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THE APPLICATION, TECHNIQUE, AND THEORY OF GISH-ROONEY INSTRUMENTS, METHODS, AND INTERPRETATION IN ELECTRICAL RESISTIVITY MEASUREMENTS

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Figure 40. Current fractions, f_1, in the surface stratum of a two-layer earth. a/h = (half of current electrode spacing)/ (surface layer thickness; k = $(1 - \bar{\sigma}) / (1 + \bar{\sigma})$ where $\bar{\sigma}$ = (conductivity of second layer) / (conductivity of surface layer).

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INTRODUCTION

The foundations of electrical prospecting were laid by Fox (1830) and Barus (1882). The former discovered that electrical currents and potentials were developed by certain ore deposits in Cornwall, England. Fox made measurements of these potentials and determined the resistivities of the minerals and surrounding rocks also. The latter, invented the non-polarizing electrode with which he overcame the polarizing effects of direct current, and was able to trace and extend the Comstock lode. According to Slichter (1934, p.234), F. Neumann proposed as early as 1884 determining the resistivity of the earth by a four-electrode method nearly like our present technique, and carefully developed the mathematical theory of the method. Wells (1914) followed the work of Barus with studies of the electric activity in ore deposits. Ambronn (1926) gives an extensive bibliography of the early work in geophysics.

The classical work of Wenner (1917) and, almost at the same time and independently, that of Schlumberger (1920) laid the foundations for resistivity measurements on the surface of the earth. While Schlumberger was actually making measurements of earth resistivity related to structural studies as early as 1912 in France, work in this country was slow in developing from the work of Wenner, but it was gaining adherents following the pioneering work of Kelley (1922) and Gish (1923). Probably the work of Gish and Rooney (1925) who redesigned the apparatus of McCollum (1921) was the largest contributing cause in this country for the sudden interest and expansion of electrical methods. Their work was followed by many others in the late 1920's, some of which are: Gish (1926); Rooney and Gish (1927); Rooney (1927); Gish and Rooney (1927); Lee (1928); Hotchkiss, Rooney, and Fisher (1929); Weaver (1929); Lundberg (1929); Leonardon and Kelley (1929); Crosby and Leonardon (1929); Carrette and Kelly (1929); Koenigsberger (1929); Hunkel (1928, 1929); Heiland (1929); ~~and many others~~.

EARLY INSTRUMENTS USED

The instruments employed by these early workers were either of the direct current measuring type, the Gish-Rooney type, or the Megger type. For the direct current type, some form of bridge or potentiometer was used to measure the potential, while a sensitive milliammeter was usually used to measure the current. The Gish-Rooney instrument was essentially a direct current measuring one with the addition of a commutator whose function was to reverse the current and potential at proper intervals. The frequency chosen for the operation of the Gish-Rooney commutator was dependent upon the widths of the non-conducting segments of the commutator and the damping and inertia of the meter movement. A constant, called a commutator factor, had to be applied to each observation to produce a reading equivalent to a direct current measurement. The megger is a self-powered instrument with internal coils so arranged that it reads directly the ratio of V/I, or ohms, on a scale by means of an indicating pointer.

DEVELOPMENT OF GISH-ROONEY TYPE INSTRUMENTS

Early Gish-Rooney instruments, such as those shown in figures 1-A and 1-B, were bulky, at times balky, but gave good readings of resistivity when they operated properly. The next later group of instruments were much smaller, figure 2; the measuring units were all combined into one case and the commutator was housed in another. Connections between the commutator and the measuring unit of the above instruments were made by shielded cables. A still further reduction in size was later accomplished when the commutator was placed in the same case with the measuring unit, figure 3. The latest models available are still more compact measuring about 11 x 12 x 16 inches. Provision is made for either manual or motor drive in the late models. Current models available have the power and drive linked by a flexible belt; but this can cause annoyance in cold weather as the belt becomes stiff, as well as the oil in the bearings, and the drag may be too heavy to allow rotation. To overcome this drag, the writer devised a gear-driven unit to replace the belt-drive and added a rheostat in the motor circuit to control the speed. This modification is seen in figure 4.

 followed by p 6

CALIBRATION OF GISH-ROONEY INSTRUMENTS

The potentiometer of the early Gish-Rooney instruments was calibrated by reference to a standard cell. This method, while quite satisfactory for a laboratory measurement, was not very reliable for a field instrument in which the standard cell was jostled, jarred, overturned at times, and subjected to wide temperature variations while transporting the instrument. This unavoidable treatment resulted in an unstable reference voltage for calibration. A later model had an arrangement whereby a fixed current could be sent through the potentiometer for calibrating. This was accomplished by short-circuiting the current binding posts, C-1 and C-2, removing the battery supply cable, and inserting in place of it a special jack and plug terminated cable which cut out the external battery and inserted the bridge battery. By setting the milliammeter on the 0 - 20 ma scale range and manipulating the potentiometer current control knob until the meter read 10 ma, the potentiometer was correctly balanced. The special cable was then removed and the external battery cable reinserted and the instrument was ready for reading.

CALIBRATION BY DUMMY EARTH

Present models of Gish-Rooney instruments have a more easily used device for calibrating called a dummy earth by Krasnow (1942). The dummy earth contains a series of resistances mounted on an insulating panel and attached to plugs which fit into the current, potential, and ground jacks of the earth resistivity instrument. Such devices are shown in figure 5, one with cover on and the other with cover removed. It will be noticed on the uncovered dummy earth that the inner resistors connected to the center or ground plug are precision wire wound non-inductive while those connected to the two outer plugs are molded metallized with 5 percent accuracy. Their resistances are shown on the figure. These resistors form a continuous circuit between the outer jacks, which are the current carrying ones, and will allow only a certain current to flow when connected to a $22\frac{1}{2}$ volt supply and this is readable on the milliammeter of the instrument. The inner set of resistors are connected across the inner jacks, located on either side of the center jack or ground, and when a current is flowing will have a certain IR or voltage drop across each one or both together. This potential is measurable by the potentiometer in the instrument.

To calibrate the instrument, the potentiometer is set to read the same in millivolts as the meter reads in milliamperes with the milliammeter set on the X 0.1 scale, the current adjusting knob of the potentiometer is then turned until the galvanometer reads zero and the instrument is balanced for reading potentials. More explicit directions will be given in a later section.

MODIFICATIONS AND IMPROVEMENTS OF GISH-ROONEY INSTRUMENTS

Some other important modifications have been incorporated into the recent models of the Gish-Rooney instrument at the suggestion of the writer and others to the maker, as well as some independently added by the maker. These improvements include a switch in the potentiometer battery circuit to lengthen its life; a departure from telephone type switches to heavy duty switches with ample current-capacity contacts; selector switch for potential electrodes combined into one switch; an extra range of X 0.1 in the potentiometer to facilitate low range readings (this does not, however, increase the accuracy of measurements); incorporation of extra ranges in the milliammeter so that it may be used for heavy currents; a switch so that either D. C. or Gish-Rooney measurements can be made; provision for inserting an external galvanometer, one either more or less sensitive than that supplied in the instrument; the addition of a compensating resistor for the commutator factor so that a factor does not have to be included in the computations; and shielded circuitry to minimize stray pick-up.

DESCRIPTION OF A RECENT MODEL GISH-ROONEY INSTRUMENT

A top view of a recent model Gish-Rooney instrument is shown in figure 6 with all the parts labeled. A below panel view in figure 7 with all the parts labelled, shows the potential side of the commutator and the brushes with their adjustments, the galvanometer, slide wire, selector switches, motor, guard-ring for the commutator, galvanometer switch, potentiometer range switch, and the bottom of the jacks. Another below panel view, figure 8, with all parts labeled, shows the current side of the commutator, the gear-drive on the commutator with idler gear for manual operation, potentiometer decade resistors, selector switches, motor-manual switch, and the current and potential reversing switch. On this particular instrument, all the potential circuit is shielded and earthed to the panel which is aluminum. This grounding, in conjunction with the copper shield between the current and potential sections of the commutator, acts as a guard ring and dissipates all stray currents set up by induction. Leakage in the circuit wiring and components is checked both before and after assembly by means of a low voltage ohm meter and a high voltage megger. The large capacitor, which is connected in the active lead of the potential circuit, is not shown, but is located in the bottom of the instrument case. This capacitor is used to block the natural earth currents and keep them from disturbing the measurements of potential. It is effective in this use because natural earth, or telluric, currents are either direct current or very slowly pulsating direct currents. Also in the bottom of the case are four "D" cells in a holder which is fastened securely to the case. These cells are connected in parallel and supply the operating current for the potentiometer. The rheostat knob for controlling the speed of the drive motor for the commutator is located in the lower right corner of the case, figure 8. The handle for manual operation is seen in the lid for the case in figures 6 and 8. The hole for inserting the handle is under the oval cover plate, figure 7, and the hole for inserting a screwdriver to enmesh the idling gear for manual operation is under the circular cover plate adjacent to the oval plate in this figure. Models of the instrument having a belt drive do not have the enmeshing knob and can be operated manually directly upon inserting the driving handle.

ACCESSORY EQUIPMENT FOR GISH-ROONEY METHOD

Conveyance for equipment. Numerous accessory items are required to enable one to perform field measurements of resistivity measurements. A conveyance of some kind is required to transport the equipment and, perhaps, all or at least part of the crew. The most convenient automobile will depend largely upon the nature of the field work, the preference of the operator, and sometimes, on what kind is available. The writer has used passenger cars, light panel trucks, heavy panel trucks, station wagons, and carryalls successfully; but has found for general use on extended assignments and long-haul trips that the heavy panel truck is exceedingly convenient.

Equipment for conveyance. Cabinets can be built and installed in a large panel truck; cabinets with sliding doors are preferred and either foam rubber or felt may be used to line the cabinets for protecting the instruments from shock under all conditions of travel. Bins or racks for wire reels can also be installed; compartments for batteries; containers for water, such as ten-gallon cream cans; drinking water supply; hammers; axes; machetes; etc.; may also be included in the cabinet installation. In a small truck, a table which folds flat against the side of the truck may be installed for setting up the instruments, and the passenger seat may be remodelled to either rotate on a base or to be lifted up and reversed and set in supports in front of the instrument. In a larger truck, a convenient arrangement is to build a desk in conjunction with the cabinets just behind the drivers seat and have a folding top on it which may be opened out and give ample room for the instruments, plotting data, etc. An extra seat may be permanently installed in front of the desk if desired for greater convenience as well as comfort during continuous operation. This seat may also be used to transport a crew member. A rack for the electrodes may also be built into the rear part of the cabinets sufficient to hold two or more sets of electrodes. Wiring for the batteries may be installed permanently and connected to a power panel which has a selector switch for obtaining the required voltage, rheostats for coarse and fine adjustment of the current, a 0 - 1,000 V. meter for ascertaining the operating voltage, jacks for connecting the operating power to the instrument, jacks for connecting the motor drive power to the instrument, switches for inside lights, and jacks for a six-volt soldering iron. The power panel is also connected to the storage battery of the truck by means of heavy cables.

Operation from a station wagon can be accomplished by setting up the instruments on the tail-gate and having the battery supply near at hand. Usually one or two more vehicles will be required to transport personnel, water, wire reels, and other miscellaneous equipment. Figures 9 to 16 inclusive show various views of the panel truck and interior equipped with cabinets and other above mentioned accessories for the resistivity equipment.

<u>Reels for wire</u>. The reels for carrying the wire are duralumin throughout except for the shaft, winding handle, and commutator, but other types of reels can be adapted for use also. The frame spiders are cast duralumin and can be used on any width reel; the bolts with nuts and tubing to tie the pair of spiders together being available in various lengths. The reel flanges are also cast duralumin and are secured to the steel shaft. Another piece of duralumin tubing, about $1\frac{1}{2}$ inches in diameter, located centrally about the axle forms the actual winding core. The original winding handles on the reels have been replaced by more convenient folding handles designed and built in the Maintainance Shop of the Geological Survey. One end of the wire goes through a hole in the reel flange and connects to a brass collector ring insulated with bakelite and fastened to the flange by bolts. The single brush of graphite impregnated copper with its holder is bolted to a piece of bakelite which in turn is bolted to the frame spider. A binding post is attached to the brush holder and connection to the electrode is accomplished by an insulated wire connected to the binding post, the wire having a large battery

clamp on the other end. Copper clamps are preferred but the last coated steel variety are more durable. The reels can be arranged for any capacity of wire up to 2,500 feet, but such a size is heavy and hard to handle. Consequently, it is more convenient to have smaller reels, 600 to 1,000 ft. capacity, and attach another reel by means of a banana plug into the hole provided on the brush holder and thus extend to any length desired. Figures 17 and 18 show views of the reels and the brush assembly.

Electrodes. Various types of electrodes and numerous kinds of materials have been used for electrodes. The Gish-Rooney method does not require non-polarizing electrodes. This is an advantage for such electrodes are fragile and require careful attention to keep in top operating condition. Measurements made on them by the author found potentials generated when they were in contact with certain geological materials. These potentials would disturb the generated potentials resulting in questionable measurements. This writer has consistently used steel rods having an outer coating of electrolytically deposited copper about 1/16 to 3/32 inch thick and having an overall diameter of 3/4 inch. This rod is pointed on the penetrating end, a hexagonal steel driving head about 2 1/2 inches long is either threaded on or swedged on to the driving end of the rod and then welded for additional strength. Figure 19 shows two electrodes in the carrying rack of the truck.

Copper also has a very high work function (F. Seitz, p. 145 ff, 1940), higher than some of the precious metals which are the next observation could be taken, a sudden thunderstorm came up and wet the area very thoroughly, but the instruments and reels were protected by the truck. Measurements were resumed after the shower and it is of interest to note on the measurements following the shower how the apparent resistivity drops continually in the same direction. The leakage current was again checked and measured more than 500 ma at the same voltage, and was caused, no doubt, by cumulative leakage through numerous defective places in the insulation.

Power supply. Power for energizing the earth is most frequently supplied by banks of dry batteries, usually radio "B" batteries of the heavy duty or extra heavy duty variety. When connected in series-parallel with two batteries in parallel, it is possible to subject these batteries to an intermittent drain of 500 ma, and under extraordinary circumstances with at least two fresh batteries in parallel, it is possible to obtain 1,500 ma intermittently. This battery connection will be noted on figure 16. Motor-generators and vibrator power supplies may be used provided that they have adequate filtering to reduce the ripple to a very low value. Such equipment introduces many phases of incidental operation not encountered with batteries, but in many instances the size and inconvenience will outweigh the convenience and bulk of batteries. Batteries should be checked at frequent intervals and, for the extra heavy duty type, discarded when the voltage drops 20 percent below the normal value. Any battery showing signs of polarization or a dead cell under a load test should be immediately replaced as such a battery may produce spurious readings of potential.

<u>Water Supply.</u> Water for wetting the electrodes may be carried in various container such as barrels, gasoline or oil cans, truck mounted tanks, and milk or cream containers.

<u>Hammers for driving electrodes.</u> Hammers selected for driving the electrodes will be more or less a personal choice. For all round service, the 30 or 32 ounce blacksmith hammer has been found very satisfactory.

<u>Measuring tapes.</u> Tapes for measuring the electrode distances should be carefully selected. During dry weather or in a dry climate, it is possible to use the metalized cloth tapes provided that the ends of the tapes are insulated from the metal tape pins. This can be done with electrical tape, adhesive tape, or rubber tape. Care must be taken that these tapes do not contact any part of measuring or energizing circuits, or lie on the wet earth.

where the electrodes have been set, for such contacts may result in effects which simulate leakage and thus result in worthless observations. A safer and better tape for use in electrical resistivity measurements is the glass fiber reinforced tape as this is non-conductive except when wet and mud covered. It is relatively inexpensive and very durable.

There are strong proponents for the measuring system which uses color coded markings on the wires. In this writers opinion, the disadvantages of this system so greatly outweigh the advantages that it has not been used, except for a single trial. Some of the main disadvantages follow. It takes a long time to apply the color coding paint for the 50 odd stations taken on a 1,000 ft interval depth profile. The color marking system is inflexible in that if extra intervals are wanted for some reason, they are not marked on the lines and considerable time will be lost in doubling back the wire to get an approximate distance. With tapes it is only a matter of a moment to locate the distance for the extra setting. Breaks in the wire or removal of a section which had been leaking upsets the color code and new markings must be applied . This means a complete set of coding paint must be available in the field. Alternatively, to maintain the correct distances for the markings on the wire, a piece of wire must be inserted for every break. This means two splices must be made and soldered thus introducing two weak points both mechanically and electrically in the wire. The chance for error in setting the electrodes is greatly increased by the color code; it is very rare to find a field assistant who cannot read numbers on a tape, but to convert colors to numbers was not so easily done by ~~some~~ some assistants encountered. Dragging through brush, mesquite, sage, water, mud and blistering heat soon removes the paint markings; tapes can be discarded without repainting.

The use of tapes is very messy during wet weather, particularly if sticky mud abounds such as over a plowed field. In dry weather it is convenient to use the winding reel type covers, but for wet weather or muddy going it is better to use tapes out of the cases and haul them in as one does a steel tape when it is not on a reel. It is quite true that tapes are effected by humidity changes, particularly so when they get old and the waterproof coating is worn off. It is easy to compare the old tapes with a new one, or with a steel tape for that matter, and discard it if it is in error. The error introduced by tapes, even those somewhat incorrect, is much less than the error introduced by winding in and around and over and under brush, rocks trees, etc., in dragging out the lines when using the color markings on the lines. Another bad feature in the use of tapes is the pin used to anchor the end; it is just impossible for some field aides to keep from losing this small item, and it is no longer easy to find blacksmiths to make these pins when you are on a field assignment. One should be provided with a more than adequate supply.

(N.B.) (All l.c. letters and insert on line as sub-head)

(TOOLS AND TESTING EQUIPMENT)

A volt-ohmeter, megger, oscilloscope, spare parts, and hand tools such as pliers, screw~~t~~drivers, etc., along with particularly critical and delicate items like meters and galvanometers, should always be conveniently at hand. Other items in the way of hand tools should be; small linemans pliers; needle-nose pliers; small screwdrivers, phillips-head screwdrivers; 110 V soldering iron; 6 V arc soldering iron, usually made from the carbon of a "D" flashlight cell; resin core solder; friction and #33 electrical tape; wire repair kit of Amp terminals: jewelers screwdriver set; headband magnifier; small metal saw; wood rasp; adjustable crescent wrench; hex-head wrenches; and perhaps others depending on the inclination of the operator to make overhauls. A very handy item devised by the writer is a wire guide which is attached to the top of a 10 gallon milk can with the spooling device immersed, thus enabling the wire to be spooled rapidly through the water in testing for leakage and insulation breaks, figure 21. It is desirable to have a battery test circuit incorporated in the circuit of the volt-ohmeter, or else a series of loading resistors should be carried so that a fixed load can be put on the battery. this will enable a no-load and fixed-load voltage reading to be taken so that poorly operating batteries can be quickly discovered and removed from the bank. It should be remembered that the connectors paralleling the various groups of batteries should be removed for testing.

Brushing equipment such as axes, machetes, and brush hooks should be standard equipment as well as an extra hydraulic jack and a tractor chain. A shovel is also a very necessary part of the equipment.

All l.c. letters & insert on line as sub-head

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COMMUNICATION WITH ASSISTANTS

Communication between the operators at the electrodes, especially the current ones in the Wenner or Lee arrangement, and the instrument men is always a problem. Several methods may be used. The standard engineers hand signals, plus a few of ones own creation, will answer for most purposes in open country. Telephones of the sound-powered variety may be used on the leads to the electrodes, but there is always the possibility that some one forgot to remove the headset plug from the circuit or that a transformer has been punctured by the high voltage. This means that a high potential may be present between the headset and ground with the electrode operator the prospective path for the current to flow when it is turned on at the instrument. With proper conditions this could be lethal.

Handitalkies or walkietalkies could be used in many areas for communication, but their line-of-sight transmission characteristics eliminates their use in many places because of hilly terrain or forests.

Portable public address systems can also be used very satisfactorily for one-way instructions. Ten watts of power will carry over a half-mile under favorable conditions, but 25 watts gives much better performance.

OPERATING THE EQUIPMENT

A description of the truck used for transporting the equipment for resistivity measurements has been given previously. In use in the field, the operator can drive the truck up to the place selected for the station center or center of the line of electrodes. In a matter of a few minutes, the equipment can all be on the ground and set up ready for reading the first interval. Each man has a set of duties to perform and quickly goes about his work of putting out his tape, setting the electrode, connecting his reel both to the electrode and the jack in the panel mounted at the rear of the truck. Figure 22 shows the panel arrangement for connecting the ends of the electrode wires to the instruments. The instrument operator by this time has set up his instrument and connected it to the power supply, the car storage battery, and the extension wires from the rear panel which come from the electrodes. A check has also been made of the potentiometer balance by means of the dummy earth. The recorder has made ready by placing a sheet of log paper and a sheet for recording notes and computations on the plotting board, affixing them by means of spring clips. The plotting board is a piece of 1/8 inch tempered masonite about 14 by 20 inches, drilled in two places at the top to receive a flexible wire which is permanently affixed by knotting the ends. This board may be placed on the truck or car steering wheel with the loop of wire over the top of the wheel to prevent it from falling or sliding, thus making a convenient desk. The spring binder clips form both a support for the 20 inch slide rule and pencils. Figure 23 shows this arrangement in place on the steering wheel.

PUBLIC ADDRESS SYSTEM FOR COMMUNICATION

A recent addition to the truck equipment was a public address system for communicating instructions to the "electrode men". This system has an output of 25 watts, and, with two speakers oppositely mounted on the front end of the truck bonnet so that they can be raised, can be heard for 500 yards or more unless the wind is in opposition, then the range may be less than half this distance, figures 9 and 10. The amplifier is mounted on the desk adjacent ot the instrument as is also the microphone, figures 11 and 12. The amplifier has a stand-by switch so that the tube filaments are always heated when the amplifier is on, but the plate supply, which is provided by a vibrator arrangement, is cut in or out as needed. The microphone is a special noise-reducing one to eliminate the excessive feed-back from such a compact installation.

ELECTRODE ALIGNMENT

It often occurs that it is difficult for the electrode men to keep their alignment straight either because of interfering vegetation, landscape irregularity, or because one or more members of the crew cannot follow a straight line or look back and align himself with the others on the line. For this difficulty, two or three cane poles with a yellow-orange flag mounted at the top are always carried in the truck, and as soon as the electrode spacing becomes large enough, are placed vertically in unoccupied electrode holes on either side of the center of the electrode line and at the center if needed.

DIRECTIONS FOR OPERATING GISH-ROONEY INSTRUMENTS

The earth resistivity apparatus has undergone numerous changes for improving its performance and reliability; most of these changes have already been mentioned as well as the components of the instrument. Consequently, it will not be possible to give specific directions for operating each model of Gish-Rooney instrument which has been made, so the directions will be restricted to the two general types; those containing a standard cell for balancing; and those balanced with the self-contained battery.

Standard cell type instrument. In order to put the standard cell type of instrument in a condition for operation, a No. 6 dry cell should be connected to the proper plus and minus terminals within the case of the instrument, or if the instrument is the later type having a built in holder for 4 D-type flashlight cells, these should be inserted, observing the correct polarity. After replacing the panel in the case, the instrument should be made level. This is most easily done with the aid of a small circular single bubble level which may be fastened to the instrument panel, if so desired. The galvanometer is next adjusted to zero, first releasing the lock if it has one, and then turning the large knurled knob on the top of the galvanometer either clockwise or counterclockwise until the pointer rests directly above the zero marking. The key or pushbutton marked S. C. is next depressed for an instant and the direction of the deflection of the galvanometer needle noted. If the needle moves to the right, the rheostat for balancing is moved clockwise; if the needle moved to the left, the rheostat is moved counterclockwise. This procedure may have to be reversed in some instruments, but it has no effect other than balance. These operations are continued until the galvanometer reads zero and the switch marked S. C. is released. The key or push-button should not be held down continuously as this causes a current to flow from the standard cell and will cause it to function improperly. The potentiometer is now ready for reading emf's. The method of measurement is the same for the voltage to be applied across the binding posts P-1 and P-2 or through the jack marked P, which indicates potential connection . The key or push-switch marked E M F is depressed and the direction of the galvanometer needle deflection noted. The large knob on the panel controlling the slide-wire of the potentiometer and the decade switch marked VOLTS are next manipulated until the galvanometer needle rests on zero. The potential is then read in millivolts by summing the two dials. In the event it is not possible to obtain a balance, either the potential is beyond the capacity of the instrument or the potential leads are reversed. On some instruments having separate switches for reversing potential and current, this condition can be corrected by changing the position of the switch marked REVERSE.

To measure earth potentials incorporating the commutator, the plugs on the cables for connecting the two units are inserted in the proper jacks, or binding posts on some models, the battery power supply connected and the jumper, or rheostat, connected across the C-1 and C-2 posts. The commutator is cranked manually at a steady rate of about 120 rpm, which speed should be just sufficient to cause no perceptible fluctuations in the needle of the milliammeter, and a reading of current in milliamperes made simultaneously with the reading of potential in millivolts, the latter having been described above. Computations of apparent resistivity are made by substituting directly in the formula

$$\rho_a = 2\pi a \frac{E}{I} \quad \text{- - - - - - - - - - - - - - - - - - -} \quad (1)$$

with a being the electrode separation in cms.

The milliammeter has a range switch with a number of positions. On position O C the meter is open-circuited; on position S C it is short-circuited and the switch should be turned to this position for transporting the instrument; on 5, 20, 200, and 500, the meter reads full scale the respective number of milliamperes. At no time should the meter be subjected to overload, for such treatment will most certainly damage the sensitive meter movement causing incorrect reading and requiring a factory overhaul. Measurements should be started on the highest range, then with the current <u>off</u> switch to the lower range which will accomodate the current in use.

<u>Dummy earth type instruments</u>. The procedure for standardizing Gish-Rooney instruments that do not have a standard cell by means of the dummy earth is somewhat different than that for one with an internal battery. The panel of the instrument is removed and 4 D-type flashlight cells are placed in the holder at the bottom of the case being sure to observe proper polarity. The battery supply is connected by the proper cable to the connecting plug marked BAT, and the driving power connected to the banana plugs in the case of the instrument with its cable. The dummy earth is inserted in the 5 jacks C-1 etc. to C-2. The instrument should be leveled with either an attached single bubble level or other convenient arrangement. Set the galvanometer on zero by adjusting the large knurled knob on the top. Place the switch marked REVERSE in either position A or B. The POTENTIOMETER switch is moved to the ON position. The DC-GR switch is moved to the GR position. The CONDENSER switch is placed in the IN position. Some instruments may be marked relative to the OFF or ON position and should be set to OFF for the capacitor to be in the circuit. Turn the VOLTAGE switch to the X 1 position. Set the meter range switch to the X 0.1 position. Place the SELECTOR switch on either the P-1 or P-2 position. Battery supply is connected to 22½ volts. After checking all the above positions, the COMMUTATOR switch can be turned to MOTOR and the speed of the motor adjusted by the control knob on the side or at the front of

the case. The speed of the commutator should be adjusted to the point where oscillations of the milliammeter needle are very small or imperceptible. The milliammeter will read somewhere between 16 and 18, disregarding the multiplying factor. Set the VOLTS knob and dial to read the same value as the milliammeter, that is between 0.160 and 0.180, then turn the GAL ADJ control knob until the galvanometer is reading zero. The instrument is now adjusted for reading potentials and may be turned off by moving the commutator switch to OFF. This balancing or standardization should be checked frequently during the course of operations. The dummy earth is next removed, the plugs attached to the ends of the wire reels are then inserted in the proper jacks. The METER RANGE switch is changed to a high range. The instrument is now ready to measure earth potentials incorporating the commutator.

The COMMUTATOR switch, or MOTOR, is placed in the MOTOR position; the METER RANGE switch adjusted to accomodate the current flowing; the current control rheostat in the external circuit adjusted to a suitable value; the VOLTS knob and dial adjusted until the galvanometer reads zero. Simultaneous readings are made of the current and potential, and the apparent resistivity computed by formula (1).

This model instrument does not have a shorting switch for the meter or a lock for the galvanometer. With reasonable caution, no difficulty, such as breaking the galvanometer suspension or damaging the movement of the meter, will be encountered.

<u>Commutator operation</u>. The commutator in the early instruments was arranged so that the drums were in the vertical. The insulation in the drums was bakelite for the potential and white marble for the current. It was very difficult to keep the brushes from "writing" on these materials, particularly so on the marble, for it acted very much as a grindstone and the tiny metallic particles ground from the brushes remained in the pores of the marble and finally set up a conduction path. Naturally this caused trouble and required that the drums be cleaned thoroughly after a very few readings. Early recommendations for cleaning were to use absolute alcohol, but this failed to remove the metallic particles which penetrated deeply into the marble. Other solvents and cleaning agents were tried. Of the ones used, ~~Carbona~~, or carbon tetrachloride proved to be much better, but with the advent of lighter fluid, this was found to be the best cleaner of all. However, the perfumed varieties should be strictly avoided. The commutator must be kept absolutely clean at all times.

In the more recent models of the earth resistivity apparatus, insulation has been removed completely from beneath the brushes in the current section, and an improved insulating material has been used in the potential section. This has removed much of the difficulty previously mentioned, and improved operation of the commutator has resulted from it as have also the measurements taken with it.

Located in the bottom of the commutator case in the early models is a large paper capacitor which is in series with the active potential leads. This capacitor will be found in the bottom of the instrument case in the later models. A switch on the panel makes it possible to place this capacitor in or out of the circuit for Gish-Rooney or D C measurements. For Gish-Rooney measurements, the capacitor removes the stray earth currents, always present in more or less amount, from the measurements of surface potentials. As explained before, these currents are either direct current or very slowly pulsating direct current, so that they are blocked by the capacitor. This is not the case for lightning as it is oscillatory as it discharges and sets up localized oscillations which makes it impossible to obtain Gish-Rooney measurements of surface potential that are within the disturbed zone.

The commutator should be oiled from time to time using a light mineral oil on the shaft bearings and a small amount of vaseline on the ball bearings and the gear teeth. When the instrument is to be stored for a period of time exceeding a few days, the steel current drum should be given a very light coating of mineral oil to prevent rusting.

There is no brush adjustment in the early model commutators other than the slotted holes in the brushes under the hold-down bolts, as a result this adjustment is a little tedious. It will be noted from the figures given herein that each drum has four brushes, two above and two below. On some models, the two brushes which are located to contact the central part of the drum are of the bent-end or bent-finger type. These are the critical brushes on each drum and require exact adjustment. First, set the potential brushes so that they will break about 2° to 5° before the current brushes break; next adjust the current brushes so that they will make about 5° to 10° before the potential brushes make. This will take ~~an~~ some trial and error procedure but can be done. Leave as much interval between the two make positions as possible This adjustment is for the elimination of transients which may be set up by the sudden introduction of current into the earth.

After the brush position is adjusted, then the brush pressure may be properly set. It is again emphasized here that both brushes and segments should be as clean as physically possible to obtain. Two methods have been used for setting the brush pressure. One is a device similar to a balance used for measuring needle pressure on a phonograph record, the other is a sensitive ohm-meter. The former needs no explanation as the pressure of each brush can be set to register the required number of grams. In using the ohm-meter, it is connected across a brush and a segment which it is contacting and the pressure adjusted until the contact resistance reaches the desired value, usually less than 0.5 ohm being satisfactory.

The commutator segments under continuous use, especially with currents above 250 ma, will be found to have burned at the edges where the current breaks particularly. Also, the brushes when adjusted tighter than necessary will write in forming deep grooves. Both of these conditions bring about poor commutator operation and should not be allowed to form if possible. Frequent, but exceedingly careful, use of 600 grit wet or dry abrasive followed by a polish of crocus cloth and this in turn followed by a very thorough cleaning, will assist in keeping the wear from bothering the measurements. However, with continued use it may be that the only way to remove the pitting is by taking the instrument to a competent machine shop and have the drums reground. Do not, under any circumstances, allow them to use a cutting tool on the drums as this will remove too much metal from the drums and will leave them too rough and ridgy. Rough, out of round, and pitted drums cause the brushes to bounce or have periodic oscillations, thus setting up transients which will ruin all readings of potential.

REDUCTION OF TRANSIENTS IN COMMUTATORS

It was proposed by Roman (1951, p. 181) that the effects of transients could be reduced by connecting capacitors across the commutator segments. Such an arrangement might apply to switch type or cam operated contact type commutators actuated at extremely low frequencies where the capacitor could completely discharge, but it will not apply to Gish-Rooney type commutators actuated at about 18 cps.

The actual operating cycle of a Gish-Rooney commutator showing both degrees and times for each portion is shown in figure 24. This figure differs from that given by Roman (1951, figure 5, p.180) in that the sloping make and break periods are not present and the times for make and break of the current and potential are different under actual operating conditions.

Any transient that may occur will be set up at either the make or break between the current brushes and the commutator segments, provided that the brushes and trueness of the drums have been accurately adjusted. These transients have a very rapid decay, as will be described later, and are non-existent during the period measurements are being made.

Now to connect a capacitor across the segments of the commutator, or preferably in a more suitable quenching position across the brush and segment, and consider the effects. Upon contact of the brush and segment, the capacitor is charged very rapidly to a voltage determined by the energizing voltage, and has also superimposed upon the capacitor charge the additional voltage charge caused by the transient. This additional voltage will probably not be very great as the earth has but little inductance. The charged capacitor is next connected across a circuit load which contains inductance, resistance, and capacity, and is always underdamped , so that both the charging and discharging is oscillatory. In both instances, that of charging and discharging of the capacitor, the oscillations will continue for an indefinite period of time, decaying some with each cycle. A more technical discussion is given by Smythe (1950, pp.328-331). It is obvious without entering into the calculations that such effects as the ones just described would, if superimposed upon the pure direct current required for the measurements by the Gish-Rooney method, completely vitiate them. Coupled with the above effects are those that are produced by the current break when the brush and segment part. These oscillations might be carried through the quiescent period of the commutator and be superimposed upon the next cycle as it begins. If it were possible to critically damp the circuit with each measurement of potential, nearly all of the oscillatory effects mentioned above could be nullified and the decay would then be exponential. By an ingenious cam-operated switching device, Crumrine (1953) discharges the capacitor, which is connected across the make and break contacts of the commutator, through a low resistance immediately following the make and break of the current. In this manner, all transients which may have been set up either by the make of the contacts or the oscillatory discharge of the capacitor, are completely dissipated before measurements of potential are made. Thus the potential is measured while the energizing field is undisturbed.

WAVE FORM ANALYSIS BY OSCILLOSCOPE

¶ The wave form produced by a commutator can be studied by means of an oscilloscope and, with some experience, troubles attendant to the wave forms produced may be diagnosed and thus easily removed. The current wave form may be obtained by placing a small resistor in the current circuit, the size will depend on the amount of current flowing and the sensitivity of the oscilloscope, but its value may be estimated from the IR drop, and connecting two wires from this resistor to the deflecting plates of the 'scope. The potential wave form may be obtained by connecting the leads from the potential electrodes directly to the deflecting plates of the 'scope. Such a study can be made for both short and long interval distances and when the commutator is in perfect adjustment the wave form for both current and potential at each distance should appear as in figure 24. A diagram of the connections is shown in figure 25 for obtaining the wave forms. If a double beam oscilloscope is available, both waves may be studied simultaneously. Such observations should be made under actual operating conditions and this requires that 110 V a c be available for operating the oscilloscope. This can easily be supplied by a gasoline engine-generator set and voltage regulator if a power line is not accessible.

Many commutators have been built and used for resistivity measurements, yet only a very few could meet the rigid requirements set forth in the preceding paragraphs. Electrical measurements of surface potential are difficult enough to interpret without adding to the difficulty by using an instrument which could not be expected to give other than erroneous measurements.

