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STOCHASTIC ANALYSIS OF PARTICLE MOVEMENT OVER A DUNE BED

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STOCHASTIC ANALYSIS OF PARTICLE MOVEMENT OVER A DUNE BED

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CONVERSION FACTORS

Factors for converting metric units to English units are shown to four significant figures. However, in the text the English equialents are shown only to the number of significant figures consistent with the values for the metric units.

Metric	Multiply by	English
metres (m)	3.281	feet (ft)
centimetres (cm)	3.281×10^{-2}	feet (ft)
tonnes per day per metres (t/day-m)	.336	tons per day per foot (ton/day-ft)
dynes per square centimetre (d/cm^2)	2.09×10^{-3}	pounds per square foot (lb/ft^2)
dynes per centimetre per second (d/cm-s)	6.85×10^{-5}	pounds per foot per second (lb/ft-s)

SYMBOLS

A, B	Constants in estimating the conditional mean of the rest periods
C, D	Constants in estimating the conditional variance of the rest periods
$C\sqrt{g}$	Dimensionless Chezy discharge coefficient
C_T	Concentration of total bed-material discharge
d	Depth of flow
d_{50}	Median sieve diameter of bed material
d_{84}	Sieve diameter of bed material for which 84 percent is smaller
d_{16}	Sieve diameter of bed material for which 16 percent is smaller
$E_{i,j,\nu}$	Event that a particle eroded from elevation y_i passes ν dune crests before it is deposited at elevation y_j
E_ν	Event that a particle passes ν dune crests before it is deposited
$\mathrm{E}[\cdot], \hat{\mathrm{E}}[\cdot]$	Mathematical expectation and its estimate, respectively
$\mathrm{F_r}$	Froude number of the flow
$f(\cdot), F(\cdot)$	Probability density and distribution functions, respectively
$f^{(n)}(\cdot)$	n-fold convolution of the probability density, $f(\cdot)$
g	Gravitation acceleration
h	Average depth of the zone in which bed-material movement occurs determined from the $y_t(x)$ record
$k_{1,y,y'}$	Scale parameter of the conditional step length distribution
$k_{2,y}$	Scale parameter of the conditional rest period distribution
L_t	Length of the $y_x(t)$ record
L_x	Length of the $y_t(x)$ record

$m_{i,j}$ Total number of bed forms contained in the $y_t(x)$ record for which the upstream side intersects the elevation y_i and the downstream side intersects the elevation y_j

$m_{i,j,\nu}$ Total number of possibilities of the event $E_{i,j,\nu}$ contained in the $y_t(x)$ record

m_i' Total number of bed forms contained in the $y_x(t)$ record and which also contain some erosion in the class interval associated with the elevation y_i

m_j Total number of bed forms contained in the $y_x(t)$ record and which also contain some deposition in the class interval associated with the elevation y_j

$m_{j,j}$ Total number of bed forms contained in the $y_x(t)$ record and which also contain both an up-crossing and a down-crossing at the elevation y_j

n Number of steps taken by a particle or number of class intervals for the realization of Y_D and Y_E

N_d Total number of particles per unit area deposited in time t

N_e Total number of particles per unit area eroded in time t

$N_d(y_i)$ Total number of particles per unit area deposited within the class interval $(\eta_i, \eta_{i+1}]$ in time t

$N_e(y_i)$ Total number of particles per unit area eroded within the class interval $(\eta_i, \eta_{i+1}]$ in time t

$N(t)$ Counting process describing number of steps taken by a particle in time t

$p(\cdot)$	Sample probability mass function
$P[\cdot]$	Probability
$\hat{q}'_B(j)$	Estimate of the mean bed-load discharge per unit width associated with the elevation y_j
$\hat{q}_T$	Estimate of the mean total bed-material discharge per unit width
$\hat{Q}_B, \hat{Q}'_B$	Estimates of the mean bed-load discharge
$\hat{Q}_S, \hat{Q}'_S$	Estimates of the mean suspended-load discharge
$\hat{Q}_T, \hat{Q}'_T, \hat{Q}''_T$	Estimates of the mean total bed-material discharge
Q_w	Water discharge
r	Number of class intervals for the realization of T
$r_{1,y,y'}, r_{2,y}$	Shape parameters for the conditional distribution of the step lengths and the rest periods, respectively
s	Number of class intervals for the realization of X
s_y	Standard deviation of bed elevation
S_e	Slope of energy-grade line
t	Measure of time
$t_{j,k}$	Statistic which measures the conditional rest periods
t_α	Class mark for the statistic, $t_{j,k}$
T	Random variable describing the rest periods of a particle
T_i	Random variable describing the duration of ith rest period
$T(n)$	Stochastic process describing the sum of n rest periods
$\overline{U}$	Mean flow velocity
$\overline{U}_*$	Mean shear velocity

$\hat{V}_B, \hat{V}_S$	Estimates of the mean transport speed of a bed-load particle and a suspended-load particle, respectively
$\hat{V}_B(j)$	Estimate of the mean transport speed of a bed-load particle associated with the elevation y_j
$\hat{V}_T, \hat{V}'_T$	Estimates of the mean transport speed of a bed-material particle
$\hat{V}_T(j)$	Estimate of the mean transport speed of a bed-material particle associated with the elevation y_j
$\mathrm{Var}[\cdot], \hat{\mathrm{Var}}[\cdot]$	Variance and its estimate, respectively
W	Width of channel
x	Measure of longitudinal distance
$\bar{x}$	Average distance traveled by bed material in time t
x_β	Class mark for the conditional step lengths
$x_{i,j,k}$	Statistic which measures the conditional step lengths of a bed-load particle
$x_{i,j,\nu,k}$	Statistic which measures the conditional step lengths of a bed-material particle
X	Random variable describing the step lengths of a particle
X_i	Random variable describing the length of ith step of a particle
$X(n)$	Stochastic process describing the longitudinal position of a particle after n steps
$\tilde{X}(t)$	Stochastic process describing the longitudinal position of a particle at time t

y, y'	Measure of vertical distance
y_i, y_j	Class marks for Y_E and Y_D
y_{max}, y_{min}	Highest and lowest elevations, respectively, at which particles are deposited or eroded
$\hat{y}_{max}, \hat{y}_{min}$	Estimates of y_{max} and y_{min}, respectively
$y_t(x)$	Elevation of the bed, y, as a function of the longitudinal coordinates, x, at a given time, t
$y_x(t)$	Elevation of the bed, y, as a function of time, t, at a fixed point, x
Y_D, Y_E	Random variables describing the elevation of particle deposition and erosion, respectively
$Y_D(n)$	Stochastic process describing the vertical position of a particle after n steps
$\tilde{y}(t)$	Stochastic process describing the vertical position of a particle at time t
Δy_j	Class width associated with y_j
$\Delta y^{+}_{j,k}$	Vertical rise of the bed in the class interval associated with y_j for the kth deposition period of the $y_x(t)$ record
$\Delta y^{-}_{j,k}$	Vertical fall of the bed in the class interval associated with y_j for the kth erosion period of the $y_x(t)$ record
γ_s	Specific weight of bed material
$\Gamma(\cdot)$	Gamma function
ζ_j, ξ_j	Effective volume ratios

η_j, η_{j+1}	Lower and upper class limits for y_j, respectively
θ	Bulk porosity of the bed material in place
$\lambda_\beta, \lambda_{\beta+1}$	Lower and upper class limits for x_β, respectively
$\hat{\rho}$	Estimate of correlation coefficient
σ_g	Geometric standard deviation of particle size
$\bar{\tau}_b$	Mean bed shear stress
$\tau_\alpha, \tau_{\alpha+1}$	Lower and upper class limits for t_α, respectively
χ_c	Critical value of chi-square statistic
Ω	Number of particles per unit volume of the bed
Ω_j	Number of particles per unit volume of the bed associated with y_j

STOCHASTIC ANALYSIS OF PARTICLE MOVEMENT

OVER A DUNE BED

By Baum K. Lee, and Harvey E. Jobson

ABSTRACT

Stochastic models are available that can be used to predict the transport and dispersion of bed-material sediment particles in an alluvial channel. These models are based on the proposition that the movement of a single bed-material sediment particle consists of a series of steps of random length separated by rest periods of random duration and, therefore, application of the models requires a knowledge of the probability distributions of the step lengths, the rest periods, the elevation of particle deposition, and the elevation of particle erosion. In the past, it has proven impossible to estimate these distributions except by use of tedious and time consuming single particle experiments.

By considering a dune bed configuration which is composed of uniformly sized particles, the probability distributions of the rest period, the elevation of particle deposition, and the elevation of particle erosion are obtained from a record of the bed elevation at a fixed point as a continuous function of time. By restricting attention to a coarse sand, where the suspended load is negligible, the probability distribution of the step length is obtained from a series of "instantaneous" longitudinal bed profiles in addition to the above information. Using these probability distributions, three bed-material transport equations and a two-dimensional stochastic model for dispersion of bed-sediment particles are developed.

The procedure was tested by determining these distributions from bed profiles formed in a large laboratory flume with a coarse sand as the bed material. The elevation of particle deposition and the elevation of particle erosion can be considered to be identically distributed, and their distribution can be described by either a "truncated Gaussian" or a "triangular" density function. The conditional probability distribution of the rest period given the elevation of particle deposition closely followed the two-parameter gamma distribution. The conditional probability distribution of the step length given the elevation of particle erosion and the elevation of particle deposition also closely followed the two-parameter gamma density function. For a given flow, the scale and shape parameters describing the gamma probability distributions can be expressed as functions of bed elevation.

The bed-material transport equations were tested for three flow conditions. The errors in the predicted mean total bed-material transport rates were -3.0, +3.5, and 80.1 percent for equation 55, and -1.7, +26.9, and +64.1 percent for equation 63. For the run with the large error, the mean total load concentration was small (8.9 milligrams per liter), and flow conditions were somewhat out of equilibrium.

INTRODUCTION

The movement of sediment in alluvial streams is so complex a process that it may never be subjected completely to a deterministic solution. It represents, in fact, an extreme degree of unsteady, nonuniform flow, since the streambed as well as the water surface may be continuously changing with time and position.

Numerous formulas and equations have been developed to predict sediment transport rates. Most of these developments ignore the actual nature of sediment movement and have assumed that the sediment transport rate can be described by a deterministic function of certain flow parameters. Unfortunately, after decades of searching, no universally accepted sediment transport equation has been found. The theories of probability, statistics, and stochastic processes have been used to describe the kinematics of a single bed-sediment particle in an alluvial channel flow and to predict the dispersion characteristics of a group of such particles. These theories have clearly demonstrated a great potential for development of stochastic models of sediment transport and dispersion.

Most of the stochastic models (Shen and Todorovic, 1971; Grigg, 1969; Yang, 1968; Sayre and Conover, 1967; Hubbell and Sayre, 1964; Crickmore and Lean, 1962; Einstein, 1937) are based on the proposition that the movement of bed-sediment particles consists of a series of steps separated by rest periods, so that determination of the probability distributions for the step lengths and the rest periods of a bed-sediment particle plays the major role in quantifying the bed-sediment transport. While this movement concept can easily be verified through laboratory observations, Einstein (1937) was the first to use it. He developed a one-dimensional probabilistic model for bedload transport. More recently, Sayre and Conover (1967) derived a two-dimensional stochastic model by introducing the probability distribution of the elevation at which a bed-sediment particle is deposited.

The probability distributions of the step lengths and the rest periods of a bed-sediment particle have been estimated from single particle experiments (Grigg, 1969) or by using a group of tracer particles (Yang, 1968; Hubbell and Sayre, 1964; Crickmore and Lean, 1962). Because of the considerable effort required to conduct such experiments, it seems clear that some way must be found to estimate the probability distributions from more readily accessible data if significant further progress is to be expected. To apply the Sayre-Conover (1967) two-dimensional stochastic model, the probability distribution of the elevation at which a bed-sediment particle is deposited must be known. A method for estimating this distribution is developed in this report.

The objectives of this study are:

1. To present a method of estimating the following probability distributions for dune bed conditions using only sounding records of the bed elevation.

A. Probability distributions (note that there are two separate distributions) of the elevation at which a bed-sediment particle is eroded and deposited

B. Conditional probability distributions of the step lengths of a bed-sediment particle given the elevations at which the particle is eroded and deposited.

A method estimating the conditional probability distribution of the rest periods of a bed-sediment particle given the elevation at which the particle is deposited has been presented by Sayre and Conover (1967).

2. To develop bed-material transport equations based on the above probability distributions and to compare the results with the experimentally measured values.

3. To derive a two-dimensional stochastic model for dispersion of bed-sediment particles as a function of the above probability distributions.

The probability distributions of the elevation at which a bed-sediment particle is eroded and deposited, and the probability distribution of the rest periods, conditioned on the elevation of deposition, will be obtained from a continuous record of the bed elevation at a particular point as a function of time. The probability distribution of the step lengths, conditioned on the elevation of erosion and the elevation of deposition, will be obtained from a series of "instantaneous" longitudinal bed profiles. With these distributions obtained, various related probability distributions of vital interest will be estimated, and a relation between the rest periods and the step lengths of a bed-sediment particle will be investigated.

Three experimental runs are analyzed and the relations between the statistics describing the postulated probability distributions and the hydraulic conditions are investigated. All data were obtained from a tilting recirculating flume of rectangular cross section 61 m (200 ft) long, 2.4 m (8 ft) wide, and 1.2 m (4 ft) deep. The bed material used in these experiments was screened river sand with a median sieve diameter equal to 1.13 mm and a geometric standard deviation equal to 1.51.

ACKNOWLEDGMENT

The data contained herein are essentially the same as those contained in a dissertation by Lee (1973). The data were collected under the general supervision of the second author. Special thanks are due E. V. Richardson, D. B. Simons, C. F. Nordin, D. C. Boes, and R. P. Osborne.

BACKGROUND

Theoretical Models

Einstein (1937) treated the movement of a single sediment particle over an alluvial bed as a stochastic process described by an alternating sequence of two independent random variables, namely, step lengths and rest periods. Considering the particle movement in the distance-time plane on a Galton's board (Parzen, 1960), Einstein derived exponential probability density functions for the step lengths and the rest periods,

$$f_X(x) = k_1 e^{-k_1 x} , \quad x > 0 \tag{1}$$

and

$$f_T(t) = k_2 e^{-k_2 t} , \quad t > 0 \tag{2}$$

respectively, where

X, T = random variables describing the step lengths and rest periods of a particle, respectively;

x, t = distance and time, respectively;

$f_X(x)$, $f_T(t)$ = common probability density functions of the step lengths and rest periods, respectively; and

k_1, k_2 = positive constants.

For a sediment particle introduced into the stream at distance $x = 0$ in such a way that it takes its first step at time $t = 0$, Einstein obtained the probability density function of the total distance traveled by the particle at time t to be

$$f(x; t) = k_1 e^{-k_1 x - k_2 t} \sum_{n=1}^{\infty} \frac{(k_1 x)^{n-1}}{\Gamma(n)} \frac{(k_2 t)^{n-1}}{\Gamma(n)}, \; x > 0, \; t > 0, \tag{3}$$

in which $\Gamma(\cdot)$ denotes the gamma function. Equation 3 also represents the concentration distribution of a group of identical sediment particles with respect to longitudinal position, x, as a function of time, t.

The probability density function for the case when the particle is initially $(t = 0)$ at rest at $x = 0$ also was obtained by a similar procedure,

$$f(x; t) = k_1 e^{-k_1 x - k_2 t} \sum_{n=1}^{\infty} \frac{(k_1 x)^{n-1}}{\Gamma(n)} \frac{(k_2 t)^{n}}{\Gamma(n+1)}, \; x > 0, \; t > 0 \; . \tag{4}$$

It should be noted that equation 4 applies only to the particle that has taken at least one step.

Einstein (1950) also developed his well-known bedload equation by considering the dynamic lift force as a random variable. The idea is that the probability of a sediment particle being eroded from the bed surface is equal to the probability that the lift force exerted on the particle exceeds its submerged weight. He obtained

$$p = 1 - \frac{1}{\sqrt{\pi}} \int_{-B_*\Psi_* - \frac{1}{\eta_0}}^{B_*\Psi_* - \frac{1}{\eta_0}} e^{-z^2} dz = \frac{A_*\Phi_*}{1 + A_*\Phi_*}, \tag{5}$$

where

p = probability of a sediment particle being eroded;

η_0, A_*, B_* = constants;

Ψ_* = intensity of shear for an individual particle size; and

Φ_* = intensity of transport for an individual particle size.

Solving equation 5 for Φ_*, which is a function of the bedload transport rate, one obtains the bedload discharge for individual particle sizes from hydraulic parameters and sediment properties.

Hubbell and Sayre (1964) presented a one-dimensional stochastic model for the longitudinal dispersion of bed-material particles in an alluvial channel. The results are identical to Einstein's (eqs. 1-4). The assumptions are:

1. The flow is in equilibrium (Simons and Richardson, 1966).

2. The particle always moves in a downstream direction with a series of alternate steps and rests.

3. The duration of movement is insignificant compared to the rest periods.

4. The stochastic processes describing the number of steps taken by a particle in a distance interval and a time interval are independent of each other and both are homogeneous Poisson processes (Parzen, 1967). These assumptions are essentially the same as those of Einstein's (1937) although stated in a different way.

Based on the concept of continuity, Hubbell and Sayre (1964) proposed the transport equation for the bed material of a certain characteristic,

$$(Q_T)_c = i_c(\gamma_s)_c(1 - \theta)Wh\left(\frac{\bar{x}}{t}\right)_c \quad , \tag{6}$$

where

Q_T = bed-material discharge in weight per unit time;

i_c = ratio of the volume of particles possessing the characteristic size to the volume of bed-material particles in the zone of particle movement;

γ_s = specific weight of the bed material;

θ = bulk porosity of the bed in place;

W = width of channel;

h = average depth of the zone in which particle movement occurs;

$\bar{x}$ = average distance traveled by bed material in time t;

t = measure of time; and

c = subscript that denotes terms associated with particles possessing a certain characteristic size.

Combining equation 6 with the result from the Hubbell-Sayre one-dimensional stochastic model gives the total bed-material discharge for all particle sizes,

$$Q_T = \sum_c i_c(\gamma_s)_c(1 - \theta)Wh\left(\frac{k_2}{k_1}\right)_c \quad , \tag{7}$$

in which k_1 and k_2 are defined in equations 1 and 2, respectively.

Sayre and Conover (1967) extended the one-dimensional stochastic model derived by Hubbell and Sayre (1964) to two dimensions by introducing the vertical level at which particles are deposited. Their analysis led to the joint probability density function for the event that a particle has, at time t, traveled a distance equal to x and is located at an elevation equal to y,

$$f(x,y;t) = f_{Y_D}(y) \sum_{n=1}^{\infty} f_X^{(n)}(x) \int_0^t f_T^{(n)}(t') \int_{t-t'}^{\infty} f_{T\backslash Y_D}(\tau\backslash y)\, d\tau dt' \quad , \tag{8}$$

where

$f_{Y_D}(y)$ = probability density function for the elevation of particle deposition;

$f_X^{(n)}(x)$, $f_T^{(n)}(t)$ = n-fold convolutions of $f_X(x)$ and $f_T(t)$, respectively;

$f_{T\backslash Y_D}(t\backslash y)$ = conditional probability density function for the rest periods given the elevation at which the particle is deposited; and

t' = sum of the first n rest periods.

If a group of identical sediment particles are released simultaneously at $x = 0$, $y = y_0$, and $t = 0$, equation 8 gives the concentration of the particles, which were initially at rest and have moved from their respective initial positions, with respect to longitudinal position, x, and vertical position, y, as a function of time, t.

In order to apply equation 8, the density functions, $f_{Y_D}(y)$, $f_{T\backslash Y_D}(t\backslash y)$, and $f_X(x)$ must be specified. The unconditional density function, $f_T(t)$ is related to $f_{T\backslash Y_D}(t\backslash y)$ and $f_{Y_D}(y)$ by the relation

$$f_T(t) = \int_{y_{\min}}^{y_{\max}} f_{T\backslash Y_D}(t\backslash y) f_{Y_D}(y)\, dy\,, \tag{9}$$

in which $y_{\max}$ and $y_{\min}$ are the highest and lowest elevations at which particles can be deposited, respectively. The marginal case of equation 8 is

$$f(x;t) = \sum_{n=1}^{\infty} f_X^{(n)}(x) \int_0^t \left[f_T^{(n)}(\tau) - f_T^{(n+1)}(\tau) \right] d\tau = \sum_{n=1}^{\infty} f_X^{(n)}(x)\; P[N(t) = n]\,, \tag{10}$$

in which $P[N(t) = n]$ denotes the probability that a particle takes n steps in a time interval t. Equation 10 is a general one-dimensional stochastic model where only longitudinal dispersion is considered. One may note that the substitution of equations 1 and 2 into equation 10 reduces to equation 4.

Yang (1968) assumed the step lengths are gamma distributed with a shape parameter, r, and the common density function,

$$f_X(x) = \frac{k_1}{\Gamma(r)} (k_1 x)^{r-1} e^{-k_1 x}, \quad x > 0, \tag{11}$$

and the rest periods are exponentially distributed with the common density function given in equation 2. Substituting equations 2 and 11 into equation 10, he obtained

$$f(x;t) = k_1 e^{-k_1 x - k_2 t} \sum_{n=1}^{\infty} \frac{(k_1 x)^{nr-1}}{\Gamma(nr)} \frac{(k_2 t)^n}{\Gamma(n+1)}, \quad x > 0,\ t > 0. \tag{12}$$

Since the gamma distribution reduces to the exponential distribution when $r = 1$, equation 4 is actually a special case of equation 12.

Shen and Todorovic (1971) generalized the Hubbell-Sayre one-dimensional model given in equations 1, 2, and 4. The essential difference between the two models is that the former was based on the nonhomogeneous Poisson processes (Parzen, 1967), while the latter was based on the homogeneous Poisson processes. In the Shen-Todorovic model, the probability density functions of the step lengths and the rest periods are, respectively,

$$f_X(x) = k_1(x)\, e^{-\int_{x_0}^{x} k_1(s)\,ds}, \quad x > 0 , \tag{13}$$

and

$$f_T(t) = k_2(t)\, e^{-\int_{t_0}^{t} k_2(s)\,ds}, \quad t > 0 , \tag{14}$$

where

$k_1(x)$, $k_2(t)$ = functions of x and t, respectively; and

x_0, t_0 = initial position and time, respectively.

The probability density function of the total travel distance of a particle, which was initially at rest and has moved from its initial position, x_0, was found to be

$$f(x;t) = k_1(x)\, e^{-\int_{x_0}^{x} k_1(s)\,ds - \int_{t_0}^{t} k_2(s)\,ds}$$

$$\cdot \sum_{n=1}^{\infty} \frac{\left[\int_{x_0}^{x} k_1(s)\,ds\right]^{n-1}}{\Gamma(n)} \frac{\left[\int_{t_0}^{t} k_2(s)\,ds\right]^{n}}{\Gamma(n+1)}, \quad x > 0,\ t > 0 . \tag{15}$$

It is seen from equation 13 and 14 that the mean number of steps taken by a particle in $(x_0, x]$ and $(t_0, t]$ are $\int_{x_0}^{x} k_1(s)\,ds$ and $\int_{t_0}^{t} k_2(s)\,ds$, respectively, whereas those of Hubbell-Sayre's model are $k_1(x - x_0)$ and $k_2(t - t_0)$, respectively. The Hubbell-Sayre (1964) one-dimensional model is a special case of the Shen-Todorovic model.

Experimental Studies

Hubbell and Sayre (1964) conducted concentration distribution experiments both in the field and laboratory to evaluate the one-dimensional stochastic model given by equation 4. The bed configurations in these experiments were large dunes in the field and ripples in the laboratory flume. Using radioactive tracer particles, a series of longitudinal concentration-distribution curves were obtained at different times for a given flow condition. The longitudinal concentration-distribution function, $\Phi(x; t)$, is defined to be the weight of tracer particles per unit volume of bed material as a function of longitudinal distance and time and is related to $f(x; t)$ by

$$\Phi(x; t) = \frac{W_T}{Wh} f(x; t) \quad , \tag{16}$$

in which W_T is the total weight of the tracer particles placed in the channel, W is the channel width, h is average depth of the zone of bed material movement, and $f(x; t)$ is given by equation 4. Based on equation 16, the parameters k_1 and k_2 were estimated. With these estimates, Hubbell and Sayre reported that the theoretical and observed concentration-distribution functions agree reasonably well.

Yang (1968) carried out a set of experiments using radioactive tracer particles to verify the model given by equation 12. Experiments were performed with ripple and dune bed conditions in a laboratory flume 1.2 m (2 ft) wide by 18.3 m (60 ft) long. He reported that the shape of the experimental longitudinal dispersion curves are fairly well represented by equation 12. Yang also made preliminary runs with a single plastic particle in a small flume and found that the step lengths very closely follow a gamma distribution with the parameter r approximately equal to 2 and that the rest periods follow an exponential distribution very closely.

The first intensive experimental study on the movement of single particles was done by Grigg (1969). The experiments were conducted in a laboratory flume with two bed material sizes. The bed configurations were ripples and dunes. Using single radioactive tracer particles, he measured the step lengths and the rest periods directly and found the step lengths to be approximately gamma-distributed and the rest periods to be approximately exponentially distributed as proposed by Yang.

Grigg found interesting correlations between: (1) Various properties of the step length distribution, the stream power (product of mean bed shear stress and mean flow velocity), and the distribution of bedform lengths; and (2) various properties of the rest period distribution and statistical properties derived from the variation of bed elevation with respect to time.

Based on an idea suggested by Hubbell and Sayre (1965), Grigg also made some progress toward experimentally testing the Sayre-Conover two-dimensional stochastic model. By analyzing a record of the bed elevation as a function of time, he showed that the conditional probability density function of the rest periods can be approximated by the exponential function,

$$f_{T \backslash Y_D}(t \backslash y) = k_3(y)\, e^{-k_3(y)\, t} \quad , \tag{17}$$

and

$$k_3(y) = \frac{1}{\alpha}\, e^{\beta y} \quad , \tag{18}$$

in which α and β are constants and y measures bed elevation in terms of the standard deviation about mean bed elevation.

Remarks

Based on the review given in the previous sections, the following remarks are offered.

1. The Sayre-Conover model given by equation 10 is the most general one-dimensional model. The rest of the one-dimensional models, which were previously discussed, can be obtained from this model by proper substitutions. Therefore, it may be rated as the best existing one-dimensional model.

2. The Sayre-Conover model given by equation 8 is the only existing two-dimensional stochastic model. The derivation of the Sayre-Conover model has been discussed by Lee (1973). To verify equation 8, a method of estimating the probabillity distribution of the elevation at which particles are deposited must be known. One of the purposes of this report is to present such a method.

3. In order for the stochastic model to serve a prediction purpose, the relation between flow conditions and the parameters describing the probability distributions must be known. Without such knowledge the stochastic models cannot contribute much to the prediction problem.

4. A great deal of effort is required to perform dispersion and single particle experiments. If another method can be developed to estimate the necessary probability distributions from more readily accessible data, considerable savings would result. The methods developed in this report require only records of bed elevation.

DEVELOPMENT OF THEORY

Characteristics of Particle Movement Over a Dune Bed

Dunes are one of the most common bed forms in alluvial channels. Field observations by Simons and Richardson (1966) indicated that dunes may form in any alluvial channel, irrespective of the size of bed material, if the stream power is sufficiently large to cause general transport of the bed material without exceeding a Froude number of unity. The longitudinal profile of a dune is approximately triangular in shape with a gentle upstream slope and a steep downstream slope. The upstream slope depends somewhat on flow conditions, whereas the downstream slope is more dependent on the angle of repose of the bed material. The length of a dune ranges from about 0.61 m (2 ft) to several hundred meters, depending on the scale of the flow system. The Chezy discharge coefficient, $C/\sqrt{g}$, ranges from 8 to 12, and the total bed-material discharge concentration ranges from 100 to 1,200 milligrams per liter for dune flow conditions. For further information readers may refer to Simons and Richardson (1966).

For dune flow conditions, a record of the bed elevation as a function of time at a particular location reveals an alternating sequence of periods during which either erosion or deposition is occurring. This type of record is commonly obtained from the output of a depth sounder which is located at a fixed location and hereafeter will be referred to as the $y_x(t)$ record; that is, the elevation of the bed, y, positive upward, as a function of time, t, at a fixed location, x. When deposition occurs, $[dy_x(t)/dt] > 0$, and when erosion occurs, $[dy_x(t)/dt] < 0$, provided these derivatives exist. An instantaneous longitudinal bed profile may be characterized by an alternating series of erosion and deposition reaches. An instantaneous logitudinal profile can be obtained by mounting a depth sounder in a boat, provided that the speed of the boat is large relative to the speed of bed forms. These bed profiles will hereafter be referred to as the $y_t(x)$ records; that is, the elevation of the bed, y, positive upward, as a function of the longitudinal coordinates, x, at a given time, t. The longitudinal coordinate will be assumed to increase in the downstream direction; therefore, the reaches with positive slopes, $[dy_t(x)/dx] > 0$, represent the upstream or stoss sides of the dunes and the reaches with negative slopes, $[dy_t(x)/dx] < 0$, represent the downstream or slip faces of the dunes. The dune crest is defined by a local maximum in the $y_t(x)$ record, and the dune trough is defined by a local minimum in the record.

Anyone who has an opportunity to observe closely the movement of sediment is aware that dunes move downstream owing to erosion from their upstream face and deposition on their downstream face. That is, the bed forms migrate downstream because deposition occurs on the downstream face, where $[dy_t(x)/dx] < 0$, and erosion occurs on the upstream face, where $[dy_t(x)/dx] > 0$. It will be assumed throughout this report that no deposition occurs on the upstream sides of dunes and no erosion occurs on the downstream faces of dunes. This assumption is not strictly true physically but is necessary for the determination of the conditional step length distributions. If the assumption is true, each sediment particle on the stoss side of a dune must make a step in the downstream direction before it is deposited on the slip face of any dune. Once deposited it rests there until the dune has migrated downstream and it becomes re-exposed on the stoss side. In other words, sediment particles are transported downstream in an alternating sequence of steps and rests of random length and duration. The frequencies and magnitudes of these steps and rests are of basic interest in understanding the nature of the movement of the sediments.

Because particles must be eroded from and deposited on the surface of the bed, the step length of a particular particle depends only on the elevation from which it is eroded, the elevation at which it is deposited, the number of dune crests which it passes before being deposited, and the scale and shape of the bed surface ($y_t(x)$ record) during the time of its movement. Likewise the rest period of a particular particle depends on the scale and shape of the $y_x(t)$ record and on the elevation at which the particle is deposited. If the bed material size is not uniform, the elevation of deposition or erosion may also depend on the size of particles because of vertical sorting.

The intimate relationship between the bed form shape, as measured by the $y_x(t)$ and $y_t(x)$ records, and the step lengths and rest periods of a bed-material particle allow the probability distributions of the step lengths and the rest periods to be estimated from the bed-form data. In the following three sections, a method of estimating the probability distributions of the rest periods, step lengths, and the elevations at which a particle is deposited or eroded using the $y_x(t)$ and $y_t(x)$ records will be presented. In the last two sections the bed-material transport equations and a general two-dimensional bed-material dispersion equation will be derived as functions of these probability distributions. In the next chapter the transport equations will be tested using data from three flume runs and the results discussed.

Estimation of the Probability Distributions of the Elevations of Deposition and Erosion

The probability that particles are deposited between the elevations η_j and η_{j+1} may be written as

$$P[\eta_j < Y_D \leq \eta_{j+1}] = \lim_{t \to \infty} \left\{ \frac{\text{number of particles deposited within the interval } (\eta_j, \eta_{j+1}] \text{ in time } t}{\text{number of particles deposited over all intervals in time } t} \right\}, \qquad (19)$$

where

$P[\cdot]$ = probability;

Y_D = random variable describing the elevations at which particles are deposited;

η_j, η_{j+1} = lower and upper class limits associated with the class mark of the elevation y_j, respectively; and

t = time during which the observations were made.

The elevation at which particles are deposited will hereafter simply be referred to as the elevation of deposition, Y_D.

If the number of particles per unit volume of the bed, Ω, is constant, the flow is stationary (statistical sense), and both erosion and deposition cannot occur at the same point at the same time, the numerator and the denominator of equation 19 can be obtained from the $y_x(t)$ record. The total number of particles deposited per unit area within the class interval $(\eta_j, \eta_{j+1}]$ in time t, denoted by $N_d(y_j)$ is given by

$$N_d(y_j) = \Omega \sum_{k=1}^{m_j} \Delta y^+_{j,k}; \; j = 1, 2, \ldots, n \;, \tag{20}$$

where

y_j = class mark for the realization of Y_D;

n = number of class intervals for the realization of Y_D;

$\Delta y^+_{j,k}$ = vertical rise of the bed in the class interval associated with y_j for the kth deposition period; and

m_j = maximum number of bed forms contained in the $y_x(t)$ record and which also contain some deposition in the class interval associated with y_j.

Figure 1 illustrates the class marks, y_j, and the vertical rise of the bed,

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$\Delta y^+_{j,k}$, within the class intervals, Δy_j, for a typical $y_x(t)$ record. It is clear that $\Delta y^+_{j,k} \leq \Delta y_j = \eta_{j+1} - \eta_j$. The total number of

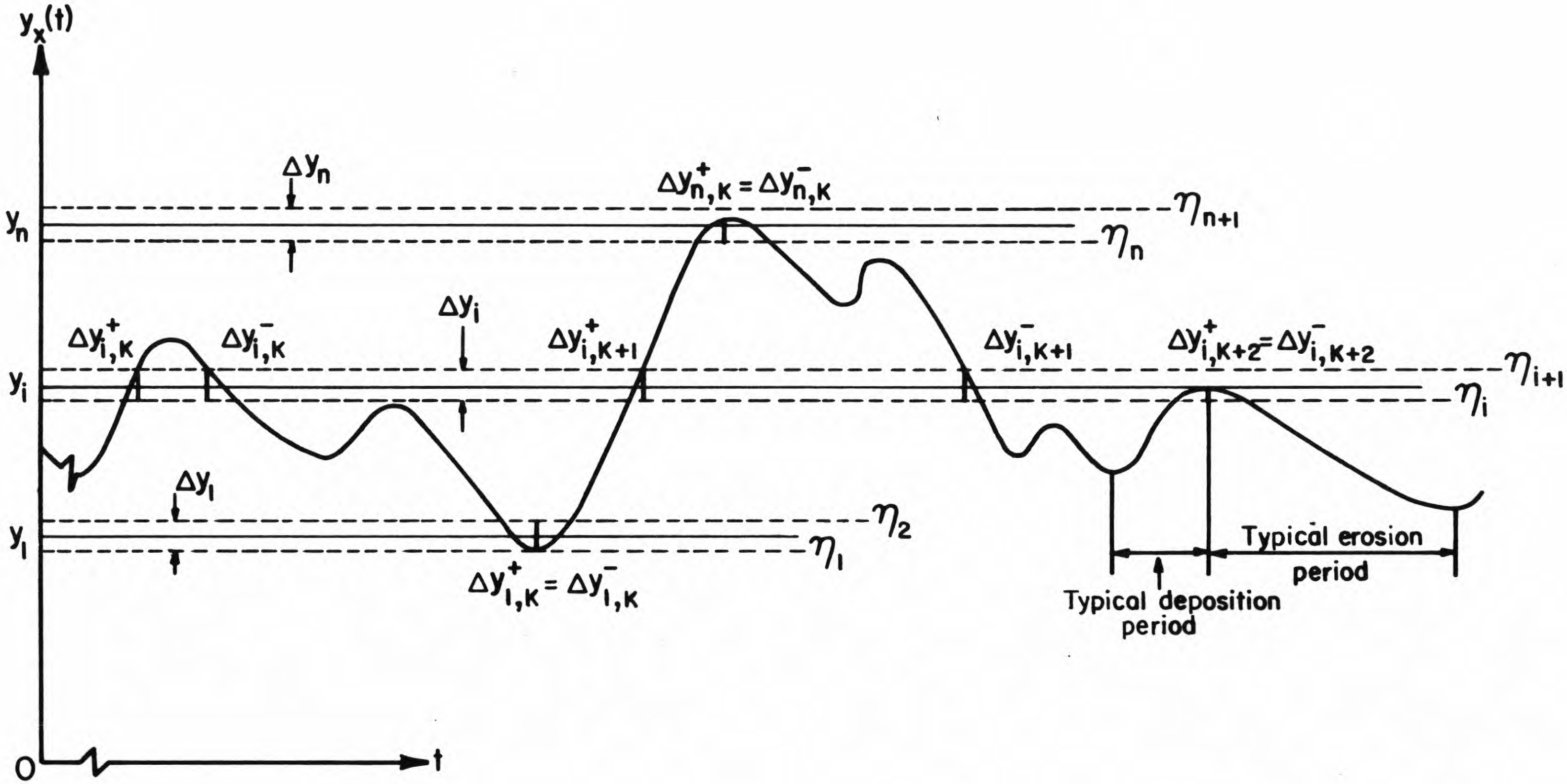

Figure 1. Typical $y_x(t)$ record illustrating the class marks for deposition and erosion.

particles per unit area deposited over all intervals, the denominator of equation 19, is designated by N_d and is obtained by summing equation 20 over all class marks

$$N_d = \sum_{j=1}^{n} N_d(y_j) = \Omega \sum_{j=1}^{n} \sum_{k=1}^{m_j} \Delta y_{j,k}^{+} \, . \tag{21}$$

Equation 19 now becomes

$$P[\eta_j < Y_D \leq \eta_{j+1}] = \lim_{m_j \to \infty} \frac{N_d(y_j)}{N_d} = \lim_{m_j \to \infty} \frac{\sum\limits_{k=1}^{m_j} \Delta y_{j,k}^{+}}{\sum\limits_{j=1}^{n} \sum\limits_{k=1}^{m_j} \Delta y_{j,k}^{+}} \, . \tag{22}$$

Similarly an analysis of the erosion periods can be used to estimate the probability that particles are eroded between the elevations η_i and η_{i+1},

$$P[\eta_i < Y_E \leq \eta_{i+1}] = \lim_{m'_i \to \infty} \frac{N_e(y_i)}{N_e} = \lim_{m'_i \to \infty} \frac{\sum\limits_{k=1}^{m'_i} \Delta y_{i,k}^{-}}{\sum\limits_{i=1}^{n} \sum\limits_{k=1}^{m'_i} \Delta y_{i,k}^{-}} \, , \tag{23}$$

where

Y_E = random variable describing the elevations at which particles are eroded;

y_i = class mark for the realization of Y_E;

η_i, η_{i+1} = lower and upper class limits of y_i, respectively;

m'_i = maximum number of bed forms contained in the $y_x(t)$ record and which also contain some erosion in the class interval associated with y_i;

$N_e(y_i)$ = total number of particles per unit area eroded from the interval $(\eta_i, \eta_{i+1}]$ centered at y_i;

N_e = total number of particles per unit area eroded over all intervals;

n = number of class intervals for Y_E; and

$\Delta \bar{y}_{i,k}$ = amount of erosion which occurred during the kth erosion period in the vertical class interval associated with y_i (fig. 1).

In the limit as m_j and m_i' approach infinity for a stationary record, the distributions of $P[\eta_j < Y_D \leq \eta_{j+1}]$ and $P[\eta_i < Y_E \leq \eta_{i+1}]$ must be identical. The elevation at which particles are eroded will hereafter simply be referred to as the elevation of erosion, Y_E.

To repeat, the following assumptions are necessary for equations 22 and 23 to be valid:

(1) Flow is in equilibrium such that both deposition and erosion processes are stationary with respect to time t.

(2) Both erosion and deposition do not occur at the same point during the same time period.

(3) The number of particles per unit volume of the bed is constant.

If the measuring equipment were sensitive enough to detect the movement of single particles, the second assumption would not be necessary because it would be physically impossible for one particle to be eroded and another to be deposited at the same point and at the same time. For $y_x(t)$ records obtained from less sensitive equipment, of course, the assumption may not be strictly true. When the bed material is not uniform in size, equations 20 through 23 are not strictly true because the number of particles per unit volume of the bed, Ω, is a function of elevation due to a vertical sorting. However, equations 22 and 23 should serve as first approximations to the true probabilities, $P[\eta_j < Y_D \leq \eta_{j+1}]$ and $P[\eta_i < Y_E \leq \eta_{i+1}]$, even for the nonuniform bed material.

If the number of particles per unit volume were known as a function of elevation, y, assumption (3) could be dropped. In this case, the counterpart of equation 22 is

$$P[\eta_j < Y_D \leq \eta_{j+1}] = \lim_{m_j \to \infty} \frac{\Omega_j \sum_{k=1}^{m_j} \Delta y^+_{j,k}}{\sum_{j=1}^{n} \Omega_j \sum_{k=1}^{m_j} \Delta y^+_{j,k}} , \tag{24}$$

in which Ω_j is the number of particles per unit volume of the bed associated with y_j. The counterpart of equation 23 would be similar. The value of Ω_j could be obtained from core-sample segments taken from different elevations within the bed. In the next section, Ω will be assumed to be a constant in estimation of $P[\eta_j < Y_D \leq \eta_{j+1}]$ and $P[\eta_i < Y_E \leq \eta_{i+1}]$. Because the bed material was very uniform in size, however, the assumption should have been very good.

