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Nonconstant Variance Regression Analysis Program

by

Marshall Strong Hellmann

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ERRATA

Changes	Equation or Line No.	Page No.
$Y_j \rightarrow \dot{Y}_j$ and $Y \rightarrow Y_j$	Line 4	6
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1. Weighted Triangulation Adjustment, by Walter L. Anderson, 1969

2. Perspective Center Determination, by John D. McLaurin, 1969

3. Nonconstant Variance Regression Analysis Program, by Marshall Strong Hellmann, 1970

Available free on application to the Chief, Computer Center Division, U. S. Geological Survey, Washington, D. C. 20242

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NONCONSTANT VARIANCE REGRESSION ANALYSIS PROGRAM

by Marshall Strong Hellmann

ABSTRACT

This program computes the estimates of various statistics using an iterative estimating technique described in the pamphlet "Non-Constant Variance Regression Analysis" by Hellmann, (1967). The estimates of the regression coefficients obtained by this iterative technique are usually the maximum likelihood estimates of the coefficients. The program is capable of performing a wide variety of regression techniques with a minimal amount of user involvement in the programming requirements. The program has several options, which allow the user a great deal of flexibility in preparing and processing the input data. Provision is made for the user to provide titles for the variables and names for the observations. The results are printed in detail and described in this paper. Confidence intervals are presented along with a histogram of the results and a scatter diagram relating the observed and estimated values of the dependent variable. The coefficients are tested to insure that the solution is a relative maximum.

INTRODUCTION

This regression analysis program was prepared to implement the regression technique described by Hellmann (1967). The technique differs from classical regression analysis techniques in that the basic assumptions are less restrictive, and the model is therefore more general and applicable to a larger class of problems.

The classical regression analysis models usually require that the standard deviations of the dependent variable are constant for all values of the independent variables. Some models allow for variations in the standard deviations of the dependent variable by introducing known weight factors to account for the variation. The regression model defined herein differs from the classical models in that it does not require that the standard deviations of the dependent variable be constant or that weighting factors be known. Rather it is assumed that the standard deviations of the dependent variable are proportional to a function of the expected value of the dependent variable.

Techniques for estimating the unknown variables for some specific cases of the above described regression model have been presented by Theil (1951), Prais (1953), Prais and Aitchison (1954), Fisher (1957), and Rao (1952). The technique presented in this model uses an iterative procedure to estimate the unknown variables, which in general produces results with a smaller unexplained variation than the techniques presented by the above authors. In fact, it can be shown that the estimates obtained by this method are a relative maximum of the likelihood function of the coefficients if the iterative procedure converges and the matrix of second order partial derivatives of the likelihood function is negative definite.

In this model the user can specify the exact form of the weighting function, which is assumed to be proportional to the standard deviations of the dependent variables, or he can specify the weighting function parametrically in terms of an additional unknown parameter.

Let the dependent variables be denoted by Y_j and the independent variables by X_{ij} for $i=1,2,...m$ and $j=1,2,...n$. The model assumes that the expected value of the variable Y_j, denoted by $E(Y_j)$, is a linear combination of the independent variables X_{ij} as follows,

$$E(Y_j)=\sum_{i=1}^{m} A_i X_{ij} \text{ for } j=1,2,...n, \tag{1}$$

where $A_1,A_2,...A_m$ are a set of real numbers to be determined.

The X_{ij} are assumed to be known mathematical variables, which are not subject to random variation. If the user wishes to apply this model to the case where the X_{ij} are random variables, then the expected values of the variables X_{ij}, $E(X_{ij})$, should be used instead of the observed values of the X_{ij}. The estimating equation would then assume the form

$$E(Y_j)=\sum_{i=1}^{m} A_i E(X_{ij}) \text{ for } j=1,2,\ldots n. \qquad (2)$$

Alternate forms of the equation (1) will result if functions of either the original independent or the dependent variables or both are used. For example if

$$E(Y_j)=\sum_{i=1}^{m} A_i \log W_{ij} \text{ for } j=1,2,\ldots n, \qquad (3)$$

where the W_{ij} denote a set of known independent variables, then equation (3) is the same as equation (1) if we make the substitution $X_{ij}=\log W_{ij}$.

In general the variables X_{ij} represent functions of known mathematical variables and the variable Y_j represents a function whose values are normally distributed with expected value given by equation (1) and whose standard deviation is proportional to some function, $F_\alpha[E(Y_j)]$, which is given parametrically in terms of a variance parameter α for each j.

Since the form of the weighting function is not fixed, the user must first decide what specific function to use and then provide three FORTRAN functions, which will compute the values of the weighting function and the first and second order derivatives of the weighting function with respect to the coefficients $A_1, A_2, \ldots A_m$, respectively. A description of four selected functions is given in Attachment 1 (p. 55).

The user also has the options of determining the accuracy of the model, the size of the critical regions for confidence intervals, initial estimates of the coefficients, A_i for $i=1,2,\ldots m$, and initial estimates for the variance parameter α.

The model uses the technique described by the author (1967) to derive the estimates of the coefficients A_i, the parameter α, the standard deviation σ_α of the dependent variables, as well as other statistics. Confidence intervals are computed for both the observed and expected values of the dependent variables for each of the observations in the sample, as well as for the regression coefficients, A_i for $i=1,2,\ldots m$, using approximate techniques.

Output from this program includes a listing of the input data, intermediate matrices used in computations, detailed results for each observation, confidence limits, a histogram of the dependent variables divided by their standard deviations, a scatter diagram relating the observed and estimated values of the dependent variable, and an evaluation of the regression coefficients.

TECHNICAL DESCRIPTION

Let Y_j denote an observed value of a random variable Y with expected value given by

$$E(Y_j) = \sum_{i=1}^{m} A_i X_{ij}, \tag{4}$$

where $(X_{1j}, X_{2j}, \ldots X_{mj})$ is a set of observed values of m non-random independent variables for j=1,2,...n, corresponding to n statistically independent trials of a set of experiments. This model is based on the assumption that the standard deviation of the random variable Y is itself variable and is equal to the product of a constant σ_α and some known positive function of $E(Y_j)$, denoted by $F_\alpha[E(Y_j)]$. The function must have continuous bounded derivatives with respect to the variables A_i for i=1,2,...m. This model is then generalized to include the case where the function F_α is given in terms of a real valued parameter α, called the "variance parameter." Let the variable R_j be defined as follows:

$$R_j = \frac{Y_j - E(Y_j)}{F_\alpha[E(Y_j)]}. \tag{5}$$

It follows that the variable R_j is normally distributed with standard deviation σ_α and expected value zero for all j.

The maximum likelihood estimates for the unknown coefficients $A_1, A_2, \ldots A_m$ and σ_α are a set of numbers which satisfy the following set of equations simultaneously:

$$\sum_{j=1}^{n} \frac{Y_j - \sum_{i=1}^{m} A_i X_{ij}}{F_\alpha(\sum_{i=1}^{m} A_i X_{ij})^3} [X_{kj} F_\alpha(\sum_{i=1}^{m} A_i X_{ij}) + (Y_j - \sum_{i=1}^{m} A_i X_{ij}) F_\alpha^k (\sum_{i=1}^{m} A_i X_{ij})] = 0 \tag{6}$$

for k=1,2,...m, where F_α^k is the partial derivative of F_α with respect to A_k, and

$$\sum_{j=1}^{n} -\frac{1}{\sigma_\alpha} + \left[\frac{Y_j - \sum_{i=1}^{m} A_i X_{ij}}{F_\alpha(\sum_{i=1}^{m} A_i X_{ij})} \right] (\frac{1}{\sigma_\alpha^3}) = 0. \tag{7}$$

The estimates of the coefficients A_i for i=1,2,...m and for σ_α shall be designated by $\hat{A}_i$ for i=1,2,...m and $\hat{\sigma}_\alpha$, respectively.

Computational Procedure for Estimating Coefficients

The solutions to the equations (6) and (7) are computed by the iterative technique described by Hellmann (1967) for each value of α specified by the user. The equations (6) can be transformed into a set of linear equations in the variables A_i by replacing the A_i by their previous estimated values in all terms except the numerator of the leading fraction. The estimate of σ_α for each set of estimates of the coefficients is computed by replacing the A_i in equation (7) by this set of estimates of the A_i. Initial estimates of the coefficients A_i can be provided by the user or by the program. The program will use the usual least squares estimates as the initial estimates in the iterative estimating technique. The iterative technique generates a sequence of estimates $(\hat{A}_{it})$ for each of the coefficients for $i=1,2,\ldots m$, where t is the iterative index. It assumes the values $1,2,\ldots T$, where T is determined as described below. If each of these sequences converge to a limit point denoted by $\hat{A}_i'$, then this set of points is assumed to be a relative maximum of the likelihood function of the coefficients. In practice it is impossible to determine these limit points exactly unless the sequences of estimates converge in a finite number of iterations. Therefore the user must specify the maximum number of iterations to be permitted on the first input control card. If the user specifies the maximum number of iterations to be zero, then the program uses the initial estimates of the coefficients as the desired solution. This option is useful for evaluating proposed solutions.

The convergence tolerance ε for the sequences of estimates must also be specified on the first input control card. This number is used to determine the limit points of these sequences. Let $S_{it}=\max(|\hat{A}_{it}|,|\hat{A}_{i(t-1)}|,1)$ for $i=1,2,...m$ and for $t=1,2,...T$, where T denotes the actual number of iterations. Let T denote the first value of t such that $|\hat{A}_{i(t-1)}-\hat{A}_{it}|-\varepsilon S_{it}\leq 0$ for $i=1,2,...m$. Then $\hat{A}_{it}$ is chosen as the estimate of the limit point of the sequence $(\hat{A}_{it})$ and is denoted by $\hat{A}_i$ for $i=1,2,...m$, respectively. Since the computer double precision word length in the IBM 360/65 is equivalent to approximately 16 decimal digits, the convergence tolerance should be no smaller than 10^{-14}. Because in some data sets significant digits may be lost due to the computational procedures, some checks are made to determine if such a loss has occurred and messages are printed to this effect. Also, if the iterative technique fails to converge within the specified tolerance, within the maximum number of iterations, then a message is written to this effect.

Estimation of the Variance Parameter α

The estimate of α is chosen in such a manner as to maximize the estimated joint frequency function of the variables Y_j by using the estimates of the coefficients $A_1, A_2, \ldots A_m$ and σ_α as computed above. For notational convenience set

$$\hat{Y}_j = \sum_{i=1}^{m} \hat{A}_i X_{ij} \text{ for } j=1,2,\ldots n. \tag{8}$$

The estimated joint frequency function is given by

$$F(Y_j, X_{ij}, \hat{A}_i, \hat{\sigma}_\alpha, \alpha) = \prod_{j=1}^{n} \frac{1}{\hat{\sigma}_\alpha F_\alpha(\hat{Y}_j)\sqrt{2\pi}} \exp - \tfrac{1}{2}\left(\frac{Y_j - \hat{Y}_j}{\hat{\sigma}_\alpha F_\alpha(\hat{Y}_j)}\right)^2 \tag{9}$$

for all values of α.

Let α_1 denote the first given value of α specified by the user for which the previously described iterative technique produces a set of estimates for the coefficients and form the ratio

$$\lambda = \frac{F(Y_j, X_{ij}, \hat{A}_i, \hat{\sigma}_{\alpha 1}, \alpha_1)}{F(Y_j, X_{ij}, \hat{A}_i, \hat{\sigma}_\alpha, \alpha)}, \tag{10}$$

where α is arbitrary.

Since

$$n\hat{\sigma}_\alpha^2 = \sum_{j=1}^{m} \left[\frac{Y_j - \hat{Y}_j}{F_\alpha(\hat{Y}_j)}\right]^2 \quad \text{for all } \alpha, \tag{11}$$

it follows that

$$\lambda = \prod_{j=1}^{n} \left[\frac{\hat{\sigma}_{\alpha} F_{\alpha}(\hat{Y}_j)}{\hat{\sigma}_{\alpha_1} F_{\alpha_1}(\hat{Y}_j)}\right]. \tag{12}$$

This ratio, denoted by λ, is called the estimated joint frequency function ratio of the estimated joint frequency functions given by equation (9). That value of α which minimizes λ maximizes the estimated joint frequency function given by equation (9). The variable λ is computed in turn for each value of α for which the iterative technique produces a solution. If there are at least three computed values of the estimated joint frequency function ratios λ, then the program fits a second degree polynomial to $\log \lambda$ to determine that estimate of α which minimizes λ. This estimate $\hat{\alpha}$ is then used to perform an additional regression analysis as performed with α_1, α_2, and so forth. Whether there are three or more computed values of λ or not, that value of α which yields the smallest value of λ is then chosen as the desired estimate of the variance parameter α and the estimates of the coefficients corresponding to that value of α are chosen as the estimates of the maximum likelihood estimates of the coefficients.

Coefficient Equations and Solutions

In order to determine if the solutions derived by the above technique are in fact solutions of the equations (6), denote the equations (6) symbolically in matrix form as

$$(D_{ki})(A_i)=(C_k), \tag{13}$$

where (D_{ki}) is the mxm matrix of coefficients of the A_i;

(A_i) is the vector of unknown coefficients;

(C_k) is the vector of constant terms.

The elements of the matrix (D_{ki}) are given by

$$D_{ki}=\sum_{j=1}^{n} X_{ij}[X_{kj}F_\alpha(\hat{Y}_j)+(Y_j-\hat{Y}_j)F_\alpha^k(\hat{Y}_j)]/F_\alpha(\hat{Y}_j)^3, \tag{14}$$

where $F_\alpha^k(\hat{Y}_j)$ is the derivative of F_α with respect to A_k evaluated at $\hat{Y}_j$ for k=1,2,...m and i=1,2,...m. The elements of the vector (C_k) are given by

$$C_k=\sum_{j=1}^{n} [Y_jX_{kj}F_\alpha(\hat{Y}_j)+(Y_j-\hat{Y}_j)F_\alpha^k(\hat{Y}_j)]/F_\alpha(\hat{Y}_j)^3 \tag{15}$$

for k=1,2,...m.

The matrix equation (13) can be solved by multiplying both sides of the equation by the inverse of (D_{ki}). Thus,

$$(A_i)=(D_{ki}^{-1})(C_k). \tag{16}$$

The program performs this operation sequentially by applying Gaussian elimination to (D_{ki}) and applying the same transformation to the vector (C_k) until the matrix (D_{ki}) has been transformed into the identity matrix. This technique usually preserves more accuracy in the results than by multiplying (C_k) by a computed inverse.

If the estimates of the coefficients are equal to the A_i as computed from equation (16), then the coefficients are indeed a solution of the equations (6). However, due to the fixed length of the numbers in the computer and based on the convergence tolerance specified by the user, these two sets of values for the A_i will, in general, differ. The difference is a measure of the computational accuracy.

Linear Relationships Between Variables

Although the independent variables are assumed to be nonrandom variables, the program computes a correlation matrix E for these variables. Whereas this matrix may provide the user with additional insight into the nature of the data, the correlations are not assumed to have any statistical significance. Set

$$\overline{X}_i=\frac{1}{n}\sum_{j=1}^{n}X_{ij} \tag{17}$$

and $$S_i=[\frac{1}{n}\sum_{j=1}^{n}(X_{ij}-\overline{X}_i)^2]^{\frac{1}{2}} \text{ for } i=1,2,\ldots m. \tag{18}$$

If S_i or S_k is equal to zero, then the elements E_{ik} of (E) are defined to be zero. Otherwise the elements E_{ik} are defined as follows,

$$E_{ik}=\frac{1}{n}\sum_{j=1}^{n}\frac{(X_{ij}-\overline{X}_i)(X_{kj}-\overline{X}_k)}{S_iS_k} \text{ for } i,\ k=1,2,\ldots m. \tag{19}$$

The matrix of elements E_{ik} is called the "arithmetical correlation matrix."

The program also computes the arithmetical linear correlation between the values of the dependent variables and the estimated values of the independent variable. Set

$$\overline{\hat{Y}}=\frac{1}{n}\sum_{j=1}^{n}\hat{Y}_j \tag{20}$$

and $$S_y=[\frac{1}{n}\sum_{j=1}^{n}(\hat{Y}_j-\overline{\hat{Y}})^2]^{\frac{1}{2}}. \tag{21}$$

If S_i or S_y is equal to zero, then the linear correlation between X_i and the estimated values of Y is defined to be zero. Otherwise the linear correlation is defined as

$$H_i=\frac{1}{n}\sum_{j=1}^{n}\frac{(X_{ij}-\overline{X}_i)(\hat{Y}_j-\overline{\hat{Y}})}{S_iS_y} \text{ for } i=1,2,\ldots m. \tag{22}$$

CONFIDENCE INTERVALS

The program computes approximate confidence intervals for the regression coefficients, for the expected values of the variables Y_j, and for the observed values of the variables Y_j. Since the variance of Y is a function of the coefficients, the derivation of confidence intervals by exact methods appears to be impossible. For large samples, confidence intervals for the variable Y_j are given approximately by

$$\hat{Y}_j \pm T_{(1-\beta)} \hat{\sigma} F_{\hat{\alpha}}(\hat{Y}_j), \tag{23}$$

where $T_{(1-\beta)}$ is the upper limit for the integral of the standardized normal distribution function f such that

$$(1-\tfrac{1}{2}\beta) = \int_{-\infty}^{T_{(1-\beta)}} f(T)\,dT \text{ for } 0 \le \beta \le 1. \tag{24}$$

If the sample size is small, then an approach similar to that used in classical (constant variance) linear regression is used. Approximate confidence intervals can be derived based on the assumptions that (1) $\hat{\alpha}=\alpha$ and (2) $F_{\hat{\alpha}}(\hat{Y}_j)=F_{\hat{\alpha}}[E(\hat{Y}_j)]$ for all j, where $\hat{Y}_j$ is computed by equation (8). Since these assumptions are not in general valid for even large samples, the resulting confidence intervals may be subject to a corresponding error. A regression analysis model slightly different from that presented herein must be used. In order to distinguish between these two models we shall denote the coefficients in the new model by $B_1, B_2, \ldots B_m$. Otherwise we shall use terminology similar to that used in the derivation of the first model. The reader must use care in keeping the terminology straight.

Let
$$W_j = F_{\hat{\alpha}}\left(\sum_{i=1}^{m} \hat{A}_i X_{ij}\right). \tag{25}$$

Then $(Y_j - \sum_{i=1}^{m} B_i X_{ij})/W_j$ is normally distributed with expected value zero and standard deviation σ for all j. Let the mxm matrix V be defined for i,k=1,2,...m, respectively, as

$$(V_{ik}) = \left(\sum_{j=1}^{n} \frac{X_{ij}X_{kj}}{W_j^2}\right)^{-1}. \tag{26}$$

The maximum likelihood estimates for the regression coefficients, B_i for i=1,2,...m, are given in matrix notation as

$$(\hat{B}_i) = (V_{ik})\left(\sum_{j=1}^{n} \frac{Y_j X_{kj}}{W_j^2}\right). \tag{27}$$

Taking the expected values of both sides of equation (27) and simplifying we have

$$E(\hat{B}_i) = B_i \text{ for } i=1,2,\ldots m. \tag{28}$$

The variance-covariance matrix of the coefficients $\hat{B}_i$ for i=1,2,...m for this model is given by $(V_{ik})\sigma^2$. It can be shown that the $\hat{B}_i$ have an m-variate normal distribution. The marginal distribution function for $\hat{B}_i$ for i=1,2,...m is given by

$$f(\hat{B}_i)=(2\pi\sigma^2 V_{ii})^{-\frac{1}{2}}\exp-\frac{1}{2}\frac{(\hat{B}_i-B_i)^2}{V_{ii}\sigma^2}\ . \tag{29}$$

The maximum likelihood estimate for σ is given by,

$$\hat{\sigma}=\left[\frac{1}{n}\sum_{j=1}^{n}\left[\frac{Y_j-\sum_{i=1}^{m}\hat{B}_i X_{ij}}{W_j}\right]^2\right]^{\frac{1}{2}}. \tag{30}$$

It can be shown in a manner exactly analagous to that of the derivation of confidence intervals for the classical model using Student's t distribution that the lengths of the $(1-\beta)$ confidence intervals of the B_i are given by,

$$2T_{(1-\beta)}\hat{\sigma}(\frac{n}{n-m}V_{ii}) \text{ for } i=1,2,...m, \tag{31}$$

where $T_{(1-\beta)}$ is the upper limit of the integral of the Student's t distribution function g with $(n-m)$ degrees of freedom such that

$$(1-\frac{1}{2}\beta)=\int_{-\infty}^{T(1-\beta)} g(T)dT \text{ for } 0\leq\beta\leq 1. \tag{32}$$

The approximate confidence intervals for the A_i are then computed using the estimate of σ computed in the first model in equation (31). (The value of $\hat{\sigma}$ computed in the first model is in general larger than the value of $\hat{\sigma}$ computed in the second model by equation (30).) The confidence intervals are centered on the $\hat{A}_i$ for $i=1,2,...m$.

We now turn our attention to deriving confidence intervals for the expected value of an arbitrary but fixed dependent variable Y_0 with associated independent variables X_{i0} for $i=1,2,\ldots m$. Since $E(\hat{B}_i)=B_i$ for $i=1,2,\ldots,m$, it follows that $E(\hat{Y}_0)=E(Y_0)$. The variance of $\frac{\hat{Y}_0}{W_0}$ is given by

$$E\left[\frac{\sum_{i=1}^{m}\hat{B}_i X_{i0}-E(Y_0)}{W_0}\right]^2=\sum_{i=1}^{m}\sum_{k=1}^{m}V_{ik}\frac{X_{i0}X_{k0}}{W_0^2}\sigma^2. \qquad (33)$$

Once again it can be shown that the length of the $(1-\beta)$ confidence interval for the $E(Y_0)$ can be derived using Student's t distribution; it is given by

$$2T_{(1-\beta)}\left[\frac{n}{n-m}\sum_{i=1}^{m}\sum_{k=1}^{m}V_{ik}\frac{X_{i0}X_{k0}}{W_0^2}\right]^{\frac{1}{2}}\hat{\sigma}W_0. \qquad (34)$$

The approximate confidence interval for $E(Y_0)$ is computed using the estimate of σ computed in the first model in equation (34) with the interval centered on $\sum_{i=1}^{m}\hat{A}_i X_{i0}=\hat{Y}_0$.