SWITCH TYPE COMMUTATOR

An ingenious switching device was devised by Dudley (1939) whereby the interval between the time the current contact was made and the time the potential contact was made was adjustable over a wide range. Such a device would be very useful for deep investigations, because of the increased time of relaxation in the earth with increasing depths. The time constant of the earth, according to Pearson (1934), is

$$k = 2.32 \times 10^{-6}$$

and this figure may also be used in computations of electrical transients.

Other switch type commutators have been developed for special types of earth resistivity measurements, but they have not been described in the literature of geophysics.

DETERMINATION OF THE COMMUTATOR FACTOR

It was mentioned previously that the commutator factor is compensated by a resistor in the recent models of the Gish-Rooney instruments, but no other explanation was made of this factor. Because the flow of current in the earth is not continuous when the commutator is employed to energize the earth, a correction must be applied to the computed resistivities to obtain the true resistivity and this is termed the commutator factor. This factor may be obtained in several ways. One way is to set up the earth resistivity apparatus for direct current measurements and actually measure two or three intervals reversing the current several times and averaging the results for each interval. The same intervals are then measured using the commutator and the resistivities computed. The compensating resistor should be disconnected, GR -DC switch must be placed in the proper place while making these measurements, and the capacitor must be out for DC measurements and in for GR measurements. By dividing ρ_a for DC by ρ_a for GR the commutator factor is obtained.

Another way to obtain the commutator factor is to make up a set of series resistors such as those shown in figure 26 and connect the leads from the four terminals to the C-1, P-1, P-2, C-2, jacks on the instrument. Resistance values other than those given on the figure may be used but the ratio should be about the same and precision resistors should be used throughout.

Measurement of the known resistance between P-1 and P-2 by both DC and GR will give the commutator factor immediately as well as a check on the accuracy of the instrument.

Still another way to get the commutator factor is by means of the dummy earth. The values of the resistors between the P-1 and P-2 plugs may be obtained with a Wheatstone bridge or with and accurate ohm-meter. The compensating resistor must be shorted with a piece of wire or removed from the circuit by unsoldering one end. The dummy earth is then placed in the plugs on the instrument panel and measurements made as in the second method given, and the commutator factor computed. The instrument is placed in its original condition and measurements made with the dummy earth in place using DC and GR. Both should be same. If not the same, or reasonably so, some part of the instrument is not functioning properly and should be corrected.

The commutator factor is variable between instruments and for correct readings should be determined for each instrument. Usually the variation is small if the instrument is made by the same manufacturer.

BRUSH ADJUSTMENT ON LATE MODELS

The later models of the Earth Resistivity Apparatus have brush adjusting screws built into the connector panel, and it is only necessary to turn these screws to move the whole brush assembly either forward or backward on the drum in order to reach the proper positions which have previously been described for the earlier models.

SPEED CONTROL FOR COMMUTATOR

None of the motor driven production models of the Earth Resistivity Apparatus have a built-in speed control. However, this can easily be added to the instrument and consists of a 10 or 25 ohm 25 W variable rheostat and associated wiring placed in one leg of the commutator drive motor circuit and mounted on the case so that the control knob of the rheostat projects outward in a convenient place. It was mentioned earlier and shown on figure 4 that the belt drive supplied on the instrument had been replaced by gears and details for this conversion will be given. All gears, except those fastened directly to the shafts, are mounted on sealed miniature ball bearings in order to reduce friction. All gears are 1/8 inch thick.

Motor drive - micarta - 1/2 inch diameter - 48 pitch - 36 tooth

Idler, motor- micarta - $2\frac{1}{2}$ inch diameter - 48 pitch - 180 tooth

Commutator drive gear, motor - brass - 2 inch diameter - 48 pitch - 144 tooth

Commutator drive gear, manual - brass - 1/2 inch diameter - 48 pitch - 36 tooth

Idler gear, manual - brass - 1 3/4 inch diameter - 48 pitch - 126 tooth

Manual drive - brass - 2 inch diameter - 48 pitch - 144 tooth

Such an installation can be successfully accomplished by a skilled instrument maker or machinist. The installation for the writer was made under the direction of Mr. H. J. Anderson, Chief of the Property Maintenance Section of the Geological Survey.

CARE OF REELS FOR CONNECTING WIRE

The collector ring on the reel requires infrequent cleaning as does also the brush assembly. Cleaning all parts with 600 grit wet or dry abrasive paper and wiping free of grit with either a dry cloth or with one saturated with lighter fluid will restore the parts to satisfactory operating condition again. An infrequent check should be made of the insulation resistance of the collector ring to be sure that it is not grounded thus allowing current to flow through the reel assembly to earth. This check may be made with either an ohm-meter or a megger and, if there is any indication of leakage, the assembly should be dismantled and each part checked individually for leakage and then reassembled after the cause is removed. At the same time, the insulation resistance of the wire on the reel can be checked, for the end of the wire must be removed from beneath the screw contact on the collector ring. This is done so that the ohm-meter or megger can be connected to either end of the wire and the reel frame for the check. Care should be taken in connecting the wire back on to the slip ring to see that no contact between the wire and the frame is present and that the insulation of the wire comes neatly through the hole in the flange.

LOCATION OF LEAKS IN CONNECTING WIRE

The wire on the reels should be checked frequently for leakage and the breaks in the insulation repaired. The wire has a normal insulation rating of 5,000 volts and it is not advisable to subject it to voltages of three to five times this value for testing. A quick leakage check can be made of all the wires connecting the electrodes to the instrument by having each man remove his rubber insulated clip from the electrode and place it on top of the roll of wire so that it lies on the rubber insulation of the coil. Then the operator of the instrument, with the milliammeter on the proper range, connects each reel in turn to the C-1 jack while the G plug remains in the C-2 jack, and to each reel he applies voltages up to 750 to determine the amount of leakage in milliamperes. Any reel leaking more than a few tenths of a milliampere should be placed under test by the procedure next described to locate and repair all leaks.

The wire is fed through the spooling device shown in figure 21 and the assembly placed in a 10 gallon milk can and firmly attached with the screw clamps provided. The milk can is filled with a salt solution, about $\frac{1}{4}$ pound of salt is sufficient, and a few tablespoons of detergent added to the solution. A megger is then connected to the dry end of the wire and to the milk can and cranked as the wire is spooled through the solution. Any break or rupture will be immediately apparent on the dial of the megger for the very slightest break in the insulation will cause the pointer of the megger to drop immediately because of the low value of resistance introduced by the contact of the water with the metal strands of the wire. By slowly moving the wire, the exact spot where there is a break can be located as it leaves the solution and immediate repair can be made.

A somewhat less effective method than the one just described for testing the insulation of the wire is to use a sensitive ohm-meter instead of the megger, one with a 20,000 ohms per volt meter movement preferred, and used on the high ohms range. The megger tests the wire at 500 volts while the ohm-meter tests it at about $7\frac{1}{2}$ volts so it is apparent that the megger testing far exceeds the ohm-meter in reliability, but the latter method may be used when a megger is not available.

An alternative method for testing the insulation leakage is to use 500 volts from the battery or motor-generator power supply, but with a current limiting resistor in the circuit so that a lethal shock could not be gotten. By using a 100 K ohm resistor and a 5 ma meter, the maximum current with an external circuit resistance of 0 ohms would not exceed the scale of the meter, and the shock under such conditions would be quite "hot" but not very injurious. Other similar arrangements would be just as effective such as a 1 ma meter and a 500 K ohm resistor.

REPAIRS TO BREAKS IN CONNECTING WIRE

The repairs made to breaks in the wire were always very unsatisfactory in the past because one had to solder the joint to give it enough strength to keep the wires from unwinding at the joint while under stress. But the soldering, done either by arc or by an iron, removed the temper from the metal strands thus weakening it and rupture was very likely to happen at the same place again when the wire was pulled harder than the joint could stand. This problem was finally solved by the use of wire end terminals, one variety being called amp terminals, and a crimping tool. And now, with the advent of No. 33 electrical tape, a splice can be made which is stronger both mechanically and electrically than the wire itself was originally.

PROCEDURE FOR SETTING ELECTRODES

In the operating practice of making resistivity measurements, it is not only very important to have the best kind of materials for electrodes, but it is also of primary importance to have these electrodes properly set in the earth. For not to do so, an operator might just as well resort to guessing and not take any observations under most conditions encountered in operating. In the first place, each electrode should be carefully wetted down and thick, sticky mud should be present in the hole from the top to the bottom of the electrode. This requirement holds except in swampy, wet-plowed, or other normally wet areas. In order to be certain that each electrode will perform exactly the same as every other electrode doing the same thing in the configuration, all electrodes should be tested with reference to the ground or center of the Lee configuration, and their settings adjusted until the amount of current flowing is the same from G to C-1 and C-2, and from G to P-1 and P-2. When properly set, the amount of current flowing to the C-electrodes will be the same, or very nearly the same, as that to the P-electrodes. The inability to make this test with porous pots, as in D C measurements, or the fact that nearly all operators of resistivity equipment have been unaware of this important principle, has doubtlessly caused much misinformation to be recorded in the literature of electrical geophysics. The theoretical aspects of this feature will be taken up later and further instructions given.

METHOD OF TESTING BATTERIES

Very few voltmeters have provisions for testing batteries under simulated load conditions, consequently one should provide such a device to make this test. A simple and convenient one is shown in the diagram of figure 27 and gives the connections for the battery and voltmeter. In use, the proper load resistor for the voltage being measured is placed across the battery after the initial no-load reading has been made. If this reading is 10 percent or more lower than the no-load voltage of the battery, the battery should be regarded with suspicion and retested again very soon during use. If the reading under load is less by 20 percent or more than the no-load reading, the battery should be replaced at once. A high resistance or polarized cell in a battery will be immediately apparent by the large drop in output voltage under load, and a battery giving such a test should be removed at once and replaced with a fresh battery.

In replacing batteries in the bank, it should be recalled that batteries to be paralleled should have the same voltage, otherwise the low-voltage battery will run down the capacity of the high-voltage battery.

ELECTRODE CONFIGURATIONS

The electrode configuration used in electrical resistivity measurements may be chosen from a rather large assortment as given by Heiland (1940), Hummel (1931), Ehrenburg and Watson (1931), Lee (1930), Jakosky (1950), and others, but the one giving the most information about electrical conditions beneath the surface of the ground is the Lee (1929) modification of the Wenner four electrode arrangement in which a fifth electrode, or third potential electrode, is added at the center of the arrangement. Apparent resistivity for the Lee arrangement is computed for the two measurements on either side of center by the formula

$$\rho_a = 4\pi a \frac{E}{I}.$$

If a modification is made of the Lee method, namely, that of taking a Wenner reading in addition to those of the right and left halves in the Lee arrangement, and computing all apparent resistivities by the Wenner formula, namely,

$$\rho_a = 2\pi a \frac{E}{I},$$

three apparent resistivity curves are obtained which plot with much better spacing on logarithmic paper. Furthermore, the well spaced plot so obtained is much easier to interpret by most techniques applicable than is the congested one using both the Lee and Wenner formulas.

RECORDING OF FIELD NOTES

Field notes for the Gish-Rooney methods have recently been recorded on $8\frac{1}{2}$ x 11 inch paper specially printed and punched so that it may be filed in standard size note book covers. The paper is printed the same on both sides and is adequate for two short depth profiles, with intervals up to about 350 feet, or for one long depth profile, with intervals up to 2,000 feet. A sample sheet is shown in figure 28. Some of the headings are, perhaps, not readily understandable and will be explained. The electrode interval is indicated as a. Under the test column, four separate columns are given, one for each of the electrodes, and the readings of current flowing to each electrode from ground G is recorded here as is also the revised current flowing after the setting of the electrodes is adjusted to the same value as all the others. Under the next column, number 6, headed E, the various applied battery voltages are recorded, i.e. the voltage impressed on the earth through the C-1 and C-2 electrodes. In the next column, number 7, headed with a larger E, the three readings of potential across the P-1, G, P-2, electrodes are recorded which are ordinarily termed the Full, across P-1 and P-2, P-1, across P-1 and G, and P-2, across P-2 and G. In column I, the current flowing in the earth under the impressed voltage is recorded. The instrument operator maintains the current in the earth at a constant value by means of the coarse and fine rheostates on the power panel of the truck. The apparent resistivity is computed by the formula applicable and recorded in the ρ_a column.

PLOTTING APPARENT RESISTIVITY CURVE

The three values of apparent resistivity computed for each interval in ohm cms are immediately plotted by the recorder on 2 x 2 cycle logarithmic paper having 5 inch cycles. 3 x 2 cycle paper may be used more conveniently for curves extending beyond the 100 ft interval. A sample plot of a resistivity curve is shown in figure 29. The full values are plotted as circles, the P-1 values as triangles, and the P-2 values as crosses. Values of apparent resistivity for intervals to two feet may be easily plotted by extending the logarithmic scale in the left margin of the paper. Pertinent information such as date, location, bearing of line of electrodes, altitudes, etc., are usually recorded in the upper right margin of the plot, leaving the remaining space for computations necessary for the interpretation. It will be noted that none of the points on the plot are joined by lines or curves; such lines tend to introduce a trend or influence in the interpretation. This will be more apparent when the techniques of interpretation are described.

The continuous plot of observations is of extreme importance to the experienced observer, for it is possible to determine from it whether the measurements are satisfactory or are being disturbed by unknown buried conductors, whether an interval electrode setting is correct, what the geological materials beneath the surface may be, to estimate the depth below the surface to the various materials, and to estimate approximately, the resistivities of the materials present. The plot also enables the operator to decide when sufficient observations have been obtained and discontinue measurements.

THEORETICAL ASPECTS OF ELECTRICAL CONDUCTION

Physical characteristics of typical conductors.

Metals. As a class, metals approach most nearly the ideal of a perfect conductor. Metals possess crystalline structure and are characterized by the presence of conduction electrons which are able to move more or less freely through the ionic crystal lattice. Thus small electric fields are transferred from one place to another very rapidly, but the motion of these conduction electrons under the influence of the applied field is partly conditioned by the forces in the crystal lattice that hold the ionic centers in position. As these forces are never perfectly isotropic, the crystal lattice, consequently, presents greater resistance to electron flow in certain crystal directions than in others. This effect is observed in large single crystals, but for metallic conductors it is of relative unimportance for these are the mosaic structures of microcrystals and the macroscopic structure offers the same resistance to current flow in any direction.

The resistivity of the substance is the ratio of the electric field within the conductor to the rate of charge per unit in the direction of the field and is usually written rho ρ. This ratio is an approximate constant of the material, and its reciprocal, the conductivity, is usually written sigma σ. Either of these parameters may be used to describe the resistance offered by a conductor to the impressed electric field. Pure silver has the lowest resistivity, or highest conductivity, of any of the metals at ordinary temperatures. Only slightly inferior to silver in this respect, however, is copper and with its relatively low cost it is the most important metal for electrical use. Aluminum with its higher resistivity, lower tensile strength, and lesser density, is valuable for certain electrical applications particularly in long, self supporting, conducting spans and in installations requiring minimum weight. Steel, even though it has a greater resistivity than any of the above metals, is often used where high mechanical strength is of the most importance. A recent advance whereby steel wire is continuously coated with successive layers of copper and aluminum will give conductors having both high conductivity and high mechanical strength.

Over a very wide range, the resistivity of a metal is independent of the current density. Gold is an excellent example of this characteristic for it shows no appreciable change in the resistivity for current densities as high as 10^6 amp cm^{-2} and only a change of a few percent at 10^7 amp cm^{-2}. Conductors for which this direct proportionality exists between the current density and the field strength are termed linear or ohmic conductors. Such a simple relation as that above indicates that the motion of a conduction electron through a metal in a field E may be likened to the motion of a visible particle through a viscous medium in which the retarding force on the particle is proportional to the velocity. If e and m are respectively the charge and mass of an electron, k is the average retarding force per unit velocity, the equation of motion in the x-direction is

$$m\frac{d^2x}{dt^2} + k\frac{dx}{dt} = eE_x \qquad (2)$$

When the steady state is reached the acceleration = 0 and the velocity becomes $(e/k)E_x$. If there are n conduction electrons per unit volume moving with this velocity, the rate of transfer of charge per unit area perpendicular to the field, i.e. current density, is

$$(ne^2/k)E_x$$

and the resistivity is

$$k/ne^2 .$$

From this discussion, the concept should be apparent of a cloud of electrons drifting through the ionic crystal lattice under the influence of an applied field and the retarding forces of the crystal lattice. For a more complete account of this subject see Seitz (1940), Slater (1934), Wilson (1936), and others.

Another important factor influencing resistivity is temperature. In all pure metals, the resistivity varies directly as the temperature, but certain alloys have a limited range over which the resistivity is essentially independent of the temperature. Some materials have a negative effect, the resistivity varies inversely as the temperature, and carbon is typical of this group. The relation of ρ and T, the temperature, is expressed over a wide range by a power series,

$$\rho = \rho_1 \left[1 + \alpha (T - T_1) + \beta (T - T_1)^2 + \gamma (T - T_1)^3 + \text{--} \right] - (3)$$

in which ρ is the resistivity at the temperature T, ρ_1 the resistivity at temperature T_1. The coefficients α, β, γ, etc., decrease in magnitude and very rapidly/for the normal range of temperature it is sufficient to neglect all but α which is known as the temperature coefficient of resistance.

Certain other physical properties of metals, such as the relation of strain to resistivity and superconductivity, have no relation to the context and will not be discussed.

Liquids. All liquids may be roughly divided into three groups on the basis of their conductivities. Certain liquids have very low conductivities of the order of 10^{-13} mho cm, and paraffin oils and certain organic liquids such as xylol are representatives. These liquids are widely used as insulators and dielectrics. Another group are the pure liquids of which alcohol and water are representatives and they have conductivities about 10^8 times as great or about 10^{-5} mho cm. The third group is represented by a solution of an acid, base, or salt, in water and they have conductivities ranging near 10^7 times that of pure water. The last two groups are generally unsatisfactory as insulators or dielectrics, and the last group, though their conductivities may seem high, are actually less by a factor of about 10^{-7} than typical metallic conductors. These solutions having large conductivities are termed electrolytic solutions and the dissolved material, or solute, is termed an electrolyte.

Electrolytic conduction follows approximately the ohmic law. However, the field is not carried by electrons as in metallic conduction, but by the more massive and slower moving positive and negative ions present in great abundance in these solutions.

Faraday was the first to make a study of the phenomena accompanying electrolytic conduction and the laws formulated by him are:

The mass of a substance liberated at an electrode by the passage of an electric current is proportional to the charge that has passed.

The mass of substance liberated at an electrode is proportional to the chemical equivalent of the substance.
The farady is then the product of the chemical equivalent and the total charge passed divided by the mass deposited.

An electrolyte may be considered as a substance whose molecules are held together largely by electrical forces. When an electrolyte is dissolved in a solution, the binding forces between the components of the molecule are weakened because of the high dielectric constant of the solvent, K for water being about 80. This results in a certain fraction of the electrolyte splitting up into positive and negative ions, and each of these primary ions may attract to itself several solvent molecules which remain firmly bound to it and the entire aggregate drifts slowly through the solution under the influence of the applied field. The negative ions which reach the anode transfer their excess electrons to that electrode and the positive ions are neutralized by the electrons from the cathode. These ions may react with the material of the electrodes or they may be liberated or deposited as neutral molecules. The equivalent conductivity

$$\lambda = \alpha F(\mu_1 + \mu_2) \qquad (4)$$

where α is the dissociation constant and μ is the mobility of the ion. The process of current flow may then be regarded as convection current under the influence of an electromagnetic field in contradistinction to conduction currents.

Varying the concentration effects both the degree of dissociation and the mobilities of the ions. Weak electrolytes such as ammonia and acetic acid depend very little on the concentration and λ may be considered to depend on c , the equivalent concentration, only through α . The dependence of α on c , in accordance with the law of mass action is given by

$$\frac{\alpha^2}{1-\alpha^2} = \frac{K}{c} \quad \text{- - - - - - - - - - - - - -} \quad (5)$$

where K is a constant containing the temperature. In the instance of strong electrolytes such as NaCl, KCl, etc., dissociation is practically complete at any ordinary concentration, i. e., $\alpha \cong 1$, and the variation of λ with concentration is largely caused by the variation in the mobilities of the ions. In fact, it may be shown in this case that the equivalent conductivity is proportional to the concentration, i.e.,

or $\simeq$

$$\lambda = \lambda_0 - a \cdot c \quad \text{- - - - - - - - - - - - -} \quad (6)$$

where λ_0 is the equivalent conductivity of the strong electrolyte at zero concentration and a is a constant.

In the above discussion it has been assumed that ohm's law holds for electrolytes. That it does hold for similar electrodes in a common solution and in the absence of complicating effects was shown by Kohlraush (see MacInnes, Chap. 3, 1939). The chief complicating factor is polarization which comes from some inhomogenity in the electrolyte caused by the electronic motion. The effect of polarization on resistance measurements can be greatly reduced by alternating current, and the reason for its use may be seen from the following analysis.

Assuming an ohmic resistance for the solution and that the gas layers give rise to a potential proportional to the total charge q passed, but of opposite direction, the potential difference between the electrodes is given by

factor, not subscript

$$V = R_i + P_q \quad \text{- - - - - - - - - - - - - - - (7)}$$

where P is a constant of proportionality. Writing $\frac{dq}{dt}$ for i and assuming an alternating potential

$$V_o \sin \omega t \quad \text{- - - - - - - - - - - - - - - (8)}$$

factor, not subscript

$$R\frac{dq}{dt} + P_q = V_o \sin \omega t \quad . \text{ - - - - - - - (9)}$$

Solution of the differential equation, neglecting the transient term, for q becomes

$$q = \frac{V_o}{1+\frac{P^2}{(R\omega)^2}}\left(\frac{1}{R\omega}\sin\omega t + \frac{P}{(R\omega)^2}\cos\omega t\right) \quad \text{- - (10)}$$

or i , which is equal to $\frac{dq}{dt}$ is given by

$$i = \frac{V_o}{R}\frac{1}{\left[1+(P/R\omega)^2\right]^{1/2}}\sin(\omega t+\varphi) \quad \text{- - - - (11)}$$

where $\varphi = \tan^{-1} (P / R\omega)$. If the frequency ω is made very large, the second term in the denominator becomes negligible and thus ohms law

$$i = V / R \qquad (12)$$

is approached. Also, φ approaches zero which means that the current and voltage are in phase.

Another effect in liquids is produced by particles in colloidal suspension, such particles cause a decrease in the resistance and usually a lowered dielectric strength. These particles possess a net charge in nearly every instance and their motion through the liquid under the influence of an electric field is known as cataphoresis.

Solids. Isotropic solids are generally dielectrics and are used as insulation and mechanical support in nearly all electrical devices. Representatives with high mechanical strength are porcelain, glass, synthetic resins, and wood. Those with very high insulating characteristics are sulphur, amber, quartz, and polyethylene. Nearly all the members of this group of solid materials have a definite crystal structure, consequently they may not be considered isotropic on a micro scale. However, the materials may be considered sensibly isotropic on a macro scale as the crystals are randomly oriented. The dielectric constants for classes of substances such as micas, glasses, etc., are not specific but vary between wide limits. These materials are seldom homogeneous and this implies surface and volume changes will appear when they are placed in an electric field. The internal molecular readjustments require a longer time in solid media than in liquids, so that generally there are appreciable time lags before a state of electrical equilibrium is reached. Bound charges and relaxation times of many minutes have been observed in dielectrics.

There are a number of interesting and important solid dielectrics that have unusual and anisotropic properties. However such properties as the formation of an oxide film on a metal in a suitable electrolytic solution, the formation of permanent electric moments, the piezoelectric effect, and the pyroelectric effect, probably have no relation to surface measurements of potential.

Continuous solids. Insofar as surface potential measurements are concerned, the earth may be considered a continuous solid, but it is not isotropic or homogeneous in the large sense. Earth materials vary over wide limits as will be evidenced in tables such as those given by Slichter and Telkes (1942). However, it is very often found that a geological material has about the same resistivity over a quite large area. Isotropic conditions are somewhat less frequently found, probably because of the great variability in the geological processes entering into the formation of the materials, the uneven weathering on the surface, the earth forces acting on the beds, and perhaps many other factors.

Electrical conduction in non-metallic earth materials takes place by both electronic and electrolytic processes, with the latter one predominating in its influence on the conduction in materials near the surface, while the former one accounts for conduction at depths in the earth. If the earth material is an insulator in its dry state, and the energy band resulting from the coalescence of the s states is separated by an energy of several electron volts from the band arising from the s states, then there is but little possibility that the distribution of thermal energy among the electrons in the s band will enable any appreciable number of them to achieve entrance to the p band and the intervening region is excluded from occupancy by electrons. If all the levels in the s band are occupied then the direction and magnitude of the motion of each electron is determined by the exclusion principle, and the application of an electric field will not alter this isotropic distribution of electron velocities. Hence there can be no directional motion of charge caused by the electric field, and the crystal is called an insulator.

If the separation of the s and p bands is very small, the crystal is a semiconductor. When the forbidden energy range is of the order of a few tenths of an electron volt, thermal energies will raise an electron from the fully occupied s band to the p band and the conductivity is dependent upon temperature. The presence of extraneous atoms in a crystal lattice may cause the conductivity of an insulator to be greatly increased, and such materials are termed impurity semi-conductors. It is conceivable that impurity semi-conductors abound in earth materials. If the local impurity levels are such that they can remove some electrons from the full s band, the conditions relating to the electron velocities are relaxed thus permitting a net flow in one direction. These materials are known as P-type semi-conductors. On the other hand, if the local impurity levels have an excess of electrons and can contribute them to the p band upon absorption of thermal energy, conduction results in that band. These semi-conductors are designated N-type and act as if they were negatively charged.

The boundary between two different crystalline materials in contact can be considered as a membrane separating two different electron concentrations. If in equilibrium, the concentrations so adjust themselves that no net work is done in moving an average mobile electron from one side to the other. Any electrical forces that may exist are counterbalanced by the non-electrical forces arising from the electron structures of the two materials. When an electric field is applied across the junction, electrons move from one side to the other and work may be done by or contra to the non-electrical band structure forces, but the average energy will not be the same in the two crystals. If no constraint is placed on the boundary levels on either side, the situation is quite symmetrical and no difference in conductivity is anticipated. If, however, the band structure influences the flow of current across the boundary, asymmetrical conduction or rectification will take place provided that one of the boundaries is metallic.

Both the near surface materials, except for metallic ores, and water are dielectrics so that upon the application of an electric field the bound electrons of the space lattice structure of the atoms, if it is in a solid, and the positive and negative aggregates, if it is a liquid, tend to separate in the direction of the applied field resulting in both mechanical and electrical distortion. This effect is termed polarization. Though the charges are not free to move through the material, a separation of charge takes place on a small scale, and all the phenomena exhibited by dielectrics are caused by polarization.

Long continued application of the electric field would result in proportionately larger electrical distortion with a correspondingly longer relaxation time after the field was removed. During this relaxation time the charges borne by the ions and electrons would diffuse and redistribute themselves until the volume density of charge becomes the same as it was at the time T = 0. If the conductivity of the dielectric is large the relaxation time is very small, but for some dielectrics the relaxation time is of the order of days. Because of their wide range of dielectric constants, earth materials may be expected to vary enormously from place to place in their relaxation times and this can be an important factor in certain techniques of measuring earth resistivities. The volume density of charge q_v may be determined for a dielectric from

$$q_v = q_{v_0} e^{\frac{\sigma t}{\kappa \kappa_0}} \text{ - - - - - - - - - - - - - - } (13)$$

in which σ is the conductivity and κ is the dielectric constant.

A more complete discussion of dielectrics will be found in Smythe (1950, 2d ed.), Harnwell (1949, 2d ed.), Von Hippel (1954), Zwikker (1954), and others.

It was previously pointed out that conduction of electricity in near surface earth materials was primarily an electrolytic phenomenon, and this is fundamentally true because of two inherent physical properties of these materials, namely, porosity and permeability. If these two properties that allow water to infiltrate and remain in the voids between the grains or crystals did not exist in the earth materials, then conduction of electricity in them would have to be entirely electronic as described in the preceding paragraphs. Water being the nearest universal solvent dissolves the soluble materials from the earth materials and in this way becomes an electrolyte containing various ions. In its pure state as obtained by distillation, water has a conductivity of 5×10^8 mho cms, and saline waters may have a conductivity as low as 6×10^{-2} mho cms, Slichter and Telkes (1942). Thus it is evident that the same earth material may have a wide range of conductivity depending both on the amount and kind of water saturation. Some studies of this effect have been made by Sundberg(1932), and Jakosky and Hopper (1937).

Conduction of electricity.

Effect of equivalent unequal source and sink. The conduction of electricity in all types of conductors, including the earth, follows the electromagnetic field theory as originally given by Maxwell (1904). Complete discussion of the theory will be found, besides in the original work, in Jeans (1946), Smythe (1950), Stratton (1941), and others. The investigation of structure below the surface of the earth is performed by observing the potential at the surface when a current is passed through the earth between two or more electrodes in contact with the soil. Smythe (1950, p. 9) shows the distribution of the lines of current flow and equipotentials in a homogeneous medium with a source and sink of equal magnitude to be symmetrical about both the source and sink reproduced as figure 30. If the source and sink are unequal, one several times the other, the distribution of lines of current flow and equi-potentials in a homogeneous medium are distorted and appear as in figure 31 taken from Maxwell (1904). Results have been reproduced from Gilchrist, Clark, and Bernholtz (1950) that show the measured values in a homogeneous medium of the lines of current flow and the equipotentials when the source and sink are of equal magnitude, figure 32, and when the equivalent effect of the source is four times that of the sink, figure 33.

Note further GC&B results if when available

It is the opinion of some workers in the field of resistivity, and completely unrecognized by others, that such a theoretical assumption as an equivalent unequal source and sink cannot exist in nature under ordinary conditions. Such an opinion was expressed by Wuenschel (1953) . But as a matter of fact, the distortion of the electrical field caused by the unequal balance of the source and sink is present in every electrical resistivity depth profile or horizontal profile unless the surface materials in which the electrodes were set was perfectly homogeneous or else there was fortuitous setting of the electrodes. This is adequately demonstrated in the field observations of two vertical depth profiles given in tables 1 and 2.

Examples of equivalent unequal source and sink.

Table 1

Field observations of a vertical depth profile showing distortion caused by an equivalent unequal source and sink.

Electrode interval feet		E millivolts	I milliamps	ohm cms	Location and remarks
3	F	470.2	0.6	451,500	
3	P-1	237.2	0.6	227,700	SE of inter-
3	P-2	230.1	0.6	221,000	section
					Marrett Road
5	F	476.3	1.3	351,500	and Massachu-
5	P-1	246.1	1.3	181,700	setts Avenue,
5	P-2	226.8	1.3	167,000	Lexington,
					Mass.
8	F	401.3	2.9	212,500	
8	P-1	215.3	2.9	114,100	
8	P-2	185.0	2.9	97,900	
10	F	213.0	3.0	136,000	
10	P-1	124.2	3.0	79,300	
10	P-2	87.0	3.0	55,620	
15	F	126.5	4.3	84,800	
15	P-1	124.2	4.3	83,200	
15	P-2	2.0	4.3	1,340	
					*Computed; instru-
20	F	77.2	4.8	61,800	ment will not
20	P-1	216.7	4.8	173,400	read negative
20	P-2	-139.5*	4.8	negative	potentials.

Table 2

Field observations of a vertical depth profile showing distortion caused by an equivalent unequal source and sink.

Electrode feet	interval	E millivolts	I milliamps	ohm cms	Location and remarks
5	F	357.3	4.5	76,230	
5	P-1	171.1	4.5	36,500	SE corner Mt.
5	P-2	187.8	4.5	40,700	Auburn Hospital
					grounds near Mt.
10	F	181.6	6.5	53,610	Auburn Street,
10	P-1	93.3	6.5	27,580	Cambridge, Mass.
10	P-2	85.9	6.5	25,390	
20	F	131.1	22.7	22,150	
20	P-1	62.0	22.7	10,480	
20	P-2	68.3	22.7	11,550	
30	F	34.8	22.6	8,870	
30	P-1	21.7	22.6	5,530	
30	P-2	12.8	22.6	3,265	
40	F	22.3	37.0	4,630	
40	P-1	18.8	37.0	3,905	
40	P-2	3.0	37.0	623	
50	F	20.5	55.0	3,580	
50	P-1	18.2	55.0	3,179	
50	P-2	2.0	55.0	349	
60	F	13.6	63.0	2,485	
60	P-1	12.6	63.0	2,303	
60	P-2	1.2	63,0	219	
70	F	10.3	76.0	1,823	
70	P-1	18.1	76.0	3,204	Computed; instru-
70	P-2	- 7.8*	76.0	negative	ment will not
					read negative
80	F	4.9	57.0	1,321	potentials.
80	P-1	20.1	57.0	5,420	
80	P-2	-15.2*	57.0	negative	

In both sets of observations, tables 1 and 2, three observations of potential were made at each electrode interval using the modified Lee configuration as used by the writer (1950); the F, P-1, and P-2 representing respectively the potential measurements across the full potential distance, the right half potential distance taken in the northerly or easterly direction, and the left half potential distance taken in the southerly or westerly direction. It was not possible to equalize the source and sink, or current electrodes, in either set of observations, but each electrode was set with care in order to get current into the very dry, sandy, almost clay or silt free, surface material. If the earth layers were homogeneous and isotropic, the P-1 and P-2 values of potential and computed resistivity would be equal and their sums equal to the F values. A certain amount of variation between the P-1 and P-2 values indicates that the electrical layering is not uniform and the amount of variation in depth or thickness of the layer may be determined from the complete resistivity curves. Such an unusual departure, which begins at the 5 ft interval in table 1 and at the 30 ft interval in table 2, indicates that something other than variation in thickness of the layers is causing the anomalous observations. In both tables 1 and 2, it will be noticed that the values of P-1 and P-2 diverge more and more, yet their sum approximates the corresponding F value, until an interval is reached where one of the P observations becomes larger than the F value and the other P observations becomes unreadable on the instrument because it is negative. This is a definite indication of distortion in the electrical field set up in the earth and is caused by the very large difference in equivalent value between the source and sink caused by the current introducing and leaving electrodes.

The distortion of the electrical field caused by unbalanced current electrodes, or equivalent unequal source and sink, was early recognized by Ehrenburg and Watson (1932, p.438) who state:

> "In certain terrains, ρ_1 changes from point to point considerably, and when the current electrode is moved, changes in ρ_1 occur which may overshadow the vertical anomaly B. Hence it is safer and sometimes necessary to use stationary current electrodes."

In the Discussion following the above article (pp. 438-442), others have made comments which not only support but amplify the above statement. L. Gilchrist remarks on equalization of the electrodes:

> "--- the dispersion of the currents due to the discontinuities in the materials of the layers themselves and the moisture content in these layers, both of which in general differ in successive layers, together with the local potential differences of the contour of the layers, are factors contributing to rapid changes in the distribution of potential on the surface and should be taken account of by the authors. --- it is suggested that the

layout by which the potential is applied to the surface should always be such that symmetry would obtain for a single homogeneous continuous layer of good conductivity. This condition is obtained by the use of ----electrodes in which the currents entering at the distant electrodes have been equalized."

S. F. Kelly amplifies and explains the authors statement: "Their suggestion for maintaining the current electrodes stationary is entirely a field procedure to avoid errors induced by changing resistivity in the surface materials. That is, as one moves the current electrodes from one place to another, the resistivity of the material in which they are placed will vary, which will cause the introduction of errors that have nothing whatever to do with the vertical changes in resistivity."

D. O. Ehrenburg offers a rebuttal to the discussion that is very clear and explanatory:

"(One of the critics)* has refused to recognize the advantage of keeping the electrodes stationary, in spite of Mr. Kelly's explanation. However a simple consideration of the lines of flow shows that a minor irregularity of the medium away from the source (such as at a probe electrode) will distort the field but slightly; whereas an irregularity at the source will distort the field considerably. This is borne out very closely by a number of field tests conducted by the authors."

*Present authors substitution.

