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THE DEVELOPMENT OF THE CONE OF DEPRESSION AROUND A PUMPED WELL IN AN INFINITE STRIP AQUIFER SUBJECT TO UNIFORM RECHARGE

By

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Complete understanding of the source of water derived from wells continues to be elusive. The fallacious idea that the cone of depression around a pumped well expands only until it has embraced an area in which the recharge balances the discharge of the well persists despite Theis' (1938 and 1940) able discussion of the factors controlling the response of an aquifer to development by wells. The purpose of this paper is to show, by numerical example and related discussion, how a particular hydrologic system, in which the natural recharge balances the natural discharge, responds to the introduction of a pumped well and ultimately reaches a new state of equilibrium.

Perhaps some misunderstanding arises from loose or imperfect interpretations of the term "cone of depression." As stated by Theis (1938, p. 891, and 1954, p. 2) the cone of depression is "--the geometric solid included between the water table or other piezometric surface after a well has begun discharging and the hypothetical position the water table or other piezometric surface would have had if there had been no discharge by the well." This definition is equally apt for the synonymous, and possibly more descriptive, term "cone of influence."

The hydrologic system devised for illustrative analysis is shown in part by the cross section and oblique view in figure 1, and is described as a water-table aquifer of width, 2a, thickness, m, and infinite length, bounded by two fully penetrating parallel perennial streams in which the stage remains constant. The aquifer must be thick enough so that the drawdowns to be created in it are negligible proportions of the original saturated thickness. The aquifer is homogeneous and isotropic and is recharged uniformly, with respect to space and time, by precipitation at the rate W inches per year. A single fully penetrating pumped well is to be introduced midway between the two streams.

The following dimensions and hydraulic constants for the system are assumed:

2a = 6 miles or 31,680 ft.
W = 6 inches per year
T = 80,000 gallons per day per foot = coefficient of transmissibility of aquifer
S = 0.20 = coefficient of storage of aquifer.

Substituting these data in equation 61 of Ground Water Note 28 (Knowles, 1955) the profile of the water table may be computed for the cross section of figure 1. The profile position is shown schematically in figure 1 and plotted to scale in figure 2 (data are given in Appendix A). Assuming the slope of the streams and the water table is quite low, in the direction normal to the plane of the cross section, it is evident that an acceptable water-table contour map (fig. 2) may be drawn as a family of straight lines paralleling the streams. The highest contour, which also marks the position of the ground-water divide, is

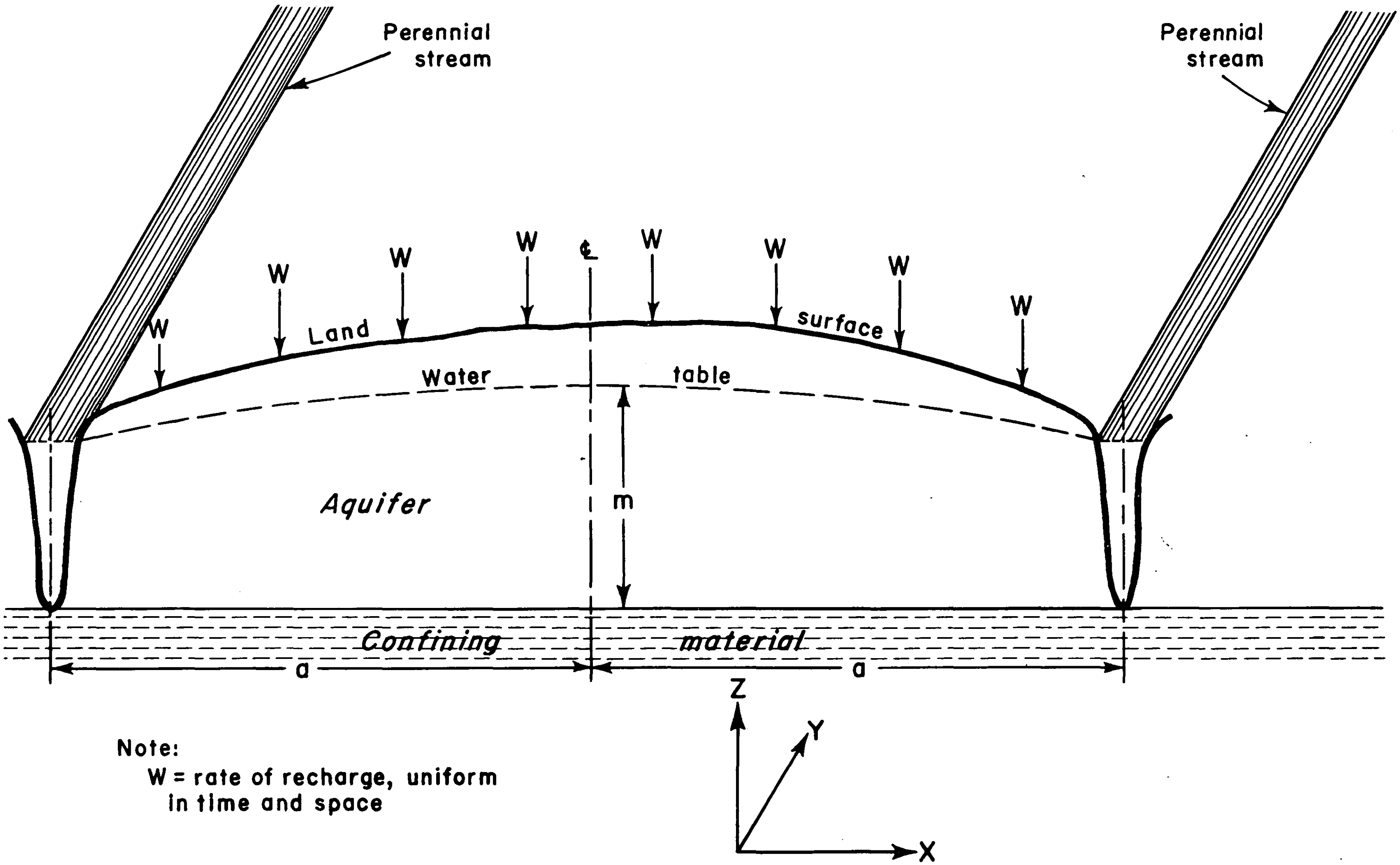

Figure 1--Cross section and oblique view of part of infinite strip aquifer bounded by two parallel streams

