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DESCRIPTION OF THE WEIGHTED REGRESSION AND QUALITY ESTIMATION USED IN THE EARTHQUAKE LOCATION PROGRAM HYPOELLIPSE

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CONTENTS

INTRODUCTION

The earthquake location program HYPOELLIPSE (Lahr, 1980) evolved out of the program HYPO71 (Lee and Lahr, 1972). One of the principal changes involved modification of the regression technique and calculation of the spatial error ellipsoid. The HYPOELLIPSE program description (Lahr, 1980) set out in Chapter III the development of the linearized least-square equations that must be solved. This report gives additional details of the development and solution of the equations, including sections on damping and error estimation. The final chapter describes the various forms of weighting available in HYPOELLIPSE.

A. DETERMINATION OF HYPOCENTER AND ORIGIN TIME

A trial-hypocenter is chosen in the vicinity of the expected final hypocenter. Corrections in latitude, longitude, depth, and origin time are then calculated in such a way that the sum of the squares of the travel-time residuals (observed travel-time minus the calculated travel-time at each station) is decreased. This iterative procedure is continued until the corrections become smaller than the value preset by the user. Termination also occurs if convergence is not reached after a predetermined number of iterations.

1.) Development of the Residual Equation for P-Wave Arrivals

Definitions:

		Program Notation
X_o	= trial longitude.	LONEP
Y_o	= trial latitude.	LATEP
Z_o	= trial depth.	Z
T_o	= trial origin time.	ORG
TP_i	= observed first P-wave arrival time at station i	TP(I)
T_i	= computed first P-wave travel time at station i	T(I)
	T_i is a function of (X_o, Y_o, Z_o).	
D_i	= sum of station P-wave time delays at station i due to:	
	a) Elevation.	ELVDLY(I)
	b) Other sources.	DLY(KNO,I)
	(KNO may be from 1 to 5 to allow up to five different sets of corrections to be used.)	
$SDLY_i$	= S-wave delay	SDLY(KNO,I)
$X_{4,i}$	= travel time residual (observed arrival time minus computed P-wave arrival time) at station i.	X(4,I)

$$X_{4,i} = TP_i - (T_o + T_i + D_i).$$

We wish to change the trial hypocenter and origin time by (dX_o, dY_o, dZ_o, dT_o) and in so doing reduce the residual $X_{4,i}$ at each station as much as possible.

Let R_i be the i^{th} station residual after this change has been made.

$$R_i = X_{4,i} + \frac{\partial X_{4,i}}{\partial X_o} dX_o + \frac{\partial X_{4,i}}{\partial Y_o} dY_o + \frac{\partial X_{4,i}}{\partial Z_o} dZ_o + \frac{\partial X_{4,i}}{\partial T_o} dT_o + \qquad \text{(A-1)}$$

higher order terms.

Assuming dX_o, dY_o, dZ_o and dT_o are small, the higher order terms can be neglected. Since TP_i, T_o and D_i are not functions of X_o, Y_o, or Z_o, this equation may be rewritten

$$R_i \cong X_{4,i} - \frac{\partial T_i}{\partial X_o} dX_o - \frac{\partial T_i}{\partial Y_o} dY_o - \frac{\partial T_i}{\partial Z_o} dZ_o - \frac{\partial X_{4,i}}{\partial T_o} dT_o \qquad \text{(A-2)}$$

For convenience in writing, the following notation is used:

	Program Notation
$\frac{\partial T_i}{\partial X_o} = X_{1,i}$	X(1,I)
$\frac{\partial T_i}{\partial Y_o} = X_{2,i}$	X(2,I)
$\frac{\partial T_i}{\partial Z_o} = X_{3,i}$	X(3,I)
$\frac{\partial X_{4,i}}{\partial T_o} = K_i$. For P arrivals $K_i = \frac{\partial T_o}{\partial T_o} = 1$.	KSMP(I)

$dX_o = Y_1$. DY1 in minutes, or Y(1) in kilometers

$dY_o = Y_2$. DY2 in minutes, or Y(2) in kilometers

$dZ_o = Y_3$. Y(3) in kilometers

$dT_o = Y_4$. Y(4) in seconds

Then the i^{th} equation becomes

$$R_i \tilde{=} X_{4,i} - X_{1,i}Y_1 - X_{2,i}Y_2 - X_{3,i}Y_3 - K_iY_4. \qquad \text{(A-3)}$$

2.) S-Wave Arrival Equation

For each S-wave arrival a similar equation may be written. If a station has P-wave and S-wave data it is assigned two values of i. The notation definitions for P given above are modified as follows:

Definitions

		Program Notation
TS_i	= Observed S-wave arrival time.	TP(I)
$X_{4,i}$	= Observed minus computed S-wave arrival time at station i.	X(4,I)

S-wave arrival times may be computed in two ways:

1) With constant P-wave velocity to S-wave velocity ratio (VPVS) throughout the structure: S-wave arrival time = T_o + VPVS *(T_i + D_i) + $SDLY_i$. Partial derivatives ($X_{j,i}$) are set equal to the values used for P arrivals multiplied by VPVS.

2) Variable VPVS within the structure: the S-wave travel time, S_i, is computed for the S velocity structure. S-wave arrival time = $T_o + S_i + SDLY_i + ELVDLY_i$. Partial derivatives ($X_{j,i}$) are computed from

the S velocity structure.

$\frac{\partial X_{4,i}}{\partial T_o} = K_i$. For S arrivals $K_i = \frac{\partial T_o}{\partial T_o} = 1.$ KSMP(I)

The residual equation is

$$R_i \cong X_{4,i} - X_{1,i}Y_1 - X_{2,i}Y_2 - X_{3,i}Y_3 - K_iY_4.$$

3.) S minus P Interval Equation

S minus P Times

If the absolute time is not available at a station where S-wave and P-arrivals can be read, the basic residual equation can be modified to use these data. Currently this option has been implemented in HYPOELLIPSE only for the case of constant VP/VS ratio throughout the structure. The above definitions are modified as follows:

Definitions: Program Notation

$X_{4,i}$ = Observed S-P interval, $TP_i - TS_i$, minus computed S-P interval. The computed S-P interval, T_{S-P}, is X(4,I)

$$T_{S-P} = VPVS*(T_i + D_i) + SDLY_i - (T_i + D_i)$$

$$= (VPVS - 1.0)*(T_i + D_i) + SDLY_i$$

Then $X_{4,i} = (TP_i - TS_i) - (VPVS - 1)*(T_i + D_i) - SDLY_i$.

The partial derivatives $(X_{j,i})$ are set equal to the values used for P arrivals multiplied by (VPVS-1.0).

$\frac{\partial X_{4,i}}{\partial T_o} = K_i = 0$ for S-P data since the S-P residual is not a function of origin time. KSMP(I)

Again the residual equation is:

$$R_i \tilde{=} X_{4,i} - X_{1,i}Y_1 - X_{2,i}Y_2 - X_{3,i}Y_3 - K_iY_4. \qquad (A-4)$$

4. Normal Equations

To summarize, we have formed for each observed P-wave arrival, S-wave arrival or S-P interval observed, one equation for the predicted residual (R_i) in terms of the four unknowns (Y_1, Y_2, Y_3, Y_4).

$$R_i \tilde{=} X_{4,i} - X_{1,i}Y_1 - X_{2,i}Y_2 - X_{3,i}Y_3 - K_iY_4.$$

With four such equations a solution with $R_i = 0$, $i = 1,4$ can always be found and will be unique as long as the determinant of the coefficients is not zero. Normally there will be data to form more than four equations yielding an overdetermined system of equations. We wish to find the changes in hypocenter and origin time (Y_1, Y_2, Y_3, Y_4) that will result in the smallest values of R_i possible. This program seeks to minimize the sum of squares Q, where

$$Q = \sum_{i=1}^{n} R_i^2.$$

Since the data will vary in quality and we will wish to weight the residuals as a function of distance, azimuth, deviation from the mean, etc., we will define a weight W_i (WT(I) in the program) and multiply each R_i^2 term by this weight. The weight should be inversely proportional to the square of the

uncertainty in the corresponding residual $X_{4,i}$.

$$Q = \sum_{i=1}^{n} W_i R_i^2. \qquad \text{(A-5)}$$

In effect, each equation of the form (A-4) is multiplied through by the square root of its weight.

The weighted equations A-4 may be written in matrix form

$$\bar{R} = \bar{r} - \bar{\bar{A}} \cdot \bar{Y} \qquad \text{(A-6)}$$

where $\bar{R}$ is the vector $\sqrt{W_i}\, R_i$ $(i = 1, n)$,

$\bar{r}$ is the vector $\sqrt{W_i}\, X_{4,i}$ $(i = 1, n)$,

$\bar{\bar{A}}$ is the matrix

$$\begin{matrix} \sqrt{W_1}X_{1,1} & \sqrt{W_1}X_{2,1} & \sqrt{W_1}X_{3,1} & \sqrt{W_1}K_1 \\ \sqrt{W_2}X_{1,2} & \sqrt{W_2}X_{2,2} & \sqrt{W_2}X_{3,2} & \sqrt{W_2}K_2 \\ \vdots & \vdots & \vdots & \vdots \\ \sqrt{W_n}X_{1,n} & \sqrt{W_n}X_{2,n} & \sqrt{W_n}X_{3,n} & \sqrt{W_n}K_n \end{matrix}$$

and $\bar{Y}$ is the vector $Y_i (i=1,4)$.

The normal equations are

$\bar{\bar{A}}'\, \bar{r} = \bar{\bar{A}}'\bar{\bar{A}}\, \bar{Y}$,

and are solved for the vector $\bar{Y}$ which will minimize Q. (Draper and Smith, 1966, pg 47).