The recent article, previously referred to, of Gilchrist, Clark, and Bernholtz(1950) gives complete details of the effects of an equivalent unequal source and sink, or unbalanced current electrodes, on the electrical field induced in the earth.

Procedure for balancing electrodes. At the time the observations given in tables 1 and 2 were taken, the writer did not fully understand how the physical processes taking place could be eliminated, and only one other such situation had been encountered before on a depth profile being measured in another state. At the first location the difficulty was eliminated by a simple procedure. The applied voltage to the earth was held constant so that the current flowing therein increased each time the electrode interval was enlarged. If the current did not increase, then it was a very simple matter with the equipment in use to find the poor electrode setting and have better contact made so that the current did increase. Later, it was realized that the theory associated with the reversal in sign of the potential, such as was shown in tables 1 and 2, was the same as that mentioned previously by Ehrenberg and Watson. As a result, a system of electrode balancing was introduced which was equivalent to balancing the source and sink in theory. All electrodes were balanced, but a close balance is not imperative for the potential electrodes, but very useful in keepin a crew on their toes in electrode setting.

Balancing of the electrodes can only be done conveniently and correctly in the Gish-Rooney, or commutated dc method, or similar methods using metal electrodes. The reason for this will be apparent from the method used in balancing the electrodes. Connection to the center electrode, or ground, is shifted to the C-1 jack on the instrument; then in turn P-1, P-2, C-1, and C-2 are placed in the C-2 jack and current passed successively through each to ground and the value of current in ma recorded. The amount of current used in balancing the electrodes should be near the value which will be used for the present reading. The C-electrode and the P-electrode having the lower values are then brought up to read the same as the C- and P-electrode high value by adding water, mud, and depth, or even attaching another electrode to the low C-electrode. This extra electrode must never be used on the P-electrode unless the interval is very large. In unusual cases, it may be necessary to bring the electrode having the higher value down to read the same as the lower one by pulling it out of the ground, or putting dry earth in the hole.

As it is not possible to pass any current through the porous pot electrodes used in the dc methods, it will be evident why it is not possible to use electrode balancing in the dc method. Therefore, all direct current measurements which have been made with the electrodes unbalanced, equivalent to the source and sink being unequal, are uncertain or misleading in many instances, and probably of little value in many other places, particularly so if made where surface resistivities were high with attendant poor electrode contact. The failure to balance the electrodes is one of the items termed by Gilchrist, Clark, and Bernholtz (1950) and Mooney (1954) a perversion of procedure.

The current electrodes may be balanced in direct current methods by locating a stake electrode at the center of the electrode configuration. It is not a recommended procedure, however, unless the potential electrodes are lifted each time the test is made in order to remove them from the high density field set up during the test. Another way would be not to position them until the balance test was completed.

Relation of work function to resistivity. The work function of metals plays a minor, but important, part in the connections and the electrodes used for resistivity measurements. In the equation,

$$I = AT^2 \epsilon^{-\frac{e\varphi}{kT}} \text{ - - - - - - - - - - - - - - } (14)$$

I represents the maximum current that may be obtained from the surface of an emitter operating at absolute temperature T; ϵ represents the base of Naperian logarithms to distinguish it from the electronic charge e; k is the Boltzman constant (1.381 x 10^{16} erg per degree C); and φ and dA are constants related to the emitting material and its surface condition. The quantity eφ in the above equation represents an energy, and φ, known as the work function and measured in volts, represents the change of potential through which an electron must go so as to gain or lose e φ ergs of energy. Values of φ can be calculated from observations of I by plotting log (I / T^2) against (1 / T), obtained from the logarithmic form of the above equation,

$$\log \frac{I}{T^2} = \log A - \frac{e\varphi}{k} \frac{1}{T} \text{ - - - - - - - - } (15)$$

so that a straight line is obtained whose slope is $- e\varphi/k$. φ thus obtained by conversion to volts ranges between 2 and 6 volts for various metals. A more complete explanation of the

~~of the~~ work function will be found in Hughes and DuBridge (1932), and a table of thermionic values is taken from their book, table 3.

Table 3

Work function of metals

Metal	Thermionic volts	Metal	Thermionic volts
Silver	4.08	Palladium	4.99
Gold	4.42	Platinum	6.27
Carbon	3.93	Rhodium	4.58
Calcium	2.24	Tantalum	4.07
Cesium	1.81	Thorium	3.35
Copper	4.38	Tungsten	4.52
Molybdenum	4.41		

Contact potentials. The potential difference between the work functions of two metals A and B may be termed the contact potential, or

$$V_c = \varphi_A - \varphi_B \quad (16)$$

The effect of an impressed field on the contact potential is not well known, but it probably would increase the effect of the spurious potentials. In consequence, therefore, to eliminate a source of spurious potentials in earth resistivity measurements, all contacts and conductors should be of the same metal and kept clean and bright in order to eliminate oxide contacts which also generate high spurious potentials. The use of iron as an electrode or other part of the circuit immediately introduces a possible spurious contact potential of approximately 0.5 volt with a copper contact such as a clip or wire. Stainless steel used for electrodes or circuit components introduces a possible spurious contact potential of 0.5 to 0.7 volt. Electrolytic action between the crystalline components of stainless steel and solutions also sets up spurious potentials.

The most satisfactory conductor would be silver, and it also would have the lowest contact potential, but it is expensive and its mechanical properties are very poor. Aluminum would be a very good conductor were it not for its tendency to form an oxide film immediately upon contact with the air and for its poor mechanical properties.

It is easy to observe that these contact potentials are present in field measurements of earth resistivity. Just leave the potential switch turned on and the field energizing switch off while a new interval is being set up and watch the galvanometer. It swings wildly back and forth with the unreeling of wire, the driving of electrodes into the earth, and the contact of the clips with the electrodes unless they are both copper. This may be termed as another major uncertainty of procedure in earth resistivity measurements which has introduced uncertain, or at least misleading, measurements into the practice of electrical resistivity.

Induced potentials in connecting wires. Potentials induced in the potential wires which are parallel to the current wires, but separated a distance of 2 or 3 feet, are small and probably negligible unless the wire carrying reels are not properly arranged. With the wire carrying reels centrally located and in close proximity, the induced potentials are quite large causing a displacement of the curve such as that shown in figure 34.

The induced potentials may be reduced to a minimum when the reels are centrally located by having the wire on them the same length and wound in opposition and by not using an iron or steel winding core. However, the inductance for each reel will probably never be exactly the same so that there will be some induced potential even if the above practice of elimination is followed.

The best procedure to adopt is to have each reel move outward as the electrode interval is increased, thus eliminating all mutual induction caused by position of the reels.

The formulas for mutual impedance of grounded wires having a finite length and based on direct current paths in a uniform earth are given by Campbell (1923). The mutual impedance of wires of a finite length, on or above a uniform earth, was discussed by Foster (1931, 1933). Mutual impedance formulas for both finite and infinite wires above an earth having an exponential variation in resistivity with depth have been derived by Gray (1933, 1934). The mutual impedance of wires of finite length on or above an earth having a two-layer stratification is treated by Riordan and Sunde (1933). The transient coupling between two wires grounded at their end points in a conducting medium of infinite extent has been applied to the collinear array of equal spacings by Wait (1953). The transient response builds up to unity in 5 x 10^{-5} sec, or 50 micro-seconds, if

$$\sigma \ell^2 \pi = 1, \text{ but}$$

$$\sigma \ell^2 \pi \text{ is}$$

nearer 3 x 10^{-4}, so $t = 1.5 \times 10^{-10}$ sec. Here σ, the conductivity, is $< 10^{-4}$ mho/meter for moist earth materials and 10^{-4} to 10^{-5} mho/meter for dry, and $\ell = 1$ meter.

The derivations are long and tedious, so the reader is referred to the original papers for them.

Induced potentials may be eliminated by the use of coaxial cable; connecting the outer shield to ground and using the axial conductor to carry the potential circuit. Care must be exercised in the use of this cable for resistivity measurements because the fine wires of the sheath conductor have a tendency to break and penetrate the insulation between it and the axial conductor thus causing very elusive shorts. Frequent replacement is therefore necessary, and is both inconvenient and expensive. The very slight improvement obtained by the use of coaxial cable in the usual practice of shallow resistivity measurements was, in the few trials made with it by the writer, hardly warranted. This was because of the large bulk, the care required to keep it from becoming a tangled mess and breaking the conductors, and the time lost at each set-up for the coaxial cable had to be completely laid out each time and taken up with care. Large diameter cores in suitable reels would have lessened these difficulties somewhat, but the continued winding and unwinding would soon have filled the cable with shorts.

Batteries as a source of energizing power.

Batteries are a very satisfactory source of power for energizing the earth with an electrical field, but they should be fresh and their individual voltages should be checked regularly. The requirements for voltages were given in a previous section.

A battery not operating properly, that is one ~~not~~ having a high internal resistance, will cause spurious readings of potential to be obtained in field measurements of earth resistivity. This fact has been checked on numerous occasions in practical field measurements of earth resistivity. For instance, when the field observations are producing a smooth apparent resistivity curve and it suddenly becomes erratic, it has been found, after checking for all other causes which might have produced such an effect and finding them absent, that there is at least one battery with a high internal resistance that is the cause of the effect. It can be located by ~~the~~ checking the individual batteries at no load and under a simulated load with a good high resistance voltmeter. Replacement of the defective battery or batteries with fresh ones having the proper rated voltage will restore the incorrect apparent resistivity curve to its smooth trend.

This unrecognized effect has probably introduced some uncertain or misleading results into apparent resistivty curves with attendant misinterpretations of the curves.

For an efficient battery power supply, the batteries in the lower voltages should be paralleled in groups of two, three, or more depending on the load requirements. It is possible to draw 1,000 to 1,500 ma for short, intermittent, and frequently repeated intervals, from a bank of batteries paralleled in groups of three or four. Above the frequently used range of voltage, it is unnecessary to have such a division of current load and the batteries can be connected in series. In the truck mounted equipment, provisions are made for a bank of 25 extra heavy duty "B" batteries so connected to give a no-load voltage of 745. While this amount is a little more than twice the rated voltage of the Gish-Rooney equipment, it has been found necessary to use such a voltage many times to obtain a field measurement of potential, disregarding the risks involved with no harmful effects as yet.

Batteries, except for their high original and replacement costs and their excessive weight and space requirements, provide the nearest to the ideal power supply for Gish-Rooney techniques. No other comercially available power supply approaches them in over-all utility. Motor-generator sets require heavy duty storage batteries, charging equipment for the storage batteries, and very efficient filtering of the output to reduce the ripple voltage below the point where it can be noted in the measurements. Vibrator power supplies require about the same amount of associated equipment. A gasoline engine-generator supply offers some promise as a convenient power supply for the Gish-Rooney equipment if the frequency is kept high, 400 cycles or more, to reduce weight and make filtering less difficult for a ripple free supply.

BASIS OF ELECTRICAL MEASUREMENTS OF EARTH RESISTIVITY

The ideal method of earth resistivity measurement would be the one determined from the distribution of potential about a point electrode located in vertically stratified ground, the ground being infinite in extent horizontally. It is, however, not possible to make such a measurement, so differences of potential between two points are measured; the potential being set up by an electrical field introduced into the earth by means of two other electrodes.

Current flow in a homogeneous stratified earth. The lines of current flow in a homogeneous earth as well as in a layered or stratified earth as given by Weaver (1929) are reproduced as figures 35 and 36. In the latter figure, the lower layer is fifty times the conductivity of the upper one, and it clearly shows how the tubes of current flow are altered by the low conductivity layer.

The fraction of the total current which penetrates below a depth D in a uniform earth is given as figure 37, after Weaver (1929). The actual surface potentials existing when an electrical field is set up in a two-layer earth, the upper layer 40 feet thick and of conductivity σ_1, the lower layer infinitely thick and of conductivity σ_2 is shown in figure 38, after Weaver

(1929). Various potential curves are given for different ratios of conductivity in this figure, and the heavy curve, $\sigma_1 = \sigma_2$, is the normal potential for a homogeneous earth. It will be apparent that there is a marked increase in the potential from its normal value when the conductivity of the upper layer is one hundred times that of the underlying layer. When the lower material is one hundred times as conducting as the upper layer there is a decrease in the potential.For ratios of higher conductivities, 1,000 or 10,000 to 1, the curves would practically coincide with those on the figure for the 100 : 1 ratio. Such an effect produced for a very large ratio of conductivities, essentially infinite, is termed the "saturation effect". Inspection of figure 38 shows that approximately half of the saturation effect is produced by the $\sigma_1 = 5\sigma_2$ ratio. Table 4 from Weaver (1929) shows the percent of saturation for the special case of a 40 ft layer over an infinite layer, an electrode spacing of 200 feet, and at a point 60 ft from the input electrode. The table has the special significance also of showing that it is possible to distinguish between layers of materials where the conductivities differ only by a small ratio.

Figure 38 also shows that the departures from the normal curve are larger when the upper layer has a higher conductivity. In general, then, potential measurements down through a good conducting layer to a poor conducting one can be made more

Table 4

Percent of saturation effect for various

ratios of conductivities

Ratio of onductivities	$\sigma_1 =$					$\sigma_2 =$				
	$100\,\sigma_2$	$10\sigma_2$	$5\,\sigma_2$	$3\,\sigma_2$	$2\,\sigma_2$	$2\,\sigma_1$	$3\,\sigma_1$	$5\,\sigma_1$	$10\,\sigma_1$	$100\,\sigma_1$
ercent of aturation ffect ealized	96	73	55	39	24	38	56	71	85	97

readily and accurately than if the reverse layering is present. Furthermore, the departures from the normal potentials caused by the presence of a lower layer are sufficiently large to give accurate measurements.

Depth of current penetration. As the electrode interval spacing is increased the depth of penetration of the current into the earth also increases, thus the effect of buried layers having either high or low conductivity can be made to show up in the surface potential measurements. However, because of the saturation effect previously noted, and the tendency for the several layers to disturb and by-pass the normal distribution of current, it is necessary to increase continuously the amount of current introduced into the earth by the current electrodes so that these two effects are overcome. Herein lies one of the primary defects of instruments used for earth resistivity measurements such as the megger and vibrator devices. These instruments cannot, under usual field conditions, accurately measure potentials at large separations, or in other words, to relatively large depths, and should be confined to measurements of near-surface anomalies.

Muskat and Evinger (1941) have presented a study of current penetration in direct current prospecting which, though it is not intended for use directly in the interpretation of direct current measurements, aids in understanding and visualizing how electrical currents are distributed in stratified electrical conducting media.

Any measurement of apparent resistivity measured at the surface will represent the resultant of a complex average of all the effects produced by the conducting beds beneath the surface. It is then to be expected that surface observations of apparent resistivity will be, in general, smooth and continuous except for errors or surface variations. If the surface layer is continuously uniform, it is not conceivable that sharp discontinuities would result from such a diffuse system such as the earth is equivalent to, namely, a current distribution through a semi-infinite space as used in direct current prospecting. Whatever sharp variations the surface observations show will depend on the particular conditions of the measurements.

It is to be anticipated that the extent of the effect caused by a certain subsurface conducting stratum upon the surface observations is directly dependent upon the fraction of the total applied current through the earth which passes through this particular stratum. In order to have the effect of this particular stratum show up on the apparent resistivity curve, the surface electrodes must be so arranged and energized that an appreciable amount of current will flow through it.

The fraction of the total current, f_1, flowing in the surface stratum of a two-layer earth is shown in figure 39, after Muskat and Evinger. This figure shows that the current fraction passing through the upper layer decreases uniformly as the electrode spacing is increased or as the upper layer thickness decreases. This rate of decrease becomes larger as the ratio of σ_2 / σ_1 increases, or as the conductivity of the lower layer increases with respect to the upper stratum. Even with the lower layer a perfect conductor, the current flowing through the upper layer will remain an appreciable fraction of the total current until the current electrode spacing exceeds six times the upper layer thickness. The variation of f_1 with the conductivity parameter k for different electrode spacings is shown in figure 40. The entire range of variation of the actual conductivity ratios σ_2 / σ_1 is covered between $k = -1$ and $k = +1$. The continuous decrease of f_1 with decreasing k implies a continuously decreasing current fraction in the upper layer as the conductivity in the lower layer increases. It is noticed that the slope of the curve with $k = +$ is steeper than with $k = -$. This feature explains the well known fact that the electrical method is able to detect a poorly conducting bed easier than it can detect a highly conducting bed.

The fraction of current passing through the uppermost layer of a three-layer earth as a function of the ratio of the electrode spacing and the top layer thickness is shown in figure 41.for several values of the conductivity contrast. In comparison with the curves for the two-layer earth, the curves appear similar; but there are two important differences. First, for the conductivity ratios $\sigma_2 / \sigma_1 < 1$ the current fractions f_1 in the top layer are in every case lower in the three-layer earth than in the two-layer earth.

Second, when $\sigma_2 / \sigma_1 > 1$ the opposite is true; the current fractions in the upper layer of the three-layer earth exceed those in the two-layer earth. This is because of the ratio of conductivit~~y~~ies chosen; the upper and third layers have the same conductivity.

The fractions of current passing through the second layer of a three-layer earth, f_2, as a function of the ratio a / h_1 are shown in figure 42. Two sets of curves are given, the dashed and solid lines having the same meaning as those of figure 41. A striking feature of these curves is that they show a maxima. The initial rise expresses the increasing overall current penetration as the electrode spacing is increased relative to the top layer thickness. The ultimate declines in the curves and intermediate maxima is an expression of the fact that as the ratio a / h_1 continues to increase the current becomes largely concentrated in the third or bottom layer. The same considerations also explain the fact that the values of a / h_1, at which the maxima appear, increase with an increasing ratio of σ_2 / σ_1. As the middle layer conductivity increased relative to that of the remaining strata, it tends to hold longer the concentration of current through it as a / h_1 is increased, than when the conductivity is lower. If the limiting case is reached with the middle layer of infinite conductivity, it blankets the bottom layer completely and the maximum f_2 is asymptotically with no loss of current to the deepest layer. Likewise, if the conductivity of the middle layer is zero, the current through it will be zero regardless of a / h_1 .

The current fractions, f_3 , in the deepest stratum of a three-layer earth are shown in figure 43. The curves show the predominant characteristic of current distribution systems through continuous and stratified earths; that of increasing penetration with increasing electrode separation. It will be noted on the figure that as the conductivity of the middle layer decreases from infinity, relative to that of the others, the current fraction f_3 increases because the blanketing effect of the conducting middle layer is lessened. If σ_2 / σ_1 decreases still more, the fraction f_3 will finally reach a maximum and then fall off to zero as $\sigma_2 / \sigma_1 \to 0$, and the middle layer becomes an insulating blanket with respect to the deepest layer.

The current fractions, f_1 , for a three layer earth are plotted in figure 44 as functions of the conductivity parameter k, for the various values of the ratio a / h_1 . While these curves are similar to those of a two-layer earth, figure 40, there is an important difference; the markedly steeper curves as k becomes less than unity. The highly conducting third layer also exerts an important influence on the slope of the curves as soon as the middle layer has any conductivity. The effect of the middle layer in forming a conductivity blanket at $k = -1$ is apparent from the convexity of the curves.

The curves for the f_2 fractions of the three-layer earth are not given here as they contribute nothing to the physical interpretation of the problem. The current fractions, f_3 , for the three-layer earth are given in figure 45 as functions of the same parameters as figure 44. Here the curves have a steep rise as k becomes less than unity and represent the complementary effect of the f_1 curves of figure 44. As the conductivity of the middle layer $\rightarrow \infty$, a blanketing effect is introduced and the current fractions approach zero as $k \rightarrow -1$. The relative positions of the a / h curves is an expression of the general increase of current penetration with increasing electrode spacing.

Curves are also given in the original article for a three-layer earth in which the deepest layer has infinite conductivity; these are not included here.

ELECTRODE ARRANGEMENTS FOR SURFACE POTENTIAL MEASUREMENTS

When an electric current I is introduced into a homogeneous, isotropic earth by means of two electrodes, C-1 and C-2, and the current flows from C-1 to C-2, the potential at any point P is

$$E_P = \frac{I}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \qquad (17)$$

where r_1 and r_2 are the distances of the point P from the electrodes. The potential at another point R where R_1 and R_2 are the distances of the point R from the electrodes is

$$E_R = \frac{I}{2\pi}\left(\frac{1}{R_1} - \frac{1}{R_2}\right). \qquad (18)$$

The potential difference between P and R is

$$E_P - E_R = \frac{I}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{R_1} + \frac{1}{R_2}\right) \qquad (19)$$

or, in terms of resistivity or conductivity, σ,

$$\frac{1}{\sigma} = \rho = 2\pi\frac{E}{I}\frac{1}{\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{R_1} + \frac{1}{R_2}\right)}. \qquad (20)$$

This expression holds for any position of either the current or potential electrodes and, as in figure 46, forms the basis for all electrode arrangement formulas. The different arrangements will be given as will the formulas for computing the apparent resistivity.

1. If the two current electrodes and the two potential electrodes are placed so as to form a straight line, as in figure 47, with the same distance between each electrode, then this is the Wenner arrangement. With the interval distance equal to a , r_1 and R_2 is equal to a , and r_2 and R_1 is equal to 2a. The expression for resistivity is then,

$$\rho = 2\pi a \frac{E}{I} \quad \text{- - - - - - - - - - - - - - - (21)}$$

2. If an additional electrode, P-c or G , is placed at the mid-point between P and R of the Wenner arrangement, the Lee partitioning arrangement is formed, figure 48. A potential measurement is made for both the right and left halves of the interval. With r_1 = a , r_2 = 2a , and R_1 and R_2 equal 3/2 a, the expression for resistivity is,

$$\rho = 4\pi a \frac{E_{PP_c}}{I} \text{ and } \rho = 4\pi a \frac{E_{RP_c}}{I} \quad \text{- - - - - - - - (23)}$$

where $E_{PP_c} \equiv E_{RP_c}$, the potential from center to each adjacent potential stake. The Lee partitioning arrangement may be modified for practical application by combining the measurements of potential obtained thereby with those of the Wenner arrangement and computing the resistivity by the Wenner formula. This procedure spreads the curves on the plot without any change in appearance or shape, and enables a more convenient inspection as well as giving an improved position for interpretations.

3. If the current electrodes are placed a great distance apart, more than five or six times the expected potential electrode distance, and the potential electrodes are kept in a line at any angle with the near-by current electrode, one of them being at a distance a and the other at a distance 2a from the current electrode, the double equidistant arrangement is formed, figure 49. Here $r_1 = a$, $R_1 = 2a$, $r_2 = \infty$, $R_2 = \infty$, and

$$\rho = 4\pi a \frac{E}{I} . \qquad (24)$$

This arrangement of electrodes has the following disadvantages: the very great distance between the current electrodes thus necessitating long current leads; the possible error created by the assumption that r_2 and $R_2 \longrightarrow \infty$; and the potential readings are only one-half those obtained with the Wenner configuration, an important factor in large depth determinations. The only advantage is the reduced number of electrodes required to be moved and set.

4. The disadvantages of arrangement 3 may be partly overcome, and the advantages of a stationary current electrode retained, by taking into consideration the influence of the stationary electrode. This is termed the asymmetrical double probe configuration, figure 50. The distance between the current electrodes is l and potential electrode interval is a, then $r_1 = a$, $r_2 = l - a$, $R_1 = 2a$, $R_2 = l - 2a$, and

$$\rho = 2\pi \frac{E}{I} \frac{2a(l-a)(l-2a)}{(l-2a)^2 + al} . \qquad (25)$$

If $\ell = 3a$ the equation reduces to the Wenner form. The disadvantages of the method are: the length of the computation required, and the somewhat smaller potentials obtained in comparison to those obtained with the Wenner arrangement.

5. By using one current electrode as a potential electrode, another arrangement is formed called the single probe method, figure 51. If the other electrode is placed at infinity and its effect is neglected, then $r_1 = 0$, $r_2 = \infty$, $R_1 = a$, $R_2 = \infty$, and

$$\rho = 2\pi a \frac{E_c - E}{I} \quad \text{- - - - - - - - - - - - - -} \quad (26)$$

where E_c is one-half the total potential difference between the current electrodes.

6. If the effect of the far current electrode is taken into consideration and the potential electrode kept along the line joining the current electrodes, as in figure 52, then $r_1 = 0$, $r_2 = \ell$, $R_1 = a$, $R_2 = \ell - a$, and

$$\rho = 2\pi \frac{E_c - E}{I} \cdot \frac{a(\ell - a)}{(\ell - 2a)} \quad \text{- - - - - - - - - - - -} \quad (27)$$

7. If the current electrodes be disposed as in arrangement 3, but the potential electrodes be spaced at right angles to and at unequal distances from a current electrode, as in figure 53, then, $r_1 = a$, $r_2 = \infty$, $R_1 = b$, $R_2 = \infty$, and

$$\rho = 2\pi \frac{E}{I} \frac{ab}{(b-a)} \quad \text{. - - - - - - - - - - - -} \quad (28)$$

Modifications of arrangements 3 to 7 are possible in order to use them similar to the manner that Lee modified the Wenner arrangement. For example, in arrangement 6 another electrode R' may be set at a distance to the left of the current electrode and the potential measured across P and R', then $r_1 = 0$, $r_2 = \ell$, $R_1 = a$, $R_2 = \ell + a$, and

$$\rho = 2\pi \frac{E_e - E}{I} = \frac{a(\ell - a)}{(\ell + 2a)} \quad \text{.- - - - - - - - - -} \quad (29)$$

Other modifications will be apparent and will not be described.

DETERMINATION OF RESISTIVITY AND APPARENT RESISTIVITY

The formulas given in the preceding section may be used to compute the values of ρ in both homogeneous and inhomogeneous materials of the earth, but for the latter case the physical meaning associated with ρ is lost. Since ρ now varies in value with a , it will be termed apparent resistivity and designated ρ_a. The value of the apparent resistivity lies somewhere between the two or more resistivities present in the ground, but its value is not a constant and is a function of the actual resistivity. This function is usually of a complicated form, but it can be expressed mathematically for some particular conditions. Its determination forms the basis for the interpretation of field results in electrical resistivity prospecting.

The differences between the potential as actually measured and as computed were shown by Weaver (1929) to eliminate resistivity and to enable conclusions to be made about the subsurface materials. It is, however, more useful to compare values which depend upon the nature of the subsurface materials and the arrangement of the electrodes, namely; the apparent, the average, and the actual, resistivities. The potential function gives the relation between the apparent and average resistivities. Potential functions for various examples of inhomogeneous ground are given by Hummel (1928, 1929) and those for two and three layer earth are also given by him (1931), and another derivation was given by Ollendorff (1928).

SUMMARY OF IMAGE THEORY

Various authors have treated the theory of images by which the formulas of the potential in a layered earth are developed. A complete discussion of image theory is given by Jeans (1946, pp. 185-200), Maxwell (1902), Stratton (1941), and others. Other treatments of the theory of images as related to a layered earth are given by Mason and Weaver (1929), Peters and Bardeen (1930), Ehrenburg and Watson (1931), Watson (1934), Heiland (1940 p. 711 et seq), Roman (1931, 1951) who also gives the history and development of the theory, and others.

Some of the underlying principles of the theory of images will be summarized here, but the reader is referred to the above references for a complete treatment.

The long form of analysis of earth conditions is based on electrical images which, in turn, is built up on the optical analogy of a mirror image. The image of a point source of light at some distance from a perfect reflector will appear to be a point equally distant behind the mirror and be of equal intensity. If the reflectivity of the mirror is not perfect, the image will be of lesser intensity than the image of the perfect reflector, and the magnitude of the image will be the product of the source intensity and the reflection coefficient. In the electrical case, the images formed at the boundary between two layers, or strata, are determined so as to find the effect of the two media having different resistivities on the potential at a point on the ground surface, the point being at a known, finite distance from the electrical source and sink. This may be shown in a simplified manner with the aid of a diagram, figure 54. It is assumed that the two media are infinite, homogeneous, isotropic, have resistivities ρ_1 and ρ_2 , and have a common interface. It is also assumed that a point charge I is present in ρ_1 , at a distance h from the boundary plane. To determine the potential at a point A in ρ_1 , it is assumed further that an image of the point charge I exists in ρ_2 at a distance $h_1 = h$ from the boundary plane, and the intensity of the image charge is $I' = k I$ where k is the reflection coefficient of the ρ_1 / ρ_2 boundary plane and the image polarity is that of the source.

The potential of point A at a distance r from the charge due to the point charge alone is

$$E_A = \rho_1 \frac{I}{4\pi r_1} \quad \text{- - - - - - - - - - - - - - - - (30)}$$

based on a form of Ohms law, and the potential of a point A at a distance r_2 from the image charge alone is

$$E_{A'} = \rho_1 \frac{I'}{4\pi r_2} \quad \text{- - - - - - - - - - - - - - (31)}$$

and substituting $I' = k I$,

$$E_{A'} = \rho_1 \frac{k I}{4\pi r_2} \quad \text{- - - - - - - - - - - - - (32)}$$

Since the potential at A is caused by both the source and image, then

$$E_A = \frac{\rho_1 I}{4\pi}\left(\frac{1}{r_1} + \frac{k}{r_2}\right). \quad (33)$$

The potential at another point B, positioned in the ρ_2 medium at a distance r_3 from the point charge I, will be effected as though the source were reduced in intensity by the amount of the reflection factor r, thus having an apparent intensity (1 - k) I. The potential at B is then

$$E_B = \frac{\rho_2 I}{4\pi r_3}(1-k) \quad (34)$$

and, as it is positioned in ρ_2, will have no image and the equation represents the total potential. It is required that there be continuity of potential at the boundary plane, thus $E_A = E_B$ and for this condition, $r_1 = r_2 = r_3 = r$, and

$$\frac{\rho_1 I}{4\pi r}(1-k) = \frac{\rho_2 I}{4\pi r}(1-k) \quad (35)$$

then,

$$K = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \quad (36)$$

This is the expression for the reflection coefficient at the boundary of the two media having resistivities ρ_1 and ρ_2, and it will be evident that it varies between + 1.0 and - 1.0, depending on the resistivities. Thus it also represents the electrification of a plane between two media of different resistivities due to a point source and its image.

The addition of another layer in the earth greatly increases the complexity of the image analysis, as does all further additions of layers. In addition to the two layers of semi-infinite extent, each being homogeneous and isotropic, a third layer must be considered, namely, the atmosphere which is also infinite, isotropic, homogeneous, and has an infinite resistivity. In figure 55, ρ_0 is the atmosphere above the two-layer earth having an infinite extent in all directions and an infinite resistivity; the near-surface layer is assumed to have a thickness h and a resistivity ρ_1 ; the bottom layer is of infinite extent and has a resistivity ρ_2 . I represents the point source and points A and B are at distances r_1 and r_2 respectively from the source; all three points lie on the interface boundary between ρ_0 and ρ_1 and in the same vertical plane. All interface boundaries are assumed parallel planes.

An infinite series of images are formed by this problem. The source will form image I_1 in the ρ_2 layer, but being at the boundary, no image will be formed in either ρ_0 or ρ_1 . The second image I_1' will be that formed by the first image I_1 in the layer as no one of the ρ_2 images can be formed in the ρ_1 layer because it is evident that the reflection of only the first ρ_2 image could fall within part of the ρ_1 layer and, consequently, would be identical with the source. The third image will be that formed by I_1' in the ρ_2 layer, I_2. The value of image $I_1 = k\,I$ where $k = (\rho_2 - \rho_1)/(\rho_2 + \rho_1)$ and its distance from the $\rho_1 - \rho_2$ boundary is h. The value of image $I_1' = k_0\,I = k_0\,k\,I$ and its distance $2\,h$ from the $\rho_0 - \rho_1$ boundary. The value of image $I_2 = k_1\,I' = k^2\,I$ since $k_1 = k$ because $k_0 = 1$, and its distance is $3\,h$ from the $\rho_1 - \rho_2$ boundary. This process may be continued to form an infinite series of terms in which the last value of the image will be $I_n = k^n\,I$ and its distance will be $2\,n\,h$ below the source in the ρ_2 layer. The potential at any point on the surface, the $\rho_0 - \rho_1$ interface, is

$$E = \frac{\rho_1}{2\pi}\left[\frac{I}{r} + \sum_{n=1}^{n=\infty}\frac{2k^n I}{r_n}\right] \qquad \text{- - - - - - - - - - - - (37)}$$

The potential of point A, figure 55, at distance r_1 from the point source I is

$$E_A = \frac{\rho_1}{2\pi}\left[\frac{I}{r_1} + \sum_{n=1}^{n=\infty}\frac{2k^n I}{\sqrt{r_1^2+(2nh)^2}}\right] \qquad \text{- - - - - - - - - - (38)}$$

Similarly, for point B at a distance r_2 from the source I is

$$E_B = \frac{\rho_1}{2\pi}\left[\frac{I}{r_2} + \sum_{n=1}^{n=\infty}\frac{2k^n I}{\sqrt{r_2^2+(2nh)^2}}\right] \qquad \text{- - - - - - - - - - (39)}$$

The potential difference between points A an B is

$$E_A - E_B = \frac{\rho_1 I}{2\pi}\left[\frac{1}{r_1} - \frac{1}{r_2} + \sum_{n=1}^{n=\infty}\frac{2k^n}{\sqrt{r_1^2+(2nh)^2}} - \sum_{n=1}^{n=\infty}\frac{2k^n}{\sqrt{r_2^2+(2nh)^2}}\right] \text{- - (40)}$$

The above derivations are all for a source I and derivations must be made for a sink, - I , of equal magnitude at distances r_3 and r_4 from points A and B respectively. The expression for the sink is

$$E_B - E_A = \frac{-\rho_1 I}{2\pi}\left[\frac{1}{r_4} - \frac{1}{r_3} + \sum_{n=1}^{n=\infty} \frac{2k^n}{\sqrt{r_4^2+(2nh)^2}} - \sum_{n=1}^{n=\infty} \frac{2k^n}{\sqrt{r_3^2+(2nh)^2}}\right] \text{- - -} \quad (41)$$

The total potential difference between points A an B is then

$$E_{AB} = (E_A - E_B) + (E_B - E_A) \quad \text{- - - - - - - - - - -} \quad (42)$$

The expression above is the general formula for the difference in potential between any two points near or in between a source and sink. It may be converted to include the electrode spacing a of the conventional Wenner arrangement if $r_1 = r_4 = a$ and $r_2 = r_3 = 2a$, and the term

$$\frac{1}{\sqrt{a^2 + (2nh)^2}}$$

reduced to the value

$$\frac{1}{a} \cdot \frac{1}{\sqrt{1 + \left(\frac{2nh}{a}\right)^2}}$$

and substituted in the expression for E_{AB} becoming

$$E_{AB} = \frac{\rho_1 I}{2\pi a}\left[1 + 4\left(\sum_{n=1}^{n=\infty} \frac{k^n}{\sqrt{1+\left(\frac{2nh}{a}\right)^2}} - \sum_{n=1}^{n=\infty} \frac{k^n}{\sqrt{4+\left(\frac{2nh}{a}\right)^2}}\right)\right] \quad \text{. -} (43)$$

Let

$$F = \left(\sum_{n=1}^{n=\infty} \frac{k^n}{\sqrt{1+\left(\frac{2nh}{a}\right)^2}} - \sum_{n=1}^{n=\infty} \frac{k^n}{\sqrt{4+\left(\frac{2nh}{a}\right)^2}}\right)$$

then

$$E_{AB} = E_A - E_B = \frac{I\rho_1}{2\pi a}\left[1 + 4F\right] \quad \text{- - - - - - - - -} \quad (44)$$

The quantity $\rho_1 (1 + 4 F)$ is an apparent resistivity and under homogeneous earth conditions $k = 0$ and $E_A - E_B = I\rho_1/2\pi a$. The ratio of apparent resistivity ρ_s, and true resistivity ρ_1, is then

$$\frac{\rho_s}{\rho_1} = \left[1 + 4\sum_{n=1}^{n=\infty} \frac{k^n}{\sqrt{1+\left(\frac{2nh}{a}\right)^2}} - 4\sum_{n=1}^{n=\infty} \frac{k^n}{\sqrt{1+\left(\frac{2nh}{a}\right)^2}}\right] \text{------} \quad (45)$$

$$= \left[1 + 4\sum_{n=1}^{n=\infty} \frac{k^n}{\sqrt{1+\left(\frac{2n}{u}\right)^2}} - 4\sum_{n=1}^{n=\infty} \frac{k^n}{\sqrt{1+\left(\frac{2n}{u}\right)^2}}\right] \text{-----} \quad (46)$$

where $u = a / h$, the ratio of electrode separation and thickness of the upper layer.

