PUMPING TESTS

Permeability tests by the Thiem method, by V. C. Fishel, engineer in charge, Division of Ground Water, Lawrence, Kans.

Synopsis

(1) The formula $P = \frac{527.7\,q}{m} \frac{\log_{10} r_2 - \log_{10} r_1}{h_2 - h_1}$ is applied under water-table conditions for determining the permeability of water-bearing materials. Data collected during the Wichita, Kansas, and the Grand Island, Nebraska, pumping tests are used to verify the appropriateness of the formula. A quick and accurate method of analysis of pumping-test data is given.

(2) A convenient method is given for obtaining the hydraulic gradient for use with the gradient formula. The hydraulic gradient is given by the derivative with respect to r of the equation for the water-table profile in the cone of depression. The equation of the draw-down curve is obtained by making use of the fact that when the altitudes of the water levels are plotted on semi-logarithmic paper against the corresponding distances of the observation wells from the pumped well the points fall on a straight line. It is shown that it is superfluous to compute the permeability by both the gradient and the Thiem formula.

(3) The data collected during the Wichita, Kansas, and the Grand Island, Nebraska, pumping tests have been analyzed and summarized in several tables to show the accuracy of the permeability determinations. An attempt is made to arrive at the simplest and most reliable method of making a pumping test. It is concluded that there should be at least 6 observation wells that are properly spaced on opposite sides of the pumped well. The observation wells can be located without regard to the direction of movement of the ground water.

Illustrations

Tables

Introduction

A formula for determining the permeability of water-bearing materials was given by J. Dupuit (1) in 1858. It was later used by Thiem in Germany (4) and in the last 20 years it has been used extensively throughout the United States and Europe. In the United States it has generally been know as the Thiem formula.

The formula may be written as follows (7, p. 79, equation 56):

$$(1)\quad P = \frac{Q\,(\log_e r_2 - \log_e r_1)}{(h_2^2 - h_1^2)}$$

Where Q is the discharge of the pumped well, in gallons a day, h_1 and h_2 are the thicknesses of the saturated material at two observation wells located at distances of r_1 and r_2, respectively, from the pumped well. From equation 1:

$$(2)\quad h_2^2 - h_1^2 = (h_2 + h_1)(h_2 - h_1) = 2m\,(h_2 - h_1)$$

where m is the thickness of the saturated water-bearing material. Then:

$$(3)\quad P = \frac{Q\,(\log_e r_2 - \log_e r_1)}{2\,m\,(h_2 - h_1)}$$

This is the general equilibrium formula given by Wenzel for artesian conditions (7, p. 79, equation 60).

An inspection of equation (1) shows that if this equation is valid a straight line curve will be obtained when h^2 is plotted against r on semilogarithmic paper. Also, an inspection of equation (3) shows that if m remains constant a straight line will be obtained when h is plotted against 4 on semilogarithmic paper. It was found from 7 pumping tests under water-table conditions (2 of which are discussed in this paper) that within the limits of experimental error a straight line results when h is plotted against r on semilogarithmic paper and hence it is concluded that equation (3) can be used for water-table conditions by using an appropriate value for m. A value of m equal to the average saturated thickness at the observation wells is used, but further investigation is needed to substantiate this accepted value.

If the rate of pumping is given in gallons a minute and the logarithms are converted to base 10, formula (3) becomes:

$$(4) \quad P = \frac{(527.7\, q)}{(m)} \frac{(\log_{10} r_2 - \log_{10} r_1)}{(h_2 - h_1)} = \frac{C \Delta \log_{10} r}{\Delta h}$$

Where m is the average thickness of the saturated material in the cone of depression, h_2 and h_1 are the altitudes of the water levels in the observation wells at r_2 and r_1 respectively, and C is a constant equal to $\frac{527.7\, q}{m}$. Formula (4) is a convenient formula for use under water-table conditions, and it is believed to be accurate within the limits of experimental error.

The standard procedure of evaluating the factors in equation (4) is given by Wenzel (7, pp. 83-84). The procedure is greatly simplified and most of the detailed work is eliminated by making use of the straight line relation that exists when the altitudes or draw-downs of the water levels in the observation wells are plotted on a natural scale against the distances of the observation wells from the pumped well plotted on a logarithmic scale. The straight line automatically takes care of any irregularities in observed water levels.