The equation $\bar{\bar{A}}'\, \bar{\bar{A}}\, \bar{Y} = \bar{\bar{A}}'\bar{r}$ is expanded in matrix form as

$$\begin{bmatrix} \Sigma W_i X_{1,i}^2 & \Sigma W_i X_{1,i} X_{2,i} & \Sigma W_i X_{1,i} X_{3,i} & \Sigma W_i X_{1,i} K_i \\ \Sigma W_i X_{2,i} X_{1,i} & \Sigma W_i X_{2,i}^2 & \Sigma W_i X_{2,i} X_{3,i} & \Sigma W_i X_{2,i} K_i \\ \Sigma W_i X_{3,i} X_{1,i} & \Sigma W_i X_{3,i} X_{2,i} & \Sigma W_i X_{3,i}^2 & \Sigma W_i X_{3,i} K_i \\ \Sigma W_i X_{1,i} K_i & \Sigma W_i X_{2,i} K_i & \Sigma W_i X_{3,i} K_i & \Sigma W_i K_i^2 \end{bmatrix} \cdot$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ Y_4 \end{bmatrix} = \begin{bmatrix} \Sigma W_i X_{1,i} X_{4,i} \\ \Sigma W_i X_{2,i} X_{4,i} \\ \Sigma W_i X_{3,i} X_{4,i} \\ \Sigma W_i K_i X_{4,i} \end{bmatrix} \qquad \text{(A-7)}$$

This set of equations is simplified by multiplying the fourth equation by $(\Sigma W_i X_{n,i} K_i)/(\Sigma W_i K_i)$ and subtracting it from the nth equation for n = 1, 2 and 3. (Note that since $K_i = 0$ or 1, $K_i^2 = K_i$)

One can show that

$$\Sigma W_i X_{j,i} X_{k,i} - \frac{\Sigma W_i X_{j,i} K_i \Sigma W_i X_{k,i} K_i}{\Sigma W_i K_i} =$$

$$\Sigma W_i \left(X_{j,i} - K_i \frac{\Sigma W_i X_{j,i} K_i}{\Sigma W_i K_i}\right) \cdot \left(X_{k,i} - K_i \frac{\Sigma W_i X_{k,i} K_i}{\Sigma W_i K_i}\right). \qquad \text{(A-8)}$$

Equations (A-7) can be rewritten.

$$\begin{bmatrix} S_{1,1} & S_{1,2} & S_{1,3} \\ S_{2,1} & S_{2,2} & S_{2,3} \\ S_{3,1} & S_{3,2} & S_{3,3} \end{bmatrix} \quad \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} S_{1,4} \\ S_{2,4} \\ S_{3,4} \end{bmatrix} \tag{A-9}$$

$$Y_4 = \tilde{X}_4 - \tilde{X}_1 Y_1 - \tilde{X}_2 Y_2 - \tilde{X}_3 Y_3 \tag{A-10}$$

where $\tilde{X}_j = \dfrac{\sum_{i=1}^{NR} W_i X_{j,i} K_i}{\sum_{i=1}^{NR} W_i K_i}$, the weighted means of the P-phase and S-phase data, and $S_{j,k} = \sum_{j=1}^{NR} W_i (X_{j,i} - K_i \tilde{X}_j)(X_{k,i} - K_i \tilde{X}_k)$, the sum of the squares matrix.

In standard methods of solution of the Equations (A-9) problems arise if the determinant of the coefficients is near zero. This occurs when in one direction, or on one plane, or perhaps in every direction, the rate of change of RMS with position is very small or zero. In step-wise multiple regression the choice of the most strongly controlled directions is limited to latitude, longitude, and depth.

In this program the IBM Scientific Subroutine EIGEN* is used to calculate the eigen values and eigen vectors of the $S_{j,k}$ (j=1,3, K=1,3) matrix. The eigen vectors form the transformation matrix which is then used to transform the column matrix $(S_{1,4}, S_{2,4}, S_{3,4})$ into the new coordinate system.

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*The subroutine EIGEN requires that the symmetric matrix S_{ij} be transformed to a vector $(ASM1_k)$ in storage mode 1. Thus a ASM1 = $(S_{1,1}, S_{1,2}, S_{1,3}, S_{2,2}, S_{2,3}, S_{3,3})$.

In the new coordinate system the Equations (A-1) become:

$$\begin{bmatrix} A_{1,1} & 0 & 0 \\ 0 & A_{2,2} & 0 \\ 0 & 0 & A_{3,3} \end{bmatrix} \begin{bmatrix} BT_1 \\ BT_2 \\ BT_3 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} \qquad \text{(A-11)}$$

Those principal directions in space with very little control will have correspondingly small eigen values. Each eigen value is the weighted sum of squares of partial derivatives of traveltime with respect to changes in the corresponding principal direction.

The BT corrections are transformed into changes in latitude, longitude, and depth, (Y_i, i=1,3). Then the origin time correction Y_4 is calculated from Equation (A-2).

After (Y_1, Y_2, Y_3, Y_4) are found it is possible to predict the weighted sum of squares of the residuals after these changes are made, assuming the higher order terms of Equation (A-1) can be neglected. This will be most nearly true for small changes in locations such as the final steps of the iteration.

Equation (A-4) in its weighted form gives the i^{th} residual after the step is made as a function of the step, the i^{th} derivatives, and the observed i^{th} residual.

$$\sqrt{W_i}R_i = \sqrt{W_i}X_{4,i} - \sqrt{W_i}X_{1,i}Y_1 - \sqrt{W_i}X_{2,i}Y_2 - \sqrt{W_i}X_{3,i}Y_3 - \sqrt{W_i}K_iY_4 \qquad \text{(A-12)}$$

The vector $\sqrt{W_i}R_i$ is orthogonal to the vectors $\sqrt{W_i}X_{j,i}$, j=1,2 or 3 and $\sqrt{W_i}K_i$. Thus an infinitesimal shift of Y_1, Y_2, Y_3, or Y_4 results in no change in magnitude of the $\sqrt{W_i}R_i$ vector which is the condition imposed by the least square Equations (A-6).

Let $\hat{X}_{4,i}$ be the predicted change in the i^{th} residual.

$$\hat{X}_{4,i} = -X_{1,i}Y_1 - X_{2,i}Y_2 - X_{3,i}Y_3 - K_iY_4$$

Using Equation (A-10) for Y_4, $\hat{X}_{4,i}$ becomes:

$$\hat{X}_{4,i} = - X_{1,i}Y_1 - X_{2,i}Y_2 - X_{3,i}Y_3 - K_i (\tilde{X}_4 - \tilde{X}_1Y_1 - \tilde{X}_2Y_2 - \tilde{X}_3Y_3).$$

Combining terms, this becomes:

$$\hat{X}_{4,i} = - Y_1(X_{1,i} - K_i\tilde{X}_1) - Y_2(X_{2,i} - K_i\tilde{X}_2) - Y_3(X_{3,i} - K_i\tilde{X}_3) - K_i\tilde{X}_4 .$$

Using this term to simplify Equation (A-12), $\sqrt{W_i}R_i = \sqrt{W_i}(X_{4,i} + \hat{X}_{4,i})$.

The predicted weighted sum of squares of residuals after the move becomes

$$PSS = \sum_{i-1}^{n} W_iR_i^2 = \Sigma (\sqrt{W_i}X_{4,i} + \sqrt{W_i}\hat{X}_{4,i})^2 = \Sigma W_i[X_{4,i}^2 + 2X_{4,i}\hat{X}_{4,i} + \hat{X}_{4,i}^2].$$

$$PSS = \Sigma W_i[X_{4,i}^2 + X_{4,i}\hat{X}_{4,i} + \hat{X}_{4,i}(X_{4,i} + \hat{X}_{4,i})].$$

$$PSS = \Sigma W_i[X_{4,i}^2 + X_{4,i}\hat{X}_{4,i} - (X_{1,i}Y_1 + X_{2,i}Y_2 + X_{3,i}Y_3 + K_iY_4) R_i].$$

Due to the orthogonality mentioned above, the sums of products involving R_i are zero and the resulting equation is

$$PSS = \Sigma W_iR_i^2 = \Sigma W_iX_{4,i}^2 + \Sigma W_i\hat{X}_{4,i}X_{4,i}.$$

Substituting for $\hat{X}_{4,i}$:

$$PSS = \Sigma W_iX_{4,i}^2 - Y_1\Sigma W_iX_{4,i}(X_{1,i} - K_i\tilde{X}_1) - Y_2\Sigma W_iX_{4,i}(X_{2,i} - K_i\tilde{X}_2)$$

$$- Y_3\Sigma W_iX_{4,i}(X_{3,i} - K_i\tilde{X}_3) - \tilde{X}_4\Sigma W_iK_iX_{4,i}.$$

Using the identity of Equation (A-8) and the definition of $\tilde{X}_4$

$$PSS = \Sigma W_i(X_{4,i} - K_i\tilde{X}_4)^2 - Y_1\Sigma W_i(X_{4,i} - K_i\tilde{X}_4)(X_{1,i} - K_i\tilde{X}_1)$$

$$- Y_2\Sigma W_i(X_{4,i} - K_i\tilde{X}_4)(X_{2,i} - K_i\tilde{X}_2) - Y_3\Sigma W_i(X_{4,i} - K_i\tilde{X}_4)(X_{3,i} - K_i\tilde{X}_3).$$

By the definition of $S_{i,j}$

$$PSS = S_{4,4} - Y_1S_{1,4} - Y_2S_{2,4} - Y_3S_{3,4}.$$

YSE, defined by

$$YSE = \sqrt{PSS/PHI} \qquad (A\text{-}13)$$

where PHI is the number of degrees of freedom, is an estimate of the standard error of the readings. PHI is equal to the number of equations, NR, minus the number of parameters estimated. For example, if 8 equations were used to calculate Δ lat, Δ lon, Δ Z, and Δ origin time, then PHI would equal 8-4 = 4.