The theoretical treatment by images for the three-layer and larger multi-layer cases is done along similar lines to the above cases; but, as pointed out by Hummel (1931, p.19), they "---become much more complicated, yet still remain elementary." A simplification in the application of images, however, has been presented by Muskat (1933).

THEORETICAL BASIS FOR INTERPRETATION OF

APPARENT RESISTIVITY CURVES

It was shown by Hummel (1931, pp. 28-29) that, by assigning limits to equation (44) and making the transpositions, the asymptotes of all three-layer resistivity curves having the limitations described must pass through the origin. The equation in this form also showed that for large electrode separations the apparent resistivities follow Kirchhoff's law for two resistances connected in parallel. The average resistivity of two infinite layers ρ_1 and ρ_2 having thicknesses h_1 and h_2 is given by

$$\frac{h_1 + h_2}{\rho_{av}} = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2} \quad \text{- - - - - - - - - - - - - - (46 a)}$$

This equation may be generalized to represent the average resistivity for any number of layers having different resistivities and thicknesses. This fact is of importance in the interpretation of apparent resistivity curves by the use of theoretical two-layer curves.

Two-layer resistivity curves. Equation (46) rewritten as

$$\rho_s = \rho_1 (1 + 4F) \quad \text{- - - - - - - - - - - - - (47)}$$

is the basis for several procedures in the interpretation of field data by theoretical methods using the Wenner, Lee, or a combination of the two, as well as other, electrode arrangements. In a resistivity depth profile, the primary purpose is the determination of the $\rho_1 - \rho_2$ interface if it is a two-layer condition in the earth. The interpretation procedure, based on equation (47), resolves itself into interpreting a series of observed values in terms of h, the thickness of the ρ_1 layer, and the relationship between ρ_s and ρ_1 and the value of the reflection coefficient obtained from

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \quad \text{- - - - - - - - - - - - - - - -} \quad (48)$$

The method proposed by Roman (1934) that uses the above principles, makes use of two families of curves in which the ratio ρ_s / ρ_1 is plotted against the ratio of h / a with k ranging in one family of curves from $+0.1$ to $+1.0$ by tenth's and in the other from -0.1 to -1.0 by tenth's. If both parameters, ρ_2 / ρ_1 and h / a, are plotted on logarithmic coordinate paper with the former the ordinate and the latter the abscissa, the curve shapes are invariant with changes in the values of the units in use. In other words, the parameters of either family may be multiplied or divided by any chosen factor and the result is that every point on the curve on one or both scales is moved the same distance parallel to the axes. The family of curves for two-layers in which $\rho_1 < \rho_2$, k = positive, is shown at the bottom of figure 56; the family for two layers in which $\rho_1 > \rho_2$, k = negative, is shown at the top of figure 56. The index for determining the thickness h of the upper layer is at unity on the above figures, but may be positioned at any point from the relation $m(h/a)$ where m is any factor. For example, some prefer the index at 8 on the chart (at the left of the index on the above figure) thus making $8h/a$ the parameter; Perret (1949). The index for determining the resistivity value for ρ_1 when $k = +1$ is the horizontal line, or abscissa, at the bottom of figure 56, and when $k =$ negative, ρ_1 is determined by the horizontal line at the top of the figure.

Interpretation by logarithmic plotting.

The two families of curves described above provide a comparative standard for observed resistivity curves plotted to the same scale on logarithmic coordinate paper. Observations of apparent resistivity by the Wenner or Lee configuration, and other configurations, ρ_s, are plotted in the vertical against electrode separation in the horizontal. Comparison, or fitting as it is usually termed, may be done by overlaying the observed resistivity curve on the proper family of theoretical two-layer curves. The depth index will determine the position of h with respect to a, in other words, the thickness of the top layer; and the horizontal line, resistivity index, top or bottom depending on whether $\rho_1 > \rho_2$ or $\rho_1 < \rho_2$, will give the resistivity of the top layer. The resistivity of the second layer ρ_2 may be determined from the value of the theoretical k-curve that is selected as the best fit for the observed curve from the relation given in equation (48). If the observed curve is a true two-layer one fitting will be readily done, the accuracy depending on the skill and experience of the operator. Observed resistivity curves of the multi-layer type may be closely fitted if $h_2 \gg h_1$.

Interpretation by linear plotting. The interpretation of two-layer curves was given in linear coordinate form by Roman (1941). In this paper, the logarithmic forms of interpretation given in previous articles (Roman 1931, 1934) have been changed to a form such that resistivity curves plotted on linear coordinate paper can be interpreted by charts and tables. The method is much longer and requires more computations than the logarithmic; the reader is referred to the original paper for details of the method.

Interpretation by Tagg's method. A method of interpreting two-layer resistivity curves that is also based on equation (47) has been described by Tagg (1934). Families of k-curves have been plotted on linear coordinate paper and with parameters ρ_s / ρ_1 plotted against h / a. The resistivity of the surface layer ρ_1 is estimated by averaging the resistivities obtained at the smaller interval distances. A set of values for ρ_s / ρ_1 at several values of a, the interval distance, are then computed. The theoretical curves are then used to obtain a set of h / a values for each corresponding ρ_s / ρ_1 value for the various k-curves, the proper family having been selected, the selection depending on whether $\rho_1 < \rho_2$, curves ascending, or $\rho_1 > \rho_2$, curves descending. As a is known for each case, then h could be determined from the h / a values of the curves, and a series of curves h plotted against k for each value of a. These curves of h vs. k should then intersect in a point which is the depth to the $h_1 - h_2$ interface. Because of the variability in the choice of ρ_1 as well as the anisotropy and inhomogeneity, the solution is seldom unique, and the depth h is chosen as the place where more of the h vs. k intersect or the central point of an area enclosed by the intersecting curves.

A revision of the above method was given by Tagg (1940) which applied to both two- and three-layer curves. The revision is based on the equation

$$\frac{\rho_{ns}}{\rho_s} = \frac{1+4F(na)}{1+4F(a)} \quad \text{- - - - - - - - - - - - -} \quad (49)$$

in which each member is identical to equation (47) except that multiples of a, designated n, are included and the two expressions divided. This relationship eliminates need for any knowledge of the upper layer of resistivity ρ_1. Families of curves are plotted for each value of n and includes curves for various values of k in which ρ_{ns} / ρ_s is plotted against h / a. From these curves, values of ρ_{ns} / ρ_s are selected for several values of a for several n's between 1.1 and 3.0 and h / a vs. k curves plotted for each value of either a or n. The intersection of these curves, or the greatest density of intersection, gives the depth to the h_2 interface.

<u>Interpretation by Pirson's method</u>. By means of successive approximations, Pirson (1934) applied the Tagg method of interpretation to three-layer resistivity curves. ρ_1, ρ_2, k_1, and h_1 are determined by a modified Tagg procedure. The depth to the third layer, h_2, is then determined by Lancaster-Jones (1930) method,

$$h_1 + h_2 = \frac{2d}{3} \qquad (50)$$

in which d is the electrode spacing a at which the inflection point occurs for the two lower maximum curvature points. The equivalent resistivity, ρ_1', for the two upper layers is then computed by equation (46 a). Taggs' method is then applied to the lower part of the curve and using the new value of ρ_1' several values of ρ_s/ρ_1 are determined near the inflection point. This gives a new value for the depth to the third layer, h_2'. From the reflection factor k_2, the resistivity of the third layer can then be computed. If the accuracy of $h_1 + h_2$ obtained is not considered of sufficient accuracy, then $h_1 + h_2$ may be used instead of $2\,d\,/\,3$ in calculating the resistivities of the two top layers. A new value for the equivalent resistivity of the two top layers is obtained ρ_1'', and a more accurate determination of $h_1 + h_2''$ is obtained. The method has limitations similar to those of Tagg along with the difficulty of choosing the inflection point correctly. Too, the value of h_2 obtained in practice is hardly altered by the new value obtained by successive approximations so that the error made in selecting the inflection point is inadequately corrected.

Three-layer resistivity curves.

All the described methods which extend two-layer methods to threelayer problems are based on an important contribution by Hummel (1931). Here he showed that in problems involving three layers, for large spacing of the electrodes, the two top layers could be combined to give a composite resistivity. Thus all three-layer problems can be reduced to two-layer problems. It was also pointed out by Hummel that three-layer resistivity curves may be represented by two-layer approximation curves. In a similar manner for four-layer curves, after three layers have been combined into a composite layer, the problem is reduced to a two-layer problem. Likewise, the four-layer curve may be represented by three two-layer approximation curves.

Extension methods of interpretation. Some three-layer curve types are more favorable to interpretation by the extension methods than others, Watson and Johnson (1938). When the second layer is a good conductor in comparison to the other layers, $h_1 + h_2$ is more readily determined and an electrode spread of two or three times the depth to the lower interface is adequate to get the help curve to track the observed curve. When the second layer is a good insulator in comparison to the other layers, $h_1 + h_2$ is obtained with difficulty and electrode spreads of inordinate dimensions are required for the help curve to approach the observed curve. The relation of the thicknesses of the two upper layers is a factor also in the favorability to interpretation by this procedure. When $h_1 \gg h_2$ a good determination for h_1 is unlikely; with $h_2 \gg h_1$ a good determination of h_2 increases in possibility as the thickness of the layer increases, and becomes increasingly better if ρ_2 is very much greater or very much less than ρ_1. However, the advantage gained by the large ratio of resistivity may be offset by the larger electrode spread required to make a good interpretation of the depths h_1 and h_2.

Four-layer resistivity curves.

A similar situation is present for the analysis of four-layer curves by extension methods. Watson and Johnson (1938) give a theoretical four-layer curve and represent it by three equivalent two-layer curves. There is, unfortunately, a graphical error in the presentation of the I''' help curve which would have been readily noticed had the theoretical curve been extended farther. When this error is corrected, as shown in figure 57, the help curve I''' approaches coincidence at very large electrode intervals with the theoretical curve. The above example, however, forcefully indicates some of the difficulties encountered in the interpretation of four layer curves by extension methods.

The theoretical four-layer curves of Mooney and Wetzel (1955) will be of much value and assistance in the interpretation of surface measurements of apparent resistivity. These curves may be used either by curve-matching methods as the authors suggest, or they may be treated in a manner similar to that proposed by Watson and Johnson (1938) using extension methods, the latter being preferable but requiring much more preparation before they can be used.

METHODS OF INTERPRETATION BASED ON POTENTIAL THEORY

The observed distribution of potential on the surface of the earth when an electrical field is set up in the earth by passing current into it through two hemispherical or cylindrical electrodes, forms the basis for the investigation of materials beneath the surface of the earth. The solution by potential theory for the simplest case of two layers is given by Gilchrist (1931) and in a more recent treatment of this case by Smythe (1950, pp 241-243); it is shown how the formula obtained by potential distribution may be converted to the same formula as obtained by image theory, Stefanesco and C. and M. Schlumberger (1930). The solutions for a number of layers in terms of Bessel functions is also given by Stefanesco and Schlumberger and by Ollendorf (1928). With the assumption that the earth consists of horizontal layers of different thickness and conductivity, Peters and Bardeen (1930) gave a direct solution of Laplace's equation by means of Bessel functions. Muskat (1933) developed a group of special formulas appropriate for computations at small and at large distances from the electrode by his study of the potential distribution on a multi-layered earth. A study similar to that of Muskat was made by King (1933). All the above references discuss a layered earth, but a study of the problem of a continuously varying conductivity was made by Slichter (1933), who also indicated the solutions for a limited number of other cases.

Direct method of Tagg, Slichter-Langer, and modification of Pekeris.

One of the early attempts at a direct solution was made by Tagg (1934) who, by means of a set of two-layer curves, showed that the depth to the second layer, as well as the resistivity of the second layer, could be determined by them from a knowledge of the potential measured at two different distances from the electrode. The formal mathematical solution of the direct problem was given by Slichter (1933) and Langer (1933, 1936) in which only a knowledge of the surface potentials is required to determine uniquely the conductivity variations with depth. The Slichter-Langer method, however, is likely to be limited in its usage as it requires that the observational data be represented as a reciprocal power series, in fact, Stevenson (1934) has shown that for certain cases failure is certain because the observed data cannot be represented by an expansion of the form postulated. By a modification of Slichter's method, that of assuming a layered earth, Pekeris (1940) developed a graphical method for determining the depths to the several layers and their conductivities which eliminated the representation of the observations by a power series. One modification has been made of the Pekeris method of interpretation but the results are not published; Mooney (1954). One of the problems in the use of the direct method of interpretation is that of transforming the field measurements of potential into kernel

values. A method has been derived by Rogers and Mooney (~~1955~~) (personal communicat
and by Schwendinger (personal communication) (~~1955~~) which will be of assistance in this respect. Keck and Colby (1942) solved the problem of determining the conductivity variation with depth by means of a perturbation method thus allowing the unperturbed function to be arbitrary.

Methods for computing kernel function.

All of the direct methods of interpreting resistivity curves require either rather laborious hand computing or the application of electronic or high speed computers to arrive at the solutions. Two methods are available for computing the kernel function; that of Slichter (1933) and the very neat method of Sunde (1949). In terms of Wetzel and McMurry (1937), the kernel of Slichter $k(\;)$

$$k(\lambda) = 1 + 2B_1(\lambda) \quad - - - - - - - - - - \quad (51a)$$

and

$$B_1(\lambda) = \frac{k_1 e^{-2\lambda h_1} + k_2 e^{-2\lambda h_2}}{1 - k_1 e^{-2\lambda h_1} - k_2 e^{-2\lambda h_2} + k_1 k_2 e^{-2\lambda (h_2 - h_1)}} \quad - - \quad (51b)$$

for three layers, in which

$$k_1 = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \; ; \quad k_2 = \frac{\rho_3 - \rho_2}{\rho_3 + \rho_2} \; ; \; h_1 , \; h_2$$

equals the distance of the layers from the surface; and λ assumes values from 0 to ∞.

Sunde derives the kernel as:

$$k_{123} = (1 - \mu_{123} e^{-2\lambda d_1}) / (1 + \mu_{123} e^{-2\lambda d_1}) \quad - - - - - \quad (51c)$$

in which $\mu_{123} = \dfrac{\rho_1 - \rho_2 k_{23}}{\rho_1 + \rho_2 k_{23}}$ - - - - - - - - - - - - - - - (51d)

and $$k_{23} = (1 - \mu_{23} e^{-2\lambda d_2}) / (1 + \mu_{23} e^{-2\lambda d_2}) \quad - - - - - \quad (51e)$$

and $$\mu_{23} = (\rho_2 - \rho_3) / (\rho_2 + \rho_3) \quad - - - - - - - - \quad (51f)$$

in which $\rho_1, \rho_2, \rho_3,$ equals resistivities of the layers,

d_1 , d_2 equals the thicknesses of the layers,

and λ varies from 0 to ∞.

An example of the computation of the kernel by each method will be given in the section on application of the methods of interpretation. There will also be included the method of reversing the solution of the kernel with an example, i.e. finding the resistivit~~y~~ies and thicknesses of the layers when only the kernel function $k_{1---n}(\lambda)$ is given.

The theoretical relation of image sources to the depth factors was investigated by Evjen (1938) as another direct method of interpreting apparent resistivity curves.

APPLICATION OF THE METHODS OF INTERPRETATION TO APPARENT RESISTIVITY CURVES OBTAINED BY ELECTRICAL DEPTH PROFILING

The theoretical and mathematical methods ~~f~~ of interpreting resistivity curves offer the nearest approach to complete objectivity and reduction of the personal equation in the analysis of surface potential measurements. Although the direct method appears to be the one which can achieve complete objectivity in the interpretation of surface potential measurements using single probe techniques, it has not yet been completely demonstrated that this worthwhile goal can actually be attained. Difficulties are certain to arise when the surface potential measurements are not closely similar to those of a theoretical case.

Interpretation by two-layer equivalents.

The analysis of the apparent resistivity curve by resolving it into its two-layer equivalents with the aid of two-, three-, and four-layer theoretical curves is a reasonably objective technique. The skill in using this method, though, will depend mainly upon the background of information and experience of the one making the interpretations.

Two-layer resistivity curves may be interpreted by the theoretical two-layer curves of Roman (1934) and personal communication. An example showing how the method is used with linear coordinates is also given by Roman (1941). Jones ~~(1936)~~ (personal communication), Wetzel and McMurry (1937), Watson and Johnson (1938), and Roman (loc. cit.) pointed out that by a slight modification the two-layer theoretical curves could be made dimensionless by plotting them on logarithmic paper. By plotting the observed curves of apparent resistivity on the same kind of paper, they may be placed over a set of the two layer curves and a fit made for a particular value of k, and the indices of the theoretical curve gives the values of ρ_1, and h_1, the resistivity and thickness of the upper layer. By applying the relation given in equation (48) or the transposed value

$$\rho_1 = \rho_2\left(\frac{1-k_1}{1+k_1}\right),$$

the resistivity may be computed for the lower layer.

Example of two-layer curve interpretation using logarithmic plotting. An example of interpretation using logarithmic plotting is shown in figure 58. The apparent resistivity curve is a two-layer curve plotted from computed observations taken at Marshfield, Wisconsin by Spicer (1953, curve 12) on the glacial deposits of the area. The computed resistivities in ohm-cms are plotted against the electrode separation in feet on K and E No. 336-J 2 x 2 cycle logarithmic paper. This particular paper is used because it is thin, strong, retains its size well, and the color is non-confusing over a light table. The theoretical two-layer curves are plotted on the same kind of paper. The observed curve of figure 58 is fitted to the theoretical curve $k = +0.98$, then h_1 is read at 6.1 feet, ρ_1 at 2,620 ohm cms, and ρ_2 is computed to be 259,300 ohm cms. The geological materials are interpreted to be :

clay with sand and gravel	0 - 6.1 feet
granite	6.1 ->130 feet.

The interpretations for the P-1 and P-2 curves are the same as for the full curve. If $\rho_2 > \rho_1$, the other group of theoretical two-layer curves would apply in the interpretation of the curve computed from the observations. The writer has never obtained a two-layer field curve of this type and, consequently, offers no example. The interpretation for this type of curve is, however, similar to the one just given.

Previous work on extension of two-layer methods to multi-layer problems. It was previously mentioned that Watson and Johnson (1938) pointed out how the two-layer methods of interpretation could be extended to three and more layers through the contribution of Hummel (1931) and the two-layer theoretical curves of Roman (loc. cit.). Another important contribution to the solution of the three layer problem were the sets of three-layer curves of Wetzel and McMurry (loc. cit) and Watson (1934). The two-layer case has been solved completely, but the three-layer case has been only partially completed according to Roman (1933). With the guidance indicated above, nearly any three-layer curve may be interpreted with reasonable assurance that the results are within an accuracy of ten percent.

The effect of varying the resistivity of the middle layer in rho a family of three-layer theoretical curves $\rho_1 : \rho_2 : \rho_3 = 1 : n : 1$ and $h_1 : d = 3 : 1$ is shown by the relationship in figure 59 where n equals 100, 10, 3, 1, 1/3, 1/10, and 1/100. A similar relation is shown by Watson (1934, figures 9 to 15). It is clearly shown by this family of curves that the inflections of the curves have no simple relation to the depth of the third layer d indicated at 8 on the abscissa in the figure.

The relationship of the family of curves $\rho_1 : \rho_2 : \rho_3 = 1 : 3 : n$ and $h_1 : d = 1 : 3$ where n = 100, 10, 3, 1, 1/3, 1/10, and 1/100 is shown in figure 60. The figure also illustrates the effect on the character of the apparent resistivity curve of the variation in the resistivity of the third layer ρ_3. It is also particularly prominent on the figure that the value of h_2, indicated at 8 on the abscissa, bears a very complex relation to the inflections of the curves. The curve 1 : 3 : 3 is a two-layer curve but fits correctly into the family of three-layer curves as it should.

The behavior of the three-layer resistivity curves in which the inflections are a maximum are difficult to interpret correctly by approximation with two-layer curves. The reasons for this are illustrated in figure 61. The curve marked 2 : 2, which is the ratio of h_1 / d, has the following resistivities: $\rho_1 : \rho_2 : \rho_3 = 1 : 10 : 1/3$. By applying Kirchhoff's law to the two upper layers the curve indicated as Asy. is obtained. The curve indicated as 0 : 4 represents the two-layer case with $\rho_1 = 10$, $\rho_2 = 1/3$, and $h_1 = 8$. The curve indicated as 4 : 0 represents the two-layer case with $\rho_1 = 1$, $\rho_2 = 1/3$, and $h_1 = 8$. The curve marked 2 : 2 will approach the curve marked Asy. at a very large value of the abscissa beyond the diagram. An attempt then to fit this curve with a two-layer curve will result in a very large error for the depth to h, in fact, h_1 will be about four times larger than it should be. By allowing either h_1 or d to approach zero, the mathematical expression for the three-layer case will reduce to the expression for a two-layer case. Allowing h_1 to vanish the curve $0 : 4$ is approached; allowing d to vanish the curve $4 : 0$ is obtained. It will be noted from the figure that these limiting two-layer cases are not envelopes of the three-layer curves. This is an unfortunate fact and, as shown on the above figure, members of the multiple-layer curve families may cross the limiting curves, thereby adding to the difficulties of extrapolating curve families.

This method of matching curves computed from field measurements with a set of theoretical curves is theoretically capable of yielding a unique answer for the resistivity function. A proof of the uniqueness theorem implicitly involved in the procedure has been given by Slichter (1933).

<u>Preparation of Wetzel - McMurry curves for use in interpretation.</u> In order to increase the scope and usefulness, as well as to provide an understanding, of the three-layer curves of Wetzel and McMurry, all thirty four families of curves should be plotted and resolved into their two-layer equivalents and the limiting curves included on the figures. Such has been done for the family $\rho_1 : \rho_2 : \rho_3 = 1 : 1/10 : 10$ and for the family $\rho_1 : \rho_2 : \rho_3 = 1 : 3 : 1/3$ as examples and are shown in figures 62 and 63 respectively.

The values of resistivity for the several layers, thicknesses of the layers, and reflection factors may be easily obtained as the curves are resolved. These values for the three h_1 / d values, 1/3, 2/2, and 3/1, are given on each figure as are the formulas by which they were computed.

Another extension of the usefulness of the three-layer Wetzel and McMurry curves may also be made with a few additional computations. Reference to figures 62 and 63 shows that the value of k_1 is the same for all ratios of h_1 / d . Using this principle, other ratios of h_1 / d may be conveniently selected such as; 1/2, 1/5, 1/7, 1/9, 1/12, 1/15, etc. Then the computations are carried out to determine the values for the resistivity, thickness and reflection factors, then the two-layer help curves are sketched on the chart of the family of curves. The family of curves $\rho_1 : \rho_2 : \rho_3 = 1 : 1/10 : 3$ are shown on figure 64 with the help curves sketched in. Shown also on the above figure is a sketched in three-layer curve for the ratio 1/7. This three-layer curve was drawn in using french curves by following the symmetry pattern of the other curves in the family. It is recognized that the drawn in curve can not be used accurately for curve matching practices, but it does serve as a valuable guide in the curve analysis procedure described. The curve is probably considerably less than 20 percent in error near the minimum inflection, the place where less control is available, and near zero error in the parts of the curve adjacent to the minimum inflection. If desired, several points or the whole curve could be computed to obtain the correct position of the curve. In a similar manner, three layer curves may be drawn for the other h_1/d ratios.

It may be easily demonstrated by the above analysis that all the help curves for this family of curves lie between the limiting curves $h_1/d = 4/0$ and $h_1/d = 0/4$. This is done by allowing the middle layer to approach zero thickness in the first instance, and the upper layer to approach zero thickness in the second. The same situation for the help curves prevails for the cases where the family of three-layer curves have minimum inflections.

The behavior of the three-layer curve for the case of the maximum inflection is not as nice as that for the minimum inflection. The three-layer curve for this case, in nearly every instance, crosses the limiting curve h_1/d, $h_1 = 0$, in an irregular manner, such that no extension three-layer curve may be drawn in for the ratios of h_1/d, $d > 3$ as given, with any reasonable degree of accuracy. This is not a major difficulty for the curve analysis procedure, but it must be recognized by the interpreter and its disadvantage overcome. This can only be done by familiarity with the theoretical curve, the computation of the three-layer curve for the h_1/d ratio, or, preferably, both.

Examples of three-layer curve interpretation using extension methods. The interpretations of several three-layer theoretical curves are given by Watson and Johnson (1938) using the extension methods, and they also show their scheme of overlaying the computed curve on the two-layer theoretical curves in order to obtain the resistivity and thickness values by means of the equivalent two-layer help curves. One interpretation of a three-layer resistivity curve is given by Wetzel and McMurry (1937) showing how a computed resistivity curve may be fitted or matched to a theoretical curve.

Example of observed three-layer curve interpretation using curve analysis method. A three-layer resistivity curve computed from observations will now be interpreted using a combination of Hummel, Watson and Johnson, Roman, and Wetzel and McMurry which has been previously mentioned as the curve analysis technique. The observational curve is number 91 from Spicer (1954) and is shown here as figure 65. The dashed line drawn through the computed points is obtained by interpolation from the Wetzel and McMurry curves $\rho_1 : \rho_2 : \rho_3 = 1 : 1/10 : 10$ and $1 : 1/10 : 3$, $h_1/d = 2/1$, then sketching in the curve by overlaying the observed curve on the three-layer theoretical curves over a light table such as the portable one shown in figure 66. Mark the positions of ρ_1, h_1, and $h_2 = h_1 + d$, values from the theoretical curves for $h_1/d = 2/1$ on the observational curve. They are respectively 8,220 ohm cms, 27 feet, and 40 feet. This is only a guide curve and accuracy in drawing is not important. However, it is essential that the two-layer curves of which the three-layer guide curve is composed be more accurately drawn. The procedure for doing this follows.

On a light table, overlay the observational curve on the two-layer theoretical curves, figure 67, so that the ρ_1 value 8,220 ohm cms corresponds with the resistivity index of the $\rho_1 > \rho_2$ theoretical curves and the h_1 value 27 corresponds with the depth index of the theoretical curves. This may be marked with a a character, such as an inverted perpendicular mark, to distinguish the fact that it indicates both of the above values. Since the resistivity ratio is 1 : 10 the k - value for the theoretical curve of the two-layers will be (1 - 10) / (1+10) = - 0.8182. Edwards (~~1950~~) (personal communication) prepared a table of these values by arguments of 0.01 for all positive and negative k - values and listed them on a card for ready reference when interpreting resistivity curves. Such a list is given in the Appendix. Next sketch in with a dotted line the k-curve = - 0. 82 which is equivalent to the help curve. Compute the ρ_2 from the equation $\rho_1 = \rho_2(1 - k)/(1 + k)$;

813? ρ_2 = 831 ohm cms. Determine the resistivity for the composite layer ρ_1 and ρ_2 using the h_2 = 40 feet in the formula

$$\frac{h_1 + d}{\rho_1''} = \frac{h_1}{\rho_1} + \frac{d}{\rho_2} = 2,112 \text{ ohm cms.}$$

Again overlay the observational curve on the two-layer $\rho_1 > \rho_2$ theoretical curves with the 40 ft line superimposed on the depth index and 2,112 ohm cms superimposed on the resistivity index line. Mark this point with a $\perp$ character to indicate both of the above values.

Sketch in the k-curve = + 0.90 as the one best fitting the computed observational points. Compute the ρ_3 value from the equation

$$\rho_1'' = \rho_3\left(\frac{1-k}{1+k}\right) ; \quad \rho_3 = 40,140 \text{ ohm cms.}$$

It is noted that the value of ρ_3 is only about one-half of what it should be for the ratio $\rho_1 : \rho_2 : \rho_3$ = 1 : 1/10 : 10 and that it is nearly twice what it should be for the ratio 1 : 1/10 : 3.

The ratio which gives the neatest fit is near $\rho_1 : \rho_2 : \rho_3 =$ $1 : 1/10 : 5$, $h_1/d = 2/1$.

It will be evident from the above figures that the P-1 and P-2 curves are almost identical to the Full curve, in fact, both would yield the same interpretation as the Full curve. Therefore it is not necessary to make a complete analysis of these curves. Thus, knowing the interpretations would be the same for the three apparent resistivity curves, it is known that the bedrock is flat within the electrode interval, 167½ feet either side of center. Furthermore, it may be assumed that there is little change in the depth to bedrock throughout the entire electrode distance because of the nearly exact similitude of the three apparent resistivity curves.

The geological materials at this depth profile as determined by the interpretations are:

soil, cobbles, clay	0 - 27 feet
clay	27 - 40 feet
granite	40 - >325 feet.

Example of four-layer curve interpretation using extension methods. A four-layer theoretical resistivity curve was interpreted by Watson and Johnson (1938, p. 20) using the help curves for two-layers. Their figure 10 for this curve has the help curve I''' incorrectly drawn and the correct presentation is given in figures 57 and 68, this paper. This four-layer curve was selected for discussion because it so nicely shows the principles, and difficulties, in the interpretation of multi-layer resistivity curves by this method.

The four-layer resistivity curve is curve I of figure 68 where $\rho_1 : \rho_2 : \rho_3 : \rho_4 = 1 : 9 : 1/3 : \infty$ and $h_1 : h_2 : h_3 : h_4 = 2 : 6 : 16 : \infty$. There is no Wetzel and McMurry three-layer curve which exactly fits the upper three layers, but their curve $\rho_1 : \rho_2 : \rho_3 = 1 : 10 : 1/3$, $h_1 / d = 1 / 3$, may be interpolated to fit adequately the upper three layers so that the general appearance of such a curve may be seen. This is curve II of the above figure. The two-layer curve for the h_1 and h_2 layers, where h_2 is assumed to be infinite in depth, is $k = +0.80$ and is curve I' in the figure. The two-layer curve for the combined h_1 and h_2 layers and the h_3 layer, where h_3 is assumed to be infinite in depth, is $k = -0.80$ and is curve I'' in the figure. The three-layer curve formed by the combination of the h_1 and h_2 layers, h_3, and h_4 is $\rho_1 : \rho_2 : \rho_3 = 1 : 1/9 : \infty$, $h_1/d = 1/2$. This is very close in value to the Wetzel and McMurry three-layer curve

$\rho_1 : \rho_2 : \rho_3 = 1 : 1/10 : 100$, $h_1/d = 1/2$, which may be obtained by interpolation, and shows the general appearance of the three-layer curve which would fit curve I if the h_1 and h_2 layers were combined. This is curve III of the above figure. The other two-layer help curve possible is obtained by combining h_1, h_2, and h_3 as the upper layer and h_4 as the lower layer where $h_4 = \infty$, $k = +1.0$, and this is curve I''' in the above figure.

For a more accurate representation of the three-layer curves used in the interpretation above, the curves $\rho_1 : \rho_2 : \rho_3 = 1 : 9 : 1/3$, $h_1/d = 1/3$, and $\rho_1 : \rho_2 : \rho_3 = 1 : 1/9 : \infty$, $h_1/d = 1/2$ could be computed. The increase in accuracy of the interpretations gained by using the computed three-layer curves instead of the interpolated curves will depend mainly upon the experience and judgement of the one making the interpretations.

Example of observed four-layer curve interpretation using curve analysis method. Application of the curve analysis methods will now be made to a four-layer resistivity curve obtained by field measuremets. The curve selected is number 89 from Spicer (195~~3~~4) and is reproduced here as figure 69. It is similar in many respects to the theoretical four-layer curve previously interpreted and described.

By comparison with the three-layer theoretical curves, the resistivity ratio for the two upper layers is easily chosen, $\rho_1 : \rho_2 = 1 : 3$, but the resistivity ratios for the lower layers are not so easily selected, especially by inspection. The third layer is strongly influenced by the high resistivity bottom layer and some other method than a three-layer curve must be used to choose the resistivity for the third layer. This selection is made by comparing the computed resistivities for the third layer on all depth profiles in the near vicinity, then choosing a mean, median, or average value for the resistivity. In this instance, 1,800 ohm cms was selected as representative which value makes the ratio $\rho_1 : \rho_2 : \rho_3 = 1 : 3 : 1/2$ for $h_1/d = 1/2$.

Using a light table overlay the computed resistivity curve on the three-layer theoretical curves $\rho_1 : \rho_2 : \rho_3 = 1 : 3 : 1/3$ for $h_1/d = 1/2$ and sketch in the three-layer curve for $\rho_1 : \rho_2 : \rho_3 = 1 : 3 : 1/3$ by comparison with the three-layer theoretical curve $\rho_1 : \rho_2 : \rho_3 = 1 : 3 : 1$, $h_1/d = 2/2$. The sketched-in curve

$\rho_1 : \rho_2 : \rho_3 =$ 1 : 3 : 1/2 , h_1 / d = 1/2 on figure 69 is marked I. Mark the depths to the second and third layers at the 26.7 and 80 lines of the theoretical three-layer curve. This will be at 6.7 and 20 feet on the observed curve. Mark ρ_1 at 3,720 ohm cms. In the next step, overlay the observed curve on the two-layer theoretical curves with $\rho_1 < \rho_2$ using the light table. Place the curve so that the depth index is at 6.7 and the resistivity index is at 3,720 and sketch in the curve $k_1 = + 0.5$ being sure to mark h_1 and ρ_1 in some manner, such as a $\perp$ mark. Compute ρ_2 from the equation $\rho_1 = \rho_2(1-k)/(1+k)$ giving $\rho_2 =$ 11,160 ohm cms. Using the depth to the interface between ρ_1 and ρ_2 = 20 feet, determine the composite resistivity of the two layers from the equation

$$\frac{h_1 + d}{\rho_1''} = \frac{h_1}{\rho_1} + \frac{d}{\rho_2},$$

giving $\rho_1'' = 6,488$ ohm cms. With the observed curve overlain on the $\rho_1 > \rho_2$ two-layer theoretical curves, and using the light table, place 6,488 on the resistivity index and 20 on the depth index and then sketch in curve $k_2 = -0.62$. Compute ρ_3 from the equation

$$\rho_1'' = \rho_3(1-k)/(1+k),$$

giving 1,814 ohm cms. The resistivities ρ_1 = 6,488 and $\rho_3 =$ 1,814 determine the three- layer curve to use for the $\rho_1'' : \rho_3 : \rho_4$ part of the computed curve, namely $\rho_1'' : \rho_3$ = 3.6 : 1 . The family 3 : 1 : > 1 is nearest this ratio. The ratio of ρ_4 is now chosen from comparative resistivity values of this layer in depth profiles nearby. The ρ_4 layer has a mean resistivity of 200,000 ohm cms giving the ratio $\rho_1'' : \rho_3 : \rho_4 = 3.6 : 1 : 105$. The three-layer curve approximating this value may be interpolated from the families of curves $\rho_1 : \rho_2 : \rho_3 = 1 : 1/3 : 10$ and $\rho_1 : \rho_2 : \rho_3 = 1 : 1/3 : 100$. The curves with the h_1/d values near 1/3 are the best fit. Mark the depth for the $h_2 - h_3$ interface at 63 feet corresponding to the 8 index on the theoretical three-layer curves. Again using the light table overlay the observed curve on the $\rho_1 < \rho_2$ theoretical curves and place 63 on the depth axis. Compute the composite resistivity for the three top layers from the Kirchhof's law as shown by Hummel, giving the value 2,011. Assuming that the observed curve is still positioned on the depth axis, move it to the value 2,011 on the resistivity index. k_3 may be determined by computation or inspection and gives the value $k_3 = + 0.98$, and this k-curve may now be sketched on the chart. Compute ρ_3 obtaining the value 199,200 ohm cms.