The value of $\frac{\Delta \log r}{\Delta h}$ is given by the slope of the straight line and hence is fairly accurately obtained.

Muskat (3, p. 370, fig. 141) used a modification of this method and obtained a straight line by plotting $h_2^2 - h_1^2$ against $\log \frac{r_e}{r}$. P.T. Bennett, Corps of Engineers, U. S. Army, (personal communication) plots on semi-logarithmic paper the altitude of the water levels against the distance of the observation well from the pumped well.

If values of $\frac{\Delta \log r}{\Delta h}$ are selected from the straight line for values of r at 10 and 100 feet (or 100 and 1,000 feet), $\Delta \log r$ becomes unity, and the formula becomes:

$$(5)\ P = \frac{C}{\Delta h}$$

Where Δh is the difference in altitude of the water levels in the observation wells at 10 and 100 feet (or 100 and 1,000 feet) and $C = \frac{(527.7\ q)}{(m)}$.

Pumping Test

The application of the formula for determining permeability will be illustrated by a pumping test near Wichita, Kansas.

A pumping test to determine the permeability of the water-bearing sand and gravel in the Arkansas River Valley near Wichita, Kansas, was made in 1937 under the direction of S. W. Lohman in cooperation with the Kansas Geological Survey, on the property of the Kansas Gas and Electric Company, about 3 miles north of Wichita (7, pp. 142-147). The well had a diameter of 20 inches and was 45.3 feet deep. The thickness of the saturated water-bearing material, as obtained from the log of the pumped well, was 26.8 feet. Observation wells were constructed at varying distances on 4 radial lines extending north, east, south, and west from the pumped well. The well was operated continuously at a rate of 1,000 ± 7 gallons a minute from 10:33 a.m., November 8, to the afternoon of November 27, at which time the pump was unavoidably stopped. Pumping was resumed on November 28 at a rate of 750 gallons a minute, and was discontinued December 8 at 9:55 a.m.

The locations of the observation wells, the altitudes of the static water levels on November 8, and the equilibrium pumping levels on November 26 and December 7 are given in Table 1.

Table 1. Water-level data collected during a pumping test near Wichita, Kansas, in 1937.

Well No. and direction from pumped well	Distance from pumped well (feet)	Static level Nov. 8, 10:30 a.m. (feet)	Pumping level Nov. 26 a/ (feet)	Pumping level Dec. 7 b/ (feet)
1 N	49.2	1309.62	1303.76	1305.06
2 N	100.7	1309.75	1305.17	1306.15
3 N	189.4	1309.91	c/1306.55	1307.20
1 S	49.0	1309.52	1304.10	1305.33
2 S	100.4	1309.58	1305.37	1306.15
3 S	190.0	1309.48	1306.34	1306.96
1 E	49.3	1309.67	1303.62	1304.80
2 E	101.3	1309.68	1304.94	1305.92
3 E	220.9	1309.73	1306.62	1307.16
4 E	500.1	1309.88	1307.97	1308.19
5 E	900.1	1309.99	1308.87	1308.88
1 W	49.3	1309.55	1304.22	1305.40
2 W	100.5	1309.53	1305.43	1306.33
3 W	139.7	1309.43	1305.98	1306.73
4 W	222.3	1309.52	1307.13	1307.62
5 W	503.5	1309.43	1308.10	1308.37
6 W	895.0	1309.63	1308.69	1309.03
OE	d/3.2	1309.62	1296.90	1300.60
e/ PW	0.0	1309.60	1294.70	1299.38

a/ Average of 2 measurements made on Nov. 26.
b/ Average of 2 measurements made on Dec. 7.
c/ Interpolated.
d/ Measured from center of pumped well (PW), situated just outside of gravel packing.
e/ Pumped well.

In Figure 1 the altitudes of the water levels on November 26 for each line of wells are plotted against the distance of the observation wells from the pumped well. The altitudes of the water levels for r at distances of 10 and 100 feet were obtained from the straight line curves and are given in Table 2.

Fig. 1. Altitudes of water levels just before pumping stopped on November 26 plotted against the distance of the observation well from the pumped well for the Wichita, Kansas, pumping test.

