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An Iterative Digital Model for Aquifer Evaluation

By

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PURPOSE

This program simulates the response of a confined or unconfined aquifer to pumping at a constant rate from one or more wells. The ground-water reservoir may be irregular in shape and non-homogeneous with infiltration from one or more lakes and streams. The program is written in FORTRAN IV for the IBM 360 system. (See Appendix for program listing).

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THEORETICAL DEVELOPMENT

The differential equation for nonsteady flow of a compressible fluid in an elastic non-homogeneous porous medium can be written (Pinder and Bredehoeft, 1968)

$$\frac{\partial}{\partial x_i}\left(T_{ij}\frac{\partial h}{\partial x_j}\right) = S\frac{\partial h}{\partial t} + W(x,y,t) \qquad (1)$$

where T_{ij} is the transmissivity tensor (L^2/T);

h is the hydraulic head (L);

S is the storage coefficient (dimensionless);

t is time (T);

W is the volume flux per unit area (L/T).

If the coordinate axes are aligned with the principal directions of the transmissivity tensor the finite-difference approximation to (1) may be written (see Fig.1)

$$T'_{xx(i-\frac{1}{2},j)}\left(\frac{h_{i-1,j,k}-h_{i,j,k}}{\Delta x_i}\right)$$

$$+\ T'_{xx(i+\frac{1}{2},j)}\left(\frac{h_{i+1,j,k}-h_{i,j,k}}{\Delta x_i}\right)$$

$$+\ T'_{yy(i,j-\frac{1}{2})}\left(\frac{h_{i,j-1,k}-h_{i,j,k}}{\Delta y_j}\right)$$

$$+\ T'_{yy(i,j+\frac{1}{2})}\left(\frac{h_{i,j+1,k}-h_{i,j,k}}{\Delta y_j}\right)$$

$$=\ S\left(\frac{h_{i,j,k}-h_{i,j,k-1}}{\Delta t}\right)$$

$$+\ \frac{q_{w(i,j)}}{\Delta x_i \Delta y_j}\ -\ K_s\left(\frac{2H_{r(i,j)}-h_{i,j,k}-h_{i,j,k-1}}{2m_{r(i,j)}}\right)$$

where

$$T'_{xx(i+\frac{1}{2},j)} = \frac{2T_{xx(i,j)}T_{xx(i+1,j)}}{T_{xx(i,j)}\Delta x_{i+1}+T_{xx(i+1,j)}\Delta x_i}$$ = the harmonic mean of

$$\frac{T_{xx(i,j)}}{\Delta x_i},\quad \frac{T_{xx(i+1,j)}}{\Delta x_{i+1}}$$

i	is the index in the x dimension;
j	is the index in the y dimension;
k	is the index in time;
K_s	is the hydraulic conductivity of the stream bottom (L/T);
m_r	is the thickness of the stream bottom (L);
$q_{w(i,j)}$	is the rate of withdrawal or injection at the node $x_i y_j$ (L^3/T);
Δt	is the time increment (T);
Δx, Δy	are space increments (L);
H_r	is the hydraulic head in the lakes and streams.

In an unconfined aquifer the transmissivity is a function of time as well as of space. In order to approximate equation (2) without generating a non-linear set of finite-difference equations, the head from the preceding time step is used, and the transmissivity is given by

$$T_{i,j,k} = K_{i,j} h'_{i,j,k-1}$$

where	K	is the hydraulic conductivity (L/T);
	h'	is the saturated thickness of the aquifer (L).

METHOD OF ANALYSIS

A rectangular net or grid as indicated in figure 1 is superposed on a plan view of the groundwater reservoir.

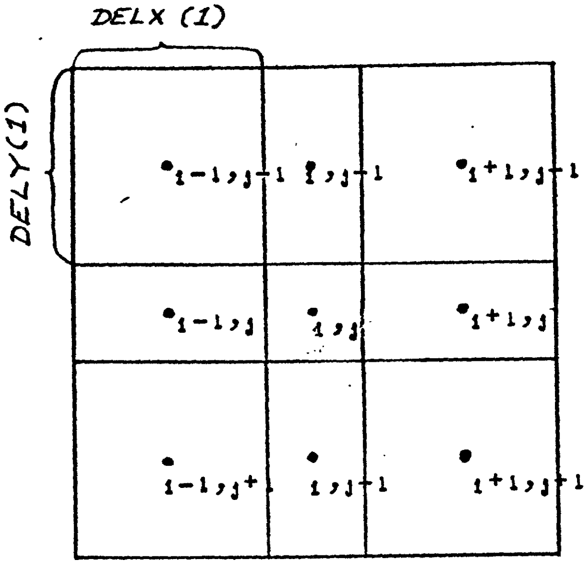


Fig. 1. Nodal array for digital model.

At each node the transmissivity, storage coefficient, and initial head are recorded. The pumping rate is recorded at each node where a well is located and the hydraulic conductivities of stream or lake beds where they occur. In the case of an unconfined aquifer the transmissivity is replaced by the hydraulic conductivity and the elevation of the base of the aquifer is also recorded. A parameter card is prepared providing the dimensions of the grid, the head in the stream, the thickness of the stream or lake beds, information concerning the maximum duration of pumping and other constants used in the computational scheme. All computer input and output is in a consistent set of units; the program is set-up to use feet and seconds.