Equations 22 and 23 are estimated by use of the sample probability mass functions which are defined to be

$$p_{Y_D}(y_j) = \frac{N_d(y_j)}{N_d} \simeq P[\eta_j < Y_D \leq \eta_{j+1}] \text{ for a large } m_j \ , \tag{25}$$

and

$$p_{Y_E}(y_i) = \frac{N_e(y_i)}{N_e} \simeq P[\eta_i < Y_E \leq \eta_{i+1}] \text{ for a large } m_i \ , \tag{26}$$

in which $p_{Y_D}(y_j)$ is used to estimate equation 22 and $p_{Y_E}(y_i)$ is used to estimate equation 23. Estimates of the mean and variance of the elevation of deposition are respectively

$$\left.\begin{aligned} \hat{E}[Y_D] &= \sum_{j=1}^{n} y_j p_{Y_D}(y_j) \ , \\ &\text{and} \\ \hat{\text{Var}}[Y_D] &= \left[\sum_{j=1}^{n} y_j^2 p_{Y_D}(y_j)\right] - \left[\sum_{j=1}^{n} y_j p_{Y_D}(y_j)\right]^2 \ , \end{aligned}\right\} \tag{27}$$

in which $\hat{E}[\cdot]$ and $\hat{\text{Var}}[\cdot]$ denote estimates of the expected value and variance, respectively. Replacing the D's with E's and the j's with i's in equation 27 gives the estimates for the mean and variance of the elevation of erosion.

The probability density functions for the elevations of deposition and erosion [$f_{Y_D}(y)$ and $f_{Y_E}(y)$] may be inferred from the probability mass functions [$p_{Y_D}(y_i)$ and $p_{Y_E}(y_i)$] by means of a statistical fitting procedure. This will be discussed later.

Estimation of the Probability Distributions of the Rest Periods

In this study the rest period of a particle is defined as the time lapse between the burial and re-exposure of the particle. This definition is consistent with the assumption that erosion and deposition do not occur at the same point during the same time period, and it is also necessary in analyzing single particle measurements because the measurement techniques cannot detect momentary rests by the particle. Using the burial definition, the $y_x(t)$ record provides a means of estimating the probability density functions of particle rest periods conditioned on the elevations of deposition. The method is illustrated schematically in figure 2, where the

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statistic $\{t_{j,k};\ j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, m_{j,j}\}$ measures the conditional rest period, the index k signifies a particular bed form. The term $m_{j,j}$ designates the maximum number of bed forms which are contained in the $y_x(t)$ record and which also contain both an up-crossing and a down-crossing at the elevation y_j. This use of the $y_x(t)$ record was first suggested by Hubbell and Sayre (1965) and was partly evaluated by Grigg (1969).

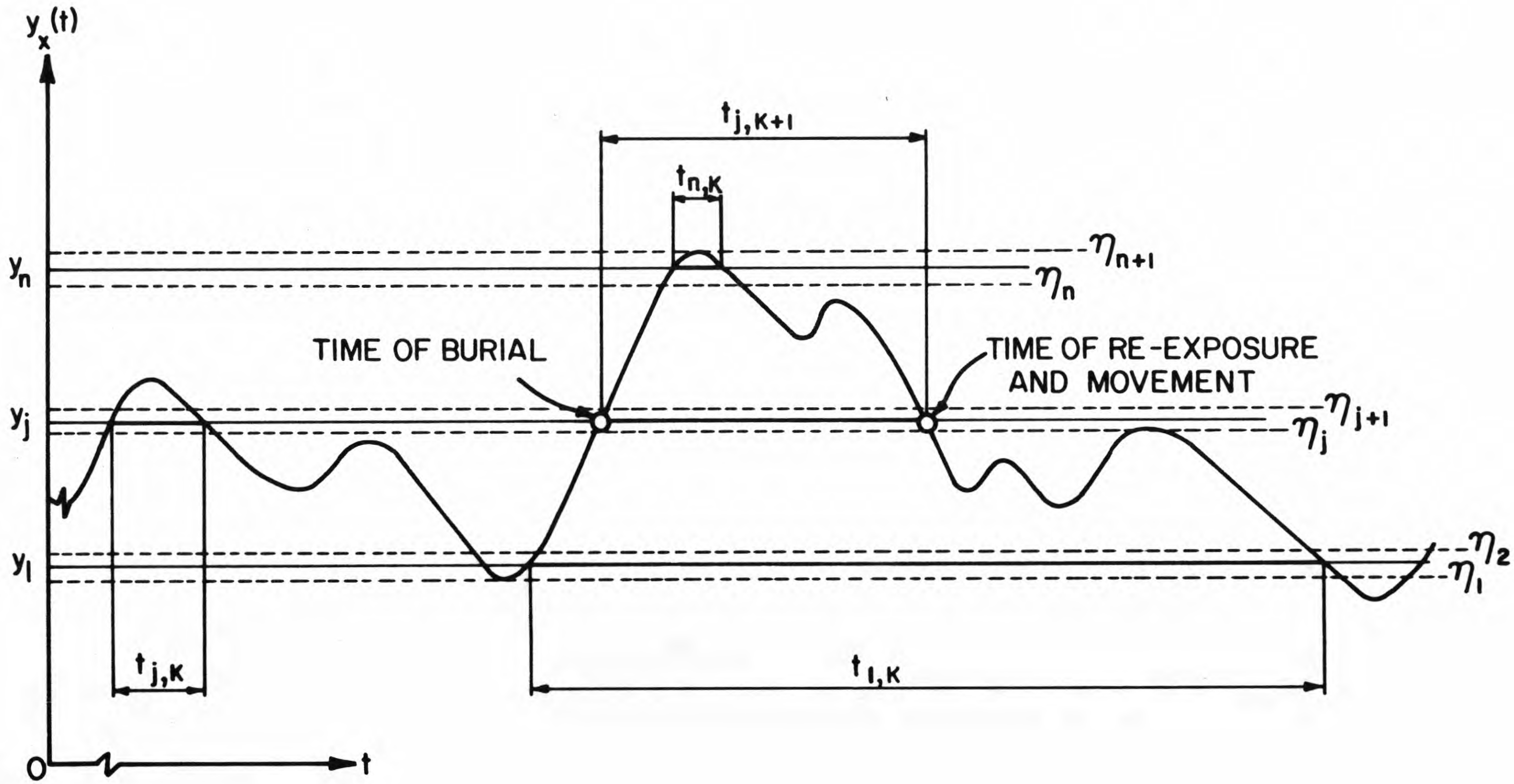

Figure 2. Typical $y_x(t)$ record illustrating the conditional rest periods of a particle.

A relative frequency analysis of the statistic $\{t_{j,k}\}$ leads to a sample conditional probability mass function of the rest periods which is defined to be

$$p_{T\backslash Y_D}(t_\alpha \backslash y_j) = P[\tau_\alpha < T \leq \tau_{\alpha+1} \backslash \eta_j < Y_D \leq \eta_{j+1}] \ ;$$

$$j = 1, 2, \ldots, n; \ \alpha = 1, 2, \ldots, r \ , \tag{28}$$

where

T = random variable describing the rest periods;

t_α, y_j = class marks for T and Y_D, respectively;

τ_α, $\tau_{\alpha+1}$ = lower and upper class limits of t_α, respectively; and

r = number of class intervals for T.

Equation 25 can be used to release the condition on equation 28 and to obtain the marginal sample probability mass function for the rest periods,

$$p_T(t_\alpha) = P[\tau_\alpha < T \leq \tau_{\alpha+1}] = \sum_{j=1}^{n} p_{T\backslash Y_D}(t_\alpha \backslash y_j)\, p_{Y_D}(y_j) \ ; \tag{29}$$

$$\alpha = 1, 2, \ldots, r \ .$$

From equations 28 and 29 the corresponding probability density functions for the conditional rest periods, $f_{T\backslash Y_D}(t\backslash y)$, and for the marginal rest periods, $f_T(t)$, may be approximated by means of a statistical fitting procedure.

The mean and variance of the conditional rest periods are estimated from the sample moments

$$\left.\begin{aligned} \hat{E}[T \backslash Y_D = y_j] &= \frac{1}{m_{j,j}} \sum_{k=1}^{m_{j,j}} t_{j,k} \ , \\ &\text{and} \\ \hat{\mathrm{Var}}[T \backslash Y_D = y_j] &= \frac{1}{m_{j,j}} \left[\sum_{k=1}^{m_{j,j}} (t_{j,k})^2 \right] - \left[\frac{1}{m_{j,j}} \sum_{k=1}^{m_{j,j}} t_{j,k} \right]^2 \ . \end{aligned}\right\} \tag{30}$$

Estimates of the mean and variance of the marginal rest periods are respectively

$$\left.\begin{aligned} \hat{E}[T] &= \sum_{j=1}^{n} \hat{E}[T \backslash Y_D = y_j] p_{Y_D}(y_j) \ , \\ &\text{and} \\ \hat{\mathrm{Var}}[T] &= \hat{E}[T^2] - (\hat{E}[T])^2 \ , \end{aligned}\right\} \tag{31}$$

in which

$$\hat{E}[T^2] = \sum_{j=1}^{n} \hat{E}[T^2 \backslash Y_D = y_j] p_{Y_D}(y_j) = \frac{1}{m_{j,j}} \sum_{k=1}^{m_{j,j}} (t_{j,k})^2 p_{Y_D}(y_j) \ .$$

The joint probability density funciton of T and Y_D, denoted by $f_{T,Y_D}(t,y)$ is estimated from the sample joint probability mass function defined to be

$$p_{T,Y_D}(t_\alpha, y_j) = P[\tau_\alpha < T \leq \tau_{\alpha+1}, \eta_j < Y_D \leq \eta_{j+1}] \quad , \tag{32}$$

where $j = 1, 2, \ldots, n$ and $\alpha = 1, 2, \ldots, r$. From equations 25 and 28, $p_{T,Y_D}(t_\alpha, y_j)$ is completely determined such that

$$p_{T,Y_D}(t_\alpha, y_j) = p_{T\backslash Y_D}(t_\alpha \backslash y_j)\, p_{Y_D}(y_j) \quad . \tag{33}$$

Finally the correlation coefficient between T and Y_D is estimated to be

$$\hat{\rho}_{T,Y_D} = \frac{\hat{E}[TY_D] - \hat{E}[T]\hat{E}[Y_D]}{\sqrt{\hat{\mathrm{Var}}[T]}\ \sqrt{\hat{\mathrm{Var}}[Y_D]}} \quad , \tag{34}$$

in which

$$\hat{E}[TY_D] = \sum_{\alpha=1}^{r} \sum_{j=1}^{n} t_\alpha y_j p_{T,Y_D}(t_\alpha, y_j) \quad ,$$

$\hat{E}[Y_D]$ and $\hat{\mathrm{Var}}[Y_D]$ are given in equation 27, and $\hat{E}[T]$ and $\hat{\mathrm{Var}}[T]$ are given in equation 31. The joint distribution expresses the relation between the rest period and the elevation of deposition and the correlation coefficient measures a degree of linear association between the rest period and the elevation of deposition.

If the shape of the $y_x(t)$ record is dependent on the flow conditions and bed-material properties, then the rest period statistics as determined by equations 28 through 34 are also functions of flow conditions and bed-material properties.

In summary, the probability distribution for the marginal rest period of a sediment particle, the rest period conditioned on the elevation of deposition, and the elevation of particle deposition and erosion can all be obtained from a continuous record of the bed elevation at a single point as a function of time. The only assumptions that are needed are:

1. Both erosion and deposition do not occur at the same point at the same time.

2. Bed elevation is stationary (in the statistical sense).

3. The number of particles per unit volume of the bed is constant.

These assumptions are not severely restrictive, and the results are equally applicable to both field and laboratory analysis.

Estimation of the Probability Distributions of the Step Lengths

The $y_x(t)$ record contained the information necessary to estimate the probability distributions of the rest periods. Both the $y_x(t)$ and the $y_t(x)$ records are necessary to determine the step length statistics. Unfortunately, more assumptions are also necessary and these assumptions may be considerably more restrictive than the ones made up to this time.

As previously mentioned, it will be assumed that each sediment particle on the stoss side of a dune makes a step in the downstream direction before it is deposited on the slip face of any dune. Once deposited it rests there until it is re-exposed on the stoss side. Let $E_{i,j,\nu}$ be the event that a particle, eroded from elevation, y_i, of the stoss side of a dune, passes ν dune crests before it is deposited at elevation, y_j. Then the statistic $\{x_{i,j,\nu,k};\ i, j = 1, 2, \ldots, n;\ \nu = 1, 2, \ldots;\ k = 1, 2, \ldots, m_{i,j,\nu}\}$ (fig.3) is the measure of the conditional step length of the event,

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$E_{i,j,\nu}$. The term $m_{i,j,\nu}$ represents the total number of possibilities of the event $E_{i,j,\nu}$ contained in the $y_t(x)$ record and the index k specifies a particular possibility. In general, the term $m_{i,j,\nu}$ will be different for each combination of values i, j, and ν.

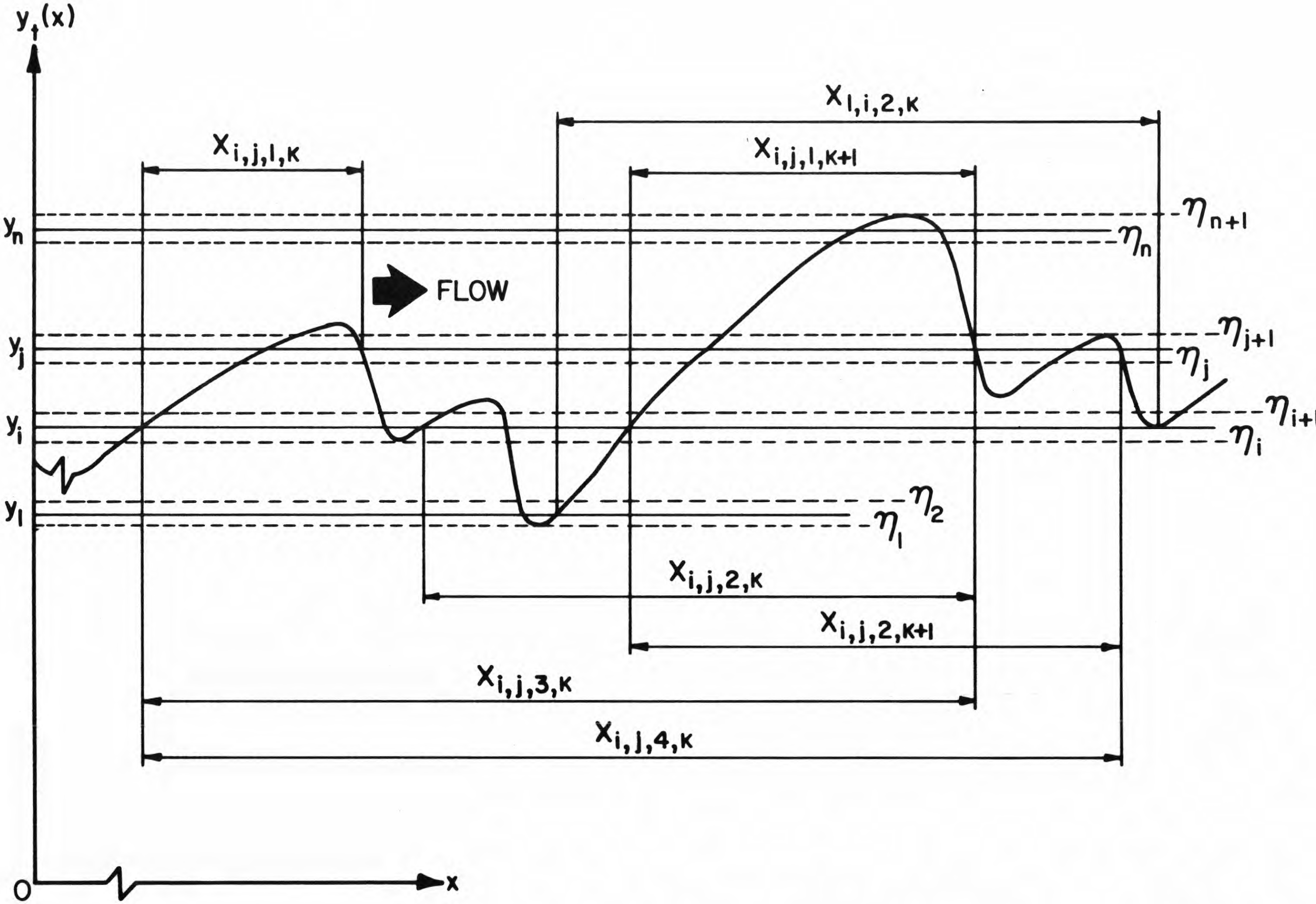

Figure 3. Statistic, $\{x_{i,j,\nu,k};\ i,j=1,2,\cdots,n;\ \nu=1,2,\cdots\cdot;\ k=1,2,\cdots,\ m_{i,j,\nu}\}$, for the step length of a particle.

A frequency analysis of the statistic $\{x_{i,j,\nu,k}\}$ gives a sample conditional probability mass function which is defined to be

$$p_{X\backslash Y_E, Y_D, E_\nu}(x_\beta \backslash y_i, y_j, \nu)$$

$$= P[\lambda_\beta < X \leq \lambda_{\beta+1} \backslash \eta_i < Y_E \leq \eta_{i+1}, \; \eta_j < Y_D \leq \eta_{j+1}, \; E_\nu] \; ;$$

$$\beta = 1, 2, \ldots, s; \; i, j, = 1, 2, \ldots, n; \; \nu = 1, 2, \ldots, \tag{35}$$

where

X = random variable describing the step lengths;

x_β = class mark for the realizations of X;

$\lambda_\beta, \lambda_{\beta+1}$ = lower and upper class limits of x_β, respectively;

s = number of class intervals for the realizations of X; and

E_ν = event that a particle passes ν dune crests before it is deposited (fig. 3).

The corresponding probability density function, $f_{X\backslash Y_E, Y_D, E_\nu}(x\backslash y', y, \nu)$, may be determined from $p_{X\backslash Y_E, Y_D, E_\nu}(x_\beta \backslash y_i, y_j, \nu)$, and its mean and variance are estimated to be

$$\hat{E}[X\backslash Y_E = y_i, \; Y_D = y_j, E_\nu] = \frac{1}{m_{i,j,\nu}} \sum_{k=1}^{m_{i,j,\nu}} x_{i,j,\nu,k} \; ,$$

and

$$\hat{\text{Var}}\,[X\backslash Y_E = y_i, \; Y_D = y_j, E_\nu] = \frac{1}{m_{i,j,\nu}} \sum_{k=1}^{m_{i,j,\nu}} (x_{i,j,\nu,k})^2 - \left[\frac{1}{m_{i,j,\nu}} \sum_{k=1}^{m_{i,j,\nu}} x_{i,j,\nu,k}\right]^2 , \tag{36}$$

in which

$$\frac{1}{m_{i,j,\nu}} \sum_{k=1}^{m_{i,j,\nu}} (x_{i,j,\nu,k})^2 = \hat{E}[X^2 \backslash Y_E = y_i,\ y_D = y_j,\ E_\nu] \ .$$

If Y_E, Y_D, and E_ν are mutually independent (it seems to be reasonable that after a particle passes the crest of a dune it has probably lost track of where it came from), the density function $[f_{X\backslash Y_D, E_\nu}(x\backslash y, \nu)]$ of the step lengths given that a particle is deposited at elevation y after passing ν dune crests is estimated from the sample conditional mass function which is given by

$$\begin{aligned} p_{X\backslash Y_D, E_\nu}(x_\beta\backslash y_j, \nu) &= P[\lambda_\beta < X \leq \lambda_{\beta+1} \backslash \eta_j < Y_D \leq \eta_{j+1},\ E_\nu] \\ &= \sum_{i=1}^{n} p_{X\backslash Y_E, Y_D, E_\nu}(x_\beta\backslash y_i, y_j, \nu)\, p_{Y_E}(y_i) \end{aligned} \qquad (37)$$

in which $p_{Y_E}(y_i)$ is given by equation 26. The mean and variance of the step lengths of a particle which is deposited at y_j after passing ν dune crests are estimated to be

$$\left.\begin{aligned} \hat{E}[X\backslash Y_D = y_j, E_\nu] &= \sum_{i=1}^{n} \hat{E}[X\backslash Y_E = y_i,\ Y_D = y_j, E_\nu]\, p_{Y_E}(y_i) \ , \\ &\text{and} \\ \hat{\mathrm{Var}}\,[X\backslash Y_D = y_j, E_\nu] &= \hat{E}[X^2\backslash Y_D = y_j, E_\nu] - (\hat{E}[X\backslash Y_D = y_j, E_\nu])^2 \ , \end{aligned}\right\} \qquad (38)$$

in which

$$\hat{E}[X^2\backslash Y_D = y_j, E_\nu] = \sum_{i=1}^{n} \hat{E}[X^2\backslash Y_E = y_i,\ Y_D = y_j, E_\nu]\, p_{Y_E}(y_i) \ .$$

Likewise, the following sample probability mass functions and corresponding means and variances are obtained:

$$\left.\begin{aligned}
p_{X\backslash Y_D}(x_\beta\backslash y_j) &= P[\lambda_\beta < X \le \lambda_{\beta+1}\backslash \eta_j < Y_D \le \eta_{j+1}] \\
&= \sum_{\nu=1}^{\infty} p_{X\backslash Y_D, E_\nu}(x_\beta\backslash y_j, \nu) P[E_\nu] \\
\hat{E}[X\backslash Y_D = y_j] &= \sum_{\nu=1}^{\infty} \hat{E}[X\backslash Y_D = y_j, E_\nu] P[E_\nu] \\
\hat{Var}[X\backslash Y_D = y_j] &= \left[\sum_{\nu=1}^{\infty} \hat{E}[X^2\backslash Y_D = y_j, E_\nu] P[E_\nu]\right] - (\hat{E}[X\backslash Y_D = y_j])^2 ;
\end{aligned}\right\} \quad (39)$$

$$\left.\begin{aligned}
p_{X\backslash E_\nu}(x_\beta\backslash \nu) &= P[\lambda_\beta < X \le \lambda_{\beta+1}\backslash E_\nu] = \sum_{j=1}^{n} p_{X\backslash Y_D, E_\nu}(x_\beta\backslash y_j, \nu) p_{Y_D}(y_j) \\
\hat{E}[X\backslash E_\nu] &= \sum_{j=1}^{n} \hat{E}[X\backslash Y_D = y_j, E_\nu] p_{Y_D}(y_j) \\
\hat{Var}[X\backslash E_\nu] &= \left[\sum_{j=1}^{n} \hat{E}[X^2\backslash Y_D = y_j, E_\nu] p_{Y_D}(y_j)\right] - (\hat{E}[X\backslash E_\nu])^2 ;
\end{aligned}\right\} \quad (40)$$

and

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\left.
\begin{aligned}
p_X(x_\beta) &= P[\lambda_\beta < X \le \lambda_{\beta+1}] = \sum_{\nu=1}^{\infty} p_{X\backslash E_\nu}(x_\beta \backslash \nu) P[E_\nu] \\
\hat{\mathrm{E}}[X] &= \sum_{\nu=1}^{\infty} \hat{\mathrm{E}}[X\backslash E_\nu] P[E_\nu] \\
\hat{\mathrm{Var}}[X] &= \left[\sum_{\nu=1}^{\infty} \hat{\mathrm{E}}[X^2\backslash E_\nu] P[E_\nu] \right] - (\hat{\mathrm{E}}[X])^2 \ .
\end{aligned}
\right\} \quad (41)
$$

The density functions, $f_{X\backslash Y_D}(x\backslash y)$, $f_{X\backslash E_\nu}(x\backslash \nu)$, and $f_X(x)$ are estimated from equation sets 39, 40, and 41, respectively.

The joint probability density function of X and Y_D, conditioned on the event E_ν, $[f_{X,Y_D\backslash E_\nu}(x,y\backslash\nu)]$ can be estimated from a sample joint probability mass function,

$$p_{X,Y_D\backslash E_\nu}(x_\beta,y_j\backslash\nu) = P[\lambda_\beta < X \leq \lambda_{\beta+1}, \eta_j < Y_D \leq \eta_{j+1}\backslash E_\nu] \tag{42}$$

and

$$\begin{aligned} p_{X,Y_D\backslash E_\nu}(x_\beta\backslash y_j,\nu) &= p_{X\backslash Y_D,E_\nu}(x_\beta\backslash y_j,\nu)\, p_{Y_D\backslash E_\nu}(y_j\backslash\nu) \\ &= p_{X\backslash Y_D,E_\nu}(x_\beta\backslash y_j,\nu)\, p_{Y_D}(y_j) \quad . \end{aligned} \tag{43}$$

Note that $p_{Y_D\backslash E_\nu}(y_j\backslash\nu) = p_{Y_D}(y_j)$ because Y_D and E_ν were assumed to be independent . The correlation coefficient of X and Y_D, conditioned on the event E_ν, is then estimated to be

$$\hat{\rho}_{X,Y_D\backslash E_\nu} = \frac{\hat{E}[XY_D\backslash E_\nu] - \hat{E}[X\backslash E_\nu]\hat{E}[Y_D]}{\sqrt{\hat{Var}[X\backslash E_\nu]}\ \sqrt{\hat{Var}[Y_D]}} \quad , \tag{44}$$

in which

$$\hat{E}[XY_D\backslash E_\nu] = \sum_{\beta=1}^{s} x_\beta \left\{ \sum_{j=1}^{n} y_j p_{X\backslash Y_D,E_\nu}(x_\beta\backslash y_j,\nu)\, p_{Y_D}(y_j) \right\} \quad , \tag{45}$$

and $\hat{E}[Y_D]$, $\hat{Var}[Y_D]$, $\hat{E}[X\backslash E_\nu]$, and $\hat{Var}[X\backslash E_\nu]$ are given by equation sets 27 and 40. Similarly, the joint probability density function of X and Y_D, $[f_{X,Y_D}(x,y)]$ and the corresponding correlation coefficient, ρ_{X,Y_D}, are estimated to be

$$p_{X,Y_D}(x_\beta, y_j) = p_{X\backslash Y_D}(x_\beta \backslash y_j)\, p_{Y_D}(y_j) \quad , \tag{45}$$

and

$$\hat{\rho}_{X,Y_D} = \frac{\hat{E}[XY_D] - \hat{E}[X]\hat{E}[Y_D]}{\sqrt{\hat{\mathrm{Var}}\,[X]}\ \sqrt{\hat{\mathrm{Var}}\,[Y_D]}} \ , \tag{46}$$

in which

$$\hat{E}[XY_D] = \sum_{\beta=1}^{s} x_\beta \left\{ \sum_{i=1}^{n} y_j p_{X\backslash Y_D}(x_\beta \backslash y_j)\, p_{Y_D}(y_j) \right\} .$$

The joint distributions (eqs. 43 and 45) express the relation between the step length and the elevation of deposition, and the correlation coefficients (eqs. 44 and 46) measure the degree of linear association between the step length and the elevation of deposition.

The American Society of Civil Engineers (Task Committee on Preparation of Sedimentation Manual, 1962) defines bedload as that material moving on or near the bed. Accepting this general definition, it would appear consistent to count any sediment particle which was able to skip across a dune trough as suspended load,since it would be extremely unlikely that a particle would be able to pass the trough while moving on or near the bed. A very precise, and admittedly restrictive, definition of bedload is used for the purpose of this report. For the purposes of this report, bedload is defined as that part of bed material which is deposited on the downstream face of the dune from which it is eroded. Then the suspended load must be that material which is not deposited on the downstream face of the dune from which it is eroded, that is, all sediment particles which pass two or more dune crests before being deposited. The same particle could be counted as bedload during one step but as suspended load during the next step. By definition then, it follows that $P[E_1]$ = probability that a particle is transported as the bedload; and $1 - P[E_1] \equiv \sum_{\nu=2}^{\infty} P[E_\nu]$ = probability that a particle is transported as the suspended load during any step. The probability distributions and moments for the step lengths of a bed-load particle may be obtained by putting $\nu = 1$ in the sets of equations 35 through 40 and 42 through 48.

For a bed material composed of coarse sand it seems reasonable to assume that all particles are transported as bedload, that is, all particles eroded from the stoss side of a dune will be deposited on the downstream side of the same dune; and therefore, $P[E_1] \equiv 1$, and $P[E_\nu] \equiv 0$ for $\nu \geq 2$. For this case, a frequency analysis of the statistic $\{x_{i,j,\nu,k};\ i,j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, m_{i,j,\nu};\ \nu = 1\}$ gives a sample conditional probability mass function which is defined to be

$$p_{X\backslash Y_E, Y_D}(x_\beta \backslash y_i, y_j) = P[\lambda_\beta < X \leq \lambda_{\beta+1} \backslash \eta_i < Y_E \leq \eta_{i+1},\ \eta_j < Y_D \leq \eta_{j+1}] ;$$

$$\beta = 1, 2, \ldots, s;\quad i, j = 1, 2, \ldots, n\ . \tag{47}$$

The corresponding probability density function, $f_{X\backslash Y_E, Y_D}(x\backslash y', y)$, may be approximated from $p_{X\backslash Y_E, Y_D}(x_\beta \backslash y_i, y_j)$. Denoting the statistic $\{x_{i,j,\nu,k};\ i,j = 1, 2, \ldots;\ k = 1, 2, \ldots, m_{i,j,\nu};\ \nu = 1\}$ simply as $\{x_{i,j,k};\ i,j = 1, 2, \ldots, n;\ k = 1, 2, \ldots, m_{i,j}\}$ (fig. 3), the corresponding mean and variance are estimated to be

$$\left.\begin{aligned} &\hat{\mathrm{E}}[X\backslash Y_E = y_i,\ Y_D = y_j] = \frac{1}{m_{i,j}} \sum_{k=1}^{m_{i,j}} x_{i,j,k}\ , \\ &\text{and} \\ &\hat{\mathrm{Var}}[X\backslash Y_E = y_i,\ Y_D = y_j] = \left(\frac{1}{m_{i,j}} \sum_{k=1}^{m_{i,j}} (x_{i,j,k})^2\right) - \left(\frac{1}{m_{i,j}} \sum_{k=1}^{m_{i,j}} x_{i,j,k}\right)^2 . \end{aligned}\right\} \tag{48}$$

The term $m_{i,j}$ represents the total number of bed forms in the sample for which the upstream side intersects the elevation y_i and the downstream side intersects the elevation y_j. In general, the term $m_{i,j}$ will be different for each combination of values of i and j (fig. 3).

Based on the statistic $\{x_{i,j,k}\}$ and assuming that Y_E and Y_D are mutually independent, the probability density functions, $f_{X\backslash Y_D}(x\backslash y)$, $f_X(x)$, and $f_{X,Y_D}(x,y)$ as well as the corresponding moments are estimated by setting $P[E_1] = 1$ in equation sets 37, 38, 39, 40, 42, and 44.

Since the bed form shape and rate of movement are dependent on the flow condition and bed material properties, it should be clear that equations 35 through 48 are also functions of the flow condition and bed material properties. The statistic $\{x_{i,j,k}\}$ will be analyzed later to estimate the various probability distributions of the step lengths for a coarse sand for three different flow conditions.

Summarizing this section, the step length distributions can be estimated by combining the information contained in the $y_x(t)$ and $y_t(x)$ records. Additional assumptions are required however. These are: (1) No deposition occurs on the upstream sides of dunes, and no erosion occurs on the downstream faces of dunes; and (2) the elevation of particle erosion, Y_E, the elevation of particle deposition, Y_D, and the event that a particle passes ν dune crests before it is deposited, E_ν, are mutually independent. The first assumption may not be strictly true especially, due to flow separation, in the neighborhood of dune trough where both deposition and erosion may occur at the same point. For dune flow conditions, however, laboratory observation shows that such an area is small enough that the results should be applicable without an appreciable error. The second assumption seems to be reasonable because as a sediment particle passes a dune crest it likely loses the memory of where it came from. Estimation of $P[E_\nu]$ would not appear to be a simple task. However, for a bed material composed of coarse sand, the assumption that all particles which are eroded from the stoss side of a dune will be deposited on the downstream side of the same dune seems to be reasonable.

Bed Material Transport Equations

The mean transport speed of a bed material particle, V_T, is estimated to be

$$\hat{V}_T = \frac{\text{Total distance traveled by a particle after } m \text{ steps}}{\text{Total time required for a particle to make } m \text{ steps}}$$

$$= \frac{m\hat{E}[X]}{m\hat{E}[T]} = \frac{\hat{E}[X]}{\hat{E}[T]} = \frac{\sum_{j=1}^{n} \hat{E}[X \backslash Y_D = y_j] p_{Y_D}(y_j)}{\sum_{j=1}^{n} \hat{E}[T \backslash Y_D = y_j] p_{Y_D}(y_j)} \quad \text{for a large } m \tag{49}$$

in which $\hat{V}_T$ denotes an estimate of the mean transport speed, V_T,

$$\hat{E}[X] = \sum_{\nu=1}^{\infty} \sum_{j=1}^{n} \sum_{i=1}^{n} \left[\frac{1}{m_{i,j,\nu}} \sum_{k=1}^{m_{i,j,\nu}} x_{i,j,\nu,k} \right] p_{Y_E}(y_i) p_{Y_D}(y_j) P[E_\nu] \ ;$$

and

$$\hat{E}[T] = \sum_{j=1}^{n} \left[\frac{1}{m_{j,j}} \sum_{k=1}^{m_{j,j}} t_{j,k} \right] p_{Y_D}(y_j) \ .$$

In equation 49, the duration of particle movement is assumed to be negligible compared to the rest period. This assumption will be used throughout this section.

The mean transport speed could also be estimated to be

$$\hat{V}_T' = \sum_{j=1}^{n} \frac{\hat{E}[X \backslash Y_D = y_j]}{\hat{E}[T \backslash Y_D = y_j]} \, p_{Y_D}(y_j) \tag{50}$$

where $\hat{V}_T'$ also estimates the mean transport speed of a bed material particle. In general it can be shown that $\hat{V}_T \neq \hat{V}_T'$ and the results of this study will indicate that $\hat{V}_T \leq \hat{V}_T'$. Now the question is: Which one will give the best estimate of the mean bed material transport rate? The difference between the two equations is the manner in which the events are averaged or weighted. So, in order to answer the question--Which is best?--one must depend upon physical arguments and reasoning. In equation 50, the average speed of a particle at each elevation is weighted by the number of particles with this speed. Equation 50 gives the best estimate of the arithmetic mean of individual particle speeds. On the other hand, equation 49 computes the estimate of the mean particle speed as the total distance traveled by a number of particles divided by the amount of time required to transport the same number of particles. In other words, the center of mass of a group of particles is translated through a distance, $\sum_{j=1}^{n} \hat{E}[X \backslash Y_D = y_j] p_{Y_D}(y_j)$ in time, $\sum_{j=1}^{n} \hat{E}[T \backslash Y_D = y_j] p_{Y_D}(y_j)$. Equation 49 will be used in this section because it is based on a mass flux concept and it gives an unbiased estimate of the mean sediment transport rate. The mean particle speed given by equation 50 will be useful in the study of bed material dispersion because it is based on individual particle speeds.

Defining the bedload and suspended load as given in the previous section, the mean transport speed of a bedload particle, V_B, is estimated to be

$$\hat{V}_B = \frac{\hat{E}[X \backslash E_1]}{\hat{E}[T]} , \tag{51}$$

where $\hat{V}_B$ is an estimate of V_B and

$$\hat{E}[X \backslash E_1] = \sum_{j=1}^{n} \sum_{i=1}^{n} \left[\frac{1}{m_{i,j}} \sum_{k=1}^{m_{i,j}} x_{i,j,k} \right] p_{Y_E}(y_i) p_{Y_D}(y_j) \quad .$$

Similarly, the mean transport speed of a suspended load particle, V_S, is estimated to be

$$\hat{V}_S = \frac{\hat{E}\left[X \,\middle|\, \bigcup_{\nu=2}^{\infty} E_\nu \right]}{\hat{E}[T]} = \frac{\sum_{\nu=2}^{\infty} \hat{E}[X \backslash E_\nu] P[E_\nu]}{\hat{E}[T] \sum_{\nu=2}^{\infty} P[E_\nu]} , \tag{52}$$

where

$\hat{V}_S$ = estimate of V_S;

$\bigcup_{\nu=2} E_\nu$ = union of event, E_ν, for $\nu \geq 2$.

$\hat{E}[X \backslash E_\nu]$ is given in equation 40. Note that

$$P\left[\bigcup_{\nu=1}^{\infty} E_\nu\right] = \sum_{\nu=1}^{\infty} P[E_\nu] = 1$$

and

$$\hat{\mathrm{E}}\left[X \,\middle|\, \bigcup_{\nu=2}^{\infty} E_\nu\right] = \frac{\sum_{\nu=2}^{\infty} \hat{\mathrm{E}}[X\backslash E_\nu]P[E_\nu]}{\sum_{\nu=2}^{\infty} P[E_\nu]} \tag{53}$$

Using equations 49, 51, 52, and 53,

$$\hat{V}_T = \frac{\hat{\mathrm{E}}[X\backslash E_1]}{\hat{\mathrm{E}}[T]} P[E_1] + \frac{\sum_{\nu=2}^{\infty} \hat{\mathrm{E}}[X\backslash E_\nu]P[E_\nu]}{\hat{\mathrm{E}}[T]}$$

$$= \hat{V}_B P[E_1] + \hat{V}_S \sum_{\nu=2}^{\infty} P[E_\nu] = \hat{V}_B P[E_1] + \hat{V}_S(1 - P[E_1]) \quad . \tag{54}$$

If all bed material particles have identical transport characteristics, which is reasonable for uniformly sized bed material, the mean total bed material discharge is obtained by use of the continuity concept,

$$\hat{Q}_T = \gamma_s (1 - \theta) W h \hat{V}_T \quad , \tag{55}$$

where

$\hat{Q}_T$ = estimate of the mean total bed material discharge in weight per unit time;

γ_s = specific weight of the bed material;

θ = porosity of the bed;

W = width of the channel;

h = average depth of the zone in which bed material movement occurs; and $\hat{V}_T$ is given in equation 49.

Similarly, estimates of the mean bedload discharge and suspended load discharge are, respectively,

$$\hat{Q}_B = \gamma_s (1 - \theta) W h \hat{V}_B P[E_1] \quad , \tag{56}$$

and

$$\hat{Q}_S = \gamma_s (1 - \theta) W h \hat{V}_S (1 - P[E_1]) \quad , \tag{57}$$

where $\hat{V}_B$ and $\hat{V}_S$ are given in equations 51 and 52, respectively. From equations 54 through 57,

$$\hat{Q}_T = \hat{Q}_B + \hat{Q}_S \quad . \tag{58}$$

Although equations 55, 56, and 57 have the form of a continuity equation, the concept of continuity applies only in a statistical sense, because particles move only when they are exposed on the stoss side of a dune or when they are in suspension. Hubbell and Sayre (1964) proposed that the average depth of the zone of bed material movement, h, be estimated from the $y_t(x)$ record. For this method, the length of the reach for which h is to be determined is divided into sections. Starting from the upstream end, each section of length ℓ_i extends from the dune trough at which the section begins to the first trough downstream that is deeper relative to a line parallel to the plane of the mean bed surface. After sectioning, a mean depth of sand above the projected line for each section, h_i is determined, and the h for the total reach, L_x, is computed as the weighted average of the h_i's for each section. Expressed mathematically,

$$h = \frac{1}{L_x} \sum_{i=1}^{m} \ell_i h_i \quad . \tag{59}$$

The reasoning behind the procedure is based upon the assumption that although the individual dunes may change shape as they progress downstream, a statistical constancy of form exists over a long reach. Hence, quantitatively the particles subject to movement are those that would move if the entire profile were to progress downstream without changing form, and the depth of bed material movement is defined by lines that are parallel to the mean bed surface and extend downstream from the deepest trough.

If all bed material particles are assumed to be transported as the bedload,

$$\hat{Q}_T = \hat{Q}_B = \gamma_s (1 - \theta) W h \hat{V}_B \ , \qquad (60)$$

where $\hat{V}_B$ is determined from equation 51. For coarse sand $P[E_1]$ is expected to be very close to unity because the suspended load is negligible compared to the bedload. For a fine sand for which $P[E_1] \neq 1$, equation 60 would give only an approximation to the total load.

Equations 55, 56, and 57 can be used with measured $y_x(t)$ and $y_t(x)$ records to compute the various transport rates. However, in order to apply the equations to the prediction of the bed material transport rate where the $y_x(t)$ and $y_t(x)$ records are not available, the relations, $\hat{E}[X \backslash Y_E = y_i, Y_D = y_j, E_\nu]$, $\hat{E}[T \backslash Y_D = y_j]$, $P[E_\nu]$, $p_{Y_D}(y_j)$, $p_{Y_E}(y_i)$, and h, to pertinent hydraulic and sediment parameters must be established.

