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MEASUREMENT OF EARTH PRESSURES IN THE IRON-ORE
MINES OF EASTERN, FRANCE

by

M. E. Tincelin

Translated by Mrs. Séverine H. Britt

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MEASUREMENT OF EARTH PRESSURES IN THE IRON-ORE
MINES OF EASTERN FRANCE

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I. INTRODUCTION

The purpose of the tests, which will be reported later, is to measure as precisely as possible the tangential stresses exerted on the periphery of a gallery cut in the Lorraine iron ore.

The values of the tangential stresses along the periphery being known, they will be compared with the values that would be given by the computations based on various hypotheses, that of elasticity, perfect plasticity, masses with internal friction, irreversibility of stresses, stratified masses, etc.

This will make it possible afterwards to determine with certainty which hypothesis is to be used for each particular case. If the end was fully attained, strictly speaking there would no longer be any hypotheses - only certainties.

It would therefore be possible to safely compute the forms and dimensions of all the underground excavations to be dug in a given material.

Research to this effect was pursued on the walls of the galleries and workings dug in the ore.

The measurements of the tangential stresses around a gallery were made by means of the flat jack test, the principle of which is briefly summarized later.

Let us take, for instance, a wall of a gallery dug in any rock, and subject to compression. A horizontal cut is made in this wall. This cut frees the rock below and over it from a part of the stresses which the rock was subject to prior to the cutting. This freeing from stresses brings about deformations of the rock which are measured by means of extensometers. These deformations are elongations.

A flat jack, Freyssinet type, is inserted in the cut. The pressure within the flat jack is raised until the deformations undergone by the rock owing to the pressure induced by the jack (deformations which are shrinkings) cancel the elongations caused by the cut. It is then assumed that the pressure induced by the jack, and which brought about this cancellation, is that which prevailed in the ground prior to the cut.

This method of measuring was conceived jointly by the engineers of the Laboratoires du Bâtiment et des Travaux Publics and the author. Its improvement was carried out in the Iron Mines, and some modifications were made. In the interpretation of the results, help and valuable advice were given by the Laboratoires du Bâtiment et des Travaux Publics.

Indeed, the tests did not give in the field all the results expected. The interpretation of divergences had to be made, and the method of measuring had to be modified and completed.

Remark - A description of the flat jack, Freyssinet type, is given in an appendix.

II. GENERAL CONDITIONS OF THE SITE ^HWERE THE TESTS WERE MADE

The strata where the measurements were made are at a depth of approximately 200 m.

They are a part of a ferruginous formation about 50 m thick where only the most mineralized deposits are mined. Consequently, there is no very clear separations between the layers and the wall-rock except in some places. These layers have a very slight dip, 1° to 4° .

This formation is of sedimentary origin and of Aalenian age Upper Liassic⁷. It is overlain by about 40 m of micaceous marl, also of Aalenian age. Over the marl, 140 m of Upper Bajocian limestone is more or less covered by Upper Bajocian marl (Gravelotte marl). Depending on state of erosion, this marl is from 0 m to 40 m thick.

These layers are mined by the classical method of rooms and pillars, with subsequent pulling of the pillars and caving of the roof.

The gray layer where the measurements are made is about 4.50 m thick. It is a calcareous layer. A chemical analysis made near the site of measurements has given the following results:

Fe	34.0%	CaO	14.93%	SiO ²	7.87%
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The compressive strength measured under the press on cubes taken in that layer varies from 486 to 740 kg/cm². The tensile strength varies from 38 to 51 kg/cm².

Measurements of the modulus of elasticity gave:

(1) By measuring the speed of sound on the ore in place: 207,000 to 440,000 kg/cm².

(2) With the jack test on the ore in place: 150,000 to 230,000 kg/cm².

(3) By measuring the deformations undergone by a cube of ore: 170,000 to 200,000 kg/cm².

(4) By measuring the frequency of vibrations of the samples: 335,000 to 495,000 kg/cm².

Poisson's ratio measurements have given:

(1) By measuring the speed of sound in the ore in place: 0.34 to 0.26.

(2) By measuring the deformation undergone by a cube of ore: 0.2.

In addition, all these measurements have proved that:

(1) The ore taken in that layer and in the section where the measurements are made has an almost elastic behavior between 10 and 140 kg/cm² for pressures of small duration.

(2) The rock under consideration is somewhat heterogeneous and anisotropic.

(3) Consequently, a measurement of the tensions of the wall of a gallery, using coefficients of elasticity E and σ will immediately be subject to uncertainties which may be as much as 50%.

III. THE TESTS AND THEIR RESULTS

In the course of the first tests, we tried to measure the stresses on the surface of a gallery wall by using only one horizontal cut under which the deformations resulting from the releasing of the stresses by the cut were cancelled by the pressure of a flat jack inserted in the cut, according to the method conceived jointly by the Laboratoires du Bâtiment et des Travaux Publics and by the author's organization.

The results obtained by means of one cut only will first be described and discussed. A cut 75 cm to 80 cm wide, 4 cm high and 75 cm deep will be considered. A flat jack, Freyssinet type, 70 cm x 70 cm is inserted in this slot cut horizontally in the vertical wall of the gallery. The jack is sealed with high-alumina or quick-setting cement. The cuts are made by drilling holes side by side. This method is long and difficult, but up to now nothing better has been found. Care must be taken not to disturb or break the rock to be tested. Augers were used in preference to percussion drills because the vibrations of the latter frequently cause the dislocation of the rocks next to the cut.

To make such a cut, and to seal the flat jack, connect the pump, etc., 2 men working 10 to 12 hours are required. The pressure cannot be applied to the flat jack until about 8 hours after the end of the sealing. In fact, the cement used for the sealing is very liquid, otherwise the insertion of the jack is impossible. The setting of the cement, or rather the absorption of the excess mixing water requires a certain time.

If the pressure is applied too soon, the jack deforms until all the mixing water is expelled, resulting usually in bursting or deformation of the jack, which make its further use impossible. In this connection, it will be mentioned that the bursting of the jack is absolutely without danger. However, care must be taken to expel the air from the jack and the pipes. This is done by bringing the jack under slight pressure without making the various fittings water-tight. Leakages are thus obtained where water and the air contained in the jack and the pipes escape under pressure.

This operation, which is made after the sealing of the jack, is arrested when the whistling due to escape of air stops.

With a new jack accurately sealed, the pressure can easily be raised up to 200 kg/cm^2 . A jack may be used about ten times but 150 kg/cm^2 can no longer be exceeded without any risk of bursting. After being used three times the jack must be repaired after each test to restore the shape and strengthen the welds.

The jacks are always recovered; this operation is difficult and requires four working hours of a skilled man.

A. Description of the test made with
only one horizontal cut

The test as a whole requires the following operations:

1) Selection of the site:

Clean the wall by making it nearly even and smooth where the cut is to be made and the extensometers set.

A very fissured zone should not be chosen, nor a place where the section of the gallery shows an anomaly or a discontinuity (bend, widening, vicinity of niche, of a shaft, etc.).

The selected even face must be parallel to the axis of the gallery (a cleavage plane should not be taken if it cuts the axis of the gallery).

The test can be made in any place of the section, even at the roof or the wall of the gallery, provided that it is subject to compression and not to tension.

The direction of the deformations obtained after the cut is made gives information in that respect. It will be recalled that the result obtained concerns the place where the cut was made and not the place where the extensometers are set.

2) Setting the extensometers and reading:

Seal the clamps if vibratory strings are used, or the plugs if dilatometers are used, either above or below the jack. It will be considered later where they should properly be set.

Let the sealing cement set. With the high-alumina or quick-setting cement, the setting takes about eight days.

A first reading is made on all the extensometers. The reading gives the initial state.

3) Cutting the slot and reading:

Protect the extensometers by a casing against the risk of shocks resulting from the cutting of the slot and sealing of the jack.

Cut the slot if possible with augers rather than percussion drills.

Make a second reading on the extensometers. This reading gives, by difference with the first reading, the deformations resulting from cutting the slot.

Make sure that the flat jack can be inserted easily in the cut.

4) Sealing of the flat jack:

Rinse the cut. The high-pressure pump is used for that work.

Fill in the cut with a very liquid cement, tamping it with a tamper. A mixture of 50 percent of cement and 50 percent of fine sand without gravel is used.

Push the flat jack into the cut. The surplus cement is expelled out of the cut.

Let the cement set for about ten hours.

Unless cement-injection equipment is available, it is not advisable to place the jack in the cut prior to the introduction of the cement. The latter method leads to the creation of very harmful voids around the

jack, which contribute to bursting of the jack in the course of measurement -- this is irremediable.

Connect the pump.

5) Putting the jack under pressure:

Raise the pressure by successive increases of 5 kg/cm^2 or of 10 kg/cm^2 .

After each increase, make a reading on the extensometers.

Exceed, if possible, by 20 to 30 kg/cm^2 , the pressure that cancels the deformations on the extensometer closest to the jack. As a rule, for the ore tested by the author, the pressure was always raised to 150 kg/cm^2 .

Return to zero, again step by step, and start anew a second cycle.

6) Recovery of the jack:

Remove the extensometers.

Cut a new slot parallel and as close as possible to the first one. This new slot is deeper and wider than the first one. But it is made without care, that is, without being rigorously rectilinear, nor perfectly continuous. It is cut below the jack.

The pressure is slightly raised in the jack. The purpose of this operation is to break the rock under the jack.

Drill, 50 cm below the jack, two holes inclined on both sides of the vertical symmetry axis of the jack. These two holes are about 1 m deep. Blast them with a minimum of explosives. Owing to the cut, the jack undergoes no damage. Get the jack out by means of gravers and pliers.

B. Results obtained

1) Tests made with two extensometers located one under the other on the vertical symmetry axis of the jack:

Among numerous results, here are two quite significant tests made in rectangular galleries with rounded corners 5 m wide and 4 m high.

First test. -- Made under a jack 70 cm wide, inserted in a cut of practically the same width (fig. 3).

String 1, located vertically along the central axis of the jack between two points sealed at elevations 21 and 41 cm under the jack, has cancelled its deformation at a pressure in the jack of $78.5 \text{ kg/cm}^2 = P_1$.

String 2, straight below string 1, but between two points sealed at elevations 45 and 65 cm, has cancelled its deformation at $68 \text{ kg/cm}^2 = P_2$.

Second test. -- Made in another place under the same jack; the strings are sealed at the same elevations as previously (fig. 4).

String 1 has cancelled its deformation at a pressure in the jack of $94.5 \text{ kg/cm}^2 = P_1$.

String 2 has cancelled its deformation at a pressure in the jack of $82.5 \text{ kg/cm}^2 = P_2$.

Remark -- All the tests made with several strings located on the same vertical have given results comparable to those mentioned. It is thus found that, in each instance, the pressure which brings about the cancellation of the deformations of the strings closest to the flat jack is stronger than that which cancels the deformations of the farthest string, and the ratio $\frac{P_1 - P_2}{P_2} \cdot 100$ is close to 15 percent.

