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COMPUTATION OF BACKWATER AT OPEN-CHANNEL CONSTRICTIONS

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PREFACE

This pamphlet was prepared under the direction of Tate Dalrymple, by J. S. Cragwall, Jr., hydraulic engineer. Walter Hofmann and Howard F. Matthai, hydraulic engineers, and others participated in its preparation.

The procedures given herein are intended to be provisional only, inasmuch as detailed experience in computing backwater at single opening constrictions is limited, and because treatment for multiple-opening constrictions is yet to be fully evolved by laboratory experimentation.

Engineers are encouraged to study and apply the alternate methods of computation presented herein, and to devise further improvements and simplifications as they gain experience in analyzing field problems.

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Introduction

As a part of our cooperative highway programs in many states, the Survey prepares reports on the hydrologic and hydraulic factors of flood flow at specific sites of existing and proposed highway bridges. In most instances, a bridge and its approach embankments create a constricting effect upon flood flow, which, in turn, causes an increase in stage upstream from the highway crossing. This increase in stage above the normal or unconstricted stage is termed backwater, and is a factor that is generally of interest to a highway department.

Laboratory work concerned with flow through single-opening constrictions in open channels has been carried on for the past several years by the Atlanta Research Unit of the Surface Water Branch. The laboratory investigations resulted in a better understanding of the behavior and definition of flow through open-channel constrictions. The relationships developed in the laboratory have been summarized in USGS Circular 284, "Computation of Peak Discharge at Contractions", and in ASCE Separate 413, "Backwater Effects of Open-Channel Constrictions". The latter pamphlet outlines a method for computing backwater at single-opening constrictions when used in conjunction with Circular 284. It is the purpose of this paper to elaborate in somewhat greater detail on the methods for computing backwater at specific bridge sites.

In the following sections of this paper an attempt is made to provide a consistent and, to a limited extent, simplified procedure for computing backwater. The suggested procedure is illustrated by an example at a specific site. Symbols and terminology are generally those used in Circular 284 and Separate 413; the few additional symbols are defined as they are introduced in the text.

At this point the reader is encouraged to review and familiarize himself with the contents of Circular 284 and Separate 413 before proceeding further.

The Backwater Formula

In Circular 284 (formula 1, p. 1) the discharge equation for flow through a contraction is given as:

$$Q = CA_3 \sqrt{2g\left(\Delta h + \alpha_1 \frac{V_1^2}{2g} - h_f\right)}$$

Transforming this equation by substituting V_3 for Q/A_3, and solving for Δh, results in formula 1, p. 3, of Separate 413, which is:

$$\Delta h = \frac{V_3^2}{2gC^2} + h_f - \alpha_1 \frac{V_1^2}{2g} \qquad (1)$$

The amount of backwater (h_1^*) is shown in Separate 413 to be a definable portion (a ratio less than unity) of the fall in water surface (Δh). This ratio, given as ($h_1^* / \Delta h$) in Separate 413, is termed the backwater ratio, and for simplification, is given the symbol C_b throughout this paper. The formula for backwater can be written:

$$h_1^* = C_b\,(\Delta h) \qquad (2)$$

Substituting the right-hand side of equation 1 for Δh in equation 2, gives:

$$h_1^* = C_b\left(\frac{V_3^2}{2gC^2} + h_f - \alpha_1 \frac{V_1^2}{2g}\right) \qquad (3)$$

Equation 3 can be considered the basic backwater formula. As defined in Separate 413, the amount of backwater (h_1^*) is the increase in water surface elevation resulting from the constriction, above the normal, unconstricted elevation at a designated section 1 upstream. Section 1 is specified (and so limited in the laboratory work) as being the natural-channel cross section located one opening width (b-distance) upstream from the constriction entrance.

Constriction-Profile Relationships

The amount of backwater (h_1^*) accounts for only a portion of the fall in water surface between sections 1 and 3 (section 3 is designated herein as the most contracted section within the constriction, usually taken to be at the downstream end of the constriction). The fall (Δh) includes other components. Not only does the constriction influence the normal profile upstream from it, but downstream as well. The profile through the constriction reach can best be described by reference to figure 1 (identical to figure 1 of Separate 413). From this figure it should be noted that at the downstream end of the constriction, section 3, the constricted water-surface profile is somewhat lower than the normal. This differential amount (a drawdown or negative backwater effect) is designated h_3^*. The third component of the fall is the normal friction loss between sections 1 and 3, designated Δh_n. From figure 1, therefore:

$$\Delta h = h_1^* + h_3^* + \Delta h_n; \text{ or } h_3^* = \Delta h - h_1^* - \Delta h_n \qquad (4)$$

Equations 3 and 4 become the tools for computing backwater at a specific site. The backwater equation (3) is sufficient by itself for computing backwater for some past flood at a bridge site where water-surface elevations are known, or can be determined from floodmarks, at sections 1 and 3 (where Δh is known). Equation 4 must necessarily be used in conjunction with equation 3 in computing backwater at proposed bridge sites where only the normal (unconstricted) water-surface profile is available.

Further inspection of figure 1 yields the following basic relationships useful in defining the water-surface profile within the constriction reach:

$$h_{1n} = h_{3n} + \Delta h_n \qquad (5)$$

$$h_1 = h_{1n} + h_1^* = h_3 + \Delta h \qquad (6)$$

$$h_3 = h_{3n} - h_3^* = h_1 - \Delta h \qquad (7)$$

Coefficients

The backwater equation (3) contains two coefficients, C and C_b. The coefficient of discharge (C) is defined by certain pertinent factors and their resulting relationships given in Circular 284. Computation of C can be accomplished by following the instructions given on p. 13 and by use of figures 22-30 of that Circular.

In Separate 413 various curves are presented for computing the backwater ratio (C_b). The backwater coefficient is shown to be primarily a function of channel contraction (m), and, to a lesser degree, to vary with channel roughness (n) and with constriction geometry.

