U. S. GEOLOGICAL SURVEY.

REPORTS - OPEN FILE SERIES, no. 1047, 1968.

UNITED STATES DEPARTMENT OF THE INTERIOR
U.S. GEOLOGICAL SURVEY
[Reports - Open file series]

CALCULATION OF CAVITY RADIUS USING AN AVERAGE POTENTIAL ENERGY FUNCTION

by

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Open-file report
1968

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Accompanies (200)
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no. 1047

U. S. GEOLOGICAL SURVEY
Washington, D. C.
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For release MAY 27, 1968

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7. Calculation of cavity radius using an average potential energy function, by G. E. Brethauer. 20 p., 3 figs., 1 table.

DEFINITION OF TERMS

WP	Point below surface where explosion occurred; called the working point.
H	Depth to working point (cm).
ρ (m)	Density of layer m (g/cm^3).
$\sum_{i=1}^{n} \rho_i h_i$	Summation of the products of density multiplied by thickness for each layer for n layers (g/cm^2).
Θ	Angle measured from a horizontal plane through the working point to any radius of the cavity (degrees).
$R_o(\Theta)$	Initial radius of the cavity at an angle Θ (cm).
$R_f(\Theta)$	Final radius of the cavity at an angle Θ.
K'	Function relating potential energy to overburden pressure.
$\bar{E}_d$	Average potential energy function (ergs).
E_t	Total energy yield of the explosion (ergs).
$K_{(m)}$	Empirical partition constant for m type of material; K is the ratio of $E_d/K'E_t$.
$\bar{F}(\Theta_1)$	Average force at an angle Θ (dynes).
$\bar{P}(\Theta_1)$	Average overburden pressure for an expanding sphere where conditions for any radius are assumed to be the same as those for a radius at the angle Θ.
$\bar{A}(\Theta_1)$	Average surface area of an expanding sphere where conditions for any radius are assumed to be the same as those for a radius at the angle Θ_1.
$\emptyset$	Symbol for Θ equals zero degrees.

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CALCULATION OF CAVITY RADIUS USING AN AVERAGE POTENTIAL ENERGY FUNCTION

By

G. E. Brethauer

ABSTRACT

The radius of a cavity formed by an underground nuclear explosion is an important variable in many problems. In this report, the equation used for calculating radii predicts nonspherical cavities. This is probably a more realistic equation than one which results in a calculated spherical cavity. The equation for calculation of nonspherical-cavity radii is derived by calculating the average potential energy function for expanding a cavity to a certain radius, $R_f(\Theta)$, and relating this energy to the total energy of the explosion by an empirically determined constant. The constant is statistically a function only of the type of medium around the explosion. The general equation expresses cavity radii as a function of density (ρ), depth of burial (H), type of medium (K), total energy of the explosion (E_t), and the angle measured from a horizontal plane to any radius ($R_f(\Theta)$). The equation for an explosion in a homogeneous medium is

$$E_t = \frac{2\pi\rho g}{3K} \left(2HR_f^3(\Theta) - R_f^4(\Theta)\sin\Theta\right).$$

Commonly a cavity radius is formed by an explosion in a layered medium and is confined to one of the layers. The approximate solution for a cavity radius in this case is

$$R_f(\Theta) = R_f(\emptyset)\left[1 - \frac{\rho_{(m)} R_f(\emptyset)\sin\Theta}{2\sum_{i=1}^{n}\rho_i h_i}\right]^{-1/3}$$

where

$$R_f(\emptyset) = \left[\frac{3E_t\,K_{(m)}}{4\pi g \sum_{i=1}^{n} \rho_i h_i}\right]^{1/3}$$

INTRODUCTION

The size and the shape of any cavity formed by a contained underground nuclear explosion are important variables. For example, calculation of chimney height, of the amount of permanent displacement of the ground surface above an explosion, and of the depth required for safe containment all require some knowledge of the calculated cavity. The methods of calculating cavity radius range from purely empirical to nearly pure theoretical approaches. As all methods require the use of empirically determined constants, which may have large variances, the theoretical approaches do not necessarily provide better answers than a purely empirical method. The equation derived in this report, though based on theory, also requires empirically determined constants. An empirical constant, K, is the calculated statistical result of the ratio of the average change of potential energy to the total energy of the explosion. The equation predicts nonspherical cavities. This result is probably more realistic than the results obtained by use of equations that predict spherical cavities.

One of the simplest methods of calculation of cavity radius is to ~~given by Rogich and Rich (1966). They~~ express cavity radius as a power function of yield times a constant (D.Rogich and C. Rich, written commun., 1966). Rogich and Rich show that parameters such as density of the medium (ρ) and depth of burial (H) are not statistically significant and that a single prediction equation could be used for explosions in either tuff or alluvium at the Nevada Test Site. However, the data are biased due to the fact that explosions of large yield are buried to a greater depth than events of small yield.

Boardman and others (1964), assume that gases produced by a contained underground nuclear explosion expand adiabatically to the overburden pressure. The derivation originated by Boardman and others is based on a comparison of two explosions under different overburden pressure and allows for differences in yield through cube-root scaling. Their equation for cavity radius is a function of the type of media, the density of the media, the depth of burial, the total yield of the explosion, and the ratio of specific heats of the vaporized rock, γ. A normal value for γ of 4/3 is assumed. The cavity pressure may not, however, be equal to the weight of the overburden for a contained nuclear explosion and, in fact, might be more or less depending on the depth of burial and other inhomogeneities in the rock. The value of 4/3 for γ might vary for different types of media.