The mean transport speed of bed material particles deposited at elevation y_j, which is denoted by $V_T(j)$, (more precisely, deposited between the elevations η_j and η_{j+1}, centered at y_j) may be estimated

$$\hat{V}_T(j) = \frac{\hat{E}[X \backslash Y_D = y_j]}{\hat{E}[T \backslash Y_D = y_j]} , \tag{61}$$

where $\hat{V}_T(j)$ estimates $V_T(j)$,

$$\hat{E}[X \backslash Y_D = y_j] = \sum_{\nu=1}^{\infty} \sum_{i=1}^{n} \left[\frac{1}{m_{i,j,\nu}} \sum_{k=1}^{m_{i,j,\nu}} x_{i,j,\nu,k} \right] p_{Y_E}(y_i) P[E_\nu] ,$$

and

$$\hat{E}[T \backslash Y_D = y_j] = \frac{1}{m_{j,j}} \sum_{k=1}^{m_{j,j}} t_{j,k} .$$

Based on equation 61, another transport equation can be developed as follows. Let ξ_j denote the percentage of volume between elevations η_j and η_{j+1} occupied by dunes over a given reach; then, ξ_j can be estimated from the $y_t(x)$ record (fig. 4),

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Figure 4 (caption on next page) belongs near here.

$$\xi_j = \frac{1}{L_x} \sum_k \lambda_{j,k} , \tag{62}$$

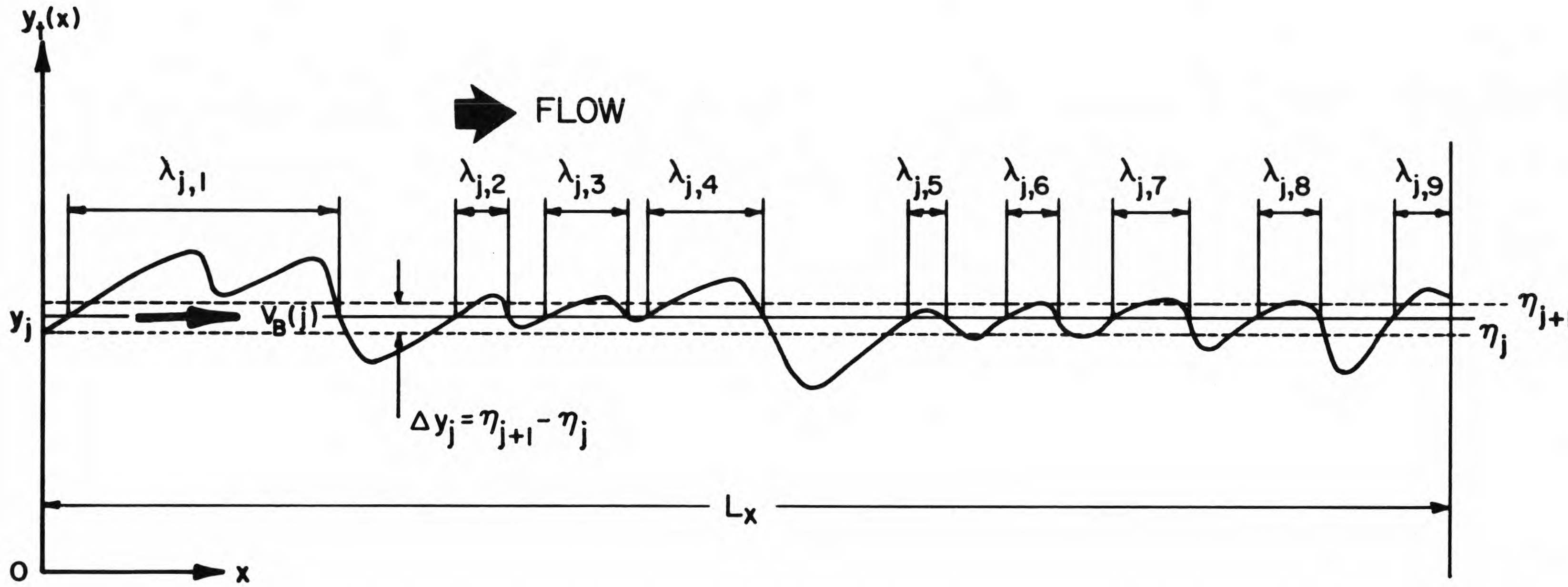

Figure 4. Method for estimating the percentage of volume occupied by dunes between elevations η_j and η_{j+1}; $\xi_j = \frac{1}{L_x}\sum_k \lambda_{j,k}$.

where L_x is the total length of $y_t(x)$ record, and $\lambda_{j,k}$ is defined in figure 4. Applying equations 61 and 62, the mean total bed material discharge can be expressed as

$$Q_T' = \gamma_s (1 - \theta) W \sum_{j=1}^{n} \hat{V}_T(j) \Delta y_j \xi_j = \gamma_s (1 - \theta) W \sum_{j=1}^{n} \frac{\hat{E}[X \backslash Y_D = y_j]}{\hat{E}[T \backslash Y_D = y_j]} \Delta y_j \xi_j \ , \qquad (63)$$

where

$\hat{Q}_T'$ = estimate of mean total bed material discharge; and

Δy_j = nonstandardized class width associated with elevation y_j ($\Delta y_j = \eta_{j+1} - \eta_j$).

Equation 63 takes into account the local variation of the depth of the zone of bed material movement with respect to the elevation of deposition, and demonstrates to what extent each elevation contributes to the total transport rate. Similarly, the mean transport speed of a bedload particle deposited at elevation y_j, which is denoted by $V_B(j)$, can be estimated from equation 61 by considering $P[E_1] = 1$ and $P[E_\nu] \equiv 0$ for $\nu \geq 2$. It can be shown, of course, that equations 54, 56, and 57 also apply at each elevation j as well as to depth-averaged values.

The third method to compute the mean total bed material discharge is based on the following reasoning:

$$Q_T = \begin{bmatrix} \text{Number of particles} \\ \text{deposited per} \\ \text{unit time and area} \end{bmatrix} \times \begin{bmatrix} \text{Weight} \\ \text{per} \\ \text{particle} \end{bmatrix} \times \begin{bmatrix} \text{Mean distance} \\ \text{traveled by} \\ \text{a particle} \end{bmatrix} \times \begin{bmatrix} \text{Width} \\ \text{of} \\ \text{channel} \end{bmatrix} \tag{64}$$

where Q_T is the mean total bed material discharge in weight per unit time. Restricting the attention to elevation y_j, the terms in equation 64 are estimated as follows:

$$\left.\begin{aligned} \begin{bmatrix} \text{Number of particles deposited per} \\ \text{unit time and area at } y_j \end{bmatrix} &= \frac{\Omega_j \sum_{k=1}^{m_j} \Delta y^+_{j,k}}{L_t} \, , \\ \begin{bmatrix} \text{Weight per particle} \\ \text{at } y_j \end{bmatrix} &= \frac{\gamma_s (1 - \theta)}{\Omega_j} \, , \text{ and} \\ \begin{bmatrix} \text{Mean distance traveled by a particle} \\ \text{which is deposited at elevation } y_j \end{bmatrix} &= \hat{E}[X \backslash Y_D = y_j] \, , \end{aligned}\right\} \tag{65}$$

where L_t is the total length of $y_x(t)$ record, and all other symbols have been defined previously. Summing the product of the terms in equation 65 over all elevations,

$$\hat{Q}''_T = \frac{\gamma_s (1 - \theta) W}{L_t} \sum_{j=1}^{n} \left\{ \hat{E}[X \backslash Y_D = y_j] \sum_{k=1}^{m_j} \Delta y^+_{j,k} \right\} \, , \tag{66}$$

where

$\hat{Q}_T''$ = estimate of mean total bed-material discharge; and

W = width of channel.

Equation 66 also illustrates the contribution of each elevation to the total transport, but its primary distinction is that the transport rate is computed from the sounding records with a minimum number of computations.

The relationship between the three transport equations, 55, 63, and 66, will now be demonstrated. First, the comparison of equations 55 and 66 is demonstrated. Combining equations 22, 25, and 66,

$$\hat{Q}_T'' = \frac{\gamma_s (1 - \theta) W}{L_t} \sum_{j=1}^{n} \left\{ \hat{E}[X \backslash Y_D = y_j] p_{Y_D}(y_j) \frac{N_d}{\Omega} \right\} = \frac{\gamma_s (1 - \theta) W N_d}{\Omega L_t} \hat{E}[X] \quad , \tag{67}$$

Multiplying and dividing by the marginal rest period and utilizing equations 20, 21, 25, 30, and 31.

$$\hat{Q}_T'' = \frac{\gamma_s (1 - \theta) W N_d}{\Omega L_t} \frac{\hat{E}[X]}{\hat{E}[T]} \sum_{j=1}^{n} \left\{ \left(\frac{1}{m_{jj}} \sum_{k=1}^{m_{jj}} t_{j,k} \right) \left(\frac{\Omega}{N_d} \sum_{k=1}^{m_j} \Delta \overset{+}{y}_{j,k} \right) \right\} \cdot \cdot \cdot \; , \tag{68}$$

As the value of Δy decreases the value of the last term can be approximated without appreciable error, as

$$\sum_{k=1}^{m_j} \Delta y^+_{j,k} \cong m_j \Delta y_j \quad . \; . \; . \qquad (69)$$

Strictly speaking $m_j \Delta y_j$ is equal to or slightly greater than the term $\sum_{k=1}^{m_j} \Delta y^+_{j,k}$ (fig. 1). Assuming a long record with small vertical class intervals such that equation 69 is valid and such that $m_j = m_{jj'}$, equation 68 reduces to

$$\hat{Q}''_T = \gamma_s (1 - \theta) W \frac{\hat{E}[X]}{\hat{E}[T]} \sum_{j=1}^{n} \frac{\Delta y_j}{L_t} \sum_{k=1}^{m_j} t_{j,k} \quad . \; . \; . \; . \; , \qquad (70)$$

which would be equivalent to equation 55 provided that the average depth of the zone in which bed-material movement occurs, h' is defined by

$$h' = \frac{1}{L_t} \sum_{j=1}^{n} \left(\Delta y_j \sum_{k=1}^{m_j} t_{j,k} \right) \; . \; . \; . \qquad (71)$$

Equation 71, is similar to equation 59 except that it is based on the time record of depth, $y_x(t)$, while equation 59 is based on the longitudinal profile, $y_t(x)$.

To investigate the relation between equations 63 and 66, we proceed as follows. The last term of equation 66 can be approximated without an appreciable error, as $m_j \Delta y_j$ using equation 69. Replacing the last term by its approximate value, and multiplying equation 66 by the right hand side of equation 30 while dividing by the left hand side,

$$\hat{Q}_T'' = \gamma_s (1 - \theta) W \sum_{j=1}^{n} \left\{ \frac{\hat{E}[X \backslash Y_D = y_j]}{\hat{E}[T \backslash Y_D = y_j]} \frac{m_j}{m_{jj}} \frac{\Delta y_j}{L_t} \right\} \sum_{k=1}^{m_{jj}} t_{j,k} \quad . \; . \; . \qquad (72)$$

The number of bed forms contained in the $y_x(t)$ record and which also contains some deposition in the class interval, m_j, should be almost equal to the total number of bed forms with both an upcrossing and a downcrossing in the interval, m_{jj}. Assuming $m_j = m_{jj}$, equation 72 is identical to equation 63 except that the percentage of the volume in the class interval occupied by dunes is computed from the $y_x(t)$ record instead of from equation 62 which is based on the $y_t(x)$ record.

In summary, three transport equations have been presented, equations 55, 63, and 66. Although the equations appear quite different in form, they are all based on similar assumptions. As the record length becomes long and the class intervals reduce to zero in the limit, the three equations would become identical provided that either the $y_x(t)$ record or the $y_t(x)$ record could be used to determine the active depth (This should be true for equilibrium flow.). In the following section, the total load will be computed for three flow conditions using all three equations, and the results will be compared.

General Two-Dimensional Stochastic Model for Dispersion of Bed-Material Sediment Particles

Let us define the following stochastic processes:

$\tilde{X}(t) = \sum_{i=1}^{N(t)} X_i$ = longitudinal position of a bed-material sediment particle at time t in which $\tilde{X}(0) = X_0 = 0$.

$N(t)$ = counting process describing number of steps taken by a bed-material sediment particle in time t.

X_i = length of ith step of a bed-material sediment particle.

$X(n) = \sum_{i=0}^{n} X_i$ = longitudinal position of a bed-material sediment particle after n steps.

$\tilde{Y}(t)$ = vertical position of a bed-material sediment particle at time t.

$Y_D(n)$ = elevation at which a bed-material sediment particle is deposited after n steps.

The probability that the particle has, at time t, traveled a distance equal to or less than x and that it is located at an elevation equal to or less than y may now be expressed as the joint distribution function

$$F(x,y;t) = P[\tilde{X}(t) \leq x, \tilde{Y}(t) \leq y] = \sum_{n=0}^{\infty} P[\tilde{X}(t) \leq x,\ \tilde{Y}(t) \leq y,\ N(t) = n] \ . \qquad (73)$$

Using the definition of conditional probability and assuming that the duration of the particle movement is negligible, equation 73 can be restated as

$$F(x,y;t) = \sum_{n=0}^{\infty} P[X(n) \leq x,\ Y_D(n) \leq y,\ N(t) = n]$$

$$= \sum_{n=0}^{\infty} P[X(n) \leq x,\ N(t) = n \backslash Y_D(n) \leq y] P[Y_D(n) \leq y]$$

$$= \int_{y_{min}}^{y} \sum_{n=0}^{\infty} P[X(n) \leq x,\ N(t) = n \backslash Y_D(n) = y'] f_{Y_D(n)}(y')\,dy' \qquad (74)$$

where

y_{min} = lowest elevation of deposition; and

$f_{Y_D(n)}(y)$ = probability density function of $Y_D(n)$.

The event, $\{N(t) = n\}$, can be expressed in terms of the rest period of a bed-material sediment particle

$$\{N(t) = n\} = \{T(n) \leq t\} \cap \{T(n+1) > t\} \qquad (75)$$

where

$\{\cdot\}$ = events;

$\cap$ = intersection of events;

$T(n) = \sum_{i=1}^{n} T_i$; and

T_i = random variable describing the duration of ith rest period of a bed-material sediment particle.

By virtue of equation 75, it follows that

$$P[N(t) = n] = P[T(n) \leq t,\ T(n+1) > t] = P[T(n) \leq t,\ T_{n+1} > t - T(n)]\ . \qquad (76)$$

For further simplification of equation 74, the following assumptions are made:

A. $X(n)$ and $N(t)$ are mutually independent for every n.

B. X_i for $i \geq 1$ are independently and identically distributed according to the probability density function $f_X(x)$, where $0 \leq x < \infty$. Outside this range, $f_X(x) = 0$.

C. X_i is independent of $Y_D(j)$ for $i \neq j$.

D. $Y_D(i)$ for $i \geq 1$ are independently and identically distributed according to the probability density function $f_{Y_D}(y)$, where $y_{\min} \leq y \leq y_{\max}$. Outside this range, $f_{Y_D}(y) = 0$.

E. T_i for $i \geq 1$ are independently and identically distributed according to the probability density function $f_T(t)$, where $0 \leq t < \infty$. Outside this range, $f_T(t) = 0$.

F. T_i is independent of $Y_D(j - 1)$ for $i \neq j$.

In other words, assumption A states that the total distance $X(n)$ traveled by a sediment particle after n steps should not depend on which time interval within $[0, t]$ that these n steps occurred. The step lengths are always positive so that the particle always moves in downstream direction (part of assumption B). Each step length depends on the elevation at which the particle is deposited at the end of that step (assumption C). The elevation at which the particle is deposited at the end of any step does not depend on the elevation at which it was deposited at the end of any previous step (assumption D). Finally, the duration of each rest period depends on the elevation at which the particle was deposited at the end of the previous step (assumption F).

Utilizing assumption A, equation 74 becomes

$$F(x,y;t) = \int_{y_{\min}}^{y} \sum_{n=0}^{\infty} \{P[X(n) \le x \backslash Y_D(n) = y']$$

$$\cdot\, P[N(t) = n \backslash Y_D(n) = y'] f_{Y_D(n)}(y')\} dy' = \int_{y_{\min}}^{y} P[X(0) \le x \backslash Y_D(0) = y']$$

$$\cdot\, P[N(t) = 0 \backslash Y_D(0) = y'] f_{Y_D(0)}(y') dy' + \int_{y_{\min}}^{y} \sum_{n=1}^{\infty} \{P[X(n) \le x \backslash Y_D(n) = y']$$

$$\cdot\, P[N(t) = n \backslash Y_D(n) = y'] f_{Y_D(n)}(y')\} dy' \; . \qquad (77)$$

Under assumptions B, C, and D, and using the concepts of joint and conditional probability,

$$P[X(n) \le x \backslash Y_D(n) = y'] = P[X(n-1) + X_n \le x \backslash Y_D(n) = y']$$

$$= \int_0^x f_{X(n-1) + X_n \backslash Y_D(n)}(x' \backslash y') dx'$$

$$= \int_0^x dx' \int_0^{x'} f_{X(n-1), X_n \backslash Y_D(n)}(\zeta, x' - \zeta \backslash y') d\zeta$$

and using assumptions C and D,

$$P[X(n) \le x \backslash Y_D(n) = y'] = \int_0^x dx' \int_0^{x'} f_{X(n-1)}(\zeta) f_{X \backslash Y_D}(x' - \zeta \backslash y') d\zeta$$

$$= \int_0^x dx' \int_0^{x'} f_X^{(n-1)}(\zeta)\; f_{X \backslash Y_D}(x' - \zeta \backslash y') d\zeta \qquad (78)$$

in which

$$X(n-1) = \sum_{i=0}^{n-1} X_i \ , \ X_0 = 0 \ ,$$

$$f_{X(n-1)}(\zeta) = f_X^{(n-1)}(\zeta) = \int_0^{\zeta} f_X^{(n-2)}(\theta) \ f_X(\zeta - \theta)\, d\theta \ ; \quad n = 3, 4, 5, \ldots$$

and (79)

$$f_X^{(n)}(\zeta) = f_X(x) \ ; \quad n = 2$$

In equations 78 and 79, $f_{X(n-1),\, X_n \backslash Y_D(n)}(\zeta, x' - \zeta \backslash y')$ denotes the joint probability density function of $X(n-1)$ and X_n, conditioned on $Y_D(n)$, $f_X^{(n-1)}(\zeta)$ is the $(n-1)$-fold convolution of the probability density function for the length of a single step, and it is equal to the probability density function for the distance traveled by the particle in $(n-1)$ steps, and $f_{X \backslash Y_D}(x \backslash y)$ is the conditional probability density function for a single step length given that the particle is deposited at elevation y.

Turning now to the other part of equation 77 and using equation 76 and assumptions D, E, and F,

$$P[N(t) = n \backslash Y_D(n) = y'] = P[T(n) \leq t,\ T_{n+1} > t - T(n) \backslash Y_D(n) = y']$$

$$= \int_0^t \int_{t-t'}^{\infty} f_{T(n),\, T_{n+1} \backslash Y_D(n)} (t', \tau \backslash y')\, d\tau dt'$$

$$= \int_0^t \int_{t-t'}^{\infty} f_{T(n)}(t') f_{T_{n+1} \backslash Y_D(n)} (\tau \backslash y')\, d\tau dt'$$

$$= \int_0^t f_T^{(n)}(t')\, dt' \int_{t-t'}^{\infty} f_{T \backslash Y_D}(\tau \backslash y')\, d\tau \ , \qquad (80)$$

in which

$$T(n) = \sum_{i=1}^{n} T_i \ ,$$

$$\left.\begin{aligned} f_{T(n)}(t') = f_T^{(n)}(t') = \int_0^{t'} f_T^{(n-1)}(\theta)\ f_T(t' - \theta)\, d\theta \ ; \quad n = 2, 3, 4, \ldots \ , \\ \text{and} \\ f_T^{(n)}(t') = f_T(t) \ ; \quad n = 1 \ . \end{aligned}\right\} \qquad (81)$$

In the above, T_{n+1} is the random variable describing the duration of $(n+1)$st rest period of a particle, $f_{T(n),T_{n+1} \backslash Y_D(n)}(t', \tau \backslash y')$ is the joint probability density function of $T(n)$ and T_{n+1}, conditioned on

$Y_D(n)$, $f_{T\backslash Y_D}(t\backslash y)$ is the conditional probability density function for the duration of a rest period given that the particle was deposited at elevation y, and $f_T^{(n)}(t)$ is the n-fold convolution of the probability density function, $f_T(t)$, for the duration of a single rest period and is equal to the probability density function for the duration of n successive rest periods.

Similarly, the terms for $n = 0$ in equation 77 become:

$$P[X(0) \leq x \backslash Y_D(0) = y'] = 1 \tag{82}$$

because $X(0) = X_0 = 0$ and $0 \leq x < \infty$, and

$$P[N(t) = 0 \backslash Y_D(0) = y'] = P[T_1 > t \backslash Y_D(0) = y'] = \int_t^\infty f_{T\backslash Y_D}(t' \backslash y')\, dt' \quad , \tag{83}$$

where T_1 is the random variable describing the duration of the first rest period in time t. It is important to note that the initial condition, $X(0) = X_0 = 0$, implies that the particle starts its first rest period at $t = 0$.

Introducing equations 78, 80, 82, and 83 into equation 77,

$$F(x,y;t) = \int_{y_{min}}^{y} f_{Y_D}(y')\,dy' \int_{t}^{\infty} f_{T\backslash Y_D}(t'\backslash y')\,dt'$$

$$+ \int_{y_{min}}^{y} f_{Y_D}(y')\,dy' \sum_{n=1}^{\infty} \left(\int_{0}^{x} dx' \int_{0}^{x'} f_X^{(n-1)}(\zeta)\; f_{X\backslash Y_D}(x' - \zeta\backslash y')\,d\zeta \right.$$

$$\left. \cdot \int_{0}^{t} f_T^{(n)}(t')\,dt' \int_{t-t'}^{\infty} f_{T\backslash Y_D}(\tau\backslash y')\,d\tau \right) . \qquad (84)$$

The first term of equation 84 represents the joint probability that the particle has not moved from its initial position and that its initial elevation is equal to or less than y, and it is not a function of x. Hence,

$$\frac{\partial^2}{\partial x \partial y} [F(x,y;t)]\Big|_{n=0} = \frac{\partial^2}{\partial x \partial y} \int_{y_{min}}^{y} f_{Y_D}(y')\,dy' \int_{t}^{\infty} f_{T\backslash Y_D}(t'\backslash y')\,dt' \equiv 0 .$$

The corresponding density function is therefore

$$f(x,y;t) = \frac{\partial^2}{\partial x \partial y} F(x,y;t) = f_{Y_D}(y) \sum_{n=1}^{\infty} \left(\int_{0}^{x} f_X^{(n-1)}(\zeta)\; f_{X\backslash Y_D}(x - \zeta\backslash y)\,d\zeta \right.$$

$$\left. \cdot \int_{0}^{t} f_T^{(n)}(t')\,dt' \int_{t-t'}^{\infty} f_{T\backslash Y_D}(\tau\backslash y)\,d\tau \right) . \qquad (85)$$

If a large number of identical particles are initially at rest at $x = 0$, $y = y_0$, equation 85 expresses the longitudinal and vertical distribution at time t of the particles which have moved from their respective initial positions. It should be noted here that $f(x, y; t)$ is not a true probability density function because

$$\int_0^\infty dx \int_{y_{\min}}^{y_{\max}} f(x,y;t)\,dy = 1 - P[N(t) = 0] < 1 \quad . \tag{86}$$

That is to say, equation 85 applies only after the particle has moved from its initial position. The expression $f(x, y; t)$ does not exist for $x = 0$.

If we assume that X_i is independent of $Y_D(j)$ for all i and j and drop assumption C, equation 85 reduces to equation 8,

$$f(x, y; t) = f_{Y_D}(y) \sum_{n=1}^{\infty} f_X^{(n)}(x) \int_0^t f_T^{(n)}(t')\,dt' \int_{t-t'}^{\infty} f_{T \backslash Y_D}(\tau \backslash y)\,d\tau \,, \tag{8}$$

which is the Sayre-Conover (1967) two-dimensional stochastic model. The difference between equations 85 and 8 is that equation 85 takes some of the dependence between X and Y_D into account whereas equation 8 is based on the independence of X and Y_D. Hence, the Sayre-Conover model is a special case of equation 85.

The marginal case of equation 85 gives the longitudinal distribution at time t of the particles which have moved from their initial positions. Integrating equation 85 over y,

$$f(x;t) = \int_{y_{min}}^{y_{max}} f(x,y;t)\,dy = \sum_{n=1}^{\infty} \int_0^x f_X^{(n-1)}(\zeta)\,d\zeta \int_0^t f_T^{(n)}(t')\,dt'$$

$$\cdot \int_{t-t'}^{\infty} d\tau \left[\int_{y_{min}}^{y_{max}} f_{X\backslash Y_D}(x-\zeta\backslash y') f_{T\backslash Y_D}(\tau\backslash y') f_{Y_D}(y')\,dy' \right] . \qquad (87)$$

By virtue of assumption A,

$$\int_{y_{min}}^{y_{max}} f_{X\backslash Y_D}(x-\zeta\backslash y') f_{T\backslash Y_D}(\tau\backslash y') f_{Y_D}(y')\,dy'$$

$$= \int_{y_{min}}^{y_{max}} f_{X,T\backslash Y_D}(x-\zeta,\tau\backslash y') f_{Y_D}(y')\,dy' = f_{X,T}(x-\zeta,\tau) = f_X(x-\zeta) f_T(\tau) \quad . \qquad (88)$$

Substituting equation 88 into equation 87 and rearranging terms,

$$f(x;t) = \sum_{n=1}^{\infty} \int_0^x f_X^{(n-1)}(\zeta)\, f_X(x-\zeta)\,d\zeta \int_0^t f_T^{(n)}(t')\,dt' \int_{t-t'}^{\infty} f_T(\tau)\,d\tau \quad .$$

Because

$$\int_0^x f_X^{(n-1)}(\zeta)\, f_X(x-\zeta)\,d\zeta = f_X^{(n)}(x) \; ,$$

and from equation 76,

$$P[N(t) = n] = \int_0^t f_T^{(n)}(t')\,dt' \int_{t-t'}^{\infty} f_T(\tau)\,d\tau \quad . \tag{89}$$

Meanwhile, equation 75 can be restated as

$$\{N(t) = n\} = \{T(n) \le t\} \bigcap \{T(n+1) > t\} = \{T(n) \le t\} - \{T(n+1) > t\}^c$$
$$= \{T(n) \le t\} - \{T(n+1) \le t\}$$

where $\{T(n+1) > t\}^c$ denotes the complement of the event $\{T(n+1) > t\}$. Because $\{T(n+1) \le t\}$ is a subevent of $\{T(n) \le t\}$, it follows that

$$P[N(t) = n] = P[T(n) \le t] - P[T(n+1) \le t] = \int_0^t f_T^{(n)}(t')\,dt' - \int_0^t f_T^{(n+1)}(t')\,dt' \quad . \tag{90}$$

From equations 89 and 90, we have the marginal probability density function,

$$f(x;t) = \sum_{n=1}^{\infty} f_X^{(n)}(x)\,P[N(t) = n] = \sum_{n=1}^{\infty} f_X^{(n)}(x) \int_0^t \left[f_T^{(n)}(t') - f_T^{(n+1)}(t') \right] dt' \quad . \tag{91}$$

Equation 91 is identical to the Sayre-Conover (1967) one-dimensional stochastic model which is given in equation 10. As with equation 85, here also $f(x;t)$ is not a true probability density function because

$$\int_0^{\infty} f(x;t)\,dx = 1 - P[N(t) = 0] < 1 \quad ,$$

where $P[N(t) = 0]$ is the probability that the particle has not moved from its initial position.

In order to apply equations 84 or 85, the probability density functions $f_{Y_D}(y)$, $f_{T \backslash Y_D}(t \backslash y)$, and $f_{X \backslash Y_D}(x \backslash y)$ must be known. These density functions are estimated from equations 25, 28, and 39. The probability density functions $f_T(t)$ and $f_X(x)$ are determined by the relations

$$f_T(t) = \int_{y_{\min}}^{y_{\max}} f_{T \backslash Y_D}(t \backslash y) f_{Y_D}(y)\, dy \quad , \tag{92}$$

and

$$f_X(x) = \int_{y_{\min}}^{y_{\max}} f_{X \backslash Y_D}(x \backslash y) f_{Y_D}(y)\, dy \quad , \tag{93}$$

where $y_{\min}$ and $y_{\max}$ are estimated from the $y_x(t)$ record. Equations 92 and 93 are the continuous forms corresponding to equations 29 and 40 (or 41), respectively. With $f_T(t)$ and $f_X(x)$ determined, the corresponding convolutions, $\overset{(n)}{f_T}(t)$ and $\overset{(n-1)}{f_X}(x)$, are determined from equations 81 and 79, respectively.

ANALYSIS AND DISCUSSION OF RESULTS

Experiment and Basic Data

Three dune runs were made in a tilting recirculating flume of rectangular cross section, 61 m (200 ft) long, 2.4 m (8 ft) wide, and 1.2 m (4 ft) deep. The flume has been described in detail by Williams (1971).

The bed material used in these experiments was a screened river sand (Cherry Creek sand), with a median sieve diameter, $d_{50} = 1.13$ mm, and a geometric standard deviation, $\sigma_g = 1.51$. The size distribution, shown in figure 5, was obtained by a sieve analysis of 3,000 grams of

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Figure 5 (caption on next page) belongs near here.

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bed material.

After an equilibrium flow, as defined by Simons and Richardson (1966), was established, the $y_x(t)$ and $y_t(x)$ records, the total bed-material discharge, and the hydraulic properties of interest were measured. The methods and procedures of the measurements have been described in detail by Lee (1969). The summary of measured and derived data are given in table 1. The values of the water discharge, depth, energy slope,

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Table 1 (p. 81of ms.) near here.

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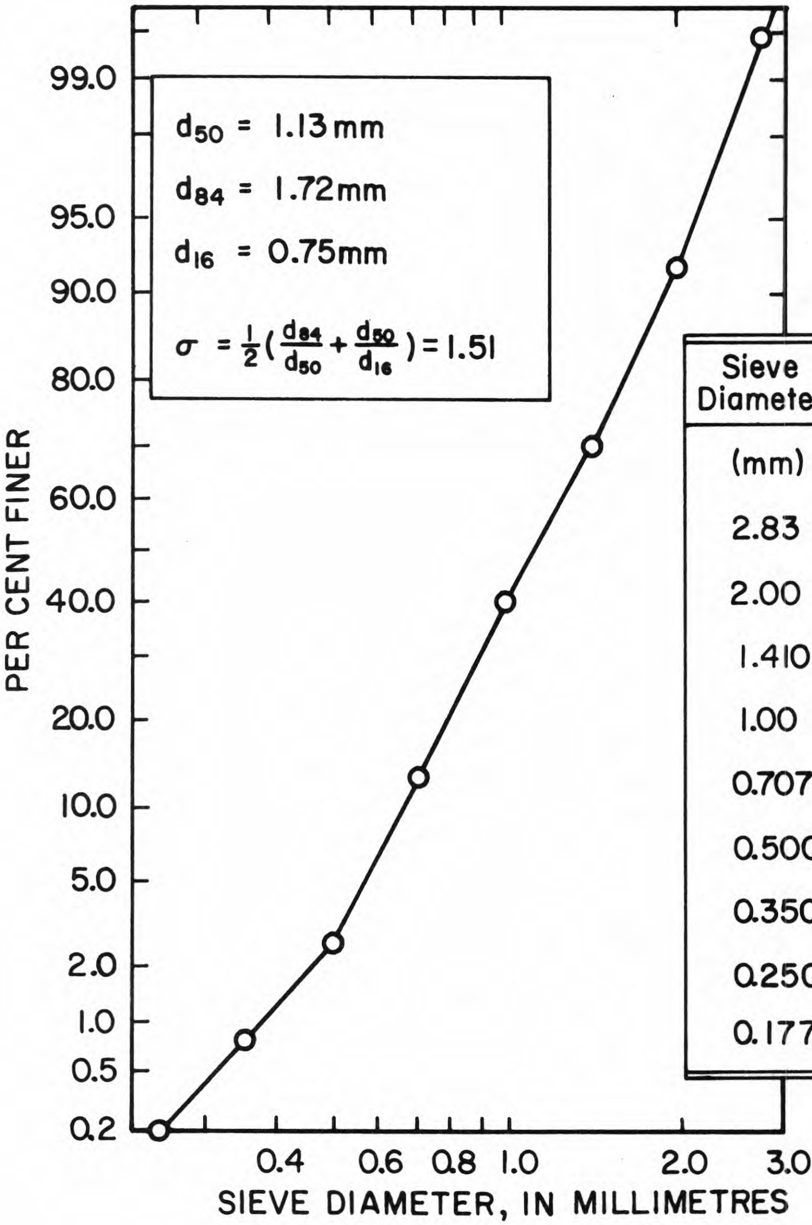
bed shape, and total bed-material concentration presented in table 1 are the average of several individual measurements. The sampled load was measured by a DH-48 sampler. The number of measurements was the same as the number of $y_t(x)$ charts.



Sieve Diameter	Percent finer than	Mean fall velocity
(mm)	—	(cm/sec)
2.83	99.35	22.60
2.00	91.60	19.00
1.410	69.12	18.10
1.00	39.86	14.60
0.707	12.87	11.10
0.500	2.63	8.45
0.350	0.78	—
0.250	0.20	—
0.177	0.05	—

Figure 5. Size distribution curve of bed material.

Table 1. *Basic data and computed parameters*

Run	Water Discharge Q_W, m^3/s		Flow Depth d, cm		Mean Flow Velocity $\overline{U}$, cm/s	Energy Slope S_e		Water Temperature T, °C	
	Mean	Standard Deviation	Mean	Standard Deviation		Mean	Standard Deviation	Mean	Standard Deviation
4A	0.464	.003	31.1	0.6	61.3	0.00167	0.00009	20.0	0.3
16	1.24	.006	90.8	0.9	55.8	0.00029	0.00021	22.8	0.2
17	1.53	.008	89.3	0.6	70.4	0.00047	0.00005	22.0	0.2

Run	Total Bed-Material Discharge			Sampled Load Concentration mg/ℓ		Mean Chezy Resistance Coefficient $C/\sqrt{g}$	Mean Bed Shear Stress $\overline{\tau}_b$, d/cm^2	Mean Shear Velocity $\overline{U}_*$, cm/s	Stream Power $\overline{\tau}_b\,\overline{U}$, d/cm-s
	Concentration C_T, mg/ℓ		Mean Total Load q_T, t/day-m						
	Mean	Standard Deviation		Mean	Standard Deviation				
4A	168.6	53.6	2.77	1.5	6.1	8.6	50.8	7.13	3110
16	8.9	3.1	0.39	0.0	0.0	12.1	25.9	4.91	1450
17	29.7	10.8	1.61	6.3	10.2	11.0	41.2	6.43	2900

Run	Mean Froude Number F_r	$y_x(t)$ Record		$y_t(x)$ Record			
		Length of Record L_t, hours	Lag Interval min	Length of Record L_x, m	Number of Charts	Time Interval of Measurements hours	Range of Lag Interval cm
4A	0.35	312	2.4	1235	31	6	3.7 - 13.0
16	0.19	80	1.2	1006	33	1	8.6 - 13.6
17	0.24	109	0.6	983	30	1	7.2 - 11.8

The $y_t(x)$ charts were obtained by mounting a sonic depth sounder on the instrument carriage and traversing it along the centerline of the flume in the upstream direction. The sonic depth sounder has been described by Karaki, Gray, and Collins (1961). Although the duration of traverse was approximately 5 minutes, the $y_t(x)$ record was considered to be instantaneous. The $y_x(t)$ record was obtained by locating a sonic depth sounder at the centerline of the flume 42.1 m (138 ft) downstream of the headbox. Both the $y_t(x)$ and $y_x(t)$ records were digitized with an analog-to-digital converter at the lag intervals shown in table 1. The lag interval on the $y_t(x)$ charts were not constant because the speed of the carriage was somewhat different for each chart. The output of the converter was to computer cards such that all statistics could be processed on the digital computer.

Probability Distributions of the Elevation of Deposition and Erosion

The sample probability mass functions for the elevation of deposition and erosion were computed using equations 25 and 26 and the $y_x(t)$ records. The results of calculations for the three flume runs are presented in table 2.

Table 2 (p. 134 of ms.) in supplemental data.

The $y_x(t)$ record of each run was standardized so that the class mark, y_i, measures the elevation of depositon or erosion in terms of the standard deviation about the mean bed elevation. The class width of 0.4 was used for all class marks. The frequency histograms for the elevation of deposition and erosion are plotted in figure 6.

Figure 6 (caption on next page) belongs near here.

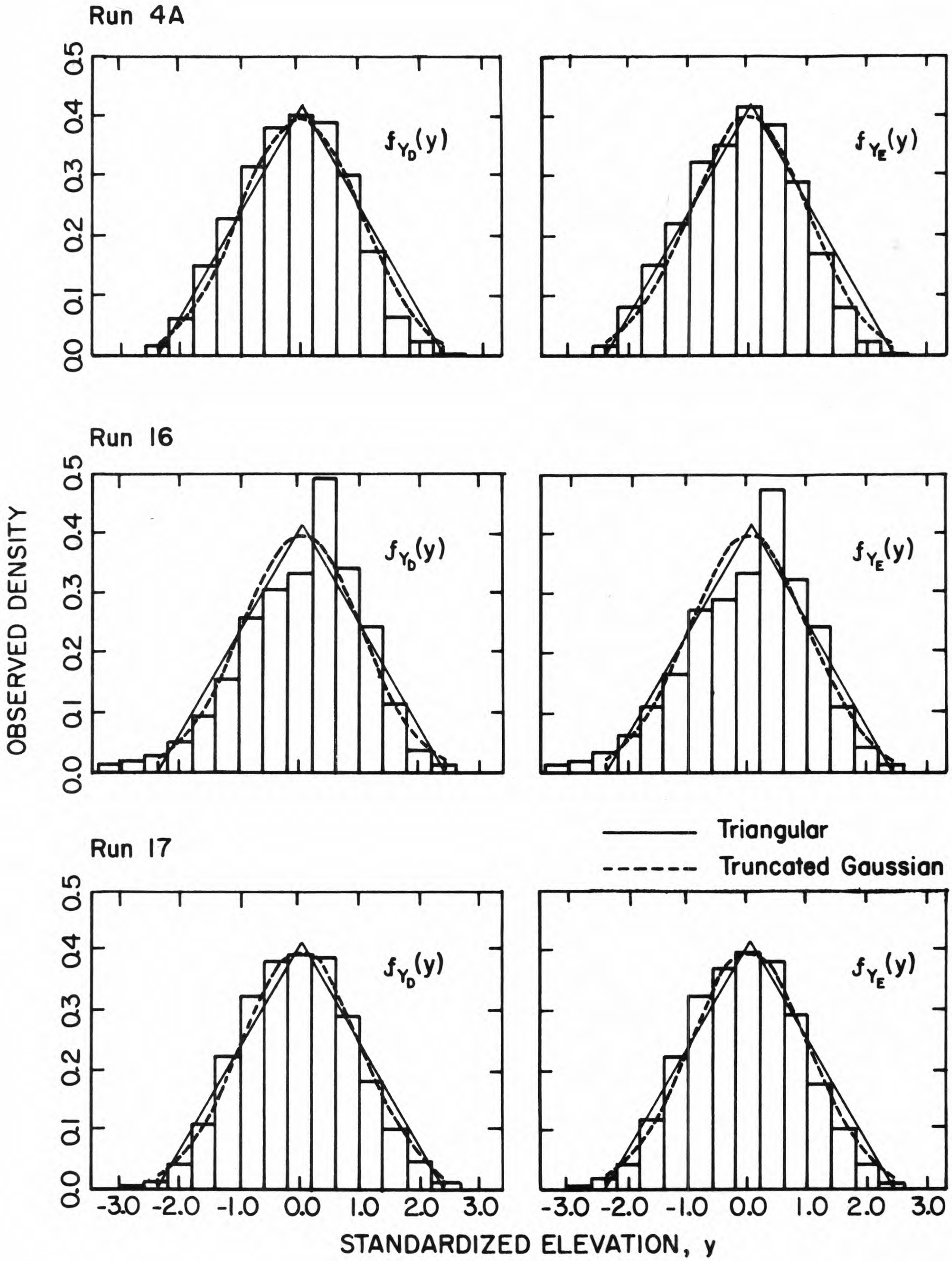


Figure 6. Frequency histograms, triangular density function, and truncated Gaussian density function for the elevation of deposition and erosion.