5.) Damped Solution

In order to assure convergence in cases of poor hypocentral control the inversion is "damped". To do this three additional equations are added to those previously described [see Equations (A-4)]. The additional equations are:

$$DY_1 = 0$$
$$DY_2 = 0$$
$$DY_3 = 0$$

where D is a constant and the hypocentral step is Y_1, Y_2, Y_3. The initial value of D is 0.001. If after a calculated step the RMS value increases, the step is recalculated using increased damping (D = D*10.). This is repeated until the RMS does decrease after the step. The damping remains at the

elevated level for two more iterations and then drops to 0.001 again.

B. ERROR ESTIMATES

1.) Joint Spatial Confidence Ellipsoid

There are many factors which may prevent the determination of true hypocentral locations. Among these are an incorrect earth velocity model, systematic reading errors, timing of the wrong phase, and random reading errors. Under the assumption that the model is correct, one may use statistical methods to estimate the effects of random reading errors which are assumed normally distributed with zero mean and σ standard error. Care must be exercised, however, not to overlook the tentative nature of these estimates, in that the effects not included may have a great effect upon the locations. The error limits are also only approximate because they are based upon the linearization of a nonlinear system.

Proceeding under these assumptions, if we take the sum of squares matrix from regression to be S as in Equation (A-1) then the distribution of errors in the calculated step $\bar{b}$, as given by Draper and Smith (1966), is

$$\bar{b} \sim N\ (\bar{\beta},\ [S]^{-1}\sigma^2)\ .$$

The estimated step is a normally distributed random variable with mean $\bar{\beta}$, the true step, and variance-covariance matrix $\sigma^2[S]^{-1}$ where σ is the standard error of the readings. The standard error in the direction given by the unit vector $\hat{p}$ is $SE = \sigma\sqrt{\hat{p}'[S]^{-1}\hat{p}}$.

The joint confidence region equation is computed because it gives further insight into the probable boundaries of the correct solution (Draper and Smith, 1966). If we take $\bar{\beta}$ to be the true step, then the sum of squares of reading deviations which would yield the step $\bar{b}$ is given by $(\bar{\beta}-\bar{b})'\ [S]\ (\bar{\beta}-\bar{b})$ and has three degrees of freedom.

If the reading standard deviation is σ then the ratio $\frac{(\bar{\beta}-\bar{b})' [S] (\bar{\beta}-\bar{b})}{\sigma^2}$

is a chi-square distribution with 3 degrees of freedom.

$\frac{(\bar{\beta}-\bar{b})' [S] (\bar{\beta}-\bar{b})}{\sigma^2} = X^2 (3, 1-\alpha)$ is the ellipsoid containing the $100(1 - \alpha)$ percent confidence region. The 68 percent value of X^2 for 3 degrees of freedom is 3.5. Therefore the 68 percent joint confidence error ellipsoid is given by $(\bar{\beta}-\bar{b})' [S] (\bar{\beta}-\bar{b}) \leq 3.5\ \sigma^2$

In the principal axis coordinate system of Equation (A-11) the error ellipsoid is

$$(\bar{\beta}-\bar{b})' [A] (\bar{\beta}-\bar{b}) = 3.5\ \sigma^2 \qquad \text{(B-1)}$$

Expanding,

$$A_{11}V_1^2 + A_{22}V_2^2 + A_{33}V_3^2 = 3.5\ \sigma^2$$

where

$$\bar{V} = (\bar{\beta}-\bar{b})$$

Then

$$V_i = \sqrt{3.5}\,\sigma/\sqrt{A_{ii}} \qquad \text{(B-2)}$$

are the principal semiaxes of the error ellipsoid.

From Equation (B-2) it is seen that the ellipsoid is only a function of the station geometry, the crustal model, and the reading standard deviation σ. If TEST(29) $\geq$ 0, then YSE of Equation (A-13) is used as an estimate of σ in Equation (B-1) as long as PHI > 0. If PHI = 0, then σ is taken to be TEST(29). When YSE is used as an estimate of σ, the F distribution should be used rather than the X^2 distribution, which yields a larger error ellipsoid. However this is not done in HYPOELLIPSE.

Using YSE for σ has a number of drawbacks. Inevitably there will be

differences between the structure of the real earth and the flat layer velocity model used to represent it. These differences will generally bias the location and may be such that the data are not compatible with any location within the model, resulting in high residuals and a high estimate of σ. Thus in many cases the residuals contain information about the incompatability of the model with the data as well as the reading errors. In order to evaluate these two effects separately, the confidence ellipsoid may be based upon an estimate of the reading standard deviation which is held fixed for all the events. If TEST(29) < 0 then ABS[TEST(29)] is always used for σ in Equation (A-6). The confidence ellipsoid is then a measure of quality of the station distribution. Evernden (1969) strongly recommends this procedure for estimating confidence bounds. In estimating solution quality with the latter option, one must consider both the confidence ellipsoid, to determine the potential accuracy of the hypocenter, and the root mean square residual (RMS), which reflects both reading errors and model incompatability. Evaluation of the RMS residual may depend upon the earthquake location. In areas where the model is known to be incompatible, large RMS values will be normal whereas the same values in an area of known compatability would indicate probable errors in the arrival time data.

2.) Derivation of confidence limits from the standard error ellipsoid

The relationship of the error ellipsoid to one and two dimensional error estimates is shown in Figure B-4. Note that if the shadow of the error ellipsoid is cast onto a plainer surface, the resulting elliptical region is larger than the 68% two parameter joint confidence region. Likewise, if the shadow of the elliptical region is cast onto a single axis, the resulting single axis limits are larger than the single variable standard deviation. The relationship between a joint two dimensional probability distribution

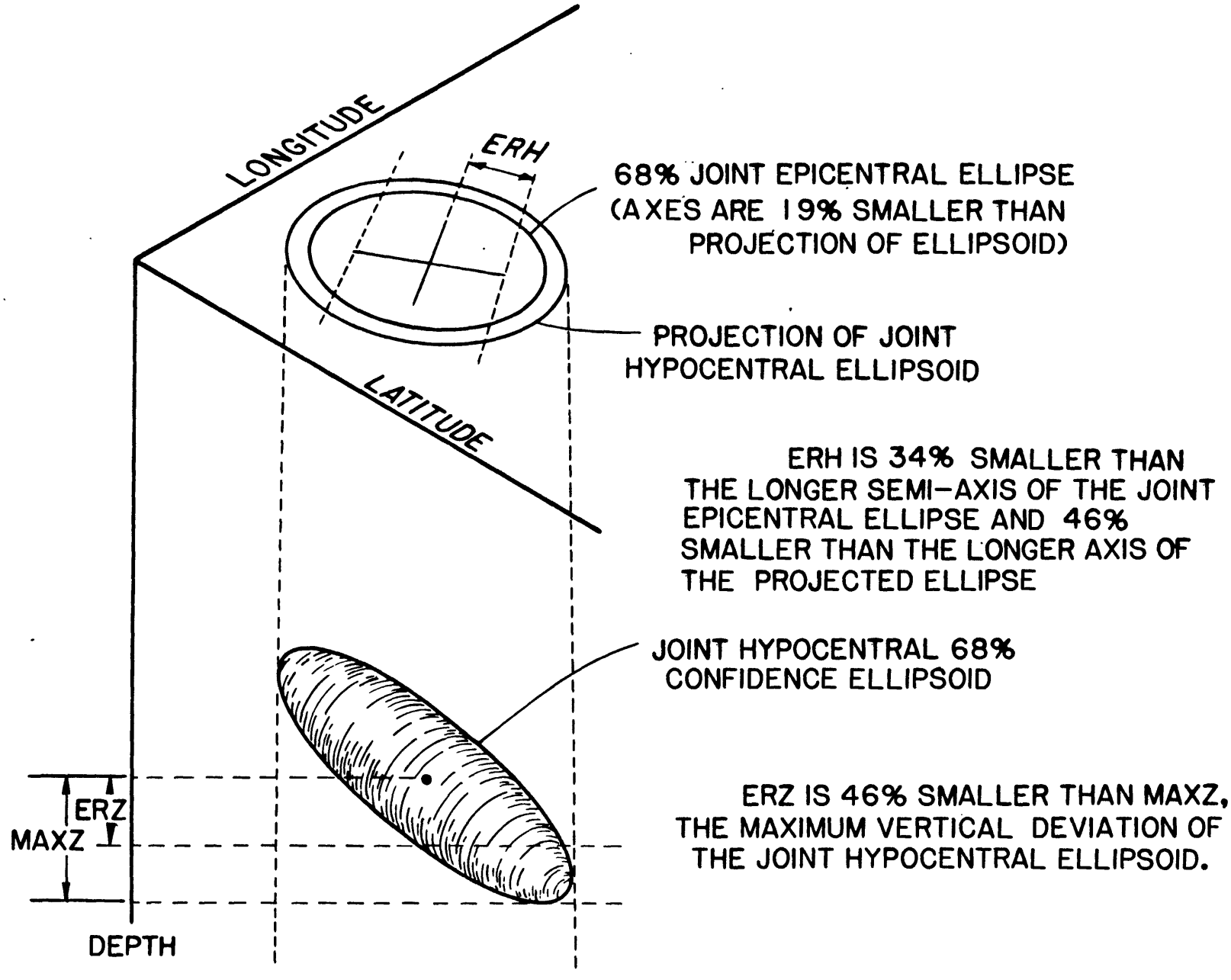

LONGITUDE
ERH
68% JOINT EPICENTRAL ELLIPSE
(AXES ARE 19% SMALLER THAN
PROJECTION OF ELLIPSOID)
PROJECTION OF JOINT
HYPOCENTRAL ELLIPSOID
LATITUDE
ERH IS 34% SMALLER THAN
THE LONGER SEMI-AXIS OF THE JOINT
EPICENTRAL ELLIPSE AND 46%
SMALLER THAN THE LONGER AXIS OF
THE PROJECTED ELLIPSE
JOINT HYPOCENTRAL 68%
CONFIDENCE ELLIPSOID
ERZ
MAXZ
ERZ IS 46% SMALLER THAN MAXZ,
THE MAXIMUM VERTICAL DEVIATION OF
THE JOINT HYPOCENTRAL ELLIPSOID.
DEPTH