Duvall and Stephenson (1965) derived an equation to calculate the radiant strain energy at any distance from the origin of a given expansion-cavity or collapse-cavity. The theoretical increase or decrease in elastic strain energy at a certain distance along a radius induced by a spherical-cavity volume change is computed using elastic theory for cavity-media interactions. The elastic work done on the medium by cavity deformation is computed. The amount of elastic work is subtracted from the change in elastic strain energy at a radial distance greater than the final cavity radius, R_f, and the difference is termed the radiant seismic energy, ΔE_s. Duvall and Stephenson do not, however, explore the possibility of a direct correlation between the theoretically calculated radiant seismic energy and the total energy of a contained underground explosion. Their equations do not include the work done on the media outside of the cavity by the plastic pulse accompanying an underground explosion. The plastic pulse might not be related to the elastic energy at any point.

DERIVATION

The equation for calculating cavity radius, as described in this report, is derived by using an average potential energy function of the cavity during its formation. This average potential energy function, $\overline{E}_d$, is a function of density of the media (ρ), depth of burial (H), and cavity radius ($R_f(\Theta)$). This energy is related to the total energy of an explosion by an empirically determined constant. The constant, K, is the ratio of average potential energy/K' to the total energy of the explosion; statistical tests indicate that the constant, K, is not the same for different media and does not differ for changes in yield or depth of burial. The equation is first derived for a homogeneous medium, then developed to a more general form that can be used to calculate cavity size and shape in layered media.

Homogeneous Model

If a contained explosion occurs at a depth of H in a specified homogeneous medium of density ρ, a preshot chamber with radius, $R_o(\Theta)$, will be expanded to a final postshot cavity with radius, $R_f(\Theta)$. The average potential energy function, which is calculated for a cavity expansion from a radius $R_o(\Theta)$ to $R_f(\Theta)$, is termed $\overline{E}_d$. A point source of energy is assumed and, therefore, the total energy of the explosion, E_t, and the energy used to change the potential energy of the cavity, $\overline{E}_d$, is distributed equally in all directions.

The potential energy function will not be the same for all angles Θ. The resistance of the media will vary with the angle Θ and will be directly related to the resistive force in the direction defined by Θ. This resistance to cavity expansion is due to two types of confining forces: (1) vertical forces due to overburden pressure and (2) confining forces due to surrounding particles. The resistance to cavity expansion due to the surrounding particles, which is related to the value of the potential energy function, is also a function of overburden pressure. Thus, resistance to cavity expansion is a function of overburden pressure and this function will be termed K'. With these assumptions in mind, a model was set up as shown in figure 1. The device is assumed to be buried in a specified homogeneous medium of density ρ, to a depth of H, at a point, WP. A plane-polar coordinate system is overlain with WP as the origin.

The potential energy change for a cavity expansion from $R_o(\Theta_1)$ to $R_f(\Theta_1)$ will be investigated: If it is assumed that the resistance to cavity expansion along any radius of the cavity is the same as that along a radius at an angle Θ_1, the energy used in displacing the mass between $R_o(\Theta_1)$ and $R_f(\Theta_1)$ would be:

$$E_d(\Theta_1) = F(\Theta_1) \cdot (R_f(\Theta_1) - R_o(\Theta_1)) \tag{1}$$

The average force overcome in expanding a sphere is:

$$\overline{F}(\Theta_1) = K'\overline{P}(\Theta_1) \cdot \overline{A}(\Theta_1) \tag{2}$$

where

$\overline{P}(\Theta_1)$ = average overburden pressure encountered from $R_o(\Theta_1)$ to $R_f(\Theta_1)$

$\overline{A}(\Theta_1)$ = average surface area of the sphere as the radius of the cavity expands from $R_o(\Theta_1)$ to $R_f(\Theta_1)$

Since $R_f(\Theta) \gg R_o(\Theta)$ for all Θ, $R_o(\Theta)$ will be assumed to be zero for Θ. Thus

$$\overline{A}(\Theta_1) = \frac{4\pi \int_{\emptyset}^{R_f(\Theta_1)} r^2 dr}{\int_{\emptyset}^{R_f(\Theta_1)} dr} = \frac{\frac{4\pi}{3} R_f^3(\Theta_1)}{R_f(\Theta_1)} = \frac{4\pi}{3} R_f^2(\Theta_1) \tag{3}$$

is the average area of a sphere whose radius expands from $R_o(\Theta_1) = \emptyset$ to $R_f(\Theta_1)$.