The pumping rate was 1,000 gallons a minute and the average saturated thickness in the cone of depression was about 22.2 feet.

Formula (4) becomes

$$(6)\ P = \frac{527.7 \times 1{,}000}{22.2} \quad \frac{(\log r_2 - \log r_1)}{(h_2 - h_1)}$$

$$= \frac{23{,}770\ \Delta \log r}{\Delta h}$$

Log r_2 - log r_1 is 1 when r_2 is 100 feet and r_1 is 10 feet or r_2 is 1000 feet and r_1 is 100 feet. Thus $\frac{\Delta \log r}{\Delta h}$ is equal to 1 divided by the difference in altitude of the water levels at 10 and 100 feet from the pumped well. The permeability as computed for each line of observation wells is given in Table 2.

Table 2. Permeability computations for equilibrium conditions on November 26 at a pumping rate of 1,000 gallons a minute. $P = \frac{23{,}770\ \Delta \log r}{\Delta h}$

Direction of observation wells from pumped well	Altitude of water level at r = 10 feet (feet)	Altitude of water level at r = 100 feet (feet)	Log r_2 - log r_1 (Δlog r)	$h_2 - h_1$ (Δh) (feet)	Coefficient of permeability
North	0.30	5.[illegible]6	1	4.96	4795
East	.65	4.95	1	4.30	5528
South	1.50	5.32	1	3.82	6223
West	1.60	5.43	1	3.83	6206
Average of north-south lines					5509
Average of east-west lines					5867
Average of all lines					5688

It will be noted on Figure 1 that the point for well 4 W falls above the curve. It is located adjacent to Little Arkansas River and the water level is probably affected by ground-water recharge. Figure 1 indicates that this method may be used qualitatively to determine ground-water recharge from rivers, and it is believed that an extension of this method could be used to determine the amount of recharge.

The draw-downs of the water levels from November 8 to November 26 were computed from Table 1 and were plotted against the distances of the observation wells from the pumped well as shown on Figure 2. It was again found that within the limits of experimental error the wells within a radius of 250 feet from the pumped well plot on a straight line. The slopes of the curves based

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Fig. 2. Draw-down of water levels between November 8 and November 26 plotted against the distance of the observation well from the pumped well for the Wichita, Kansas, pumping test.

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on draw-downs are slightly different from the curves based on altitudes. The data and computations are given in Table 3.

Table 3. Permeability computations based on draw-down of water levels between November 8 and November 26 at a pumping rate of 1,000 gallons a minute.

$$P = \frac{23,770 \Delta \log r}{\Delta h}$$

Direction of observation wells from pumped well	Draw-down of water level at r = 10 feet (feet)	Draw-down of water level at r = 100 feet (feet)	Log r_2 - log r_1 (Δlog r)	$s_1 - s_2$ (Δs) (feet)	Coefficient of permeability
North	8.84	4.55	1	4.29	5541
East	9.20	4.70	1	4.50	5282
South	8.16	4.24	1	3.92	6064
West	8.27	4.05	1	4.22	5633
Average of north-south lines					5803
Average of east-west lines					5458
Average of all lines					5630

Gradient Formula

Values of r (distance of observation well from the pumped well) and corresponding values of h (altitude of water level) were obtained from Figure 1 from the curve of the east line of wells and were plotted in Figure 3 to show the draw-down curve on coordinate paper. The altitudes of the water levels

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Fig. 3. Draw-down curve for the east line of wells on November 26 for the Wichita, Kansas, pumping test. The curve was drawn from data obtained from Figure 1B.

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for November 26 as given in Table 1 were then plotted to show their relation to the draw-down curve as determined by the straight line relationship in Figure 1.

If the water-table or piezometric surface in a homogeneous formation is horizontal before pumping begins, water will percolate toward a pumped well equally from all directions, and the same quantity of water will percolate toward the pumped well through each of the infinite series of concentric cylindrical sections around the well. The area of each concentric cylindrical section is $A = 2\pi r m$, where r is the radius of the cylinder and m is the thickness of the saturated water-bearing material. Thus, according to our usual interpretation of Darcy's law:

(8) $Q = P i A = 2\pi P i r m$

Where Q is the discharge of the pumped well, P is the coefficient of permeability and i is the hydraulic gradient.

