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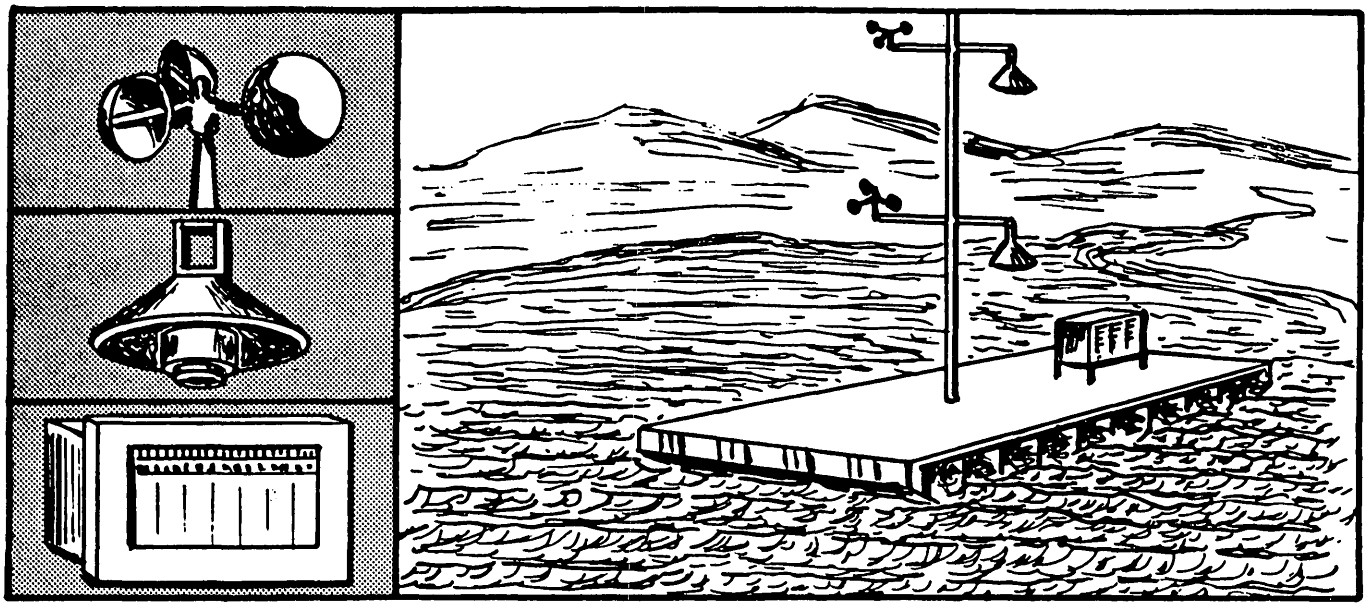

EVALUATION OF TURBULENT TRANSFER LAWS USED IN COMPUTING EVAPORATION RATES

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CONTENTS

SYMBOLS

Symbol	Definition
A	Area of reservoir in acres.
A_T	Constant defined by equation 71.
A_u	Constant defined by equation 80.
B	Coefficient defined by equation 41.
B'	Coefficient defined by equation 44.
B_T	Constant defined by equation 72.
B_u	Constant defined by equation 81.
B_q	Constant defined by equation 84.
C_D	Drag coefficient.
C_p	Coefficient of specific heat at constant pressure.
C_T	Constant of integration for the temperature profile.
C_q	Constant of integration for the specific humidity profile.
$D_{\ell z}$	Constant defined by equation 51.
$D_{t\ell z}$	Constant defined by equation 78.
D_{tz}	Constant defined by equation 77.
$D_{u\ell z}$	Constant defined by equation 50.
$D_{q\ell z}$	Constant defined by equation 54.
D_{qz}	Constant defined by equation 86.
D_z	Constant defined by equation 75.
$D_{z\ell z}$	Constant defined by equation 76.
E	Rate of evaporation.

Symbol	Definition
e_a	Partial pressure of water vapor.
$\overline{e}_a$	Arithmetic mean of the partial pressure of water vapor measured at the 2, 4, and 8-meter levels.
e_o	Saturation vapor pressure of air which is at a temperature equal to that of the water surface.
e_{sp}	Saturation vapor pressure of air.
F_G	Buoyant force.
$f(u)$	Coefficient in Dalton's law or the wind function.
g	Acceleration of gravity.
H	Vertical flux of sensible heat.
L	Monin and Obukhov length scale.
N	Coefficient of proportionality in the wind function or the empirical mass-transfer coefficient.
P	Atmospheric pressure.
R	Gas constant.
r	Coefficient of correlation.
S	Atmospheric stability as defined by equation 68.
S T	Stability parameter defined by equation 56.
T	Absolute temperature of air.
T_o	Water temperature
T'	Virtual temperature defined by equation 20.
T_*	Parameter defined by equation 26.
T_8	Temperature of air at 8 meters.
T_w	Wet-bulb temperature.
U	Velocity of air.

Symbol	Definition
u	Mean value of the horizontal component of the wind velocity.
u'	Horizontal component of the turbulent velocity fluctuation.
u_*	Shear velocity.
u_*'	Error in the apparent value of the shear velocity.
u_*^a	Apparent value of the shear velocity.
$\tilde{u}_*$	True value of the shear velocity.
u_2	Wind velocity at 2 meters.
u_8	Wind velocity at 8 meters.
v'	Vertical component of the turbulent velocity fluctuation.
Q_*	Parameter defined by equation 13.
Q_*'	Error in the apparent value of Q_*.
Q_*^a	Apparent value of Q_*.
$\tilde{Q}_*$	True value of Q_*.
q	Specific humidity of air.
q_o	Specific humidity of saturated air which has a temperature equal to that of the water surface.
x	Horizontal coordinate in the direction of the wind.
y	Horizontal coordinate normal to the direction of the wind.
z	Vertical coordinate direction.

Symbol	Definition
z_o	Roughness length.
α	Monin - Obukhov coefficient.
Γ	Adiabatic lapse rate.
δ	Thickness of the boundary layer.
ε_H	Turbulent transfer coefficient for sensible heat.
ε_m	Turbulent transfer coefficient for momentum.
ε_Ω	Turbulent transfer coefficient for a scalar quantity.
θ	Potential temperature.
θ_0	Potential temperature of the water surface.
κ	Von Karman coefficient.
ρ	Density of air.
σ	Standard deviation.
τ	Shear stress.
Ω	Scalar quantity.
Ω'	Fluctuating component of a scalar quantity.

ILLUSTRATIONS

Page

Page

ABSTRACT

Although the process of evaporation has received the attention of hydrologists, meteorologists, and agriculturalists for many years, the measurement or estimation of the rate of evaporation from water surfaces still is not an easy matter. The aerodynamic method of computing evaporation rates has a number of significant advantages over other methods and in certain situations it is about the only way in which the evaporation rate can be measured. In order to use the aerodynamic method some functional form describing the variation of wind velocity with elevation must be assumed. Although the logarithmic law appears to be an adequate description when atmospheric conditions are neutrally stable, no wind law has been found which is satisfactory under all conditions of atmospheric stability. The log+linear law, proposed a number of years ago, was specifically designed to extend the applicability of the log law to conditions which are at least nearly neutrally-stable.

A massive set of data collected at Hefner, Oklahoma, were used to evaluate the theoretical correctness and practicality of the log+linear law for computing evaporation rates by the aero-dyamic method. The theoretical correctness of the log+linear law was evaluated by comparing its results with those obtained by use of the log law and the mass-transfer method was used as a reference from which the practicality of the log+linear law can be judged.

The use of the log+linear law produced more accurate predictions of evaporation rates than could be obtained by use of the log law. There was a strong indication that the log+linear law at least partly accounted for the atmospheric stability effects. The results of the log+linear law were found to be almost independent of the assumed value of the Monin - Obukhov coefficient, α , as long as α was within the range 1 to 3. Provided that the measurement errors in the velocities are averaged out in a prescribed manner, the log+linear law can be expected to provide monthly evaporation rates which are accurate to within 17 percent. This accuracy approaches that which can be expected from the mass-transfer method.

INTRODUCTION

The process of evaporation has received the attention of hydrologists, meteorologists, and agriculturalists for many years. For example, Roberts (1969, p. 669) reports that Benjamin Franklin attempted to reduce the evaporation from a small pond in 1765 by spreading a thin film on its surface. In 1908 the Weather Bureau, in cooperation with the Reclamation Service and the U. S. Geological Survey, began a project to measure the evaporation from the Salton Sea, California (Roberts, 1969, p. 667). Since evaporation is a very large factor in the hydrologic cycle, there is great need for accurate evaporation information for water resources planning.

Increasing industrialization in this country has caused a dramatic increase in the quantity of water which is used for cooling purposes. Almost one half of all the water used in the United States is utilized for cooling (FWPCA, 1968, p. 5), and a major part of the excess energy which is added to a water system by the cooling water is ultimately transferred to the atmosphere as a result of increased evaporation. Evaporation is a major factor in the determination of the effect of thermal loading on water systems. Unfortunately, the measurement or estimation of the rate of evaporation from water surfaces is by no means an easy matter.

At least six methods are currently used in order to measure the evaporation rate from different water systems. These are the water-budget method, the energy-budget method, the empirical mass-transfer method, the aerodynamic or gradient method, the evaporation-pan method, and the eddy-correlation method. This report will concern itself with only the aerodynamic and the empirical mass-transfer methods; however, in order that these methods may be put into perspective, each method will be discussed very briefly in what follows.

The least complicated is the water-budget method, which applies the simple conservation of matter principle to a control volume. It involves a simple equation which states that the evaporation is equal to the total inflow minus the outflow,plus or minus any change in storage. While the method is simple, in practice it is almost impossible to measure the terms of the equation with sufficient accuracy to determine a reasonable value for the evaporation. The difficulty is that in most cases the inflow and outflow terms are large in comparison to the evaporation term so that the evaporation must be computed from the difference of two large numbers which are nearly equal in size. Only rarely can this method be successfully applied to real situations.

A variation of the water-budget method is a method in which the control volume is the air above the water. In this method the flux of water vapor approaching the lake and leaving the lake is determined by measurements of wind velocity and air humidity, upwind and downwind of the lake. The difference in these fluxes is the rate of evaporation from the lake. This method would not be expected to be generally applicable, but in certain special cases it may be quite useful. For example, Wiersma (1970, p. 50) found this method to be the most accurate way to determine the magnitude of evaporation losses from a sprinkled field of bromus grass.

The energy-budget method, like the water-budget method, applies the conservation principle to the water body. Energy is the conservative quantity in the energy-budget method, and the terms are more numerous and difficult to measure, but the computed evaporation is not as sensitive to small errors in the mass inflow and outflow terms. For lakes of moderate size and with reasonably small water inflows and outflows, the energy-budget is probably the most accurate and practical method of determining yearly or monthly evaporation rates.

The mass-transfer method is based on Dalton's law, which states that the evaporation rate is proportional to the vapor pressure gradient between the evaporating surface and air above the surface. The constant of proportionality, often called the wind function, has been assumed to take many forms, but in any case it is primarily dependent upon wind speed. The mass-transfer method requires relatively few, simple measurements and is quite accurate if the value of the wind function is known. The wind function varies in a complex manner with many variables and in general must be determined independently for each reservoir.

The aerodynamic or gradient method relates the velocity and humidity gradients of the air in the vertical direction to the rate of evaporation from the underlying surface. A mixing coefficient is determined from the velocity gradient and an assumed functional relationship between wind speed and elevation. The vertical flux of water vapor is then determined from the humidity gradient, the mixing coefficient, and an assumed functional relationship between humidity and elevation. The evaporation rate is considered equal to the vertical flux of water vapor. Many different functional forms of the relation between wind speed and elevation have been proposed and tested. The aerodynamic method requires accurate measurements of velocity and humidity gradients which are difficult to obtain. It also suffers because the form of the functional relation between wind speed and elevation has defied accurate definition.

The most common method of estimating evaporation rates is to measure the evaporation from an evaporation pan. While this method is convenient, the relation between the rate of evaporation from the pan and the rate of evaporation from a neighboring body of water is difficult to estimate.

The eddy-correlation method determines the vertical flux of water vapor from the correlation of the turbulent components of the variation in absolute humidity at a point and the vertical component of the turbulent fluctuations in wind speed at the same point. Instrumentation problems involved in the application of this method are almost insurmountable.

As can be inferred from even this brief description of the various methods, each method has advantages and disadvantages and none of the methods can be said to be the best under all circumstances. The method used depends entirely upon the situation under consideration.

In certain situations the aerodynamic method may be the only way in which evaporation can be measured. For example, in estuaries the inflow and outflow terms are too large for either the water-budget or the energy-budget method to be applied,and both the mass-transfer and the evaporation-pan methods should have some independent measure of the evaporation rate in order to determine required empirical coefficients. In situations like this, one is forced to accept the disadvantages of the aerodynamic method and to attempt to use it. In addition, the aerodynamic method has a number of significant advantages over the other methods. First, its application requires no empirical coefficient as is necessary for the mass-transfer or evaporation-pan methods. Second, the evaporation rate can be determined for very short time periods while the energy and water-budget methods can only give evaporation rates which represent long term average values. Third, all measurements are made in the air away from the surface, so that the character of the underlying surface is immaterial. Also, the aerodynamic method can theoretically be used to determine the evaporation rate from a relatively small portion of a large body of water; whereas the energy-budget and water budget methods can only be used to determine the average rate of evaporation from the entire body of water.

For these reasons, as well as others, there has been a great interest in improving the aerodynamic method for many years. As a result, many functional forms of the relation between wind speed and elevation have been proposed. After careful analysis, all of these forms are found to be deficient in fully accounting for the effect of atmospheric stability.

Monin and Obukhov (1959) proposed a functional form that is known as the log+linear law and which was designed to account for stability effects, at least under near-neutral conditions. This law appears to be theoretically sound, and while it has been tested under limited conditions (Webb, 1970), few sets of data are available which are extensive enough to determine its general applicability under widely varying conditions.

A joint project (U.S. Geological Survey, 1954b) undertaken by the Weather Bureau and four other government agencies at Lake Hefner near Oklahoma City, Oklahoma, provided a data set which is extensive enough to determine the general applicability of the law. This comprehensive evaporation research project provided very good estimates of the daily evaporation rates from a water-budget method as well as measurements of wind velocity, temperature, and humidity at four elevations over the center of the lake every 30 minutes for an entire 15-month period.

The purpose of this report is to make use of this massive set of data, in order to evaluate the theoretical correctness and practicality of the log+linear law for computing evaporation rates. This purpose will be partly fulfilled by answering two questions. First, does the use of the log+linear law, instead of other laws, improve the estimate of evaporation when the atmosphere departs from the neutrally stable condition? Second, what accuracy can be expected when the log+linear law is used in conjunction with data of the type and quality collected at Lake Hefner? In answering the second question, the shortcomings of the aerodynamic method will be illustrated and some ways in which these shortcomings may be minimized will be demonstrated in the discussion of the first question.

In order to apply the log+linear law, the value of α, a constant in the Monin Obukov model, was first determined from the Lake Hefner data using a method proposed by Deacon (1962). The evaporation rate is then determined for each 30-minute period by using the log+linear law and an average daily evaporation is computed from these figures. These daily evaporation quantities are then compared to the evaporation rate which was determined from the water-budget. By assuming the water-budget evaporation to be exact, the error in the aerodynamic method can be determined for each day of record. This error is then correlated with average wind velocity, specific humidity and atmospheric stability.

In order to serve as a basis for comparison, the same type of analysis is performed using the logarithmic (log) law and the mass-transfer method.

Following a method proposed by Pasquill (Sutton, 1953, p. 311) a procedure for reducing the measurement and model errors is devised and tested using both the log and log+linear laws.

The theoretical development of both the log and log+linear laws will be presented in brief form initially, as well as the empirical wind function which will be used to serve as a basis of comparison. Following this, a brief description of the Lake Hefner data will be given. The method of computations which was used in the application of the log law will then be discussed and the results of these computations presented. This will be followed by a similar discussion of the methods used in the application of the log+linear law and these results will be presented. The results obtained from the direct application of the log law will be compared to those obtained from the direct application of the log+linear law. The effectiveness of the modified Pasquill approach will then be discussed and its effectiveness when applied to the data will be evaluated. Using the Pasquill approach, results obtained from the log+linear law will then be compared to the results obtained from the log law. Finally, the log+linear law, as modified by the Pasquill approach, will be compared to the empirical mass-transfer approach and the practicality of the aerodynamic method will be discussed.

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THEORETICAL DEVELOPMENT

In this section some basic concepts are discussed which are involved when any velocity law is used in the aerodynamic method of computing evaporation rates. Following the discussion of these concepts a brief development and discussion of the logarithmic velocity law is presented, which in turn, is followed by an equally abbreviated derivation and discussion of the log+linear law. Finally, the empirical mass-transfer formula is presented along with a discussion of the necessary empirical wind function.

As a real fluid flows past a solid boundary the effects of viscosity produce a velocity profile which is characterized by a zero velocity at the solid surface and a velocity gradient which generally decreases with increasing distance above the surface. The exact nature of the velocity profile is governed by the character of the underlying surface, both immediately below the point of observation and for a considerable distance upstream of this point. The influence of the underlying surface on the velocity profile decreases with increasing distance from the surface. The zone in which the velocity profile is primarily governed by the underlying surface is known as the boundary layer.

The boundary layer phenomena may be most easily visualized by considering an infinite fluid with a constant velocity U flowing past a semi-infinite flat plate. This process is illustrated on figure 1. The velocity must be zero at the plate but for regions very near to the beginning of the plate, the velocity is equal to its initial value of U at small distances above the plate. Therefore a very large velocity gradient exists near the beginning of the plate . This large velocity gradient results in a large shearing stress which exerts a retarding force on the surrounding fluid particles. The shearing stress retards the flow at distances further and further from the boundary, so that the thickness of the layer of retarded fluid increases in the downstream direction. By definition, the boundary layer thickness, δ, is the thickness of the layer of fluid which has been retarded by the boundary.

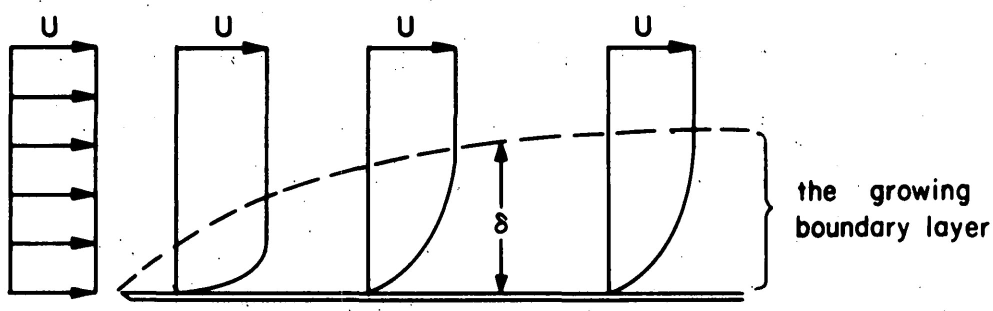

Figure 1. Growth of a boundary layer along one side of a flat plate (δ is the thickness of the boundary layer).

For turbulent flow, almost the entire retarding force of the boundary is transmitted through the fluid by the turbulence stresses. Because Osborn Reynolds was the first to demonstrate the importance of the turbulence stress terms, they are often called Reynolds stresses (Hinze, 1959, p. 19). The turbulence stress term of interest here results from the vertical flux of momentum and is related to the turbulence components of the velocity by the relation

$$\tau = -\rho \, \overline{u' \, v'} \qquad (1)$$

where τ = shear stress; ρ = density of the fluid; u' = horizontal component of the turbulent velocity fluctuation; v' = vertical component of the turbulent velocity fluctuation; and the overbar indicates a time averaged value. After comparing the turbulence stress terms in the equation of motion to the corresponding stress terms caused by viscosity effects, Bousinesq introduced the concept of an "apparent" or "turbulence" or "eddy" viscosity, ε_m (Hinze, 1959, p. 20). The turbulence stress is given by

$$\tau = \varepsilon_m \, \rho \, \frac{\partial u}{\partial z} \qquad (2)$$

where ε_m = turbulent transfer coefficient for momentum; u = mean value of the horizontal velocity; and z = vertical coordinate. Equation 2 indicates that the turbulence stress results from the diffusive transfer of momentum and consequently, that the boundary layer growth can be visualized as the diffusive spread of the momentum deficit caused by the boundary.

The diffusion of scalar quantities such as thermal energy or matter can be considered in a similar manner. Considering the vertical diffusive flux of a scalar quantity, Ω, in an incompressible turbulent flow, the flux can be determined as

$$\text{Flux} = \rho\, \overline{\Omega' v'} \tag{3}$$

where Ω' is the fluctuating component of the scalar quantity Ω. Introducing the Bousinesq coefficient for turbulent diffusion, the vertical flux is

$$\text{Flux} = -\rho\, \varepsilon_\Omega \frac{\partial \Omega}{\partial z} \tag{4}$$

where ε_Ω is the turbulent transfer coefficient for the scalar quantity Ω. The negative sign is used because the diffusive flux is always in the direction of a decreasing magnitude of Ω. It is generally assumed that the value of ε_Ω is proportional or equal to the value of ε_m. The relation of ε_Ω to ε_m has received wide attention, but at present there seems to be little reason to assume the value of ε_Ω to be different than the value of ε_m (Deacon and Swinbank, 1958; Brutsaert, 1965; Webb, 1970).

The resisting stress exerted by a water surface on a moving layer of air is generally different than that which a land surface exerts on the moving air. The change in shear stress, which occurs as the air passes from land to water, causes a boundary layer to be developed over the water similar to that which is developed over a flat plate. The thickness of the boundary layer over the lake represents the distance above the water surface for which the velocity distribution is governed by the resistance characteristics of the lake surface.

As air passes over the water surface it absorbs water vapor. This vapor is diffused upward into the air stream by turbulence, in the same manner as the momentum perturbation caused by the resistance characteristics of the water surface is diffused upward. A vapor blanket or vapor boundary layer thus exists over a water surface which is similar to the momentum boundary layer. The development of the vapor boundary layer is illustrated on figure 2. The path line of a typical evaporated particle is also shown on figure 2. At very low elevations relative to the water surface the air can be considered saturated with water vapor at all times so that the rate of change of specific humidity with respect to distance in the direction of the wind is zero except at the waters edge. This rate of change of specific humidity increases with elevation and should obtain its maximum value somewhere near the upper extremity of the boundary layer. In computing the evaporation rate by use of the aerodynamic method, one must generally assume that the gradient of humidity in the direction of the wind is zero. This is the constant flux assumption and states simply that the total evaporation is diffused vertically through the air. It is easily seen that the validity of the constant flux assumption decreases with increasing elevation above the water surface. It is mandatory, therefore, that measurements be made well within the vapor blanket when the aerodynamic method is used. Because the thickness of the boundary layer increases rather slowly, the aerodynamic method is impractical when short fetches are involved.

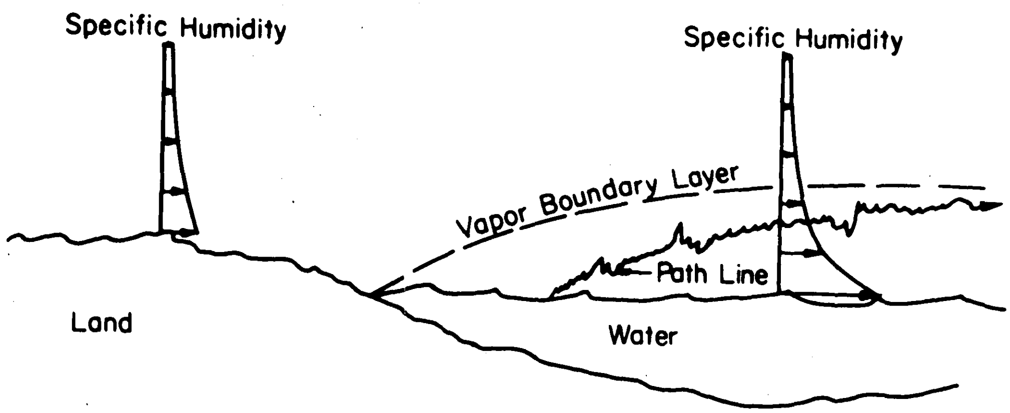

Figure 2. Schematic representation of the development of a vapor boundary layer.

In the analysis of wind structure in the lowest layers of the atmosphere, it is advantageous to regard the air as part of a fully developed turbulent boundary layer in which both the Coriolis force and changes in the gradient of pressure in the direction of the wind are negligible. For steady motion this implies that the shearing stress is invariant with elevation. These assumptions restrict the analysis to motion in a layer of depth not exceeding a few tens of meters. A system of axes is defined in which x is measured in the direction of the wind, y across the wind, and z vertically.

For a fully turbulent, neutrally stable atmosphere, the air can be considered of uniform density, and the velocity gradient should be a function only of elevation, air density, and shearing stress. Or symbolically

$$\frac{\partial u}{\partial z} = f(z, \rho, \tau) \qquad (5)$$

Dimensional analysis yields

$$\frac{\partial u}{\partial z} = f\left(\frac{u_*}{z}\right) \qquad (6)$$

where u_* is the shear velocity defined as the square root of the shear stress divided by the density. The simplest possible functional form for equation 6 is

$$\frac{\partial u}{\partial z} = \frac{u_*}{\kappa z} \qquad (7)$$

where κ is the familiar Von Karmon coefficient which has a value of about 0.4. Upon integration equation 7 yields

$$u = \frac{u_*}{\kappa} \ln \frac{z}{z_0} \qquad (8)$$

where z_0 is a constant length scale characterizing the roughness of the underlying surface which is often called the roughness height

Physically z_0 is the elevation at which the velocity, as computed from equation 8, is zero. Because the flow must be fully turbulent for equation 8 to be valid, the relation probably breaks down for elevations approaching z_0. The suggestion that the natural wind profile should conform to equation 8 for conditions of neutral stability appears to have originated with Prandtl in 1932 (Sutton, 1953, p. 232).

It is convenient to introduce a drag coefficient, C_D for later use

$$C_D = \left(\frac{u_*}{u}\right)^2 \tag{9}$$

The turbulent transfer coefficient for momentum is easily determined from equations 2 and 7

$$\epsilon_m = \kappa \, u_* \, z \tag{10}$$

Assuming that the turbulent transfer coefficient for water vapor is equal to that for momentum, equation 10 can be substituted into equation 4 to determine the vertical flux of water vapor

$$\text{Flux} = -\rho \, \kappa \, u_* \, z \frac{\partial q}{\partial z} \tag{11}$$

where q is the specific humidity of the air. In the region of the boundary layer where the constant flux assumption is valid, the flux is independent of elevation and is therefore equal to the rate of evaporation, E. Within this region of the boundary layer equation 11 can be integrated to give

$$q = Q_* \ln z + C_q \tag{12}$$

where C_q is a constant of integration and

$$Q_* = - \frac{E}{\rho \, \kappa \, u_*} \tag{13}$$

In equation 12, the value of q approaches infinity as the value of z approaches zero. Physically, as z approaches some small value, say z_0, the value of q must approach the specific humidity of saturated air which has a temperature equal to that of the water surface. This saturation specific humidity, which will be called q_0, is a reference humidity just as zero is a reference velocity in the velocity distribution. Forcing the value of q to approach the reference humidity, q_0, as z approaches z_0 equation 12 takes a form which is analogous to the equation for the velocity profile

$$q - q_0 = Q_* \ln \frac{z}{z_0} \tag{14}$$

Equation 14 is as general as equation 12,and one might expect that the values of z_0 in equations 8 and 14 to be the same. Unfortunately, such is not the case. Apparently, the validity of equation 14 also breaks down for values of z approaching z_0. Therefore the values of z_0 in equations 8 and 14 must be considered simply as constants of integration and no known relationship exists between them.

Since the value of u_* and z_0 are constant for any velocity profile, the velocity at two levels is required in order to determine the value of u_*. If more than two values of the velocity are available, the value of u_* can be determined such that equation 8 best fits the data. Likewise, two values of specific humidity are required in order to determine Q_* from equation 12 because the value of C_q in this equation is not related to the value of z_0 obtained from the velocity profile. Again, if more than two values of specific humidity are available for any profile, the value of Q_* can be determined, using the method of least squares or some similar procedure, such that equation 12 best fits the measured values of specific humidity. The evaporation rate can then be computed from the values Q_* and u_* using equation 13 .

Evaluating equation 12 at two levels and subtracting, the value of C_q can be eliminated, and

$$Q_* = \frac{q_1 - q_2}{\ln z_1 / z_2} \qquad (15)$$

Evaluating equation 8 at two levels

$$u_* = \kappa \frac{u_1 - u_2}{\ln z_1/z_2} \qquad (16)$$

Combining equations 13, 15, and 16 and assuming the values of z_1 and z_2 are the same for both equations 15 and 16

$$E = \frac{\rho \kappa^2 (u_1 - u_2)(q_1 - q_2)}{\left[\ln z_1/z_2\right]^2} \qquad (17)$$

Equation 17 was originally derived by Thornthwaite and Holzman in 1939 (Priestley, 1959, p. 90).

Marciano and Harbeck (1954, p. 64) have applied this equation to data obtained at the 2- and 4- meter levels of Lake Hefner and found that, while the results tend to scatter around a line of perfect correlation, the computed evaporation rate for individual days was often considerably in error. Marciano and Harbeck attributed much of this scatter to the difficulty in making accurate measurements of the small differences in wind velocity and specific humidity. This statement may be easily justified. The average rate of evaporation for Lake Hefner is about 0.41 cm/day (centimeters per day). Harbeck (1954, p. 7) gives the median wind speed to be about 450 cm/sec (centimeters per second), for which the median shear velocity would be about 29 cm/sec. Combining these figures with equations 15 and 16, typical values of $(u_2 - u_1)$ and $(q_1 - q_2)$ are 73 cm/sec and 0.00065. Both of these differences are small relative to the mean values of the individually measured quantities. A small error in the measurement of any of the four quantities causes a large error in the computed value of E.

Given a neutrally stable atmosphere, all mixing of the air occurs as a result of turbulence and it is generally agreed that equation 8 adequately describes the velocity profile. A neutrally stable atmosphere occurs when the vertical temperature distribution is such that a parcel of air experiences no change in density relative to the surrounding air as it is displaced from one level to another in an adiabatic manner. An atmosphere is neutrally stable when the temperature decreases with increasing elevation at a rate equal to the adiabatic lapse rate.