The truncated Gaussian probability density function, defined by

$$
\left.
\begin{aligned}
& f_{Y_D}(y) = f_{Y_E}(y) = \frac{\dfrac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}}{\dfrac{1}{\sqrt{2\pi}} \displaystyle\int_{-2.4}^{2.4} e^{-\frac{1}{2}y^2} dy} \cong 1.017 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2} \\
& \text{for } -2.4 \leq y \leq 2.4 \\
& f_{Y_D}(y) = f_{Y_E}(y) = 0 \text{ otherwise} ,
\end{aligned}
\right\} \qquad (94)
$$

appears to fit the data reasonably well. A symmetric triangular density function defined by

$$
\left.
\begin{aligned}
& f_{Y_D}(y) = f_{Y_E}(y) = -\frac{1}{2.4^2} y + \frac{1}{2.4} \text{ for } 0 \leq y \leq 2.4 \\
& f_{Y_D}(y) = f_{Y_E}(y) = \frac{1}{2.4^2} y + \frac{1}{2.4} \text{ for } -2.4 \leq y \leq 0 \\
& f_{Y_D}(y) = f_{Y_E}(y) = 0 \text{ otherwise} ,
\end{aligned}
\right\} \qquad (95)
$$

also appears to fit the data reasonably well. Equations 94 and 95 are both plotted in figure 6. In equations 94 and 95, $f_{Y_D}(y)$ and $f_{Y_E}(y)$ are the probability density functions of the elevation of deposition and erosion, respectively, and y is the standardized elevation.

Both distributions assume nonzero values only for $-2.4 \leq y \leq 2.4$ and the two models postulate that Y_D and Y_E are identically distributed. The truncation limits on these distributions are rather arbitrary.

The mean and variance for the truncated Gaussian density are

$$\left.\begin{aligned} & E[Y_D] = E[Y_E] = 0 \ , \\ \text{and} & \\ & \mathrm{Var}[Y_D] = \mathrm{Var}[Y_E] = E[Y_D^2] = E[Y_E^2] \doteq 1.017 \int_{-2.4}^{2.4} y^2 g(y)\, dy = 0.891 \end{aligned}\right\} \tag{96}$$

where $g(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}y^2}$. For the triangular density

$$\left.\begin{aligned} & E[Y_D] = E[Y_E] = 0 \\ \text{and} & \\ & \mathrm{Var}[Y_D] = \mathrm{Var}[Y_E] = E[Y_D^2] = E[Y_E^2] = \frac{2.4^2}{6} = 0.960 \end{aligned}\right\} . \tag{97}$$

The variances of these distributions are quite sensitive to the assumed truncation limits.

A goodness of fit test using the chi-square statistic indicated that neither model would be rejected for runs 16 and 17 at a significance level of 0.05. For run 4A, however, both models were rejected at the same level of significance. As seen in figure 6, the truncated Gaussian density appears to give a slightly better approximation to $f_{Y_D}(y)$ and $f_{Y_E}(y)$; but, the triangular density is much easier to handle in analytical treatments. The variance of the triangular distribution is even more sensitive to the assumed limits than is the variance of the truncated Gaussian distribution. Therefore, the triangular distribution probably should not be used in predicting the variance.

For stationary processes, continuity requires that the probability of erosion equal the probability of deposition for all elevations. Therefore, the density functions for the elevations of deposition and erosion must be identical. The mean and variance of sample histograms as well as the total number of points available for analysis, Σm_i, are shown in table 2. Little data were available for run 16, only 134 crossings compared to 2,167 for run 4A and 708 for run 17. Although run 16 was continued for 33 hours, the very low transport rate (table 1) and slow movement of the bed forms limited the number of crossings available for analysis. It should also be pointed out that equilibrium flow was never attained for this flow which was barely above the initiation of motion stage.

Rest Period Distributions

The sample conditional probability mass function of the rest periods were computed by determining the difference between the time of re-exposure and movement and the time of burial of the center of each class mark for each crossing event, $m_{j,j}$, that occurred in the $y_x(t)$ record (fig. 2). The results of the measurements are presented in tables 3 through 5, and

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Tables 3-5 (p. 135-137 of ms.) in supplemental data.

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examples of the mass functions are presented in figures 7, 8, and 9.

Figures 7,8,9 (captions on next page) belong near here.

The standardized $y_x(t)$ record was used and the class width of the elevation was taken to be the same as that used in determining the probability distribution for the elevation of deposition, 0.4.

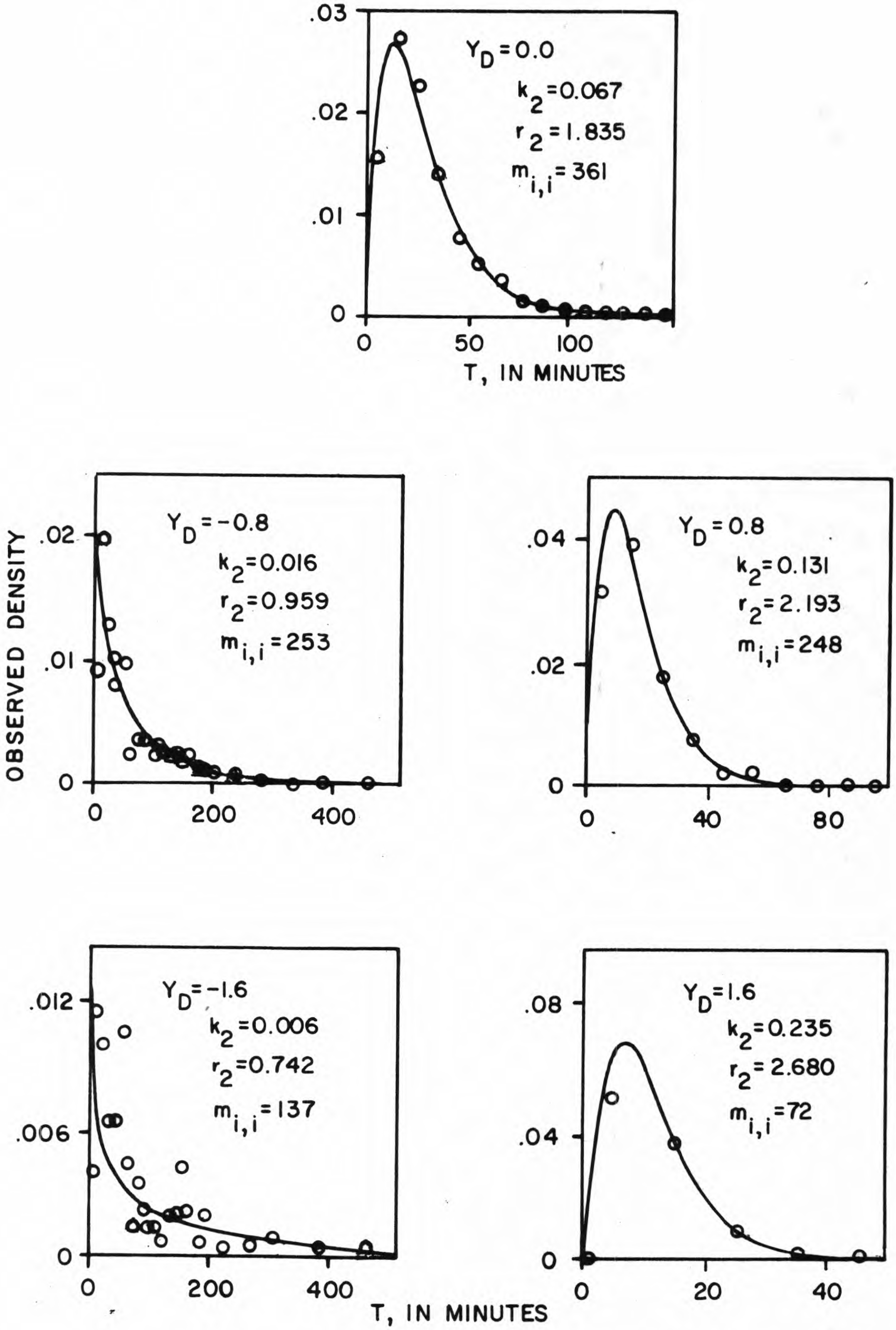

Figure 7. Sample probability mass functions of the conditional rest periods with fitted two-parameter Gamma functions (run 4A).

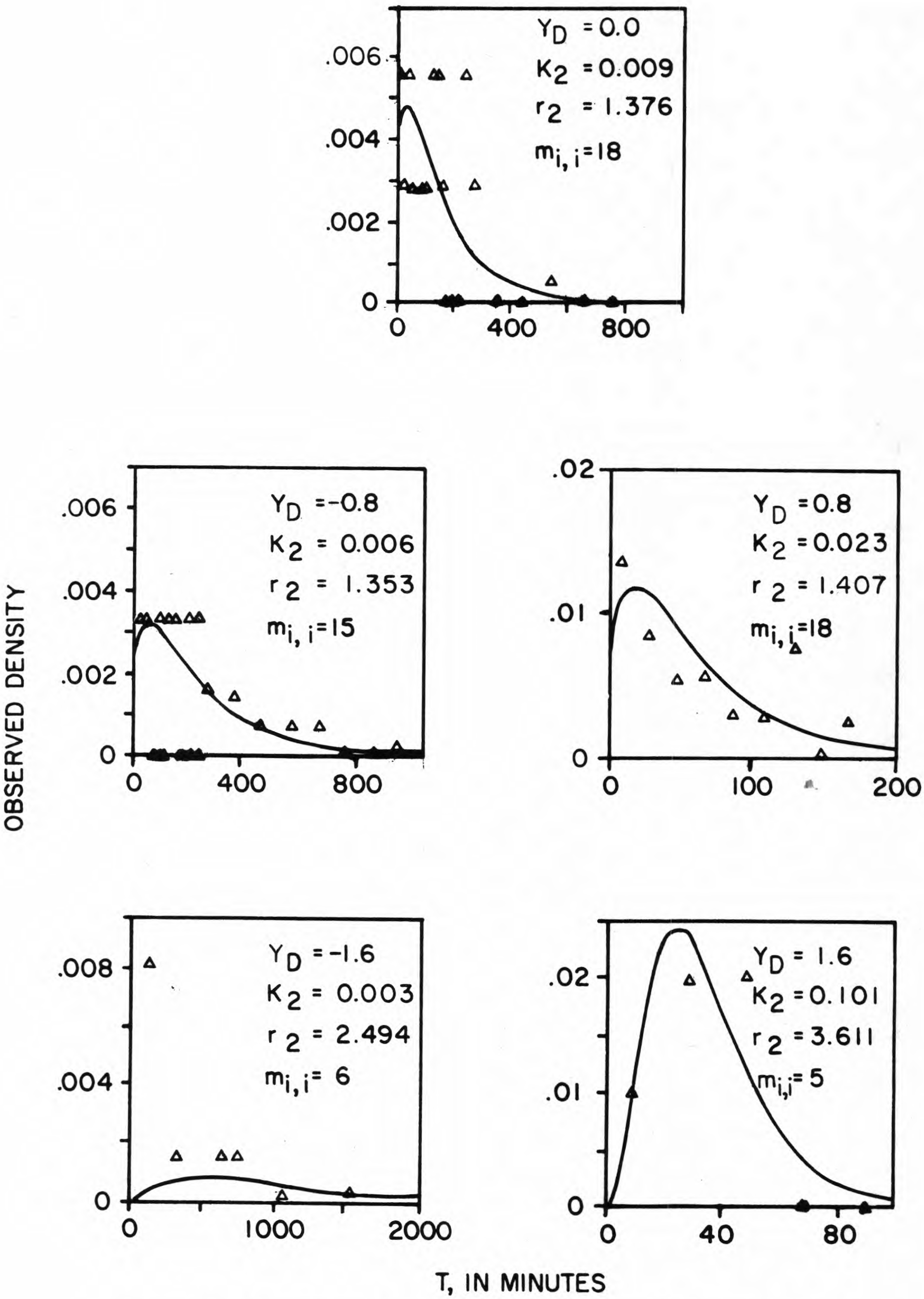

Figure 8. Sample probability mass functions of the conditional rest periods with fitted two-parameter Gamma functions (run 16).

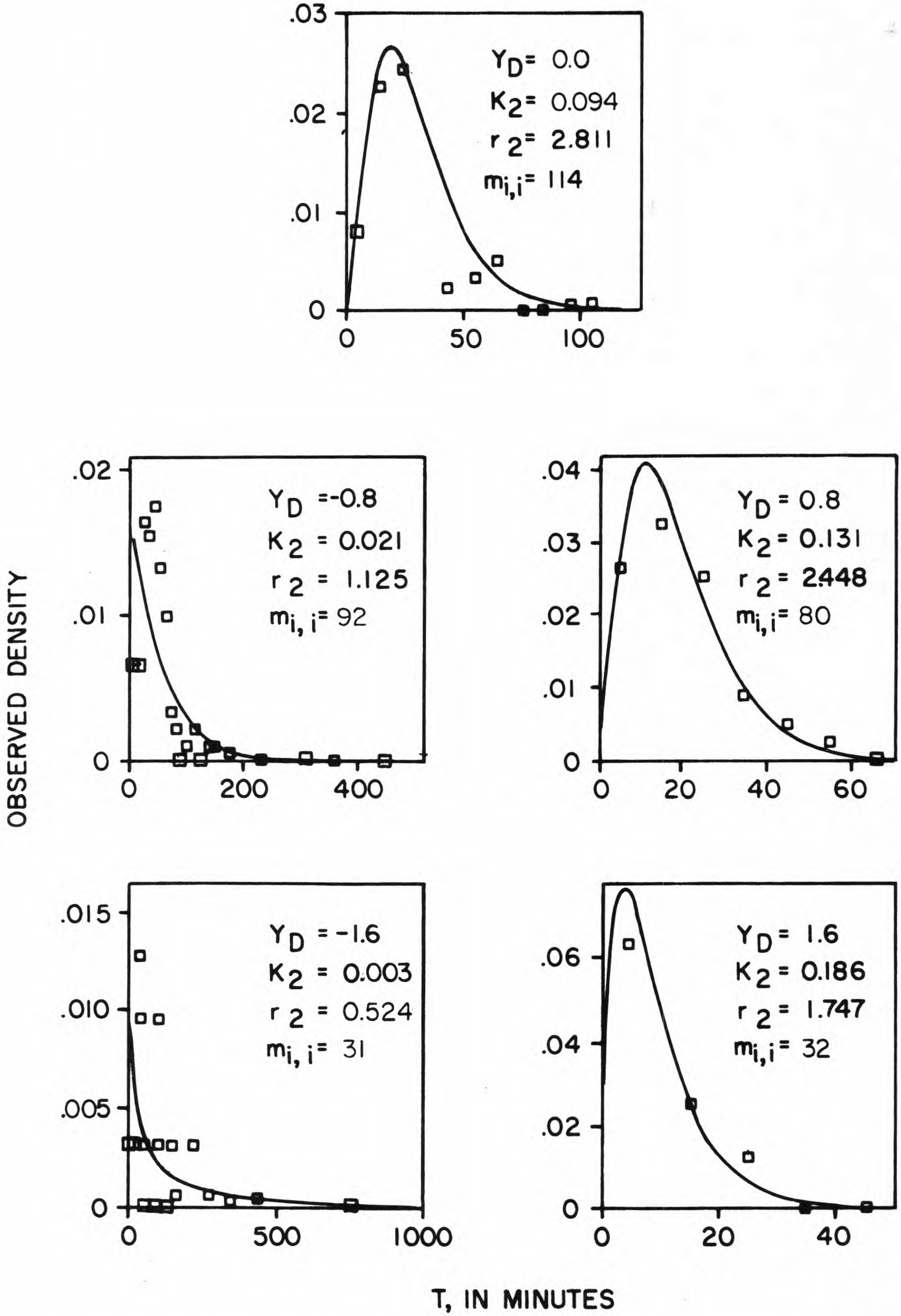

Figure 9. Sample probability mass functions of the conditional rest periods with fitted two-parameter Gamma functions (run 17).

The mean and variance of the conditional rest periods were computed using equation 30, and the results are presented in table 6. These results

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Table 6 (p.138 of ms.) in supplemental data.

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are also plotted as a function of bed elevation in figure 10. As can be

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Figure 10 (caption on next page) belongs near here.

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seen from figure 10, both the conditional mean and variance of the rest periods decrease with increasing elevation of deposition. Inspection of figure 2 indicates that the conditional mean should decrease with increasing elevation of deposition. However, the decrease of the variance is not so obvious. Because the mean value is decreasing with increasing elevation, the decrease in the variance is not too meaningful. The coefficient of variation (standard deviation-mean) is probably a better measure of the variability of the rest periods. Restricting our attention to runs 4A and 17, for reasons to be discussed later, the coefficient of variation remained roughly constant in the range of 0.6-0.75 for elevations above the mean bed elevation, and it increased with decreasing elevation to a value of about 1.5 at 2.4 standard deviations below the mean bed elevation. Thus the variability of the rest period, as measured relative to its mean, also decreases with increasing elevation at least up to the mean bed elevation.

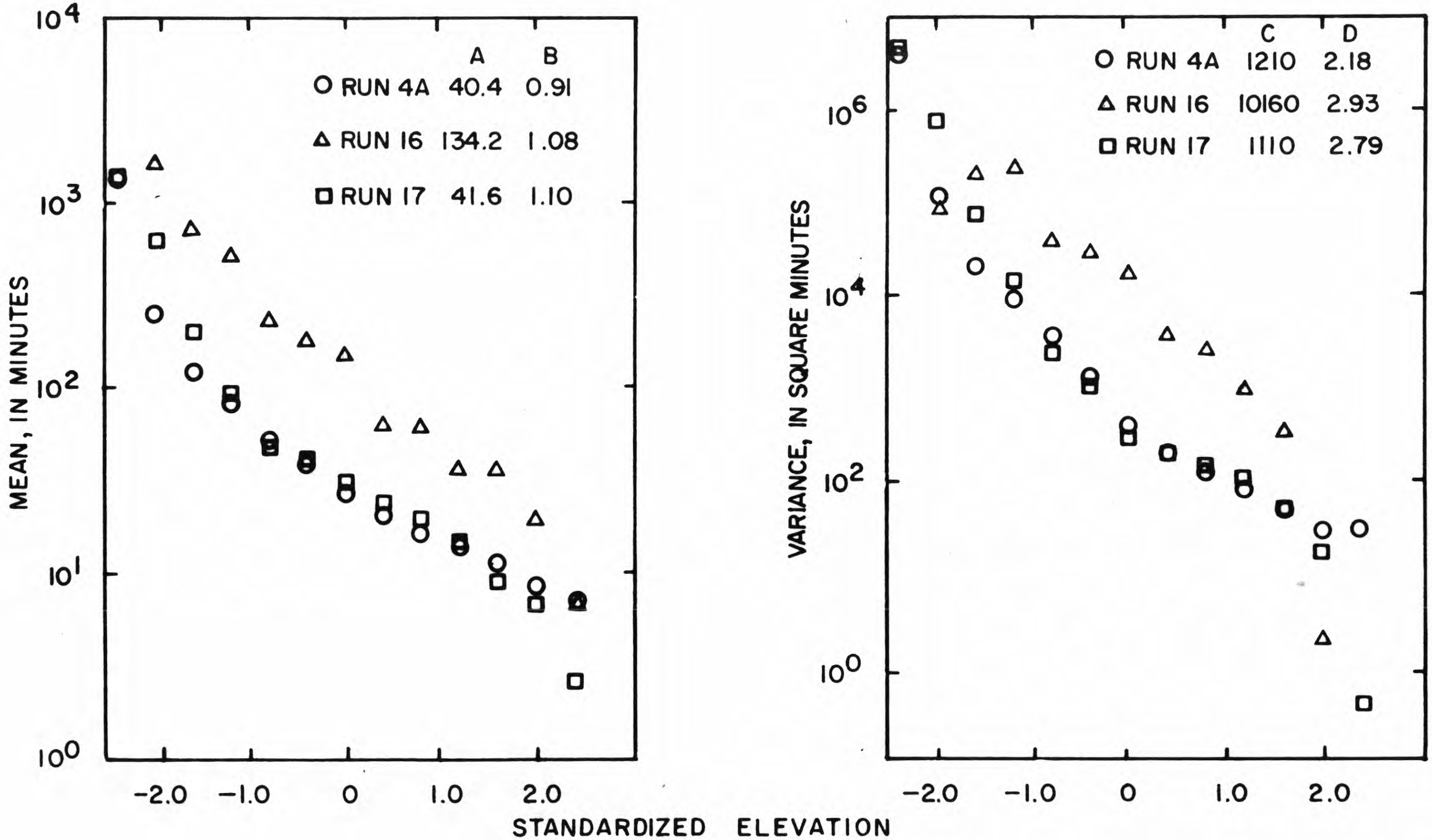

Figure 10. Variation of the conditional mean and variance of rest periods with bed elevation.

As seen from figure 10, both the mean and variance of the conditional rest periods may be approximated by an expression of the form,

$$\left.\begin{aligned} \hat{E}[T \backslash Y_D = y] &= Ae^{-By} , \\ &\text{and} \\ \hat{\mathrm{Var}}[T \backslash Y_D = y] &= Ce^{-Dy} \end{aligned}\right\} . \tag{98}$$

The constants A, B, C and D in equation 98 were determined by a regression analysis of the data plotted on figure 10, and the resulting values are presented on the figure. The values A and C represent measures of the mean and variance of the rest period, respectively, for the mean bed elevation. The values of B and D are measures of the rate of change of the mean and variance of the rest period with bed elevation, respectively.

The distributions of the conditional rest periods were approximated by the two-parameter gamma probability density function which has the form,

$$f_{T\backslash Y_D}(t\backslash y) = \frac{k_{2,y}}{\Gamma(r_{2,y})}(k_{2,y}t)^{r_{2,y}-1} e^{-(k_{2,y})t} \tag{99}$$

where

$\Gamma(\cdot)$ = gamma function; and

$k_{2,y}$, $r_{2,y}$ = scale and shape parameters, respectively.

The scale and shape parameters were estimated by the method of moments,

$$\left.\begin{aligned} k_{2,y} &= \frac{\hat{\mathrm{E}}[T\backslash Y_D = y]}{\hat{\mathrm{Var}}[T\backslash Y_D = y]}, \\ &\text{and} \\ r_{2,y} &= \frac{(\hat{\mathrm{E}}[T\backslash Y_D = y])^2}{\hat{\mathrm{Var}}[T\backslash Y_D = y]} = k_{2,y}\hat{\mathrm{E}}[T\backslash Y_D = y] \end{aligned}\right\}, \tag{100}$$

and the data contained in table 6 . The variation of $k_{2,y}$ and $r_{2,y}$ with bed elevation are presented in table 7 along with the results of a chi-square

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Table 7 (p. 139 of ms.) in supplemental data.

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goodness of fit test. The ability of the two-parameter gamma distribution to fit the measured mass functions is illustrated on figures 7, 8, and 9.

From table 7, as well as from figures 7, 8, and 9, both the scale and shape parameters increase with increasing bed elevation, with a few exceptions for the shape parameter. The shape of the conditional density of the rest periods (figs. 7, 8, 9) approaches a J-shape and becomes more peaked as bed elevation decreases. Therefore, the exponential density might fit better than the two-parameter gamma density below the mean bed elevation ($y < 0$). The exponential density function is a special case of the gamma density with $r_{2,y} = 1$. The better fit of the exponential density seems to be consistent with the fact that all rejections of the chi-square test (6 rejections out of 22 at a significant level of 0.05) occurred below the mean bed elevation (table 7). It would appear that the exponential form for the conditional rest period as proposed by Grigg (1969) is only valid for elevations below the mean bed elevation.

A major factor in determining the degree of fit between the measured density functions and the fitted curves in figures 7, 8, and 9 appears to be the number of points available from which the distribution was constructed. In general, if more than 100 points were available, $m_{i,i}$, the fit is pretty good. The weakness of the data for run 16 is very apparent. Even at the mean bed elevation, only 18 crossing events were observed.

Combining equations 98 and 100, the scale and shape parameters can be estimated using only the constants A, B, C, and D.

$$\left.\begin{aligned} k_{2,y} &= \frac{Ae^{Dy}}{Ce^{By}} \\ &\text{and} \\ r_{2,y} &= \frac{A^2e^{Dy}}{Ce^{2By}} = k_{2,y}Ae^{-By} \end{aligned}\right\} . \tag{101}$$

The sample joint probability mass functions of the rest period and the elevation of deposition were computed from equation 33 using the results presented in tables 2 through 5. The results of these computations are presented in tables 8, 9, and 10. The correlation coefficients were

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Tables 8,9,10 (p. 140-142 of ms.) in supplemental data.

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computed by using equation 34, along with the data contained in tables 2, 6, 8, 9, and 10. The values of the correlation coefficients were -0.27, -0.53, and -0.26 for runs 4A, 16, and 17, respectively. The rest period and the elevation of deposition are negatively correlated, but the degree of their linear association is not strong.

The sample marginal probability mass functions, $p_T(t_\alpha)$, were computed by use of equation 29 and the data contained in tables 2, 3, 4, and 5. The results of these computations are also presented in tables 8, 9, and 10. The sample frequency histograms for the marginal rest periods are plotted in figure 11. The mean and variance of the marginal rest periods were

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Figure 11 (caption on next page) belongs hear here.

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computed by use of equation 31. These results are also presented in figure 11. The variance values appear to be extremely large. For example, the standard deviation for run 4A is almost four times the mean value. The computed variance values are extremely dependent on the long rest periods, the extreme events generally occur at low bed elevations. For example, by ignoring rest periods of greater than 2,000 minutes, which have a probability of occurance of only 0.0015, the variance is reduced from 42,000 to 12,000.

Also shown in figure 11 are exponential density functions with a mean equal to the computed marginal mean. The exponential density function fits the data reasonably well; however, there would appear to be room for improvement. A gamma density fitted by the method of moments would be an extremely poor fit of the data. A gamma distribution, estimated by the maximum likelyhood method may fit the data reasonably well.

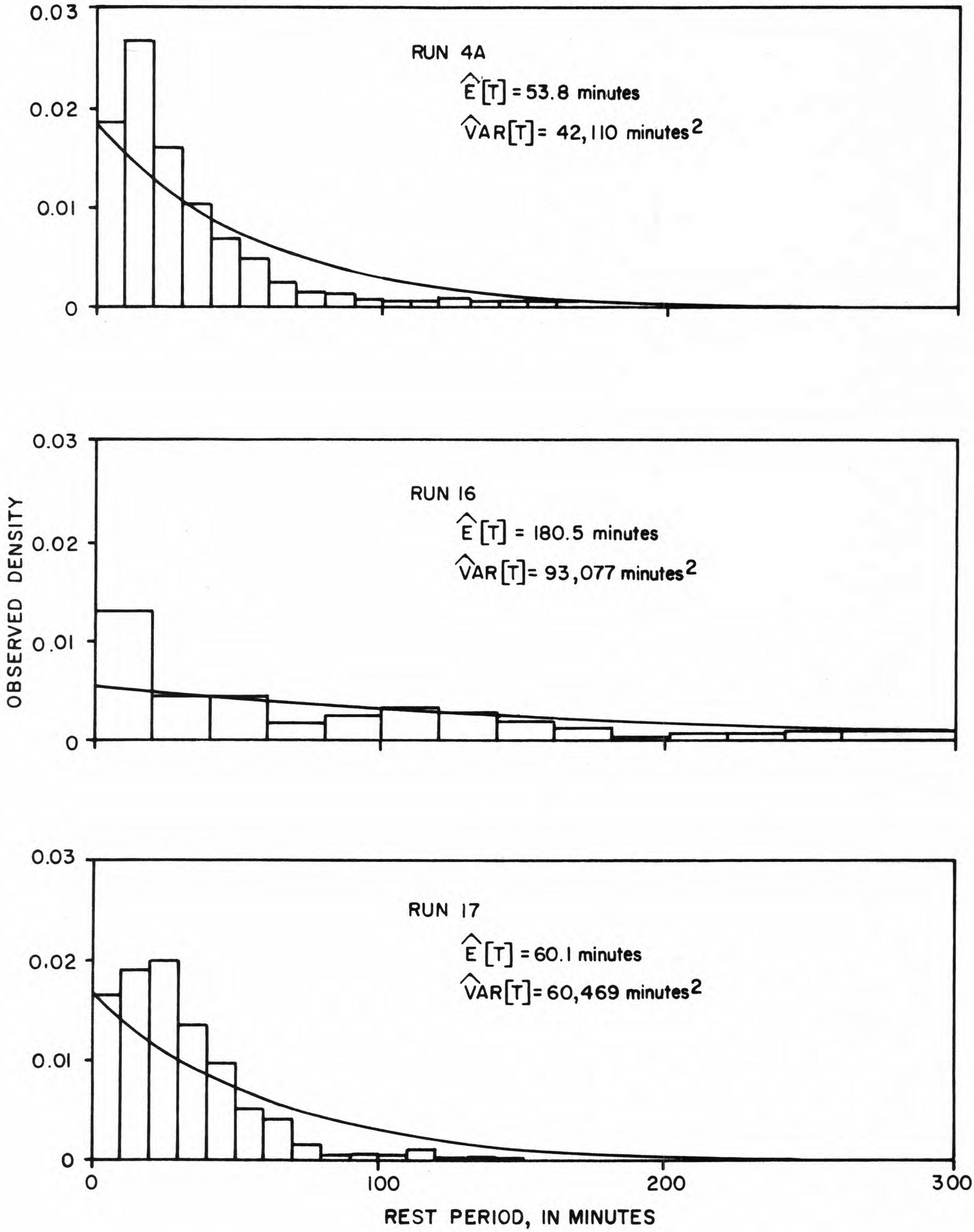

Figure 11. Frequency histograms for the marginal rest period and exponential fits.

The marginal distribution of the rest periods could also be estimated by

$$f_T(t) = \int_{-2.4}^{2.4} f_{T\backslash Y_D}(t\backslash y)\, f_{Y_D}(y)\, dy \tag{102}$$

where $f_{T\backslash Y_D}(t\backslash y)$ is the two-parameter gamma density (eq. 101) with parameters given by equation 101, and $f_{Y_D}(y)$ is given by equation 94, or it could be obtained by fitting the frequency histograms contained in figure 11 with some assumed distribution.

Step Length Distributions

The $y_t(x)$ record was standardized after removing a straight line trend. The trend determined by the method of least squares accounted for the possibility that the sand bed in the flume was not parallel to the instrument carriage rails supporting the sonic sounder. In standardizing the $y_t(x)$ record, the standard deviation obtained from the $y_x(t)$ record was used. With these standardized data, the statistic $\{x_{i,j,k}\}$ was analyzed (fig. 3) to estimate various probability distributions of the step lengths.

The sample probability mass functions given the elevations of deposition and erosion were computed by using equation 47, and the results are presented in tables 11 through 56. Examples of these mass functions

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Tables 11-56 (p.143-188 of ms.) in supplemental data.

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are shown in figures 12, 13, and 14. The corresponding means and

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Figures 12, 13, and 14 (captions on next page) belong near here.

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variances were estimated by equation 48 and summarized in tables 57, 58, and 59.

Tables 57, 58, 59 (p. 189-191 of ms.) in supplemental data.

It can be seen from tables 57, 58, and 59, as well as on figures 12, 13, and 14, that the conditional mean of the step length decreases with an increase in either the elevation of deposition or of erosion. This result could be expected simply from the typical shape of the dunes. Likewise the conditional variance of the step length tends to decrease with an increase in the elevation of either deposition or erosion. The above statements essentially imply that longer step lengths are associated with lower elevations at which a sediment particle is eroded and deposited, and vice versa.

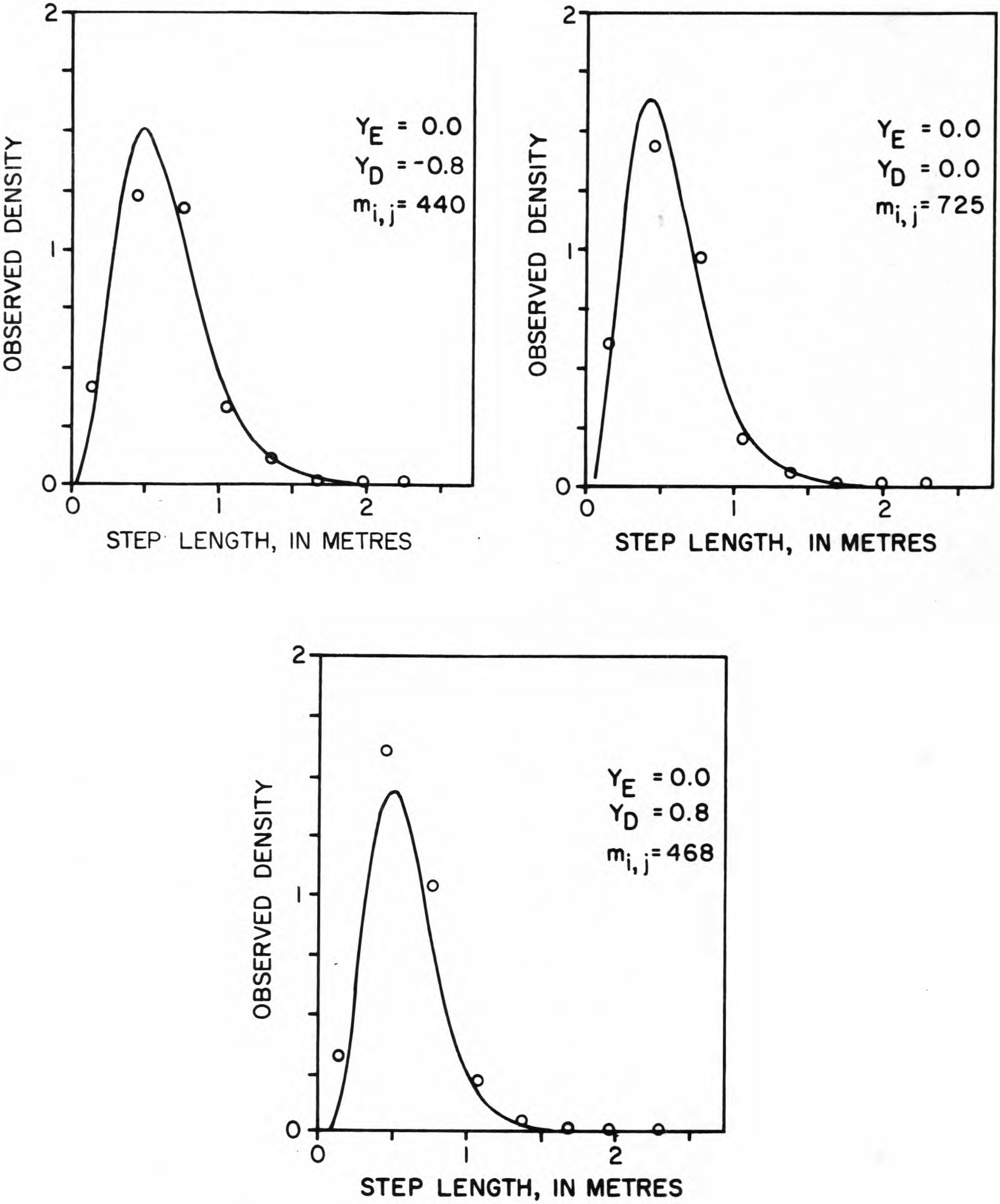

Figure 12. Sample probability mass functions of the conditional step lengths given the elevation of erosion is 0.0 with Gamma fits (run 4A).

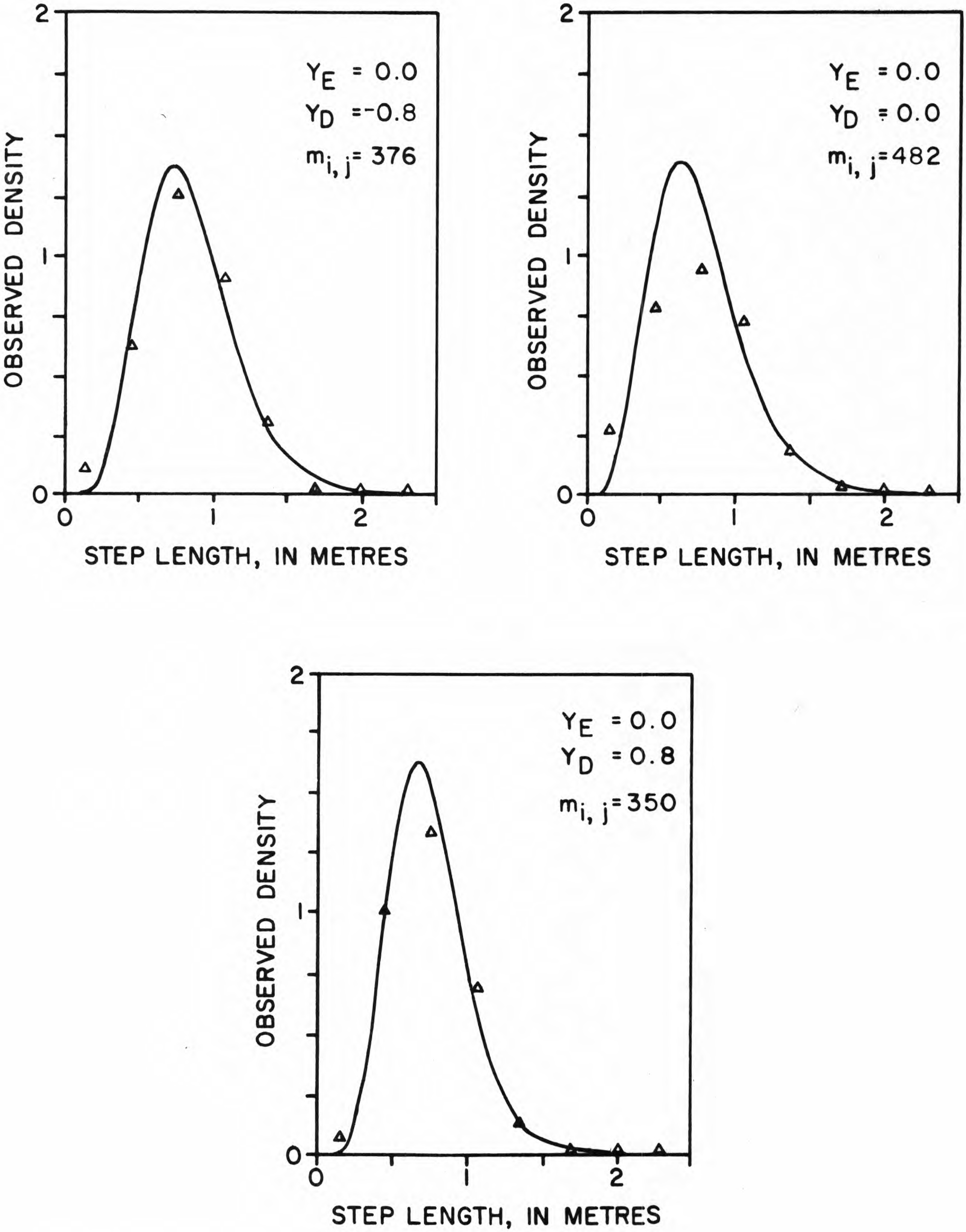

Figure 13. Sample probability mass functions of the conditional step lengths given the elevation of erosion is 0.0 with Gamma fits (run 16).

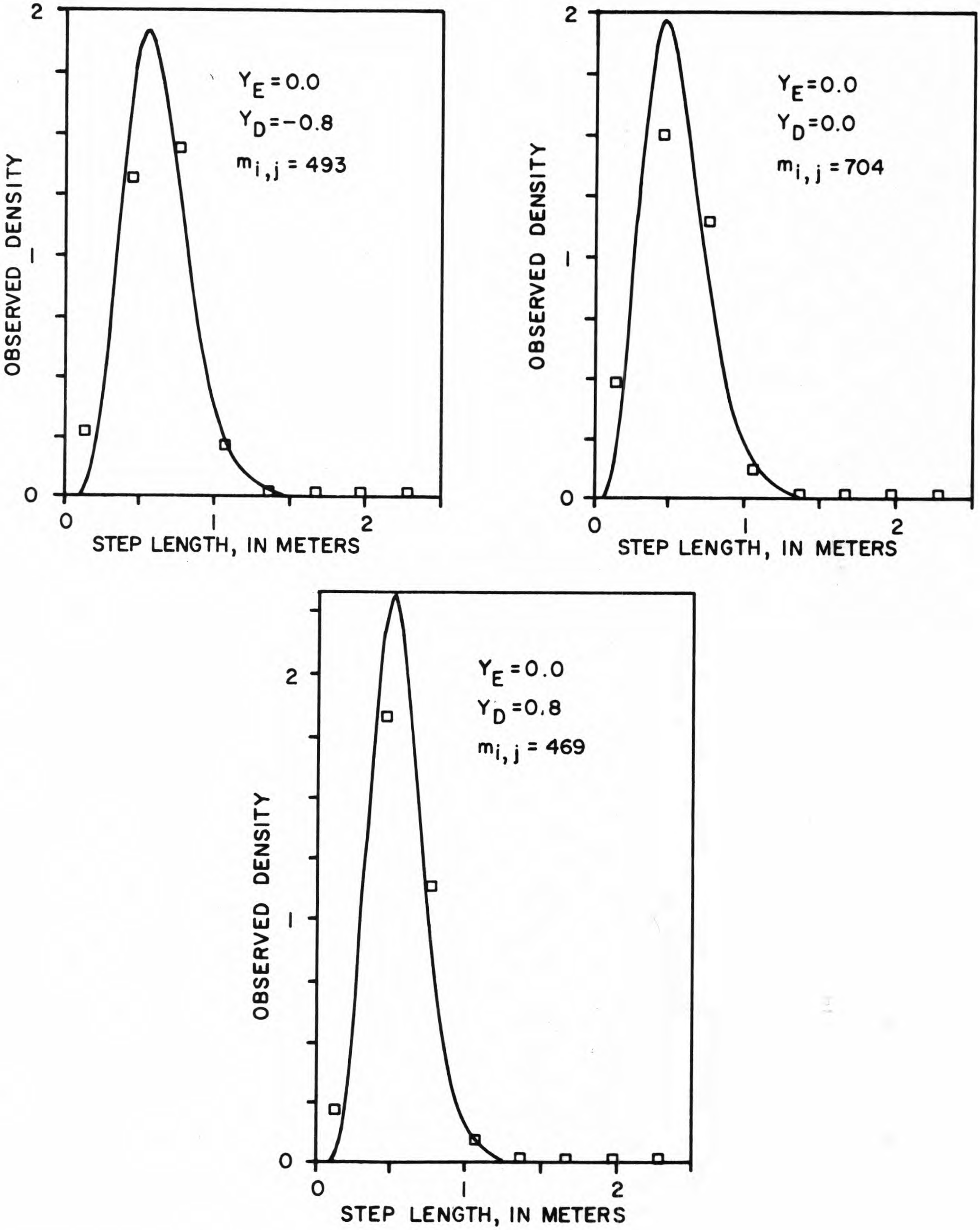

Figure 14. Sample probability mass functions of the conditional step lengths given the elevation of erosion is 0.0 with Gamma fits (run 17).