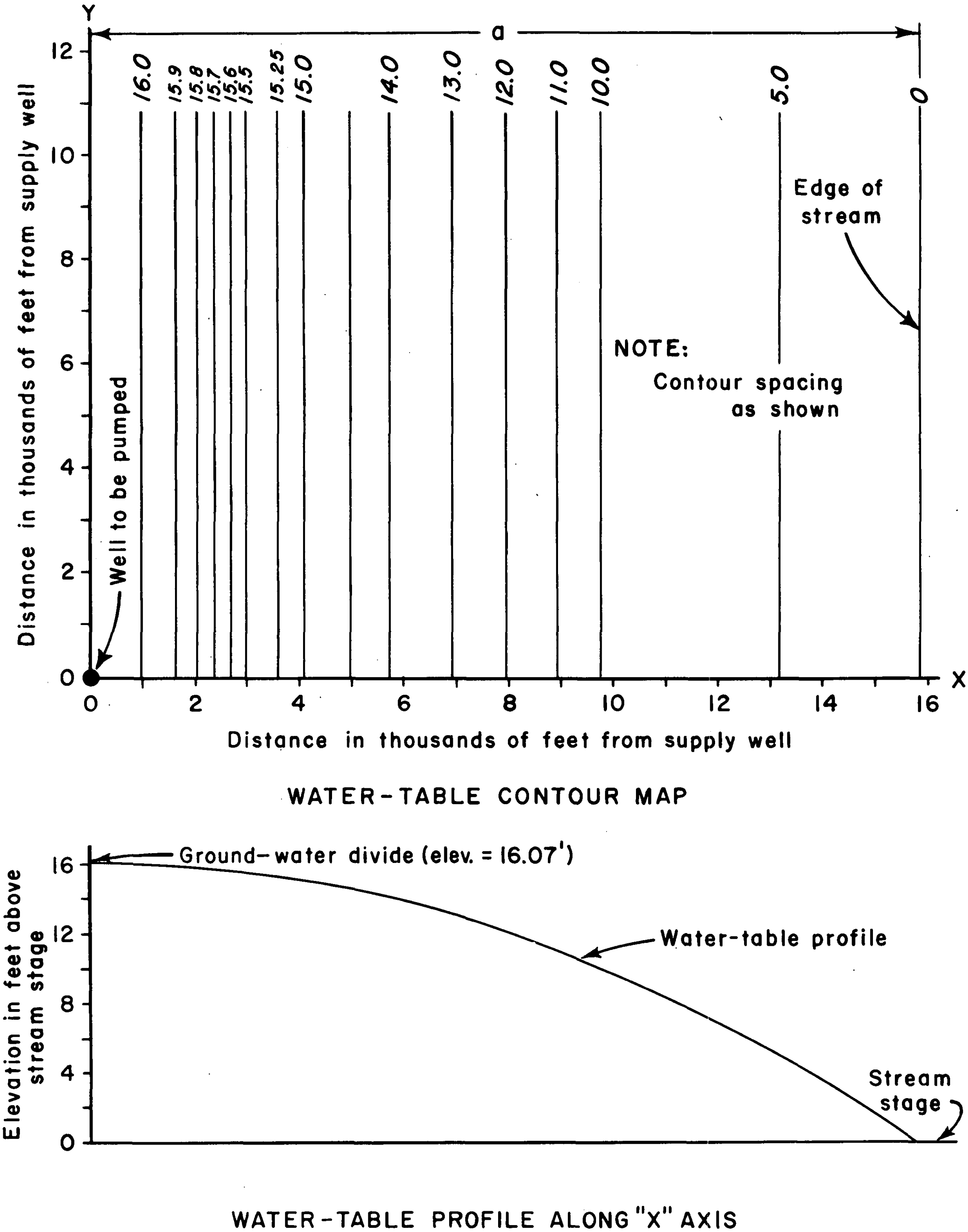

Figure 2.--Contour map and profile of water table before pumping begins

midway between the streams. Because of symmetry only one quadrant of the aquifer need be contoured.

The map and profile of the water table, as shown in figure 2, depict the initial state of balance between natural recharge to and discharge from the aquifer for this particular hydrologic system. If the rate of recharge does not vary, if the stream level does not change, and if no other recharge to or discharge from the system is postulated, then the shape and position of the water table are unvarying with time and remain as shown. From symmetry the aquifer obviously discharges equally into the two streams. The natural discharge into one stream, for the assumed dimensions and constants of this hydrologic system, expressed in gallons per day per foot of stream channel, is

$$\frac{Wa\ (7.48)}{12(365)} = \frac{6(15{,}840)\ 7.48}{12(365)} = 162$$

Before analyzing the changes that will take place in the hydrologic system through the introduction of a pumped well, consider briefly the nature of the initial flow field, the flow field that will be introduced, and the resultant flow field. Referring to the orientation of the three coordinate axes shown in figure 1, obviously the initial conditions in the hydrologic system are such that two-dimensional flow prevails which can be described entirely by reference to the XZ plane; no flow components exist in the third, or "Y", direction. Assuming the drawdowns to be created in the aquifer are a very small proportion of the saturated thickness the flow field related only to the pumped well is satisfactorily approximated as two dimensional (radial flow), and can be described entirely by reference to the XY plane. When the two respective flow fields are combined, therefore, the resultant field will be three dimensional, requiring reference to all three coordinate axes for proper description.