If B, i, r, and m are known the gradient formula can be used to determine the amount of water that moves toward the well past a given concentric cylinder. Or, if Q, i, r, and m can be determined the formula may be used to determine the coefficient of permeability.

(9) Thus $P = \frac{Q}{2 \pi i r m}$

Q, r, and m can be readily determined. The hydraulic gradient (i) was determined by Wenzel as follows (7, pp. 85-86):

> A profile of the cone of depression is first constructed by plotting the elevations of the water levels in the observation wells located on a straight line through the discharging well against the distances of the wells from the discharging well and by connecting the plotted points with a smooth curve, thus showing the cone of depression from the farthest upgradient well to the farthest downgradient well at some time, t, after the discharge is started. A profile of the initial water table or piezometric surface should be constructed also. From these profiles the draw-down of the water level and the altitude of the water level at any time may be determined for any distance upgradient and downgradient.
>
> The hydraulic gradient at any distance, r, from the discharging well is approximately equal to the difference in altitude between two points at distance b on each side of point r divided by the distance between the points - that is 2 b. Thus
>
> $$i = \frac{f(r + b) - f(r - b)}{2b}$$
>
> in which f (r - b) is the altitude of the water level at the distance r - b from the pumped well and f (r + b) is the altitude of the water level at the distance r + b from the pumped well.

The hydraulic gradient or the slope of the curve in Figure 3 can be determined approximately by another graphical method. A straight line drawn tangent to the curve gives the slope of the surve at the point of tangency. In Figure 3 a straight line is drawn tangent to the curve at a point where r is equal to

97 feet. The slope (i) of the line is then given by the equation

$$i = \frac{h_2 - h_1}{r_2 - r_2} = \frac{9 - 3}{310 - 0} = .019$$

The hydraulic gradient can be obtained more readily by making use of the straight line relation on semi-logarithmic paper between r and h. The equation of the straight line is given by

$$(10)\quad h = a \log_{10} r + b$$

where a is the slope of the line and b is the intercept of the line with the h axis. The hydraulic gradient is then given by the derivative of equation 10 with respect to r.

$$(11)\quad \frac{d\,h}{d\,r} = i = \frac{a \log_{10} e}{r} = \frac{0.4343a}{r}$$

The slope (a) of the straight line is given by $a = \frac{h_2 - h_1}{\log_{10} r_2 - \log_{10} r_1} = \frac{\Delta h}{\Delta \log_{10} r}$ From Figure 1 and Table 2, a is equal to 4.30 for the east line of wells on November 26. At a distance of 97 feet from the pumped well $i = \frac{0.4343 \times 4.30}{97} = 0.019$ which is the same hydraulic gradient as determined graphically above. If $i = \frac{0.4343\ a}{r}$ is substituted in the gradient formula, the formula becomes

$$(12)\quad P = \frac{1440\ Q}{(2\pi r\ m)\left(\frac{0.4343a}{r}\right)}$$

where Q is measured in gallons a minute and P is Meinzer's coefficient of permeability. r cancels out of the denominator and equation (12) reduces to

$$(13)\quad P = \frac{527.7\ Q}{m\ a}$$

$$(14)\quad = \frac{527.7\ Q}{m} \quad \frac{\log_{10} r_2 - \log_{10} r_1}{h_2 - h_1}$$

It is thus seen that the gradient formula becomes identical with equation (4) and if advantage is taken of the straight line relation between r and h, it becomes superfluous to compute the permeability by both methods. If the data do not plot on a straight line then the conditions for equation (4) are not fulfilled and this method of computing the coefficient of permeability will give only approximate results.

Summary of Analyses

A pumping test was made by Wenzel (7, pp. 117-127) in 1931 in the Platte River Valley near Grand Island, Nebraska. About 80 observation wells were constructed in the vicinity of an existing irrigation well. Forty-seven of these observation wells were constructed in 4 lines (A, B, C, and D) extending out from the pumped well at right angles to each other. Line B extends upstream and line D downstream. The data collected during this test have been analyzed according to the method used for the test near Wichita but for purposes of brevity the analyses have been omitted in this paper. The results of the permeability determinations for the two tests are summarized in order to show the accuracy of the methods and to arrive at the most rational method of analyzing and using pumping-test data. The coefficients of permeability for the Wichita and Grand Island tests are summarized in Table 4.