Translator's note: The positions of P_1 and P_2 in figures 3 and 4 should apparently be exchanged to agree with the text.7

2) Tests made with several extensometers sealed at the same elevations
with regard to the jack:

A test made with a flat jack 70 cm wide, set horizontally, and under which four strings were each sealed between the elevations 20 and 34 cm (fig. 5) gave the following results:

String 1 has cancelled its deformation at a pressure in the jack of 82.5 kg/cm^2 .

String 2 has cancelled its deformation at a pressure in the jack of 86.5 kg/cm^2 .

String 3 has cancelled its deformation at a pressure in the jack of 117 kg/cm^2 .

String 4 has cancelled its deformation at a pressure in the jack of 175 kg/cm^2 .

The explanation of this phenomenon will be given later. This phenomenon corresponds to a stress field which does not have a vertical axis of symmetry.

Remark -- On the other hand, other tests made under the same conditions have given comparable results for strings sealed symmetrically with regard to the vertical axis of symmetry of the flat jack. In this instance, the earth pressure was vertical.

3) Results obtained by varying the width of the cut, the width of the jack remaining constant:

Let a jack, Freyssinet type, 70 cm wide, be inserted in a horizontal cut of about the same width. Under the cut, two vibratory strings have been sealed, one under the other, on the vertical axis of symmetry of the jack, between points located at elevations 21 and 41 cm, and at elevations 45 and 65 cm.

By raising the pressure in the jack, string 1 has cancelled its deformation at a pressure in the jack of 94 kg/cm^2 , and string 2 at a pressure in the jack of 83 kg/cm^2 .

The rock was left at rest for two days, then the cut was widened by 15 cm at both sides of the jack. The cut was consequently 1 m wide. The jack had not moved. The widened sections of the cut remained empty (fig. 6).

The deformations 0 - 1 and 0 - 2 correspond to the deformations resulting from the cutting of the 70 cm wide slot.

The deformations 0 - 1' and 0 - 2' correspond to the deformations resulting from the cutting of the slot widened to 1 m.

String 1 underwent an increase of deformation considerably less than that of string 2.

By again raising the pressure in the jack, string 1 cancels its deformation at a pressure in the jack of 108 kg/cm^2 , and string 2 at a pressure of 102 kg/cm^2 .

Remarks -- 1) Owing to the widening of the cut, the pressure of cancellation of the deformations given by string 1 has increased about 15 percent and for string 2 about 23 percent.

2) Prior to the widening, the ratio $\left(\frac{P_1 - P_2}{P_2}\right) 100 \approx 15$ percent.
After the widening the ratio $\left(\frac{P_1 - P_2}{P_2}\right) 100 \approx 6$ percent.

C. Phenomena related to the tests

1) Repeated putting under pressures: cycles stress/strain. Creep.

The method assumes the reversibility of the stresses and deformations. It is assumed that the body behaves always in the same way between each stage of pressures. This does not necessarily imply the hypothesis of elasticity, because the deformation may very well not be linear.

Now, the tests show that the curves stresses/strains are different from one cycle to another and that the modulus of elasticity in compression increases with the number of cycles. Later it has been found that the modulus of elasticity in expansion is always greater than that obtained by compression. In other words, displaced cycles are obtained, such as those shown in fig. 7.

If it is so, the second cycle, which is made under the jack, must bring the cancellation of the deformation at a pressure lower than that of the first cycle.

Here are the results of the first tests:

Pressure which cancelled the deformation 1st cycle	Pressure which cancelled the deformation 2nd cycle	Percentage of difference
kg/cm ²	kg/cm ²	%
87	80	- 8
79	73	- 8.9
82	80	- 2.5
87	79	- 9.2
38	35	- 7.9
35	35	0
94	79	- 16
82.5	78	- 5.4
125	125	0
75	61	- 18.7

Two tests which gave percentages of difference of - 23 percent and - 41 percent were discarded. The writer has good ground to believe that these tests were aberrant, indeed for other reasons.

The average percentage of difference is - 7.6 percent.

These tests demonstrate that the irreversibility of stresses and strains exists almost always in the ore subjected to the test.

One should not forget that the cycles so accomplished are obtained by instantaneous loading, not maintained in time.

A cycle such as OBA is then obtained. If a new cycle is started again immediately AB'A' is obtained.

But on the other hand, if two or three days elapse after the first cycle is made, it is frequently noted that point A has a tendency to merge again into point O. Consequently, creep of the rock is occurring.

By making a new cycle, it tends to mingle with cycle OBA. Likewise, by maintaining the stress giving point B, this point is displaced with time on a parallel to the x axis passing through B.

The straight OB becomes inclined and the modulus of deformation, or modulus of elasticity decreases; in other words, a measurement made under a lasting pressure brings about a decrease of the cancellation pressure of the deformations obtained under an instantaneous pressure.

In the calcareous and resistant rocks where the tests were made, it seems that the influence of creep may be disregarded, considering that the tests last approximately from 16 to 24 hours, and that the deformations caused by the cutting of the slot increase during that time from 5 percent to 8 percent in relation to their initial value.

Consequently, due to creep an error in excess of the same order occurs.

During this same lapse of time, the creep has also interfered to reduce the phenomenon of stress irreversibility; in other words, in this particular case and in calcareous minerals, it seems that by neglecting the irreversibility of stresses and strains, as well as the creep, no error higher than \pm 8 percent is made.

On the other hand, in some rocks, the precision may decrease considerably.

2) Results obtained with double vibratory strings:

At the start, all the tests were made by using vibratory strings as extensometers. There were two vibratory strings stretched between each pair of sealing pins.

When the slot was cut, the strings A and B underwent an elongation. It was always found, and this without any exception, that string A underwent a larger elongation than string B (fig. 8).

Thus, it seems that the area surrounding M is more acted upon than that directly at the surface, that is close to O.

When the pressure rises in the jack, the elongation of the strings A and B is cancelled by a certain value of the pressure. But the increase of pressure never succeeds to equilibrate the deformations on the strings A and B. The initial difference is always maintained quite roughly, and at times is even increased; the cancellation pressure of the elongations is consequently higher in M than at point O.

The following table gives the results of the tests made with double vibratory strings.

Elongation of string A	Elongation of string B	$\frac{(\Delta A - \Delta B)}{\Delta B} \times 100$	Cancellation pressure in O	Cancellation pressure in M	$\frac{(P_M - P_O)}{P_O} \times 100$
μ/m	μ/m	%	kg/cm^2	kg/cm^2	%
+ 500	+ 402	24	82	120	46
+ 256	+ 100	156	88	122	45
+ 330	+ 80	310	80	110	37
+ 528	+ 492	7	95	141	48
+ 312	+ 163	91	79	117	48
+ 654	+ 504	30	69	105	38

Three tests were eliminated, whose results are obviously unreliable, but which, however, give a $\Delta A > \Delta B$ and $P_M > P_O$.

On the average, a cancellation pressure near M is found 43.5 percent higher than at the surface, that is near O.

It is rather puzzling to explain this phenomenon, which is not due to the bending of the sealing pins.

These pins have a 400 mm^2 (20 x 20) section. Moreover, as the pressure in the jack increases, if the pins bent, this condition would little by little come to an end, and at a given time the deformations on the two strings A and B would be equilibrated. Yet, it is never the case.

One can also allege that the ore in the immediate vicinity of the gallery has lost its elasticity, either through plasticity, or through perturbations resulting from blasting at the time of the excavation of the gallery. First of all, prior to making a test, the wall is stripped as far as possible, either with the pick or the gad, or with the pick-hammer, to a depth of 20 to 40 cm.

One should consequently assume that in each test the ore has recuperated its elastic properties 15 cm from the place where the stripping was stopped. The stripping is stopped not only when the solid is attained, but when a vertical plane surface parallel to the axis of the gallery is reached. These conditions frequently make it necessary to strip or cut into the solid rock excessively. Yet the same variation of the deformations and cancellation pressures is always found. Lastly, the stress/strain curves taken at point O and point M show that the moduli of elasticity and of deformation are practically equivalent. Consequently, it seems that this explanation is not valid.

The following hypotheses can be made:

(1) Granting that the tested material, whether viscous or pulverulent, possesses the characteristics of media with internal friction, the law of variation of the vertical stress undergone by the material is different from that obtained for an elastic material (fig. 9).

In an elastic medium the vertical stress is maximum at the edge of the gallery and then tends to decrease.

In a non-elastic medium, the vertical stress tends to increase at first, goes to a maximum, then decreases.

It can thus be assumed that the ore under consideration follows the rules of the non-elastic media. It is possible.

But the 40 percent variation of the vertical stress within a 15 cm depth allows one to assume that the maximum is always near the wall of the gallery (15 to 30 cm). The disturbed and non-elastic zone is consequently very thin.

If this zone is removed, the elastic medium should be found. Yet, it is not the case. After removing 20 to 30 cm of material, the same law of variation is found again, with about 40 percent increase of the vertical stress at a depth of 15 cm. Considering these results, this hypothesis was not retained, especially because the figures given by the tests at point M considerably exceed the value of the tangential stress given at the surface by the photoelastic tests.

(2) It can also be assumed that the cut is not deep enough and that the release of stresses does not develop as well in depth as at the surface. But the release of stresses occurs more energetically in depth than at the surface, seeing that string A stretches out more than string B. Consequently, this phenomenon is not to be taken into consideration.

(3) The rock near the gallery undergoes an increase of compression which brings about a flexion tending to bend strips of ore delimited by natural fractures (mine veins or cleavage plane), or produced by the stresses to which the rock is subjected. These strips of ore line the vertical walls of the gallery, and eventually break loose when the pressure is sufficient (fig. 10).

It is believed that this last hypothesis is the right one. Evidently, one cannot assert with certainty that this point of view is correct. However, it is based on other results which do not belong to the present study, and which confirm this hypothesis (measurement of the speed of sound propagation, complete cutting all along one section of the gallery, etc.).

Each strip of ore behaves thus independently of the next one, but each one would be more compressed on the face toward the gallery. The variation law or graph of the pressure would consequently have a "saw-tooth" behavior (fig. 11).

Supplementary tests will be made to attempt to delineate, if possible, this method of working by vertical ore slice.

In any case, the measurement obtained at the surface is the one sought, but, knowing the value of this surface stress, one cannot get an immediate idea of the distribution of the stresses in depth. It seems at least to be rather aleatory in the present state of the tests.

IV. INTERPRETATION OF RESULTS

We will proceed in the attempt to analyze the results of the tests described in the previous paragraphs, and to show how to modify the flat jack test in order to make a more precise and particularly a more complete measurement.

In consequence of all the tests we have just described, we raised the following questions:

(1) How far from the cut should the extensometers be sealed, the results being different according to this distance?

(2) How to make the non-verticality of the principal stresses come into play?

(3) Should one use a big or a little jack?

(4) Should the slot be of a dimension just sufficient to insert the jack or should it be larger? The results vary, in fact, with the dimension of the slot for a given jack.

Neglecting or omitting to take these four factors into consideration leads to errors.

In addition, we know that we disregard creep and the irreversibility of stresses. In other words, from the start we acknowledge in our case an error of ± 7 percent to 8 percent.