As in a number of areas of mathematical physics, in studying problems in the flow of fluids through porous media the principle of superimposition is often used. Superimposition is especially helpful in analyzing a three-dimensional problem, such as the one developed herein, which is recognizable as the resultant of two or more component flow fields, each involving simply one- or two-dimensional flow. When the principle is applicable, therefore, it permits individual analysis of, say, the head distribution throughout the component flow fields, followed by their successive superimposition and algebraic summation yielding finally the head distribution throughout the resultant field. The prime criterion for applicability of this analytical technique is that the differential equations describing the component flow fields be linear (Jacob, 1950, p. 361). A differential equation is said to be linear if it is of the first degree with respect to the dependent variable and its derivatives (Ingersoll and Zobel, 1948, p. 11-12; Phillips, 1934, p. 38 and 49). Waiving explanation of the notation, the particular differential equations that describe the initial steady-state flow field and the unsteady state field to be superimposed with the introduction of the pumped well are, respectively, of the form

$$Wx = -Ph\frac{dh}{dx} \text{ (After Jacob, 1950, p. 381.)}$$

and

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \text{ (Jacob, 1950, p. 366.)}$$

In both equations h is the dependent variable and nowhere does it, or any of its derivatives appear with an exponent higher than one. Both equations are therefore of the first degree and linear, justifying application of the principle of superimposition.

Let a fully penetrating well be constructed at the center of the aquifer, that is, midway between the two boundary streams. The center-point location, which is also coincident with a point on the ground-water divide, is chosen primarily to simplify the ensuing computations; the choice does not prejudice the significance of the discussion. Let the well be pumped continuously at the steady rate of 200 gpm (gallons per minute). Of interest, now, is the resultant flow field at selected times after pumping begins and at the ultimate new steady state.

With the propriety of using the technique of superimposition firmly established the problem is restated, in terms of the two previously recognized constituent flow fields, as follows:

1. The initial flow field reflects simply uniform recharge, over the entire infinite strip aquifer, in balance with the aquifer discharge to the two parallel boundary streams. Steady-state conditions prevail, which is to say the flow field is unvarying with time. Thus if this flow field is to be added algebraically to some other it makes no difference what time or times are selected for viewing the resultant field; the configuration of the initial flow field enters into each analysis exactly as depicted, in plan and profile, in figure 2.

2. The introduced flow field develops in the manner predictable for a pumped well at the center of an infinite strip aquifer which is subject to <u>no recharge</u> and is bounded by two parallel streams whose stages are held constant. Obviously this is an unsteady flow field which expands with elapsed pumping time. Thus its configuration and extent depends upon the time selected for viewing; the delineation of its ultimate steady state (and some intermediate states) involves recourse to image-well theory

to satisfy the boundary conditions of
no drawdown at the streams.

With the initial flow field defined, and identified as unvarying with time, the problem of defining the resultant of the two described constituent flow fields, at selected times, becomes primarily an exercise in describing the unsteady field developed by the pumped well. Fortunately the mathematical theory required for such an exercise has been presented by Theis (1935) and by Hantush and Jacob (1955, p. 106-107) and need not be redeveloped here. However, limited discussion of the mechanics of accomplishing the exercise, employing the above cited works, is appropriate.

To meet the stipulation that no drawdown can occur along the aquifer boundaries image wells are required as shown in figure 3. Each image well, whether recharging or discharging, operates at 200 gpm simultaneously and continuously with the real pumping well. Thus along either boundary the effects of all wells on one side are seen to annul exactly the effects of the wells correspondingly placed on the other side, and the condition of no drawdown is fulfilled. Observe, however, (fig. 3) that the pattern of image wells repeats itself out to infinity in two opposite directions -- a fact bearing directly on the question "How long will it take for the flow field developed by the pumping well to reach a steady state?" Because the system of image wells extends out to infinity in both directions an infinite period of time is required for the superimposed flow field, and hence the resultant flow field, to become steady. In other words, infinite time is needed for the effects from the most distant image wells to reach the real aquifer. However, the degree to which different parts of the flow field approach stabilization in finite times can be judged by examining the illustrations that follow.