The approximate confidence interval for the variable Y_0 is computed using the second model. The variance of

$\frac{Y_0-\sum_{i=1}^{m}\hat{B}_i X_{i0}}{W_0}$ is given by

$$E\left[\frac{Y_0-\sum_{i=1}^{m}\hat{B}_i X_{i0}}{W_0}\right]^2=E\left[\frac{Y_0-\sum_{i=1}^{m}B_i X_{i0}}{W_0}\right]^2+E\left[\frac{\sum_{i=1}^{m}(B_i-\hat{B}_i)X_{i0}}{W_0}\right]^2$$

$$+2E\left(\left[\frac{Y_0-\sum_{i=1}^{m}B_i X_{i0}}{W_0}\right]\left[\frac{\sum_{i=1}^{m}(B_i-\hat{B}_i)X_{i0}}{W_0}\right]\right). \qquad (35)$$

But since Y_0 and $\hat{B}_i$ are statistically independent, the last term in equation (35) is equal to zero. Therefore

$$E\left[\frac{Y_0-\sum_{i=1}^{m}\hat{B}_i X_{i0}}{W_0}\right]^2=\sigma^2(1+\sum_{i=1}^{m}\sum_{k=1}^{m}V_{ik}\frac{X_{i0}X_{k0}}{W_0^2}). \tag{36}$$

Also the $E\left[\frac{Y_0-\sum_{i=1}^{m}\hat{B}_i X_{i0}}{W_0}\right]=0.$

Therefore we can once again use Student's t distribution to compute the length of the $(1-\beta)$ confidence interval for Y_0. See Mood and Graybill (1963, p. 351-352) for derivation. This is given by

$$2T_{(1-\beta)}\left[\frac{n}{n-m}(1+\sum_{i=1}^{m}\sum_{k=1}^{m}V_{ik}\frac{X_{i0}X_{k0}}{W_0^2})\right]^{\frac{1}{2}}\hat{\sigma}W_0. \tag{37}$$

(Equations 26.7.5 and 26.2.23 of Abramowitz and Stegun (1964, p. 925-949) are used to compute $T_{(1-\beta)}$.) The approximate confidence interval for Y_0 is computed using the estimate of σ computed in the first model in equation (37) with the interval centered on $\sum_{i=1}^{m}\hat{A}_i X_{i0}=\hat{Y}_0$.

Fisher (1925) contains a dissertation on some basic concepts in the theory of statistical estimation and in particular a section on the efficiency of weighting, which is pertinent to the above derivations of confidence intervals.

ANALYSIS OF THE RANDOM VARIABLE R

R is the fundamental random variable in the model and is defined by equation (5). The n sample values of R are first classified into M classes of equal width, where M is the minimum of ([n/m],20). If [n/m] is less than or equal to 2, then none of the computations on R in this section are performed. A histogram of R is printed out together with the interval boundaries and frequency of occurrence expressed as a percentage of the total sample size. Since the expected value of R is zero, the sample statistics are computed about zero rather than the sample mean. The sample mean, sample variance, and coefficients of skewness and kurtosis are computed by the following formulas.

Sample mean value, $$\overline{R}=\frac{1}{n}\sum_{j=1}^{n} R_j \tag{38}$$

Sample variance, $$S_R^2=\frac{1}{n}\sum_{j=1}^{n} R_j^2 \tag{39}$$

Coefficient of skewness, $$\gamma_1=\frac{1}{n}\sum_{j=1}^{n} R_j^3/S_R^3 \tag{40}$$

Coefficient of kurtosis, $$\gamma_2=\frac{1}{n}(\sum_{j=1}^{n} R_j^4/S_R^4)-3 \tag{41}$$

The statistics in this model are based on the assumption that R is normally distributed with mean zero and constant variance σ_α^2. In order to determine the appropriateness of these assumptions, the sample values R_j are once again classified into M classes, where each class has the probability of occurrence of 1/M. Therefore the expected frequency of occurrence E_k for each of the intervals is given by n/M. Let O_k denote the number of observations in the kth interval. Then the distribution of the variable

$$U^2=\sum_{k=1}^{M}(O_k-E_k)^2/E_k \tag{42}$$

is given approximately by the chi-square distribution with M-2 degrees of freedom. (See Mood and Graybill, 1963, p. 308-311.) The $(1-\beta)$ confidence interval for U^2 is chosen as the interval $(0,T^2_{(1-\beta)})$ which corresponds to good agreement between the observed and expected frequencies of occurence. $T^2_{(1-\beta)}$ is chosen such that

$$\int_0^{T^2_{(1-\beta)}} F(X^2)\,dX^2=1-\beta, \tag{43}$$

where $F(X^2)$ is the chi-square distribution function. The sample value for U^2 is computed by equation (42) using $E_k=n/M$. Since the variable R is normally distributed with mean zero and variance σ_α^2, it follows that the variable $\bar{R}$ given by equation (38) is normally distributed with mean zero and variance σ_α^2/n. Also the variable nS_R^2/σ_α^2 has a chi-square distribution with n degrees of freedom and is statistically independent of $\bar{R}$. Therefore the variable

$$T=\bar{R}/S_R\sqrt{n} \tag{44}$$

has Student's t distribution with n degrees of freedom. The $(1-\beta)$ confidence interval for $\overline{R}$ is given by

$$-T_{(1-\beta)}S_R/\sqrt{n} \le \overline{R} \le T_{(1-\beta)}S_R/\sqrt{n}, \tag{45}$$

where $T_{(1-\beta)}$ is given by equation (32).

Since nS_R^2/σ_α^2 has a chi-square distribution with n degrees of freedom a $(1-\beta)$ confidence interval for the standard deviation of R is given by

$$\sqrt{nS_R^2/T^2_{(1-\frac{1}{2}\beta)}} \le \sigma_\alpha \le \sqrt{nS_R^2/T^2_{\frac{1}{2}\beta}}, \tag{46}$$

where $T^2_{\frac{1}{2}\beta}$ and $T^2_{(1-\frac{1}{2}\beta)}$ are chosen such that (see equation 26.4.18, Hoel, 1958, p. 925-949)

$$\int_0^{T^2_{\frac{1}{2}\beta}} F(x^2)\,dx^2 = \tfrac{1}{2}\beta, \tag{47}$$

and

$$\int_{T^2_{(1-\frac{1}{2}\beta)}}^{\infty} F(x^2)\,dx^2 = \tfrac{1}{2}\beta. \tag{48}$$

EVALUATION OF REGRESSION COEFFICIENTS

The program performs three checks on the coefficients $A_1, A_2, \ldots A_m$ to determine if the likelihood function has a relative maximum when evaluated at the point $(\hat{A}_1, \hat{A}_2, \ldots \hat{A}_m)$.

The first check determines if the coefficients are a simultaneous solution of the set of equations (13) obtained by setting the partial derivative of the log of the likelihood function with respect to the A_i equal to zero for $i=1,2,...m$. The estimates $\hat{A}_1,\hat{A}_2,...\hat{A}_m$ are a solution of these m equations if

$$\sum_{i=1}^{m} D_{ki}\hat{A}_i - C_k = 0 \qquad (49)$$

for $k=1,2,...m$, where D_{ki} and C_k are defined by equations (14) and (15), respectively. Due to the fact that the subtraction operation on fixed length numbers in the computer loses precision when the numbers are very nearly equal, the test for determining whether the estimates are a solution of the equations (49) is made as follows. Let B(k) be the sum of all the positive addends in equation (49) and T(k) be the sum of all the negative addends in equation (49) for each k. Then

$$B(k)+T(k) = \sum_{i=1}^{m} D_{ki}\hat{A}_i - C_k \qquad (50)$$

for each k.

The accuracy of the solution $\hat{A}_1,\hat{A}_2,...\hat{A}_m$ can be determined by testing B(k) and -T(k) for the number of significant digits which are equal. For example if B(1)=1.432167 and -T(1)=1.432201, then the program would determine that these two numbers were equal to four significant digits. The program tests the B(k) and -T(k) for equality in significant digits for $k=1,2,...m$. Let NSD denote the minimum number of significant digits between the B(k) and -T(k). If NSD is zero, then the program writes a statement that the coefficients are not a simultaneous solution of the partial derivative equations. Otherwise the program writes a statement that the coefficients are a simultaneous solution of the partial derivative equations to NSD significant digits.

The second check determines if the sequences of coefficient estimates have converged within the tolerance specified for each sequence. Let A_i' be computed as follows:

$$A_i' = \sum_{k=1}^{m} D_{ki}^{-1} C_k \tag{51}$$

for $i=1,2,\ldots m$, where the D_{ki}^{-1} are the elements of the inverse of the matrix (D_{ki}) given by equation (14). Let $S_i = \mathrm{Max}(|A_i'|,|\hat{A}_i|,1)$. If $|A_i' - \hat{A}_i| \leq \varepsilon S_i$ for $i=1,2,\ldots m$, then a message is written in the output that the coefficients are within the specified tolerance. Otherwise a message is written that the coefficients are not within the specified tolerance.

A final check is made to determine if the likelihood function exhibits a relative maximum at the point $(\hat{A}_1,\hat{A}_2,\ldots\hat{A}_m)$. The program computes the matrix of second order partial derivatives of the log of the likelihood function with respect to the coefficients $A_1,A_2,\ldots A_m$, evaluated at the point $(\hat{A}_1,\hat{A}_2,\ldots\hat{A}_m)$. If this matrix is negative definite, then this point is a relative maximum of the likelihood function. The program computes the eigen vectors and eigen values of the matrix. If all of the eigen values are negative, then the matrix is negative definite. The eigen values are printed out so that the user can determine if the solution is a relative maximum. As a check on the eigen vector subroutine, the determinant of the matrix is computed and compared to the product of the eigen values. These two numbers should be equal within the error tolerance of this subroutine. This subroutine determines the minimum number of significant digits in the eigen vectors and eigen values and this number is printed out. The program terminates with the message "END OF EXECUTION".

INPUT

This program requires a considerable number of input parameters in addition to the usual regression data matrix. The input can have a fixed length record format of 80 characters or a variable length record format. The user is referred to IBM, Systems Reference Library (1968) for FORTRAN details.

Input Specifications

The first record is always read in from a card and must have the following format.

Characters	Format	Meaning
1- 2	I2	Number of independent variables.
3- 5	I3	Number of observations.
6-15	D10.3	Convergence tolerance.
16-25	D10.3	Lower bound for weight function.
26-27	I2	Number of parameters given for weight function.
28-31	I4	Maximum number of iterations for each value of the parameters.
32-41	D10.3	Parameter for weighted curve fitting technique.
42-44	F3.2	Size of critical regions for computing confidence intervals.
45	I1	Data format indicator.
46	I1	Coefficient indicator.
47-48	I2	FORTRAN data set reference no.
49	I1	Symmetric coefficient equations indicator.

The rest of the input records are read from the data set indicated by characters 47-48 on the above card. If this number is less than 9 or greater than 98, then it is set equal to 5, which corresponds to the card reader.

The next set of input records must contain the given values of the parameter α for the variance weighting function. The format of these records is (3D22.15). There must be at least one value of the parameter and not more than 20. The first value of α for which the estimating technique produces an answer serves a special purpose as described in equations (10-12).

If the coefficients indicator is zero or blank, then the initial estimates for the regression coefficients are set equal to the standard least squares estimates of the coefficients. No initial estimates should be included.

If the coefficient indicator is not zero or blank, then the program assumes that the user will provide initial estimates of these coefficients. The format of these coefficients is (3D22.15). There must be the same number of coefficients as the number of independent variables specified on the first input control card.

If the symmetric coefficient equations indicator is zero or blank, then the program assumes that the matrix of coefficients of the variables $A_1, A_2, \ldots A_m$ given by the equations (6) is symmetric for each iteration. A significant amount of computer time can be saved by using this indicator when the equations are symmetric.

In order to allow use of input data not prepared specifically for this program, the format of the data is specified by the user. This object time format is used to read in the data matrix for the regression analysis. This matrix can be read in either the standard or transposed mode as indicated by the data format indicator on the first input control card.

The next set of input records must contain the name of the dependent variable followed by the names of the independent variables. These records must have the format (6(A8,2X),20X). If names are not desired, the records should contain blanks.

Standard Mode

If the data format indicator is zero or blank then an 80 character record is required to define the format specifications for both the dependent and independent variables. Both of these variables must have the same format. For example, this format statement might be (6D12.5,8X).

The next set of input records must contain the names for each observation of the dependent variable. The format for these records is (6(A8,2X),20X). There must be the same number of names as the number of observations. If names are not desired, the records should contain blanks.

The next set of input records must contain the values of the dependent variable Y for each observation. The format of this data is given by the object time format statement. This data is stored in double precision.

The next set of input records must contain the values of the independent variables for each observation. The format of this data is given by the object time format statement. This data is stored in double precision and referred to in the output as X(1),X(2),...X(m), where m is less than or equal to 10.

<u>Transposed Mode</u>

If the data format indicator is not a zero or blank, then the following 80 character record is required to define the format of the input data matrix and the names of each observation of the dependent variable. The first data field in this statement refers to the name of the observation and should be in one of the following format codes A1-A8. The second data field refers to the value of the dependent variable for this observation and must be a floating point code which describes the input data. This number is stored in double precision. The remaining data fields refer to the independent variables and must be in floating point format codes. These numbers are also stored in double precision. For example, a typical format statement might be (2X,A4,5X,D12.5,5D10.3). This format statement would correspond to the following input record.

Characters	Format	Meaning
3- 6	A4	Name of observation.
12-23	D12.5	Value of dependent variable for this observation.
24-33	D10.3	Value of first independent variable for this observation.
34-43	D10.3	Value of second independent variable for this observation.
44-53	D10.3	Value of third independent variable for this observation.
54-63	D10.3	Value of fourth independent variable for this observation.
64-73	D10.3	Value of fifth independent variable for this observation.

OUTPUT

The output from this program consists of tables, a scatter diagram, a histogram, sets of confidence limits, and related statistics. (See the attached example output, p. 62-78.) Some variation in the output will occur as a result of choosing different values of the input parameters. A brief description of each of the printouts follows.

Input Data and Parameters

The program prints out the values of the input data and parameters together with appropriate identifying labels (p. 64-65). At the end of the input listing is printed the message "END OF INITIALIZATION", which signifies that the program has read in all of the input data without detecting any errors.

Intermediate Results

The program then begins the iteration technique for each given value of the weighting function parameter. A summary of the results for each value of this parameter for which the technique produces a solution is printed out. If any error conditions occur during the iterative procedure or if the sequences of estimates do not converge within the tolerance specified, then messages are printed out stating the nature of the error condition. The printout (p. 66) contains the estimating equation derived for the given values of the variance parameter α, $\hat{\sigma}_\alpha$ defined by equation (11), the logarithm of λ given by equation (12), the number of iterations required for convergence, and the value of the variance parameter.

Results of Weighted Curve Fitting Technique

If there are three or more values of λ, then the program performs a weighted parabolic curve fitting technique using the values of $\log\lambda$ as the dependent variable and the corresponding values of α as the independent variable. A detailed table of results containing the values of the dependent variable, the estimates of the dependent variable, the difference between these two values, the value of a built-in weighting function, the ratio of the difference to the weight, the standard deviation of the observed values about the estimated values, and the names of each of the variables is printed out (p. 67). The estimating equation is also printed out as well as the number of iterations and the value of the variance parameter. The program also computes that value of α corresponding to the minimum of the fitted curve which is labeled "ESTIMATE OF VARIANCE PARAMETER ALPHA" (p. 67).

Nonconstant Variance Regression Analysis Results

This printout (p. 68) contains the detailed and summary output referred to above corresponding to that value of α for which the $\log\lambda$ is minimal. This printout contains the final estimates of the regression coefficients, standard deviation of the ratios, and estimate of α. The remaining printouts and results all correspond to this regression analysis and corresponding estimate of α.

Joint Frequency Function Ratios

This printout (p. 69) contains the computed values of the joint frequency function ratios, the λ's given by (12), for each of the values of the variance parameter α for which the iterative technique produced a solution. Since the range of these numbers can be quite large, the exponent is printed using the FORTRAN format I5 instead of the standard two-digit exponent of the FORTRAN D format.

Coefficient Equations and Solutions

This printout (p. 70-71) contains the coefficient matrix D_{ki} and the vector of constant terms C_k defined by equation (13) as well as the values of the inverse of the coefficient matrix and estimates of the coefficients A_i given by equation (16). These coefficients should not differ from those estimated by the iterative technique by more than the convergence tolerance times the value of the corresponding coefficients. If they do, then the iterative technique has not converged within the tolerance specified.

Linear Relationships Between Variables

This printout (p. 72) contains the arithmetical correlation matrix for the independent variables. These values are computed as described in equations (17), (18), and (19). The correlation matrix is not assumed to have any statistical significance and is included only as an aid in evaluating the independent variables. This printout also contains the arithmetical linear correlation between the estimated values of Y and the independent variables as computed by equation (22).

Confidence Intervals

Three different sets of approximate confidence intervals are computed using the variance-covariance matrix of the regression coefficients given by equation (26), which is also written in this printout (p. 73-75). The approximate confidence intervals for the coefficients are computed by equation (31). The approximate confidence intervals for the expected values of the variables Y_j for $j=1,2,\ldots n$ are computed by equation (34) and the approximate confidence intervals for the variables Y_j for $j=1,2,\ldots n$ are computed by equation (37).

Statistics for Distribution of Ratios

This printout (p. 76) contains a histogram of the sample values of R as well as the sample statistics of this variable computed about zero. The data is classified into intervals of equal expectation and a chi-square value is computed, which is a measure of how well the sample values of R fit the hypothesized distribution. These intervals and the observed and expected frequencies are also printed. Confidence intervals for testing the goodness of fit as well as for the sample mean and the standard deviation of R are also printed.

Graph of Observed and Estimated Values of Y

This printout (p. 77) contains a scatter diagram relating the observed and estimated values of the Y variable. The asterisks on this graph represent the points $(Y_j, \hat{Y}_j)$ for $j=1,2,...n$. The periods represent the upper and lower limits of the confidence interval for each of the observations in the sample. These limits are computed by adding and subtracting the quantity computed by equation (37) to the estimate Y_j for each j. Two X's are plotted on the graph. The line joining these two X's represents the line of means for Y.

Analysis of Regression Coefficients

This printout (p. 78) is described in the section of this paper titled "Evaluation of regression coefficients" (p. 22). The matrix of second order partial derivatives of the log of the likelihood function is given by

$$(E_{ik})=\left(\frac{\partial^2 \sum_{j=1}^{n}\left[\frac{Y_j-\hat{Y}_j}{\hat{\sigma}_{\hat{\alpha}}F_{\hat{\alpha}}(\hat{Y}_j)}\right]^2}{\partial A_i\,\partial A_k}\right). \tag{52}$$

This matrix should be a real symmetric matrix. If the coefficients are a relative maximum, then the matrix should be negative definite.

FORTRAN FUNCTIONS REQUIRED BY PROGRAM

This program requires the user to supply three FORTRAN function subprograms which compute the estimated values of the function F_α and the first and second order partial derivatives of F_α with respect to the coefficients $A_1,A_2,\ldots A_m$. There is a great deal of flexibility available to the user in the coding and use of these functions. If the functions are not restricted to be functions of the coefficients $A_1,A_2,\ldots A_m$, the first and second order derivatives would be zero. The functions must conform to the following specifications:

X is a real variable double precision matrix containing the values of the independent variables.

A is a real variable double precision vector containing the estimates of the coefficients $A_1, A_2, \ldots A_m$ for the preceding iteration.

ALPHA is a real double precision variable, which is given the value of the variance parameter α by the main program.

J is an integer variable denoting the observation for which the weighting function is to be evaluated.

I is an integer variable denoting that the partial derivative is to be computed with respect to the coefficient A_I.

K is an integer variable denoting that the second order partial derivative is to be computed with respect to the coefficient A_K.

NX is an integer variable representing the number of independent variables.

NY is an integer variable representing the number of observations of the dependent variable.

EY_J is equal to $\sum_{i=1}^{m} A_i X_{iJ}$.

Function EYF

The FORTRAN function EYF can be coded usina the following statements to define this function.

```
FUNCTION EYF(X,A,ALPHA,J,NX,NY)
DOUBLE PRECISION X(10,500),A(1),ALPHA,EYF
- - -
```

Insert FORTRAN statements to compute $EYF=F_{\alpha}(EY_{J})$ here.

```
- - -
RETURN
END
```

The only external variable which should be changed by this function is the real variable EYF. The function must contain at least one statement setting the value of EYF.

Function DEYF

The FORTRAN function DEYF can be coded using the following statements to define this function.

```
FUNCTION DEYF(X,A,ALPHA,J,I,NX,NY)
DOUBLE PRECISION X(10,500),A(1),ALPHA,DEYF
- - -
```

Insert FORTRAN statements to compute $DEYF=\frac{\partial F_{\alpha}(EY_{J})}{\partial A_{I}}$ here.

```
- - -
RETURN
END
```

The only external variable which should be changed by this function is the real variable DEYF. The function must contain at least one statement setting the value of DEYF.

<u>Function DDEYF</u>

The FORTRAN function DDEYF can be coded using the following statements to define this function.

```
FUNCTION DDEYF(X,A,ALPHA,J,I,K,NX,NY)
DOUBLE PRECISION X(10,500),A(1),ALPHA,DDEYF
- - -
```

Insert FORTRAN statements to compute DDEYF=$\frac{\partial^2 F_\alpha(EY_J)}{\partial A_I \partial A_K}$ here.

```
- - -
RETURN
END
```

The only external variable which should be changed by this function is the real variable DDEYF. The function must contain at least one statement setting the value of DDEYF.

See examples in Attachment 1 (p. 55) for those weighting functions presently available on the USGS IBM 360/65 computer.

TIMING, STORAGE REQUIREMENTS AND OPERATING NOTES

Timing on IBM 360/65, Release 16 with HASP II

The CPU time required for executing this program fluctuates greatly depending on the number of independent variables, the number of observations, and the convergence tolerance specified on the first control card. The program requires approximately mn/400 seconds per iteration, where m is the number of independent variables and n is the number of observations. The user can determine an approximate upper bound for the overall execution time by the formula

$$T = 10 + K(N_\alpha + 2)\,smn/400 \text{ seconds,} \tag{53}$$

where N_α is the number of given values of the variance parameter;

s is equal to .5 if the symmetric coefficients indicator is zero or blank, to 1 otherwise;

K is the maximum number of iterations specified on the first input card.

The total run time varies depending on the manner in which the job is processed. Approximately three minutes should be added to the estimated execution time if the entire program is being compiled and link-edited although this mode of operation is not recommended.

