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AN ELECTRICAL DEVICE FOR COMPUTING THEORETICAL DRAWDOWNS OF GROUND-WATER LEVELS

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Abstract

The construction, calibration, and use of an electrical "slide rule" for computing theoretical drawdowns of ground-water levels are described. The instrument facilitates the computation of drawdowns under given conditions of discharge or recharge by means of the Theis nonequilibrium equation. It is simple to construct and use and can be a valuable aid in ground-water studies.

Introduction

It is desirable to predict drawdowns under given conditions of discharge or recharge during many ground-water investigations. This is usually done by applying the Theis nonequilibrium equation (Theis, 1935, p. 519-524), which may be written:

$$(1) \qquad s = \frac{114.6\ Q}{T}\ W(u),$$

where:

$$(2) \qquad W(u) = \int_{u}^{\infty} \frac{e^{-u}}{u}\ du,$$

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and:

$$(3) \qquad u = \frac{1.87\, r^2\, S}{Tt}$$

s is the predicted drawdown in feet at any point in the vicinity of a discharging well.

Q is the discharge rate of the pumped well in gallons per minute.

T is the coefficient of transmissibility expressed as the rate of flow of water in gallons per day through a vertical strip of aquifer, 1 foot wide and extending the full saturated height, under a hydraulic gradient of 100 percent at the prevailing temperature of the water. It is usually determined by applying the nonequilibrium formula to pumping-test data (Brown, 1953, p. 844-866).

S is the coefficient of storage of the aquifer or the volume of water it releases from or takes into storage per unit surface area of the aquifer per unit change in the component of head normal to that surface. It is also determined by applying the nonequilibrium formula to data from pumping tests.

r is the distance in feet from the discharging well to the point where s is to be computed.

t is the time in days since pumping began.

e is the natural-logarithm base.

The drawdown, s, in an aquifer is readily computed for a given set of values of Q, T, S, r, and t. The appropriate values are substituted into expression (3), and u is determined. Then, the corresponding value of W(u) is determined from tables relating W(u) and u (Wenzel, 1942, opp. p. 88), and expression 1 is then solved for the drawdown, s.

It is often desirable to determine how drawdowns in a given aquifer will vary with pumping rate, with time, or with distance from the discharging well. Successive values of s must then be computed for different values of Q, t, or r, and the repeated solution of the equations becomes time consuming so that the use of computational aids becomes desirable.

Nomographs and computing devices have been devised to facilitate the use of the nonequilibrium formula. Drawdowns under given conditions of discharge or recharge may be computed by means of the chart of Theis (1939), the nomograph of Commons (1942) or of Chow (1951, p. 48-49), by the slide rule of Theis and Brown (1951), or by the electrical computer described in this paper. Computing devices are also available for solving aquifer tests for values of T and S. They are nomograph of Jeffords (Jeffords, R. M., U. S. Geological Survey, personal communication) and the nomographs of Remson and van Hylckama (in preparation).

The design and construction of electronic computers of high accuracy and wide flexibility is difficult and expensive. However, a specialized computer of acceptable though not great accuracy, such as that described in this paper, can be constructed by the application of simple principles and from inexpensive electrical components. The junior author has constructed a number of such electrical computers to solve equations used in micrometeorology and climatology (Halstead, 1954).

Construction of the Computer

Figure 1 is the wiring diagram of the electrical computer designed to solve the nonequilibrium formula for drawdown. Figure 2 shows the calibrated face of the instrument. It consists of two electrically independent circuits. Circuit 1 is designed to fit expression 3,

$$(3) \qquad u = \frac{1.87\ S}{T} \cdot r^2 \cdot \frac{1}{t} .$$

Circuit 2 is designed to fit expression 1,

$$(1) \qquad s = \frac{114.6}{T} \cdot Q \cdot W(u).$$

In expression 3, t, r, and S/T can vary over extremely large ranges. For this reason, circuit 1 is based upon the addition of logarithms rather than upon the direct multiplication of values. It consists of a microammeter and three similar units, each one consisting of a battery, a battery control to adjust for changes in battery strength, a switch, and a variable voltage divider or potentiometer. Voltage dividers of the same magnitude should be used in circuit 1. Voltage dividers of the particular values shown in figure 1 were used in the computer discussed here because they were readily available.