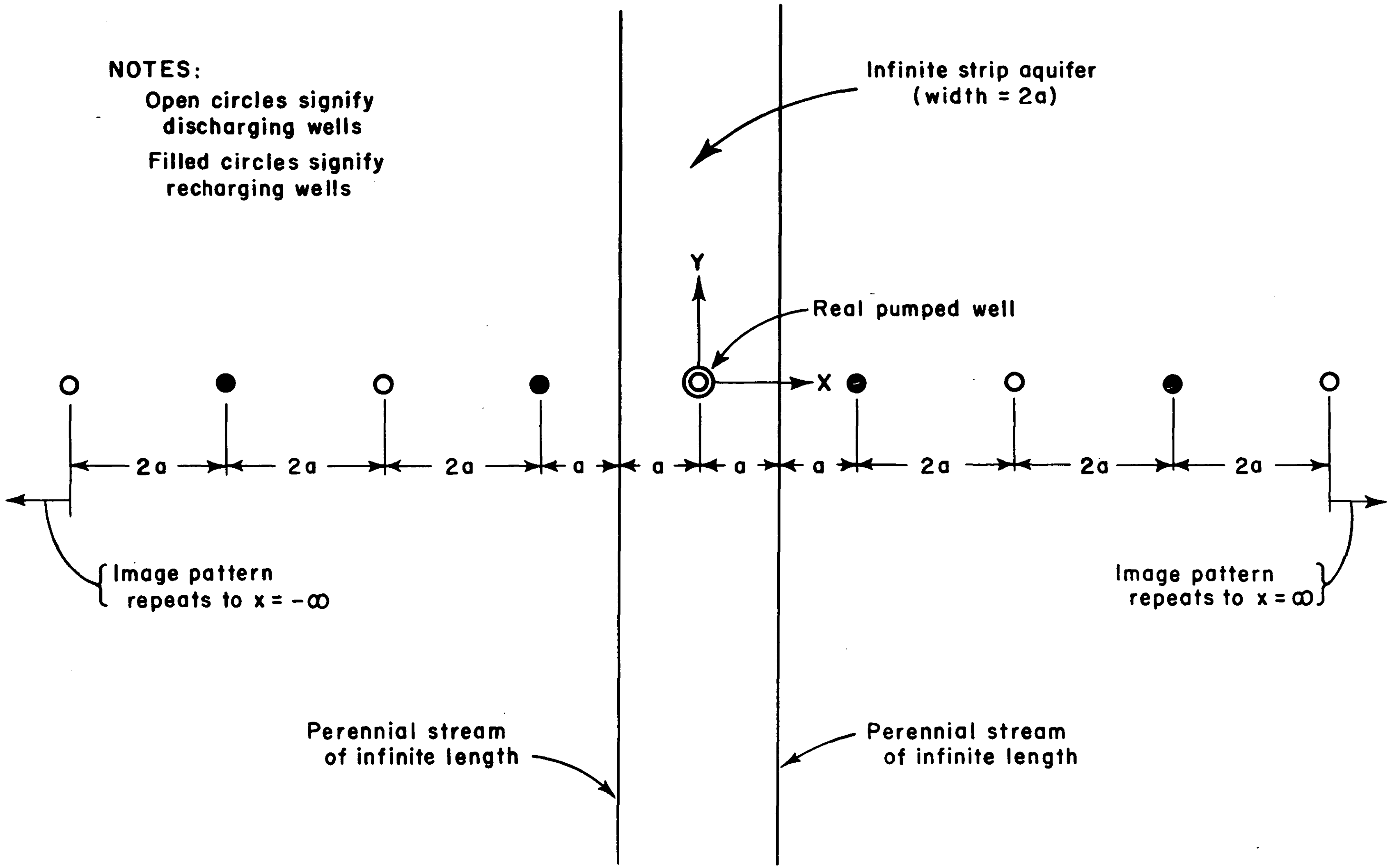

Figure 3.--Image-well array for single pumped well at center of infinite strip aquifer bounded by perennial streams

The times (since pumping started) arbitrarily selected for viewing the composite or resultant flow field are 30 days, 500 days, and when the new steady-state conditions are realized, that is, at time $t = \infty$. The viewing expedients, for each of the three times, are to be a water-table profile and a water-table contour map similar to those shown in figure 2.

In its early stages of development the flow field related to the pumping well at the aquifer center is easily described by computing the profile shape of the cone of depression using Theis' (1935, p. 520, or 1952, p. 3, eq. 4) familiar nonequilibrium equation. Not until enough time has elapsed for the cone of depression to reach the boundary streams will it be necessary to consider the image wells and resort to Hantush and Jacob's equation which properly sums all image- and real-well effects.

At the end of 30 days' pumping Theis' equation yields the drawdowns at selected distances from the pumping well given in table 1. A drawdown of 0.01 foot is arbitrarily taken as the sensible limit of the cone of depression. For the 30-day period, this drawdown occurs at a distance of 3,800 feet from the pumping well. A contour map of the cone of depression (for $t = 30$ days), considering only the flow field being developed by the pumping well, is thus a family of circles whose common center is the pumping well. These contours are indicated, in one quadrant only, by light short dashes in the map portion of figure 4. In the same map portion the straight parallel lines, shown in part by light long dashes, are water-table contours depicting the initial steady-state flow field (see also fig. 2).

Table 1.-- Drawdown data for an infinite aquifer tapped by a single pumping well.

Drawdown s (ft)	Distance r (ft)a/	Distance r (ft)b/	Drawdown s (ft)	Distance r (ft)a/	Distance r (ft)b/
0.01	3,800	15,400	0.30	1,260	5,150
.02	3,340	13,700	.40	1,020	4,180
.04	2,860	11,700	.60	690	2,820
.05	2,700	11,000	.80	480	1,950
.07	2,450	10,000	1.0	330	1,370
.10	2,170	8,880	1.5	140	570
.15	1,840	7,540	2.0	60	240
.20	1,600	6,570	3.0		40
.25	1,420	5,790			

a/ Period of pumping = 30 days

b/ Period of pumping = 500 days

r = radial distance from pumping well to points in aquifer at which indicated drawdown occurs.

Distances computed using the Theis nonequilibrium formula.