The adiabatic lapse rate, therefore, supplies a criterion from which atmospheric stability can be determined. If the temperature gradient exceeds the adiabatic lapse rate, a parcel of air which is displaced upward by an infinitesimal amount from a level at which it had had the same temperature and pressure as the surrounding air, will be at a higher temperature than the surrounding air at the new level. It will, therefore, be of a lower density than the surrounding air. The buoyancy force which results from this condition, tends to make the parcel continue to rise. An atmosphere with such a temperature gradient must be statically unstable and a lapse condition is said to prevail. Similarly, in an atmosphere for which the temperature gradient is less than the adiabatic lapse rate, a parcel of air which has been forced upward will be more dense than the surrounding air, and it will tend to sink back to its original level. This type of an atmosphere is statically stable and an inversion condition is said to prevail.

If the temperature gradient is equal to the adiabatic lapse rate, turbulent velocity fluctuations will cause no net flux of sensible heat. The gradient term in equation 4 must be modified when the diffusion of sensible heat is to be considered. It is convenient to define a temperature that can be used directly with equation 4 in order to determine the flux of sensible heat. The potential temperature θ, of dry air is such a temperature and is defined as the temperature which a volume of air assumes when brought adiabatically from its existing pressure to a standard pressure, generally that at the surface (Sutton, 1953, p. 10). The gradient of potential temperature may be expressed in terms of the gradient of absolute temperature, T, and the adiabatic lapse rate, T, as

$$\frac{\partial \theta}{\partial z} = \frac{\partial T}{\partial z} + \Gamma \qquad (18)$$

where $\Gamma = -9.86 \times 10^{-5}$ °C/cm (Celsius degrees per centimeter). For practical cases to be considered here, equation 18 is valid for moist as well as dry air.

In a non-adiabatic atmosphere, either stable or unstable, buoyant forces become important factors in the mixing process. The density of moist air can be computed using the formula (Sutton, 1953, p. 3)

$$\rho = \frac{P}{R\,T'} \tag{19}$$

where P = the atmospheric pressure; R = the gas constant for dry air (2.876×10^6 square centimeters per degree Celsius per second squared); and T' = the virtual temperature. The virtual temperature is defined as the temperature at which a sample of dry air would have the same density as the mixture, providing that both are at the same pressure. It is computed from

$$T' = \frac{T}{(1 - 3e_a/8P)} \tag{20}$$

where e_a is the partial pressure of the water vapor. The buoyant force on a unit volume of air, F_G, is given by

$$F_G = \rho\, g \tag{21}$$

where g is the acceleration of gravity. Combining equations 19 and 21

$$F_G = \frac{P}{R}\,\frac{g}{T'} \tag{22}$$

Because the atmospheric pressure and the gas constant are almost invariant, the buoyant force is proportional to g/T'. The effect of humidity on the buoyant force is included in equation 22 through the use of the virtual temperature.

In order to account for buoyancy effects, the velocity gradient must be a function of g/T', the coefficient of specific heat at constant pressure, C_p, and the vertical flux of sensible heat, H, as well as τ, ρ, and z. By including these added variables and using the basic hypothesis of similitude, Monin and Obukhov (1959) showed that

$$\frac{\partial u}{\partial z} = \phi_1 \left(\frac{u_*}{\kappa z}\right) \phi_2 \left(\frac{z}{L}\right) \qquad (23)$$

where L is the Monin and Obukhov length scale given by

$$L = \frac{-u_*^3}{\kappa (g/T')(H/C_p \rho)} \qquad (24)$$

Using the same methods they showed

$$\frac{\partial \theta}{\partial z} = \phi_3 \left(\frac{T_*}{z}\right) \phi_4 \left(\frac{z}{L}\right) \qquad (25)$$

where

$$T_* = \frac{-H}{\kappa u_* C_p \rho} \qquad (26)$$

The length, L, has a physical interpretation. Because the values of κ, g, C_p, ρ, and T' are nearly constant (note that the value of T' is on the absolute scale) the value of L can be considered to be proportional to the shear velocity cubed, divided by the vertical flux of sensible heat. For a stable atmosphere, the inversion condition, the flux of sensible heat is downward (negative H) therefore, because u_* is positive, the value of L is positive. Likewise, for an unstable atmosphere, the lapse condition, the value of H is positive; therefore, the value of L is negative. The sign of L is determined by the temperature gradient of the atmosphere in relation to the adiabatic lapse rate.

As the temperature gradient of the atmosphere approaches the adiabatic lapse rate, the vertical flux of sensible heat approaches zero and the absolute value of L approaches infinity. As the value of L approaches infinity the value of z/L, of course, approaches zero for all z. Because it is known that equation 7 is a satisfactory description of the velocity gradient under the condition of neutral stability, the value of ϕ_2 (0) must be unity, and ϕ_1 must be given by

$$\phi_1\left(\frac{u_*}{Kz}\right)=\frac{u_*}{Kz} \qquad (27)$$

Likewise it is assumed that ϕ_4 (0) is unity and

$$\phi_3\left(\frac{T_*}{z}\right)=\frac{T_*}{z} \qquad (28)$$

It is seen that ϕ_2 and ϕ_4 are correction functions which are applied to the log law in order to account for buoyancy effects. As the value of H approaches zero, the functions ϕ_2 and ϕ_4 must approach unity. It can be shown that $\phi_2 = \phi_4$ if one assumes that the fluxes can be described by equations 2 and 4 and that the turbulent transfer coefficients for momentum and heat are the same. The assumption of the equilivance of the transfer coefficients for momentum and heat is subject to more doubt than is the assumption of the equilivance of the transfer coefficients for momentum and vapor. Webb (1970) states that the coefficients for heat and momentum as well as those for vapor and momemtum are equal over a fairly large range of stabilities. Businger, and other (1971) state that the ratio of transfer coefficients for heat and momentum at neutrality is approximately 1.35 and that this ratio varies significally with atmospheric stability. Equilivance of the coefficients is assumed for the purpose of this report.

Because the value of ϕ_2 is known for values of z/L equal to zero, it is assumed that ϕ_2 can be described by a power series expansion about this point of the form

$$\phi_2 = 1 + \alpha(\frac{z}{L}) + \beta(\frac{z}{L})^2 + \ldots \tag{29}$$

For small values of $\frac{z}{L}$, conditions of near neutral stability, ϕ_2 can be represented by retaining only the first two terms of equation 29. Combining equations 27 and 29 with equation 23

$$\frac{\partial u}{\partial z} = \frac{u_*}{\kappa z} (1 + \alpha \frac{z}{L}) \tag{30}$$

and combining equations 28 and 29 with equation 25

$$\frac{\partial \theta}{\partial z} = \frac{T_*}{z} (1 + \alpha \frac{z}{L}) \tag{31}$$

Equations 30 and 31 then should be valid for conditions which are at or near the neutrally stable condition. These equations illustrate quite clearly that the log+linear law applies a linear correction, to the standard log law which accounts for atmospheric stability.

The value of α, called the Monin-Obukhov coefficient, can only be determined from physical measurements. Monin and Obukhov (1959, p.21) originally proposed a value of 0.6 . This value is considerably at variance with the findings of later workers, who find values of α nearly ten times as great as that proposed by Monin and Obukhov (Deacon, 1962, p. 3170). Using the observations of others taken over the sea, Deacon (1962, p. 3171) found the value of α to range from 3.3 to 3.7. Webb (1970) analyzed 89 profiles which had been chosen very carefully such that they were taken under near neutrally stable conditions and which contained no " funny looking " anomalies. For inversion conditions he found that the log+linear law fit the data quite well for a fairly large range of stabilities . For lapse conditions the range was considerably smaller. Webb suggested that the value of α is between 4.5 and 5.2 but found that a large range in values of α produced an acceptable fit of the velocity profiles.

Integrating equation 30

$$u = \frac{u_*}{\kappa} \{\ln \frac{z}{z_0} + \frac{\alpha}{L}(z-z_0)\} \tag{32}$$

and integrating equation 31

$$\theta = T_* \{\ln z + \frac{\alpha}{L} z\} + C_T \tag{33}$$

where C_T is the constant of integration. Forcing θ to the temperature of the water surface, θ_0, as the value z approaches z_0

$$\theta - \theta_0 = T_* \{\ln \frac{z}{z_0} + \frac{\alpha}{L}(z - z_0)\} \tag{34}$$

In an analogous manner the equation for the specific humidity profile can be derived (Monin and Obukhov, 1959, p. 16) as

$$q = Q_* \{\ln z + \frac{\alpha}{L} z\} + C_q \tag{35}$$

Forcing q to approach q_0 as z approaches z_0,

$$q - q_0 = Q_* \{\ln \frac{z}{z_0} + \frac{\alpha}{L} (z - z_0)\} \tag{36}$$

As for the log law, the values of z_0 in equations 32, 34, and 36 represent little more than constants of integration, so that there is little reason to hope that they be uniquely related to each other. Equations 32, 33, and 35 must be considered to contain six unknowns u_*, H, E, z_0, C_T, an C_q. Measurements of at least two levels of velocity, temperature, and humidity are needed in order to solve for the unknowns. Computation of u_*, H, and E from the log+linear law must be based on differences between measured values of velocity, temperature, and humidity, and therefore measurement errors are amplified greatly.

Perhaps the most accepted method of estimating the rate of evaporation from water surfaces is the mass-transfer method. The evaluation of any other method of computing evaporation should thus use the mass-transfer method as at least one basis of comparison. As mentioned previously, the mass-transfer method is based directly on Dalton's law which can be written in the form

$$E = f(u)(e_o - e_a) \qquad (37)$$

where f (u) is a coefficient which is some function of the wind speed, e_o is the saturation vapor pressure of air at the temperature of the water surface, and e_a is the partial pressure of water vapor in the air. Many forms of the wind function, f (u), have been proposed. Most of these, however, have been deduced from pan evaporation data. Harbeck (1962) is one of very few who has presented a form of the wind function which has been derived from lake evaporation measurements. The wind function suggested by Harbeck is

$$f(u) = N\, u_2 \qquad (38)$$

where u_2 is the wind velocity measured 2 meters above the water surface; and N is a dimensional coefficient of proportionality. The value of N is a function of many variables and in general should be determined independently for each site. Harbeck (1962, p. 104) does give an empirical expression which can be used to estimate N. This expression is

$$N = \frac{0.00338}{A^{0.05}} \qquad (39)$$

where A is the area of the lake in acres and the units of N are inches hour per day mile millibar. Harbeck estimates that the standard error of estimate for equation 39 is 16 percent (1962, p. 104).

Although the value of N is never known exactly when equations 37 and 38 are used in order to compute evaporation rates, in general these equations give a much better estimate of the evaporation rate than does the Thornthwaite and Holzman equation 17 . This is because equation 37 does not amplify the measurement errors nearly as much as does equation 17. Equation 38 does not depend upon a velocity difference, and although 37 does contain a vapor pressure difference term, the value of e_a is usually much smaller than the value of e_o.

Yen and Landvatter (1970) studied the evaporation from a heated moist surface into a very cold stream of air in a wind tunnel. They found that equation 37 with f(u) of the form given by equation 38 remains valid even under extreme conditions of instability.

It has been pointed out by Pasquill (Sutton, 1953, p. 311) that the Thornthwaite and Holzman equation can be written in a form similar to the empirical mass-transfer equation as

$$E = B\, u_2\, (q_1 - q_2) \tag{40}$$

where the subscripts simply designate reference elevations and

$$B = \frac{\kappa^2\, \rho(1 - u_1/u_2)}{(\ln z_2/z_1)^2} \tag{41}$$

The value of u_1/u_2 is constant for a neutrally stable atmosphere. If the value of B in equation 41 is determined from the average of many measurements, equation 40 could be used in place of equation 17 and a much smaller measurement error magnification would be expected.

Carrying the lead of Pasquill one step further, evaluating equation 14 at two levels and subtracting

$$Q_* = \frac{(q_1 - q_0) - (q_2 - q_0)}{\ln z_1/z_2} \qquad (42)$$

Factoring out $q_2 - q_0$ and changing the sign

$$Q_* = \frac{B'}{\ln z_1/z_2} (q_0 - q_2) \qquad (43)$$

where

$$B' = 1 - \frac{q_1 - q_0}{q_2 - q_0} \qquad (44)$$

The value of B' should be constant for a neutrally stable atmosphere just as should the value of B. Combining equations 13, 16, 41, and 43

$$E = B\, B'\, u_2\, (q_0 - q_2) \qquad (45)$$

which is identical in form to the empirical mass-transfer formula proposed by Harbeck, except that it is written in terms of specific humidity instead of vapor pressure.

The empirical mass-transfer equation with the wind function given by equation 38, appears to be entirely consistant with the log law formulation of the aerodynamic method. Brutsaert and Yeh (1970) have shown how this same expression can be obtained from a power law distribution of velocity.

THE DATA

In order to test the general applicability of the log+ linear law under the widely varying conditions likely to be found in field applications, a massive set of data is needed. This set of data must contain accurate estimates of the actual evaporation rate as well as numerous measurements of the aerodynamic parameters of wind speed, temperature, and humidity at two or more levels, all of which are well within the boundary layer of the water surface. Massive sets of data which meet all these requirements are rare indeed. The Lake Hefner data is perhaps the most extensive set of data available which meets the requirements.

The Lake Hefner study was an outgrowth of a cooperative investigation at Lake Mead in Arizona and Nevada. The investigation at Lake Mead was undertaken in late 1947, 1948, and early 1949 in order to determine average monthly evaporation values which could be used for planning purposes and to establish operating procedures which would minimize evaporation losses from a chain of reservoirs (U.S. Geological Survey, 1954b, p. 1). It was found that Lake Mead was not suitable for the basic and detailed investigation which was needed. After considering the advantages and disadvantages of many lakes and reservoirs the cooperating agencies chose Lake Hefner as the place to launch a detailed and integrated attack on the water loss problem. The purpose of the investigation at Lake Hefner was to develop an improved method or methods for the determination and prediction of water losses by evaporation using the aerodynamic and energy-budget theories (U. S. Geological Survey, 1954b p. xii).

Lake Hefner was chosen as the site of the investigation because its characteristics allowed an accurate determination of evaporation rates using the water-budget method and because it met certain requirements of size, shape, depth, topographic setting, and climate. These requirements were imposed so that the energy-budget and aerodynamic methods could be evaluated. Because one of the purposes of collecting the Lake Hefner data was to investigate the use of the aerodynamic theory, these data are ideally suited for evaluating the log+linear law.

The physical and climatological characteristics of Lake Hefner have been given by Harbeck (1954, p. 6). Only a brief summary of these characteristics is presented here. The climate of Lake Hefner has been classed as subhumid. The normal annual rainfall is about 78 centimeters with the period of April-June representing the period of greatest rainfall. The average wind speed at 2 meters above the water surface is about 480 cm/sec. The wind speed is within the range 250-1000 cm/sec during 90 percent of the time. The mean air temperature ranges from a high of about 27°C in July to a low of about 4°C in January. The lake's shape is fairly regular and circular, and it has a mean area of about 9 million square meters. The natural drainage area into the lake is only about 30 percent larger than the area of the lake itself. Most of the lake's inflow results from diversion from the North Canadian River. The topography surrounding the lake is flat to gently rolling with sparse vegetation.

The daily evaporation, as determined by the water budget, was used as the control for the entire project. Every effort was made to measure each water-budget term to a precision consistent with its significance in the resulting evaporation and to evaluate the errors inherent in the measurements. Harbeck and Kennon (1954, p. 17-34) describe the measurement procedures used to determine each term of the water budget. They estimated the approximate magnitude of the error in measuring each water-budget term for each day of record and combined these errors in order to estimate the standard error of the computed evaporation for each day. Each daily figure of evaporation was classified on the basis of the estimated standard error into four groups. Sixty-two percent of the daily evaporation figures were classified as either A or B meaning that the standard error of the computed evaporation was less that 9.9 acre-feet. Based on the average area of the lake, 9.9 acre-feet of water represents 0.133 cm. of depth. In this report, only those days of record which were classified as either A or B are analyzed. The water-budget evaporation terms which are used herein were obtained from table 1 of the Lake Hefner Base Data Report (U.S. Geological Survey, 1954a, p. 9-16).

The meteorological variables that were measured at Lake Hefner included the air temperature, wet bulb temperature, and wind speed at 2, 4, 8, and 16 meters above the lake surface as well as the wind direction and the temperature of the water surface. These measurements were made from a barge which was anchored near the center of the lake. Identical measurements were made at three shore stations but since these measurements were not necessarily within the lake boundary layer they are not used in this report. A complete description of the equipment which was used to make the measurements has been given by Anderson (1954, p. 35-45).

The average wind speed during each 30-minute interval was recorded. Temperatures were measured with wet and dry thermocouples. All temperatures were recorded on one recorder by use of a switching arrangement. Each temperature was recorded during a 3-minute interval in each 30-minute period. The order of recording the temperatures was as follows; water temperature, 2-meter dry then wet bulb temperatures, 4-meter dry then wet, 8-meter dry then wet, 16-meter dry then wet, and finally a reference zero was recorded (G. E. Harbeck Jr., oral communication, 6/22/71). Therefore, all temperatures represent a 3-minute average and all temperatures were not taken at the same time. The meteorologic data for each 30-minute interval was determined from the analog charts and the values were punched on computer cards. The information on these cards was transferred to magnetic tape for the purpose of the present investigation. The 30-minute data were too numerous to be published, but average values for 3-hour periods have been published in the Base Data Report (U. S. Geoogical Survey, 1954a).

COMPUTATIONS AND PRESENTATION OF RESULTS

In this section the methods used in the computations and the results of these will be presented. Initially the procedures which are common to all computations are presented. Then those involved in the use of the log law are described and the results of these computations presented. Following this, the computations and results derived from the log+linear law are presented. Finally the results obtained when the empirical mass-transfer equation is used are presented.

Computations common to both laws

Only sets of data which met certain requirements of accuracy and completeness were analyzed. The first requirement was that the daily water-budget evaporation had to be rated as either A or B with regard to accuracy. These ratings were obtained from the Base Data Report (U.S. Geological Survey, 1954a, p. 9). If the accuracy of the water-budget evaporation was not rated either A or B the entire day was ignored. Although data were collected continuously for a 15-month interval, there were many 30-minute periods for which some of the data were missing. Equipment malfunction was the principle cause of gaps in the data. A 30-minute period was ignored if any of the 2 -, 4-, or 8-meter data were missing. The second requirement to be met by a day's data was that it contain at least 20 acceptable sets of 30-minute data. It is believed that at least 20 sets of 30-minute data are necessary in order to provide a reasonable estimate of conditions prevailing during the day. Finally, all the data for the months of June and July, 1950 were eliminated. The data obtained during these 2 months were suspected to be of poor quality because of difficulties encountered in setting up such a complex data collection system. Preliminary analysis of the results, using the data for all 15 months, tended to confirm this suspicion. A total of 222 days met these three requirements. These 222 days contained 8,793 sets of acceptable 30-minute data, or on the average, each day contained 39 out of a possible 48 sets of acceptable 30-minute data.

All the aerodynamic equations have been presented in terms of the specific humidity so this quantity was determined from the wet and dry bulb temperatures for every set of data before further analyses were performed. The saturation vapor pressure of the air, e_{sp}, was first determined from the wet bulb temperature by use of the Kirchhoff-Rankine-Dupre formula (Sutton, 1953, p. 4)

$$e_{sp} = \exp\{63.042 - 7139.6 / T_w - 6.2558 \ln T_w\} \quad (46)$$

where e_{sp} is in mb (millibars) and T_w is wet bulb temperature in degrees Kelvin. The constants in this equation were determined, by the method of least squares, such that the equation is most accurate within the range of 0 to 30°C (degrees Celsius). Within this range the maximum error of 0.38 percent occurs at 30°C. The vapor pressure of the air, e_a , was next determined from the psychrometric equation (Hodgman, 1951, p. 2094)

$$e_a = e_{sp} - 0.00066\, P(T - T_w)\{1 + 0.00115\, (T - T_w)\} \quad (47)$$

where T is the dry bulb temperature in degrees Kelvin and P is the barometric pressure, in millibars. Based on the mean temperature and elevation at Lake Hefner, it was assumed that the barometric pressure was 973.3 mb (Hodgman, 1951, p. 2083). Finally the specific humidity of the air was determined from

$$q = \frac{0.622\, e_a}{973.3 - 0.378\, e_a} \tag{48}$$

In order to simplify the computations a special tape was written which contained only the data which passed the three requirements just discussed above and in which the wet bulb temperature was replaced by the computed specific humidity.

The following procedure was used to evaluate each method of computing the evaporation rate. The measured values of wind speed, temperature and specific humidity were used in conjunction with the appropriate prediction equation to compute an evaporation rate during every available 30-minute interval of a given day. The average of these evaporation rates will be called the computed evaporation rate for the day. The measured evaporation rate for the day was defined as the evaporation rate determined from the water-budget. The error for a day was defined as the measured evaporation rate minus the computed evaporation rate. Actually, this error results from errors in both the computed and measured evaporation rates. Nevertheless, the method which gives the smallest values of error was considered to be the most accurate. The best method is then the method which gives the required accuracy at the least total cost.

Error Analysis

Any evaluation of an evaporation prediction equation must be concerned with some form of error analysis. The difference between the measured and computed value of the evaporation rate can result from at least three factors; measurement errors, model errors, and coefficient errors.

The difference between the recorded value of a physical quantity and its true mean value during the period of interest is defined as a measurement error. Measurement errors are inevitable in any physical measurement and can be classified as either random or systematic. Measurement errors, as defined here, can result from two separate causes, first the usual equipment and personal errors and second, errors caused by not properly averaging over the time period. Errors due to the second cause, which could be called averaging or sampling errors, were most critical for the recorded temperatures. Each temperature represents the average of only 3 minutes of record during the 30-minute interval. One can only hope that the systematic part of the measurement errors was small. Generally, if equipment is properly maintained and calibrated, systematic errors in measurements can be reduced to near zero values. It is believed that, except for possibly the 4-meter dry-bulb temperatures and all 16-meter data, the systematic errors in the Lake Hefner data are small. The random part of measurement errors is much more difficult to control because of the great number of possible causes. The effect of random errors can be reduced to as small a value as is desirable, at least theoretically, by averaging the results of many observations. The number of observations can be increased by making simultaneous measurements at many levels or by making measurements at only two levels but averaging the results over time. A combination of both averaging procedures is employed here. Because only three levels of acceptable data were available, averaging the results over time was the most effective way of reducing the effect of random measurement errors. Unfortunately, as has been pointed out before, the aerodynamic method tends to magnify the effect of measurement errors greatly and so averaging over many observations is necessary.

No mathematical model can possibly account for all the factors which contribute to the shape of the velocity, temperature, and humidity profiles. The model error is defined as that error which results in the computed evaporation because the mathematical model is incapable of adequately describing the physical process. For example, if the true velocity and specific humidity are not proportional to the logarithm of z, there will probably be an error in the computed evaporation rate no matter how accurately the values of velocity and humidity are known.

The coefficient error is defined as the error in the computed evaporation which results from the improper choice of an empirical coefficient. The values of κ and α in the aerodynamic methods are not considered as empirical coefficients in this definition. The value of κ has been assumed to have a value of 0.40 a priori. If this assumption is incorrect, the resulting error will be considered to be included in the model error. The value of α could also be assumed a priori, or its value can be deduced at a particular site from measurements of velocity and temperature profiles without knowing the actual rate of evaporation. Any errors caused by the improper choice of α are also included in the model errors. It is, therefore, assumed that the coefficient errors in the aerodynamic models are zero. The empirical mass-transfer approach which is used here contains one coefficient, N. It is assumed that the exact value of N

is that value for which the long term computed evaporation rate would be exact, provided all measurements were exact. This value cannot be determined a priori. The coefficient error for the empirical mass-transfer method is equal to the error in the value of N.

The log law

Marciano and Harbeck (1954, p. 46-70) have applied the Thornthwaite-Holzman equation to the 2- and 8-meter data of Lake Hefner. Because the Thornthwaite - Holzman equation is based directly on the log law, further analysis using this law may in some ways seem redundant. The log law is to be used as a standard from which any improvements resulting from the use of the log+linear law must be gaged, so it is necessary that the best possible results be obtained from the log law and that these results be obtained under the same conditions of sorting and analysis as is to be used for the log+linear law. In order to reduce the measurement errors as much as possible and at the same time make use of all possible data the log law was generalized such that all three levels of data (2-, 4-, and 8-meter) can be used in determining the evaporation rate.

The first step in the application of the log law to a particular set of data is to determine the shear velocity. The values of u_*/κ and $\ln z_o$ were determined by use of equation 8 and the measured velocities such that the sum of the squares of the differences between the measured and computed velocities was a minimum. Minimizing the sum of the squares the value of u_*/κ is obtained from

$$\frac{u_*}{\kappa} = \frac{D_{u\ell z}}{D_{\ell z}} \tag{49}$$

where

$$D_{u\ell z} = \sum_{i=1}^{3} u_i \ln z_i - \frac{1}{3}\left[\sum_{i=1}^{3} u_i\right]\left[\sum_{i=1}^{3} \ln z_i\right] \tag{50}$$

and

$$D_{\ell z} = \sum_{i=1}^{3} \ln^2 z_i - \frac{1}{3}\left[\sum_{i=1}^{3} \ln z_i\right]^2 \quad . \tag{51}$$

In these expressions, the value of u_i is the measured velocity in cm/sec and z_i is the elevation of the measurement in meters. The value of u_* was then determined for each 30-minute interval of each day by assuming $\kappa=0.4$, a total of 8,793 times. Regression analysis yielded the relation

$$u_* = -1.85 + 0.0731\ u_2 \tag{52}$$

where u_2 is the 2-meter wind velocity in centimeters per second and u_* is in centimeters per second. The value of u_2/u_* should be reasonably constant for any particular body of water. Therefore, this ratio was used as a means of discarding obviously incorrect data. The entire 30-minute period was ignored if

u_2/u_* was greater than 137 or less than 1.37. These limits are extremely broad so no reasonable data were ignored. About 4 percent of the data were eliminated by this restriction.

For each profile that contained reasonable wind data, the value of Q_* was computed from the specific humidity profile. The values of Q_* and C_q were determined from equation 12, and the measured values of specific humidity. These computations were similar to those used for the velocity profile. The sum of the squares of the differences between the measured and computed specific humidity was minimized. The computation equation is

$$Q_* = \frac{D_{q\ell z}}{D_{\ell z}} \tag{53}$$

where

$$D_{q\ell z} = \sum_{i=1}^{3} q_i \ln z_i - \frac{1}{3}\left[\sum_{i=1}^{3} q_i\right]\left[\sum_{i=1}^{3} \ln z_i\right] . \tag{54}$$

The density of the air was estimated in the following manner. A representative vapor pressure was assumed to be given by the arithmetic average of the measured specific humidities and was calculated from

$$\bar{e}_a = \frac{\frac{P}{3}\sum_{i=1}^{3} q_i}{0.622 + \frac{0.378}{3}\sum_{i=1}^{3} q_i} \tag{55}$$

where P is the atmospheric pressure, 973.3 mb. The density of moist air was then computed from equations 19 and 20. The arithmetic mean of the air temperatures was used in equation 20. The effect of humidity and temperature on the air density was accounted for, but the effect of varying barometric pressure on density was ignored.

The evaporation rate for each 30-minute period was then computed by use of equation 13. From equations 13 and 49 it is seen that the value of E is proportional to κ^2 and had the value of κ been assumed to be 0.38 instead of 0.4 each evaporation rate would have been 9.8 percent smaller. Values of κ as small as 0.38 are often assumed, but these smaller values are usually associated with atmospheric conditions that are not neutrally stable. Any evaporation rate computed for a 30-minute period which had a value larger than 10 cm/day or smaller than -2 cm/day was ignored in computing the daily average evaporation rate. These limits were arbitrary but they were believed to be large enough so that no reasonable data were rejected. For example, the mean evaporation rate from Lake Hefner was 0.41 cm/day, the maximum daily evaporation rate as determined from the water-budget method was 1.44 cm/day, and the minimum daily evaporation rate was -0.674 cm/day. Only about one or two profiles per thousand were rejected because of this restriction. The mean computed evaporation rate for the day was then determined as the average of all acceptable 30-minute evaporation rates.

The calculations just described result from the direct application of the log law to each individual profile of data. Some of the results of these computations are presented on figure 3. The ordinate of figure 3 is the measured evaporation for the day as determined from the water-budget method and the abscissa of the figure is the mean computed evaporation rate for the day. The points tend to scatter about a line of perfect correlation but there are significant errors on individual days. A large amount of averaging has already occurred in the results presented on figure 3. Each point represents the average of several profiles and each profile uses the results of measurements obtained at three levels. The standard deviation, σ, of the daily errors is shown on the figure as well as the coefficient of correlation, r, between the computed and measured values of evaporation. The coefficient of correlation is a measure of the linear correlation between these two values. A value of one for r indicates perfect correlation and a value of zero indicates no correlation.

The standard deviation of daily errors is perhaps the best measure of the accuracy of the log law for computing daily evaporation values. Approximately 2/3 of the daily errors should be smaller than the standard deviation. The error in the daily evaporation was less than 0.383 cm approximately 2/3 of the time. This error is to be compared to an average daily evaporation rate of 0.41 cm and an estimated maximum standard error in the water-budget evaporation of 0.133 cm.

