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HYDRAULICS

COMPUTATION OF DRAWDOWNS AT EQUILIBRIUM CAUSED BY WELLS DRAWING WATER FROM AN AQUIFER FED BY A FINITE STRAIGHT LINE SOURCE

By
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The following brief informal paper was originally prepared by C. V. Theis in 1941 as a memorandum in answer to an inquiry from W. N. White, then District Engineer (GW) at Austin, Tex. It shows how much the computed drawdowns at equilibrium, assuming an infinite line source, would have to be increased to take into account the faulting in the outcrop of the Carrizo sand in the Lufkin area, Texas. A more detailed map and discussion of the area referred to on page 3 are given in a paper by W. F. Guyton entitled "Results of pumping tests of the Carrizo sand in the Lufkin area, Texas," which appeared in the Transactions of the American Geophysical Union, part 1, pages 40-48, 1942. Theis' memorandum is reproduced herewith as another in the Hydraulics series of Ground Water Notes for distribution to Branch personnel. It will serve as a useful complement to Ground Water Note No. 14 (Hydraulics), which considered the situation of a discharging well diverting water from a nearby infinite line source. Theis' original memorandum form is maintained, except that an abbreviated table giving values for hyperbolic cosines is appended.

The method given here follows Muskat (1937, p. 186-192). I have not been able to follow all of Muskat's steps but I believe the formula given here is correct.

The finite line source has a length of 2c and is taken on the x-axis. The y-axis bisects the line source. Hence the source extends from -c to +c on the x-axis. The coordinates of the well and of the point at which the drawdown is desired are measured in terms of the length c; thus the coordinates of the well are x_o/c, y_o/c, and of the point at which the drawdown is desired are x/c, y/c.

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Muskat transforms these coordinates into another system of orthogonal coordinates which appear to be the stream lines and the equipotential lines from the line source. The transformation is according to the following equations which are rewritten from Muskat's equation 4 (op. cit., p. 188):

$$x/c = \cosh \xi \cos \eta$$

$$y/c = \sinh \xi \sin \eta$$

Cosh ξ and sinh ξ are respectively the hyperbolic cosine and hyperbolic sine of ξ, tables of which may be found, among other places, in the Handbook of Chemistry and Physics (1952-1953, p. 180-186). An abbreviated table is appended giving values of the hyperbolic cosine. Cos η and sin η are of course in natural or radian measure.

A chart is appended (figure 2) giving the values of ξ and η for values of x/c and y/c. To use this chart simply find the point corresponding to x/c, y/c using the regular graph paper coordinates and read the values of ξ and η given by the two sets of curves.

Muskat's Eq. 11 is:

$$p = p_e + \frac{q}{2} \ln^{*} \left[\frac{\cosh(\xi + \xi_o) - \cos(\eta - \eta_o)}{\cosh(\xi - \xi_o) - \cos(\eta - \eta_o)}\right] \cdot \left[\frac{\cosh(\xi + \xi_o) - \cos(\eta + \eta_o)}{\cosh(\xi - \xi_o) - \cos(\eta + \eta_o)}\right]$$

in which p = the pressure at the point ξ, η and p_e = the pressure at the source, and q = what Muskat calls the "strength" of the well. Inasmuch as we are interested only in the drawdown, p_e may be disregarded, and the long second term only used. The value of q may be found from Muskat's Eq. 15 and turns out to be $q = Q'/2\pi T$, in which Q' is the quantity pumped by the well in gallons a <u>day</u>.

$$s = \frac{2.3Q'}{4\pi T} \log^{**} \left[\frac{\cosh(\xi + \xi_o) - \cos(\eta - \eta_o)}{\cosh(\xi - \xi_o) - \cos(\eta - \eta_o)}\right] \cdot \left[\frac{\cosh(\xi + \xi_o) - \cos(\eta + \eta_o)}{\cosh(\xi - \xi_o) - \cos(\eta + \eta_o)}\right]$$

When the source is infinite in length ($c = \infty$), this equation can be shown to reduce to

$$s = (2.3Q'/4\pi T) \log r_i^2/r_r^2$$

in which r_i is the distance from the point in question to the image well and r_r is the distance to the real well. This expression can also be derived from Lubin's equation when the time is infinite and the system has reached equilibrium. This agreement is a partial check on the equation given here.

In using this equation keep in mind the properties of the hyperbolic and natural cosines, as follows:

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* ln signifies the natural logarithm, or, as it is sometimes written, $\log_e$.

