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Use of slide rule in solving ground-water problems involving application of the nonequilibrium formula

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During many ground-water investigations the quantitative phases of the work frequently require application of the Theis nonequilibrium formula. Existing literature on ground-water hydraulics describes in detail the manner in which this formula is to be used in determining the coefficients of transmissibility and storage of an aquifer. However, when quantitative analyses move into the realm of predictions, or treatment of multiple-well systems, the conventional computational processes become unduly burdensome and time consuming. A desire to eliminate repetitious routine in these processes, without sacrificing any accuracy in the final results prompted the development, by the senior author, of a simple modification of the slide rule permitting direct solution of the nonequilibrium formula without recourse to charts or tables.

The basic Theis nonequilibrium formula is given as

$$s = \frac{114.6Q}{T} \int_{\frac{1.87r^2S}{Tt}}^{\infty} \left(\frac{e^{-u}}{u}\right) du = \frac{114.6Q}{T} W(u) \quad \text{where } u = \frac{1.87r^2S}{Tt}$$

This can be represented as:

$s = \frac{Q}{T} F$, in which F is termed the "drawdown factor"

$F = 114.6W(u)$ and is a function of u or $\frac{1.87r^2S}{Tt}$

The value of u, or $\frac{1.87r^2S}{tT}$, can be computed easily on the slide rule. Once this computation is performed the value of W(u) is immediately fixed inasmuch as it is a function of u. Obviously, therefore, it is possible to develop a special scale giving the values of F, the drawdown factor, that correspond to values of u. Thus, instead of reading u on the ordinary slide rule scale, the equivalent drawdown factor F is read on the special scale.

To perform the operation it is only necessary that the value of t on the B scale be set opposite the value of $\frac{S}{T}$ on the A scale. Opposite r on the C scale, ordinarily, there would be read the value of $\frac{r^2S}{t\ T}$ on the D scale. If the D scale were displaced to the left by $\sqrt{1.87}$, there would be read the value of $\frac{1.87r^2S}{tT}$. If instead of using a displaced D scale, there is prepared a special scale of corresponding values of the integral W(u) multipled by 114.6, the drawdown factor can be read directly. As the range of useful values of F is far beyond the limits of the slide rule, it is necessary to have several scales. Two slips have been prepared for attachment to a 20-inch duplex rule, each slip bearing four such scales marked by certain numbers at the ends, the significance of which will be discussed.

In the prepared scales the dimensions to be used, following the usual notation, are:

- Q in gallons a minute,
- T in Meinzer's units---gallons per day per foot,
- r in feet for the scales marked "feet-days" and in miles for those marked "miles-years,"
- S is a dimensionless fraction,
- t in days for the scales marked "feet-days" and in years for those marked"mile-years,"
- s in feet for both sets of scales.

Procedure for use of the slide rule and special scales:

1. Express $\frac{S}{T}$ as 1 to 100 times an even power of 10; i.e., 0.000002 is 2×10^{-6} and 0.00002 is 20×10^{-6}. Set this figure on the A scale of the slide rule, recognizing that the left half of the scale provides for numbers 1 to 10 and the right half 10 to 100.

2. Express t as 1 to 100 times an even power of 10; i.e., 3 days or 3 years is 3×10^{0}, 0.2 days or years 20×10^{-2}, etc. Set this value on the B scale opposite the value for $\frac{S}{T}$ on the A scale.

3. Express r as 1 to 10 times any power of 10; i.e., $2{,}000 = 2 \times 10^{3}$. Find this on the C scale.

4. Opposite the value of r on the C scale read the value of F on the special scales.

To determine which special scale to use when reading the value of F: Substract algebraically the power of 10 used for t from the power of 10 used for $\frac{S}{T}$ (always negative). Add twice the power of 10 used for r, inasmuch as r appears as the square. The result is the number that appears beside the special scale to be used. In other words, the number beside the special scale represents the characteristic of the logarithm of the value obtained by resolving $\frac{r^2S}{tT}$, and is determined by algebraically summing the exponents of the quantities in the expression. This summation of exponents is thus: $(\frac{S}{T} - t + 2r)$. Sometimes, after setting t and $\frac{S}{T}$, it is necessary to shift the slide in order to read the value of r. This is equivalent to going up or down two powers of 10. If the slide must be shifted to the Right, Rise vertically one scale (the new scale will therefore be 2 larger in absolute value and 2 less in algebraic value than the old). If the slide must be shifted to the Left, Lower vertically one scale.

The chief value of the slide rule over charts and tables is the ability to read quickly the values for drawdown at a great many distances for a given aquifer at a given time. In this case after the first drawdown is computed the proper scale is either indicated or easily chosen for all other values of r. For a time it may be desirable to check against charts the values found by slide rule, as the matter of working with exponents of 10 may prove to be a little disconcerting when the mind is endeavoring to concentrate on the main problem.

The special scales, printed on plasticized paper, are available by writing to the Albuquerque, N. Mex., ground-water district office. In their present form they are for attachment only to a 20-inch duplex-type slide rule and are to be regarded as of a temporary or preliminary nature pending more widespread field testing of their accuracy and suitability for use. The Albuquerque office will welcome any corrections, criticisms, and suggestions directed toward improving these scales. If there is sufficient demand, it is contemplated that after an appropriate trial period the scales can then be more precisely reproduced in suitable permanent form. Extra copies of this article may be obtained by writing the Washington office.