Interpreting the above resistivity curve determinations in terms of the geological materials gives;

clay soil, sand	0 - 6.7 feet
gravel, sand, clay	6.7 - 20 feet
clay	20 - 63 feet
granite	63 -> 325 feet.

Interpretation of observed multi-layered curves using curve analysis methods. Apparent resistivity curves computed from observational data of more than four layers may be interpreted also by the curve analysis procedures previously described for three and four layer conditions. An excellent example of a five-layer curve interpreted by this method is given by Spicer (1953), number 18E at Marshfield, Wisconsin and reproduced here as figure 70. This resistivity curve was selected from the same source as the curve previously interpreted and was the one indicating the best place to drill a well for a city water supply. Subsequent drilling at the center of the depth profile proved the interpretations of the resistivity curves, but it will be evident from the comparison of the drill log and the resistivity interpretations on the above figure that there is some variance. This variance is not surprising or different, it more frequently happens that such a condition prevails because the electrical horizons are not always the lithological ones. The hardpan reported by the driller appears to be the same as clay on the apparent resistivity curve. The sand and gravel differentiation given by the drill log makes no appearance on the apparent resistivity curve, but the change from dry to saturated sand and gravel is very evident. The bedrock interpretation is the same by both drilling and resistivity. The complete interpretation of the computed apparent resistivity curve may be readily performed by those interested, using the curve analysis methods previously given.

For the interpretations of curves having more complex layering, such will be found in the papers by Edwards (1951) and Spicer (1950, 1952).

Interpretation of observed resistivity curves by the method of Ebert.

The Wetzel and McMurry curves (loc. cit.) may be placed in four groups as shown by the type curves of figure 71. Curves of type 1 are quite easily interpreted by the use of Roman two-layer curves and the help-curves of Hummel. The three other types give very erratic interpretations for the depth to the deep layer if one is not familiar with the method. This is because the help-curves do not follow the curve being interpreted but are displaced to one side or the other, the amount of displacement being dependent upon the resistivities and thicknesses of the two upper layers. A method for overcoming this difficulty was presented by Ebert (1942) which will be summarized using the terminology of Ebert.

HS-1 curves of Ebert are two-layer curves of $\mu > 1$ and $\mu < 1$, figures 4 and 5 of Ebert, and are the same as the Roman two-layer curves except that they extend to their asymptotes. However, Roman (1954) has unpublished curves comparable to these. The m-1 line is the depth axis, the $\mu = \rho_2/\rho_1$ ratio value 1 is the resistivity of the upper layer, and their intersection is LK, the "left cross".

Application to Type 1 curves. Type 1 curves of figure 71 may be interpreted with the aid of the HK curves, figure 72, which are the same as those of Ebert figure 8. The abscissa values ξ_H are the fictive resistivity for the combined ρ_{1+2} layer, and the ordinate values $\Lambda_H = m_1 + m_2$ are the depths to the third or lower layer.

The HK curves are calculated by methods pointed out by Hummel (loc. cit.),

$$\rho_{1+2} = \xi_H = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}} \quad \text{- - - - - - - - - - - - - - -} \quad (52)$$

and

$$\Lambda_H = m_1 + m_2 \quad \text{- - - - - - - - - - - - - - -} \quad (53)$$

where m_1 , m_2 $\overset{(\equiv)}{\equiv}$ h_1 , d of Wetzel and McMurry. Points on the μ —curves are obtained by choosing a ratio for μ , for example 1/3, and solving for a series of ratios for $\nu = m_2 / m_1$, such as 1/1, 1/2, 1/3, 1/5, ----, 1/99.

<u>Interpretation of a Type 1 three-layer curve</u>. The theoretical curve $\rho_1 : \rho_2 : \rho_3 = 1 : 1/3 : 3$, $h_1 : d = 1 : 3 = m_1 : m_2$ was selected from Wetzel and McMurry (loc. cit.) three-layer curves for interpretation, and is shown completely interpreted in figure 73. The detailed interpretation procedure is as follows. It is assumed that all curves are plotted on the same kind of logarithmic paper. Overlay the curve to be interpreted on the two-layer theoretical curves, $\mu < 1$ or $\rho_1 > \rho_2$, and sketch in the curve which matches the left part of the curve, $k_1 = -0.5$ in this example. Also indicate the resistivity ρ_1 and depth ~~and depth~~ $m_1 = h_1$ with a symbol such as T . ρ_2 may be computed from either the μ or k value. The coordinates ρ_1, m_1 locate point T the "left cross" of

Ebert. Next, overlay the curve being interpreted on the HK-curves, figure 72, with the point (ρ_1, m_1) at the origin (1, 1) and sketch on the μ-curve ρ_2 / ρ_1 = 1 / 3. Again place the curve over the two-layer curves this time over the $\rho_2 > \rho_1$ set and in such manner that the μ-curve lies over the "left cross", or intersection of the depth axis and the ρ_1 axis. Keeping the axes both parallel, shift the figure along the μ-curve until the right portion of the curve covers a theoretical curve, or one interpolated logarithmically, on the two-layer chart. Sketch in the two-layer curve covered, indicate the k_2 value and mark the depth to the third layer, $m_1 + m_2$ at the origin of the HK curves; this curve will be $k_2 \doteq + 0.765$. Compute ρ_3 from the fictive resistivity, and value of k_2.

Application to Type 3 curves. The formulas for obtaining the curves to interpret Type 3 curves are obtained from the relations of anisotropy maintaining among the layers. The anisotropy coefficient

$$\lambda = \sqrt{\frac{\rho_t}{\rho_\ell}} \quad \text{- - - - - - - - - - - - - - - - - (54)}$$

in which

$$\rho_t = \frac{m_1 \rho_1 + m_2 \rho_2 + \cdots + m_n \rho_n}{m_1 + m_2 + \cdots + m_n} \quad \text{- - - - - - - (55)}$$

and

$$\rho_\ell = \frac{m_1 + m_2 + \cdots + m_n}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} + \cdots + \frac{m_n}{\rho_n}} \quad \text{.- - - - - - - (56)}$$

The coordinates for the anisotropic point are

$$\xi_A = \sqrt{\frac{m_1\rho_1 + m_2\rho_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}} \qquad (57)$$

and

$$\Lambda = \sqrt{(m_1\rho_1 + m_2\rho_2)\left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}\right)} \qquad (58)$$

The AK diagram, figure 10 Ebert, is computed from the just cited formulas. The fictive resistivity, ξ_A, and fictive thickness, Λ_A, are computed for both the μ-curves and the ν-curves. To obtain the μ-curves, the ratio ρ_2 / ρ_1 is kept constant and the ratio m_2 / m_1 is varied. For example, ρ_2 / ρ_1 is taken as 3/1, $m_1 = 1$, and m_2 used at successive values such as 0, 1, 2, ---, 99. To obtain the ν-curves, the ratio m_2 / m_1 is kept constant and the ratio ρ_2 / ρ_1 is varied. For example, m_2 / m_1 is taken at 3/1, $\rho_1 = 1$, and ρ_2 used at successive values such as 1, 2, 5, ---, 1,000.

The computed AK curves are plotted on logarithmic paper and it will be noted that all are asymptotic to a 45° line through the origin, figure 74.

Interpretation of a Type 3 three-layer curve. A curve from the Wetzel and McMurry group was again selected for interpretation, $\rho_1 : \rho_2 : \rho_3 = 1 : 3 : 10$, $h_1/d = 1/3 = m_1 / m_2$, and this curve is shown completely interpreted in figure 75. Place the curve to be interpreted over a set of two-layer theoretical curves on a light table using the $\rho_1 < \rho_2$ series, and sketch in the help curve for the two upper layers. Indicate the location of the intersection of ρ_1 and h_1 with the adopted symbol. Next, overlay the curve being interpreted on the AK curves of figure, using the light table, and place the point (h_1 , ρ_1) over the origin. Sketch in the μ-curve, $\mu = \rho_2 / \rho_1 = 3$. The curve being interpreted is now placed over the two-layer theoretical curves in a manner so that the point (h_1, ρ_1) falls on the μ-curve, and the curve being interpreted is shifted along this curve, the axes being kept parallel, until the right portion of the curve covers a theoretical curve or a logarithmically interpreted curve. Sketch in this curve and indicate its value, $k = + 0.78$. Return the curve to the AK diagram and the intersection of the ν-curve, $m_2/ m_1 = 1/3$, with the abscissa gives the depth of the third layer. The anisotropic point of Ebert is to the right and below the Hummel point. Values of other constants not specifically mentioned are computed as in the first example.

It is possible, of course, to compute the values for the coordinates of the anisotropic point (ξ_A , Λ_A) from the formulas, but the depth to the third layer must be found in some way beforehand.

Application to Type 2 three-layer curves. The formulas for obtaining the curves to interpret Type 2 three-layer curves ars also obtained from the anisotropic relations present among the layers. The "displaced anisotropic point" (ξ_{VA} , Λ_{VA}) lies in the vertical as the anisotropic triangle is shifted to the right. The amount of the displacement is dependent on the anisotropic coefficient λ , and is represented by ξ . Because of this relation, $\xi_A \equiv \xi_{VA}$. The constant Λ_{VA} differs from Λ_A by the value of function ξ . The coordinates for the displaced anisotropic point are;

$$\xi_{VA} = \sqrt{\frac{m_1 \rho_1 + m_2 \rho_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}} \quad - - - - - - - - - - - - - (59)$$

and

$$\Lambda_{VA} = \sqrt{(m_1 \rho_1 + m_2 \rho_2 \left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} \right)} \; . - - - - - - - - - (60)$$

The VAK curves of Ebert figure 11, are obtained from these formulas. The fictive resistivity, ξ_{VA}, is the same as that of the AK curves for both the μ-curves and ν-curves. Before computations can be made to obtain Λ_{VA} for μ-curves, it is necessary to compute the series of λ-values to get the function ξ . This calculation is made for each set of values of m_1, m_2, ρ_1 , and ρ_2 that have been used in the computations for the $\Lambda_A$values; for example m_1, m_2, ρ_1 , ρ_2 = 1 ; 1.5 , 1,000 , 1,500, etc. For the final value, $\Lambda_{VA} = \lambda \Lambda_A$. The computed series of curves are plotted on logarithmic paper with ξ_{VA} as ordinate values and Δ_{VA} as abscissa values. The series of curves obtained are tangent to a function related to λ .

The values of the ξ function for the ν-curves, as used by Ebert in his figure 11, are different from the computed values, but he states that his figure is not exact and only gives a relative picture. The λ-values are computed as explained previously and applied to the Δ values so that $\Delta_{VA} = \xi \Delta_A$. The series of ν-curves obtained are plotted on the same sheet of logarithmic paper with the μ-curves. The ν-curves when plotted will be found to be a little different from those given by Ebert, particularly in the smaller values.

The VAK curves of figure 76 have been modified by a graphical procedure so that they are identical to those given by Ebert. The curves when plotted are tangent to the function related to λ .

<u>Interpretation of a Type 2 three-layer curve.</u> The theoretical curve $\rho_1 : \rho_2 : \rho_3 = 1 : 3 : 1/3$, $h_1/d = 1/3 = m_1/m_2$ was selected from the Wetzel and McMurry curves for interpretation using the VAK curves. The curve to be interpreted is placed on a light table over a set of two-layer curves of $\rho_1 < \rho_2$ in a manner such that the help curve $k = +0.5$, or $\mu = 3$, can be sketched. The values of $h_1 = m_1$ and ρ_1 are indicated by the adopted symbol at their intersection and the k-value recorded. The curve being interpreted is next overlain on the VAK curves, figure 76, with the point (m_1, ρ_1) over the origin. Determine the μ-value, $\mu = \rho_2/\rho_1$, equals 3/1 in this example, and sketch on the μ-curve (3). Return the curve being interpreted to the two-layer theoretical curves and place it over the $\rho_1 > \rho_2$ curves so that the intersection of the depth axis m_1 and the ρ_1 axis lies in coincidence with the μ-curve (3). Shift the curve being interpreted along the μ-curve, the axes remaining parallel, and the point (h_1, ρ_1) of the two-layer curves coinciding with the μ-curve, until the right portion of the curve overlies a theoretical curve or one interpolated logarithmically. Sketch on the k-curve, $k = -0.72$ for this example, and indicate the fictive depth to the third layer displaced anisotropic point. Compute ρ_3 from the k or μ value. Again place the curve over the VAK curves and extend the ν-curve $m_2/m_1 = 3$ to the abscissa arriving at the depth 8. There is a small disagreement here in the k_2 values obtained by the Ebert method, probably because of the reason mentioned previously; [illegible] curves are only representative. The complete [illegible] is given in figure 7[illegible].

Application to Type 4 three-layer curves. The "shifted Hummel's point" is used to obtain the curves for interpreting the Type 4 three-layer curves. This point will lie in the vertical and below as the anisotropic triangle is shifted to the right. The amount of shifting required is dependent upon μ and ν, or

$$\eta = f(\mu, \nu) . \qquad (61)$$

The formulas for obtaining the values of the coordinates for the shifted Hummel's point are;

$$\xi_{VH} = \frac{1+\nu}{\eta\left(1+\frac{\nu}{\mu}\right)} \qquad (62)$$

and

$$\Lambda_{VH} = \frac{1+\nu}{\eta} . \qquad (63)$$

The VHK curves of Ebert figure 12 were obtained from these formulas.

The values of $f(\eta)$ is not given by Ebert, but from the anisotropy triangle relation, it is apparently exponential. Measurements from the asymptotes of the μ-curves, Ebert figure 12, and plotting the results, reveals that $f(\eta)$ may be represented by an equation of the type $y = a\,x^m$. By least squares, the solution $y = 1.02\,x^{3.91}$ is obtained for $f(\eta)$.

The ξ_H values from the HK curves, figure 72, are used for calculating the VHK μ-curves and when multiplied by $1/\eta$ give

ξ_{VH}. Near the origin the μ-curve is obtained by graphical application of the anisotropic triangle, otherwise unusable values are obtained.

The Δ_H values were computed for the HK curves, but the Δ_{VH} values for the VHK curves were obtained by the following method. The ν-curves are distributed along the abscissa from the origin by the equation, obtained by least squares,

$$y = 1.545\, x^{1.118}\, e^{0.002x} \quad \text{--------} \quad (64)$$

The ν-curves are then computed from the following equations obtained by least squares:

$$\left.\begin{array}{ll} \nu = 1, & y = 1.720\, e^{0.003x} \\ \nu = 2, & y = 2.819\, e^{0.004x} \\ \nu = 3, & y = 4.309\, e^{0.003x} \\ \nu = 5 & y = 7.731\, e^{0.002x} \\ \nu = 9, & y = 16.35\, e^{0.001x} \\ \nu = 24, & y = 53.28\, e^{0.0004x} \end{array}\right\} \quad \text{---} \quad (65)$$

Curves for other values of ν may be obtained either by interpolation from a plot of the above constants or by logarithmic interpolation and the equations above for locating the curve intersection on the abscissa.

The μ and ν curves are plotted on the same sheet of logarithmic paper and with the same coordinates in figure 78.

Interpretation of a Type 4 three-layer curve. Another curve was selected from the Wetzel and McMurry group, $\rho_1 : \rho_2 : \rho_3 =$ 1 : 1/10 : 1/100 , $h_1/d =$ 1/3, for interpretation using the VHK curves, and is shown completely interpreted in figure 79. This curve to be interpreted is overlain on the two layer theoretical curves on a light table in a way that the help curve for the two upper layers may be drawn. This is the curve for $k_1 = -0.5$ in this example and is sketched on the sheet having the curve. Mark the point (ρ_1, h_1 = 100, 60) with a symbol such as used previously. Next overlay the curve on the VHK curves, figure 78, with the Hummels' point (τ) above the origin. Sketch in the μ-curve ρ_2/ρ_1 = 1/10, obtained by interpolation. Again overlay the curve on the two-layer curves with the point (ρ_1 , h_1) for the $\rho_1 > \rho_2$ curves coinciding with the μ-curve. Shift the curve being interpreted along the μ-curve, the axes remaining parallel, until the extreme lower part of the curve being interpreted covers a two-layer theoretical curve or one interpolated logarithmically. In this example it will be the interpolated curve $k_2 = -0.94$. Indicate the position of the shifted Hummel's point, which is the point on the μ-curve above the point (ρ_1 , h_1) by an (X) as in figure 79. Return the curve to overlay the VHK curves. The shifted Hummel's point (X) will lie on the curve ν = 1/3 and the correct depth to the third layer is read on the abscissa axis at 8 where it intersects the ν = 1/3 curve. Compute ρ_3 with the aid of k_2.

Extension of the method to multi-layer curves by Ebert.

It is also possible to interpret four-layer curves by this procedure. Eight different forms may be taken by the four-layer curves and the interpretation procedures for the three-layer curves may be repeatedly applied until a complete solution is obtained. This will incur the application of two help points besides the one for the two upper layers.

In the multi-layer case of two or more layers, if the deepest layer has infinite resistivity, an accurate and rapid depth determination is possible. The rising portion of the curve is at an angle of 45° with the abscissa, and by extending the constructed asymptote to the abscissa axis, the "length conductivity" s is obtained,

$$s = s_1 + s_2 = \frac{m_1}{\rho_1} + \frac{m_2}{\rho_2} \quad \text{(66)}$$

m_2 is also obtained and with it $h_2 = m_1 + m_2$, the depth of the third layer, figure 80.

Interpretation of observed resistivity curves by the method of Rosenzweig.

Other methods than those given previously have been proposed for interpreting two, three, and more layer resistivity curves that are based on theoretical and mathematical principles. In this group may be placed the method of Rosenzweig (1938) in which two sets of curves are prepared, one showing the relationship between x_1 and x_2 by given constant reflection factors k and the other showing similar relationships but by constant values of relative depth χ. Here $x_1 = \rho_{a_1}/\rho_a$, $x_2 = \rho_{a_2}/\rho_a$, and $\chi = h/a$. A more suitable set of curves than the above for graphic interpolation were computed by using difference ratios of the first and second order substituted in the equations for x_1 and x_2 and the equations solved.

Rosenzweig gives the interpretation of a resistivity curve by his method and checks the results by the method of Tagg (1937, 1940). The two results differ by about 10 percent. Apparent resistivity curve 91 from the Marshfield, Wisconsin study, Spicer (1954), was selected to test the application of the method, see figure 65. The notation of Rosenzweig is used in the computations.

The first point for test is at 80 feet.

1. $a = 80$ feet.

2. $\xi = 0.2$, $\Delta a = 0.2 \times 80 = 16$ feet

   $a_1 = a - \Delta a = 64$ feet, $a_2 = a + \Delta a = 96$ feet.

3. From the ρ_a curve: $\rho_a = 5{,}520$ ohm cms

$\rho_{a_1} = 5{,}000$ ohm cms

$\rho_{a_2} = 6{,}500$ ohm cms.

4. Difference ratios:

$$\Delta_1 = \frac{6500 - 5000}{5520} = +0.272$$

$$\Delta_2 = \frac{6500 + 5000 - 2 \times 5520}{5520} = +0.083.$$

5. Point P, having coordinate of Δ_1 and Δ_2 on the master diagram, lies outside of the curves so no solution is possible.

Moving the selected point to 30 feet.

1. a = 30 feet.

2. $\xi = 0.2$, $\Delta a = 0.2 \times 30 = 6$ feet.

$a_1 = a - \Delta a = 24$ feet, $a_2 = a + \Delta a = 36$ feet.

3. From the ρ_a curve: $\rho_a = 5{,}920$ ohm cms

$\rho_{a_1} = 6{,}500$ ohm cms

$\rho_{a_2} = 5{,}350$ ohm cms.

4. Difference ratios:

$$\Delta_1 = \frac{5350 - 6500}{5920} = -0.194$$

$$= \frac{5350 + 6500 - 2 \times 5920}{5920} = +0.002 .$$

5. From the master diagram, point P with coordinates Δ_1 and Δ_2, yields: $k = -0.45$ and $\chi = 0.9$, then

$h = 0.9 \times 30 = 27$ feet.

This is the same depth to the first layer as obtained by curve analysis but the k value differs , the one by curve analysis being $k = -0.82$.

Choosing a point near the interface of the next pair of layers below the one above.

1. $a = 45$ feet

2. $\xi = 0.2$, $\Delta a = 0.2 \times 45 = 9$ feet

   $a_1 = a - \Delta a = 36$ feet, $a_2 = a + \Delta a = 54$ feet.

3. From the ρ_a curve; $\rho_a = 4925$ ohm cms

   $\rho_{a_1} = 5360$ ohm cms

   $\rho_{a_2} = 4880$ ohm cms.

4. Difference ratios:

$$\Delta_1 = \frac{4880 - 5360}{4925} = -0.097$$

$$\Delta_2 = \frac{4880 + 5360 - 2 \times 4925}{4925} = +0.079.$$

5. Point P, coordinates Δ_1 and Δ_2 yields from the master diagram $k = -0.57$ and $\chi = 0.19$

so $h = 0.19 \times 45 = 9$ feet.

This is an impossible solution.

Selecting a point for test in the opposite direction from the interface in the above trial.

1. $a = 35$ feet

2. $\xi = 0.2$, $\Delta a = 0.2 \times 35 = 7$ feet

   $a_1 = a - \Delta a = 28$ feet, $a_2 = a + \Delta a = 42$ feet.

3. From the ρ_a curve; $\rho_a = 5{,}440$ ohm cms

   $\rho_{a_1} = 6{,}000$ ohm cms

   $\rho_{a_2} = 5{,}020$ ohm cms.

4. Difference ratios :

$$= \frac{5020 - 6000}{5440} = -0.180$$

$$= \frac{5020 + 6000 - 2 \times 5440}{5440} = +0.026.$$

5. Point P, coordinates Δ_1 and Δ_2, from the master diagram gives for a solution $k = -0.28$ and $\chi = 0.6$ then $h = 35 \times 0.6 = 21.0$ feet.

This is also an impossible solution.

Apparent resistivity curve 91 is of a completely different type than the one used by Rosenzweig as an example, and this is probably the reason for his method failing to give a satisfactory interpretation of this curve. It seems that care must be exexcised in choosing the type of resistivity curve that will be suitable for interpretation by this method, and only curves of the type having a maximum inflection may be used.

Interpretation of observed resistivity curves by the method of Longacre.

A somewhat similar treatment of the above problem was made by Longacre (1941) except that his master diagrams are modified Tagg curves, and only two h - k curves are required by this method to obtain a solution instead of the large number required by the Tagg method. The method is rather unpredictable for multi-layer conditions; it may or may not give a solution which fact has also been pointed out by the author (1950).

Interpretation of observed curve by the method of Tagg.

The method proposed by Tagg (1934) has been applied to the interpretation of apparent resistivity curves by Heiland (1940), Jakosky (1950), and Dobrin (1952) in their text books, and by Rao (1942) in an article. Others, doubtlessly, have applied the method but their complete results are not published. Later, Tagg (1935, 1937, 1940) revised his method so that the effects of near-surface materials were eliminated from the interpretations. In order to use the Tagg method, "master curves" for the k-values in his tables need to be plotted, 150 in all. Values for plotting these master curves will be found in the 1937 and 1940 articles. An example showing the application of the method is given in the 1935 paper.

The same curve, No. 91 from Marshfield, Wisconsin figure 65, will now be interpreted by the revised Tagg method. This curve is noted to be a three-layer one and will require that an interpretation be made of the upper part to determine the first boundary, and an interpretation of the lower part to find the second boundary.

According to Tagg, when $\rho_1 > \rho_2$,

$$\frac{\rho_{na}}{\rho_a} = \frac{1 + 4F(na)}{1 + 4F(a)},$$

and this applies to the upper part of curve 91 in figure 65. Select a value for a , say 10 feet, and read off the apparent resistivity from the observed curve, 8,020 ohm cms. These values will be found in columns 1 and 2 of table 5.

Table 5

Determinations of ρ_{na}/ρ_a values by Tagg method for upper part of curve 91, figure 65.

a feet	ρ_a ohm cms	n	na feet	ρ_{na} ohm cms	ρ_{na}/ρ_a
10	8,020	1.5	15	7,550	0.942
10	8,020	2.0	20	6,850	0.854
10	8,020	2.5	25	6,300	0.786
10	8,020	3.0	30	5,920	0.738

Values of n are now chosen, column 3, and na values computed, column 4. Apparent resistivities at corresponding na values are read from the observed resistivity curve, column 5. ρ_{na}/ρ_a ratios are ~~now~~ then computed, column 6.

The master curves for $\rho_1 > \rho_2$ are now used to find the k and h/a values for each ρ_{na}/ρ_a and to compute the h values. These values are given in table 6.

Table 6

Determination of h by Tagg method

for upper part of curve 91, figure 65.

a = 10, n = 1.5, $\rho_{na}/\rho_a = 0.942$

k	h/a	h
- 0.2	0.348	3.48
- 0.2	1.260	12.60
- 0.3	1.560	15.60
- 0.4	1.772	17.72

a = 10, n = 2.0, $\rho_{na}/\rho_a = 0.854$

k	h/a	h
- 0.8	0.232	2.32
- 0.7	0.264	2.64
- 0.6	0.272	2.72
- 0.5	0,312	3.12
- 0.4	0.340	3.40
- 0.3	0.409	4.09
- 0.3	1.440	14.40
- 0.4	1.720	17.20

a = 10, n = 2.5, $\rho_{na}/\rho_a = 0.786$

k	h/a	h
- 0.9	0.240	2.40
- 0.8	0.264	2.64
- 0.7	0.272	2.72
- 0.6	0.312	3.12
- 0.5	0.348	3.48
- 0.4	0.401	4.01
- 0.3	0.512	5.12
- 0.3	1.400	14.00
- 0.4	1.780	17.80

a = 10, n = 3.0, $\rho_{na}/\rho_a = 0.738$

k	h/a	h
- 0.7	0.300	3.00
- 0.6	0.344	3.44
- 0.5	0.395	3.95
- 0.4	0.467	4.67
- 0.3	0.620	6.20
- 0.3	1.380	13.80
- 0.4	--	- -

The h and k values are now used as abscissa and ordinate values respectively to plot four curves. The intersection of these four curves is the solution for the depth to the second layer, namely 16.5 feet, on figure 81 and the value of $k = -0.34$.

The same procedure is applied now to the lower part of the observed curve. Values are tabulated in tables 7 and 8. It will be noted that conductivities rather than resistivities must be used for this part of the curve and master curves for $\rho_1 < \rho_2$ used to find h/a and k values.

Table 7

Determination of σ_{na}/σ_a values by Tagg method for lower part of curve 91, figure 65.

a feet	ρ_a ohm cms	n	na feet	ρ_{na} ohm cms	σ_{na}/σ_a
100	6,500	1.3	130	7,850	0.828
100	6,500	1.5	150	8,850	0.734
100	6,500	2.0	200	11,100	0.575
100	6,500	2.5	250	13,600	0.478

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Table 8

Determination of h by Tagg method

for lower part of curve 91, figure 65.

$a = 100,\ n = 1.3,\ \sigma_{na}/\sigma_a = 0.828$			$a = 100,\ n = 1.5,\ \sigma_{na}/\sigma_a = 0.734$		
k	h/a	h	k	h/a	k
0.9	0.197	19.7	0.8	--	--
0.8	0.356	35.6	0.9	0.190	19.0
0.8	0.700	70.0	0.9	0.956	95.6
0.9	0.900	90.0	1.0	1.175	117.5
1.0	1.110	110.0			

$a = 100,\ n = 2.0,\ \sigma_{na}/\sigma_a = 0.734$			$a = 100,\ n = 2.5,\ \sigma_{na}/\sigma_a = 0.478$		
k	h/a	h	k	h/a	k
0.8	--	--	0.8	--	--
0.9	0.380	38.0	0.9	0.344	34.4
0.9	0.952	95.2	0.9	0.972	97.2
1.0	1.200	120.0	1.0	1.282	128.2

With k-values as ordinate and h-values as abscissa, four curves are plotted as shown on figure 82. The intersection of the four curves at 94 feet gives the depth to the third layer and the value of $k = +0.92$.

While theoretically a solution should be possible for any point selected on the curve by the Tagg method, it frequently happens that difficulty is experienced in obtaining a solution, perhaps because of unsmoothed irregularities in the observed curve or other causes. Such was the case here as points at 20 feet, 25 feet, and 80 feet were tried and did not give a solution. Furthermore, the solution obtained by the Tagg method is at variance with the one obtained by curve analysis, namely 27 feet and 40 feet, and the depth to bedrock from drilling information nearby. The k-values obtained by curve analysis methods are -0.82 and $+0.90$.

Interpretation of non-uniform, non-parallel, and dipping beds.

A few methods have been given for interpreting the non-uniform layering conditions that are based on theoretical or mathematical considerations. Maillet and Doll (1932) and Maillet (1947) discuss the effects of anisotropy on the apparent resistivity curve and give methods for including this effect in the interpretations. Pirson (1935) shows how the anisotropy coefficient may be determined and applied to the depth interpretations. But Slichter (1933) in his evaluation of the problem shows that the anisotropic condition always produces a shallower depth than would be had for the isotropic case. C. and M. Schlumberger and Leonardon (1934) show how the coefficient of anisotropy may be obtained and the average resistivity determined therefrom, and what the effects of anisotropy are on the apparent resistivity curve. While the above presentations of much interest from a theoretical point of view, they are quite limited, at present, in their practical application.

The effects of non-parallel or dipping beds have also been studied by numerous workers, and the subject is completely and ably discussed by Van Nostrand and Cook (1955) in a discussion following the article by Maeda (1955) on the subject of dipping beds. The reader is referred to these papers as they are too long for inclusion here and too important to be neglected for reading.

Empirical methods of interpreting resistivity curves.

The interpretation of apparent resistivity curves by empirical methods was used by Gish and Rooney (1925) who proposed that the depth to horizontal discontinuities is equal to the electrode separation at which a point of maximum curvature appears on the apparent resistivity curve. Lancaster-Jones (1930) proposed that the depth to an interface be obtained by using two-thirds of the electrode spacing for which an inflection point occurs in the apparent resistivity curve. Lugeon and Schlumberger (1933) proposed that three-fourths of the same distance be used as the depth to a horizontal discontinuity. It was recognized by Thies and Watson (personal communication) (~~1936~~) from their field work that such an arbitrary rule for selecting the horizontal interface was not correct. They adjusted the interface distance on the basis of the k-value of the curve: if the k-value ranged from 0.1 to 0.4 the control point was 0.80 Of the interval to the curve inflection; if k ranged from 0.5 to 0.7 the control point was 0.85 of the interval to the curve inflection; and if k ranged from 0.7 to 1.0 the control point was 0.90 of the inflection distance. The control point is actually the interpreted depth to the interface.

As to be expected, all the above empirical rules were found to fail except under certain conditions. The conditions required for success in using the rule, though not then recognized, were the proper layer thickness and layer resistivity to meet the conditions of the rule and these points are nicely illustrated by Watson (1934).

The empirical methods used by F. W. Lee and associates in interpreting resistivity curves by so-called "breaks" have not been fully described in any publication known to the writer. An idea of one of the principles Lee (1939) fostered in explaining the so-called breaks may be had from his description of the breaking down of electrical horizons in the ground which take the form of an electrical double layer. That there is no effect upon the electrical field produced by the contact potentials where they exist at the common bounding surfaces is stated by Planck (1932),

> "--- an abrupt change of potential at the common boundary surface of two substances,no matter how great it may be, has not the slightest influence on the constitution of the electrostatic field."

If such a condition exists in the earth, it would not be possible to measure its effect, Smythe (1950, pp. 14, 58) , Planck (1932, p. 73). Another principle employed by Lee, according to Roman (1951, p. 208), was the weighting of resistivity curves by various methods, an artifice which Roman states proved useful for correlation and for depth determinations.

Evjen (1938) described a method of interpreting resistivity curves by depth factors, which, properly defined, are maximum weights expressed as a fraction of the electrode spread. The measured results are interpreted in terms of a fictitious quantity termed image strength rather than reality, resistivity. Universal depth factors exist which may be calculated once and for all for any particular electrode arrangement. This has been done for the Wenner, the modified Wenner, and the potential drop ratio method and are explained by the author.

The justification for the method of "breaks", according to Roman (1951, p.204), is an intuitive one, and is very useful in interpretation as a first approximation. However, electrical theory will not provide the abrupt changes in the character of the curve as Lee describes them, Heiland (1940), Watson (1934), Wetzel and McMurry (1937), Watson and Johnson (1938), nor will field measurements, provide that they have been done with the proper techniques and instrumentation. The writer has many apparent resistivity curves to demonstrate the validity of the above statement regarding field measurements, some of which are included herin.

The use of a cumulative, or integral, curve obtained from the apparent resistivity curve was proposed by Moore (1944) as a method for obtaining the depths to the interfaces in the layered problem. Ruedy (1945) offers an explanation of why Moore's method works. Muskat (1945) presents a mathematical analysis of the above method pointing out the correct features as well as the incorrect ones. Muskat concludes,

> "It appears, therefore, that while the method of Moore does suggest an interesting approach to the problem of interpretation of resistivity data, it is beset with such practical

difficulties as would necessitate extreme caution and care when applying it as a geophysical method of prospecting."

Roman, in his discussion (1945, pp. 214-215) of Moore's method, does not analyze it on the basis of mathematical or physical principles but assumes a philosophical viewpoint and states,

"Any available method is satisfactory if it leads to a suitable interpretation.---- Although the speaker leans strongly to the theoretical analyses, he is not opposed to empirical attacks by those interested in them."

Heiland, in his discussion (1945, pp. 217-220) of the above paper, primarily on an analytical basis,states,

> "At first glance this method is deceivingly simple; further investigation shows, however, that : (1) the basic assumption of straight lines is not justified; an (2) that in practice the location of the intersections may be so arbitrary as to seriously limit the usefulness of the straight line approximation."

In the concluding remarks by Heiland, he states,

> "In no case was a curve found with any abrupt breaks, and depth interpretations were made by the conventional methods of analysis with speed and accuracy. Abrupt changes in resistivity curves are generally due to variations of contact resistance and may be discovered and eliminated by modifying the technique . . . ".

Application of empirical methods to the interpretation of theoretical and observed curves.

Some theoretical and measured curves will now be examined to test the application of the above empirical methods. The same curve of Hummel (1931, figure 8) that Heiland used in the discussion above, but plotted here with ohm cms against a , is shown in figure 83. The Gish-Rooney rule would give the depths to the layers at 12 feet and 25 feet as compared to 10 feet and 40 feet that are the correct values; errors of 20 and 37 percent respectively. The Lancaster-Jones two-thirds rule would give depths of 8 feet and 17 feet thus increasing the errors to 20 and 58 percent. The Lugeon Schlumberger three-fourths rule would lie between the two values above with depths of 9 feet and 19 feet; the errors being 10 and 53 percent. The derivative curve for the Hummel curve computed for 5-foot intervals is shown as curve I in the upper part of figure 84. It quite accurately portrays the depths of 10 feet and 40 feet. The integral curve, or Moore summation method, is shown as curve II in the lower part of figure 84 and is also computed with 5-foot intervals. It is apparent upon inspection that the integral curve is not a series of straight lines but actually is curved throughout it's extent and has two inflections. This is, of course, as it should be and any straight line drawn touching or tangent to the curve will only be a part of the unlimited number of tangents possible. Nevertheless, a solution is shown and was accomplished by drawing straight lines through the points which approach that condition. The solution in this manner gives depths of 7.5 feet and 52.5 feet, errors of 25 and 31 percent respectively.