Table 4. Summary of coefficients of permeability for the pumping tests near Wichita, Kansas, and Grand Island, Nebraska.

Method of Determination	Coefficient of permeability						
	Direction of observation wells from pumped well						
	N	E	S	W	Average		
					N and S	E and W	All
(Wichita)							
Altitude Nov. 26	4,795	5,528	6,223	6,206	5,509	5,867	5,688
Altitude Dec. 7	4,831	4,938	6,147	6,043	5,489	5,491	5,490
Draw-down Nov. 8 to 26	5,541	5,282	6,064	5,633	5,803	5,458	5,630
(Grand Island)	(A)	(D)	(C)	(B)	(A and C)	(D and B)	
Altitude July 31	1,000	1,192	983	864	992	1,028	1,010
Draw-down July 29 to 31	993	1,075	986	938	990	1,007	998

The departures of the coefficients of permeability given in Table 4 from the average coefficient obtained from the 4 lines of wells and the percentage departure are given in Table 5.

Table 5. Departure of coefficients of permeability for each line of observation wells from the average coefficient for all four lines and the percentage of departure.

Method of Determination	Departure from average					Percentage departure from average				
	N	E	S	W	N-S or E-W	N	E	S	W	N-S or E-W
(Wichita)										
Altitude Nov. 26	-893	-160	+545	+518	±179	13.9	2.8	9.6	9.1	3.1
Altitude Dec. 7	-659	-552	+657	+553	± 1	12.0	10.1	12.0	10.1	.02
Draw-down Nov. 8 to 26	- 89	-348	+434	+ 3	±173	1.8	6.2	7.7	.05	3.1
(Grand Island)										
Altitude July 31	- 10	+182	- 27	-146	± 18	1.0	18.0	2.7	14.5	2.3
Draw-down July 29 to 31	- 5	+ 77	- 12	- 60	± 8	.5	7.7	1.2	6.0	.8

By using only one line of wells it is shown in Tables 4 and 5 that at Wichita the departure of the coefficient of permeability from the average coefficient of all 4 lines of wells ranges from a minimum of 0.05 to a maximum of 13.9 percent and at Grand Island from 0.5 to 18.0 percent. The average departure for the Wichita test for 12 determinations is 7.9 percent. The average departure for the Grand Island test for 8 determinations is 6.3 percent. Thus, based on only 2 tests, the use of only one line of wells would give the permeability within about 8 percent of the average, but the departure may range from 0.05 to 18.0 percent. By using the average of the lines of wells on opposite sides of the pumped well at Wichita the departure from the average of all 4 lines ranged from 0.02 to 3.1 percent and at Grand Island from 0.8 to 2.3 percent. The average departure of the 3 computations for Wichita and 2 for Grand Island was 2.3 percent.

Wenzel (7, p. 83) found from a study of the data collected during the Grand Island test that consistent results could be obtained by following an empirical procedure described by him as follows:

Using for the draw-down of the water level s_1 the average of the draw-downs on opposite sides of the pumped well - preferably upgradient and downgradient - at the distance r_1 from the pumped well. Similarly the draw-down s_2 is taken as the average of the draw-downs at the distance r_2 on opposite sides of the pumped well.

The differences in altitude or draw-down for a radius of 10 and 100 feet for each line of wells and are summarized in Table 6.

Table 6. Difference in altitude or draw-down at distances of 10 and 100 feet from the pumped well and coefficients of permeabilities determined from average differences of altitude or draw-down on opposite sides of the well.