Two modes of operation are recommended on the USGS computer. Mode 1 requires approximately 30 seconds plus execution time for operating the program. Mode 2 requires approximately 50 seconds plus execution time for operating the program. See "Operating notes" (p. 41) for a description of these two modes of operation.

Storage Requirements

The program requires approximately 220,000 bytes of core storage on the IBM 360/65 computer. In addition, core storage is required for input-output buffers as well as for the operating system. The program is presently operating in a 230K region under Release 16 of the operating system.

Operating Notes

Two different modes are recommended for operating the program on the USGS computer depending on whether a stored weighting function or a user-supplied weighting function is used.

Mode 1 - Stored Weighting Functions

JOB CARD (See USGS job card instruction sheet.)

```
//STEP1 EXEC LINKFORT,REGION.GO=230K
//LKED.LIB DD DSN=SYS1.LOADLIB,DISP=SHR
//LKED.SYSIN DD *
  INCLUDE LIB(W8251#1,W8251#n)  - See note below. -
  ENTRY REGRES
```

```
/*
//GO.FTnnF001 DD   -  See note below.  -
//GO.SYSIN DD *
Control card as specified in INPUT.
Additional data cards as neccessary.
/*
```

Note: n is the number of the weighting function desired. (See "Weighting functions presently available", p. 55.) nn is the FORTRAN data set reference number in columns 47-48 of the control card. If the FORTRAN data set reference number is 5, then the //GO.FTnnF001 DD card should be omitted and the data placed following the control card in the reader. The remaining fields in this DD statement are used to describe the input data set.

Mode 2 - User-Supplied Weighting Functions

```
    JOB CARD (See USGS job card instruction sheet.)
//STEP1 EXEC FORTHCLG,REGION.GO=230K
//FORT.SYSIN DD *
- - FORTRAN FUNCTIONS EYF,DEYF, and DDEYF - -
/*
//LKED.LIB DD DSN=SYS1.LOADLIB,DISP=SHR
//LKED.SYSIN DD *
  INCLUDE LIB(W8251#1)
```

```
  ENTRY REGRES
/*
//GO.FTnnF001 DD   -  See note above. -
//GO.SYSIN DD *
Control card as specified in INPUT.
Additional data cards as neccessary.
/*
```

DIAGNOSTIC MESSAGES

The following set of diagnostic messages may be printed in the output from this program. The first message is printed when any one of several different conditions are encountered by the program. It is referred to as Message #1 in this paper.

CHECK INPUT DATA, INITIAL ESTIMATES OF COEFFICIENTS, AND PARAMETERS OF WEIGHTING FUNCTION.

END OF EXECUTION

Cause: This message is printed out if any one of several different conditions are encountered, which preclude further execution of the program. The user should check his input data to determine the cause of the trouble. If an additional diagnostic message is printed in the output, then the user should try to determine the cause of the trouble on the basis of all the messages printed.

Action: Program terminates and a STOP 1 may be written in the HASP system log.

NUMBER OF OBSERVATIONS IS LESS THAN NUMBER OF INDEPENDENT VARIABLES.

Cause: The number of observations must be less than or equal to the number of independent variables on the first input control card.

Action: Message #1 is printed in the output. The program terminates and a STOP 1 may be written in the HASP system log.

END OF INITIALIZATION

BEGIN EXECUTION

Cause: All of the input data has been successfully read into the program. (See "Input data and parameters", p. 31.)

Action: The program enters the execution phase.

POSSIBLE LOSS IN NUMBER OF SIGNIFICANT FIGURES. VARIANCE PARAMETER = 0.xxxxxD±nn

Cause: The subroutine WORK, which solves the set of linear equations obtained from equations (6) and (7) by the iteration technique described in the section "Computational procedures for estimating the coefficients", (p. 8), has returned an error code indicating that a loss in the number of significant figures may have occurred for the stated variance parameter.

Action: Execution continues. User should check results for loss in accuracy. A switch is set so that this message will be printed no more than once for each value of the variance parameter.

THE FUNCTION F(EY) IS LESS THAN 0.xxxxxD±nn ON ITERATION nnn. VARIANCE PARAMETER = 0.xxxxxD±nn

Cause: The weighting function is less than the lower bound for the weight function specified on the first input control card for the stated variance parameter.

Action: Program execution continues but results for stated variance parameter are not included in further computations. If the iterative technique fails to converge for any reason for all values of the variance parameter, then Message #1 is printed in the output and a STOP 1 may be printed in the HASP system log. Execution is terminated.

ITERATION TECHNIQUE FAILED TO CONVERGE WITHIN TOLERANCE SPECIFIED. VARIANCE PARAMETER = 0.xxxxxD±nn

Cause: The sequences of estimates of the coefficients failed to converge within tolerance specified, within the maximum number of iterations, indicated on the first input control card for stated variance parameter. (See "Input specifications", p. 25, and "Computational procedures for estimating the coefficients", p. 8.)

Action: Same as for previous message.

COEFFICIENT MATRIX IS SINGULAR. VARIANCE PARAMETER = 0.xxxxxD±nn

Cause: Subroutine WORK, which solves the set of linear equations obtained from equations (6) and (7) by the iterative technique described in the section "Computational procedures for estimating the coefficients" (p. 8), has determined that the set of equations are singular for the stated variance parameter.

Action: Same as for previous two messages.

END OF EXECUTION

Cause: Normal end of program.

Action: Program terminates and a STOP 0 may be printed in the HASP system log.

POSSIBLE LOSS IN NUMBER OF SIGNIFICANT FIGURES.

Cause: The subroutine WORK, which solves the set of linear equations given by equation (13) has returned an error code indicating that a loss in significant figures may have occurred.

Action: Program execution continues. Condition may also be indicated by the number of significant digits for the inverse being equal to zero.

COEFFICIENT MATRIX IS SINGULAR

Cause: The subroutine WORK, which solves the set of linear equations given by equation (13) has determined that the matrix (D_{ki}) given by equation (14) is singular. This means that the procedures used to calculate the inverse of the coefficient matrix may not be accurate.

Action: Same as for previous message.

THE FUNCTION F(EY) IS LESS THAN 1.0D-10 FOR THE nnnTH OBSERVATION.

Cause: The subroutine F2DEYF, which computes the matrix of second order partial derivatives of the likelihood function with respect to the coefficients $A_1, A_2, \ldots A_m$, has encountered the stated condition, where F(EY) denotes the weighting function $F_{\hat{\alpha}}(E\hat{Y}_j)$ for the final estimate of α and j=nnn. When this condition exists, the matrix of second order derivatives is usually unreliable. This condition can be overcome by insuring that the function F_α is chosen such that it is bounded below by at least 10^{-10}.

Action: The value of 10^{-10} is used in place of $F_{\hat{\alpha}}(E\hat{Y}_j)$ for the nnnth observation to prevent overflow conditions from taking place. For some data sets the overflow condition may still persist. If it does, the message shows the probable cause of the overflow.

COEFFICIENTS ARE NOT WITHIN TOLERANCE SPECIFIED.

Cause: See paragraph (p. 24) containing equation (51).

Action: Execution continues.

COEFFICIENTS ARE WITHIN TOLERANCE SPECIFIED.

Cause: See paragraph (p. 24) containing equation (51).

Action: Execution continues.

COEFFICIENTS ARE NOT A CRITICAL POINT OF THE LOG OF THE LIKELIHOOD FUNCTION.

Cause: See the section of this paper titled "Evaluation of regression coefficients" (p. 22) and in particular equations (49) and (50).

Action: Execution continues.

COEFFICIENTS ARE A SIMULTANEOUS SOLUTION OF THE PARTIAL DERIVATIVE EQUATIONS OF THE LOG OF THE LIKELIHOOD FUNCTION TO nn SIGNIFICANT DIGITS.

Cause: See the section of this paper titled "Evaluation of regression coefficients" (p. 22) and in particular equations (49) and (50).

Action: Execution continues.

POSSIBLE LOSS IN NUMBER OF SIGNIFICANT DIGITS

Cause: The subroutine WORK, which solves the set of linear equations given by equations (13), (14), and (15), has returned an error code indicating that the set of coefficients used to check the estimates of the coefficients may be subject to error. See equation (51).

Action: Execution continues.

COEFFICIENTS ARE A RELATIVE MAXIMUM WITHIN TOLERANCE SPECIFIED IF ALL OF THE EIGEN VALUES OF THE ABOVE MATRIX ARE NEGATIVE AND IF THE COEFFICIENTS ARE A SIMULTANEOUS SOLUTION OF THE PARTIAL DERIVATIVE EQUATIONS OF THE LOG OF THE LIKELIHOOD FUNCTION.

Cause: See "Evaluation of regression coefficients" (p. 22).

Action: Execution continues.

HIGHER ORDER DERIVATIVES WOULD HAVE TO BE ANALYZED TO DETERMINE IF THESE COEFFICIENTS ARE A RELATIVE MAXIMUM.

END OF EXECUTION

Cause: The determinant of the matrix of second order partial derivatives is equal to zero.

Action: Execution is terminated. STOP 0 may be printed in the HASP system log.

COMPUTED EIGEN VALUES OR VECTORS MAY NOT BE RELIABLE.

Cause: The minimum number of significant digits for the eigen values or vectors computed by the subroutine, EIGEN, is equal to zero.

ACTION: Execution is complete at this point. Normal end of execution may be indicated by a STOP 0 printed in the HASP system log.

MINIMUM NUMBER OF SIGNIFICANT DIGITS FOR EIGEN VALUES OR VECTORS = nn

Cause: EIGEN subroutine has a built-in checking procedure to determine accuracy of results.

Action: Execution continues.

In addition to the above-listed diagnostic messages, the standard IBM System/360 operating system completion codes and error messages may be written in the output. If any of these messages are written in the output, it usually is due to the input data not conforming to the input specifications. It may also be due to the iterative technique generating a set of values, which are not compatible with the mathematical model or are out of range of the computer's number system. If any error messages or completion codes are contained in the output then the results from the program may be in error. The user is warned to examine the output from this program closely for error messages.

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ATTACHMENTS

1. WEIGHTING FUNCTIONS PRESENTLY AVAILABLE

This attachment contains a listing of each of the sets of three FORTRAN functions of the type required by the Nonconstant Variance Regression Analysis program, which are presently available on the USGS IBM 360/65 computer. Each of these sets is stored in load module form with the member names W8251#2, W8251#3, W8251#4, and W8251#5 in the SYS1. LOADLIB library on the PERM01 disk pack. These functions were compiled using the IBM FORTRAN-IV (H Level) compiler. Subsequent sets will be stored as need and use dictate.

It is the user's responsibility to make sure that these functions possess the required properties for the particular data set being processed and in particular for the values of the variance parameter α used in the program.

MEMBER W8251#2

The library member W8251#2 contains the three FORTRAN functions EYF,DEYF, and DDEYF given below.

$$F_{\alpha}(EY_j)=Z+|EY_j|^{\alpha}$$ (See note, p. 57.)

```
      FUNCTION EYF (X,A,ALPHA,J,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,EYF,Z
      DATA SWITCH/0/
      IF (SWITCH) 3,1,3
1     SWITCH=1
      READ(5,2) Z
2     FORMAT (E12.5)
3     CONTINUE
      EYF=0
      DO 4 I=1,NX
4     EYF=EYF+A(I)*X(I,J)
      IF (ALPHA) 5,6,5
5     EYF=Z+DABS(EYF)**ALPHA
      RETURN
6     EYF=1+Z
      RETURN
      END
```

$$\frac{\partial F_{\alpha}(EY_j)}{\partial A_i}=X_{ij}\,\mathrm{sgn}(EY_j)\,\alpha|EY_j|^{\alpha-1}$$

```
      FUNCTION DEYF (X,A,ALPHA,J,I,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,DEYF,Z1
      DEYF=0
      Z1=1
      DO 1 I1=1,NX
1     DEYF=DEYF+A(I1)*X(I1,J)
      IF (DEYF) 2,3,3
2     Z1=-1
3     CONTINUE
      IF (ALPHA-1) 4,5,4
4     DEYF=ALPHA*X(I,J)*(DABS(DEYF))**(ALPHA-1)
      DEYF=DEYF*Z1
      RETURN
5     DEYF=X(I,J)*Z1
      RETURN
      END
```

$$\frac{\partial^2 F_\alpha(EY_j)}{\partial A_i \partial A_k} = X_{ij} X_{kj} \alpha(\alpha-1) |EY_j|^{\alpha-2}$$

```
      FUNCTION DDEYF (X,A,ALPHA,J,I,K,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,DDEYF
      DDEYF=0
      DO 1 I1=1,NX
1     DDEYF=DDEYF+A(I1)*X(I1,J)
      IF (ALPHA-2) 2,3,2
2     DDEYF=(ALPHA-1)*ALPHA*X(K,J)*X(I,J)*(DABS(DDEYF))
     1 **(ALPHA-2)
      RETURN
3     DDEYF=X(K,J)*X(I,J)*ALPHA*(ALPHA-1)
      RETURN
      END
```

Note: The value of Z is arbitrary and must be provided by the user in columns 1-12 of a data card. It should be punched in the format E12.5 and the card should be placed at the end of the set of input cards immediately preceding the last /* control card.

MEMBER W8251#3

The library member W8251#3 contains the three FORTRAN functions EYF, DEYF, AND DDEYF given below.

$$F_\alpha(EY_j) = (Z+|EY_j|)^\alpha \quad \text{(See note above.)}$$

```
      FUNCTION EYF (X,A,ALPHA,J,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,EYF,Z
      COMMON Z
      DATA SWITCH/0/
      IF (SWITCH) 3,1,3
1     SWITCH=1
      READ (5,2) Z
2     FORMAT (E12.5)
3     CONTINUE
      EYF=0
      DO 4 I=1,NX
4     EYF=EYF+A(I)*X(I,J)
      IF (ALPHA) 5,6,5
5     EYF=(Z+DABS(EYF))**ALPHA
      RETURN
6     EYF=1
      RETURN
      END
```

$$\frac{\partial F_\alpha(EY_j)}{\partial A_i} = X_{ij}\,\mathrm{sgn}(EY_j)\,\alpha(Z+|EY_j|)^{\alpha-1}$$

```
      FUNCTION DEYF (X,A,ALPHA,J,I,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,DEYF,Z1,Z
      COMMON Z
      DEYF=0
      DO 1 I1=1,NX
1     DEYF=DEYF+A(I1)*X(I1,J)
      Z1=1
      IF (DEYF) 2,3,3
2     Z1=-1
3     CONTINUE
      IF (ALPHA-1) 4,5,4
4     DEYF=ALPHA*X(I,J)*(DABS(DEYF)+Z)**(ALPHA-1)
      DEYF=DEYF*Z1
      RETURN
5     DEYF=X(I,J)*Z1
      RETURN
      END
```

$$\frac{\partial^2 F_\alpha(EY_j)}{\partial A_i\,\partial A_k} = (\alpha-1)\,\alpha X_{ij} X_{kj}(Z+|EY_j|)^{\alpha-2}$$

```
      FUNCTION DDEYF (X,A,ALPHA,J,I,K,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,DDEYF,Z
      COMMON Z
      DDEYF=0
      DO 1 I1=1,NX
1     DDEYF=DDEYF+A(I1)*X(I1,J)
      IF (ALPHA-2) 2,3,2
2     DDEYF=(ALPHA-1)*ALPHA*X(K,J)*X(I,J)*(DABS(DDEYF)+Z)
     1**(ALPHA-2)
      RETURN
3     DDEYF=X(K,J)*X(I,J)*ALPHA*(ALPHA-1)
      RETURN
      END
```

MEMBER W8251#4

The library member W8251#4 contains the three FORTRAN functions EYF, DEYF, and DDEYF given below.

$$F_\alpha(EY_j)=(Z+(EY_j-\alpha)^2)^{-1}$$ (See note, p. 57.)

```
      FUNCTION EYF (X,A,ALPHA,J,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,EYF,Z
      COMMON Z
      DATA SWITCH/0/
      IF (SWITCH) 3,1,3
1     SWITCH=1
      READ (5,2) Z
2     FORMAT (E12.5)
3     CONTINUE
      EYF=0
      DO 4 I=1,NX
4     EYF=EYF+A(I)*X(I,J)
      EYF=1/(Z+(EYF-ALPHA)**2)
      RETURN
      END
```

$$\frac{\partial F\alpha(EY_j)}{\partial A_i}=\frac{-2(EY_j-\alpha)X_{ij}}{[Z+(EY_j-\alpha)^2]^2}$$

```
      FUNCTION DEYF (X,A,ALPHA,J,I,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,DEYF,DX,Z
      COMMON Z
      DEYF=0
      DO 1 I1=1,NX
1     DEYF=DEYF+A(I1)*X(I1,J)
      DX=DEYF-ALPHA
      DEYF=(-2*DX*X(I,J))/(Z+DX**2)**2
      RETURN
      END
```

$$\frac{\partial^2 F_\alpha(EY_j)}{\partial A_i \partial A_k} = \frac{X_{ij}X_{kj}[-2Z+6(EY_j-\alpha)^2]}{[Z+(EY_j-\alpha)^2]^3}$$

```
      FUNCTION DDEYF (X,A,ALPHA,J,I,K,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,DDEYF,Z,F
      COMMON Z
      DDEYF=0
      DO 1 I1=1,NX
1     DDEYF=DDEYF+A(I1)*X(I1,J)
      DDEYF=DDEYF-ALPHA
      F=Z+DDEYF**2
      DDEYF=X(I,J)*X(K,J)*(-2*F+8*DDEYF**2)/F**3
      RETURN
      END
```

MEMBER W8251#5

The library member W8251#5 contains the three FORTRAN functions EYF, DEYF, and DDEYF given below.

$F_\alpha(EY_j)=W_j$ (See note, p. 61.)

```
      FUNCTION EYF (X,A,ALPHA,J,NX,NY)
      DOUBLE PRECISION X(10,500),A(1),ALPHA,EYF,W(500)
      DATA SWITCH/0/
      IF (SWITCH) 3,1,3
1     READ (5,2) (W(J),J=1,NY)
2     FORMAT (6E12.5)
      SWITCH=1
3     EYF=W(J)
      RETURN
      END
```

$$\frac{\partial F_{\alpha}(EY_j)}{\partial A_i}=0$$

```
FUNCTION DEYF (X,A,ALPHA,J,I,NX,NY)
DOUBLE PRECISION X(10,500),A(1),ALPHA,DEYF
DEYF=0
RETURN
END
```

$$\frac{\partial^2 F_{\alpha}(EY_j)}{\partial A_i \partial A_k}=0$$

```
FUNCTION DDEYF (X,A,ALPHA,J,I,K,NX,NY)
DOUBLE PRECISION X(10,500),A(1),ALPHA,DDEYF
DDEYF=0
RETURN
END
```

Note: The numbers W_j are known positive weights read in from the card reader. There should be one value of W_j for each observation. The cards should be punched in the format (6E12.5) and placed at the end of the set of input cards immediately preceding the last /* control card.

2. SAMPLE OUTPUT

The following set of output typifies that produced by this program. Some variation in the output will occur as the result of choosing different parameters and of course the results may look very different for different data sets.