A four-layer theoretical curve after Watson and Johnson (1938, figure 10, p.20) is shown in figure 85 and will be interpreted next by the empirical methods. Application of the Gish-Rooney rule gives depths to the interface boundaries at 0.75 feet, 12.5 feet, and 43 feet instead of at 2 feet, 8 feet, and 24 feet; errors of 62.5, 56, and 79 percent respectively. It is evident, without presenting the figures, that the application of the Lancaster-Jones two-thirds rule or the Lugeon-Schlumberger three-fourth rule would also give very large errors for the depths. The derivative curve figure 86 for the four-layer theoretical curve fails to determine correctly all the depths to the interfaces giving 3, 8, and 21 feet instead of 2, 8, and 24 feet. The integral curve obtained by Moore's method is a continuous curve, figure 86. Drawing tangents to this curve through the portions which approach nearest to straight lines gives depths to the interfaces of 4.7, 21.5, and 41.7 feet for the depths to the actual boundaries of 2, 8, and 24 feet.

Turning now from theoretical curves to an observed resistivity curve, figure 87, for the application of empirical methods. This curve was obtained near Marshfield, Wisconsin (Spicer, 1954) and the site was later drilled for a city water supply well. The drilling log, given on the figure, shows that ten different materials were recognized by the driller. The Gish-Rooney rule gives the depths to the layers as 5 feet, 27.5 feet, and 82 feet; the two-thirds rule of Lancaster-Jones gives these depths as 3 feet, 18 feet, and 55 feet; the three-fourths rule of Lugeon-Schlumberger gives 3 3/4 feet, 21 feet, and 62 feet. The derivative curve, when smoothed, figure 88, gives depths of 5 feet, 12.5 feet, 30 feet, and 82 feet. The integral curve is a continuous smooth curve and it is not possible to make an interpretation, rather it should be stated you can make any interpretation you choose. The depths given by the above methods are lacking in consistency, and neither agree with the drilling log nor with the curve analysis interpretations briefly summarized on figure 88.

Another observed resistivity curve No.89 from the Marshfield, Wisconsin area, figure 89, is also interpreted by the application of empirical methods. Depths to the interfaces by the Gish-Rooney rule are 14 feet, 35 feet, and 72 feet; by the Lancaster-Jones rule are 9 feet, 23 feet, and 48 feet; by the Lugeon-Schlumberger rule 10.5 feet, 26 feet, and 54 feet. On figure 90, the step function curve shows the layering and depths to the interfaces by curve analysis and the computed resistivities of the different layers. The smoothed differential curve on this same figure shows interfaces at 7.5 feet, 25 feet, and 155 feet. By the use of the Moore integral curve method, another curve is obtained to which an infinite number of tangents may be drawn to obtain an unlimited number of solutions.

Attempting a solution, however, with the nearest straight line portions of the curve gives depths to the interfaces of 43 feet and 187 feet, figure 90. These results show a great divergence among themselves and do not correlate well with the interpreted results shown on the step function curve, figure 90.

Interpretation of curves by the method of Barnes.

In an article applied to soil investigations, particularly borrow sites for highway construction, Barnes (1952) develops a method for interpreting resistivity measurements by means of a layer-value determination. The method is stated to be based on the Wenner equation (loc. cit.) for the four electrode configuration and Kirchhoff's law for resistances in parallel. The resisitivities are computed by the usual formula for resistances in parallel for the arbitrary layers selected, usually the layers are equal to the interval distances used.

Barnes shows in his paper that the actual thicnesses of the layers are not determined, and the results are approximations to the thicknesses. It is evident, that except under ideally chosen conditions the determinations would not be comparable to those obtained by the application of theoretical methods of interpretation.

The modification by Pekeris of the direct method of Slichter and Langer.

The basic theory of Slichter and Langer (loc. cit.) for the direct method of interpretation of apparent resistivity curves was modified by Pekeris (1940) to give his graphical method and is derived in the paper. The Pekeris method is based essentially on the fact that for large values of λ

$$F_m = k_m \xi_m \left[1 + \frac{k_2}{k_1} \cdot \frac{1}{e^{2\lambda d_m}} \right] \tag{67}$$

where: F_m is a function assuming different values as m goes from zero to m;

k_m is the reflection factor for m as it goes from zero to m;

ξ_m is the descending exponential function.

The paper mentioned above has been revised and extended here so as to make this interpretive procedure available to those who would like to use it. The revised procedure is applied to an example. The reversal of this method is also explained using an example.

Explanation of the method ~~of Pekeris~~. It is assumed first that the medium consists of layers having thicknesses d_1, d_2, - - d_n and resistivities ρ_1, ρ_2, - - - ρ_n. Also let

$$k_i = (\rho_{i+1} - \rho_i)/(\rho_{i+1} + \rho_i) \qquad (68)$$

be the reflection factor. Then the constants of thickness and resistivity for each layer can be determined as follows.

(1) Determine the potential ϕ_r around a single electrode as a function of the distance r from the electrode. ϕ_r can be obtained directly by measuring the potential distribution around one electrode with the other electrode removed to a great distance. If the electrodes are at a distance L apart, and ψ_r denotes the potential measured along a line passing through the electrodes at a point which is at a distance r from the positive electrode and $(r + L)$ from the negative, then

$$\phi_r = \sum_{n=0}^{\infty} \psi(r + nL) \qquad (69)$$

(2) Determine the kernel after Slichter (loc. cit.) from

$$k(\lambda) = \lambda \int_0^{\infty} \phi_r \, J_0(\lambda r)\, r \, dr \qquad (70)$$

$k(\lambda)$ can be obtained either by numerical integration or by mechanical integration, the latter with the aid of a planimeter. For theoretical studies, the kernel can be computed in a relatively short time by either of the two methods given in ~~the~~ a previous section, that of Wetzel and McMurry (loc. cit.) after Slichter, and that of Sunde (loc. cit.). The latter method may also be applied to any number of layers. For the general case of n layers, the mutual resistance between points on the surface is,

$$Q(r) = \frac{\rho_1}{2\pi}\int_0^\infty k_{12\cdots n}(\lambda)\, J_0(\lambda r)\, d\lambda \quad \text{------} \quad (71)$$

in which

$$k_{(m-1)m\cdots n} = \frac{1-\mu_{(m-1)m\cdots n}\, e^{-2\lambda d_{m-1}}}{1+\mu_{(m-1)m\cdots n}\, e^{-2\lambda d_{m-1}}} \quad \text{---} \quad (72)$$

and

$$\mu_{(m-1)m\cdots n} = \frac{\rho_{(m-1)} - \rho_m k_{m(m+1)\cdots n}}{\rho_{(m-1)} + \rho_m k_{m(m+1)\cdots n}} \quad \text{---} \quad (73)$$

and $k_{m(m+1)}$ is the two-layer kernel function for an upper layer of resistivity ρ_{m-1} and a depth d_{m-1} and a lower layer of resistivity ρ_m and of infinite depth.

Specifically, for a two layer case:

$$k_{12} = \frac{1-\mu_{12}\, e^{-2\lambda d_1}}{1+\mu_{12}\, e^{-2\lambda d_1}} \quad \text{------------} \quad (74)$$

where

$$\mu_{12} = (\rho_1 - \rho_2)/(\rho_1 + \rho_2). \quad \text{---------} \quad (75)$$

For a three layer case:

$$k_{123} = \frac{1-\mu_{123}\, e^{-2\lambda d_1}}{1+\mu_{123}\, e^{-2\lambda d_1}} \quad \text{---------} \quad (76)$$

$$\mu_{123} = \frac{\rho_1 - \rho_2 k_{23}}{\rho_1 + \rho_2 k_{23}} \quad \text{---------} \quad (77)$$

$$k_{23} = \frac{1-\mu_{23}e^{-2\lambda d_2}}{1+\mu_{23}e^{-2\lambda d_2}} \qquad (78)$$

$$\mu_{23} = \frac{\rho_2-\rho_3}{\rho_2+\rho_3} \qquad (79)$$

For a four-layer case:

$$\left.\begin{aligned}
\mu_{34} &= \frac{\rho_3-\rho_4}{\rho_3+\rho_4} \\
k_{34} &= \frac{1-\mu_{34}e^{-2\lambda d_3}}{1+\mu_{34}e^{-2\lambda d_3}} \\
\mu_{234} &= \frac{\rho_2-\rho_3 k_{34}}{\rho_2+\rho_3 k_{34}} \\
k_{234} &= \frac{1-\mu_{234}e^{-2\lambda d_2}}{1+\mu_{234}e^{-2\lambda d_2}} \\
\mu_{1234} &= \frac{\rho_1-\rho_2 k_{234}}{\rho_1+\rho_2 k_{234}} \\
k_{1234} &= \frac{1-\mu_{1234}e^{-2\lambda d_1}}{1+\mu_{1234}e^{-2\lambda d_1}}
\end{aligned}\right\} \qquad (80)$$

(3) From the theory underlying the method,

$$f_1(\lambda) = \frac{k(\lambda)+1}{k(\lambda)-1} \qquad (81)$$

Then plot $\ln|f_1(\lambda)|$ against λ. For large values of λ the points will lie on a straight line whose slope is $2d_1$ and whose intercept on the ordinate axis is $\ln(1/k_1)$ or $\ln(-1/k_1)$ depending ~~on~~

on whether $f_1(\lambda)$ is positive or negative for large values of λ. The plot of $f_1(\lambda)$ may be made on semi-logarithmic paper with λ as the linear abscissa and $f_1(\lambda)$ the logarithmic ordinate, and in this manner the intercept of the asymptotic straight line with the ordinate axis is $1/k_1$ or $-1/k_1$ depending on the sign of the large $f_1(\lambda)$. Compute the value of k. Determine the values of ρ_1 and ρ_2 from the relation

$$\frac{\rho_2}{\rho_1} = \frac{1+k_1}{1-k_1}. \qquad (82)$$

Determine the slope of the asymptotic straight line. If this is done on linear coordinate paper, the same linear scale should be used for both the ordinate and abscissa. On semi-logarithmic paper, two points are chosen with coordinates (x_1 , $\ln y_1$) and (x_2 , $\ln y_2$). Then

$$2d_1 = \frac{1}{x_2-x_1}(\ln y_2 - \ln y_1). \qquad (83)$$

If all the values of $f_1(\lambda)$ lie on the straight line, then $d_2 = \infty$, otherwise proceed with the next step.

(4) With k_1 and d_1 determined as above, compute

$$f_2(\lambda) = \frac{1-k_1^2}{1-k_1\xi_1 f_1(\lambda)} \qquad (84)$$

where $\xi_1 = e^{-2\lambda d_1}$

and plot $\ln|f_2(\lambda)-1|$ against λ on linear coordinate paper or on semi-logarithmic paper with λ the linear abscissa. For large λ, the points will again lie on a straight line, the line having a slope of $2d_2$ and an intercept with the ordinate axis of $\ln(k_1/k_2)$ or $\ln(-k_1/k_2)$ depending on the sign of large $|f_2(\lambda)-1|$. Determine k_2 from the value of the intercept and compute ρ_3 from the relation,

$$\frac{\rho_3}{\rho_2} = \frac{1+k_2}{1-k_2}. \qquad (85)$$

If all the points of $|f_2(\lambda)-1|$ lie on a straight line, then $d_3 = \infty$, otherwise proceed with the next step.

(5) With k_2 and d_2 determined as above, compute

$$f_3(\lambda) = \frac{1-k_2^2}{1+y_2-y_2 f_2(\lambda)} \qquad (86)$$

where

$$y_2 = \frac{k_2}{k_1} e^{-2\lambda d_2}$$

and plot $\ln|f_3(\lambda)-1|$ against λ on linear coordinate paper or on semi-logarithmic paper with λ the linear abscissa. For large λ, the points will again lie on a straight line having a slope of $2d_3$ and an intercept with the ordinate axis of $\ln(k_2/k_3)$ or $\ln(-k_2/k_3)$ depending on the sign of $|f_3(\lambda)-1|$. As above, if $|f_3(\lambda)-1|$ is plotted on semi-logarithmic paper, k_2/k_3 may be read directly and k_3 computed from the value of the intercept. Compute ρ_4 from the relation,

$$\frac{\rho_4}{\rho_3} = \frac{1+k_3}{1-k_3} \quad \text{- - - - - - - - - - - - - -} \quad (87)$$

If all the points of $|f_3(\lambda) - 1|$ lie on a straight line, then $d_4 = \infty$, otherwise continue with the next step.

(6) Continue, if necessary, with $f_4(\lambda)$ and the $f(\lambda)'s$ of higher order where

$$f_m(\lambda) = \frac{1 - k^2_{(m-1)}}{1 + y_{(m-1)} - y_{(m-1)}\, f_{(m-1)}} \quad \text{- - - - - - - - -} \quad (88)$$

In each step, $2d_m$ is the slope of the asymptotic line and $\ln|k_{(m-1)}/k_m|$ is the intercept with the ordinate axis.

The above procedures describe how to determine ρ_2 and ρ_3 in in sections 3 and 4, which are not given in the original paper, and also clarifies several other points. In table 1 of the original paper, d_3 for the kernel $k_2(\lambda)$ should read $d_3 = \infty$ instead of $d_3 = \rho_1$, and ρ_3 for the same kernel should read $\rho_3 = 0$ instead of $\rho_3 = \infty$. This latter value was determined both by analysis and by computation of the kernels for the constants $\rho_1 : \rho_2 : \rho_3 = 1 : 20 : 0$ and $d_1 : d_2 : d_3 = 1 : 2 : \infty$, and $\rho_1 : \rho_2 : \rho_3 = 1 : 20 : \infty$ and $d_1 : d_2 : d_3 = 1 : 2 : \infty$. Several errors in the last decimal place of this kernel as given in the original table were also found. Other omissions or points of variance will be pointed out in the explanation of the examples.

Not having any suitable single electrode field measurements which could be used for a complete study and analysis of the direct method of interpretation, the writer was obliged to use kernel functions computed for certain assumed constants. This was an advantage, in one respect, for it enabled a complete check to be made of the direct method from the computations of the kernel to the values obtained by the solution using the Pekeris method.

A complete computation of the kernel $k_1(\lambda)$, as given by Pekeris in his table 1 column 2, is given here in table 9 and the method of Sunde (loc. cit.) was used for the calculation. The column headed k_{123} is the kernel function for the various values of λ given in the first column. Comparison with the values given by Pekeris shows some differences in the last figure. The graph of the function is shown in figure 91.

.003	.03	.97045	.8780	.1220	1.8780	.0649	1.298	- .298	2.298	- .1297	.006	.99402	- .1288
.005	.05	.95123	.8604	.1396	1.8604	.0750	1.500	- .500	2.500	- .2000	.010	.99005	- .1980
.007	.07	.93239	.8440	.1560	1.8440	.0846	1.692	- .692	2.692	- .2571	.014	.98610	- .2536
0.01	0.10	0.90484	0.8186	0.1814	1.8186	0.0997	1.994	- 0.994	2.994	-0.3320	0.020	0.98020	-0.3254
.02	.20	.81873	.7406	.2594	1.7406	.1490	2.980	- 1.980	3.980	- .4973	.040	.96079	- .4775
.03	.30	.74082	.6703	.3297	1.6703	.1973	3.946	- 2.946	4.946	- .5955	.060	.94176	- .5606
.04	.40	.67032	.6065	.3935	1.6065	.2449	4.898	- 3.898	5.898	- .6607	.080	.92312	- .6102
.05	.50	.60653	.5488	.4512	1.5488	.2913	5.826	- 4.826	6.826	- .7065	.100	.90484	- .6395
0.06	0.60	0.54881	0.4967	0.5033	1.4967	0.3362	6.724	- 5.724	7.724	-0.7413	0.120	0.88692	-0.6565
.08	.80	.44933	.4067	.5933	1.4067	.4215	8.430	- 7.430	9.430	- .7880	.160	.85214	- .6720
.10	1.00	.36788	.3328	.6672	1.3328	.5003	10.006	- 9.006	11.006	- .8183	.200	.81873	- .6700
.15	1.50	.22313	.2022	.7978	1.2022	.6635	13.27	-12.270	14.270	- .8597	.300	.74082	- .6370
.20	2.00	.13534	.1224	.8776	1.1224	.7820	15.640	-14.640	16.640	- .8796	.400	.67032	- .5897
0.30	3.00	0.04979	0.0451	0.9549	1.0451	0.9135	18.270	-17.270	19.270	-0.8960	0.600	0.54881	-0.4917
.45	4.50	.01111	.0101	.9899	1.0101	.9795	19.590	-18.590	20.590	- .9030	.900	.40657	- .3672
.60	6.00	.00248	.0022	.9978	1.0022	.9962	19.924	-18.924	20.924	- .9040	1.200	.30119	- .2722
.80	8.00	.00034	.0003	.9997	1.0003	.9994	19.988	-18.988	20.988	- .9050	1.600	.20190	- .1827
1.00	10.00	.00004	.00004	.99996	1.00004	.9999	19.998	-18.998	20.998	- .9050	2.000	.13534	- .1225

After Sunde: $d_1 = 1.0$ $\quad \rho_1 = 1.0$
$d_2 = 5.0$ $\quad \rho_2 = 20.0$
$d_3 = \infty$ $\quad \rho_3 = 1.0$

Pekeris, C. L. Table 1, p. 3

$$Q = \frac{\varphi r}{c} = \text{ratio of potential to current}$$

$$Q_r = \frac{\rho_1}{2\pi} \int_0^\infty k_{123}(\lambda)\, J_0(\lambda r)\, d\lambda$$

$$k_{123} = (1 - \mu_{123} e^{-2\lambda d_1}) / (1 + \mu_{123} e^{-2\lambda d_1})$$

$$\mu_{123} = \frac{\rho_1 - \rho_2 k_{23}}{\rho_1 + \rho_2 k_{23}}$$

$$k_{23} = (1 - \mu_{123} e^{-2\lambda d_2}) / (1 + \mu_{23} e^{-2\lambda d_2})$$

Example 1. The solution of the kernel function $k_1(\lambda)$ by the Pekeris method to obtain the resistivity curve constants is given in table 10 and figure 92. The procedure in detail follows.

Table 10

Solution of Pekeris $k_1(\lambda)$

$\rho_1 : \rho_2 : \rho_3 = 1 : 20 : 1$, $d_1 : d_2 : d_3 = 1 : 5 : \infty$

λ	$k_{123}(\lambda)$	$k_{123}(\lambda)+1$	$k_{123}(\lambda)-1$	$f_1(\lambda) = \frac{k_{123}(\lambda)+1}{k_{123}(\lambda)-1}$	2λ	$\xi_1 = e^{-2\lambda d_1}$	$[k_1 \xi_1 f_1 \lambda]$	$[1 - k_1\xi_1 f_1(\lambda)]$	$f_2(\lambda)$	$f_2(\lambda)-1$
0.00	1.000	2.000	0.000	∞	0.000	1.0000	∞	-∞	0.0000	-1.0
.001	1.100	2.100	.100	21.000	0.002	.9980	19.070	-18.070	-0.0096	-1.0096
.003	1.297	2.297	.297	7.730	.006	.9940	6.983	- 5.983	- .0290	-1.0290
.005	1.494	2.494	.494	5.047	.010	.9901	4.543	-3.543	- .0490	-1.0490
.007	1.679	2.679	.679	3.944	.014	.9861	3.535	-2.535	- .0685	-1.0685
0.01	1.966	2.966	0.966	3.070	0.02	0.9802	2.737	-1.737	-0.1000	-1.1000
.02	2.828	3.828	1.828	2.096	.04	.9608	1.831	-0.831	- .2116	-1.2116
.03	3.552	4.552	2.552	1.784	.06	.9418	1.528	- .528	- .3289	-1.3289
.04	4.131	5.131	3.131	1.639	.08	.9231	1.376	- .376	- .4620	-1.4620
.05	4.546	5.546	3.546	1.565	.10	.9048	1.288	- .288	- .6028	-1.6028
0.06	4.821	5.821	3.821	1.523	0.12	0.8869	1.228	-0.228	-0.7618	-1.7618
.08	5.097	6.097	4.097	1.489	.16	.8521	1.154	- .154	-1.128	-2.128
.10	5.060	6.060	4.060	1.493	.20	.8187	1.112	- .112	-1.551	-2.551
.15	4.508	5.508	3.508	1.571	.30	.7408	1.058	- .058	-2.994	-3.994
.20	3.874	4.874	2.874	1.696	.40	.6703	1.033	- .033	-5.262	-6.252
0.30	2.934	3.934	1.934	2.034	0.60	0.5488	1.016	-0.016	-10.86	-11.86
.45	2.160	3.160	1.160	2.723	.90	.4066	1.007	- .007	-24.81	-25.81
.60	1.724	2.724	0.724	3.763	1.20	.3012	1.031	- .031	- 5.602	- 6.602
.80	1.447	2.447	.447	5.472	1.60	.2019	1.004	- .004	-43.40	-44.40
1.00	1.279	2.279	.279	8.167	2.00	.1353	1.003	- ,003	-57.88	-58.88

See curve figure 92 for d_1 and ξ_1.

$1 - k_1^2 = +0.1737$ $(k_1 = 0.909)$

See curve figure 92 for d_2, ρ_2, ρ_3, and remainder of the solution.

In column (1) tabulate the values of λ from 0.0 to 1.0 or 2.0 by increments such that detail will be obtained throughout the curve. In column (2) tabulate the values of the kernel function k_{123}. Columns (3), (4), and (5) yield the $f_1(\lambda)$ function, equation (81).

Referring next to the figure 92, values of $f_1(\lambda)$ are shown plotted as small circles on semi-logarithmic paper with the ordinate having values from 1.0 to 100, and the abscissa having values from 0.0 to 1.4. The slope of $f_1(\lambda)$ is positive. The intercept on the ordinate axis is 1.1 and $1/k_1 = +1.10$ then $k_1 = +0.909$. It follows then from $\rho_2/\rho_1 = (1 - k_1)/(1 + k_1)$ that $\rho_2/\rho_1 = 1/20$.

The slope is next determined by taking two points on the asymptotic straight line. For example: $\lambda = 0.4$ and $\lambda = 0.7$ and the $\ln f_1(\lambda)$ for the points are 2.43 and 4.43 by figure 92. (Pekeris gives 2.47 and 4.51 for the same points). Then from equation (83),

$$2d_1 = \frac{1}{0.7 - 0.4}(\ln 4.43 - \ln 2.43)$$

$$= \frac{1}{0.3}(1.48840 - 0.88789) = 2.0017$$

$$d_1 = 1.00 \text{ and } \xi_1 = e^{-2\lambda} \text{ since } d = 1.$$

The slope must be taken on the linear part of the $f_1(\lambda)$ curve.

Returning to table 10. Column (7) is obtained by multiplying column (1) by column (2). Column 8, the ξ_1 function, or $e^{-2\lambda d_1}$, is obtained from any volume of tables giving the descending exponential function such as those of the National Bureau of Standards (1939). The constant value for column 9 is obtained from equation (82) and is the numerator for the fraction representing $f_2(\lambda)$. Columns 10 and 11 give the denominator for $f_2(\lambda)$, and column 12 is then $f_2(\lambda)$. Column 13 is the function $f_2(\lambda) - 1$. Completion of the solution, as indicated by column 13, is on figure 75.

The slope of $f_2(\lambda) = 2d_2$ and the intercept on the ordinate axis 1.0 . $\overset{(k_1/k_2)}{-k_1/k_2} = 1$ and $k_1 = k_2$ so $k_2 = -0.909$. Substitution in equation (85) gives $\rho_3 = \rho_1 = 1$.

Next determine the slope by selecting two points on the straight line portion of $f_2(\lambda) - 1$, $\lambda = 0.0$ and $\lambda = 0.02$. The $\ln f_2(\lambda)$ for these points are 0.0 and 0.19228 .

$$2\,d_2 = \frac{1}{0.02}\left(\ln 1.2116 - \ln 1\right)$$

$$2\,d_2 = \frac{1}{0.02}\left(0.19228\right) = 9.614$$

$$d_2 = 4.807 \doteq 5.0$$

Because of the limited number of significant figures carried in the computation, the curve of $f_2(\lambda) - 1$ is not a complete straight line as it should be. The resulting solution of the $k_1(\lambda)$ function by the Pekeris method is $\rho_1 : \rho_2 : \rho_3 = 1 : 20 : 1$ and $d_1 : d_2 : d_3 = 1 : 5\ (4.807) : \infty$.

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Example 2. The computation for the kernel function $k_2(\lambda)$, as given by Pekeris in column 3 of table 1, is given here in table 11, and in graphical form in figure 93. With one or two exceptions, the values of the table are the same as those given in the above article. However, additional small values of λ have been computed and added in the table.

The kernel function $k_2(\lambda)$ is solved completely using the method of Pekeris in table 12 and figure 94. The explanation given for the headings of table 10 applies as well to this table.

The intercept of $f_1(\lambda)$ on the ordinate axis is 1.11, then $1/k_1 = 1.11$ giving $k_1 = 0.9009$. $\rho_2 / \rho_1 = (1 + k_1)/(1 - k_1) = 19.18$ or $\rho_1 : \rho_2 = 1 : 20$. The slope of $2d_1$ is positive. Then

$$2d_1 = \frac{1}{0.4-0.2}(\ln 2.45 - \ln 1.65)$$

$$2d_1 = \frac{1}{0.2}(0.39531)$$

$$2d_1 = 1.9766$$

$$d_1 = 1.00 \quad \text{and} \quad \xi_1 = e^{-2\lambda} \quad \text{since } d_1 = 1 .$$

The value of d_1 cannot be obtained from the single value of $\ln|f_1|$ at $\lambda = 1.1$ as indicated by Pekeris, but must be combined with another value of $\ln|f_1|$ at $\lambda = 0.0$, or any other value between the two, in a manner similar to that in the previous example. It should also be mentioned again that $\rho_3 = 0$ and

Table 11

Computation of kernel $k_2(\lambda)$ of table 1, Pekeris.

$[1-\mu_{23}e^{-2\lambda d_2}]$	$[1+\mu_{23}e^{-2\lambda d_2}]$	k_{23}	$\rho_2 k_{23}$	$[\rho_1-\rho_2 k_{23}]$	$[\rho_1+\rho_2 k_{23}]$	μ_{123}	$2\lambda d_1$	$e^{-2\lambda d_1}$	$\mu_{123}e^{-2\lambda d_1}$	$[1-\mu_{123}e^{-2\lambda d_1}]$	$[1+\mu_{123}e^{-2\lambda d_1}]$	k_{123}
-0.0000	2.0000	0.0000	0.0000	+0.000	+ 1.000	+1.0000	0.000	1.00000	+1.0000	0.0000	+2.0000	0.0000
- .0040	1.9960	.0020	.0401	+ .960	+ 1.040	+0.9222	.002	0.99800	+0.9210	.0790	1.9210	.0411
- .0119	1.9881	.0060	.1197	+ .880	+ 1.120	+ .7861	.006	.99402	+ .7815	.2185	1.7815	.1227
- .0190	1.9802	.0100	.2000	+ .800	+ 1.200	+ .7500	.010	.99005	+ .7425	.2575	1.7425	.1478
- .0276	1.9724	.0140	.2798	+ .720	+ 1.280	+ .5628	.014	.98610	+ .5554	.4446	1.5554	.2859
-0.0392	1.9608	0.0200	0.4000	+0.600	+ 1.400	+0.4285	0.02	0.98020	+0.4200	0.5800	1.4200	0.4084
- .0769	1.9231	.0398	.7960	+ .204	+ 1.796	+ .1136	.04	.96079	+ .1092	.8908	1.1092	.8033
- .1131	1.8869	.0599	1.1980	- .198	+ 2.198	- .0901	.06	.94176	- .0848	1.0848	0.9152	1.185
- .1479	1.8521	.0799	1.5970	- .597	+ 2.597	- .2300	.08	.92312	- .2123	1.2123	.7877	1.535
- .1813	1.8187	.0998	1.9950	- .995	+ 2.995	- .3322	.10	.90484	- .3013	1.3013	.6987	1.861
-0.2134	1.7866	0.1195	2.390	-1.390	+ 3.390	-0.4100	0.12	0.88692	-0.3638	1.3638	0.6362	2.144
- .2738	1.7262	.1587	3.174	-2.174	+ 4.174	- .5207	.16	.85214	- .4450	1.4450	.5550	2.607
- .3297	1.6703	.1973	3.946	-2.946	+ 4.946	- .5955	.20	.81873	- .4877	1.4877	.5123	2.903
- .4512	1.5488	.2913	5.826	-4.826	+ 6.826	- .7068	.30	.74082	- .5219	1.5219	.4781	3.182
- .5507	1.4493	.3800	7.600	-6.600	+ 8.600	- .7673	.40	.67032	- .5143	1.5143	.4857	3.120
-0.6988	1.3012	0.5367	10.734	-9.734	+11.734	-0.8297	0.60	0.54881	-0.4552	1.4552	0.5448	2.671
- .8347	1.1653	.7161	14.322	-13.322	+15.322	- .8692	.90	.40657	- .3525	1.3525	.6475	2.089
- .9093	1.0907	.8336	16.672	-15.672	+17.672	- .8865	1.20	.30119	- .2671	1.2671	.7329	1.729
- .9592	1.0408	.9217	18.434	-17.434	+19.434	- .8966	1.60	.20190	- .1812	1.1812	.8188	1.442
- .9917	1.0183	.9737	19.574	-18.574	+20.574	- .9030	2.00	.13534	- .1223	1.1223	.8777	1.279
-0.9997	1.0003	0.9960	19.920	-18.920	+20.920	-0.9040	4.00	0.01832	-0.0166	1.0166	0.9834	1.033
-1.0000	1.0000	1.0000	20.000	-19.000	+21.000	- .9047	5.00	.00674	-0.0061	1.0061	.9939	1.012

- 2, $d_3 = \infty$, $\rho_1 = 1$, $\rho_2 = 20$, $\rho_3 = 0$, $k_{123} = (1-\mu_{123}e^{-2\lambda d_1})/(1+\mu_{123}e^{-2\lambda d_1})$

$\mu_{123} = (\rho_1 - \rho_2 k_{23})/(\rho_1+\rho_2 k_{23})$

$k_{23} = (1+\mu e^{-2\lambda d_2})/(1+\mu e^{-2\lambda d_2})$

$\mu_{23} = (\rho_2-\rho_3)/(\rho_2+\rho_3) = 1.0$

Table 12.

Solution of Pekeris $k_2(\lambda)$

λ	$k_1\lambda$	$k_1\lambda+1$	$k_1\lambda-1$	$f_1(\lambda)$	See plotted curve for other values, fig. 94.	2λ	$e^{-2\lambda d_1}$	$1-k_1^2 = 0.1884$	$[k_1\xi_1 f_1\lambda]$	$[1-k_1\xi_1 f_1\lambda]$	$f_2(\lambda)$	$f_2\lambda-1$
0.00	1.000	2.000	0.000	∞		0.00	1.0000		∞	- ∞	0.0000	- 1.0000
.05	1.85	2.85	.85	3.353		.10	0.9048		2.734	-1.734	- .1087	- 1.1087
.10	2.91	3.91	1.91	2.048		.20	.8187		1.512	-0.512	- .3680	- 1.3680
.15	3.19	4.19	2.19	1.913		.30	.7408		1.277	- .277	- .6800	- 1.680
.20	3.12	4.12	2.12	1.944		.40	.6703		1.175	- .175	-1.076	- 2.076
0.25	2.91	3.91	1.91	2.048		0.50	0.6065		1.119	-0.119	-1.583	- 2.583
.30	2.67	3.67	1.67	2.197		.60	.5488		1.087	- .087	-2.166	- 3.166
.35	2.45	3.45	1.45	2.379		.70	.4966		1.064	- .064	-2.944	- 3.944
.40	2.28	3.28	1.28	2.562		.80	.4493		1.037	- .037	-5.093	- 6.093
.45	2.09	3.09	1.09	2.834		.90	.4066		1.038	- .038	-4.958	- 5.958
0.50	1.952	2.952	0.952	3.102		1.00	0.3679		1.028	-0.028	-6.725	- 7.725
.60	1.730	2.730	.730	3.739		1.20	.3012		1.015	- .015	-12.56	-13.56
.70	1.565	2.565	.565	4.537		1.40	.2466		1.008	- .008	-23.55	-24.55
.80	1.442	2.442	.442	5.526		1.60	.2019		1.006	- .006	-31.40	-32.40
.90	1.349	2.349	.349	6.730		1.80	.1653		1.003	- .003	-62.80	-63.80
1.00	1.278	2.278	.278	8.193		2.00	0.1353		1.000	0.000	- ∞	- ∞

See curve for remainder of solution, figure 94.