Curves of backwater ratio (C_b) versus percent of channel contraction (m), with channel roughness (n) as a secondary factor are shown in figure 2 (similar to figure 9 of Separate 413). These curves are for one constriction type only, termed the base type, which corresponds to Type I geometry of Circular 284 with square-edged, vertical-faced abutments. That is why the ordinate in figure 2 is labeled $C_{b(I)}$, meaning backwater ratio for the base type. Note, also, that a channel roughness of 0.050 is the maximum presented. Laboratory experimentation led to the conclusion that an n value of 0.050 was the limiting (maximum) value influencing C_b.

Figure 3 (identical to figure 12 of Separate 413) defines the effect of constriction geometry on the backwater ratio. Constriction geometry for the constriction under study is related to constriction geometry for the base type by the ratio of their respective C's. The abscissa of figure 3 is in these terms. The coefficient for the base type is selected using an m and L/b identical to those of the constriction type under study. The C's for each type should be determined in the manner described in Circular 284, taking into account all applicable secondary effects such as k_f, k_y, k_r, k_w, k_ϕ, k_θ, k_x, k_e, k_t, and k_j. The ordinate scale on figure 3 is an adjustment factor (k_c), which when multiplied by $C_{b(I)}$ from figure 2, gives the backwater ratio (C_b) for the constriction type under study.

The foregoing procedure for determining the backwater ratio (C_b) may appear to the reader to be unnecessarily tedious; however, once familiarity with the process has been achieved, computation of the coefficient is relatively simple. Attempts to simplify the backwater-ratio curves met with some success, particularly for constriction types III and IV. The simplification, however, involved averaging processes with some sacrifice in accuracy without appreciable savings in time. In view of our inexperience with the effect of these averaging and rounding processes on any particular backwater result, it was decided not to include any of the simplified backwater-ratio curves at the present time.

Computation of Backwater at Proposed Bridge Sites

In our highway bridge-site reports certain features pertaining to the stream and proposed structure are usually furnished or specified by the highway department. Among these are (1) a cross section of the natural channel along the proposed highway centerline, (2) location and size of the proposed opening, and (3) design flood or floods. In addition, for computing backwater some data concerning geometry must be obtained in order to classify the constriction as to type.

In the average or typical bridge-site report, the following factors are computed by the Survey: (1) frequency curve, (2) stage-discharge relation, (3) distribution of conveyance and discharge laterally across the approach section (section 1), and (4) stage-area and stage-conveyance curves for the opening (section 3).

With the above data available, backwater created by the proposed constriction for a given flood frequency or discharge can be computed. The assumption is made that the stage-discharge relationship applies to the normal (unconstricted) channel at section 3, which is the general case in most site reports. (The term normal is used to describe conditions pertaining to the unconstricted, natural channel; the term constricted refers to conditions with the constriction in place.) Referring again to figure 1, it can be seen that because of the drawdown at section 3, induced by the constriction, the process of computing backwater, starting with a known normal elevation at section 3, becomes one of trial and error, if an exact answer is to be obtained. Limited studies have indicated that some simplification can be made without any great sacrifice in accuracy of the computed result. Employing these simplifications, reasonably direct methods of computing backwater have been evolved. An explanation of these methods, labeled "trial method", and "semi-direct method" follows.

Trial Method

The backwater formula is repeated:

$$h_1{*} = C_b\left(\frac{V_3^2}{2gC^2} + h_f - \alpha_1\frac{V_1^2}{2g}\right) \qquad (3)$$

This formula may be written in terms of h_3* as:

$$h_3{*} = (1 - C_b)\left(\frac{V_3^2}{2gC^2} + h_f - \alpha_1\frac{V_1^2}{2g}\right) - \Delta h_n \qquad (8)$$

The term Δh_n should be computed on basis of the normal condition. The other terms of equation 8 (C_b, C, V_3, h_f, and V_1), strictly speaking, should be computed on basis of the constricted condition; hence, the apparent necessity for a trial-and-error solution of h_1* or h_3* when only the normal conditions are known. It has been found, however, that the terms h_f and ($\alpha_1 V_1^2/2g$) are generally a small part, quantitatively, of the expression; are subject to only slight change in computed value between the normal and constricted profiles; and in most cases, therefore, can be computed for the normal stages (h_{1n} and h_{3n}) without much sacrifice in accuracy of the final result. The same is true for the coefficients C and C_b. This is the simplification referred to in the foregoing section.

Equation 8 then can be written:

$$h_3^* = (1 - C_b) \frac{V_3^2}{2gC^2} + (1 - C_b) \left(h_f - \alpha_1 \frac{V_1^2}{2g}\right) - \Delta h_n \text{ ; or}$$

$$h_3^* = (1 - C_b) \frac{V_3^2}{2gC^2} + Y \text{ ; where} \tag{9}$$

$$Y = (1 - C_b) \left(h_f - \alpha_1 \frac{V_1^2}{2g}\right) - \Delta h_n \tag{10}$$

In the above simplified expression for h_3^* (equation 9), the term V_3 applies to the constricted condition (elevation h_3 at section 3) and is usually significantly greater than V_3 for the normal condition (elevation h_{3n}) because of the drawdown at section 3 induced by the constriction. The value of V_3 for the normal elevation at section 3 is known, and is:

$V_{3\,(norm)} = Q/A_3$, where A_3 is the area of constricted section 3 at the normal elevation (h_{3n}). The area at section 3 for the constricted condition (A_c) can be written as: $A_c = A_3 - b_t h_3^*$, where b_t is the top width of flow at elevation h_{3n}. This expression is exact for a constricted section with vertical abutments, and is sufficiently accurate for sloping abutments.

Formula 9 is a convenient tool for computing h_3^* and, from it h_1^*, by a successive trial-and-error procedure. The first trial value of h_3^* can be based on a value of V_3 computed for the normal elevation at section 3 (h_{3n}). A second trial value of h_3^* is obtained using a value of V_3 based on the lower elevation at section 3 indicated by the first h_3^* value, and so on (see sample problem). When h_3^* computed equals the h_3^* value assumed in computing V_3, the equation is satisfied.