UNITED STATES GEOLOGICAL SURVEY
COMPUTER CENTER DIVISION

NONCONSTANT VARIANCE REGRESSION ANALYSIS PROGRAM

PROGRAM NO. W8251

DESIGNED AND PROGRAMMED BY

MARSHALL STRONG HELLMANN

```
EQUIPMENT:  IBM SYSTEM 360-65
LANGUAGE:   FORTRAN-IV H-LEVEL
DATE:       DECEMBER 18, 1968
REFERENCE:  PAMPHLET TITLED "NON-CONSTANT VARIANCE REGRESSION ANALYSIS", BY MARSHALL STRONG HELLMANN, 1967, LIBRARY OF CONGRESS
            CATALOG CARD NUMBER 67-23725
```

```
                                        INPUT DATA AND PARAMETERS

          NUMBER OF INDEPENDENT VARIABLES =    5

          NUMBER OF OBSERVATIONS =    50

          CONVERGENCE TOLERANCE =  0.100D-07% OF EACH COEFFICIENT.

          LOWER BOUND FOR WEIGHT FUNCTION =  0.100D-14

          NUMBER OF PARAMETERS GIVEN FOR WEIGHT FUNCTION =    6

          MAXIMUM NUMBER OF ITERATIONS =    30

          VARIANCE PARAMETER FOR CURVE FITTING FUNCTION =  0.20000D 00

          SIZE OF CRITICAL REGIONS = 0.05

          DATA FORMAT INDICATOR = 0

          COEFFICIENTS INDICATOR = 0

          FORTRAN DATA SET REFERENCE NUMBER =  9

          SYMMETRIC COEFFICIENT EQUATIONS SWITCH = 0


          VALUES OF GIVEN PARAMETERS OF WEIGHT FUNCTION
 0.5000000000000000D 00 0.6000000000000000D 00 0.7000000000000000D 00 0.9999999999999999D 00 0.4000000000000000D 00 0.3000000000000000D 00

          NAMES OF DEPENDENT VARIABLES
OBS.-001  OBS.-002  OBS.-003  OBS.-004  OBS.-005  OBS.-006  OBS.-007  OBS.-008  OBS.-009  OBS.-010  OBS.-011  OBS.-012  OBS.-013
OBS.-014  OBS.-015  OBS.-016  OBS.-017  OBS.-018  OBS.-019  OBS.-020  OBS.-021  OBS.-022  OBS.-023  OBS.-024  OBS.-025  OBS.-026
OBS.-027  OBS.-028  OBS.-029  OBS.-030  OBS.-031  OBS.-032  OBS.-033  OBS.-034  OBS.-035  OBS.-036  OBS.-037  OBS.-038  OBS.-039
OBS.-040  OBS.-041  OBS.-042  OBS.-043  OBS.-044  OBS.-045  OBS.-046  OBS.-047  OBS.-048  OBS.-049  OBS.-050

          DEPENDENT VARIABLE          Y TEST

-0.9921426735978270 01-0.9460491270252710 01-0.7755246206892970 01-0.7640675463414360 01-0.4803921347476170 01-0.5138730956871900 01
-0.4424974695298560 01-0.3415597108464370 01-0.3411183784928300 01-0.4142958521224590 01-0.4417486769119610 01-0.4623043930963410 01
-0.5145785897120550 01-0.5322421307929090 01-0.3609516782903150 01-0.4970741338889630 01-0.3853615981239460 01-0.3518276345560900 01
-0.3203610385951990 01-0.1550551131311850 01-0.1759963209908410 01-0.1694090498800840 01-0.1971617336301820 01-0.2714574858904860 01
-0.3780424374943350 01-0.6258319054463710 01-0.6879562916834850 01-0.7502486222555360 01-0.8207756553305490 01-0.7676927034136780 01
-0.7308023640358060 01-0.4876117443580890 01-0.4624055492942200 01-0.1895373154584690 01-0.5479667497483850-01 0.1326071217601200 01
 0.2688000159509230 01 0.6881200071761510 01 0.7780870457268340 01 0.7456818434904670 01 0.1011880558675230 02 0.1071448195693190 02
 0.1072206637165950 02 0.7615375261792600 01 0.1031130064957800 02 0.7863510257338220 01 0.6067155655140830 01 0.3533025197174260 01
 0.2195400566201480 01-0.1695371942302080 01
```

```
        INDEPENDENT VARIABLES

        VARIABLE  1            TEST 1
0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00
0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00
0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00
0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00
0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00
0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00
0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00
0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00 0.999999999999999D 00
0.999999999999999D 00 0.999999999999999D 00
        VARIABLE  2            TEST 2
0.210573768615723D 01 0.242073535919189D 01 0.261678123474121D 01 0.267939281463623D 01 0.262211036682129D 01 0.248477268218994D 01
0.212460784912109D 01 0.220273590087891D 01 0.216954612731934D 01 0.225258636474609D 01 0.245000457763672D 01 0.273112010955811D 01
0.304405117034912D 01 0.332849311828613D 01 0.353055095672607D 01 0.361599826812744D 01 0.357869243621826D 01 0.344212532043457D 01
0.325371074676514D 01 0.307319545745850D 01 0.295806789398193D 01 0.294949340820312D 01 0.306227302551269D 01 0.328122329711914D 01
0.356491279602051D 01 0.385583209991455D 01 0.409447193145752D 01 0.423391628265381D 01 0.425132846832275D 01 0.415356826782227D 01
0.397551441192627D 01 0.377152252197266D 01 0.360214519500732D 01 0.351930046081543D 01 0.355356788635254D 01 0.370671081542969D 01
0.395125675201416D 01 0.423721504211426D 01 0.450433063507080D 01 0.469684123992920D 01 0.477713298797607D 01 0.473495197296143D 01
0.459992004394531D 01 0.438673591613769D 01 0.418423938751221D 01 0.404098987579346D 01 0.400095653533936D 01 0.408274173736572D 01
0.427503490447998D 01 0.453937244415283D 01
        VARIABLE  3            TEST 3
0.255146217346191D 01 0.290295696258545D 01 0.319397258758545D 01 0.341179084777832D 01 0.354922199249268D 01 0.360619640350342D 01
0.359023952484131D 01 0.351592922210693D 01 0.340347862243652D 01 0.327665424346924D 01 0.316025257110596D 01 0.307745265960693D 01
0.304732227325439D 01 0.308276462554932D 01 0.318914222717285D 01 0.336371231079102D 01 0.359595584869385D 01 0.386877059936523D 01
0.416035938262939D 01 0.444665431976318D 01 0.470397472381592D 01 0.491166400909424D 01 0.505435085296631D 01 0.512367057800293D 01
0.511919403076172D 01 0.504847431182861D 01 0.492622852325439D 01 0.477271938323975D 01 0.461154174804687D 01 0.446702671051025D 01
0.436155414581299D 01 0.431312084197998D 01 0.433333873748779D 01 0.442617416381836D 01 0.458751010894775D 01 0.480561351776123D 01
0.506241321563721D 01 0.533549976348877D 01 0.560060310363769D 01 0.583425807952881D 01 0.601639270782471D 01 0.613253879547119D 01
0.617542934417725D 01 0.614580821990967D 01 0.605232524871826D 01 0.591061496734619D 01 0.574156475067139D 01 0.556901550292969D 01
0.541713523864746D 01 0.530777645111084D 01
        VARIABLE  4            TEST 4
0.286260318756103D 01 0.319668579101562D 01 0.349554157257080D 01 0.375242233276367D 01 0.396173667907715D 01 0.411982727050781D 01
0.422545623779297D 01 0.428004646301269D 01 0.428771686553955D 01 0.425509643554687D 01 0.419098186492920D 01 0.410583972930908D 01
0.401118087768555D 01 0.391889190673828D 01 0.384050273895264D 01 0.378649806976318D 01 0.376568698883057D 01 0.378466224670410D 01
0.384740352630615D 01 0.395503902435303D 01 0.410579776763916D 01 0.429510879516602D 01 0.451590156555176D 01 0.475906085968018D 01
0.501398563385010D 01 0.526925849914551D 01 0.551335525512695D 01 0.573537635803223D 01 0.592573165893555D 01 0.607671070098877D 01
0.618300628662109D 01 0.624201774597168D 01 0.625402450561523D 01 0.622217178344726D 01 0.615228843688965D 01 0.605250358581543D 01
0.593275833129883D 01 0.580418968200684D 01 0.567843818664551D 01 0.556691932678223D 01 0.548012638092041D 01 0.542697143554687D 01
0.541424083709717D 01 0.544615840911865D 01 0.552413940429687D 01 0.564670753479004D 01 0.580957126617432D 01 0.600589275360107D 01
0.622673988342285D 01 0.646160697937012D 01
        VARIABLE  5            TEST 5
0.311852550506592D 01 0.342813396453857D 01 0.371445369720459D 01 0.397388362884521D 01 0.420299625396728D 01 0.439892768859863D 01
0.455963706970215D 01 0.468409824371338D 01 0.477240276336670D 01 0.482582283020019D 01 0.484679603576660D 01 0.483885192871094D 01
0.480651187896728D 01 0.475510973698730D 01 0.469060230255127D 01 0.461935806274414D 01 0.454789733886719D 01 0.448264884948730D 01
0.442969512939453D 01 0.439455699920654D 01 0.438195419311523D 01 0.439563560485840D 01 0.443824481964111D 01 0.451119518280029D 01
0.461463528222656D 01 0.474744606018066D 01 0.490726661682129D 01 0.509063529968262D 01 0.529307556152344D 01 0.550932884216309D 01
0.573352718353271D 01 0.595944786071777D 01 0.618075942993164D 01 0.639128494262695D 01 0.658523178100586D 01 0.675743198394775D 01
0.690355587005615D 01 0.702028465270996D 01 0.710543251037598D 01 0.715804672241211D 01 0.717844295501709D 01 0.716820144653320D 01
0.713007831573486D 01 0.706792926788330D 01 0.698654365539551D 01 0.689145374298096D 01 0.678869819641113D 01 0.668462371826172D 01
0.658558845520019D 01 0.649773693084717D 01

END OF INITIALIZATION

BEGIN EXECUTION
```

INTERMEDIATE RESULTS

```
E(Y) =  -0.1982037001191990 02X(1)  -0.1845918287532220 01X(2) + 0.3752204137017200 01X(3)
        -0.4841919785213860 01X(4) + 0.5903054212573250 01X(5)
STANDARD DEVIATION OF THE RATIOS IS  0.298990 00     LOGARITHM OF JOINT FREQUENCY FUNCTION RATIO =  0.0
NUMBER OF ITERATIONS =   21     VARIANCE PARAMETER =  0.500000 00

E(Y) =  -0.1988890919630540 02X(1)  -0.1877904017468150 01X(2) + 0.3783965442853300 01X(3)
        -0.4859394900564330 01X(4) + 0.5927825152643500 01X(5)
STANDARD DEVIATION OF THE RATIOS IS  0.259250 00     LOGARITHM OF JOINT FREQUENCY FUNCTION RATIO =  0.601210-01
NUMBER OF ITERATIONS =   24     VARIANCE PARAMETER =  0.600000 00

ITERATION TECHNIQUE FAILED TO CONVERGE WITHIN TOLERANCE SPECIFIED.  VARIANCE PARAMETER =  0.700000 00
E(Y) =  -0.1997144265253410 02X(1)  -0.1903276271020730 01X(2) + 0.3820633179390100 01X(3)
        -0.4889566691951820 01X(4) + 0.5957605839663920 01X(5)
STANDARD DEVIATION OF THE RATIOS IS  0.226660 00     LOGARITHM OF JOINT FREQUENCY FUNCTION RATIO =  0.353880 00
NUMBER OF ITERATIONS =   31     VARIANCE PARAMETER =  0.700000 00

ITERATION TECHNIQUE FAILED TO CONVERGE WITHIN TOLERANCE SPECIFIED.  VARIANCE PARAMETER =  0.100000 01
E(Y) =  -0.2039635151761710 02X(1)  -0.1981809184899130 01X(2) + 0.3962333463070740 01X(3)
        -0.5063386295079480 01X(4) + 0.6128599874639980 01X(5)
STANDARD DEVIATION OF THE RATIOS IS  0.158860 00     LOGARITHM OF JOINT FREQUENCY FUNCTION RATIO =  0.263360 01
NUMBER OF ITERATIONS =   31     VARIANCE PARAMETER =  0.100000 01

E(Y) =  -0.1975467474853090 02X(1)  -0.1804899270382650 01X(2) + 0.3726412994545520 01X(3)
        -0.4838640028656660 01X(4) + 0.5879847496351300 01X(5)
STANDARD DEVIATION OF THE RATIOS IS  0.347470 00     LOGARITHM OF JOINT FREQUENCY FUNCTION RATIO =  0.121920 00
NUMBER OF ITERATIONS =   18     VARIANCE PARAMETER =  0.400000 00

E(Y) =  -0.1969491768608020 02X(1)  -0.1769777228472690 01X(2) + 0.3702829128774440 01X(3)
        -0.4841344000340750 01X(4) + 0.5864720688979740 01X(5)
STANDARD DEVIATION OF THE RATIOS IS  0.406880 00     LOGARITHM OF JOINT FREQUENCY FUNCTION RATIO =  0.369790 00
NUMBER OF ITERATIONS =   13     VARIANCE PARAMETER =  0.300000 00
```

RESULTS OF WEIGHTED CURVE FITTING TECHNIQUE

VARIABLE	OBSERVED	ESTIMATED	DIFFERENCE	WEIGHT	RATIO	DEVIATION
0.50000D 00	0.0	-0.31795D-02	-0.31795D-02	0.52375D 00	-0.60708D-02	0.22201D-02
0.60000D 00	0.60121D-01	0.97405D-01	0.37284D-01	0.24918D 01	0.14963D-01	0.10563D-01
0.70000D 00	0.35388D 00	0.40748D 00	0.53594D-01	0.33013D 01	0.16234D-01	0.13994D-01
0.10000D 01	0.26336D 01	0.25946D 01	-0.38996D-01	0.47742D 01	-0.81681D-02	0.20237D-01
0.40000D 00	0.12192D 00	0.10572D 00	-0.16193D-01	0.25317D 01	-0.63960D-02	0.10732D-01
0.30000D 00	0.36979D 00	0.42411D 00	0.54322D-01	0.33276D 01	0.16325D-01	0.14105D-01

RATIO = 0.26362D 01 + -0.10516D 02ALPHA + 0.10474D 02ALPHA**2
NUMBER OF ITERATIONS = 8 VARIANCE PARAMETER = 0.20000D 00

ESTIMATE OF VARIANCE PARAMETER ALPHA = 0.50199D 00

NONCONSTANT VARIANCE REGRESSION ANALYSIS RESULTS

VARIABLE	OBSERVED	ESTIMATED	DIFFERENCE	WEIGHT	RATIO	DEVIATION
OBS.-001	-0.99214D 01	-0.95860D 01	0.33548D 00	0.31100D 01	0.10787D 00	0.97718D 00
OBS.-002	-0.94605D 01	-0.86384D 01	0.82204D 00	0.29517D 01	0.27849D 00	0.87999D 00
OBS.-003	-0.77552D 01	-0.76652D 01	0.90085D-01	0.27798D 01	0.32407D-01	0.82873D 00
OBS.-004	-0.76407D 01	-0.66757D 01	0.96502D 00	0.25935D 01	0.37209D 00	0.77318D 00
OBS.-005	-0.48039D 01	-0.57151D 01	-0.91115D 00	0.23989D 01	-0.37982D 00	0.71517D 00
OBS.-006	-0.51387D 01	-0.48564D 01	0.28228D 00	0.22107D 01	0.12769D 00	0.65905D 00
OBS.-007	-0.44250D 01	-0.41833D 01	0.24170D 00	0.20511D 01	0.11784D 00	0.61149D 00
OBS.-008	-0.34156D 01	-0.37667D 01	-0.35106D 00	0.19459D 01	-0.18041D 00	0.58012D 00
OBS.-009	-0.34112D 01	-0.36432D 01	-0.23201D 00	0.19136D 01	-0.12124D 00	0.57050D 00
OBS.-010	-0.41430D 01	-0.37992D 01	0.34379D 00	0.19543D 01	0.17591D 00	0.58263D 00
OBS.-011	-0.44175D 01	-0.41663D 01	0.25119D 00	0.20469D 01	0.12271D 00	0.61024D 00
OBS.-012	-0.46230D 01	-0.46308D 01	-0.77413D-02	0.21585D 01	-0.35864D-02	0.64350D 00
OBS.-013	-0.51458D 01	-0.50543D 01	0.91476D-01	0.22554D 01	0.40558D-01	0.67240D 00
OBS.-014	-0.53224D 01	-0.53032D 01	0.19267D-01	0.23105D 01	0.83390D-02	0.68882D 00
OBS.-015	-0.36095D 01	-0.52783D 01	-0.16688D 01	0.23051D 01	-0.72398D 00	0.68720D 00
OBS.-016	-0.49707D 01	-0.49401D 01	0.30643D-01	0.22297D 01	0.13743D-01	0.66473D 00
OBS.-017	-0.38536D 01	-0.43208D 01	-0.46713D 00	0.20847D 01	-0.22408D 00	0.62150D 00
OBS.-018	-0.35183D 01	-0.35218D 01	-0.35455D-02	0.18813D 01	-0.18846D-02	0.56088D 00
OBS.-019	-0.32036D 01	-0.26960D 01	0.50758D 00	0.16452D 01	0.30852D 00	0.49047D 00
OBS.-020	-0.15506D 01	-0.20169D 01	-0.46635D 00	0.14222D 01	-0.32792D 00	0.42398D 00
OBS.-021	-0.17600D 01	-0.16430D 01	0.11694D 00	0.12831D 01	0.91138D-01	0.38252D 00
OBS.-022	-0.16941D 01	-0.16837D 01	0.10411D-01	0.12989D 01	0.80156D-02	0.38724D 00
OBS.-023	-0.19716D 01	-0.21740D 01	-0.20242D 00	0.14767D 01	-0.13707D 00	0.44025D 00
OBS.-024	-0.27146D 01	-0.30650D 01	-0.35039D 00	0.17546D 01	-0.19970D 00	0.52309D 00
OBS.-025	-0.37804D 01	-0.42293D 01	-0.44891D 00	0.20624D 01	-0.21766D 00	0.61486D 00
OBS.-026	-0.62583D 01	-0.54840D 01	0.77433D 00	0.23497D 01	0.32954D 00	0.70051D 00
OBS.-027	-0.68796D 01	-0.66219D 01	0.25768D 00	0.25830D 01	0.99762D-01	0.77005D 00
OBS.-028	-0.75025D 01	-0.74480D 01	0.54486D-01	0.27400D 01	0.19885D-01	0.81686D 00
OBS.-029	-0.82078D 01	-0.78116D 01	0.39613D 00	0.28064D 01	0.14115D 00	0.83665D 00
OBS.-030	-0.76769D 01	-0.76278D 01	0.49101D-01	0.27730D 01	0.17707D-01	0.82671D 00
OBS.-031	-0.73080D 01	-0.68860D 01	0.42206D 00	0.26342D 01	0.16022D 00	0.78532D 00
OBS.-032	-0.48761D 01	-0.56430D 01	-0.76691D 00	0.23837D 01	-0.32173D 00	0.71063D 00
OBS.-033	-0.46241D 01	-0.40060D 01	0.61807D 00	0.20070D 01	0.30795D 00	0.59834D 00
OBS.-034	-0.18954D 01	-0.21075D 01	-0.21215D 00	0.14539D 01	-0.14592D 00	0.43344D 00
OBS.-035	-0.54797D-01	-0.81995D-01	-0.27198D-01	0.28493D 00	-0.95455D-01	0.84944D-01
OBS.-036	0.13261D 01	0.19535D 01	0.62738D 00	0.13995D 01	0.44828D 00	0.41723D 00
OBS.-037	0.26880D 01	0.39080D 01	0.12200D 01	0.19822D 01	0.61549D 00	0.59095D 00
OBS.-038	0.68812D 01	0.57165D 01	-0.11647D 01	0.23992D 01	-0.48547D 00	0.71526D 00
OBS.-039	0.77809D 01	0.73296D 01	-0.45124D 00	0.27181D 01	-0.16601D 00	0.81032D 00
OBS.-040	0.74568D 01	0.87016D 01	0.12448D 01	0.29625D 01	0.42017D 00	0.88321D 00
OBS.-041	0.10119D 02	0.97775D 01	-0.34132D 00	0.31411D 01	-0.10866D 00	0.93644D 00
OBS.-042	0.10714D 02	0.10488D 02	-0.22631D 00	0.32537D 01	-0.69554D-01	0.97001D 00
OBS.-043	0.10722D 02	0.10754D 02	0.31475D-01	0.32948D 01	0.95532D-02	0.98225D 00
OBS.-044	0.76154D 01	0.10496D 02	0.28808D 01	0.32549D 01	0.88505D 00	0.97038D 00
OBS.-045	0.10311D 02	0.96612D 01	-0.65009D 00	0.31223D 01	-0.20821D 00	0.93083D 00
OBS.-046	0.78635D 01	0.82391D 01	0.37558D 00	0.28824D 01	0.13030D 00	0.85932D 00
OBS.-047	0.60672D 01	0.62834D 01	0.21622D 00	0.25158D 01	0.85946D-01	0.75003D 00
OBS.-048	0.35330D 01	0.39198D 01	0.38677D 00	0.19852D 01	0.19482D 00	0.59184D 00
OBS.-049	0.21954D 01	0.13407D 01	-0.85471D 00	0.11586D 01	-0.73774D 00	0.34539D 00
OBS.-050	-0.16954D 01	-0.12137D 01	0.48163D 00	0.11021D 01	0.43700D 00	0.32857D 00

E(Y) = -0.1982168469387130D 02X(1) -0.1846671649331940D 01X(2) + 0.3752773104521410D 01X(3)

-0.4842118343267060D 01X(4) + 0.5903541233820730D 01X(5)

STANDARD DEVIATION OF THE RATIOS IS 0.29813D 00 . LOGARITHM OF JOINT FREQUENCY FUNCTION RATIO = -0.89015D-03
NUMBER OF ITERATIONS = 21 VARIANCE PARAMETER = 0.50149D 00

COEFFICIENT EQUATIONS AND SOLUTIONS

COEFFICIENT MATRIX

C(K,1)	D(K,2)	D(K,3)	D(K,4)	D(K,5)
0.227367781752849D 02	0.799804746734800D 02	0.103697589690399D 03	0.125574004824250D 03	0.134722362965330D 03
0.799804746734800D 02	0.288070693581039D 03	0.370980785629447D 03	0.449604311789206D 03	0.482253788720777D 03
0.103697589690399D 03	0.370980785629447D 03	0.482709360902291D 03	0.580887998653932D 03	0.622908558516082D 03
0.125574004824250D 03	0.449604311789206D 03	0.580887998653932D 03	0.714114389770918D 03	0.764100382563762D 03
0.134722362965330D 03	0.482253788720777D 03	0.622908558516082D 03	0.764100382563762D 03	0.821053663359480D 03

VECTOR OF CONSTANT TERMS

EQUATION 1 -0.219305647422371D 02
EQUATION 2 -0.551451773191452D 02
EQUATION 3 -0.644039922888953D 02
EQUATION 4 -0.863472779289673D 02
EQUATION 5 -0.760944360243780D 02

INVERSE OF COEFFICIENT MATRIX

D'(K,1)	D'(K,2)	D'(K,3)	D'(K,4)	D'(K,5)
0.245189386647772D 01	-0.107917663982042D 00	-0.304426537398209D 00	-0.113443607170484D-01	-0.974152004035651D-01
-0.107917663982033D 00	0.454760516188730D 00	-0.209908991644920D 00	-0.312717230144299D-01	-0.610462123344471D-01
-0.304426537398217D 00	-0.209908991644915D 00	0.252134196202009D 00	-0.236010713657661D-01	0.392119062288582D-02
-0.113443607170626D-01	-0.312717230144145D-01	-0.236010713657722D-01	0.349847923972636D 00	-0.287445739236350D 00
-0.974152004035509D-01	-0.610462123344638D-01	0.392119062289320D-02	-0.287445739236348D 00	0.317590239137064D 00

MINIMUM NUMBER OF SIGNIFICANT DIGITS FOR INVERSE = 14

COEFFICIENT CHECK

```
A( 1) = -0.198216846939241D 02
A( 2) = -0.184667164945670D 01
A( 3) =  0.375277310449609D 01
A( 4) = -0.484211834321621D 01
A( 5) =  0.590354123389041D 01
```

JOINT FREQUENCY FUNCTION RATIOS

RATIO	VARIANCE PARAMETER
1.000000D 0	0.500000D 00
1.14847D 0	0.600000D 00
2.25882D 0	0.700000D 00
4.30130D 2	0.100000D 01
1.32408D 0	0.400000D 00
2.34309D 0	0.300000D 00
0.99795D 0	0.50199D 00

LINEAR RELATIONSHIPS BETWFEN VARIABLES

ARITHMETICAL CORRELATION MATRIX FOR INDEPENDENT VARIABLFS

	X(1)	X(2)	X(3)	X(4)	X(5)
X(1)	1.0000	0.26194D-14	0.19273D-14	0.20713D-14	0.32222D-14
X(2)	0.26194D-14	1.0000	0.81767	0.75750	0.79978
X(3)	0.19273D-14	0.81767	1.0000	0.72278	0.79237
X(4)	0.20713D-14	0.75750	0.72278	1.0000	0.82928
X(5)	0.32222D-14	0.79978	0.79237	0.82928	1.000D