$\rho_1 : \rho_2 : \rho_3 = 1 : 20 : 0$, $d_1 : d_2 : d_3 = 1 : 2 : \infty$

Table 13

Computation of kernel $k_{1234}(\lambda)$

by the method of Sunde

λ	$2\lambda d_3$	$e^{-2\lambda d_3}$	$\mu_{34} e^{-2\lambda d_3}$	$[1 - \mu_{34} e^{-2\lambda d_3}]$	$[1 + \mu_{34} e^{-2\lambda d_3}]$	k_{34}	$\rho_3 k_{34}$	$[\rho_2 - \rho_3 k_{34}]$	$[\rho_2 + \rho_3 k_{34}]$	μ_{234}	$2\lambda d_2$	$e^{-2\lambda d_2}$	$\mu_{234} e^{-2\lambda d_2}$
0.000	0.000	1.00000	-0.81818	1.81818	0.18182	10.00000	$\rho_3 = 1$ ∴ same as k_{34}	10.00000	30.00000	0.33333	0.00	1.00000	0.33333
.001	.006	0.99402	- .81329	1.81329	.18671	9.71180		10.28820	29.71180	.34627	.02	0.98020	.33941
.003	.018	.98216	- .80358	1.80358	.19642	9.18226		10.81774	29.18226	.37070	.06	.94176	.34911
.005	.030	.97045	- .79400	1.79400	.20600	9.70874		11.29126	28.70874	.39330	.10	.90484	.35587
.007	.042	.95887	- .78453	1.78453	.21547	8.28203		11.81797	28.28203	.41786	.14	.86936	.36327
0.010	0.06	0.94176	-0.77053	1.77053	0.22947	7.71574		12.28426	27.71574	0.44322	0.20	0.81873	0.36288
.015	.09	.91393	- .74776	1.74776	.25224	6.92896		13.07104	26.92896	.48539	.30	.74082	.35959
.020	.12	.88692	- .72566	1.72566	.27434	6.29022		13.70978	26.29022	.52148	.40	.67032	.34956
.030	.18	.83527	- .68340	1.68340	.31660	5.31712		14.68288	25.31712	.57996	.60	.54881	.31829
.040	.24	.78663	- .64630	1.64630	.35370	4.65451		15.34549	24.65451	.62242	.80	.44933	.27967
0.050	0.30	0.74082	-0.60612	1.60612	0.39388	4.07769		15.92231	24.07769	0.66129	1.00	0.36788	0.24328
.060	.36	.69768	- .57083	1.57083	.42917	3.66016		16.33983	23.66016	.69061	1.20	.30119	.20800
.070	.42	.65705	- .53759	1.53759	.46421	3.32517		16.67483	23.32517	.71489	1.40	.24660	.17629
.080	.48	.61878	- .50627	1.50627	.49373	3.05080		16.94920	23.05080	.73530	1.60	.20190	.14846
.090	.54	.58275	- .47679	1.47679	.52321	2.82256		17.17744	22.82256	.75265	1.80	.16530	.12441
0.10	0.60	0.54881	-0.44903	1.44903	0.55097	2.62996		17.37004	22.62996	0.76757	2.00	0.13534	0.10388
.20	1.20	.30119	- .24643	1.24643	.75357	1.65403		18.34597	21.65403	.84723	4.00	.01832	.01552
.30	1.80	.16530	- .13524	1.13524	.86476	1.31278		18.68722	21.31278	.87681	6.00	.00248	.00217
.40	2.40	.09072	- .07422	1.07422	.92578	1.16034		18.83966	21.16034	.89033	8.00	.00034	.00030
.50	3.00	.04979	- .04074	1.04074	.95926	1.08494		18.91506	21.08494	.89709	10.00	.000045	.00004
0.60	3.60	0.02732	-0.02235	1.02235	0.97765	1.04572		18.95428	21.04572	0.90062	12.00	0.000006	0.00000
.80	4.80	.00823	- .00673	1.00673	.99327	1.01355		18.98645	21.01355	.90353	16.00	.000000	.00000
.90	5.40	.00452	- .00370	1.00370	.99630	1.00743		18.99257	21.00743	.90409	18.00	.000000	.00000
1.00	6.00	.00248	- .00203	1.00203	.99797	1.00407		18.99593	21.00407	.90439	20.00	.000000	.00000
2.00	8.00	.00034	- .00028	1.00028	.99972	1.00056		18.99944	21.00056	.90471	40.00	.000000	.00000

$$\mu_{234} = \frac{\rho_2 - \rho_3 k_{34}}{\rho_2 + \rho_3 k_{34}}$$

$$k_{1234} = \frac{1 - \mu_{1234} e^{-2\lambda d_1}}{1 + \mu_{1234} e^{-2\lambda d_1}} \qquad k_{34} = \frac{1 - \mu_{34} e^{-2\lambda d_3}}{1 + \mu_{34} e^{-2\lambda d_3}}$$

$\rho_1 : \rho_2 : \rho_3 : \rho_4 = 1 : 20 : 1 : 10$

$$\mu_{1234} = \frac{\rho_1 - \rho_2 k_{234}}{\rho_1 + \rho_2 k_{234}} \qquad \mu_{34} = \frac{\rho_3 - \rho_4}{\rho_3 + \rho_4} = -0.81818$$

$d_1 : d_2 : d_3 : d_4 = 1 : 10 : 3 : \infty$

$$k_{234} = \frac{1 - \mu_{234} e^{-2\lambda d_2}}{1 + \mu_{234} e^{-2\lambda d_2}}$$

$[1 + \mu_{234} e^{-2\lambda d_2}]$	$[1 + \mu_{234} e^{-2\lambda d_2}]$	k_{234}	$\rho_2 k_{234}$	$[\rho_1 - \rho_2 k_{234}]$	$[\rho_1 + \rho_2 k_{234}]$	μ_{1234}	$2\lambda d_1$	$e^{-2\lambda d_1}$	$[\mu_{1234} e^{-2\lambda d_1}]$	$[1 - \mu_{1234} e^{-2\lambda d_1}]$	$[1 + \mu_{1234} e^{-2\lambda d_1}]$	k_{1234}
0.66667	1.33333	0.50000	10.0000	- 9.0000	11.0000	-0.81818	0.000	1.00000	-0.81818	1.81818	0.18182	10.0000
.66059	1.33941	.49319	9.8638	- 8.8638	10.8638	- .81590	.002	0.99800	- .81427	1.81427	.18573	9.7683
.65089	1.34911	.48246	9.6492	- 8.6492	10.6492	- .81219	.006	.99402	- .80733	1.80733	.19267	9.3804
.64413	1.35587	.47507	9.5014	- 8.5014	10.5014	- .80955	.010	.99005	- .80149	1.80149	.19851	9.0751
.63673	1.36327	.46706	9.3412	- 8.3412	10.3412	- .80660	.014	.98610	- .79539	1.79539	.20461	8.7747
0.63712	1.36288	0.46748	9.3496	- 8.3496	10.3496	-0.80676	0.020	0.98020	-0.79079	1.79079	0.20921	8.5598
.64041	1.35959	.47103	9.4206	- 8.4206	10.4206	- .80807	.030	.97045	- .78419	1.78419	.21581	8.2674
.65044	1.34956	.48196	9.6392	- 8.6392	10.6392	- .81202	.040	.96079	- .78018	1.78018	.21982	8.0984
.68171	1.31829	.51712	10.3424	- 9.3424	11.3424	- .82367	.060	.94176	- .77570	1.77570	.22430	7.9166
.72033	1.27967	.56290	11.2580	-10.2580	12.2580	- .83684	.080	.92312	- .77250	1.77250	.22750	7.7912
0.75672	1.24328	0.60865	12.1730	-11.1730	13.1730	-0.84817	0.10	0.90484	-0.76746	1.76746	0.23254	7.6007
.79200	1.20800	.65563	13.1126	-12.1126	14.1126	-0.85828	.12	.88692	- .76123	1.76123	.23877	7.3763
.82371	1.17629	.70026	14.0052	-13.0052	15.0052	- .86671	.14	.86936	- .75348	1.75348	.24652	7.1129
.85154	1.14846	.74146	14.8292	-13.8292	15.8292	- .87365	.16	.85214	- .74447	1.74447	.25553	6.8269
.87559	1.12441	.77871	15.5742	-14.5742	16.5742	- .87933	.18	.83527	- .73448	1.73448	.26552	6.5324
0.89612	1.10388	0.81179	16.2358	-15.2358	17.2358	-0.88396	0.20	0.81873	-0.72372	1.72372	0.27628	6.2390
.98448	1.01552	.96943	19.3886	-18.3886	20.3886	- .90191	.40	.67032	- .60457	1.60457	.39543	4.0578
.99783	1.00217	.99567	19.9134	-18.9134	20.9134	- .90437	.60	.54881	- .49633	1.49633	.50367	2.9709
.99970	1.00030	.99950	19.9900	-18.9900	20.9900	- .90472	.80	.44933	- .40652	1.40652	.59348	2.3700
.99996	1.00004	.99992	19.9984	-18.9984	20.9984	- .90475	1.00	.36788	- .33284	1.33284	.66716	1.9978
0.99999	1.00001	0.99998	19.9996	-18.9996	20.9996	- .90476	1.20	0.30119	-0.27250	1.27250	0.72750	1.7491
1.00000	1.00000	1.00000	20.0000	-19.0000	21.0000	- .90476	1.60	.20190	- .18267	1.18267	.81733	1.4470
1.00000	1.00000	1.00000	20.0000	-19.0000	21.0000	- .90476	1.80	.16530	- .14956	1.14956	.85044	1.3517
1.00000	1.00000	1.00000	20.0000	-19.0000	21.0000	- .90476	2.00	.13534	- .12245	1.12245	.87755	1.2791
1.00000	1.00000	1.00000	20.00000	-19.0000	21.0000	- .90476	4.00	.01832	- .01658	1.01658	.98342	1.0337

Application to a four-layer kernel function. In order to test further the application of the Pekeris soultion to more complex layering, a four layer problem was chosen and the kernel k_{1234} computed for $\rho_1 : \rho_2 : \rho_3 : \rho_4 = 1 : 20 : 1 : 10$ and $d_1 : d_2 : d_3 : d_4 = 1 : 10 : 3 : \infty$ by the method of Sunde (loc. cit.) . The computations are shown as table 13 and the kernels obtained in the computations for two, three and four layers are shown in figure 95.

N.B. (All caps. in heading)

Relation between terms in the kernel function.

It will be apparent from analysis and inspection of the computations of table 13 above that the following are true.

μ_{34} is a constant and equals $(\rho_3 - \rho_4)/(\rho_3 + \rho_4) = -0.81818$.

$\mu_{34} e^{-2\lambda d_3}$ at $\lambda = 0$ equals $\mu_{34} = -0.81818$ and at $\lambda = \infty$ equals $e^{-2\lambda} = 0$.

$k_{34}(\lambda)$ at $\lambda = 0$ equals $\rho_4 = 10$ and at $\lambda = \infty$ equals $\rho_3 = 1$.

μ_{234} at $\lambda = 0$ equals $\dfrac{\rho_2\rho_3 - \rho_3\rho_4}{\rho_2\rho_3 + \rho_3\rho_4} = 0.33333$ and at $\lambda = \infty$ equals $\dfrac{\rho_2 - \rho_3}{\rho_2 + \rho_3} = 0.90476$.

k_{234} at $\lambda = 0$ equals $\dfrac{\rho_3\rho_4}{\rho_2\rho_3} = 0.50000$ and at $\lambda = \infty$ equals 1.

μ_{1234} at $\lambda = 0$ equals $\dfrac{\rho_1\rho_2\rho_3 - \rho_2\rho_3\rho_4}{\rho_1\rho_2\rho_3 + \rho_2\rho_3\rho_4} = -0.81818$ and at $\lambda = \infty$ equals $\dfrac{\rho_1 - \rho_2}{\rho_1 + \rho_2} = -0.90476$.

k_{1234} at $\lambda = 0$ equals $\dfrac{\rho_2\rho_3\rho_4}{\rho_1\rho_2\rho_3} = 10$ and at $\lambda = \infty$ equals 1.

The above relations will be of help in understanding the part played by the resistivities of the layers in the formation of the kernel. The variation with λ from $0 \rightarrow \infty$ is controlled by d the exponent of the descending exponential thus producing, together with the resistivities, the character of the kernel. Use will also be made of these facts to assist in reversing the solution of k_{1234} the kernel, i.e., determining the resistivity constants from the values of the kernel and its argument.

In addition to the explanation given of the Pekeris theory and the analysis just preceeding, a few other points should be made clear. Since

$$k(\lambda) = 1 - 2\beta_1(\lambda), \quad \mu_{123} e^{-2\lambda d_1} = \frac{\beta_1(\lambda)}{1+\beta_1(\lambda)} = \frac{1-k(\lambda)}{1+k(\lambda)},$$

and $f_1(\lambda) = \dfrac{k(\lambda)+1}{k(\lambda)-1} = -\dfrac{1}{\mu_{123}\, e^{-2\lambda d_1}}$,

it follows that

$$\begin{aligned} 2d_2 &= \frac{1}{\lambda_2-\lambda_1}\left[\ln f_1(\lambda)_{\lambda=2} - \ln f_1(\lambda)_{\lambda=1}\right] \\ &= \frac{1}{\lambda_2-\lambda_1}\left[\ln \frac{1}{-\mu_{123}\, e^{-2\lambda d_1}}_{\lambda=2} - \ln \frac{1}{-\mu_{123}\, e^{-2\lambda d_1}}_{\lambda=1}\right] \\ &= \frac{1}{\lambda_2-\lambda_1}\left[\ln - \frac{\mu_{123}\, e^{-2\lambda d_1}\,{}_{\lambda=2}}{\mu_{123}\, e^{-2\lambda d_1}\,{}_{\lambda=1}}\right] \\ &= \frac{1}{\lambda_2-\lambda_1} \ln - \frac{N_2}{N_1} \end{aligned}$$

where N_1 and N_2 are successive values of $\mu_{123} e^{-2\lambda d_1}$.

The slope of the asymptotic line is the value for 2d .

$$\ln \mu_{123} e^{-2\lambda d_1} = \ln \mu_{123}\, \lambda - 2d_1 .$$

Using two successive values of λ , then

$$\ln \frac{\mu_2 \lambda_2 e^{-2\lambda_2 d_1} (\lambda = 2)}{\mu_2 \lambda_1 e^{-2\lambda_2 d_1} (\lambda = 1)} = \frac{\ln \mu \, (\lambda = 2)}{\ln \mu \, (\lambda = 1} - 2(\lambda_2 - \lambda_1) d_1 \, ,$$

Solving for d_1 ,

$$2d_1 = \frac{1}{\lambda_2 - \lambda_1} \ln \frac{\mu \, (\lambda = 2)}{\mu \, (\lambda = 1)}$$

$$d_1 = \frac{1}{2(\lambda_2 - \lambda_1)} \ln \frac{\mu \, (\lambda = 2)}{\mu \, (\lambda = 1)} \, .$$

Solution for the reversal of the kernel function.

The complete solution of the reversal of the kernel K1234, page 265 and figure 95 is given in table 14, and detailed directions for the computations follow.

Table 14
Reversal of Pekeris direct solution
of the kernel $k(\lambda)$ to obtain
the constants

1. K_{1234}	2. $\frac{1-x}{1+x} = k_{1234}$, $x = [\mu_{1234} e^{-2\lambda d_1}]$	3. 2λ	4. Determination of d_1 — Second method: $\frac{\mu_{1234} e^{-2\lambda d_1}\ (\lambda=n)}{\mu_{1234} e^{-2\lambda d_1}\ (\lambda=n-1)}$	4. Second method: log of values in left col. times $\frac{1}{2(\lambda_2-\lambda_1)}$	4a. $e^{-2\lambda d_1}$	5. μ_{1234}
10.0000	-0.81818	0.000			1.00000	-0.81818
9.7683	- .81427	.002			0.99800	- .81590
9.3804	- .80732	.006			.99402	- .81218
9.0751	- .80149	.010			.99005	- .80954
8.7747	- .79539	.014			.98610	- .80660
8.5598	-0.79079	0.020			0.98020	-0.80676
8.2674	-0.78419	.030			.97045	- .80807
8.0984	- .78018	.040			.96079	- .81202
7.9166	- .77570	.060			.94176	- .82367
7.7912	- .77250	.080			.92312	- .83684
7.6007	-0.76746	0.10	0.9935	0.652	0.90484	-0.84817
7.3763	- .76123	.12			.88692	- .85828
7.1129	- .75348	.14			.86936	- .86670
6.8269	- .74447	.16			.85214	- .87365
6.5324	- .73448	.18	0.9866	0.736	.83527	- .87933
6.2390	-0.72372	0.20			0.81873	-0.88395
4.0578	- .60457	.40			.67032	- .90191
2.9709	- .49634	.60			.54881	- .90439
2.3700	- .40653	.80			.44933	- .90475
1.9978	- .33284	1.00	0.8187	1.002	.36788	- .90475
1.7491	-0.27249	1.20		$d_1 = 1$	0.30119	-0.90475
1.4470	- .18267	1.60			.20190	- .90475
1.3517	- .14955	1.80			.16530	- .90475
1.2791	- .12246	2.00			.13534	- .90475
1.0337	- .01657	4.00			.01832	- .90475

4. Determination of d_1 — First method

Select two points where λ is large,
2λ = 1.60 and 1.20.

For λ = 0.80
$-1/f_1(\lambda) = -0.18267$
and for λ = 0.60
$-1/f_1(\lambda) = -0.27249$.

Then $2d_1$ =

$$\frac{-1}{0.80-0.60}(\ln -0.18267 - \ln -0.27249) =$$

$$\frac{-1}{0.20}(-1.70007 - (-1.30016))$$

$2d_1 = 1/0.20(0.39991) = 1.9996$

d_1 = 1.00

6. ρ_1, ρ_2, k_1

$$\frac{\rho_1-\rho_2}{\rho_1+\rho_2} = -0.9048.$$

$\rho_2 = 20\rho_1$

$\therefore \rho_1 = 1$
$\rho_2 = 20.$

$\rho_2/\rho_1 = \frac{1+k_1}{1-k_1}$ and $k_1 = \frac{-19}{-21} = +0.9048$

7. $1+\mu_{1234}$	7a. $[20(1+\mu_{1234})]$	7b. $1-\mu_{1234}$	7c. K_{234}	8. $1-k_{234}$	8a. $1+k_{234}$	8b. $\mu_{234} e^{-2\lambda d_2}$
0.18182	3.6364	1.81818	0.49999	0.50001	1.49999	0.33334
.18410	3.6820	1.81590	.49318	.50682	1.49318	.33942
.18782	3.7564	1.81218	.48242	.51758	1.48242	.34915
.19046	3.8092	1.80954	.47504	.52496	1.47504	.35590
.19340	3.8680	1.80660	.46706	.53294	1.46706	.36327
0.19324	3.8648	1.80676	0.46749	0.53251	1.46749	0.36287
.19193	3.8386	1.80807	.47102	.52898	1.47102	.35960
.18798	3.7596	1.81202	.48197	.51803	1.48197	.34955
.17633	3.5266	1.82367	.51712	.48288	1.51712	.31829
.16316	3.2632	1.83684	.56290	.43710	1.56290	.27967
0.15183	3.0366	1.84817	0.60863	0.39137	1.60863	0.24329
.14172	2.8344	1.85828	.65562	.34438	1.65562	.20801
.13330	2.6660	1.86670	.70019	.29981	1.70019	.17634
.12635	2.5270	1.87365	.74145	.25855	1.74145	.14847
.12067	2.4134	1.87933	.77871	.22129	1.77871	.12441
0.11605	2.3210	1.88395	0.81170	0.18830	1.81170	0.10394
.09809	1.9618	1.90191	.96947	.03053	1.96947	.01550
.09561	1.9122	1.90439	.99592	.00408	1.99592	.00204
.09525	1.9050	1.90475	.99986	.00014	1.99986	.00007
.09525	1.9050	1.90475	.99987	.00013	1.99987	.00006
0.09525	1.9050	1.90476	0.99997	0.00003	1.99997	0.00000
.09525	1.9050	1.90476	.99997	.00003	1.99997	.00000

Determination of d_2

9 First Method

Select two points in the range where is not small;

λ = 0.09 and 0.10 .

$$\underset{\lambda=0.09}{f_2(\lambda)} = \frac{1-(0.9048)^2}{1-(0.9048)(0.83527)(1/(-0.73348)}$$

$\underset{\lambda=0.09}{f_2(\lambda)} = -5.97299$

$\underset{\lambda=0.09}{f_2(\lambda)-1} = -6.97299$.

$$\underset{\lambda=0.10}{f_2(\lambda)} = \frac{1-(0.9048)^2}{1-(0.9048)(0.81873)(1/(-0.72372)}$$

$\underset{\lambda=0.10}{f_2(\lambda)-1} = -8.68716$.

$$2d_2 = \frac{-1}{0.01}(\ln -8.68716 - \ln -6.97299)$$

$= 21.98$

$d_2 = 10.99 \doteq 10$

9a. Second Method: $\frac{\mu_{234} e^{-2\lambda d_2}\ (\lambda=n)}{\mu_{234} e^{-2\lambda d_2}\ (\lambda=n-1)}$	9a. ln of values in left col. x $\frac{1}{2(\lambda_2-\lambda_1)}$	10. $2\lambda d_2$	10a. $e^{-2\lambda d_2}$	11. μ_{234}	13. $1-\mu_{234}$	13a. $20(1-\mu_{234})$	13b. $1+\mu_{234}$	13c. k_{34}
		0.00	0.00000	0.33334	0.66666	13.3333	1.33334	9.99985
		.02	.98020	.34628	.65372	13.0744	1.34628	9.71150
		.06	.94176	.37074	.62926	12.5852	1.37074	9.18132
		.10	.90484	.39333	.60667	12.1334	1.39333	8.70820
		.14	.86936	.41786	.58214	11.6428	1.41786	8.21153
		0.20	0.81873	0.44321	0.55679	11.1358	1.44321	7.71599
		.30	.74082	.48541	.51459	10.2918	1.48541	6.92859
		.40	.67032	.52147	.47853	9.5706	1.52147	6.29036
		.60	.54881	.57996	.42004	8.4008	1.57996	5.31710
		.80	.44933	.62242	.37758	7.5516	1.62242	4.65453
		1.00	0.36788	0.66133	0.33867	6.7734	1.66133	4.07709
0.8512	8.055	1.2	.30119	.69063	.30937	6.1874	1.69063	3.65982
.8475	8.273	1.4	.24660	.71468	.28532	5.7064	1.71468	3.32797
.8418	8.611	1.6	.20190	.73536	.26464	5.2928	1.73536	3.04997
.8377	8.855	1.8	.16530	.75263	.24737	4.9474	1.75263	2.82284
0.8374	8.873	2.0	0.13534	0.76799	0.23201	4.6402	1.76799	2.62456
.1494	8.620	4.0	.01832	.84607	.15393	3.0786	1.84607	1.66765
.1317	9.251	6.0	.00248	.82291	.17709	3.5418	1.94294	1.94294
	10.0							

12.

Compute ρ_4 from k_{234} =

$$\frac{\rho_3 \rho_4}{\rho_2 \rho_3} = 0.5.$$

Compute ρ_3 from

$$\frac{\rho_2\rho_3 - \rho_3\rho_4}{\rho_2\rho_3 + \rho_3\rho_4} =$$

ρ_3 = 1.

ρ_4 = 10

14. $1-k_{34}$	14a. $1+k_{34}$	14b. $[\mu_{34} e^{-2\lambda d_3}]$
-8.99985	10.99985	-0.81818
-8.71150	10.71150	- .81328
-8.18132	10.18132	- .80356
-7.70820	9.70820	- .79399
-7.21153	9.21153	- .78288
-6.71599	8.81599	-0.77054
-5.92859	7.92859	- .74775
-5.29036	7.29036	- .72567
-4.31710	6.31710	- .68340
-3.65453	5.65453	- .64630
-3.07709	5.07709	- .60607
-2.65982	4.65982	- .57080
-2.32797	4.32797	- .53789
-2.04997	4.04997	- .50617
-1.82284	3.82284	- .47683
-1.62456	3.62456	-0.44821
-0.66765	2.66765	- .25027
- .94294	2.94294	- .32041

Determination of d_3

15a. First method

Select two points where λ is small;
λ = 0.01 and 0.04.

Compute $f_3(\lambda)$.

$$f_3(\lambda) = \frac{1-k_2^2}{1+\frac{k_2}{k_1}e^{-2\lambda d_2} - \frac{k_2}{k_1}e^{-2\lambda d_2} f_2(\lambda)}$$

$$\underset{\lambda=0.04}{f_3(\lambda)} = \frac{1-(-0.9048)^2}{1-0.44933+(0.44933)(-2.23270)}$$

$= -1.40071$.

$$\underset{\lambda=0.01}{f_3(\lambda)} = \frac{1-(-0.9048)^2}{1-0.81873+(0.81873)(-1.4924)}$$

$= -1.17426$.

$$2d_3 = \frac{-1}{0.04-0.01}(\ln -1.40071 - \ln -1.17426)$$

$d_3 = 2.939 \doteq 3.0$

Since d_3 is linear, or nearly so, at small values of (λ) the curve of $f_3(\lambda)$ is a straight line and $d_4 = \infty$.

15b. Second Method: $\frac{\mu_{34} e^{-2\lambda d_3}\ (\lambda=n)}{\mu_{34} e^{-2\lambda d_3}\ (\lambda=n-1)}$	ln of values in left column
0.9386	3.169
.9418	2.998
.9418	2.998
.9417	3.003
.9417	3.003

Column 1. Tabulated values of the kernel $k_{1234}(\lambda)$.

Column 2. Let $x = \mu_{1234}\, e^{-2\lambda d_1}$

then

$$\frac{1-x}{1+x} = k_{1234} \text{ and the vaule of } \mu_{1234}\, e^{-2\lambda d_1} \; ;$$

or,

$$\mu_{1234}\, e^{-2\lambda d_1} = \frac{1-k_{1234}}{1+k_{1234}} \, .$$

Column 3. Tabulated values of 2λ from the kernel function argument.

Column 4. Select two points in the range where λ is large (between $\lambda = 0.5$ and $\lambda = 1.0$, for example $\lambda = 0.80$ and $\lambda = 0.60$.

Now

$$\frac{k(\lambda)+1}{k(\lambda)-1} = f_1(\lambda)$$

and

$$\frac{1-\mu_{1234}\, e^{-2\lambda d_1}}{1+\mu_{1234}\, e^{-2\lambda d_1}} = k(\lambda),$$

therefore, letting $ce = \mu_{1234}\, e^{-2\lambda d_2}$

$$f_1(\lambda) = \frac{\dfrac{1-ce}{1+ce}+1}{\dfrac{1-ce}{1+ce}-1} = \frac{1}{-ce} = \frac{1}{\mu_{1234}\, e^{-2\lambda d_1}} \, .$$

For $\lambda = 0.8$, $\mu_{1234}\, e^{-2\lambda d_1} = -\,0.18267$ and

for $= 0.6$, $\mu_{1234}\, e^{-2\lambda d_1} = -\,0.27249$.

Then $f_1(\lambda)$ at $\lambda = 0.8$ equals

$$\frac{-1}{-0.18267} = 5.47435 \, , \quad \text{and at } \lambda = 0.6,$$

$$f_1(\lambda) = \frac{-1}{-0.27249} = 3.66986 .$$

The slope of the line is

$$\frac{1}{0.8-0.6}(\ln 5.47435 - \ln 3.66987) = 2d_1 =$$

$$\frac{1}{0.2}(1.70006 - 1.39991) = 1.9996 \text{ and }$$

$$d_1 = 1.00 .$$

Since $f_1(\lambda)$ is the negative reciprocal function of $\mu_{1234}e^{-2\lambda d_1}$, it can also be shown that the slope can be determined from the values of $\mu_{1234}e^{-2\lambda d_1}$ directly as follows. Using the same points as above,

$$2d_1 = \frac{1}{0.80 - 0.60} (\ln -0.18267 - \ln (-0.27249))$$

$$= \frac{1}{0.2} (-1.7007 - (-1.30016))$$

$$= 1.9996$$

$$d_1 = 1.00 .$$

An alternate procedure for determining d_1 is as follows. Start with values of $\mu_{1234}e^{-2\lambda d_1}$, λ being near the upper range of the computation, and divide the value for $\lambda = n$ by the value of $\lambda = n - a$ where a is the interval between successive values regardless of the amount. Determine the natural logarithm of this quotient. Then,

$$d_1 = -\frac{1}{2(\lambda_2 - \lambda_1)} (\ln \frac{\mu_{(\lambda=n)}}{\mu_{(\lambda=n-a)}}) .$$

As λ becomes large, d_1 approaches its true value.

Column 4a. Tabulate the values of either $e^{-2\lambda d_1}$ or $e^{2\lambda d_1}$ from a table.

Column 5. Compute μ_{1234} either by division with $e^{-2\lambda d_1}$ values or by multiplication with $e^{2\lambda d_1}$ values depending on which were tabulated.

Column 6. Since μ_{1234} approaches $(\rho_1 - \rho_2)/(\rho_1 + \rho_2)$ at large values of λ, then at $\lambda = 1.0$, $(\rho_1 - \rho_2)/(\rho_1 + \rho_2) = -0.9048$.

Solving,

$$1.9048\,\rho_1 = 0.0952\,\rho_2 \text{ , or}$$

$$\rho_2 = 20\,\rho_1 \text{ . Therefore, } \rho_1 = 1 \text{ and } \rho_2 = 20 \text{ .}$$

Determine k_1 from $\rho_2/\rho_1 = (1 + k)/(1 - k)$; $k_1 = 0.9048$.

As a check, the intercept on the ordinate axis may be computed using the equation,

$$\log \frac{y}{b} = M\,x$$ where M is the slope determined from the use of uniform scales in plotting.

Columns 7, 7a, 7b, 7c, μ_{1234} equals $(\rho_1 - \rho_2 k_{234})/(\rho_1 + \rho_2 k_{234})$; let $x = k_{234}$, $c = \mu_{1234}$, $\rho_1 = 1$, and $\rho_2 = 20$.

Then,

$$\frac{1 - 20\,x}{1 - 20\,x} = c \text{ , and } x = \frac{1 - c}{20(1 - c)} \text{ or}$$

$$k_{234} = \frac{1 - \mu_{1234}}{20(1 + \mu_{1234})} \text{ .}$$

The values tabulated will be evident from the above explanation.

Columns 8, 8a, 8b. The following explanation will clarify the tabulations in these columns.

$$k_{234} = \frac{1 - \mu_{234} e^{-2\lambda d_2}}{1 + \mu_{234} e^{-2\lambda d_2}} \; ; \quad \text{let} \quad ce = \mu_{234} e^{-2\lambda d_2} ,$$

then,

$$k_{234} = \frac{1 - ce}{1 + ce} .$$ Transposing, solving for ce,

and substituting,

$$\mu_{234} e^{-2\lambda d_2} = \frac{1 - k_{234}}{1 + k_{234}} .$$

Column 9. Since $f_2(\lambda)$ equals

$$\frac{1 - k_1^2}{1 - k_1 f_1(\lambda)} ,$$ and $f_1(\lambda)$ equals

$$- \frac{1}{\mu_{1234} e^{-2\lambda d_1}}$$ (the negative reciprocal of $\mu_{1234} e^{-2\lambda d_1}$) .

Using the negative reciprocal value for $f_1(\lambda)$,

$$f_2(\lambda) = \frac{1 - k_1^2}{1 - k_1 \xi_1 \left(\frac{1}{\mu_{1234} e^{-2\lambda d_1}}\right)} .$$

Select two points in the range where λ is not small; $\lambda = 0.09$ and $\lambda = 0.10$.

$$f_2(\lambda)_{\lambda=0.09} = \frac{1 - (0.9048)^2}{1 - (0.9048)(0.81873)\left(\frac{1}{0.73448}\right)} = -5.97299$$

$$f_2(\lambda)_{\lambda=0.09} - 1 = -6.97299 .$$

$$f_2(\lambda)_{\lambda=0.10} = \frac{1 - (0.9048)^2}{1 - (0.9048)(0.81873)\left(\frac{1}{0.72372}\right)} = -7.68716$$

$$f_2(\lambda)_{\lambda=0.10} - 1 = -8.68716 .$$

$$2d_2 = \frac{-1}{0.10-0.09}\left(\ln -8.68716 - \ln(-6.97299)\right)$$

$$2d_2 = \frac{-1}{0.01}(-2.16184 - 1.94204)$$

$$= 21.980$$

$$d_2 \doteq 10.0 .$$ (or $\simeq$)

Since $f_2(\lambda)$ is negative, $\ln -(k_1/k_2)$ is used and the intercept on the ordinate axis determined as in column 4.

Column 9a. An alternate procedure for determining d_2 is the same as the one described for d_1 except that values for $\mu_{234} e^{-2\lambda d_2}$ are used. Values for $\lambda = n$ are divided by values for $\lambda = n - a$ where a is as before the interval between successive values. λ is again selected in the range of larger values. The natural logarithm of the quotient is then obtained from a table. The value of the logarithm approaches the value of d_2 reaching this value at large values of λ ; $d_2 = 10$.

Column 10. Tabulations of $2\lambda d_2$, $d_2 = 10$.

Column 10a. Tabulate either $e^{-2\lambda d_2}$ or $e^{2\lambda d_2}$.

Column 11. Compute μ_{234} either by division using the descending exponential or by multiplication using the ascending exponential.

Column 12. Compute ρ_4. $k_{234} = (\rho_3\rho_4)/(\rho_2\rho_3) = 0.5$. Therefore, $\rho_3\rho_4 = 0.5\,\rho_2\rho_3$ and $\rho_4 = 0.5\,\rho_2$ or 10 .

Compute ρ_3. $\mu_{234} = (\rho_2\rho_3 - \rho_3\rho_4)/(\rho_2\rho_3 + \rho_3\rho_4) = 0.33333$,

$\rho_3 = 1$.

Columns 13, 13a, 13b, 13c. Compute k_{34} .

$$\mu_{234} = \frac{\rho_2 - \rho_3 k_{34}}{\rho_2 + \rho_3 k_{34}} \quad . \quad \text{Let } k_{34} = x \text{ and } \mu_{234} = c;$$

$\rho_2 = 20$ and $\rho_3 = 1$.

$$\text{Then,} \quad c = \frac{20 - x}{20 + x} \quad ; \quad x = \frac{20(1 - c)}{1 + c} = k_{34} = \frac{20(1 - \mu_{234})}{1 + \mu_{234}} \; .$$

The values tabulated will be evident from the formula for k_{34} .

Columns 14, 14a, 14b. Obtain $\mu_{34} e^{-2\lambda d_3}$.

$$k_{34} = \frac{1 - \mu_{34} e^{-2\lambda d_3}}{1 + \mu_{34} e^{-2\lambda d_3}} \; . \quad \text{Let } k_{34} = c \,, \; \mu_{34} e^{-2\lambda d_3} = ue,$$

then,

$$c = \frac{1 - ue}{1 + ue} \,, \quad ue = \frac{1 - c}{1 + c} \quad \text{or}$$

$$\mu_{34} e^{-2\lambda d_3} = \frac{1 - k_{34}}{1 + k_{34}} \quad .$$

The relation of this equation to the tabultion is evident.

Column 15. Select two points where λ is small, for example, $\lambda = 0.01$ and $\lambda = 0.04$. Compute $f_3(\lambda)$ for each.

$$f_3(\lambda) = \frac{1 - k_2^2}{1 - (k_2/k_1) e^{-2\lambda d_2} - (k_2/k_1) e^{-2\lambda d_2} f_2(\lambda)}$$

where $f_2(\lambda)$ and $f_1(\lambda)$ are the same as given in columns 9 and 4 above.

$$\underset{\lambda = 0.04}{f_3(\lambda)} = \frac{1 - (-0.9048)^2}{1 - 0.44933 + (0.44933)(-2.23270)}$$

$$= -0.40071$$

$$f_3(\lambda)_{\lambda=0.04} - 1 = -1.40071 \; .$$

$$f_3(\lambda)_{\lambda=0.01} = \frac{1-(-0.9048)^2}{1-0.81873+(0.81873)(-1.4924)}$$

$$= -0.17426 \; .$$

$$f_3(\lambda)_{\lambda=0.01} - 1 = -1.17426 \; .$$

Determine d_3 from

$$2d_3 = \frac{-1}{0.04 - 0.01} (\ln -1.4007 - (\ln -1.17426))$$

$$= \frac{-1}{0.03} (-0.33697 - (-0.16063))$$

$$= 5.878$$

$$d_3 = 2.939 \doteq 3.0 \; .$$

(or $\simeq$)

λ was selected at small values to determine if the $f_3(\lambda)$ function was linear and, it being so, then $d_4 = \infty$.

Column 15a. To compute d_3 by an alternate method, the procedure outlined for the alternate computation of column 4 and the computation of column 9a is used except that values for $\mu_{34}e^{-2\lambda d_3}$ are substituted in the equation,

$$d_3 = -\frac{1}{2(\lambda_2 - \lambda_1)} \ln \frac{\mu(\lambda=2)}{\mu(\lambda=1)} \; .$$

The entire group of values may be computed for d_3, but if values near $\lambda = 0$ are constant, it may be assumed that all values lie on the asymptotic straight line for which d_3 is the slope; therefore $d_4 = \infty$. However, if the values near $\lambda = 0$ are not constant the computations must be continued to determine the remaining resistivity constants for the subsequent layers.

rho The solution for the reversal of the kernel k_{1234} has given $\rho_1 : \rho_2 : \rho_3 : \rho_4 = 1 : 20 : 1 : 10$ and $d_1 : d_2 : d_3 : d_4 = 1 : 10 : 5 : \infty$, the same values as started with in the originally computed kernel of table 13.

Use of depth factors of Evjen in interpretation of resistivity curves.

The use of depth factors in the interpretation of resistivity depth profile curves was reviewed by Evjen (1938) and the method placed on a theoretical basis by the use of image strengths. A depth factor may be defined as the depth of maximum weight expressed as a definite part of the electrode spread. There is, however, no universal depth factor such that the depth of penetration of the exploring current into the ground is given as a certain fraction of the electrode spread. Instead, by the use of image strengths, a universal depth factor does exist for a given electrode arrangement, and the resistivities at various depths may be calculated from the image strengths.

It is clearly shown that the success of the method depends on the accuracy of the observations so that calculations of the first and second derivatives of W, the observed weight, may be justified. In general, the accuracy with which the resistivities at depth may be calculated from the observations increases with the number of derivatives which can be justifiably computed. Usually the second derivative cannot be computed with much confidence because both observational errors and irregular variations in the resistivity of the surface layer.

The weight function, Q_o, of Evjen exhibits a broad feature and is a smooth curve. As such, it is a theoretical impossibility, as well as physical, for the apparent resistivity curve to show sharp features as a result of changes of resistivity with depth. Any ripple on the curve of "wave length" much less than the electrode separation at which the disturbance occurs must be ascribed to observational errors or to irregular variations in the resistivity of the surface layer.

The weight function aP_1 for the Wenner electrode arrangement is given in figure 96 and shows the distribution of weights with depth. The weight function aP_2 for the modified Wenner arrangement is also given on this figure.

As the images associated with the interface appear at twice the depth of the interface, then the depth factor of V is approximately 0.108 when referred to the total spread.

$V = a\Delta_1 - e_o$ where a is the Wenner interval, Δ_1 is the potential difference of the Wenner arrangement, and e_o is the point source.