The distribution of conditional step lengths were approximated by the two parameter gamma probability density functions,

$$f_{X\backslash Y_E, Y_D}(x\backslash y, y') = \frac{k_{1,y,y'}}{\Gamma(r_{1,y,y'})} (xk_{1,y,y'})^{\left(-1 + r_{1,y,y'}\right)} e^{-(k_{1,y,y'})x} \tag{103}$$

where

y and y' = arguments of Y_E and Y_D, respectively; and

$k_{1,y,y'}$ and $r_{1,y,y'}$ = scale and shape parameters, respectively.

The parameters $k_{1,y,y'}$ and $r_{1,y,y'}$ were estimated by the method of moments, using data contained in tables 57 , 58 , and 59 and the expressions

$$k_{1,y,y'} = \frac{\hat{E}[X\backslash Y_E = y,\ Y_D = y']}{\hat{Var}[X\backslash Y_E = y,\ Y_D = y']}$$

and (104)

$$r_{1,y,y'} = \frac{\left(\hat{E}[X\backslash Y_E = y,\ Y_D = y']\right)^2}{\hat{Var}[X\backslash Y_E = y,\ Y_D = y']} = \hat{E}[X\backslash Y_E = y,\ Y_D = y']k_{1,y,y'}$$

The variation of $k_{1,y,y'}$ and $r_{1,y,y'}$ with the elevations of erosion and deposition are shown in tables 60 , 61 , and 62 . These approximations

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Tables 60,61,62 (p. 192-194 of ms.) in supplemental data.

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are also shown in figures 12, 13, and 14.

The chi-square test for goodness of fit was used to test these gamma approximations. The results of these tests are summarized in tables 63, 64, and 65. None of the 81 distributions tested could be rejected at the

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Tables 63, 64, 65 (p. 195-197 of ms.) in supplemental data.

0.05 level of significance. In other words, there is no good statistical reason to reject the hypothesis that the probability density functions for the step lengths, given the elevation of deposition and erosion, are distributed according to the two-parameter gamma distribution. The fitted gamma distributions are also plotted in the example mass functions presented in figures 12, 13, and 14. These figures also help to illustrate the ability of the two parameter gamma distributions to fit the measured conditional step length distributions.

The sample conditional mass functions, given the elevation of deposition, were computed based on equation 37 and the data contained in tables 11 through 56 and 2. These mass functions are presented in tables 66, 67, and 68. The corresponding conditional means and variances

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Tables 66, 67, 68 (p. 198-200of ms.) in supplemental data.

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were computed using equation 38 and are presented in table 69 as well

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Table 69 (p. 201 of ms.) in supplemental data.

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as being plotted in figure 15. Again, the general decrease in the expected

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Figure 15 (caption on next page) belongs near here.

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value of the step length with an increase in the elevation of deposition is apparent.

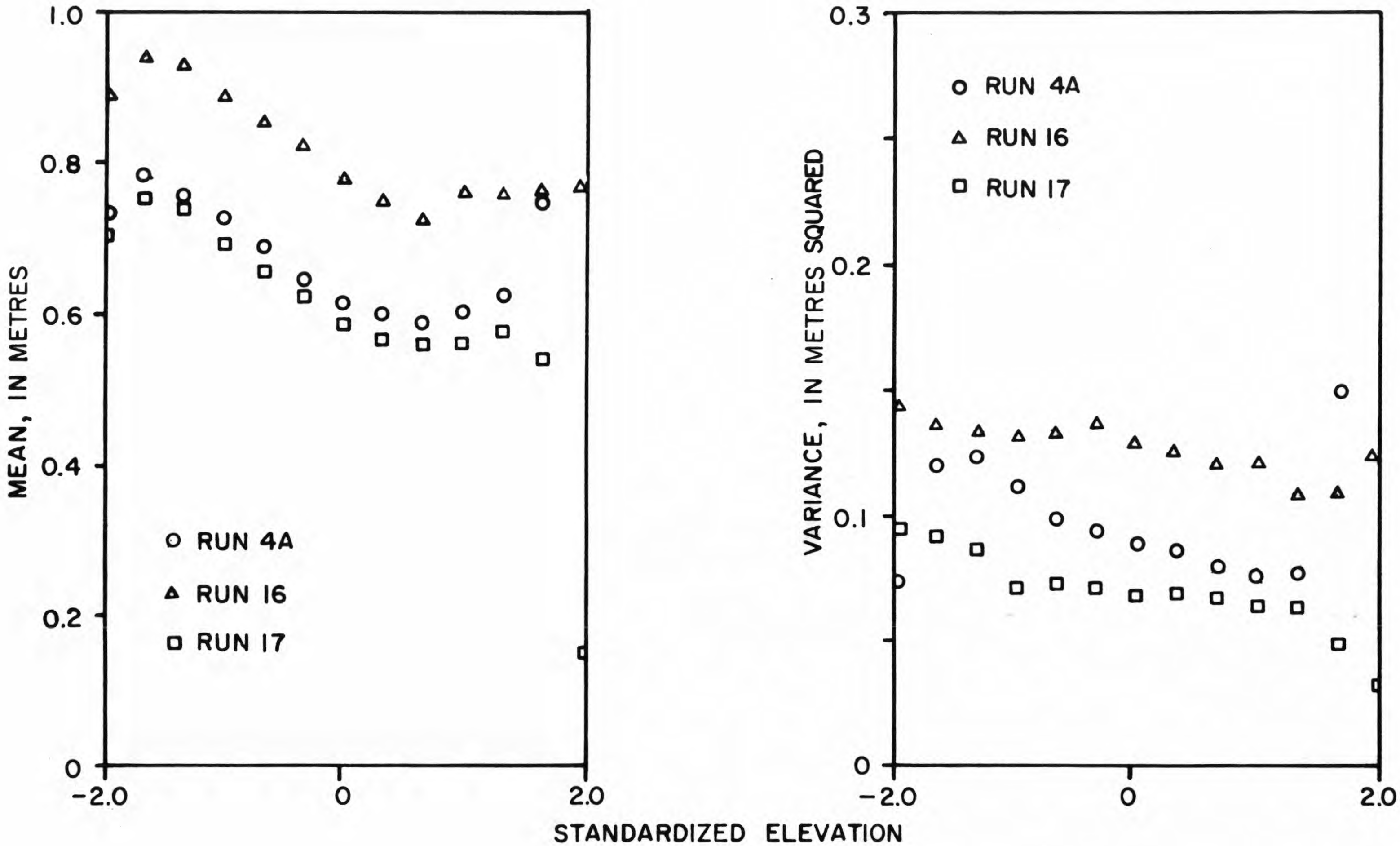

Figure 15. Variation of the conditional mean and variance of step lengths with bed elevation; $\hat{E}[X\backslash Y_D = y]$.

The sample joint probability mass function of the step length and the elevation of deposition was computed by equation 43, and the results are shown in tables 70, 71, and 72. The correlation coefficients

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Tables 70,71,72 (p. 202-204 of ms.) in supplemental data.

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were computed by equation 44, and their values were -0.15, -0.15, and -0.20 for runs 4A, 16, and 17, respectively. This indicates that the step length and the elevation of deposition are negatively correlated, but the degree of their linear association is not strong. The sample marginal probability mass functions, $p_X(x_\beta)$, computed using equation 40, are also shown in tables 70, 71, and 72. The sample frequency histograms for the marginal rest periods are plotted in figure 16. The mean and variance

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Figure 16 (caption on next page) belongs near here.

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of the marginal rest periods were computed by use of equation 41. These results are also presented in figure 16. The range of the means is fairly small, only 0.610 to 0.799 m (2.0 to 2.62 ft). The mean dune lengths, as measured by the distance between trough points, for the three runs were 1.19, 1.66, and 1.23 m (3.90, 5.45, and 4.04 ft) respectively for runs 4A, 16, and 17. The mean step lengths were, therefore, 54, 48, and 49 percent of the mean dune lengths. Grigg (1969) found the mean step lengths of single tagged particles to be about 60 percent of the mean dune length. Of course,

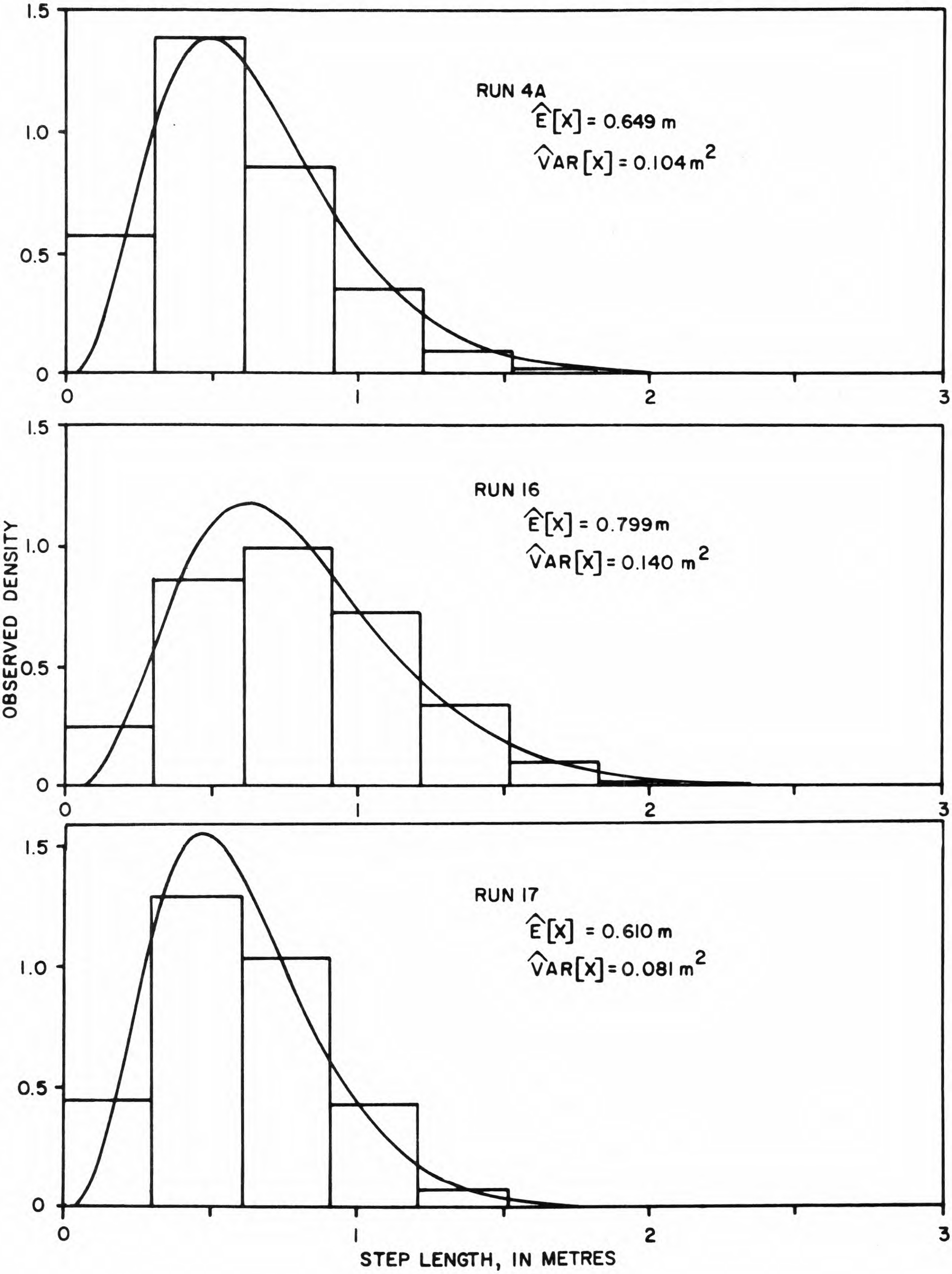

Figure 16. Frequency histograms for the marginal step length with Gamma fits.

Grigg was working with a much finer sand, .33 to .45 mm, as compared to 1.15 mm for this study. Also shown in figure 16 are gamma density functions for which the parameters k and r were determined from the mean and variance shown in the figure. The gamma functions appear to fit the data very well for all three runs. The value of the parameter r ranged from 4.05 for run 4A to 4.59 for run 17. This is slightly more than twice the value estimated by Yang (1968) from the step length distribution of a single plastic particle.

Bed-Material Transport

The following assumptions and conditions were used to estimate the mean total bed-material transport rate by equations 55, 63, and 66:

1. Because the bed material was coarse sand (fig. 5), all sediment particles are assumed to be transported as bed load. Expressed mathematically, $P[E_1] = 1$.

2. $\gamma_s(1 - \theta) = 1602\ kg/m^3$.

3. $\Delta y_j = 0.4 s_y$ everywhere.

By virtue of item 1, it follows that $\hat{V}_T = \hat{V}_B$, $\hat{V}_T(j) = \hat{V}_B(j)$, and $\hat{Q}_T = \hat{Q}_B$. In item 3, s_y is the standard deviation of the bed elevation computed from the $y_x(t)$ record.

All parameters and statistics which are required by equations 55, 63, and 66 are summarized in tables 73 and 74. The average depth of

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Table 73 (p. 113 of ms.) near here. Table 74 (p. 205 of ms) in supplemental data.

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the zone of bed material movement, h, was determined by equation 59. It was found that one chart of the $y_t(x)$ record (about 34 m (111.5 ft)) is sufficient to obtain a reliable value of h, although over 30 charts of the $y_t(x)$ record were used in this study. Each chart contained about ten dunes. Using equation 62, ξ_j, the percentage of volume between elevations η_j and η_{j+1} occupied by dunes (hereafter will be referred to as the effective volume ratio) was obtained from the $y_t(x)$ record. The results are presented in table 74 and plotted in figure 17. As shown in figure 17

Figure 17 (caption on next page) belongs near here.

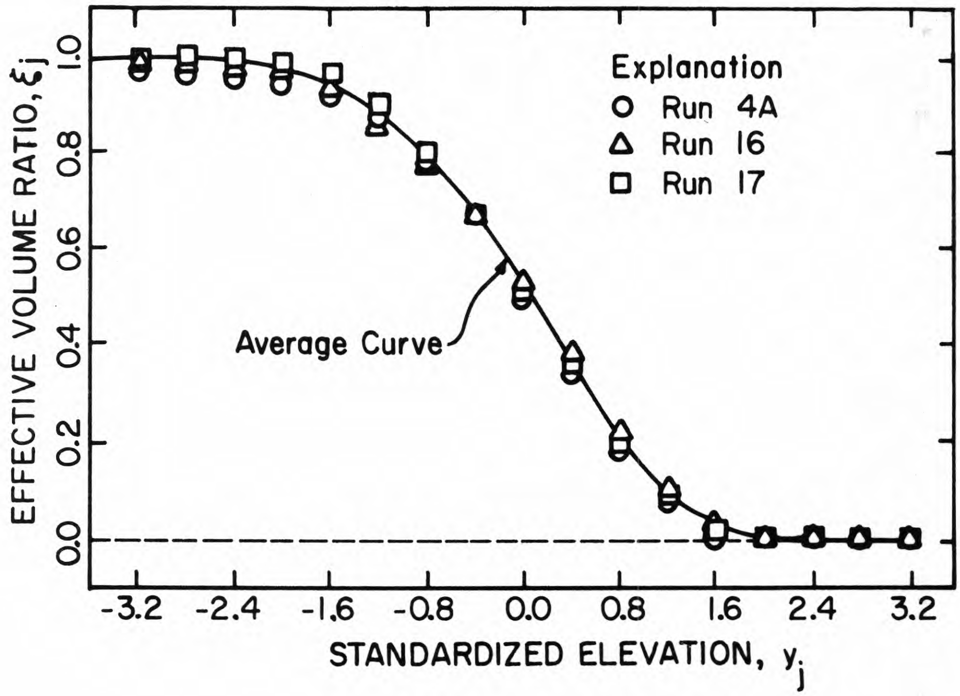

Figure 17. Effective volume ratio as a function of bed elevation, y_j; $\xi_j = (\sum_k \lambda_{j,k}/L_x)$.

Table 73. *Variation of various statistics with stream power*

Run	$\overline{\tau}_b \overline{U}$ (d/cm-s)	$\hat{E}[X]$ (m)	$\hat{Var}[X]$ (m^2)	$\hat{E}[T]$ (min)	$\hat{Var}[T]$ (min^2)	$\hat{V}_T = \frac{\hat{E}[X]}{\hat{E}[T]}$ (cm/s)	$\hat{V}'_T$ (cm/s)	h (cm)	s_y (cm)
4A	3,110	0.649	0.104	53.8	42,110	0.0201	0.0378	9.66	4.26
16	1,450	0.799	0.140	180.5	93,077	0.0073	0.0166	6.89	3.01
17	2,900	0.610	0.081	60.1	60,469	0.0169	0.0372	7.13	3.61

ξ_j is nearly independent of flow condition. As long as the bed forms are dunes, ξ_j does not change appreciably. It is also shown in figure 17 that ξ_j is nearly unity and zero at $y_j = -2.4$ and $y_j = +2.4$, respectively. This is partial justification for the upper and lower limits of the elevations of erosion and deposition used in equations 94 and 95.

Another effective volume ratio can be obtained from the $y_x(t)$ record. Denoting this ratio by ζ_j,

$$\zeta_j = \frac{1}{L_t} \sum_{k=1}^{m_j} t_{j,k} \tag{105}$$

where

L_t = total length of $y_x(t)$ record;

m_j = maximum number of bed forms contained in the $y_x(t)$ record which also contain some deposition at elevation y_j; and

$t_{j,k}$ = measurement of the conditional rest period.

There is no significant difference between ξ_j and ζ_j (table 74) except for depths greater or less than 2.0 standard deviations from the mean. The longitudinal profiles ($y_t(x)$ records) appear to contain a larger number of extreme events than the time record at a given point (the $y_x(t)$). The explanation for this is probably that the flow was fairly stationary but that it was not longitudinally uniform.

A comparison of measured and computed total bed-material transport rates is shown in table 75 . It is seen that:

Table 75 (p. 116 of ms.) near here.

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1. For run 4A, all three equations provide excellent estimates to the observed mean total bed-material discharges.

2. Equation 55 provided an excellent estimate to the mean total bed-material discharge for run 17. However, the other two equations overestimated the discharge by more than 25 percent. The reason for the differences in the equations is not understood.

3. None of the equations gave good estimates of the mean total bed-material discharge for run 16. The consistently overestimated discharge ranged from 64 percent for equation 63 to 80 percent for equation 55. It should be remembered, however, that the mean total bed-material discharge was less than 9 mg/ℓ during this run, that the flow was not in equilibrium as illustrated by the large variation of energy slope (table 1), and that very few rest period statistics were available for analysis (fig. 8).

Table 75. *Comparison of measured and computed total bed-material transport rates*

Run	Measured total bed-material discharge (t/day-m)				
	Number of measurements	Maximum	Minimum	Standard deviation	Mean
4A	54	5.30	0.91	0.88	2.77
16	32	0.72	0.18	0.14	0.40
17	32	3.04	0.59	0.59	1.61

Run	Computed mean, $\hat{q}_T$ (t/day-m)			Computed mean/measured mean		
	Eq. 55	Eq. 63	Eq. 66	Eq. 55	Eq. 63	Eq. 66
4A	2.68	2.72	2.94	0.970	0.983	1.066
16	0.70	0.64	0.67	1.801	1.641	1.725
17	1.67	2.05	2.18	1.035	1.269	1.354

Taken as a whole, the results are very encouraging. Although equation 55 gave the most accurate results for run 17, it should be noted that equations 63 and 66 gave very consistent results for all runs when they are compared one with the other. The discharge predicted by equation 66 was 8.3, 8.4, and 8.5 percent larger than that predicted by equation 63 for runs 4A, 16, and 17, respectively. Although equation 66 is probably simpler to evaluate than equation 63, it appears that some accuracy has been sacrificed. The main difference between equations 63 and 66 is the way in which the effective depth or effective volume ratio (eq. 62, 71) is computed, and these functions were similar (table 74); therefore, the consistency of their final result was expected. Equation 55 had the lowest average absolute error for all three runs, however, equation 63 gave the most accurate result on two out of three runs. Because of the similarity of equations 55 and 63, it would be difficult to say one was more accurate than the other. Their relative accuracy probably depends on chance occurrence of extreme events in one or the other records of bed elevation.

In table 75, $\hat{q}_T$ is the mean total bed-material discharge in weight per width and time, and it was obtained by dividing equations 55, 63, and 66 by the width of the channel, W.

If we define $q'_B(j)$ as the mean bed-load discharge associated with elevation, y_j, then based on equation 63,

$$\hat{q}'_B(j) = \gamma_s (1 - \theta) \hat{V}_B (j) \xi_j \Delta y_j \tag{106}$$

where $\hat{q}'_B(j)$ estimates $q'_B(j)$ and $\hat{V}_B (j)$ is an estimate of the mean transport speed of a bed-load particle at elevation y_j. The mean transport speed, $\hat{V}_B (j)$, is given by equation 61 provided that the suspended load is negligible. With equation 106, the variation of bed-load discharge with bed elevation may be investigated. This variation is shown in figure 18 for

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Figure 18 (caption on next page) belongs near here.

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all three runs. It is seen that the maximum bed-load discharge is associated with the mean bed elevation and that an insignificant portion of the bed-load movement appears to occur for $y_j \leq -2.4$ and $y_j \geq +2.4$.

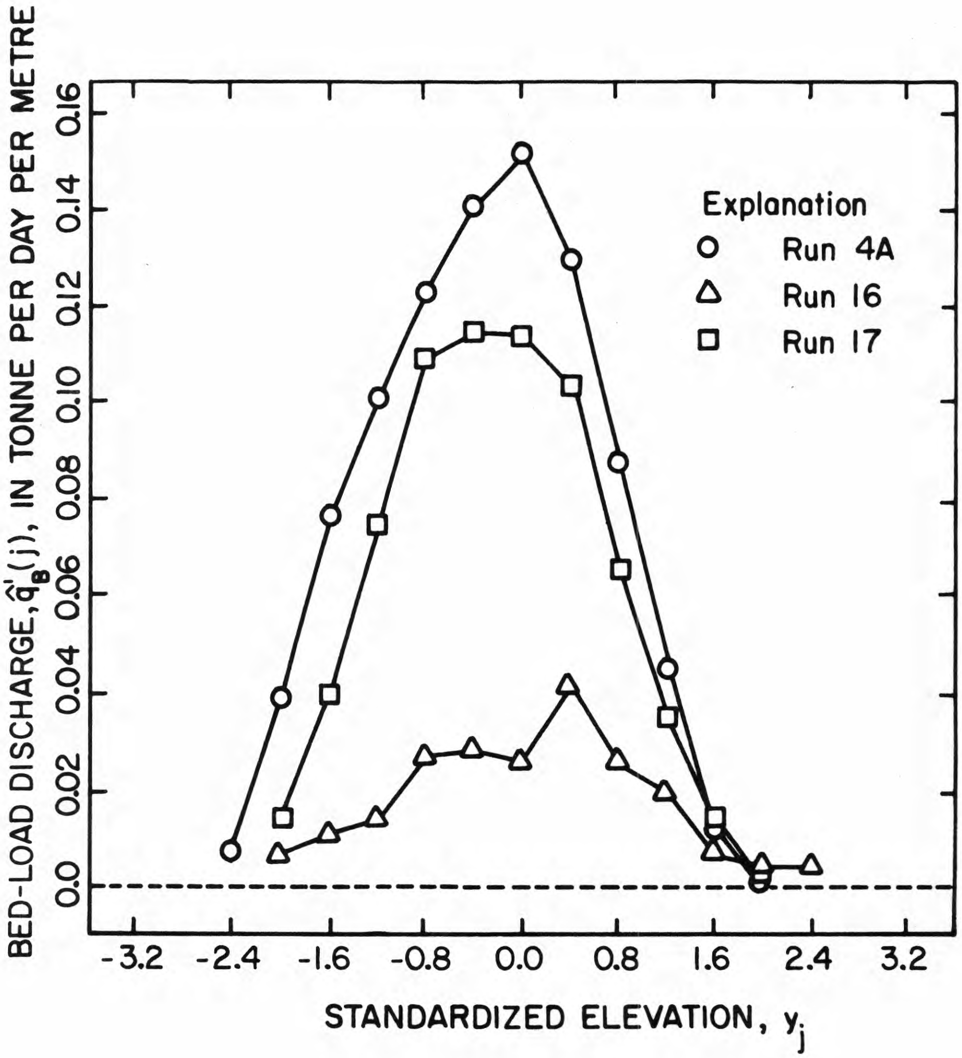

Figure 18. Variation of bed-load discharge with bed elevation.

Variation of Various Statistics with Flow Condition and a Relation between the Step Length and the Rest Period

Although three flume runs are not sufficient to establish a reliable relation between the various statistics and flow conditions, some qualitative trends can be determined from table 73 . The stream power (product of mean bed shear stress and mean flow velocity) was used as a measure of the flow conditions. From table 73, it is seen that:

1. The mean transport speed of a bed-material particle ($\hat{V}_T$, $\hat{V}'_T$), the average depth of the zone in which bed material movement occurs (h), and the standard deviation of the bed (s_y), appear to increase with increasing stream power ($\overline{\tau}_b \overline{U}$).

2. The marginal mean of the step lengths ($\hat{E}[X]$), the marginal variance of the step lengths ($\hat{Var}[X]$), the marginal mean of the rest periods ($\hat{E}[T]$), and the marginal variance of the rest periods ($\hat{Var}[T]$), appear to decrease with increasing stream power within the range of stream power investigated here.

The variation of the ratios of the conditional mean step length to the conditional mean rest period $(\hat{V}_B(y) = (\hat{E}[X \backslash Y_D = y])/(\hat{E}[T \backslash Y_D = y]))$ and of the conditional variance of the step length to the conditional variance of the rest period $(\hat{Var}[X \backslash Y_D = y]/\hat{Var}[T \backslash Y_D = y])$ with bed elevation, y, is shown in figures 19 and 20. From these figures it is seen that both ratios increase with increasing bed elevation.

Figures 19 and 20 (captions on next page) belong near here.

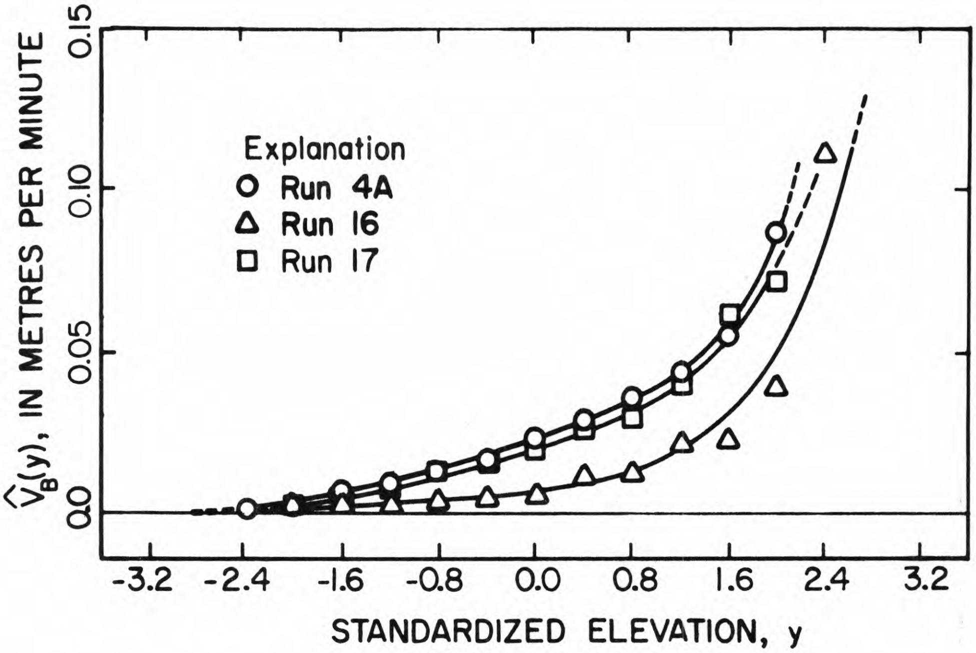

Figure 19. Mean transport speed of a bed-load particle as a function of bed elevation, y.

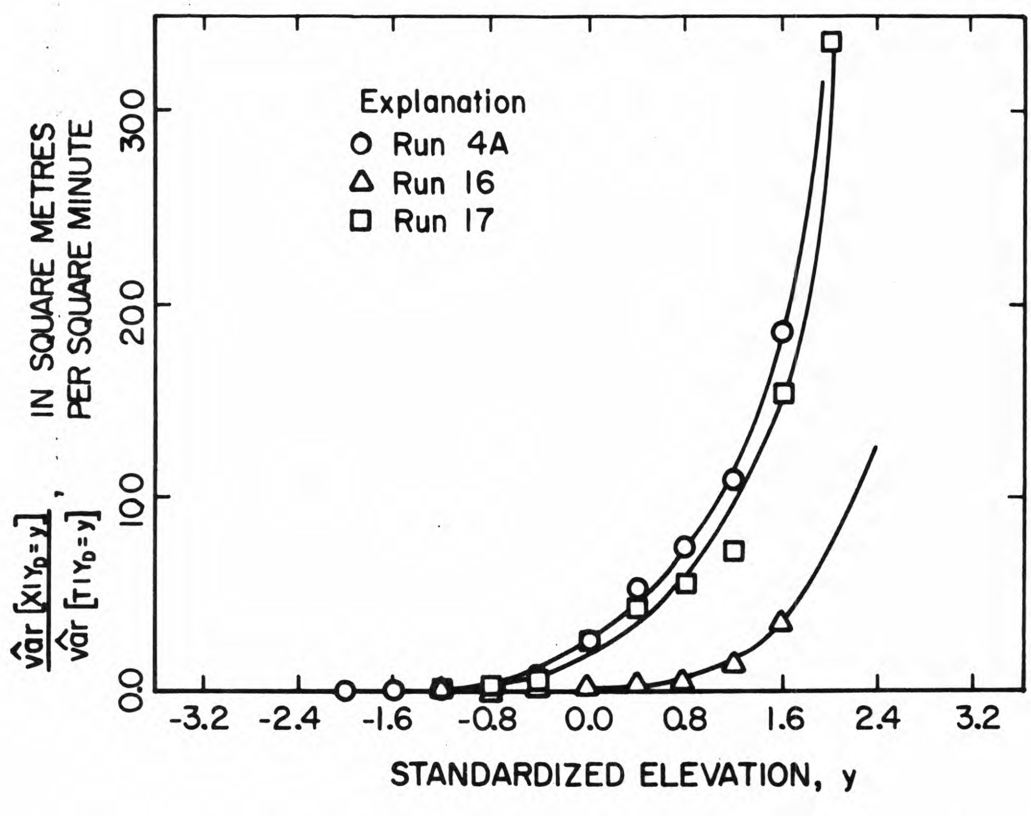

Figure 20. Ratio of the conditional variance of step lengths to the conditional variance of rest periods as a function of bed elevation.

Two-Dimensional Stochastic Model for Dispersion of Bed-Material Sediment Particles

A two-dimensional stochastic model for dispersion of bed-material sediment particles was derived earlier and was given by equation 85,

$$f(x,y;t) = f_{Y_D}(y) \sum_{n=1}^{\infty} \left(\int_0^x f_X^{(n-1)}(\zeta) \; f_{X \backslash Y_D}(x-\zeta \backslash y)\, d\zeta \cdot \int_0^t f_T^{(n)}(t')\, dt' \int_{t-t'}^{\infty} f_{T \backslash Y_D}(\tau \backslash y)\, d\tau \right) . \tag{85}$$

The one-dimensional model as a marginal case of equation 85 was

$$f(x;t) = \sum_{n=1}^{\infty} f_X^{(n)}(x) \int_0^t \left[f_T^{(n)}(t') - f_T^{(n+1)}(t') \right] dt' . \tag{91}$$

Note that y is the standardized elevation. In order to apply equations 85 and 91, the probability density functions, $f_{Y_D}(y)$, $f_{T \backslash Y_D}(t \backslash y)$, $f_T(t)$, $f_T^{(n)}(t)$, $f_T^{(n+1)}(t)$, $f_{X \backslash Y_D}(x \backslash y)$, $f_X(x)$, $f_X^{(n-1)}(x)$, and $f_X^{(n)}(x)$ must be specified.

Although probability density functions for all these distributions have not been determined in this report, the measured probability mass functions have been presented in tables 2, 3-5, 8-10, and 66-68, respectively. Equations for determining the n-fold convolutions of $p_T(t)$ and $p_X(x)$ can be obtained from equations 82 and 79 with proper substitutions (Parzen, 1967). Further progress in the solution of either equation 85 or 91 could proceed along either of two lines. First, all probability density functions could be replaced with the corresponding sample probability mass functions, the integrals approximated by summations, and the solutions obtained numerically. Alternately, the mass functions could be fitted by density functions of some assumed form and an analytical solution attempted. Lee (1973) used various fitting procedures to obtain all the probability density functions required to solve equation 85, but the integration of the equation appears quite formidable.

SUMMARY AND CONCLUSIONS

Stochastic models were developed which can be used to predict the transport and dispersion of bed-material sediment particles in an alluvial channel. These models are based on the proposition that the movement of bed-material sediment particles consists of a series of steps separated by rest periods and, therefore, their application requires a knowledge of the probability distributions of the step lengths, the rest periods, and the elevation of particle deposition and erosion.

The probability distribution of the rest periods, conditioned on the elevation of particle deposition and the probability distributions of the elevation of particle erosion and deposition, were obtained from a record of the bed elevation at a fixed point as a continuous function of time [$y_x(t)$ record]. The necessary assumptions were: (1) Equilibrium flow; (2) both erosion and deposition do not occur at the same point during the same time period; and (3) the number of particles per unit volume of the bed is constant.

The probability distribution of the step lengths, conditioned on the elevation of particle erosion and the elevation of particle deposition, was obtained from a series of instantaneous longitudinal bed profiles [$y_t(x)$ record]. The required assumptions were: (1) All bed-material sediment particles which are eroded from the upstream face of a dune will be deposited on the downstream side of the same dune; and (2) no deposition occurs on the upstream sides of dunes, and no erosion occurs on the downstream faces of dunes. These assumptions appeared to be reasonable at least for a dune-covered bed composed of a coarse sand.

Introducing an additional assumption that the elevation of particle erosion and the elevation of particle deposition are mutually independent, various related probability distributions were obtained. These distributions included: (1) The marginal distributions of the rest periods and the step lengths; (2) the joint distribution of the rest periods and the elevation of particle deposition; and (3) the joint distribution of the step lengths and the elevation of particle deposition.

A two-dimensional stochastic model for disperison of bed-sediment particles was then derived (eq. 85). In order to apply the model, the probability distributions of (1) the step lengths given the elevation of particle deposition; (2) the rest periods given the elevation of particle deposition; and (3) the elevation of particle deposition, must be known. The mass functions of these distributions were estimated; however, the integrations required by the model remained unsolved.

Applying the concept of continuity, three bed-material transport models were presented. Application of these models requires the estimation of: (1) The conditional means of the rest periods and the step lengths; (2) the probability distribution of the elevation of deposition; (3) the average depth of the zone of bed-material movement; and (4) the effective volume ratio. These were all obtained from the $y_x(t)$ and $y_t(x)$ records. In the derivation of the models, the bed load was defined as that part of bed material which is deposited on the downstream face of the dune from which it is eroded, and the suspended load was defined as that part of bed material which passes two or more dune crests before being deposited. These definitions are very precise compared to the definitions prepared by the Task Committee on Preparation of Sedimentation Manual (1962).

Based on flume experiments with a coarse sand, the following conclusions were drawn:

1. The elevation of particle erosion and the elevation of particle deposition can be considered to be identically distributed, and their distribution can be approximated by either a truncated Gaussian density function or a symmetric triangular density function. In general, the truncated Gaussian density provides slightly better results; although the triangular density is much easier to handle analytically.

2. The conditional probability distribution of the rest periods, given the elevation of deposition, can be well described by the two-parameter gamma density function. The shape of the conditional density approaches a J-shape and becomes more peaked as bed elevation decreases.

A. Both the conditional mean and variance of the rest periods increase with decreasing bed elevation. These relations can be expressed by exponential functions.

B. Both the scale and shape parameters for the conditional distribution of the rest periods increase wtih increasing bed elevation, and they can be described by exponential functions of bed elevation.

C. The correlation coefficient between the rest periods and the elevation of deposition indicated that the rest periods and the elevation of deposition are negatively correlated, but the degree of their linear association is not strong.

3. The conditional probability distribution of the step lengths, given the elevation of deposition and the elevation of erosion, can be approximated by the two-parameter gamma distribution. The shape of the conditional density is strongly dependent on the elevation of deposition and erosion.

A. For a fixed elevation of deposition, both the double conditional mean and variance of the step lengths increase with decreasing elevation of erosion. In other words, longer step lengths are associated with lower elevation at which a sediment particle is eroded or deposited and vice versa.

B. The correlation coefficient between the step lengths and the elevation of deposition indicates that they are negatively correlated, but the degree of their linear associated is not strong.

4. All three bed-material transport models are found to be quite satisfactory except for run 16.

A. The effective volume ratio can be obtained from either the $y_t(x)$ record or the $y_x(t)$ record, and it appears to be nearly independent of flow condition.

B. The maximum bed-load movement is associated with mean bed elevation, and little movement occurs for $y \leq -2.4$ and $y \geq +2.4$.

5. The mean transport speed of a bed-material particle, the average depth of the zone of bed material movement, and the standard deviation of bed elevation increased with increasing stream power, whereas the marginal means and variances of the rest periods and the step lengths decreased with increasing stream power.

Figures 10 and 15 suggest that the step lengths and the rest periods are positively correlated in an average sense, but the degree of linear association was not strong.

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SUPPLEMENTAL DATA

(All elevations are standardized.)

Table 2. *Sample probability mass functions of elevations of deposition and erosion*

Standardized Elevation y_i	Run 4A $p_{Y_D}(y_i)$	Run 4A $p_{Y_E}(y_i)$	Run 16 $p_{Y_D}(y_i)$	Run 16 $p_{Y_E}(y_i)$	Run 17 $p_{Y_D}(y_i)$	Run 17 $p_{Y_E}(y_i)$	Triangular Density	Truncated Gaussian
-3.6	0.000	0.000	0.000	0.000	0.000	0.000	-----	-----
-3.2	.000	.000	.006	.006	.000	.000	-----	-----
-2.8	.000	.000	.007	.007	.001	.002	-----	-----
-2.4	.006	.006	.011	.013	.005	.006	0.004	0.006
-2.0	.025	.032	.019	.025	.017	.017	.028	.022
-1.6	.060	.060	.038	.044	.044	.047	.056	.046
-1.2	.091	.088	.062	.065	.089	.089	.083	.079
-0.8	.125	.129	.104	.109	.129	.129	.111	.118
-0.4	.152	.140	.122	.116	.153	.148	.139	.148
0.0	.160	.166	.133	.134	.154	.157	.158	.162
0.4	.156	.154	.197	.189	.154	.153	.139	.148
0.8	.120	.116	.137	.130	.117	.118	.111	.118
1.2	.070	.068	.097	.097	.073	.072	.083	.079
1.6	.025	.032	.046	.044	.041	.041	.056	.046
2.0	.009	.009	.016	.016	.019	.017	.028	.022
2.4	.001	.001	.005	.005	.004	.004	.004	.006
2.8	.000	.000	.000	.000	.000	.000	-----	-----
$\sum_i m_i$ or $\sum_i m_i'$	2,167	2,167	134	134	708	708	-----	-----
$\hat{E}[Y_D]$ or $\hat{E}[Y_E]$	-.130	-.133	.055	.006	-.039	-.043	-----	-----
$\widehat{Var}[Y_D]$ or $\widehat{Var}[Y_E]$	.813	.850	.999	1.046	.863	.870	-----	-----

Note: m_i is the total number of bed forms contained in the $y_x(t)$ record and which also contain some deposition in the class interval associated with the elevation y_i.

m_i' is the total number of bed forms contained in the $y_x(t)$ record and which also contain some erosion in the class interval associated with the elevation y_i.