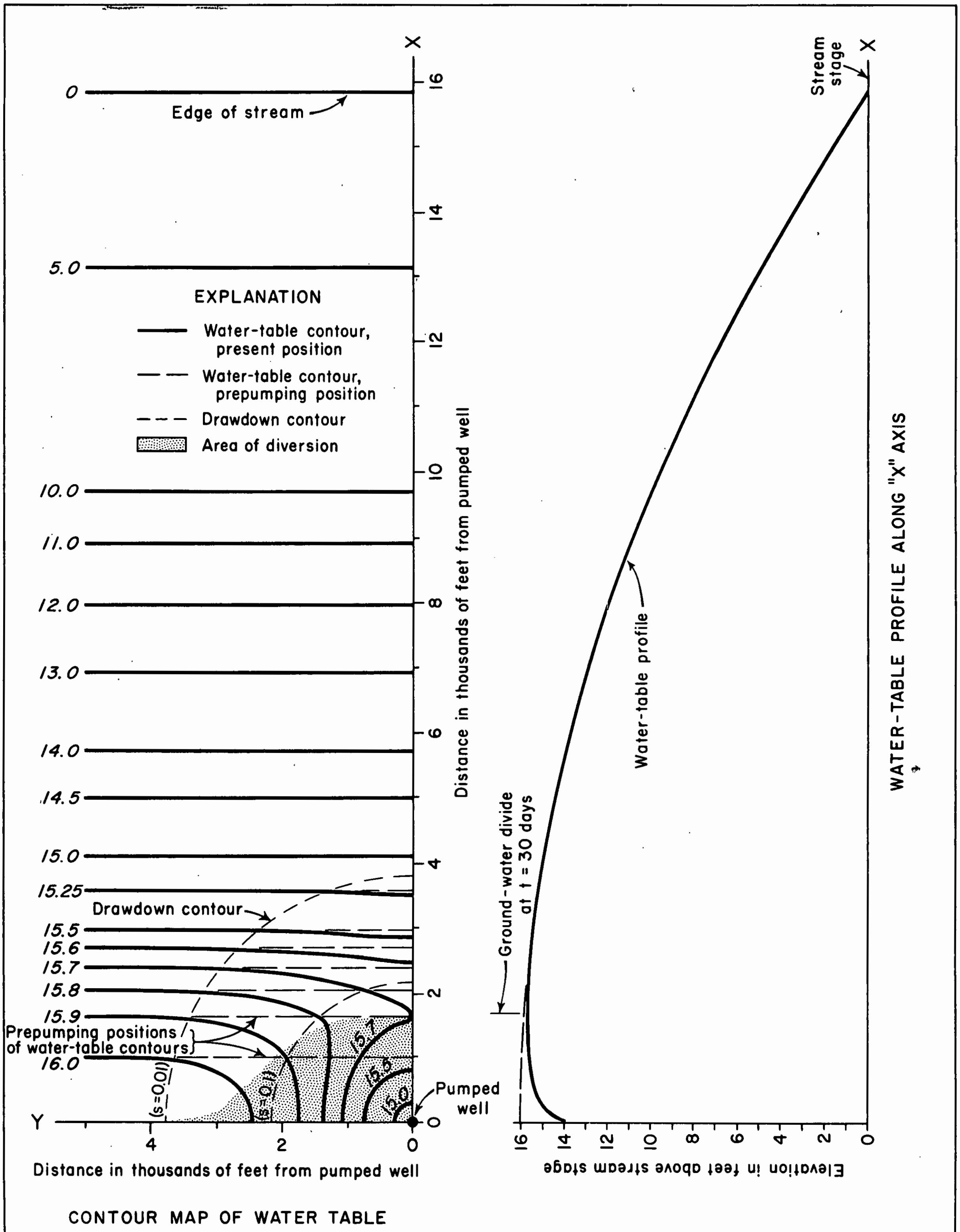

Figure 4.--Contour map and profile of water table when period of pumping equals 30 days.

Each point used in constructing the map and profile of the resultant flow field, represented by the heavy solid lines in figure 4, was determined by subtracting the computed or interpolated drawdown for that point from the initial water-table elevation of the same point. Because of symmetry only part of one quadrant of the aquifer need be illustrated. Note especially the size and configuration of the shaded area, enclosed by the shifted ground-water divide which, in its new position, and at the indicated instant of time, marks the division between that part of the flow field contributary to the well and that part contributary to the stream. More will be said of this later.

Analysis of the flow field, related only to the pumping well, at the end of a 500-day pumping period reveals that the cone of depression may be represented satisfactorily by contours which again are a family of concentric circles. Table 1 contains drawdown values computed by using Theis' nonequilibrium equation; the sensible limit of the cone of depression, as marked by the position of the 0.01 foot drawdown value, is seen to be about 15,400 feet from the pumping well. Thus the cone has expanded so that measureable drawdowns are beginning to be produced at distances from the real pumped well roughly equal to the aquifer half-width "a"; that is to say the cone has almost reached the streams. As pumping continues beyond 500 days, therefore, the buildup effects of the first pair of recharging image wells begin to offset the drawdown effect from the real well, and proper analysis of the cone of depression requires something more than simple application of Theis' equation to a single pumped well. Figure 5 was prepared for t = 500 days using the same procedures and conventions described for figure 4. Again, note the size and configuration of the shaded area bounded by the ground-water divide.

Description of the ultimate steady-state configuration of the cone of depression, after lapse of an infinite period of pumping, requires computations (of drawdown) involving reference to Hantush and Jacob's work, particularly to their equations 21 and 23 with the related explanations of notation. At any point (x, y) in an infinite strip aquifer, tapped by a single pumped well at the coordinate origin at the center of the aquifer, the general expression for the steady state drawdown (when $t \to \infty$) is written as follows:

$$s = \frac{Q}{\pi T} \sum_{n=1}^{\infty} \frac{1}{n} \left\{ \sin \left[\frac{n\pi(x-a)}{2a} \right] \sin \left(- \frac{n\pi}{2} \right) \right\} e^{-\pi n y/2a} \quad \ldots(1)$$

where the coordinate axes are oriented as shown in figure 1 and where

s = drawdown at any point (x, y)
Q = rate of discharge of real well
T = coefficient of transmissibility of aquifer
a = half width of aquifer.

Inspection of equation 1 shows that when n is an <u>even</u> number the term $\sin(-n\pi/2)$ is zero. Therefore only <u>odd</u> numbers need be considered, and as successive odd numbers are assigned to n, in evaluating the series summation, the value of the term $\sin(-n\pi/2)$ changes alternately from -1 to +1.

Two simplified forms of equation 1 are helpful in performing the desired drawdown computations. Note that for a transverse profile coincident with the X axis (see fig. 1) any point on the cone of depression has the general space coordinates (x,0). For y = 0 the exponential function in equation 1 equals 1. Thus the equation is shortened to the form

$$s = \frac{Q}{\pi T} \sum_{n=1}^{\infty} \frac{1}{n} \left\{ \sin \left[\frac{n\pi(x-a)}{2a} \right] \sin \left(- \frac{n\pi}{2} \right) \right\} \quad \ldots\ldots\ldots\ldots(2)$$

Substituting the values specified in this problem for Q, T, and a equation 2 becomes

$$s = \frac{200(1440)}{\pi(80,000)} \sum_{n=1}^{\infty} \frac{1}{n} \left\{ \sin \left[n\pi \left(\frac{x}{31,680} - \frac{1}{2} \right) \right] \sin \left(- \frac{n\pi}{2} \right) \right\}$$

or

$$s = (1.146) \sum_{n=1}^{\infty} \frac{1}{n} \left\{ \sin \left[n\pi \left(\frac{x}{31,680} - \frac{1}{2} \right) \right] \sin \left(- \frac{n\pi}{2} \right) \right\} \quad \text{..(2a)}$$

Similarly, for a longitudinal profile coincident with the Y axis (see fig. 1) any point on the cone of depression has the general space coordinates (0,y). When x = 0 the first sine term in equation 1 becomes identical to the second, and even though their respective values change alternately from -1 to +1 their product is always +1. Equation 1 is therefore shortened to the form

$$s = \frac{Q}{\pi T} \sum_{n=1}^{\infty} \frac{1}{n} \; e^{-\pi n y/2a} \quad \text{.....(3)}$$

Substituting appropriately for Q, T, and a equation 3 becomes

$$s = (1.146) \sum_{n=1}^{\infty} \frac{1}{n} \; e^{-\pi n y/31,680} \quad \text{.....(3a)}$$

Equations 2a and 3a are used to compute steady-state drawdowns for selected points along the transverse profile (X-axis) and longitudinal profile (Y-axis) respectively; results appear in table 2, and sample detailed computations for a selected point on each profile appear in appendix B. These particular computations in turn offer a ready means

Table 2.-- Steady state drawdown in feet at indicated coordinate points in infinite strip aquifer.