Method of determination	Difference in altitude $(h_2 - h_1)$ or draw-down $(s_1 - s_2)$ (feet)						Coefficient of permeability		
					Average				
	N	E	S	W	N-S	E-W	N-S	E-W	Average
(Wichita)									
Altitude Nov. 26	4.96	4.30	3.82	3.83	4.390	4.065	5415	5847	5631
Altitude Dec. 7	3.69	3.61	2.90	2.95	3.295	3.280	5411	5435	5423
Draw-down Nov. 8 to 26	4.29	4.50	3.92	4.22	4.360	4.105	5452	5790	5621
(Grand Island)	(A)	(D)	(C)	(B)					
Altitude July 31	2.85	2.39	2.90	3.30	2.875	2.875	991	1002	996.5
Draw-down July 29 to 31	2.87	2.65	2.89	3.04	2.880	2.845	989	1002	995.5

The departure from average of the permeability given in Table 6 determined by two opposite lines of wells and the percentage of departure is given in Table 7.

Table 7. The departure from average of the permeability dete[illegible]ed by the average altitude of draw-down by two opposite lines of wells, and the percentage of departure.

Method of determination	Departute of N-S or E-W coefficient of permeability from average coeff.	Percentage of departure
(Wichita)		
Altitude Nov. 26	216	3.8
Altitude Dec. 7	12	.2
Draw-down Nov. 8 to 26	169	3.0
(Grand Island)		
Altitude July 31	5.5	.5
Draw-down July 29-31	6.5	.6

The five determinations shown in Tables 6 and 7 had a departure from the average ranging from 0.2 to 3.8 percent and averaged 1.6 percent. The departure for the Grand Island test was less by Wenzel's method (0.5 percent in Table 7 in comparison with 1.5 percent in Table 5) than by computing each line separately and averaging the two lines on opposite sides of the well, but the departure for the Wichita test was greater by Wenzel's method (2.3 percent in Table 7 in comparison with 2.9 percent in Table 5). Either method gives consistent results, and both are easy to apply.

Wenzel (7, pp. 113-114) states that: From three to six observation wells located on a line through the discharge well should be constructed upgradient from the well and an equal number should be constructed on the same line downgradient. The distance from the discharge well to each upgradient well should be equal to the distance from the discharge well to a corresponding downgradient well.

The approximate slope of the natural water table or piezometric surface should be determined before the observation wells are constructed. The observation wells should be located on a line through the discharge well perpendicular to the contour lines, that is, on a line parallel to the maximum slope of the water table.

The results given in the above tables indicate, however, that the lines of observation wells may parallel the direction of the natural movement of the ground water or may intersect it at any angle. Other considerations such as convenience of constructing or measuring the observation wells would seem to be of more paramount importance in determining the direction of the lines of observation wells than the direction of the movement of the ground water.

Comparison with other methods

The pumping tests at Wichita, Kansas, and Grand Island, Nebraska, have also been studied and the coefficients of permeability computed by Wenzel (7) and by C. E. Jacob (unpublished report). The results obtained by Wenzel, by Jacob, and by this method are given in Table 8.

Table 8. A comparison of the coefficients of permeability given in this paper with those obtained by other methods.

Line or lines	Coefficient of Permeability		
	Wenzel	Jacob	Fishel
Grand Island, Neb.			
B		900	864
SW		910	864
W		1,000	937
A		1,020	1,000
N		1,000	1,052
D		1,000	1,192
C		940	983
Average A-C	997	980	992
Average B-D		950	1,028
Average A-B-C-D		965	1,010
Average all lines		965	985
Wichita (pumping rate, 1,000 g.p.m.)			
N			4,795
E			5,528
S			6.223
W			6,206
Average N-S	5,805	5,750	5,509
Average E-W			5,867
Average N-E-S-W			5,688

As would be expected, the three methods give somewhat different values, but it is believed that the results obtained by each of the three methods are within the limits of experimental error. Uncontrollable factors encountered during the pumping test will probably cause greater errors than the method of computing the permeability.

References

(1) Jules Dupuit, Theoretical studies on the movement of water, 275 pp., Paris, 1848.

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(3) Morris Muskat, Flow of homogeneous fluids through porous media, McGraw-Hill Book Co., New York, 763 pp., 1937.

(4) G. Thiem, Hydrologic methods, J. M. Gebhardt, Leipzig, 56 pp., 1906.

(5) L. K. Wenzel, Recent investigations of Thiem's method for determining permeability of water-bearing materials, Amer. Geophys. Union Trans. pp. 313-317, 1932.