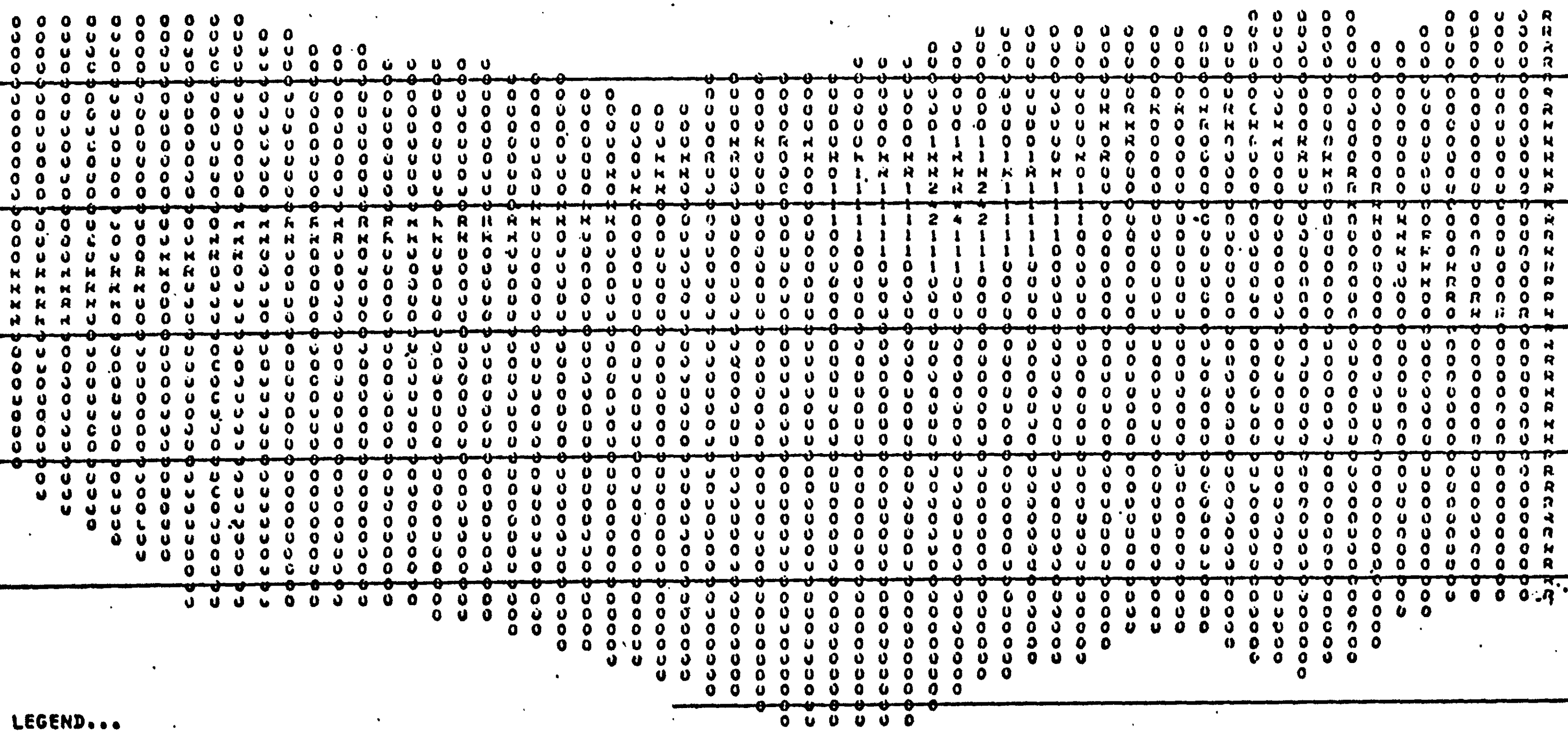
The objective of the program is to solve equation (2) implicitly for the desired boundary conditions at each node on the grid after Δt seconds of pumping. These values are then used as initial conditions for the calculation of the head values at time $2\Delta t$; this procedure is continued until the reservoir has been pumped the desired length of time. The size of the time step is increased exponentially. Numerical values and an alphameric contour map of the drawdown in the aquifer are printed at selected time steps (see Fig. 2).

Figure 2 -- Alphameric contour map of drawdown at Antigonish
Nova Scotia

In order to determine the precision of the calculations, a mass balance may also be computed at this time if there is no infiltration into the system.

```
                    ITERATION NUMBER          15
              SIZE OF TIME STEP IN SECONDS=  2.33
DURATION OF PUMPING AT THIS PRINTOUT IN SECONDS=  17.5
                                         MINUTES= 0.291
                                         HOURS= 0.485E-02
                                         DAYS= 0.202E-03
```

```
                    ALPHABETIC CONTOURS FOR DRAWDOWN
```



```
LEGEND...
DRAWDOWN FROM 0-35 FEET REPRESENTED BY SYMBOLS 0-Z
DRAWDOWN GREATER THAN 35 FEET REPRESENTED BY THE SYMBOL *
CONE OF IMPRESSION INDICATED BY SYMBOL >
LETTER R  INDICATES AREA OF RECHARGE
LETTER W  INDICATES LOCATION OF A WELL
```

Fig. 2

The iterative alternating-direction implicit procedure is the mathematical technique used to solve the N simultaneous equations where N is the number of nodes in the matrix. Hydraulic head values are calculated for each node in the matrix by solving alternately the equations for rows and columns implicitly. Equation (2) for the calculation of rows is

$$
\begin{aligned}
& T'_{xx(i-\frac{1}{2},j)} \left(\frac{h^{n}_{i-1,j,k} - h^{n}_{i,j,k}}{\Delta x_{i}}\right) \\
& + T'_{xx}(i+\tfrac{1}{2},j) \left(\frac{h^{n}_{i+1,j,k} - h^{n}_{i,j,k}}{\Delta x_{i}}\right) \\
& + T'_{yy(i,j-\frac{1}{2})} \left(\frac{h^{n-1}_{i,j-1,k} - h^{n-1}_{i,j,k}}{\Delta y_{j}}\right) \\
& + T'_{yy(i,j+\frac{1}{2})} \left(\frac{h^{n-1}_{i,j+1,k} - h^{n-1}_{i,j,k}}{\Delta y_{j}}\right) \\
& = S \left(\frac{h^{n}_{i,j,k} - h_{i,j,k-1}}{\Delta t}\right) + \frac{q_{w(i,j)}}{\Delta x_{i}\ \Delta y_{j}} \\
& - K_{s} \left(\frac{2H_{r(i,j)} - h^{n}_{i,j,k} - h_{i,j,k-1}}{2m_{r(i,j)}}\right) \\
& + I(h^{n}_{i,j,k} - h^{n-1}_{i,j,k})
\end{aligned}
$$

where I is a normalized iteration parameter;

n is the index indicating iteration number.

At each time step the row and column equations are solved alternately until the greatest head difference between a row and column computation at any node is less than a prescribed error criteria. When this closure is achieved the program begins solving for the head at the new time $t + \Delta t$. The procedure is continued until the desired period of analysis has been simulated or the aquifer becomes dewatered. When any condition arises which terminates computation the head matrix and the elapsed simulation period are punched on cards. These cards can be used as input if it is necessary to extend the period of analysis at a later time.

For a detailed explanation of the technique used for solving the finite-difference equations see Douglas and Rachford (1956).

APPLICATION

The following section describes the preparation of parameter and data cards. The arrangement of the assembled program deck is shown in figure 3.

Figure 3. Assembled program deck.

Parameter cards

All values are right justified in the data field as indicated in Fig. 4.

Figure 4. Sample coding form.