Figure B-4

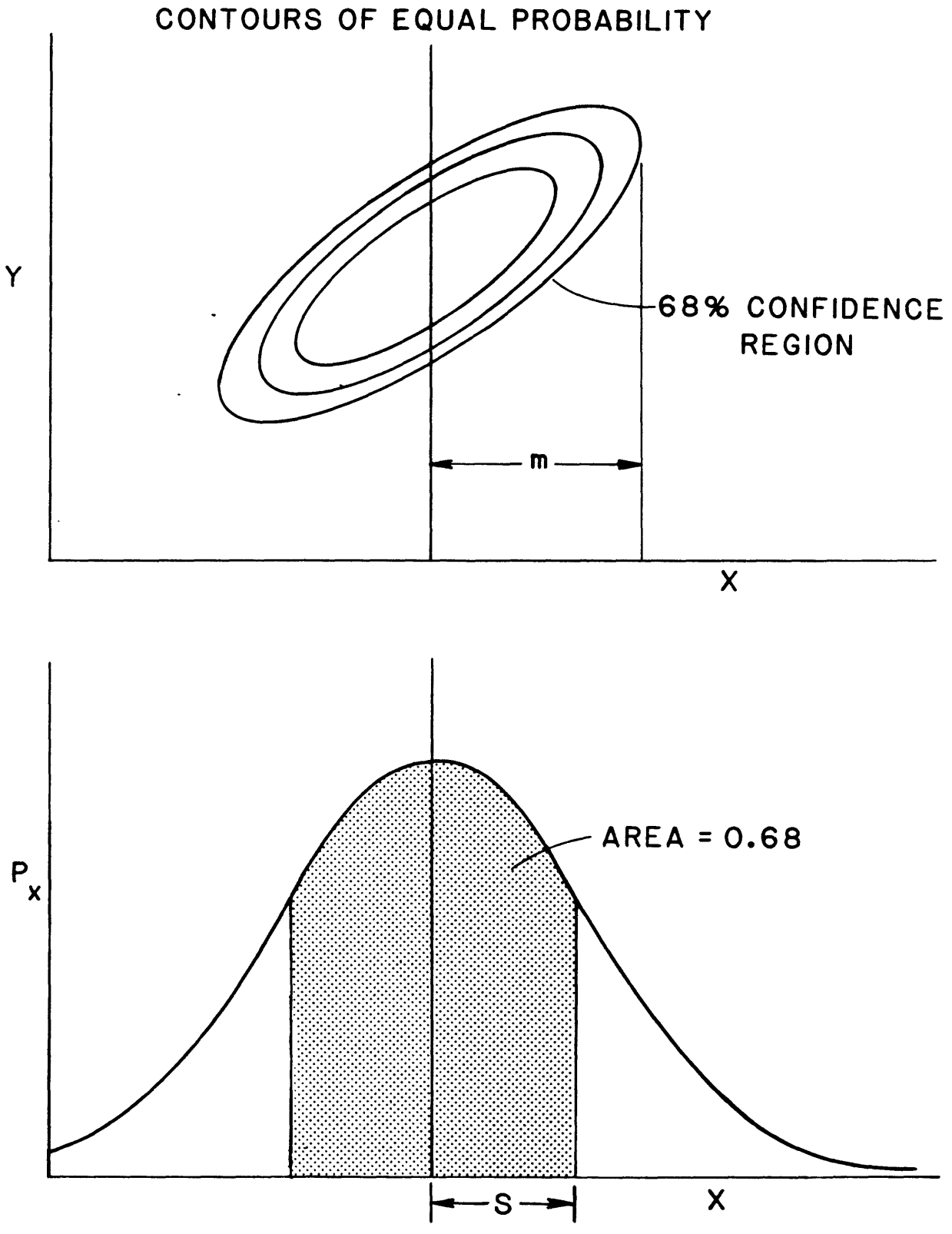

CONTOURS OF EQUAL PROBABILITY
Y
68% CONFIDENCE REGION
m
X
AREA = 0.68
P_x
S
X

Figure B-5

(P_{xy}) and a one dimensional distribution (P_x) is illustrated in Figure B-5. For each value of x, P_x is equal to the integral over y of the joint probability function P_{xy}. The ratio between s, the standard deviation of x, and m, the maximum deviation of the 68% joint confidence ellipse in the x direction, is equal to the square root of the ratio of the 68% value of chi-square with one degree of freedom to the 68% value of chi-square with two degrees of freedom. Similarly, the scaling relationship between the shadow of the joint hypocentral ellipsoid and the joint epicentral region is based on chi-square values for two and three degrees of freedom. Figure B-6 shows chi-square for P = 1, 2 and 3 for probabilities of 0.1 to .95. The program listing for ELLIPSE in the Appendix shows how to compute and plot a two-dimensional one standard deviation region if given the three-dimensional error ellipsoid. ELLIPSE is modified from the program GPP3 (Lahr, 1975).

3.) Computation of the standard error of origin time

The equation for origin time correction, (A-10) is

$$Y_4 = \tilde{X}_4 - \tilde{X}_1 Y_1 - \tilde{X}_2 Y_2 - \tilde{X}_3 Y_3$$

The variance of Y_4 is

$$VAR(Y_4) = VAR(\tilde{X}_4) + \tilde{X}_1^2 VAR(Y_1) + \tilde{X}_2^2 VAR(Y_2) + \tilde{X}_3^2 VAR(Y_3) + \text{covariance terms.}$$

If this equation is developed in the principal axis coordinate system, then the covariance terms are zero. The standard error of origin time is then

$$SE_{ORIG} = (\text{Reading Standard Error}) * SQRT(VAR(Y_4))$$

where Reading Standard Error is defined by YSE, Equation (A-13), or TEST(29), depending on the value of TEST(29) and the number of degrees of freedom.

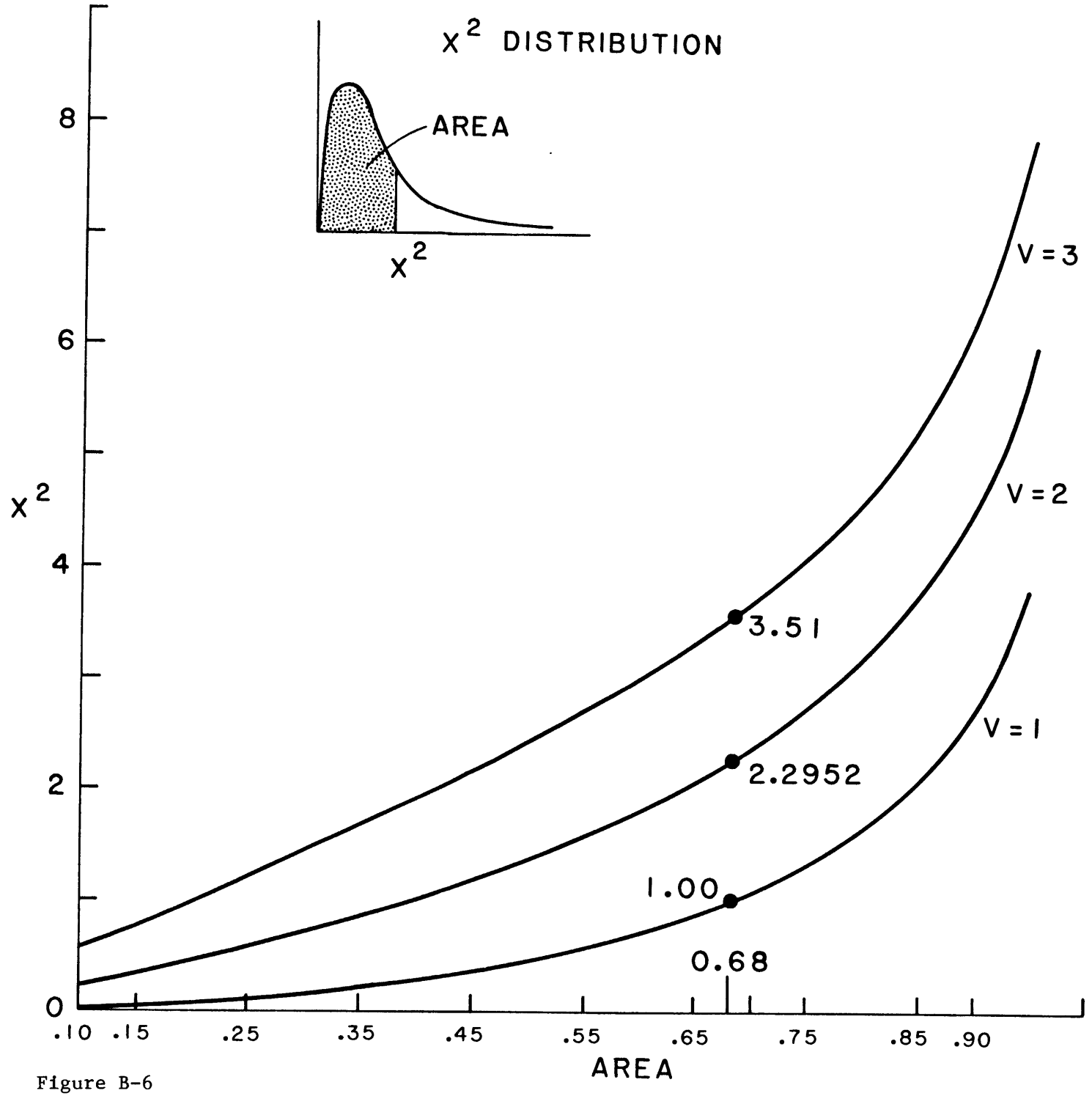

X² DISTRIBUTION
AREA
X²
8
6
4
2
0
X²
V = 3
V = 2
V = 1
3.51
2.2952
1.00
0.68
.10 .15 .25 .35 .45 .55 .65 .75 .85 .90
AREA

Figure B-6

C. WEIGHTING

1. Introduction

Geiger's least-squares method of earthquake location reduces the data to a set of linear equations, with one equation representing each reading of P-phase time, S-phase time, or S-P interval time. The four unknowns are the changes in latitude, longitude, depth and origin time. Clearly a set of at least four equations with the rank of the coefficient matrix greater than or equal to four is required to solve for the unknowns. When there are more than four equations a solution satisfying all equations is not always possible and the method of least squares minimizes the sum of the squares of the residuals from each equation.