The depth factor refers to the sum of the images associated with an interface but, under certain favorable conditions, has a more direct relation to the real depths to the interfaces. For example, if a layer is present beneath the surface such that the apparent resistivity curve taken at the surface has a maximum or minimum, this maximum or minimum usually will come at a total electrode spread which is approximately nine times the depth to the upper interface.

The resolving power of the method is inadequate to give detail comparable to that which can be obtained from a well log. The resolving power also decreases rapidly for increasing electrode spreads, hence, increasing depths. This latter condition prevails because the "half-width" of the weight function is directly proportional to the electrode spread which is related to the depth explored. The "half-width" is approximately one and one-half times the depth explored. Therefore, in general two electrical interfaces of discontinuity cannot be resolved by surface measurements of electrical resistivity unless one is at least twice as deep as the other. However, if the upper interface has a higher resistivity than the lower one, the resolving power is somewhat better than if the opposite condition prevails.

Interpretation of horizontal profile resistivity curves.

None of the theoretical methods used for the interpretation of vertical, or depth, profile resistivity curves may be used to interpret the curves obtained when the electrical resistivity measurements are taken at a single spaced electrode setting, the setting then moved at a certain interval distance along a straight line on the surface of the earth. Such observational measurements are termed horizontal resistivity profiling.

The interpretive procedure for the horizontal resistivity profile is usually based on a comparison with observations over a known area or over areas that are similar, or else with measurements made on samples of the known subsurface materials. The sample measurements may be from either laboratory tests or in situ tests, preferably the latter. For the location of faults, dikes, or certain other geologic features, there are other aids to interpretation which will be described later in this section.

__Combination with vertical depth profile curves.__ Horizontal resistivity profiling results may be greatly improved and made more meaningful by combining the method with vertical depth profiling. For example, at a given station observations are taken for a series of electrode intervals, 10, 20, 30, 40, 50, 60, 80, 100 ft, etc., and then at the next station on the horizontal profile the same set of intervals are taken. This is continued along the entire horizontal profile distance. In this way, a series of horizontal resistivity profiles may be obtained as well as a series of vertical depth profiles, and, from them both, changes of the materials and the depths to the materials may be more accurately determined. Such procedure supplies the observer with a more complete and satisfactory portrayal of the character of the materials beneath the surface of the ground because the resistivities and depths to the different layers can be determined. An example of such resistivity measurements are shown in figure 97 where both horizontal and vertical resistivity profiles were made along the traverse in order to locate sand and gravel deposits for highway construction in northwestern Minnesota.

It is not always a safe estimate to assume that a resistivity anomaly, which appears on a horizontal resistivity profile taken at a fixed depth, is actually at that depth. Much better determinations of depth and resistivity will be possible with the procedure outlined above, and if an anomaly consistently appears at a certain depth only on a certain horizontal profile of related stations, then it is definitely located near that depth and can be actually determined throughout the traversed distance.

Interpretation of geologic features such as faults, dikes, etc., from horizontal profiles. The interpretation of other geologic features such as faults, dikes, and other three dimensional bodies, by means of horizontal resistivity profiles is discussed by Heiland (1940, pp. 718-721), by Jakosky (loc. cit., pp. 498 - 507), by Roman (loc. cit., pp. 201-202), and by others. Tagg (1930) gave four sets of computed curves to assist in the interpretation of the observed resistivity curves obtained both in horizontal and vertical electrical profiling. Two sets of the curves are for the case where the constant interval electrode system moves along one material, across the boundary of the fault plane one electrode at a time, and over into the other material. One set of the curves, figure 98, are for σ_a/σ_1 or the reciprocal ρ_a/ρ_1 against ρ_1/ρ_a d/a with k positive where d is the distance from the fault and a is the interval distance; and the other set is for ρ_a/ρ_1 with k negative, figure 99.

It will be seen from the accompanying figures that when an electrode system crosses a fault, each electrode produces a discontinuity in the curve so that there are four in number for the Wenner array of electrodes. When the electrode arrangement is expanding, the figures show that there are only two discontinuities, one for each electrode crossing the fault boundary.

In the same article, Tagg also shows the effect of a fault plane parallel to the line of electrodes with a set of curves. It will be seen from figure 102 that an electrode system must be at a distance of four times the electrode interval a from a vertical fault for it to have a negligible effect on the observations of apparent resistivity.

Cook and Van Nostrand (1954) have made both theoretical and applied studies of the location of shale sinks in limestone areas by means of horizontal electrical profiling using direct current methods and the Lee arrangement of electrodes, and they also have studied the effect on the depth profile curve of the distance of the sink from the electrode configuration. Measurements were also made with the center of the electrode configuration placed at various distances from the center of the filled sink, and the observations madeeby the vertical depth profile method. Their paper is not readily condensed and the reader is referred to the original article for complete information.

Logn (1954) applied the one electrode configuration to the study of discontinuities and developed approximation formulas for the computation of related resistivity curves. Like the previous paper referred to, this article does not condense readily and the reader will find the original work valuable.

The problem of determining the potential distribution about a point source near one or more parallel plane boundaries separating two or more homogeneous isotropic materials has been solved by Guyod (1944, pt. 5), Jakosky (1950, p.483), Smythe (loc. cit., pp. 182-183), and probably others. Resistivity measurements on the surface of the earth are a particular case of the above problem when the location of dikes, faults, and other geologic features with vertical boundaries are encountered. Buckner (1954) applies the problem specifically to the case of the bore hole normal to the beds as required in well-logging, and develops a function for which curves are given to assist in the interpretation of the problem. The curves given by him are of similar character to those given in the previous references.

Application of Gish-Rooney instruments. Gish-Rooney type equipment and procedures associated with its use may be readily applied to horizontal electrical profiling. Application of the Lee arrangement of electrodes to the horizontal profiling procedure will enable checks to be made of the electrode balance with consequent higher validity for the measurements of surface potential and resulting apparent resistivity. Also, changes in the character of the materials and their depths beneath the surface will be more readily apparent with the use of the above mentioned electrode array. It is, therefore, advisable to obtain potential measurements with both the Wenner and Lee array in order to check completely the surface potential measurements.

Correlation of resistivity observations with geological information

The successful interpretation of apparent resistivity curves depends, to a large degree, upon the correlation available or obtainable with the geological materials of the immediate area in which the measurements were made. Too much reliance cannot be placed on so-called drillers logs of drill holes or wells. If the samples from the drill holes, or the cores, have been examined by a geologist and the materials identified, then the logs are of much more value. Even so, it will be found that the electrical characteristics of the earth materials frequently do not closely parallel the lithological characteristics described in the log, in fact, it is often the case that the correlation is poor or even non-existant.

Two examples show this condition particularly well. The one is in the glacial area of Wisconsin, the other in the sedimentary area of New Mexico, figures 103 and 104 respectively.

Both holes were carefully logged from the samples by a geologist. The figures serve to emphasize the study and judgement the interpreter must give to the evaluation of the correlation between the apparent resistivity curve and the drilling log.

It has been shown by Pekeris (loc. cit., pp. 36-38) and others, that a thin layer at depth cannot be detected by resistivity methods. If the thin layer lies beneath one surface layer, and $k(\lambda)$ the kernel, is known only to five percent, the imbedded layer could not be detected with certainty by surface resistivity methods. Should the thin layer lie below two surface layers, then the relevant function is $f_2(\lambda)$, which is necessarily known less accurately than $k(\lambda)$, there would be consequently a still smaller possibility of detecting the thin layer under this condition. With the thin layer below three surface layers, the possibility for detecting it by surface measurements becomes still less because $f_3(\lambda)$ is known with much less accuracy than $f_2(\lambda)$. Comparison of the apparent resistivity curve with the drilling log in figure 104 (Spicer, 1941), makes it obvious that thin layers cannot be detected at depth by surface resistivity methods. Instead, the thin layers tend to act in conjunction and appear as an average value on the curve of apparent resistivity. In general, the deeper a layer is embedded beneath a series of layers the thicker it must be to cause an indication on the apparent resistivity curve obtained by surface potential measurements.

Much valuable information for correlating the interpretations of the apparent resistivity curves may be obtained by in situ measurements on the exposed geological materials themselves. This is done by setting up the resistivity equipment in the same manner as if a depth profile was to be obtained, and taking measurements at small intervals with the Wenner configuration or the Lee modification of the same. In this manner, an apparent resistivity curve is obtained that may be interpreted giving computed resistivities of the materials over which the measurements were made. If several such measurements are made over different outcrops an approximate range of resistivity will be obtained.

Some precautions should be carefully observed, however, in making in situ measurements. If the exposed outcrop is solid rock into which the electrodes cannot be driven, small "mud-pies" may be made on the rock and short, small diameter, copper electrodes placed in the mud-pie at proper distances. If the electrodes can be driven into the earth materials, penetration should never be more than one percent of the electrode interval being measured. As before, short, small diameter, copper or copper-clad steel, electrodes should be used. Greater penetration of the electrodes than this amount will tend to disrupt the point source and sink resulting in a badly distorted electrical field in the ground, and as a consequence the measurements of potential will be distorted and of questionable worth. The area of the outcrop should be carefully chosen. In particular, it should be flat and have both vertical and horizontal dimensions of such an extent that the electrical field will be contained therein for the largest interval measured.

The apparent resistivity curves obtained by the above tests will be one, two, or perhaps three layer curves. If curves having a more complex layering are obtained, then the formation, or material, test is probably of little value and another location should be sought. The complex curves may be caused by numerous conditions such as: irregular bedding or layering; too many unrecognized electrical layers below the material being tested; poor electrode contact; poorly adjusted instrument; and perhaps other more local conditions which the observer alone must be able to recognize as interference.

It frequently happens that resistivity measurements must be made in an area where there is no knowledge of the geological materials beneath the surface of the earth, or in an area where the geological information is confined to logs from shallow wells and inferred knowledge of the subsurface materials. Under such conditions, the geophysicist must rely upon his breadth of experience to interpret the materials correctly from the apparent resistivity curves.

The author has found it very helpful to keep a continuous record of the complete interpretations of the apparent resistivity curves arranged in a related manner. A portion of such a record is shown in table 15 which was made in conjunction with the interpretations of the apparent resistivity curves for the Marshfield, Wisconsin project (Spicer, loc. cit.)

Table 19.

Portion of a continuous record of
complete interpretations of apparent resistivity curves,
Marshfield, Wisconsin.

Soil and surface materials 6 feet			Near surface materials			Mid-depth materials sand and gravel		
Materials	Depth feet	Computed resistivity in ohm cms	Materials	Depth feet	Computed resistivity in ohm cms	Materials	Depth feet	Computed resistivity in ohm cms
sdy cl soil	0 - 1.15	16,000	cl and hard pan	1.15- 4.6	5,333	sd gvl	4.6 - 18	15,770
cl	0 - 5.9	5,390	cl	5.9 - 12.0	1,797	sd gvl	12 - 47	64,060
cl and gvl	0 - 1.6	22,600	cl	1.6 - 3.2	2,235	gvl	3.2 - 34	21,350
cl and gvl	0 - 1.4	9,550	--	--	--	gvl	1.4 - 5.7	28,650
sd and cl	0 - 0.8	5,550	cl	0.8 - 3.1	1,833	--	--	--

Mid-depth materials Sand - gravel - clay			Mid-depth materials clay			Bedrock granite		
Materials	Depth feet	Computed resistivity in ohm cms	Materials	Depth feet	Computed resistivity in ohm cms	Materials	Depth feet	Computed resistivity in ohm cms
--	--	--	cl	18 - 46	3,610	granite	46 - 200	488,500
sd gvl cl	47 - 79	8,340	--	--	--	granite	79 - 200	586,500
gvl some cl	34 - 74	9,150	--	--	--	granite	74 - 200	549,500
cl some gvl	5.7- 52	7,850	--	--	--	granite	52 - 200	551,200
cl sd gvl	3.1- 52	5,510	cl	12 - 52	3,240	granite	52 - 200	447,800

Field procedures, case histories, and external disturbances

It is not possible to include herein all the resistivity studies which the writer has made during the past two decades. However, nearly all of these studies, along with the observations or interpretations, are available in the form of reports, open file reports, or in publications of the Geological Survey. Therefore, a selection will be made from the various kinds of studies, and the material will be chosen so as to either typify the work done or else emphasize the important or unusual feature encountered.

Methods of setting electrodes. Most workers have always followed the practice of wetting or mudding in each electrode at each interval, but this is not necessary for certain conditions of the near surface materials. If observations are being made following a soaking rain, it will be found that the earth is usually wet enough to make satisfactory contact with the metal electrodes provide that they are tightly tamped after driving. However, it is emphasized that it still remains necessary to balance the electrodes in order to keep the electrical fields set up in the earth undistorted. There are, of course, certain places such as over gravels, sands, or similar high resistivity materials, where it is not possible to obtain contact easily under any conditions and such areas are not considered in the above general statement.

In other areas, such as those encountered in the western part of the United States, where the near-surface materials contain a large proportion of adobe clay, it is possible in selected areas to set the electrodes dry and obtain generally satisfactory measurements. The same precautions for electrode balancing as mentioned in the preceding paragraph applies here as well. The measurements shown in figure 105 were taken with both dry and wet electrode settings on alluvial fill along the Little Colorado River near St. Johns, Arizona. Only five points on this curve were checked with dry set electrodes, four were exactly the same and one was slightly lower. This difference in the one value may have been caused by having the electrodes set too deep for a small interval distance.

<u>Measurements of unbalanced and balanced electrodes.</u> The effect produced on the measurements of potential with the electrodes unbalanced and then balanced is shown in table 16.

Table 16

Effect on measurements of potential
with electrodes unbalanced and balanced, depth profile
No. Fr-76A, near Columbus, Ohio.

Electrode interval a feet	Electrode balance test in milliamperes				Potential E		Interval
	P-1	P-2	C-1	C-2	Before balance	After balance	
45	32.3* 31.2**	25.9 30.0	29.2 32.8	34.1 30.2	28.2 13.9 14.2	28.1 14.1 13.9	Full P-1 P-2
50	28.0* 27.0**	29.0 28.0	30.9 31.1	31.8 31.8	28.2 13.8 14.0	28.2 13.8 14.4	Full P-1 P-2
100	58* 58**	58 58	62 62	52 62	20.3 10.2 10.0	20.3 10.0 10.2	Full P-1 P-2
240	178* 178**	170 170	168 159	158 157	27.8 13.8 13.9	27.7 14.8 12.8	Full P-1 P-2
280	175* 175**	195 178	143 143	193 145	30.7 15.3 15.3	30.4 16.3 13.8	Full P-1 P-2

* Before balancing; ** After balancing.

The values in the table are excerpts from a complete depth pro- made in Franklin County near Columbus, Ohio. The measurements of potential and current were all made with the commutator operating so as not to disturb the natural earth conditions, insofar as possible.

The distortion in the electrical field, which was discussed from a theoretical point of view in a previous section, is clearly evident from a comparison of all the tabulated values of potential before and following balance. It will be immediately apparent that a comparison of the Full values alone does not reveal the effects of electrode unbalance. At the 45 ft interval, the P's had reversed from their order in the previous interval measured, but when the electrodes were balanced the order reversed to the previous one. Upon reading the instrument for the 45 ft interval, the sum of the P's, which should equal the Full value, was too low but after balancing the electrodes this difference was located in the unbalanced P-readings. The P's are again reversed by balancing in the 100 ft interval measurements. The effect of a greater unbalance is shown in the 240 and 280 ft interval measurements. What appeared to be excellent readings were actually poor ones because of this large unbalance. A still greater difference in the unbalance causes a greater effect on the Full measurement as shown by the last interval of the table. As this unbalance between the electrodes becomes still larger the effect on the three potential readings becomes greater until the electrical field in the earth is so distorted that one of the P's becomes larger than the Full potential measurement and the other P becomes negative. This condition was described and illustrated in an earlier section and need not be extended.

Thus it is apparent that measurements made by using only the Lee electrode arrangement, without balancing the electrodes and without checking by the Wenner measurement, may be seriously in error. The same is also true in the reverse sense. In fact, doubt is cast upon the reliability of all measurements made in either of these ways.

Use of porous pots with Gish-Rooney measurements. The Gish-Rooney type earth resistivity equipment does not require the use of porous pots for measuring potential, only metal stakes. However, if and when they are used with this equipment, one or the other of the following arrangements should be used so that none of the intense current flow near the surface of the earth passes through them. The first is to have a thick pad of felt, or similar material, saturated with copper sulfate solution on which to place the porous pot electrode to elevate it above the surface of the earth. Actual contact between the earth and the porous electrode is then made through this pad, and any current flow does not disturb the chemical balance of the electrode. It is possible to build this pad integral with the electrode such as Bleil (1948) did. It is also possible to surround the electrode with an insulating material, such as suggested by Bleil (1953) to eliminate any polarization caused by the energizing field. Furthermore, it should also be mentioned that the contact of the porous pot electrode with the earth should be as small as possible in the horizontal dimension, particularly for the smaller interval measurements. Perhaps it is needless to state that the current electrodes should be balanced by means of a central copper electrode.

Effect of chemicals in the soil on electrodes. Because of the chemicals frequently present in the earth materials, and many of these have the property of generating an electrical potential upon contact with certain pairs of the electromotive series, it is inadvisable to use dissimilar metals for electrodes in Gish-Rooney measurements. The use of dissimilar metals may cause spurious potentials to be measured which can, in turn, produce incorrect or even valueless resistivity determinations. It has been reported by Krasnow (1940) that stainless steel and certain other iron alloys will very frequently be found to generate a potential when used as electrodes for Gish-Rooney work. This is particularly true, apparently, for alloys which develop crystal structure rather than those where the alloying metal enters the lattice structure.

Iron electrodes corrode badly according to Casagrande (1949), the corrosion causing an immediate reduction in the current flow because of the inert salts formed on the surface of the iron. Unless the corrosion forms uniformly on both current electrodes, an unbalanced electrode condition would be started at once. This electrode corrosion would be somewhat less using Gish-Rooney methods, the amount of corrosion depending on the time and strength of current flow.

Effect of contact potentials on measurements. Many pairs of metals upon contact develop a potential which may also add to the errors in measurements of surface potential. It is, therefore, advisable to use the same metal throughout the electrical circuit. It has always been the writers practice to use copper in resistivity measurements to eliminate the above physical effect. A discussinn of this subject is given by Planck (loc.cit., chap. 2).

Effect of battery polarization on measurements. Yet another effect, which is probably of a chemical nature, should be eliminated from potential measurements; namely, battery polarization. For reasons of age, current withdrawn, chemical decomposition, and perhaps others, a dry cell develops a high resistance to the withdrawal of current from it, but the open circuit voltage of the cell remains the same. If such a cell is present in a battery which is being used to energize the earth for resistivity measurements, a condition similar in effect to that of unbalanced electrodes may be set up. The measurements of surface potential, and computed resistivity, will vary in a manner similar to the behavior of the unbalanced electrodes but no amount of balancing will eliminate the condition. The cause of the trouble can be easily eliminated by frequent and regular testing of the battery voltage under load, and discarding all batteries showing an excessive voltage drop.

Effect of induction on measurements. The effects of induction have been eliminated, in-so-far as possible, in the Gish-Rooney equipment. In the instrument proper, including both the measuring unit and the commutator, either all wires carrying current are shielded or else all the potential circuit wires are shielded. In either case, the shields are at ground potential. The stray current pick-up between the potential and current sections of the commutator is eliminated by a guard ring, or grounded shield. Other possible minute circulating currents are eliminated by the aluminum panel of the instrument, it also being at ground potential. For shallow work, the value of shielding is not readily apparent, but for deep investigations it is of primary importance and usually is a determining factor in the success or failure of the resistivity measurements.

When the wire carrying reels are placed at the center of the electrode array, it is possible to get some inductive pick-up between them. In consequence, should it be necessary to keep the reels at the center of the line of electrodes, the reels should be arranged so that no induction effects could take place. The writers' preference is to move the reels with the electrodes as the electrode array expands outward, thus further eliminating any possible inductive effects from the reels of wire.

There is ,of course, some inductive effects possible from the wires connecting the electrodes to the instrument because of their close proximity. This can be minimized by the operators setting the electrodes if given instructions to keep the wires a certain distance apart. The inductive effect which may exist between the wires and the earth cannot be eliminated, that is, by any practical means.

The larger effects caused by induction mentioned above take place in Gish-Rooney equipment only during the make and break of the current; a period of about a millisecond for the make and about 8 to 10 milliseconds for the break. When the actual measurements are being made, only pure direct current is flowing, and it does not create the variable inductive effects of alternating current or rapidly oscillating transients.

Effect of improper care of commutator on measurements. Another very important item remains in regard instrument maintainance, in this instance the commutator. The contact area of the brushes on the commutator and the pressure on the brushes obviously cannot be designed for optimum performance, such as is used on a motor commutator. Consequently, oxidation proceeds very rapidly on both the commutator segments and brush faces, and it must be removed both frequently and thoroughly. Failure to do so means erratic measurements resulting in poor apparent resistivity curves which are frequently interpreted for materials which are not present in the earth at all, but are actually caused by the high resistivity oxides on the commutator segments and brushes.

Bounce of the brushes must never be permitted to occur either. Its happening produces transients during the period when measurements are being made that cause unpredictable potentials to be set up and measured. This results in apparent resistivity curves which cannot be interpreted to give the true conditions present in the earth.

In a general way, the precautions given previously for the instrument commutator apply also to the commutator on the wire reel. Although the brush pressure and brush area are entirely adequate for the current carried by the commutator on the wire reel, dirt and oxides accumulate under the brush and must be removed once in a while. The frequency of removal will depend largely upon working conditions, but the operator can easily remove the accumulation in just a few minutes, and it should be done regularly as a matter of routine maintainance.

External disturbances to Gish-Rooney measurements. Some of the external disturbances to Gish-Rooney electrical resistivity measurements will be discussed next and illustrations given of the effects upon apparent resistivity curves. These disturbances are termed noise by some writers. One group of these disturbances results from surface geological features encountered in difficult places where a curve must be obtained. Another group of these disturbances to electrical resistivity measurements results from the advancement of man in improving his comforts, conveniences and mode of living. To such advances as the following the operator must be keenly alert for the anomalous results obtained. Metal fences with or without metal posts; buried pipe lines such as gas or water mains, sewers, concrete irrigation pipes; buried cables, such as telephone cables and tower grounds; power lines, both the usual type and the more recent R. E. A. type using an earth return; and electric railways.

Effect of a gully on a resistivity curve. In the investigation of foundations for dams or investigations of the character and amount of fill at a dam site, it frequently happens that the only area adequate in size for completeing a vertical electrical depth profile is cut by deep washes or gullies. It seems reasonable to assume that such surface characteristics would be expressed in some manner on the apparent resistivity curve for the depth profile obtained over such an area. Figure 106 shows the apparent resistivity curve obtained where such a situation was encountered. The observations were made on the alluvial fill of the right bank of the Zuni River near St. Johns, Arizona. The center of the line of observations was positioned so that the deep gully located about 90 feet upstream from the center would interfere least with the observations required to get the character of fill and the depth to bedrock.

It is apparent from figure 106, that as the current electrode crosses the gully, the resistivity curve is effected both on the P-1 and P-2 halves but the effect is more pronounced on the P-2 side. Then as the potential electrodes reach the gully the Full curve immediately drops about 350 ohm cms and the P-1 and P-2 curves drop in a proportional manner. The complete explanation for this behavior in the curve was not determined by measurements in the field, but it is evident that the gully caused the potential field to be distorted in such a manner that the effect of the low resistivity material above the bedrock was emphasized on the apparent resistivity curves, thus causing the curves to drop suddenly. It would be very interesting as well as instructive to determine the potential field under such a condition, and to have the actual field known with its' distortion caused by a gully, or void in the surface layers. A stream, such as a creek or river will usually cause the same effect as a gully on the apparent resistivity curve, but the magnitude is usually less because of the large difference in resistivity of the materials filling the void.

Effect of a narrow remnant of alluvium on a resistivity curve. Another condition that is often encountered in dam site investigations, is to have a narrow remnant of alluvium in the stream channel, such as shown in figure 107, where the thickness of fill and depth to bedrock has to be determined.

The alluvial fill is not continuous and has a cliff-like appearance along the side towards the present stream. It also has an unknown type of contact with the rock forming the walls of the stream channel, i.e., it may be vertical, horizontal, sloping, or an irregular contact.

In either instance, the surface potential measurements will be greatly disturbed by the influence of the infinite resistivity of the air on the one side of the remnant of alluvium and the high resistivity of rock wall on the other side. A depth profile curve obtained on the remnant of alluvium shown in the previous figure clearly depicts the distortion produced, figure 108.

The ascending part of the curve is much steeper than the dotted help curve shown in the figure for a theoretical three-layer curve with the bottom layer of very high resistivity. Even with $k = +1.0$, the slope of the help curve would change inappreciably.

While the above illustration represents a very extreme condition obtainable in dam site investigations, there are many other variations in layering at dam sites of such a nature that their presence can only be suspected on the apparent resistivity curves. It is possible that other geophysical methods in some instances, such as seismic, could determine the subsurface conditions; but if they do not the only recourse is by drilling.

Effect of metal fences on measurements. Barbed wire fences on dry wood posts cause no disturbances in surface potential measurements. If the fence is supported on concrete posts, they should be tested for leakage as they may contain steel mesh or rods which are in contact with the earth. Old and dilapidated fences, where the strands of barbed wire or other metal fence have fallen down and have been buried by grass and soil, forms an excellent conductor that will badly distort the energizing field set up in the earth. Barbed wire or other metal fence on metal stake fence posts will cause the energizing field to be distorted so much that correct measurements of surface potential cannot be obtained. Another unsuspected cause of electrical field distortion is that of hog-wire fences on wood or concrete posts where grasses and wind-blown soil have accumulated around the lower strands of wire. Except in the very arid parts of the country, this accretion and the wire strands forms an excellent conductor thus producing a large amount of distortion in the electrical field which predominates in the surface potential measurements obtained nearby.

The effect produced on an actual apparent resistivity curve by a hog-wire fence is shown in figure 109.

The positioning of the line of electrodes with respect to the fences is shown in the insert of the above figure. The leakage current, measured between the electrode at the center of the line of electrodes G and the fence, when the C-1 electrode reached the fence was 50 ma at $22\frac{1}{2}$ volts, and when P-1 reached the fence, the leakage current was 185 ma at 84 volts. The actual distortion of the energizing field in the earth caused by the grounded fence was not determined, because this is a very time consuming operation and does not assist greatly in the interpretation of the results. It will be obvious to anyone familiar with electrical resistivity measurements, however, that the energizing field is so badly distorted that the apparent resistivity measurements are completely unreliable for purposes of interpretation.

The effects from metal fences on the measurements of surface potential can never be passed by or ignored. Figure 110 shows an apparent resistivity depth profile curve that was obtained when the line of electrodes was midway between two parallel fences spaced about 400 ft apart.

The leakage current to these fences measured low, the amount is not recorded in the notes but was probably very small or the observations would have been stopped, and it was considered when the measurements were made that a fairly good curve had been obtained. The curve was interpreted and a depth to bedrock was found which seemed reasonable, 127 feet. It was found out later by drilling that the depth here to bedrock is over 300 feet. What happened is obvious; the curve is distorted and resulted in valueless interpretations because of the unsuspected distortion created in the energizing electrical field by a metal fence. A safe rule about grounded metal fences would be: never attempt measurements for a depth profile curve whose center or any part of the electrode array will come within ~~twice and preferably~~ three ~~or~~ to four times the maximum interval distance which will be used in the measurements. This same rule applies as well to other buried conductors in the earth. Some disturbances on apparent resistivity curves created by buried pipe lines will be illustrated next.

Effect of a water supply main on measurements. The effect of a city water supply main on a depth profile curve is shown in figure 111. These measurements were taken in a section of the city of Binghamton, New York that had not been built up, but the area was surrounded by water mains. It was supposed that the bedrock beneath the area was shale and its expected depth was about 150 feet. Because of the distortion in the curve, no interpretation of the curve was attempted.

Effect of a gas pipe line on measurements. The disturbance produced on an apparent resistivity depth profile curve by a cross country gas supply line is shown in figure 112. It was not known what had caused the erratic behavior of the curve until a search was made following the completion of the field measurements, and it was learned that the gas pipe line crossed the field where the measurements were completed. The bedrock of the area is probably limestone, but there is a possibility that it is overlain by a bed of shale. The depth of the bedrock beneath the surface is not known, but may be more than 11 feet. The measurements were taken a few miles southwest of Urbana, Ohio in the search for a buried pre-glacial stream channel. Not even a qualitative interpretation could be satisfactorily made of the curve.

Effect of a concrete irrigation pipe on measurements.

Frequently, in areas having irrigation systems in the western part of the United States, the water for irrigation is transported in large concrete pipes buried in the earth. It is not always possible to avoid these pipes in locating the electrode line for a resistivity depth profile, and, as explained, they cause an anomalous distortion of the apparent resistivity curve. An instance of such distortion is shown on figure 113. The measurements were taken in the valley of the Santa Ynez River near Lompoc, California. The bedrock is probably shale and may be less than 100 feet beneath the surface, but it was not possible to interpret the curve and obtain any depths or kinds of material other than for the soil at the surface.

Effect of a street sewer main on measurements. In another open area within the city limits of Binghamton, New York, a resistivity depth profile curve was distorted by a street sewer main along which the line of electrodes, unfortunately, were placed. Obviously, no interpretation could possibly be made of the curve obtained, which is shown in figure 114.

Effect of buried telephone cable on measurements. While making an investigation for salt-water intrusion in the Cutler area near Miami, Florida, one location where information was needed was found to have a 3/4 inch lead telephone cable buried under the surface of the earth at a depth of a few inches. With the hope that the cable was completely insulated from the earth, a resistivity depth profile was completed. However, there was some breakdown in the insulation of the cable for the apparent resistivity curve obtained was disturbed as will be noted in figure 115. The bedrock here is oolite limestone and was known to be near the surface, however, the depth to bedrock could not be ascertained with any certainty from the curve. Beyond the 50 ft interval, it will be noticed, the distortion caused by the cable made the observations worthless.

Effect of power lines on measurements. High voltage power line towers are grounded for protection from lightning, and frequently a heavy cable is used to tie them together. This ground-tie may be found either on the poles or buried below the surface of the earth. In either case, taking of surface potential measurements is not possible near them because of the distortion that will be introduced on the curves. No example can be given as these situations have always been avoided.

Power distribution lines as usually encountered will, ordinarily, give no trouble in measurements of surface potential unless they carry a common ground connection. Observations have been made successfully near single phase power lines having voltages as high as 44,000 in some places. Yet in other places the observations were valueless as in the instance shown in figure 116.

Beginning with the 15 ft interval, the galvanometer started to oscillate while measurements were being made and continued until the last reading was completed. Investigation of the area adjoining that where the measurements were taken revealed that the entire area was surrounded by an earth-return power line system which had caused all the oscillations in the galvanometer.

The type of power line mentioned above is becoming more and more prevalent throughout the rural areas and the operator should be keenly aware of the effects produced and be alert for its presence. If such an electrical distribution system is not suspected in an area and observations are attempted, but in doing so the galvanometer is found to oscillate, perhaps a small amount or even to the point where it is unreadable, one may be assured that such a distribution system is present. It would be advisable to seek immediately a new location for taking measurements as it will not be possible to obtain a useable apparent resistivity curve under conditions such as described.

Again, there may be locations where R. E. A. lines, which use earth returns, are known to be in the immediate vicinity of the place where the resistivity measurements are being made, but there is no oscillation or wandering of the galvanometer. This does not necessarily mean, though, that no effects on the measurements are being introduced by the ground-return power line. Instead, the effect is there but more difficult to determine. An illustration of an apparent resistivity curve obtained under such conditions as have just been described is shown in figure 117. The R.E. A. power line was located about 500 ft beyond the place where the C-1 electrode was last set. The distortion on the apparent resistivity curve is first noted with certainty at the 50 ft interval distance, becomes indeterminate between the 80 and 120 ft interval distances, and makes a more recognizable appearance from there to the end of the observations. While the measurements of surface potential were being made, it was also found that the electrodes could not be easily balanced. This is a possible, though not a definite, characteristic of earth conditions in an area near earth-return power lines.

Effect of electric railways on measurements. Electric railways are a source of disturbance to earth resistivity measurements and disturb them very much the same way as buried water mains, metal fences, or other near surface grounded conductors. The transients set up by the arcing between the trolley roller, or slide, and the trolley, between the brushes and the commutator, and between the contacts of the controller, will cause distortion in the Gish-Rooney measurements of surface potential. The effects produced by an electric railway operating by alternating current are only noticed while the train is operating in the near vicinity of the place where measurements are being made. During the period of time while the train is approaching, passing, and receding, the galvanometer needle swings erratically. Measurements, as expected, are not possible during this event and an illustration cannot be given.

APPENDIX

Table

Positive and negative values of (1 - k)/(1 + k)

	$\frac{1-k}{1+k}$ $k = +$	$\frac{1-k}{1+k}$ $k = -$	k	$\frac{1-k}{1+k}$ $k = +$	$\frac{1-k}{1+k}$ $k = -$	k	$\frac{1-k}{1+k}$ $k = +$	$\frac{1-k}{1+k}$ $k = -$	k	$\frac{1-k}{1+k}$ $k = +$	$\frac{1-k}{1+k}$ $k = -$
00	0.0000	∞	0.75	0.1429	7.000	0.50	0.3333	3.000	0.25	0.6000	1.667
99	.0050	199.000	.74	.1494	6.692	.49	.3423	2.922	.24	.6129	1.632
98	.0101	99.01	.73	.1561	6.407	.48	.3514	2.846	.23	.6260	1.597
97	.0152	65.67	.72	.1628	6.143	.47	.3605	2.774	.22	.6393	1.564
96	.0204	49.00	.71	.1696	5.897	.46	.3699	2.704	.21	.6529	1.532
95	0.0256	39.00	0.70	0.1765	5.667	0.45	0.3793	2.636	0.20	0.6667	1.500
94	.0309	32.33	.69	.1834	5.452	.44	.3889	2.571	.19	.6807	1.469
93	.0363	27.57	.68	.1905	5.250	.43	.3986	2.509	.18	.6949	1.439
92	.0417	24.00	.67	.1976	5.061	.42	.4085	2.448	.17	.7094	1.410
91	.0471	21.22	.66	.2048	4.882	.41	.4184	2.390	.16	.7241	1.381
90	0.0526	19.00	0.65	0.2121	4.714	0.40	0.4286	2.333	0.15	0.7391	1.353
89	.0582	17.18	.64	.2195	4.556	.39	.4388	2.279	.14	.7544	1.326
88	.0638	15.67	.63	.2270	4.405	.38	.4493	2.226	.13	.7699	1.299
87	.0695	14.38	.62	.2346	4.263	.37	.4599	2.175	.12	.7857	1.273
86	.0753	13.29	.61	.2422	4.128	.36	.4706	2.125	.11	.8018	1.247
85	0.0811	12.33	0.60	0.2500	4.000	0.35	0.4815	2.077	0.10	0.8182	1.222
84	.0870	11.50	.59	.2579	3.878	.34	.4925	2.030	.09	.8349	1.198
83	.0929	10.76	.58	.2658	3.762	.33	.5038	1.985	.08	.8519	1.174
82	.0989	10.11	.57	.2739	3.651	.32	.5152	1.941	.07	.8692	1.151
81	.1050	9.526	.56	.2821	3.545	.31	.5267	1.899	.06	.8868	1.128
80	0.1111	9.000	0.55	0.2903	3.444	0.30	0.5385	1.857	0.05	0.9048	1.105
79	.1173	8.524	.54	.2987	3.348	.29	.5504	1.816	.04	.9231	1.083
78	.1236	8.091	.53	.3072	3.255	.28	.5625	1.778	.03	.9417	1.062
77	.1299	7.696	.52	.3158	3.167	.27	.5748	1.740	.02	.9608	1.041
76	.1364	7.333	.51	.3245	3.082	.26	.5873	1.703	.01	.9802	1.020

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