The question which preoccupies us is the following:

What is the precision of the measurement using the flat jack sealed in only one horizontal cut?

To answer this question, we have attempted to explain by calculation the phenomena which occur when the slot is cut and when the pressure is raised in the jack.

We will start from two hypotheses:

(a) We will grant that the ore behaves according to the laws of elasticity.

(b) We will grant that the cut, having the shape of a very elongated slot, may be compared to an ellipse whose long axis equals the width, and small axis equals the height of the slot.

The computations will give us an approximate idea of the way the stresses and deformations react during the test. It is not a question of finding quantitative laws, but rather qualitative laws. It will be seen that the qualitative laws found by computation govern the flat jack test perfectly.

The deformations can then be calculated starting from the tensions present in the mass prior to the opening of the elliptic cut. When a flat jack is introduced within the cut, the jack produces a uniform pressure on the lower and upper edges of the horizontal ellipse; but it cannot press on the vertical edges of the ellipse (fig. 12), that is in the directions of arrows A and B.

We consequently have a uniform pressure only between points S and R, and a practically null pressure at points A and B.

To calculate the stresses and deformations produced in the rock along the y axis, we can compare the system to a load evenly distributed over an infinite edge, the AB edge. This comparison is about right for the points on the y axis, but it cannot be used in calculating stresses near the points S and R.

The development of the calculations corresponding to the chosen hypotheses will be found in the appendix.

These calculations give the stresses N_x and N_y which result from the cutting of the slot at points b_1, b_2, b_3, b_n , located on the y axis.

The resulting difference of stress is obtained easily. In fact, prior to the cutting of the slot, the rock was subjected to a compression P in a vertical direction, and to a compression KP in a horizontal direction. K is the coefficient of transmission of stresses:

$$K = \frac{\sigma}{1 - \sigma}$$

σ is the coefficient of Poisson of the tested material.

The stress difference between this initial state which has just been defined and the state resulting from the opening of the cut takes this initial state into account.

This difference is

$$\Delta N_y = -P + N_y;$$

$$\Delta N_x = -KP + N_x.$$

One then goes easily to the corresponding deformations

$$e_y = \frac{\Delta N_y}{E} - \sigma \frac{\Delta N_x}{E} = \frac{P}{E} (M_n)$$

E is the modulus of elasticity of the ore;

M_n , an integral or fractional number given by the calculation for points $b_1, b_2, b_3, \dots b_n$.

P, the initial vertical pressure which we try to measure.

The calculation is also made of stresses N'_y and N'_x on the y axis at points $b_1, b_2, b_3, \dots b_n$, produced by the pressure provided by the flat jack; this pressure is taken as equal to P.

The corresponding deformations are also obtained

$$e'_y = \frac{N'_y}{E} - \sigma \frac{N'_x}{E} = \frac{P}{E} (M'_n)$$

M'_n being an integral or fractional number given by the calculation for points $b_1, b_2, b_3, \dots b_n$.

The test consists in cancelling the deformations which result from the cutting of the slot by means of those obtained by raising the pressure in the jack.

In other words, at each point $b_1, b_2, b_3, \dots b_n$, we make

$$e_y + e'_y = 0$$

In absolute value, this gives

$$|e_y| = |e'_y|$$

or

$$\frac{P}{E} (M_n) = \frac{P}{E} (M'_n)$$

or

$$(M_n) = (M'_n)$$

Now the calculation shows that (M_n) is almost always different from (M'_n) when P in the ground is equal to P in the jack.

Consequently, if during the test the pressure is raised so as to obtain the cancellation of the deformations, the cancellation pressure P' will be different from the pressure originally present in the ground P.

The value of the ratio $\frac{P}{P'}$, at each point of the y axis will be determined in the following cases:

- 1) Jack, 0.40 m wide, sealed in a cut of same dimensions.

Case where Poisson's coefficient:

$$\sigma = 0.1$$

$$\sigma = 0.2$$

$$\sigma = 0.33$$

$$\sigma = 0.5$$

- 2) Jack, 1 m wide, sealed in a cut of same dimensions.

Case where Poisson's coefficient:

$$\sigma = 0.1$$

$$\sigma = 0.2$$

$$\sigma = 0.33$$

$$\sigma = 0.5$$

- 3) 1 m wide cut in which a narrower jack has been sealed.

The error inherent to this method of testing will thus be determined and also the factors which have to be modified to reduce this error.

Finally, we will see if the phenomena brought out by calculation are found also during the tests in the field.

1) Interpretation of results obtained on extensometers sealed on the vertical, but at different elevations.

Curves showing the difference of deformations along the y axis after cutting the slot are given below. On these curves, traced in solid lines, curves in dashed lines were superposed, which represent the difference of the deformations obtained by raising the pressure in the jack to a value P equal to that present in the mass prior to the cutting of the slot.

The curves of fig. 13 correspond to a 0.40 m jack and a Poisson's coefficient of 0.2.

The curves of fig. 14 correspond to a 1 m jack and a Poisson's coefficient of 0.2.

In abscissae are the distances measured along the y axis and evaluated as a function of L, width of the cut.

In ordinates are the deformations or rather the quantities M_n . To obtain the deformations, M_n has to be multiplied by $\frac{P}{E}$.

It is seen that the curves in solid lines and the curves in dashed lines cross each other at only one point B. For some values of σ , there is another intersection point A on the axis of the ordinates.

At these two points, the measurement with the flat jack gives the initial pressure quite well. But at the other points, one immediately sees that this is not so.

By gradually increasing the pressure in the jack, it is seen that the intersection points of the two curves are displaced.

At the intersection point, one has:

$$\frac{P}{E} (M_n) = \frac{P}{E} (M'_n)$$

Because M_n and M'_n are not equal except at the two points A and B, P has to be different from P', the pressure in the jack.

Consequently, when by increasing the pressure in the jack, the deformations produced by the cutting of the slot are cancelled, the value of the pressure present in the ground prior to the cutting of the slot is obtained only at the intersection point. Everywhere else an error is made, which we will attempt to evaluate.

It is impossible to foresee the exact location of the intersection point, the location of which varies with the value of Poisson's coefficient. Besides, the calculations are exact only within the framework of the chosen hypotheses, that is: elasticity of the rock and elliptic cut. The reality is certainly so different that the location of the intersection points cannot be computed.

However, to understand the variation of the error made by cancelling the deformations at the various points of the y axis or vertical axis of symmetry of the jack, the values of the following ratio were calculated:

$$\frac{P}{P'} = \frac{\text{pressure which was present in the ground}}{\text{pressure in the jack which cancels the deformations}}$$

along the y axis, at points $\frac{L}{10}, \frac{L}{5}, \dots, L$, the width of the cut being L. The results obtained were summarized in two tables.

- (1) Case of a 0.40 m wide jack sealed in a cut of same width (table I).
- (2) Case of a 1 m jack sealed in a cut of same width (table II).

Table I

0.40 m jack	$\frac{P}{P_1}$	$\frac{P}{P_1}$	$\frac{P}{P_1}$	$\frac{P}{P_1}$
$\sigma =$	0.1	0.2	0.333	0.5
$y = 0$	0.98	0.98	0.95	0.83
$\frac{L}{10}$	0.98	0.97	0.95	0.87
$\frac{L}{5}$	0.99	0.95	0.95	0.88
$\frac{L}{3.33}$	1.01	0.99	0.97	0.92
$\frac{L}{2.5}$	1.08	1.05	1.03	0.98
$\frac{L}{2}$	1.17	1.13	1.11	1.06
$\frac{L}{1.66}$	1.27	1.23	1.21	1.17
$\frac{L}{1.43}$	1.40	1.34	1.30	1.26
$\frac{L}{1.25}$	1.51	1.47	1.43	1.39
$\frac{L}{1.11}$	1.63	1.58	1.56	1.52
L	1.77	1.68	1.67	1.65

Table II

1 m jack	$\frac{P}{P'}$	$\frac{P}{P'}$	$\frac{P}{P'}$	$\frac{P}{P'}$
$\sigma =$	0.1	0.2	0.333	0.5
$y = 0$	0.99	0.99	0.98	0.92
$\frac{L}{10}$	0.98	0.98	0.94	0.89
$\frac{L}{5}$	1	0.98	0.93	0.88
$\frac{L}{3.33}$	1.04	1.01	0.98	0.93
$\frac{L}{2.5}$	1.10	1.08	1.02	1.005
$\frac{L}{2}$	1.20	1.17	1.11	1.08
$\frac{L}{1.66}$	1.31	1.27	1.24	1.18
$\frac{L}{1.43}$	1.44	1.39	1.37	1.30
$\frac{L}{1.25}$	1.58	1.52	1.48	1.38
$\frac{L}{1.111}$	1.71	1.69	1.61	1.46
L	1.87	1.83	1.73	1.53

The following conclusions are drawn from these two tables:

(a) The extensometer the farthest from the jack will cancel its deformation at a pressure in the jack smaller than that which cancels the deformation of an extensometer closer to the jack. This statement has been verified by the tests on the rock (cf. III-B-1).

(b) The pressure cancelling the deformations which was recorded by an extensometer distant from the jack is more subject to errors than that recorded by an extensometer close to the jack.

(c) With a jack sealed in a cut of same dimension as the jack, the most favorable zone where the extensometers must be sealed is that close to $L/3.33$, whatever the dimensions of the jack may be.

Theoretically, in that zone the error is smaller than 10 percent, whatever the value of Poisson's coefficient may be.

In other words, an extensometer sealed between 20 and 40 cm, for instance, on the vertical symmetry axis of a 40 cm wide jack will give useless results. But an extensometer sealed between 20 and 40 cm on the vertical symmetry axis of a 100 cm wide jack will give good results.

Consequently, the dimension L of the jack (sealed in a cut of same dimension) has no influence on the measurements providing, however, that the extensometers are sealed on the vertical symmetry axis of the jack and at elevations, with regard to the jack, close to $L/3.33$.

2) Interpretation of the results obtained with a jack of dimensions different from those of the cut.

The stress distribution has also been calculated in the case where the cut is larger than the flat jack sealed into it.

(a) Cut 1 m wide in which a 0.80 m flat jack is sealed. Poisson's coefficient = 0.2.

(b) Cut 1 m wide in which a 0.70 m jack is sealed. Poisson's coefficient = 0.2.

In fig. 15, the curves obtained when the jack had a 1 m width identical to that of the cut were first plotted.

Curve 1 represents the difference of the deformations obtained by the opening of the 1 m cut.

Curve 2 represents the difference of the deformations obtained by the stresses induced by the 1 m jack when a pressure P in the jack is identical to that which prevailed in the ground.

Moreover curves 3 and 4 have been superposed on the two preceding curves.

Curve 3 represents the difference of deformations obtained by the stresses induced by the 0.80 m wide jack sealed in the 1 m cut when a pressure P in the jack is identical to that which prevailed in the ground.

Curve 4 refers, under the same conditions, to a 0.70 m wide jack.

We see that curves 1 and 2 are displaced from one another. As a result, the errors of measurements may be great.