Consider, for example, an earthquake with five recorded P-phase times. There are five possible combinations of four of the readings and, in general, each combination would have a different solution. The least-square solution would be somewhere among these five solutions and would best represent all of the readings as a whole. However, if one reading was known to have twice the standard error of the others, then of the five solutions the one based upon the other four readings would have the greatest chance of being correct. This information is properly included in the least-square method by including the same equation more than or less than once depending upon the estimated reading errors. In the example just given the equation with twice the standard error of the others should be multiplied by a weight of one half, while the others are multiplied by weights of one.

From Equation (A-2)

$$\omega_i R_i = \omega_i \left[X_{4,i} + \frac{\partial X_{4,i}}{\partial X} \, dX_o + \frac{\partial X_{4,i}}{\partial Y_o} \, dY_o + \frac{\partial X_{4,i}}{\partial Z_o} \, dZ_o + \frac{\partial X_{4,i}}{\partial T_o} \, dT_o \right],$$

where ω_i is the equation weight.

Then when the residuals are squared and summed:

$$Q = \sum_{i=1}^{n} \omega_i^2 R_i^2 .$$

This equation is usually written

$$Q = \sum_{i=1}^{n} W_i R_i^2 \quad \text{where } W_i = \omega_i^2 .$$

To summarize, the equation weights, ω_i, should be set inversely proportional to the standard error of the corresponding arrival time reading. Note that if there are just four equations, weighting will not affect the solution. Similarly, if there are more than four equations but one equation is critical in that without it the rank of the coefficient matrix is three, then the weight given that equation will not affect the solution.

2.) Assigned weight codes

In the program each reading can be assigned a weight code of 0, 1, 2, 3, or 4. These correspond to $W_i = \omega_i^2 = 1$, $(\frac{3}{4})^x$, $(\frac{1}{2})^x$, $(\frac{1}{4})^x$ and 0, respectively, and X is set equal to Test(36). The code should be assigned so that ω_i is inversely proportional to the estimated standard error of the reading.

For S and S-P readings the weights are also multiplied by TEST(39). This allows the relative importance of the P and S readings to be varied with ease.

3.) Weight out large residuals

The program has the option of giving zero weight to equations with residuals greater than a specified number of seconds (TEST(15)) from the mean. This is particularly designed for reading errors larger than 5 or 10

seconds. The residuals are removed one at a time with recalculation of the mean.

4.) Boxcar Weighting

Residuals greater than a specified number (TEST(17)) of standard deviations from the mean may be given zero weight. This is done in one step. It may also be done by quadrants with the quadrants defined as for Jeffreys' weighting.

5.) Distance Weighting

The user specifies two distances, D1 = TEST(11) and D2 = TEST(12) which define the ramp function for distance weighting, as shown in Figure 1 below.

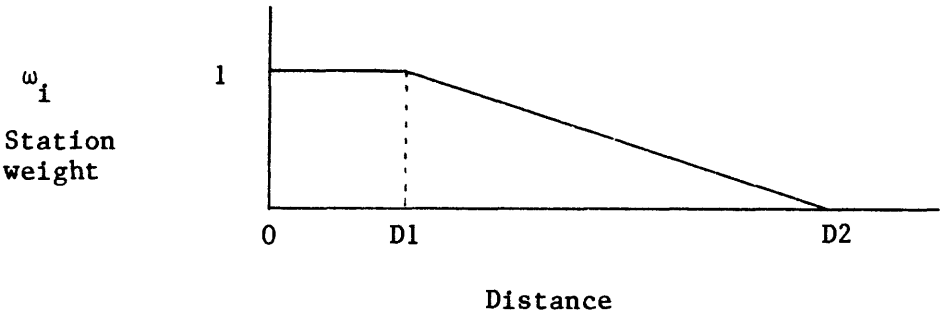


Figure C-1

6. Azimuthal Weighting

The subroutine AZWTOS modifies the weight for each station to be a function of azimuth. The region around the epicenter is divided into four quadrants and the stations in each quadrant are weighted so that the sum of the azimuthal weights for each of the four quadrants are equal.

The azimuths are sorted in order of increasing azimuth and the maximum gap in azimuth (i.e., the maximum difference between two consecutive azimuths in the sorted list) is determined. The region around the epicenter is then divided into four quadrants such that one of the dividing lines bisects the maximum gap. The other dividing line is then perpendicular to that direction.

The weight is calculated from

$$W_i = \frac{NRWT}{NOQ*NRQ_i}$$

where NRWT is the total number of P, S and S-P readings with weight greater than zero.

NOQ is the total number of quadrants occupied by weighted stations.

NRQ_i is the number of stations in the quadrant containing the i^{th} station.

7.) Jeffreys' weighting

Jeffreys' weighting is also based on the residuals. As explained in the next section it takes account of the wide tails on the otherwise normal distribution of errors of arrival time readings. Care must be taken in its use when the earthquake is displaced from its correct location. In that situation the residuals will have an azimuthal dependence as shown in Figure 2.

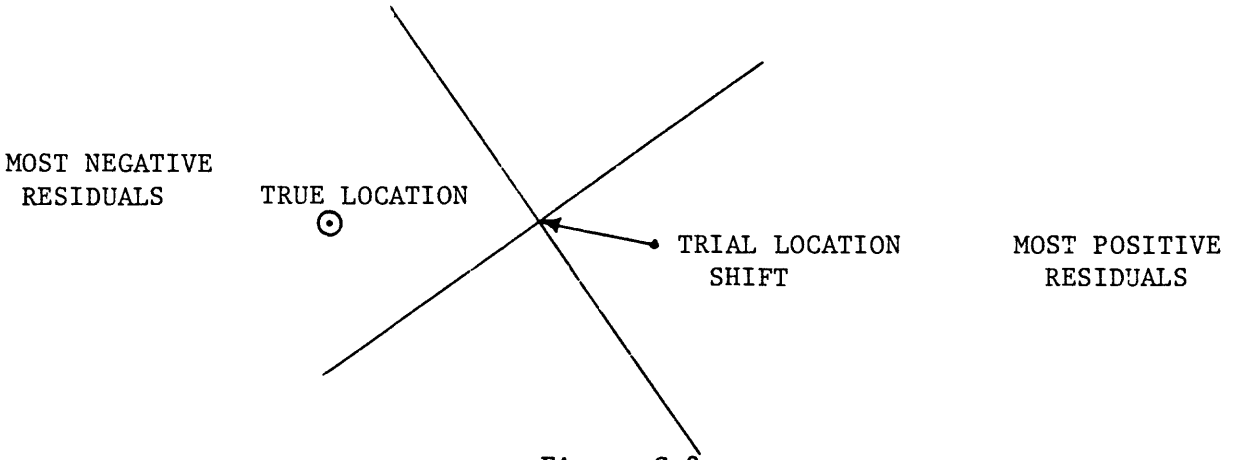


Figure C-2

For this reason there is a program option to apply Jeffreys' weighting within each of four quadrants separately. The arrow in Figure 2 indicates the move from the previous trial hypocenter. This direction is used to specify four

quandrants into which the stations are divided, since it will often be oriented as indicated in Figure (2). Then the mean is calculated for each quadrant and Jeffreys' weighting is applied to each separately. A minimum of five readings is required in a quadrant for Jeffreys' weighting to be applied quadrant by quadrant.

8.) Details of the Jeffreys' weighting calculation

Jeffreys (1961) describes a method of weighting travel-time residuals that accounts not only for the small errors in reading the correct phase, but also for the large errors introduced when occasionally the wrong phase is read. He assumes that most observers will agree on where a particular phase begins and that the reading errors for these observers will be normally distributed about the true arrival time. In addition, however, he assumes that there is a very small number of large errors caused by reading the wrong phases.

The normal distribution $f_\eta(X)$ of the residuals of correctly picked phases is

$$f_\eta(X) = \frac{1}{\sigma\sqrt{2\pi}} \quad EXP\left[-\frac{(X - M)^2}{2\sigma^2}\right] \qquad (C\text{-}1)$$

where σ is the standard deviation of the residuals,

$$\sigma = \frac{\Sigma W_i (X_i - M)^2}{\Sigma W_i}, \qquad (C\text{-}2)$$

and M = the weighted mean of the residuals,

$$M = \frac{\Sigma W_i X_i}{\Sigma W_i} \qquad (C\text{-}3)$$

The frequency distribution (f_m) of the travel-time residuals of the

misidentified phases is assumed to be

$$f_m = ag - \frac{a}{\sigma\sqrt{2\pi}} \quad EXP\left[-\frac{(X - M)^2}{2\sigma^2}\right] \tag{C-4}$$

where a is a constant and g is an unknown function of X. M and σ are the same as in equation C-1.