Card 1

Column	Variable	Contents
1 - 10	TMAX	simulation period in hours
11 - 20	DIML	number of nodes in a column of the matrix *
21 - 30	DIMW	number of nodes in a row of the matrix *
31 - 40	NUMT	maximum number of time steps *
41 - 50	QRE	vertical leakage into the aquifer in feet per second
51 - 60	DELT	initial time increment in seconds

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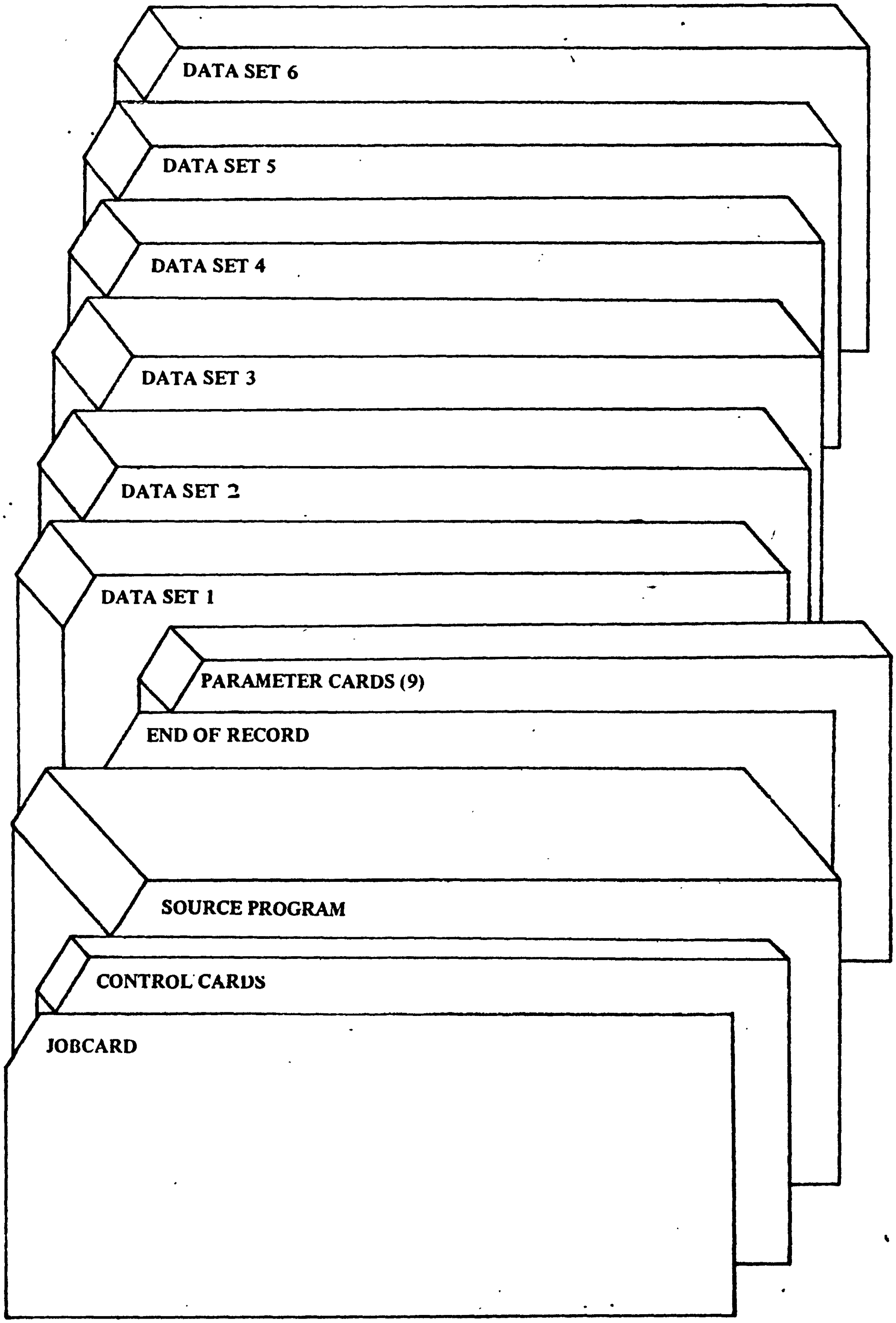
DATA SET 6
DATA SET 5
DATA SET 4
DATA SET 3
DATA SET 2
DATA SET 1
PARAMETER CARDS (9)
END OF RECORD
SOURCE PROGRAM
CONTROL CARDS
JOBCARD

Fig. 3

PROGRAMMER DIVISION U. S. DEPARTMENT OF THE INTERIOR GEOLOGICAL SURVEY NUMBER SHEET 1 OF 2
LOCATION PHONE COMPUTER CODING FORM PROJECT

IDENTIFICATION INFORMATION: ITERATIVE DIGITAL MODEL FOR AQUIFER EVALUATION
SAMPLE CODING FORM

TMAX	DIML	DIMW	NUMT	QRE	DELT		
10000.	5	5	5	0.	10.0		
M	KTH	FACTOR	LENGTH	ERR	FACS	FACB	FACP
10.	1	.001	4	.001	.001	1.	.001
FACT	RIVER	SPACNG					
1.	100.	1.					
SUM							
0.							

PUNCH

WATERTABLE

CØNTØUR

NUMERIC

CHECK

-DATA SET 1; DELX,DELY-

100.	100.	50.	100.	100.

-DATA SET 2; STRT-

100.	100.	100.	100.	100.

FORM 9-633A JULY 1967 0 = ZERO Ø = ALPHA O 1 = ONE I = ALPHA I 2 = TWO Z = ALPHA Z / = SLASH | = VERT. BAR - = MINUS _ = HORZ. BAR DATA CODING FORM

Fig. 4

DIVISION
PHONE

U S DEPARTMENT OF THE INTERIOR
GEOLOGICAL SURVEY
COMPUTER CODING FORM

PROGRAM NO.
PROJECT

SHEET 2 OF 2

ENTIFICATION ITERATIVE DIGITAL MODEL FOR AQUIFER EVALUATION
NFORMATION SAMPLE CODING FORM

STATEMENT IDENTIFICATION

```
-DATA SET 3; RATE
 0.  0. -1.  0.  0.
-DATA SET 4; PERM
 0.  1.  1.  1.  0.
-DATA SET 5; BOTTOM
 0.  0.  0.  0.  0.
-DATA SET 6; S
 1.  1.  1.  1.  1.
```

0 = ZERO O = ALPHA O 1 = ONE I = ALPHA I 2 = TWO Z = ALPHA Z / = SLASH | = VERT. BAR - = MINUS _ = HORZ. BAR

PROGRAM CODING FORM

Fig. 4

GPO : 1968 O - 266-594

Card 2

1 - 10	M	thickness of stream or lake beds in feet
11 - 20	KTH	number of time steps between printouts *
21 - 30	FACTOR	multiplier for values of hydraulic conductivity of stream or lake beds
31 - 40	LENGTH	number of iteration parameters *
41 - 50	ERR	error criteria for closure
51 - 60	FACS	multiplier for storage coefficient
61 - 70	FACB	multiplier for aquifer base bottom elevation
71 - 80	FACP	multiplier for hydraulic conductivity of aquifer

Card 3

1 - 10	FACT	multiplier for transmissivity
11 - 20	RIVER	hydraulic head in river in feet
21 - 30	SPACNG	contour interval in feet

Card 4

1 - 10	SUM	elapsed time at beginning of computations (usually 0. unless provided as punched output)

Card 5

1 - 5	PUNCH	indicator for punched output; if punched output is desired at termination of computations write PUNCH, otherwise leave card blank.