Curves 1 and 3 are closer to one another, and the errors decrease.

Curves 1 and 4 also approach each other, and the errors again decrease.

We have calculated as previously the ratios P/P' . The letter L refers to the width of the cut which remains constant and equal to 1 m.

Table III

	P/P' when the jack is 1 m wide	P/P' when the jack is 0.80 m wide	P/P' when the jack is 0.70 m wide
0	0.99	0.99	0.99
$\frac{L}{10}$	0.98	0.98	0.98
$\frac{L}{5}$	0.98	0.97	0.96
$\frac{L}{3.33}$	1.01	0.98	0.95
$\frac{L}{2.5}$	1.08	1.02	0.97
$\frac{L}{2}$	1.17	1.07	1
$\frac{L}{1.66}$	1.27	1.12	1.03
$\frac{L}{1.43}$	1.39	1.21	1.11
$\frac{L}{1.25}$	1.52	1.32	

The conclusions to be drawn are:

With the 1 m jack, the place where the extensometers can be sealed with less than 10 percent of error extends from 0 to $\frac{L}{2.5}$.

With the 0.80 m jack, it extends from 0 to $\frac{L}{2}$.

With the 0.70 m jack, it extends from 0 to $\frac{L}{1.66}$.

By lessening the width of the jack in relation to that of the cut the extensometers will cancel their deformation at higher pressures in the jack than those obtained when the jack and the cut had the same dimensions.

Likewise, the extensometers next to the jack will show, at the cancellation of the deformations, a smaller increase of pressure than that shown by the more distant extensometers. The increase was noted in relation to the results found when the jack and the cut had identical dimensions.

As an example:

Let us assume that P in the ground is equal, prior the opening of the cut, to 100 kg/cm^2 .

When the jack had the same dimensions as the cut, one found with the test:

$$\text{at point } \frac{L}{5} : P'_1 = \frac{100}{0.97} = 103 \text{ kg/cm}^2.$$

$$\text{at point } \frac{L}{2} : P'_2 = \frac{100}{1.17} = 85.5 \text{ kg/cm}^2.$$

This gives a percentage of increase ranging between 85.5 and 103.

$$\left(\frac{P'_1 - P'_2}{P'_2} \right) 100 = \text{approximately 20 percent.}$$

When the 0.70 m jack is sealed in a 1 m cut, one has:

$$\text{at point } \frac{L}{5} : P'_1 = \frac{100}{0.96} = 104 \text{ kg/cm}^2.$$

$$\text{at point } \frac{L}{2} : P'_2 = \frac{100}{1} = 100 \text{ kg/cm}^2.$$

And we obtain:

$$\left(\frac{P'_1 - P'_2}{P'_2} \right) 100 = 4\%.$$

Moreover, one sees that at point $\frac{L}{5}$, the result did not increase much (it went from 103 to 104), but that it has considerably increased at point $\frac{L}{2}$ (by going from 85.5 to 100).

These results and these conclusions are quantitatively comparable to what we have found by our tests previously mentioned (cf. III B.3).

3) Interpretation of the results obtained with a row of extensometers sealed at the same elevations with regard to the jack.

We have pointed out (cf. III B.2) that the strings sealed at a same elevation with regard to the jack, showed in some cases, values of cancellation of the pressure P very different from each other.

One could expect to find the same pressure value of cancellation of the deformations at least for the couples of strings sealed symmetrically with regard to the y axis. Yet, one frequently sees that this cancellation pressure increases regularly from one string to the other, either by taking them from left to right, or from right to left.

Our cutting tests on the walls and on the roof have shown positively that the main deformations were not always in the plane normal to the gallery. This verification makes us think that either the pressure P does not act vertically, which is hard to conceive, or that vertical pressure P acts on sections of our galleries which are not vertical.

These sections are delimited by cleavage planes which are not always strictly vertical. Moreover, the principal deformations are inclined in relation to the vertical or to the horizontal by an angle included between 0° and 30° with an average of around 10° . The explanation that we have

just given of the non-verticality of the greatest of the principal stresses on the gallery walls is only a hypothesis. There are locally other causes; in particular, a limestone nodule embedded in the ore causes the orientation of the principal stresses to deviate, or at least that of the principal deformations. Nothing allows us to assert that the principal deformations have the same orientation as the principal stresses. But we are certain that the principal deformations acting on a gallery wall are frequently non-vertical.

However, we assume for the calculation that the principal pressure stress does not act vertically. This hypothesis allows us to compute the distribution of the stresses around an elliptic cut non-vertically loaded.

We will get a fairly precise idea of the phenomena which occur by computing only the tangential tension on the edge of the cut.

To be precise, one should calculate the stresses by using the formulas given in the appendix. But, anyway we would find a law of variation similar to that found for the tangential stress on the edge of the ellipse.

We have made the calculation for an ellipse of horizontal long axis, the ratio of the axes being $0.46 \tanh \xi_0 = 0.5$.

The formula which gives the tangential stresses on the edge of the elliptic cut is:

$$M_n = \frac{P}{\cosh 2\xi_0 - \cos 2\eta} \cdot \frac{1}{(1+K)\sinh 2\xi_0 + (1-K)\cos 2\alpha - (1-K)\cos 2(\eta-\alpha)e^{2\xi_0}}.$$

We have found on fig. 16 the variation curve of this tangential stress:

In the case where $\alpha = \frac{\pi}{2}$ that is when the pressure P acts perpendicular to the long axis.

In the case where $\alpha = \frac{\pi}{4}$ that is when the pressure P forms a 45° angle in relation to the vertical; we have taken $K = 0.25$.

When $\alpha = \frac{\pi}{2}$, the curve is almost horizontal under a large part of the ellipse.

When $\alpha = \frac{\pi}{4}$, the curve rises from right to left. The deformations will have the same behavior. In order to cancel the deformations by means of a uniform pressure acting on the edges of the ellipse, it will be necessary for the pressure to be different at each point. It will increase or decrease regularly when going from right to left or from left to right.

The experiments and results to which reference was made confirm what has been established by computation.

The tests previously referred to (cf. III - B.2) however, brought out a significant practical conclusion: it seems that not only is one of the principal deformations not vertical in many cases, but also neither is one of the principal stresses.

Moreover, the computation shows that along the y axis the distribution of the deformations resulting from the cutting of the slot is strongly modified in relation to the result found when the stress P acts vertically. On the other hand, the deformations induced by the jack do not change. The values of the ratio $\frac{P}{P_1}$ show that errors then increase considerably.

We have shown on fig. 17 the differences of the deformations undergone by the ore when the greatest principal stress P acts at 45° in relation to the vertical, the cut being horizontal.

Curve (1) relates to the deformations produced by the cutting of the 1 m wide slot when P acts vertically and KP acts horizontally, that is at 90° in relation to P.

Curve (2) relates to the deformations produced by the cutting of the slot when P acts at 45° in relation to the cut and KP acts at 90° in relation to P (the cut remaining horizontal).

Curve (5) is identical to curve (2) but P acts then at 60° in relation to the cut.

Curve (3) refers to the deformations caused by the pressure induced in the 1 m wide jack when this pressure is equal to P.

Curve (4) refers to the deformations caused by the pressure induced in the 1 m wide jack, this pressure being equal to the component of P evaluated on a direction perpendicular to the cut, when P acts at 45° in relation to the cut. This component is obtained from the ellipse of tensions and has the value: $P'' = 0.625 P$ in the case where Poisson's coefficient is equal to 0.2, and in the case where P acts in a direction inclined at 45° in relation to the cut.

Curve (6) is identical to curve (4) but refers to the case where P acts at 60° in relation to the cut: $P'' = 0.8125 P$.

We note immediately that the determination of the value of P when P acts non-vertically is impossible by this method. Errors are too big. We have computed as previously the value of the ratio $\frac{P}{P'}$ in the cases where P acts in a direction inclined at 75° , 60° , or 45° in relation to the cut.

Pressure present in the ground and acting in a direction P = inclined at 75° , 60° , or 45° .

$\frac{P}{P'}$ Pressure in the jack which cancels the deformations

Moreover, we have calculated the ratio

$\frac{P''}{P'}$ Component of P acting in a direction perpendicular to the cut
Pressure in the jack which cancels the deformations

The values of the ratio $\frac{P}{P'}$ show that it is impossible to determine P. But the measurement allows the determination of P'' , component of the pressure P, with a precision comparable to what we have seen previously.

The word component of P is not proper; in fact, it is a question of the value of the stress acting perpendicularly to the cut, P'' and P being connected by the equations defining the ellipse of tensions.

When $\alpha = 90^\circ$, P acts perpendicular and KP parallel to the cut.

When $\alpha = 75^\circ$, P acts in a direction forming a 75° angle with the long axis of the ellipse or of the cut. KP acts perpendicular to this last direction,

and so on.

Briefly, by placing the extensometers on the vertical symmetry axis of the jack at about $\frac{L}{3.33}$ theoretically, when cancelling the deformations with the use of the jack, the value of the stress acting perpendicularly to the cut is obtained.

On the other hand, without any doubt, the measurement does not make it possible to get the value of the principal stress P acting on the cut, because it is difficult, if not impossible, to determine the orientation of the principal stresses by means of extensometers set under one cut only.

We are, however, rather cautious about the certainty of getting by this method the value of the stress acting perpendicularly to the cut. If the medium is elastic and isotropic, we can obtain this stress, but when the medium is not elastic or is anisotropic, one can only assert that the value found is closer to the component of P acting perpendicularly to the cut than to the value of P .

These deductions are valid only on the y axis or symmetry axis of the jack and of the cut perpendicular to the symmetry axis. To the left and the right of this y axis values of cancellation of the deformations are obtained which have nothing in common with P or with P' .

Table IV

$\sigma = 0.2$	$\frac{P}{P'}$	$\frac{P}{P'}$	$\frac{P''}{P'}$	$\frac{P}{P'}$	$\frac{P''}{P'}$	$\frac{P}{P'}$	$\frac{P''}{P'}$	
α	90°	75°	75°	60°	60°	45°	45°	
0	0.99	1.04	0.99	1.09	0.90	1.58	0.99	for $\alpha = 75^\circ$ $P'' = 0.9497 P$
$\frac{L}{10}$	0.98	1.02	0.96	1.17	0.97	1.56	0.98	
$\frac{L}{5}$	0.98	1.03	0.97	1.20	1	1.56	0.98	for $\alpha = 60^\circ$ $P'' = 0.8125 P$
$\frac{L}{3.33}$	1.01	1.07	1.01	1.25	1.03	1.63	1.02	
$\frac{L}{2.5}$	1.08	1.14	1.08	1.34	1.09	1.75	1.09	for $\alpha = 45^\circ$ $P'' = 0.625 P$
$\frac{L}{2}$	1.17	1.27	1.20	1.46	1.20	1.92	1.18	
$\frac{L}{1.66}$	1.27	1.44	1.36	1.64	1.33	2.08	1.28	
$\frac{L}{1.43}$	1.39							
$\frac{L}{1.25}$	1.52							
$\frac{L}{1.111}$	1.69							
L	1.83							

4) Conclusion of the interpretations of our results obtained in the flat jack test.