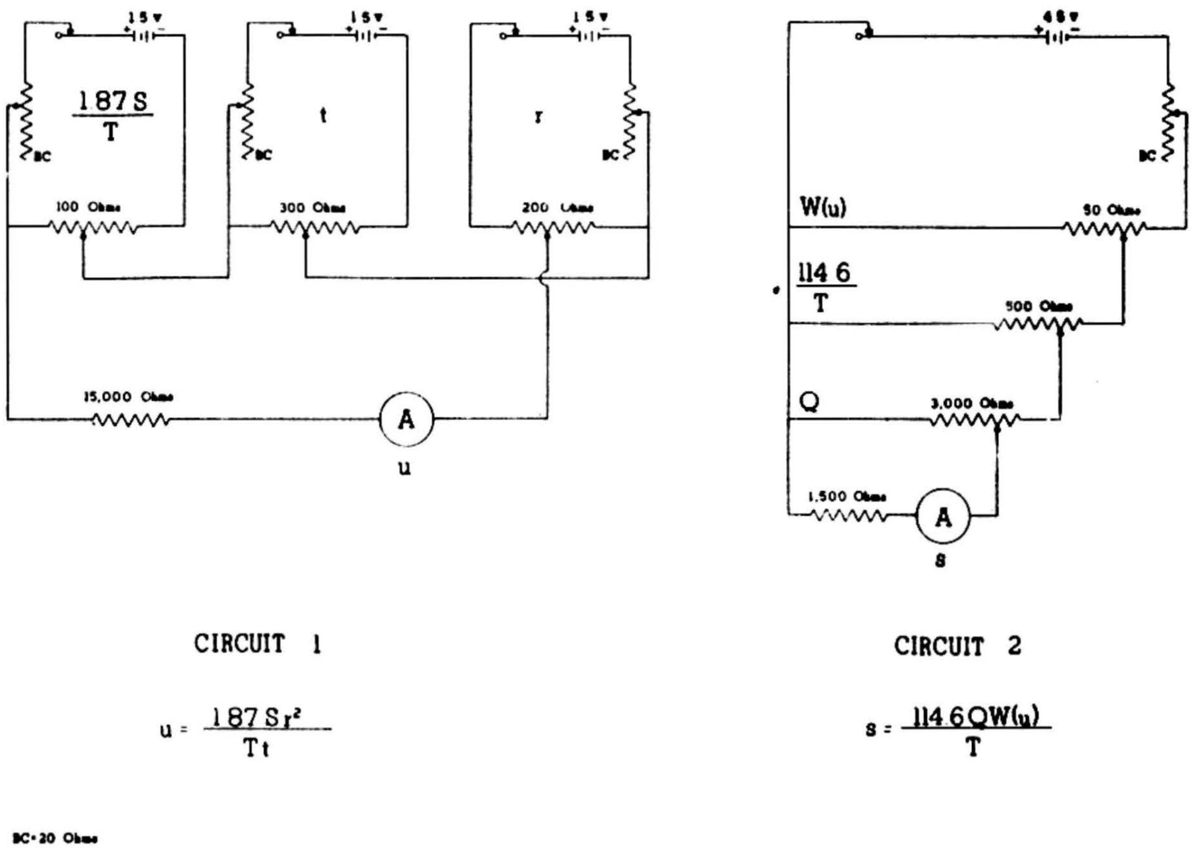

Figure 1: Wiring diagram of the computer.