** log signifies logarithm to the base 10, that is, $\log_{10}$.

$$\cosh z = \cosh(-z)$$

$$\cos z = \cos(-z)$$

$$\cos(\pi - z) = -\cos z$$

In words, the hyperbolic cosine is always positive no matter whether the argument is positive or negative, the cosine of an argument (angle) is positive in the 1st and 4th quadrants and negative in the 2nd and 4rd.

In applying this equation to the Carrizo sand and the pumping at Lufkin I have computed the final drawdown at Nacogdoches in the following way, adopting the physical situation shown in the accompanying sketch (figure 1). Actual dimensions are not given inasmuch as we are interested only in the relative position of all the elements of the problem.

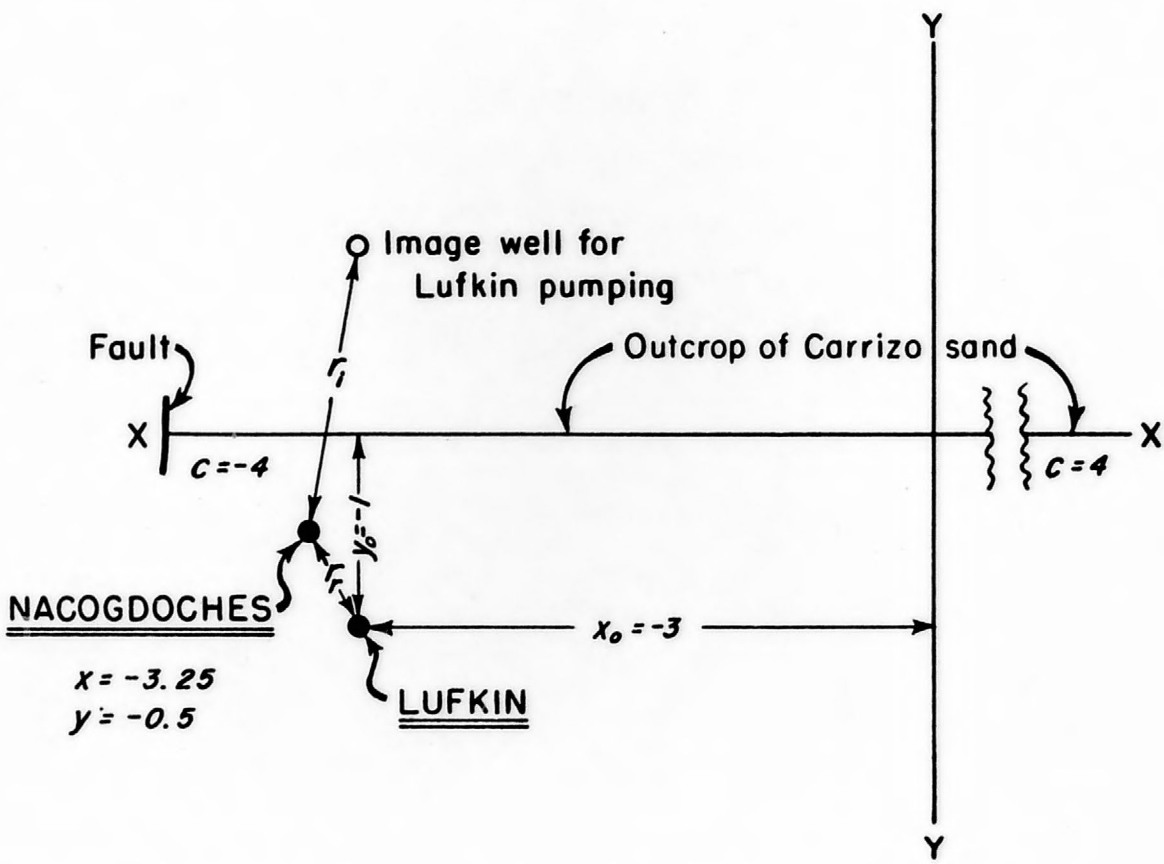

Figure 1.--Generalized location plan of Lufkin area

Then, from the sketch

$$\left.\begin{array}{l} x_o/c = .75 \\ y_o/c = .25 \end{array}\right\} \quad \text{Equivalent to (from chart)} \quad \left\{\begin{array}{l} \xi_o = .35 \\ \eta_o = .78 \end{array}\right.$$

$$\left.\begin{array}{l} x/c = .812 \\ y/c = .125 \end{array}\right\} \quad \left\{\begin{array}{l} \xi = .20 \\ \eta = .65 \end{array}\right.$$

