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Nonequilibrium Type Curves Modified for Two-Well Systems

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The Theis nonequilibrium formula [a/] is now used generally by the Ground Water Branch in analyzing ground-water movement in the vicinity of a pumping well. However, development of the Theis formula postulates homogeneous formations of wide areal extent and few formations fulfill these assumptions. In order to broaden the scope of its application, the Theis formula can be modified for use in situations where the pumped formation is adjacent to another formation having a considerably different transmissibility.

The modification is made by an application of the image well theory, which can be explained rather briefly. Consider a pumping well in formation A. If it is assumed that this formation is abutted by a relatively impermeable formation B, located at distance a from the pumping well, and the boundary between the formations approximates a straight line, water cannot flow from the impermeable zone to the well. This condition can be simulated by the supposition that formation A is infinite in extent and that an image well is located at distance a from the contact between the two formations and on the opposite side from the pumping well. If the image well begins pumping at the same time and at the same rate as the pumping well, a theoretical ground-water divide is developed along the line of contact between the two formations. Thus, the situation

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a/ Theis, C. V., The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage: Am. Geophys. Union Trans., pt. 2, pp. 519-524, 1935.

found in nature has been duplicated hydraulically by assuming an infinite aquifer and using a pumping image well to give the required boundary effect.

Similarly, if Formation B has a transmissibility much greater than formation A it can be assumed that there is an unlimited source of water from region B. Therefore, the condition is imposed that no drawdown occurs along the line of contact. This can be simulated again by an image well in the same location as before with the exception that the image well must recharge at the same rate as the well that is pumped in formation A.

The net effect of the image well and the pumping well on an observation well in the pumped formation is the algebraic sum of the respective draw-down caused by each well. The basic Theis formula b/ is:

$$s = \frac{114.6Q}{T} W(u) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where

$$W(u) = -0.577216 - \log_e u + u - \frac{u^2}{2x2!} + \frac{u^3}{3x3!} - \frac{u^4}{4x4!} \ldots\ldots$$

and $u = \frac{1.87\ r^2S}{Tt}$

In the above expressions T is the transmissibility in gallons a day per foot under a unit hydraulic gradient; Q is the well discharge in gallons a minute; s is the drawdown in the observation well in feet; t is time in days since pumping began; r is the distance from the pumping well to the observation well in feet; S is the storage coefficient, which is the cubic feet of water released from a vertical prism of the aquifer one square foot in cross-sectional area as the head in the formation is reduced one foot. Writing

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b/ Wenzel, L. K., Methods for determining permeability of water-bearing materials with special reference to discharging-well methods: U. S. Geol. Survey Water Supply Paper 887, p. 88, 1942.

equation 1 for a two-well pumping system, where one of the pumping wells is the image set up to reproduce either the barrier or source type of boundary conditions, gives the following relation:

$$s_o = s_p \pm s_i = \frac{114.6Q}{T}\left[W(u)_p \pm W(u)_i\right] = \frac{114.6Q}{T}\sum W(u) \quad . \; . \; . \; (2)$$

where s_o is the observed drawdown in the observation well, s_p is the component of the drawdown in the observation well caused by the pumping well, s_i is the component of the drawdown in the observation well caused by the pumping of the image well, $W(u)_p$ is the value of $W(u)$ for the pumping well and $W(u)_i$ is the value of $W(u)$ for the image well.

This procedure is similar to the one followed by R. G. Kazmann [c/]. Values of u_p and u_i corresponding to values of $W(u)_p$ and $W(u)_i$, are

$$u_p = \frac{1.87 r_p^2 S}{Tt} ; \quad u_i = \frac{1.87 r_i^2 S}{Tt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (3)$$

u_p and u_i are related as follows:

$$u_p = \left(\frac{r_p}{r_i}\right)^2 u_i; \text{ or } u_p = K^2 u_i \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

in which

$$K = \frac{r_p}{r_i} \quad . \; (5)$$

Thus it should be apparent that a whole family of modified type curves can be constructed where the shape of each curve depends only on the relative position of the observation well with respect to the pumping and image wells.

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c/ Kazmann, R. G., Notes on determining the effective distance to a line of recharge: Am. Geophys. Union Trans., p. 854, December, 1946.

A selection of these modified type curves has been prepared by assuming various values of K and u_p and computing $\sum W(u)$ in equation (2) utilizing equation (4) and the table d/ of values of W(u) and (u). For convenience these modified Theis curves have been prepared on log paper as plots of $\sum W(u)$ versus $\frac{1}{u_p}$. (See accompanying illustration.)

Plotting on log paper each drawdown determined in an observation well, versus the corresponding elapsed time since pumping started, results in a curve analogous to one of the family of type curves. If the formation pumped is of wide areal extent, the curve of observed data is the same curve given by the original Theis type curve, which is shown as the "parent curve" in the middle of the family of modified type curves. However, if conditions near the pumping well approximate those assumed in the image-well theory, the curve of observed data will follow the original Theis type curve until the effect of the image well is felt at the observation well. At that time the curve of the observed data will begin to deviate from the parent curve and follow one of the modified curves.