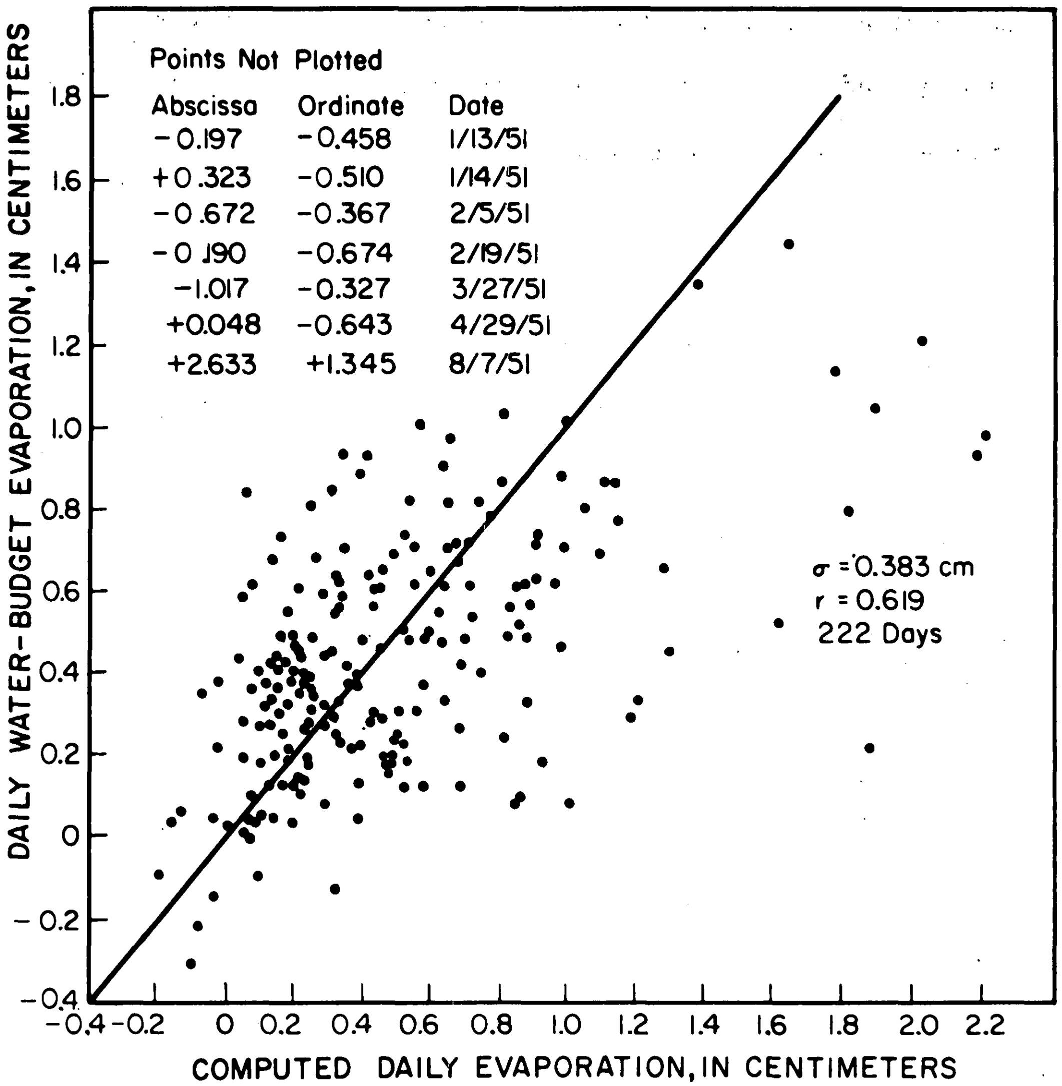

Figure 3. Experimental test of the direct application of the log law.

The daily error is defined as the water-budget evaporation rate minus the average computed evaporation rate for the day and the values on figure 3 are very large. Approximately 1/3 of the days had an error in the computed evaporation in excess of 92 percent of the mean daily evaporation. In an attempt to isolate the cause of these large errors, the magnitude of the error was correlated with daily average values of various meteorologic parameters.

The log law is postulated to be valid only under conditions of neutral stability. Therefore, it is logical to expect the error to be strongly correlated with atmospheric stability. Perhaps the most commonly used measure of atmospheric stability is the gradient form of the Richardson number (Priestley, 1959, p. 9). However, the Richardson number must be computed using exactly the same data used to compute the evaporation rate. The measure of atmospheric stability which is used in this report is the one proposed by Marciano and Harbeck (1954, p. 52)

$$S\,T = \frac{T_8 - T_o}{u_8{}^2} \qquad (56)$$

where T_8 is the dry bulb air temperature at 8 meters, u_8 is the wind speed at 8 meters, and T_o is the water temperature. This term is closely related to the Richardson number and is not a function of gradients which must be measured in the air. It also has the advantage of being dependent upon the water temperature, a quantity which was not used in the determination of the computed evaporation. Neglecting the lapse rate, the value of the stability parameter is zero if the atmosphere is neutrally stable. The value of the stability parameter was computed for each 30-minute interval of the day and the values were averaged in order to determine the average value of ST for the day. The correlation of the daily error values with the stability parameter is demonstrated on figure 4. Also shown on figure 4 is the regression line and the correlation coefficient. The results of 6 days gave extremely large absolute values of $S\,T$ and were not used in the correlation analysis.

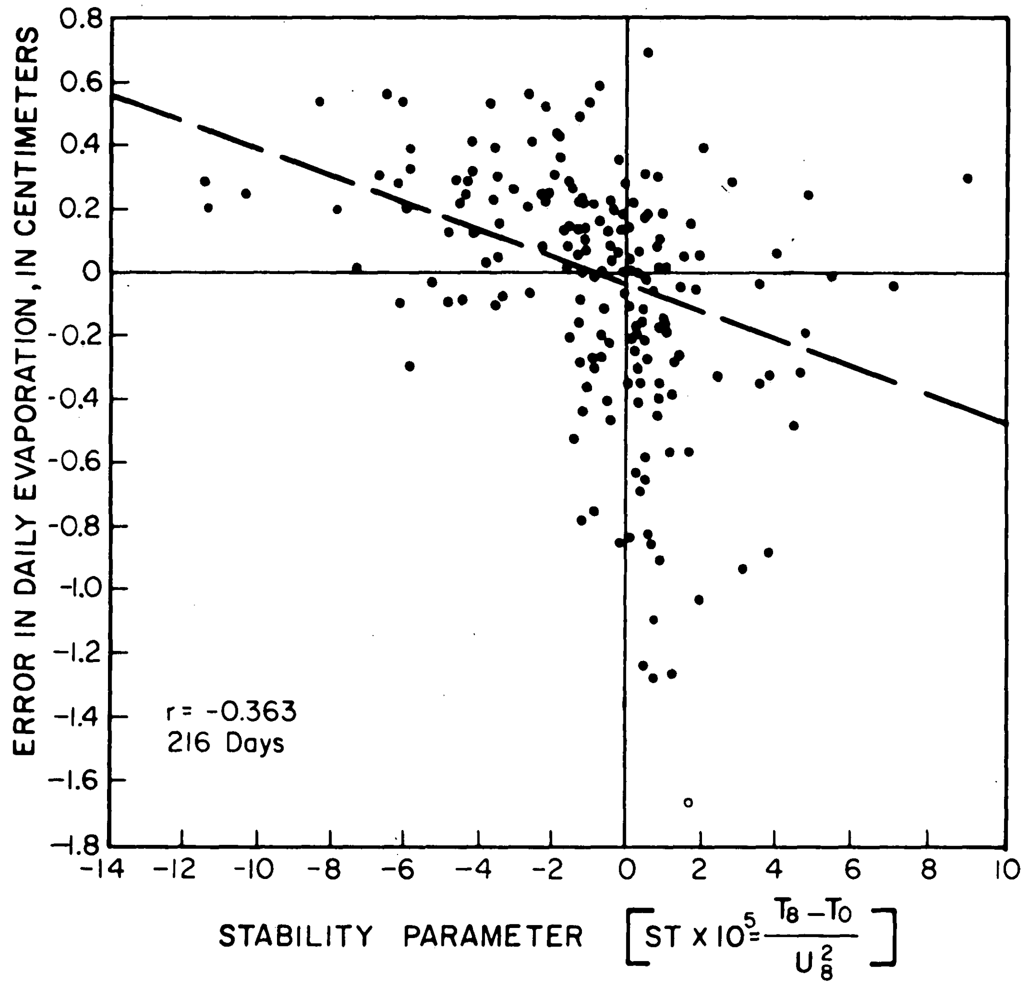

Figure 4. Correlation of the daily error values with the stability parameter for the direct application of the log law.

The distribution of the points on figure 4 suggest that the error decreases with increasing values of the stability parameter, but it is not obvious that this association is significant. Assuming the population distribution is bivariant normal and using a transformation given by R. A. Fisher (Cooper, 1969, p. 212), the confidence limits for the population correlation coefficient can be obtained. Accepting a 0.99 level as the significance criterion, the true value of the coefficient of correlation between the error and stability parameter has a value between the limits -0.285 and -0.505. The probability that the true correlation coefficient is as large as zero is very small, and it can be concluded that there exists a very significant correlation between the error and atmospheric stability. This result was, of course, expected and one may be surprised that the scatter on figure 4 is so large. This scatter can be partially explained as follows. The stability parameter varies throughout the day and is usually negative at night and positive during the daylight hours. Therefore the mean value of S T for the day is often very nearly equal to zero even though its magnitude would probably have been large throughout the day. If the measured evaporation rate during all 30-minute intervals had been available so that averaging throughout the day was not necessary, it is expected that the correlation would have been much better. Nevertheless, the data illustrated on figure 4 demonstrate that the accuracy of the log law decreases as the atmosphere departs from the condition of neutral stability.

The correlation between the daily error values and the daily average value of the 8-meter wind speed is illustrated on figure 5. Applying Fisher's z transformation, the 0.99 confidence limits for the population correlation coefficient are estimated to be -0.038 and -0.369. The error is also very significantly correlated with wind velocity.

The correlation between the daily error values and the daily average value of the 8-meter specific humidity is illustrated on figure 6. Applying the z transformation, the 0.99 confidence limits on r are +0.302, -0.036, and the 0.95 confidence limits on r are +0.264, +0.005. Therefore, the error is probably correlated with the specific humidity, but the probability is not as large as was the case for wind velocity and stability parameter.

Assuming that the population correlation coefficients for velocity and (or) specific humidity are different than zero, the interpretation of the meaning of the result is difficult. It could mean that the measurements contain systematic errors or that there is some basic model error in the log law. Because it is a foregone conclusion that the log law is not completely adequate, it will be assumed these correlations are not an indication of systematic errors in the measurements.

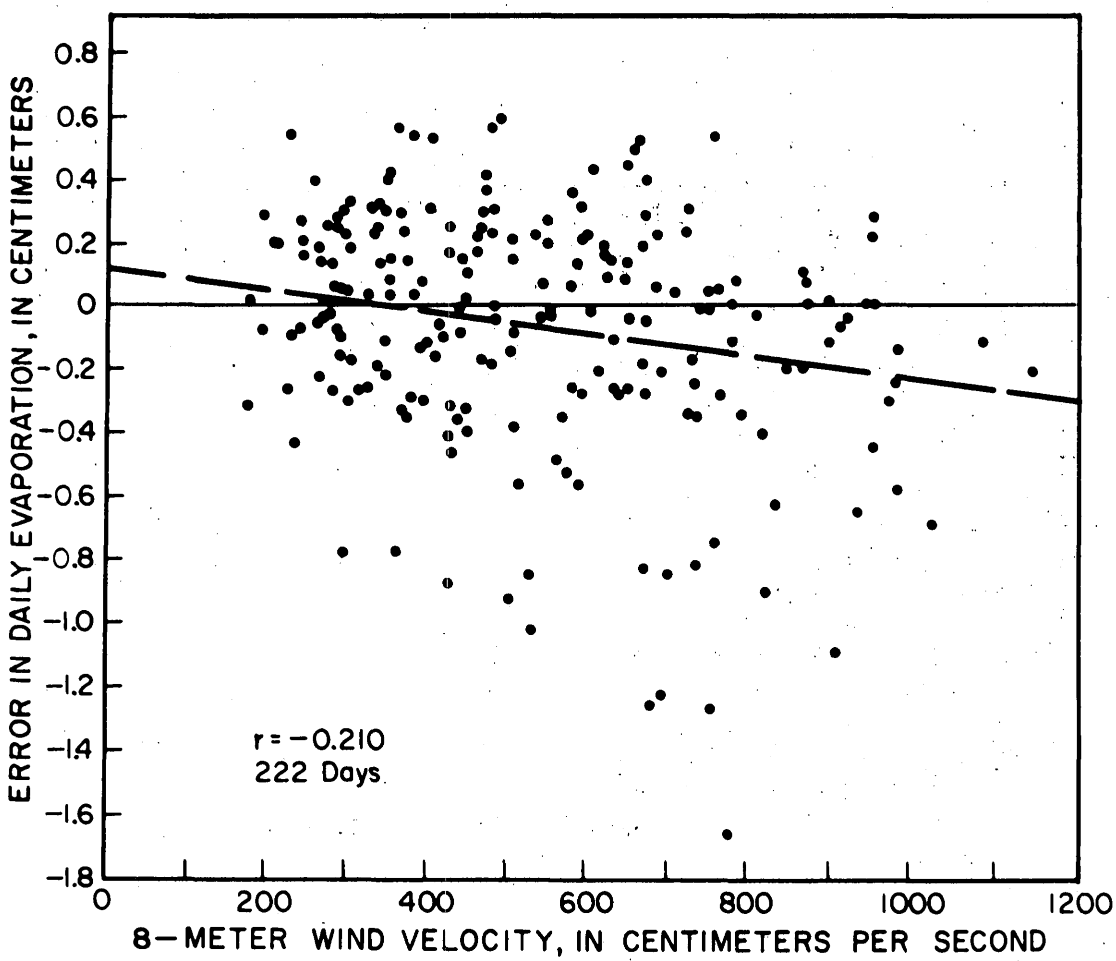

Figure 5. Correlation of the daily error values with the wind velocity for the direct application of the log law.

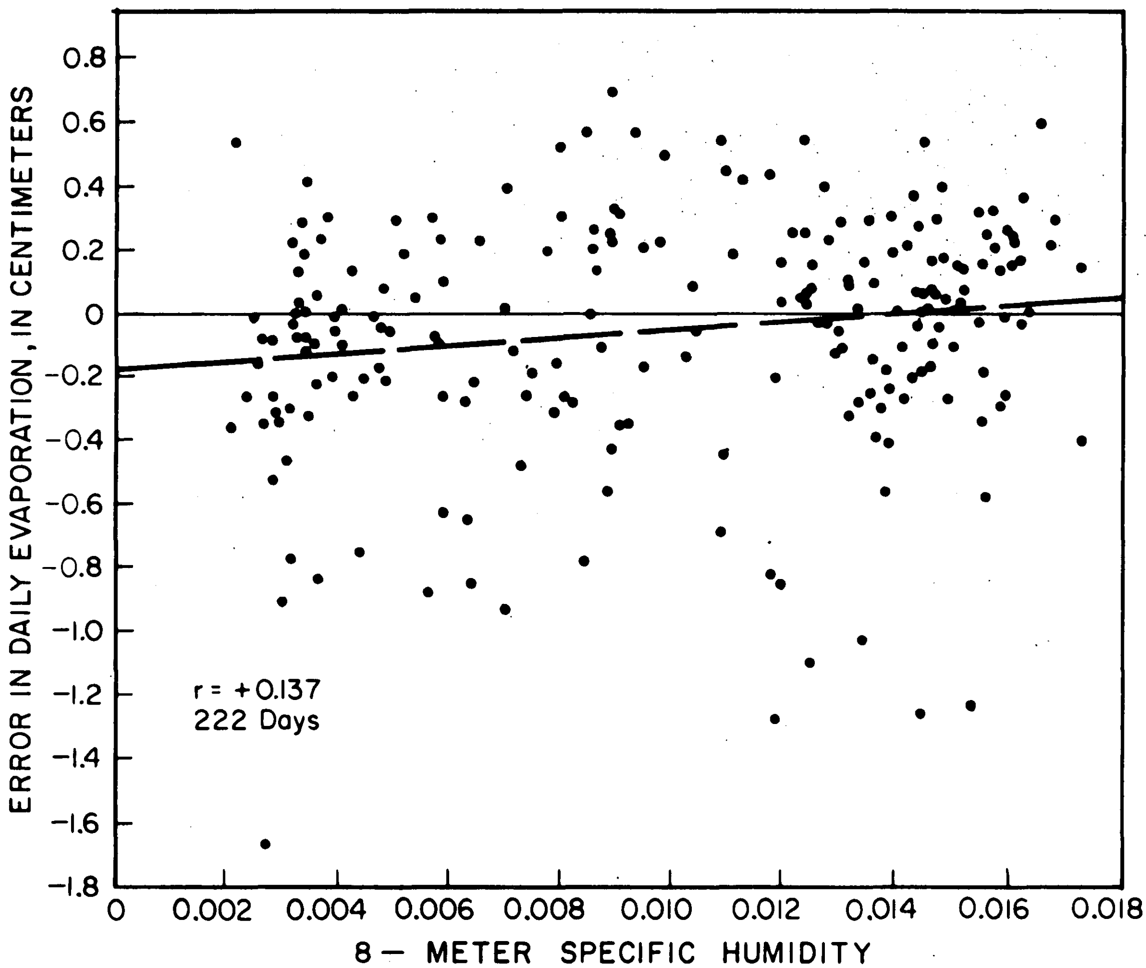

Figure 6. Correlation of the daily error values with the specific humidity of the air for the direct application of the log law.

The temperature difference between the air and the water surface is an important factor in determining the atmospheric stability and, therefore, the accuracy of the log law. This temperature difference tends to vary seasonally. The seasonal nature of the error in the log law is demonstrated on figure 7. The ordinate of figure 7 represents the cumulative error in the computed evaporation and the abscissa represents time in days. The cumulative error represents the sum of all previous errors and is, therefore, the integrated effect of all previous errors. This integration process smoothes the results and makes seasonal trends more obvious. The instantaneous error is given by the slope of the curve. The number of days on the abscissa represents the number of days of acceptable record. There were fewer days of acceptable data during the winter months than during the summer months.

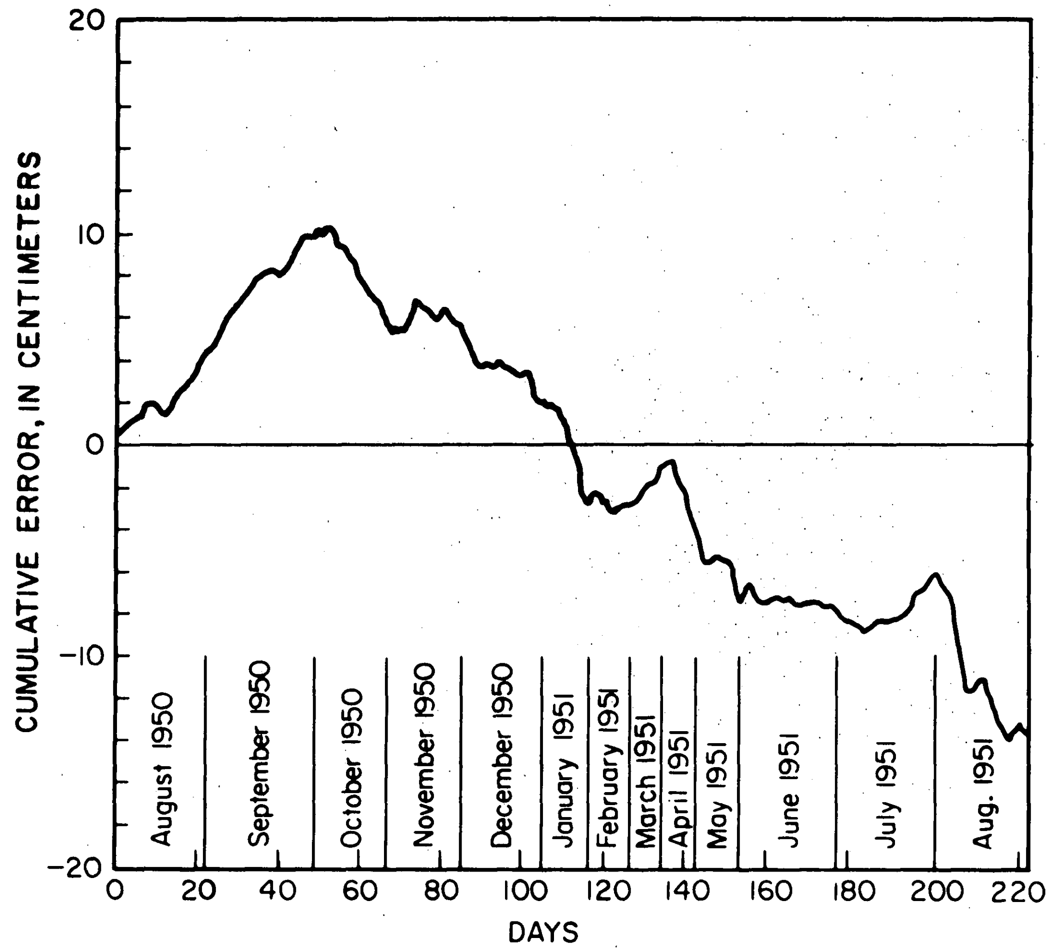

Figure 7. Seasonal variation of error values resulting from the direct application of the log law.

The log law generally underestimated the evaporation rate during the months of August and September 1950, but it generally overestimated the evaporation rate during the other 11 months. The total measured evaporation for the 222 days of record was 90.93 cm. When averaged over the entire 222-day period, the log law tended to overestimate the evaporation rate by 15 percent. The average error for the entire 222 days is not representative of the accuracy which can be expected from the direct application of the log law. During the months of August and September, 1950, the evaporation rate was underpredicted by an average of 41.5 percent and during the rest of the time the evaporation rate was overpredicted by an average of 38.1 percent.

The results presented on figures 3 through 7 were based on curves fitted to three levels of data (2-, 4-, and 8-meter). To serve as a basis for comparison from which the value of the third level of data can be approximated, the Thornthwaite - Holzman equation was used to determine the evaporation rate for each 30-minute period. Otherwise the computation procedures were the same as was used above.

The coefficient of correlation and the standard deviation of daily errors were 0.646 and 0.390 cm when the 2-and 8-meter levels were used. The seasonal variation of the errors is shown on figure 8. These results are very similar to those obtained using all three levels. The standard deviations of the daily errors were nearly the same and, when averaged over the entire 222 days, the Thornthwaite-Holzman equation overpredicted the evaporation by 8.6 percent compared to a value of 15 percent for all three levels. During the months of August and September, 1950, the 2- and 8-meter data underpredicted the actual evaporation by an average of 40.2 percent as compared to 41.5 percent for the three levels of data and during the rest of the time it overpredicted the evaporation by an average of 28.6 percent as compared to 38.1 percent for the three levels of data. It would appear from a comparison of the results obtained using the 2- and 8-meter levels with those obtained from the use of all three levels that the addition of the 4-meter data does not increase the accuracy by very much.

The results obtained with the 2-and 4-meter data are quite different than the results obtained by use of all three levels. The coefficient of correlation and the standard deviation of daily errors were 0.461 and 0.592 cm when the 2-and 4-meter levels were used and the seasonal variation of the errors is shown on figure 9. This standard deviation of daily errors is more than 50 percent larger than the value obtained by use of all three levels. Although the 2- and 4- meter data predict the evaporation pretty accurately during the months of August and September 1950, these data overpredicted the evaporation by an average of 77 percent during the rest of the time.

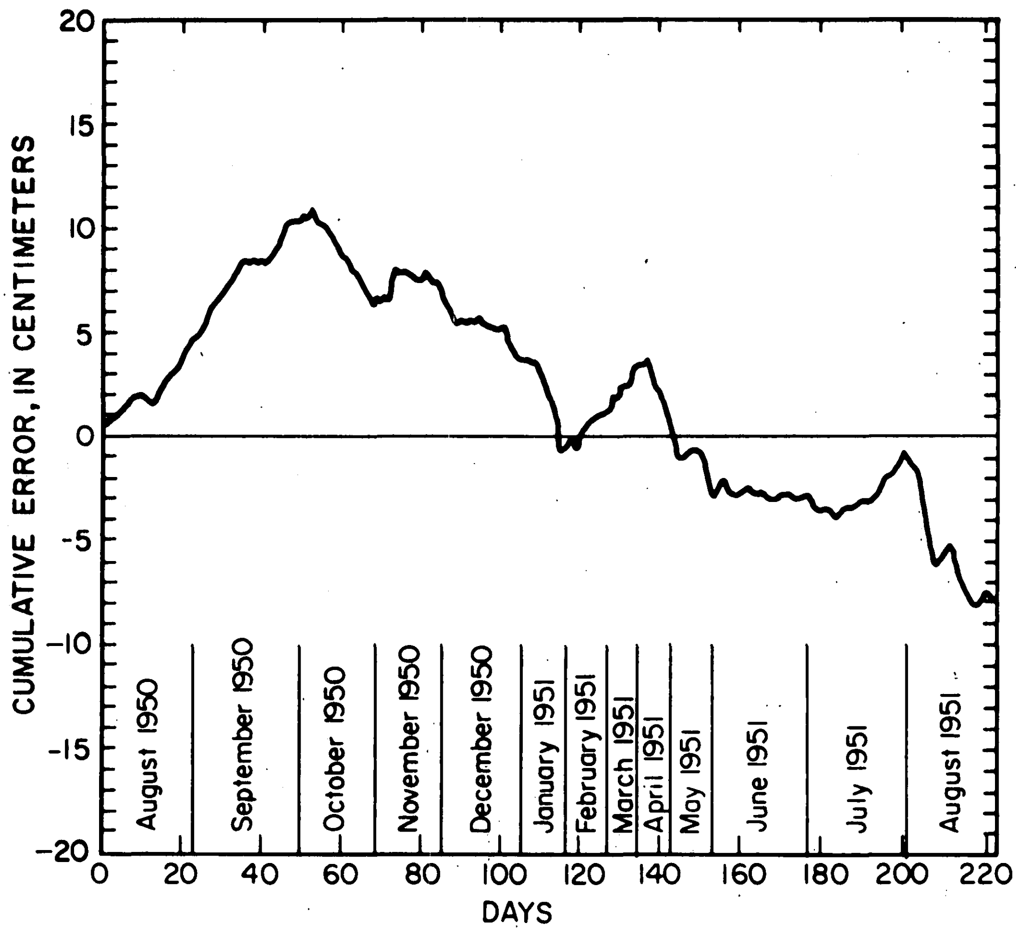

Figure 8. Seasonal variation of error values resulting from the use of the Thornthwaite-Holzman equation with the 2- and 8-meter data.

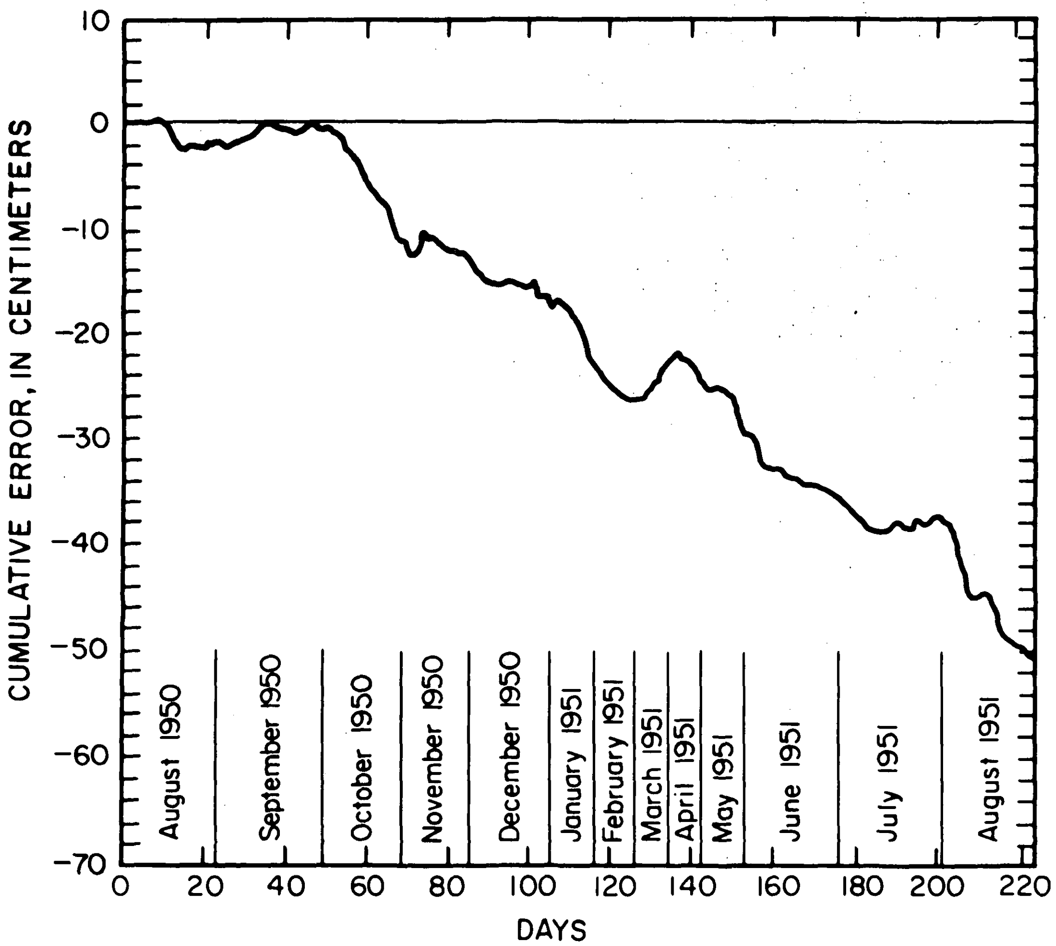

Figure 9. Seasonal variation of error values resulting from the use of the Thornthwaite-Holzman equation with the 2- and 4-meter data.

The large divergence of the results obtained by use of the 2-and 4-meter data from the results obtained by use of all three levels of data and by use of the 2-and 8-meter data suggest possible systematic errors in the 4-meter data.

The consistency of the 4-meter velocity data was investigated first. The distribution of the points on figure 4 suggests that "on the average" the atmospheric conditions over Lake Hefner tend to be neutrally stable. For neutrally stable conditions the ratio of the velocities at any two levels should be constant. The ratios of the 8-meter to 2-meter velocity and the 4-meter to 2-meter velocity were computed for all 8,793 sets of data and the distribution functions for these ratios are shown on figure 10. The values of the ratios range between wide limits because of measurement errors and because of atmospheric conditions which are not neutrally stable. But the median value of these ratios should be practically independent of random measurement errors, and because "on the average", the atmosphere is neutrally stable, the median values of these ratios should be consistent with the log law. The median value of the ratio of the 8-meter to the 2-meter wind velocity is 1.226. The value of z_o, which can be computed from this ratio and equation 8, is 0.410 cm. The value of z_o which is determined from the median value of the ratio of the 4-meter to 2-meter wind velocity is 0.431 cm. The relatively good agreement between the values of z_o as determined from the two ratios indicates that the velocity data contain no significant systematic errors.