First of all, we have drawn from these interpretations very significant observations which have nothing to do with the testing method under consideration.

The phenomena described during our tests in the field have been wholly explained by the calculation based on the elastic hypotheses. We may say that the elastic hypotheses, consequently the photoelastic tests, give for our calcareous ores, a sufficient approximation on the distribution of stresses around our mine openings.

If we note discrepancies between the calculation and the reality, it is because very frequently, the defining conditions are false; particularly one uses values of Poisson's coefficient which are a little far from reality. Yet, this coefficient has an enormous influence on the stress distribution. On the other hand, the third stress is always neglected, whose role, we believe, is basic. Finally the mass is stratified and jointed.

On the basis of results of the computation made according to elastic hypotheses, it was possible to explain all the phenomena qualitatively. It would be vain to try to obtain quantitative laws; particularly the errors resulting from the flat jack test cannot be calculated. The computation can give the minimum error but it cannot give the maximum error.

However, we can state that:

- (1) The extensometers have to be sealed in a zone close to $\frac{L}{6}$.
- (2) The extensometers must be sealed on the symmetry axis of the jack, perpendicular to the cut. Moreover, the symmetry axis of the jack must coincide with the symmetry axis of the cut. It is the y axis.

(3) The test gives a result concerning the stress acting perpendicularly to and at the level of the cut. The test does not give information about the direction of the principal stress at the level of the cut.

(4) In our calcareous ore, the error of measurement, considering the irreversibility of creep stresses, is included between - 20 percent and + 10 percent. . It is an acceptable result, because the precision is quite superior to that given by the other methods with which we are acquainted. The precision will still be better when the tests will be made on more resistant and more elastic rocks than the calcareous iron ore (cf. II). But in softer rocks, whose behavior deviates notably from elasticity, the precision is reduced. If the rock shows a significant plasticity, the method can no longer be applied.

Remark 1 -- To make sure of the reliability of the method, the following test was made:

A large slot was cut in which a large jack was inserted. The slot was much larger than the jack (fig. 18).

The pressure in the jack was raised to a known and constant value. A new and smaller cut was made below and a jack of same dimension was sealed into it. We thus knew the pressure in the ground; it was given by the upper jack. The small cut modified the deformations which were cancelled by means of the small jack sealed in this cut.

In a first test, the result found was exact at + 1 percent on the extensometer located on the y axis, and on the other extensometers sealed vertically.

In another test, the result showed an error of + 10 percent, except on a vibratory string where the error was of 40 percent, but whose readings were incoherent, which permits one to assume that there was some sort of anomaly.

Remark 2 -- Now to measure the deformations, a dilatometer (system L'Hermite and Meynier) was used, which measures the deformations between the studs sealed in the rock. The precision is slightly smaller than that obtained with the vibratory strings ($10 \mu/m$ instead of $5 \mu/m$), but it is quite sufficient in our ore. Lastly, the stud setting and the readings are easier and the equipment is less fragile.

In fig. 16, studs are sealed on the vertical symmetry axis of the jack, that is on the y axis. Unfortunately, they can hardly be discerned.

Remark 3 -- Jacks at least 0.50 m wide and 0.50 m deep are advisable. We use 0.70 m x 0.70 m jacks. The small jacks are easy to place; on the other hand, the zone near $\frac{L}{3}$ is too close to the cut. Moreover, the influence of the jack edges can no longer be negligible.

Remark 4 -- This method of measuring can be applied with the present "Freysinet" jack only in shallow mines (depth smaller than 250 m).

As a matter of fact, the pressure cannot practically be raised in the jack to more than 160 or 180 kg/cm², otherwise it busts. These pressures are attained easily at a 250 m depth in rectangular galleries.

In galleries of circular section, one can consider making measurements down to 300 m depth.

For greater depths, the "Freyssinet" jack should, if possible, be strengthened and modified. We expect to be able to do it by partly replacing the steel sheets by rubber. Indeed, near the pillar-drawing faces, the stresses are high and the tangential stresses cannot then be measured.

Lastly, we recall that the method of measurement is applicable only when the rock is subjected to compression. In the case of tension, the measurement is impossible.

V. SUGGESTED MODIFICATIONS

We have noted the relative lack of precision of the flat jack test made as it was just described.

We also note the following points:

(a) This test does not give information on the orientation of the principal stresses.

(b) It gives us the vertical tangential stress which existed at the level of the cut, and not at the level of the extensometers. Consequently, one test can give one result only.

(c) It is impossible to obtain with this test the values of the horizontal tangential stress, the values of Poisson's coefficient and of the modulus of deformation or of the modulus of elasticity.

In other words, stress/strain curves are obtained which can serve only to determine a cancellation point of the deformations. The value of the stress induced by the jack is known, but the stress induced at the level of the extensometers is not known. Yet, the stress/strain curves read on extensometer rosettes should allow the determination of Poisson's coefficient and of the modulus of deformation. To avoid these drawbacks, the test was modified as follows:

A. Description of the test.

We cut out, by means of slots made as previously described, a block of ore, 70 cm x 70 cm, provided with at least two rosettes of extensometers (1) and (2) (fig. 19).

We get the A, B, C and D slots.

The reading on the extensometers allows the determination of the direction of the principal stresses at the level of the extensometer rosettes (1) and (2).

Then, a flat jack is introduced in cut A, and a flat jack in cut B.

The cuts D and C are filled up with cement.

By raising the pressure in the upper jack V_s , the modulus of deformation corresponding to a vertical stress, say E_y , and Poisson's coefficient σ_y were obtained.

By raising the pressure in the lateral jack V_L , the coefficients E_x and σ_x are likewise obtained.

By means of the elasticity equations:

$$e_y = \frac{N_y}{E_y} - \sigma_x \frac{N_x}{E_x}$$
$$e_x = \frac{N_x}{E_x} - \sigma_y \frac{N_y}{E_y}$$

N_y and N_x can be determined. The pressure is then raised to N_y in V_s and to N_x in V_L . Then, the data of the computation, which assume the elastic hypotheses, are checked to find out if they correspond to the cancellation on the vertical and horizontal extensometers. It is found that the values of the pressure given in V_s and V_L always have to be slightly modified.

But, up to now, this modification has not, in our ore, exceeded 5 percent of this value.

This time, we note that the tangential stresses obtained concern places where the extensometers are set, and no longer the places where the cuts were made.

Consequently, it is desirable to use as many extensometer rosettes as possible.

B. Results

The results obtained by a complete cutting are less abundant than those obtained by a horizontal cut only. So, we will not yet speak of an average.

However, we give below the results obtained in two of our tests.

- 1) Test made in a 4.50 m wide and 3.50 m high gallery at half-height on a vertical wall (fig. 20).

We started by cutting the slot A only, and sealing V_s . In other words, we had the flat jack test in one horizontal cut only.

This test gave us the pressure cancelling the deformations:

On the studs 1	94.5 kg/cm ²
On the studs 2	88.0 kg/cm ²
On the studs 4	?

We then proceeded to the complete cutting and sealed V_L .

The pressures in the two jacks which produced the cancellation of the vertical and horizontal extensometers are:

On the studs 1	100 kg/cm ² = N_y
On the studs 2	95.5 kg/cm ² = N_y
On the studs 4	92 kg/cm ² = N_x

We have obtained:

Rosette (1):	E_y	132,700 kg/cm ²
	σ_y	0.45
Rosette (2):	E_y	131,400 kg/cm ²
	σ_y	0.44

• The angle of the largest principal stress with the vertical was 27°.

2) Test made in a 5 m wide and 4 m high gallery, at half-height, on a vertical wall.

We did not make a test with only one horizontal cut. We immediately made the complete cutting. Here are the results:

N_y on studs 1	82 kg/cm ²
N_y on studs 2	85 kg/cm ²
N_x on studs 4	54 kg/cm ²
E_y	160,700 kg/cm ²
σ_y	0.27
E_x	230,700 kg/cm ²
σ_x	0.34

The angle of the largest principal stress with the vertical was 10°.

C. Discussion of the results

The precision of the tests is increased.

The deformations produced by the cutting of the slots are cancelled at any point of the cut block by the pressures in the jacks such as the ratio $\frac{P}{P_1} = 1$.

The lack of precision due to the non-superposition of the curves (1) and (2), which has been referred to previously (cf. IV-1) no longer exists.

On the other hand, the error due to the irreversibility of stresses and to creep remains. The same holds true for the error due to the reading of the measurement apparatus.

In other words, the results obtained with two flat jacks are exact to ± 10 percent in our ore. In a perfectly elastic rock, one would get ± 2 percent.

As a conclusion, with the test made with two jacks, one can:

(1) Obtain the value of the vertical and horizontal tangential stresses to ± 10 percent, and also the value of the principal stresses and their orientation. It is understood that to obtain these principal stresses and their orientation, we use the Mohr circle, and consequently hypotheses that refer to it.

(2) Obtain the value of the vertical and horizontal moduli of deformation E_y and E_x with the same precision.

(3) Obtain the value of Poisson's coefficients σ_y and σ_x with the same precision: σ_y is obtained on a horizontal extensometer when the stress acts vertically; σ_x is obtained on a vertical extensometer when the stress acts horizontally.

(4) Obtain all these values on each extensometer rosette located on the block cut by the slots. As a matter of fact, the tangential stresses may and must normally vary along a block 70 cm high located on a gallery wall.

* * *

On the other hand, the test became much more complicated, specially in its practical application. In particular, 15 working-hours of two skilled men are required to make the cuts and seal the jacks.

But the amount of information it provides fully justifies the increase in complication. The determination of the modulus of deformation by means of the jack-driving test, described in the *Annales de l'Institut Technique du Bâtiment et des Travaux Publics* (Sols et fondations, no. 3, September 1950, "Détermination du module d'élasticité des roches en place" [Determination of the modulus of elasticity of the rocks in place], by

Mr. Habib) is practically impossible in the iron mines of the East. The sections of galleries are such that it is much more difficult to carry out the jack-driving test than the test which has just been described, with complete cutting on four faces and sealing of the two jacks.

Moreover, the jack-driving test does not give information on the value of the tangential stresses and may, with difficulty, give Poisson's coefficient. Besides, the jack-driving test cannot practically take into account the anisotropy of the ore. However, to the advantage of the jack-driving test, mention will be made of its considerable stress power. Much higher stress rates can be applied than those given by the Freyssinet jacks.

To overcome this disadvantage, we pursue our research toward increasing the possibilities of the flat jacks. We believe the jacks have to be modified. We hope to attain the expected results soon.

In a forthcoming communication, we will talk about the considerations that may be drawn from the knowledge of the vertical and horizontal stresses of E_y , E_x , σ_y and σ_x . These considerations are very significant and deserve a full account.

Thanks are here presented to our Assistant, Mr. Leonet, who has helped us constantly in the practical carrying out of these tests. His advice and suggestions have always been very valuable to us.

Appendix I

Calculation of tensions and deformations produced by cutting of a horizontal slot compared to an ellipse.