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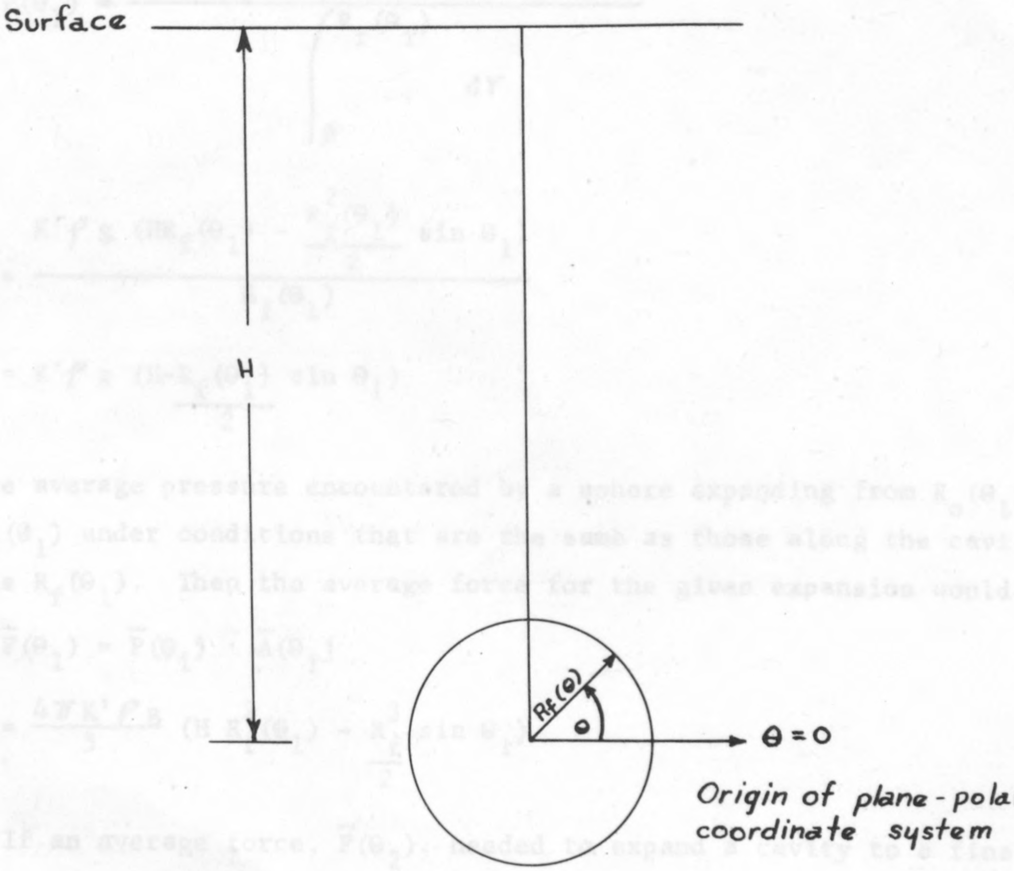

Figure 1.--Schematic diagram of average potential energy function model

Now,

$$\overline{P}(\theta_1) = \frac{K'\rho g \int_{\emptyset}^{R_f(\theta_1)} (H - \sin\theta_1)\, d\gamma}{\int_{\emptyset}^{R_f(\theta_1)} d\gamma} \tag{4}$$

$$= \frac{K'\rho g \left(HR_f(\theta_1) - \frac{R_f^2(\theta_1)}{2} \sin\theta_1\right)}{R_f(\theta_1)}$$

$$= K'\rho g \left(H - \frac{R_f(\theta_1)}{2} \sin\theta_1\right)$$

is the average pressure encountered by a sphere expanding from $R_o(\theta_1) = \emptyset$ to $R_f(\theta_1)$ under conditions that are the same as those along the cavity radius $R_f(\theta_1)$. Then the average force for the given expansion would be:

$$\overline{F}(\theta_1) = \overline{P}(\theta_1) \cdot \overline{A}(\theta_1) \tag{5}$$

$$= \frac{4\pi K'\rho g}{3} \left(H\, R_f^2(\theta_1) - \frac{R_f^3}{2} \sin\theta_1\right).$$

If an average force, $\overline{F}(\theta_2)$, needed to expand a cavity to a final radius of $R_f(\theta_2)$ is calculated, it will be different from $\overline{F}(\theta_1)$. The two average forces are different because the average overburden pressure along a ray at the angle θ_1 is different from that along a ray at the angle θ_2. $\overline{E}_d$ does not change with angle. From equation (1) it can be seen that, if the average force changes with angle, the final cavity radius must change with angle.

In the following, a homogeneous model is investigated where rays from the origin at different angles meet different resistance. If the average value of the average pressure around the sphere overcome in forming the cavity can be calculated, the average force and cavity radius can be computed as follows:

$$(\bar{P}(\Theta))_{ave} = \frac{K'\rho g \int_{\emptyset}^{2\pi} (H - \frac{R_f(\Theta)}{2} \sin\Theta \; d\Theta}{\int_{\emptyset}^{2\pi} d\Theta} \tag{6}$$

$$= \frac{K'\rho g \,[2H\Theta - R_f(\Theta) \cos\Theta]_{\emptyset}^{2\pi}}{2\,[\Theta]_{\emptyset}^{2\pi}}$$

$$= \frac{K'\rho g\,(4\pi H - R_f(\Theta) - \emptyset + R_f(\Theta)}{4\pi}$$

$$= K'\rho g H$$

Thus, the average value of the average pressure around the sphere overcome in forming the cavity is that pressure which is encountered by a ray along the angle $\Theta = \emptyset$. The average value of average area will be:

$$(\bar{A}(\Theta))_{ave} = \frac{4\pi}{3} R_f^2(\emptyset) \tag{7}$$

The average of the forces for the incremental arcs $d\Theta$ would be:

$$(F_{ave})_{ave} = \frac{4\pi K'\rho g}{3} H R_f^2(\emptyset) \tag{8}$$

and

$$\bar{E}_d = \frac{4\pi K'\bar{\rho} g}{3} H R_f^3(\emptyset) \tag{9}$$

where $R_f(\emptyset)$ is the final cavity radius along the horizontal axis at $\Theta = \emptyset$ (zero).

In the foregoing discussion axial symmetry has been assumed around the Y-axis.

Evaluation of the relation between $\bar{E}_d$ and E_t

To relate the calculated quantity $\bar{E}_d$, the average potential energy function, to E_t, the total energy of the explosion, a functional relationship was assumed and tested.

First, scatter diagrams of $(\bar{E}_d/K'E_t) \times 100$ against E_t are plotted (figs. 2 and 3) for 52 events in alluvium and 30 events in tuff. From these plots a linear constant relationship was assumed. A least-squares analysis is used to test the hypothesis that $\bar{E}_d/K'$ is a constant fraction of E_t by assuming $[(\bar{E}_d/K'E_t) \times 100] = K\% = a_0 + a_1 E_t$. K% is the ratio of $\bar{E}_d/K'$ to E_t in percent. The literals a_0 and a_1 are intercept and slope constant, respectively. For both sets of data a_1 is very close to zero which shows that K% is not a function of the most important variable, E_t (see Appendix). Thus, the cavity radius prediction equation could be written:

$$E_t = \frac{4\pi\rho g}{3K_{(m)}} \left(H R_f^3(\theta) - \frac{R_f^4(\theta) \sin\theta}{2}\right) \tag{10}$$

where $K_{(m)}$ is an empirical partition constant.