X ↓ / Y →	0	500	1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000	10,000	12,000	14,000	16,000	20,000
0		2.12	1.73	1.33	1.10	0.93	0.81	0.71	0.63	0.56	0.50	0.45	0.36	0.29	0.24	0.16
500	2.00	1.93	1.66	1.31	1.09	0.93										
1,000	1.65	1.66	1.53	1.26	1.07	.92	.80	.70	.62	.55	.50					
2,000	1.27	1.31	1.26	1.12	0.99	.87	.77	.68	.60	.54	.48	.43				
3,000	1.06	1.08	1.06	0.98	.89	.80	.72	.65	.58	.52	.47	.42	.34			
4,000	0.94	0.91	0.90	.86	.80	.73	.66	.60	.54	.49	.44	.40	.32	.27		
5,000	.78	.79	.78	.75	.70	.65	.60	.55	.50	.45	.41	.37	.31	.25	.21	.14
6,000	.74	.72	.69	.65	.62	.58	.54	.50	.45	.42	.38	.35	.29	.23	.19	.13
7,000	.58	.59	.58	.56	.54	.51	.48	.44	.41	.38	.35	.32	.26	.22	.18	.12
8,000	.50		.50	.48	.47	.44	.42	.39	.36	.33	.31	.28				
9,000	.42			.41	.40	.38	.36	.34	.32	.29	.27	.25	.21	.17	.14	.10
10,000	.36			.34	.33	.32	.30	.29	.27	.25	.23	.21	.18	.15	.12	.08
11,000	.29															
12,000	.23															
13,000	.16			.16		.15		.14		.12		.10	.09	.07	.06	.04
14,000	.11			.10		.10		.09		.08		.07	.06	.05	.04	.03
15,000	.06															
15,840	0															

Notes:

Drawdowns result from pumping a single well, at center of aquifer, at steady rate of 200 gpm until time t ∞.
Computations involve use of equations 1, 2a, and 3a as appropriate; sample computations appear in Appendix B.
Drawdowns accurate only to nearest tenth of a foot.

for determining the drawdown at any point whose coordinates comprise any of the x and y values chosen for points on the transverse and longitudinal profiles. That the solutions for particular points on the two coordinate axes may be combined to yield solutions for points between those axes follows if it is recognized that this represents merely recombining equations 2 and 3 to revert back to the general equation 1. The computation requires only a term by term multiplication of the series expressions developed for the appropriate X-direction and Y-direction solutions, followed by summation of the products and multiplication by the factor 1.146 (see Appendix B, last column). Results, for selected points, also appear in table 2.

The steady-state drawdowns computed for selected points whose coordinates are indicated in Table 2 are subtracted from the initial steady-state water-table elevations of the same set of points. The resultant elevations are the basis for the water-table profiles and contour map in figure 6.

Comparison of the profile and plan representations of the resultant water-table positions, after pumping periods of 30 days, 500 days, and $t \rightarrow \infty$, shows plainly how this hydrologic system responds to newly imposed discharge (the pumping well) and reveals the degree and promptness with which different parts of the system approach new stabilization. The migration of the ground-water divide, from its initial position (fig. 2) coincident with the longitudinal centerline of the aquifer to its final position enclosing the shaded area of figure 6 is evident.

Introduction of the term "area of diversion" is appropriate in labeling and discussing the shaded areas shown in figures 4, 5, and 6. Inasmuch as the ground-water divide is the outer limit of the shading it follows that precipitation anywhere on the shaded areas must eventually reappear as well discharge. In other words each of the three shaded areas represents, for the indicated period of pumping, the area in which the recharge is being diverted to the pumped well instead of escaping from the aquifer entirely as discharge into the two boundary streams. The relative sizes of the three areas warrant comparison and added comment; the sizes are determined through inspection of figures 4, 5, and 6 and are given in table 3. Given in the last column of the table are the values for the total recharge being received by each area, in gallons per minute. Thus, after the pumped well has operated 30 days it has succeeded in developing an area of diversion of 15.2 x 10^6 sq. ft. Uniform recharge of 6 inches per year on this area is equivalent to pouring water into the aquifer at the rate of 110 gpm. Obviously this is not enough to offset the steady rate at which water is being removed through the well. Therefore the area of diversion must continue to expand, until, at $t = \infty$, the intercepted recharge exactly balances the rate at which the well is being pumped.

In the period when the area of diversion is still expanding the well discharge is made up in part by intercepted recharge and the balance by withdrawal from storage in the aquifer. Although theoretically the withdrawal from storage does not cease short of $t = \infty$ it should be apparent, from figures 5 and 6 and table 3, that virtual stabilization of the entire flow system with cessation of significant draft upon storage occurs at some finite time not greatly in excess of 500 days.

The fallacy expressed in the opening paragraphs of this paper is perhaps most easily controverted, for this particular hydrologic system at least, through substitution of 'area of diversion' for 'cone of depression'. It is the area of diversion which must expand until it has captured

Table 3. - Extent of area of diversion and computed recharge on that area

Pumping period (days)	Area of diversion (millions of sq. ft.)		Total recharge on area, converted to equivalent rate of pumping (gpm)
	In 1 quadrant	In 4 quadrants	
30	3.80	15.2	110
500	6.15	24.6	175
∞	6.95	27.8	200

enough recharge to counterbalance the newly imposed discharge. The only limits to expansion of the cone of depression are the physical boundaries of the system.

Review of the demonstrated response of this hydrologic system to the imposed well discharge should make it indisputably clear that the steady pumping of 200 gpm has been satisfied in the long run only by a decrease of 200 gpm in the discharge from the aquifer to the streams. Although some ground water was taken from storage, in developing the flow field needed to meet the demands of the pumping well, in the final analysis the system reaches its new steady state through diminution of the natural discharge to the streams, the diminution being exactly equal to the newly imposed discharge.