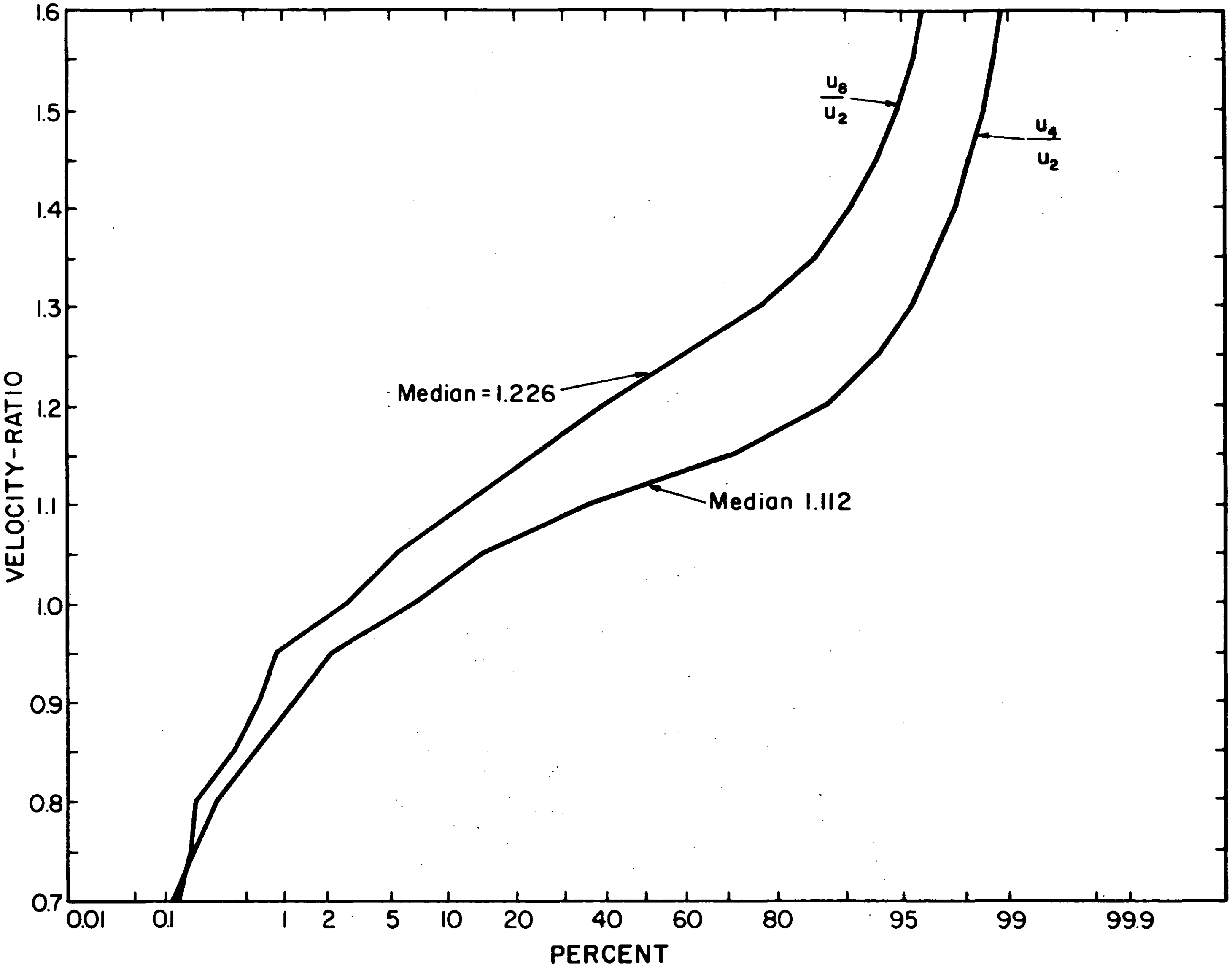

Figure 10. Cumulative distribution functions of velocity ratios.

The consistency of the 4-meter specific humidity data was investigated next. The quantity $q-q_o$ in equation 14 is defined as the humidity deficit. The ratios of the 8-meter to 2-meter humidity deficit and the 4-meter to 2-meter humidity deficit were computed for all profiles and the distribution functions of these ratios are shown on figure 11. During 7 percent of the time the humidity deficit at 4 meters was larger than that at 8 meters. This makes one suspect that the data were not consistent during a fairly large part of the time. The median values of the ratios are 1.0927 and 1.0651 for the 8- to 2- and 4- to 2-meter data respectively. These ratios and equation 14 can be used to determine values of z_o which apply to the "average" specific-humidity profile. The values of z_o are 1.06×10^{-3} cm and 4.6×10^{-3} cm for the 8-2 and 4-2 meter data respectively. It is of no particular significance that these values of z_o are much smaller than were the values of z_o which were determined from the velocity profiles. However, the large difference in the two values of z_o which were determined from the specific humidity data indicates that there was probably some systematic error in the measurement of specific humidity. This systematic error could have occurred at any or all of the levels. However, the previous analyses, as illustrated on figures 7, 8, and 9, suggest that the principle error occurred in the 4-meter data. So, it is suspected that the 4-meter specific humidity was underpredicted during a fairly large percentage of the time.

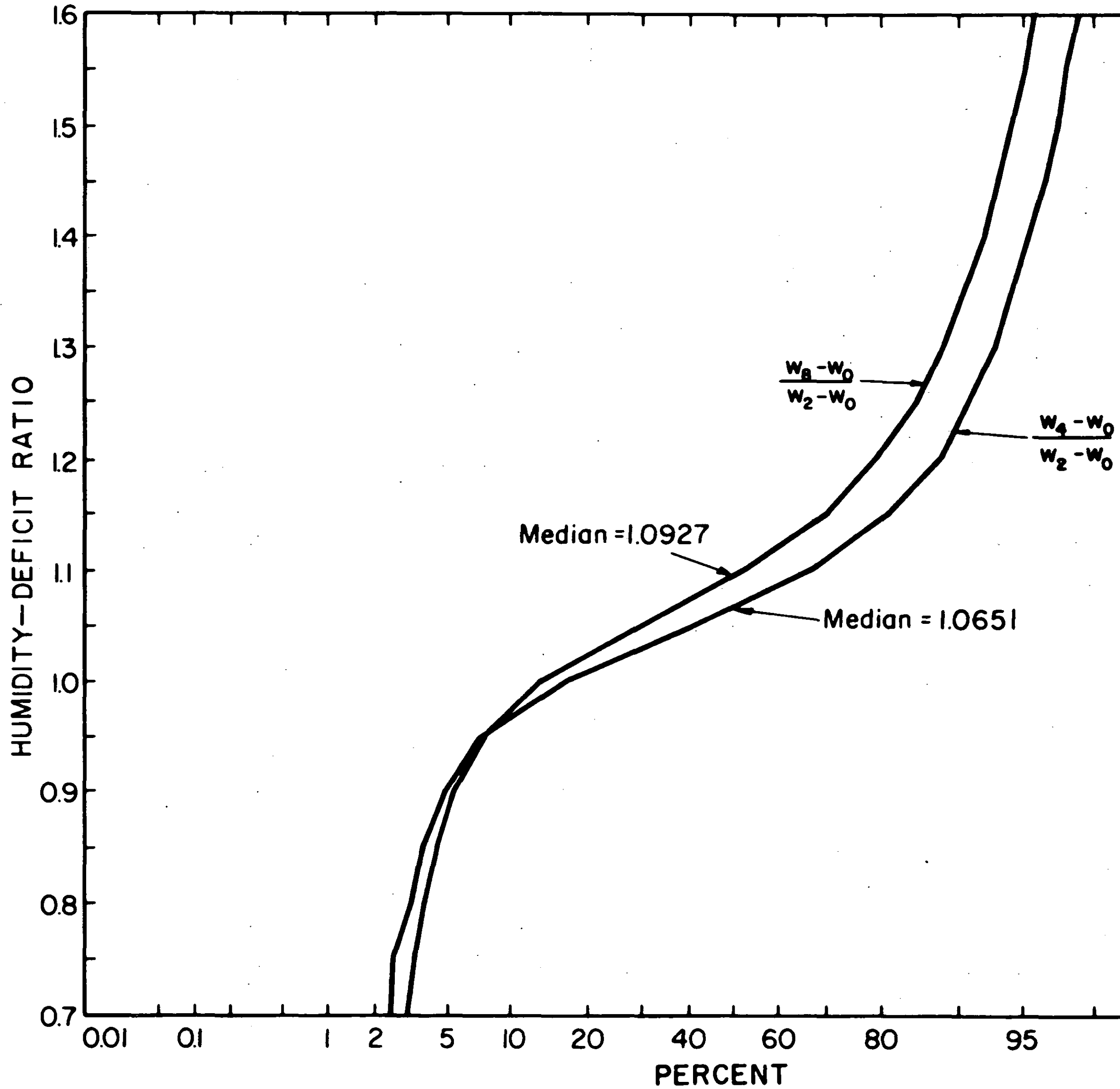

Figure 11. Cumulative distribution functions of humidity-deficit ratios.

The specific humidity is a quantity which was determined from the measured values of the wet and dry-bulb temperatures. The consistency of the dry-bulb temperatures was investigated next. The distribution of the potential temperature which is consistent with the log law can be obtained from equation 34 by setting α equal to zero. The quantity $\theta-\theta_o$ in equation 34 is defined as the temperature deficit. The distribution functions for the temperature deficit ratios are shown on figure 12. The median values of the ratios are 1.0773 and 1.0421 and the values of z_o are 3.23×10^{-6} cm and 3.94×10^{-5} cm for the 8- and 2- meter and 4- and 2- meter data respectively. Again the small value of z_o is of no particular concern. The large difference between the two values of z_o determined from the potential temperature profiles indicates that the dry bulb temperature measurements contain some systematic errors. Because the potential-temperature profiles appear to be more inconsistent than do the specific-humidity profiles, it is suspected that the 4-meter dry-bulb temperatures are the only measurements that contain significant systematic errors. Even with these systematic errors in the 4-meter specific-humidity values, the results which are based on all three levels appear to be at least slightly better than the results which are only based on the 2- and 8-meter data.

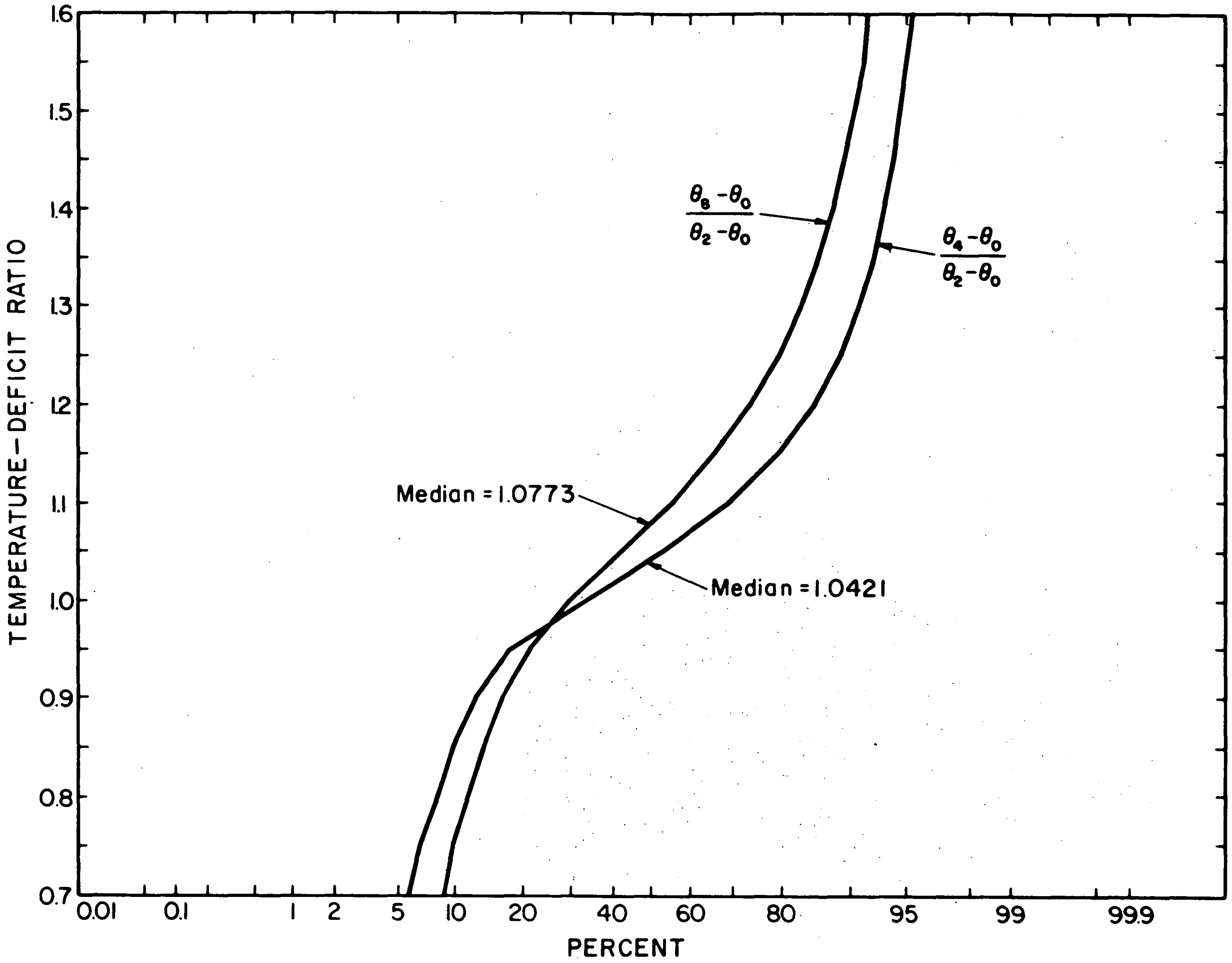

Figure 12. Cumulative distribution functions of the temperature-deficit ratios.

Because of the large expected value of the error in the evaporation rate which is computed from one set of profiles, any conclusions must be based on the average of many observations. The evaporation rate for each 30-minute period has been computed from equation 13 in which the value of u_* has been determined from the velocity profile, and the value of Q_* has been determined from the specific-humidity profile. When the Thornthwaite-Holzman equation is used, all three operations are combined into one. An error in the computed evaporation results from errors in the values of u_* and Q_*. It will be shown next how errors in u_* and Q_* tend to accumulate when the average of many observations is used. A method of analysis is developed which will minimize this undesirable accumulation.

Let us represent the true value of the shear velocity, $\tilde{u}_*$, by

$$\tilde{u}_* = u_*^a + u_*' \tag{57}$$

where u_*^a is the apparent value of the shear velocity, such as that determined from equation 49, and u_*' is the error in u_*^a . Likewise represent the true value of Q_* , $\tilde{Q}_*$ by

$$\tilde{Q}_* = Q_*^a + Q_*' \tag{58}$$

where Q_*^a is the apparent value of Q_* , such as that determined by equation 53, and Q_*' is the error in Q_*^a . Assuming that the density of air is constant and using the overbar to represent the average of several quantities, the true average evaporation rate is determined as

$$\overline{E} = \rho\ \kappa\ \overline{\tilde{u}_*\ \tilde{Q}_*} \tag{59}$$

and using equations 57 and 58

$$\overline{E} = \rho\ \kappa \left\{ \overline{u_*^a\ Q_*^a} + \overline{u_*^a\ Q_*'} + \overline{u_*'\ Q_*^a} + \overline{u_*'\ Q_*'} \right\} \quad . \tag{60}$$

Because the true values of u_* and Q_* are never known the average evaporation rate must be estimated by using only the first term of equation 60. The last three terms in equation 60 approach zero as the number of observations increase indefinitely if, and only if, three restrictions on the distribution of the errors are satisfied. First, the mean value of the errors must be zero. Second, the error in the shear velocity must be independent of the apparent magnitude of Q_* and the error in Q_* must be independent of the apparent shear velocity; and third, the errors in u_* and Q_* must not be correlated with each other. The true evaporation rate can be predicted by use of the aerodynamic method if and only if these three restrictions on the distribution of the errors in u_* and Q_* are satisfied.

The coefficient errors in the aerodynamic methods have been included in the model errors; therefore, the errors in u_* and Q_* as determined by the aerodynamic method result from only two causes. The aerodynamic method contains only measurement errors and model errors. Because the measurement and model errors are additive, each type must be subjected to the above stated restrictions. If the data contain no systematic errors, it is probable that the measurement errors satisfy these restrictions because there is no particular reason to believe that an error in the measurement of velocity is correlated with either the magnitude of specific humidity or the error in its measurement. Similarly, there is no particular reason to believe that the measurement error in the specific humidity is in any way related to the shear velocity.

The model errors are probably not quite so random. It has been observed that "on the average" the atmosphere behaves as if it were neutrally stable; therefore, model errors for the log law probably satisfy the first requirement quite well. It is perhaps even reasonable to expect that they satisfy the second restriction. However, it has been demonstrated rather conclusively, both on figure 4 and elsewhere, that the log law is not valid under atmospheric conditions that are not neutrally stable. Therefore, it is reasonable to expect that rather large model errors will occur in both u_* and Q_* when the atmosphere departs from neutral stability. Because the errors in u_* and Q_* are both correlated with atmospheric stability, it is reasonable to expect that they are highly correlated with each other. This implies that restriction number 3 is not satisfied and that the value of the fourth term in equation 60 will not approach zero as the number of observations increases.

If some method could be found to predict either u_* or Q_* such that the error in this prediction was not correlated with atmospheric stability, then the log law could be used to predict the other value, either u_* or Q_*, and long term evaporation rates could be accurately predicted by the aerodynamic method.

Fortunately there is a method by which this can be accomplished at least to a large extent. The roughness of a water surface is dominated by the structure of the water waves and the water wave structure is dependent upon the wind velocity. The shear velocity, therefore, is a direct function of the wind velocity. The functional form of this relation is not known, but the commonly assumed square resistance law would imply that the shear velocity is proportional to the first power of the wind velocity.

Perhaps the best estimate of the functional relation between the shear velocity and wind speed has already been presented in equation 52. This equation is based on all the data which is used in the report, so, provided that the mean value of errors in u_* was zero and the square resistance law is valid, the values of u_* as determined from equation 52 should be quite accurate. Marciano and Harbeck (1954, p. 49) have presented a relationship between the shear velocity and the 8-meter wind speed which is based only on data obtained under atmospheric conditions which were neutrally stable. The relationship that they presented can be expressed as

$$u_* = 0.0557\, u_8 + 3.8\times10^{-6}\, u_8^2 \qquad (61)$$

where u_8 is the 8-meter wind velocity and both u_* and u_8 are expressed in centimeters per second. Both of these expressions, equations 52 and 61, have been used to determine the computed evaporation but only the results obtained by the use of equation 52 will be presented in detail.

The modified log law will be the term used to refer to the method of computing the evaporation rate from equation 13 wherein the value of Q_* has been determined from equation 53 and the value of u_* has been determined from an equation similar to equation 52. The modified log law has at least two advantages over the log law. First, equation 52 does not magnify the measurement errors as does equation 49 and second, it is believed that the model error in equation 52 is less correlated with stability than is the model error in equation 49. While the modified log law reduces the magnification of the measurement errors in the wind velocity and is believed to reduce the last cross product term in equation 60, it is entirely consistent with the log law because the coefficients in equation 52 were determined directly by use of the log law.

The evaporation rate was computed using equation 52 and the modified log law in exactly the same manner as for the log law. Figure 13 is a plot of the measured against the computed evaporation rate and is similar to figure 3. The use of the modified log law has reduced the standard deviation of the daily errors from 0.383 cm to 0.270 cm. Part of this reduction must be the result of a reduced magnification of the measurement errors in velocity, but part of it is believed to result from a reduction of the model error in the determination of u_*.

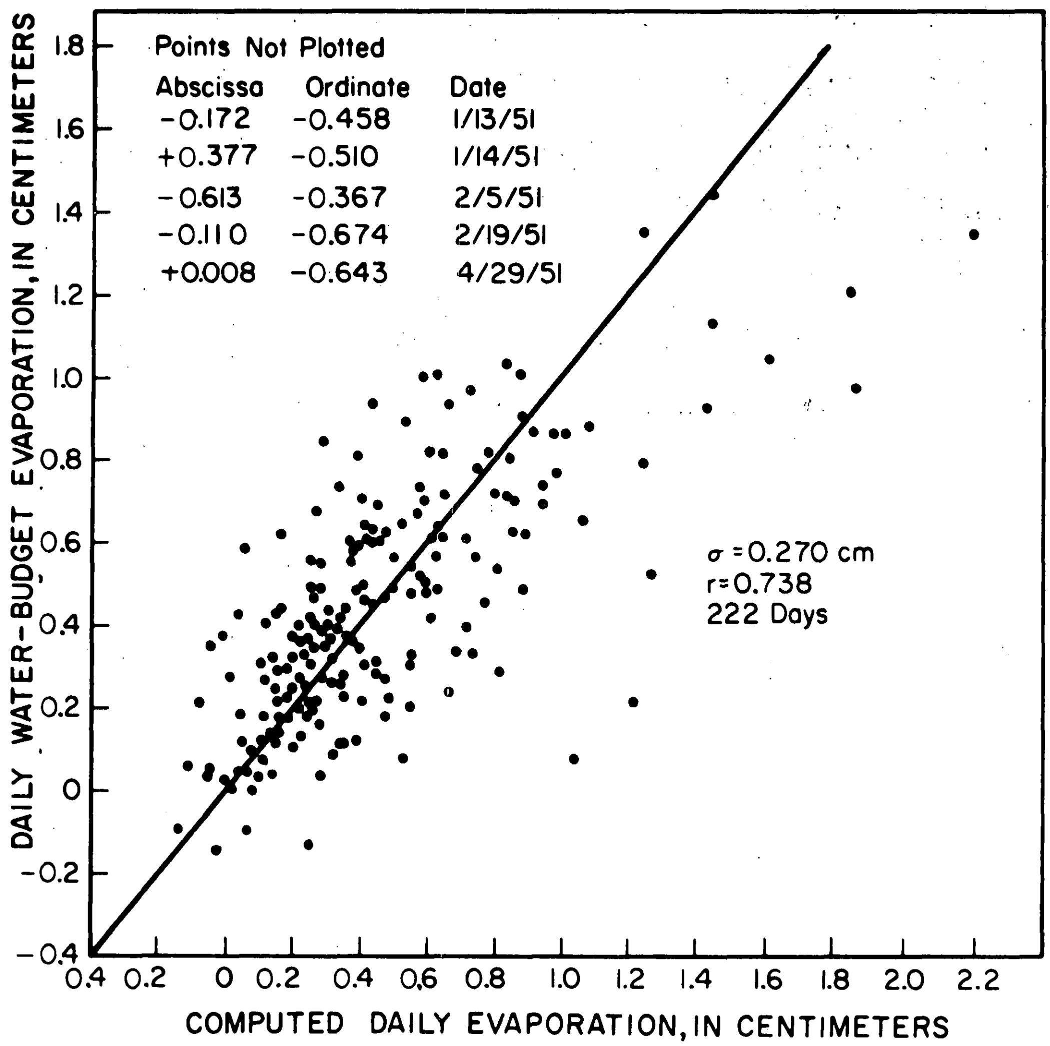

Figure 13. Experimental test of the modified log law.

The correlation of the daily error values resulting from the use of the modified log law with the stability parameter is demonstrated on figure 14. The 0.99 level confidence limits on the population correlation coefficient are -0.460 and -0.143. The correlation of the daily error values resulting from the use of the modified log law with the 8-meter wind velocity is demonstrated on figure 15. The 0.99 level confidence limits on the population correlation coefficient are -0.400 and -0.073, and the correlation of daily errors with the 8-meter specific humidity is demonstrated on figure 16. The 0.99 level confidence limits on this population correlation coefficient are -0.187 and +0.157.

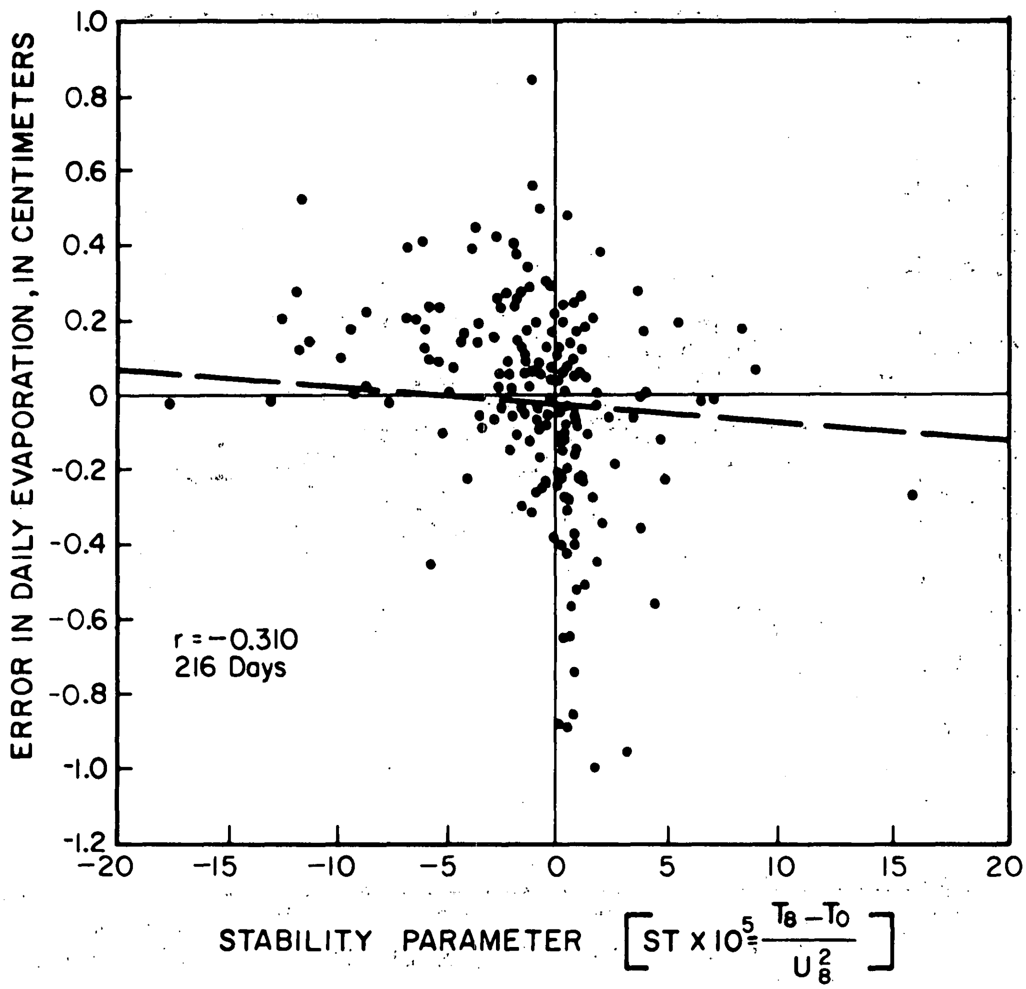

Figure 14. Correlation of the daily error values with the stability parameter for the application of the modified log law.

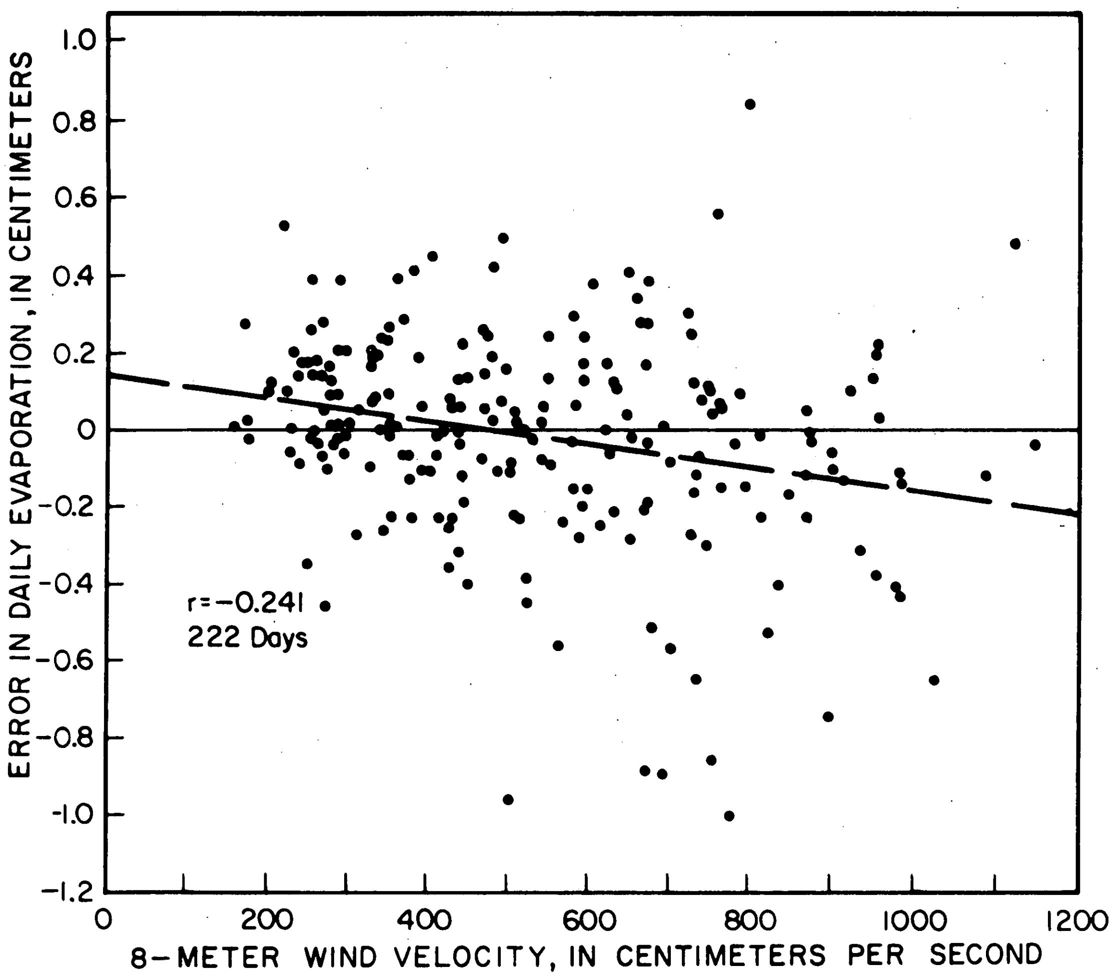

Figure 15. Correlation of the daily error values with wind velocity for the application of the modified log law.