Once the final value of h_3^* has been computed, h_1^* can be solved by:

$$h_1^* = \frac{h_3^* + \Delta h_n}{1/C_b - 1} \tag{11}$$

Knowing h_1^*, h_3^*, h_{3n}, and Δh_n, constricted elevations h_1 and h_3 are simply computed by formulas 5, 6, and 7.

Semi-Direct Method

From the preceeding section, "Trial Method", equation 9 and section-3 area and velocity relationships are repeated:

$$h_3^* = (1 - C_b)\frac{V_3^2}{2gC^2} + Y \text{ ; and} \tag{9}$$

$$A_c = A_3 - b_t h_3^*$$

The value of V_3 for the constricted section may be written as:

$$V_{3(constr.)} = \frac{Q}{A} = \frac{Q}{A_3 - b_t h_3^*}$$

Substituting this latter expression for V_3 in formula 9 gives:

$$h_3^* = \frac{(1 - C_b)}{2gC^2}\left(\frac{Q}{A_3 - b_t h_3^*}\right)^2 + Y$$

This expression results in a third-degree equation and is subject to a semi-direct solution involving trigonometric functions. The solution for h_3^* was found to be:

$$h_3^* = \frac{2}{3}(Y - d_m)\cos\left(\frac{u}{3}\right) + \frac{Y + 2d_m}{3} \tag{12}$$

The terms of equation 12 are defined as follows:

Y is computed by formula 10;

d_m is the mean depth at section 3 for the normal elevation (h_{3n}) and is equal to (A_3/b_t);

u is an angle between 0° and 90°, the cosine of which is positive and is given by the formula:

$$\cos u = 1 + \frac{27\,(1 - C_b)\,Q^2}{4gC^2 b_t^2\,(Y - d_m)^3} \text{ ; and} \tag{13}$$

cos (u/3) may be selected from the curve of figure 4, from the known value of cos u computed by formula 13.

Once the value of h_3^* has been computed, h_1^* can be solved by:

$$h_1^* = \frac{h_3^* + \Delta h_n}{1/C_b - 1} \qquad (11)$$

Knowing h_1^*, h_3^*, h_{3n}, and Δh_n constricted elevations h_1 and h_3 are simply computed by formulas 5, 6, and 7.

Limitations of Methods

The value of h_3^* given by either the trial method or the semi-direct method is not exact but will be very close to the true value in most cases. It is not exact because certain factors in the backwater formula (C, C_b, h_f, and $\alpha_1 \frac{V_1^2}{2g}$) were computed on basis of normal-profile elevations.

Should the values of h_1^* and h_3^* be very large with respect to depth of flow, a second computation of h_3^* and h_1^* might be desirable. The factors C, C_b, h_f, and $\alpha_1 V_1^2/2g$ would be recomputed on basis of the constricted elevations (h_1 and h_3) arrived at in the first computation; these new values would be substituted in formulas 9, 10 and 11 (trial method) or in formulas 10, 11, 12, and 13 (semi-direct method) to yield new final values of h_3^* and h_1^*. This second computation will not ordinarily be necessary.

Computation Steps

Trial Method

1. Discharge coefficient (C). - Compute C for the proposed opening, C_I for the equivalent base-type opening, and the ratio C/C_I. The form, "Contracted-opening coefficients" (9-193-A), is convenient for this purpose. The coefficients C and C_I can be computed on basis of the normal elevation at section 3 (h_{3n}).

   No details are given herein for computing C except as shown in the sample computation. The coefficient should be computed as outlined in Circular 284, including all appropriate k factors as described and defined in the Circular.

The factor for effect of piling (k_j) should be included. If piling or pier details are unknown, a reasonable estimate of j should be made.

In most bridge-site reports the design flood discharge has been laterally distributed across the approach section prior to computing backwater. From such a distribution diagram the channel-contraction ratio (m) can be computed directly as $(\frac{q_a + q_b}{Q})$, or $(1 - \frac{q}{Q})$.

2. Backwater ratio (C_b). - Compute the backwater ratio (C_b) for the proposed opening. The factor C_b equals the product of $C_{b(I)}$ (from figure 2) and k_c (from figure 3).

3. Normal friction loss (Δh_n). - Compute normal friction loss between sections 1 and 3, by the formula:

$$\Delta h_n = (\frac{Q}{K_1})^2 \; (L + L_w)$$

4. Constricted friction loss (h_f).- Compute constricted friction loss between sections 1 and 3, by:

$$h_f = \frac{Q^2 \; L_w}{K_1 \; K_3} + (\frac{Q}{K_3})^2 L$$

Only the cross section along the centerline of the proposed highway (nearest to section 3) is ordinarily available. It is assumed, unless other information is available, that the channel is uniform through the reach being considered, and the same cross-section shape applies at both sections 1 and 3. For this reason, section properties (A and K) for 1 and 3 may be computed using the one available cross section and the water-surface elevation h_{3n}. Where an approach cross section is defined by field survey, section properties at 1 and 3 are computed using water-surface elevations h_{1n} and h_{3n}, respectively.

Conveyance K_3 should be computed, according to Circular 284, using piles or piers; however, in most bridge-site reports these structural details are unknown. In such cases, the value of n for section 3 should be increased by 0.005 to 0.015, depending upon estimated pier density, in computing conveyance of the gross section.

The length L_w is taken as equal to constriction width b. The length L is taken as defined in Circular 284 for the various constriction types, and has already been computed in step 1, computation of C.

5. Approach-section velocity head ($\alpha_1 V_1^2/2g$). - Compute the approach section velocity head ($\alpha_1 V_1^2/2g$) where;

$$V_1 = \frac{Q}{A_1}$$

$$\alpha_1 = \frac{\Sigma(\text{subsection } K_1^3/a_1^2)}{(\text{total } K_1^3/A_1^2)} = \frac{(v^2 q)}{V^2 Q}$$

For simplification and because of its relatively slight effect upon the final value of backwater, this term can be based on the normal elevation.