Recommended for supplemental review are Jacob's (1950, p. 381-383) brief observations on flow to a well in an infinite strip aquifer subject to uniform recharge and bounded by two parallel streams. Jacob's comments suggest the manner in which the configuration and extent of the area of diversion are related to the well discharge, the rate of recharge, and the well position with respect to the streams.

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Appendix A. -- Data for plotting water-table profile for cross section of aquifer oriented in X direction.

X' (ft)	X (ft)	h_o (ft)	X'	X	h_o
15,840	0	16.07	7,840	8,000	11.97
15,340	500	16.05	6,840	9,000	10.88
14,840	1,000	16.00	5,840	10,000	9.66
13,840	2,000	15.81	4,840	11,000	8.32
12,840	3,000	15.49	3,840	12,000	6.85
11,840	4,000	15.04	2,840	13,000	5.25
10,840	5,000	14.47	1,840	14,000	3.52
9,840	6,000	13.76	840	15,000	1.66
8,840	7,000	12.93	0	15,840	0

Notes:

X' = distance from left-hand stream (see fig. 1)

X = distance from midpoint of aquifer (see fig. 1)

h_o = elevation of water table above stream stage

Computations of h_o were made using the relation:

$$h_o = 6.4041 \times 10^{-2} (31.68 \underline{X} - \underline{X}^2)$$

where $\underline{X} = X'/1000$

Data tabulated above were used in plotting water-table profile shown in fig. 2.

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Appendix B. -- Sample computations of steady-state drawdown at indicated coordinate points in infinite strip aquifer.

	(x = 1,000, y = 0)				x = 0, y = 1,000			x = 1,000 y = 1,000
n	nA	sin (nA)	$\sin(-\frac{n\pi}{2})$	$\frac{1}{n}\sin(nA)\sin(-\frac{n\pi}{2})$	nA'	$e^{-nA'}$	$\frac{1}{n}e^{-nA'}$	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(5) x (7)
1	- 1.472	-0.995	-1	+0.995	0.0992	0.906	0.906	+0.901
3	- 4.416	+ .956	+1	+ .319	.2975	.743	.248	+ .237
5	- 7.360	- .880	-1	+ .176	.4958	.609	.122	+ .107
7	-10.304	+ .770	+1	+ .110	.6942	.499	.071	+ .055
9	-13.248	- .630	-1	+ .070	.8925	.410	.046	+ .029
11	-16.192	+ .466	+1	+ .042	1.0908	.336	.031	+ .014
13	-19.136	- .282	-1	+ .022	1.2892	.275	.021	+ .006
15	-22.080	+ .089	+1	+ .006	1.4875	.226	.051	+ .001
17	-25.024	+ .109	-1	- .006	1.6858	.185	.011	- .001
19	-27.968	- .301	+1	- .016	1.8842	.152	.008	- .002
21	-30.912	+ .483	-1	- .023	2.0825	.125	.006	- .003
23	-33.856	- .646	+1	- .028	2.2808	.102	.004	- .003
25	-36.800	+ .782	-1	- .031	2.4792	.084	.003	- .003
27	-39.744	- .097	+1	- .004	2.6775	.069	.003	.000
29	-42.688	+ .962	-1	- .032	2.8758	.056	.002	- .002
31	-45.632	- .997	+1	- .032	3.0742	.046	.001	- .001
33	-48.576	+ .993	-1	- .030	3.2725	.038	.001	- .001
35	-51.520	- .950	+1	- .027	3.4708	.031	.001	- .001
37	-54.464	+ .870	-1	- .024	3.6692	.025	.001	- .001
39	-57.408	- .758	+1	- .019			Σ = 1.501	.000
41	-60.352	+ .615	-1	- .015				Σ = 1.332
43	-63.296	- .448	+1	- .010				
45	-66.240	+ .263	-1	- .006				
47	-69.184	- .069	+1	- .001				
49	-72.128	- .129	-1	+ .003				
				Σ = 1.439				

$A = \pi\left(\frac{x}{2a} - \frac{1}{2}\right)$

$A = \pi\left(\frac{1,000}{31,680} - \frac{1}{2}\right) = -1.4716$

s = (1.146) (1.439) = <u>1.65'</u>
(See equation 2a.)

$A' = \frac{\pi y}{2a} = \frac{1,000\pi}{31,680} = 0.099166$

s = (1.46) (1.51) = <u>1.73'</u>
(See equation 3a.)

s = (1.146) (1.332) = <u>1.53'</u>

Note: The three drawdown values computed above are entered in Table 2.