Let an ellipse be cut in an infinite plate of homogeneous and isotropic elastic material.

We will use a system of elliptic coordinates defined by

$$y = c \cosh \xi \cos \eta$$

$$x = c \sinh \xi \sin \eta$$

The coordinate curves $\xi = \text{const.}$ are a family of ellipses all having the same value of $c^2 = a^2 - b^2$ (a and b are the semi-axes of the ellipses).

The coordinate curves $\eta = \text{constant}$ are hyperbolas orthogonal to the preceding ellipses.

We put the plate, cut by the ellipse whose long axis is horizontal and parallel to the x axis, under a system of stress defined by:

- (1) A stress P acting at an angle α measured from the x axis.
- (2) A stress KP acting at an angle $\alpha + \frac{\pi}{2}$ also measured from the x axis. K is the coefficient of transmission of stresses defined by $\frac{\sigma}{1 - \sigma}$ where σ is Poisson's ratio.

The Airy function for the case where $K = 0$ has been determined by M. Theodor Pöschl /. For $K \neq 0$, we have obtained:

/ Theodor Pöschl, "Über eine partikuläre Lösung des biharmonischen Problems für den Aussenraum der Ellipse". Zeitschrift für angewandte Mathematik und Mechanik. Band 2, 1922, p. 89-96, 4 fig.

$$F = \frac{Pc^2}{8} \left\{ (1+K) \sinh 2\xi + (1-K) \cos 2\alpha e^{-2(\xi-\xi_0)} \right. \\
- 2 \left[(1+K) \cosh 2\xi_0 - (1-K) \cos 2\alpha \right] \xi \\
\left. - (1-k) \left[\cosh 2(\xi - \xi_0) - 1 \right] e^{2\xi_0} \cos 2(\eta - \alpha) \right\}$$

The components of tension in elliptical coordinates are:

$$N_\xi = \frac{1}{h_1^2} \frac{\partial^2 F}{\partial \eta^2} + \frac{1}{h_1^2 h_2} \frac{\partial h_2}{\partial \xi} \frac{\partial F}{\partial \xi} - \frac{1}{h_1^2} \frac{\partial h_2}{\partial \eta} \frac{\partial F}{\partial \eta}$$

$$N_\eta = \frac{1}{h_1^2} \frac{\partial^2 F}{\partial \xi^2} - \frac{1}{h_1^2} \frac{\partial h_1}{\partial \xi} \frac{\partial F}{\partial \xi} + \frac{1}{h_1 h_2^2} \frac{\partial h_1}{\partial \eta} \frac{\partial F}{\partial \eta}$$

$$N_{\xi\eta} = -\frac{1}{h_1 h_2} \frac{\partial^2 F}{\partial \xi \partial \eta} + \frac{1}{h_1^2 h_2} \frac{\partial h_1}{\partial \eta} \frac{\partial F}{\partial \xi} + \frac{1}{h_1 h_2^2} \frac{\partial h_2}{\partial \xi} \frac{\partial F}{\partial \eta}$$

where

$$h_1^2 = h_2^2 = \frac{c^2}{2} (\cosh 2\xi - \cos 2\eta).$$

One may obtain expressions for N_ξ and N_η from F and the preceding equations.

$$N_\xi = \frac{P(1-k)}{\cosh 2\xi - \cos 2\eta} \left[\cosh 2(\xi - \xi_0) - 1 \right] e^{2\xi_0} \cos 2(\eta - \alpha) \\
+ \frac{P}{2} \frac{\sinh 2\xi}{(\cosh 2\xi - \cos 2\eta)^2} \left[(1+K) \cosh 2\xi - (1-k) \cos 2\alpha e^{-2(\xi-\xi_0)} - (1+K) \cosh 2\xi_0 \right. \\
\left. + (1-K) \cos 2\alpha - (1-K) \sinh 2(\xi - \xi_0) e^{2\xi_0} \cos 2(\eta - \alpha) \right] \\
- \frac{P}{2} \frac{\sin 2\eta}{(\cosh 2\xi - \cos 2\eta)^2} (1-K) \left[\cosh 2(\xi - \xi_0) - 1 \right] e^{2\xi_0} \sin 2(\eta - \alpha)$$

$$N_\eta = \frac{P}{\cosh 2\xi - \cos 2\eta} \left[(1+K) \sinh 2\xi + (1-K) \cos 2\alpha e^{-2(\xi-\xi_0)} - (1-K) \cosh 2(\xi - \xi_0) \right. \\
\left. \times e^{2\xi_0} \cos 2(\eta - \alpha) \right] \\
+ \frac{P}{2} \frac{\sin 2\eta}{(\cosh 2\xi - \cos 2\eta)^2} (1-K) \left[\cosh 2(\xi - \xi_0) - 1 \right] e^{2\xi_0} \sin 2(\eta - \alpha) \\
- \frac{P}{2} \frac{\sinh 2\xi}{(\cosh 2\xi - \cos 2\eta)^2} \left[(1+K) \cosh 2\xi - (1-K) \cos 2\alpha e^{-2(\xi-\xi_0)} \right. \\
\left. - (1+K) \cosh 2\xi_0 + (1-K) \cos 2\alpha - (1-K) \sinh 2(\xi - \xi_0) e^{2\xi_0} \cos 2(\eta - \alpha) \right]$$

For $\xi = \xi_0$, we obtain the tangential stresses on the ellipse: $N_\xi = 0$

$$N_\eta = \frac{P}{\cosh 2\xi_0 - \cos 2\eta} \left[(1+K) \sinh 2\xi_0 + (1-K) \cos 2\alpha - (1-K) \cos 2(\eta - \alpha) e^{2\xi_0} \right]$$

We proceed to calculate the radial stress for a greatly elongated ellipse resembling the cut into which the flat jack will be put.

We have $\frac{b}{a} = \tanh \xi_0$, b and a being the semi axes of the ellipse.

We want to compute the stresses along the y axis, that is, on the prolongation of the small axis.

In that case: $\eta = \frac{\pi}{2}$

Moreover, we assume that the pressure P acts vertically, therefore: $\alpha = \frac{\pi}{2}$

We then obtain the stresses along the y axis:

$$N_\eta = \frac{P}{\cosh 2\xi + 1} \left[(1+K) \sinh 2\xi - (1-K) e^{-2(\xi - \xi_0)} - (1-K) \cosh 2(\xi - \xi_0) e^{2\xi_0} \right]$$

$$- \frac{P}{2} \frac{\sinh 2\xi}{(\cosh 2\xi + 1)} \left[(1+K) \cosh 2\xi + (1-K) e^{-2(\xi - \xi_0)} - (1+K) \cosh 2\xi_0 - (1-K) \sinh 2(\xi - \xi_0) e^{2\xi_0} \right]$$

$$N_\xi = \frac{P(1-K)}{\cosh 2\xi + 1} \left[\cosh 2(\xi - \xi_0) - 1 \right] e^{2\xi_0} + \frac{P}{2} \frac{\sinh 2\xi}{(\cosh 2\xi + 1)} \left[(1+K) \cosh 2\xi \right.$$

$$\left. + (1-K) e^{-2(\xi - \xi_0)} - (1+K) \cosh 2\xi_0 - (1-K) \sinh 2(\xi - \xi_0) e^{2\xi_0} \right]$$

Appendix II

Calculation of the tensions induced in a semi-plane subjected to a normal load evenly distributed over a length RS of the edge X'X.

We have seen (fig. 12) that in order to compute the deformations produced by the pressure caused by the flat jack, we had compared the system to a normal load evenly distributed over a length RS. We have assumed this comparison on the Y axis, and remarked that it could not be a question of retaining it around points R and S.

In fact, the edge XX' in the mass is not a free edge.

The tensions have been computed by A. Picard in his book, *La Photoelasticité*, Dunod editor, p. 189.

Timoschenko gives them too:

$$N_x = -\frac{P}{\pi} [\varphi_1 - \varphi_2 + \frac{1}{2} (\sin 2\varphi_1 - \sin 2\varphi_2)]$$

$$N_y = -\frac{P}{\pi} [\varphi_1 - \varphi_2 - \frac{1}{2} (\sin 2\varphi_1 - \sin 2\varphi_2)]$$

$$T = \frac{P}{2\pi} (\cos 2\varphi_1 - \cos 2\varphi_2).$$

In every point of the y axis, $\varphi_1 = \pi - \varphi_2$

Consequently one has:

$$N_x = -\frac{P}{\pi} [\varphi_1 - \varphi_2 + \sin 2\varphi_2]$$

$$N_y = -\frac{P}{\pi} [\varphi_1 - \varphi_2 - \sin 2\varphi_2]$$

$$T = 0$$

The deformations along the y axis are easily obtained:

$$e_y = \frac{N_y}{E} - \sigma \frac{N_x}{E}$$

Appendix III

Freyssinet jacks

The flat jacks, Freyssinet type, have been conceived mainly for the needs of the "Travaux Publics", particularly with the purpose of economically pre-stressing concrete.

These jacks are essentially made of soft steel plates, 15/10 gauge, electrically welded together so as to form a hermetic pocket in which the liquid under pressure is injected (generally water). One may, besides, inject a liquid product which becomes solid after injection.

A perspective section is given in fig. 22, which shows the construction of these jacks.

The dotted lines represent the weld lines. These jacks may be circular, rectangular, annular, or of any shape.

The normal range of travel of these jacks is 2 cm. If a larger one is wanted, several jacks must be superposed.

The highest advisable internal pressure is 150 kg/cm^2 . During our tests, we have raised it several times to 200 kg/cm^2 .

These jacks are always embedded in cement or wedged between two steel plates.

They are patented. A French company has the exclusive exploitation of these patents. It is able to supply the jacks and all the accessories necessary for their operation.

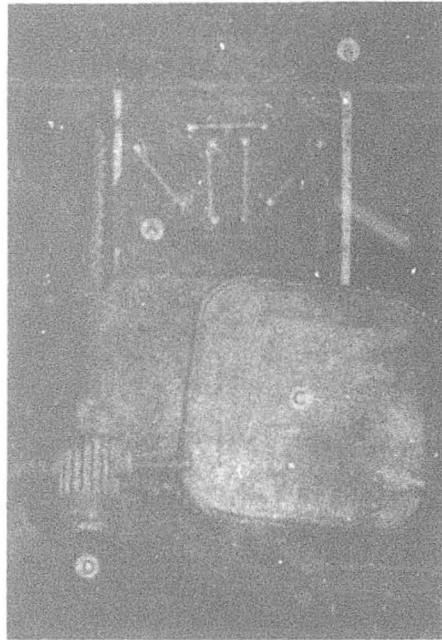


Fig. 1. General view prior to sealing the jack.

- A. Rosette of double vibratory strings.
- B. Cut.
- C. Freyssinet flat jack, 70 x 70 cm.
- D. Electrical auger.

