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THE SPACING OF PUMPED WELLS

By
C. V. Theis

The problem of spacing pumped wells properly is generally one of economics--the farther apart they are the less their interference but the greater the cost of connecting pipeline and electrical installation. For a simple case of two wells pumping at the same rate from a thick and areally extensive aquifer the problem is not difficult. The cost of an installation of two such wells, insofar as it is affected by the spacing of the wells, may be reduced to an annual unit charge consisting of (a) the cost of lifting the water against the additional head owing to the drawdown in each well caused by pumping the other, expressed as a cost per year, and (b) the cost of connecting pipeline between the two wells and electrical installation, including maintenance, depreciation, and capital charges on the cost of installation, which may be expressed as a cost per foot of intervening distance per year. Considering average cost values over a year's time, and adopting the units commonly used by the Geological Survey, let

s = drawdown, in feet, in one pumped well caused by pumping the other well

c = cost, in dollars, to raise a gallon of water 1 foot, consisting largely of power charges, but also properly including some additional charges on the equipment

k = capitalized cost, in dollars per <u>year</u> per foot of intervening distance, for maintenance, depreciation, original cost of pipeline, etc.

Q = pumping rate of <u>each</u> well, in gpm

r = distance between the two pumped wells, in feet

C = total yearly cost, in dollars

Then the general expression for total yearly cost may be written in the form

$$
\begin{aligned}
C &= 2 \times 365 \times 1{,}440\, cQs + kr \\
&= 1.05 \times 10^{6}\, cQs + kr \quad \ldots\ldots\ldots\ldots (1)
\end{aligned}
$$

In the region between and for some distance around the pumped wells it can be assumed that steady-state conditions have prevailed throughout most of the 1-year pumping period. Thus the drawdown in either pumped well resulting from pumping the other well is given by the following approximate expression (Cooper and Jacob, 1946, p. 527)

$$
s = \frac{Q}{4\pi T}\left(-0.577 - \log_e \frac{r^2 S}{4Tt}\right) \quad \ldots\ldots\ldots (2)
$$

Converting to the usual Survey units there follows

$$
\begin{aligned}
s &= \frac{114.6\,Q}{T}\left(-0.577 - \log_e \frac{1.87\, r^2 S}{Tt}\right) \\
&= \frac{114.6\,Q}{T}\left(-0.577 - \log_e \frac{1.87\, S}{Tt} - \log_e r^2\right) \quad \ldots (3)
\end{aligned}
$$

in which

T = coefficient of transmissibility of the aquifer, in gallons per day per foot

S = coefficient of storage of the aquifer

t = elapsed time of pumping, in days

Other terms are as previously defined. Substituting in equation (1) the value for s as given in equation (3) there results

$$
\begin{aligned}
C &= 1.05 \times 10^{6} \left[\frac{114.6\, Q^2 c}{T}\left(-0.577 - \log_e \frac{1.87\, S}{Tt} - 2\log_e r\right)\right] + kr \\
&= \frac{1.20 \times 10^{8}\, c\, Q^2}{T}\left(-0.577 - \log_e \frac{1.87\, S}{Tt} - 2\log_e r\right) + kr \ldots (4)
\end{aligned}
$$

The minimum cost will correspond to the point at which the first derivative of C with respect to r equals zero. Differentiating equation (4) and equating the resulting expression to zero yields

$$\frac{dC}{dr} = \frac{1.2 \times 10^8\ c\,Q^2}{T}\left(-\frac{2}{r}\right) + k = 0$$

from which the optimum well spacing, r_o , is found to be

$$r_o = \frac{2.4 \times 10^8\ c\,Q^2}{kT} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \qquad (5)$$

As a final test of the propriety of using the approximate relation given by equation (2) the value of u, as computed from the relation $u = 1.87\, r_o^2\, S/Tt$, should prove to be less than 0.02.

To illustrate the use of equation (5) assume the availability of electrical power at 1 1/2 cents per kwh and an overall efficiency of 50 per cent. The power charge (c) for lifting water will then be about 1×10^{-7} dollars per gallon per foot. Assuming the cost of pipeline and electric wiring as $10 per foot and capitalizing this at 10 percent gives a cost (k) of $1 per foot per year for capital charges, depreciation, and maintenance. If the aquifer transmissibility (T) is assumed as 50,000 gpd/ft and the discharge (Q) as 500 gpm from each well, then

$$r_o = \frac{2.4 \times 10^8 \times 10^{-7} \times 25 \times 10^4}{1 \times 5 \times 10^4}$$

$$= 120 \text{ feet.}$$

Inspection of equation (5) shows that for fixed cost factors (c and k) the optimum spacing, r_o , of two pumped wells will vary directly with the square of the proposed pumping rate, Q, or inversely with the aquifer transmissibility, T. Thus r_o will be greater for an aquifer of low transmissibility than it will for an aquifer of high transmissibility; or within the same aquifer r_o will be greater for a high pumping rate than for a low rate. The only aquifer property involved in determining r_o is seen to be the transmissibility. This means that for a water-table and an artesian aquifer of the same transmissibility the values of r_o will be identical. However, when equation (4) is used to compute the respective total annual costs, obviously it will cost less to pump from the water-table aquifer (large value of S) than from the artesian aquifer (small value of S).

The curve of total annual cost vs. distance between the two pumped wells is quite flat in the neighborhood of the minimum value, so that not much is lost by deviating somewhat from the minimum value given by equation (5). Therefore, in extensive aquifers the governing principle for spacing wells generally is convenience of operation and not hydrologic conditions. This, of course, is not to say that wells should be located at opposite corners of the pumphouse. An analysis similar to that given in the foregoing example, using appropriate constants, should be made whenever comparable field problems may require some investigation of criteria for spacing wells that are to be pumped, but considerable latitude should be allowed for judgment of convenience, security, and other factors that cannot be evaluated quantitatively.

Obviously, if aquifer boundaries are present or more than two wells are pumping, the foregoing analysis will have to be modified to take into account the added conditions.

REFERENCES

Cooper, H. H., Jr. and Jacob, C. E., 1946, A generalized graphical method for evaluating formation constants and summarizing well-field history: Am. Geophys. Union Trans., v. 27, no. 4, p. 526-534, August, (Reprinted as U. S. Geol. Survey Ground Water Note 7, January 1953.)