$K_{(m)}$ for alluvium = .012

$K_{(m)}$ for tuff = .0206

Thus, if the total energy of an explosion, the type and density of the medium, and the depth of burial are given, the cavity radius in any direction from the WP can be predicted for an explosion in a homogeneous medium.

Layered Model

Most underground nuclear explosions are set off at depth in a medium which is made up of layers, each of a different material (that is, rock type), density, and thickness. The length of each cavity radius, $R_f(\theta)$, formed in a layered medium will depend on (1) the total energy of the explosion, (2) the density and thickness of each of the layers through which the cavity radius extends, (3) the angle of incidence at which the

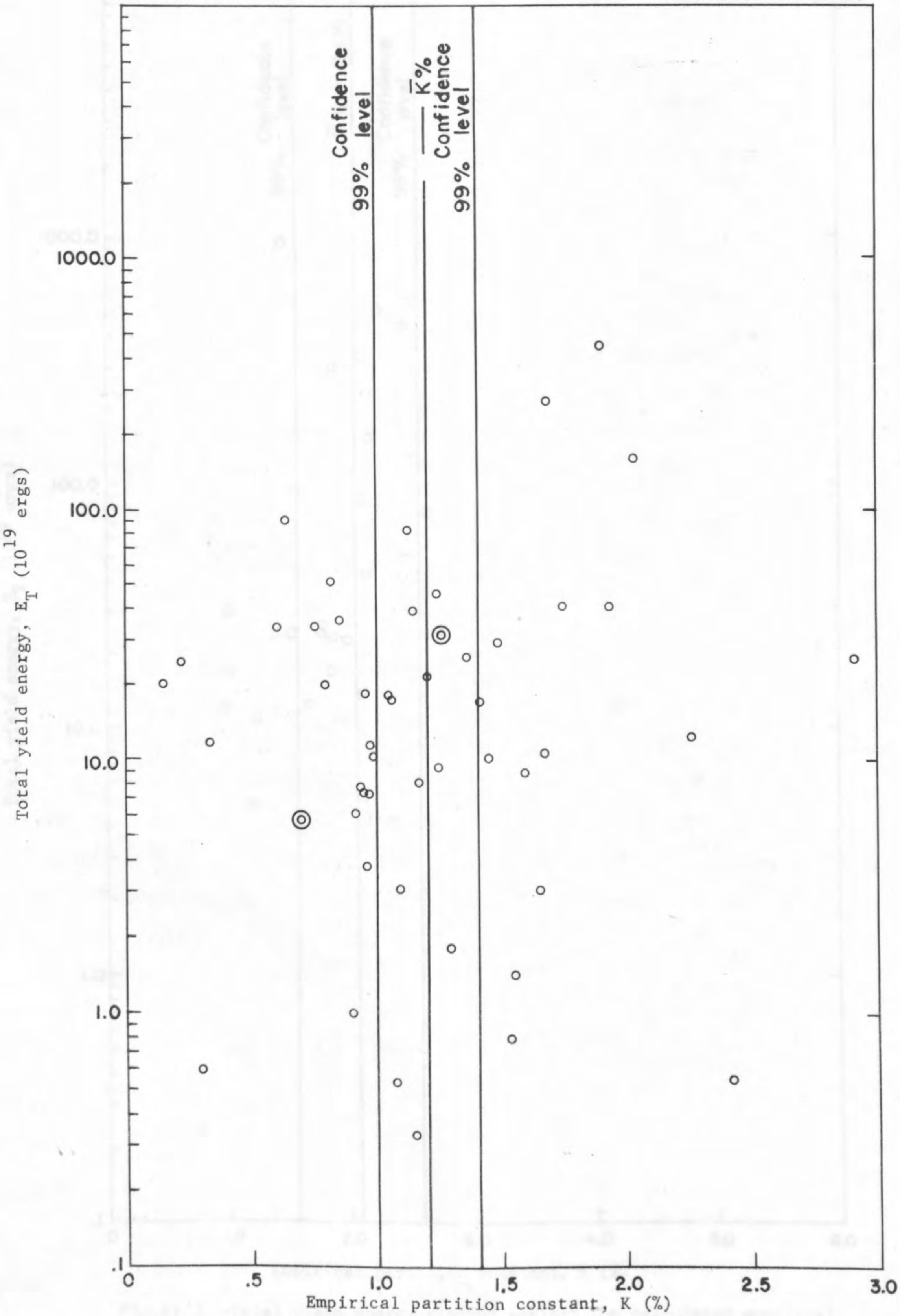

Figure 2.--Total yield energy plotted against the calculated empirical constant, K, in percent, for 52 events in alluvium (modified after Roland F. Beers, Inc., written commun., 1966)

