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METHOD OF DETERMINING THE COEFFICIENT OF STORAGE FROM STRAIGHT-LINE PLOTS WITHOUT EXTRAPOLATION

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It was shown by Jacob (1944, 1947) and by Cooper and Jacob (1946) that of the infinite series resulting from the solution of the Theis (1935) exponential integral, only the first two terms need be used for values of u less than about 0.01 and that the equation may be simplified as follows:

$$T = \frac{Q}{4\pi s}\left[-0.5772 - \log_e \frac{r^2 S}{4\,Tt} + \cdot\cdot\cdot\right] \quad (1)$$

$$= \frac{Q}{4\pi s}\left[\log_e 0.562 + \log_e \frac{4\,Tt}{r^2 S}\right]$$

$$= \frac{2.3\,Q}{4\pi s} \log_{10} \frac{2.25\,Tt}{r^2 S} \quad (2)$$

The straight-line solutions of Cooper and Jacob (1946) are obtained by differentiating s or s/Q (in equation 2) with respect to $\log_{10} t$ or $\log_{10} t/r^2$. The resulting differential is graphically shown by the slope of a semilog plot of s or s/Q versus $\log_{10} t$ or $\log_{10} t/r^2$. Similarly, the straight-line solution of Jacob and Lohman (1952) for the nonsteady flow to a flowing artesian well of constant drawdown is

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obtained by differentiating s_w/Q with respect to $\log_{10} t/r_w^2$, observing as before that the differential is graphically shown by the slope of the plot of s_w/Q versus $\log_{10} t/r_w^2$. The Thiem equation for steady-state flow also can be obtained in this manner, in which s is plotted against $\log_{10} r$.

In each of the cited solutions the plot is made on semilogarithmic paper, and the value of the storage coefficient (S) generally is determined by extrapolating the straight-line part of the plot to the point where s, s/Q, or $s_w/Q = 0$. Then, from equation (1)

$$s = \frac{Q}{4\pi T}\left[\log_e \frac{4\,Tt}{r^2 S} - 0.5772\right] = 0$$

$$\log_e \frac{4\,Tt}{r^2 S} = 0.5772 = \log_e 1.781$$

$$\frac{4\,Tt}{r^2 S} = 1.781$$

$$S = 2.25\,T\,t/r^2 \text{ or } 2.25\,T\,t/r_w^2 \qquad (3)$$

In units commonly employed by the Geological Survey, where T is expressed in gpd/ft (gallons per day per foot), r is in feet, and t is in days, equation (3) may be written

$$S = 0.30\,T\,t/r^2 \text{ or } 0.30\,T\,t/r_w^2 \qquad (4)$$

or

$$S = 2.1 \times 10^{-4}\,T\,t/r^2 \text{ or } 2.1 \times 10^{-4}\,T\,t/r_w^2 \qquad (5)$$

in which t is in minutes.

Equation (3), (4), or (5) gives the desired results satisfactorily if the straight line has sufficient slope so that its zero arithmetic coordinate can be found within the confines of the semilogarithmic plot. If, however, the slope is quite flat, so that the zero arithmetic coordinate occurs outside the confines of the plot, error may result from graphical extrapolation onto an adjoining sheet of graph paper. Although a mathematical rather than a graphical extrapolation may be used, it is the purpose of this note to describe a simple method by which S may be determined within the data region of the straight-line plot without the need for any extrapolation.

For the type solution involving a semilog plot of s_w/Q values versus log t/r_w^2 values, Jacob and Lohman (1952, p. 567) showed that equation (2) can be solved for S to give

$$S = \frac{2.25\,T\,t/r_w^2}{\log_{10}^{-1}\left[\frac{4\pi T\,s_w/Q}{2.3}\right]} \tag{6}$$

Differentiating equation (2) with respect to the plotted variables yields the simple relation

$$T = \frac{2.3}{4\pi\Delta(s_w/Q)} \tag{7}$$

where $\Delta(s_w/Q)$ is the change over one log cycle of the ratio t/r_w^2. Combining equations (6) and (7) gives

$$S = \frac{2.25T\ \ t/r_w^2}{\log_{10}^{-1}\left[\frac{s_w/Q}{\Delta(s_w/Q)}\right]} \tag{8}$$

Thus to compute the value of S, select any convenient point on the semilog plot and substitute its coordinates s_w/Q and t/r_w^2 in equation (8). The value of T is taken from previous computations, and the value of $\Delta(s_w/Q)$ is as defined for equation (7).

Similarly, for solutions involving semilog plots of s versus $\log_{10}t$ or $\log_{10}t/r^2$, the simplified form of equation (6) becomes

$$S = \frac{2.25\,T\,t/r^2}{\log_{10}^{-1}\left[\frac{s}{\Delta s}\right]} \tag{9}$$

where Δs is the change in drawdown over one log cycle of t or t/r^2. For the analysis of a semilog plot of s versus $\log_{10} r$ equation (6) becomes

$$S = \frac{2.25\, T\, t/r^2}{\log_{10}^{-1}\left[\frac{-2s}{\Delta s}\right]} \tag{10}$$

where the negative sign in the bracketed term reflects the fact that drawdown <u>decreases</u> as the distance from the discharging well increases. Thus in equation (10) when substituting for Δs, which is the change in drawdown over one log cycle of r, the numerical value should be prefixed with a minus sign to recognize properly the negative slope of the data plot in the straight-line region of interest. The two negative signs then combine to make the bracketed term positive.