Thus in solving a simple boundary problem, the plot of the observed data is superimposed on the family of type curves, keeping the axes of the two graphs parallel (that is, maintain the $\sum W(u)$ axis parallel with the s axis and the $1/u_p$ axis parallel with the t axis). When the two graphs are matched, a point common to both can be selected to obtain values of $\sum W(u)$ and $1/u_p$ corresponding with values of t and s. Equation 2 can then be solved for T, and equation 3 can be solved for S (r_p is known). By

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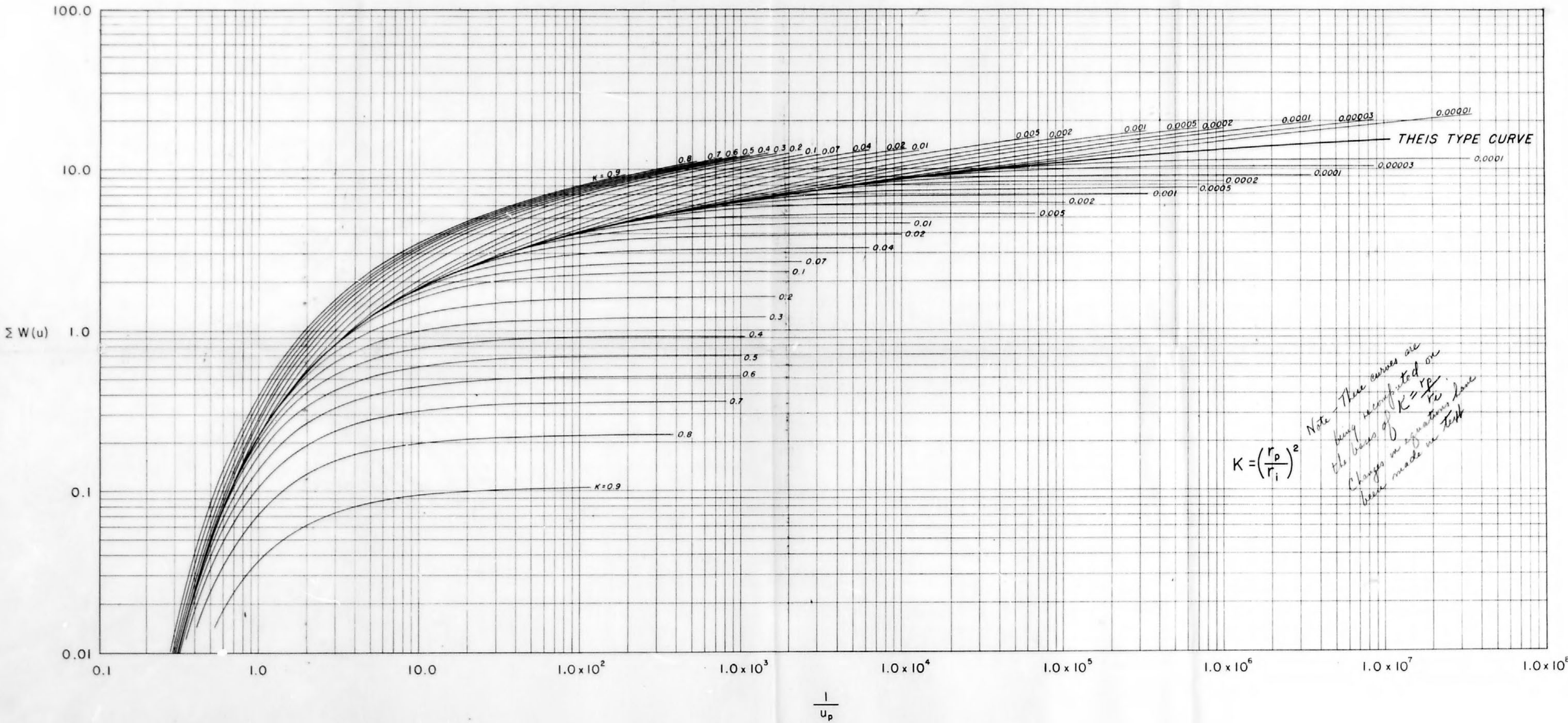
d/ Wenzel, L. K., op. cit., table, p. 89.

noting the particular modified curve that matched the observed data the value of K is found and therefore the value of r_i, as expressed by equation 5, may be computed.

If the position of the contact or boundary line between the two formations is unknown, it can be determined by data from three observation wells, provided the ratio K is sufficiently small and the wells are not too close together. The observation wells must be on the same side of the contact as the pumping well, and for best results K should preferably be less than 0.4 but greater than 0.01.

The determination of the position of the contact or boundary is a simple application of the geometry of the image-well theory. The line of contact between the two formations is midway between the image and pumping wells and oriented in a direction normal to the line drawn through these two wells. The intersection of arcs of circles drawn at radii r_{ai}, r_{bi}, and r_{ci} from observation wells a, b, and c, respectively, determines the position of the image well. The wells should be located so that a sharp intersection of the arcs results.



NON-EQUILIBRIUM (MODIFIED) TYPE CURVES FOR TWO-WELL SYSTEMS