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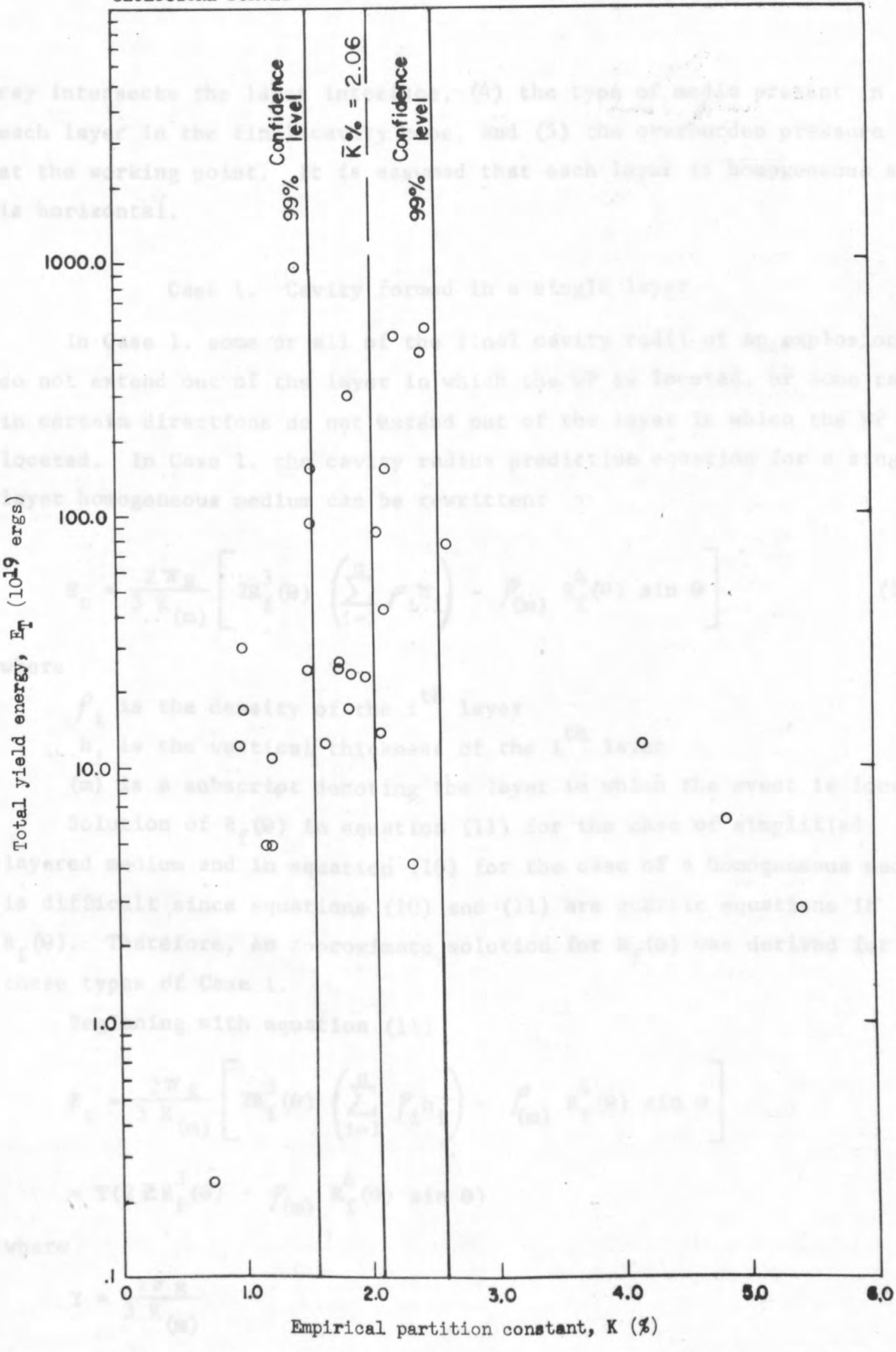

Figure 3.--Total yield energy plotted against the calculated empirical constant, K, in percent, for 30 events in tuff (modified after Roland F. Beers, Inc., written commun., 1966)

ray intersects the layer interface, (4) the type of media present in each layer in the final cavity zone, and (5) the overburden pressure at the working point. It is assumed that each layer is homogeneous and is horizontal.

Case 1. Cavity formed in a single layer

In Case 1. some or all of the final cavity radii of an explosion do not extend out of the layer in which the WP is located, or some radii in certain directions do not extend out of the layer in which the WP is located. In Case 1. the cavity radius prediction equation for a single-layer homogeneous medium can be rewritten:

$$E_t = \frac{2\pi g}{3\,K_{(m)}}\left[2R_f^3(\Theta)\left(\sum_{i=1}^{n}\rho_i h_i\right) - \rho_{(m)}\,R_f^4(\Theta)\,\sin\Theta\right] \tag{11}$$

where

ρ_i is the density of the i^{th} layer

h_i is the vertical thickness of the i^{th} layer

(m) is a subscript denoting the layer in which the event is located.

Solution of $R_f(\Theta)$ in equation (11) for the case of simplified layered medium and in equation (10) for the case of a homogeneous media is difficult since equations (10) and (11) are quartic equations in $R_f(\Theta)$. Therefore, an approximate solution for $R_f(\Theta)$ was derived for these types of Case 1.

Beginning with equation (11)

$$E_t = \frac{2\pi g}{3\,K_{(m)}}\left[2R_f^3(\Theta)\left(\sum_{i=1}^{n}\rho_i h_i\right) - \rho_{(m)}\,R_f^4(\Theta)\,\sin\Theta\right]$$

$$= T(2\,Z\,R_f^3(\Theta) - \rho_{(m)}\,R_f^4(\Theta)\,\sin\Theta)$$

where

$$T = \frac{2\pi g}{3\,K_{(m)}}$$

$$Z = \sum_{i=1}^{n} \rho_i h_i$$

Upon rewriting, equation (11) becomes

$$R_f^3(\Theta) = \frac{E_T}{2ZT\left[1 - \dfrac{\rho_{(m)}\, R_f(\Theta)\, \sin\Theta}{2Z}\right]} \tag{12}$$

$$R_f(\Theta) = \left[\frac{E_T}{2ZT\left[1 - \dfrac{\rho_{(m)}\, R_f(\Theta)\, \sin\Theta}{2Z}\right]}\right]^{1/3} \tag{13}$$

$$= \left(\frac{E_T}{2ZT}\right)^{1/3} \left[1 - \frac{\rho_{(m)}\, R_f(\Theta)\, \sin\Theta}{2Z}\right]^{-1/3}$$