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Card 6

1 - 10	WATER	indicator of water-table conditions, if water-table conditions are encountered write WATERTABLE: otherwise leave card blank.

Card 7

1 - 7	CONTR	indicator of contoured printout; if an alphameric contour map of hydraulic head is desired write CONTOUR; otherwise leave this card blank.

Card 8

1 - 7	NUM	indicator of numerical printout; if the numerical head values are desired write NUMERIC, otherwise leave this card blank.

Card 9

1 - 5	CHCK	indicator of computational check; if a mass balance on the system is desired write CHECK; otherwise leave card blank.

* values are fixed point; they must be right justified and not include a decimal point.

Data Cards

Set 1

Distance between nodes in the prototype in feet: the distance between adjacent nodes is recorded as indicated in figure 1. The values in the x direction, *DELX*, are recorded first and are followed, beginning on a new card, by the values in the y direction, *DELY*. As indicated in figure 1 the values for *DELY* and *DELX* must remain constant for each row and column respectively. Each value may occupy 10 spaces including the decimal point.

Set 2

Initial hydraulic head, STRT: the initial hydraulic head values are recorded from left to right along the rows beginning with the top row. Each row begins on a new card and each value may occupy 10 spaces including a decimal point (there will be a maximum of eight values recorded on each card). The hydraulic head values punched at the termination of the program can be read as initial hydraulic head values.

Set 3

Locations of wells, lakes, and streams, RATE: the locations of wells, lakes, and streams are recorded left to right along the rows, beginning with the top row. Each row begins on a new card and each value may occupy only 4 spaces including a negative sign and decimal point (no sign is necessary for a positive number). At any node where a lake or stream occurs the hydraulic conductivity of the bed divided by the hydraulic conductivity multiplier is recorded. At any node where a well is located the pumping rate in cfs is recorded preceded by a negative sign.

Set 4

Hydraulic conductivity: PERM or transmissivity T: this data set consists of the hydraulic conductivity values in the case of a water table problem or transmissivity values in a confined aquifer situation. The values divided by the appropriate multiplier are recorded left to right along the rows beginning with the top row. Each row begins on a new card and each value may occupy up to 4 spaces including the decimal point.

Set 5

Aquifer base elevation, BOTTOM: this data set consists of the elevation of the base of the aquifer measured from the same reference datum as the initial head, divided by an appropriate multiplier. This data set is omitted if a confined-aquifer problem is considered. The values are recorded left to right along the rows beginning with the top row. Each row begins on a new card and each value may occupy up to 4 spaces including the decimal point.

Set 6

Storage coefficient values, S: this data set consists of the storage coefficient at each node, divided by the appropriate multiplier. The values are recorded left to right along the rows, beginning with the top row. Each row begins on a new card and each value may occupy up to 4 spaces including the decimal point.

PROGRAM FEATURES

Number of nodes:

The dimensions of the nodal array considered in an aquifer problem generally depend upon the storage capacity in the computer and available funds. The larger the nodal array the more computer time is required to simulate a specified period of analysis. This program permits arrays up to 50 X 50 nodes.

Drawdown near the pumping well:

Withdrawal from a well is assumed to occur over the area of influence of the well node. Drawdown values within one node of the well, therefore, will not accurately represent field responses. The use of smaller space increments near the well will improve the accuracy of the solution.

Advantages of iterative technique:

Certain limitations inherent in the alternating direction implicit technique are overcome by introducing a relaxation factor and iteration. Some of the important advantages of an iterative analysis are:

i. abrupt changes in transmissivity between nodes is permitted,

ii. variable space increments may be used,

iii. any time increment Δt may be used,

iv. the elliptic form of the flow equation can be treated directly (the steady-state problem),

v. the iterative procedure requires less computer time when periods extending tens or hundreds of years are simulated.

Multiplication Constants

Multiplication constants (FACTOR, FACB, FACS, FACP, FACT) are used to decrease the number of data cards when very small or very large numbers must be read. For example, if it was necessary to read a storage coefficient value of .0001 the multiplication factor FACS could be set equal to .0001 and 1. would be recorded on the data card; the program would convert the 1. to .0001 during execution.

Program Diagnostics

Two program diagnostics may be generated if an abnormal program termination occurs. The message EXCEEDED PERMITTED NUMBER OF ITERATIONS is printed when convergence is not achieved within 100 iterations. This situation generally arises when an error in data input results in an impossible physical problem. The message WELL GOES DRY occurs when the cone of depression in an unconfined aquifer drops below the impermeable aquifer base.

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Iteration Parameters

The choice of the number of iteration parameters, LENGTH, and the convergence criteria, ERR, depends upon the physical problem considered. Three to seven iteration parameters and a value for ERR of .001 should provide satisfactory results in most problems.

Mass Balance

The CHECK option permits the calculation of a mass balance when there is no infiltration into the system. The volume of water removed from the aquifer is computed by integration over the cone of depression and by calculating the volume removed from the wells. These values are compared and the deviation expressed as a percent of the volume pumped. If infiltration does occur the deviation is a measure of the volume of water entering the system.

This program is not designed to simulate all of the complicated hydrologic problems encountered in the field. It is intended as a starting point from which more complex models can be developed.

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SELECTED REFERENCES

Douglas, J., and Rachford, H. H., Jr., 1956, On the numerical solution of head conduction problems in two or three space variables, Trans. Amer. Math. Soc., 82, p. 421-439.

Pinder, G. F., and Bredehoeft, J. D., 1968, Application of Digital Computer for Aquifer Evaluation; Water Resources Research, Vol. 4, No. 5, pp. 1069-1093.