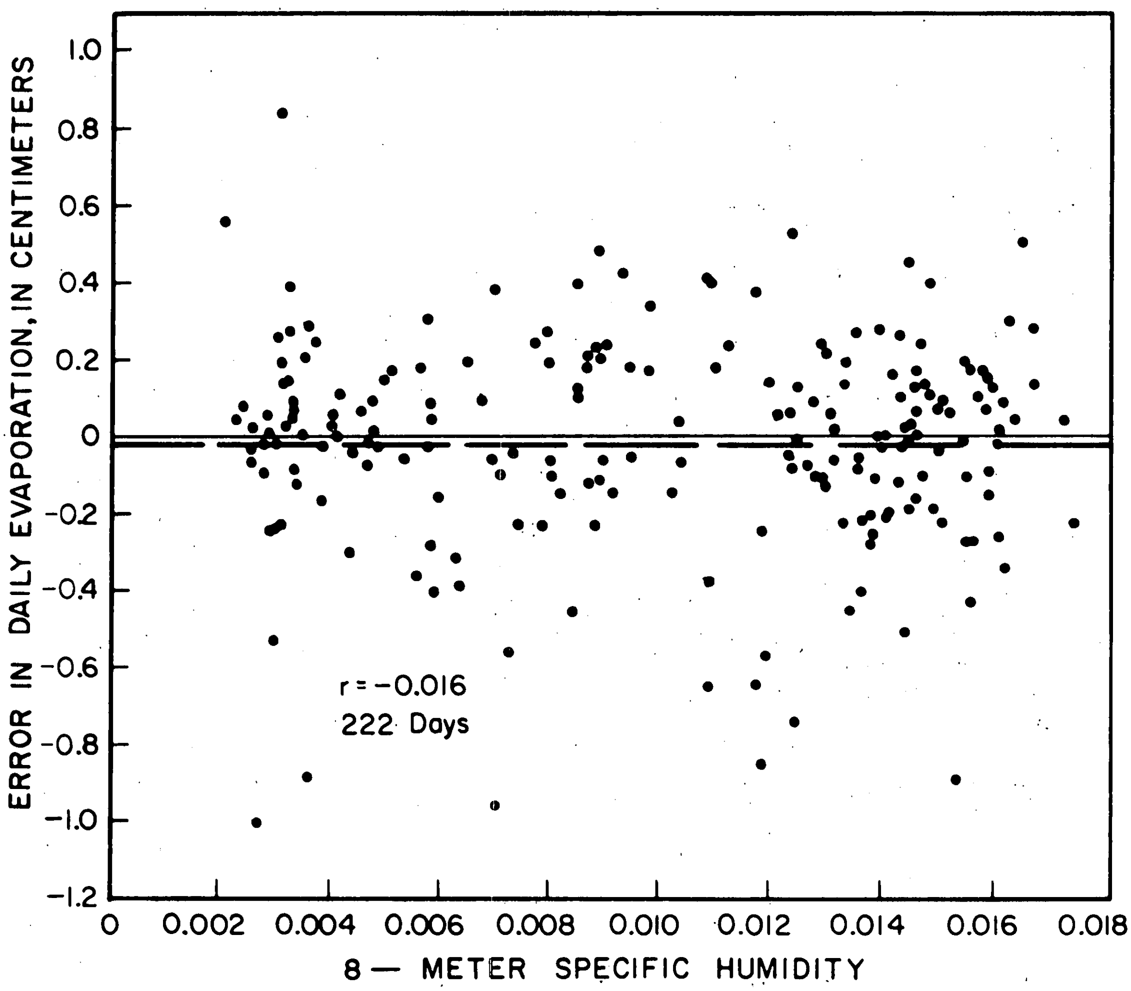

Figure 16. Correlation of the daily error values with specific humidity for the application of the modified log law.

The primary purpose of developing the modified log law was to reduce the cumulative effect of the correlation of model errors, the fourth term in equation 60. Therefore, the graph of the cumulative errors in the computed evaporation as a function of time should offer the best means of determining whether or not the goal of the method has been achieved. Figure 17 is such a plot. As illustrated on figure 17, the modified log law using equation 52 to estimate the value of u_* overpredicted the total evaporation during the 222 days by 4.19 cm or 4.6 percent. The shape of the curve on figure 17 is very similar to the shape of the curve on figure 7 which indicates that the seasonal nature of the errors in the log law and the modified log law are similar. During the months of August and September 1950, the modified log law underpredicted the evaporation by 25.8 percent whereas the direct log law underpredicted it by 41.5 percent. During the rest of the time the modified log law overpredicted the evaporation by 16.8 percent whereas the direct log law overpredicted the evaporation by 38.1 percent. Because the mean value of the shear velocity for the 222-day period is the same whether computed from equation 49 or 52 and the value of Q_* was computed from equation 53 in both the log and the modified log laws, the improved accuracy indicated on figure 17 must have resulted from reductions in one or all of the last three terms in equation 60.

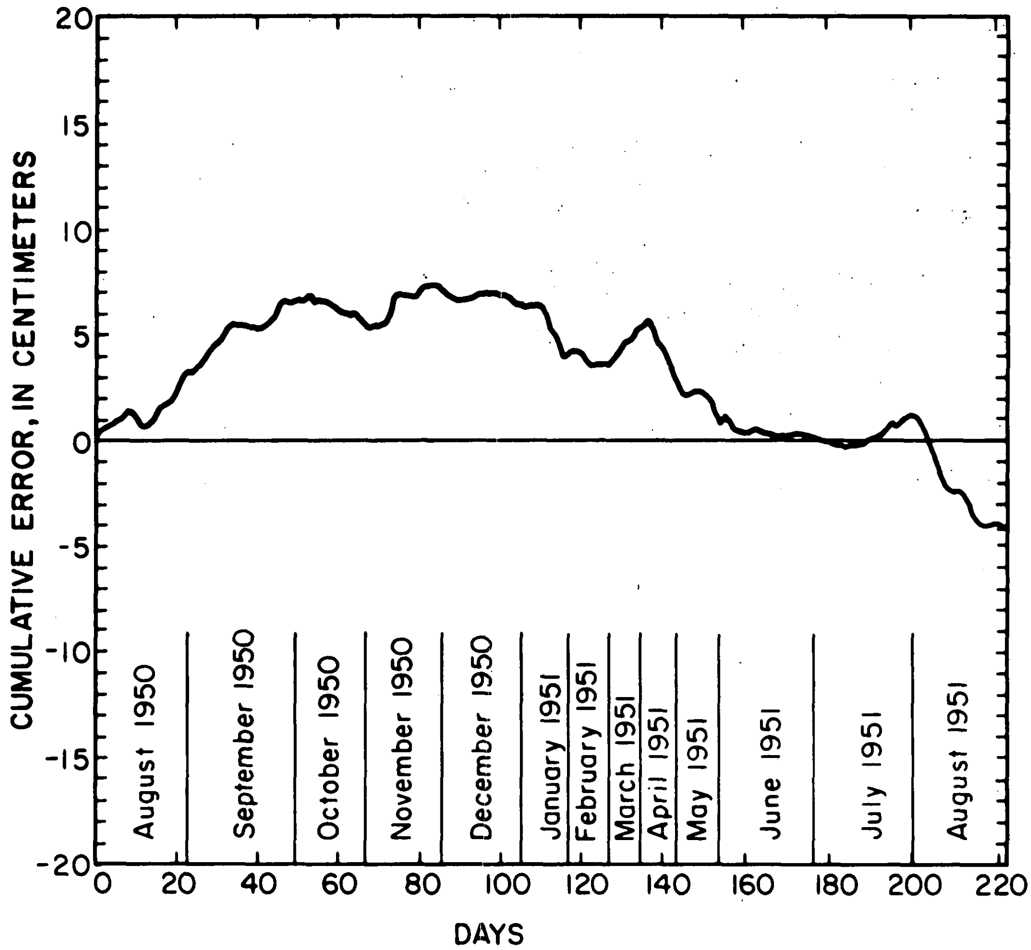

Figure 17. Seasonal variation of error values resulting from the application of the modified log law.

The data were also analyzed using the modified log law wherein the shear velocity was determined from equation 61. The results of this analysis are presented in brief form. The standard deviation of the daily errors was 0.302 cm and the coefficient of correlation of the daily errors with the stability parameter, wind speed, and specific humidity were -0.310, -0.241, and -0.008 respectively. The accumulated values of the daily errors varied with time in a manner which was very similar to those illustrated on figure 7 except that the accumulated errors for the modified log law were only about 2/3 as large as those shown on figure 7. The relationship between shear velocity and mean wind velocity which is based upon all data instead of only data obtained when the atmosphere is neutrally stable appears to give more accurate results.

The log+linear law

The purpose of this section is to describe the computation procedure and the results obtained when the log+linear law is used to describe the data. Before this description can be applied the value of α must be determined. The value of α was determined from the data using a method proposed by Deacon (1962) and which will be described before further analysis is performed. After a value for α has been determined, the results obtained as a result of the direct application of the log+linear law to the data will be presented. Then, using a method similar to that used in the modified log law, the data will be analyzed again using what is called the modified log+linear law.

The assumption is made in this section that all wind, temperature, and humidity profiles can be described by equations of the form given in equations 32, 33, and 35. The validity of this assumption will be assessed by comparing the computed to the measured evaporation values. The first problem is to find a value for α which will make equations 32 and 33 best fit the measured wind and temperature profiles. After Deacon (1962), apply equation 32 to two levels, say the 8-meter and 2-meter levels, and subtract to give

$$u_8 - u_2 = \frac{u_*}{\kappa} \ln 4 + \alpha \frac{6 u_*}{\kappa L} \qquad (62)$$

Divide both sides by a reference velocity. The 8-meter velocity will be used here because it is believed to be the most reliable. This gives

$$\frac{u_8 - u_2}{u_8} = \frac{u_*}{\kappa\, u_8} \ln 4 + \alpha \frac{6\, u_*}{u_8\, \kappa L} \qquad (63)$$

From equation 24 and by use of the definition of u_*

$$\frac{u_*}{\kappa L} = - \left(\frac{g}{T'}\right)\left(\frac{H}{\tau\, C_p}\right) \qquad (64)$$

From equations 2 and 4

$$\frac{H}{C_p\, \tau} = - \frac{\varepsilon_H}{\varepsilon_m} \left(\frac{\partial\theta/\partial z}{\partial u/\partial z}\right) \qquad (65)$$

where ε_H is the turbulent transfer coefficient for sensible heat. Combining equations 64 and 65

$$\frac{u_*}{\kappa L} = \left(\frac{g}{T'}\right)\left(\frac{\varepsilon_H}{\varepsilon_m}\right)\left(\frac{\partial\theta/\partial z}{\partial u/\partial z}\right) \quad . \qquad (66)$$

The reference virtual temperature is assumed to be measured at 8 meters. Like the reference velocity, the elevation of the measurement of this temperature is arbitrary. If the functional forms of the equations which describe the temperature and velocity profiles are the same, even if they are not described by equations 30 and 31, the ratio of the gradients can be replaced by a difference quotient so

$$\frac{u_*}{\kappa L} = \left(\frac{g}{T'_8}\right)\left(\frac{\varepsilon_H}{\varepsilon_m}\right)\left(\frac{\theta_8 - \theta_2}{u_8 - u_2}\right) \quad . \qquad (67)$$

A measure of atmospheric stability, S, is now defined

$$S = \left(\frac{g}{T'_8}\right)\left(\frac{6}{u_8}\right)\left(\frac{\theta_8 - \theta_2}{u_8 - u_2}\right) \quad . \qquad (68)$$

With this definition, equation 63 can be rewritten in the convenient form

$$\frac{u_8 - u_2}{u_8} = \frac{u_*}{\kappa\, u_8} \ln 4 + \alpha\left(\frac{\varepsilon_H}{\varepsilon_m}\right) S \quad . \qquad (69)$$

At least under conditions of near neutral stability, the value of $u_*/\kappa\, u_8$ should be nearly constant, equation 8. If, therefore, the left side of equation 69 were plotted as a function of S for each set of profiles, the results should define a straight line in the neighborhood of $S=0$. The slope of this straight line is the quantity $\alpha(\varepsilon_H/\varepsilon_m)$. To be consistent with the assumptions made in the derivation of equation 32, the value of ε_H will be assumed to be equal to the value of ε_m .

The values of $(u_8 - u_2)/u_8$ and S were computed for all 8,793 sets of profiles. In order to reduce the scatter of individual points, groups of these values were averaged. The value of S determined the group to which individual points belonged. Figure 18 is a plot of the resulting averages. The number of profiles represented by each plotted point ranges from 131 for $S = -0.0546$ to 584 for $S = +0.0123$. A smooth curve through the data points is shown. The slope of this curve in the region of $S=0$ is not well defined. It is possible to draw a reasonable curve through the data which has a slope at $S=0$ with any value between 2.1 and 4.2. This rather indeterminate slope is consistent with Webb's (1970) observation that a large range in values of α produce acceptable fits of velocity profiles. At this point the value of α was assumed to be 3.0.

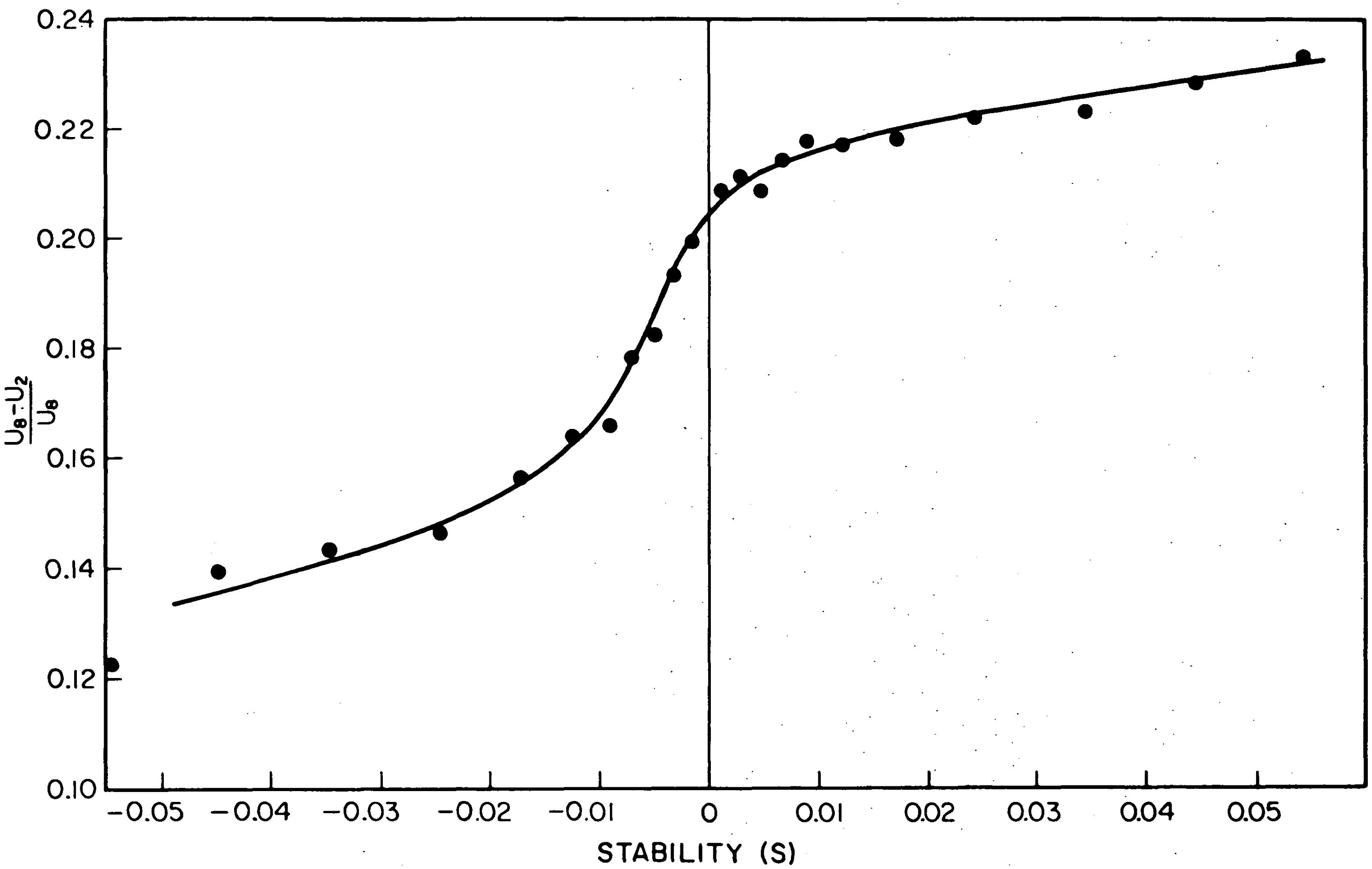

Figure 18. Profile data plotted to give α in the log+linear law.

Once the value of α is assumed the problem reduces to the determination of the values of shear velocity, heat flux, and vapor flux for each set of profiles which make equations 32, 33, and 35 best fit the measured data. This task is more complicated than it was for the log law because the equations are more complicated and because the shear velocity is dependent on the heat flux. The equations became so complicated that an iterative technique became necessary. This technique will be described in detail in what follows, but briefly it involved first, estimating a value for u_* from equation 52, then determining a value for H such that equation 33 best fit the temperature data. Using this value of H a new value of u_* was determined such that equation 32 best fit the velocity data . The second value of u_* was compared to the first value of u_* and if they did not agree well, the process was repeated using the new value of u_* as a starting point. After successive values of u_* converged, the value of E was determined such that equation 35 best fit the humidity data.

Keeping this overall procedure in mind the specific steps will be described in detail. The value of u_* as determined by equation 52 was considered to be the best estimate that could be obtained a priori; therefore, it was used as a starting point.

Combining equation 33 with the definitions of T_* and L (equations 26, 24), the expression for the potential temperature can be written as

$$\theta = A_T H \ln z + B_T H^2 z + C_T \tag{70}$$

where the constant

$$A_T = (\kappa u_* C_p \rho)^{-1} \tag{71}$$

and the constant

$$B_T = \frac{\alpha g}{T' C_p^2 \rho^2 u_*^4} \tag{72}$$

The virtual temperature, T', was determined from

$$T' = \frac{T_4}{1 - \dfrac{0.375\ q\ w}{0.622 + 0.378\ q_4}} \tag{73}$$

This virtual temperature is on the absolute scale and includes the effect of water vapor on the buoyancy term. The potential temperature was referenced to the 2-meter level so that no correction needed to be made to that temperature. The potential temperatures at the 4- and 8-meter levels were obtained by simply adding 0.02°C and 0.06°C to the respective absolute temperatures. The value of the density of air, ρ, was calculated in the same manner as for the log law. Minimizing the sums of the squares of the differences between the measured potential temperature and the potential temperature as determined by equation 70

$$2 B_T^2 D_z H^3 - 3 A_T B_T D_{z\ell z} H^2 + [A_T^2 D_{\ell z} - 2 B_T D_{tz}] H + A_T D_{t\ell z} = 0 \tag{74}$$

where

$$D_z = \sum_{i=1}^{3} z_i^2 - \frac{1}{3}\left[\sum_{i=1}^{3} z_i\right]^2 \tag{75}$$

$$D_{z\ell z} = \sum_{i=1}^{3} z_i \ln z_i - \frac{1}{3}\left[\sum_{i=1}^{3} z_i\right]\left[\sum_{i=1}^{3} \ln z_i\right] \tag{76}$$

$$D_{tz} = \sum_{i=1}^{3} \theta_i z_i - \frac{1}{3}\left[\sum_{i=1}^{3} \theta_i\right]\left[\sum_{i=1}^{3} z_i\right] \tag{77}$$

$$D_{t\ell z} = \sum_{i=1}^{3} \theta_i \ln z_i - \frac{1}{3}\left[\sum_{i=1}^{3} \theta_i\right]\left[\sum_{i=1}^{3} \ln z_i\right] \quad . \tag{78}$$

All real solutions to equation 74 were determined and the solution which gave the minimum value of the squared error was selected as the best value of H.

Using the best value of H, the value of u_* which minimized the sum of the squares of the differences between the measured velocity and the velocity computed by equation 32 was determined. To do this equation 32 was rewritten in the form

$$u = u_* A_u \ln z + B_u z + C_u \tag{79}$$

where the constant

$$A_u = 1/\kappa \tag{80}$$

and the constant

$$B_u = - \frac{\alpha g H}{u_*^2 T' C_p \rho} \quad . \tag{81}$$

In the determination of B_u the previous estimate of u_* was used. Minimizing the sum of the squares of the errors in equation 79

$$u_* = \frac{D_{u\ell z} - B_u D_{z\ell z}}{A_u D_{\ell z}} \quad . \tag{82}$$

This value of u_* was called the new value of u_* and its value was compared to the previous value of u_*. If the difference between the two values of u_* was greater than 0.07 cm/sec, another iteration was performed by using the new value of u_* as a starting point with equations 71, 72, and 74. The arbitrary difference of 0.07 cm/sec was selected as being approximately equal to 1 percent of the minimum expected value for u_* as determined from the minimum wind speed expected at Lake Hefner.

In general the iteration procedure converged rapidly, seldom requiring more than five or six iterations. In certain cases, however the process did not converge. If convergence was not obtained within 20 iterations it was arbitrarily assumed that the process was unstable and the entire set of profiles was discarded. Thirty-three percent of the profiles failed to converge and so were eliminated. This caused the loss of a considerable part of the data. The failure of the process to converge, however, would tend to indicate that the data were in error. The process is believed to sort out the "bad" data. If the process converged, the value of u_2/u_* was computed, and to be consistent with the analysis of the log law, the set of profiles was ignored if the value of u_2/u_* was less than 1.37 or greater than 137. Very few profiles failed this test.

If an acceptable value of u_* was obtained, the evaporation, E was then determined such that the sum of the squares of the differences between the measured specific humidity and the specific humidity computed from equation 35 was a minimum. Rewriting equation 35

$$q = q_* \ln z + Q_* B_q z + C_q \qquad (83)$$

where

$$B_w = \frac{\kappa \alpha g H}{T' C_p u_*^3} \quad . \qquad (84)$$

The value of H corresponding to the last iterative process was used. Minimizing the error in equation 83

$$Q_* = \frac{B_q D_{qz} - D_{q\ell z}}{D_{\ell z} - 2 B_q D_{z\ell z} + B_q^2 D_z} \qquad (85)$$

where

$$D_{qz} = \sum_{i=1}^{3} q_i z_i - \frac{1}{3}\left[\sum_{i=1}^{3} q_i\right]\left[\sum_{i=1}^{3} z_i\right] \quad . \qquad (86)$$

The value of the evaporation was then computed from equation 13. To be consistent with the analysis of the log law, values of E which were larger than 10 cm/day or less than -2 cm/day were considered unacceptable and the average daily evaporation rate was computed as the average of all the acceptable evaporation rates for the day. In this case, because the iteration procedure often did not converge, there were often less than 20 periods of acceptable record for the day. No day's results were thrown out because the final average was based on less than 20 sets of profiles.

These calculations result from the direct application of the log+linear law to each individual set of profile data. Some of the results of these computations are presented on figure 19. Figure 19 is directly comparable to figures 3 and 13. The standard deviation of daily errors, illustrated on figure 19, is 0.272 cm. This value is to be compared with the value of 0.383 cm obtained from the direct application of the log law. Although there seems to be a definite improvement it is well to remember that the standard daily error represented on figure 19 is still 65 percent of the mean daily evaporation.

The sensitivity of the results to the assumed value of α was investigated at this point. Various values of α, ranging from 1.0 to 5.0 were assumed and the standard deviation of daily error values were computed in the same manner for each assumed value of α. The standard deviation remained constant, to within three significant figures, for values of α within the range 1.5 to 2.5 and it increased slowly as values departed in either direction from this range. This confirms that the results are quite insensitive to the assumed numerical value of α, as was observed by Webb (1970) and predicted by the use of figure 18. The assumed value of 3 for α, which is used in this study, is close to the optimum value.

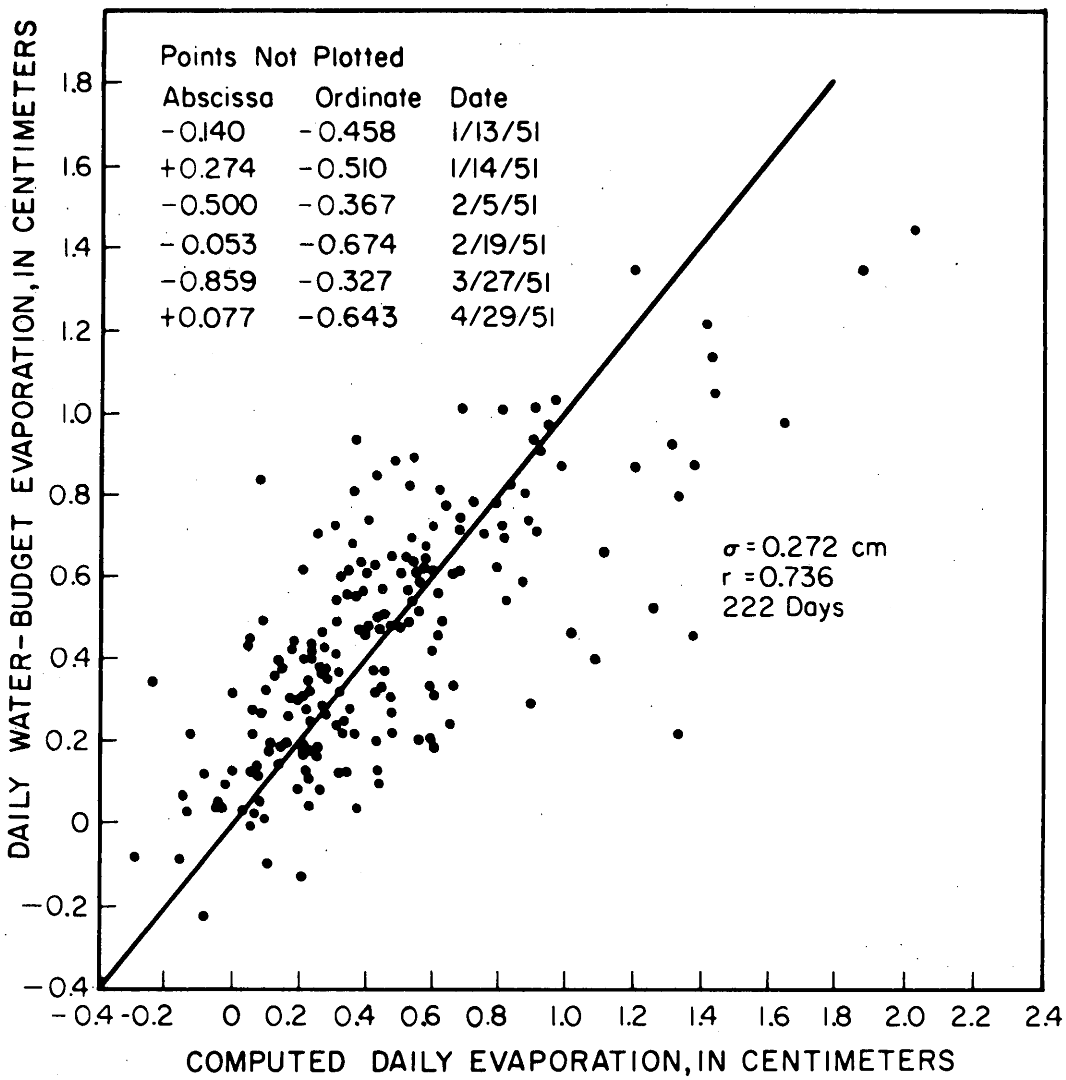

Figure 19. Experimental test of the direct application of the log+linear law.

The log+linear law was specifically designed to account for atmospheric stability. Figure 20 is a plot of the daily error values against the stability parameter, S T . Also included is the regression line. Using the same procedure as for the log law case, the 0.99 level confidence interval for the correlation coefficient is -0.069, -0.396. Therefore, the daily error values are still highly correlated with stability.

The correlation between the daily error values and the daily average value of the 8-meter wind speed is illustrated on figure 21. The 0.99 level confidence limits on the population correlation coefficient are -0.076, -0.402. The daily error value is significantly correlated with the 8-meter velocity.

The correlation between the daily error values and the daily average value of the 8-meter specific humidity is illustrated on figure 22. The 0.99 level confidence limits on the population correlation coefficient are -0.012, +0.324. Likewise, the 0.95 confidence limits are +0.030, +0.286. As for the log law, the error is probably correlated with the specific humidity, but the probability is not as large as was the case for the wind velocity and stability parameter.

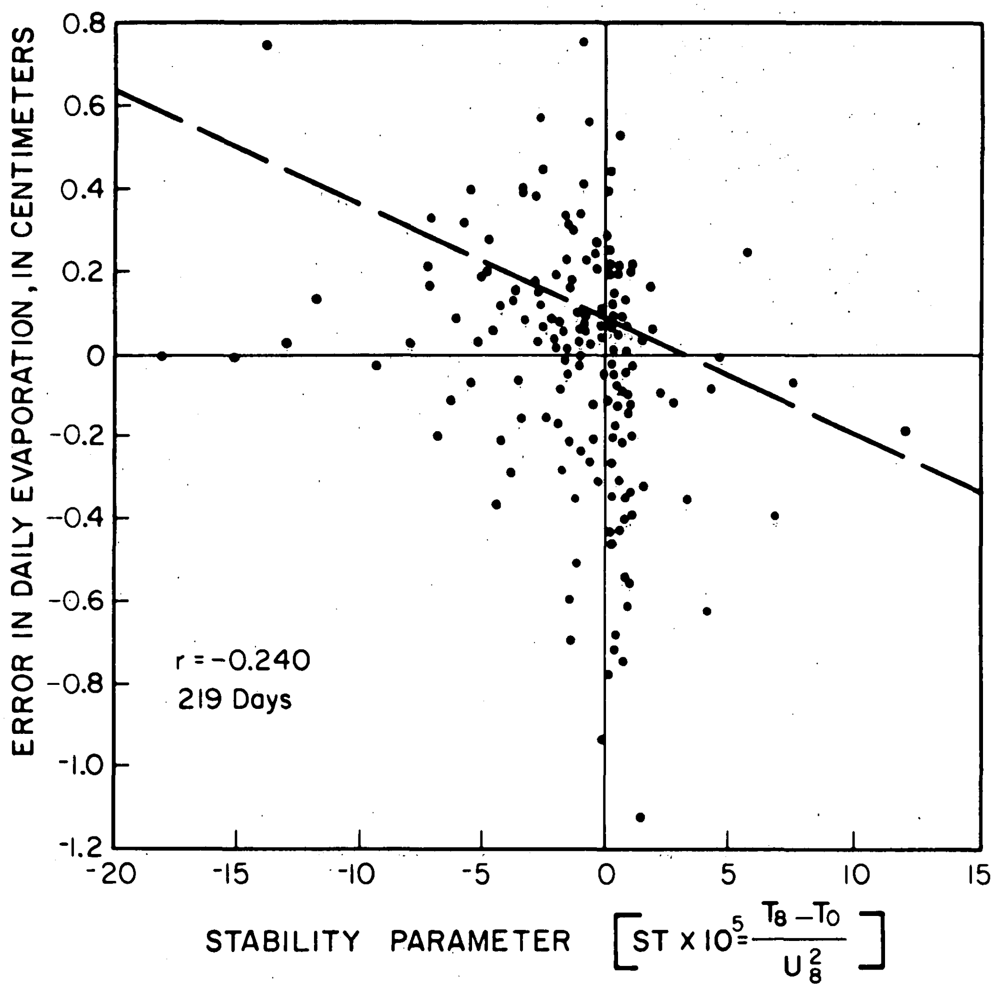

Figure 20. Correlation of the daily error values with the stability parameter for the direct application of the log+linear law.