Substituting in equation given,

$$s = \frac{2.3Q'}{4\pi T} \log \left[\frac{\cosh(.55) - \cos(-.13)}{\cosh(-.15) - \cos(-.13)}\right] \cdot \left[\frac{\cosh(.55) - \cos(1.43)}{\cosh(-.15) - \cos(1.43)}\right]$$

$$= \frac{2.3Q'}{4\pi T} \log \left[\frac{1.1551 - .9915}{1.0113 - .9915}\right] \cdot \left[\frac{1.1551 - .1399}{1.0113 - .1399}\right]$$

$$= (2.3Q'/4\pi T) \log 9.64 = .985\,(2.3Q'/4\pi T)$$

The drawdown under the assumption of an infinite straight line would be

$$s = (2.3Q'/4\pi T) \log (r_i^2/r_r^2)$$

$$= (2.3Q'/4\pi T) \log (1.5^2 + .25^2)/(.5^2 + .25^2)$$

$$= (2.3Q'/4\pi T) \log 7.4$$

$$= .87\,(2.3Q'/4\pi T)$$

Comparing the two log terms, inasmuch as the other factors are the same, the ratio of the drawdown under the assumption of a finite source to the drawdown under the assumption of an infinite source is, at equilibrium

$$.985/.87 = 1.13$$

Hence if the distances assumed are approximately correct the drawdown at Nacogdoches will start out as Guyton has computed it, in the early stages of pumping, and will gradually approach a value 13 per cent greater than Guyton's computation as equilibrium is approached.

November 28, 1941

REFERENCES

Muskat, M., 1937, The flow of homogeneous fluids through porous media: McGraw-Hill Book Co., p. 186-192.

Hodgman, C. D., Editor in chief, 1952-1953, Handbook of chemistry and physics (34th edition): Chemical Rubber Publishing Co., p. 180-186.