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```
C        TO PROVIDE THE POTENTIAL DISTRIBUTION IN A CONFINED AQUIFER AFTER
C       A DESIGNATED PERIOD OF TIME.
C       ***************************************************************
C
C       METHODO
C        THE ALTERNATING DIRECTION IMPLICIT TECHNIQUE IS USED TO SOLVE
C       THE LINEAR EQUATIONS.
C
C       DESCRIPTION OF PARAMETERSO
C       A=COEFFICIENT IN FINITE DIFFERENCE EQUATION
C       B=COEFFICIENT IN FINITE DIFFERENCE EQUQTION
C       BE(I)=DUMMY VARIABLE DEFINED AS C/W
C       BOTTOM(I,J)=ELEVATION OF BOTTOM
C       C=COEFFICIENT IN FINITE DIFFERENCE EQUATION
C       D=DUMMY VARIABLE REPRESENTING PARAMETERS OF EQUATION WHOSE VALUES
C       ARE KNOWN FROM PREVIOUS ITERATION
C       DAYS=NUMBER OF DAYS SINCE PUMPING STARTED
C       DELT= LENGTH OF INITIAL TIME STEP (SECONDS)
C       DELX=DISTANCE BETWEEN NODES IN THE PROTOTYPE IN THE X DIRECTION
C            (FEET)
C       DELY=DISTANCE BETWEEN NODES IN THE PROTOTYPE IN THE Y DIRECTION
C            (FEET)
C       DIML=NUMBER OF NODES IN COLUMN OF MATRIX
C       DIMW=NUMBER OF NODES IN ROW OF MATRIX
C       ERR=CLOSURE CRITERIA FOR ITERATION
C       FACTOR=MULTIPLICATION FACTOR FOR ADJUSTING STORAGE COEFFICIENT
C       FACB=MULTIPLICATION FACTOR FOR BOTTOM ELEVATION
C       FACP=MULTIPLICATION FACTOR FOR  HYDRAULIC CONDUCTIVITY
C       FACS=MULTIPLICATION FACTOR FOR STORAGE COEFFICIENT
C       FACT=MULTIPLICATION FACTOR FOR TRANSMISSIVITY
C       G=DUMMY VARIABLE DEFINED AS (D-A*G(J-1))/W
C       HRS=NUMBER OF HOURS SINCE PUMPING STARTED
C       IMK=ITERATION PARAMETER
C       KEEP(I,J)=STORAGE MATRIX FOR PHI VALUES
C       KTH=NUMBER OF TIME STEPS BETWEEN PRINTOUTS
C       M=THICKNESS OF STREAM BED OR LAKE BOTTOM (FEET)
C       NUMT=MAXIMUM NUMBER OF TIME STEPS
C       PHI(I,J)=HEAD IN AQUIFER AT NODE (I,J) IN FEET
C       PRNT(I,J)=ARRAY FOR PRINTING OUT HEAD MATRIX IN CONTOUR FORM
C       QRE= RECHARGE FLUX FROM THE CONFINING LAYER (CUBIC FEET PER SECOND
C       RATE(I,J)=RATE OF PUMPING IN CFS IF NEGATIVE
C                 HYDRAULIC CONDUCTIVITY OF STREAM OR LAKE BOTTOM IF
C       POSITIVE
C       RHO=DUMMY VARIABLE DEFINED AS S(I,J)/DELT
C       RIVER=HEAD IN RIVER
C       RR=VARIABLE ACCOUNTING FOR VERTICAL LEAKAGE
C       RW=VARIABLE ACCOUNTING FOR PUMPING
C       S(I,J)=STORAGE COEFFICIENT (DIMENSIONLESS)
C       STRT(I,J)=INITIAL VALUE OF HYDRAULIC HEAD IN AQUIFER
C       SUM=DURATION OF PUMPING IN SECONDS
```

```
C     SYM(J)=LINEAR ARRAY CONTAINING SYMBOLS FOR PRINTING OUT HEAD
C               MATRIX
C     T(I,J)=TRANSMISSIVITY (FEET**2/SECOND)
C     TEMP(I)=TEMPORY LOCATION OF CURRENTLY CALCULATED VALUES OF HEAD
C     TMAX=MAXIMUM ALLOTTED PERIOD OF PUMPING
C     W=DUMMY VARIABLE DEFINED AS B-A*BE(J-1)
C
C     ..............................................................
C
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      DIMENSION S(50,50), PERM(50,50), BOTTOM(50,50), RATE(50,50), KEEP(
     150,50), G(50), TEMP(50), BE(50), RHOP(25), CHK(10), STRT(50,50), T
     2(50,50), PHI(50,50)
      DIMENSION DELX(50), DELY(50)
      REAL MINS,M,K
      REAL*8 KEEP,IMK,NUM
      INTEGER DIML,DIMW
      DOUBLE PRECISION PHI,KEEP,D,G,TEMP,BE,W,T1,T2,T3,T4,RHO,A,B,C,DELT
     1,RHOP,PARAM,IMK,RIVER,CHK,PNCH,WATER,CONTR,NUM,CHCK
C
C     ..............................................................
C
      DATA CHK(1)/5HPUNCH/,CHK(2)/8HWATERTAB/,CHK(3)/7HCONTOUR/,CHK(4)/7
     1HNUMERIC/,CHK(5)/5HCHECK/
C
      READ (5,520) TMAX,DIML,DIMW,NUMT,QRE,DELT,M,KTH,FACTOR,LENGTH,ERR,
     1FACS,FACB,FACP,FACT,RIVER,SPACNG
      READ (5,610) SUM
      READ (5,600) PNCH,WATER,CONTR,NUM,CHCK
      READ (5,680) (DELX(J),J=1,DIMW)
      READ (5,680) (DELY(I),I=1,DIML)
      DO 10 I=1,DIML
      READ (5,620) (STRT(I,J),J=1,DIMW)
      DO 10 J=1,DIMW
   10 PHI(I,J)=STRT(I,J)
      DO 20 I=1,DIML
      READ (5,530) (RATE(I,J),J=1,DIMW)
      DO 20 J=1,DIMW
   20 IF (RATE(I,J).GT.0) RATE(I,J)=RATE(I,J)*FACTOR
      IF (WATER.NE.CHK(2)) GO TO 50
      DO 30 I=1,DIML
      READ (5,530) (PERM(I,J),J=1,DIMW)
      DO 30 J=1,DIMW
   30 PERM(I,J)=PERM(I,J)*FACP
      DO 40 I=1,DIML
      READ (5,530) (BOTTOM(I,J),J=1,DIMW)
      DO 40 J=1,DIMW
   40 BOTTOM(I,J)=BOTTOM(I,J)*FACB
      GO TO 70
   50 DO 60 I=1,DIML
```