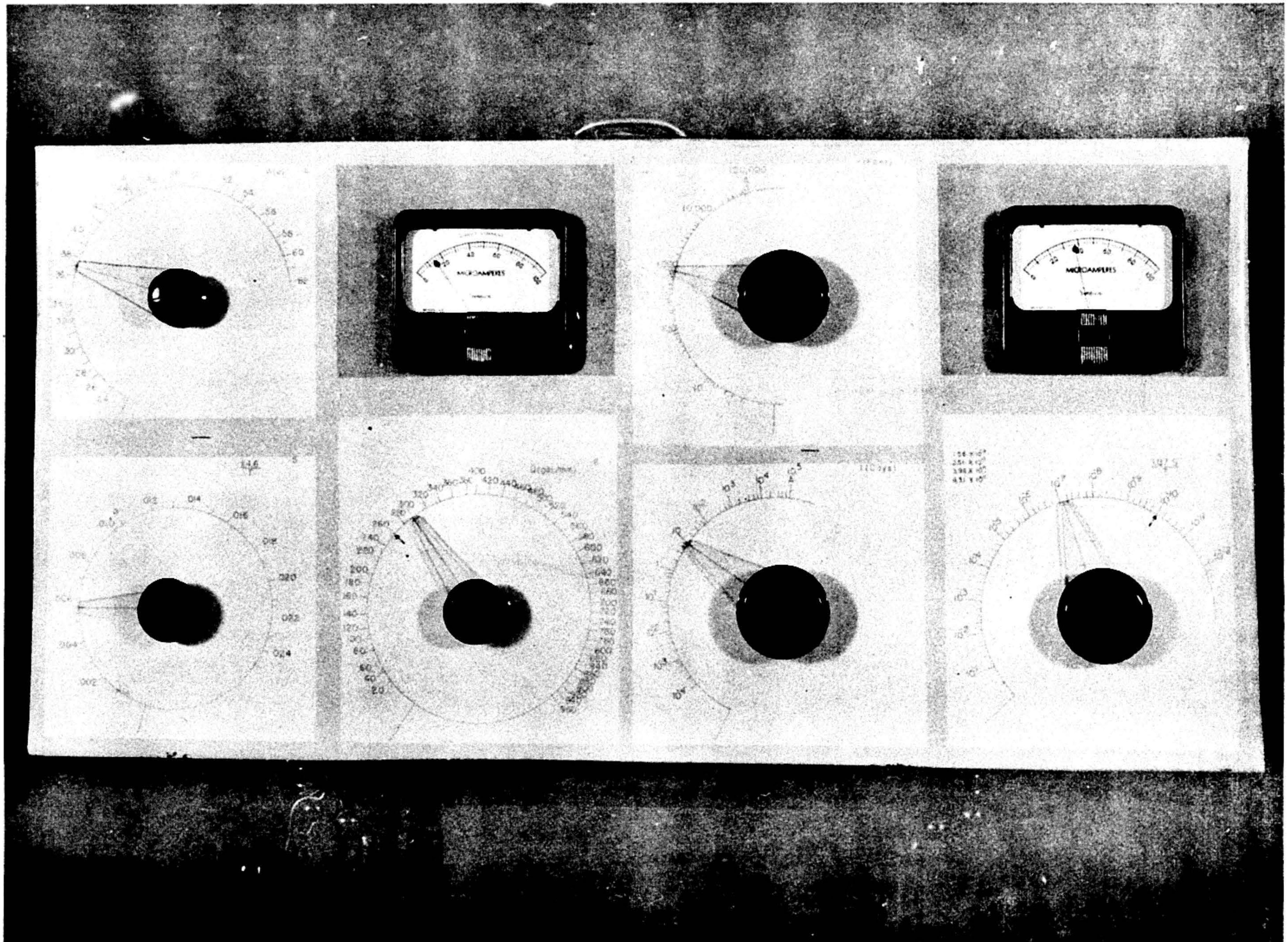

Figure 2.--Calibrated face of the computer. Circuit 1 is to the right and circuit 2 is to the left.

Other units with resistances of the same magnitude could have been used instead.

The meter reading in circuit 1 is proportional to 1/u. The wiring used in the instrument described here is such that clockwise movement of the t pointer increases the value of t, the value of 1/u, and the meter reading. A clockwise movement of the r pointer increases the value of r but decreases the value of 1/u and the meter reading. A clockwise movement of the 1.87S/T pointer decreases the value of 1.87S/T but increases the value of 1/u and the meter reading. These relationships are in accord with expression 3.

Circuit 2, which is based upon expression 1, is a direct multiplication circuit. A logarithmically additive circuit could have been used instead. Each of the voltage dividers in this circuit must have a total resistance five to ten times as large as the one preceding it in order to minimize interactions between them. The voltage dividers represent W(u), 114.6/T, and Q, respectively, and the microammeter reading is proportional to s. A clockwise movement of any pointer in circuit 2 increases the value of the respective term, the value of s, and the microammeter reading. These relationships are in accord with expression 1.

The reading on the microammeter in circuit 1 is a function of 1/u. A value of W(u) must be put on the appropriate dial in circuit 2. The conversion from u to W(u) is normally done with the aid of expression 2 or the tables already referred to. The calibration of this instrument permits the conversion to be accomplished automatically. Because u and W(u) are inverse functions, the microammeter of circuit

1 is arranged to read as a function of $1/u$. This makes the $W(u)$ scale and the microammeter in circuit 1 increase in the same direction and facilitates the calibration.