ARITHMETICAL CORRFLATION BETWEEN THE ESTIMATED VALUES AND THE INDEPENDFNT VARIABLES

X(1)	0.41914D-15
X(2)	0.62324
X(3)	0.80837
X(4)	0.45429
X(5)	0.83401

CONFIDENCE INTERVALS

COVARIANCE MATRIX V(I,K) TO BE USED FOR COMPUTING CONFIDENCE INTERVALS

```
 0.24977D 01-0.14399D 00-0.30112D 00-0.20098D-01-0.78590D-01
-0.14399D 00 0.45769D 00-0.20625D 00-0.19397D-01-0.69434D-01
-0.30112D 00-0.20625D 00 0.24695D 00-0.25177D-01 0.67666D-02
-0.20098D-01-0.19397D-01-0.25177D-01 0.34697D 00-0.28920D 00
-0.78590D-01-0.69434D-01 0.67666D-02-0.28920D 00 0.31813D 00
```

APPROXIMATE 95% CONFIDENCE INTERVALS FOR REGRESSION COEFFICIENTS

COEFFICIENT	CONFIDENCE INTERVAL
A(1)	(-0.20822D 02,-0.18821D 02)
A(2)	(-0.22750D 01,-0.14184D 01)
A(3)	(0.34382D 01, 0.40674D 01)
A(4)	(-0.52150D 01,-0.44692D 01)
A(5)	(0.55465D 01, 0.62606D 01)

APPROXIMATE 95% CONFIDENCE INTERVALS FOR THE EXPECTED VALUE OF Y

VARIABLE	LOWER LIMIT	ESTIMATED	UPPER LIMIT
OBS.-001	-0.10030D 02	-0.95860D 01	-0.91215D 01
OBS.-002	-0.90438D 01	-0.86384D 01	-0.82331D 01
OBS.-003	-0.80202D 01	-0.76652D 01	-0.73101D 01
OBS.-004	-0.69859D 01	-0.66757D 01	-0.63654D 01
OBS.-005	-0.59975D 01	-0.57151D 01	-0.54326D 01
OBS.-006	-0.51458D 01	-0.48564D 01	-0.45671D 01
OBS.-007	-0.45081D 01	-0.41833D 01	-0.38585D 01
OBS.-008	-0.41266D 01	-0.37667D 01	-0.34067D 01
OBS.-009	-0.40150D 01	-0.36432D 01	-0.32714D 01
OBS.-010	-0.41571D 01	-0.37992D 01	-0.34412D 01
OBS.-011	-0.45073D 01	-0.41663D 01	-0.38253D 01
OBS.-012	-0.49887D 01	-0.46308D 01	-0.42729D 01
OBS.-013	-0.54719D 01	-0.50543D 01	-0.46368D 01
OBS.-014	-0.57904D 01	-0.53032D 01	-0.48159D 01
OBS.-015	-0.58074D 01	-0.52783D 01	-0.47493D 01
OBS.-016	-0.54614D 01	-0.49401D 01	-0.44188D 01
OBS.-017	-0.47829D 01	-0.43208D 01	-0.38586D 01
OBS.-018	-0.38935D 01	-0.35218D 01	-0.31501D 01
OBS.-019	-0.29912D 01	-0.26960D 01	-0.24009D 01
OBS.-020	-0.23027D 01	-0.20169D 01	-0.17311D 01
OBS.-021	-0.19752D 01	-0.16430D 01	-0.13108D 01
OBS.-022	-0.20614D 01	-0.16837D 01	-0.13060D 01
OBS.-023	-0.25690D 01	-0.21740D 01	-0.17791D 01
OBS.-024	-0.34558D 01	-0.30650D 01	-0.26741D 01
OBS.-025	-0.46224D 01	-0.42293D 01	-0.38363D 01
OBS.-026	-0.59074D 01	-0.54840D 01	-0.50605D 01
OBS.-027	-0.70944D 01	-0.66219D 01	-0.61493D 01
OBS.-028	-0.79597D 01	-0.74480D 01	-0.69363D 01
OBS.-029	-0.83298D 01	-0.78116D 01	-0.72935D 01
OBS.-030	-0.81121D 01	-0.76278D 01	-0.71435D 01
OBS.-031	-0.73035D 01	-0.68860D 01	-0.64684D 01
OBS.-032	-0.59779D 01	-0.56430D 01	-0.53081D 01
OBS.-033	-0.42618D 01	-0.40060D 01	-0.37501D 01
OBS.-034	-0.23025D 01	-0.21075D 01	-0.19126D 01
OBS.-035	-0.24552D 00	-0.81995D-01	0.81531D-01
OBS.-036	0.17725D 01	0.19535D 01	0.21344D 01
OBS.-037	0.36558D 01	0.39080D 01	0.41603D 01
OBS.-038	0.53631D 01	0.57165D 01	0.60699D 01
OBS.-039	0.68725D 01	0.73296D 01	0.77867D 01
OBS.-040	0.81595D 01	0.87016D 01	0.92437D 01
OBS.-041	0.91820D 01	0.97775D 01	0.10373D 02
OBS.-042	0.98742D 01	0.10488D 02	0.11102D 02
OBS.-043	0.10151D 02	0.10754D 02	0.11356D 02
OBS.-044	0.99275D 01	0.10496D 02	0.11065D 02
OBS.-045	0.91444D 01	0.96612D 01	0.10178D 02
OBS.-046	0.77935D 01	0.82391D 01	0.86846D 01
OBS.-047	0.59289D 01	0.62834D 01	0.66378D 01
OBS.-048	0.36562D 01	0.39198D 01	0.41834D 01
OBS.-049	0.10944D 01	0.13407D 01	0.15870D 01
OBS.-050	-0.15618D 01	-0.12137D 01	-0.86570D 00

APPROXIMATE 95% CONFIDENCE INTERVALS FOR THE VARIABLE Y

VARIABLE	LOWER LIMIT	OBSERVED	UPPER LIMIT
OBS.-001	-0.11609D 02	-0.99214D 01	-0.75630D 01
OBS.-002	-0.10551D 02	-0.94605D 01	-0.67263D 01
OBS.-003	-0.94605D 01	-0.77552D 01	-0.58699D 01
OBS.-004	-0.83466D 01	-0.76407D 01	-0.50047D 01
OBS.-005	-0.72598D 01	-0.48039D 01	-0.41703D 01
OBS.-006	-0.62856D 01	-0.51387D 01	-0.34273D 01
OBS.-007	-0.55218D 01	-0.44250D 01	-0.28447D 01
OBS.-008	-0.50501D 01	-0.34156D 01	-0.24832D 01
OBS.-009	-0.49105D 01	-0.34112D 01	-0.23759D 01
OBS.-010	-0.50871D 01	-0.41430D 01	-0.25112D 01
OBS.-011	-0.55063D 01	-0.44175D 01	-0.28263D 01
OBS.-012	-0.60434D 01	-0.46230D 01	-0.32182D 01
OBS.-013	-0.65420D 01	-0.51458D 01	-0.35666D 01
OBS.-014	-0.68449D 01	-0.53224D 01	-0.37614D 01
OBS.-015	-0.68306D 01	-0.36095D 01	-0.37261D 01
OBS.-016	-0.64449D 01	-0.49707D 01	-0.34353D 01
OBS.-017	-0.57191D 01	-0.38536D 01	-0.29224D 01
OBS.-018	-0.47695D 01	-0.35183D 01	-0.22741D 01
OBS.-019	-0.37786D 01	-0.32036D 01	-0.16135D 01
OBS.-020	-0.29615D 01	-0.15506D 01	-0.10723D 01
OBS.-021	-0.25206D 01	-0.17600D 01	-0.76544D 00
OBS.-022	-0.25886D 01	-0.16941D 01	-0.77878D 00
OBS.-023	-0.31889D 01	-0.19716D 01	-0.11592D 01
OBS.-024	-0.42425D 01	-0.27146D 01	-0.18874D 01
OBS.-025	-0.55929D 01	-0.37804D 01	-0.28658D 01
OBS.-026	-0.70306D 01	-0.62583D 01	-0.39373D 01
OBS.-027	-0.83240D 01	-0.68796D 01	-0.49197D 01
OBS.-028	-0.92566D 01	-0.75025D 01	-0.56394D 01
OBS.-029	-0.96623D 01	-0.82078D 01	-0.59610D 01
OBS.-030	-0.94489D 01	-0.76769D 01	-0.58067D 01
OBS.-031	-0.86051D 01	-0.73080D 01	-0.51668D 01
OBS.-032	-0.71888D 01	-0.48761D 01	-0.40973D 01
OBS.-033	-0.53021D 01	-0.46241D 01	-0.27099D 01
OBS.-034	-0.30484D 01	-0.18954D 01	-0.11667D 01
OBS.-035	-0.32547D 00	-0.54797D-01	0.16148D 00
OBS.-036	0.10492D 01	0.13261D 01	0.28577D 01
OBS.-037	0.26280D 01	0.26880D 01	0.51881D 01
OBS.-038	0.41570D 01	0.68812D 01	0.72759D 01
OBS.-039	0.55492D 01	0.77809D 01	0.91101D 01
OBS.-040	0.67493D 01	0.74568D 01	0.10654D 02
OBS.-041	0.77017D 01	0.10119D 02	0.11853D 02
OBS.-042	0.83388D 01	0.10714D 02	0.12638D 02
OBS.-043	0.85824D 01	0.10722D 02	0.12925D 02
OBS.-044	0.83585D 01	0.76154D 01	0.12634D 02
OBS.-045	0.76181D 01	0.10311D 02	0.11704D 02
OBS.-046	0.63607D 01	0.78635D 01	0.10117D 02
OBS.-047	0.46517D 01	0.60672D 01	0.79151D 01
OBS.-048	0.26356D 01	0.35330D 01	0.52039D 01
OBS.-049	0.56698D 00	0.21954D 01	0.21144D 01
OBS.-050	-0.19935D 01	-0.16954D 01	-0.43402D 00

STATISTICS FOR DISTRIBUTION OF RATIOS

HISTOGRAM OF PERCENT FREQUENCY FOR 10 CLASSES

```
                                 INTERVAL BOUNDARIES       % FREQ
|XXXX                       -0.738D 00 TO -0.575D 00      4.00
|XX                         -0.575D 00 TO -0.413D 00      2.00
|XXXXXX                     -0.413D 00 TO -0.251D 00      6.00
|XXXXXXXXXXXXXXXXXXXXXX     -0.251D 00 TO -0.886D-01     22.00
|XXXXXXXXXXXXXXXXXXXXXX     -0.886D-01 TO  0.737D-01     22.00
|XXXXXXXXXXXXXXXXXXXXXXXX    0.737D-01 TO  0.236D 00     24.00
|XXXXXXXXXX                  0.236D 00 TO  0.398D 00     10.00
|XXXXXX                      0.398D 00 TO  0.560D 00      6.00
|XX                          0.560D 00 TO  0.723D 00      2.00
|XX                          0.723D 00 TO  0.885D 00      2.00
```

SAMPLE STATISTICS OF RATIOS COMPUTED ABOUT ZERO

SAMPLE MEAN = 0.25041D-01 SAMPLE VARIANCE = 0.88879D-01 SKEWNESS = 0.22282D 00 KURTOSIS = 0.40657D 01

TEST FOR GOODNESS OF FIT TO NORMAL DISTRIBUTION

INTERVAL	OBSERVED	EXPECTED
- INFINITY TO -0.38212D 00	3.00	5.00
-0.38212D 00 TO -0.25086D 00	3.00	5.00
-0.25086D 00 TO -0.15622D 00	6.00	5.00
-0.15622D 00 TO -0.75406D-01	5.00	5.00
-0.75406D-01 TO 0.0	3.00	5.00
0.0 TO 0.75406D-01	8.00	5.00
0.75406D-01 TO 0.15622D 00	9.00	5.00
0.15622D 00 TO 0.25086D 00	3.00	5.00
0.25086D 00 TO 0.38212D 00	5.00	5.00
0.38212D 00 TO + INFINITY	5.00	5.00

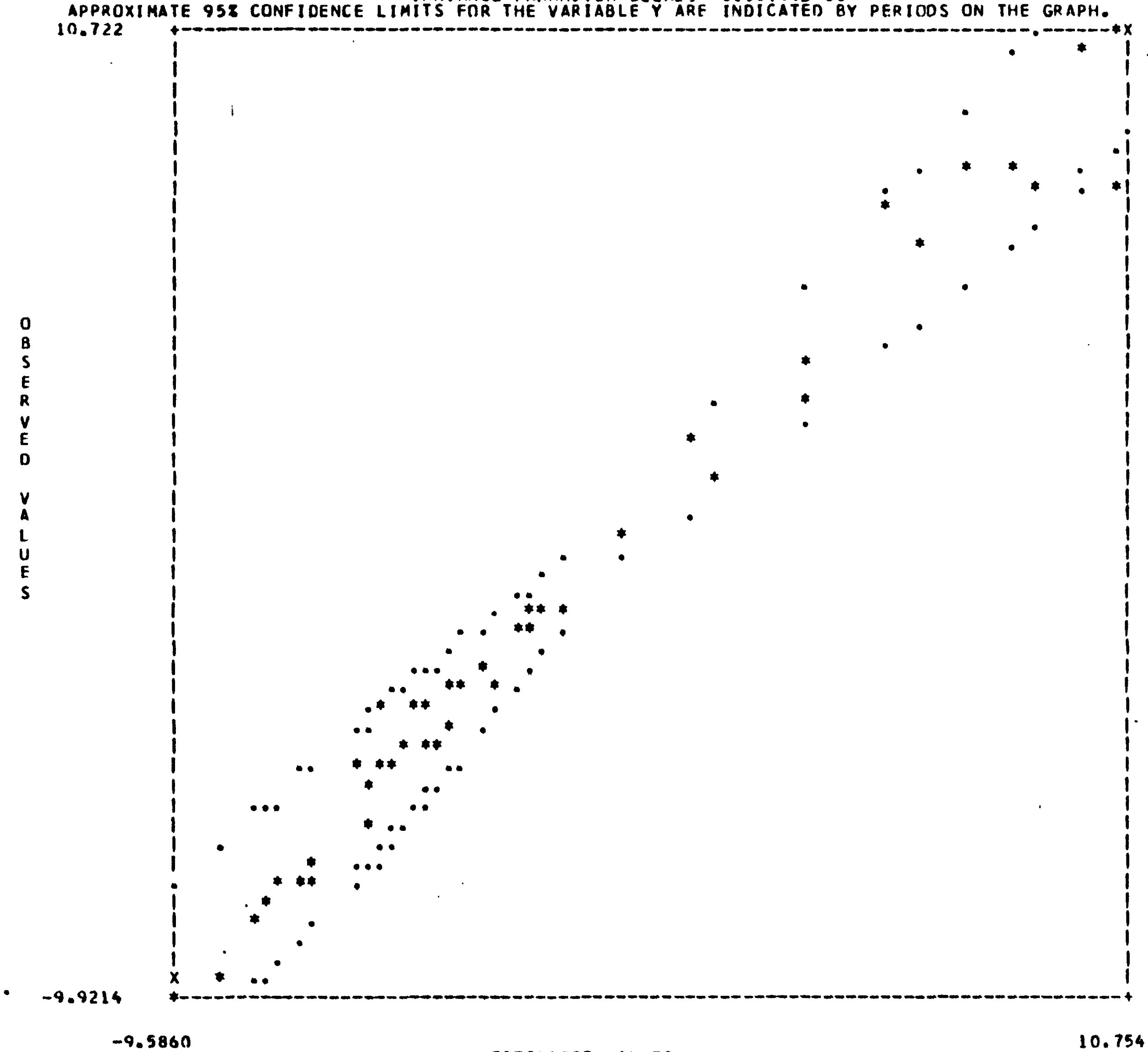
95% CONFIDENCE INTERVAL FOR TESTING GOODNESS OF FIT IS (0.0, 0.15112D 02)
CHI SQUARE VALUE FOR SAMPLE DISTRIBUTION = 0.84000D 01

95% CONFIDENCE INTERVAL FOR SAMPLE MEAN IS (-0.84703D-01, 0.84703D-01)
95% CONFIDENCE INTERVAL FOR STANDARD DEVIATION IS (0.35305D-01, 0.52479D-01)

GRAPH OF OBSERVED AND ESTIMATED VALUES OF Y

VARIANCE PARAMETER EQUALS 0.50199D 00

APPROXIMATE 95% CONFIDENCE LIMITS FOR THE VARIABLE Y ARE INDICATED BY PERIODS ON THE GRAPH.



LINE JOINING TWO X'S REPRESENTS THE LINE OF MEANS

ANALYSIS OF REGRESSION COEFFICIENTS

COEFFICIENTS ARE A SIMULTANEOUS SOLUTION OF THE PARTIAL DERIVATIVE EQUATIONS OF THE LOG OF THE LIKELIHOOD FUNCTION TO 10 SIGNIFICANT DIGITS.

COEFFICIENTS ARE WITHIN TOLERANCE SPECIFIED.

SECOND ORDER PARTIAL DERIVATIVES OF THE LOG OF THE LIKELIHOOD FUNCTION

```
-0.191951690029267D 02-0.686673075394069D 02-0.885616429719523D 02-0.104230254372486D 03-0.111411479777472D 03

-0.686673075394069D 02-0.253561558546410D 03-0.324163785960983D 03-0.381955273278249D 03-0.407781348164228D 03

-0.885616429719523D 02-0.324163785960983D 03-0.419281084800784D 03-0.490160457649980D 03-0.523398164633633D 03

-0.104230254372486D 03-0.381955273278249D 03-C.490160457649980D 03-0.585435166855877D 03-0.623454542930291D 03

-0.111411479777472D 03-0.407781348164228D 03-0.523398164633633D 03-0.623454542930291D 03-0.667451347449112D 03
```

COEFFICIENTS ARE A RELATIVE MAXIMUM WITHIN TOLERANCE SPECIFIED IF ALL OF THE EIGEN VALUES OF THE ABOVE MATRIX ARE NEGATIVE AND IF THE COEFFICIENTS ARE A SIMULTANEOUS SOLUTION OF THE PARTIAL DERIVATIVE EQUATIONS OF THE LOG OF THE LIKELIHOOD FUNCTION.