(6) L. K. Wenzel, The Thiem method for determining permeability of water-bearing materials, U. S. Geol. Survey Water-Supply Paper 679, pp. 1-57, 1935.

(7) L. K. Wenzel, Methods for determining permeability of water-bearing materials, U. S. Geol. Survey Water-Supply Paper 887, pp. 1-190, 1942.

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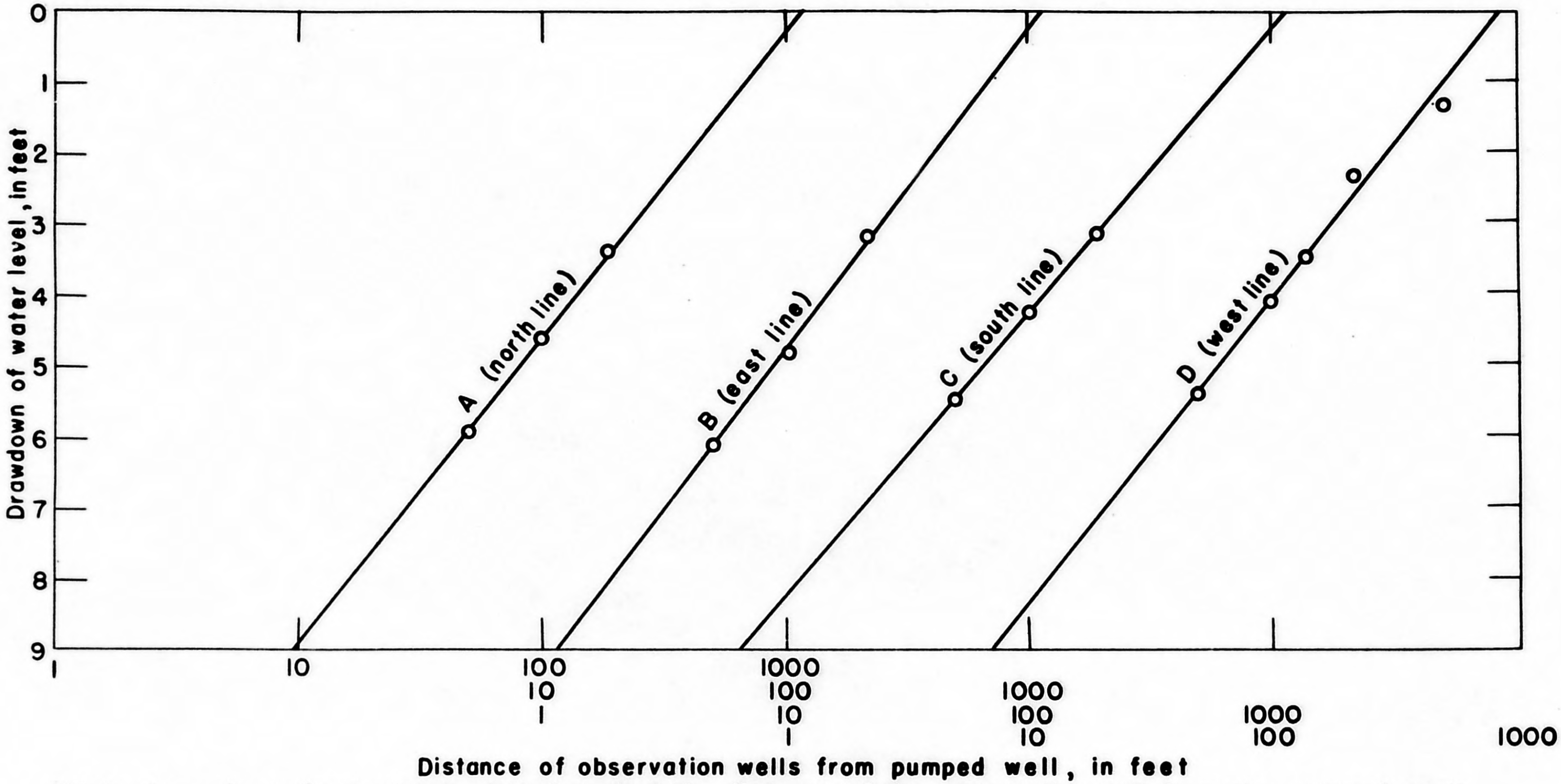

FIG. 2. Draw-down of water levels plotted against the distance of the observation well from the pumped well.