Calibration of the Computer

Account must be taken of the variation of battery strength with time before calibrating or using the computer. The battery outputs must be the same every time the instrument is used. This is accomplished by means of the arrows and A marks on the respective dials and the battery controls, which are operated by knobs on the side of the instrument and are not shown in figure 2. For example, the t subcircuit is properly adjusted when the pointer is set on the arbitrarily chosen reading, A, and the microammeter reads 50 with all other parts of circuit 1 switched off.

All calibration is done by comparison with the microammeter readings. Circuit 1, which is arranged logarithmically, is calibrated first.

Step 1: The $1.87S/T$ subcircuit is switched on, and the pointer is advanced until the microammeter reads 5. That position of the pointer is made 10^{-1}, for example. The pointer is then advanced until the microammeter reads 10, and that position of the pointer is made 10^{-2}. The procedure is continued until the desired range of values has been placed on the dial.

Step 2: With the $1.87S/T$ and r subcircuits off, the t scale is calibrated in a similar fashion.

Step 3: With the 1.87S/T subcircuit off and the r pointer turned completely counterclockwise, the t pointer is adjusted to give a microammeter reading of 50. The r pointer is then advanced until the microammeter reading has decreased to 40. That position of the r pointer is labelled 10. The r pointer is then moved until the microammeter reading has been decreased to 30, and this new position of the r pointer is then labelled 100. The procedure is continued until a satisfactory range of values is obtained.

The procedure outlined gives values to powers of 10 because of its logarithmic nature. The intermediate values can be interpolated logarithmically. For demonstration purposes, the smaller divisions on the 1.87S/T scale were spaced uniformly, as shown on figure 2, and the corresponding values were placed in the corner of the scale. The r and t scales were spaced logarithmically in the normal manner.

Circuit 2 is a simple multiplication circuit, and it is calibrated in the following way:

Step 1: The Q scale is inserted linearly. The Q pointer is turned completely clockwise, and the W(u) and 114.6T pointers are then adjusted until the meter reads 100. This position of the Q pointer is chosen as 1,000. The Q pointer is then turned counterclockwise until the microammeter reads 90. This position of the pointer is 900. The procedure is used to obtain the complete calibration of this scale.

Step 2: It is decided where to put a figure in the desired range, .001 for example, on the 114.6/T scale. With the pointer in that position, the W(u) and Q pointers are adjusted to give a random reading on the microammeter. The 114.6T pointer is then advanced until the microammeter reading is doubled. This position of the pointer will then be .002. The 114.6/T pointer can then be advanced until the original microammeter reading is multiplied by 10, and the value of 114.6/T equal to .01 will have been determined. The entire scale is calibrated in this manner.

Step 3: To calibrate the W(u) scale, a trial problem is set up and solved for s by means of the equations. The same values of r, t, and 1.87S/T used in the problem are set on the respective dials in circuit 1 and give a reading on the microammeter. Assume that this reading is 50. The same values of Q and 114.6/T used in the problem are then set on the respective dials of circuit 2. The W(u) pointer is finally adjusted until the value of s obtained from the equations appears on the microammeter of circuit 2. The computer is now adjusted so that it completely "simulates" the trial problem. Therefore, circuits 1 and 2 are linked by giving the position of the W(u) pointer the value shown by the microammeter in circuit 1. In this case, the value is 50. In similar fashion, other values can be determined for the W(u) scale by simulating other trial problems. In essence, this scale is calibrated by making the computer solutions coincide with the mathematical solutions.

Use of the Computer

The operation of the computer is simple and rapid. Values of 114.6/T and 1.87S/T are first computed for an aquifer and placed on the appropriate dials. The values of r, t, and Q are then placed on the respective dials. Finally, the W(u) pointer is adjusted to the reading shown on the microammeter in circuit 1. The drawdown is then read from the microammeter in circuit 2. In the model shown in figure 2, a satisfactory range of values was obtained by adjusting the microammeter to read twice the value of the drawdown. Once the 114.6/T and 1.87S/T dials are set, they need not be changed for any computations that involve the same aquifer. Successive values of s for different values of Q, r, or t can then be determined in a matter of seconds.

The computer has great value as a demonstration device. The effect upon drawdown of pumping rate, time, or any of the other factors involved can be demonstrated visually by advancing the appropriate dials.

Conclusions

The computer described in this paper is simple in its construction and calibration and easy to operate. Similar computers can be constructed to solve other ground-water problems. For example, it is possible to build one that will solve pumping-test data for values of T and S. It is believed that this type of instrument will prove a valuable aid in ground-water studies.

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