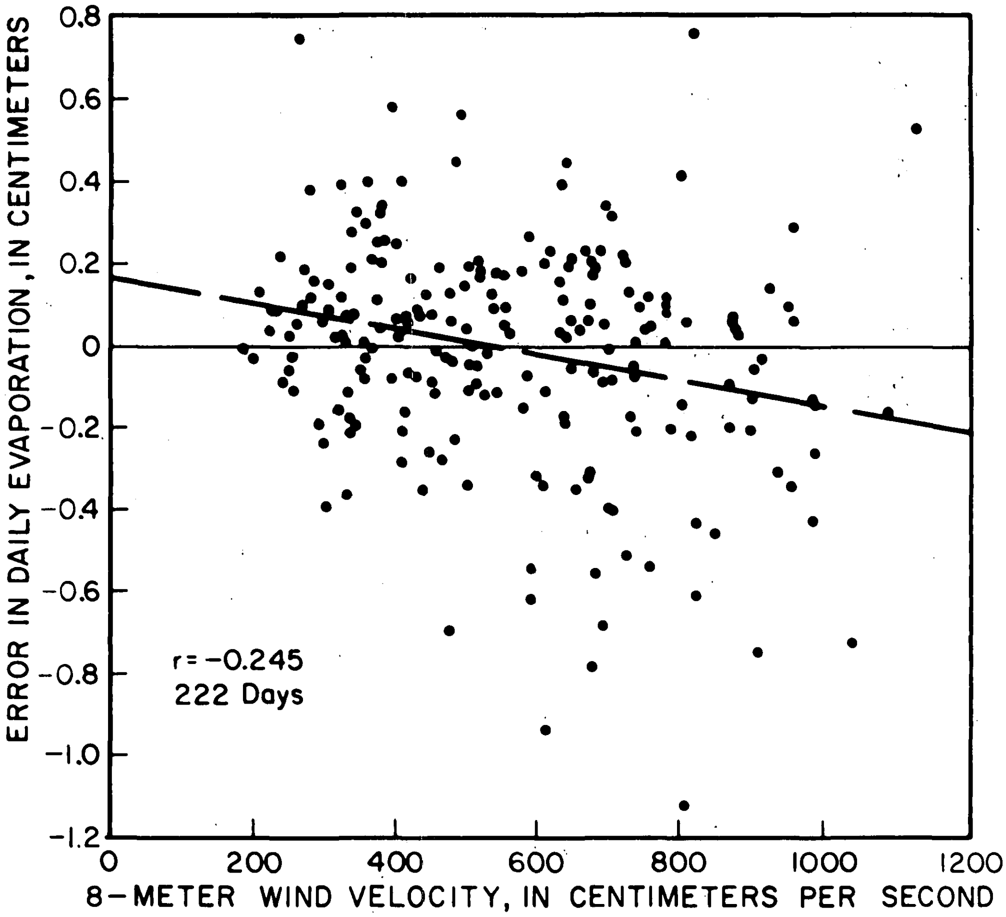

Figure 21. Correlation of daily error values with the wind velocity for the direct application of the log+linear law.

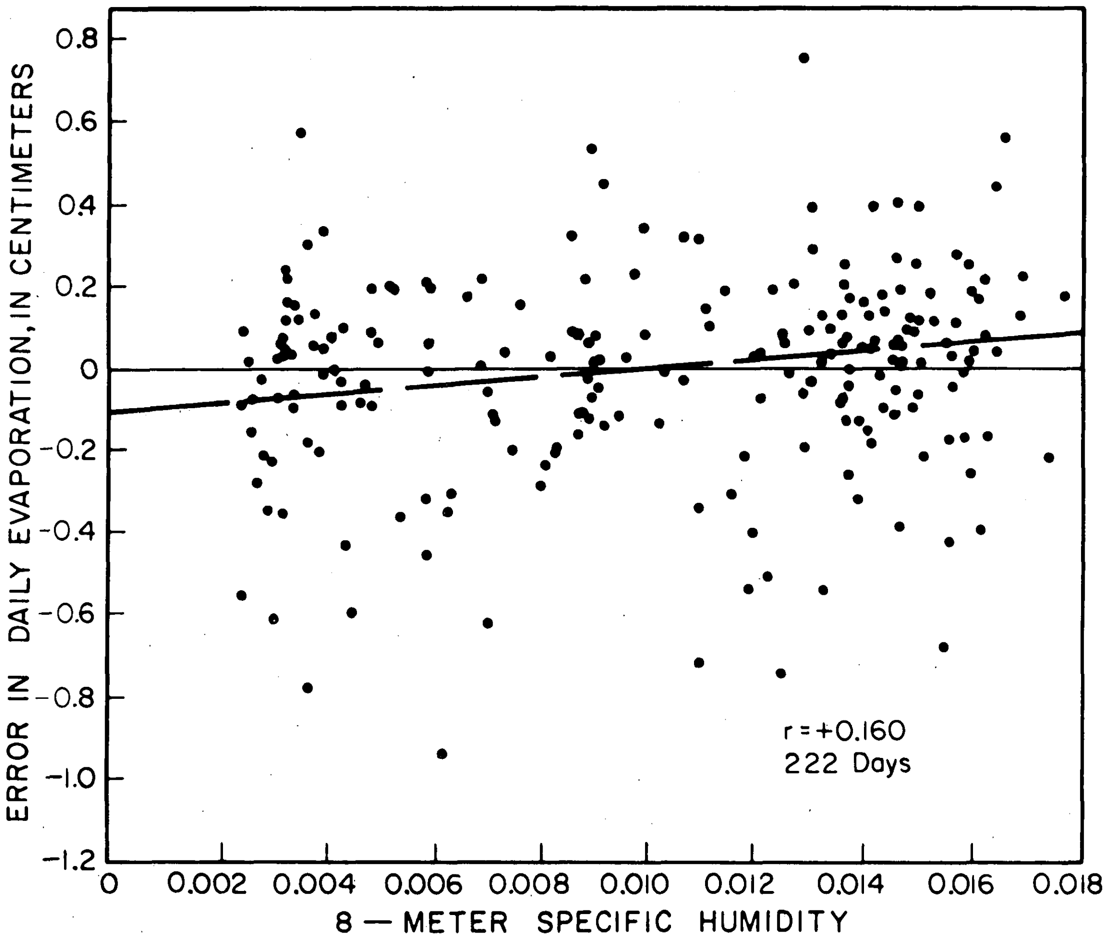

Figure 22. Correlation of the daily error values with the specific humidity of the air for the direct application of the log+linear law.

The seasonal nature of the errors in the log+linear law is demonstrated on figure 23. Comparing figure 23 to figure 7, the reduction in the cumulative error which results from the use of the log+linear law instead of the log law is clearly evident. At the end of the 222 days, the cumulative error in the computed evaporation is only -3.01 cm indicating that on the average the log+linear law overpredicted the evaporation by 3.3 percent. Unfortunately this 3.3 percent error is not generally representative of the accuracy of the log+linear law. During the months of August and September 1950, the log+linear law underpredicted the evaporation by an average 31.2 percent while during the months of October 1950, through August 1951, it overpredicted the evaporation by an average 17.6 percent.

As indicated by the high negative correlation of errors with the stability parameter on figure 20, the direct application of the log+linear law does not fully account for stability effects. A modified approach, therefore, such as that used on the log law, would appear to be fruitful. The results of such an approach will be presented next.

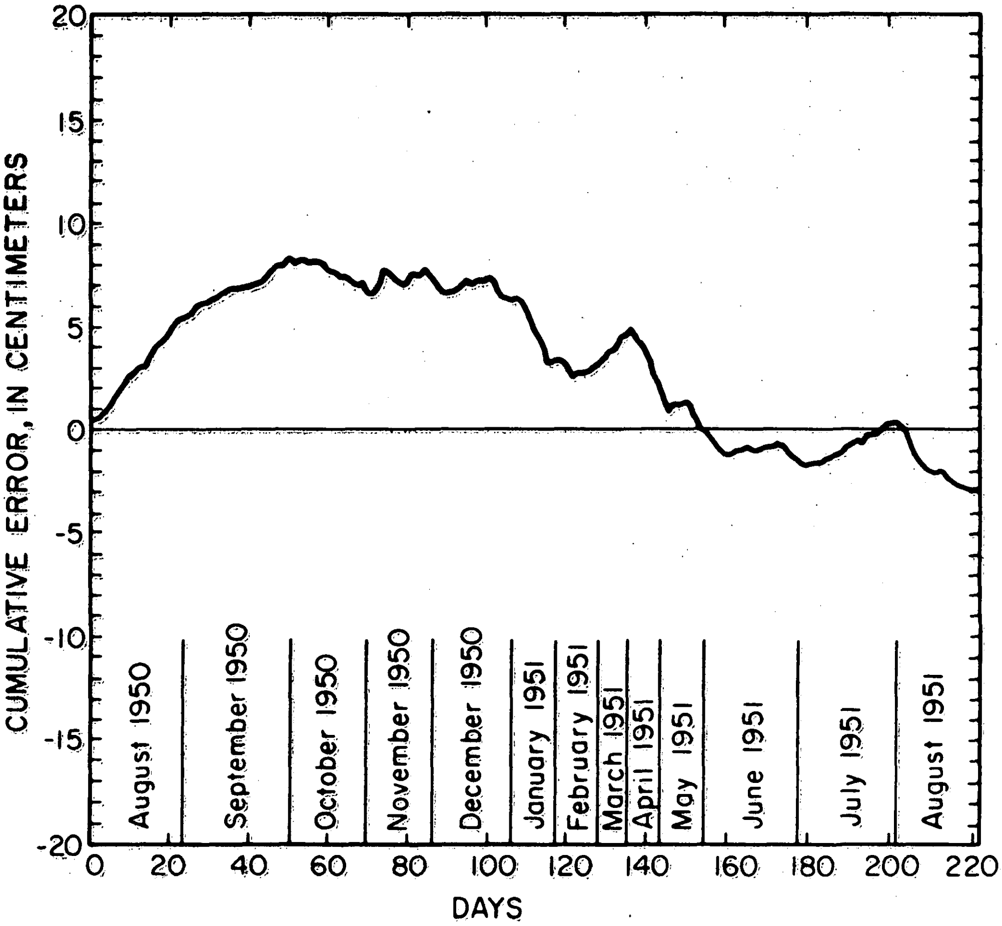

Figure 23. Seasonal variation of error values resulting from the direct application of the log+linear law.

The first step was to find a prediction equation for u_* consistent with the log+linear law, just as equation 52 is consistent with the log law. This equation is assumed to have the same form as equation 52 but with different coefficients. First the shear velocity was computed using the method described for the log+linear law. The coefficients were then determined by regression. The resulting equation was

$$u_* = -1.15 + 0.0712\, u_2 \quad . \tag{87}$$

Equation 52 predicts lower shear values than does equation 87, but the differences are only 1.7 and 8.9 percent at 1,000 and 100 cm/sec respectively.

The evaporation was computed in the same manner as before except that equation 87, instead of equation 82, was used to compute u_*. No iteration was needed so no profiles were discarded because of non-convergence. This procedure is called the modified log+linear procedure.

The computed and water-budget evaporation for each day is presented on figure 24. The standard deviation of daily error values, figure 24, is 6 percent lower than the standard deviation obtained by use of either the modified log law or the direct log+linear law. Because the errors resulting from the direct use of the log+linear law were highly correlated with stability, figure 20, it was expected that the modified log+linear law would reduce the standard deviation by a larger amount. One explanation for this rather poor showing of the modified log+linear law may be that profiles for which the iteration process did not converge were not ignored on figure 24 while they were on figure 19. The quality of the data which were used to check the modified log+linear law may have been poorer than that used to check the direct log+linear law because there is reason to believe that the data for which equations 74 and 82 did not converge were of poor quality. The modified log+linear law was then applied to only the profiles for which equations 74 and 82 converged. The standard deviation of daily errors for these data was 0.241 cm which represents a 12 percent reduction over the value obtained by the direct application of the log+linear law. Further results obtained from the modified log+linear law are based on all profiles, whether or not they converged, so that they will be consistent with those illustrated on figure 24 and so that the quality of the data will be consistent with that used for the log law.

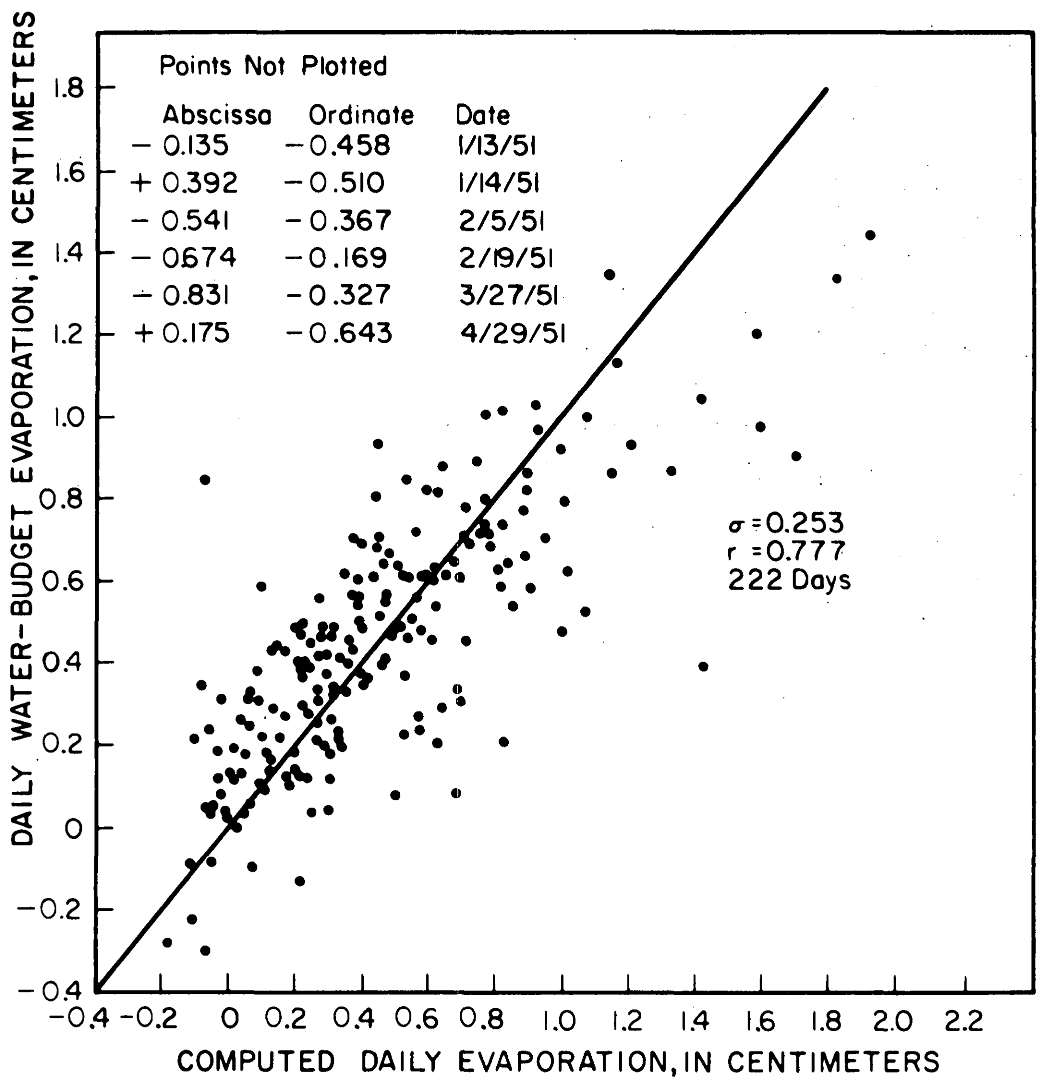

Figure 24. Experimental test of the modified log+linear law.

The correlation between the daily error values and the daily average stability parameter is illustrated on figure 25. The use of equation 87 has reduced the correlation coefficient on stability from -0.240 to -0.073. The 0.99 level confidence limits for the correlation coefficient presented on figure 25 are +0.100, -0.245. Using only the convergent data the correlation coefficient was -0.297.

The correlation between the daily error values and the daily average 8-meter wind speed is illustrated on figure 26. The 0.99 level confidence limits for the correlation coefficient presented on figure 26 are -0.033, -0.365. The errors are highly correlated with wind velocity.

The correlation between the daily error values and the daily average specific humidity at 8 meters is illustrated on figure 27. The 0.99 level confidence limits for the correlation coefficient presented on figure 27 are -0.198, +0.146. The errors are not significantly correlated with the specific humidity.

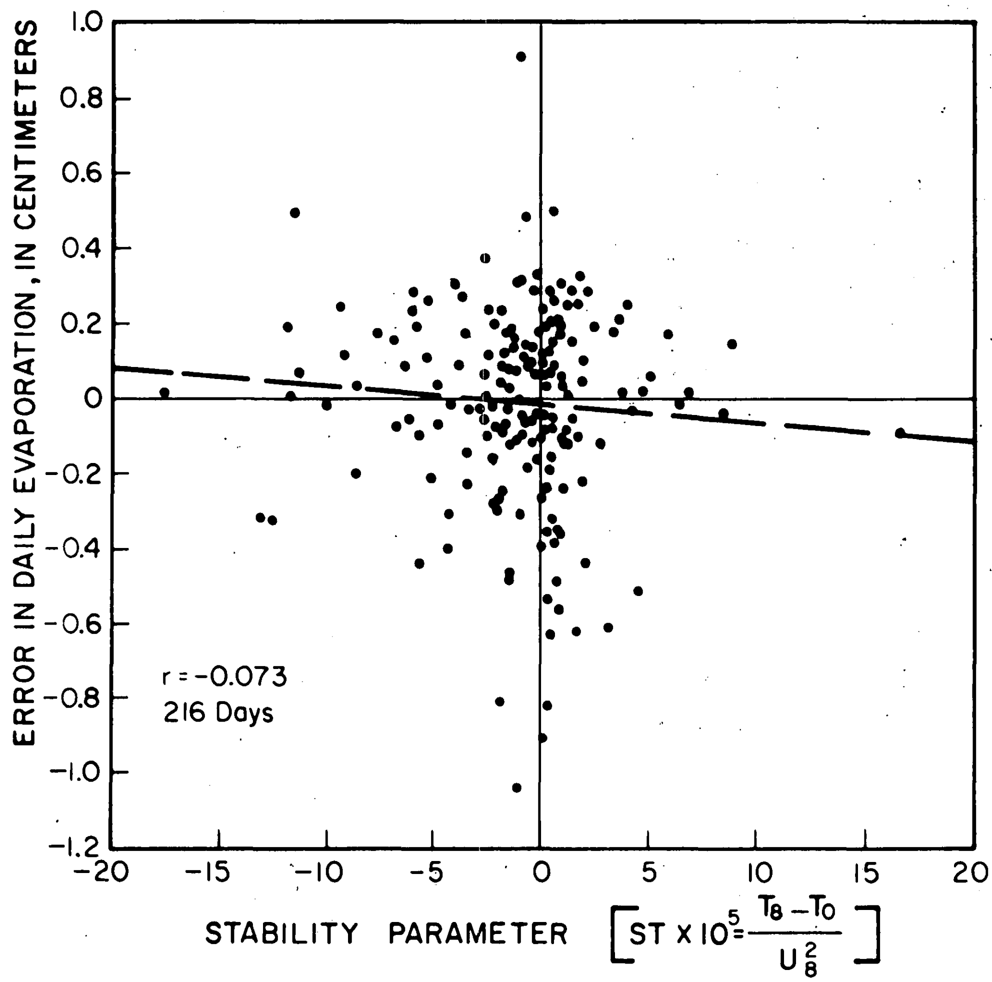

Figure 25. Correlation of the daily error values with the stability parameter for the application of the modified log+linear law.

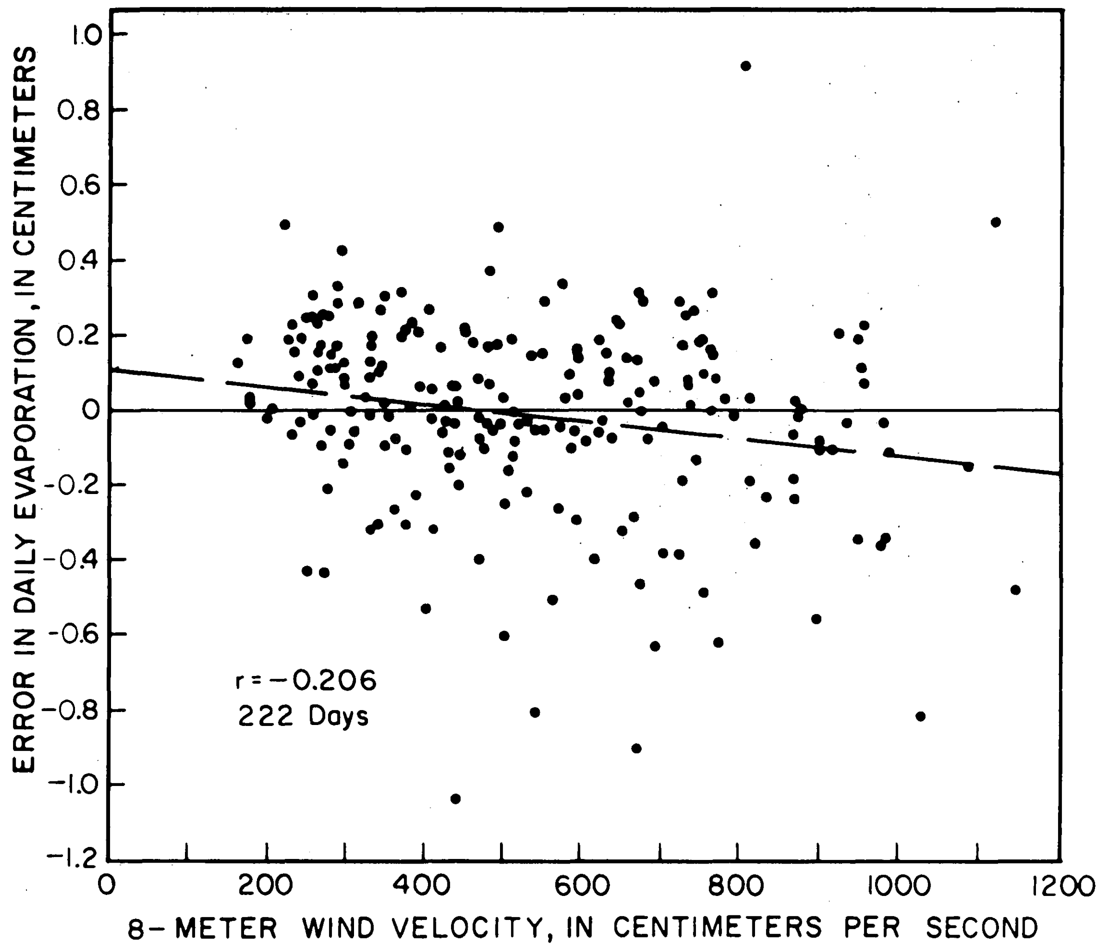

Figure 26. Correlation of the daily error values with the wind velocity for the modified log+linear law.

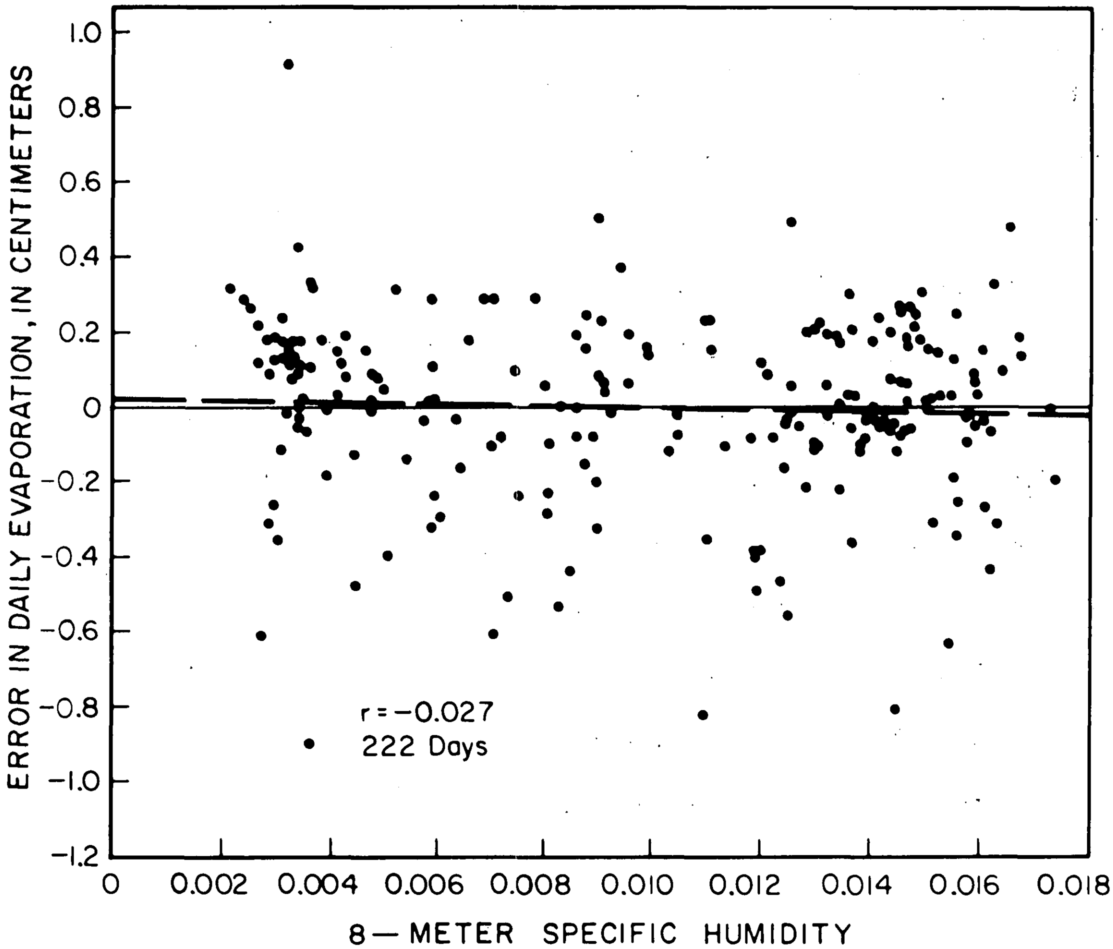

Figure 27. Correlation of daily error values with the specific humidity of the air for the application of the modified log+linear law.

The seasonal nature of the errors in the modified log+linear law is demonstrated on figure 28. At the end of 222 days the cumulative error in the computed evaporation is -0.85 percent. During the months of August and September 1950, the evaporation was underpredicted by an average 14.4 percent and during the months of October 1950, through August 1951, the evaporation was overpredicted by an average 6.8 percent. On figure 28 the maximum cumulative error occurred about January 1, 1951. During the months of August 1950 through December 1950, the evaporation was underpredicted by an average 14.8 percent and during the months of January through August 1951, the evaporation was overpredicted by an average 16.4 percent.

When only the convergent profiles were used the resulting seasonal variation of errors was very similar to that presented on figure 23. The total error at the end of the 222 days of record was -3.93 cm and the error, as of October 1, 1950, was +5.48 cm. The average errors were 4.2 percent overprediction for the entire period, 20.4 percent underprediction during the months of August and September 1950, and 13.7 percent overprediction during the rest of the time.

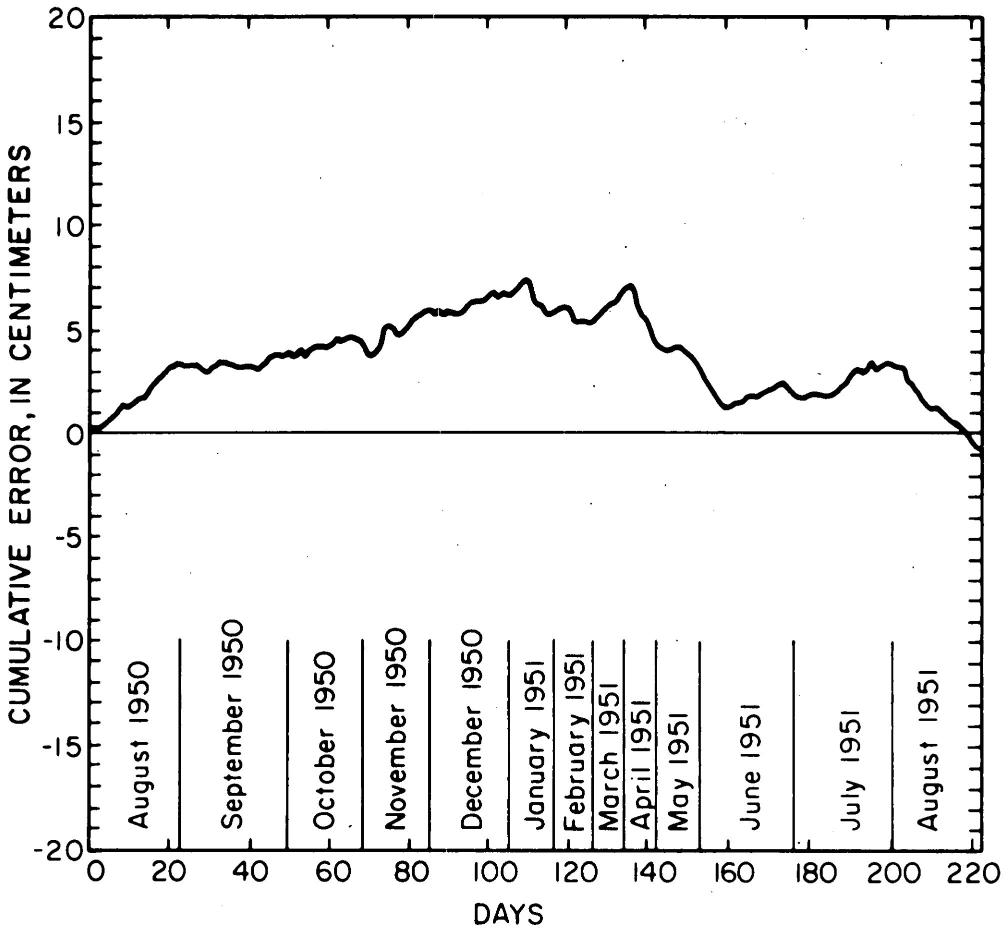

Figure 28. Seasonal variation of error values resulting from the application of the modified log+linear law.