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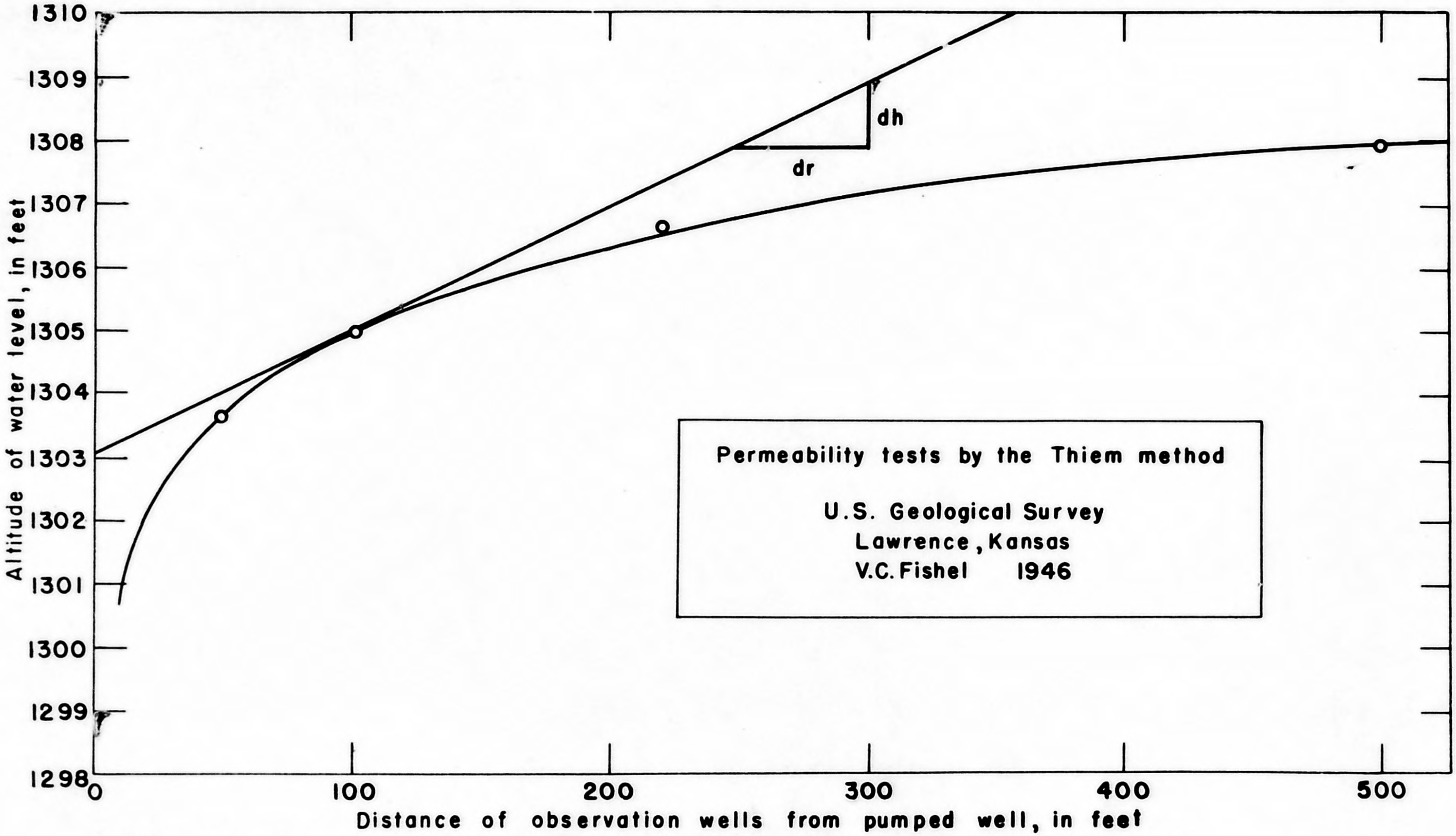

FIG. 3. Draw-down curve for line B. This curve becomes IB on semi-logarithmic paper.

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