The total assumed frequency distribution is then

$$f = f_n + f_m = \frac{(1-a)}{\sigma\sqrt{2\pi}} \quad EXP\left[-\frac{(X - M)^2}{2\sigma^2}\right] + ag \tag{C-5}$$

Figure 3 is a plot of f, f_n and f_m.

Assuming the frequency distribution of equation C-5, Jeffreys (1961, p. 215) shows that the weighting function W is

$$W = \frac{1}{1 + \mu \quad EXP\left[\frac{(X - M)^2}{2\sigma^2}\right]} \tag{C-6}$$

where

$$\mu = \frac{ag}{\left[\frac{1 - a}{\sigma\sqrt{2\pi}}\right]} = \frac{ag}{\left[ag + \frac{(1 - a)}{\sigma\sqrt{2}}\right] - ag} \tag{C-7}$$

μ is the density of data points with large residuals divided by the density of data points with small residuals minus the density at large residuals. Thus μ can be estimated from the data.

Note that the weights are a function of σ and M which are in turn functions of the weights, equations (C-2) and (C-3).

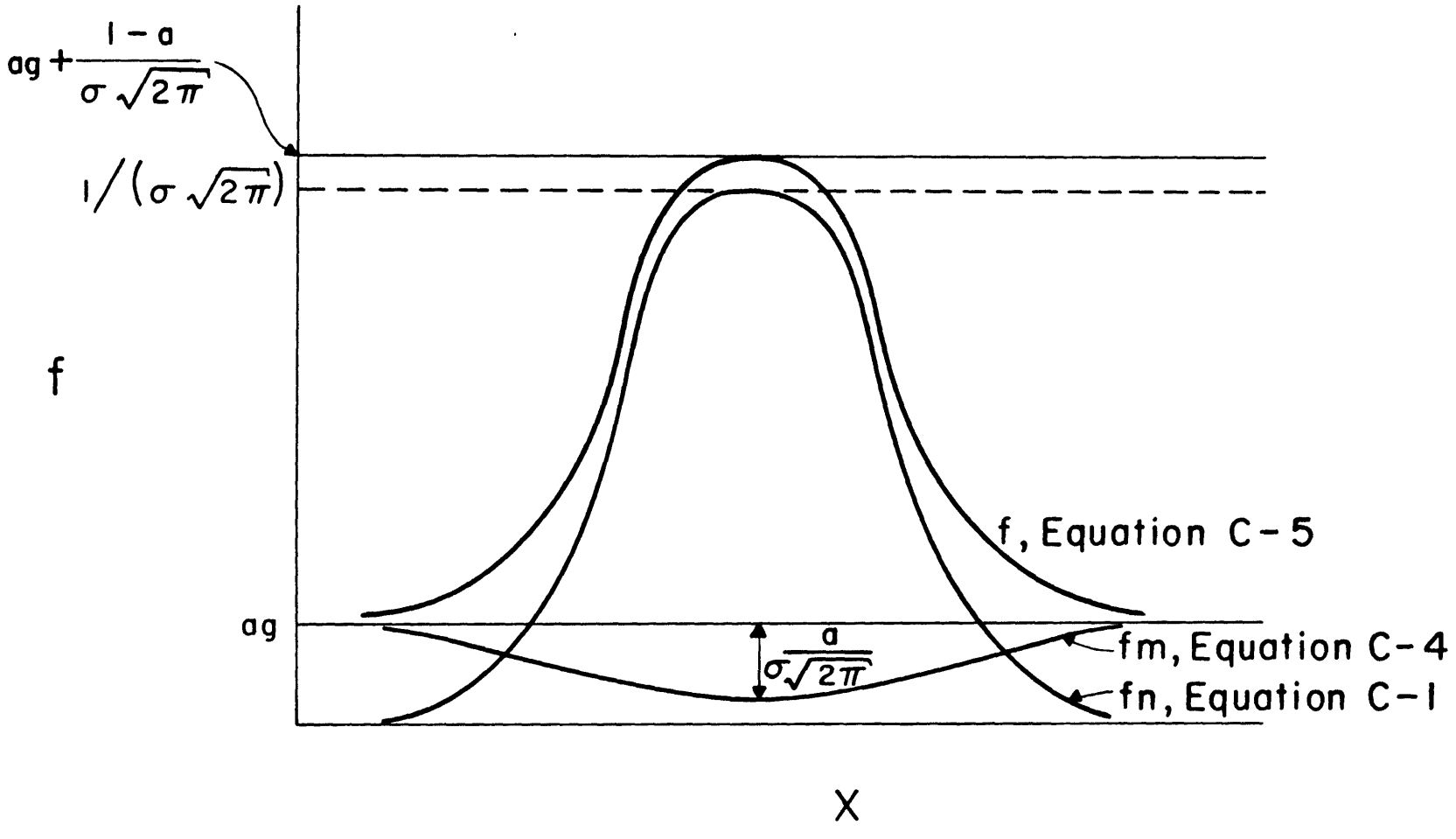


Figure C-3. Jeffreys' weighting.

Thus a method of successive approximations could be used to evaluate the weights. Jeffreys (1961, p. 216) states, however, that more often than not the second approximation almost repeats the first. In this program the first approximation is used.

Bolt (1960) found that for teleseismic data, setting μ and σ equal to 0.02 and $\sqrt{10}$, respectively, rather than calculating them from each data set yielded generally satisfactory results. In this program the weights are calculated from the formula:

$$WT(I) = \frac{1 + TEST(20)}{1 + TEST(20)\ EXP\left[\frac{1}{2}\left[\frac{X-M}{TEST(34)}\right]\right]2}$$

TEST(20) corresponds to μ and the default value is 0.02. σ corresponds to the standard deviation of the current earthquake. If σ is less than TEST(34) then TEST(34) is used in place of σ.

To save computation time a weighting function array, WF, is defined such that

$$WF(K) = \frac{1 + TEST(20)}{1 + TEST(20)\ EXP\left[[0.1\ (K-1)]^2/2\right]} \quad , K = 1,51$$

WF corresponds to the weights for residuals from zero to five standard deviations from the mean in units of tenths of a standard deviation. Beyond five standard deviations the weight is set equal to zero. The index K is calculated in the program as

$$K = \frac{ABS(X(4,i) - AVRPS * KSMP(i)) * 10.0}{\sigma} + 1.5$$

X(4,i) = travel time residual

AVRPS = average of the P and S residuals

KSMP(i) = 1 normally

KSMP(i) = 0 if this is a residual for S-P data

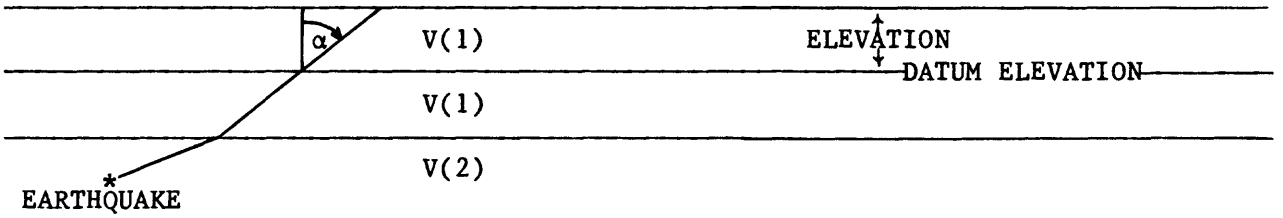
D. ELEVATION DELAY

The effect on travel time of variations in elevation specified for each station may be included by setting TEST(2) in HYPOELLIPSE to a non-negative value. The elevation correction, as detailed below, is based on an approximation which works reasonably well for over a wide range of angle of incidence.

1.) TEST(2) = 0.0

In this case the travel time is increased by an amount equal to the time for a wave to pass through a layer with thickness equal to the elevation and with a velocity equal to the velocity of the top crustal layer. The path length is computed from the angle of incidence of the ray at the surface.

ELVDLY = ELEVATION*COS(α)/v(1)

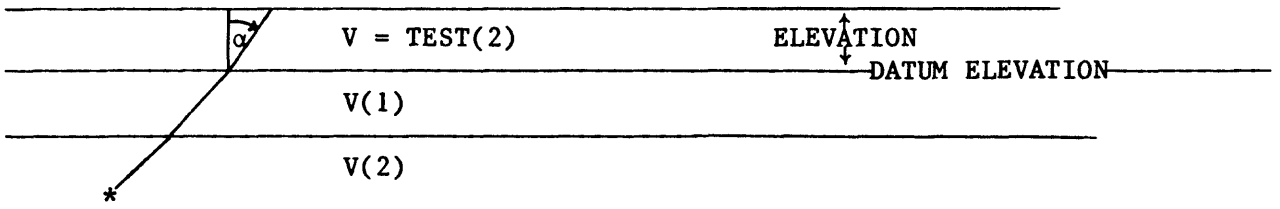


This formulation is an approximation which works reasonably well over a wide variation in α.

2.) TEST(2) GREATER THAN ZERO.

In this case, the velocity above the datum elevation is set to TEST(2). Again, the angle of incidence at the surface is used in computing the elevation delay.

ELVDLY = ELEVATION*COS(α)/TEST(2)



For the case of variable Vp/Vs ratio within the model, the elevation delay for S is computed from

ELVDLY(S) = ELEVATION*COS(α)*TEST(1)/TEST(2), where

TEST(1) is the global value of Vp/Vs ratio.