The empirical mass-transfer approach

In order to serve as a basis for comparison,the empirical mass-transfer method was also used to compute the evaporation rate for each time period. Equation 37,with the wind function given by equation 38, was used for the calculations. One of the major advantages of equation 37 is that it does not tend to magnify measurement errors as does the aerodynamic methods. This advantage is at least partially offset because equation 38 contains an unknown coefficient error in the value of N . It was possible to eliminate this coefficient error in the results presented here because the evaporation for the total period was known from the water budget. The value of N in equation 38 was determined such that the total computed evaporation for the 222 days was exactly equal to the measured evaporation. This procedure eliminated the coefficient error from the value of N , at least for the time period of interest here. The resulting value of N was 1.302×10^{-4} which gives the evaporation rate in centimeters per day when the velocity is in centimeters per second and the vapor pressure in millibars.

Using this value for N , the evaporation was computed for each 30-minute time period of each day and the average evaporation for each of the 222 days was determined. In these computations, the wind velocity at the 2-meter level was used for u_2 , the value of e_o was determined from the water temperature and equation 46, and the value of e_a was determined from equation 48 and the specific humidity at the 2-meter level.

The computed and water-budget evaporation for each day is presented on figure 29. The standard deviation of the daily error values shown on figure 29 is 0.141 cm. The estimated maximum value of the standard deviation of the errors in the water-budget evaporation was 0.133 cm. Figure 29 was constructed using a value for N which was "exact". If the value of N had been chosen such that the total error for 13 months was not zero, the value of the standard deviation would have been considerably larger. For example, a 1 percent decrease in the value of N would increase the value of the standard deviation by about 1.5 percent. Nevertheless, the reason for the wide acceptance of the empirical mass-transfer method is clear.

The correlation between the daily error values for the empirical mass-transfer method and the daily average values of the stability parameter is illustrated on figure 30. The 0.99 level confidence limits for the correlation coefficient presented on figure 30 are +0.007, -0.333, and the 0.95 confidence limits are -0.036, -0.296. The error is probably correlated with stability.

The correlation between the daily error values and the daily average 8-meter wind speed is illustrated on figure 31. The 0.99 level confidence limits for the correlation coefficient are -0.003, -0.338. The error is highly correlated with wind velocity.

The correlation between the daily error values and the daily average 8-meter specific humidity is illustrated on figure 32. The 0.99 level confidence limits for the correlation coefficient are +0.165, -0.179. The error is not significantly correlated with specific humidity.

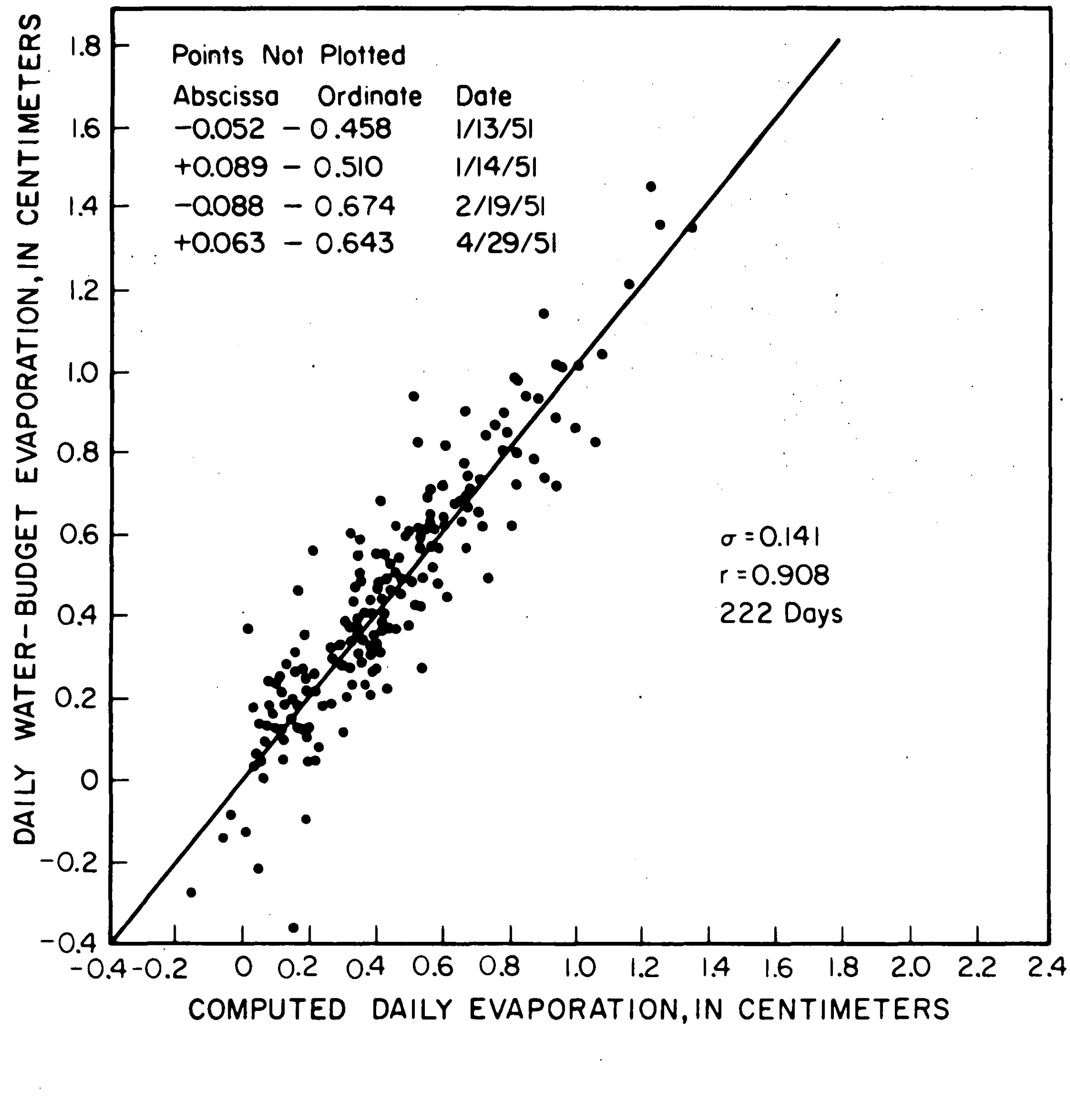

Figure 29. Experimental test of the empirical mass-transfer formula.

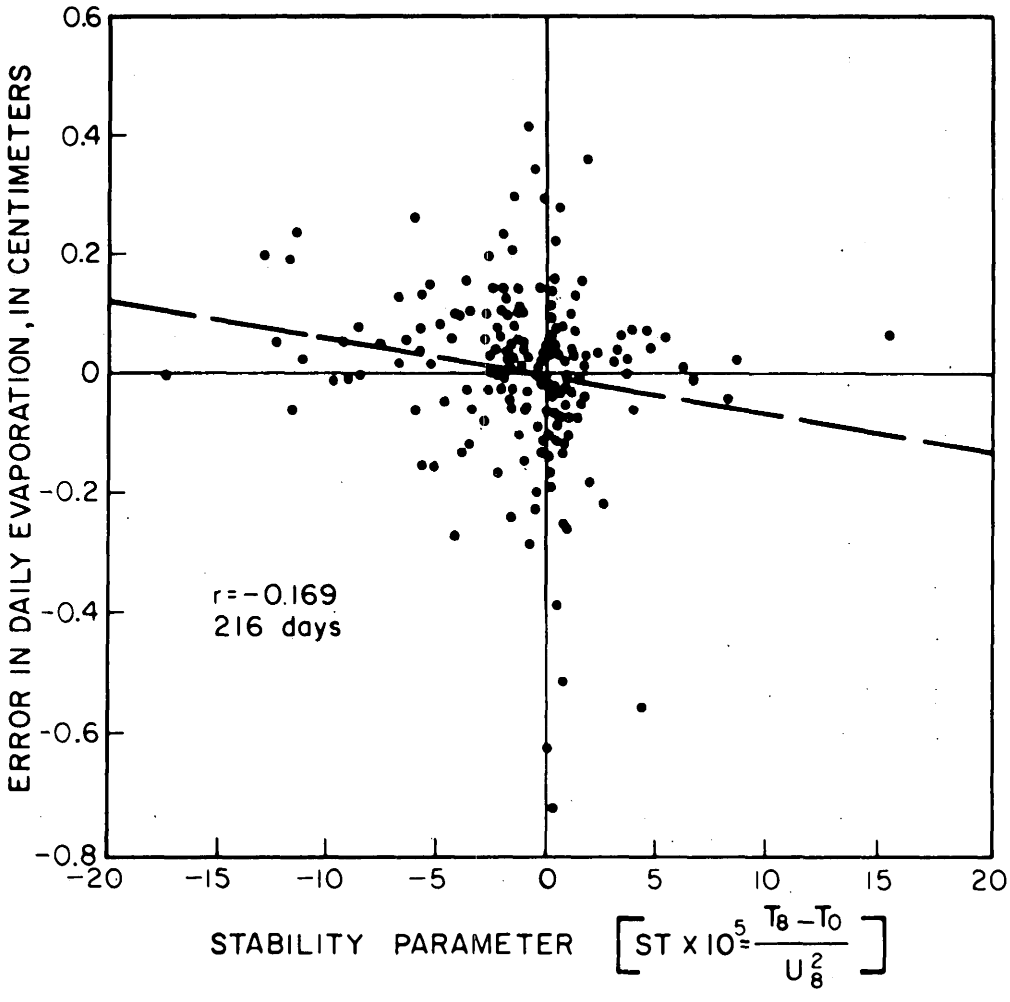

Figure 30. Correlation of the daily error values with the stability parameter for the empirical mass-transfer method.

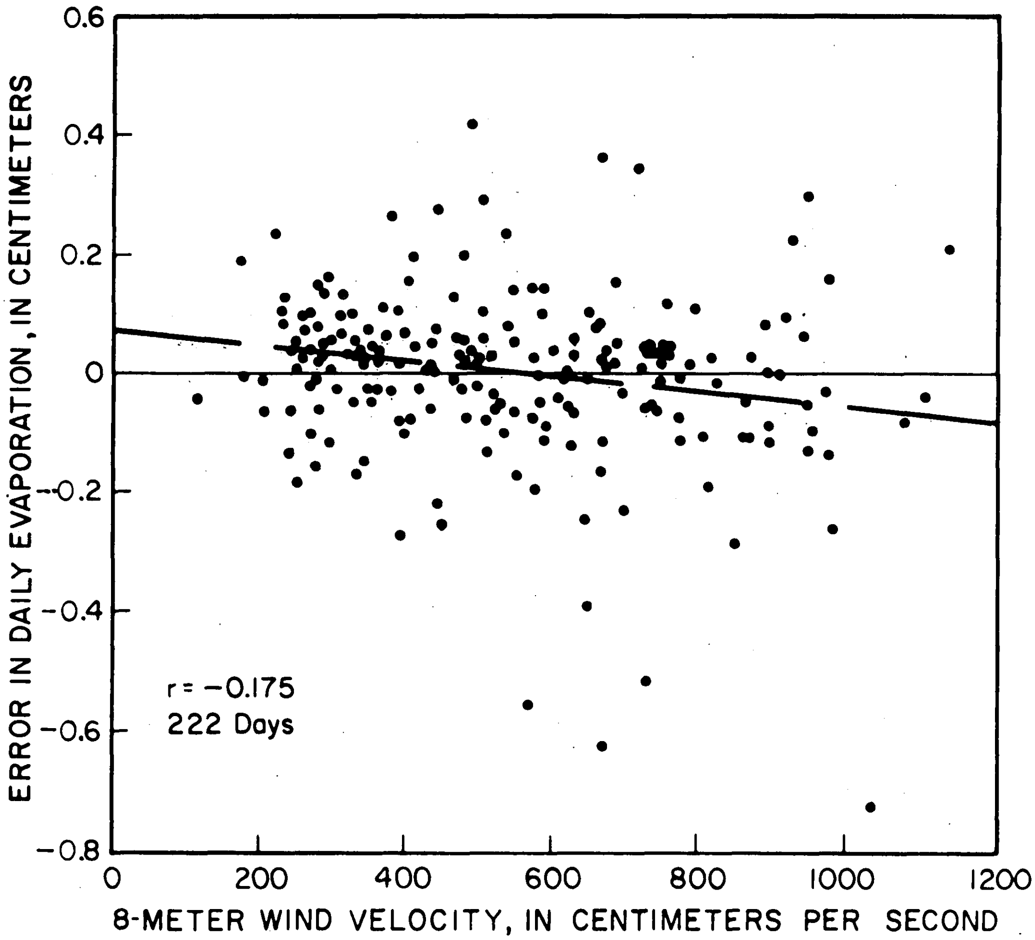

Figure 31. Correlation of the daily error values with wind velocity for the empirical mass-transfer method.

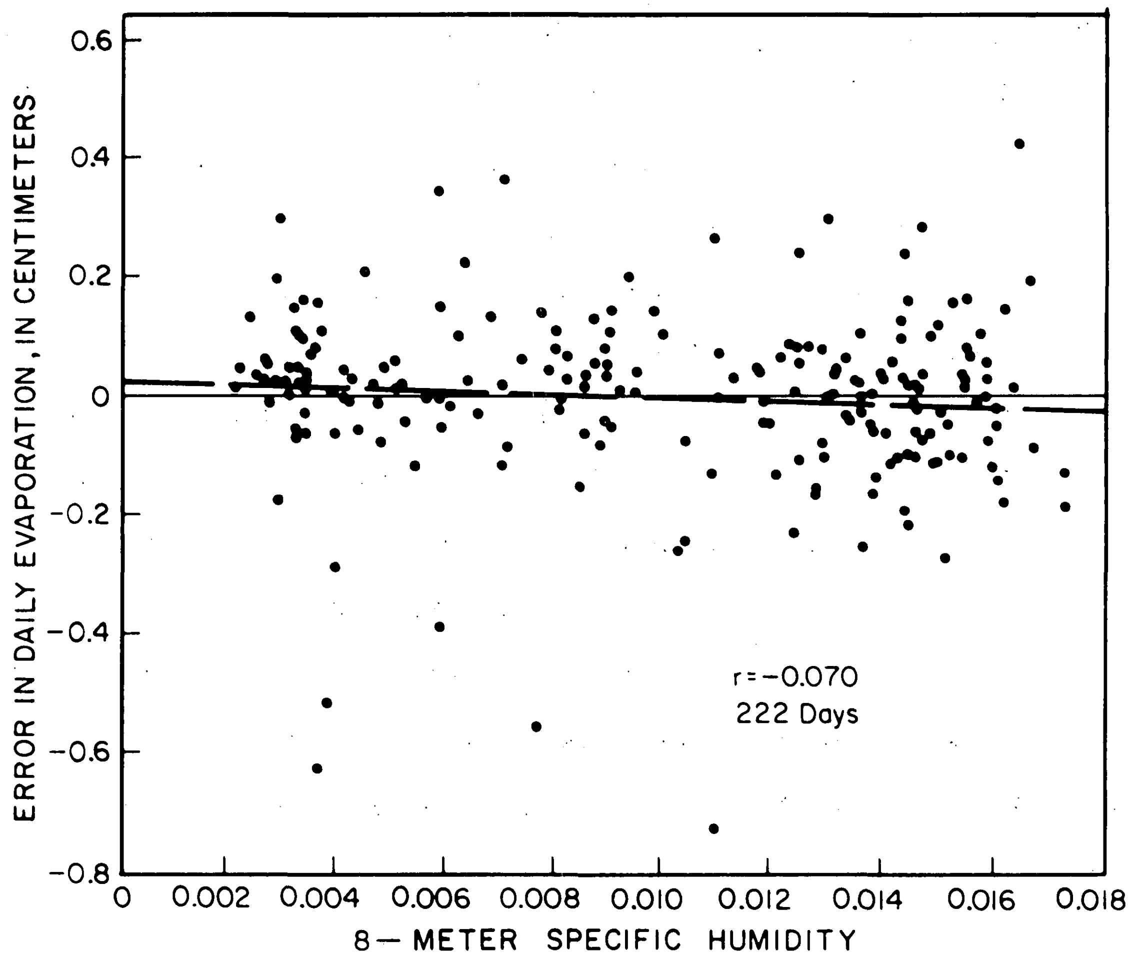

Figure 32. Correlation of the daily error values with the specific humidity of the air for the empirical mass-transfer method.

The seasonal nature of the errors in the empirical mass-transfer method is demonstrated on figure 33. The overall similarity in the shapes of all the seasonal variation curves (figures 7, 17, 23, 28, 33) is remarkable. For example, the local maximum in mid-April and during the first part of August 1951 is prominent on all figures. The local minimum in July 1951, is also common to all curves. The empirical curve (figure 33) resembles the modified log+linear curve (figure 28) more than it does the other curves in that it does not show a maximum about the first of October 1951. Like the modified log+linear law, the empirical mass-transfer method consistently underpredicted the evaporation during the months of September through December, 1951. The empirical equation underpredicted the evaporation during this period by about 10.5 percent while the modified log+linear law underpredicted it by 14.8 percent. If the values of N were increased by 4.3 percent the two methods would have been very comparable during this period of time. During the months of January through July 1951, the empirical equation overpredicted the evaporation by an average of 5.8 percent compared to an overprediction of 6.4 percent by the modified log+linear law.

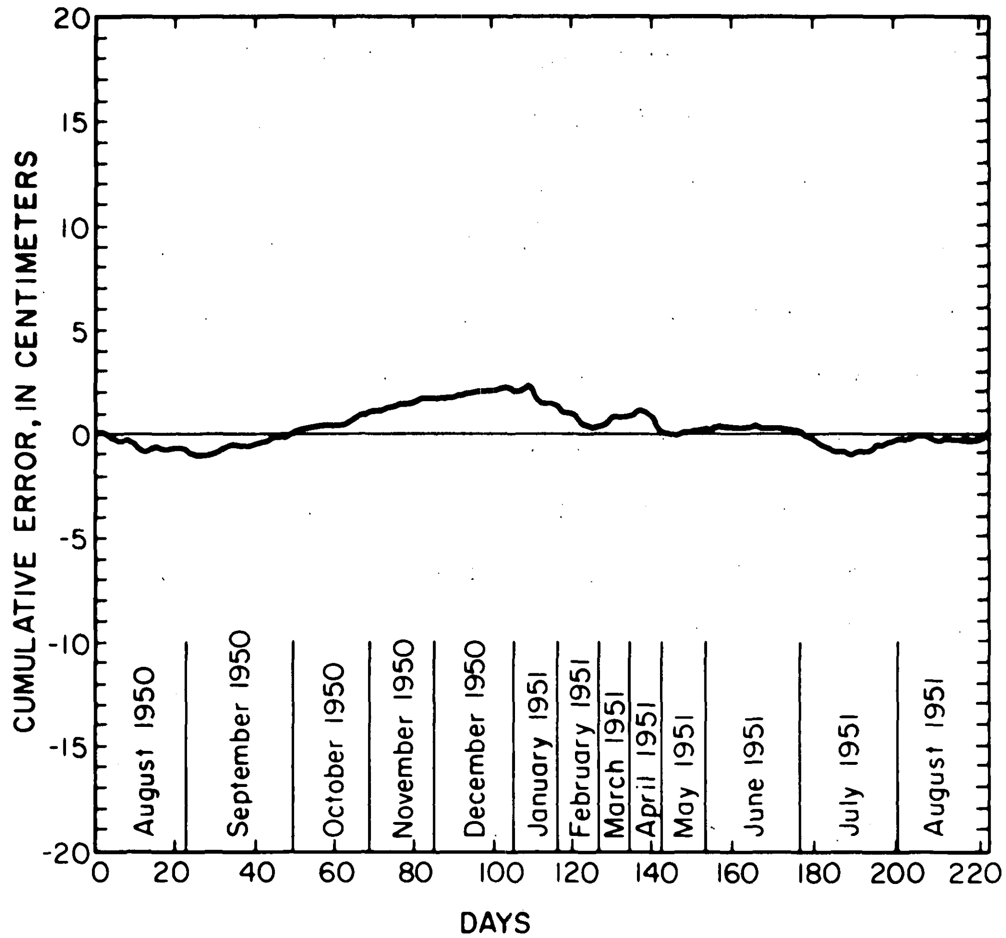

Figure 33. Seasonal variation of error resulting from the empirical mass-transfer method.

DISCUSSION OF RESULTS

General

Several methods were used to compute the evaporation from Lake Hefner. The theoretical soundness and practicality of the log+linear law as a method of computing evaporation will now be considered. This objective will be fulfilled primarily by comparing the results obtained from the log+linear law with the results obtained by other methods. The theoretical soundness will be inferred from a comparison with the log law, which is assumed to be theoretically correct under conditions of neutral stability. The practicality will be illustrated by a comparison with the empirical mass-transfer method.

After a brief discussion of the philosophy of the comparisons, the direct log+linear law will be compared to the direct log law. The effectiveness of the modified procedures is then discussed and the modified log+linear law is compared to the modified log law. Finally, the modified log+linear law is compared to the empirical mass-transfer method and the general practicality of the aerodynamic method is discussed.

The ultimate practical purpose of the methods investigated here is to allow the computation of the flux of momentum (shear), heat, or vapor (evaporation) from a surface using measurements of velocity, temperature, and humidity which are obtained at positions above the surface. If a law accurately represents the measured velocity, temperature, and humidity profiles but fails to improve the users ability to estimate the shear, heat flux, or evaporation, it will have failed in the practical sense. The criterion which will be used to test the success of each method or law will be its accuracy in predicting the evaporation. Ideally, the method should accurately predict the shear velocity and heat flux as well as evaporation, but no independent measurements of these quantities are available. It is quite probable however, that the method which best predicts the evaporation will also be most accurate in the prediction of shear velocity and heat flux at least on a micrometeorological scale.

Consider the evaporation rate as a random variable which fluctuates continually with time as does wind velocity, temperature, and specific humidity. Because no model can possibly account for all the factors which effect the evaporation rate, this rate contains random components in addition to those caused by the random nature of the independent variables. If the random nature of the quantities involved is accepted, the success or failure of any of the models during a single observation event has little statistical or practical significance. Any conclusions about the models must be based on outcomes of many individual measurements and upon the laws of statistics.

The quantities of interest here are the probable error which can be expected on an individual application of the method and the probable error in the total evaporation which is obtained from many applications. Because the water-budget evaporation value could not be determined for a period of time shorter than 1 day, it was impossible to measure the error which results from the single application of any of the laws or methods directly. However in general, the daily error value represents the mean of about 39 individual errors. If one assumes that the individual errors are mutually independent, the standard deviation of the individual errors can be computed as the standard deviation of the daily errors multiplied by the square root of the number of individual observations during the day (Cooper, 1969, p. 46). Therefore on the average, one would expect that the standard deviation of the individual errors to be about $\sqrt{39} = 6.24$ times as large as those presented on figures 3, 8, 13, 24, and 29. Only about 67 percent of the profiles were used with the direct log+linear law, so the standard deviation of the individual errors for this method are expected to be about $\sqrt{26} = 5.1$ times as large as that shown on figure 19.

Direct comparison of the log+linear to the log law

The log+linear law was developed to account, at least partly, for the effects of atmospheric stability on the velocity, temperature, and humidity profiles. Theoretically this law appears to be sound for an atmosphere which is nearly neutrally-stable, but in truth, the atmosphere is often far from neutrally stable. The question arises; does the law have any value for conditions that are likely to occur in the field? The purpose of this section is to demonstrate the success or failure of the log+linear law in improving the accuracy of the computed evaporation in comparison with that computed from the log law.

Probably the best measure of the general accuracy which can be expected from application of a method is the standard deviation of the daily error values. Approximately 32 percent of the computed daily evaporation rates should be in error by an amount larger than the standard deviation but only about 5 percent of the rates should be in error by an amount greater than twice the standard deviation. The standard deviation of daily errors for the log law was 0.383 cm while that for the log+linear law was 0.272 cm. The log+linear law gave more accurate results for this particular set of measurements.

The 222 error values presented on figure 3 represent a sample of an infinite number of possible error values which could be obtained by use of the log law if it were applied to all possible sets of data which are comparable to those obtained at Lake Hefner. This infinite number of possible error values is called a population and has a definite mean and standard deviation. The standard deviation of the error values on figure 3 is only a sample standard deviation but it is also the best estimate available for the population standard deviation. Likewise, the sample standard deviation presented on figure 19 is the best estimate of the standard deviation of the population of all possible error values which could be obtained by use of the log+linear law if it were applied to all possible sets of data comparable with those obtained at Lake Hefner. If the standard deviation of the log+linear population is smaller than the standard deviation of the log population, one could say with certainty that the log+linear law is more accurate, as measured by the standard deviation, than the log law when applied to data which compare with those obtained at Lake Hefner.

The true values of the standard deviations for the populations can not be determined, so it can not be said with certainty which law is the most accurate. However, confidence limits for a population standard deviation can be obtained if one is willing to assume that the population distribution is normal (Cooper, 1969, p.77). Assuming the populations are normal, the 0.99 confidence limits for the standard deviations of the daily error values are 0.243, 0.311 and 0.342, 0.438 cm for the log+linear and the log laws respectively. For the log+linear law error values, these confidence limits imply that the probability that the population standard deviation is larger than 0.311 cm is less than 0.01 while the probability that the population standard deviation for the log law error values is less than 0.342 cm is also less than 0.01. Therefore it is almost certain that the log+linear law is more accurate than the log law when these laws are used on data comparable with those collected at Lake Hefner.

Why is the log+linear law more accurate than the log law? It is natural to assume that the improved accuracy is the result of a better accounting for the effects of atmospheric stability, but it is necessary to demonstrate this. The correlation coefficients between the error values and the stability parameters are given on figures 4 and 20. It has been shown that the 0.99 level confidence limits on the correlation coefficient for the log+linear law are -0.069, -0.396 while the same limits for the log law are -0.285, -0.505. The overlapping of these limits demonstrate that there is a definite possibility that the errors in the log+linear law are just as correlated with the stability parameter as are the errors in the log law. Hypothesizing that the population correlation coefficients are the same, Fischer's Z transformation can be generalized to test for a significant difference in the sample values of the correlation coefficients (Snedecor, 1956, p. 178). Using the one tailed test, the correlation coefficient for the log+linear law is significantly less than that for the log law at the 0.90 confidence level but it is not at the 0.95 level. This is not an extremely high probability, but it is large enough to imply that the stability correction in the log+linear law does improve the accuracy of the results, especially when it is remembered that only a very crude measure of the atmospheric stability was used.

The errors in the log+linear law are correlated with stability just as were the errors in the log law. This implies that the first two terms in the expansion of equation 29 are not sufficient to completely describe the function ϕ_2 under all conditions found in the field. The originator of the log+linear law only expected the correction to work for cases where the atmosphere is nearly neutrally-stable. The analysis presented here suggests that the correction, while not perfect, does improve the daily predicted evaporation even under situations that are far from neutrally stable.

While the expected value of the error in the computed evaporation for a day is important, it is also important to know something about the expected value of the error in the computed evaporation for time periods longer than 1 day. For the purposes of water resources management, it is seldom necessary to know the total evaporation which occured in time intervals as short as 1 day. Even if the expected value of the error in daily evaporation values were large, a method would be considered useful if these errors were random so that the errors in the weekly or monthly evaporation values were small. The purpose of figures 7 and 23 was to demonstrate the accuracy of the laws when long term evaporation rates are considered. If the errors from day to day were completely random the cumulative error on these figures should fluctuate randomly about zero. The method of presentation smoothed the results and made seasonal trends more obvious. The slopes of these curves during any time interval represents the accuracy of the method during that time interval.

It is apparent that the log+linear law performed better in predicting long term evaporation rates, as well as daily values, than did the log law because the slope of the curve on figure 23 is generally less than the slope of the curve on figure 7. During the entire period the log+linear law was in error by only 3.3 percent compared to 15.0 percent for the log law. During the months of August and September 1950, when both laws tended to underpredict the actual evaporation, it reduced the average error from 41.5 to 31.2 percent and during the rest of the time, when both methods tended to overpredict the actual evaporation, it reduced the average error from 38.1 to 17.6 percent.

The direct application of the log+linear law to the Lake Hefner data gave better results than did the direct application of the log law. The standard deviation of the daily error values was reduced from 0.383 to 0.272 cm or about 34 percent. There is a fairly strong indication that the errors were less correlated to atmospheric stability and the average error was reduced appreciably during all seasons of the year. It would appear that the log+linear law is theoretically more correct than the log law even though it can not completely account for the effects of atmospheric stability when conditions are far from neutral.

Effectiveness of the modified procedure

Both the log law and the log+linear law utilize equation 13 in the computation of the evaporation rate. Both laws determine the evaporation rate as the product of u_*, which is related to the slope of the velocity profile, times Q_*, which is related to the slope of the specific humidity profile. The two laws assume different functional forms for the velocity profile, but in either case the forms of the temperature and specific humidity profiles are assumed to be the same as that of the velocity profile. Neither functional form is correct if the atmosphere is not near neutral stability. The functional forms of the temperature and specific humidity profiles are likely to represent the measured values poorly if the functional form of the velocity profile fails to fit the measured velocity profile. Errors in the computed value of u_* are very likely correlated with errors in the computed value of $Q*$ because both are correlated with atmospheric stability. Equation 13 tends to compound these errors when the evaporation rate is computed and the last term in equation 60 is not zero.