```
      READ (5,530) (T(I,J),J=1,DIMW)
      DO 60 J=1,DIMW
   60 T(I,J)=T(I,J)*FACT
   70 DO 80 I=1,DIML
      READ (5,530) (S(I,J),J=1,DIMW)
      DO 80 J=1,DIMW
   80 S(I,J)=S(I,J)*FACS
C
C     .............................................................
C
C     PRINT PARAMETER VALUES
      WRITE (6,550) DELT,TMAX,NUMT,QRE,DIML,DIMW,LENGTH,ERR,FACTOR,FACS,
     1FACB,FACP,FACT,RIVER,M,KTH
      WRITE (6,690) (DELX(J),J=1,DIMW)
      WRITE (6,700) (DELY(I),I=1,DIML)
      TMAX=TMAX*3600.
      IF (WATER.EQ.CHK(2)) GO TO 100
      WRITE (6,560)
      DO 90 I=1,DIML
   90 WRITE (6,570) I,(T(I,J),J=1,DIMW)
  100 WRITE (6,590)
      DO 110 I=1,DIML
  110 WRITE (6,570) I,(S(I,J),J=1,DIMW)
      WRITE (6,540)
      DO 120 I=1,DIML
  120 WRITE (6,570) I,(RATE(I,J),J=1,DIMW)
      IF (WATER.NE.CHK(2)) GO TO 150
      WRITE (6,630)
      DO 130 I=1,DIML
  130 WRITE (6,570) I,(PERM(I,J),J=1,DIMW)
      WRITE (6,640)
      DO 140 I=1,DIML
  140 WRITE (6,570) I,(BOTTOM(I,J),J=1,DIMW)
  150 JNO1=DIMW-1
C
C     .............................................................
C
C     COMPUTE ITERATION PARAMETERS
C     COMPUTE HMIN
      HMIN=2.
      XVAL=3.1415**2/(2.*DIMW**2)
      YVAL=3.1415**2/(2.*DIML**2)
      DO 160 I=2,DIML
      DO 160 J=2,DIMW
      IF (T(I,J).EQ.0.) GO TO 160
      XPART=XVAL*(1/(1+DELX(J)**2/DELY(I)**2))
      YPART=YVAL*(1/(1+DELY(I)**2/DELX(J)**2))
      HMIN=AMIN1(HMIN,XPART,YPART)
  160 CONTINUE
      ALPHA=EXP(ALOG(1/HMIN)/(LENGTH-1))
```

```
      RHOP(1)=HMIN
      DO 170 NTIME=2,LENGTH
  170 RHOP(NTIME)=RHOP(NTIME-1)*ALPHA
      PARAM=RHOP(1)
      WRITE (6,650) (RHOP(J),J=1,LENGTH)
      KT=0
      TEST=0
C
C     ..................................................................
C
C     IF TEST EQUALS 1 CONTINUE ITERATION, IF TEST EQUALS 0 GO TO NEXT
C     TIME STEP
  180 IF (TEST.EQ.1) GO TO 250
      IFINAL=0
      IF (KT.GT.NUMT.OR.SUM.GT.TMAX) IFINAL=1
      NTH=0
      IF (MOD(KT,KTH).NE.0.OR.KT.EQ.0.AND.IFINAL.NE.1) GO TO 210
      WRITE (6,580) KT,DELT,SUM,MINS,HRS,DAYS,KOUNT
      IF (CONTR.EQ.CHK(3)) CALL PRNTA
      IF (NUM.EQ.CHK(4)) CALL PRNT1
      IF (CHCK.EQ.CHK(5)) CALL CHECK
      IF (IFINAL.NE.1) GO TO 210
      IF (PNCH.NE.CHK(1)) STOP
  190 DO 200 I=1,DIML
  200 WRITE (7,620) (PHI(I,J),J=1,DIMW)
      WRITE (7,610) SUM
      STOP
  210 CONTINUE
      KT=KT+1
      KOUNT=0
      DO 220 I=1,DIML
      DO 220 J=1,DIMW
  220 KEEP(I,J)=PHI(I,J)
      IF (WATER.NE.CHK(2)) GO TO 240
      DO 230 I=1,DIML
      DO 230 J=1,DIMW
      T(I,J)=PERM(I,J)*(PHI(I,J)-BOTTOM(I,J))
      IF (T(I,J).GE.0.) GO TO 230
      WRITE (6,660)
      GO TO 190
  230 CONTINUE
  240 DELT=DELT+DELT
      SUM=SUM+DELT
      HRS=SUM/3600.
      MINS=HRS*60.
      DAYS=HRS/24.
      GO TO ~~290~~ 270
  250 IF (KOUNT.LT.100) GO TO 260
      WRITE (6,670)
      GO TO 190
```

```
  260 KOUNT=KOUNT+1
      IF (MOD(KOUNT,LENGTH)) 270,270,280
  270 NTH=0
  280 NTH=NTH+1
      PARAM=RHOP(NTH)
      TEST=0.
  290 DO 300 J=1,DIMW
  300 TEMP(J)=PHI(1,J)
      DO 400 I=2,DIML
      DO 360 J=2,JNO1
C
C     ...............................................................
C
C     DETERMINE WHETHER NODE IS OUTSIDE AQUIFER BOUNDARY
      IF (T(I,J)) 310,360,310
C
C     ...............................................................
C
  310 RHO=S(I,J)/DELT
C
C     ...............................................................
C
C     CALCULATE AVERAGE VALUES OF T BETWEEN ADJACENT NODES
C     NODE CONFIGURATION  T1=LEFT, T2=RIGHT, T3=UPPER, T4=LOWER
      T1=((2.*T(I,J-1)*T(I,J))/(T(I,J)*DELX(J-1)+T(I,J-1)*DELX(J)))/DELX
     1(J)
      T2=((2.*T(I,J+1)*T(I,J))/(T(I,J)*DELX(J+1)+T(I,J+1)*DELX(J)))/DELX
     1(J)
      T3=((2.*T(I-1,J)*T(I,J))/(T(I,J)*DELY(I-1)+T(I-1,J)*DELY(I)))/DELY
     1(I)
      T4=((2.*T(I+1,J)*T(I,J))/(T(I,J)*DELY(I+1)+T(I+1,J)*DELY(I)))/DELY
     1(I)
      IMK=PARAM*(T1+T2+T3+T4)
      K=0.0
C
C     ...............................................................
C
C     CHECK WHETHER NODE IS ALONG A STREAM OR ON A LAKE
      IF (RATE(I,J)) 330,330,320
  320 K=RATE(I,J)
C
C     ...............................................................
C
C     CALCULATE VALUES FOR PARAMETERS A,B,C, AND BE
  330 B=-T1-T2-RHO-K/(2.*M)-IMK
      A=T1
      C=T2
      W=B-A*BE(J-1)
      BE(J)=C/W
C
```