Equations (9) and (10) are applied in a manner similar to that described for equation (8). In Geological Survey units, the 2.25 in equations (8), (9), and (10) becomes 0.30 (for t in days) or 2.1×10^{-4} (for t in minutes).

The example given in figure 1 will suffice to illustrate the method. Note that point A was conveniently chosen to give an even value of t/r^2 and to coincide with one of the points that was used in computing Δs (1.31). The coordinates of point A are evidently 3.25 and 1. In determining the antilog of $\left[\frac{3.25}{1.31} = 2.48\right]$ (which can be done readily by the use of a log-log slide rule) remember that the .48 is the mantissa, which establishes the digits 3, 0, and 2; and that the 2.-- is the characteristic, which fixes the position of the decimal point. Thus the antilog sought is 302.

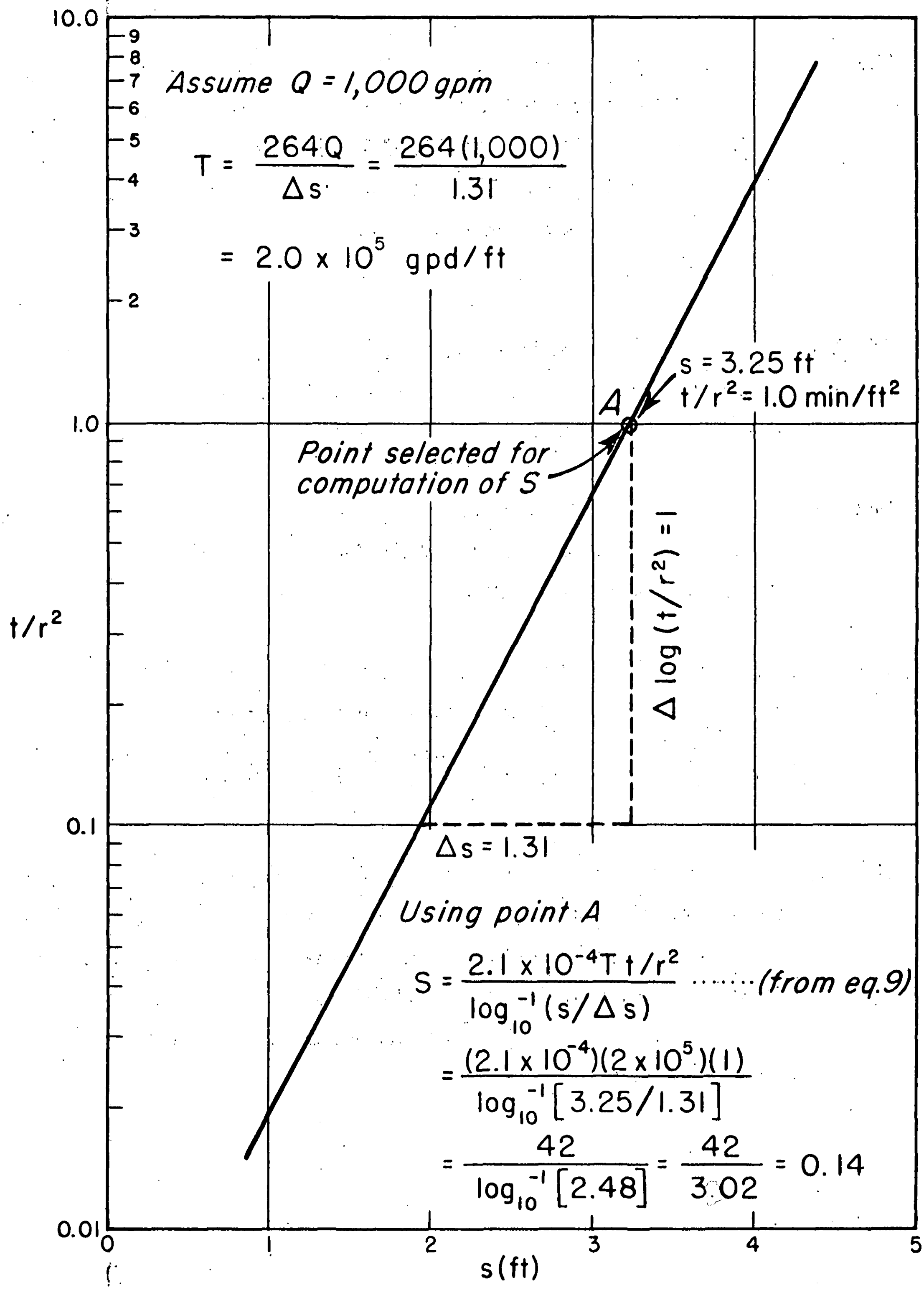

Figure 1-- Semilogarithmic plot of aquifer-test data and sample computations of coefficients of transmissibility and storage

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