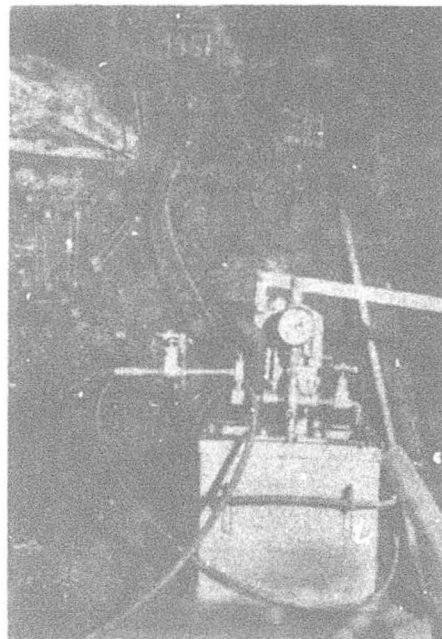


Fig. 2. General view of the test, jack sealed, pump connected.

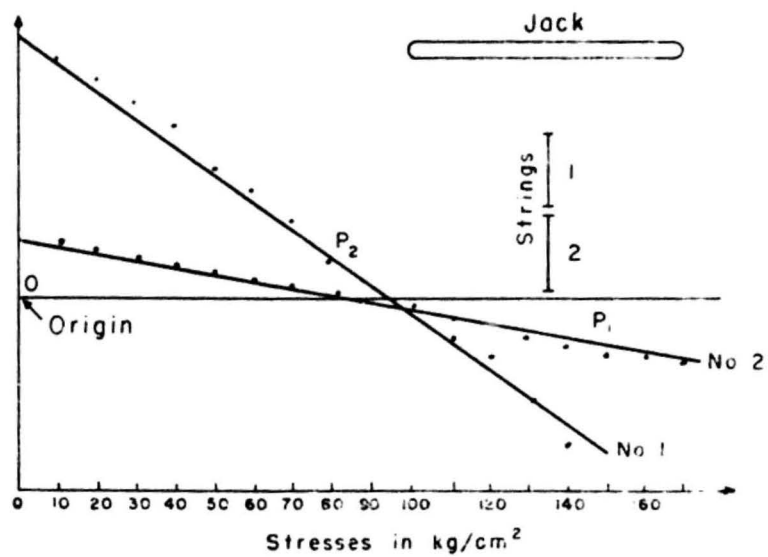
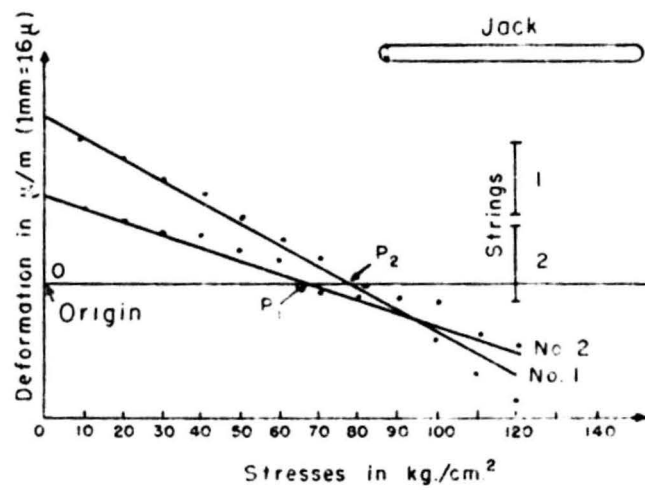


Fig. 3. and 4.

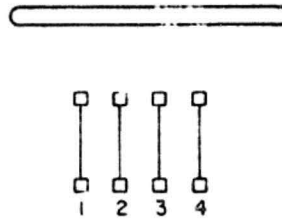


Fig. 5.

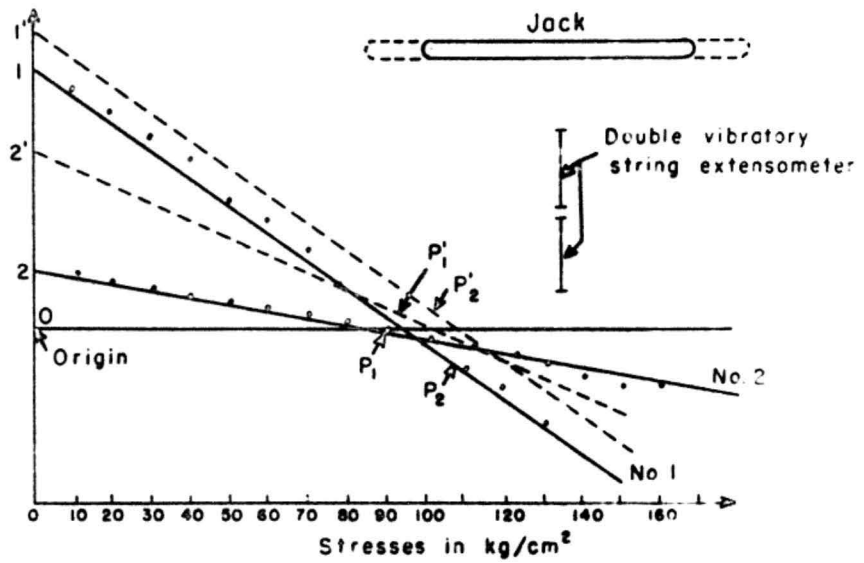


Fig. 6. Double vibratory string extensometers.

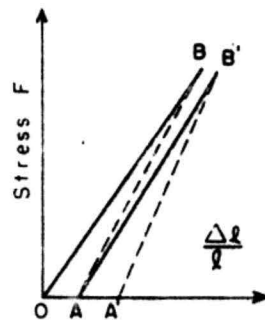


Fig. 7.

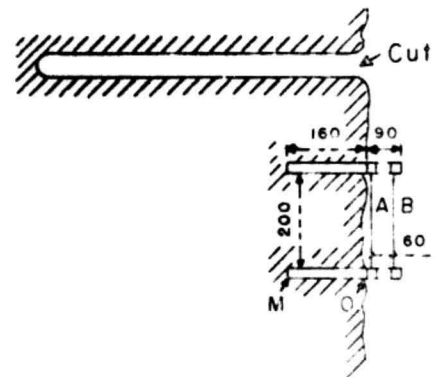


Fig. 8.

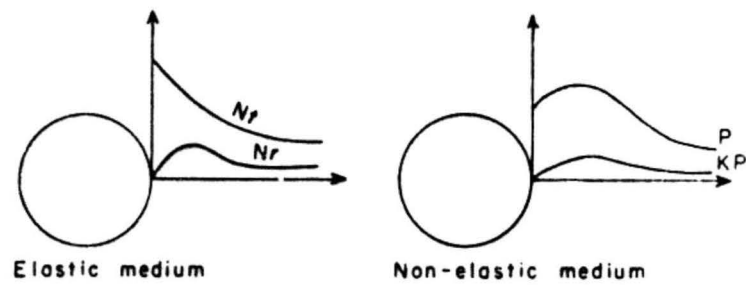


Fig. 9.

N_t = Tangential stress measured along Ox , that is vertical stress.
 N_r = Radial stress measured along Ox , that is, horizontal stress.



Fig. 10.

A - Cleavage plane prior to cutting the gallery.
 B - Cleavage plane after cutting the gallery (Bending exaggerated).

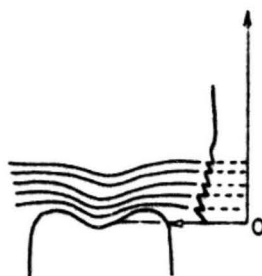


Fig. 11.

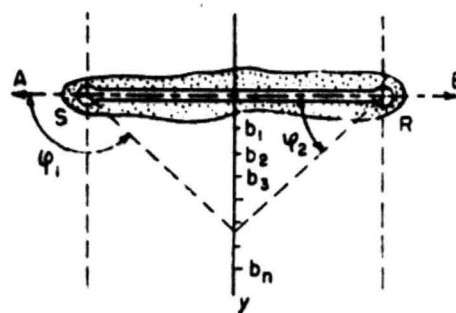


Fig. 12.

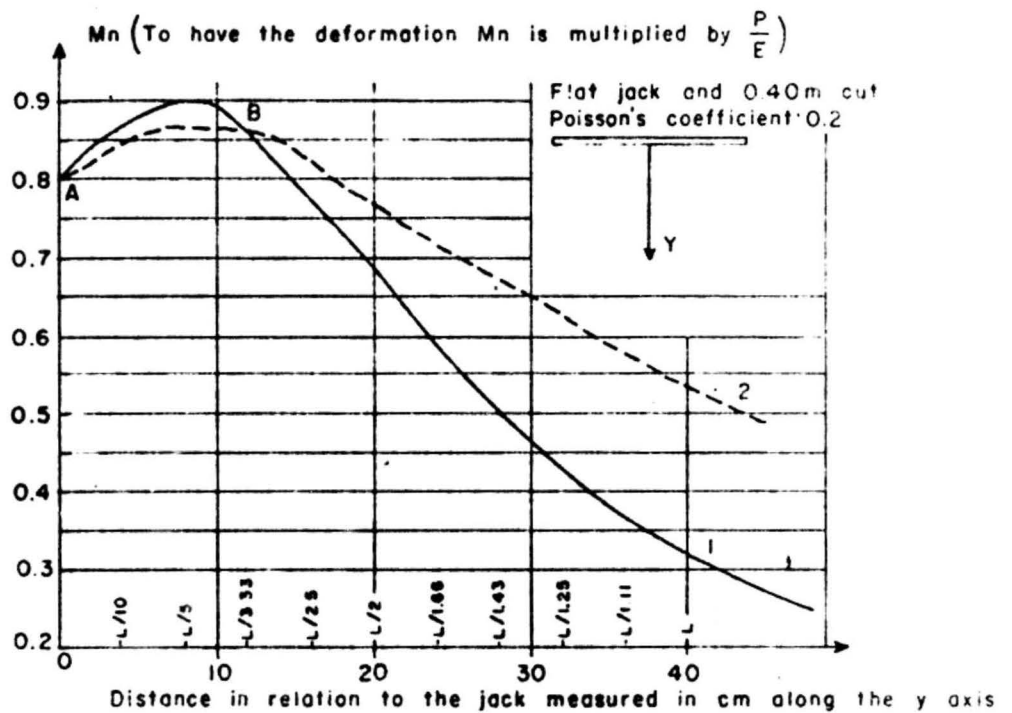


Fig. 13.

1. Curve related to the deformations produced by cutting the slot.
2. Curve related to the deformations produced by the pressure induced by the jack, when this pressure is identical to that which was present in the ground prior to cutting the slot.