Table 3. *Sample conditional probability mass function of rest periods,* $p_{T|Y_D}(t_\alpha|y_i)$ (Run 4A)

τ_α	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140
$\tau_{\alpha+1}$	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
t_α	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145
Standardized elevation, y_i															
-2.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2.4	0	0	0	0	0	0	0	0	0	0	0	0	0	.0909	0
-2.0	.0448	.0746	.1045	.0896	.0448	.0298	.0149	.0298	.0298	.0149	0	.0298	.0149	.0149	.0149
-1.6	.0411	.1241	.1022	.0657	.0657	.1095	.0438	.0146	.0365	.0219	.0146	.0146	.0073	.0219	.0219
-1.2	.1031	.1443	.1186	.0723	.0979	.0670	.0412	.0258	.0361	.0103	.0154	.0052	.0309	.0206	.0361
-0.8	.0909	.2016	.1265	.0988	.0830	.0988	.0237	.0316	.0316	.0237	.0277	.0237	.0198	.0158	.0158
-0.4	.1019	.2229	.1337	.1752	.117[illegible]	.0637	.0478	.0350	.0223	.0159	.0127	.0096	.0159	.0064	.0032
0.0	.1579	.2742	.2271	.1385	.0803	.0499	.0360	.0111	.0083	.0054	0	.0028	.0083	0	0
0.4	.2189	.3639	.2041	.1036	.0710	.0148	.0177	.0059	0	0	0	0	0	0	0
0.8	.3145	.3952	.1694	.0766	.0202	.0242	0	0	0	0	0	0	0	0	0
1.2	.4067	.3533	.1867	.0467	.0067	0	0	0	0	0	0	0	0	0	0
1.6	.5139	.3750	.0833	.0278	0	0	0	0	0	0	0	0	0	0	0
2.0	.7368	.2105	.0526	0	0	0	0	0	0	0	0	0	0	0	0
2.4	.6667	.3333	0	0	0	0	0	0	0	0	0	0	0	0	0
2.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

τ_α	150	160	170	180	190	200	250	300	350	400	500	600	1,000	2,000	$m_{i,i}$
$\tau_{\alpha+1}$	160	170	180	190	200	250	300	350	400	500	600	1,000	2,000	8,000	
t_α	155	165	175	185	195	225	275	325	375	450	550	800	1,500	5,000	
Standardized elevation, y_i															
-2.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2.4	0	0	0	0	0	.2727	.0909	0	0	.0909	0	.0909	.0909	.2727	11
-2.0	0	0	.0149	.0448	0	.0746	.0597	.0149	.0298	.0298	.0448	.0896	.0448	0	67
-1.6	.0438	.0219	.0146	.0073	.0219	.0219	.0219	.0438	.0219	.0292	.0 19	.0146	0	0	137
-1.2	.0258	.0052	.0052	.0103	.0155	.0309	.0464	.0155	.0052	.0154	0	0	0	0	194
-0.8	.0158	.0198	.0079	.0079	.0039	.0198	.0079	0	0	.0039	0	0	0	0	253
-0.4	.0032	.0032	.0032	0	0	.0032	.0032	0	0	0	0	0	0	0	314
0.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	361
0.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	338
0.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	248
1.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	150
1.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	72
2.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	19
2.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
2.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note: τ_α, $\tau_{\alpha+1}$, and t_α are in minutes.

$m_{i,i}$ is the total number of bed forms contained in the $y_x(t)$ record and which also contain both an up-crossing and a down-crossing at the elevation y_i.

Table 4. *Sample conditional probability mass function of rest periods,* $p_{T|Y_D}(t_\alpha|y_i)$ (Run 16)

	τ_α	0	20	40	60	80	100	120	140	160	180	200
	$\tau_{\alpha+1}$	20	40	60	80	100	120	140	160	180	200	220
	t_α	10	30	50	70	90	110	130	150	170	190	210
Standardized elevation, y_i	-2.8	0	0	0	0	0	0	0	0	0	0	0
	-2.4	0	0	0	0	0	0	0	0	0	0	0
	-2.0	0	0	0	0	0	0	0	0	0	0	0
	-1.6	0	0	0	0	0	0	0	.1667	0	0	0
	-1.2	.2500	0	0	0	0	0	0	.1250	0	0	0
	-0.8	.1333	.0667	.0667	0	0	.0667	.0667	.0667	0	0	.0666
	-0.4	.1765	.0588	0	.0588	.0588	.0588	.0588	.0588	.0588	0	.0588
	0.0	.1111	.0556	.1111	.0556	.0556	.0556	.1111	.1111	.0555	0	0
	0.4	.4138	.1034	.0690	.0345	.0345	.2069	.0345	0	.0345	.0345	0
	0.8	.2778	.1667	.1111	.1111	.0556	.0556	.1666	0	.0556	0	0
	1.2	.5385	0	.2308	0	.2308	0	0	0	0	0	0
	1.6	.2000	.4000	.4000	0	0	0	0	0	0	0	0
	2.0	.5000	.5000	0	0	0	0	0	0	0	0	0
	2.4	1.0000	0	0	0	0	0	0	0	0	0	0
	2.8	0	0	0	0	0	0	0	0	0	0	0

	τ_α	220	240	260	300	400	500	600	700	800	1,300	$m_{i,i}$
	$\tau_{\alpha+1}$	240	260	300	400	500	600	700	800	1,300	1,800	
	t_α	230	250	280	350	450	550	650	750	1,050	1,550	
Standardized elevation, y_i	-2.8	0	0	0	0	0	0	0	0	0	0	0
	-2.4	0	0	0	0	0	0	0	0	0	0	0
	-2.0	0	0	0	0	0	0	0	0	0	1.0000	2
	-1.6	0	0	0	.1667	0	0	.1667	.1667	.1666	.1666	6
	-1.2	0	0	.1250	0	.1250	0	0	.1250	.1250	.1250	8
	-0.8	.0666	0	.0666	.1333	.0667	.0667	.0667	0	0	0	15
	-0.4	0	.0588	.1176	.0588	.0588	0	.0588	0	0	0	17
	0.0	0	.1111	.1111	0	0	.0555	0	0	0	0	18
	0.4	.0345	0	0	0	0	0	0	0	0	0	29
	0.8	0	0	0	0	0	0	0	0	0	0	18
	1.2	0	0	0	0	0	0	0	0	0	0	13
	1.6	0	0	0	0	0	0	0	0	0	0	5
	2.0	0	0	0	0	0	0	0	0	0	0	2
	2.4	0	0	0	0	0	0	0	0	0	0	1
	2.8	0	0	0	0	0	0	0	0	0	0	0

Note: τ_α, $\tau_{\alpha+1}$, and t_α are in minutes.

$m_{i,i}$ is the total number of bed forms contained in the $y_x(t)$ record and which also contain both an up-crossing and a down-crossing at the elevation y_i.

Table 5. *Sample conditional probability mass function of rest periods,* $p_{T|Y_D}(t_\alpha|y_i)$ (Run 17)

τ_α / $\tau_{\alpha+1}$ / t_α	0 / 10 / 5	10 / 20 / 15	20 / 30 / 25	30 / 40 / 35	40 / 50 / 45	50 / 60 / 55	60 / 70 / 65	70 / 80 / 75	80 / 90 / 85	90 / 100 / 95	100 / 110 / 105	110 / 120 / 115	120 / 130 / 125
Standardized elevation, y_i: -2.8	0	0	0	0	0	0	0	0	0	0	0	0	0
-2.4	0	0	0	0	0	0	0	0	0	0	0	0	0
-2.0	0	0	0	0	.1000	.1000	0	0	0	0	0	.1000	.1000
-1.6	.0322	.0322	.0322	.1290	.0968	.0322	0	.0322	0	.0323	.0323	.0968	0
-1.2	.0328	.0164	.1311	.0984	.1639	.0492	.1639	.0656	.0328	.0328	.0328	.0164	.0164
-0.8	.0652	.0652	.1630	.1522	.1739	.1304	.0978	.0326	.0217	0	.0109	.0217	0
-0.4	.0385	.1442	.2404	.2115	.1250	.1058	.0288	.0385	.0096	.0192	.0096	0	0
0.0	.0789	.2281	.2544	.2105	.1228	.0351	.0526	0	0	.0088	.0088	0	0
0.4	.1892	.3063	.2703	.1261	.0631	.0270	.0180	0	0	0	0	0	0
0.8	.2625	.3250	.2500	.0875	.0500	.0250	0	0	0	0	0	0	0
1.2	.4286	.2857	.1786	.0893	.0179	0	0	0	0	0	0	0	0
1.6	.6250	.2500	.1250	0	0	0	0	0	0	0	0	0	0
2.0	.7273	.2727	0	0	0	0	0	0	0	0	0	0	0
2.4	1.0000	0	0	0	0	0	0	0	0	0	0	0	0
2.8	0	0	0	0	0	0	0	0	0	0	0	0	0

τ_α / $\tau_{\alpha+1}$ / t_α	130 / 140 / 135	140 / 150 / 145	150 / 200 / 175	200 / 250 / 225	250 / 300 / 275	300 / 400 / 350	400 / 500 / 450	500 / 1,000 / 750	1,000 / 2,000 / 1,500	2,000 / 3,000 / 2,500	3,000 / 4,000 / 3,500	$m_{i,i}$
Standardized elevation, y_i: -2.8	0	0	0	0	0	0	0	0	0	0	0	0
-2.4	0	.3333	0	.3334	0	0	0	0	0	0	.3333	3
-2.0	.1000	0	0	0	.1000	0	.1000	.1000	.1000	.1000	0	10
-1.6	.0322	.0322	.0322	.1613	.0322	.0322	.0645	.0645	0	0	0	31
-1.2	.0164	0	.0328	.0328	0	.0328	.0164	.0164	0	0	0	61
-0.8	.0109	.0109	.0217	0	.0109	.0109	0	0	0	0	0	92
-0.4	0	0	0	0	.0096	0	0	0	0	0	0	104
0.0	0	0	0	0	0	0	0	0	0	0	0	114
0.4	0	0	0	0	0	0	0	0	0	0	0	111
0.8	0	0	0	0	0	0	0	0	0	0	0	80
1.2	0	0	0	0	0	0	0	0	0	0	0	56
1.6	0	0	0	0	0	0	0	0	0	0	0	32
2.0	0	0	0	0	0	0	0	0	0	0	0	11
2.4	0	0	0	0	0	0	0	0	0	0	0	3
2.8	0	0	0	0	0	0	0	0	0	0	0	0

Note: τ_α, $\tau_{\alpha+1}$, and t_α are in minutes.

$m_{i,t}$ is the total number of bed forms contained in the $y_x(t)$ record and which also contain both an up-crossing and a down-crossing at the elevation y_i.

Table 6. *Variation of conditional mean and variance of rest periods with elevation of deposition;* $\hat{E}[T|Y_D=y]$ and $\widehat{\mathrm{Var}}[T|Y_D=y]$

Standardized Elevation y_i	$\hat{E}[T\|Y_D=y_i]$, min			$\widehat{\mathrm{Var}}[T\|Y_D=y_i]$, min^2		
	Run 4A	Run 16	Run 17	Run 4A	Run 16	Run 17
-2.8	-------	-------	-------	---------	-------	-----------
-2.4	1,400.8	-------	1,443.7	4,179,378	-------	4,671,616
-2.0	252.2	1,550.3	610.6	109,148	84,679	762,079
-1.6	120.5	714.8	198.0	19,552	204,840	74,793
-1.2	83.4	505.6	92.5	8,696	246,658	14,193
-0.8	58.2	230.9	53.5	3,529	39,416	2,548
-0.4	40.2	185.8	40.4	1,288	28,686	1,129
0.0	27.4	151.7	29.8	409	16,715	316
0.4	21.0	64.5	22.3	193	4,030	188
0.8	16.7	61.5	19.0	127	2,691	145
1.2	13.9	36.0	14.0	82	1,039	107
1.6	11.4	35.6	9.4	49	351	50
2.0	8.5	19.8	7.5	30	2	18
2.4	7.1	6.9	2.6	29	-------	0.4
2.8	-------	-------	-------	---------	-------	-----------
$\hat{E}[T]$	53.8	180.5	60.1	---------	-------	-----------
$\widehat{\mathrm{Var}}[T]$	-------	-------	-------	42,110	93,077	60,469
$\sum_i m_{i,i}$	2,167	134	708	2,167	134	708

Table 7. *Estimates of parameters and the results of goodness of fit test for the conditional rest periods (two-parameter gamma)*

Standardized Elevation y_i	Run 4A				Run 16				Run 17			
	$k_{2,y}$ 1/ min^{-1}	$r_{2,y}$ 2/	$m_{i,i}$	Goodness of Fit Test 3/	$k_{2,y}$ min^{-1}	$r_{2,y}$	$m_{i,i}$	Goodness of Fit Test	$k_{2,y}$ min^{-1}	$r_{2,y}$	$m_{i,i}$	Goodness of Fit Test
-2.4	0.0003	0.470	11	-------	-----	-----	0	-------	------	-----	3	-------
-2.0	.0023	.583	67	-------	-----	-----	2	-------	0.0008	0.489	10	-------
-1.6	.006	.742	137	$x^2 > x_c^2$	0.003	2.494	6	-------	.003	.524	31	$x^2 < x_c^2$
-1.2	.010	.800	194	$x^2 > x_c^2$	.002	1.036	8	-------	.007	.604	61	$x^2 > x_c^2$
-0.8	.016	.959	253	$x^2 < x_c^2$	.006	1.353	15	-------	.021	1.125	92	$x^2 > x_c^2$
-0.4	.031	1.255	314	$x^2 > x_c^2$	.006	1.204	17	$x^2 < x_c^2$	.036	1.444	104	$x^2 > x_c^2$
0.0	.067	1.835	361	$x^2 < x_c^2$	.009	1.376	18	$x^2 < x_c^2$	.094	2.811	114	$x^2 < x_c^2$
0.4	.109	2.276	338	$x^2 < x_c^2$	.016	1.031	29	$x^2 < x_c^2$	.118	2.638	111	$x^2 < x_c^2$
0.8	.131	2.193	248	$x^2 < x_c^2$	.023	1.407	18	$x^2 < x_c^2$	.131	2.488	80	$x^2 < x_c^2$
1.2	.157	2.361	150	$x^2 < x_c^2$	.035	1.247	13	-------	.131	1.827	56	$x^2 < x_c^2$
1.6	.235	2.680	72	$x^2 < x_c^2$	.101	3.611	5	-------	.186	1.747	32	$x^2 < x_c^2$
2.0	.283	2.408	19	-------	-----	-----	2	-------	.417	3.125	11	-------
2.4	------	-----	3	-------	-----	-----	1	-------	------	-----	3	------

1/ $k_{2,y} = \dfrac{\hat{E}[T|Y_D=y]}{\widehat{\text{Var}}[T|Y_D=y]}$

2/ $r_{2,y} = \dfrac{\left(\hat{E}[T|Y_D=y]\right)^2}{\widehat{\text{Var}}[T|Y_D=y]}$

3/ x_c^2 = critical chi-square value at a significant level of 0.05

Table 8. *Sample joint probability mass function of rest periods and elevation of deposition,* $p_{T,Y_D}(t_\alpha, y_i)$ (Run 4A)

τ_α	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140
$\tau_{\alpha+1}$	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
t_α	5	15	25	35	45	55	65	75	85	95	105	115	125	135	145
Standardized elevation, y_i															
-2.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2.4	0	0	0	0	0	0	0	0	0	0	0	0	0	.0005	0
-2.0	.0011	.0019	.0027	.0023	.0011	.0008	.0004	.0008	.0008	.0004	0	.0008	.0004	.0004	.0004
-1.6	.0025	.0075	.0062	.0040	.0040	.0066	.0026	.0009	.0022	.0013	.0009	.0009	.0004	.0013	.0013
-1.2	.0093	.0131	.0107	.0065	.0089	.0061	.0037	.0023	.0033	.0009	.0014	.0005	.0028	.0019	.0033
-0.8	.0114	.0253	.0158	.0124	.0104	.0124	.0030	.0040	.0040	.0030	.0035	.0030	.0025	.0020	.0020
-0.4	.0155	.0338	0203	.0266	.0179	.0097	.0072	.0053	.0034	.0024	.0019	.0014	.0024	.0010	.0005
0.0	.0253	.0440	.0365	.0222	.0129	.0080	.0058	.0018	.0013	.0009	0	.0004	.0013	0	0
0.4	.0341	.0567	.0318	.0161	.0111	.0023	.0028	.0009	0	0	0	0	0	0	0
0.8	.0377	.0473	.0203	.0092	.0024	.0029	0	0	0	0	0	0	0	0	0
1.2	.0286	.0248	.0131	.0033	.0005	0	0	0	0	0	0	0	0	0	0
1.6	.0127	.0093	.0021	.0007	0	0	0	0	0	0	0	0	0	0	0
2.0	.0064	.0018	.0004	0	0	0	0	0	0	0	0	0	0	0	0
2.4	.0005	.0003	0	0	0	0	0	0	0	0	0	0	0	0	0
2.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$p_T(t_\alpha)$	.1857	.2658	.1600	.1033	.0691	.0487	.0255	.0160	.0150	.0089	.0077	.0070	.0098	.0070	.0074

τ_α	150	160	170	180	190	200	250	300	350	400	500	600	1,000	2,000
$\tau_{\alpha+1}$	160	170	180	190	200	250	300	350	400	500	600	1,000	2,000	8,000
t_α	155	165	175	185	195	225	275	325	375	450	550	800	1,500	5,000
Standardized elevation, y_i														
-2.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2.4	0	0	0	0	0	.0015	.0005	0	0	.0005	0	.0005	.0005	.0015
-2.0	0	0	.0004	.0011	0	.0019	.0015	.0004	.0008	.0008	0011	.0023	.0011	0
-1.6	.0026	.0013	.0009	.0004	.0013	.0013	.0013	.0026	.0013	.0018	.0013	.0009	0	0
-1.2	.0023	.0005	.0005	.0009	.0014	.0028	.0043	.0014	.0005	.0014	0	0	0	0
-0.8	.0020	.0025	.0010	.0010	.0005	.0025	.0010	0	0	0	0	0	0	0
-0.4	.0005	.0005	.0005	0	0	.0005	.0005	0	0	.0005	0	0	0	0
0.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2.8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$p_T(t_\alpha)$	.0074	.0048	.0032	.0035	.0032	.0105	.0090	.0044	.0026	.0049	.0024	.0037	.0017	.0015

Note: τ_α, $\tau_{\alpha+1}$, and t_α are in minutes.

Table 9. *Sample joint probability mass function of rest periods and elevation of deposition,* $p_{T,Y_D}(t_\alpha, y_i)$ (Run 16)

	τ_α	0	20	40	60	80	100	120	140	160	180	200
	$\tau_{\alpha+1}$	20	40	60	80	100	120	140	160	180	200	220
	t_α	10	30	50	70	90	110	130	150	170	190	210
Standardized elevation, y_i	-2.8	0	0	0	0	0	0	0	0	0	0	0
	-2.4	0	0	0	0	0	0	0	0	0	0	0
	-2.0	0	0	0	0	0	0	0	0	0	0	0
	-1.6	0	0	0	0	0	0	0	.0063	0	0	0
	-1.2	.0151	0	0	0	0	0	0	.0075	0	0	0
	-0.8	.0138	.0069	.0069	0	0	.0069	.0069	.0069	0	0	.0069
	-0.4	.0216	.0072	0	.0072	.0072	.0072	.0072	.0072	.0072	0	.0072
	0.0	.0148	.0074	.0148	.0074	.0074	.0074	.0148	.0148	.0074	0	0
	0.4	.0817	.0204	.0136	.0068	.0068	.0409	.0068	0	.0068	.0068	0
	0.8	.0381	.0229	.0153	.0153	.0076	.0076	.0229	0	.0076	0	0
	1.2	.0524	0	.0225	0	.0225	0	0	0	0	0	0
	1.6	.0093	.0185	.0185	0	0	0	0	0	0	0	0
	2.0	.0082	.0082	0	0	0	0	0	0	0	0	0
	2.4	.0048	0	0	0	0	0	0	0	0	0	0
	2.8	0	0	0	0	0	0	0	0	0	0	0
$p_T(t_\alpha)$		.2598	.0915	.0916	.0367	.0515	.0700	.0586	.0427	.0290	.0068	.0141

	τ_α	220	240	260	300	400	500	600	700	800	1,300
	$\tau_{\alpha+1}$	240	260	300	400	500	600	700	800	1,300	1,800
	t_α	230	250	280	350	450	550	650	750	1,050	1,550
Standardized elevation, y_i	-2.8	0	0	0	0	0	0	0	0	0	0
	-2.4	0	0	0	0	0	0	0	0	0	0
	-2.0	0	0	0	0	0	0	0	0	0	.0191
	-1.6	0	0	0	.0063	0	0	.0063	.0063	.0063	.0063
	-1.2	0	0	.0075	0	.0075	0	0	.0075	.0075	.0075
	-0.8	.0069	0	.0069	.0138	.0069	.0069	.0069	0	0	0
	-0.4	0	.0072	.0144	.0072	.0072	0	.0072	0	0	0
	0.0	0	.0148	.0148	0	0	.0074	0	0	0	0
	0.4	.0068	0	0	0	0	0	0	0	0	0
	0.8	0	0	0	0	0	0	0	0	0	0
	1.2	0	0	0	0	0	0	0	0	0	0
	1.6	0	0	0	0	0	0	0	0	0	0
	2.0	0	0	0	0	0	0	0	0	0	0
	2.4	0	0	0	0	0	0	0	0	0	0
	2.8	0	0	0	0	0	0	0	0	0	0
$p_T(t_\alpha)$		.0137	.0220	.0436	.0273	.0216	.0143	.0204	.0138	.0138	.0329

Note: τ_α, $\tau_{\alpha+1}$, and t_α are in minutes.

Table 10. *Sample joint probability mass function of rest periods and elevation of deposition,* $p_{T,Y_D}(t_\alpha, y_i)$ (Run 17)

	τ_α	0	10	20	30	40	50	60	70	80	90	100	110
	$\tau_{\alpha+1}$	10	20	30	40	50	60	70	80	90	100	110	120
	t_α	5	15	25	35	45	55	65	75	85	95	105	115
Standardized elevation, y_i	-2.8	0	0	0	0	0	0	0	0	0	0	0	0
	-2.4	0	0	0	0	0	0	0	0	0	0	0	0
	-2.0	0	0	0	0	.0017	.0017	0	0	0	0	0	.0017
	-1.6	.0014	.0014	.0014	.0056	.0042	.0014	0	.0014	0	.0014	.0014	.0042
	-1.2	.0029	.0015	.0017	.0088	.0146	.0044	.0146	.0058	.0029	.0029	.0029	.0015
	-0.8	.0085	.0085	.0211	.0197	.0226	.0169	.0127	.0042	.0028	0	.0014	.0028
	-0.4	.0059	.0220	.0367	.0323	.0191	.0162	.0044	.0059	.0015	.0029	0	.0029
	0.0	.0121	.0351	.0392	.0324	.0189	.0054	.0081	0	0	.0013	.0013	0
	0.4	.0294	.0476	.0420	.0196	.0098	.0042	.0028	0	0	0	0	0
	0.8	.0309	.0382	.0294	.0103	.0059	.0029	0	0	0	0	0	0
	1.2	.0312	.0208	.0130	.0065	.0013	0	0	0	0	0	0	0
	1.6	.0256	.0102	.0051	0	0	0	0	0	0	0	0	0
	2.0	.0137	.0051	0	0	0	0	0	0	0	0	0	0
	2.4	.0035	0	0	0	0	0	0	0	0	0	0	0
	2.8	0	0	0	0	0	0	0	0	0	0	0	0
$p_T(t_\alpha)$		.1651	.1904	.1996	.1352	.0981	.0531	.0426	.0173	.0072	.0085	.0070	.0131

	τ_α	120	130	140	150	200	250	300	400	500	1,000	2,000	3,000
	$\tau_{\alpha+1}$	130	140	150	200	250	300	400	500	1,000	2,000	3,000	4,000
	t_α	125	135	145	175	225	275	350	450	750	1,500	2,500	3,500
Standardized elevation, y_i	-2.8	0	0	0	0	0	0	0	0	0	0	0	0
	-2.4	0	0	.0018	0	.0018	0	0	0	0	0	0	.0018
	-2.0	.0017	.0017	0	0	0	.0017	0	.0017	.0017	.0017	.0017	0
	-1.6	0	.0014	.0014	.0014	.0070	.0014	.0014	.0028	.0028	0	0	0
	-1.2	.0015	.0015	0	.0029	.0029	0	.0029	.0015	.0015	0	0	0
	-0.8	0	.0014	.0014	.0028	0	.0014	.0014	0	0	0	0	0
	-0.4	.0015	0	0	0	0	.0015	0	0	0	0	0	0
	0.0	0	0	0	0	0	0	0	0	0	0	0	0
	0.4	0	0	0	0	0	0	0	0	0	0	0	0
	0.8	0	0	0	0	0	0	0	0	0	0	0	0
	1.2	0	0	0	0	0	0	0	0	0	0	0	0
	1.6	0	0	0	0	0	0	0	0	0	0	0	0
	2.0	0	0	0	0	0	0	0	0	0	0	0	0
	2.4	0	0	0	0	0	0	0	0	0	0	0	0
	2.8	0	0	0	0	0	0	0	0	0	0	0	0
$p_T(t_\alpha)$		.0047	.0060	.0046	.0071	.0117	.0060	.0057	.0060	.0060	.0017	.0017	.0018

Note: τ_α, $\tau_{\alpha+1}$, and t_α are in minutes.

Table 11. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D = y_j = -3.2$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	------	------	------	0
-2.0	------	------	------	------	------	------	------	------	0
-1.6	------	------	------	------	------	------	------	------	0
-1.2	------	1.0000	------	------	------	------	------	------	1
-0.8	------	------	1.0000	------	------	------	------	------	2
-0.4	------	------	------	------	------	------	------	------	2
0.0	------	.5000	.5000	------	------	------	------	------	1
0.4	------	1.0000	------	------	------	------	------	------	0
0.8	------	------	------	------	------	------	------	------	0
1.2	------	------	------	------	------	------	------	------	0
1.6	------	------	------	------	------	------	------	------	0
2.0	------	------	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	0.000	.652	.348	------	------	------	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 12. - *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D = y_j = -2.8$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	0.5000	0.5000	------	------	------	2
-2.0	------	------	0.5000	.5000	------	------	------	------	2
-1.6	------	------	.6667	.3333	------	------	------	------	3
-1.2	------	0.3333	.3333	.3333	------	------	------	------	6
-0.8	------	.3000	.5000	.2000	------	------	------	------	10
-0.4	------	.3333	.3333	.2500	.0834	------	------	------	12
0.0	------	.2857	.5714	------	.1429	------	------	------	7
0.4	------	.4000	.4000	.2000	------	------	------	------	5
0.8	------	1.0000	------	------	------	------	------	------	2
1.2	------	------	------	------	------	------	------	------	0
1.6	------	------	------	------	------	------	------	------	0
2.0	------	------	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	0.000	.451	.358	.151	.040	------	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 13. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D = y_j = -2.4$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	0.5000	0.5000	------	------	------	------	2
-2.0	------	------	.2000	.4000	0.4000	------	------	------	5
-1.6	------	0.1818	.2728	.3636	.1818	------	------	------	11
-1.2	------	.2778	.2778	.3333	.1111	------	------	------	18
-0.8	------	.2333	.4666	.2668	.0333	------	------	------	30
-0.4	------	.3421	.2632	.3421	.0526	------	------	------	38
0.0	0.0294	.3529	.4706	.1471	------	------	------	------	34
0.4	------	.3500	.5000	.1500	------	------	------	------	20
0.8	------	.3077	.6154	------	.0769	------	------	------	13
1.2	------	1.0000	------	------	------	------	------	------	2
1.6	------	1.0000	------	------	------	------	------	------	1
2.0	------	------	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.004	.450	.364	.155	.027	------	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 14. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D = y_j = -2.0$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	1.0000	------	------	1
-2.4	------	------	0.2500	0.2500	------	.2500	------	0.2500	4
-2.0	------	------	.1538	.3077	0.3078	.0769	0.1538	------	13
-1.6	------	0.1905	.2857	.1429	.2380	.1429	------	------	21
-1.2	------	.1795	.3590	.2820	.1282	.0513	------	------	39
-0.8	------	.2154	.4461	.2154	.1027	.0154	------	------	65
-0.4	0.0361	.2771	.3735	.2169	.0603	.0120	.0241	------	83
0.0	.0135	.3649	.4189	.1622	------	.0405	------	------	74
0.4	------	.3958	.3958	.1458	.0208	.0418	------	------	48
0.8	.0690	.3793	.4483	------	.1034	------	------	------	29
1.2	------	.5000	.3750	------	.1250	------	------	------	8
1.6	------	1.0000	------	------	------	------	------	------	2
2.0	------	------	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.018	.388	.372	.122	.071	.025	.004	.000	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 15. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D = y_j = -1.6$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	1.0000	------	------	------	------	1
-2.8	------	------	0.2000	.2000	0.4000	0.2000	------	------	5
-2.4	------	------	.1539	.3847	.3077	.0768	0.0769	------	13
-2.0	------	------	.2143	.3571	.3214	.1072	------	------	28
-1.6	------	0.0870	.4131	.2609	.1522	.0652	.0216	------	46
-1.2	0.0122	.1341	.3779	.2804	.1341	.0366	.0122	0.0122	82
-0.8	.0078	.1953	.4218	.2890	.0391	.0391	------	.0078	128
-0.4	.0526	.3289	.3158	.2303	.0395	.0197	.0066	.0066	152
0.0	.0616	.3630	.3972	.1233	.0274	.0205	.0068	------	146
0.4	.0278	.4908	.3148	.1204	.0278	.0093	.0093	------	108
0.8	.1167	.4167	.3501	.0333	.0666	.0167	------	------	60
1.2	------	.7392	.0870	.0435	.0870	.0435	------	------	23
1.6	.2500	.5000	------	.2500	------	------	------	------	4
2.0	------	------	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.058	.403	.300	.153	.054	.025	.005	.002	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 16. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D=y_j = -1.2$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	0.5000	0.5000	------	------	------	------	2
-2.8	------	------	.3750	.1250	0.2500	0.2500	------	------	8
-2.4	------	0.1666	.1111	.3333	.2222	.1111	0.0555	------	18
-2.0	------	.0750	.3500	.2750	.2000	.1000	------	------	40
-1.6	0.0267	.0931	.3600	.2932	.1600	.0666	------	------	75
-1.2	.0220	.1323	.3675	.2307	.0956	.0367	.0073	0.0073	136
-0.8	.0461	.2443	.3550	.2812	.0461	.0230	------	.0046	217
-0.4	.0720	.3144	.3258	.2083	.0606	.0151	------	.0038	264
0.0	.1064	.3384	.3764	.1255	.0380	.0076	.0076	------	263
0.4	.0857	.4474	.3284	.0904	.0333	.0095	.0048	------	210
0.8	.1209	.4916	.2821	.0564	.0403	.0081	------	------	124
1.2	.0651	.6293	.1519	.1302	------	.0217	------	------	46
1.6	.2222	.5555	------	.2222	------	------	------	------	9
2.0	------	1.0000	------	------	------	------	------	------	1
2.4	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.087	.420	.282	.150	.042	.016	.002	.001	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 17. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D=y_j$ = -0.8

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	0.5000	------	0.5000	------	------	------	------	2
-2.8	------	.2222	0.1111	.2222	0.2222	0.2222	------	------	9
-2.4	------	.1540	.2310	.2310	.2695	.0770	0.0385	------	26
-2.0	------	.1127	.3542	.2254	.2737	.0322	------	------	62
-1.6	0.0175	.1140	.3596	.3333	.1403	.0351	------	------	114
-1.2	.0240	.1440	.3792	.3216	.1009	.0288	.0048	0.0048	210
-0.8	.0477	.2593	.3397	.2771	.0596	.0149	------	.0030	336
-0.4	.0679	.3440	.3416	.1895	.0491	.0047	------	.0023	427
0.0	.1293	.3723	.3564	.0999	.0318	.0045	.0045	------	440
0.4	.1646	.4213	.3153	.0725	.0195	.0028	.0028	------	358
0.8	.1454	.5487	.2204	.0516	.0281	.0047	------	------	213
1.2	.0990	.6490	.1870	.0550	------	.0110	------	------	91
1.6	.2500	.6250	.0625	.0625	------	------	------	------	16
2.0	------	1.0000	------	------	------	------	------	------	2
2.4	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.112	.440	.275	.126	.037	.008	.001	.001	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 18. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D = y_j = -0.4$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	0.3333	------	0.3333	0.3333	------	------	------	3
-2.8	------	.1667	0.3333	.1667	.1667	0.1667	------	------	12
-2.4	------	.1428	.2857	.2571	.2000	.0857	0.0286	------	35
-2.0	0.0130	.0909	.4156	.2727	.1688	.0390	------	------	77
-1.6	.0207	.1241	.3931	.3172	.1103	.0345	------	------	145
-1.2	.0189	.1396	.4453	.2717	.1019	.0151	.0038	.0038	265
-0.8	.0331	.2861	.4043	.2104	.0591	.0071	------	.0024	423
-0.4	.0860	.3835	.3459	.1326	.0484	.0018	------	.0018	558
0.0	.1443	.4312	.3134	.0862	.0199	.0017	.0033	------	603
0.4	.2082	.4676	.2515	.0530	.0157	.0020	.0020	------	509
0.8	.2157	.5654	.1503	.0490	.0163	.0033	------	------	306
1.2	.2016	.5581	.1783	.0542	------	.0078	------	------	129
1.6	.3846	.3846	.1538	.0769	------	------	------	------	26
2.0	------	1.0000	------	------	------	------	------	------	3
2.4	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.154	.439	.264	.105	.031	.005	.001	.001	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 19. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D=y_j = 0.0$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	1.0000	------	------	------	------	2
-2.8	------	0.0714	0.4286	.2857	0.1429	0.0714	------	------	14
-2.4	------	.0769	.3846	.2308	.2051	.0769	0.0256	------	39
-2.0	0.0112	.1011	.3933	.3033	.1573	.0337	------	------	89
-1.6	.0060	.1198	.4251	.3174	.1138	.0180	------	------	167
-1.2	.0068	.1689	.4426	.2939	.0676	.0169	------	0.0034	296
-0.8	.0219	.3107	.4026	.2144	.0416	.0066	------	.0022	457
-0.4	.0678	.4017	.3179	.1256	.0281	.0033	.0017	------	605
0.0	.1848	.4372	.2938	.0648	.0152	.0014	.0028	------	725
0.4	.2396	.4984	.2093	.0399	.0096	.0032	------	------	626
0.8	.2953	.5335	.1315	.0298	.0074	.0025	------	------	403
1.2	.3529	.4920	.1176	.0321	.0053	------	------	------	187
1.6	.3846	.5128	.0769	.0256	------	------	------	------	39
2.0	.5000	.5000	------	------	------	------	------	------	4
2.4	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.209	.435	.238	.092	.021	.004	.001	.000	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 20. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D=y_j$ = 0.4

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	1.0000	------	------	------	------	2
-2.8	------	0.0769	0.3846	.3846	0.1538	------	------	------	13
-2.4	------	.0488	.4146	.3171	.1463	0.0488	0.0244	------	41
-2.0	0.0111	.0889	.4222	.3111	.1444	.0222	------	------	90
-1.6	.0123	.1104	.4601	.3190	.0920	.0061	------	------	163
-1.2	.0070	.1789	.4456	.2737	.0807	.0105	------	0.0035	285
-0.8	.0143	.3024	.4310	.2071	.0381	.0048	.0024	------	420
-0.4	.0328	.4197	.3887	.1332	.0182	.0055	.0018	------	548
0.0	.1381	.4850	.2973	.0646	.0120	------	.0030	------	666
0.4	.3138	.4864	.1498	.0314	.0071	.0014	------	------	701
0.8	.3736	.4835	.1099	.0264	.0066	------	------	------	455
1.2	.4574	.4350	.0852	.0179	.0045	------	------	------	223
1.6	.5238	.3571	.1190	------	------	------	------	------	42
2.0	.5000	.5000	------	------	------	------	------	------	4
2.4	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.242	.413	.237	.085	.020	.002	.001	.000	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 21. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D=y_j = 0.8$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	1.0000	------	------	------	------	2
-2.8	------	0.0909	0.3636	0.3636	0.1818	------	------	------	11
-2.4	------	.0278	.3889	.3889	.1389	0.0278	0.0278	------	36
-2.0	------	.0875	.4375	.3500	.1000	.0250	------	------	80
-1.6	------	.1314	.5255	.2847	.0511	.0073	------	------	137
-1.2	0.0044	.2267	.4444	.2444	.0622	.0178	------	------	225
-0.8	.0065	.2932	.4658	.2020	.0261	.0065	------	------	307
-0.4	.0435	.3862	.4246	.1253	.0205	------	------	------	391
0.0	.1026	.4936	.3162	.0684	.0171	.0021	------	------	468
0.4	.2266	.5742	.1660	.0293	.0039	------	------	------	512
0.8	.4936	.4149	.0766	.0149	------	------	------	------	470
1.2	.4056	.2755	.0310	.0062	------	------	------	------	232
1.6	.6000	.3555	.0444	------	------	------	------	------	45
2.0	.7500	.2500	------	------	------	------	------	------	4
2.4	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.257	.403	.243	.080	.015	.002	.000	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 22. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D{=}y_j = 1.2$

$Y_E{=}y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	0.5000	0.5000	------	------	------	------	2
-2.8	------	------	.5000	.5000	------	------	------	------	8
-2.4	------	------	.4583	.3750	0.1250	0.0417	------	------	24
-2.0	------	0.0980	.5098	.3137	.0588	.0196	------	------	51
-1.6	0.0114	.1364	.5227	.2955	.0228	.0114	------	------	88
-1.2	.0072	.2086	.4676	.2446	.0576	.0144	------	------	139
-0.8	.0055	.2928	.4420	.2210	.0387	------	------	------	181
-0.4	.0138	.3825	.4332	.1521	.0184	------	------	------	217
0.0	.0395	.5257	.3399	.0751	.0158	.0040	------	------	253
0.4	.1304	.6051	.2246	.0362	.0036	------	------	------	276
0.8	.4014	.4930	.0915	.0141	------	------	------	------	284
1.2	.6555	.3067	.0336	.0042	------	------	------	------	238
1.6	.6957	.2609	.0435	------	------	------	------	------	46
2.0	.7500	.2500	------	------	------	------	------	------	4
2.4	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.242	.407	.250	.086	.013	.002	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 23. - *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D\!=\!y_j = 1.6$

$Y_E\!=\!y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	0.7500	0.2500	------	------	------	------	4
-2.4	------	0.0769	.4615	.2308	0.1538	0.0769	------	------	13
-2.0	------	.1000	.4500	.4000	------	.0500	------	------	20
-1.6	------	.1515	.4848	.3030	.0303	.0303	------	------	33
-1.2	------	.2045	.4773	.2500	.0455	.0227	------	------	44
-0.8	------	.2708	.4583	.2083	.0625	------	------	------	48
-0.4	0.0189	.3585	.4151	.1509	.0566	------	------	------	53
0.0	.0357	.4286	.4643	.0357	.0357	------	------	------	56
0.4	.0794	.5556	.3333	.0317	------	------	------	------	63
0.8	.2836	.5821	.1045	.0299	------	------	------	------	67
1.2	.5735	.3971	.0147	.0147	------	------	------	------	68
1.6	.8261	.1304	.0435	------	------	------	------	------	46
2.0	.7500	.2500	------	------	------	------	------	------	4
2.4	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.213	.395	.284	.083	.022	.003	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 24. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 4A)