6. Compute Y using formula 10. -

$$Y = (1-C_b)\ (h_f - \alpha_1 \frac{V_1^2}{2g}) - \Delta h_n$$

7. Compute h_3^* by successive trials using formula 9. -

$$h_3^* = \frac{(1-C_b)\ V_3^2}{2gC^2} + Y \quad ; \text{where}$$

$$V_3 = \frac{Q}{A_c} = \frac{Q}{A_3 - b_t\, h_3^*}$$

8. Compute h_1^* using formula 11. -

$$h_1^* = \frac{h_3^* + \Delta h_n}{1/C_b - 1}$$

9. Compute constricted elevations h_1 and h_3. -

$$h_1 = h_{3n} + \Delta h_n + h_1^*$$

$$h_3 = h_{3n} - h_3^*$$

10. If a second computation of backwater is thought necessary, repeat steps 1, 2, 4-9, using first results of step 9 for h_1 and h_3. Step 1 will rarely need recomputing except for Type I and Type IV (2:1), where C is dependent upon F, or for Type II where C is dependent upon the depth coefficient (k_y). The values of m, L/b, and other

secondary k-factors will scarcely be affected by changes in elevation at sections 1 and 3. The value of C_b as computed in step 2 might be changed a slight amount due to some change in the (C/C_I) ratio.

Semi-Direct Method

Proceed with steps 1-6 as given in the trial method.

7. Compute d_m at section 3 for the normal elevation (h_{3n}). -

$$d_m = (A_3/b_t) \text{ ; where}$$

A_3 = area of section 3 at normal elevation (h_{3n}), and

b_t = top width of water surface at section 3 at normal elevation (h_{3n}).

8. Compute cos u by formula 13. -

$$\cos u = 1 + \frac{27(1-C_b)\, Q^2}{4gC^2 b_t^{\,2}(Y-d_m)^3}$$

9. Select value of $\cos(\frac{u}{3})$ from curve of figure 4, corresponding to value of cos u computed in step 8.

10. Compute drawdown $(h_3{}^*)$ at section 3. -

$$h_3{}^* = \frac{2}{3}(Y - d_m)\cos\left(\frac{u}{3}\right) + \frac{Y + 2d_m}{3}$$

11. Compute backwater $(h_1{}^*)$ at section 1. -

$$h_1{}^* = \frac{h_3{}^* + \Delta h_n}{1/C_b - 1}$$

12. Compute constricted elevations h_1 and h_3.

$$h_1 = h_{3n} + \Delta h_n + h_1{}^*$$

$$h_3 = h_{3n} - h_3{}^*$$

13. If a second computation of backwater is thought necessary, repeat steps 1-2, 4-6, 8-9, and 10-12. In recomputation of steps 8 and 9, note that the terms b_t and d_m remain unchanged - these values are for the normal elevation (h_{3n}).

The Backwater Result

The foregoing computation procedures yield by either a trial or semi-direct solution, a final value of backwater (h_1^*) for a given discharge at a specific site. This computed backwater figure is by definition the increase in stage above the normal at section 1 resulting from the constriction.

The highway engineer is, of course, interested in this figure of backwater, but even more important to him generally is the matter of water-surface elevation along the upstream side of the embankment. To what height will the design flood reach on the upstream side of the fill back away from the waterway opening?

This question can be answered only on basis of theoretical considerations. No laboratory work has been done to answer this question specifically.

The maximum theoretical height that will be reached on the upstream side of the fill in the stagnation corner will be the water-surface elevation in an approach section located upstream approximately at the beginning of drawdown, plus the velocity head in that approach section. The foregoing disregards any friction loss between that approach section and highway embankment, and, moreover, assumes full (100 percent) transformation of kinetic energy (velocity head) to potential energy (static head). It is unlikely that the water-surface elevation against the fill will, in any natural condition, reach the theoretical height (full-stagnation head). The theoretical height does, however, give a figure conservatively safe for use by the highway engineer in deciding upon road-grade requirements across the stream channel.

In the laboratory work dealing with flow through a constriction, the approach section was taken as one opening width ($\underline{b}$-distance) upstream from the constriction entrance in order to be at or above the beginning of drawdown. Backwater (h_1^*) has been defined on that basis, and, hence, should be computed in the same way. It should be noted, however, that the beginning of drawdown upstream from the constriction and along the embankment face depends upon the percent of channel contraction (m). This is illustrated by figure 2, p. 3, of Circular 284, wherein the distances, (D) upstream and (x) along the embankment, to beginning of drawdown are defined in terms of opening width (b). The authors of Circular 284 have stressed the accuracy limitations of these curves as being too poor to use in pinpointing the beginning of drawdown. Yet the relationships can be used to give a more reasonable estimate of water-surface elevation against the fill than by arbitrarily using the elevation (h_1) of the approach section b-distance upstream. Maximum probable water-surface elevation against the fill in the stagnation corner (h_s) can be estimated as:

$$h_s = h_1 - \left(\frac{Q}{K_1}\right)^2 L_w \left(1 - \frac{D}{b}\right) + \alpha_1 V_1^2 / 2g \qquad (14)$$

The ratio D/b can be selected from figure 2 of Circular 284. Other terms in equation 14 have been computed in the foregoing computation of backwater. For channel contractions greater than 50 percent the second term on the right side of equation 14 would be negligible and can be disregarded.

The distance x, defined in figure 2 of Circular 284, might serve as a rough criterion to the highway engineer for required length of embankment paving, where such paving is deemed necessary.

After backwater (h_1^*) has been computed for a proposed opening, the question might arise as to whether or not the low steel of the proposed structure will clear the constricted water surface of the design flood. Again no quantitative method for evaluating the fall between section 1 and the constriction entrance was defined by laboratory experimentation. According to Circular 284 (p. 5), a safe rule-of-thumb estimate would be to assume that one-half the total fall (Δh) occurs between section 1 and the upstream side of the bridge.

Mean Velocity in the Contracted Section

In the past in bridge-site reports, mean velocity in a proposed opening has been computed on basis of the normal elevation (h_{3n}) at the contracted section. In constriction problems involving high velocity at low depths (high Froude numbers) or relatively low channel-contraction ratios, the constricted elevation at section 3 (h_3) may be considerably lower than the normal (h_{3n}), and hence would give a mean velocity considerably larger. Which figure of mean velocity should be shown in the report?