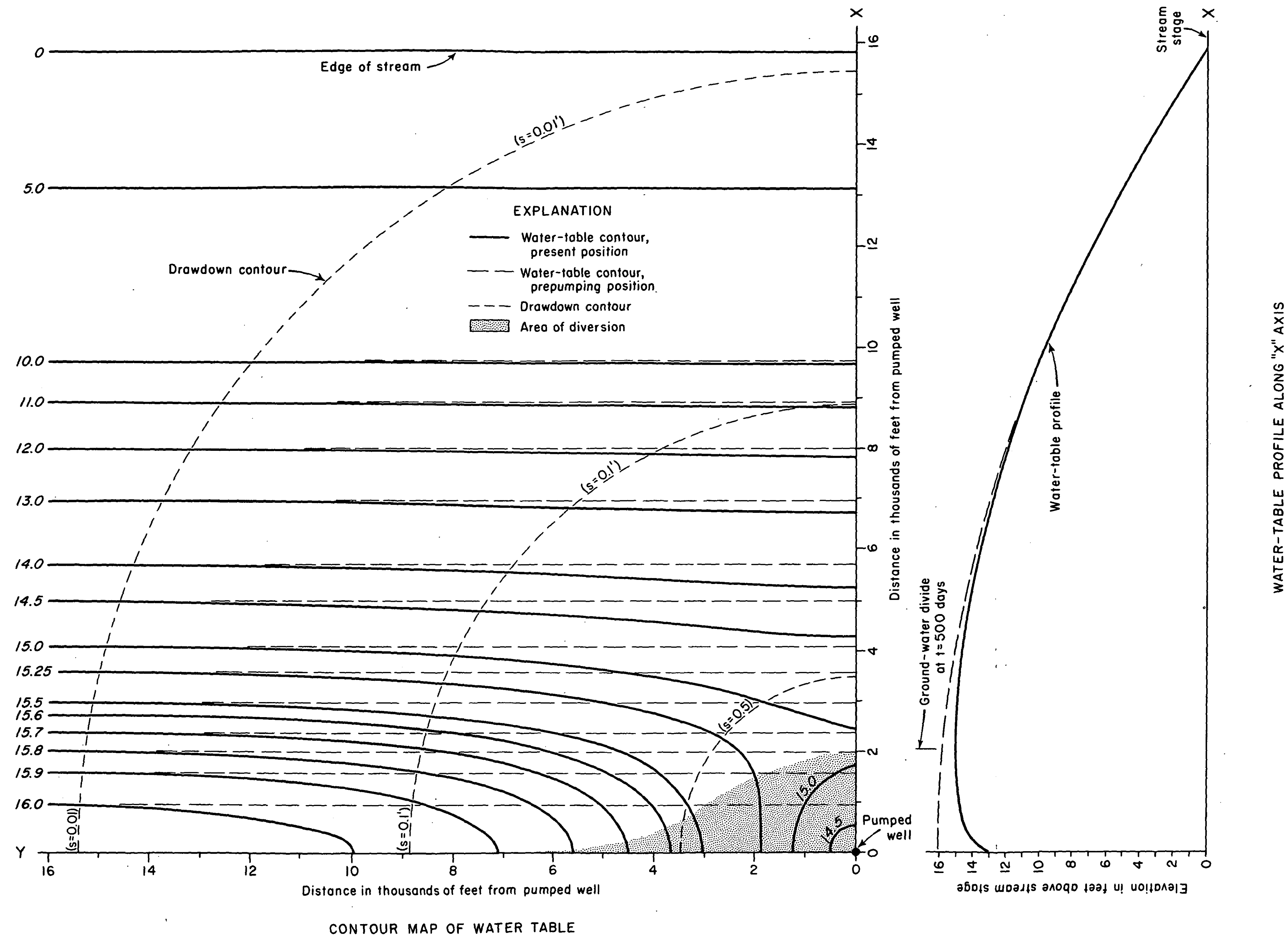

Figure 5.-- Contour map and profile of water table when period of pumping equals 500 days.

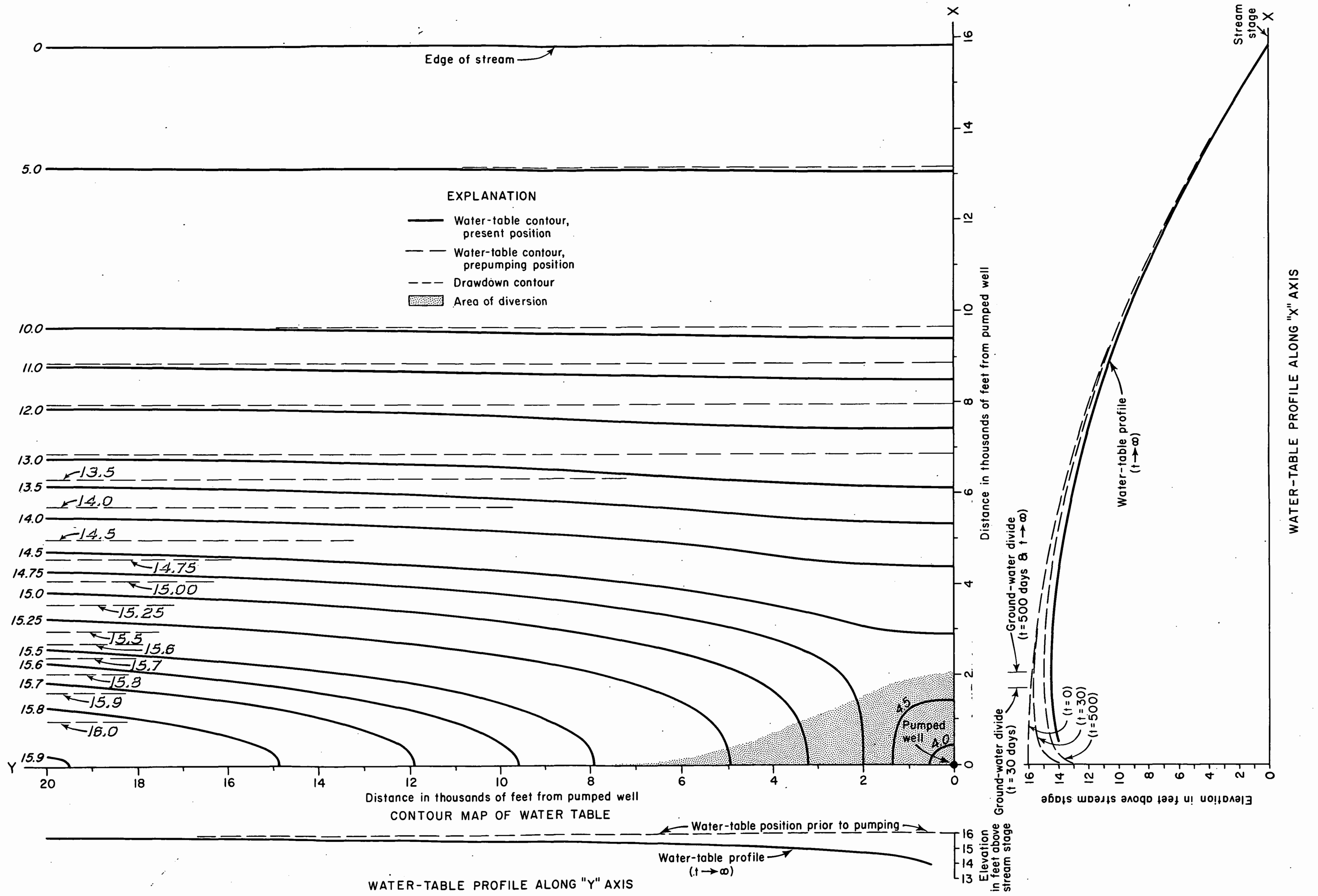

Figure 6.--Contour map and profiles of water table when period of pumping approaches infinity.