$Y_D = y_j = 2.0$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	1.0000	------	------	1
-2.0	------	------	------	0.5000	------	.5000	------	------	2
-1.6	------	------	0.5000	.2500	0.2500	------	------	------	4
-1.2	------	------	.7500	------	.2500	------	------	------	4
-0.8	------	------	.7500	------	.2500	------	------	------	4
-0.4	------	0.5000	.2500	------	.2500	------	------	------	4
0.0	------	.5000	.2500	------	.2500	------	------	------	4
0.4	------	.7500	------	.2500	------	------	------	------	4
0.8	------	.7500	------	.2500	------	------	------	------	4
1.2	0.5000	.2500	------	.2500	------	------	------	------	4
1.6	.7500	------	.2500	------	------	------	------	------	4
2.0	.7500	.2500	------	------	------	------	------	------	4
2.4	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.177	.374	.137	.160	.149	.003	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 25.- *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D = y_j = -3.6$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	------	------	------	0
-2.0	------	------	------	------	------	------	------	------	0
-1.6	------	------	------	------	------	------	------	------	0
-1.2	------	------	------	------	------	------	------	------	0
-0.8	------	------	------	1.0000	------	------	------	------	1
-0.4	------	------	------	1.0000	------	------	------	------	1
0.0	------	------	1.0000	------	------	------	------	------	1
0.4	------	------	1.0000	------	------	------	------	------	1
0.8	------	------	1.0000	------	------	------	------	------	1
1.2	------	1.0000	------	------	------	------	------	------	1
1.6	------	------	------	------	------	------	------	------	0
2.0	------	------	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	------	------	0
3.2	------	------	------	------	------	------	------	------	0
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	------	.125	.583	.292	------	------	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 26. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D=y_j = -3.2$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	------	------	------	0
-2.0	------	------	------	------	------	------	------	------	0
-1.6	------	------	------	------	------	------	------	------	0
-1.2	------	------	------	------	------	------	------	------	0
-0.8	------	------	------	1.0000	------	------	------	------	1
-0.4	------	------	------	.5000	------	0.5000	------	------	2
0.0	------	------	0.5000	------	0.5000	------	------	------	2
0.4	------	------	.5000	------	.5000	------	------	------	2
0.8	------	------	.5000	.5000	------	------	------	------	2
1.2	------	1.0000	------	------	------	------	------	------	1
1.6	------	------	------	------	------	------	------	------	0
2.0	------	------	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	------	------	0
3.2	------	------	------	------	------	------	------	------	0
3.6	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	------	.125	.292	.300	.208	.075	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 27. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D=y_j = -2.8$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	1.0000	------	------	------	1
-2.4	------	------	------	------	1.0000	------	------	------	3
-2.0	------	------	------	0.2500	.7500	------	------	------	4
-1.6	------	------	------	.3333	.5000	------	------	0.1667	6
-1.2	------	------	------	.5714	.2857	------	0.1429	------	7
-0.8	------	------	------	.6250	.2500	------	.1250	------	8
-0.4	------	------	0.2222	.5556	------	0.2222	------	------	9
0.0	------	------	.5556	.2222	.1111	.1111	------	------	9
0.4	------	0.1111	.5556	.2222	.1111	------	------	------	9
0.8	------	.4286	.2857	.2857	------	------	------	------	7
1.2	0.3333	.6667	------	------	-------	------	------	------	3
1.6	------	------	------	------	------	------	------	------	0
2.0	------	------	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	------	------	0
3.2	------	------	------	------	------	------	------	------	0
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.034	.153	.260	.323	.154	.044	.025	.007	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 28. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D = y_j = -2.4$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	1.0000	------	------	------	1
-2.4	------	------	------	0.2500	.5000	0.2500	------	------	4
-2.0	------	------	------	.4286	.2857	.2857	------	------	7
-1.6	------	------	------	.4000	.3000	.2000	------	0.1000	10
-1.2	------	------	------	.3333	.4444	.1111	0.1111	------	18
-0.8	------	------	0.1429	.4762	.2381	.0952	.0476	------	21
-0.4	------	0.0333	.3333	.3000	.1667	.1667	------	------	30
0.0	------	.1290	.4194	.2581	.1613	.0323	------	------	31
0.4	------	.1786	.4643	.2857	.0714	------	------	------	28
0.8	0.0455	.3182	.4091	.2273	------	------	------	------	22
1.2	.1429	.2857	.5714	------	------	------	------	------	7
1.6	------	.3333	.6667	------	------	------	------	------	3
2.0	------	1.0000	------	------	------	------	------	------	1
2.4	------	------	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	------	------	0
3.2	------	------	------	------	------	------	------	------	0
3.6	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.020	.154	.332	.256	.142	.080	.012	.004	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 29. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i \cdot y_j)$ (Run 16)

$Y_D = y_j = -2.0$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	1.0000	------	------	------	------	1
-3.2	------	------	------	1.0000	------	------	------	------	1
-2.8	------	------	------	.5000	0.5000	------	------	------	2
-2.4	------	------	------	.3846	.3077	0.2308	------	0.0769	13
-2.0	------	------	0.0357	.2143	.4643	.1429	.1429	-------	28
-1.6	------	------	.0488	.3171	.3659	.1951	.0244	.0488	41
-1.2	------	------	.0508	.4068	.3390	.1186	.0847	------	59
-0.8	------	------	.1594	.4493	.2029	.1449	.0435	------	69
-0.4	------	0.0357	.3214	.3095	.2143	.1071	.0119	------	84
0.0	0.0118	.1059	.3882	.2824	.1529	.0588	------	------	85
0.4	------	.2051	.4231	.2821	.0897	------	------	------	78
0.8	.0308	.3692	.4308	.1538	.0154	------	------	------	65
1.2	.0357	.3929	.4643	.1071	------	------	------	------	28
1.6	------	.7000	.3000	------	------	------	------	------	10
2.0	------	1.0000	------	------	------	------	------	------	1
2.4	------	------	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	------	------	0
3.2	------	------	------	------	------	------	------	------	0
3.6	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.009	.191	.309	.268	.145	.059	.016	.003	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 30. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D=y_j = -1.6$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	1.0000	------	------	------	------	1
-3.2	------	------	------	1.0000	------	------	------	------	1
-2.8	------	------	------	.2500	0.5000	------	0.2500	------	4
-2.4	------	------	------	.3000	.4000	0.2500	------	0.0500	20
-2.0	------	------	0.0227	.2045	.4091	.2955	.0682	------	44
-1.6	------	------	.0349	.3256	.3256	.2326	.0698	.0116	86
-1.2	------	------	.1016	.3281	.3281	.1563	.0859	------	128
-0.8	0.0065	------	.1806	.4258	.2129	.1484	.0258	------	155
-0.4	------	0.0829	.2818	.3481	.2099	.0663	.0110	------	181
0.0	.0109	.1739	.3233	.2989	.1539	.0217	.0054	------	184
0.4	.0124	.1988	.4410	.2857	.0559	.0062	------	------	161
0.8	.0242	.3387	.4839	.1290	.0242	.0081	------	------	124
1.2	.0308	.4615	.4154	.0769	.0154	------	------	------	65
1.6	.0385	.5769	.3846	------	------	------	------	------	26
2.0	------	.6000	.4000	------	------	------	------	------	5
2.4	------	1.0000	------	------	------	------	------	------	2
2.8	------	------	------	------	------	------	------	------	0
3.2	------	------	------	------	------	------	------	------	0
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.012	.200	.316	.258	.139	.060	.014	.001	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 31. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D=y_j = -1.2$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	1.0000	------	------	------	------	1
-3.2	------	------	------	1.0000	------	------	------	------	1
-2.8	------	------	------	.2000	0.6000	------	0.2000	------	5
-2.4	------	------	0.0400	.2800	.4000	0.2000	------	0.0800	25
-2.0	------	------	.0149	.2537	.5373	.1343	.0597	------	67
-1.6	------	------	.0391	.3828	.3125	.2031	.0547	.0078	128
-1.2	------	0.0105	.1105	.3737	.3053	.1474	.0526	------	190
-0.8	0.0084	.0126	.2134	.4184	.2259	.1046	.0126	.0042	239
-0.4	.0036	.1103	.2989	.3310	.1922	.0498	.0107	.0036	281
0.0	.0306	.1565	.4048	.2857	.0986	.0204	.0034	------	294
0.4	.0266	.2395	.4677	.2129	.0418	.0076	.0038	------	263
0.8	.0250	.4050	.4450	.1000	.0200	.0050	------	------	200
1.2	.0492	.5492	.3033	.0738	.0246	------	------	------	122
1.6	.1200	.5800	.2800	.0200	------	------	------	------	50
2.0	------	.6667	.3333	------	------	------	------	------	15
2.4	------	.8000	.2000	------	------	------	------	------	5
2.8	------	1.0000	------	------	------	------	------	------	1
3.2	------	1.0000	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.024	.227	.316	.242	.130	.046	.013	.002	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 32. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D=y_j = -0.8$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	1.0000	------	------	------	------	1
-3.2	------	------	------	1.0000	------	------	------	------	2
-2.8	------	------	------	.2857	0.5714	------	0.1429	------	7
-2.4	------	------	0.0370	.3704	.3333	0.1852	.0741	------	27
-2.0	------	------	.0270	.2432	.5405	.1486	.0405	------	74
-1.6	------	------	.0592	.3553	.3421	.1776	.0658	------	152
-1.2	------	0.0087	.1179	.4148	.2969	.1179	.0437	------	229
-0.8	0.0068	.0341	.2287	.3993	.2389	.0785	.0102	0.0034	293
-0.4	.0113	.1155	.3352	.3099	.1803	.0423	.0028	.0028	355
0.0	.0372	.1941	.3830	.2766	.0957	.0106	.0027	------	376
0.4	.0359	.3024	.4162	.2006	.0359	.0090	------	------	334
0.8	.0551	.4173	.4016	.1102	.0118	.0039	------	------	254
1.2	.0927	.5298	.3046	.0662	.0066	------	------	------	151
1.6	.1500	.6167	.2166	.0167	------	------	------	------	60
2.0	.0455	.7727	.1818	------	------	------	------	------	22
2.4	.1250	.8750	------	------	------	------	------	------	8
2.8	------	1.0000	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.038	.250	.299	.237	.125	.039	.011	.001	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 33. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D=y_j = -0.4$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	1.0000	------	------	------	------	1
-3.2	------	------	------	.6667	0.3333	------	------	------	3
-2.8	------	------	------	.3750	.5000	0.1250	------	------	8
-2.4	------	------	0.0357	.3929	.3214	.1786	0.0714	------	28
-2.0	------	------	.0370	.2963	.4568	.1358	.0617	0.0123	81
-1.6	------	------	.0632	.3908	.3161	.1667	.0517	.0115	174
-1.2	------	0.0074	.1338	.4498	.2639	.1004	.0372	.0074	269
-0.8	0.0029	.0411	.2786	.3578	.2346	.0674	.0147	.0029	341
-0.4	.0146	.1335	.3617	.2864	.1553	.0437	.0049	------	412
0.0	.0520	.2240	.3710	.2715	.0633	.0158	.0023	------	442
0.4	.0530	.3283	.4015	.1843	.0253	.0076	------	------	396
0.8	.0808	.4714	.3603	.0774	.0067	.0034	------	------	297
1.2	.1193	.5511	.2784	.0511	------	------	------	------	176
1.6	.2571	.5857	.1571	------	------	------	------	------	70
2.0	.1154	.7692	.1154	------	------	------	------	------	26
2.4	.1111	.8889	------	------	------	------	------	------	9
2.8	------	1.0000	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.056	.257	.298	.228	.112	.037	.010	.002	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 34. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D = y_j = 0.0$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	1.0000	------	------	------	------	1
-3.2	------	------	------	1.0000	------	------	------	------	3
-2.8	------	------	0.1250	.3750	0.3750	0.1250	------	------	8
-2.4	------	------	.0714	.3571	.3571	.1786	0.0357	------	28
-2.0	------	------	.0595	.3214	.4524	.1190	.0476	------	84
-1.6	------	0.0055	.1215	.3812	.2928	.1602	.0331	0.0055	181
-1.2	------	.0106	.1837	.4629	.2191	.1131	.0106	------	283
-0.8	0.0027	.0603	.3205	.3425	.2192	.0438	.0082	.0027	365
-0.4	.0182	.1663	.3736	.2733	.1390	.0273	.0023	------	439
0.0	.0871	.2407	.2859	.2220	.0519	.0104	.0021	------	482
0.4	.0837	.3814	.3651	.1488	.0163	.0047	------	------	430
0.8	.1136	.4984	.3218	.0599	.0063	------	------	------	317
1.2	.2270	.5081	.2324	.0324	------	------	------	------	185
1.6	.2877	.5616	.1507	------	------	------	------	------	73
2.0	.2222	.6667	.1111	------	------	------	------	------	27
2.4	.3000	.7000	------	------	------	------	------	------	10
2.8	------	1.0000	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.084	.284	.292	.206	.097	.031	.005	.001	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 35. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D = y_j = 0.4$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	1.0000	------	------	------	------	1
-3.2	------	------	0.3333	.6667	------	------	------	------	3
-2.8	------	------	.2500	.2500	0.3750	0.1250	------	------	8
-2.4	------	------	.1034	.3793	.3103	.1724	0.0355	------	29
-2.0	------	------	.1111	.3457	.4198	.0864	.0370	------	81
-1.6	------	0.0059	.1302	.4497	.2249	.1775	.0118	------	169
-1.2	------	.0150	.1992	.4737	.2331	.0752	.0038	------	266
-0.8	------	.0504	.3501	.3442	.2166	.0326	.0030	0.0030	337
-0.4	0.0075	.1421	.4190	.2993	.1147	.0150	.0025	------	401
0.0	.0608	.2635	.4144	.2185	.0360	.0068	------	------	444
0.4	.1272	.4108	.3326	.1161	.0112	.0022	------	------	448
0.8	.1723	.5108	.2646	.0462	.0062	------	------	------	325
1.2	.2857	.4868	.1958	.0317	------	------	------	------	189
1.6	.3288	.5479	.1233	------	------	------	------	------	73
2.0	.2593	.6296	.1111	------	------	------	------	------	27
2.4	.3000	.7000	------	------	------	------	------	------	10
2.8	1.0000	------	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.103	.287	.292	.203	.087	.025	.003	.000	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 36. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D = y_j = 0.8$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	1.0000	------	------	------	------	1
-3.2	------	------	0.6667	.3333	------	------	------	------	3
-2.8	------	------	.2500	.3750	0.2500	0.1250	------	------	8
-2.4	------	------	.1923	.3462	.3846	.0385	0.0385	------	26
-2.0	------	------	.1791	.3731	.3433	.1045	------	------	67
-1.6	------	0.0068	.1575	.4726	.2466	.1027	.0137	------	146
-1.2	------	.0179	.2646	.4305	.2377	.0448	.0045	------	223
-0.8	------	.0441	.3566	.3713	.1912	.0331	.0037	------	272
-0.4	------	.1285	.4608	.2947	.1003	.0157	------	------	319
0.0	0.0200	.3114	.4086	.2143	.0400	.0057	------	------	350
0.4	.1044	.4478	.3352	.0989	.0137	------	------	------	364
0.8	.3021	.4471	.2115	.0393	------	------	------	------	331
1.2	.3105	.5105	.1737	.0053	------	------	------	------	190
1.6	.3973	.4932	.1096	------	------	------	------	------	73
2.0	.2592	.6667	.0741	------	------	------	------	------	27
2.4	.4000	.6000	.0741	------	------	------	------	------	10
2.8	1.0000	------	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.115	.290	.298	.195	.082	.018	.002	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 37. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D=y_j = 1.2$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	1.0000	------	------	------	------	1
-3.2	------	------	1.0000	------	------	------	------	------	1
-2.8	------	------	.2000	.6000	0.2000	------	------	------	5
-2.4	------	------	.1538	.3846	.3846	------	0.0769	------	13
-2.0	------	------	.1290	.3226	.4516	0.0968	------	------	31
-1.6	------	------	.1216	.4459	.2973	.1081	.0270	------	74
-1.2	------	------	.2232	.4107	.3125	.0536	------	------	112
-0.8	------	0.0205	.2808	.4452	.2055	.0479	------	------	146
-0.4	------	.0629	.4229	.3714	.1314	.0114	------	------	175
0.0	------	.2284	.4822	.2284	.0609	------	------	------	197
0.4	0.0195	.4195	.4195	.1317	.0098	------	------	------	205
0.8	.1691	.5314	.2415	.0580	------	------	------	------	207
1.2	.4091	.4545	.1364	------	------	------	------	------	198
1.6	.5135	.4054	.0811	------	------	------	------	------	74
2.0	.3333	.6296	.0370	------	------	------	------	------	27
2.4	.5000	.5000	------	------	------	------	------	------	10
2.8	1.0000	------	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.096	.263	.305	.218	.099	.017	.002	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 38. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D=y_j = 1.6$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	1.0000	------	------	------	------	1
-2.0	------	------	0.1429	.7143	0.1429	------	------	------	7
-1.6	------	------	.1905	.4286	.3333	0.0476	------	------	21
-1.2	------	------	.2162	.4054	.3243	.0541	------	------	37
-0.8	------	------	.2500	.5192	.2115	.0192	------	------	52
-0.4	------	0.0625	.3906	.4375	.0938	.0156	------	------	64
0.0	------	.1429	.5714	.2338	.0519	------	------	------	77
0.4	------	.3580	.4691	.1728	------	------	------	------	81
0.8	0.1084	.5422	.2892	.0602	------	------	------	------	83
1.2	.2874	.5517	.1609	------	------	------	------	------	87
1.6	.6081	.3108	.0811	------	------	------	------	------	74
2.0	.4074	.5556	.0370	------	------	------	------	------	27
2.4	.7000	.3000	------	------	------	------	------	------	10
2.8	1.0000	------	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.080	.245	.325	.259	.081	.010	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 39. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D=y_j = 2.0$

$Y_E=y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	1.0000	------	------	------	------	1
-2.0	------	------	0.5000	.5000	------	------	------	------	2
-1.6	------	------	.2000	.4000	0.2000	0.2000	------	------	5
-1.2	------	------	.4444	.1111	.4444	------	------	------	9
-0.8	------	------	.2632	.4210	.2632	.0526	------	------	19
-0.4	------	0.0476	.4286	.3333	.1905	------	------	------	21
0.0	------	.2222	.5185	.2222	.0370	------	------	------	27
0.4	0.0345	.3793	.4483	.1379	------	------	------	------	29
0.8	.0667	.4333	.4000	.1000	------	------	------	------	30
1.2	.3125	.4688	.2188	------	------	------	------	------	32
1.6	.4062	.4375	.1563	------	------	------	------	------	32
2.0	.5556	.4074	.0370	------	------	------	------	------	27
2.4	.7000	.3000	------	------	------	------	------	------	10
2.8	1.0000	------	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.077	.239	.367	.206	.095	.016	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 40. *Sample conditional probability mass function of step lengths*, $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D = y_j = 2.4$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	------	------	------	0
-2.0	------	------	------	------	------	------	------	------	0
-1.6	------	------	------	0.3333	0.3333	0.3333	------	------	3
-1.2	------	------	0.1667	.1667	.6666	------	------	------	6
-0.8	------	------	.3000	.4000	.3000	------	------	------	10
-0.4	------	------	.5455	.3636	.0909	------	------	------	11
0.0	------	0.2857	.4286	.2857	------	------	------	------	14
0.4	------	.2500	.5625	.1875	------	------	------	------	16
0.8	------	.3125	.6250	.0625	------	------	------	------	16
1.2	0.1250	.6250	.2500	------	------	------	------	------	16
1.6	.1875	.6250	.1875	------	------	------	------	------	16
2.0	.3750	.6250	------	------	------	------	------	------	16
2.4	.8000	.2000	------	------	------	------	------	------	10
2.8	1.0000	------	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.032	.237	.405	.204	.107	.015	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 41. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D = y_j = 2.8$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	------	------	------	0
-2.0	------	------	------	------	------	------	------	------	0
-1.6	------	------	------	------	1.0000	------	------	------	1
-1.2	------	------	0.5000	0.5000	------	------	------	------	2
-0.8	------	------	.5000	.5000	------	------	------	------	2
-0.4	------	------	.5000	.5000	------	------	------	------	2
0.0	------	0.5000	------	.5000	------	------	------	------	2
0.4	------	.5000	.5000	------	------	------	------	------	2
0.8	------	.5000	.5000	------	------	------	------	------	2
1.2	------	.5000	.5000	------	------	------	------	------	2
1.6	------	1.0000	------	------	------	------	------	------	2
2.0	0.5000	.5000	------	------	------	------	------	------	2
2.4	.5000	.5000	------	------	------	------	------	------	2
2.8	1.0000	------	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.011	.347	.372	.224	.046	------	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 42. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 16)

$Y_D = y_j = 3.2$

$Y_E = y_i$	Class Mark, x_β, ft								$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	------	------	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	------	------	------	0
-2.0	------	------	------	------	------	------	------	------	0
-1.6	------	------	------	------	------	------	------	------	0
-1.2	------	------	1.0000	------	------	------	------	------	1
-0.8	------	------	1.0000	------	------	------	------	------	1
-0.4	------	1.0000	------	------	------	------	------	------	1
0.0	------	1.0000	------	------	------	------	------	------	1
0.4	------	1.0000	------	------	------	------	------	------	1
0.8	------	1.0000	------	------	------	------	------	------	1
1.2	------	1.0000	------	------	------	------	------	------	1
1.6	1.0000	------	------	------	------	------	------	------	1
2.0	1.0000	------	------	------	------	------	------	------	1
2.4	1.0000	------	------	------	------	------	------	------	1
2.8	1.0000	------	------	------	------	------	------	------	1
3.2	1.0000	------	------	------	------	------	------	------	1
3.6	------	------	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.072	.735	.193	------	------	------	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 43. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D=y_j$ = -2.8

$Y_E=y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	------	0
-2.0	------	------	------	------	------	------	0
-1.6	------	------	------	------	------	------	0
-1.2	------	------	------	------	------	------	0
-0.8	------	------	------	------	------	------	0
-0.4	------	------	------	------	------	------	0
0.0	------	------	------	1.0000	------	------	1
0.4	------	0.5000	0.5000	------	------	------	2
0.8	------	1.0000	------	------	------	------	1
1.2	------	------	------	------	------	------	0
1.6	------	------	------	------	------	------	0
2.0	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	------	.452	.179	.368	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 44. *Sample conditional probability mass function of step lengths*, $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D = y_j$ = -2.4

$Y_E=y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	------	0
-2.0	------	------	------	------	------	------	0
-1.6	------	------	0.3333	0.3333	0.3333	------	3
-1.2	------	0.1429	.4286	.2857	.1429	------	7
-0.8	------	.4286	.1428	.2857	.1429	------	7
-0.4	------	.4286	.1428	.4286	------	------	7
0.0	------	------	.5000	.5000	------	------	6
0.4	------	.4286	.5714	------	------	------	7
0.8	0.2500	.2500	.5000	------	------	------	4
1.2	------	1.0000	------	------	------	------	1
1.6	------	1.0000	------	------	------	------	1
2.0	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.030	.356	.333	.231	.050	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 45. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D=y_j$ = -2.0

$Y_E=y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	0
-2.4	------	------	------	1.0000	------	------	1
-2.0	------	------	------	.6667	0.3333	------	3
-1.6	------	------	0.1818	.4546	.3636	------	11
-1.2	------	0.0455	.3182	.3636	.2727	------	22
-0.8	------	.2059	.2353	.4118	.1176	0.0294	34
-0.4	------	.2105	.3947	.3421	.0526	------	38
0.0	0.0513	.1538	.5128	.2308	.0513	------	39
0.4	.0555	.3333	.5556	.0555	------	------	36
0.8	.0417	.5417	.3750	.0417	------	------	24
1.2	------	.9167	.0833	------	------	------	12
1.6	------	1.0000	------	------	------	------	3
2.0	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.022	.316	.349	.229	.080	.004	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 46. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D=y_j$ = -1.6

$Y_E=y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	0
-2.4	------	------	------	1.0000	------	------	1
-2.0	------	------	0.1111	.5556	0.3333	------	9
-1.6	0.0286	------	.1429	.4571	.3143	0.0571	35
-1.2	------	0.0395	.2632	.4342	.2632	------	76
-0.8	------	.1651	.3211	.3853	.1284	------	109
-0.4	------	.1797	.4531	.3281	.0391	------	128
0.0	.0465	.2481	.4961	.2016	.0078	------	129
0.4	.0286	.3714	.5429	.0571	------	------	105
0.8	.0147	.5882	.3676	.0294	------	------	68
1.2	------	.8214	.1786	------	------	------	28
1.6	.1111	.7778	.1111	------	------	------	9
2.0	------	------	------	------	------	------	0
2.4	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.016	.317	.372	.223	.069	.003	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 47. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D=y_j = -1.2$

$Y_E=y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	0
-2.4	------	------	0.2500	0.2500	0.2500	0.2500	4
-2.0	------	------	.3478	.4783	.1304	.0435	23
-1.6	0.0119	0.0238	.2738	.3810	.2976	.0119	84
-1.2	.0111	.0611	.3611	.4278	.1389	------	180
-0.8	.0077	.1969	.3629	.3745	.0579	------	259
-0.4	.0235	.2584	.4631	.2350	.0201	------	298
0.0	.0532	.3522	.4884	.1030	.0033	------	301
0.4	.0560	.5080	.4000	.0360	------	------	250
0.8	.0440	.7233	.2138	.0189	------	------	159
1.2	.0896	.7612	.1492	------	------	------	67
1.6	.1579	.6316	.2105	------	------	------	19
2.0	------	1.0000	------	------	------	------	2
2.4	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.041	.389	.352	.173	.042	.003	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 48. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D = y_j = -0.8$

$Y_E = y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	1.0000	------	1
-2.4	------	------	0.1250	0.5000	.3750	------	8
-2.0	------	------	.3793	.4138	.2069	------	29
-1.6	0.0088	0.0442	.2655	.4690	.2124	------	113
-1.2	.0074	.0852	.3963	.4222	.0778	0.0111	270
-0.8	.0123	.2094	.4236	.3202	.0296	.0049	406
-0.4	.0396	.2792	.4750	.1875	.0167	.0021	480
0.0	.0811	.4037	.4381	.0669	.0081	.0020	493
0.4	.1087	.5411	.3188	.0290	.0024	------	414
0.8	.1236	.6891	.1760	.0075	.0037	------	267
1.2	.2069	.6810	.0948	.0172	------	------	116
1.6	.2667	.6667	.0666	------	------	------	30
2.0	------	1.0000	------	------	------	------	2
2.4	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.078	.402	.329	.156	.033	.002	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 49. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D=y_j = -0.4$

$Y_E=y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	1.0000	------	------	2
-2.4	------	------	0.3077	.6154	0.0769	------	13
-2.0	------	0.0250	.3250	.4500	.1750	0.0250	40
-1.6	------	.0597	.3358	.4552	.1493	------	134
-1.2	------	.1302	.4159	.3714	.0794	.0032	315
-0.8	0.0103	.2320	.4538	.2710	.0287	.0041	487
-0.4	.0614	.3217	.4594	.1443	.0116	.0017	603
0.0	.0981	.4351	.3956	.0649	.0063	------	632
0.4	.1541	.5468	.2697	.0275	.0018	------	545
0.8	.1989	.6835	.1064	.0112	------	------	357
1.2	.2830	.6415	.0629	.0126	------	------	159
1.6	.3556	.6000	.0444	------	------	------	45
2.0	.2500	.7500	------	------	------	------	4
2.4	------	------	------	------	------	------	0
2.8	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.113	.411	.310	.141	.024	.001	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 50. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D = y_j = 0.0$

$Y_E = y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	0.5000	0.5000	------	------	2
-2.4	------	------	.3077	.6154	0.0769	------	13
-2.0	------	0.0455	.4318	.3636	.1591	------	44
-1.6	------	.0725	.4058	.4130	.1087	------	138
-1.2	------	.1683	.4222	.3492	.0571	0.0032	315
-0.8	0.0120	.2371	.5020	.2271	.0199	.0020	502
-0.4	.0563	.3772	.4554	.1033	.0078	------	639
0.0	.1491	.4574	.3509	.0398	.0028	------	704
0.4	.2210	.5645	.1984	.0161	------	------	620
0.8	.2725	.6448	.0730	.0097	------	------	411
1.2	.3968	.5556	.0423	.0053	------	------	189
1.6	.5273	.4364	.0364	------	------	------	55
2.0	.6000	.4000	------	------	------	------	5
2.4	1.0000	------	------	------	------	------	1
2.8	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.164	.406	.297	.115	.017	.001	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 51. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D = y_j = 0.4$

$Y_E = y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	1.0000	------	------	------	1
-2.4	------	0.1111	.2222	0.5556	0.1111	------	9
-2.0	------	.0513	.4615	.3077	.1795	------	39
-1.6	------	.0650	.3984	.4636	.0732	------	123
-1.2	------	.1382	.4545	.3600	.0473	------	275
-0.8	0.0022	.2405	.5367	.2004	.0178	0.0022	449
-0.4	.0316	.4088	.4684	.0825	.0088	------	570
0.0	.1360	.5008	.3338	.0294	------	------	647
0.4	.3067	.5462	.1346	.0125	------	------	639
0.8	.3553	.5765	.0635	.0047	------	------	425
1.2	.5101	.4495	.0404	------	------	------	198
1.6	.7167	.2667	.0167	------	------	------	60
2.0	.6000	.4000	------	------	------	------	5
2.4	1.0000	------	------	------	------	------	1
2.8	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.196	.390	.292	.107	.015	.000	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 52. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D=y_j = 0.8$

$Y_E=y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	1.0000	------	------	------	1
-2.4	------	0.1250	.3750	0.3750	0.1250	------	8
-2.0	------	.1071	.3929	.3929	.1071	------	28
-1.6	------	.0619	.3814	.5155	.0412	------	97
-1.2	------	.1337	.4802	.3416	.0446	------	202
-0.8	0.0031	.2346	.5741	.1759	.0123	------	324
-0.4	.0341	.3683	.5195	.0732	.0049	------	410
0.0	.0682	.5565	.3454	.0299	------	------	469
0.4	.2360	.6294	.1284	.0062	------	------	483
0.8	.4707	.4754	.0515	.0023	------	------	427
1.2	.6050	.3700	.0250	------	------	------	200
1.6	.7581	.2258	.0161	------	------	------	62
2.0	.8000	.2000	------	------	------	------	5
2.4	1.0000	------	------	------	------	------	1
2.8	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.201	.382	.304	.102	.011	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 53. *Sample conditional probability mass function of step lengths*, $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D = y_j$ = 1.2

$Y_E = y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	0
-2.4	------	------	0.5000	0.2500	0.2500	------	4
-2.0	------	------	.5882	.2941	.1176	------	17
-1.6	------	0.0444	.4889	.4222	.0444	------	45
-1.2	------	.1414	.5455	.2828	.0303	------	99
-0.8	------	.2258	.6258	.1419	.0065	------	155
-0.4	0.0050	.3682	.5274	.0945	.0050	------	201
0.0	.0553	.5660	.3447	.0340	------	------	235
0.4	.1347	.7143	.1429	.0082	------	------	245
0.8	.4211	.5223	.0526	.0040	------	------	247
1.2	.7500	.2300	.0200	------	------	------	200
1.6	.8387	.1452	.0161	------	------	------	62
2.0	.8000	.2000	------	------	------	------	5
2.4	1.0000	------	------	------	------	------	1
2.8	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.187	.384	.327	.092	.010	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 54. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_E = y_j = 1.6$

$Y_E = y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	0
-2.4	------	------	------	0.5000	0.5000	------	2
-2.0	------	------	0.5714	.2857	.1429	------	7
-1.6	------	------	.4667	.5333	------	------	15
-1.2	------	0.1081	.6486	.2162	.0270	------	37
-0.8	------	.2453	.5660	.1887	------	------	53
-0.4	------	.4203	.4493	.1159	.0145	------	69
0.0	0.0375	.5375	.3500	.0750	------	------	80
0.4	.1071	.7143	.1667	.0119	------	------	84
0.8	.3176	.5882	.0941	------	------	------	85
1.2	.6353	.3412	.0235	------	------	------	85
1.6	.9032	.0806	.0161	------	------	------	62
2.0	.8000	.2000	------	------	------	------	5
2.4	1.0000	------	------	------	------	------	1
2.8	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.161	.399	.323	.107	.010	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 55. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i=y_j)$ (Run 17)

$Y_D=y_j = 2.0$

$Y_E=y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	0
-2.4	------	------	0.5000	0.5000	------	------	2
-2.0	------	------	.6667	.3333	------	------	3
-1.6	------	------	.6667	.3333	------	------	3
-1.2	------	0.2000	.6000	.2000	------	------	5
-0.8	------	.5000	.3750	.1250	------	------	8
-0.4	------	.6250	.3750	------	------	------	8
0.0	------	.6000	.4000	------	------	------	10
0.4	------	.8182	.1818	------	------	------	11
0.8	0.4167	.5000	.0833	------	------	------	12
1.2	.5000	.5000	------	------	------	------	12
1.6	.8333	.1667	------	------	------	------	12
2.0	1.0000	------	------	------	------	------	5
2.4	1.0000	------	------	------	------	------	1
2.8	------	------	------	------	------	------	0
$p_{X\|Y_D}(x_\beta\|y_j)$	.141	.497	.304	.058	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 56. *Sample conditional probability mass function of step lengths,* $p_{X|Y_E,Y_D}(x_\beta|y_i,y_j)$ (Run 17)

$Y_D = y_j = 2.4$

$Y_E = y_i$	Class Mark, x_β, ft						$m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.6	------	------	------	------	------	------	0
-3.2	------	------	------	------	------	------	0
-2.8	------	------	------	------	------	------	0
-2.4	------	------	------	------	------	------	0
-2.0	------	------	------	------	------	------	0
-1.6	------	------	------	------	------	------	0
-1.2	------	------	------	------	------	------	0
-0.8	------	------	------	------	------	------	0
-0.4	------	------	------	------	------	------	0
0.0	------	------	------	------	------	------	0
0.4	------	1.0000	------	------	------	------	1
0.8	------	1.0000	------	------	------	------	1
1.2	1.0000	------	------	------	------	------	1
1.6	1.0000	------	------	------	------	------	1
2.0	1.0000	------	------	------	------	------	1
2.4	1.0000	------	------	------	------	------	1
2.8	------	------	------	------	------	------	0
$p_{X|Y_D}(x_\beta|y_j)$	.333	.667	------	------	------	------	----

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 57. *Conditional means and variances of step lengths;* $\hat{E}[X|Y_E = y_i,\ Y_D = y_j]$ and $\widehat{Var}[X|Y_E = y_i,\ Y_D = y_j]$ (Run 4A)

$Y_E = y_i$ \ $Y_D = y_j$	-3.6	-3.2	-2.8	-2.4	-2.0	-1.6	-1.2	-0.8	-0.4	0.0	0.4	0.8	1.2	1.6	2.0	2.4
-3.6	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	4.189 -----	4.057 -----	4.000 -----	3.934 -----	3.857 -----	----- -----	----- -----	----- -----
-3.2	----- -----	----- -----	----- -----	----- -----	----- -----	3.974 -----	3.038 1.246	2.842 1.417	3.150 1.235	3.647 .101	3.549 .141	3.440 .191	3.283 .302	----- -----	----- -----	----- -----
-2.8	----- -----	----- -----	----- -----	----- -----	3.792 -----	4.172 1.570	3.823 2.148	3.620 1.918	3.385 1.529	3.239 1.021	3.207 .839	3.195 .941	2.965 .360	2.768 .395	----- -----	----- -----
-2.4	----- -----	----- -----	3.636 .373	3.215 .132	4.730 3.957	4.024 1.500	3.605 1.849	3.476 1.768	3.409 1.684	3.459 1.357	3.357 1.156	3.309 .966	3.184 .743	3.199 .985	5.346 -----	----- -----
-2.0	----- -----	----- -----	3.298 .605	3.843 .978	4.214 1.409	3.771 .986	3.407 1.133	3.261 1.074	3.133 1.069	3.102 .872	3.036 .783	2.935 .665	2.909 .557	3.000 .707	4.308 1.505	----- -----
-1.6	----- -----	----- -----	2.848 .507	2.944 1.114	3.417 1.665	3.239 1.175	3.173 1.176	3.076 .991	2.989 .974	2.956 .779	2.898 .714	2.804 .599	2.773 .555	2.820 .559	3.485 .987	----- -----
-1.2	----- -----	2.861 -----	2.704 .766	2.848 1.009	3.018 1.125	3.136 1.439	3.063 1.390	2.976 1.210	2.854 1.046	2.808 .872	2.785 .806	2.717 .726	2.710 .682	2.733 .669	3.175 1.055	-----
-0.8	----- -----	2.611 .054	2.478 .392	2.588 .620	2.747 .942	2.785 1.137	2.653 1.145	2.583 1.063	2.505 .887	2.498 .759	2.507 .683	2.466 .623	2.503 .603	2.562 .669	2.879 1.155	----- -----
-0.4	----- -----	2.082 .098	2.434 .836	2.506 .796	2.589 1.251	2.519 1.229	2.455 1.197	2.336 1.001	2.180 .925	2.170 .772	2.197 .656	2.205 .602	2.275 .563	2.360 .653	2.604 1.225	----- -----
0.0	----- -----	1.892 -----	2.557 .609	2.300 .584	2.400 1.011	2.274 1.069	2.221 1.084	2.083 .950	1.929 .854	1.797 .813	1.843 .666	1.916 .581	2.006 .542	2.081 .540	2.350 1.296	----- -----
0.4	----- -----	----- -----	2.312 .943	2.289 .401	2.365 .985	2.159 .986	2.054 .910	1.903 .771	1.728 .723	1.595 .652	1.437 .637	1.521 .495	1.661 .435	1.774 .429	2.087 1.192	----- -----
0.8	----- -----	----- -----	1.692 .010	2.133 .892	2.257 1.076	2.108 1.091	1.933 .837	1.775 .690	1.604 .634	1.439 .550	1.306 .485	1.127 .416	1.244 .367	1.394 .357	1.829 1.106	----- -----
1.2	----- -----	----- -----	----- -----	1.731 .020	2.279 .755	2.188 1.127	1.903 .889	1.756 .626	1.581 .608	1.373 .501	1.203 .429	1.036 .313	.870 .301	.968 .297	1.508 1.035	----- -----
1.6	----- -----	----- -----	----- -----	1.430 -----	1.518 .241	1.779 1.693	1.687 .942	1.558 .539	1.503 .646	1.288 .424	1.142 .397	.932 .273	.812 .275	.595 .270	1.081 .829	----- -----
2.0	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	1.670 -----	1.377 .000	1.375 .084	1.086 .120	.954 .125	.869 .128	.785 .132	.687 .147	.460 .203	----- -----
2.4	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----
$\hat{E}[X\|Y_D = y_j]$	-----	1.294	2.172	2.397	2.577	2.486	2.384	2.257	2.119	2.025	1.972	1.938	1.983	2.036	2.458	-----
$\widehat{Var}[X\|Y_D = y_j]$	-----	1.398	1.232	.884	1.376	1.415	1.277	1.141	1.094	1.035	1.000	.935	.892	.915	1.671	-----

Note: Upper values are the means, in feet, and lower values are the variances, in feet squared.

Table 58. *Conditional means and variances of step lengths;* $\hat{E}[X|Y_E = y_i,\ Y_D = y_j]$ and $\widehat{Var}[X|Y_E = y_i,\ Y_D = y_j]$ (Run 16)

$Y_E = y_i$ \ $Y_D = y_j$	-3.6	-3.2	-2.8	-2.4	-2.0	-1.6	-1.2	-0.8	-0.4	0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2
-3.6	-----	-----	-----	-----	3.655	3.592	3.531	3.469	3.407	3.338	3.263	3.186	3.111	-----	-----	-----	-----	-----
	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
-3.2	-----	-----	-----	-----	3.442	3.379	3.318	3.496	3.594	3.427	3.102	3.042	2.897	-----	-----	-----	-----	-----
	-----	-----	-----	-----	-----	-----	-----	.115	.164	.091	.026	.051	-----	-----	-----	-----	-----	-----
-2.8	-----	-----	4.600	4.516	3.852	4.551	4.447	4.293	4.142	4.027	3.833	3.696	3.830	-----	-----	-----	-----	-----
	-----	-----	-----	-----	.678	1.068	1.179	.930	.832	.868	1.047	.983	.573	-----	-----	-----	-----	-----
-2.4	-----	-----	4.334	4.415	4.548	4.612	4.621	4.480	4.363	4.217	4.063	3.905	3.909	3.098	3.051	-----	-----	-----
	-----	-----	.032	.467	1.059	.979	1.077	1.053	1.001	.869	.892	.913	.843	-----	-----	-----	-----	-----
-2.0	-----	-----	4.185	4.477	4.623	4.626	4.514	4.437	4.399	4.223	4.083	3.931	4.012	3.591	3.126	-----	-----	-----
	-----	-----	.067	.545	.809	.760	.699	.752	.904	.746	.728	.716	.656	.199	.223	-----	-----	-----
-1.6	-----	-----	4.649	4.646	4.438	4.524	4.369	4.327	4.261	4.075	3.981	3.834	3.987	3.851	3.816	4.430	4.379	-----
	-----	-----	1.942	1.396	1.074	1.032	.931	.950	1.067	.953	.829	.790	.733	.620	1.163	.858	-----	-----
-1.2	-----	-----	4.390	4.513	4.295	4.269	4.113	4.039	3.942	3.783	3.694	3.562	3.685	3.689	3.568	3.952	3.243	2.391
	-----	-----	1.397	1.076	1.038	1.079	.960	.918	.978	.880	.748	.705	.674	.665	1.074	.842	.904	-----
-0.8	3.410	3.256	3.919	4.020	3.961	3.878	3.726	3.632	3.553	3.399	3.356	3.287	3.474	3.481	3.519	3.580	2.933	2.112
	-----	-----	1.062	.837	.982	1.077	1.131	1.100	1.082	1.012	.833	.783	.747	.609	.840	.507	.824	-----
-0.4	3.197	4.391	3.823	3.633	3.608	3.443	3.294	3.188	3.079	2.936	2.927	2.917	3.072	3.116	3.108	3.129	2.698	1.913
	-----	3.638	1.307	1.236	1.108	1.069	1.121	1.126	1.131	1.059	.869	.762	.656	.524	.603	.483	.733	-----
0.0	2.917	3.802	3.384	3.076	3.132	2.969	2.851	2.731	2.617	2.446	2.436	2.457	2.638	2.674	2.582	2.657	2.459	1.726
	-----	2.159	.958	1.092	1.020	1.042	1.028	1.021	1.050	1.032	.851	.714	.583	.462	.584	.502	.611	-----
0.4	2.534	3.273	2.893	2.727	2.769	2.692	2.559	2.437	2.316	2.155	1.963	1.960	2.174	2.262	2.296	2.421	2.219	1.574
	-----	1.598	.860	.695	.737	.745	.745	.760	.756	.733	.766	.675	.516	.411	.498	.424	.433	-----
0.8	2.266	2.844	2.413	2.317	2.285	2.300	2.207	2.104	1.987	1.846	1.718	1.519	1.684	1.824	1.938	2.116	1.999	1.409
	-----	1.072	.739	.576	.599	.591	.629	.617	.582	.558	.551	.612	.465	.367	.459	.358	.337	-----
1.2	1.728	1.574	1.431	2.067	2.127	2.058	1.966	1.844	1.713	1.598	1.498	1.388	1.175	1.358	1.544	1.788	1.769	1.197
	-----	-----	.213	.601	.428	.481	.527	.510	.468	.434	.427	.410	.416	.349	.433	.312	.308	-----
1.6	-----	-----	-----	2.125	1.808	1.780	1.687	1.628	1.481	1.385	1.297	1.215	1.114	.946	1.195	1.438	1.512	.998
	-----	-----	-----	.041	.130	.311	.373	.371	.342	.335	.338	.340	.340	.383	.390	.252	.223	-----
2.0	-----	-----	-----	1.590	1.371	1.811	1.718	1.676	.1507	1.419	1.340	1.269	1.198	1.109	.960	1.054	1.257	.814
	-----	-----	-----	-----	-----	.160	.297	.229	.242	.232	.232	.234	.232	.237	.276	.183	.139	-----
2.4	-----	-----	-----	-----	-----	1.734	1.729	1.414	1.269	1.168	1.080	1.014	.949	.879	.802	.672	.923	.652
	-----	-----	-----	-----	-----	.020	.029	.072	.067	.080	.084	.083	.083	.083	.084	.096	.017	-----
2.8	-----	-----	-----	-----	-----	-----	1.604	1.291	1.195	1.096	1.000	.936	.885	.832	.780	.727	.675	.496
	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
3.2	-----	-----	-----	-----	-----	-----	1.284	.971	.875	.777	.681	.671	.565	.512	.460	.407	.355	.176
	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
$\hat{E}[X\|Y_D = y_j]$	2.073	2.512	3.008	2.924	3.097	3.049	2.927	2.825	2.716	2.567	2.473	2.402	2.517	2.500	2.516	2.550	2.308	1.495
$\widehat{Var}[X\|Y_D = y_j]$	1.463	3.470	2.290	3.635	1.530	1.512	1.499	1.509	1.547	1.477	1.432	1.381	1.392	1.234	1.232	1.409	1.164	.366

Note: Upper values are the means, in feet, and lower values are the variances, in feet squared.