Because of the great diversity of problems encountered and our limited experience in backwater computations, it is felt that no definite recommendations need be outlined at this time. If a site report is being prepared where backwater obviously will be slight and of no interest to the highway engineer, mean velocity figures based on the normal elevation at section 3 (h_{3n}) would be satisfactory. If, however, backwater computations are made, it would be desirable to show mean velocity based on the constricted elevation at section 3 (h_3). The latter gives the higher figure which naturally is more conservative from the design engineer's viewpoint.

Computation of Backwater at Multiple-Opening Sites

The foregoing sections have dealt only with backwater, its definition and computation, at single-opening constrictions. Laboratory experimentation on multiple openings is, at this writing, just getting under way. Defined computational procedures for computing backwater at multiple-opening constrictions undoubtedly are several years away.

In the meantime, where backwater is of interest at multiple-opening sites, the following computational procedure is suggested: (1) isolate each opening with its corresponding approach section and computed discharge, (2) compute backwater for each opening by same procedure as for single opening, and (3) show either the computed backwater for each opening, or as maximum probable backwater, the largest computed backwater value from the several openings.

General Flood Curves

In many bridge-site reports it is necessary to show general curves relating discharge, frequency, mean velocity, backwater, etc., to stage. This method of summarizing hydrologic and hydraulic data in a report has been encouraged and recommended by the Technical Standards Section for some time. To prepare backwater curves for a range in stage and discharge, or for several opening proposals, involves a considerable volume of routine computation if the procedure given herein is followed rigorously for a number of points. A suggested shortcut is to compute backwater for two discharges and corresponding stages, as, say, for the 10- and 50-year floods. Relate the final backwater figures to their corresponding mean velocity-heads, computed on basis of normal elevation at section 3, in terms of a coefficient times velocity head. For example, suppose $h_1^* = 1.25\,(V_3^2/2g)$ at the 50-year flood, and $h_1^* = 1.12\,(V_3^2/2g)$ at the 10-year flood. The stage-backwater curve between these extremes could be defined as the velocity-head coefficient, interpolated with respect to $(V_3^2/2g)$, multiplied by the $(V_3^2/2g)$ corresponding to the particular stage or discharge under consideration.

Sample Backwater Computation

Sample backwater computations are included for the site report, "Missouri River at Jefferson City, Mo.", a report prepared in August 1952, and distributed to all district offices.

Computations in the sample problem are numbered and titled to agree with the numbered computation steps given in the text.

Sample problem.- Computation of backwater, Missouri River at Jefferson City, Mo.

A bridge-site report was prepared for this location in August 1952, a copy of which was distributed to each district office. The proposed location, layout, and other factors pertaining to the hydrology and hydraulics of the site are given in that report.

The design flood for the proposed crossing was taken as the flood of July 1951; peak discharge, 540,000 cfs; normal stage, 553.9 ft. at proposed centerline; recurrence interval, 47 years.

From the bridge-site report, or computations based thereon, the following factors are given:

$Q = 540{,}000$ cfs — $h_{3n} = 553.9$ ft.

$K_1 = 40{,}800{,}000$ @ 553.9 ft. — $K_3 = {}^{*}28{,}500{,}000$ @ 553.9 ft.

$A_1 = 116{,}200$ sq. ft. @ 553.9 ft. — $A_3 = 68{,}900$ sq. ft. @ 553.9 ft.

$\alpha_1 = 2.21$

The problem: To compute backwater created by the proposed crossing and probable maximum upstream stage along highway fill.

Computation of Backwater

Trial Method

1. Discharge coefficients (C).-

From attached "Comp. of coefs." sheet (5 of 5)

$C = \underline{0.756}$ $C_I = \underline{0.715}$ $C/C_I = \underline{1.06}$

2. Backwater ratio (C_b).-

$m = 18$ $n = 0.032$ (estimated from main-channel $n = 0.030$, and flood-plain $n = 0.035$).

$C/C_I = 1.06$

From figure 2: $C_{b(I)} = 0.35$

From figure 3: $k_c = 0.71$

$C_b = 0.35(0.71) = \underline{0.25}$

* Pier or piling details unknown; K_3 revised from value shown in report, by raising main-channel n from 0.030 to 0.035; overbank from 0.035 to 0.045

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Sample problem--continued.

Trial Method--Continued.

3. Normal friction loss (Δh_n).-

$$\Delta h_n = \left(\frac{Q}{K_1}\right)^2 (L + L_w) = \left(\frac{540{,}000}{40{,}800{,}000}\right)^2 (51 + 2900) = \underline{\underline{0.52}} \text{ ft.}$$

4. Constricted friction loss (h_f).-

$$h_f = \frac{Q^2 L_w}{K_1 K_3} + \left(\frac{Q}{K_3}\right)^2 L = \frac{(540{,}000)^2 (2900)}{(40{,}800{,}000)(28{,}500{,}000)} + \left(\frac{540{,}000}{28{,}500{,}000}\right)^2 51$$

$$h_f = 0.727 + 0.018 = \underline{\underline{0.74}} \text{ ft.}$$

5. Approach-section velocity head ($\alpha_1 V_1^2/2g$).-

$$\alpha_1 \frac{V_1^2}{2g} = \frac{2.21}{64.3}\left(\frac{540{,}000}{116{,}200}\right)^2 = \underline{\underline{0.74}} \text{ ft.}$$

6. Factor Y by formula 10.-

$$Y = (1 - C_b)(h_f - \alpha_1 V_1^2/2g) - \Delta h_n = (1 - 0.25)(0.74 - 0.74) - 0.52 = -\underline{\underline{0.52}} \text{ ft.}$$

7. Drawdown (h_3^*) by successive trials (formula 9).-

First trial: Assume $h_3^* = 0$ $A_3 = 68{,}900$ $V_3 = \frac{540{,}000}{68{,}900} = 7.83$

$$h_3^* = \frac{(1 - C_b) V_3^2}{2g C^2} + Y = \frac{0.75(7.83)^2}{64.3(.756)^2} - 0.52 = 1.25 - 0.52 = 0.73 \text{ ft.}$$