Let

$$R_f(\emptyset) = \left(\frac{E_T}{2ZT}\right)^{1/3} = \left[\frac{3\, E_T\, K_{(m)}}{4\pi g \sum_{i=1}^{n} \rho_i h_i}\right]^{1/3}$$

then

$$\frac{R_f^3(\Theta)}{R_f^3(\emptyset)} = \left[\frac{2Z}{2Z - \rho_{(m)}\, R_f(\Theta)\, \sin\Theta}\right]^{1/3} = \left[\frac{2Z}{2Z - \rho_{(m)}\, (R_f(\emptyset) + \delta)\, \sin\Theta}\right]^{1/3} \tag{14}$$

where

$\delta = R_f(\Theta) - R_f(\emptyset)$ and

$\delta \ll R_f(\emptyset)$.

Upon expansion of the denominator of the right-hand member of equation (14) the following approximation is calculated

$$2Z - \rho_{(m)}(R_f(\emptyset) + \delta)\sin\Theta = 2Z - \rho_{(m)}\, R_f(\emptyset)\sin\Theta - \rho_{(m)}\,\delta\sin\Theta \tag{15}$$

$$\doteq 2Z - \rho_{(m)}\, R_f(\emptyset)\sin\Theta$$

When using this approximation, equation (13) becomes

$$R_f(\Theta) = R_f(\emptyset)\left[\frac{2Z}{2Z - \rho_{(m)} R_f(\emptyset) \sin\Theta}\right]^{1/3} \tag{16}$$

$$= R_f(\emptyset)\left[\frac{1}{1 - \dfrac{\rho_{(m)} R_f(\emptyset) \sin\Theta}{2Z}}\right]^{1/3}$$

which is an approximate solution for equations (10) and (11).

Case 2. Cavity formed in more than one layer

Under more usual conditions the final cavity zone encompasses more than a single layer of material. Thus, to calculate $R_f(\Theta)$ for certain Θ, the fact that the cavity is formed in layers of different media, which have varying thicknesses and densities, must be taken into account. If the cavity radius, $R_f(\Theta_1)$ at an angle Θ_1 extends through m layers and partially through the m + 1 layer ($m \geq 1$), the cavity radius prediction equation must be divided into two parts. The first part will compute the amount of energy necessary to form a cavity radius in a specific direction through m layers of material. The second part will evaluate the energy needed to extend the cavity radius in a specified direction from the m layer—m+1 layer interface to the final cavity radius in the m + 1 layer

Thus

$$\sum_{J=1}^{m} E_T(J) = \frac{4\pi g}{3}\sum_{J=1}^{m}\frac{1}{K_J}\left\{\left[\sum_{J=1}^{m} R_J(\Theta)\right]^3\left[Z - \frac{\rho_J}{2}\sum_{J=1}^{m} t_J\right] - \left[\sum_{J=1}^{m} R_{J-1}(\Theta)\right]^3\left[Z - \frac{\rho_{J-1}}{2}\sum_{J=1}^{m} t_{J-1}\right]\right\} \tag{17}$$

where

t_J is the thickness of each layer measured along the ray at $\Theta = 90°$.

$$R_{J=\emptyset}(\Theta) = \emptyset \text{ (zero)}$$
$$t_{J=\emptyset} = \emptyset \text{ (zero)}$$

gives the energy used in forming a cavity radius through m layers at an angle Θ. The energy used in forming a cavity radius at an angle Θ through m layers is less than the total energy, E_T. The energy required to expand the cavity at an angle Θ through m + 1 layers would be greater than E_T. The total energy minus the energy used in forming a cavity radius at an angle Θ through m layers would be the residual energy available to extend the cavity at an angle Θ in the $(m + 1)^{th}$ layer. This residual energy, $E_{T(m+1)}$ can be used to find $R_{m+1}(\Theta)$, the length of the cavity radius at an angle Θ in the $(m + 1)^{th}$ layer because

$$E_{T(m+1)} = E_T - \sum_{J=1}^{m} E_{T(J)} = \frac{4\pi g}{3K_{m+1}} \left\{ Z\left[\sum_{J=1}^{m+1} R_J(\Theta)\right]^3 - \frac{\rho_{(m+1)}}{2}\sin\Theta\left[\sum_{J=1}^{m+1} R_J(\Theta)\right]^4 - Z\left[\sum_{J=1}^{m} R_J(\Theta)\right]^3 + \frac{\rho_{(m+1)}\sin\Theta}{2}\left[\sum_{J=1}^{m} R_J(\Theta)\right]^4 \right\} \tag{18}$$

Combining equations (17) and (18)

$$E_T = \frac{4\pi g}{3}\left[\left\{\sum_{J=1}^{m} \frac{1}{K_J}\left[\left(\sum_{J=1}^{m} R_J(\Theta)\right)^3\left(Z - \frac{\rho_J}{2}\sum_{J=1}^{m} t_J\right) - \left(\sum_{J=1}^{m} R_{J-1}(\Theta)\right)^3\left(Z - \frac{\rho_{J-1}}{2}\sum_{J=1}^{m} t_{J-1}\right)\right]\right\} + \frac{1}{K_{m+1}}\left\{Z\left[\sum_{J=1}^{m+1} R_J(\Theta)\right]^3 - \frac{\rho_{m+1}\sin\Theta}{2}\left[\sum_{J=1}^{m+1} R_J(\Theta)\right]^4 - Z\left[\sum_{J=1}^{m} R_J(\Theta)\right]^3 + \frac{\rho_{m+1}\sin\Theta}{2}\left[\sum_{J=1}^{m} R_J(\Theta)\right]^4\right\}\right] \tag{19}$$

Equation (19) is the cavity radius prediction equation for the general case, Case 2., in a layered medium.