DETERMINANT OF THE ABOVE MATRIX = -0.162996935084750D 05

PRODUCT OF COMPUTED EIGEN VALUES = -0.162996935084748D 05

MINIMUM NUMBER OF SIGNIFICANT DIGITS FOR EIGEN VALUES OR VECTORS = 10

EIGEN VALUE # 1 = -0.426192283504782D 00

EIGEN VALUE # 2 = -0.160779828740070D 01

EIGEN VALUE # 3 = -0.173470859790728D 01

EIGEN VALUE # 4 = -0.708997245830210D 01

EIGEN VALUE # 5 = -0.193406566502799D 04

END OF EXECUTION

3. PROGRAM LISTING

The following program was compiled using the IBM FORTRAN-IV (G – Level) compiler and the FORTRAN-IV (H-Level) compiler with optimization equal to zero. Since the IBM compilers have some features not available on other manufacturers' compilers, the program may have to be modified to work on other compilers.

```
C      NONCONSTANT VARIANCE REGRESSION PROGRAM BY MARSHALL S. HELLMANN  RGS
      DIMENSION X(10,500), Y(500), W(500), A(10), B(10), EY(500), DY(500RGS
     1), V(10,500), C(10), D(10,10), RDY(500), ALPHA(20), A0(10), U(500)RGS
     2, U0(500), RL(20), X1(3,20), Y1(20), SLABEL(20), LABEL(500)       RGS
      DIMENSION SIGN(10), F(500), G(500), FMT(20), VLABEL(10)           RGS
      DOUBLE PRECISION XNY, LABEL, SLABEL, F, G, VLABEL, YLABEL         RGS
      DOUBLE PRECISION A1,A2,ALPHM,FEY,FRL,FMIN,SRDY,ALPHA,DET,DEL,DELTARGS
     1,EPSLON,DEYF,EYF,FEYF,FDEYF,A0,EPSL,X,Y,W,A,B,EY,DY,V,C,D,U,RDY,U0RGS
     2,RL,X1,Y1,PA,ALPHM1,DELT1                                         RGS
      DATA BLNK/' '/,PLUS/'+'/                                          RGS
      DATA A0/10*0.0D0/                                                 RGS
      DELT1=1.0D-30                                                     RGS
      EPS=1.0D-15                                                       RGS
      I2=6                                                              RGS
      READ (5,1) NX,NY,EPSLON,DELTA,NA,NITE,A2,PA,NO,IRUN,I1,ISYM       RGS
    1 FORMAT (I2,I3,2D10.3,I2,I4,D10.3,F3.2,2I1,I2,I1)                  RGS
      IF (I1-8) 3,3,2                                                   RGS
    2 IF (I1-99) 4,3,4                                                  RGS
    3 I1=5                                                              RGS
    4 WRITE (I2,159)                                                    RGS
      XNY=NY                                                            RGS
      WRITE (I2,5)                                                      RGS
    5 FORMAT (1H153X25HINPUT DATA AND PARAMETERS)                       RGS
      WRITE (I2,6) NX                                                   RGS
    6 FORMAT (1H010X34HNUMBER OF INDEPENDENT VARIABLES = I3/)           RGS
      WRITE (I2,7) NY                                                   RGS
    7 FORMAT (1H010X25HNUMBER OF OBSERVATIONS = I4/)                    RGS
      EPSL=EPSLON*100                                                   RGS
      WRITE (I2,8) EPSL                                                 RGS
    8 FORMAT (1H010X24HCONVERGENCE TOLERANCE = E10.3,22H% OF EACH COEFFIRGS
     1CIENT./)                                                          RGS
      WRITE (I2,9) DELTA                                                RGS
    9 FORMAT (1H010X34HLOWER BOUND FOR WEIGHT FUNCTION = E10.3/)        RGS
      WRITE (I2,10) NA                                                  RGS
   10 FORMAT (1H010X49HNUMBER OF PARAMETERS GIVEN FOR WEIGHT FUNCTION = RGS
     1I4/)                                                              RGS
      WRITE (I2,11) NITE                                                RGS
   11 FORMAT (1H010X31HMAXIMUM NUMBER OF ITERATIONS = I4/)              RGS
      WRITE (I2,12) A2                                                  RGS
   12 FORMAT (1H0,10X,'VARIANCE PARAMETER FOR CURVE FITTING FUNCTION = 'RGS
     1,D12.5/)                                                          RGS
      WRITE (I2,13) PA,NO,IRUN                                          RGS
   13 FORMAT (1H010X27HSIZE OF CRITICAL REGIONS = F4.2//1H010X24HDATA FORGS
     1RMAT INDICATOR = ,I1//1H010X25HCOEFFICIENTS INDICATOR = ,I1/)     RGS
      WRITE (I2,14) I1,ISYM                                             RGS
   14 FORMAT (1H0,10X,'FORTRAN DATA SET REFERENCE NUMBER = ',I2//1H0,10XRGS
     1,'SYMMETRIC COEFFICIENT EQUATIONS SWITCH = ',I1/)                 RGS
      WRITE (I2,15)                                                     RGS
   15 FORMAT (1H )                                                      RGS
      IF (NY-NX) 16,18,18                                               RGS
   16 WRITE (I2,17)                                                     RGS
   17 FORMAT (1H0,5X,'NUMBER OF OBSERVATIONS IS LESS THAN NUMBER OF INDERGS
     1PENDENT VARIABLES.')                                              RGS
      GO TO 157                                                         RGS
   18 CONTINUE                                                          RGS
      READ (I1,19) (ALPHA(K),K=1,NA)                                    RGS
   19 FORMAT (3D22.15)                                                  RGS
      WRITE (I2,20)                                                     RGS
```

```
20 FORMAT (1H010X45HVALUES OF GIVEN PARAMETERS OF WEIGHT FUNCTION/)
   WRITE (I2,21) (ALPHA(K),K=1,NA)
21 FORMAT (1H ,6D22.15)
   IF (IRUN) 22,26,22
22 READ (I1,19) (A0(I),I=1,NX)
   WRITE (I2,23)
23 FORMAT (1H010X33HINITIAL ESTIMATES OF COEFFICIENTS/)
   DO 24 I=1,NX
24 WRITE (I2,25) I,A0(I)
25 FORMAT (1H 10X2HA(,I2,4H) = ,D22.15)
26 READ (I1,27) FMT
27 FORMAT (20A4)
   READ (I1,31) (YLABEL,(VLABEL(I),I=1,NX))
   IF (NO) 28,30,28
28 DO 29 J=1,NY
29 READ (I1,FMT) (LABEL(J),Y(J),(X(I,J),I=1,NX))
   GO TO 33
30 READ (I1,31) (LABEL(I),I=1,NY)
31 FORMAT ((6(A8,2X),20X))
   READ (I1,FMT) (Y(J),J=1,NY)
   DO 32 I=1,NX
32 READ (I1,FMT) (X(I,J),J=1,NY)
33 WRITE (I2,34)
34 FORMAT (1H010X28HNAMES OF DEPENDENT VARIABLES/)
   WRITE (I2,35) (LABEL(I),I=1,NY)
35 FORMAT ((1X,13(A8,2X)))
   WRITE (I2,36) YLABEL
36 FORMAT (1H0,10X,'DEPENDENT VARIABLE',10X,A8/)
   WRITE (I2,40) (Y(J),J=1,NY)
   WRITE (I2,37)
37 FORMAT (1H010X22HINDEPENDENT VARIABLES /)
   DO 39 I=1,NX
   WRITE (I2,38) I,VLABEL(I)
38 FORMAT (1H 10X,9HVARIABLE I2,10X,A8)
39 WRITE (I2,40) (X(I,J),J=1,NY)
40 FORMAT (1H ,6D22.15)
   WRITE (I2,41)
41 FORMAT (1H0,21HEND OF INITIALIZATION//,1H ,'BEGIN EXECUTION'/,1H1,
  156X,'INTERMEDIATE RESULTS'//)
   SWITCH=0.0
   MNA=NA
   DO 42 J=1,NY
42 U0(J)=1
   K1=0
43 K1=K1+1
44 DO 45 I=1,NX
45 A(I)=A0(I)
   DO 46 J=1,NY
   W(J)=1.0D0
   DO 46 I=1,NX
46 V(I,J)=0.0D0
   A1=ALPHA(K1)
   NITER=0
   SRDY=0
   RL(K1)=0
   SKIP=1
   IF (IRUN.EQ.0) GO TO 55
47 DO 50 J=1,NY
```

```
      IF (SWITCH-1) 49,48,49
   48 W(J)=FEYF(X,A,A2,J,NX,ALPHM)
      GO TO 50
   49 W(J)=EYF(X,A,A1,J,NX,NY)
   50 CONTINUE
      DO 51 J=1,NY
      FEY=DABS(W(J))-DELTA
      IF (FEY) 81,81,51
   51 CONTINUE
      DO 54 I=1,NX
      DO 54 J=1,NY
      IF (SWITCH-1) 53,52,53
   52 V(I,J)=0
      GO TO 54
   53 V(I,J)=DEYF(X,A,A1,J,I,NX,NY)
   54 CONTINUE
   55 DO 56 J=1,NY
      EY(J)=0
      DO 56 I=1,NX
   56 EY(J)=EY(J)+A(I)*X(I,J)
      DO 57 K=1,NX
      C(K)=0
      DO 57 J=1,NY
   57 C(K)=C(K)+Y(J)*X(K,J)/W(J)**2+(Y(J)-EY(J))*V(K,J)*Y(J)/W(J)**3
      DO 61 K=1,NX
      LIMIT=K
      IF (ISYM) 58,59,58
   58 LIMIT=NX
   59 DO 61 I=1,LIMIT
      D(I,K)=0.0D0
      DO 60 J=1,NY
   60 D(I,K)=D(I,K)+X(I,J)*(X(K,J)*W(J)+(Y(J)-EY(J))*V(K,J))/(W(J))**3
   61 CONTINUE
      IF (ISYM) 64,62,64
   62 DO 63 I=1,NX
      DO 63 K=1,I
   63 D(I,K)=D(K,I)
   64 CONTINUE
      CALL WORK (C,D,NX,IER,DET)
      IF (IER) 85,65,65
   65 IF (IER-NX) 66,69,66
   66 IF (SKIP) 67,69,67
   67 WRITE (I2,68) ALPHA(K1)
   68 FORMAT (1H0,10X,46HPOSSIBLE LOSS IN NUMBER OF SIGNIFICANT FIGURES,
     13H.    ,21HVARIANCE PARAMETER = E12.5)
      SKIP=0
   69 DO 70 I=1,NX
   70 B(I)=C(I)
      IF (SWITCH-1) 75,71,75
   71 IF (B(3)-DELT1) 72,72,74
   72 DO 73 I=1,NX
   73 B(I)=A(I)
      ALPHM=ALPHM1
      GO TO 95
   74 ALPHM1=ALPHM
      ALPHM=-B(2)/(2*B(3))
   75 DO 78 I=1,NX
      DEL=DABS(A(I)-B(I))
```

```
      DET=DABS(A(I))
      IF (DET-DABS(B(I))) 76,76,77
   76 DET=DABS(B(I))
   77 IF (DEL-EPSLON*DET) 78,78,79
   78 CONTINUE
      GO TO 95
   79 IF (NITE.EQ.0) GO TO 95
      DO 80 I=1,NX
   80 A(I)=B(I)
      NITER=NITER+1
      IF (NITER-NITE) 47,47,83
   81 WRITE (I2,82) DELTA,NITER,ALPHA(K1)
   82 FORMAT (1H0,'THE FUNCTION F(EY) IS LESS THAN ',E10.3,' ON ',10HITE
     1RATION ,I3,3H.  ,21HVARIANCE PARAMETER = ,E12.5)
      GO TO 87
   83 WRITE (I2,84) ALPHA(K1)
   84 FORMAT (1H0,10X,38HITERATION TECHNIQUE FAILED TO CONVERGE,28H WITH
     1IN TOLERANCE SPECIFIED.,2X,21HVARIANCE PARAMETER = ,E12.5)
      GO TO 95
   85 WRITE (I2,86) ALPHA(K1)
   86 FORMAT (1H010X31HCOEFFICIENT MATRIX IS SINGULAR.2X21HVARIANCE PARA
     1METER = E12.5)
   87 IF (SWITCH) 89,88,143
   88 MK=MNA-1
      NA=NA-1
   89 IF (SWITCH) 90,91,143
   90 RL(K1)=RL(1)
      ALPHA(K1)=ALPHA(1)
      GO TO 147
   91 IF (K1-MNA) 92,94,94
   92 DO 93 K=K1,MK
   93 ALPHA(K)=ALPHA(K+1)
   94 MNA=MNA-1
      K1=K1-1
      GO TO 128
   95 DO 96 I=1,NX
   96 B(I)=A(I)
      DO 97 J=1,NY
      EY(J)=0
      DO 97 I=1,NX
   97 EY(J)=EY(J)+A(I)*X(I,J)
      DO 100 J=1,NY
      IF (SWITCH-1) 99,98,99
   98 W(J)=FEYF(X,A,A2,J,NX,ALPHM)
      GO TO 100
   99 W(J)=EYF(X,A,A1,J,NX,NY)
      IF (W(J)) 81,81,100
  100 SRDY=0.0D0
      DO 101 J=1,NY
      DY(J)=EY(J)-Y(J)
      RDY(J)=DY(J)/W(J)
  101 SRDY=SRDY+RDY(J)**2
      SRDY=DSQRT(SRDY/XNY)
      IF (SRDY) 103,102,103
  102 SRDY=10D-50
  103 CONTINUE
      DO 104 J=1,NY
  104 U(J)=SRDY*W(J)
```

```
    IF (SWITCH-1) 107,105,107
105 WRITE (I2,106)
106 FORMAT (1H1,40X,'RESULTS OF WEIGHTED CURVE FITTING TECHNIQUE')
    WRITE (I2,115)
    GO TO 136
107 CONTINUE
    IF (K1-1) 110,108,110
108 DO 109 J=1,NY
109 U0(J)=U(J)
110 DO 111 J=1,NY
111 RL(K1)=RL(K1)+DLOG10(U(J))-DLOG10(U0(J))
    IF (SWITCH) 112,119,127
112 IF (SWITCH+1) 113,147,127
113 CONTINUE
    WRITE (I2,114)
114 FORMAT (1H145X,'NONCONSTANT VARIANCE REGRESSION ANALYSIS RESULTS')
    WRITE (I2,115)
115 FORMAT (1H03X8HVARIABLE4X10HOBSERVED  10X10HESTIMATED 10X10HDIFFER
   1ENCE10X,9HWEIGHT   11X,9HRATIO    11X10HDEVIATION )
    IF (SWITCH-1) 116,136,116
116 DO 117 J=1,NY
117 WRITE (I2,118) LABEL(J),Y(J),EY(J),DY(J),W(J),RDY(J),U(J)
118 FORMAT (1H 3X,A8,6(4X,E12.5,4X))
119 WRITE (I2,120)
120 FORMAT (1H015X7HE(Y) = )
    DO 123 I=1,NX
    IF (A(I)) 122,121,121
121 SIGN(I)=PLUS
    GO TO 123
122 SIGN(I)=BLNK
123 CONTINUE
    SIGN(1)=BLNK
    WRITE (I2,124) (SIGN(I),A(I),I,I=1,NX)
124 FORMAT (1H+,22X3(A1,D22.15,2HX(,I1,2H) )/1H0,22X3(A1,D22.15,2HX(,I
   11,2H) )/1H0,22X,3(A1,D22.15,2HX(,I1,2H) )/5(1H0,22X,3(A1,D22.15,2H
   2X(,I2,2H) )/))
    WRITE (I2,125) SRDY,RL(K1)
125 FORMAT (1H015X36HSTANDARD DEVIATION OF THE RATIOS IS E12.5,5X,'LOG
   1ARITHM OF JOINT FREQUENCY FUNCTION RATIO = ',D12.5)
    WRITE (I2,126) NITER,ALPHA(K1)
126 FORMAT (1H 15X,'NUMBER OF ITERATIONS = ',I4,5X,'VARIANCE PARAMETER
   1 = ',D12.5//)
    IF (SWITCH+2) 127,152,128
127 CALL DUMP (40000,120000,0)
128 IF (K1-MNA) 43,129,129
129 CONTINUE
    IF (MNA) 130,157,130
130 IF (NA-2) 131,131,132
131 KM=MINJ(RL,NA)
    ALPHM=ALPHA(KM)
    GO TO 146
132 CONTINUE
    DO 135 K=1,NA
    DO 133 I=1,3
    V(I,K)=0.0D0
133 X1(I,K)=X(I,K)
    DO 134 I=2,3
134 X(I,K)=(ALPHA(K))**(I-1)
```

```
      X(1,K)=1                                                          RGS 291
      Y1(K)=Y(K)                                                        RGS 292
      Y(K)=FRL(RL(K))                                                   RGS 293
  135 W(K)=1                                                            RGS 294
      A(1)=1                                                            RGS 295
      A(2)=0                                                            RGS 296
      A(3)=0                                                            RGS 297
      MX=NX                                                             RGS 298
      MY=NY                                                             RGS 299
      NX=3                                                              RGS 300
      NY=NA                                                             RGS 301
      K1=NA+1                                                           RGS 302
      ALPHA(K1)=A2                                                      RGS 303
      ALPHM=ALPHA(MINJ(Y,NA))                                           RGS 304
      ALPHM1=ALPHM                                                      RGS 305
      NITER=0                                                           RGS 306
      SKIP=1                                                            RGS 307
      NA1=NA                                                            RGS 308
      SWITCH=1.0                                                        RGS 309
      GO TO 55                                                          RGS 310
  136 DO 137 J=1,NY                                                     RGS 311
  137 WRITE (I2,138) ALPHA(J),Y(J),EY(J),DY(J),W(J),RDY(J),U(J)         RGS 312
  138 FORMAT (1H ,E12.5,6(3X,E12.5,5X))                                 RGS 313
      WRITE (I2,139) (B(I),I=1,3)                                       RGS 314
  139 FORMAT (1H015X8HRATIO = E12.5,3H + E12.5,8HALPHA + E12.5,8HALPHA**RGS 315
     12)                                                                RGS 316
      IF (B(3)-DELT1) 141,141,140                                       RGS 317
  140 ALPHM=-B(2)/(2*B(3))                                              RGS 318
  141 WRITE (I2,126) NITER,ALPHA(K1)                                    RGS 319
      WRITE (I2,142) ALPHM                                              RGS 320
  142 FORMAT (1H0,15X,'ESTIMATE OF VARIANCE PARAMETER ALPHA = ',E12.5)  RGS 321
  143 CONTINUE                                                          RGS 322
      NX=MX                                                             RGS 323
      NY=MY                                                             RGS 324
      DO 145 K=1,NA1                                                    RGS 325
      DO 144 I=1,3                                                      RGS 326
  144 X(I,K)=X1(I,K)                                                    RGS 327
  145 Y(K)=Y1(K)                                                        RGS 328
  146 K1=NA+1                                                           RGS 329
      ALPHA(K1)=ALPHM                                                   RGS 330
      MNA=K1                                                            RGS 331
      SWITCH=-1.0                                                       RGS 332
      GO TO 44                                                          RGS 333
  147 FMIN=RL(1)                                                        RGS 334
      NMIN=1                                                            RGS 335
      DO 149 I=2,NA                                                     RGS 336
      IF (FMIN-RL(I)) 149,149,148                                       RGS 337
  148 FMIN=RL(I)                                                        RGS 338
      NMIN=I                                                            RGS 339
  149 CONTINUE                                                          RGS 340
      NAR=NA+1                                                          RGS 341
      IF (FMIN-RL(NAR)) 151,151,150                                     RGS 342
  150 SWITCH=-2                                                         RGS 343
      GO TO 113                                                         RGS 344
  151 ALPHM=ALPHA(NMIN)                                                 RGS 345
      K1=NAR                                                            RGS 346
      ALPHA(K1)=ALPHM                                                   RGS 347
      SWITCH=-2                                                         RGS 348
```

```
      GO TO 44                                                          RGS 349
  152 CALL WRAPUP (NA,NY,NX,Y,RL,EY,ALPHA,U,X,A1,C,A,V,W,EPSLON,UO,PA,LARGS 350
     1BEL,F,G)                                                          RGS 351