Second trial: Assume $h_3^* = 0.80$ $A_3 = 68{,}900 - 0.80(2908) = 66{,}580$ $V_3 = 8.11$

$$h_3^* = \frac{0.75(8.11)^2}{36.7} - 0.52 = 1.34 - 0.52 = 0.82 \text{ ft.}$$

Third trial: Assume $h_3^* = 0.82$ $A_3 = 68{,}900 - 0.82(2908) = 66{,}510$ $V_3 = 8.12$

$$h_3^* = \frac{0.75(8.12)^2}{36.7} - 0.52 = 1.34 - 0.52 = \underline{\underline{0.82}} \text{ ft. (ok as assumed)}$$

8. Backwater (h_1^*) by formula 11.-

$$h_1^* = \frac{h_3^* + \Delta h_n}{1/C_b - 1} = \frac{0.82 + 0.52}{1/0.25 - 1} = \underline{0.45} \text{ ft.}$$

Sample problem--continued.

Trial Method--Continued.

9. Constricted elevations h_1 and h_3.-

$$h_1 = h_{3n} + \Delta h_n + h_1^* = 553.9 + 0.52 + 0.45 = \underline{554.87} \text{ ft.}$$

$$h_3 = h_{3n} - h_3^* = 553.9 - 0.82 = \underline{553.08} \text{ ft.}$$

Semi-Direct Method

Steps 1-6 identical to trial method.

7. Mean depth (d_m) at section 3 for elevation h_{3n}.-

$$d_m = A_3/b_t = \frac{68,900}{2,908} = \underline{23.7} \text{ ft.}$$

8. Cosine u by formula 13.-

$$\cos u = 1 + \frac{27(1-C_b)Q^2}{4gC^2b_t^2(Y-d_m)^3} = 1 + \frac{27(1-.25)(540,000)^2}{4(32.16)(.756)^2(2908)^2(-0.52-23.7)^3} = 1 - 0.667 = \underline{0.333}$$

9. Cosine (u/3) from curve of figure 4.-

$$\cos(u/3) = \underline{0.917}$$

10. Drawdown (h_3^*) by formula 12.-

$$h_3^* = \frac{2}{3}(Y-d_m)\cos\left(\frac{u}{3}\right) + \frac{Y+2d_m}{3} = \frac{2}{3}(-0.52-23.7)(0.917) + \frac{-0.52+2(23.7)}{3}$$

$$h_3^* = -14.80 + 15.62 = \underline{0.82 \text{ ft.}}$$

Computations of h_1^*, h_1 and h_3 identical to steps 8 and 9 of trial method.

Summary of Results--Approximate Methods

Backwater (h_1^*) = 0.45 ft. Drawdown (h_3^*) = 0.82 ft.
Elevation (h_1) = 554.87 ft. Elevation (h_3) = 553.08 ft.

Sample problem--continued.

Results by Exact Computation

The foregoing results given by either the trial or semi-direct methods will be close to results obtained by a rigorous arithmetical procedure. In order to make a comparison, all factors were recomputed, starting with elevations h_1 and h_3 equal to 554.87 and 553.08 ft., respectively. The computations are not shown herein, but the various factors and results are tabulated.

$h_1 = 554.87$ ft } assumed	**Final profile results**
$h_3 = 553.08$ ft. } assumed	$h_1^* = 0.49$ ft.
$C = 0.756$	$h_3^* = 0.87$ ft.
$C_I = 0.719$	$h_1 = 554.91$ ft. } close enough
$C/C_I = 1.05$	$h_3 = 553.03$ ft. } to assumed
$C_b = 0.26$	
$h_f = 0.75$ ft.	
$\alpha_1 V_1^2/2g = 0.68$ ft.	

It is seen that results obtained on page 3, using the simplified, approximate methods, are in close agreement with results obtained by a fully-developed rigorous analysis.

Probable Maximum Upstream Stage

Probable maximum height against fill at left edge of flood plain would be:

$$h_5 = h_1 - \left(\frac{Q}{K_1}\right)^2 L_w\left(1-\frac{D}{b}\right) + \alpha_1\frac{V_1^2}{2g} = 554.87 - \left(\frac{540{,}000}{40{,}800{,}000}\right)^2 2900(1-0.5) + 0.74$$

$$h_5 = 554.87 - 0.25 + 0.74 = \underline{\underline{555.36}} \text{ ft.}$$

[Note.- Had exact values from above been used, h_5 would compute as 555.36 ft.]

(January 1954)
Comp. of coefs.

UNITED STATES DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
WATER RESOURCES DIVISION

File ______

Sample problem -- continued.

Computation of coefficients for *Missouri River at Jefferson City, Mo.*
for flood of *540,000 cfs, normal stage 553.9 ft.*

APPROACH SECTION PROPERTIES

Subsection	n	$\frac{1.486}{n}$	a	w.p.	r	$r^{2/3}$	$K = \frac{1.486}{n} a r^{2/3}$
Main channel (1581-2700)	*0.030*	*49.5*	*48,550*	*1,140*	*42.6*	*12.23*	*29,400,000*
Over bank (2700-11,213)	*.035*	*42.4*	*67,670*	*8,517*	*7.95*	*3.98*	*11,420,000*
			116,220	*9,657*			*40,820,000*
							α = 2.21
Total							

CHARACTERISTICS OF CONSTRICTION

Embankment and abutment slope *1:1* Type of abutment *III*

ITEMS AND RATIOS *AT ELEV. 553.9 FT.*

Items		Ratios	
Q	*540,000*	$m = 1 - (K_q/K_{total})$	$= \frac{97,000}{540,000} = .18$
b	*2,900*	L/b	*0.018*
L	*51*	r/b	–
r	–	W/b	–
W	–	x/b	*0.004*
x	*11*	$(y_a+y_b)/2b$	–
y_a	–	$t/(y_3+\Delta h)$	–
y_b	–	$F = V_3/\sqrt{gy_3}$	$= \frac{7.84}{5.67 \times 4.88} = 0.28$ *(equiv. Type I)*
b_t	*2908*	$j = A_j/A_3$	*estimated 0.03*
y_3	*23.7 (23.8 for equiv. type I)*	$e = K_a/K_b$	*0*
A_j *(estimated j)*		ϕ	*0°*
A_3	*68,900*	θ	–
K_a *q_a*	*0*		
K_b *q_b*	*97,000*		