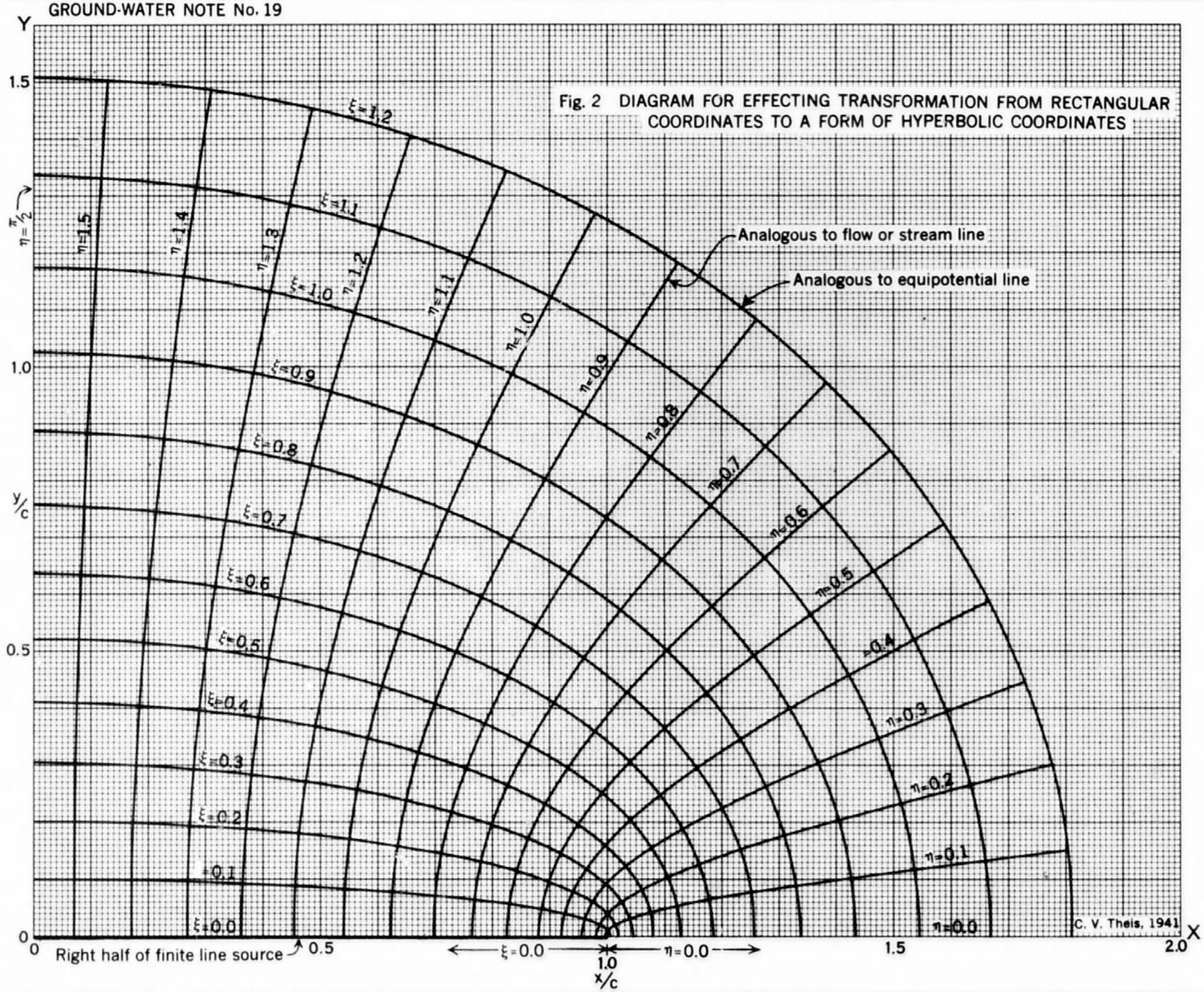

Fig. 2 DIAGRAM FOR EFFECTING TRANSFORMATION FROM RECTANGULAR COORDINATES TO A FORM OF HYPERBOLIC COORDINATES

Table 1--Hyperbolic Cosines $\cosh x = 1/2\,(e^{x} + e^{-x})$ 1/

x	0	1	2	3	4	5	6	7	8	9
0.0	1.0000	1.0000	1.0002	1.0004	1.0008	1.0012	1.0018	1.0024	1.0032	1.0040
0.1	1.0050	1.0061	1.0072	1.0085	1.0098	1.0113	1.0128	1.0145	1.0162	1.0181
0.2	1.0201	1.0221	1.0243	1.0266	1.0289	1.0314	1.0340	1.0367	1.0395	1.0424
0.3	1.0453	1.0484	1.0516	1.0550	1.0584	1.0619	1.0655	1.0692	1.0731	1.0770
0.4	1.0811	1.0852	1.0895	1.0939	1.0984	1.1030	1.1077	1.1125	1.1174	1.1225
0.5	1.1276	1.1329	1.1383	1.1438	1.1494	1.1551	1.1609	1.1669	1.1730	1.1792
0.6	1.1855	1.1919	1.1984	1.2051	1.2119	1.2188	1.2258	1.2330	1.2402	1.2476
0.7	1.2552	1.2628	1.2706	1.2785	1.2865	1.2947	1.3030	1.3114	1.3199	1.3286
0.8	1.3374	1.3464	1.3555	1.3647	1.3740	1.3835	1.3932	1.4029	1.4128	1.4229
0.9	1.4331	1.4434	1.4539	1.4645	1.4753	1.4862	1.4973	1.5085	1.5199	1.5314
1.0	1.5431	1.5549	1.5669	1.5790	1.5913	1.6038	1.6164	1.6292	1.6421	1.6552
1.1	1.6685	1.6820	1.6956	1.7093	1.7233	1.7374	1.7517	1.7662	1.7808	1.7956
1.2	1.8107	1.8258	1.8412	1.8568	1.8725	1.8884	1.9045	1.9208	1.9373	1.9540
1.3	1.9709	1.9880	2.0053	2.0228	2.0404	2.0583	2.0764	2.0947	2.1132	2.1320
1.4	2.1509	2.1700	2.1894	2.2090	2.2288	2.2488	2.2691	2.2896	2.3103	2.3312
1.5	2.3524	2.3738	2.3955	2.4174	2.4395	2.4619	2.4845	2.5074	2.5305	2.5538
1.6	2.5775	2.6014	2.6255	2.6499	2.6746	2.6995	2.7247	2.7502	2.7760	2.8020
1.7	2.8283	2.8549	2.8818	2.9090	2.9364	2.9642	2.9922	3.0206	3.0492	3.0782
1.8	3.1075	3.1370	3.1669	3.1972	3.2277	3.2585	3.2897	3.3212	3.3530	3.3852
1.9	3.4177	3.4506	3.4838	3.5173	3.5512	3.5855	3.6201	3.6551	3.6904	3.7261
2.0	3.7622	3.7986	3.8355	3.8727	3.9103	3.9483	3.9867	4.0255	4.0647	4.1043

1/ Handbook of Chemistry and Physics (34th Edition, p. 180-186).