```
C     ...............................................................
C
C     CHECK NODE FOR POSSIBLE WELL LOCATION
      RR=QRE
      RW=0.0
      IF (RATE(I,J)) 340,350,350
  340 RW=-RATE(I,J)/(DELX(J)*DELY(I))
  350 D=-T3*PHI(I-1,J)+(T4+T3-IMK)*PHI(I,J)-T4*PHI(I+1,J)-RHO*KEEP(I,J)-
     1(K/M)*(RIVER-KEEP(I,J)/2.)+RW-RR
      G(J)=(D-A*G(J-1))/W
  360 CONTINUE
C
C     ...............................................................
C
C     CALCULATE HEAD VALUES FOR ROWS OF MATRIX AND PLACE THEM IN
C     TEMPORARY LOCATION TEMP
      NO3=DIMW-2
      DO 390 KNO4=1,NO3
      NO4=DIMW-KNO4
      PHI(I-1,NO4)=TEMP(NO4)
      IF (T(I,NO4)) 380,370,380
  370 TEMP(NO4)=PHI(I,NO4)
      GO TO 390
  380 TEMP(NO4)=G(NO4)-BE(NO4)*TEMP(NO4+1)
  390 CONTINUE
  400 CONTINUE
C
C     ***************************************************************
C
C     FOLLOW SIMILIAR PROCEDURE FOR COLUMNS OF MATRIX AS THAT CONSIDERED
C     FOR ROWS
      DO 410 I=1,DIML
  410 TEMP(I)=PHI(I,1)
      INO1=DIML-1
      DO 510 J=2,DIMW
      DO 470 I=2,INO1
      IF (T(I,J)) 420,470,420
C
C     ...............................................................
C
  420 RHO=S(I,J)/DELT
C
C     ...............................................................
C
C     CALCULATE AVERAGE VALUES OF T BETWEEN ADJACENT NODES
      T1=((2.*T(I,J-1)*T(I,J))/(T(I,J)*DELX(J-1)+T(I,J-1)*DELX(J)))/DELX
     1(J)
      T2=((2.*T(I,J+1)*T(I,J))/(T(I,J)*DELX(J+1)+T(I,J+1)*DELX(J)))/DELX
     1(J)
      T3=((2.*T(I-1,J)*T(I,J))/(T(I,J)*DELY(I-1)+T(I-1,J)*DELY(I)))/DELY
```

```
     1(I)
      T4=((2.*T(I+1,J)*T(I,J))/(T(I,J)*DELY(I+1)+T(I+1,J)*DELY(I)))/DELY
     1(I)
      IMK=PARAM*(T1+T2+T3+T4)
      K=0.0
C
C     ..............................................................
C
C     CHECK WHETHER NODE IS ALONG A STREAM OR ON A LAKE
      IF (RATE(I,J)) 440,440,430
  430 K=RATE(I,J)
C
C     ..............................................................
C
C     CALCULATE VALUES FOR PARAMETERS A,B,C,AND BE
  440 A=T3
      C=T4
      B=-T4-T3-RHO-K/(2.*M)-IMK
      W=B-A*BE(I-1)
      BE(I)=C/W
C
C     ..............................................................
C
C     CHECK NODF FOR POSSIBLE WELL LOCATION
      ~~RR=0.0~~ QRE
      RW=0.0
      IF (RATE(I,J)) 450,460,460
  450 RW=-RATE(I,J)/(DELX(J)*DELY(I))
  460 D=-T1*PHI(I,J-1)+(T1+T2-IMK)*PHI(I,J)-T2*PHI(I,J+1)-RHO*KEEP(I,J)-
     1(K/M)*(RIVFR-KEEP(I,J)/2.)+RW-RR
      G(I)=(D-A*G(I-1))/W
  470 CONTINUE
C
C     ..............................................................
C
C     CALCULATE HEAD VALUES FOR COLUMNS OF MATRIX AND PLACE IN TEMPORARY
C     LOCATION TEMP
      NO3=DIML-2
      DO 500 KNO4=1,NO3
      NO4=DIML-KNO4
      PHI(NO4,J-1)=TEMP(NO4)
      IF (T(NO4,J)) 490,480,490
  480 TEMP(NO4)=PHI(NO4,J)
      GO TO 500
  490 TEMP(NO4)=G(NO4)-BE(NO4)*TEMP(NO4+1)
      IF (DABS(TEMP(NO4)-PHI(NO4,J)).GT.ERR) TEST=1.
  500 CONTINUE
  510 CONTINUE
      GO TO 180
C
```

```
C     ******************************************************************
C
C
C
  520 FORMAT (F10.2,3I10,2F10.2/F10.2,I10,F10.2,I10,4F10.2/3F10.2)
  530 FORMAT (20F4.0)
  540 FORMAT (1H1,61X,11HRATE MATRIX)
  550 FORMAT (1H1,60X,16HINPUT PARAMETERS//40H LENGTH OF INITIAL TIME ST
     1EP IN SECONDS=,E10.3//46H MAXIMUM PERMITTED PERIOD OF PUMPING IN H
     2OURS=,E10.3//40H MAXIMUM PERMITTED NUMBER OF TIME STEPS=,I4//60H F
     3LUX DUE TO VERTICAL LEAKAGE FROM A CONFINING LAYER IN CFS=,E10.3//
     437H NUMBER OF NODES IN COLUMN OF MATRIX=,I4//34H NUMBER OF NODES I
     5N ROW OF MATRIX=,I4//32H NUMBER OF ITERATION PARAMETERS=,I4//28H E
     6RROR CRITERIA FOR CLOSURE=,E10.3//53H MULTIPLIER FOR HYDRAULIC CON
     7DUCTIVITY OF STREAM BED=,E10.3//36H MULTIPLIER FOR STORAGE COEFFIC
     8IENT=,E10.3//34H MULTIPLIER FOR BOTTOM ELEVATION=,E10.3//39H MULTI
     9PLIER FOR HYDRAULIC CONDUCTIVITY=,E10.3//31H MULTIPLIER FOR TRANSM
     $ISSIVITY=,E10.3//15H HEAD IN RIVER=,E10.3//25H THICKNESS OF STREAM
     $ BED=,E10.3//40H NUMBER OF TIME STEPS BETWEEN PRINTOUTS=,I4)
  560 FORMAT (1H1,64X,23HTRANSMISSIBILITY MATRIX)
  570 FORMAT (1H0,I5,(1H ,14E9.1))
  580 FORMAT (1H1,55X,17HTIME STEP NUMBER=,I10/50X,29HSIZE OF TIME STEP
     1IN SECONDS=,E10.3/40X,48HDURATION OF PUMPING AT THIS PRINTOUT IN S
     2ECONDS=,E10.3/86X,8HMINUTES=,E10.3/86X,6HHOURS=,E10.3/86X,5HDAYS=,
     3E10.3/55X,17HITERATION NUMBER=,I10)
  590 FORMAT (1H1,54X,26HSTORAGE COEFFICIENT MATRIX)
  600 FORMAT (A5/A8/A7/A7/A5)
  610 FORMAT (F10.3)
  620 FORMAT (8F10.4)
  630 FORMAT (1H1,52X,29HHYDRAULIC CONDUCTIVITY MATRIX)
  640 FORMAT (1H1,46X,40HELEVATION OF IMPERMEABLE BASE OF AQUIFER)
  650 FORMAT (1H1,56X,20HITERATION PARAMETERS///(1H ,10E12.3))
  660 FORMAT (1H0,13HWELL GOES DRY)
  670 FORMAT (1H0,39HEXCEEDED PERMITTED NUMBER OF ITERATIONS)
  680 FORMAT (8F10.0)
  690 FORMAT (1H1,40X,40HGRID SPACING IN PROTOTYPE IN X DIRECTION///(1H0
     1,12F10.3))
  700 FORMAT (1H0,40X,40HGRID SPACING IN PROTOTYPE IN Y DIRECTION///(1H0
     1,12F10.3))
      END
```