APPENDIX

Statistical Tests

To find the empirical relation between the total energy, E_t, and the derived quantity, $\frac{\bar{E}_d}{K'}$, scatter diagrams of $(\bar{E}_d/K'E_t) \times 100$ versus E_t for 52 events in alluvium and 30 events in tuff (Roland F. Beers, Inc., written commun., 1966) were plotted on Ln-Ln graph paper (fig. 2 and 3). Because the calculated quantity $(\bar{E}_d/K'E_t) \times 100$ for each graph appeared to be a constant for all E_t, it was assumed that $\bar{E}_d/E_t$ is equal to K. The average value for K was then found for the events in alluvium and tuff.

$K_{alluvium} = .012$

$K_{tuff} = .0206$

To test if $K = K(E_t)$ the method of least squares was used assuming

$$K\% = a_{\emptyset} E_t^{a_1} \qquad (1)$$

to be the relation between K and E_t. For the 52 events in alluvium this method showed that

$$a_1(\text{alluvium}) = 1.742 \times 10^{-22} \qquad (2)$$

and for the 30 events in tuff

$$a_1(\text{tuff}) = -2.26 \times 10^{-23} \qquad (3)$$

These values for the slope show that the empirical partition constant, K, is not a function of the total energy of the event. By using a student's T test for data of events in alluvium and tuff, the range and percent range of K for confidence levels of 99 percent, 95 percent, 80 percent, and 50 percent were evaluated.

Tables 1 and 2 show that for the 99 percent confidence level, the $\bar{K}$ (tuff) and $\bar{K}$ (alluvium) are not of the same population; thus, the type of medium is statistically significant. The range of K for both sets of data is affected by the range in the values given for E_t and R_f.

Table 1

$\overline{K}$ (alluvium) = .012

Confidence level (percent)	Percent range	K maximum	K minimum
99	±17.0	0.014	0.0100
95	±12.5	0.0135	0.0105
80	± 8.4	0.013	0.011
50	± 4.2	0.0125	0.0115

Table 2

$\overline{K}$ (tuff) = .0206

Confidence level (percent)	Percent range	K maximum	K minimum
99	±25.2	0.0258	0.0154
95	±18.8	0.0245	0.0167
80	±12.1	0.0231	0.0181
50	± 6.3	0.0219	0.0193

The range of estimate for $R_f(\emptyset)$ was next calculated assuming the accuracy of E_t to be ±50 percent. The equations used were:

$$R_+(\emptyset)\% = \left[\frac{R_{max}(\emptyset) - R(\emptyset)}{R(\emptyset)}\right] \times 100$$

$$= \left[\frac{E_{T\,max}^{1/3} - E_T^{1/3}}{E_T^{1/3}}\right] \times 100 \qquad (4)$$

$$R_-(\emptyset)\% = \left[\frac{R_{min}(\emptyset) - R(\emptyset)}{R(\emptyset)}\right]$$

$$= \left[\frac{E_{T\,min}^{1/3} - E_T^{1/3}}{E_T^{1/3}}\right] \tag{5}$$

Now

$$E_{t\,max} = 1.5\ E_t \tag{6}$$

and

$$E_{t\,min} = 0.5\ E_t \tag{7}$$

So equations (4) and (5) can be written

$$R_+(\emptyset)\% = \left[\frac{(1.5)^{1/3}\ E_T^{1/3} - E_T^{1/3}}{E_T^{1/3}}\right] \times 100$$

$$= \frac{(1.414-1)}{1} \times 100 = 14.4 \tag{8}$$

$$R_-(\emptyset)\% = \left[\frac{(0.5)^{1/3} E_T^{1/3} - E_T^{1/3}}{E_T^{1/3}}\right] \times 100$$

$$= \frac{0.749-1}{1} \times 100 = -20.6 \tag{9}$$

Equations (8) and (9) give the range of error inherent in predicting an average cavity radius, assuming ±50 percent error in the given value for the total energy of the explosion.

If the average cavity radius is obtained in postshot measurements, the total energy of the explosion can be predicted. If the measured cavity radius is accurate within ±10 percent then the range of the estimate for a predicted E_t can be calculated using the following equations:

$$E_{T+}\% = \left[\frac{R_{max}^3(\emptyset) - R^3(\emptyset)}{R^3(\emptyset)}\right] \times 100 \tag{10}$$

$$E_{T-}\% = \left[\frac{R^3_{min}(\emptyset) - R^3(\emptyset)}{R^3(\emptyset)}\right] \times 100 \tag{11}$$

Since

$$R_{max}(\emptyset) = 1.1\ R(\emptyset) \tag{12}$$

and

$$R_{min}(\emptyset) = 0.9\ R(\emptyset) \tag{13}$$

equations (10) and (11) can be written

$$E_{T+}\% = \left[\frac{(1.1)^3 - 1}{1}\right] \times 100 = 33.0 \tag{14}$$

$$E_{T-}\% = \left[\frac{(0.9)^3 - 1}{1}\right] \times 100 = -27.1 \tag{15}$$

Equations (14) and (15) give the inherent error in predicting the total energy of the explosion from the measured postshot cavity radius, assuming $R(\emptyset)$ is accurate within ±10 percent.