      CALL CONFI (X,A,NX,NY,V,W,SRDY,PA,Y,EY,LABEL,F,G)                 RGS 352
      NC=20                                                             RGS 353
      IF (NY/5-NC) 153,154,154                                          RGS 354
  153 NC=NY/5                                                           RGS 355
      IF (NC-2) 155,155,154                                             RGS 356
  154 CONTINUE                                                          RGS 357
      CALL HISTO (RDY,NY,NC,PA,NX)                                      RGS 358
  155 CALL PLOT (NA,NY,NX,Y,RL,EY,ALPHA,U,X,A1,C,A,V,W,EPSLON,UO,PA,LABERGS 359
     1L,F,G)                                                            RGS 360
      WRITE (I2,156)                                                    RGS 361
  156 FORMAT (1HO,'END OF EXECUTION')                                   RGS 362
      STOP 0                                                            RGS 363
  157 WRITE (I2,158)                                                    RGS 364
  158 FORMAT (1HO,5X,'CHECK INPUT DATA, INITIAL ESTIMATES OF COEFFICIENTRGS 365
     1S, AND PARAMETERS OF WEIGHTING FUNCTION.'/1HO,'END OF EXECUTION') RGS 366
      STOP 1                                                            RGS 367
C                                                                       RGS 368
  159 FORMAT (1H1/////////51X,'UNITED STATES GEOLOGICAL SURVEY'/55X,'COMRGS 369
     1PUTER CENTER DIVISION'//////////42X, 'NONCONSTANT VARIANCE REGRESSRGS 370
     2ION ANALYSIS PROGRAM'///58X,'PROGRAM NO. W8251'//54X,'DESIGNED ANDRGS 371
     3 PROGRAMMED BY'//55X,'MARSHALL STRONG HELLMANN'//////////5X,'EQUIPRGS 372
     4MENT   IBM SYSTEM 360-65'/5X,'LANGUAGE    FORTRAN-IV H-LEVEL'/5X,'RGS 373
     5DATE         DECEMBER 18, 1968'/5X'REFERENCE   PAMPHLET TITLED  NONRGS 374
     6-CONSTANT VARIANCE REGRESSION ANALYSIS , BY MARSHALL STRONG HELLMARGS 375
     7NN, 1967, LIBRARY OF CONGRESS'/17X,'CATALOG CARD NUMBER 67-23725')RGS 376
      END                                                               RGS 377-

      SUBROUTINE WRAPUP (NA,NY,NX,Y,RL,EY,ALPHA,U,X,A1,C,A,V,W,EPSLON,UOWUP    1
     1,PA,LABEL,F,G)                                                    WUP    2
      DIMENSION A(10), Y(500), RL(20), EY(500), ALPHA(20), U(500), X(10,WUP    3
     1500), UO(500), C(10), V(10,500), W(500), IRL(20), EYS(500), YS(500WUP    4
     2), T1(500), T(500), F(500), G(500), LABEL(500)                    WUP    5
      REAL IMAGE(2000)                                                  WUP    6
      DOUBLE PRECISION Y,RL,EY,ALPHA,U,X,A1,C,A,V,W,EPSLON,YBAR,YBARS,RLWUP    7
     1YBAR,UO,PR,PA,LABEL,F,G                                           WUP    8
      I2=6                                                              WUP    9
      WRITE (I2,1)                                                      WUP   10
    1 FORMAT (1H150X31HJOINT FREQUENCY FUNCTION RATIOS///)              WUP   11
      NA=NA+1                                                           WUP   12
      YBAR=0.0D0                                                        WUP   13
      DO 2 J=1,NY                                                       WUP   14
    2 YBAR=YBAR+Y(J)                                                    WUP   15
      YBAR=YBAR/NY                                                      WUP   16
      YBARS=0.0D0                                                       WUP   17
      DO 3 J=1,NY                                                       WUP   18
    3 YBARS=YBARS+(Y(J)-YBAR)**2                                        WUP   19
      YBARS=DSQRT(YBARS/NY)                                             WUP   20
      IF (YBARS) 5,4,5                                                  WUP   21
    4 RLYBAR=100                                                        WUP   22
      IRLY=111111                                                       WUP   23
      GO TO 7                                                           WUP   24
    5 RLYBAR=0.0D0                                                      WUP   25
      DO 6 J=1,NY                                                       WUP   26
    6 RLYBAR=RLYBAR+DLOG10(UO(J))-DLOG10(YBARS)                         WUP   27
```

```
      RLYBAR=RLYBAR+RL(NA)
      IRLY=RLYBAR
      RLYBAR=RLYBAR-IRLY
      RLYBAR=10**RLYBAR
    7 CONTINUE
      DO 8 K=1,NA
      IRL(K)=RL(K)
    8 RL(K)=10**(RL(K)-IRL(K))
      WRITE (I2,9)
    9 FORMAT (1H047X5HRATIO13X18HVARIANCE PARAMETER/)
      WRITE (I2,10) (RL(K),IRL(K),ALPHA(K),K=1,NA)
   10 FORMAT (20(1H 45X,F7.5,1HD,I5,13X,E12.5/))
      RETURN
      ENTRY PLOT(NA,NY,NX,Y,RL,EY,ALPHA,U,X,A1,C,A,V,W,EPSLON,U0,PA,LABE
     1L,F,G)
      I2=6
      PR=PA/2
      XMAX=EY(1)
      XMIN=EY(1)
      YMIN=Y(1)
      YMAX=Y(1)
      DO 19 J=1,NY
      IF (XMAX-EY(J)) 11,12,12
   11 XMAX=EY(J)
   12 IF (XMIN-EY(J)) 14,14,13
   13 XMIN=EY(J)
   14 IF (YMAX-Y(J)) 15,16,16
   15 YMAX=Y(J)
   16 IF (YMIN-Y(J)) 18,18,17
   17 YMIN=Y(J)
   18 CONTINUE
      EYS(J)=EY(J)
      YS(J)=Y(J)
   19 CONTINUE
      CALL PLOT1 (1,50,1,83)
      CALL PLOT2 (IMAGE,XMAX,XMIN,YMAX,YMIN)
      NDF=NY-NX
      IF (NDF) 22,22,20
   20 DO 21 J=1,NY
      T1(J)=F(J)
   21 T(J)=G(J)
      CALL PLOT3 (1H.,EYS,T1,NY)
      CALL PLOT3 (1H.,EYS,T,NY)
   22 CALL PLOT3 (1H*,EYS,YS,NY)
      IF (XMAX-YMAX) 24,23,23
   23 XMAX=YMAX
   24 IF (XMIN-YMIN) 25,25,26
   25 XMIN=YMIN
   26 CONTINUE
      CALL PLOT3 (1HX,XMIN,XMIN,1)
      CALL PLOT3 (1HX,XMAX,XMAX,1)
      WRITE (I2,27)
   27 FORMAT (1H1,45X,'GRAPH OF OBSERVED AND ESTIMATED VALUES OF Y')
      WRITE (I2,28) ALPHA(NA)
   28 FORMAT (1H051X26HVARIANCE PARAMETER EQUALS E12.5)
      IF (NDF) 31,31,29
   29 CONTINUE
      IR=(1-PA)*100
```

```
      WRITE (I2,30) IR
   30 FORMAT (1H 21X,12HAPPROXIMATE ,I2,'% CONFIDENCE LIMITS FOR THE VAR
     1IABLE Y ARE INDICATED BY PERIODS ON THE GRAPH.')
   31 CALL PLOT4 (30,30H                  OBSERVED VALUES)
      WRITE (I2,32)
   32 FORMAT (1H 58X,'ESTIMATED VALUES'/)
      WRITE (I2,33)
   33 FORMAT (42X,'LINE JOINING TWO X''S REPRESENTS THE LINE OF MEANS')
      CALL MAXMUM (X,Y,A1,NX,NY,EY,C,A,V,W,EPSLON)
      RETURN
      END


      FUNCTION XMAX (X,N)
      DIMENSION X(1)
      DOUBLE PRECISION X,XMAX
      XMAX=X(1)
      DO 3 I=1,N
      IF (XMAX-X(I)) 1,1,2
    1 XMAX=X(I)
    2 CONTINUE
    3 CONTINUE
      RETURN
      END


      FUNCTION XMIN (X,N)
      DIMENSION X(1)
      DOUBLE PRECISION X,XMIN
      XMIN=X(1)
      DO 3 I=1,N
      IF (XMIN-X(I)) 2,1,1
    1 XMIN=X(I)
    2 CONTINUE
    3 CONTINUE
      RETURN
      END


      FUNCTION TNORM (A)
      DOUBLE PRECISION C0,C1,C2,DP,D1,D2,D3,T,XP,TNORM,A
      C0=2.515517D0
      C1=.802853D0
      C2=.010328D0
      D1=1.432788D0
      D2=.189269D0
      D3=.001308D0
      T=DLOG(1/(A**2))
      T=DSQRT(T)
    1 XP=C0+C1*T+C2*T**2
      DP=D1*T+D2*T**2+D3*T**3+1
      TNORM=T-XP/DP
      RETURN
      END
```

```
      FUNCTION MINJ (X,N)
      DIMENSION X(1)
      DOUBLE PRECISION X,Y
      MINJ=1
      Y=X(1)
      DO 2 I=1,N
      IF (Y-X(I)) 2,1,1
    1 Y=X(I)
      MINJ=I
    2 CONTINUE
      RETURN
      END

      FUNCTION TDIST (A,N)
      DOUBLE PRECISION TNORM,TDIST,A,X,G1,G2,G3,G4
      X=TNORM(A)
      G1=(X**3+X)/4
      G2=(5*X**5+16*X**3+3*X)/96
      G3=(3*X**7+19*X**5+17*X**3-15*X)/384
      G4=(79*X**9+776*X**7+1482*X**5-1920*X**3-945*X)/92160
      TDIST=X+G1/N+G2/(N**2)+G3/(N**3)+G4/(N**4)
      RETURN
      END

      SUBROUTINE CHISQ (N,A,C,D)
      DOUBLE PRECISION XP,H,HV,X,XN,TNORM,A,C,D
      SWITCH=1
      XP=TNORM(A)
    1 XP=-XP
      H=HV(N,XP)
      XN=N
      X=XN*(1-2/(9*XN)+(XP-H)*(2/(9*XN))**.5)**3
      IF (SWITCH-1) 3,2,3
    2 C=X
      SWITCH=0
      GO TO 1
    3 D=X
      RETURN
      END

      FUNCTION HV (N,XP)
      DIMENSION X(15), H(15)
      DOUBLE PRECISION X,H,XP,H60,DELX,DELH,HV,XN
      DATA H/-.0118D0,-.0067D0,-.0033D0,-.0010D0,+.0001D0,+.0006D0,+.000
     16D0,+.0002D0,-.0003D0,-.0006D0,-.0005D0,+.0002D0,+.0017D0,+.0043D0
     2,+.0082D0/
      DATA X/-3.5D0,-3.0D0,-2.5D0,-2.0D0,-1.5D0,-1.0D0,-0.5D0,+0.0D0,+0.
     15D0,+1.0D0,+1.5D0,+2.0D0,+2.5D0,+3.0D0,+3.5D0/
      XN=N
      IF (XP-X(1)) 2,2,1
    1 IF (XP-X(15)) 3,3,4
    2 H60=H(1)
      GO TO 7
    3 H60=H(15)
      GO TO 7
    4 DO 5 I=2,15
```

```
      IF (XP-X(I)) 6,6,5
    5 CONTINUE
    6 DELX=(XP-X(I))*2
      DELH=H(I)-H(I-1)
      H60=H(I)+DELH*DELX
    7 HV=60*H60/XN
      RETURN
      END

      SUBROUTINE MAXMUM (X,Y,A1,NX,NY,EY,C,A,V,W,EPSLON)
      DIMENSION X(10,500), Y(500), B(10), D(10,10), EY(500), W(500)
      DIMENSION A(10), C(10), U(500), V(10,500)
      DOUBLE PRECISION E,EPSLON,W,X,Y,C,D,EY,U ,V,A1,F2DEYF,B,EYF,DEYF,D
     1ET,DET2,T(10),A
      NSD=15
      I2=6
      WRITE (I2,1)
    1 FORMAT (1H148X, 'ANALYSIS OF REGRESSION COEFFICIENTS'///)
      DO 2 J=1,NY
      W(J)=EYF(X,A,A1,J,NX,NY)
    2 CONTINUE
      DO 3 I=1,NX
      DO 3 J=1,NY
      V(I,J)=DEYF(X,A,A1,J,I,NX,NY)
    3 CONTINUE
      DO 4 K=1,NX
      C(K)=0.0D0
      DO 4 J=1,NY
    4 C(K)=C(K)+Y(J)*X(K,J)/W(J)**2+(Y(J)-EY(J))*V(K,J)*Y(J)/W(J)**3
      DO 11 K=1,NX
      B(K)=0
      T(K)=0
      IF (C(K)) 5,6,6
    5 B(K)=-C(K)
      GO TO 7
    6 T(K)=-C(K)
    7 CONTINUE
      DO 11 I=1,NX
      D(I,K)=0.0D0
      DO 8 J=1,NY
    8 D(I,K)=D(I,K)+X(I,J)*(X(K,J)*W(J)+(Y(J)-EY(J))*V(K,J))/(W(J))**3
      IF (A(I)*D(I,K)) 10,9,9
    9 B(K)=B(K)+A(I)*D(I,K)
      GO TO 11
   10 T(K)=T(K)+A(I)*D(I,K)
   11 CONTINUE
      DO 12 K=1,NX
      T(K)=-T(K)
   12 NSD=MIN(NSD,LISD(T(K),B(K),15))
      IF (NSD) 13,13,15
   13 WRITE (I2,14)
   14 FORMAT (1H0,'COEFFICIENTS ARE NOT A CRITICAL POINT OF THE LOG OF T
     1HE LIKELIHOOD FUNCTION.')
      GO TO 17
   15 CONTINUE
      WRITE (I2,16) NSD
   16 FORMAT (1H048HCOEFFICIENTS ARE A SIMULTANEOUS SOLUTION OF THE ,'PA
```

```
  1RTIAL DERIVATIVE EQUATIONS OF THE LOG OF THE LIKELIHOOD FUNCTION TMAX
  20'/1H ,I2,' SIGNIFICANT DIGITS.')                                MAX
17 CONTINUE                                                         MAX
   CALL WORK (C,D,NX,IER,DET)                                       MAX
   IF (IER-NX) 18,20,20                                             MAX
18 WRITE (I2,19)                                                    MAX
19 FORMAT (1H0,'POSSIBLE LOSS IN NUMBER OF SIGNIFICANT DIGITS')     MAX
20 DO 25 K=1,NX                                                     MAX
   E=DABS(A(K)-C(K))                                                MAX
   DET=DABS(A(K))                                                   MAX
   IF (DABS(A(K))-DABS(C(K))) 21,21,22                              MAX
21 DET=DABS(C(K))                                                   MAX
22 IF (E-EPSLON*DET) 25,25,23                                       MAX
23 WRITE (I2,24)                                                    MAX
24 FORMAT (1H0,'COEFFICIENTS ARE NOT WITHIN TOLERANCE SPECIFIED.')  MAX
   GO TO 27                                                         MAX
25 CONTINUE                                                         MAX
   WRITE (I2,26)                                                    MAX
26 FORMAT (1H0,'COEFFICIENTS ARE WITHIN TOLERANCE SPECIFIED.')      MAX
27 WRITE (I2,28)                                                    MAX
28 FORMAT (1H031X51HSECOND ORDER PARTIAL DERIVATIVES OF THE LOG OF THMAX
  1E ,20HLIKELIHOOD FUNCTION )                                      MAX
   DO 29 I=1,NX                                                     MAX
   DO 29 K=1,NX                                                     MAX
29 D(I,K)=F2DEYF(Y,EY,A,A1,NX,X,I,K,NY)                             MAX
   DO 30 I=1,NX                                                     MAX
30 WRITE (I2,31) (D(I,K),K=1,NX)                                    MAX
31 FORMAT (1H0,6D22.15/1X,6D22.15)                                  MAX
   WRITE (I2,32)                                                    MAX
32 FORMAT (1H0,'COEFFICIENTS ARE A RELATIVE MAXIMUM WITHIN TOLERANCE MAX
  1SPECIFIED IF ALL OF THE EIGEN VALUES OF THE ABOVE MATRIX ARE NEGATMAX
  2IVE AND IF'/1H ,'THE COEFFICIENTS ARE A SIMULTANEOUS SOLUTION OF TMAX
  3HE PARTIAL DERIVATIVE EQUATIONS OF THE LOG OF THE LIKELIHOOD FUNCTMAX
  4ION.')                                                           MAX
   M=1                                                              MAX
   DO 33 I=1,NX                                                     MAX
   M=M+I-1                                                          MAX
   DO 33 K=1,I                                                      MAX
   J=M+K-1                                                          MAX
33 W(J)=D(I,K)                                                      MAX
   NSD=0                                                            MAX
   CALL WORK (C,D,NX,IER,DET)                                       MAX
   WRITE (I2,34) DET                                                MAX
34 FORMAT (1H034HDETERMINANT OF THE ABOVE MATRIX = E22.15)          MAX
   IF (DET) 37,35,37                                                MAX
35 WRITE (I2,36)                                                    MAX
36 FORMAT (1H010X51HHIGHER ORDER DERIVATIVES WOULD HAVE TO BE ANALYZEMAX
  1D ,58HTO DETERMINE IF THESE COEFFICIENTS ARE A RELATIVE MAXIMUM.) MAX
   GO TO 47                                                         MAX
37 CALL EIGEN (W,U,EY,NX,NSD)                                       MAX
   J=0                                                              MAX
   DO 38 I=1,NX                                                     MAX
   J=J+I                                                            MAX
38 B(I)=W(J)                                                        MAX
   DET2=1.0D0                                                       MAX
   DO 39 I=1,NX                                                     MAX
39 DET2=DET2*B(I)                                                   MAX
   WRITE (I2,40) DET2                                               MAX
```

```
40 FORMAT (1H0,35HPRODUCT OF COMPUTED EIGEN VALUES = D22.15)
   WRITE (I2,41) NSD
41 FORMAT (1H067HMINIMUM NUMBER OF SIGNIFICANT DIGITS FOR EIGEN VALUE
  1S OR VECTORS = I2)
   DO 42 M=1,NX
42 WRITE (I2,43) M,B(M)
43 FORMAT (1H014HEIGEN VALUE # I2,2X,2H= ,E22.15)
   IF (NSD) 44,45,47
44 CALL DUMP (40000,120000,0)
45 WRITE (I2,46)
46 FORMAT (1H0,53HCOMPUTED EIGEN VALUES OR VECTORS MAY NOT BE RELIABL
  1E.)
47 RETURN
   END


   SUBROUTINE CONFI (X,A,NX,NY,V,W,SRDY,PA,Y,EY,LABEL,F,G)
   DIMENSION C(10,10), D(10,10), S(10), V(10,500), X(10,500), A(10),
  1W(500), Y(500), EY(500), BL(10), SX(10), XM(10), E(10,10), H(10),
  2F(500), B(10), LABEL(500), G(500)
   DOUBLE PRECISION T,C,D,S,SRDY,DET,PA,PR,TDIST,C1,C2,W,V,X,A,Y,EY,H
  1,EYM,SEYM,F,B,XNNX,XNYX,LABEL,G,E,SX,XM
   DATA BL/10*' '/
   I2=6
   DO 1 K=1,NX
   S(K)=0
   DO 1 J=1,NY
 1 S(K)=S(K)+Y(J)*X(K,J)/W(J)**2+(Y(J)-EY(J))*V(K,J)*Y(J)/W(J)**3
   DO 2 I=1,NX
   DO 2 K=1,NX
   D(I,K)=0.0D0
   DO 2 J=1,NY
 2 D(I,K)=D(I,K)+X(I,J)*(X(K,J)*W(J)+(Y(J)-EY(J))*V(K,J))/(W(J))**3
   WRITE (I2,3)
 3 FORMAT (1H1,49X,'COEFFICIENT EQUATIONS AND SOLUTIONS')
   WRITE (I2,4)
 4 FORMAT (1H0,57X,'COEFFICIENT MATRIX'/)
   WRITE (I2,5) (BL(I),I,I=1,NX)
 5 FORMAT (1H0,6(A1,4X,'D(K,'I1,')',11X)/1H+,3(A1,10X,'/D(K,'I1,')',4
  1X),A1,10X'/D(K,'I2,')',3X)
   DO 6 K=1,NX
 6 WRITE (I2,7)(D(I,K),I=1,NX)
 7 FORMAT (1H0,6D22.15/1H ,6D22.15)
   WRITE (I2,8)
 8 FORMAT (1H0,54X,'VECTOR OF CONSTANT TERMS'/)
   DO 9 K=1,NX
 9 WRITE (I2,10) K,S(K)
10 FORMAT (1H ,47X,'EQUATION 'I2,5X,D22.15)
   DO 11 I=1,NX
   DO 11 K=1,NX
11 E(I,K)=D(I,K)
   CALL WORK (S,E,NX,IER,DET)
   IF (IER) 13,12,12
12 IF (IER-NX) 15,17,17
13 WRITE (I2,14)
14 FORMAT (1H15X,'COEFFICIENT MATRIX IS SINGULAR')
   GO TO 17
15 WRITE (I2,16)
```

```
16 FORMAT (1H15X,'POSSIBLE LOSS IN NUMBER OF SIGNIFICANT FIGURES.')
17 CONTINUE
   CALL DINV (C,D,NX,IER,DET,NSD)
   WRITE (I2,18)
18 FORMAT (//1H1,51X,'INVERSE OF COEFFICIENT MATRIX')
   WRITE (I2,19) (BL(I),I,I=1,NX)
19 FORMAT (1H0,6(A1,4X,'D''(K,'I1,')',10X)/1H+,3(A1,11X,'/D''(K,'I1,'
  1)',2X),A1,11X'/D''(K,'I2,')',1X)
   DO 20 K=1,NX
20 WRITE (I2,7)(C(I,K),I=1,NX)
   WRITE (I2,21) NSD
