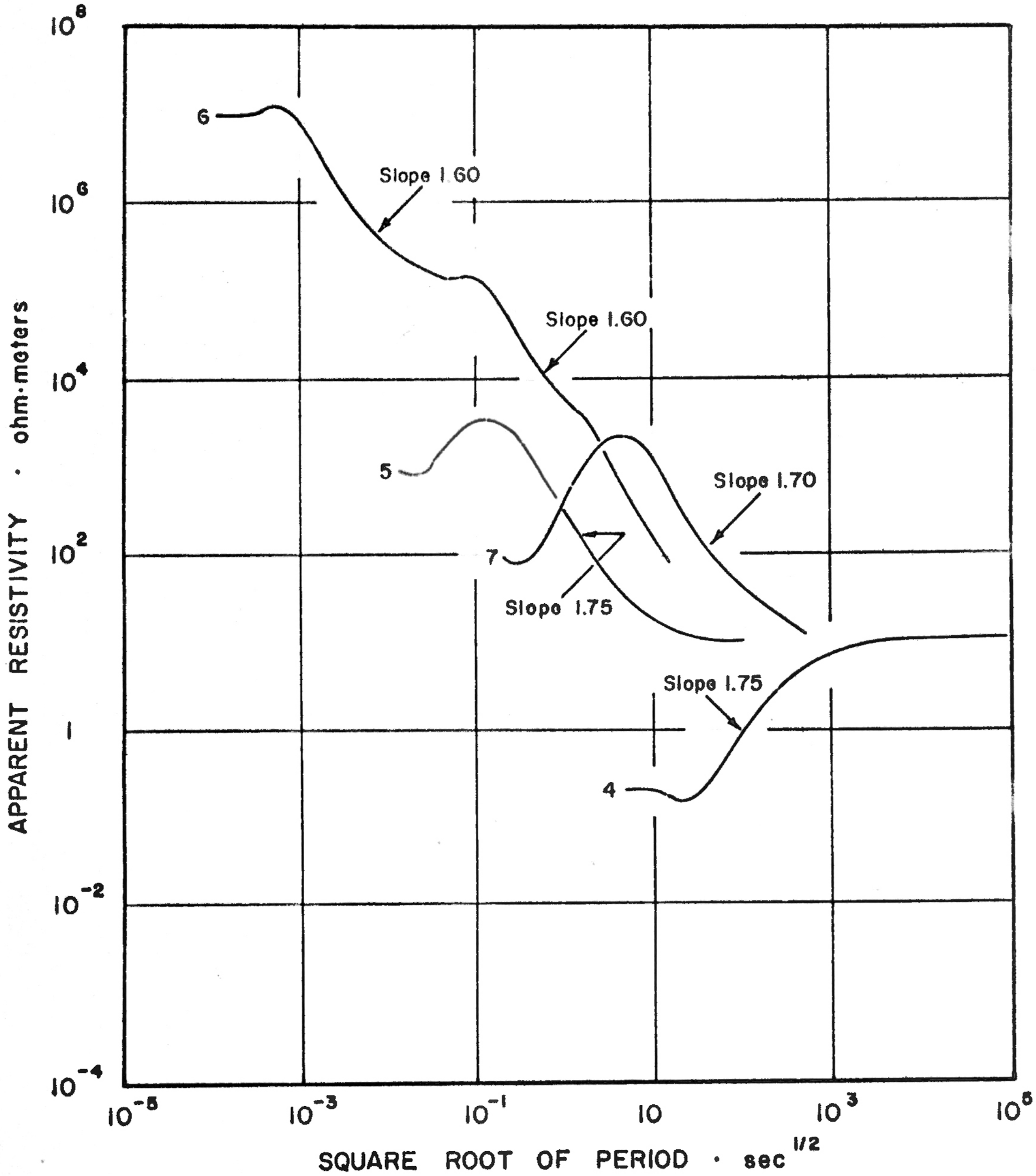

FIGURE 5: Magneto-telluric resistivity curves – models 4,5,6,& 7

The mountain model (model 5) curve does not differ greatly in shape from the curves in Fig. 3. The maximum is greater in amplitude and the frequencies required to define the entire curve range up into audio frequencies.

Models 6 and 7 show the effect of a thin low-resistivity cover over a high-resistivity section such as limestone. The detail, apparent on curve 6, is lost in curve 7, showing the attenuation of electromagnetic waves in highly-conductive strata. The frequencies required for the investigation of model 6 are far higher than those ordinarily used in magneto-telluric surveys.

CONCLUSIONS

For the earth models investigated, layers within the crust had assumed resistivities at least 100-times larger than the resistivities of the upper crustal layers and the mantle had an assumed constant resistivity. Computations of the apparent resistivity which would be observed with the magneto-telluric method for these models indicate that a three-layer model approximation would provide all the information that is available. Furthermore, the obtainable information from the magneto-telluric model would be limited to a determination of the overall thickness of the resistant portion of the crust. The magneto-telluric method seems to be useful only if supplemented with electrical techniques of investigation such as the galvanic-resistivity method and/or the measurement of mutual coupling between induction loops.

Table 1.

Resistivity sections for special models

Model	Type	Layer	Resistivity, ohm-meters	Layer thickness, kilometers	Layer type
4	Sea	1	0.2	5	Sea water
		2	$2x10^3$	10	Labradorite
		3	10^2	3	Eclogite
		4	10	Infinite	Eclogite
5	Mountain	1	10^3	4	Wet granite
		2	10^6	16	Dry granite
		3	10^3	30	Labradorite
		4	10	Infinite	Eclogite
6	Limestone	1	10^7	1.0	Limestone
		2	10^5	15	Granite
		3	10^3	20	Labradorite
		4	10	Infinite	Eclogite
7	Sedimentary rocks with limestone	1	10^2	0.5	Sedimentary
		2	10^7	0.5	Limestone
		3	10^5	15	Granite
		4	10^3	20	Labradorite
		5	10	Infinite	Eclogite

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