APPENDIX

PROGRAM TO COMPUTE THE TWO-DIMENSIONAL PROJECTION OF AN ERROR ELLIPSOID

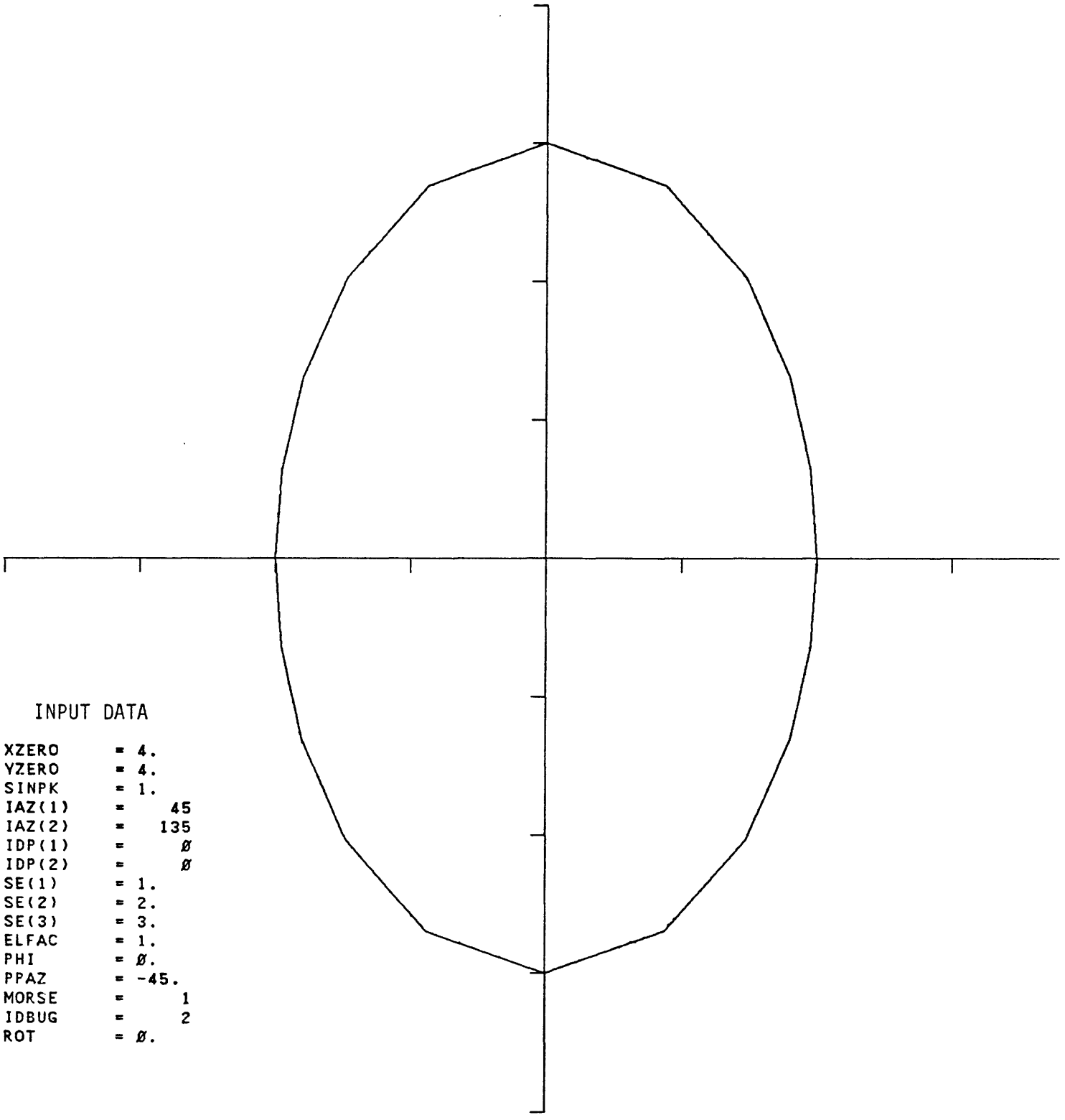
```
C------ ELLIPSE.FOR     J. C. LAHR     4/6/84
C------      BASED ON GPP3 COMPUTATIONS
C------      THIS PROGRAM WILL PLOT ONE ERROR ELLIPSE
C
C
      DIMENSION IAZ(2),IDP(2),SE(3)
C
      CALL PLOTS(Ø.,Ø.,Ø)
      CALL PLOT(1.,1.,-3)
C
      XZERO     = 3.Ø
      YZERO     = 3.Ø
      SINPK     = 1.
      IAZ(1)    =    45
      IAZ(2)    =   135
      IDP(1)    =     Ø
      IDP(2)    =     Ø
      SE(1)     = 4.
      SE(2)     = 2.
      SE(3)     = Ø.
      ELFAC     = 1.
      PHI       = Ø.
      PPAZ      = -45.
      MORSE     =     1
      IDBUG     =     2
      ROT       = Ø.
C
      ICMOUT = 6
      RPD = 1.745329251994E-2
C
C------ OPTIONAL READ STATEMENT
   11 READ(9,1ØØ,END=999) XZERO,YZERO,SINPK,
     1       IAZ,IDP,SE,ELFAC,PHI,PPAZ,MORSE,IDBUG,ROT
  1ØØ FORMAT(3(18X,F1Ø.2/),4(18X,I5/),6(18X,F1Ø.2/),18X,I5/,18X,I5,
     1        /18X,F1Ø.2)
C
C------ CONVERT PHI AND PPAZ TO RADIANS
      PHI = PHI*RPD
      PPAZ = PPAZ*RPD
C
      CALL PLOTEL(XZERO,YZERO,SINPK,
     1                  IAZ,IDP,SE,ELFAC,PHI,ROT,PPAZ,MORSE,IDBUG)
      GO TO 11
C
  999 STOP
      END
      SUBROUTINE PLOTEL(XZERO,YZERO,SINPK,
     1                  IAZ,IDP,SE,ELFAC,PHI,ROT,PPAZ,MORSE,IDBUG)
C
C------- PLOT ONE STANDARD DEVIATION ERROR ELLIPSE. ------------------
C
C
C                    INPUT
C
C   XZERO     X POSITION OF CENTER OF ELLIPSE
C   YZERO     Y POSITION OF CENTER OF ELLIPSE
C   SINPK     INCHES PER KM
C   IAZ       AZIMUTH OF FIRST TWO ELLIPSOID AXES
C   IDP       DIP OF FIRST TWO ELLIPSOID AXES
C             (THE THIRD AXIS IS COMPUTED BY CROSS MULTIPLICATION)
C   SE        STANDARD ERROR OF ALL THREE ELLIPSOID AXES
C   ELFAC     A SCALE FACTOR FOR ELLIPSES.  SET EQUAL TO ONE
C               FOR THE JOINT 2-DIMENSIONAL 68% CONFIDENCE REGION
```

```
C   PHI        ANGLE LOCAL MERIDIAN MAKES WITH Y PLOT AXIS
C   ROT        ROTATE THE ELLIPSE CLOCKWISE BY THIS AMOUNT
C   PPAZ       AZIMUTH OF THE CROSS SECTION PLAIN
C   MORSE      Ø FOR MAP PROJECTION
C              1 FOR CROSS SECTION PROJECTION
C   IDBUG      .GE. 2 FOR TRACE OF COMPUTATIONS
C                       OUTPUT
C
      DIMENSION IAZ(2),IDP(2),SE(3)
C
      RPD = 1.745329251994E-2
C
      CALL ELLIPS(IAZ,IDP,SE,ELFAC,PHI,PPAZ,MORSE,IDBUG,
     1                    AEL,BEL,CEL)
C
C------- THE ELLIPSE EQUATION IS:
C         1 = AEL*X**2 + BEL*X*Y + CEL*Y**2
C        OR IN TERMS OF R AND THETA:
C         1 = AEL*R**2*COS(THETA)**2 + BEL*R**2*COS(THETA)*SIN(THETA) +
C          CEL*R**2*SIN(THETA)**2
C
C
C------- PLOT A GRID
      AMAX = 1./SQRT(AEL)
      IF(1./SQRT(CEL) .GT. AMAX) AMAX = 1./SQRT(CEL)
      CALL PLOT(Ø.,Ø.,-998)
      IMAX = AMAX + 1.5
      XI = -IMAX + XZERO
      CALL PLOT(XI, YZERO, 3)
      DO 1ØØ I = -IMAX, IMAX
      XI = I + XZERO
      CALL PLOT(XI, YZERO, 2)
      CALL PLOT(XI, YZERO+.1, 2)
      CALL PLOT(XI, YZERO, 2)
  1ØØ CONTINUE
      YI = -IMAX + YZERO
      CALL PLOT(XZERO, YI, 3)
      DO 2ØØ I = -IMAX, IMAX
      YI = I + YZERO
      CALL PLOT(XZERO, YI, 2)
      CALL PLOT(XZERO+.1, YI, 2)
      CALL PLOT(XZERO, YI, 2)
  2ØØ CONTINUE
      IF(AEL .EQ. CEL) GO TO 65Ø
      THETA = Ø.5*ATAN(BEL/(AEL-CEL))
      GO TO 675
  65Ø THETA = 45.Ø*RPD
  675 DTHET = 18.Ø*RPD
      DO 7ØØ I = 1,21
      CS = COS(THETA)
      SN = SIN(THETA)
      RA =  1.Ø/SQRT(AEL*CS**2+BEL*CS*SN+CEL*SN**2)
      X = RA*COS(THETA-RPD*ROT)*SINPK + XZERO
      Y = RA*SIN(THETA-RPD*ROT)*SINPK + YZERO
      IF(I .EQ. 1) CALL PLOT(X,Y,3)
      CALL PLOT(X,Y,2)
  7ØØ THETA = THETA + DTHET
      CALL PLOT(X,Y,3)
      CALL PLOT(Ø.,Ø.,-998)
      RETURN
      END
      SUBROUTINE ELLIPS(IAZ,IDP,SE,ELFAC,PHI,PPAZ,MORSE,IDBUG,
     1                    AEL,BEL,CEL)
C------- PROGRAM TO FIND SHADOW PROJECTION OF THE ERROR ELLIPSE ON
```

```
C          A HORIZONTAL PLANE OR ON ANY VERTICAL PLANE.
C
C                         INPUT
C
C    IAZ        AZIMUTH OF FIRST TWO ELLIPSOID AXES (DEGREES)
C    IDP        DIP OF FIRST TWO ELLIPSOID AXES (DEGREES)
C    SE         STANDARD ERROR OF ALL THREE ELLIPSOID AXES (KILOMETERS)
C    ELFAC      A SCALE FACTOR FOR ELLIPSES.  SET EQUAL TO ONE
C                 FOR THE JOINT 2-DIMENSIONAL 68% CONFIDENCE REGION
C    PHI        ANGLE THE LOCAL MERIDIAN MAKES WITH Y PLOT AXIS (RADIANS)
C    PPAZ       AZIMUTH OF THE CROSS SECTION PLAIN (RADIANS)
C    MORSE      Ø FOR MAP PROJECTION
C               1 FOR CROSS SECTION PROJECTION
C    IDBUG      .GE. 2 FOR TRACE OF COMPUTATIONS
C                         OUTPUT
C
C    THE RESULTING ELLIPSE IS>  AEL*X**2 + BEL*X*Y + CEL*Y**2 = 1
C
C
C
      DIMENSION IAZ(2),IDP(2),SE(3)
      DIMENSION PAUV(3,3),TPACS(3,3),TEACS(3,3)
C
      DATA RPD/.Ø17453292/
      NOFILE = 1Ø
C
      IF(IDBUG .GE. 2) WRITE(NOFILE,9ØØ)
  9ØØ FORMAT(/,'   *** SUBROUTINE ELLIPSE ***          ')
   5Ø DO 15Ø I = 1,3
      DO 15Ø J = 1,3
      TPACS(I,J) = Ø.Ø
      PAUV(I,J) = Ø.Ø
  15Ø TEACS(I,J) = Ø.Ø
C
C------- FIND ELLIPSE PRINCIPAL AXIS VECTORS IN TERMS OF THE PROJECTION
C         PLANE COORDINATES.  Z OF THE PROJECTION PLANE COORDS IS THE
C         PROJECTION DIRECTION.
      IF(MORSE .EQ. 1) GO TO 23Ø
C
C------- FOR MAP PROJECTION. ------------------------------------------
      DO 225 I = 1,2
      DP = IDP(I)*RPD
      AZ = IAZ(I)*RPD
      APP = AZ + PHI
      IF(IDBUG .LE. 1) GO TO 22Ø
      WRITE(NOFILE,1ØØØ) IAZ(I),IDP(I)
 1ØØØ FORMAT(' AZ = ',I3,7X,' DP = ',I3,7X)
  22Ø PAUV(I,1) = COS(DP)*SIN(APP)
      PAUV(I,2) = COS(DP)*COS(APP)
  225 PAUV(I,3) = -SIN(DP)
      GO TO 24Ø
C
C------- FOR X-SECTION PROJECTION. ------------------------------------
  23Ø DO 235 I = 1,2
      DP = IDP(I)*RPD
      AZ = IAZ(I)*RPD
      APP = AZ + PHI
      IF(IDBUG .LE. 1) GO TO 233
      WRITE(NOFILE,1ØØØ) IAZ(I),IDP(I)
  233 PAUV(I,1) = COS(DP)*COS(APP-PPAZ)
      PAUV(I,2) = -SIN(DP)
  235 PAUV(I,3) = COS(DP)*SIN(APP-PPAZ)
C
C------- CROSS MULTIPLY TO FIND THIRD UNIT VECTOR.
```

```
  24Ø PAUV(3,1) = PAUV(1,2)*PAUV(2,3)-PAUV(2,2)*PAUV(1,3)
      PAUV(3,2) = PAUV(2,1)*PAUV(1,3)-PAUV(1,1)*PAUV(2,3)
      PAUV(3,3) = PAUV(1,1)*PAUV(2,2)-PAUV(2,1)*PAUV(1,2)
C
      IF(IDBUG .LE. 1) GO TO 25Ø
      WRITE(NOFILE,1Ø15)
 1Ø15 FORMAT(/1X,'TRANSFORMATION TENSOR FROM PRINCIPAL AXIS TO ',
     1 'PROJECTION PLANE COORDINATES'/)
      DO 26Ø I = 1,3
  26Ø WRITE(NOFILE,1Ø2Ø) (PAUV(I,J),J=1,3)
 1Ø2Ø FORMAT(3F12.2)
C
C------- SET UP TENSOR REPRESENTING ELLIPSOID IN PRINCIPAL COORDINATES.
  25Ø DO 275 I = 1,3
  275 TPACS(I,I) = 1.Ø/(.ØØØØØØ1+(SE(I)*ELFAC)**2)
C
C------- TRANSFORM THIS TENSOR TO THE PROJECTION COORDINATE SYSTEM.
      DO 375 I = 1,3
      DO 375 J = I,3
      DO 35Ø K = 1,3
      DO 35Ø L = 1,3
      TEACS(I,J) = TEACS(I,J) + PAUV(K,I)*PAUV(L,J)*TPACS(K,L)
  35Ø CONTINUE
  375 TEACS(J,I) = TEACS(I,J)
      IF(IDBUG .LE. 1) GO TO 4ØØ
      WRITE(NOFILE,1Ø5Ø)
 1Ø5Ø FORMAT(/' ERROR ELLIPSOID IN PRINCIPAL COORDINATES',
     1   ' AND PROJECTION COORDINATES'/)
      DO 38Ø I = 1,3
  38Ø WRITE(NOFILE,1Ø75) (TPACS(I,J),J=1,3),(TEACS(I,J),J=1,3)
 1Ø75 FORMAT(3F1Ø.2,1ØX,3F1Ø.2)
C
C------- ELLIPSOID IS X.TEACS.X - 1 = F = Ø
C------- GRAD F IS A NORMAL VECTOR.
C------- GRAD F .K = Ø DEFINES THE PLANE>
C------- TEACS(1,3)*X + TEACS(2,3)*Y + TEACS(3,3)*Z = Ø
C------- SOLVE FOR Z AND SUBSTITUTE IN F EQUATION TO OBTAIN ELLIPSE. ---
C------- THE ELLIPSE IS>  AEL*X**2 + BEL*X*Y + CEL*Y**2 = 1
C
C------- SQRT(CHI-SQUARE) FOR 3 DEGREES OF FREEDOM AND 68% IS 1.87
C        SQRT(CHI-SQUARE) FOR 2 DEGREES OF FREEDOM AND 68% IS 1.515
C        RATIO = 1.515/1.87 = .81
C        RATIO SCALES THE ELLIPSE FOR TWO DEGREES OF FREEDOM, MAINTAINING 68%
C        FOR THIS PROGRAM, LEAVE RATIO SET TO 1.Ø
         RATIO = 1.
C
  4ØØ AEL = RATIO*(TEACS(1,1)-TEACS(1,3)*TEACS(1,3)/TEACS(3,3))
      BEL = RATIO*2.Ø*(TEACS(1,2)-TEACS(1,3)*TEACS(2,3)/TEACS(3,3))
      CEL = RATIO*(TEACS(2,2)-TEACS(2,3)*TEACS(2,3)/TEACS(3,3))
      IF(IDBUG .LE. 1) RETURN
      WRITE(NOFILE,11ØØ) AEL,BEL,CEL
 11ØØ FORMAT(/' AEL =',E1Ø.4,' BEL =',E1Ø.4,' CEL =',E1Ø.4)
      RETURN
      END
```