Table 59. *Conditional means and variances of step lengths;* $\hat{E}[X|Y_E = y_i,\ Y_D = y_j]$ and $\widehat{Var}[X|Y_E = y_i,\ Y_D = y_j]$ (Run 17)

$Y_E = y_i$ \ $Y_D = y_j$	-3.2	-2.8	-2.4	-2.0	-1.6	-1.2	-0.8	-0.4	0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.8
-3.2	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
-2.8	-----	-----	-----	-----	-----	-----	4.657	3.553	3.157	2.792	2.663	-----	-----	-----	-----	-----
	-----	-----	-----	-----	-----	-----	-----	.343	.113	-----	-----	-----	-----	-----	-----	-----
-2.4	-----	-----	-----	3.984	3.909	4.026	3.773	3.354	3.188	3.191	3.048	3.206	3.580	3.308	-----	-----
	-----	-----	-----	-----	-----	1.057	.427	.407	.414	.504	.608	.491	.365	.340	-----	-----
-2.0	-----	-----	-----	4.118	3.699	3.441	3.319	3.285	3.104	3.051	3.000	3.030	3.225	2.868	-----	-----
	-----	-----	-----	.395	.335	.725	.575	.596	.546	.531	.542	.456	.360	.245	-----	-----
-1.6	-----	-----	3.655	3.693	3.686	3.471	3.320	3.210	3.076	3.056	2.983	2.932	2.979	2.633	-----	-----
	-----	-----	.456	.320	.824	.751	.623	.565	.546	.475	.421	.327	.208	.247	-----	-----
-1.2	-----	-----	3.066	3.369	3.408	3.117	3.008	2.922	2.818	2.816	2.769	2.704	2.707	2.515	-----	-----
	-----	-----	1.202	.700	.632	.611	.629	.589	.552	.477	.453	.411	.416	.706	-----	-----
-0.8	-----	-----	2.656	3.013	3.017	2.790	2.662	2.574	2.499	2.476	2.465	2.441	2.411	2.259	-----	-----
	-----	-----	1.053	.935	.733	.638	.681	.610	.530	.452	.408	.342	.330	.391	-----	-----
-0.4	-----	-----	2.516	2.772	2.709	2.461	2.354	2.232	2.141	2.129	2.151	2.187	2.219	1.998	-----	-----
	-----	-----	.833	.664	.613	.583	.616	.608	.529	.445	.383	.354	.407	.287	-----	-----
0.0	-----	3.396	2.878	2.523	2.379	2.153	2.041	1.934	1.792	1.765	1.814	1.877	1.942	1.798	-----	-----
	-----	-----	.223	.563	.522	.496	.516	.522	.502	.435	.351	.312	.358	.220	-----	-----
0.4	-----	2.110	2.131	2.123	2.091	1.896	1.763	1.652	1.513	1.377	1.421	1.521	1.625	1.527	1.624	-----
	-----	.965	.344	.457	.370	.360	.398	.405	.382	.403	.318	.260	.288	.181	-----	-----
0.8	-----	1.108	1.636	1.888	1.846	1.696	1.573	1.464	1.332	1.219	1.069	1.135	1.291	1.396	1.234	-----
	-----	-----	.439	.230	.237	.245	.289	.279	.257	.256	.272	.241	.247	.182	-----	-----
1.2	-----	-----	1.817	1.584	1.621	1.526	1.400	1.280	1.146	1.048	.941	.786	.891	.934	.967	-----
	-----	-----	-----	.090	.220	.253	.245	.248	.214	.210	.203	.212	.202	.147	-----	-----
1.6	-----	-----	1.226	1.285	1.520	1.540	1.289	1.190	1.038	.945	.841	.725	.545	.639	.753	-----
	-----	-----	-----	.106	.248	.254	.155	.183	.174	.168	.161	.157	.161	.133	-----	-----
2.0	-----	-----	-----	-----	-----	1.557	1.366	1.088	.948	.866	.793	.714	.629	.494	.546	-----
	-----	-----	-----	-----	-----	.060	.053	.108	.150	.148	.147	.139	.123	.102	-----	-----
2.4	-----	-----	-----	-----	-----	-----	-----	-----	.941	.796	.682	.577	.494	.411	.328	-----
	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
2.8	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----
$\hat{E}[X\|Y_D = y_j]$	-----	.939	2.311	2.473	2.440	2.271	2.153	2.042	1.925	1.866	1.841	1.847	1.899	1.777	.505	-----
$\widehat{Var}[X\|Y_D = y_j]$	-----	1.015	1.104	1.069	1.010	.945	.961	.949	.911	.910	.797	.752	.765	.600	.424	-----

Note: Upper values are the means, in feet, and lower values are the variances, in feet squared.

Table 60. *Estimates of parameters describing two-parameter gamma distribution for conditional step lengths* (Run 4A)

$Y_E = y_i$ \ $Y_D = y_j$	-3.2	-2.8	-2.4	-2.0	-1.6	-1.2	-0.8	-0.4	0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.8
-3.2	-------	------	------	------	------	2.438	2.006	2.551	36.109	25.170	18.010	10.871	------	------	------	------
	-------	------	------	------	------	7.407	5.700	8.034	131.689	89.329	61.956	24.818	------	------	------	------
-2.8	-------	------	------	------	2.657	1.780	1.887	2.214	3.172	3.822	3.395	8.236	7.008	------	------	------
	-------	------	------	------	11.086	6.804	6.832	7.494	10.275	12.258	10.848	24.420	19.397	------	------	------
-2.4	-------	9.748	24.356	1.195	2.683	1.950	1.966	2.024	2.549	2.904	3.425	4.285	3.248	------	------	------
	-------	35.444	78.305	5.654	10.795	7.029	6.834	6.901	8.817	9.749	11.335	13.644	10.389	------	------	------
-2.0	-------	5.451	3.929	2.991	3.825	3.007	3.036	2.931	3.557	3.877	4.414	5.222	4.243	2.866	------	------
	-------	17.978	15.120	12.603	14.422	10.245	9.901	9.182	11.035	11.772	12.954	15.193	12.730	12.348	------	------
-1.6	-------	5.617	2.643	2.052	2.756	2.698	3.104	3.069	3.795	4.059	4.681	4.996	5.045	3.531	------	------
	-------	15.998	7.780	7.012	8.929	8.561	9.548	9.173	11.217	11.762	13.126	13.855	14.226	12.305	------	------
-1.2	-------	3.530	2.822	2.683	2.179	2.204	2.460	2.728	3.220	3.405	3.742	3.974	4.085	3.009	------	------
	-------	9.545	8.039	8.096	6.834	6.750	7.319	7.787	9.042	9.623	10.168	10.768	11.328	9.555	------	------
-0.8	48.352	6.321	4.174	2.916	2.449	2.317	2.430	2.824	3.291	3.671	3.958	4.151	3.830	2.493	------	------
	126.247	15.664	10.803	8.011	6.822	6.147	6.276	7.074	8.221	9.202	9.761	10.390	9.811	7.176	------	------
-0.4	21.245	2.911	3.148	2.070	2.050	2.051	2.334	2.357	2.811	3.349	3.663	4.041	3.614	2.126	------	------
	44.232	7.086	7.889	5.358	5.163	5.035	5.451	5.138	6.100	7.358	8.076	9.193	8.529	5.535	------	------
0.0	-------	4.199	3.938	2.374	2.127	2.049	2.193	2.259	2.210	2.767	3.298	3.701	3.854	1.813	------	------
	-------	10.736	9.058	5.697	4.837	4.551	4.567	4.357	3.972	5.100	6.319	7.424	8.020	4.261	------	------
0.4	-------	2.452	5.078	2.401	2.190	2.257	2.468	2.390	2.446	2.256	3.073	3.818	4.135	1.751	------	------
	-------	5.668	13.066	5.678	4.727	4.636	4.697	4.130	3.902	3.242	4.674	6.342	7.336	3.654	------	------
0.8	-------	------	2.391	2.098	1.932	2.309	2.572	2.530	2.616	2.693	2.709	3.390	3.905	1.654	------	------
	-------	------	5.101	4.734	4.073	4.464	4.566	4.058	3.765	3.517	3.053	4.217	5.443	3.025	------	------
1.2	-------	------	------	3.018	1.941	2.171	2.805	2.600	2.741	2.804	3.310	2.890	3.259	1.457	------	------
	-------	------	------	6.879	4.248	4.190	4.926	4.111	3.763	3.373	3.429	2.515	3.155	2.197	------	------
1.6	-------	------	------	6.299	1.051	1.791	2.891	2.327	3.038	2.877	3.414	2.953	2.204	1.304	------	------
	-------	------	------	9.562	1.869	3.021	4.503	3.497	3.913	3.285	3.182	2.398	1.311	1.410	------	------
2.0	-------	------	------	------	------	------	------	16.369	9.050	7.632	6.789	5.947	4.673	2.266	------	------
	-------	------	------	------	------	------	------	22.507	9.828	7.281	5.900	4.668	3.211	1.042	------	------
2.4	-------	------	------	------	------	------	------	------	-------	------	------	------	------	------	------	------
	-------	------	------	------	------	------	------	------	-------	------	------	------	------	------	------	------
2.8	-------	------	------	------	------	------	------	------	-------	------	------	------	------	------	------	------
	-------	------	------	------	------	------	------	------	-------	------	------	------	------	------	------	------

Note: Upper values are $k_{1,y,y'}$, and lower values are $r_{1,y,y'}$.

Table 61. *Estimates of parameters describing two-parameter gamma distribution for conditional step lengths* (Run 16)

$Y_E = y_i$ \ $Y_D = y_j$	-3.6	-3.2	-2.8	-2.4	-2.0	-1.6	-1.2	-0.8	-0.4	0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2
-3.6	-----	-----	-------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	-----
	-----	-----	-------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	-----
-3.2	-----	-----	-------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	-----
	-----	-----	-------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	-----
-2.8	-----	-----	-------	------	5.681	4.261	3.772	4.616	4.978	4.639	3.661	3.760	6.684	------	------	------	------	-----
	-----	-----	-------	------	21.885	19.393	16.773	19.817	20.620	18.683	14.032	13.897	25.600	------	------	------	------	-----
-2.4	-----	-----	135.437	9.454	4.295	4.711	4.291	4.254	4.359	4.853	4.555	4.227	4.637	------	------	------	------	-----
	-----	-----	586.986	41.739	19.532	21.727	19.827	19.060	19.017	20.464	18.507	16.702	18.126	------	------	------	------	-----
-2.0	-----	-----	62.463	8.215	5.714	6.087	6.458	5.900	4.866	5.661	5.608	5.490	6.116	18.045	14.018	------	------	-----
	-----	-----	216.406	36.777	26.418	28.158	29.150	26.179	21.406	23.906	22.900	21.582	24.537	64.800	43.820	------	------	-----
-1.6	-----	-----	2.394	3.328	4.132	4.384	4.693	4.555	3.993	4.276	4.802	4.853	5.439	6.211	3.281	5.163	------	-----
	-----	-----	11.129	15,462	18.339	19.832	20.503	19.708	17.016	17.424	19.117	18.607	21.686	23.920	12.521	22.873	------	-----
-1.2	-----	-----	3.142	4.194	4.138	3.956	4.284	4.400	4.031	4.299	4.938	5.052	5.435	5.547	3.322	4.694	3.587	-----
	-----	-----	13.795	18.928	17.772	16.890	17.622	17.771	15.889	16.263	18.243	17.997	20.028	20.464	11.853	18.549	11.634	-----
-0.8	-----	-----	3.690	4.803	4.034	3.601	3.294	3.302	3.284	3.359	4.029	4.198	4.651	5.716	4.189	7.061	3.559	-----
	-----	-----	14.462	19.308	15.977	13.964	12.275	11.992	11.667	11.416	13.521	13.799	16.156	19.897	14.742	25.279	10.440	-----
-0.4	-----	1.207	2.925	2.939	3.256	3.221	2.938	2.831	2.722	2.772	3.368	3.828	4.683	5.946	5.154	6.478	3.681	-----
	-----	5.300	11.182	10.678	11.749	11.089	9.679	9.026	8.362	8.140	9.859	11.166	14.386	18.529	16.019	20.270	9.931	-----
0.0	-----	1.761	3.532	2.817	3.070	2.849	2.773	2.675	2.492	2.370	2.862	3.441	4.509	5.788	4.421	5.293	4.024	-----
	-----	6.695	11.954	8.665	9.617	8.460	7.907	7.305	6.523	5.797	6.973	8.455	11.896	15.477	11.416	14.063	9.896	-----
0.4	-----	2.048	3.364	3.918	3.757	3.613	3.435	3.206	3.063	2.940	2.563	2.904	4.213	5.504	4.610	5.710	5.125	-----
	-----	6.704	9.732	10.685	10.403	9.727	8.790	7.814	7.095	6.336	5.030	5.691	9.159	12.449	10.586	13.824	11.372	-----
0.8	-----	1.671	3.265	4.022	3.815	3.892	3.509	3.410	3.414	3.308	3.118	2.482	3.622	4.970	4.222	5.911	5.932	-----
	-----	4.752	7.879	9.320	8.716	8.951	7.744	7.175	6.784	6.107	5.357	3.770	6.099	9.065	8.183	12.507	11.858	-----
1.2	-----	-----	6.718	3.439	4.970	4.279	3.730	3.616	3.660	3.682	3.508	3.385	2.824	3.891	3.566	5.731	5.744	-----
	-----	-----	9.614	7.109	10.570	8.805	7.334	6.667	6.270	5.884	5.255	4.699	3.319	5.284	5.506	10.247	10.160	-----
1.6	-----	-----	-------	------	13.908	5.723	4.523	4.388	4.330	4.134	3.837	3.574	3.276	2.470	3.064	5.706	6.780	-----
	-----	-----	-------	------	25.145	10.188	7.630	7.144	6.413	5.726	4.977	4.342	3.650	2.336	3.662	8.206	10.252	-----
2.0	-----	-----	-------	------	------	11.319	5.784	7.319	6.227	6.116	5.776	5.423	5.164	4.679	3.478	5.760	9.043	-----
	-----	-----	-------	------	------	20.498	9.938	12.266	9.384	8.679	7.740	6.882	6.186	5.189	3.339	6.070	11.367	-----
2.4	-----	-----	-------	------	------	------	------	19.639	------	14.600	12.857	12.217	11.434	10.590	9.548	7.000	------	-----
	-----	-----	-------	------	------	------	------	27.769	------	17.053	13.886	12.387	10.851	9.309	7.657	4.704	------	-----
2.8	-----	-----	-------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	-----
	-----	-----	-------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	-----
3.2	-----	-----	-------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	-----
	-----	-----	-------	------	------	------	------	------	------	------	------	------	------	------	------	------	------	-----

Note: Upper values are $k_{1,y,y'}$, and lower values are $r_{1,y,y'}$.

Table 62. *Estimates of parameters describing two-parameter gamma distribution for conditional step lengths* (Run 17)

$Y_E = y_i$ \ $Y_D = y_j$	-3.2	-2.8	-2.4	-2.0	-1.6	-1.2	-0.8	-0.4	0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.8
-3.2	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----
-2.8	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	10.359 36.804	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----
-2.4	----- -----	----- -----	----- -----	----- -----	----- -----	3.809 15.335	8.836 33.338	8.241 27.640	7.700 24.594	6.331 20.203	5.013 15.280	6.530 20.934	9.808 35.113	9.729 32.185	----- -----	----- -----
-2.0	----- -----	----- -----	----- -----	10.425 42.931	11.042 40.844	4.746 16.332	5.772 19.158	5.512 18.106	5.685 17.646	5.746 17.530	5.535 16.605	6.645 20.134	8.953 28.855	11.706 33.573	----- -----	----- -----
-1.6	----- -----	----- -----	8.015 29.296	11.541 42.620	4.473 16.488	4.622 16.042	5.329 17.692	5.681 18.237	5.634 17.329	6.434 19.661	7.086 21.136	8.966 26.280	14.322 42.666	10.660 28.068	----- -----	----- -----
-1.2	----- -----	----- -----	2.551 7.820	4.813 16.214	5.392 18.377	5.101 15.901	4.782 14.385	4.961 14.496	5.105 14.386	5.904 16.624	6.113 16.926	6.579 17.790	6.507 17.615	3.562 8.959	----- -----	----- -----
-0.8	----- -----	----- -----	2.522 6.699	3.222 9.709	4.116 12.418	4.373 12.201	3.909 10.406	4.220 10.861	4.715 11.783	5.478 13.563	6.042 14.893	7.137 17.422	7.306 17.615	5.777 13.031	----- -----	----- -----
-0.4	----- -----	----- -----	3.020 7.599	4.175 11.572	4.419 11.972	4.221 10.388	3.821 8.996	3.671 8.194	4.047 8.665	4.784 10.186	5.616 12.080	6.178 13.511	5.452 12.098	6.962 13.909	----- -----	----- -----
0.0	----- -----	----- -----	12.906 37.143	4.481 11.306	4.557 10.842	4.341 9.346	3.955 8.073	3.705 7.165	3.570 6.397	4.057 7.161	5.168 9.375	6.016 11.292	5.425 10.535	8.173 14.694	----- -----	----- -----
0.4	----- -----	2.186 4.614	6.195 13.201	4.646 9.862	5.651 11.817	5.267 9.986	4.430 7.809	4.079 6.739	3.961 5.993	3.417 4.705	4.468 6.350	5.850 8.898	5.642 9.169	8.436 12.882	----- -----	----- -----
0.8	----- -----	----- -----	3.727 6.097	7.933 14.977	7.789 14.379	6.922 11.740	5.443 8.562	5.247 7.682	5.183 6.904	4.762 5.805	3.930 4.201	4.710 5.345	5.227 6.748	7.121 9.229	----- -----	----- -----
1.2	----- -----	----- -----	----- -----	17.600 27.878	7.368 11.944	6.032 9.204	5.714 8.000	5.161 6.606	5.355 6.137	4.990 5.230	4.635 4.362	3.708 2.914	4.411 4.327	6.694 6.587	----- -----	----- -----
1.6	----- -----	----- -----	----- -----	12.123 15.578	6.129 9.316	6.063 9.337	8.316 10.719	6.503 7.738	5.966 6.192	5.625 5.316	5.224 4.393	4.618 3.348	3.385 1.845	4.804 3.070	----- -----	----- -----
2.0	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	10.074 10.960	6.320 5.991	5.851 5.067	5.394 4.278	5.137 3.668	5.114 3.216	4.843 2.392	----- -----	----- -----
2.4	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----
2.8	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----	----- -----

Note: Upper values are $k_{1,y,y'}$, and lower values are $r_{1,y,y'}$.

Table 63. *Results of goodness of fit test for conditional step lengths* (Run 4A)

$Y_D=y_j$	$Y_E=y_i$ = -0.8				$Y_E=y_i$ = 0.0				$Y_E=y_i$ = 0.8			
	$k_{1,y,y'}$[1/] ft^{-1}	$r_{1,y,y'}$[2/]	$m_{i,j}$	Goodness of Fit Test[3/]	$k_{1,y,y'}$ ft^{-1}	$r_{1,y,y'}$	$m_{i,j}$	Goodness of Fit Test	$k_{1,y,y'}$ ft^{-1}	$r_{1,y,y'}$	$m_{i,j}$	Goodness of Fit Test
-1.6	2.449	6.822	128	$x^2 < x_c^2$	2.127	4.837	146	$x^2 < x_c^2$	1.932	4.073	60	$x^2 < x_c^2$
-1.2	2.317	5.147	217	$x^2 < x_c^2$	2.049	4.551	263	$x^2 < x_c^2$	2.309	4.464	124	$x^2 < x_c^2$
-0.8	2.430	6.276	336	$x^2 < x_c^2$	2.193	4.567	440	$x^2 < x_c^2$	2.257	4.566	213	$x^2 < x_c^2$
-0.4	2.824	7.074	423	$x^2 < x_c^2$	2.259	4.357	603	$x^2 < x_c^2$	2.530	4.048	306	$x^2 < x_c^2$
0.0	3.291	8.221	457	$x^2 < x_c^2$	2.210	3.972	725	$x^2 < x_c^2$	2.616	3.765	403	$x^2 < x_c^2$
0.4	3.671	9.202	420	$x^2 < x_c^2$	2.767	5.100	666	$x^2 < x_c^2$	2.693	3.517	455	$x^2 < x_c^2$
0.8	3.958	9.761	307	$x^2 < x_c^2$	3.298	6.319	468	$x^2 < x_c^2$	2.709	3.053	470	$x^2 < x_c^2$
1.2	4.151	10.390	181	$x^2 < x_c^2$	3.701	7.424	253	$x^2 < x_c^2$	3.390	4.217	284	$x^2 < x_c^2$
1.6	3.830	9.811	48	$x^2 < x_c^2$	3.854	8.020	56	$x^2 < x_c^2$	3.905	5.443	67	$x^2 < x_c^2$

1/ $$k_{1,y,y'} = \frac{\hat{E}[X|Y_E=y,\ Y_D=y']}{\hat{Var}[X|Y_E=y,\ Y_D=y']}$$

2/ $$r_{1,y,y'} = \frac{\left(\hat{E}[X|Y_E=y,\ Y_D=y']\right)^2}{\hat{Var}[X|Y_E=y,\ Y_D=y']}$$

3/ x_c^2 = critical chi-square value at a significant level of 0.05

Table 64. *Results of goodness of fit test for conditional step lengths* (Run 16)

$Y_D=y_j$	$Y_E=y_i$ = -0.8				$Y_E=y_i$ = 0.0				$Y_E=y_i$ = 0.8			
	$k_{1,y,y'}$ 1/ ft^{-1}	$r_{1,y,y'}$ 2/	$m_{i,j}$	Goodness of Fit Test 3/	$k_{1,y,y'}$ ft^{-1}	$r_{1,y,y'}$	$m_{i,j}$	Goodness of Fit Test	$k_{1,y,y'}$ ft^{-1}	$r_{1,y,y'}$	$m_{i,j}$	Goodness of Fit Test
-1.6	3.601	13.964	155	$x^2 < x_c^2$	2.849	8.460	184	$x^2 < x_c^2$	3.892	8.951	124	$x^2 < x_c^2$
-1.2	3.294	12.275	239	$x^2 < x_c^2$	2.773	7.907	294	$x^2 < x_c^2$	3.509	7.744	200	$x^2 < x_c^2$
-0.8	3.302	11.992	293	$x^2 < x_c^2$	2.675	7.305	376	$x^2 < x_c^2$	3.410	7.175	254	$x^2 < x_c^2$
-0.4	3.284	11.667	341	$x^2 < x_c^2$	2.492	6.523	442	$x^2 < x_c^2$	3.414	6.784	297	$x^2 < x_c^2$
0.0	3.359	11.416	365	$x^2 < x_c^2$	2.370	5.797	482	$x^2 < x_c^2$	3.308	6.107	317	$x^2 < x_c^2$
0.4	4.029	13.521	337	$x^2 < x_c^2$	2.363	6.973	444	$x^2 < x_c^2$	3.118	5.357	325	$x^2 < x_c^2$
0.8	4.198	13.799	272	$x^2 < x_c^2$	3.441	8.455	350	$x^2 < x_c^2$	2.482	3.770	331	$x^2 < x_c^2$
1.2	4.651	16.156	146	$x^2 < x_c^2$	4.509	11.896	197	$x^2 < x_c^2$	3.622	6.099	207	$x^2 < x_c^2$
1.6	5.716	19.897	52	$x^2 < x_c^2$	5.788	15.477	77	$x^2 < x_c^2$	4.970	9.065	83	$x^2 < x_c^2$

1/ $k_{1,y,y'} = \dfrac{\hat{E}[X|Y_E=y,\ Y_D=y']}{\widehat{Var}[X|Y_E=y,\ Y_D=y']}$

2/ $r_{1,y,y'} = \dfrac{\left(\hat{E}[X|Y_E=y,\ Y_D=y']\right)^2}{\widehat{Var}[X|Y_E=y,\ Y_D=y']}$

3/ x_c^2 = critical chi-square value at a significant level of 0.05

Table 65. *Results of goodness of fit test for conditional step lengths* (Run 17)

$Y_D=y_j$	$Y_E=y_i$ = -0.8				$Y_E=y_i$ = 0.0				$Y_E=y_i$ = 0.8			
	$k_{1,y,y'}$ [1/] ft^{-1}	$r_{1,y,y'}$ [2/]	$m_{i,j}$	Goodness of Fit Test [3/]	$k_{1,y,y'}$ ft^{-1}	$r_{1,y,y'}$	$m_{i,j}$	Goodness of Fit Test	$k_{1,y,y'}$ ft^{-1}	$r_{1,y,y'}$	$m_{i,j}$	Goodness of Fit Test
-1.6	4.116	12.418	109	$x^2 < x_c^2$	4.557	10.842	129	$x^2 > x_c^2$	7.789	14.379	68	$x^2 < x_c^2$
-1.2	4.373	12.201	259	$x^2 > x_c^2$	4.341	9.346	301	$x^2 < x_c^2$	6.922	11.740	159	$x^2 < x_c^2$
-0.8	3.909	10.406	406	$x^2 > x_c^2$	3.955	8.073	493	$x^2 < x_c^2$	5.443	8.562	267	$x^2 < x_c^2$
-0.4	4.220	10.861	487	$x^2 < x_c^2$	3.705	7.165	632	$x^2 < x_c^2$	5.247	7.682	357	$x^2 < x_c^2$
0.0	4.715	11.783	502	$x^2 < x_c^2$	3.570	6.397	704	$x^2 > x_c^2$	5.183	6.904	411	$x^2 < x_c^2$
0.4	5.478	13.563	449	$x^2 < x_c^2$	4.057	7.161	647	$x^2 > x_c^2$	4.762	5.805	425	$x^2 < x_c^2$
0.8	6.042	14.893	324	$x^2 < x_c^2$	5.168	9.375	469	$x^2 < x_c^2$	3.930	4.201	427	$x^2 < x_c^2$
1.2	7.137	17.422	155	$x^2 < x_c^2$	6.016	11.292	235	$x^2 < x_c^2$	4.710	5.345	247	$x^2 < x_c^2$
1.6	7.306	17.615	53	$x^2 < x_c^2$	5.425	10.535	80	$x^2 < x_c^2$	5.227	6.748	85	$x^2 < x_c^2$

1/ $$k_{1,y,y'} = \frac{\hat{E}[X|Y_E=y,\ Y_D=y']}{\widehat{Var}[X|Y_E=y,\ Y_D=y']}$$

2/ $$r_{1,y,y'} = \frac{\left(\hat{E}[X|Y_E=y,\ Y_D=y']\right)^2}{\widehat{Var}[X|Y_E=y,\ Y_D=y']}$$

3/ x_c^2 = critical chi-square value at a significant level of 0.05

Table 66. *Sample conditional probability mass function of step lengths,* $p_{X|Y_D}(x_\beta|y_j)$ (Run 4A)

$Y_D=y_j$	Class Mark, x_β, ft								$\sum_i m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-2.8	0.000	0.451	0.358	0.151	0.040	0.000	0.000	0.000	49
-2.4	.004	.450	.364	.155	.027	.000	.000	.000	174
-2.0	.018	.388	.372	.122	.071	.025	.004	.000	387
-1.6	.058	.403	.300	.153	.054	.025	.005	.002	796
-1.2	.087	.420	.282	.150	.042	.016	.002	.001	1,413
-0.8	.112	.440	.275	.126	.037	.008	.001	.001	2,306
-0.4	.154	.439	.264	.105	.031	.005	.001	.001	3,094
0.0	.209	.435	.238	.092	.021	.004	.001	.000	3,653
0.4	.242	.413	.237	.085	.020	.002	.001	.000	3,653
0.8	.257	.403	.243	.080	.015	.002	.000	.000	2,920
1.2	.242	.407	.250	.086	.013	.002	.000	.000	1,811
1.6	.213	.395	.284	.083	.022	.003	.000	.000	519
2.0	.177	.374	.137	.160	.149	.003	.000	.000	43
2.4	-----	-----	-----	-----	-----	-----	-----	-----	0
									$\sum_i \sum_j m_{i,j} = 20{,}818$

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 67. *Sample conditional probability mass function of step lengths,* $p_{X|Y_D}(x_\beta|y_j)$ (Run 16)

$Y_D=y_j$	Class Mark, x_β, ft								$\sum_i m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	
-3.6	0.000	0.125	0.583	0.292	0.000	0.000	0.000	0.000	6
-3.2	.000	.125	.292	.300	.208	.075	.000	.000	10
-2.8	.034	.153	.260	.323	.154	.044	.025	.007	66
-2.4	.020	.154	.332	.256	.142	.080	.012	.004	183
-2.0	.009	.191	.309	.268	.145	.059	.016	.003	565
-1.6	.012	.200	.316	.258	.139	.060	.014	.001	1,187
-1.2	.024	.227	.316	.242	.130	.046	.013	.002	1,888
-0.8	.038	.250	.299	.237	.125	.039	.011	.001	2,347
-0.4	.056	.257	.298	.228	.112	.037	.010	.002	2,735
0.0	.084	.284	.292	.206	.097	.031	.005	.001	2,918
0.4	.103	.287	.292	.203	.087	.025	.003	.000	2,813
0.8	.115	.290	.298	.195	.082	.018	.002	.000	2,412
1.2	.096	.263	.305	.218	.099	.017	.002	.000	1,478
1.6	.080	.245	.325	.259	.081	.010	.000	.000	623
2.0	.077	.239	.367	.206	.095	.016	.000	.000	246
2.4	.032	.237	.405	.204	.107	.015	.000	.000	136
2.8	.011	.347	.372	.224	.046	.000	.000	.000	23
3.2	.072	.735	.193	.000	.000	.000	.000	.000	12
3.6	.000	.000	.000	.000	.000	.000	.000	.000	0

$\sum_i \sum_j m_{i,j} = 19,607$

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 68. *Sample conditional probability mass function of step lengths,* $p_{X|Y_D}(x_\beta|y_j)$ (Run 17)

$Y_D=y_j$	Class Mark, x_β, ft						$\sum_i m_{i,j}$
	0.5	1.5	2.5	3.5	4.5	5.5	
-3.2	-----	-----	-----	-----	-----	-----	0
-2.8	0.000	0.452	0.179	0.368	0.000	0.000	4
-2.4	.030	.356	.333	.231	.050	.000	43
-2.0	.022	.316	.349	.229	.080	.004	223
-1.6	.016	.317	.372	.223	.069	.003	697
-1.2	.041	.389	.352	.173	.042	.003	1,646
-0.8	.078	.402	.329	.156	.033	.002	2,696
-0.4	.113	.411	.310	.141	.024	.001	3,336
0.0	.164	.406	.297	.115	.017	.001	3,638
0.4	.196	.390	.292	.107	.015	.000	3,440
0.8	.201	.382	.304	.102	.011	.000	2,717
1.2	.187	.384	.327	.092	.010	.000	1,516
1.6	.161	.399	.323	.107	.010	.000	585
2.0	.141	.497	.304	.058	.000	.000	92
2.4	.333	.667	.000	.000	.000	.000	6
2.8	-----	-----	-----	-----	-----	-----	0
							$\sum_i \sum_j m_{i,j} = 20{,}639$

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 69. *Variation of conditional mean and variance of step lengths with elevation of deposition;* $\hat{E}[X|Y_D=y]$ and $\widehat{Var}[X|Y_D=y]$

Standardized Elevation y_j	$\hat{E}[X\|Y_D=y_j]$, ft			$\widehat{Var}[X\|Y_D=y_j]$, ft^2		
	Run 4A	Run 16	Run 17	Run 4A	Run 16	Run 17
-3.6	-----	2.073	-----	-----	1.463	-----
-3.2	1.194	2.512	-----	1.396	3.476	-----
-2.8	2.172	3.008	0.989	1.232	2.290	1.815
-2.4	2.397	2.924	2.311	.884	1.635	1.104
-2.0	2.577	3.097	2.473	1.376	1.530	1.069
-1.6	2.486	3.049	2.440	1.415	1.512	1.010
-1.2	2.384	2.927	2.271	1.277	1.499	.845
-0.8	2.257	2.825	2.153	1.141	1.509	.861
-0.4	2.119	2.716	2.042	1.084	1.547	.849
0.0	2.025	2.567	1.925	1.035	1.477	.811
0.4	1.972	2.473	1.866	1.008	1.432	.818
0.8	1.938	2.402	1.841	.935	1.381	.797
1.2	1.983	2.517	1.847	.892	1.392	.762
1.6	2.056	2.500	1.899	.915	1.234	.765
2.0	2.458	2.516	1.777	1.671	1.252	.606
2.4	-----	2.550	.506	-----	1.409	.424
2.8	-----	2.308	-----	-----	1.164	-----
3.2	-----	1.495	-----	-----	.368	-----
3.6	-----	-----	-----	-----	-----	-----

Table 70. *Sample joint probability mass function of step lengths and elevation of deposition,* $p_{X,Y_D}(x_\beta, y_j)$ (Run 4A)

$Y_D=y_j$	Class Mark, x_β, ft							
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
-3.2	------	------	------	------	------	------	------	------
-2.8	------	0.0000	0.0000	0.0000	0.0000	------	------	------
-2.4	0.0000	.0025	.0020	.0009	.0002	------	------	------
-2.0	.0005	.0099	.0095	.0031	.0018	0.0006	0.0001	------
-1.6	.0035	.0244	.0182	.0093	.0033	.0015	.0003	0.0001
-1.2	.0079	.0381	.0255	.0136	.0038	.0014	.0002	.0001
-0.8	.0140	.0551	.0345	.0158	.0046	.0010	.0001	.0001
-0.4	.0234	.0666	.0400	.0159	.0047	.0008	.0002	.0002
0.0	.0335	.0698	.0382	.0148	.0034	.0006	.0002	------
0.4	.0377	.0644	.0369	.0133	.0031	.0003	.0002	------
0.8	.0308	.0483	.0291	.0096	.0016	.0002	------	------
1.2	.0170	.0286	.0176	.0060	.0009	.0001	------	------
1.6	.0053	.0098	.0070	.0021	.0005	.0001	------	------
2.0	.0015	.0032	.0012	.0014	.0013	.0000	------	------
2.4	------	------	------	------	------	------	------	------
$p_X(x_\beta)$	.175	.421	.260	.106	.029	.007	.001	.001

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 71. *Sample joint probability mass function of step lengths and elevation of deposition,* $p_{X,Y_D}(x_\beta, y_j)$ (Run 16)

$Y_D = y_j$	Class Mark, x_β, ft							
	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5
-3.2	0.0000	0.0008	0.0019	0.0013	0.0005	0.0005	------	------
-2.8	.0003	.0012	.0020	.0025	.0012	.0003	0.0002	0.0001
-2.4	.0002	.0017	.0036	.0027	.0015	.0009	.0001	.0000
-2.0	.0002	.0037	.0059	.0051	.0028	.0011	.0003	.0001
-1.6	.0005	.0075	.0119	.0097	.0052	.0023	.0005	.0000
-1.2	.0014	.0137	.0190	.0146	.0078	.0028	.0008	.0001
-0.8	.0039	.0258	.0309	.0245	.0129	.0040	.0011	.0001
-0.4	.0068	.0314	.0364	.0279	.0137	.0045	.0012	.0002
0.0	.0112	.0378	.0389	.0274	.0129	.0041	.0007	.0001
0.4	.0203	.0566	.0576	.0401	.0172	.0049	.0006	------
0.8	.0158	.0398	.0409	.0268	.0113	.0025	.0003	------
1.2	.0093	.0256	.0297	.0212	.0096	.0017	.0002	------
1.6	.0037	.0114	.0151	.0120	.0038	.0005	------	------
2.0	.0013	.0039	.0060	.0035	.0016	.0003	------	------
2.4	.0002	.0011	.0019	.0010	.0005	.0001	------	------
2.8	------	------	------	------	------	------	------	------
$p_X(x_\beta)$	.075	.262	.302	.221	.103	.030	.006	.001

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 72. *Sample joint probability mass function of step lengths and elevation of deposition,* $p_{X,Y_D}(x_\beta, y_j)$ (Run 17)

$Y_D = y_j$	Class Mark, x_β, ft						
	0.5	1.5	2.5	3.5	4.5	5.5	6.5
-3.2	------	------	------	------	------	------	------
-2.8	0.0000	0.0007	0.0003	0.0006	------	------	------
-2.4	.0002	.0019	.0018	.0012	0.0003	------	------
-2.0	.0004	.0055	.0061	.0040	.0014	0.0001	------
-1.6	.0007	.0138	.0162	.0097	.0030	.0001	------
-1.2	.0037	.0347	.0314	.0154	.0037	.0003	------
-0.8	.0101	.0520	.0425	.0202	.0043	.0003	------
-0.4	.0173	.0627	.0473	.0215	.0037	.0002	------
0.0	.0252	.0623	.0456	.0176	.0026	.0002	------
0.4	.0302	.0601	.0450	.0165	.0023	------	------
0.8	.0236	.0448	.0357	.0120	.0013	------	------
1.2	.0136	.0280	.0238	.0067	.0007	------	------
1.6	.0066	.0163	.0132	.0044	.0004	------	------
2.0	.0026	.0093	.0057	.0011	------	------	------
2.4	.0012	.0023	------	------	------	------	------
2.8	------	------	------	------	------	------	------
$p_X(x_\beta)$	.135	.394	.315	.131	.024	.001	------

Note: The class widths for step lengths are equal to 1 foot for all β.

Table 74. *Comparison of the effective volume ratios at elevation* y_j; ξ_j, *from* $y_t(x)$ *record and* ζ_j, *from* $y_x(t)$ *record*

$Y_D = y_j$	Run 4A		Run 16		Run 17	
	ξ_j	ζ_j	ξ_j	ζ_j	ξ_j	ζ_j
-2.8	0.968	-----	1.000	-----	1.000	-----
-2.4	.963	-----	.995	-----	.998	-----
-2.0	.949	0.903	.975	-----	.989	-----
-1.6	.925	.883	.935	0.890	.964	0.943
-1.2	.877	.865	.865	.840	.900	.867
-0.8	.791	.787	.778	.719	.801	.756
-0.4	.668	.675	.670	.656	.669	.645
0.0	.512	.529	.535	.567	.522	.522
0.4	.344	.379	.385	.388	.364	.380
0.8	.189	.222	.230	.230	.202	.234
1.2	.078	.112	.103	.097	.079	.120
1.6	.015	.044	.036	.037	.020	.046
2.0	.001	.009	.012	.008	.002	.013
2.4	.000	-----	.004	.001	.000	.001
2.8	-----	-----	.001	-----	.000	-----