Better results may be expected if a method can be found to determine either u_* or Q_* for which the errors were not as strongly correlated with stability. Because the water surface roughness is so strongly dependent upon the wind speed, an equation similar to equation 52 might be found such that its errors are less correlated with stability than are the errors in either equation 49 or 82. The coefficients in equation 52 were determined by use of all data, equation 49, and a least squares regression. Equations 49 and 52 are, therefore, based upon consistent assumptions, but the measurement, and perhaps the model, errors in equation 52 have been reduced through the averaging process. Likewise equations 82 and 87 are consistent with the assumptions in the log+linear law. The averaged shear velocity equations, 52 and 87, should provide a better estimate of the instantaneous shear velocity than do equations 49 and 82 because the averaged equations do not magnify individual measurement errors. Elimination of the magnification factor alone should reduce the error values in the daily evaporation rates provided that the model errors in the averaged expressions are no larger than are the model errors in equations 49 and 82. If the model errors in the averaged expressions are either smaller or less correlated with stability than are the model errors in equations 49 and 82, an added bonus is received. The reduction in the last term of equation 60 reduces the error in an averaged evaporation rate.

The consistency of equations 52 and 87 can be accessed by use of a drag coefficient. Combining equations 9 and 52 to find the 2-meter drag coefficient consistent with the log law yields

$$C_D = 0.00534 - \frac{0.270}{u_2} + \frac{3.4}{u_2^2} \qquad (88)$$

Within the range of wind velocities measured at Lake Hefner, the value of the 2-meter drag coefficient varied from a minimum of 0.00298 for a wind speed of 100 cm/sec to a maximum of 0.00507 for a wind speed of 1,000 cm/sec. Converting these coefficients to the 10-meter level, they range from 0.00200 to 0.00307. These values for the drag coefficient are an average of about 1.8 times as large as those which would be predicted by a relation suggested by Deacon (Roll, 1965, p. 160). However Deacon's relation was derived for neutrally stable conditions over the sea. The drag coefficient at Lake Hefner increased with wind speed about twice as fast as does the value predicted by Deacon's relation.

Similar results were obtained by use of the log+ linear law. Combining equations 9 and 87 the expression for the 2-meter drag coefficient is

$$C_D = 0.00507 - \frac{0.164}{u_2} + \frac{1.3}{u_2^2} \qquad (89)$$

Within the range of 100 to 1000 cm/sec the 2-meter drag coefficient varied from 0.00356 to 0.00491. Converting these to 10-meter values using the log law they range from 0.00232 to 0.00299. The value of C_D determined from equation 89 averages about 1.9 times that predicted by Deacon's equation but the slope of equation 89 with respect to the wind velocity nearly matches that predicted by Deacon.

The standard deviation of daily errors for the direct and modified log law are given on figures 3 and 13 respectively. The 0.99 level confidence limits of the standard deviations are 0.342, 0.438 and 0.242, 0.309 for direct and modified methods respectively. The 35 percent reduction in the standard deviation of daily error values obtained by use of the modified procedure with the log law is very significant, both in the engineering and the statistical sense.

For the purpose of evaluating the effect of using the modified procedure with the log+linear law, both methods should be applied to data of the same quality. Therefore the results presented on figure 24 will not be used. Instead, the results which were obtained using only the profiles for which equations 74 and 82 converged are used. The 0.90 level confidence limits of the standard deviation of the daily errors for the modified log+linear law are 0.224, and 0.263 while those for the direct log+linear law (figure 19), are 0.253, and 0.297 These limits overlap even at this relatively low confidence level. Assuming the error values are normally distributed and using the F-test (Cooper, 1969, p. 96) it can be shown that the standard deviation for the modified method is smaller than that for the direct method with a probability larger than 0.90. This reduction of 12 percent is not as large as that obtained with the log law but is still probably significant in the statistical sense. The much smaller improvement for the log+linear law is probably the result of two factors. First, the model errors in equations 82 and 85 are smaller than those in equations 49 and 53 so the correlation of the errors in equation 60 is smaller to begin with. Second, when the log+linear law is applied, three fluxes, momentum, heat and vapor, are computed so the model errors are propagated in three computations instead of two.

It is interesting to compare the results presented on figure 24 with the results obtained using only the data for which the computed shear velocities converged. Did the profiles for which the computations did not converge contain poor quality data? The standard error for an individual profile can be estimated as the standard error in the daily values times the square root of the number of profiles analyzed in determining the daily average. When all profiles were used (figure 24) the estimated standard error for an individual profile measurement is $0.253 \times \sqrt{39} = 1.58$ cm, because on the average 39 profiles were used in computing the daily average evaporation. When only convergent data were used an average of 26 profiles per day were available, so the estimated standard error for an individual profile measurement was $0.241 \times \sqrt{26} = 1.23$ cm. These figures indicate that the data which would not converge was of poor quality because the estimated standard error for an individual profile measurement using convergent data was about 25 percent less than the standard error using all profiles. Much of the advantage gained by sorting the data was lost in the final analysis becasue the daily evaporation figures had to be estimated from a smaller set of data. For example, the standard deviation of daily errors on figure 24 is only about 5 percent larger than the value which was obtained by the use of the convergent data.

The reduction in the standard error for the daily computed evaporation values which resulted from the use of the modified procedure could occur simply because the measurement errors in the velocity profiles are not amplified. It is also possible that the averaged shear velocity equation contains smaller model errors than either equation 49 or 82 or that these errors are less correlated with stability. The sample correlation coefficient for the direct application of the log law is given on figure 4 as -0.363 and the 0.99 level confidence limits on this coefficient were -0.285 and -0.505. Likewise the sample correlation coefficient for the modified log law is given on figure 14 as -0.310 with 0.99 level confidence limits of -0.143, and -0.460. The modified procedure did reduce the sample correlation coefficient but the reduction does not appear statistically significant. Using Fischer's z transformation the difference still does not appear to be significant.

The sample correlation coefficient between the daily errors and the stability parameter for the direct application of the log+linear law was given on figure 20 as -0.240 and the value for the modified log+linear law using only convergent data was -0.297. In this case, use of the modified procedure appears to have actually increased the correlation of errors with the stability parameter. The increase is not significant in the statistical sense.

Although it can not be proven beyond a reasonable doubt, it would appear that the model errors in the averaged shear velocity equation are smaller than the model errors in the log equation, equation 49, and larger than the errors in the log+linear equation, equation 82. If this is true, it would help to explain the relatively small improvement in the standard deviation of daily errors which resulted from the use of the modified procedure with the log+linear law. The modified log law has benefited from a reduction in the amplification of the measurement errors as well as a reduction in the correlation between the model errors in the shear velocity with the stability parameter. For the modified log+linear law, however, the advantage which was gained because of the reduction in the amplification of the measurement errors was reduced somewhat by the relatively larger model errors in the averaged shear velocity equation.

The total effect of using the modified procedure can perhaps be illustrated best by comparing the resulting long term average errors. Use of the modified procedure with the log law reduced the average error for all seasons of the year. The procedure reduced the average error from 15 to 4.6 percent for the entire period, from 41.5 to 25.8 percent during the months of August and September, and from 38.1 to 16.8 percent during the rest of the time. Use of the modified procedure with the log+linear law also reduced the average error during each season. It reduced the average error from 31.2 to 20.4 percent during the months of August and September 1950 and reduced it from 17.6 to 13.7 percent during the rest of the time. However it increased the error for the entire period from 3.3 to 4.2 percent.

An averaged shear velocity relation, (that is, the modified procedure) is recommended for use with either the log or the log+linear law. Improvements will be greater when it is used with the log law. The method obviously reduces the magnification of measurement errors in the velocity profile. The model errors in the averaged relation are apparently either reduced or at least less correlated with stability than are the errors in the direct log law. The model errors in the averaged relation are apparently larger than those in the direct log+linear equation, but in this case at least, the improved accuracy which resulted from the smaller magnification of measurement errors more than compensated for the increase in the model errors.

Comparison of the modified log+linear to the modified log law

The most valid comparison of the two laws should be obtained by comparing the results of the modified log+linear law to those of the modified log law because the most valid comparison should result when the most accurate formulation of each law is used. This comparison is presented now. The results of the modified log+linear law will be those which were obtained by use of all data, including that which did not converge for the direct log+ linear formulation. The use of all profiles will assure that both sets of results are based on data of the same quality.

The standard deviation of the daily error values for the modified log law has been given on figure 13 as 0.270 cm and that for the modified log+linear law has been given on figure 24 as 0.253 cm. The 0.99 level confidence limits on the standard deviation for the modified log law have been given as 0.242 and 0.309 cm. In the same manner the 0.99 level confidence limits for the modified log+linear law are found to be 0.226 and 0.290 cm. Assuming that the error values are normally distributed and using the F-test (Cooper, 1969, p. 96), statistically it is seen that there is little reason to believe that the population standard deviations are different for the two laws. Although for this particular set of data the modified log+linear law has reduced the standard deviation of daily errors by 6.5 percent over that obtained by use of the modified log law, it can not be proven that the modified log law would not be just as accurate or perhaps even more accurate if another set of data of comparable quality were analyzed.

The coefficient of correlation between the stability parameter and the daily error values for the modified log law was -0.310 (figure 14). It was -0.073 for the modified log+linear law (figure 25). The first value is highly significant statistically while the last is not. It would appear, at first glance, that the modified log+linear law is significantly less correlated with stability than is the modified log law, but this conclusion is not entirely justified. The coefficient of correlation obtained by use of the modified log+linear law with the convergent data was -0.297. Because the daily average value of the stability parameter is so near zero and is such a poor measure of the individual values of the parameter, it is not too surprising that these seemingly inconsistant results are obtained.

The long term average errors for the two methods have been presented on figures 17 and 28. When averaged over the entire period of record the modified log law overpredicted the average evaporation by an average of 4.6 percent, while the modified log+linear law overpredicted it by an average of 0.85 percent. During the months of August and September 1950 the modified log law underpredicted the evaporation by an average of 25.8 percent while the modified log+linear law underpredicted it by an average of 14.4 percent. During the months of October 1950 through August 1951, both methods overpredicted the evaporation. The average error was 16.8 percent for the modified log law and 6.8 percent for the modified log+linear law. The long term average error was always less for the modified log+linear law.

Most of the evidence indicates that the modified log+linear law is a more accurate description of the evaporation process than is the modified log law. The difference is small. Apparently the average shear velocity relation was so much more effective when used with the log law than it was when used with the log+linear law that the differences between the two methods have been reduced to the point where it can no longer be proven to exist with a high degree of confidence.

Comparison of the modified log+linear law to the empirical mass-transfer formulation

The ultimate purpose of any aerodynamic method is to accurately predict evaporation rates from water surfaces. All aerodynamic methods suffer from two disadvantages: they require a tremendous amount of data and they greatly magnify any errors in the measurement of these data. These disadvantages are present but are not nearly as acute when the empirical mass-transfer method is used. The disadvantage of the empirical mass-transfer method is that it is dependent upon the value of an empirical coefficient. Because both the aerodynamic and the empirical mass-transfer methods are designed to accomplish the same purpose and because both have their advantages and disadvantages, it is enlightening to compare the results obtained from the modified log+linear law to those obtained from the empirical mass-transfer approach. Only the results of the modified log+linear law are used for this comparison because these results appear to be the most accurate obtainable from any of the aerodynamic methods which were investigated.

An empirical coefficient, N, for the mass-transfer method can be estimated by use of equation 39. Harbeck (1962, p. 104) suggests that the standard error for this estimate is about 16 percent. For the purposes of this report, the value of N was determined such that the average error for the entire 222 days of record was zero. This value of N is "exact" for this set of data, so that in the comparison it must be remembered that any errors in an estimated value of N would add to the errors determined here. In other words, for the purpose of this comparison, the major disadvantage of the empirical mass-transfer method has been eliminated. However, the consequences of errors in N will be pointed out as the results are presented.

The standard deviation of daily error values for the empirical mass-transfer method was 0.141 cm (figure 29). Comparing this value with 0.253 cm which was obtained by use of the modified log+linear law (figure 24), it is easily seen why the empirical method is so widely accepted. It has been estimated that the standard deviation of daily errors in the water-budget evaporation could be as large as 0.133 cm. The small standard deviation of daily error values is somewhat misleading because it is based on an "exact" value of N. The standard deviation of error values increases rapidly if the value of N is in error. For example a 1 percent decrease in the value of N causes an increase in the standard deviation of approximately 1.5 percent.

The primary reason for the accuracy of the empirical mass-transfer equation is probably that it does not greatly magnify errors in the measurement of velocity or vapor pressure. The vapor pressure gradient in equation 37 is large because it is determined as the difference between the value for air and the saturation value at a temperature equal to the water surface temperature. Also the saturation vapor pressure is dependent only upon the water temperature, which is relatively easy to measure, and not on the difference between a wet-bulb and a dry-bulb temperature measurement.

The coefficient of correlation between daily error values and the stability parameter for the empirical mass-transfer method was -0.169 (figure 30). This correlation coefficient is significantly less than zero at the 0.99 confidence level but it would seem meaningless to compare it to the value obtained by use of the modified log+linear law since the modified log+linear law gave a coefficient of correlation of -0.073 when all data were considered and a value of -0.297 when only convergent data were considered. It does appear that the error in the empirical mass-transfer formula is correlated with atmospheric stability.

The seasonal variation of error values for the empirical mass-transfer method and the modified log+linear law are illustrated on figures 33 and 28 respectively. Although the shapes of the curves on these two figures have many similarities, differences are also apparent. During the month of August 1950, the empirical law over-predicted the average evaporation by 6.0 percent while the modified log+linear law underpredicted it by an average of 27.2 percent. During the months of July and August 1951, the empirical law predicted the average evaporation nearly correctly while the modified log+linear law overpredicted it by an average of 9.2 percent. During the rest of the time, however, the curves were very similar. During the months of September through December 1950, both laws tended to underpredict the evaporation, the empirical law by 10.5 percent and the log+linear law by 14.8 percent. For the months of January through June 1951, both tended to overpredict the evaporation, the empirical method by 10.8 percent and the log+linear method by 28.3 percent.

Except for the month of August 1950, the maximum difference between the average errors for the two methods was 17.5 percent. If the expected value of the error in the value of N is as large as 16 or 17 percent then the modified log+linear law will probably produce values of average evaporation which are as accurate as those that can be obtained by use of the empirical mass-transfer equation for time intervals longer than 1 month.

It has been shown how the Thornthwaite-Holzman equation can be expressed in a form that is equivalent to the empirical mass-transfer formula, equation 45. The constants in equation 45 can be evaluated by use of the data contained on figures 10 and 11. Assuming the median of the ratios to be the most representative value and working between the 2-and 8-meter levels the values of B and B' can be determined. Using the median value of the velocity ratio (figure 10), equation 41, and assuming the density of air to be 1.2×10^{-3} g/cm^3 (gram per cubic centimeter), the value of B can be determined to be -2.27×10^{-5} g/cm^3. Likewise by use of the median humidity-deficit ratio (figure 11) the value of B' can be determined by use of equation 45 to be -0.0927. The resulting value of N is 2.10×10^{-6}. Converting units to those used previously the value of N is 1.16×10^{-4}. This value is 10.8 percent smaller than the value necessary to make the computed and the water-budget evaporation values identical for the entire 222 days of record.

The close agreement between the value of N which was determined by use of equation 45 and the value which was determined by use of the water-budget control strengthens ones confidence in the empirical mass-transfer formula and suggests a possible method of determining a value of N which would be applicable to large lakes, estuaries or the ocean.

Practicality of the aerodynamic method

One of the major disadvantages of the aerodynamic method is that all calculations are based on the vertical gradient of quantities within the atmosphere. These gradients are usually small and must be computed from differences in measured quantities as illustrated by equation 17. Equation 17 greatly magnifies any measurement errors and the severity of this magnification can be illustrated. Working between the 2- and 8-meter levels the median velocity ratio was 1.226 (figure 10). A 1 percent measurement error in either velocity would result in an error of about 5 percent for equation 17. Therefore measurement errors in velocity are magnified by a factor of about 5 in equation 17 when the 2- and 8-meter levels are used. Using the 2- and 4-meter levels the errors are magnified by a factor of about 10. The magnification factor for errors in the specific humidity are even worse. In this case the factor depends upon the magnitude of the quantities involved as well as the median ratio of the humidity deficit (figure 11). For typical conditions during the month of January, an error in the measurement of the specific humidity at 8-meters is amplified about 12 times and an error at the 4-meter level is amplified about 18 times. Likewise for conditions typical of July, errors in the 8-meter specific humidity are amplified about 26 times and those for the 4-meter level are amplified nearly 40 times.

Limitations of the aerodynamic method are apparent. The magnification problem can only be reduced by making measurements further apart. However, towers which are taller than 8 meters are expensive to construct over the water. In addition to this, all measurements must be made well within the boundary layer and the boundary layer grows in thickness very slowly with fetch. The upper limit of measurements may often be limited by the thickness of the vapor blanket.

The effect of error magnification can be reduced by averaging the results of many observations provided that the measurement errors are random. The averaging can be accomplished in two ways. Measurements can be made at many levels or repetitive measurements can be obtained at only two levels and the results averaged over time. A combination of both ways was used in this report.

By obtaining data at many levels, more accurate instantaneous gradients should be obtained. However the instrumentation cost is very high. In addition, the magnification factor increases as the levels of observation become closer together. It would appear that the value of each additional level of data decreases rapidly. Although fitting the curves using data obtained at multiple levels will theoretically increase the accuracy, the Lake Hefner data did not contain enough levels to determine how much improvement can be expected from each added level of information. There is a definite advantage to having data from more than two levels in that it allows one to evaluate the consistency of the data from each level by determining values of z_0 for each pair of levels. This procedure led to the suspicion that the 4-meter dry-bulb temperatures of the Lake Hefner data were in error.

The other way of reducing the effect of measurement errors, averaging over time, was evaluated. This procedure does not increase the instrumentation requirements but it does eliminate the ability to make accurate short term determinations of evaporation.

The modified procedure which has been developed in this report is one means of partly averaging out errors in velocity measurements before evaporation calculations are made. After a long term record has been obtained and an equation of the form given by equation 52 is developed, better short term values can be obtained than would be possible by the direct use of the aerodynamic method. Necessarily, a certain amount of empiricism has crept into the method. The really large error magnification factor appears to be associated with the humidity profiles, so that the results which can be obtained by the modified procedure are definitely limited.

The next step would appear to be the averaging of errors in the humidity profiles by use of equation 44 and the median value of the humidity deficit ratio (figure 11). One could argue the merits of using either the mean or the median value of the ratios. The median value was chosen here because it is unaffected by the magnitude of extreme values, which are suspected to have little physical significance. At this point the aerodynamic method has nearly reverted to the empirical mass-transfer approach.

Harbeck and Meyers (1970) concluded that the energy budget method is a reliable method for the determination of evaporation from lakes and reservoirs for periods of time ranging from a week to a month. Because the required equipment is expensive and data processing is time-consuming, they concluded that the energy budget should ordinarily be used only long enough to permit determining the mass-transfer coefficient, N, with adequate accuracy. The energy budget equipment can then be moved to a second reservoir, and evaporation measurements continued indefinitely at the first reservoir using the mass-transfer method at a minimum of expense.

It is not proposed here that the aerodynamic method is competitive with the energy budget method for determining evaporation from lakes and reservoirs. There exists, however, water bodies to which the energy-budget determination of N can not be applied such as very large lakes, and oceans. The aerodynamic method might be used in the same manner as the energy budget to determine the mass-transfer coefficient. In this case, long term averaging is necessary in order to reduce the effect of measurement errors and to cancel the effect of seasonal trends.

How long must the aerodynamic method be used before a specified accuracy in the value of N can be expected? First, because of the rather large seasonal effect illustrated on figure 28, complete years should be used. Assuming that the modified log+linear method is used and that the data will be comparable to those collected at Lake Hefner, one can estimate the number of days of record which will be necessary. For example, assume that the daily error values are drawn from a normal population with zero mean (figure 24), with a standard deviation of 0.253 cm (figure 24), and that the average evaporation rate will be 0.41 cm per day. Then 1,020 days of acceptable record are required in order to establish the value of N to within 5 percent at the 0.99 confidence level. Three years of record would be needed.

Three years of record taken every 30 minutes at three levels would be a massive undertaking. It is tempting to suggest that the value of N be determined from equation 39 and to abandon all efforts to use the aerodynamic method. It must be remembered, however, that equation 39 was developed empirically using only data from lakes and reservoirs where the energy budget had been applied. The applicability of this equation for estuaries or the oceans has never been adequately checked. In these situations it is perhaps necessary to make a realistic assessment of the shortcomings of the aerodynamic method and learn to minimize its disadvantages to the greatest possible extent. One object of this report has been to illustrate these shortcomings and perhaps indicate some ways in which the disadvantages of the aerodynamic method can be minimized.

Summary and conclusions

The process of evaporation has received the attention of hydrologists, meteorologists, and agriculturalists for many years. Because it is a major factor in the transfer of excess heat from water systems to the atmosphere, and because of its importance in water short areas, interest in the process is increasing. Unfortunately, the measurement or estimation of the rate of evaporation from water surfaces is by no means an easy matter. The methods which may currently be used to measure the evaporation rate include the water-budget method, the energy-budget method, the empirical mass-transfer method, the aerodynamic method, the evaporation pan method, and the eddy-correlation method. The method which should be used depends entirely upon the situation under consideration because each method has advantages and disadvantages so that none of them can be said to be the best. This report has analyzed only the aerodynamic and the empirical mass-transfer methods.

In certain situations the aerodynamic method is about the only way in which the evaporation can be measured. In these situations one is forced to accept its disadvantages and to attempt to use it. In addition, the aerodynamic method has a number of significant advantages over the other methods. It requires no empirical coefficient, "instantaneous" rates are theoretically possible, all measurements are in the air away from the surface, and measurement of the evaporation rate from a relatively small area of a large body of water is possible. There has been a great interest in improving the aerodynamic method and many functional forms of the relation between wind speed and elevation have been proposed. All are found to be deficient in properly accounting for the effects of atmospheric stability.

In 1954, Monin and Obukhov (1959) proposed the log+linear law which was designed to account for stability effects. While the law has received considerable testing under limited conditions, few sets of data are available which are extensive enough to determine its general applicability under widely varying conditions. The purpose of this report was to make use of the massive set of data collected at Lake Hefner, Oklahoma during the years of 1950 and 1951 (U.S. Geological Survey, 1954b) in order to evaluate the theoretical correctness and practicality of the log+linear law as a method of computing evaporation rates. Under conditions of neutral atmospheric stability, the log law appears to be theoretically sound. Because the log+linear law represents a linear correction term to the basic log law, the theoretical soundness of this correction was inferred by comparing the results which were obtained from the log+linear law to similar results which were obtained from the log law. The empirical mass-transfer method is probably the most practical and accurate method available for computing evaporation rates provided the value of the empirical coefficient is known. Because accurate values of the daily evaporation rates were available from the water-budget method, it was possible to determine the "exact" value of the empirical coefficient. The practicality of the log+linear law was inferred by comparing its results with those obtained from the mass-transfer method. Following a suggestion proposed by Pasquill (Sutton, 1953, p. 311), a method of reducing the effect of measurement errors was devised and tested on both the log and log+linear laws.

None of the models accounted for all the factors which effect the evaporation rate, therefore this rate contained random components in addition to those caused by the random nature of the independent variables and by the random errors in the measurement of the variables. As a result, the success or failure of any of the methods during a single observation event was given little statistical or practical significance. Instead the conclusions about the various methods were based on the outcome of many individual observations and upon the laws of statistics. This method of analysis required a massive set of rather high quality data.

By use of a method proposed by Deacon (1962) the value of the constant α in the log+linear law was determined beforehand from the measured data. A value of 3.0 was used throughout the report, but the results are not very sensitive to the assumed value of α as long as it is within the range of 1.0 to 3.0.

The direct application of the log+linear law produced more accurate predictions of evaporation rates than were produced by the direct application of the log law. The standard deviation of daily error values was reduced by 34 percent, a value which was very significant statistically. The error in the seasonal average evaporation rate was reduced appreciably during all seasons of the year. There was a fairly strong indication that these improvements were the result of a better accounting for the atmospheric stability effects by the log+linear law, because the daily error values were less correlated with the daily average value of the stability parameter. Even though the log+linear law does not completely eliminate the correlation of the error with stability, it does provide more accurate results than can be expected from the direct application of the log law when applied to field data comparable to those collected at Lake Hefner.

The modified procedure, that is the use of an averaged shear velocity relation, is recommended for use with either the log or the log+linear laws. The expected improvement to be gained by the use of this procedure is much larger when it is applied to the log law than when it is applied to the log+linear law. When applied to either law, the averaged shear velocity relation reduces the effect of measurement errors in the velocity. Apparently the averaged shear velocity relation contains errors which are smaller, or at least less correlated with stability, than are the model errors in equation 49 (log law) but larger than those in equation 82 (log+linear law).

Although most of the evidence indicates that the modified log+linear law is a more accurate description of the evaporation process than is the modified log law, the difference in the accuracies of the two is not large enough to be statistically significant. Apparently the averaged shear velocity relation is so much more effective on the log law that the differences between the two methods has been reduced to the point where it can no longer be proven to exist with a high degree of confidence.

If the value of the empirical mass-transfer coefficient is known, the mass-transfer method is much more accurate than the aerodynamic method. The excellent accuracy of the mass-transfer method apparently results from the small magnification of measurement errors. Except for the month of August 1950, the maximum difference between the long term average evaporation rates as determined by the mass-transfer and the modified log+linear methods was 17.5 percent. If the value of N can only be estimated to within 17 percent the modified log+linear law approaches the accuracy of the mass-transfer method as a means of determining average evaporation rates for time intervals longer than 1 month. It was possible to predict the empirical mass-transfer coefficient to within 10.8 percent of the value obtained from the water-budget control by use of the median values of the velocity and humidity deficit ratios obtained between the 8- and 2-meter levels.

One of the major disadvantages of the aerodynamic method is that the computation procedure tends to amplify measurement errors greatly. The effect of this amplification can only be eliminated by an averaging process. The modified procedure is one way to partly achieve this goal. It only reduces the amplification, however, of errors in the measured velocity and errors in the measured specific humidity apparently contain the largest amplification factors.

It is suggested that the modified log+linear law can be used to determine the mass-transfer coefficient in situations where the energy-budget method is not applicable. The method of approach should be the same as that suggested by Harbeck and Meyer (1970) except that the modified log+linear law is used instead of the energy-budget method. For conditions similar to those encountered at Lake Hefner, it is estimated that 3 years of record would be necessary in order to establish the value of the mass-transfer coefficient to within 5 percent.

REFERENCES

Anderson, L. J., 1954, Instrumentation for mass-transfer and energy-budget studies, *in* Water-loss investigations: Lake Hefner studies, technical report: U.S. Geol. Survey Prof. Paper 269, p. 35-45.

Brutsaert, Wilfried, 1965, Equations for vapor flux as a fully turbulent diffusion process under diabatic conditions: Bull. Internat. Assoc. Sci. Hydorlogy, v. 10, no. 2, p. 11-21.

Brutsaert, Wilfried, and Yeh, Gour-Tsyh, 1970, Implications of a type of empirical evaporation formula for lakes and pans: Water Resources Research, v. 6, no. 4, p. 1202-1208.

Businger, J. A., Wyngaard, Y. I., and Bradley, E. F., 1971, Flux-profile relationships in the atmospheric surface layer: Journal of the Atmospheric Sciences, v. 28, p. 181-189.

Cooper, B. E., 1969, Statistics for experimentalists: New York, Pergamon Press, 336 p.

Deacon, E. L., 1962, Aerodynamic roughness of the sea: Jour. Geophys. Research, v. 67, no. 8, p. 3167-3172.

Deacon, E. L., and Swinbank, W. C., 1958, Comparison between momentum and water vapor transfer: Arid Zone Research - XI, Climatology and microclimatology, Unesco, p. 38-41.

Federal Water Pollution Control Administration, 1968, Industrial waste guide on thermal pollution: Pacific Northwest Water Lab. Rept., Corvallis, Oregon, 112 p.

Harbeck, G. E., Jr., 1954, General description of Lake Hefner, *in* Water-loss investigations: Lake Hefner studies, technical report: U. S. Geol. Survey Prof. Paper 269, p. 5-9.

Harbeck, G. E., Jr., 1962, A practical field technique for measuring reservoir evaporation utilizing mass-transfer theory: U. S. Geol. Survey Prof. Paper 272-E, p. 101-105.

Harbeck, G. E., Jr., and Kennon, F. W., 1954, The water-budget control, *in* Water-loss investigations: Lake Hefner studies, technical report: U.S. Geol. Survey Prof. Paper 269, p. 17-34.

Harbeck, G. E., Jr., and Meyers, J. S., 1970, Present day evaporation measurement techniques: Am. Soc. Civil Engineers Proc., v. 96, no. HY7, p. 1381-1390.

Hinze, J. O., 1959, Turbulence: New York, McGraw-Hill Book Co., 586 p.

Hodgman, C. D. (editor), 1951, Handbook of chemistry and physics: Cleveland, Chemical Rubber Publishing Co., 2894 p.

Marciano, J. J., and Harbeck, G. E., Jr., 1954, Mass-transfer studies, *in* Water-loss investigations: Lake Hefner studies, technical report: U.S. Geol. Survey Prof. Paper 269, p. 46-70.

Monin, A. S., and Obukhov, A. M., 1959, Basic laws of turbulent mixing in the ground layer of the atmosphere [translated by John Miller]: Federal and Technical Inf. Clearinghouse, Springfield, Virginia, 30 p.

Priestley, C. H. B., 1959, Turbulent transfer in the lower atmosphere: Chicago, University of Chicago Press, 130 p.

Roberts, W. J., 1969, Significance of evaporation in hydrologic education, *in* The progress of hydrology: Internat. Seminar for Hydrol. Professors, 1st, proc., Univ. of Illinois [Urbana], v. 2, p. 666-693.

Roll, H. U., 1965, Physics of the marine atmosphere: New York, Academic Press, 426 p.

Snedecor, G. W., 1956, Statistical methods: Ames, Iowa State University Press, 534 p.

Sutton, O. G., 1953, Micrometeorology: New York, McGraw-Hill Book Co., 333 p.

U.S. Geological Survey, 1954a, Water-loss investigations: Lake Hefner studies, base data report: U.S. Geol. Survey Prof. Paper 270, 300 p.

_____1954b, Water-loss investigations: Lake Hefner studies, technical report: U.S. Geol. Survey Prof. Paper 269, 150 p.

Webb, E. K., 1970, Profile relationships: the log-linear range, and extension to strong stability: Royal Meteorol. Soc. Quart. Jour., v. 96, no. 408, p. 67-90.

Wiersma, J. L., 1970, Influence of low rates of water application by sprinklers on the microclimate: South Dakota State Univ. Completion Rept., Proj. A-006-SDAK, 84 p.

Yen, Y. C., and Landvatter, G. R., 1970, Evaporation of water into a sub-zero air stream: Water Resources Research, v. 6, no. 2, p. 430-439.