```
C     ******************************************************************
      SUBROUTINE PRNTA
C     ******************************************************************
C     THIS SUBROUTINE PRINTS OUT THE HEAD MATRIX AS ALPHABETIC CONTOURS
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      INTEGER DIML,DIMW
      REAL K
      DIMENSION RATE(50,50), SYM(39), PRNT(60), PHI(50,50), STRT(50,50),
     1 BLANK(60), S(50,50)
      DIMENSION DELY(50), DELX(50)
      DOUBLE PRECISION PHI
      WRITE (6,50)
      DATA SYM/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1HA,1HB,1HC,1HD,1HE,1
     1HF,1HG,1HH,1HI,1HJ,1HK,1HL,1HM,1HN,1HO,1HP,1HQ,1HR,1HS,1HT,1HU,1HV
     2,1HW,1HX,1HY,1HZ,1H0,1H ,1H*,1HG/,BLANK/60*1H /
      IND=(65-DIMW)/2
      DO 40 IB=1,DIML
      DO 30 JB=1,DIMW
      K=STRT(IB,JB)-PHI(IB,JB)
      K=K/SPACNG
      IF (K.LT.0) GO TO 10
      K=AMOD(K,36.)
      IF (K.LT.1.) PRNT(JB)=SYM(36)
   10 IF (K.LT.0) PRNT(JB)=SYM(39)
      IF (PHI(IB,JB).EQ.STRT(IB,JB)) PRNT(JB)=SYM(37)
      N=K
      IF (N.LT.1) GO TO 20
      PRNT(JB)=SYM(N)
   20 IF (RATE(IB,JB).GT.0.) PRNT(JB)=SYM(27)
      IF (RATE(IB,JB).LT.0.) PRNT(JB)=SYM(32)
   30 CONTINUE
   40 WRITE (6,60) (BLANK(I),I=1,IND),(PRNT(JB),JB=1,DIMW)
      WRITE (6,70) SPACNG
      RETURN
C
C     ******************************************************************
C
C
C
   50 FORMAT (1H0,50X,32HALPHABETIC CONTOURS FOR DRAWDOWN,////)
   60 FORMAT (1H ,65A2)
   70 FORMAT (10H0LEGEND***/18H0CONTOUR INTERVAL=,F10.3/32H0LOCATION OF
     1RECHARGE BOUNDARY=R/16H0WELL LOCATION=W/21H0CONE OF IMPRESSION=G)
      END
```

```
C     ******************************************************************
      SUBROUTINE PRNT1
C     ******************************************************************
C     THIS SUBROUTINE PRINTS OUT THE HEAD MATRIX IN NUMERICAL FORM
C     ..................................................................
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG
      DIMENSION RATE(50,50), DDN(60), PHI(50,50), S(50,50), STRT(50,50)
      DIMENSION DELY(50), DELX(50)
      INTEGER DIML,DIMW
      DOUBLE PRECISION PHI
      WRITE (6,30)
      DO 20 I=1,DIML
      DO 10 J=1,DIMW
   10 DDN(J)=STRT(I,J)-PHI(I,J)
   20 WRITE (6,40) I,(DDN(K),K=1,DIMW)
      RETURN
C
C     ******************************************************************
C
C
C
   30 FORMAT (1H1,58X,16HDRAWDOWN IN FEET//)
   40 FORMAT (1H0,I5,(1H ,11E11.3))
      END
```

```
      ****************************************************************** D   1
      SUBROUTINE CHECK                                                   D   2
      ****************************************************************** D   3
      THIS SUBROUTINE COMPUTES THE ERROR IN THE SOLUTION ON A MASS       D   4
      BALANCE BASIS                                                      D   5
      ***WARNING - USE THIS SUBROUTINE ONLY WHEN THERE IS NO INFILTRATIO D   6
      INTO THE SYSTEM***                                                 D   7
      COMMON PHI,SUM,DELX,DELY,DIML,DIMW,RATE,S,STRT,SPACNG              D   8
      INTEGER DIML,DIMW                                                  D   9
      DIMENSION RATE(50,50), S(50,50), PHI(50,50), STRT(50,50)           D  10
      DIMENSION DELY(50), DELX(50)                                       D  11
      DOUBLE PRECISION PHI                                               D  12
      TOTL=0.0                                                           D  13
      PUMP=0.0                                                           D  14
      DO 10 IC=2,DIML                                                    D  15
      DO 10 JC=2,DIMW                                                    D  16
      TOTL=TOTL+S(IC,JC)*DELX(JC)*DELY(IC)*(STRT(IC,JC)-PHI(IC,JC))      D  17
   10 IF (RATE(IC,JC).LT.0.0) PUMP=PUMP-RATE(IC,JC)*SUM                  D  18
      DIFF=PUMP-TOTL                                                     D  19
      PERCNT=(DIFF/PUMP)*100.                                            D  20
      WRITE (6,20) TOTL,PUMP,DIFF,PERCNT                                 D  21
      RETURN                                                             D  22
C                                                                        D  23
C     ****************************************************************** D  24
C                                                                        D  25
C                                                                        D  26
C                                                                        D  27
   20 FORMAT (63H0QUANTITY PUMPED ACCORDING TO CONE OF DEPRESSION IN CUB D  28
     1IC FEET=,E20.10//59H QUANTITY PUMPED ACCORDING TO WELL DISCHARGE I D  29
     2N CUBIC FEET=,E20.10//51H ESTIMATE FROM PUMPING LESS ESTIMATE FROM D  30
     3 DRAWDOWN=,E20.10//42H DIFFERENCE AS A PERCENT OF VOLUME PUMPED=,E D  31
     420.10)                                                             D  32
      END                                                                D  33-
```