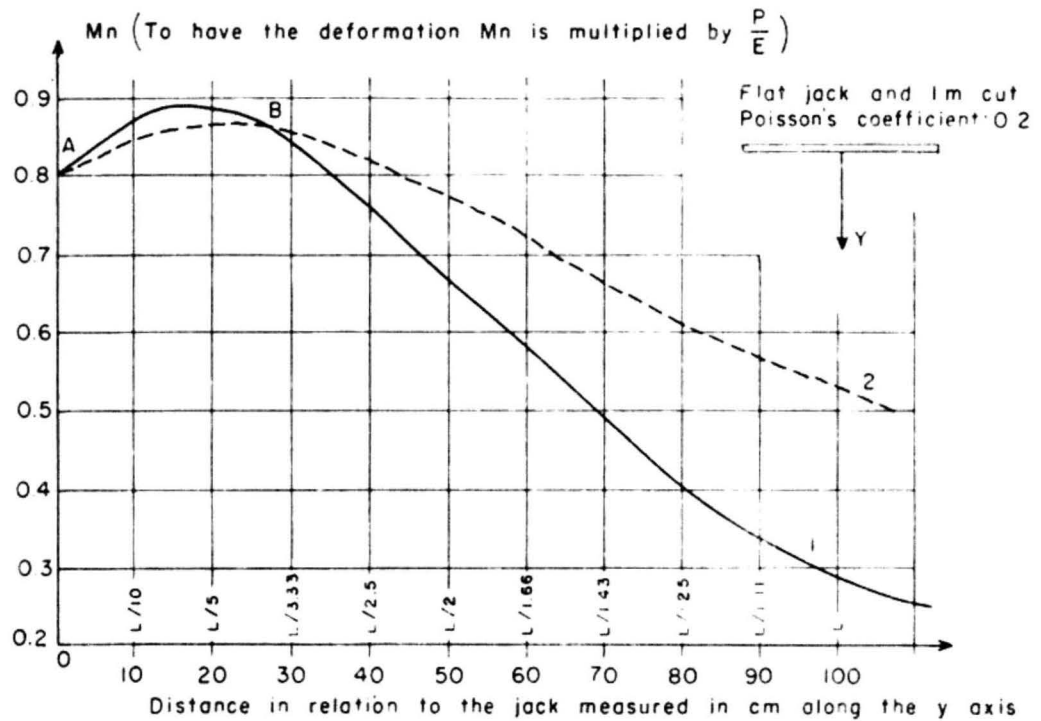


Fig. 14.

1. Curve related to the deformations produced by cutting the slot.
2. Curve related to the deformations produced by the pressure induced by the jack, when this pressure is identical to that which was present in the ground prior to cutting the slot.

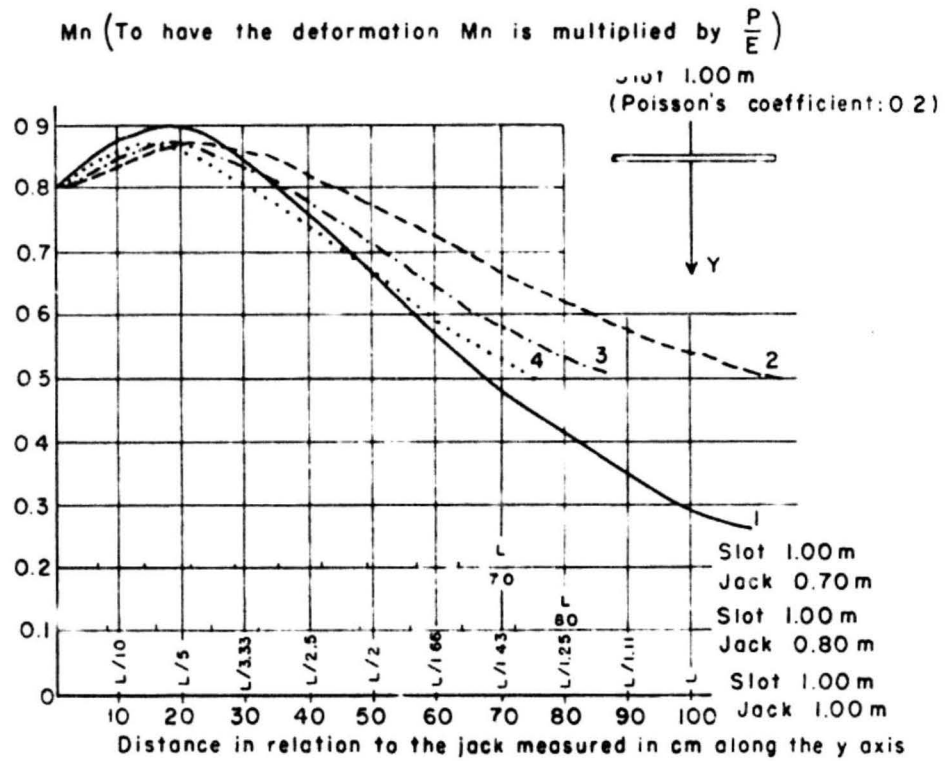


Fig. 15.

1. Curve related to the deformations produced by cutting the slot.
2. 3. 4. Curve related to the deformations produced by the pressure induced by the jack, when this pressure is identical to that which was present in the ground prior to cutting the slot.

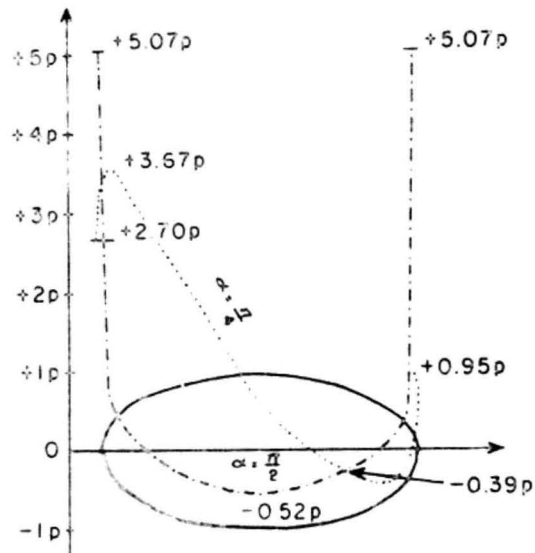


Fig. 16.

Distribution of stresses around an elliptic gallery
whose large axis is horizontal.

Ratio of the ellipse axes = 0.46.

p = pressure acting at infinity in direction α .

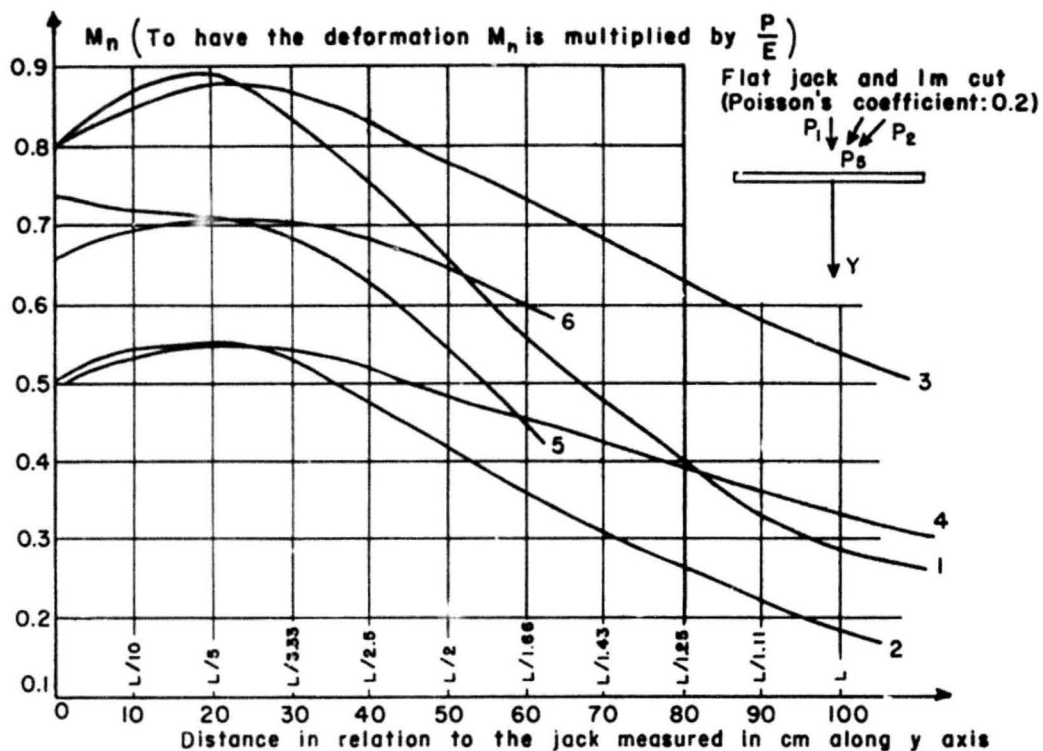
In direction $\alpha + \frac{\pi}{2}$ acts a pressure Kp .

--- Case of vertical pressure $\alpha = \frac{\pi}{2}$.

..... Case of inclined pressures----- $\alpha = \frac{\pi}{4}$.

† Compression.

- Traction.



1. Curve related to the deformation produced by cutting the 1 m slot when P acts vertically.
2. Curve related to the deformations produced by cutting the slot when P acts as 45° .
3. Curve related to the deformations caused by the pressure induced by the jack when this pressure is equal to P .
4. Curve related to the deformations caused by the pressure induced by the jack when this pressure is equal to $P'' = 0.625 P$.
5. Curve related to the deformations produced by cutting the slot when P acts at 60° .
6. Curve related to the deformations produced by the pressure induced by the jack, when this pressure is equal to $P'' = 0.8125 P$.

Fig. 17

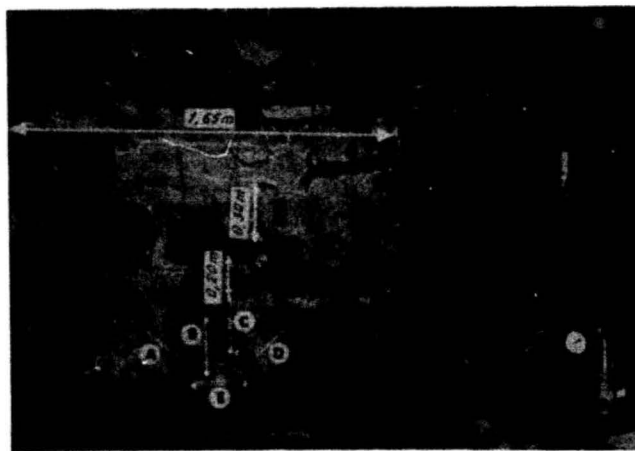


Fig. 18.

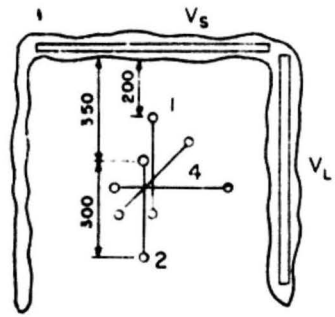


Fig. 20.

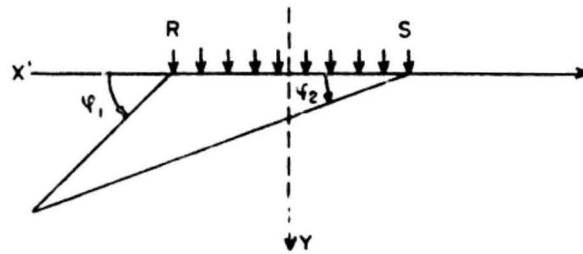


Fig. 21.



FIG. 22.