		C′		k_ϕ		k_x		k_j		k_e		k_F		
This	C =	*0.80*	×	*1.00*	×	*1.005*	×	*0.985*	×	*0.955*	×	*1.00*	=	*0.756*
Base	C_I =	*0.80*	×	*1.00*	×	*1.00*	×	*0.985*	×	*0.955*	×	*0.950*	=	*0.715*

$\frac{C}{C_I} = 1.06$

Sheet No. *5* of *5* Sheets. Prepared by *JSC* Date *8/12/54* Checked by *WH* Date *8/13/54*

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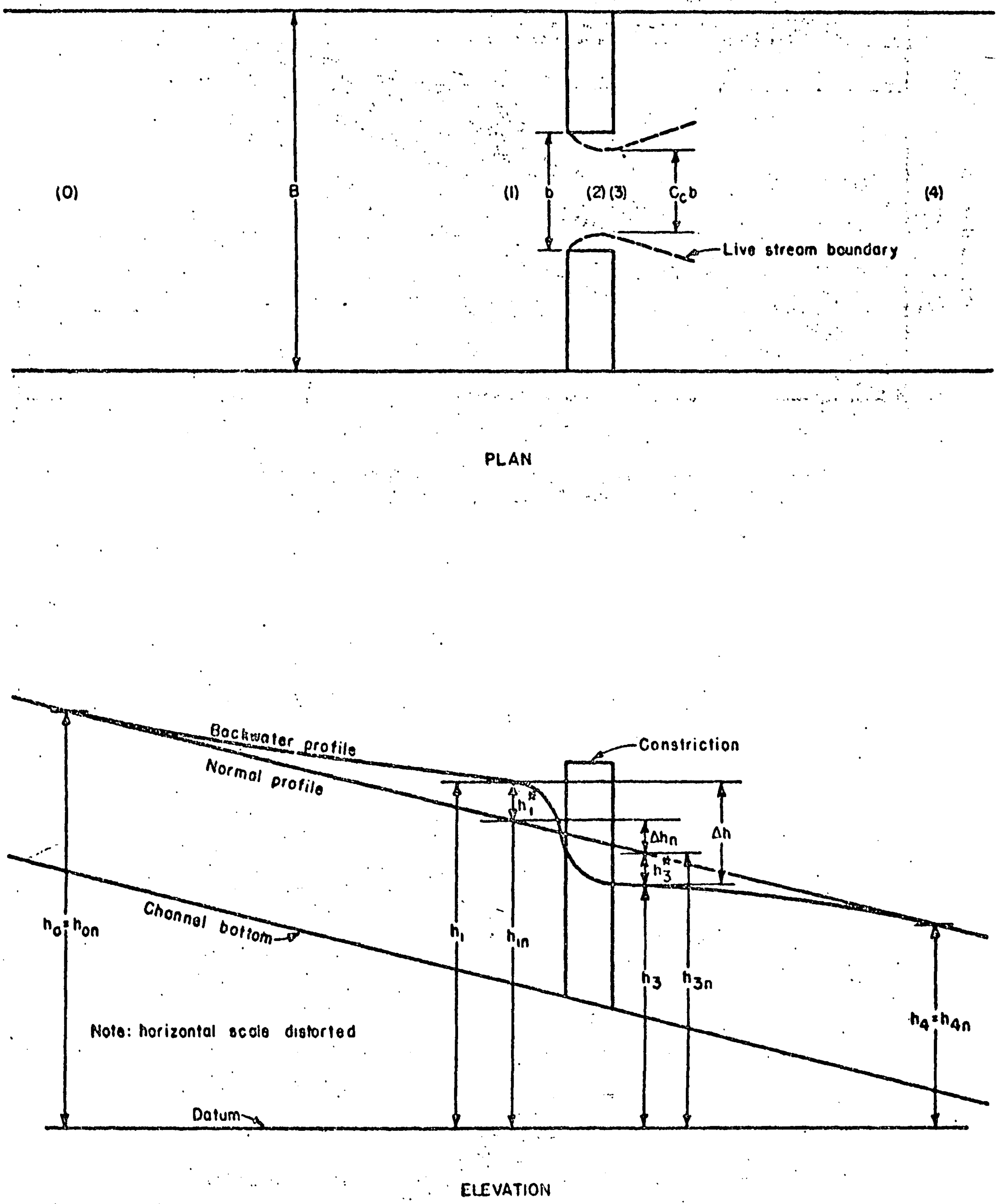