The range of K was not considered in determining the range of the estimate for either predicted $R(\emptyset)$ or predicted E_t because (1) the error in K is assumed to be due to the error in the given values of $R(\emptyset)$ and E_t in the two sets of data that were used to calculate K, and (2) no matter what the value of K, it will have the same value for the estimated range of E_t or $R(\emptyset)$ and it will cancel out of equations (8) and (9) and equations (14) and (15).

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U. S. GOVERNMENT PRINTING OFFICE : 1959 O - 511171
867-100

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CALCULATION OF CAVITY RADIUS USING AN AVERAGE POTENTIAL ENERGY FUNCTION--SUPPLEMENT 1

by

G. E. Brethauer

Open-file report
1968

This report is preliminary and has not been edited or reviewed for conformity with U.S. Geological Survey standards.

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CALCULATION OF CAVITY RADIUS USING AN AVERAGE POTENTIAL ENERGY FUNCTION--SUPPLEMENT 1

By

G.E. Brethauer

Additional explanation of certain concepts used in the derivation of the cavity radius equation in the open-file report (released May 27, 1968) on "Calculation of cavity radius using an average potential energy function," by G. E. Brethauer is given below. Errors in the report are shown in corrected form in this supplement.

CORRECTIONS

Change all K's in the report to $K_{(m)}$'s; K%'s in the report to $K_{(m)}$%'s.

A term $R_f(\Theta)$ was omitted in equation(4), page 7. The equation should read:

$$\overline{P}(\Theta_1) = \frac{\rho g \int_{\emptyset}^{R_f(\Theta_1)} (H - R_f(\Theta) \sin \Theta_1)\, dr}{\int_{\emptyset}^{R_f(\Theta_1)} dr} \tag{4}$$

$$= \frac{\rho g \left[H\, R_f(\Theta_1) - \frac{R_f^2(\Theta_1)}{2} \sin \Theta_1 \right]}{R_f(\Theta_1)}$$

$$= \rho g \left[H - \frac{R_f(\Theta_1)}{2} \sin \Theta_1 \right]$$

In equation(5), page 7, K' was omitted. The equation should read:

$$\overline{F}(\theta_1) = K'\overline{P}(\theta_1)\overline{A}(\theta_1) = \frac{4\pi K' \rho g}{3} \left[HR_f^2(\theta_1) - \frac{R_f^3(\theta_1)}{2} \sin \theta_1 \right] \tag{5}$$

In equation(6), page 8, K' should be omitted. The equation should read:

$$\left[\overline{P}(\theta)\right]_{ave} = \frac{\rho g \int_{\emptyset}^{2\pi} (H - \frac{R_f(\theta)}{2} \sin \theta) d\theta}{\int_{\emptyset}^{2\pi} d\theta} \tag{6}$$

$$= \frac{\rho g \left[2H\theta - R_f(\theta) \cos \theta \right]_{\emptyset}^{2\pi}}{2 \left[\theta \right]_{\emptyset}^{2\pi}} = \frac{\rho g \left[4\pi H - R_f(\theta) - \emptyset + R_f(\theta) \right]}{4\pi}$$

$$= \rho gH$$

Equation (17), page 14, should read:

$$\sum_{J=1}^{m} E_{t(J)} = \frac{4\pi g}{3} \sum_{J=1}^{m} \frac{1}{K_{(J)}} \left\{ Z \left\{ \left[\sum_{n=1}^{J} R_n(\theta) \right]^3 - \left[\sum_{n=1}^{J-1} R_n(\theta) \right]^3 \right\} - \frac{\rho_{(J)} \sin \theta}{2} \left\{ \left[\sum_{n=1}^{J} R_n(\theta) \right]^4 - \left[\sum_{n=1}^{J-1} R_n(\theta) \right]^4 \right\} \right\} \tag{17}$$

The quantity t_J is eliminated.

Equation (19), page 15, should read:

$$E_t = \frac{4\pi g}{3} \sum_{J=1}^{m} \frac{1}{K_J} \left\{ Z \left\{ \left[\sum_{n=1}^{J} R_n(\theta) \right]^3 - \left[\sum_{n=1}^{J-1} R_n(\theta) \right]^3 \right\} \right.$$

$$- \frac{\rho_{(J)} \sin\theta}{2} \left\{ \left[\sum_{n=1}^{J} R_n(\theta) \right]^4 - \left[\sum_{n=1}^{J-1} R_n(\theta) \right]^4 \right\} \Bigg\} \Bigg\}$$

$$+ \frac{4\pi g}{3K_{(m+1)}} \left\{ Z \left\{ \left[R_f(\theta) \right]^3 - \left[\sum_{n=1}^{m} R_n(\theta) \right]^3 \right\} \right.$$

$$\left. - \frac{\rho_{(m+1)} \sin\theta}{2} \left\{ \left[R_f(\theta) \right]^4 - \left[\sum_{n=1}^{m} R_n(\theta) \right]^4 \right\} \right\} \qquad (19)$$

Figure 2, page 10, and figure 3, page 11. Ordinate axis label should read: Total Yield Energy, $E_t(10^{19})$.

Page 15, line 2. The equation $t_{J=\emptyset} = \emptyset$ (zero) should be omitted.

SUPPLEMENTARY INFORMATION

K' is a function relating the overburden pressure to the resistance that the medium has to being displaced. The value of K' was found to be a constant. Thus K' divided into the average change of the potential energy function gives a relation between the total yield energy and the change in potential energy.

This relation is expressed mathematically as:

$$E_T = P_{(m)} \Delta \bar{\Phi} = P_{(m)} \frac{K' \Delta \bar{\Phi}}{K'} = \frac{P_{(m)} \bar{E}_d}{K'}$$

Let $P_{(m)} = 1/K_{(m)}$

$$K_{(m)} = \left(\bar{E}_d / K' \right) / E_T$$