PLOT OUTPUT FROM SAMPLE RUN



INPUT DATA

```
XZERO     = 4.
YZERO     = 4.
SINPK     = 1.
IAZ(1)    =    45
IAZ(2)    =   135
IDP(1)    =     Ø
IDP(2)    =     Ø
SE(1)     = 1.
SE(2)     = 2.
SE(3)     = 3.
ELFAC     = 1.
PHI       = Ø.
PPAZ      = -45.
MORSE     =     1
IDBUG     =     2
ROT       = Ø.
```

PRINTED OUTPUT FROM SAMPLE RUN

```
  *** SUBROUTINE ELLIPSE ***
AZ =  45        DP =   Ø
AZ = 135        DP =   Ø

TRANSFORMATION TENSOR FROM PRINCIPAL AXIS TO PROJECTION PLANE COORDINATES

       Ø.ØØ        Ø.ØØ        1.ØØ
      -1.ØØ        Ø.ØØ        Ø.ØØ
       Ø.ØØ       -1.ØØ        Ø.ØØ

ERROR ELLIPSOID IN PRINCIPAL COORDINATES AND PROJECTION COORDINATES

     1.ØØ      Ø.ØØ      Ø.ØØ                Ø.25      Ø.ØØ      Ø.ØØ
     Ø.ØØ      Ø.25      Ø.ØØ                Ø.ØØ      Ø.11      Ø.ØØ
     Ø.ØØ      Ø.ØØ      Ø.11                Ø.ØØ      Ø.ØØ      1.ØØ

AEL =Ø.2500E+ØØ BEL =Ø.ØØØØE+ØØ CEL =Ø.1111E+ØØ
```

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