21 FORMAT (1H0'MINIMUM NUMBER OF SIGNIFICANT DIGITS FOR INVERSE = ',I
  12)
   WRITE (I2,22)
22 FORMAT (//58X,'COEFFICIENT CHECK'/)
   DO 23 K=1,NX
23 WRITE (I2,24) (K,S(K),K=1,NX)
24 FORMAT (1H ,51X,'A('I2,') = ',D22.15)
   NNX=NY-NX
   XNNX=NNX
   XNYX=NY/XNNX
   DO 26 I=1,NX
   XM(I)=0.0D0
   DO 25 J=1,NY
25 XM(I)=XM(I)+X(I,J)
26 XM(I)=XM(I)/NY
   DO 29 I=1,NX
   SX(I)=0.0D0
   DO 27 J=1,NY
27 SX(I)=SX(I)+(X(I,J)-XM(I))**2
   IF (SX(I)) 29,28,29
28 SX(I)=NY
29 SX(I)=DSQRT(SX(I)/NY)
   DO 30 I=1,NX
   DO 30 K=1,NX
   D(I,K)=0.0D0
   DO 30 J=1,NY
30 D(I,K)=D(I,K)+(X(I,J)-XM(I))*(X(K,J)-XM(K))/(SX(I)*SX(K)*NY)
   WRITE (I2,31)
31 FORMAT (1H1,47X,'LINEAR RELATIONSHIPS BETWEEN VARIABLES'///39X,'AR
  1ITHMETICAL CORRELATION MATRIX FOR INDEPENDENT VARIABLES')
   WRITE (I2,32) (BL(I),I,I=1,NX)
32 FORMAT (1H0,10X,9(A1,3X,'X(',I1,')',4X),A1,3X,'X(',I2,')')
   DO 33 K=1,NX
33 WRITE (I2,34) (K,(D(I,K),I=1,NX))
34 FORMAT (1H0,5X,'X(',I2,')',10G12.5)
   EYM=0
   DO 35 J=1,NY
35 EYM=EYM+EY(J)
   EYM=EYM/NY
   SEYM=0
   DO 36 J=1,NY
36 SEYM=SEYM+(EY(J)-EYM)**2
   IF (SEYM) 44,44,37
37 SEYM=DSQRT(SEYM/NY)
   DO 40 I=1,NX
   H(I)=0
   IF (SX(I)) 40,40,38
```

```
38 DO 39 J=1,NY
39 H(I)=H(I)+(EY(J)-EYM)*(X(I,J)-XM(I))
   H(I)=H(I)/(NY*SX(I)*SEYM)
40 CONTINUE
   WRITE (I2,41)
41 FORMAT (1H0, 22X,'ARITHMETICAL CORRELATION BETWEEN THE ESTIMATED V
  1ALUES AND THE INDEPENDENT VARIABLES'/)
   DO 42 I=1,NX
42 WRITE (I2,43) I,H(I)
43 FORMAT (1H056X,'X(',I2,')',3X,G12.5)
44 DO 45 I=1,NX
   DO 45 J=1,NY
45 V(I,J)=X(I,J)/W(J)
   DO 46 I=1,NX
   DO 46 K=1,NX
   D(I,K)=0
   DO 46 J=1,NY
46 D(I,K)=D(I,K)+V(I,J)*V(K,J)
   CALL DINV (C,D,NX,IER,DET,NSD)
   WRITE (I2,47)
47 FORMAT (1H1,56X,'CONFIDENCE INTERVALS'//32X,'COVARIANCE MATRIX V(I
  1,K) TO BE USED FOR COMPUTING CONFIDENCE INTERVALS')
   DO 48 I=1,NX
48 WRITE (I2,49) (C(I,K),K=1,NX)
49 FORMAT (1H0,12X,10D12.5)
   IP=(1-PA)*100
   WRITE (I2,50) IP
50 FORMAT (///35X,'APPROXIMATE ',I2,'% CONFIDENCE INTERVALS FOR REGRE
  1SSION COEFFICIENTS'//50X,'COEFFICIENT',8X,'CONFIDENCE INTERVAL')
   DO 51 I=1,NX
51 S(I)=SRDY*DSQRT(C(I,I)*XNYX)
   PR=PA/2
   T=TDIST(PR,NNX)
   DO 52 I=1,NX
   C1=A(I)-T*S(I)
   C2=A(I)+T*S(I)
52 WRITE (I2,53) I,C1,C2
53 FORMAT (1H0,52X,'A(',I2,')',7X,'(',E12.5,',',E12.5,')')
   DO 55 J=1,NY
   DO 54 K=1,NX
   B(K)=0
   DO 54 I=1,NX
54 B(K)=B(K)+V(I,J)*C(I,K)
   F(J)=0
   DO 55 K=1,NX
55 F(J)=F(J)+B(K)*V(K,J)
   WRITE (I2,56) IP
56 FORMAT (1H1,34X,'APPROXIMATE ',I2,'% CONFIDENCE INTERVALS FOR THE
  1EXPECTED VALUE OF Y')
   WRITE (I2,57)
57 FORMAT (1H030X,'VARIABLE',10X,'LOWER LIMIT',11X,'ESTIMATED',13X,'U
  1PPER LIMIT')
   DO 58 J=1,NY
   G(J)=DSQRT(F(J)*XNYX)*SRDY*T*W(J)
   C1=EY(J)-G(J)
   C2=EY(J)+G(J)
58 WRITE (I2,59) LABEL(J),C1,EY(J),C2
59 FORMAT (1H 30X,A8,10X,3(D12.5,10X))
```

```
      WRITE (I2,60) IP
   60 FORMAT (1H139X,'APPROXIMATE ',I2,'% CONFIDENCE INTERVALS FOR THE V
     1ARIABLE Y')
      WRITE (I2,61)
   61 FORMAT (1H030X,'VARIABLE',10X,'LOWER LIMIT',11X,'OBSERVED'14X,'UPP
     1ER LIMIT')
      DO 62 J=1,NY
      F(J)=DSQRT((F(J)+1)*XNYX)*SRDY*T*W(J)
      C1=EY(J)-F(J)
      C2=EY(J)+F(J)
      F(J)=C1
      G(J)=C2
   62 WRITE (I2,59) LABEL(J),C1,Y(J),C2
      RETURN
      END


      FUNCTION F2DEYF (Y,EY,A,A1,NX,X,I,K,NY)
      DIMENSION X(10,500), A(10), Y(500), EY(500)
      DOUBLE PRECISION EYF,DEYF,DDEYF
      DOUBLE PRECISION R,S,T,U,V,W,X,Y,A,F,A1,EY,F2DEYF
      F2DEYF=0.0D0
      DO 4 J=1,NY
      R=EYF(X,A,A1,J,NX,NY)
      S=DEYF(X,A,A1,J,I,NX,NY)
      T=DEYF(X,A,A1,J,K,NX,NY)
      U=DDEYF(X,A,A1,J,I,K,NX,NY)
      F=Y(J)-EY(J)
      Z=1.0E-10
      IF (R-Z) 1,1,3
    1 WRITE (6,2) J
    2 FORMAT (1H0//1H0'THE FUNCTION F(EY) IS LESS THAN 1.0E-10 FOR THE '
     1,I3,'TH OBSERVATION.')
      R=1.0E-10
    3 W=(-X(K,J)*R-3*T*F)/(R**4)
      V=X(I,J)*R+F*S
      W=W*V
      V=X(I,J)*T-X(K,J)*S+F*U
      V=V*F/(R**3)
    4 F2DEYF=F2DEYF+W+V
      RETURN
      END


      SUBROUTINE PRPLOT
      IMPLICIT LOGICAL*1(W),LOGICAL*1(K)
      DIMENSION ABNOS(26), X(1), Y(1)
      LOGICAL*1 IMAGE(1),CH,LABEL(1)
      LOGICAL*1 VC,HC,NC,BL
      DATA HC/'-'/,NC/'+'/,BL/' '/
      DATA VC/Z4F/,I3/6/
      ENTRY PLOT1(NHL,NSBH,NVL,NSBV)
      NH=IABS(NHL)
      NSH=IABS(NSBH)
      NV=IABS(NVL)
      NSV=IABS(NSBV)
      NVM=NV-1
      NVP=NV+1
```

```
      NDH=NH*NSH
      NDHP=NDH+1
      NDV=NV*NSV
      NDVP=NDV+1
      NIMG=(NDHP*NDVP)
      RETURN
      ENTRY PLOT2(IMAGE,XMAX,XMIN,YMAX,YMIN)
      YMX=YMAX
      DH=(YMAX-YMIN)/FLOAT(NDH)
      DV=ABS(XMAX-XMIN)/FLOAT(NDV)
      DO 1 I=1,NVP
    1 ABNOS(I)=(XMIN+FLOAT((I-1)*NSV)*DV)
      DO 2 I=1,NIMG
    2 IMAGE(I)=BL
      DO 6 I=1,NDHP
      I2=I*NDVP
      I1=I2-NDV
      KNHOR=MOD(I-1,NSH).NE.0
      IF (KNHOR) GO TO 4
      DO 3 J=I1,I2
    3 IMAGE(J)=HC
    4 CONTINUE
      DO 6 J=I1,I2,NSV
      IF (KNHOR) GO TO 5
      IMAGE(J)=NC
      GO TO 6
    5 IMAGE(J)=VC
    6 CONTINUE
      XMIN1=XMIN-DV/2.
      YMIN1=YMIN-DH/2.
      RETURN
      ENTRY PLOT3(CH,X,Y,N3)
      DO 13 I=1,N3
      IF (DV) 7,8,7
    7 DUM1=(X(I)-XMIN1)/DV
      GO TO 9
    8 DUM1=0
    9 IF (DH) 10,11,10
   10 DUM2=(Y(I)-YMIN1)/DH
      GO TO 12
   11 DUM2=0
   12 CONTINUE
      IF (DUM1.LT.0..OR.DUM2.LT.0.) GO TO 13
      IF (DUM1.GE.NDVP.OR.DUM2.GE.NDHP) GO TO 13
      NX=1+INT(DUM1)
      NY=1+INT(DUM2)
      J=(NDHP-NY)*NDVP+NX
      IMAGE(J)=CH
   13 CONTINUE
      RETURN
      ENTRY PLOT4(NL,LABEL)
      DO 17 I=1,NDHP
      WL=BL
      IF (I.LE.NL) WL=LABEL(I)
      I2=I*NDVP
      I1=I2-NDV
      IF (MOD(I-1,NSH).EQ.0) GO TO 15
      WRITE (I3,14)WL,(IMAGE(J),J=I1,I2)
```

```
14 FORMAT (18X,A1,12X,101A1)
   GO TO 17
15 CONTINUE
   ORDNO=(YMX-FLOAT(I-1)*DH)
   IF (I.EQ.NDHP) ORDNO=YMIN
   WRITE (I3,16)WL,ORDNO,(IMAGE(J),J=I1,I2)
16 FORMAT (18X,A1,G12.5,101A1)
17 CONTINUE
   WRITE (I3,18)(ABNOS(J),J=1,NVP)
18 FORMAT (1H0,24X,G12.5,71X,G12.5)
   RETURN
   END
```

```
   FUNCTION FEYF (X,A,A1,J,NX,ALPHM)
   DIMENSION X(10,500), A(10)
   DOUBLE PRECISION X,A,FEYF,Z,A1,ALPHM
   Z=1.0D-6
   FEYF=100*(X(2,J)-ALPHM)
   FEYF=(FEYF**2+Z)**A1
   RETURN
   END
```

```
   SUBROUTINE HISTO (X,N,M,PA,NX)
   DIMENSION X(1), B(3), ILINE(101), FREQ(52), PCT(52)
   DOUBLE PRECISION XDEL,XMEAN,XSKW,XVAR,XKRT,X,XMIN,XMAX,S
   DOUBLE PRECISION TNORM,Z,T,TDIST,PR,CHIL,CHIU,B,FREQ
   DOUBLE PRECISION DEL,CHI2,XNM,XM,C1,C2,SU,PA
   DATA IB/' '/,IX/'X'/,IBAR/Z4F404040/
   I2=6
   B(1)=XMIN(X,N)
   B(2)=M
   B(3)=XMAX(X,N)
   DO 1 I=1,M
 1 PCT(I)=0
   DEL=(B(3)-B(1))/M
   IF (DEL) 44,44,2
 2 FREQ(1)=B(1)+DEL
   WRITE (I2,3)
 3 FORMAT (1H148X37HSTATISTICS FOR DISTRIBUTION OF RATIOS)
   FREQ(1)=B(1)+DEL
   DO 4 J=2,M
 4 FREQ(J)=FREQ(J-1)+DEL
   FREQ(M)=B(3)
   DO 6 J=1,N
   DO 5 I=1,M
   IF (X(J)-FREQ(I)) 6,6,5
 5 CONTINUE
 6 PCT(I)=PCT(I)+1
   DO 7 I=1,M
 7 PCT(I)=PCT(I)*100./N
   XMEAN=0
   XVAR=0.0D0
   XSKW=0.0D0
   XKRT=0.0D0
   DO 8 J=1,N
 8 XMEAN=XMEAN+X(J)
```

```
      XMEAN=XMEAN/N
      DO 9 J=1,N
      XDEL=X(J)
      XVAR=XVAR+XDEL**2
      XSKW=XSKW+XDEL**3
    9 XKRT=XKRT+XDEL**4
      XVAR=XVAR/N
      XSKW=XSKW/N
      XKRT=XKRT/N
      S=DSQRT(XVAR)
      WRITE (I2,10) M
   10 FORMAT (1H0,'HISTOGRAM OF PERCENT FREQUENCY FOR ',I3,' CLASSES'/)
      WRITE (I2,11)
   11 FORMAT (1H ,102X,19HINTERVAL BOUNDARIES,5X,6H% FREQ)
      T=B(1)
      DO 16 I=1,M
      DO 12 J=1,100
      ILINE(J)=IB
   12 CONTINUE
      IK=PCT(I)+0.5
      IK=IK+1
      DO 13 J=1,IK
      ILINE(J)=IX
   13 CONTINUE
      ILINE(1)=IBAR
      IF (I.EQ.1) GO TO 14
      T=FREQ(I-1)
   14 WRITE (I2,15) ((ILINE(K),K=1,100),T,FREQ(I),PCT(I))
   15 FORMAT (1H ,100A1,E10.3,' TO ',E10.3,F8.2)
   16 CONTINUE
      WRITE (I2,17)
   17 FORMAT (1H0,43X,'SAMPLE STATISTICS OF RATIOS COMPUTED ABOUT ZERO')
      IF (S) 19,18,19
   18 XSKW=0
      XKRT=0
      GO TO 20
   19 XSKW=XSKW/S**3
      XKRT=XKRT/XVAR**2
   20 CONTINUE
      WRITE (I2,21) XMEAN,XVAR,XSKW,XKRT
   21 FORMAT (1H0,5X,'SAMPLE MEAN = ',E12.5,5X,'SAMPLE VARIANCE = ',E12.
     15,5X,'SKEWNESS = ',E12.5,5X,'KURTOSIS = ',E12.5)
      XM=.5D0
      XNM=DFLOAT(N)/M
      M1=M-1
      T=0
      DO 25 I=1,M1
      T=DFLOAT(I)/M
      IF (T-XM) 23,24,22
   22 Z=1-T
      FREQ(I)=S*TNORM(Z)
      GO TO 25
   23 FREQ(I)=-S*TNORM(T)
      GO TO 25
   24 FREQ(I)=0
   25 CONTINUE
      DO 26 I=1,M
   26 PCT(I)=0
```

```
      DO 28 J=1,N
      DO 27 I=1,M
      IF (X(J)-FREQ(I)) 28,28,27
   27 CONTINUE
   28 PCT(I)=PCT(I)+1
      CHI2=0
      DO 29 I=1,M
   29 CHI2=CHI2+(PCT(I)-XNM)**2
      CHI2=CHI2/XNM
      N1=M1-1
      CALL CHISQ (N1,PA,CHIL,CHIU)
      CHIL=0
      IP=(1-PA)*100
      WRITE (I2,30)
   30 FORMAT (//44X,'TEST FOR GOODNESS OF FIT TO NORMAL DISTRIBUTION')
      WRITE (I2,31)
   31 FORMAT (1H0,49X,8HINTERVAL,13X,8HOBSERVED,7X,8HEXPECTED)
      WRITE (I2,32) FREQ(1),PCT(1),XNM
   32 FORMAT (1H ,39X,16H- INFINITY   TO ,E12.5,5X,F6.2,9X,F6.2)
      IF (M-2) 36,36,33
   33 DO 34 I=2,M1
   34 WRITE (I2,35) (FREQ(I-1),FREQ(I),PCT(I),XNM)
   35 FORMAT (1H ,39X,E12.5,4H TO ,E12.5,5X,F6.2,9X,F6.2)
   36 WRITE (I2,37) FREQ(M1),PCT(M),XNM
   37 FORMAT (1H ,39X,E12.5,14H TO + INFINITY,7X,F6.2,9X,F6.2)
      WRITE (I2,38) IP,CHIL,CHIU
   38 FORMAT (1H029X,I2,54H% CONFIDENCE INTERVAL FOR TESTING GOODNESS OF
     1 FIT IS (,F3.1,1H,,E12.5,1H))
      WRITE (I2,39) CHI2
   39 FORMAT (1H 38X,43HCHI SQUARE VALUE FOR SAMPLE DISTRIBUTION = E12.5
     1)
      N1=N
      SU=S/SQRT(FLOAT(N1))
      PR=PA/2
      C1=SU*TDIST(PR,N1)
      C2=-C1
      WRITE (I2,40) IP,C2,C1
   40 FORMAT (1H031X,I2,42H% CONFIDENCE INTERVAL FOR SAMPLE MEAN IS (,E1
     12.5,1H,,E12.5,1H))
      CALL CHISQ (N1,PR,C1,C2)
      IF (C1) 41,42,41
   41 C1=XVAR/C1
      C2=XVAR/C2
   42 C1=DSQRT(C1)
      C2=DSQRT(C2)
      WRITE (I2,43) IP,C2,C1
   43 FORMAT (1H 28X,I2,49H% CONFIDENCE INTERVAL FOR STANDARD DEVIATION
     1IS (,E12.5,1H,,E12.5,1H))
   44 RETURN
      END
```

```
      FUNCTION FRL (RL)
      DOUBLE PRECISION FRL,RL
      FRL=RL
      RETURN
      END
```

```
      SUBROUTINE EIGEN (A,R,B,N,NSD)
      DIMENSION A(1), R(1), B(1)
      DOUBLE PRECISION A,R,ANORM,ANRMX,THR,X,Y,SINX,SINX2,COSX,COSX2,SINE
     1CS,RANGE,B,W,W1,Z,Q
      MSDR=15
      MSD=15
      MV=NSD
      RANGE=1.0D-14
      NX2=N*(N+1)/2
      DO 1 I=1,NX2
    1 B(I)=A(I)
      IF (MV-1) 2,5,2
    2 IQ=-N
      DO 4 J=1,N
      IQ=IQ+N
      DO 4 I=1,N
      IJ=IQ+I
      R(IJ)=0.0D0
      IF (I-J) 4,3,4
    3 R(IJ)=1.0D0
    4 CONTINUE
    5 ANORM=0.0D0
      DO 7 I=1,N
      DO 7 J=I,N
      IF (I-J) 6,7,6
    6 IA=I+(J*J-J)/2
      ANORM=ANORM+A(IA)*A(IA)
    7 CONTINUE
      IF (ANORM) 33,33,8
    8 ANORM=DSQRT(2*ANORM)
      ANRMX=ANORM*RANGE/DFLOAT(N)
      IND=0
      THR=ANORM
    9 THR=THR/DFLOAT(N)
   10 L=1
   11 M=L+1
   12 MQ=(M*M-M)/2
      LQ=(L*L-L)/2
      LM=L+MQ
      IF (DABS(A(LM))-THR) 26,13,13
   13 IND=1
      LL=L+LQ
      MM=M+MQ
      Z=A(LL)
      Q=A(MM)
      X=(Z-Q)/2
      Y=-A(LM)/DSQRT(A(LM)*A(LM)+X*X)
      IF (X) 14,15,15
   14 Y=-Y
   15 SINX=Y/DSQRT(2*(1+(DSQRT(1-Y*Y))))
      SINX2=SINX*SINX
      COSX2=1-SINX2
      COSX=DSQRT(COSX2)
      SINCS=SINX*COSX
      ILQ=N*(L-1)
      IMQ=N*(M-1)
      DO 25 I=1,N
```

```
   IQ=(I*I-I)/2
   IF (I-L) 16,23,16
16 IF (I-M) 17,23,18
17 IM=I+MQ
   GO TO 19
18 IM=M+IQ
19 IF (I-L) 20,21,21
20 IL=I+LQ
   GO TO 22
21 IL=L+IQ
22 X=A(IL)*COSX
   W=A(IM)*SINX
   Y=A(IL)*SINX
   W1=-A(IM)*COSX
   A(IM)=Y-W1
   A(IL)=X-W
23 IF (MV-1) 24,25,24
24 ILR=ILQ+I
   IMR=IMQ+I
   Z=R(ILR)*COSX
   Q=R(IMR)*SINX
   X=Z-Q
   Z=R(ILR)*SINX
   Q=-R(IMR)*COSX
   R(IMR)=Z-Q
   R(ILR)=X
25 CONTINUE
   X=2*A(LM)*SINCS
   Z=A(LL)*COSX2
   Q=-A(MM)*SINX2
   Y=Z-Q
   Z=A(LL)*SINX2
   Q=-A(MM)*COSX2
   W=Z-Q
   W1=-W
   A(LM)=0
   A(LL)=Y-X
   A(MM)=W+X
26 IF (M-N) 27,28,27
27 M=M+1
   GO TO 12
28 IF (L-(N-1)) 29,30,29
29 L=L+1
   GO TO 11
30 IF (IND-1) 32,31,32
31 IND=0
   GO TO 10
32 IF (THR-ANRMX) 33,33,9
33 IQ=-N
   DO 37 I=1,N
   IQ=IQ+N
   LL=I+(I*I-I)/2
   JQ=N*(I-2)
   DO 37 J=I,N
   JQ=JQ+N
   MM=J+(J*J-J)/2
   IF (A(LL)-A(MM)) 34,37,37
34 X=A(LL)
```

```
      A(LL)=A(MM)
      A(MM)=X
      IF (MV-1) 35,37,35
   35 DO 36 K=1,N
      ILR=IQ+K
      IMR=JQ+K
      X=R(ILR)
      R(ILR)=R(IMR)
   36 R(IMR)=X
   37 CONTINUE
      IF (MV-1) 38,48,38
   38 CONTINUE
      DO 45 I2=1,N
      Y=A(I2*(I2+1)/2)
      I1=N*(I2-1)
      DO 43 L=1,N
      X=0
      DO 42 M=1,N
      IF (M-L) 39,40,40
   39 I=L*(L-1)/2+M
      GO TO 41
   40 I=M*(M-1)/2+L
   41 K=I1+M
   42 X=X+B(I)*R(K)
      Z=Y*R(I1+L)
      NSD=LISD(X,Z,15)
   43 MSD=MIN(NSD,MSD)
      MSDR=MIN(MSD,MSDR)
      K1=I2*(I2+1)/2+1
      IF (I2-N) 44,46,46
   44 A(K1)=MSD
   45 MSD=15
   46 DO 47 I=1,N
      K=I*(I+1)/2
      L=K+1
   47 B(K)=A(L)
      B(NX2)=MSD
      NSD=MSDR
   48 CONTINUE
      RETURN
      END


      FUNCTION LISD (X,Y,N)
      DOUBLE PRECISION X,Y,EPS,T
      LISD=0
      IF (DABS(Y)-DABS(X)) 1,2,2
    1 T=X
      IF (T) 3,7,3
    2 T=Y
      IF (T) 3,7,3
    3 EPS=DLOG10(DABS(T))
      IX=EPS
      IF (EPS) 4,6,6
    4 IF (EPS-IX) 5,6,6
    5 IX=IX-1
    6 IF (Y-X) 8,7,8
    7 LISD=N
```

```
      GO TO 12
    8 T=DLOG10(DABS(Y-X))
      IT=T
      IF (T) 9,11,11
    9 IF (T-IT) 10,11,11
   10 IT=IT-1
   11 LISD=IX-IT-1
      LISD=MAX(LISD,0)
   12 CONTINUE
      RETURN
      END


      SUBROUTINE DINV (C,A,NX,IER,DET,NSD)
      DIMENSION C(10,10), A(10,10), S(10,10)
      DOUBLE PRECISION C,A,S,T,PIV,TOL,B,DET,XN,T2,EPS
      EPS=1.0D-15
      DET=1.0D0
      IER=NX
      PIV=DABS(A(1,1))
      DO 3 K=1,NX
      DO 3 I=1,NX
      S(I,K)=A(I,K)
      C(I,K)=0.0D0
      IF (PIV-DABS(A(I,K))) 1,1,2
    1 PIV=DABS(A(I,K))
      TOL=EPS*PIV
    2 CONTINUE
    3 CONTINUE
      DO 4 I=1,NX
    4 C(I,I)=1.0D0
      DO 21 L=1,NX
      PIV=DABS(A(L,L))
      DO 7 I=L,NX
      IF (PIV-DABS(A(I,L))) 5,5,6
    5 PIV=DABS(A(I,L))
      IP=I
    6 CONTINUE
    7 CONTINUE
      IF (PIV) 11,10,8
    8 IF (PIV-TOL) 9,12,12
    9 IER=IER-1
      GO TO 12
   10 IER=-1
      GO TO 22
   11 CALL DUMP (40000,120000,0)
   12 CONTINUE
      PIV=A(IP,L)
      DET=DET*PIV
      DO 13 J=1,NX
      C(IP,J)=C(IP,J)/PIV
   13 A(IP,J)=A(IP,J)/PIV
      A(IP,L)=1.D0
      DO 16 I=1,NX
      B=A(I,L)
      IF (I-IP) 14,16,14
   14 DO 15 J=1,NX
      C(I,J)=C(I,J)-B*C(IP,J)
```

```
15 A(I,J)=A(I,J)-B*A(IP,J)
   A(I,L)=0.0D0
16 CONTINUE
   DO 17 J=1,NX
   T=C(IP,J)
   C(IP,J)=C(L,J)
   C(L,J)=T
   T=A(IP,J)
   A(IP,J)=A(L,J)
17 A(L,J)=T
   IF (IP-L) 18,19,18
18 XN=-1.0D0
   GO TO 20
19 XN=1.0D0
20 DET=DET*XN
21 CONTINUE
22 CONTINUE
   IF (IER) 23,24,24
23 DET=0.0D0
   NSD=0
   RETURN
24 NSD=15
   DO 30 I=1,NX
   DO 30 K=1,NX
   T=0.0D0
   T2=0
   A(I,K)=0.0D0
   DO 27 J=1,NX
   IF (C(I,J)*S(J,K)) 25,26,26
25 T2=T2-C(I,J)*S(J,K)
   GO TO 27
26 A(I,K)=A(I,K)+C(I,J)*S(J,K)
27 CONTINUE
   IF (I-K) 29,28,29
28 T=1.0D0
29 T2=T2+T
   NSD=MIN(NSD,LISD(A(I,K),T2,15))
30 CONTINUE
   RETURN
   END

   SUBROUTINE WORK (C,A,NX,IER,DET)
   DIMENSION C(10), A(10,10)
   DOUBLE PRECISION C,A,T,PIV,TOL,B,DET,XN,EPS
   DET=1.0D0
   EPS=1.0D-15
   IER=NX
   PIV=DABS(A(1,1))
   DO 3 K=1,NX
   DO 3 I=1,NX
   IF (PIV-DABS(A(I,K))) 1,1,2
 1 PIV=DABS(A(I,K))
   TOL=EPS*PIV
 2 CONTINUE
 3 CONTINUE
   DO 20 L=1,NX
   PIV=DABS(A(L,L))
```

```
      DO 6 I=L,NX
      IF (PIV-DABS(A(I,L))) 4,4,5
    4 PIV=DABS(A(I,L))
      IP=I
    5 CONTINUE
    6 CONTINUE
      IF (PIV) 10,9,7
    7 IF (PIV-TOL) 8,11,11
    8 IER=IER-1
      GO TO 11
    9 IER=-1
      GO TO 21
   10 CALL DUMP (40000,120000,0)
   11 CONTINUE
      PIV=A(IP,L)
      DET=DET*PIV
      DO 12 J=L,NX
   12 A(IP,J)=A(IP,J)/PIV
      C(IP)=C(IP)/PIV
      A(IP,L)=1.D0
      DO 15 I=1,NX
      B=A(I,L)
      IF (I-IP) 13,15,13
   13 DO 14 J=L,NX
   14 A(I,J)=A(I,J)-B*A(IP,J)
      A(I,L)=0.0D0
      C(I)=C(I)-B*C(IP)
   15 CONTINUE
      IF (IP-L) 16,18,16
   16 XN=-1.0D0
      DO 17 J=1,NX
      T=A(IP,J)
      A(IP,J)=A(L,J)
   17 A(L,J)=T
      T=C(IP)
      C(IP)=C(L)
      C(L)=T
      GO TO 19
   18 XN=1.0D0
   19 DET=DET*XN
   20 CONTINUE
   21 CONTINUE
      IF (IER) 22,23,23
   22 DET=0.0D0
   23 CONTINUE
      RETURN
      END

      FUNCTION MIN(M,N)
      IF(M-N) 2,2,1
    1 MIN=N
      RETURN
    2 MIN=M
      RETURN
      END
```

```
      FUNCTION MAX(M,N)
      IF(M-N) 1,1,2
    1 MAX=N
      RETURN
    2 MAX=M
      RETURN
      END
```