FIG. 1—DEFINITION SKETCH OF THE BACKWATER REACH

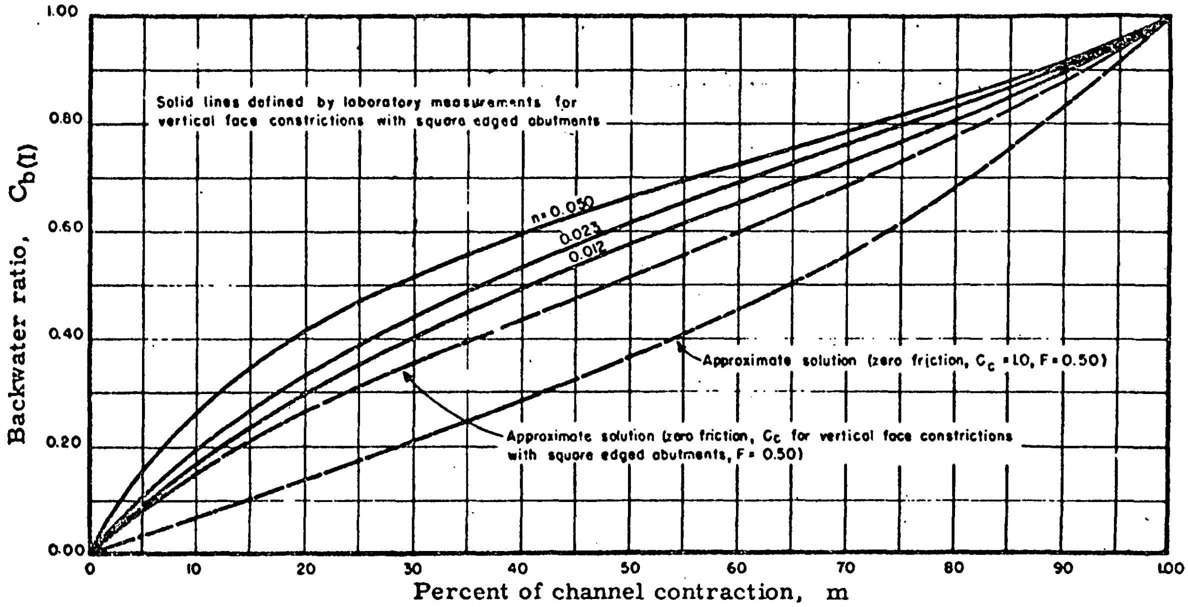

FIG. 2 - CURVES OF BACKWATER RATIO FOR VERTICAL FACE CONSTRICTIONS WITH SQUARE EDGED ABUTMENTS

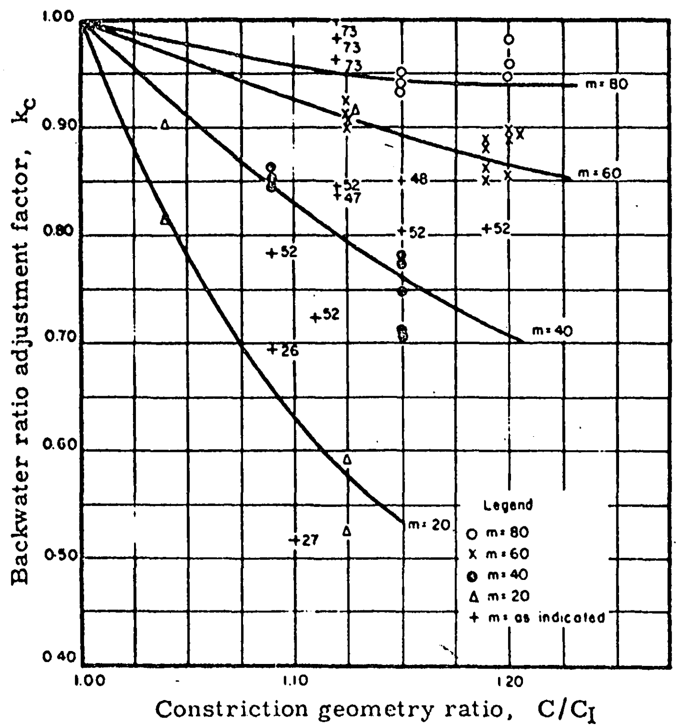

FIG. 3 - THE EFFECT OF CONSTRICTION GEOMETRY ON THE BACKWATER RATIO

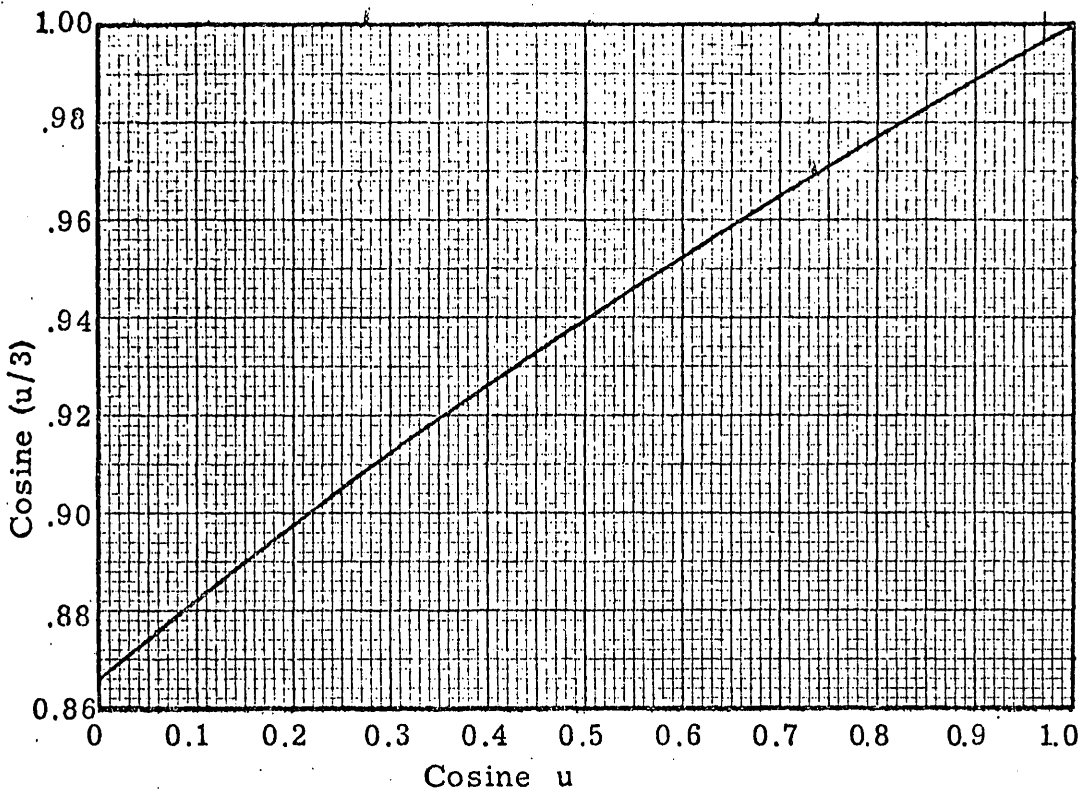

FIG. 4 - COSINE (u/3) VS COSINE u, WHERE u IS BETWEEN 0 AND 90 DEG.